Fault Tolerant Quantum Computation

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Overview

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Introduction

- The goal of fault-tolerant quantum computing is to operate a large-scale (quantum) computer reliably, even though the components of the computer are noisy.
- Reliability can be enhanced by encoding the computer's state in the blocks of a quantum error-correcting code.
- Each "logical" qubit is stored nonlocally, shared by many physical qubits, and can be protected if the noise is sufficiently weak and also sufficiently weakly correlated in space and time.

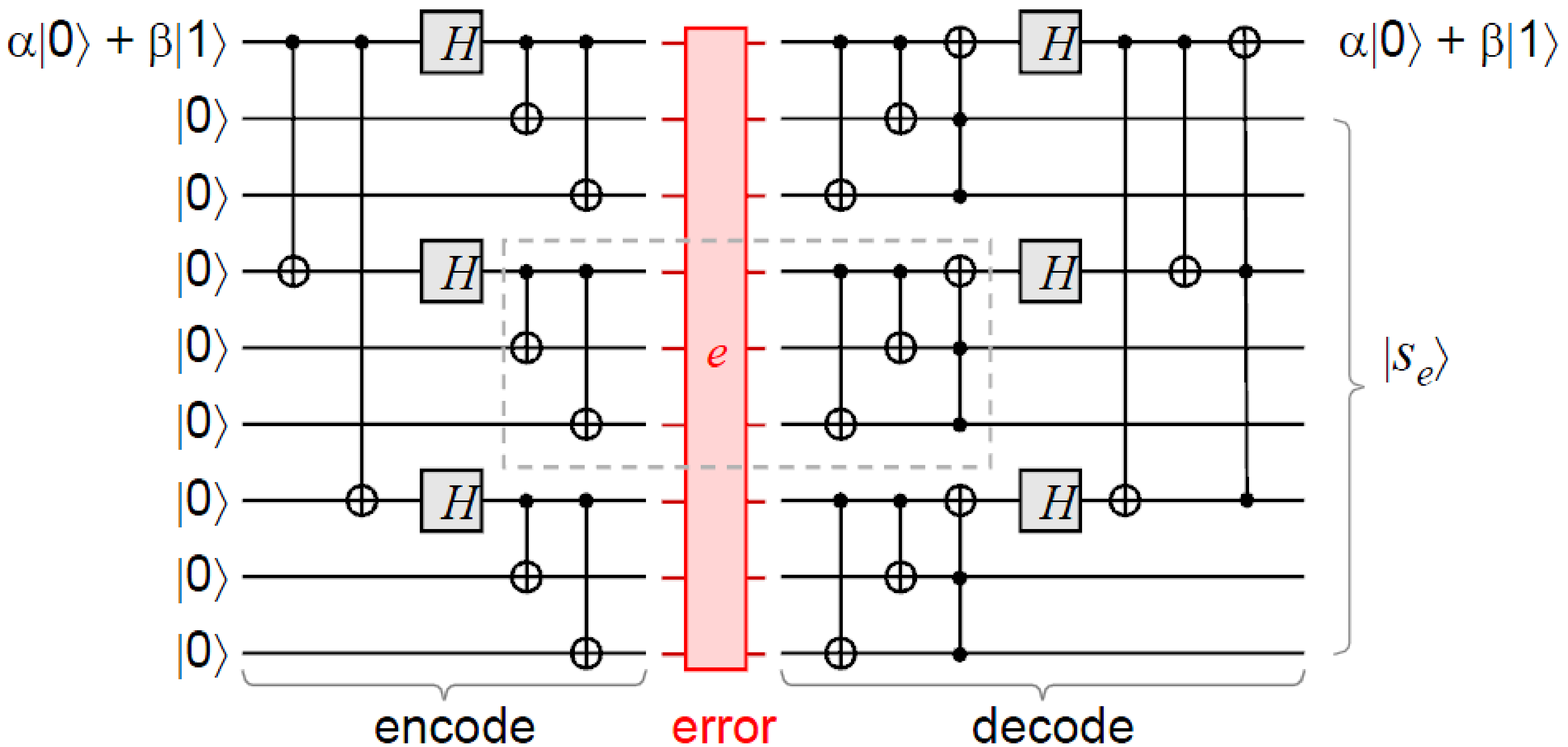
Two central questions are:

1)For what noise models does fault-tolerant quantum computing work effectively?

2)For a given noise model, what is the overhead cost of simulating an ideal quantum computation with noisy hardware?

