

LOGIC-BASED EXPLAINABLE ARTIFICIAL INTELLIGENCE

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Lecture 03

Recapitulate second lecture

- Rigorous definitions of abductive and contrastive explanations

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- Example algorithm for finding one AXp/CXp

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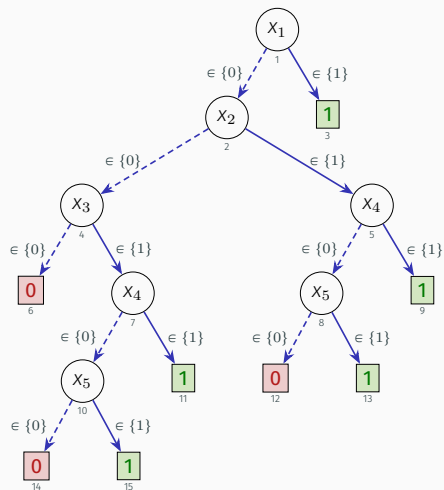
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- Explanations for monotonic classifiers

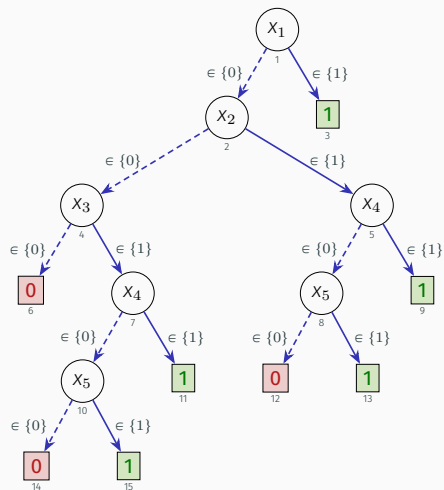
Recap AXps/CXps: DT example

- Instance: $((0, 0, 1, 0, 0), 0)$



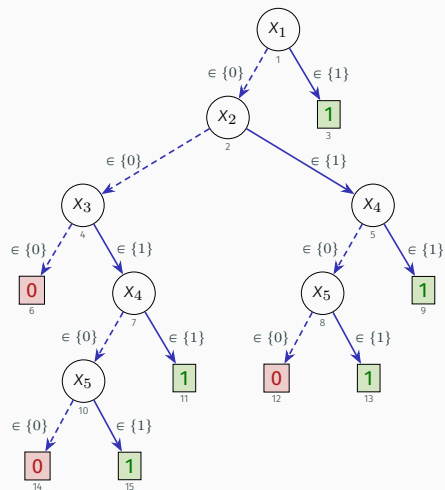
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- Instance: $((0, 0, 1, 0, 0), 0)$
- One AXp: $\{1, 4, 5\}$



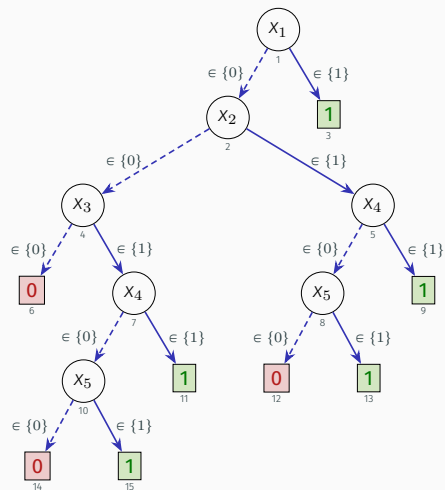
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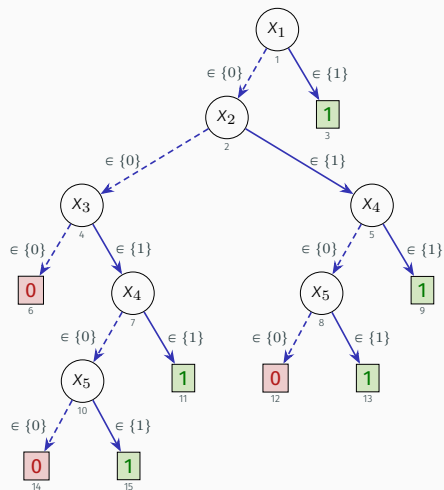
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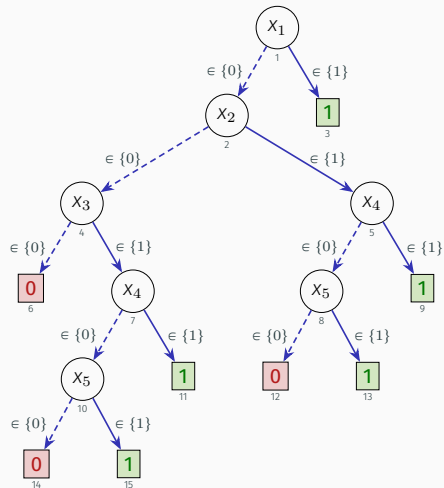
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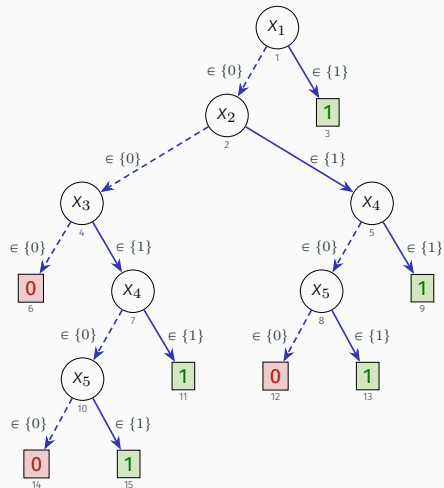
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 - $l_3: \{2, 5\}$



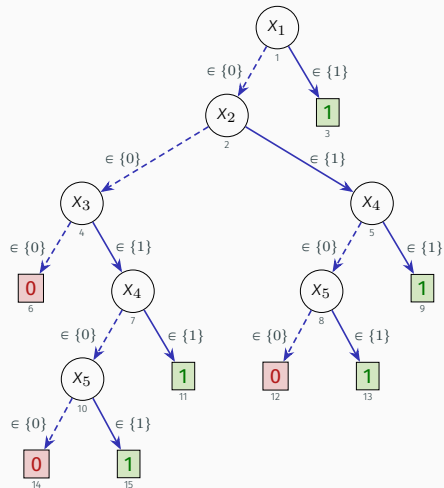
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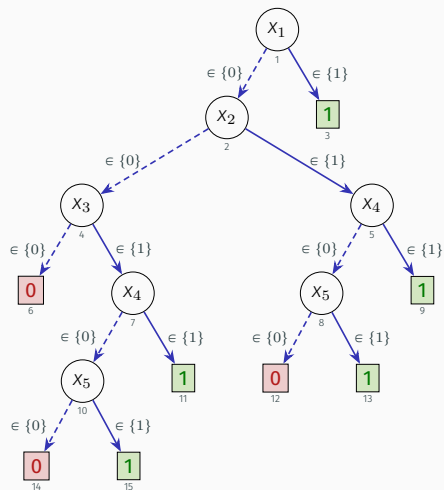
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 - $l_5: \{1\}$



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 - $l_4: \{2, 4\}$
 - $l_5: \{1\}$
 - $\mathcal{L} = \{\{1\}, \{4\}, \{5\}\}$



Recap AXps/CXps: DL example

R_1 : IF $(x_1 = 1)$ THEN 0
 R_2 : ELSE IF $(x_2 = 1)$ THEN 1
 R_3 : ELSE IF $(x_4 = 1)$ THEN 0
 R_{DEF} : ELSE THEN 1

Entry	x_1	x_2	x_3	x_4	Rule	$\kappa_1(\mathbf{x})$
00	0	0	0	0	R_{DEF}	1
01	0	0	0	1	R_3	0
02	0	0	0	2	R_{DEF}	1
03	0	0	1	0	R_{DEF}	1
04	0	0	1	1	R_3	0
05	0	0	1	2	R_{DEF}	1
06	0	1	0	0	R_2	1
07	0	1	0	1	R_2	1
08	0	1	0	2	R_2	1
09	0	1	1	0	R_2	1
10	0	1	1	1	R_2	1
11	0	1	1	2	R_2	1
12	1	0	0	0	R_1	0
13	1	0	0	1	R_1	0
14	1	0	0	2	R_1	0
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18	1	1	0	0	R_1	0
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- Instance: $(\mathbf{v}, c) = ((0, 0, 1, 2), 1)$
- AXp's: $\{1, 4\}$ (prediction unchanged)
- CXp's:
 - $\{1\}$, by flipping the value of feature 1
 - $\{4\}$, by flipping the value of feature 4
 - But also, $\{\{1\}, \{4\}\}$ by MHS duality

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Plan for this course

- Lecture 01 – unit(s):
 - #01: Foundations
- Lecture 02 – unit(s):
 - #02: Principles of symbolic XAI – feature selection
 - #03: Tractability in symbolic XAI (& myth of interpretability)
- Lecture 03 – unit(s):
 - #04: Intractability in symbolic XAI (& myth of model-agnostic XAI)
 - #05: Explainability queries
- Lecture 04 – unit(s):
 - #06: Recent, emerging & advanced topics
- Lecture 05 – unit(s):
 - #07: Principles of symbolic XAI – feature attribution (& myth of Shapley values in XAI)
 - #08: Corrected feature attribution – nuSHAP
 - #09: Conclusions & research directions

Some necessary comments...

