Logic-Based Explainable Artificial Intelligence

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Lecture 05

Recapitulate fourth lecture

- Monotonic classifiers vs. weighted voting games

- Advanced topics:
  - Sample-based explanations
  - Inflated explanations
  - Probabilistic explanations
  - Constrained explanations
  - Distance-restricted explanations
  - Explanations using surrogate models
  - Certified explainability

Monotonicity & WCGs

- Every WVG $\mathcal{G}$, described by $[q; n_1, \ldots, n_m]$, can be represented as a monotonically increasing boolean classifier $\mathcal{M} = (\mathcal{F}, \{0,1\}^m, \{0,1\}, \kappa)$, such that:
  - Each voter i is mapped to a boolean feature i, such that feature i takes value 1 if voter i votes Yes; otherwise it takes value 0;
  - The classification function $\kappa : \mathbb{F} \rightarrow \{0,1\}$ is defined by:

$$\kappa(\mathbf{x}) = \begin{cases} 1 & \text{if } \sum_{i=1}^{m} n_i x_i \geqslant q \\ 0 & \text{otherwise} \end{cases}$$

  - The target instance is $(\mathbb{1}, 1)$; and
  - Each minimal winning coalition $\mathcal{C}$ corresponds to an AXp of $\mathcal{E} = (\mathcal{M}, (\mathbb{1}, 1))$

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$\therefore$ WVGs can be analyzed by studying the AXps/CXps of monotonically increasing boolean classifiers

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- **Q:** How should features be ranked in terms of importance?

Plan for this course – light at the end of the tunnel...

- Lecture 01 – unit(s):
  - #01: Foundations
- Lecture 02 – unit(s):
  - #02: Principles of symbolic XAI – feature selection
  - #03: Tractability in symbolic XAI (& myth of interpretability)
- Lecture 03 – unit(s):
  - #04: Intractability in symbolic XAI (& myth of model-agnostic XAI)
  - #05: Explainability queries
- Lecture 04 – unit(s):
  - #06: Recent, emerging & advanced topics
- Lecture 05 – unit(s):
  - #07: Principles of symbolic XAI – feature attribution (& myth of Shapley values in XAI)
  - #08: Corrected feature attribution – nuSHAP
  - #09: Conclusions & research directions

Unit #07

Principles of Symbolic XAI – Feature Attribution

Detour: Standard SHAP Intro (from another course...)

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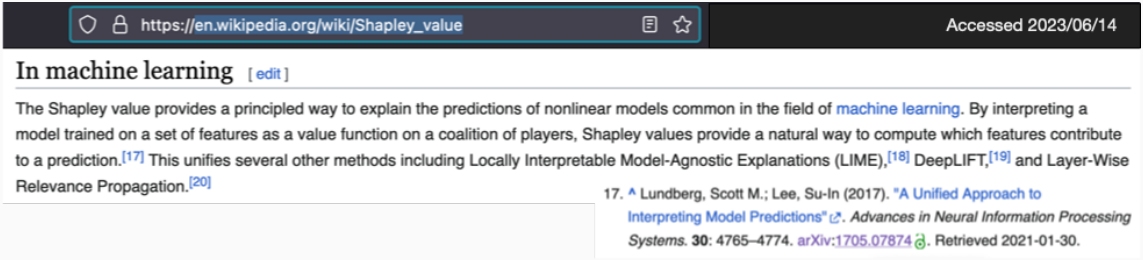
- Shapley values are becoming ubiquitous in XAI... – E.g. see slides from other XAI course...



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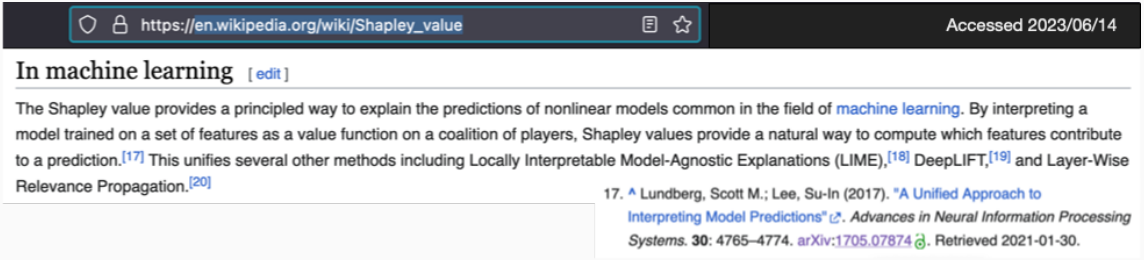

- Q: Do Shapley values for XAI **really** provide a rigorous measure of feature importance?