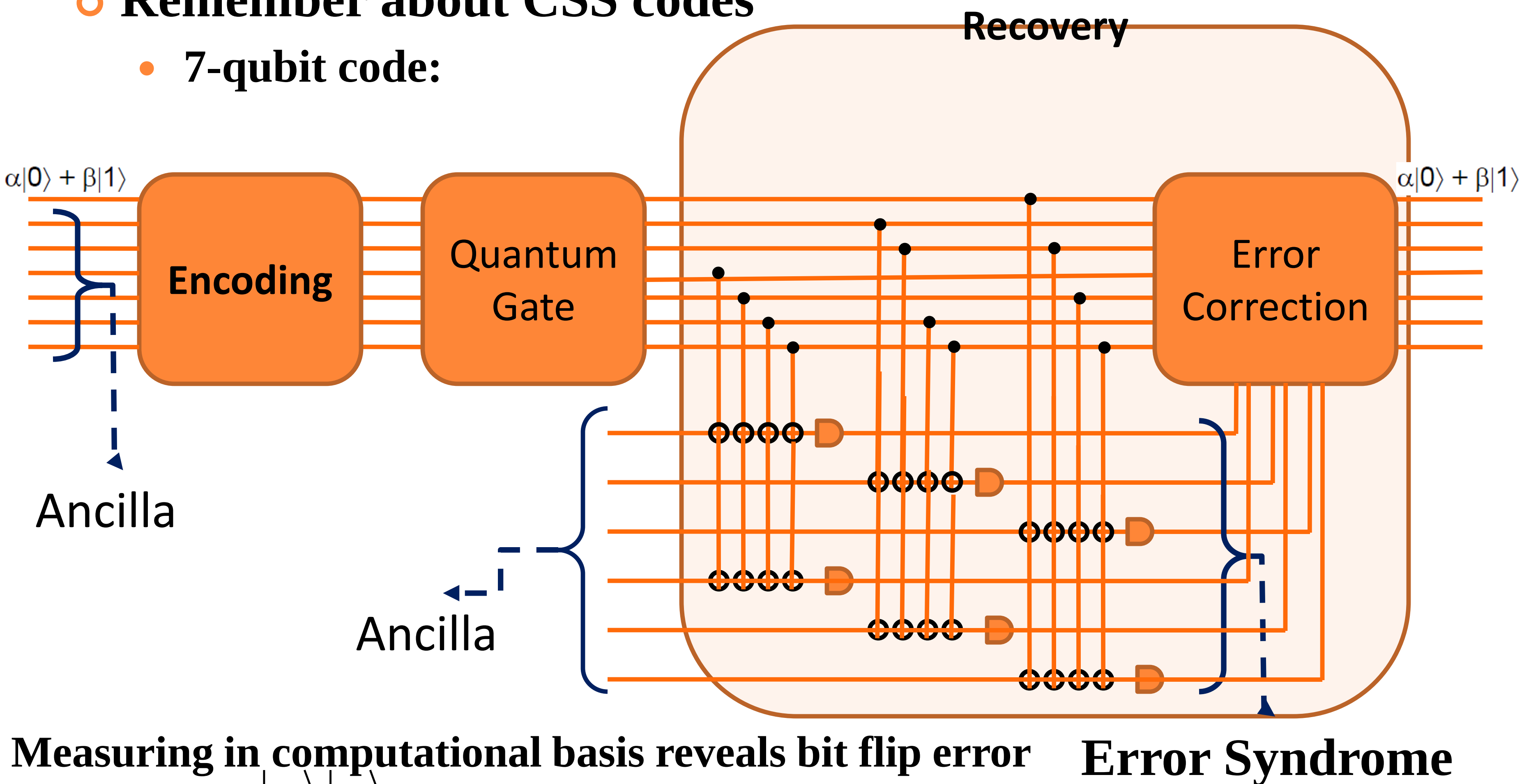
Quantum Error Correction codes

- **Remember from the course about Shor's 9-qubit code.**



Quantum Error Correction codes

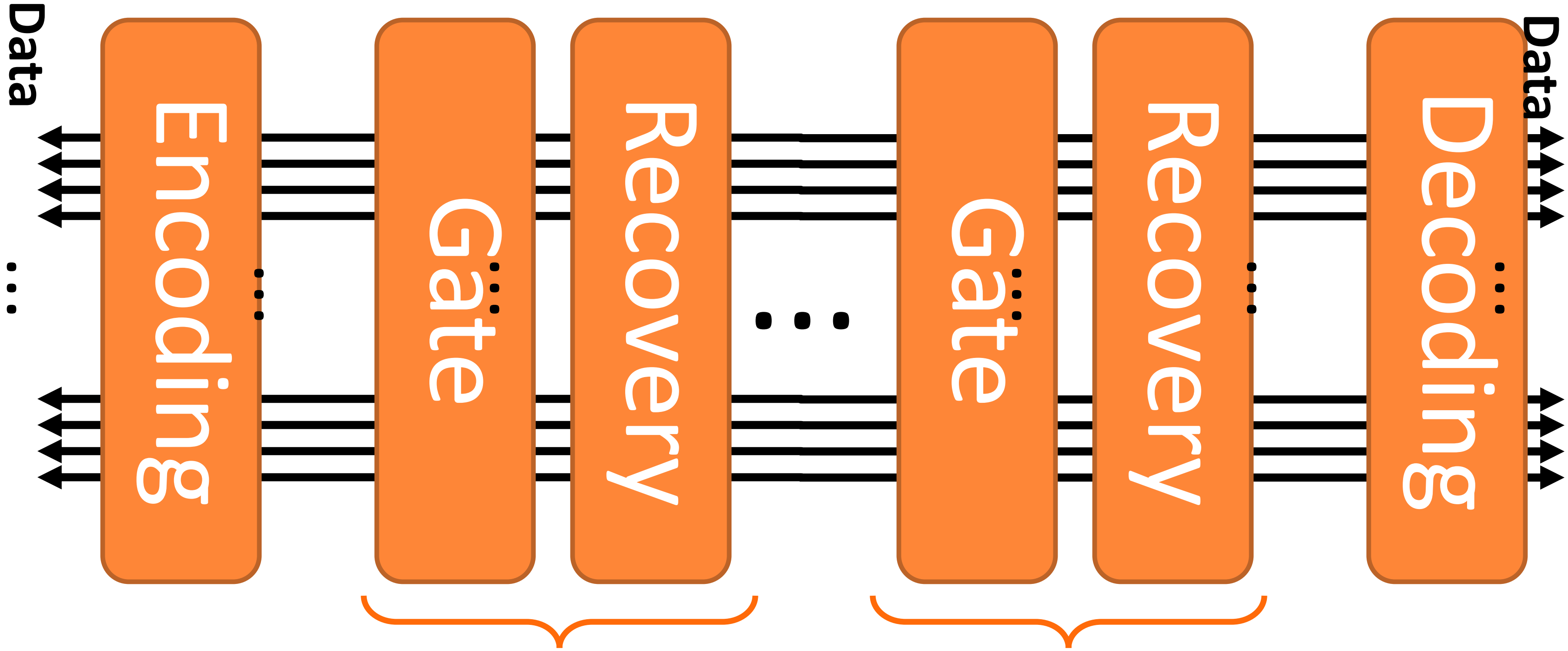
- **Remember about CSS codes**
  - **7-qubit code:**