- Std question: Can we apply symbolic XAI to this highly complex ML model XYZ?

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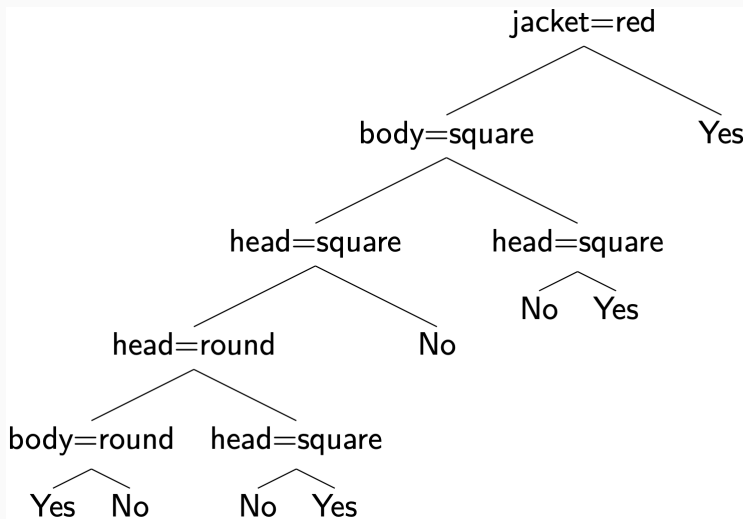
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 - Would you use an ML model that you cannot explain with rigor, and whose heuristic explanations can be incorrect, and so debugging/understanding with rigor is all but impossible?

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- What is the bottom line?
 - For high-risk and safety-critical domains, one **ought** to deploy models that can be explained with rigor
 - If that means using a fairly unexciting NN with up to 100K neurons, that is the cost of trust; **for anything else, one is trying his/her luck, in situations that could become catastrophic!**

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 - More examples next...



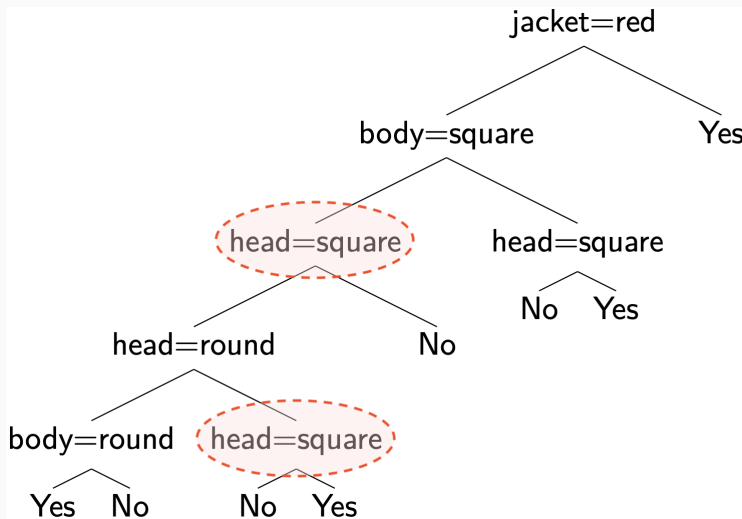
Source: Xiyang Hu, Cynthia Rudin, Margo I. Seltzer:

Optimal Sparse Decision Trees.

NeurIPS 2019: 7265-7273

Priceless optimal sparse decision trees (OSDT) – & non-optimality!...

[HRS19]



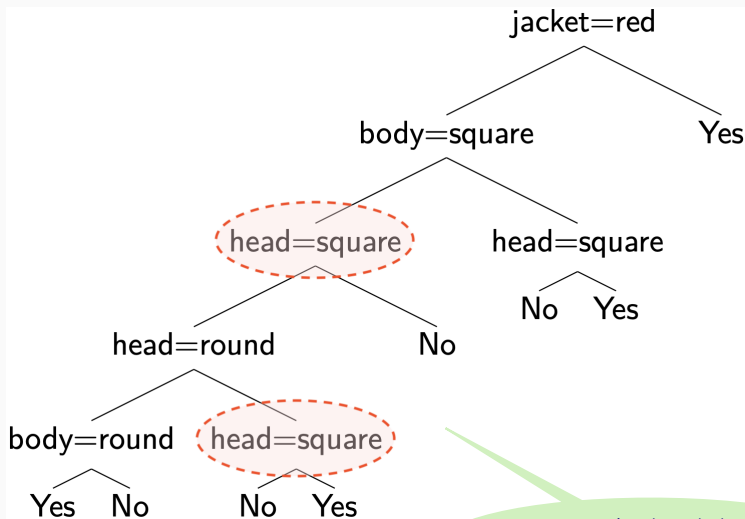
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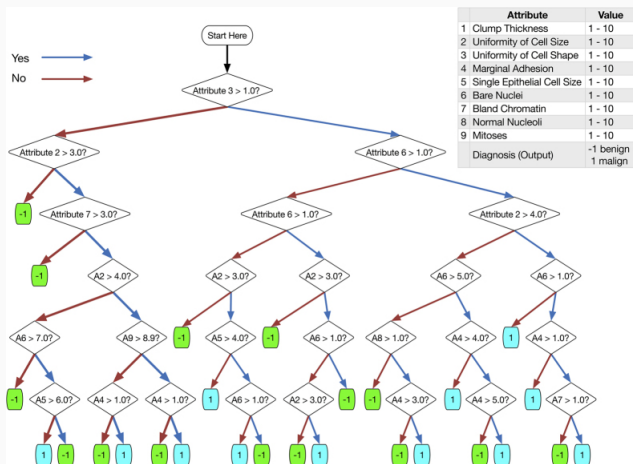
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An optimal tool that
produces **non-optimal** DTs...!?

BTW, highly problematic decision trees also in precision medicine...



Example Interpretable Rules Induced by MediBoost:

$A3 \text{ Uniformity of Cell Shape} \leq 1.0 \wedge A2 \text{ Uniformity of Cell Size} > 3.0 \wedge A7 \text{ Bland Chromatin} \leq 3.0 \Rightarrow \text{predict benign}$

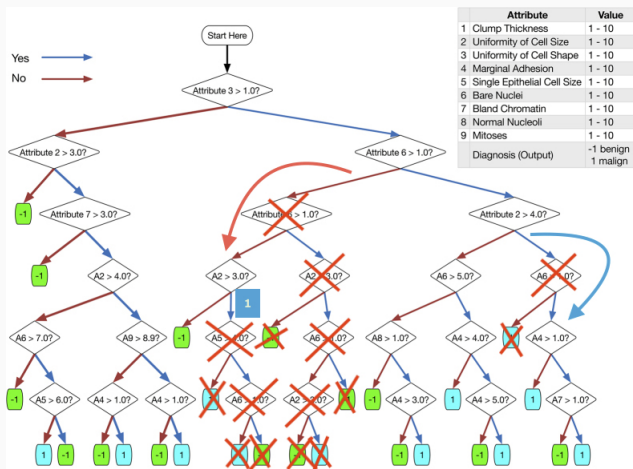
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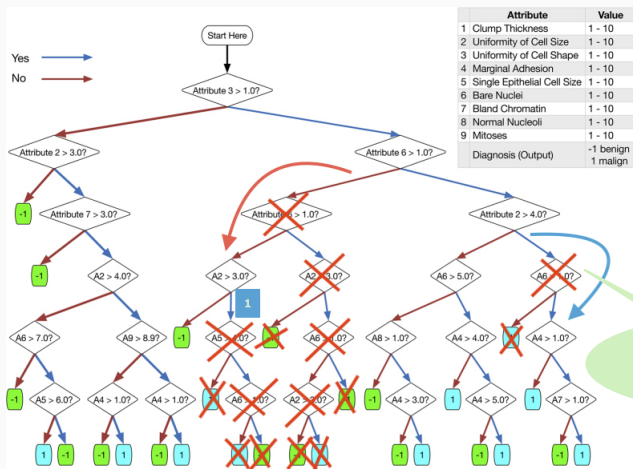
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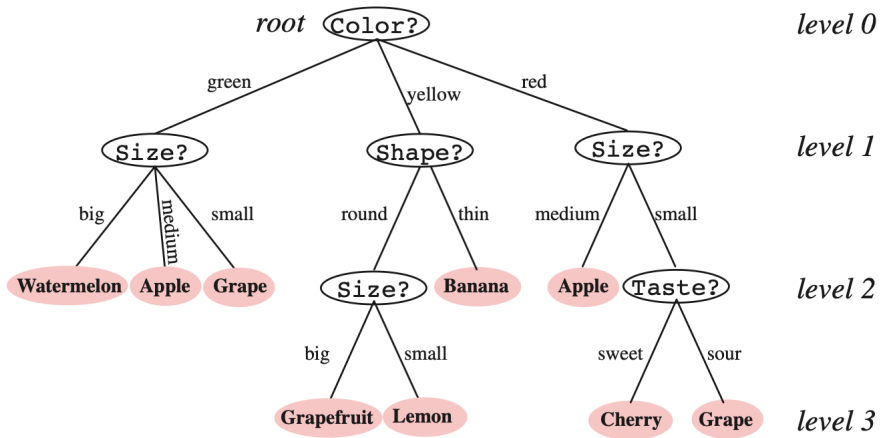
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- And how to debug complex ML models if heuristic explanations are also incorrect (more later)?

