Sets and Indices:

- Suppliers, indexed by i
- Plants, indexed by j
- Retailers, indexed by k
- Collection centres, indexed by m
- Disassembly centres, indexed by d
- Remanufacturing centres, indexed by r
- Parts, indexed by c
- Periods, indexed by p
- Architectures, indexed by a
- R imperatives, indexed by e
- Design alternatives, indexed by s

Parameters:

- Flow cost from suppliers to plants flow_cost_suppliers_plants$[i, j]$
- Purchase cost from suppliers purchase_cost_suppliers$[c, d]$
- Flow cost from plants to retailers flow_cost_plants_retailers$[j, k]$
- Flow cost from retailers to collection centres flow_cost_retailers_collection_centres$[k, m]$
- Flow cost from collection centres to plants flow_cost_collection_centres_plants$[m, j]$
- Flow cost from collection centres to retailers flow_cost_collection_retailer$[m, k]$
- Flow cost from collection centres to remanufacturing flow_cost_collection_centres_remanufac
- Flow cost from remanufacturing to refurbishing flow_cost_remanufacturing_refurbishing$[r, j]$
- Flow cost from remanufacturing to recycling flow_cost_remanufacturing_reclycling$[r, j]$
- Flow cost from remanufacturing to remanufacturing flow_cost_remanufacturing_remanufactu
- Opening cost for collection centres opening_cost_collection$[m]$
- Opening cost for reprocessing centres opening_cost_reprocessing$[r]$
- Opening cost for suppliers opening_cost_supplier$[i]$

Variables:

- $x[i, j, c, p]$: Flow from suppliers to plants
- $y[j, k, p]$: Flow from plants to retailers
- $w[k, m, p]$: Flow from retailers to collection centres
- $a[m, j, p]$: Flow from collection/disassembly centres to plants
- $b[m, k, p]$: Flow from collection/disassembly centres to retailers
- $dk[k, p]$: Disposal flow from retailers
- $dm[m, c, p]$: Disposal flow from collection/disassembly centres
- $f[m, r, c, p]$: Flow from collection/disassembly centres to remanufacturing centres
- $erf[r, j, c, p]$: Flow from disassembly centre to remanufacturing centres due to refurbishing
- $erm[r, j, c, p]$: Flow from disassembly centre to remanufacturing centres due to remanufacturing
- $er[r, j, c, p]$: Flow from disassembly centre to remanufacturing centres due to recycling
- $opm[m]$: Binary variable for opening collection centre m
- $opr[r]$: Binary variable for opening reprocessing centre r
- $ops[i]$: Binary variable for opening supplier i
- $ar[a]$: Binary variable, 1 if product follows architecture a
- $de[s, c]$: Binary variable, 1 if design alternative s is used for part c
- $rimp[e]$: Binary variable, if R imperative e is possible
- $monetary\_costs$: Variable to define the save the objective function value

Minimization Objective Function:

Min**: monetary_costs**

Objective Function Relationship:

$$\begin{aligned}\text{monetary\_costs} \geq & \sum_{i,j,c,p} x[i,j,c,p] \cdot \text{flow\_cost\_suppliers\_plants}[i,j] + \\ & \sum_{i,j,c,p,d} x[i,j,c,p] \cdot \text{purchase\_cost\_suppliers}[c,d] + \\ & \sum_{j,k,p} y[j,k,p] \cdot \text{flow\_cost\_plants\_retailers}[j,k] + \\ & \sum_{k,m,p} w[k,m,p] \cdot \text{flow\_cost\_retailers\_collection\_centres}[k,m] + \\ & \sum_{m,j,p} a[m,j,p] \cdot \text{flow\_cost\_collection\_centres\_plants}[m,j] + \\ & \sum_{m,k,p} b[m,k,p] \cdot \text{flow\_cost\_collection\_retailer}[m,k] + \\ & \sum_{m,r,c,p} f[m,r,c,p] \cdot \text{flow\_cost\_collection\_centres\_remanufacturing}[m,r] + \\ & \sum_{r,j,c,p} erf[r,j,c,p] \cdot \text{flow\_cost\_remanufacturing\_refurbishing}[r,j] + \\ & \sum_{r,j,c,p} erm[r,j,c,p] \cdot \text{flow\_cost\_remanufacturing\_remanufacturing}[r,j] + \\ & \sum_{r,j,c,p} er[r,j,c,p] \cdot \text{flow\_cost\_remanufacturing\_reclycling}[r,j] + \\ & \sum_{m} opm[m] \cdot \text{opening\_cost\_collection}[m] + \\ & \sum_{r} opr[r] \cdot \text{opening\_cost\_reprocessing}[r] + \\ & \sum_{i} ops[i] \cdot \text{opening\_cost\_supplier}[i].\end{aligned}$$

Capacity Constraints:

- **constraint 1, Suppliers:**

$$\sum_{j} x[i,j,c,p] \leq \text{suppliers\_capacity}[i,c], \quad \forall i,c,p$$

- **constraint 2, Plants:**

$$\sum_{k} y[j,k,p] \leq \text{plants\_capacity}[j], \quad \forall j,p$$

- **constraint 4: Collection/Disassembly Centres:**

$$\sum_{j,p} (a[m,j,p] + b[m,k,p]) \leq \text{collection\_centres\_capacity}[m,c] \cdot \text{opm}[m], \quad \forall m,c,p$$

- **constraint 5, Remanufacturing Centres:**

$$\sum_{j,p}(erf[r,j,c,p]+erm[r,j,c,p]+er[r,j,c,p]) \leq \text{remanufacturing\_centres\_capacity}[r,c]\cdot\text{opr}[r$$

Flow Constraints:

- **constraint 6, Plants:**

$$\sum_{i,c,p} x[i,j,c,p]+\sum_{r,c,p}(erf[r,j,c,p]+erm[r,j,c,p]+er[r,j,c,p]) = \sum_{k,c,p} y[j,k,p], \quad \forall j$$

- **constraint 7, Retailers:**

$$\sum_{j} y[j,k,p] + \sum_{m} b[m,k,p] = \sum_{m} w[k,m,p] + dk[k,p], \quad \forall k,p$$

- **constraint 8, Collection/Disassembly Centres:**

$$\sum_{k,p} w[k,m,p] - \sum_{j,p}(a[m,j,p] + b[m,k,p]) = \sum_{r,c,p} f[m,r,c,p], \quad \forall m$$

- **constraint 9, flow of remanufacturing centres**

$$\sum_{m} f[m,r,c,p] = \sum_{j}(erf[r,j,c,p] + erm[r,j,c,p] + er[r,j,c,p]), \quad \forall r,c,p$$

Constraints 10 and 18, Selection of architecture and design Constraints:

$$\sum_{a} \text{ar}[a] = 1 \quad \text{(One and only one architecture must be selected)}$$

$$\sum_{s} \text{de}[s,c] = 1, \quad \forall c \quad \text{(One and only one design per part)}$$

Other constraints:

- **constraint 3, Demand Constraints for Retailers:**

$$\sum_{j} y[j,k,p] \geq \text{retailer\_demand}[k,p], \quad \forall k,p$$

- **constraint ,Reusing Constraints:**

$$\sum_{j} (a[m,j,p] + b[m,k,p]) \leq \sigma \cdot \sum_{k} w[k,m,p], \quad \forall m,p$$

- **Disposal Rate Constraints:**

$$dm[m,c,p] \geq \lambda \cdot \sum_{k} w[k,m,p], \quad \forall m,c,p$$

- **Remanufacturing Rates:**

$$\sum_{j,c} (erf[r,j,c,p] + erm[r,j,c,p]) \leq \beta \cdot \sum_{m,c} f[m,r,c,p], \quad \forall r,p$$

- **Refurbishing Constraints:**

$$\sum_{j,c} erf[r,j,c,p] \leq \alpha \cdot \sum_{m,c} f[m,r,c,p], \quad \forall r,p$$

- **Opening Costs Constraints:**

$$w[k,m,p] \leq \text{big\_m} \cdot \text{opm}[m], \quad \forall k,m,p$$
$$f[m,r,c,p] \leq \text{big\_m} \cdot \text{opr}[r], \quad \forall m,r,c,p$$

- **Constraints 11 to 15, Restriction Constraints based on R Imperatives:**

$$\begin{aligned}
\text{erf}[r,j,c,p] &\leq \text{rimp}[0] \cdot \text{big_m}, && \text{(Refurbishing possible)} \\
\text{erm}[r,j,c,p] &\leq \text{rimp}[1] \cdot \text{big_m}, && \text{(Remanufacturing possible)} \\
\text{er}[r,j,c,p] &\leq \text{rimp}[2] \cdot \text{big_m}, && \text{(Recycling possible)} \\
b[m,k,p] &\leq \text{rimp}[3] \cdot \text{big_m}, && \text{(Reusing possible)} \\
a[m,j,p] &\leq \text{rimp}[4] \cdot \text{big_m}, && \text{(Repackaging possible)}
\end{aligned}$$

- **Design to Part Relations:**

$$\sum_{s} de[s,c] \cdot \text{designs\_of\_parts}[c,s] = 1, \quad \forall c$$

- **Cosntraints 16 and 17, R Imperative Compliance with Architecture and Design:**

$$\text{rimp}[e] \cdot \text{ar}[a] \leq \text{r\_imperatives\_of\_architecture}[a,e], \quad \forall e,a$$
$$\text{rimp}[e] \cdot \sum_{c} \text{de}[s,c] \leq \text{r\_imperatives\_of\_designs}[s,e], \quad \forall e,s$$