Measuring in computational basis reveals bit flip error
Measuring in $|+\rangle, |-\rangle$ **basis reveals phase flip error**

Quantum Error Correction codes

- A computation block diagram



Overview of Errors

- Errors are far more common in quantum computers than classical computers [1]
- The main sources of the errors
  - Decoherence as a result of environment-system interaction
  - Imperfect logic gates

- The future prospects for quantum computing received a tremendous boost from the discovery that **quantum error correction** is really possible in principle.
- Not only we need to store the information but also we need to process the information
- Processing information induces error propagation. So we need **fault tolerant gates**

[1] Preskill, J. (1997) Fault-Tolerant Quantum Computation. arXiv:quant-ph/9712048v1 19 Dec 1997

What is a “Fault Tolerant” device?

“A device is fault-tolerant if it works reliably despite of having imperfect basic components.”

Basics of Fault-Tolerance

• The purpose of fault-tolerance is to enable reliable quantum computations when the computer's basic components are unreliable.

• To achieve this, the qubits in the computer are encoded in blocks of a quantum error-correcting code, which allows us to correct the state even when some qubits are wrong.

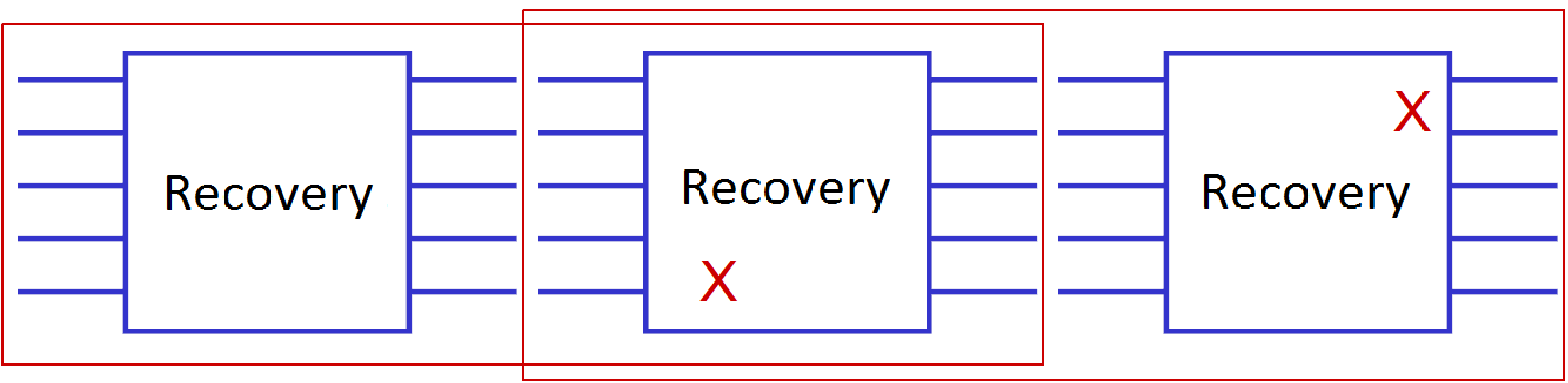
• A fault-tolerant protocol [2] prevents catastrophic error propagation by ensuring that a single faulty gate or time step produces only a single error in each block of the quantum error-correcting code.

[2] Gottesman, D. (2007) Fault-Tolerant Quantum Computation. arXiv:quant-ph/0701112v2 30 Aug 2007

Fault-Tolerant Recovery

- Let us now assume that the recovery block is not perfect. Suppose we need to store data in a quantum memory. A fault is a location in a circuit where a gate or storage error occurs and an error is a qubit in a block that deviates from the ideal state.

A quantum memory fails only if two faults occur in some "extended rectangle."

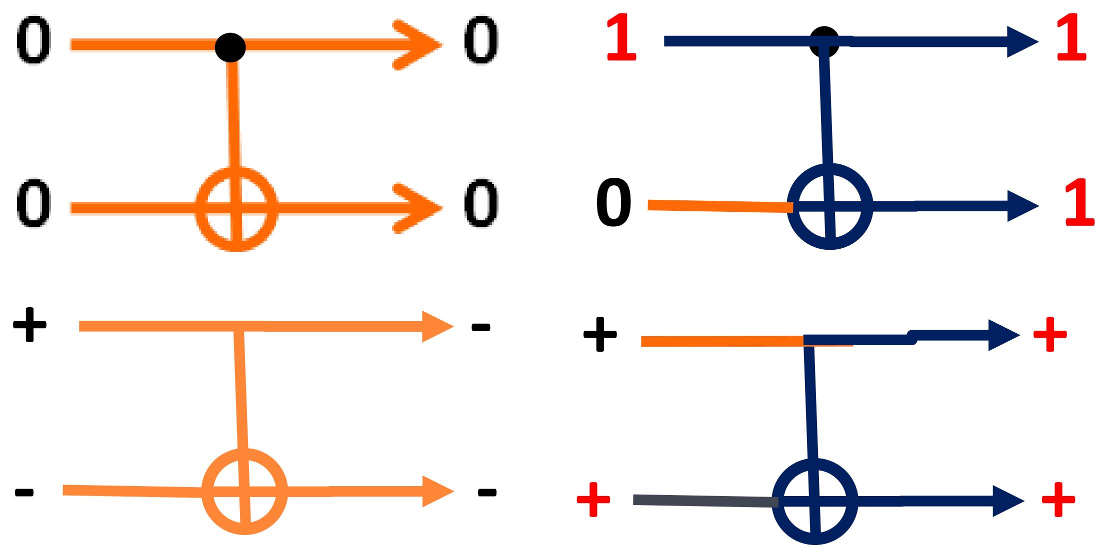


Fault-Tolerant Recovery

- We need to ensure that the recovery part does not cause an error with probability of $O(\varepsilon)$
- The biggest concern in designing the recovery block:
  - Propagation of the error from the source qubit to the target qubit and vice versa