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- However, it is relatively simple to implement tree learners
- Can one really trust the operation of more complex ML models, even those subject to extensive testing?
- And how to debug complex ML models if heuristic explanations are also incorrect (more later)?
- **For trustworthy AI, there exists no alternative to rigorous logic-based explanations!**

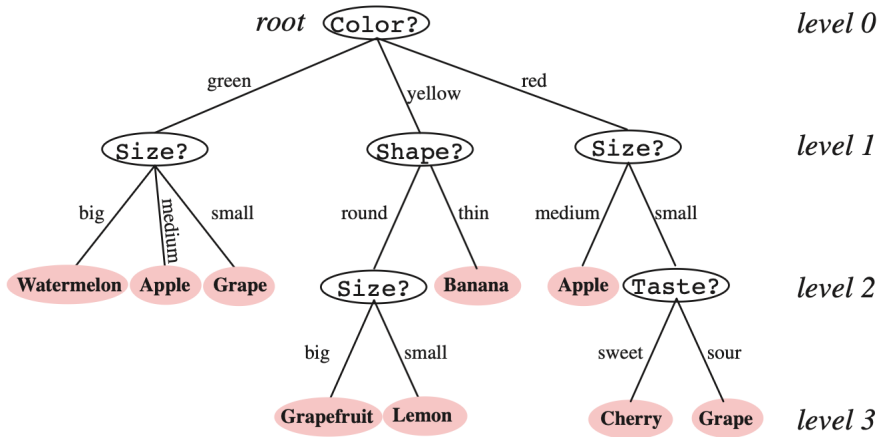
BTW, problematic DTs even in books...

[dud01]



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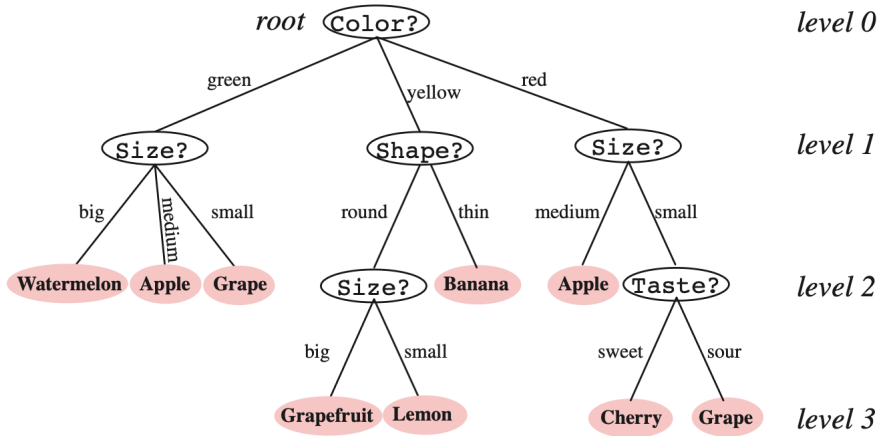
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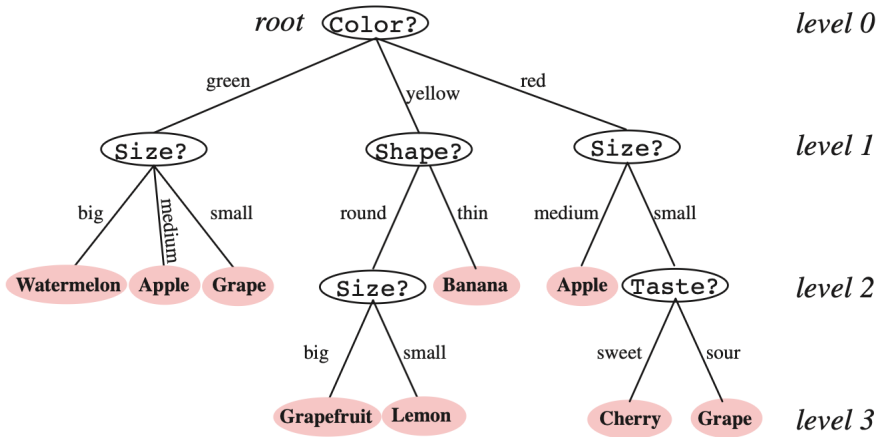
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- Or, what if $\text{Color} = \text{red} \wedge \text{Size} = \text{big}??$
- Easy to envision more serious use-cases...

Unit #04

(Efficient) Intractability in Symbolic XAI

An encoding for DLs – components

R_1 :	IF	(τ_1)	THEN	d_1
R_2 :	ELSE IF	(τ_2)	THEN	d_2
		...		
R_j :	ELSE IF	(τ_j)	THEN	d_j
		...		
R_n :	ELSE IF	(τ_n)	THEN	d_n
R_{DEF} :	ELSE		THEN	d_{n+1}

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- Clauses for encoding ϕ : $\mathfrak{E}_\phi(z_1, \dots)$, such that $z_1 = 1$ iff $\phi = 1$
- For τ_j : $\mathfrak{E}_{\tau_j}(t_j, \dots)$
- For $x_i = v_i$: $\mathfrak{E}_{x_i=v_i}(l_i, \dots)$
- Let $e_j = 1$ iff d_j matches c
- Prediction change with rule up to R_j (with $d_j \neq c$), if $\tau_j \not\models \perp$ and $\tau_k \models \perp$, for $1 \leq k < j$, with $e_k = 1$:

$$\left[f_j \leftrightarrow \left(t_j \wedge \bigwedge_{1 \leq k < j, e_k=1} \neg t_k \right) \right]$$

An encoding for DLs – components

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- For $x_i = v_i$: $\mathfrak{E}_{x_i=v_i}(l_i, \dots)$
- Let $e_j = 1$ iff d_j matches c
- Require that at least one f_j , with $e_j = 0$ and $1 \leq j \leq n$, to be consistent (i.e. some rule up to j with prediction other than c to fire):

$$\left(\bigvee_{1 \leq j \leq n, e_j = 0} f_j \right)$$

An encoding for DLs – components

R_1 :	IF	(τ_1)	THEN	d_1
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		\dots		
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		\dots		
R_n :	ELSE IF	(τ_n)	THEN	d_n
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- The set of soft clauses is given by: $\mathcal{S} \triangleq \{(l_i), i = 1, \dots, m\}$
- The set of hard clauses is given by:

$$\mathcal{B} \triangleq \bigwedge_{1 \leq i \leq m} \mathfrak{E}_{x_i=v_i}(l_i, \dots) \wedge \bigwedge_{1 \leq j \leq n} \mathfrak{E}_{\tau_j}(t_j, \dots) \wedge \\ \bigwedge_{1 \leq j \leq n, e_j=0} \left(f_j \leftrightarrow \left(t_j \wedge \bigwedge_{1 \leq k < j, e_k=1} \neg t_k \right) \right) \wedge \left(\bigvee_{1 \leq j \leq n, e_j=0} f_j \right)$$

- $\mathcal{B} \cup \mathcal{S} \models \perp$
 - MUSes are AXp's & MCSes are CXp's

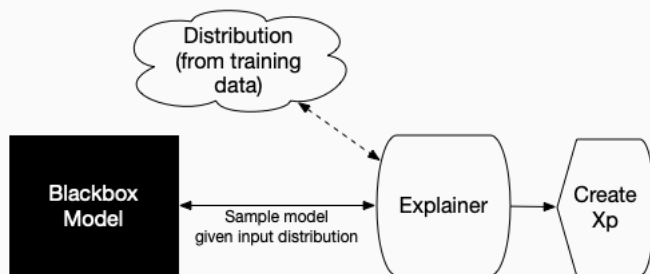
Outline – Unit #04

Explaining Decision Lists

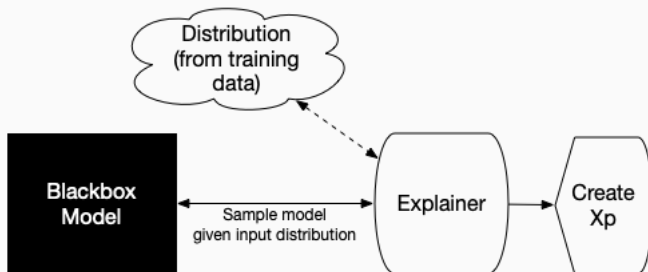
Myth #02: Model-Agnostic Explainability

Progress Report on Symbolic XAI

What is model-agnostic explainability?



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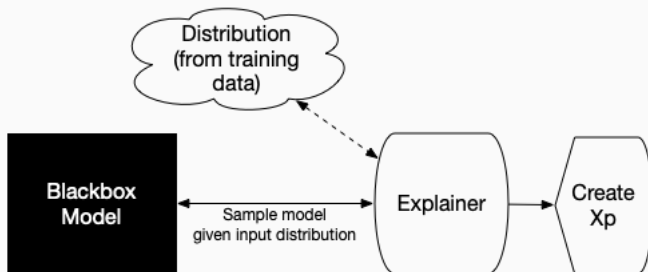
- Wildly popular XAI approach
 - **Feature attribution**: LIME, SHAP, ...
 - **Feature selection**: Anchors, ...

