Common graph theory metrics:

*Nodes and edges*

An *NxN* binary graph *G*, consists of *N* nodes and undirected edges which represent functional connectivity between regions. This adjacency matrix is constructed by thresholding a correlation matrix, with edges defined as:

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where *T* is the threshold and *z(i,j)* is the measure of association between nodes *i* and *j* (correlation, partial correlation, z-transformed correlation). If the measure of correlation between two nodes exceeds some set threshold, a functional connection (either positive or negative) is said to exist between them.

*Subgraph*

A subgraph  is the set of all nodes directly connected to node *i*. The degree of each node,  , is the number of nodes directly connected to node *i*, i.e. the number of nodes in .

*Mean degree*

The mean degree, or degree of connectivity, is the average of the degrees of all the nodes in *G*:

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which is a metric for the overall density of connections in the network.

*Cost*

The cost of a network is defined as the number of existing edges divided by the number of possible edges in a fully connected network, where every node is connected to every other node:

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*Connectivity strength*

The connectivity strength of the *i*th node is the sum of the connections divded by the degree of the node, a measure of the average strength of the connections in each subgraph:

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*Mean connectivity strength*

An expression for the overall strength of connectivity in the graph as a whole:

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*Node clustering coefficient*

The clustering coefficient of the *i*th node is the ratio of the existing connections to the number of possible connections in the subgraph :

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*Network clustering coefficient*

The clustering coefficient of the network is the average of the nodal clustering coefficients:

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*Characteristic path length*

The characteristic path length of the *i*th node is:

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*Mean characteristic path length*

The mean characteristic path length summarizes the overall connectivity of the network; a more efficient network has shorter paths between nodes.

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*Local efficiency*

The local efficiency of the *i*th node is calculated as:

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*Mean efficiency*

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*Global efficiency*

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Relationship between cost, efficiency, and fault tolerance

Maybe average centrality measures, rank them, and assign colors based on quantile? Could be a good visualization scheme.

*Small-world Properties*

The primary features of a small-world network, compared to a random network, are similar path lengths but higher clustering coefficients. This is expressed as:

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Measures of centrality

Degree is the number of nodes connected to another node. The degree of a node is the sum of its row (or column) in the adjacency matrix.

Betweenness: The number of shortest paths in the graph that pass through the node divided by the total number of shortest paths.



that’s the list of nodes included in the path.

Nodes with a high betweenness centrality are of interest because they control information flow in a network, may be required to carry more information (cite this)

Closeness:

Normalized inverse of the sum of topological distances in the graph



distance is the number of edges between 2 nodes. N is the number of nodes. For node i.

determines how central a node is in a network, ie how important it is at spreading information from it to the other nodes in the network.

Most central is determined by all of these measures of centraility

Ref.

Aldous & Wilson, Graphs and Applications.An introductory approach. Springer, 2000.