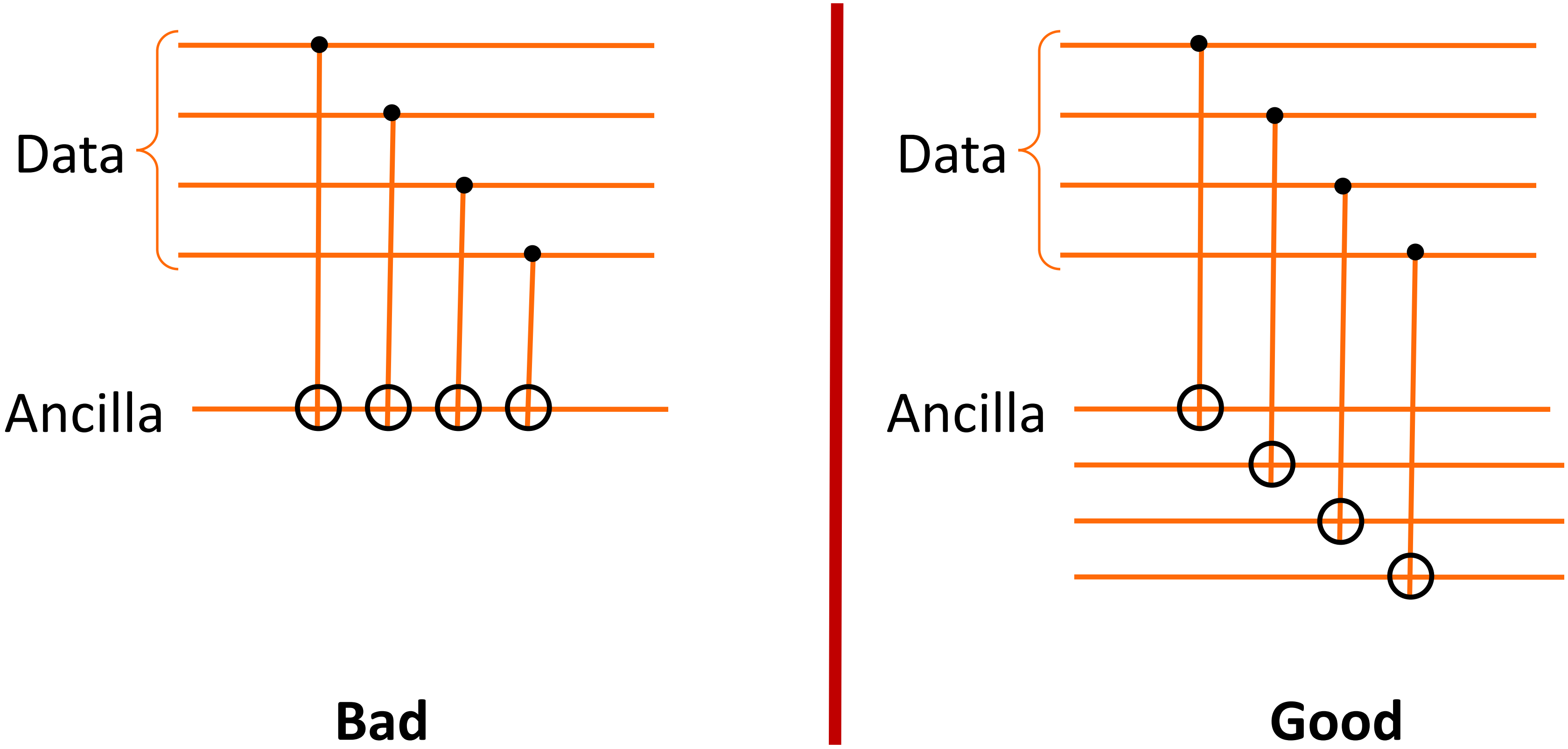
From Source to Target

From Target to Source



Fault-Tolerant Recovery

- Solution:
  - First step is to use each ancilla only for one 2-qubit operation



Fault-Tolerant Recovery

- However, adding ancilla creates another difficulty.
  - How to prepare the state such that
    - We do not feed multiple phase errors into the data
    - By measuring the ancilla, we obtain information about the errors and no information about the data
  - One way to overcome this problem is to use the 4-qubit Shor state or 7-qubit Steane state
    - **Shor State :**