[RSG16, LL17, RSG18]

[RSG16, LL17]

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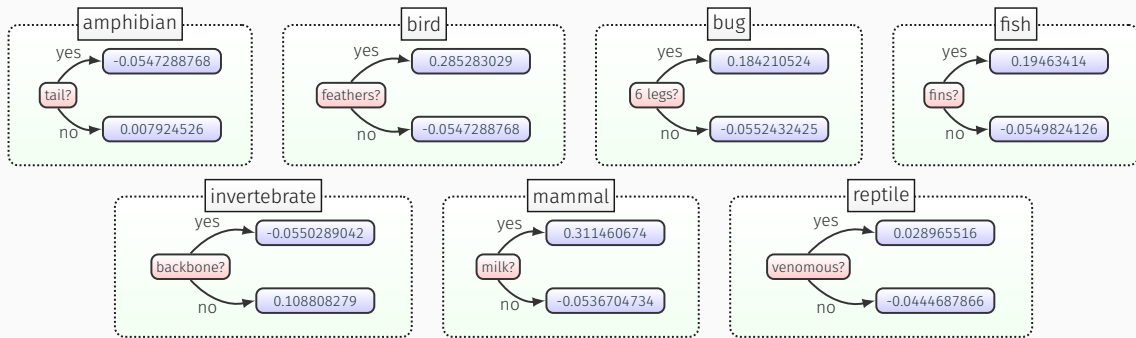
[RSG16, LL17]

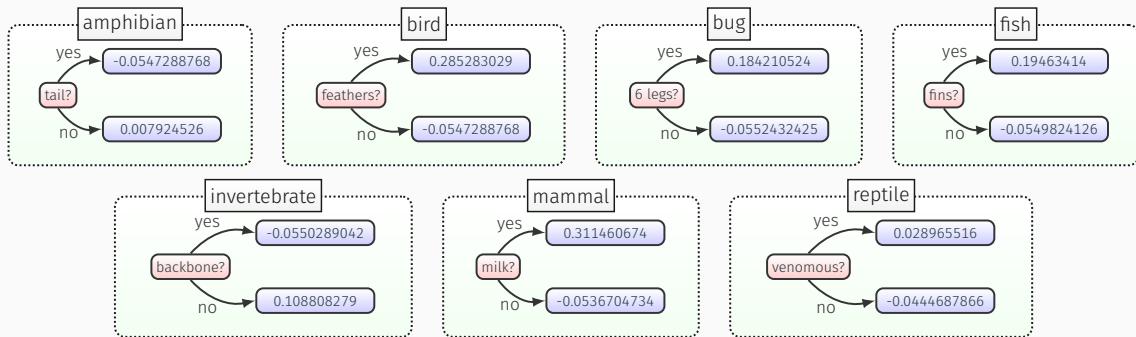
[RSG18]

- **Q:** Are model-agnostic explanations rigorous?

Easy to spot problems – BT for zoo dataset

[INM19b, Ign20]

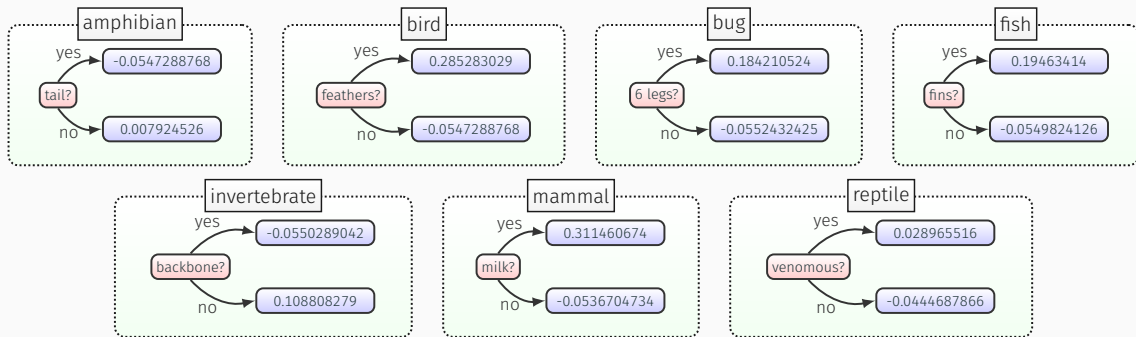




- Example instance:

IF (animal_name = pitviper) \wedge \neg hair \wedge \neg feathers \wedge eggs \wedge \neg milk \wedge \neg airborne \wedge \neg aquatic \wedge predator \wedge \neg toothed \wedge backbone \wedge breathes \wedge venomous \wedge \neg fins \wedge (legs = 0) \wedge tail \wedge \neg domestic \wedge \neg catsize

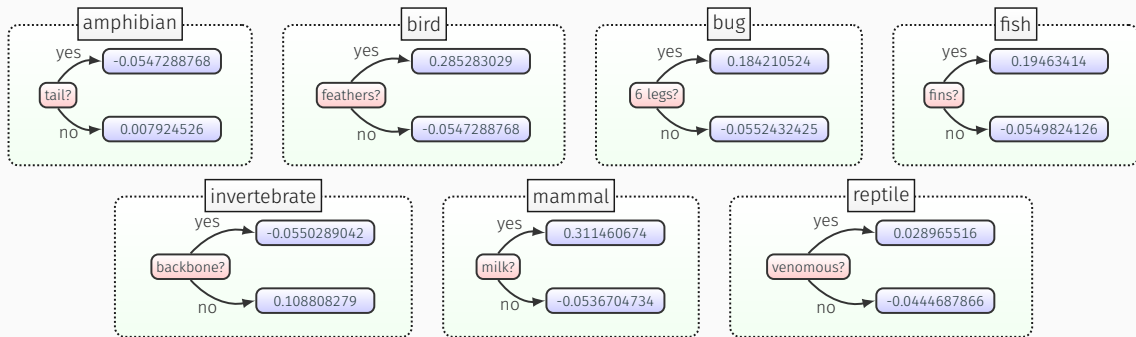
THEN (class = reptile)



- Example instance (& **Anchor** picks):

[RSG18]

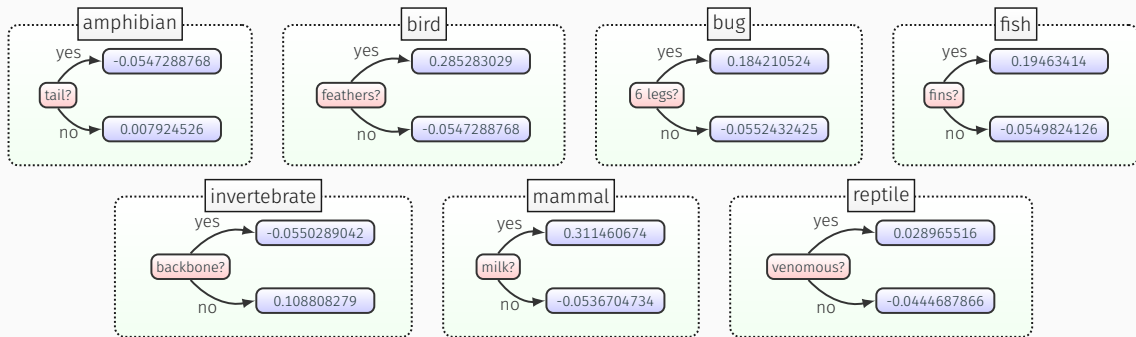
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 venomous \wedge \neg **fins** \wedge (legs = 0) \wedge tail \wedge \neg domestic \wedge \neg catsize
THEN (class = **reptile**)



- Explanation obtained with **Anchor**:

[RSG18]

IF $\neg hair \wedge \neg milk \wedge \neg toothed \wedge \neg fins$
THEN (class = reptile)



- But, explanation **incorrectly “explains”** another instance (from **training data!**)

IF (animal_name = toad) \wedge \neg hair \wedge \neg feathers \wedge eggs \wedge \neg milk \wedge
 \neg airborne \wedge \neg aquatic \wedge \neg predator \wedge \neg toothed \wedge backbone \wedge breathes \wedge
 \neg venomous \wedge \neg fins \wedge (legs = 4) \wedge \neg tail \wedge \neg domestic \wedge \neg catsize
 THEN (class = amphibian)

Incorrect explanations:

Classifier for deciding bank loans

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Two samples: Bessie := (v_1 , **Y**) and Clive := (v_2 , **N**)

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Explanation X: age = 45, salary = 50K

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And,

X is consistent with Bessie := (v_1 , **Y**)

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∴ different outcomes & same explanation !?

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 1. \mathcal{X} is sufficient for prediction:

$$\forall(\mathbf{x} \in \mathbb{F}). \bigwedge_{j \in \mathcal{X}} (x_j = v_j) \rightarrow (\kappa(\mathbf{x}) = c)$$

2. And, \mathcal{X} is subset-minimal:

$$\forall(t \in \mathcal{X}). \exists(\mathbf{x} \in \mathbb{F}). \bigwedge_{j \in (\mathcal{X} \setminus \{t\})} (x_j = v_j) \rightarrow (\kappa(\mathbf{x}) \neq c)$$

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- Approach is bounded by scalability of rigorous explanations...

How serious is the lack of rigor of model-agnostic explanations?

- Obs: Lack of rigor of model-agnostic explanations known since 2019

[INM19b, Ign20, YIS⁺23]

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[INM19b, Ign20, YIS⁺23]

[CG16]

[RSG18]

Dataset	% Incorrect	% Redundant	% Correct
adult	80.5%	1.6%	17.9%
lending	3.0%	0.0%	97.0%
rcdv	99.4%	0.4%	0.2%
compas	84.4%	1.7%	13.9%
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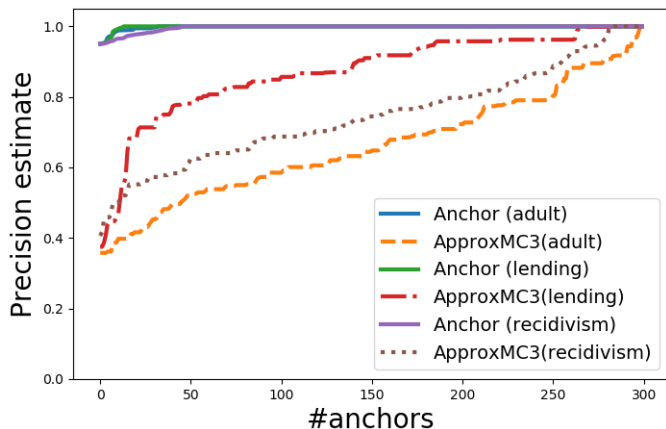
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[NSM⁺19]

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- For feature attribution we proposed different ways of assessing rigor

[INM19b, NSM⁺19, Ign20, YIS⁺23]

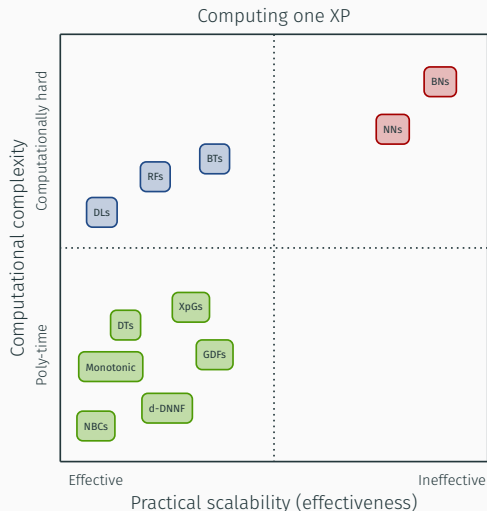


Outline – Unit #04

Explaining Decision Lists

Myth #02: Model-Agnostic Explainability

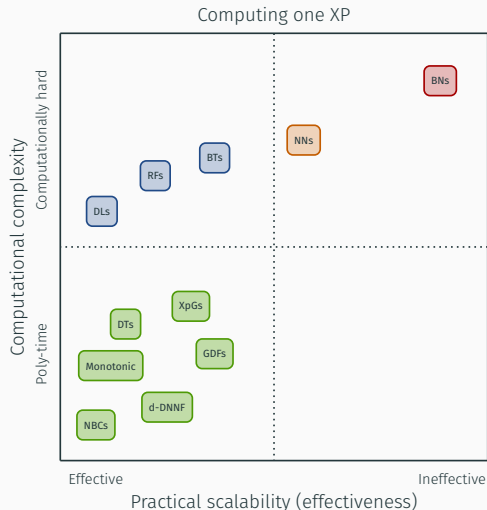
Progress Report on Symbolic XAI



[INM19b, Ign20, IIM20, MGC⁺20, MGC⁺21, HIIM21, IMS21, IM21, CM21, HII⁺22, IISMS22]

Formal explanations efficient for several families of classifiers

- Polynomial-time:
 - Naive-Bayes classifiers (NBCs) [MGC⁺20]
 - Decision trees (DTs) [IIM20, HIIM21]
 - XpG's: DTs, OBDDs, OMDDs, etc. [HIIM21]
 - Monotonic classifiers [MGC⁺21]
 - Propositional languages (e.g. d-DNNF, ...) [HII⁺22]
 - Additional results [CM21, HII⁺22]
- Comp. hard, but **effective** (efficient in practice):
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- Comp. hard, and **ineffective** (hard in practice):
 - Neural networks (NNs) [INM19a]
 - Bayesian networks (BNs) [SCD18]



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- Comp. hard, and ineffective (hard in practice):
 - Bayesian networks (BNs) [SCD18]

Results for RFs in 2021 (with SAT)

[IMS21]

Dataset	#F	#C	#I	RF			CNF		SAT oracle				AXp (RFxpL)				Anchor	
				D	#N	%A	#var	#cl	MxS	MxU	#S	#U	Mx	m	avg	%w	avg	%w
ann-thyroid	(21	3	718)	4	2192	98	17854	29230	0.12	0.15	2	18	0.36	0.05	0.13	96	0.32	4
appendicitis	(7	2	43)	6	1920	90	5181	10085	0.02	0.02	4	3	0.05	0.01	0.03	100	0.48	0
banknote	(4	2	138)	5	2772	97	8068	16776	0.01	0.01	2	2	0.03	0.02	0.02	100	0.19	0
biodegradation	(41	2	106	5	4420	88	11007	23842	0.31	1.05	17	22	2.27	0.04	0.29	97	4.07	3
heart-c	(13	2	61)	5	3910	85	5594	11963	0.04	0.02	6	7	0.07	0.01	0.04	100	0.85	0
ionosphere	(34	2	71)	5	2096	87	7174	14406	0.02	0.02	22	11	0.11	0.02	0.03	100	12.43	0
karhunen	(64	10	200)	5	6198	91	36708	70224	1.06	1.41	35	29	14.64	0.65	2.78	100	28.15	0
letter	(16	26	398	8	44304	82	28991	68148	1.97	3.31	8	8	6.91	0.24	1.61	70	2.48	30
magic	(10	2	381)	6	9840	84	29530	66776	0.51	1.84	6	4	2.13	0.07	0.14	99	0.91	1
new-thyroid	(5	3	43)	5	1766	100	17443	28134	0.03	0.01	3	2	0.08	0.03	0.05	100	0.36	0
pendigits	(16	10	220)	6	12004	95	30522	59922	2.40	1.32	10	6	4.11	0.14	0.94	96	3.68	4
ring	(20	2	740	6	6188	89	19114	42362	0.27	0.44	11	9	1.25	0.05	0.25	92	7.25	8
segmentation	(19	7	42)	4	1966	90	21288	35381	0.11	0.17	8	10	0.53	0.11	0.31	100	4.13	0
shuttle	(9	7	116	3	1460	99	18669	29478	0.11	0.08	2	7	0.34	0.05	0.14	99	0.42	1
sonar	(60	2	42)	5	2614	88	9938	20537	0.04	0.06	36	24	0.43	0.04	0.09	100	23.02	0
spectf	(44	2	54)	5	2306	88	6707	13449	0.07	0.06	20	24	0.34	0.02	0.07	100	8.12	0
texture	(40	11	550)	5	5724	87	34293	64187	0.79	0.63	23	17	3.24	0.19	0.93	100	28.13	0
twonorm	(20	2	740	5	6266	94	21198	46901	0.08	0.08	12	8	0.28	0.06	0.10	100	5.73	0
vowel	(13	11	198)	6	10176	90	44523	88696	1.66	2.11	8	5	4.52	0.15	1.15	66	1.67	34
waveform-40	(40	3	500	5	6232	83	30438	58380	0.50	0.86	15	25	7.07	0.11	0.88	100	11.93	0
wdbc	(33	2	78)	5	2432	76	9078	18675	1.00	1.53	20	13	5.33	0.03	0.65	79	3.91	21

Dataset			Minimal explanation			Minimum explanation		
			size	SMT (s)	MILP (s)	size	SMT (s)	MILP (s)
australian	(14)	m	1	0.03	0.05	—	—	—
		a	8.79	1.38	0.33	—	—	—
		M	14	17.00	1.43	—	—	—
backache	(32)	m	13	0.13	0.14	—	—	—
		a	19.28	5.08	0.85	—	—	—
		M	26	22.21	2.75	—	—	—
breast-cancer	(9)	m	3	0.02	0.04	3	0.02	0.03
		a	5.15	0.65	0.20	4.86	2.18	0.41
		M	9	6.11	0.41	9	24.80	1.81
cleve	(13)	m	4	0.05	0.07	4	—	0.07
		a	8.62	3.32	0.32	7.89	—	5.14
		M	13	60.74	0.60	13	—	39.06
hepatitis	(19)	m	6	0.02	0.04	4	0.01	0.04
		a	11.42	0.07	0.06	9.39	4.07	2.89
		M	19	0.26	0.20	19	27.05	22.23
voting	(16)	m	3	0.01	0.02	3	0.01	0.02
		a	4.56	0.04	0.13	3.46	0.3	0.25
		M	11	0.10	0.37	11	1.25	1.77
spect	(22)	m	3	0.02	0.02	3	0.02	0.04
		a	7.31	0.13	0.07	6.44	1.61	0.67
		M	20	0.88	0.29	20	8.97	10.73

First rigorous approach
for explaining NNs !

			Minimal explanation			Minimum explanation		
			size	SMT (s)	MILP (s)	size	SMT (s)	MILP (s)
australian	(14)	m	1	0.03	0.05	—	—	—
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cleve	(13)	m	4	0.05	0.07	4	—	0.07
		a	8.62	3.32	0.32	7.89	—	5.14
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hepatitis	(19)	m	6	0.02	0.04	4	0.01	0.04
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Scales to (a few)
tens of neurons...