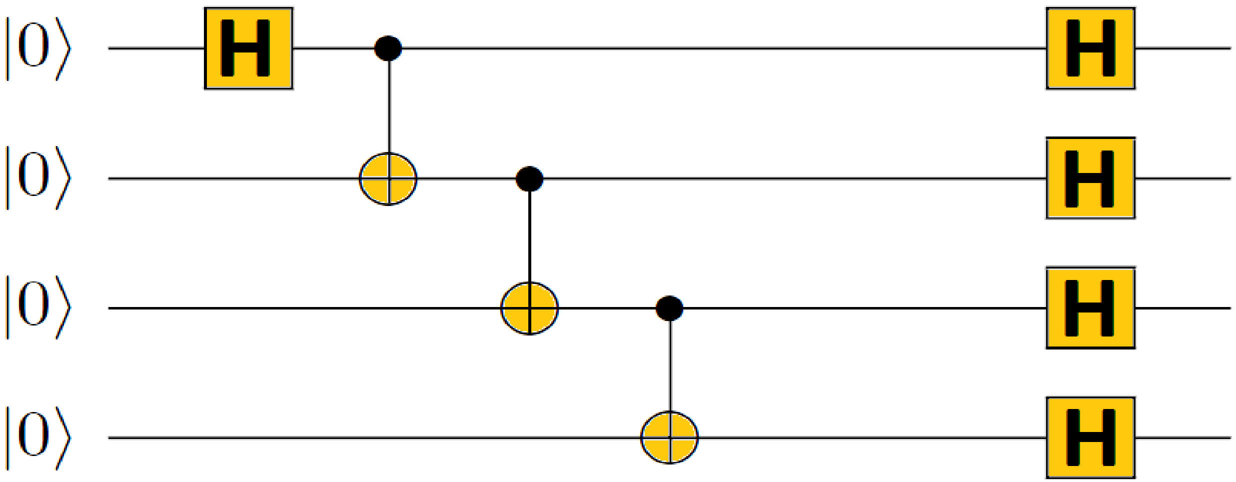
$$\left|Shor\right\rangle_{anc} = \frac{1}{\sqrt{8}} \sum_{v\ even} \left|v\right\rangle_{anc}$$

    - **Steane state:**

$$\left|Steane\right\rangle_{anc} = \frac{1}{4} \sum_{v \in Hamming} \left|v\right\rangle_{anc}$$

Fault-Tolerant Recovery: Preparing the ancilla (For shor's state)

- Shor's state [3] preparation:

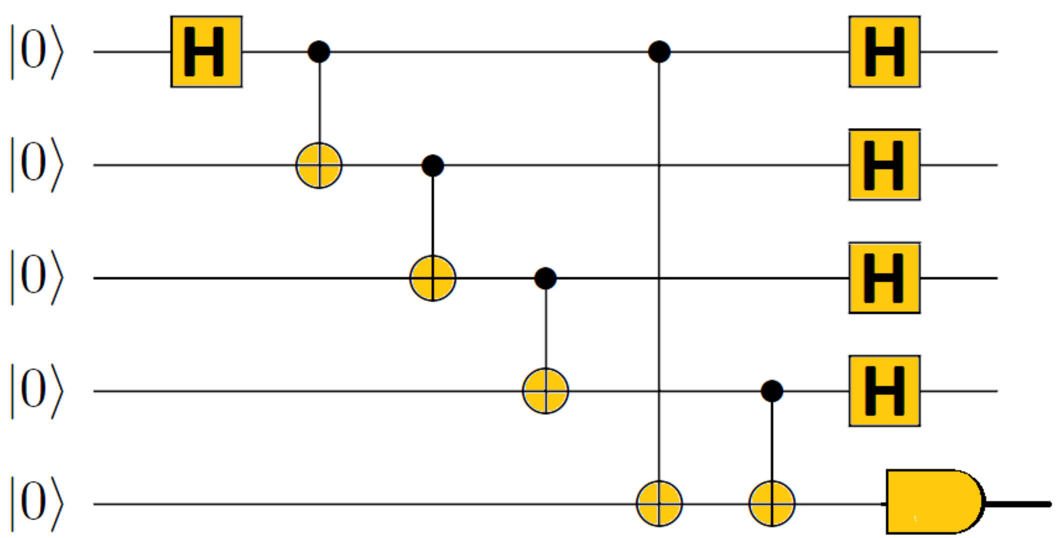


- However, using this circuit, our procedure is yet not fault tolerant. Any single error in this process causes two phase errors in the recovery block which cannot be corrected

[3] Peter W. Shor (1997) Fault Tolerant Quantum Computation. arXiv:quant-ph/9605011v2 5 Mar 1997

Fault-Tolerant Recovery: verifying the ancilla (For shor's state)

- To overcome this problem ,we must verify the ancilla. If it is good then we will keep it and if it is not, we will discard it and prepare another one
- Verifying the ancilla which is prepared in Shor's state: In faulty instances, first and last bits are not equal. Therefore, we add two XOR gates followed by a measurement