Results for NNs in 2023 (using Marabou [KHI⁺19])

[HM23]

DNN	points	AXp	#Calls	Time	#TO	AXp	#Calls	Time	#TO
$\epsilon = 0.1$					$\epsilon = 0.05$				
ACASXU_1_5	#1	3	5	185.9	0	2	5	113.8	0
	#2	2	5	273.8	0	1	5	33.2	0
	#3	0	5	714.2	0	0	5	4.3	0
ACASXU_3_1	#1	0	5	2219.3	0	0	5	14.2	0
	#2	2	5	4263.5	1	0	5	1853.1	0
	#3	1	5	581.8	0	0	5	355.9	0
ACASXU_3_2	#1	3	5	13739.3	2	1	5	6890.1	1
	#2	3	5	226.4	0	2	5	125.1	0
	#3	2	5	1740.6	0	2	5	173.6	0
ACASXU_3_5	#1	4	5	43.6	0	2	5	59.4	0
	#2	3	5	5039.4	0	2	5	4303.8	1
	#3	2	5	5574.9	1	2	5	2660.3	0
ACASXU_3_6	#1	1	5	6225.0	1	0	5	51.0	0
	#2	3	5	4957.2	1	2	5	1897.3	0
	#3	1	5	196.1	0	1	5	919.2	0
ACASXU_3_7	#1	3	5	6256.2	0	4	5	26.9	0
	#2	4	5	311.3	0	1	5	6958.6	1
	#3	2	5	7756.5	1	1	5	7807.6	1
ACASXU_4_1	#1	2	5	12413.0	2	1	5	5090.5	1
	#2	1	5	5035.1	1	0	5	2335.6	0
	#3	4	5	1237.3	0	4	5	1143.4	0
ACASXU_4_2	#1	4	5	15.9	0	4	5	12.1	0
	#2	3	5	1507.6	0	1	5	111.3	0
	#3	2	5	5641.6	2	0	5	1639.1	0

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	#2	2	5	4263.5	1	0	5	1853.1	0
	#3	1	5	581.8	0	0	5	355.9	0
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	#2	3	5	226.4	0	2	5	125.1	0
	#3	2	5	1740.6	0	2	5	173.6	0
ACASXu_3_5	#1	4	5	43.6	0	2	5	59.4	0
	#2	3	5	5039.4	0	2	5	4303.8	1
	#3	2	5	5574.9	1	2	5	2660.3	0
ACASXu_3_6	#1	1	5	6225.0	1	0	5	51.0	0
	#2	3	5	4957.2	1	2	5	1897.3	0
	#3	1	5	196.1	0	1	5	919.2	0
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	#3	2	5	7756.5	1	1	5	7807.6	1
ACASXu_4_1	#1	2	5	12413.0	2	1	5	5090.5	1
	#2	1	5	5035.1	1	0	5	2335.6	0
	#3	4	5	1237.3	0	4	5	1143.4	0
ACASXu_4_2	#1	4	5	15.9	0	4	5	12.1	0
	#2	3	5	1507.6	0	1	5	111.3	0
	#3	2	5	5641.6	2	0	5	1639.1	0

Scales to a few
hundred neurons

Model	Deletion							SwiftXplain						
	avgC	nCalls	Len	Mn	Mx	avg	TO	avgC	nCalls	Len	FD%	Mn	Mx	avg
gtsrb-dense	0.06	1024	448	52.0	76.3	63.1	0	0.23	54	447	77.4	10.8	14.0	12.2
gtsrb-convSmall	0.06	1024	309	59.2	82.6	65.1	0	0.22	74	313	39.7	15.1	19.5	16.2
gtsrb-conv	—	—	—	—	—	—	100	96.49	45	174	33.2	3858.7	6427.7	4449.4
mnist-denseSmall	0.28	784	177	190.9	420.3	220.4	0	0.77	111	180	15.5	77.6	104.4	85.1
mnist-dense	0.19	784	231	138.1	179.9	150.6	0	0.75	183	229	11.5	130.1	145.5	136.8
mnist-convSmall	—	—	—	—	—	—	100	98.56	52	116	21.3	4115.2	6858.3	5132.8

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Scales to **tens of thousands** of neurons!

More recent results (from 2024)...

[IHM⁺24]

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Scales to **tens of thousands** of neurons!

Largest for MNIST: **10142** neurons
Largest for GSTRB: **94308** neurons

Unit #05

Queries in Symbolic XAI

Enumeration of Explanations

Feature Necessity & Relevancy

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 - Recall: for DTs, enumeration of CXp's is in P

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[MGC⁺21]

[HIIM21, IIM22]

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- No known algorithms for **direct** enumeration of AXp's [MM20]
 - Akin to enumerating MUSes
- Enumeration of MCSes + dualization often not realistic [LS08, FK96]
 - There can be too many CXp's...
- Best solution is a MARCO-like algorithm (for enumerating MUSes) [LPMM16]
 - On-demand enumeration of AXp's/CXp's

Recall computing one AXp/CXp – oneXP

Input: Predicate \mathbb{P} , parameterized by \mathcal{T}, \mathcal{M}

Output: One XP \mathcal{S}

1: **procedure** oneXP(\mathbb{P})

2: $\mathcal{S} \leftarrow \mathcal{F}$

3: **for** $i \in \mathcal{F}$ **do**

4: **if** $\mathbb{P}(\mathcal{S} \setminus \{i\})$ **then**

5: $\mathcal{S} \leftarrow \mathcal{S} \setminus \{i\}$

6: **return** \mathcal{S}

▷ Initialization: $\mathbb{P}(\mathcal{S})$ holds

▷ Loop invariant: $\mathbb{P}(\mathcal{S})$ holds

▷ Update \mathcal{S} only if $\mathbb{P}(\mathcal{S} \setminus \{i\})$ holds

▷ Returned set \mathcal{S} : $\mathbb{P}(\mathcal{S})$ holds

Generic oracle-based enumeration algorithm

Input: Parameters $\mathbb{P}_{\text{axp}}, \mathbb{P}_{\text{cxp}}, \mathcal{T}, \mathcal{F}, \kappa, \mathbf{v}$

```
1:  $\mathcal{H} \leftarrow \emptyset$ 
2: repeat
3:    $(\text{outc}, \mathbf{u}) \leftarrow \text{SAT}(\mathcal{H})$ 
4:   if  $\text{outc} = \text{true}$  then
5:      $\mathcal{S} \leftarrow \{i \in \mathcal{F} \mid u_i = 0\}$ 
6:      $\mathcal{U} \leftarrow \{i \in \mathcal{F} \mid u_i = 1\}$ 
7:     if  $\mathbb{P}_{\text{cxp}}(\mathcal{U}; \mathcal{T}, \mathcal{F}, \kappa, \mathbf{v})$  then
8:        $\mathcal{P} \leftarrow \text{oneXP}(\mathcal{U}; \mathbb{P}_{\text{cxp}}, \mathcal{T}, \mathcal{F}, \kappa, \mathbf{v})$ 
9:        $\text{reportCxp}(\mathcal{P})$ 
10:       $\mathcal{H} \leftarrow \mathcal{H} \cup \{(\vee_{i \in \mathcal{P}} \neg u_i)\}$ 
11:    else
12:       $\mathcal{P} \leftarrow \text{oneXP}(\mathcal{S}; \mathbb{P}_{\text{axp}}, \mathcal{T}, \mathcal{F}, \kappa, \mathbf{v})$ 
13:       $\text{reportAXp}(\mathcal{P})$ 
14:       $\mathcal{H} \leftarrow \mathcal{H} \cup \{(\vee_{i \in \mathcal{P}} u_i)\}$ 
15: until  $\text{outc} = \text{false}$ 
```

$\triangleright \mathcal{H}$ defined on set $U = \{u_1, \dots, u_m\}$; initially no constraints

\triangleright Use SAT oracle to pick assignment s.t. known constraints in \mathcal{H}

$\triangleright \mathcal{S}$: fixed features

$\triangleright \mathcal{U}$: universal features; $\mathcal{F} = \mathcal{S} \cup \mathcal{U}$

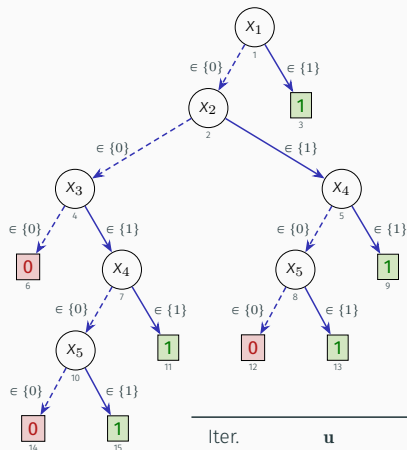
$\triangleright \mathcal{U} = \mathcal{F} \setminus \mathcal{S} \supseteq \text{some Cxp}$

$\triangleright \mathcal{P} \subseteq \mathcal{U}$: one 1-value variable must be 0 in future iterations

$\triangleright \mathcal{S} \supseteq \text{some AXP}$

$\triangleright \mathcal{P} \subseteq \mathcal{S}$: one 0-value variable must be 1 in future iterations