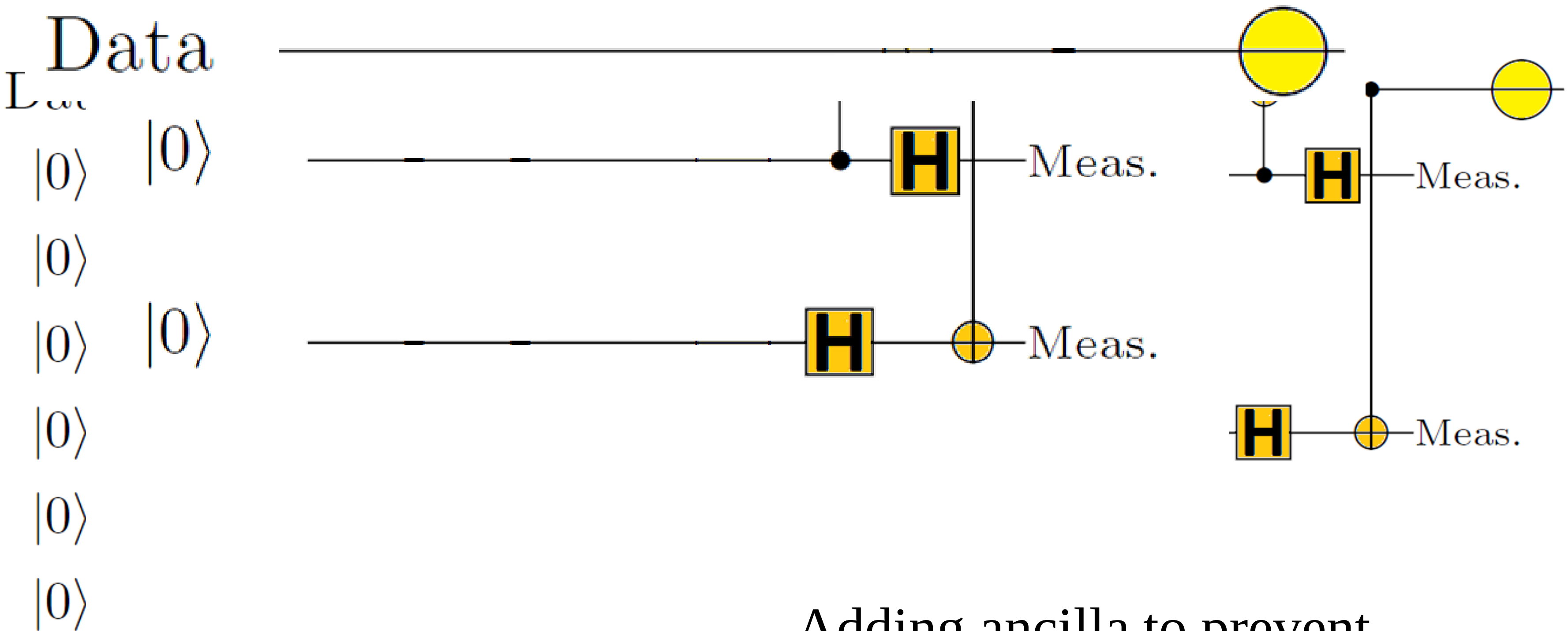


Fault-Tolerant Recovery: verifying the syndrome

- We are not done yet: A single bit-flip error in the syndrome will result in a faulty syndrome.
- To overcome this, we can repeat computing the syndromes to check if they agree.
- If they didn't, we can continue computing the syndromes until we get two agreeing in a row

Fault-Tolerant Recovery: The complete Circuit

- The complete circuit for Steane error recovery:



Adding ancilla to prevent error propagation

Steane's 7-qubitCode

Verifying the syndrome

Verifying the ancilla prepared in Steane's state (Encoded 0's are prepared then verified)

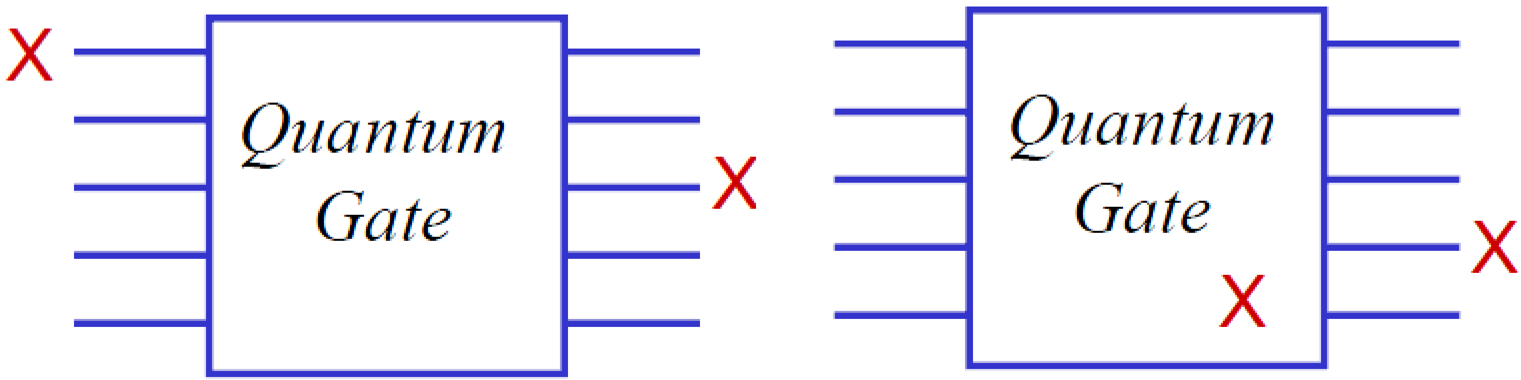
Fault-Tolerant quantum gates

- Coding can protect quantum information. But we need more than just storing, we need to process the information as well

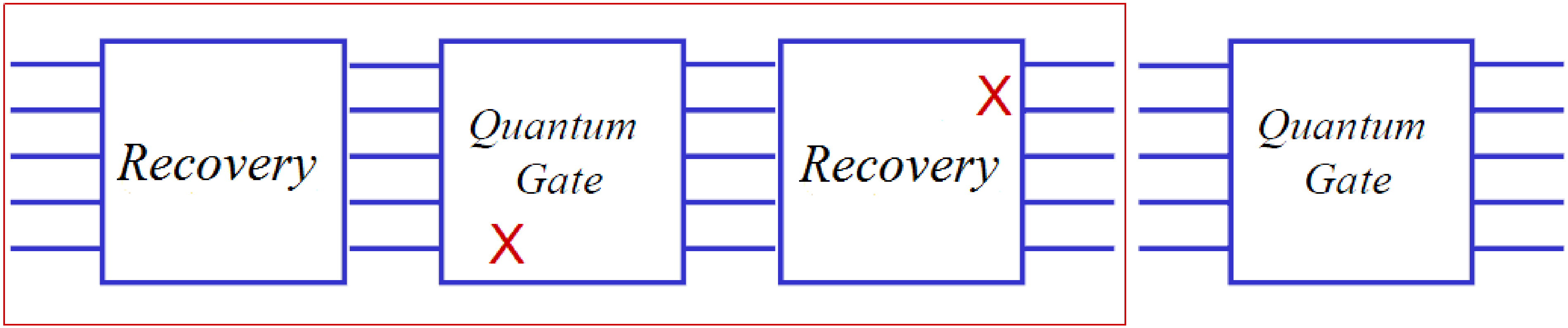
- Decoding, performing the gate and then re-encoding is not a good idea. Since the procedure temporarily exposes the quantum information to harm.

- What do we expect from a fault-tolerant quantum gate?
  - To prevent the propagation of the error

Fault-Tolerant quantum gates

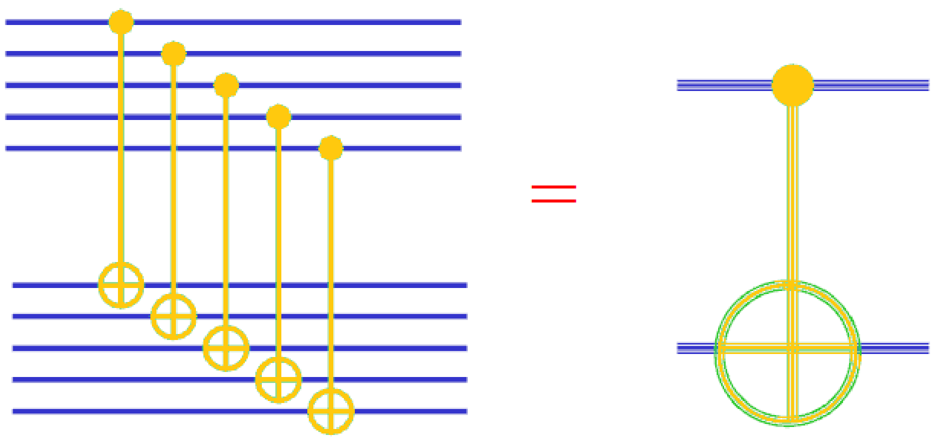


- Each gate is preceded by an error correction step. The circuit simulation fails only if two faults occur in some "extended rectangle."



Fault-Tolerant quantum gates

- There are a number of gates which can be easily implemented with Steane's 7-qubit code: $H, CNOT, P = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$

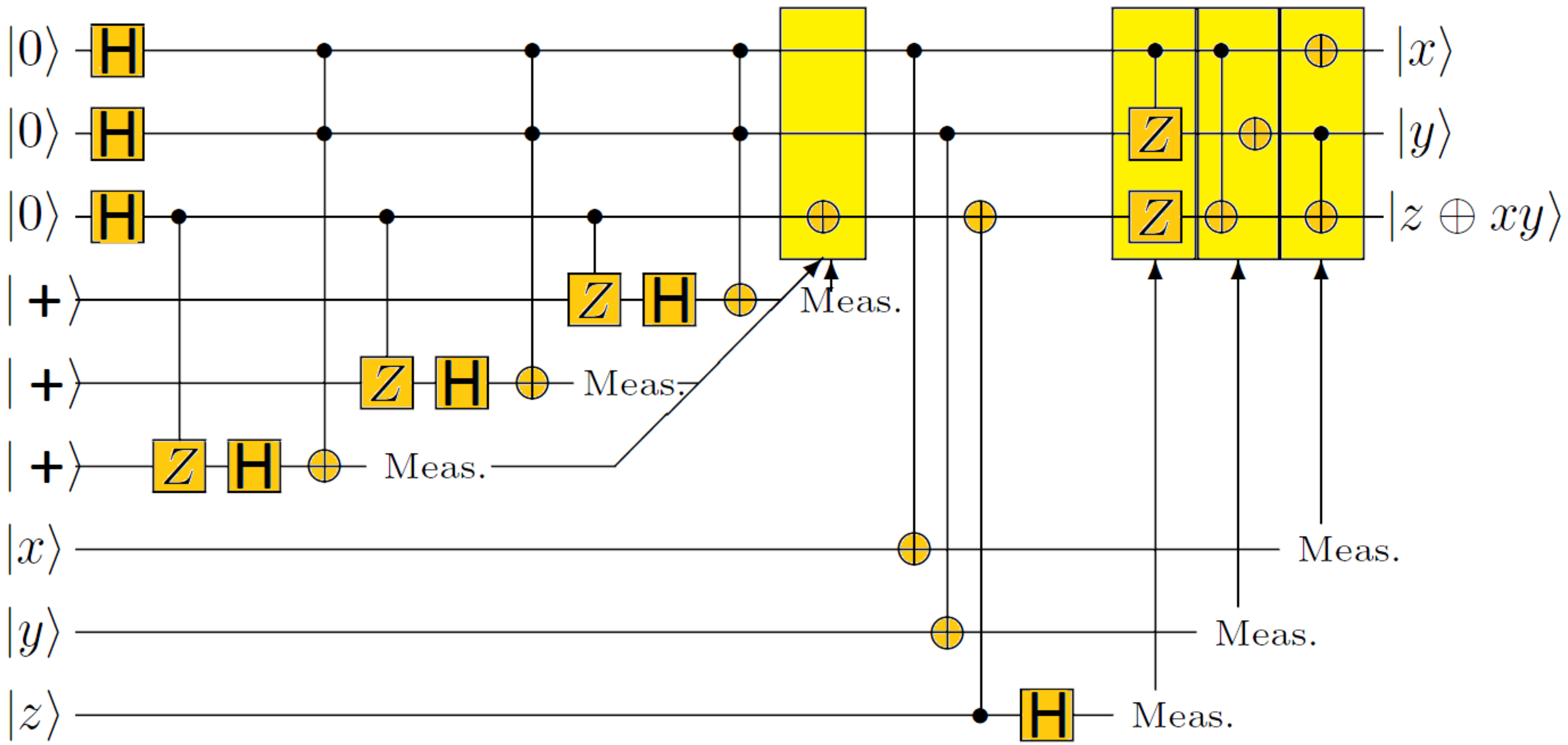


- Unfortunately they do not form a universal set of gates [5], following Shor, we can add "Toffoli" gate

[5] Preskill, J. (2006, Mar). Pedagogical Lecture: Fault Tolerant Quantum Computation . Retrieved from http://online.kitp.ucsb.edu/online/qubit06/preskill

Fault-Tolerant quantum gates

- Shor’s fault-tolerant Toffoli gate
  - It is so carefully designed to minimize the propagation of error. i.e. Two errors must occur in the block to fail us.