DT classifier – example run of enumerator



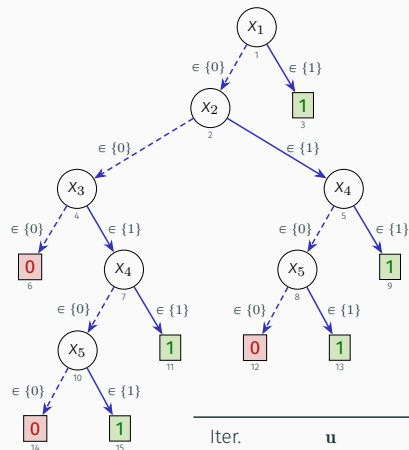
• Instance: $(\mathbf{v}, c) = ((0, 0, 1, 0, 1), 1)$

X_3	X_5	X_1	X_2	X_4	$\kappa_2(\mathbf{x})$
1	1	0	0	0	1
1	1	0	0	1	1
1	1	0	1	0	1
1	1	0	1	1	1
1	1	1	0	0	1
1	1	1	0	1	1
1	1	1	1	0	1
1	1	1	1	1	1

X_3	X_5	X_1	X_2	X_4	$\kappa_2(\mathbf{x})$
0	0	0	0	0	0
0	1	0	0	0	0
1	0	0	0	0	0
1	1	0	0	0	1

Iter.	\mathbf{u}	\mathcal{S}	$\mathbb{P}_{\text{CXP}}(\cdot)$	AXp	CXp	Clause	Resulting \mathcal{H}
1	$(1, 1, 1, 1, 1)$	\emptyset	1	–	$\{3\}$	$(\neg u_3)$	$\{(\neg u_3)\}$
2	$(1, 1, 0, 1, 1)$	$\{3\}$	1	–	$\{5\}$	$(\neg u_5)$	$\{(\neg u_3), (\neg u_5)\}$
3	$(1, 1, 0, 1, 0)$	$\{3, 5\}$	0	$\{3, 5\}$	–	$(u_3 \vee u_5)$	$\{(\neg u_3), (\neg u_5), (u_3 \vee u_5)\}$
5	$[\text{outc} = \text{false}]$	–	–	–	–	–	$\{(\neg u_3), (\neg u_5), (u_3 \vee u_5)\}$

DT classifier – another example run of enumerator



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1	$(0, 0, 0, 0, 0)$	$\{1, 2, 3, 4, 5\}$	0	$\{3, 5\}$	–	$(u_3 \vee u_5)$	$\{(u_3 \vee u_5)\}$
2	$(0, 0, 1, 0, 0)$	$\{1, 2, 4, 5\}$	1	–	$\{3\}$	$(\neg u_3)$	$\{(u_3 \vee u_5), (\neg u_3)\}$
3	$(0, 0, 0, 0, 1)$	$\{1, 2, 3, 4\}$	1	–	$\{5\}$	$(\neg u_5)$	$\{(u_3 \vee u_5), (\neg u_3), (\neg u_5)\}$
5	$[\text{outc} = \text{false}]$	–	–	–	–	–	$\{(u_3 \vee u_5), (\neg u_3), (\neg u_5)\}$

DTs admit more efficient algorithms

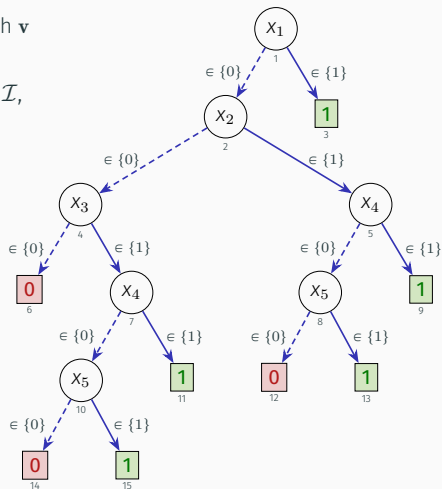
- Recall:
 - Given instance (\mathbf{v}, c) , create set \mathcal{I}
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 - Let I_k denote the features with literals inconsistent with \mathbf{v}
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- \mathcal{I} is the set of CXp's – algorithm runs in poly-time

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 - Obs: starting hypergraph is poly-size!
 - And each MHS is an AXp**
- Example:
 - $I_1 = \{3\}$
 - $I_2 = \{5\}$
 - $I_3 = \{2, 5\}$
 - \therefore keep I_1 and I_2
 - AXp's: MHSes yield $\{\{3, 5\}\}$



Enumeration of Explanations

Feature Necessity & Relevancy

(Conditioned) Classifier Decision Problem ((C)CDP)

[HCM⁺23]

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Feature necessity

- Consider instance (\mathbf{v}, c)
- Sets of all AXp's & CXp's:

$$\mathbb{A} := \{\mathcal{X} \subseteq \mathcal{F} \mid \text{AXp}(\mathcal{X})\}$$

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More on feature necessity

[HCM⁺23]

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 - I.e. this is the case for DTs, DGs, and monotonic classifiers, among others

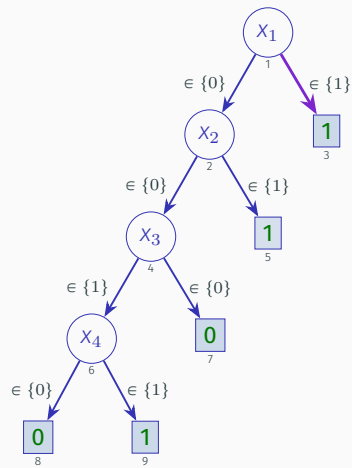
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 - **This holds for any classifier!**
 - Let \mathbf{u} be obtained from \mathbf{v} by replacing the constant v_t by some variable $u_t \in \mathcal{D}_t$
 - Feature t is AXp-necessary if $\kappa(\mathbf{u}) \neq \kappa(\mathbf{v})$ for some value $u_t \in \mathcal{D}_t$

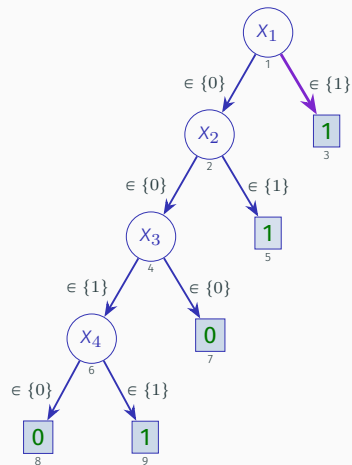
An example

- Instance $(\mathbf{v}, c) = ((0, 0, 0, 0), 0)$



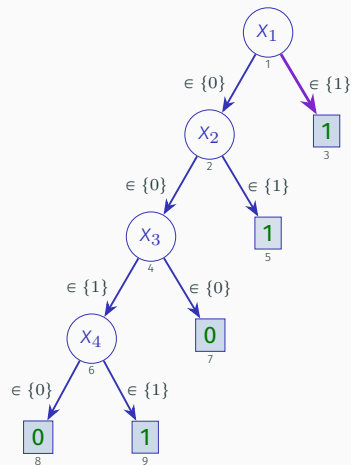
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- Is feature 1 AXp-necessary?



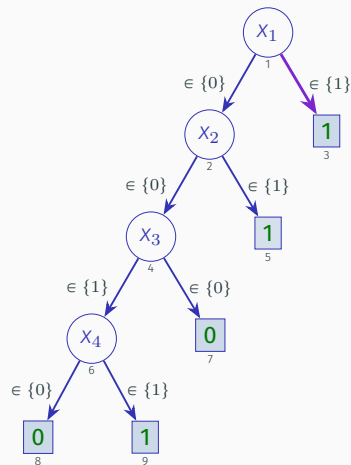
An example

- Instance $(\mathbf{v}, c) = ((0, 0, 0, 0), 0)$
- Is feature 1 AXp-necessary?
 - Does there exist u_1 , such that $\kappa(u_1, 0, 0, 0) \neq \kappa(0, 0, 0, 0)$?



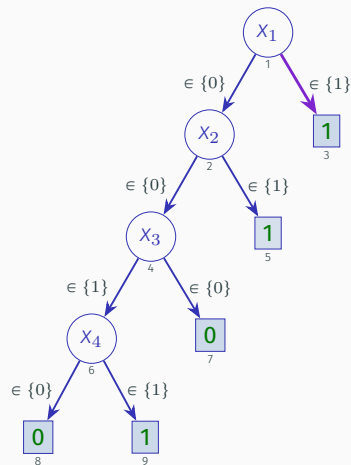
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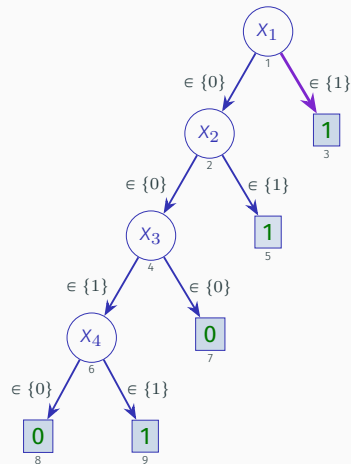
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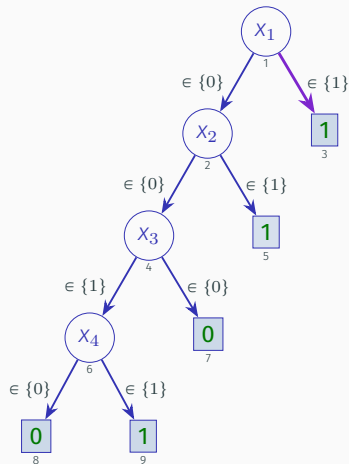
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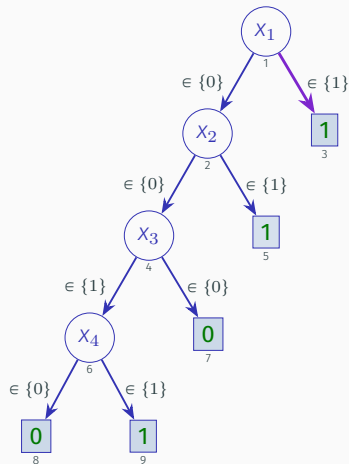
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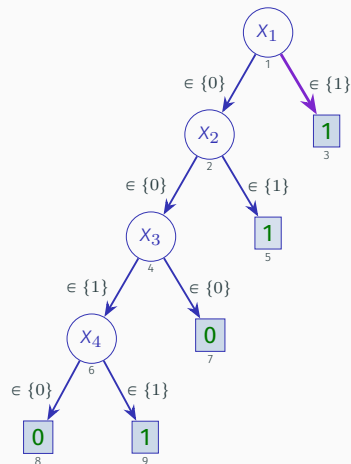
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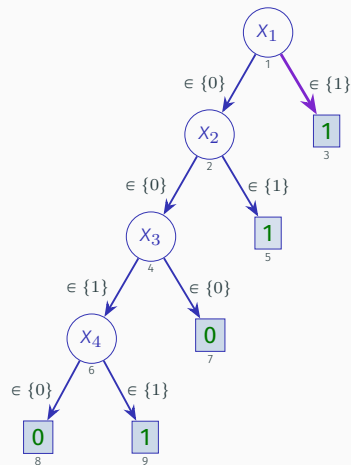
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- Features 1, 2, 3 are **irrelevant**, since there are not included in any AXp/CXp
- Obs: irrelevant features are **absolutely unimportant!**

We could propose some other explanation by adding features 1, 2 or 3 to AXp {4}, but prediction would remain unchanged for **any** value assigned to those features

- And we aim for **irreducibility** (**Occam's razor is a mainstay of AI/ML**)

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- General case: best solution is to exploit **abstraction refinement**

Abstraction refinement for feature relevancy

- **Claim:** $\mathcal{X} \subseteq \mathcal{F}$ and $t \in \mathcal{X}$. If $\text{WAXp}(\mathcal{X})$ holds and $\text{WAXp}(\mathcal{X} \setminus \{t\})$ does not hold, then any $\text{AXp } \mathcal{Z} \subseteq \mathcal{X} \subseteq \mathcal{F}$ must contain feature t .

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- Block counterexamples in both cases

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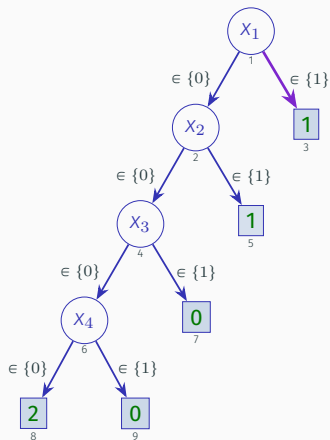
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A general abstraction refinement algorithm

Input: Instance \mathbf{v} , Target Feature t ; Feature Set \mathcal{F} , Classifier κ

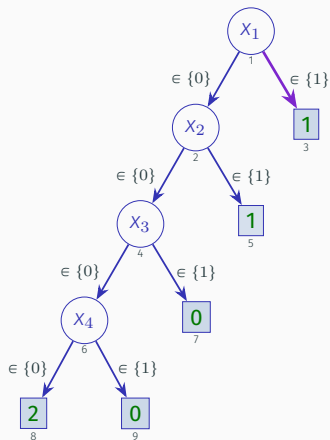
```
1: function FRPCGR( $\mathbf{v}, t; \mathcal{F}, \kappa$ )
2:    $\mathcal{H} \leftarrow \emptyset$  ▷  $\mathcal{H}$  overapproximates the subsets of  $\mathcal{F}$  that do not contain an AXp containing  $t$ 
3:   repeat
4:      $(\text{outc}, s) \leftarrow \text{SAT}(\mathcal{H}, s_t)$  ▷ Use SAT oracle to pick candidate WAXp containing  $t$ 
5:     if  $\text{outc} = \text{true}$  then
6:        $\mathcal{P} \leftarrow \{i \in \mathcal{F} \mid s_i = 1\}$  ▷ Set  $\mathcal{P}$  is the candidate WAXp, and  $t \in \mathcal{P}$ 
7:        $\mathcal{D} \leftarrow \{i \in \mathcal{F} \mid s_i = 0\}$  ▷ Set  $\mathcal{D}$  contains the features not included in  $\mathcal{P}$ 
8:       if  $\neg \text{WAXp}(\mathcal{P})$  then ▷ Is  $\mathcal{P}$  not a WAXp?
9:          $\mathcal{H} \leftarrow \mathcal{H} \cup \text{newPosCl}(\mathcal{D}; t, \kappa)$  ▷  $\mathcal{P}$  is not a WAXp; must pick some non-picked feature
10:      else ▷  $\mathcal{P}$  is a WAXp
11:        if  $\neg \text{WAXp}(\mathcal{P} \setminus \{t\})$  then ▷  $\mathcal{P}$  without  $t$  not a WAXp?
12:          reportWeakAXp( $\mathcal{P}$ ) ▷ Feature  $t$  is included in any AXp  $\mathcal{X} \subseteq \mathcal{P}$ 
13:          return true
14:         $\mathcal{H} \leftarrow \mathcal{H} \cup \text{newNegCl}(\mathcal{P}; t, \kappa)$  ▷ WAXp( $\mathcal{P} \setminus \{t\}$ ) holds; some feature in  $\mathcal{P}$  must not be picked
15:   until  $\text{outc} = \text{false}$ 
16:   return false ▷ If  $\mathcal{H}$  becomes inconsistent, then there is no AXp that contains  $t$ 
```

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- Is $t = 1$ relevant?

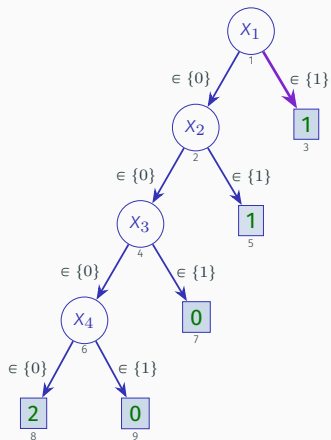
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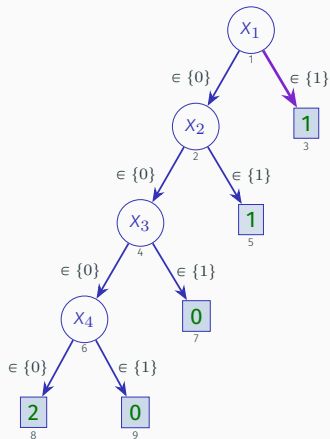
$t = 1$					
s	\mathcal{P}	WAXp(\mathcal{P})	WAXp($\mathcal{P} \setminus \{t\}$)	Return?	Clause
$(1, 1, 1, 1)$	$\{1, 2, 3, 4\}$	✓	✓	---	$(\neg u_2 \vee \neg u_3 \vee \neg u_4)$
$(1, 1, 0, 1)$	$\{1, 2, 4\}$	✓	✓	---	$(\neg u_2 \vee \neg u_4)$
$(1, 1, 0, 0)$	$\{1, 2\}$	✓	✓	---	$(\neg u_2)$
$(1, 0, 0, 0)$	$\{1\}$	✓	✗	true	---

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$t = 4$					
s	\mathcal{P}	$\text{WAXp}(\mathcal{P})$	$\text{WAXp}(\mathcal{P} \setminus \{t\})$	Return?	Clause
$(1, 1, 1, 1)$	$\{1, 2, 3, 4\}$	✓	✓	---	$(\neg u_1 \vee \neg u_2 \vee \neg u_3)$
$(1, 1, 0, 1)$	$\{1, 2, 4\}$	✓	✓	---	$(\neg u_1 \vee \neg u_2)$
$(1, 0, 0, 1)$	$\{1, 4\}$	✓	✓	---	$(\neg u_1)$
$(0, 1, 0, 1)$	$\{2, 4\}$	✓	✓	---	$(\neg u_2)$
$(0, 0, 0, 1)$	$\{4\}$	✗	—	---	$(u_1 \vee u_2 \vee u_3)$
$(0, 0, 1, 1)$	$\{3, 4\}$	✗	—	---	$(u_1 \vee u_2)$
$[\text{outc} = \text{false}]$	---	—	—	false	---

Questions?

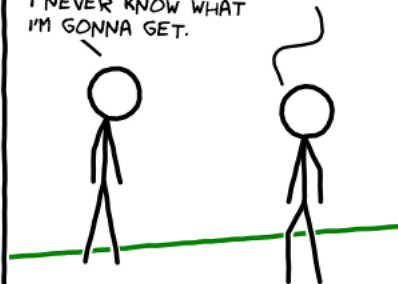
BLACK BOX MODELS

MY ML MODEL...

IS LIKE A
(BLACK) BOX OF
CHOCOLATES.

I NEVER KNOW WHAT
I'M GONNA GET.

BUT WHY?



<https://arxiv.org/abs/1901.01686> & <http://cmx.io/edit/>

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