Fault-Tolerant quantum gates

- Fault Tolerant gates form a discrete set which is not that desirable, but it is also an unavoidable feature of any fault tolerant scheme

  - The reason: We cannot avoid making an error by applying the wrong gate, a gate that differs from the intended one by a small amount

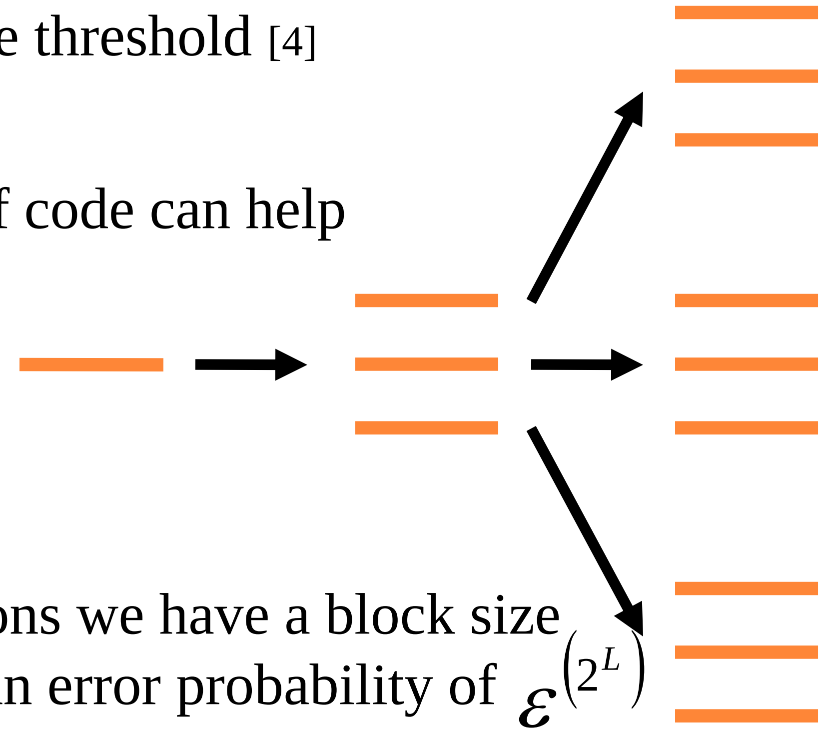
Does an Accuracy Threshold for Quantum Computation exist?

- At first glance it is not clear that such threshold exists
  - There exist error correcting codes which correct t errors per unit of time. So the error probability is ε^{t+1}
  - In contrast, such error correcting codes are so complex increasing the total error probability. Using them, we can carry T cycles of error correction before an error is likely if

our gates have an accuracy of $\varepsilon \approx \left(\frac{1}{\log(T)}\right)^{b}$

The Accuracy Threshold for Quantum Computation

- $\varepsilon \approx \left(\frac{1}{\log(T)}\right)^{b}$ is not a very desirable threshold [4]
- Fortunately using a special kind of code can help
  - Concatenated codes



  - Using this code with L recursions we have a block size of 7^{L} (for 7-qubit code), but an error probability of $\varepsilon^{\left(2^{L}\right)}$

[4] Knill, E., Laflamme, R., & Zurek, W. (1996) Accuracy Threshold for Quantum Computation. quant-ph/9610011 v3 15 Oct 1996

What is meant by the accuracy threshold?

- It doesn't mean :

  once our gate perform better than some threshold, we can do any desired size of computation [7]

- It means :

  with little cost (L recursions), we can do a huge computation (still its size is bounded)

[7] C. H. Bennett, E. Bernstein, G. Brassard and U. Vazirani, Strengths and weaknesses of quantum computation, SIAM J. Computing,Oct. 1997

The value of the threshold

- It sets a target value of accuracy that experimentalists will try to achieve.
- However, citing a single number as the threshold value is a bit deceptive, as there are a large number of variables involved
- Thresholds are derived for specific quantum error correction codes and fault-tolerant protocols and with specific assumptions about the physical implementation
- Thresholds are computed using Simulations or mathematical proves

Conclusion

- Discovery of the quantum error correction codes gave a huge boost to quantum information field: a single qubit cannot survive decoherence, but a set of qubits can; “together we stand, divided we fall”
- The propagation of the error is prevented by using fault tolerant recovery after storing or processing the information, and by using fault tolerant gates for processing the information.
- Thanks to the existence of “Concatenated code”, we can do a huge computation with relatively small number of qubits

References

[1] Preskill, J. (1997) Fault-Tolerant Quantum Computation. arXiv:quant-ph/9712048v1 19 Dec 1997.

[2] Gottesman, D. (2007) Fault-Tolerant Quantum Computation. arXiv:quant-ph/0701112v2 30 Aug 2007.

[3] Peter W. Shor (1997) Fault Tolerant Quantum Computation. arXiv:quant-ph/9605011v2 5 Mar 1997

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[7] C. H. Bennett, E. Bernstein, G. Brassard and U. Vazirani, Strengths and weaknesses of quantum computation, SIAM J. Computing,Oct. 1997.

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