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- ϕ: $2^{\mathcal{F}} \rightarrow \mathbb{R}$ defined by,

$$\phi(\mathcal{S}) = 1/2^{|\mathcal{F} \setminus \mathcal{S}|} \sum_{\mathbf{x} \in \Upsilon(\mathcal{S})} \kappa(\mathbf{x}) \ = v_e(\mathcal{S})$$

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Marginal contribution (in SHAP lingo)!

[ABBM21, ABBM23]

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How are Shapley values computed in practice?

- Exact evaluation is computationally (very) hard [VLSS21, ABBM21, VLSS22, ABBM23, HMS24]

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- Polynomial-time algorithm for deterministic decomposable boolean circuits [ABBM21]
- Polynomial-time algorithm for boolean functions represented with a truth-table [HM23a]

What do Shapley values tell in terms of feature importance?

- [SK10] reads:
  "*According to the 2nd axiom, if two features values have an identical influence on the prediction they are assigned contributions of equal size. The 3rd axiom says that if a* ***feature has no influence*** *on the prediction* ***it is assigned a contribution of 0****.*"
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  - Qs: can we have **irrelevant** features with a non-zero Shapley value, and/or **relevant** features with a Shapley of zero?
    - Recall: **relevant** features occur in **some** AXp/CXp; **irrelevant** features do **not** occur in **any** AXp/CXp

Outline – Unit #07

Exact Shapley Values for XAI

Myth #03: Shapley Values for XAI

Corrected SHAP Scores

A Correct New SHAP – nuSHAP

Voting Power & Power Indices

Feature Importance Scores

Shapley values vs. feature (ir)relevancy – identified issues [HM23a, HM23b, HM23c, MH23, HMS24, MSH24]

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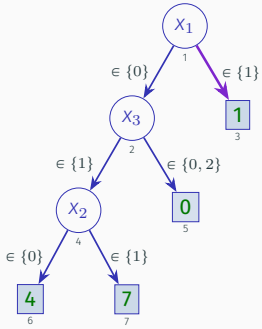
Any of these issues is a cause of (**serious**) concern per se**!**

Some stats – all boolean functions with 4 variables

[HM23a, HM23b, HM23c, MH23, HMS24, MSH24]

Issue-related metric	Value	Recap issue
# of functions	65536	
# number of instances	1048576	
# of I1 issues	781696	
# of functions with I1 issues	65320	
% I1 issues / function	99.67	$[\text{Irrelevant}(i) \wedge (\text{Sv}(i) \neq 0)]$
# of I2 issues	105184	
# of functions with I2 issues	40448	
% I2 issues / function	61.72	$[\text{Irrelevant}(i_1) \wedge \text{Relevant}(i_2) \wedge (\lvert\text{Sv}(i_1)\rvert > \lvert\text{Sv}(i_2)\rvert)]$
# of I3 issues	43008	
# of functions with I3 issues	7800	
% I3 issues / function	11.90	$[\text{Relevant}(i) \wedge (\text{Sv}(i) = 0)]$
# of I4 issues	5728	
# of functions with I4 issues	2592	
% I4 issues / function	3.96	$[\text{Irrelevant}(i_1) \wedge (\text{Sv}(i_1) \neq 0)] \wedge [\text{Relevant}(i_2) \wedge (\text{Sv}(i_2) = 0)]$
# of I5 issues	1664	
# of functions with I5 issues	1248	
% I5 issues / function	1.90	$[\text{Irrelevant}(i) \wedge \forall_{1 \leqslant j \leqslant m, j \neq i} (\lvert\text{Sv}(j)\rvert < \lvert\text{Sv}(i)\rvert)]$

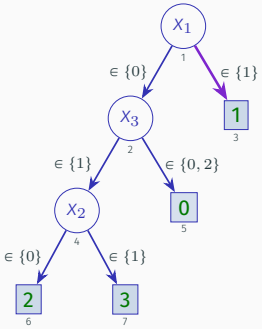
Previous results do matter! Let's go non-boolean...



DT1

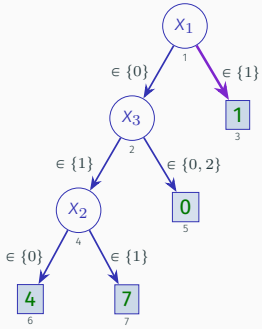
row #	X_1	X_2	X_3	$\kappa_1(\mathbf{x})$	$\kappa_2(\mathbf{x})$
1	0	0	0	0	0
2	0	0	1	4	2
3	0	0	2	0	0
4	0	1	0	0	0
5	0	1	1	7	3
6	0	1	2	0	0
7	1	0	0	1	1
8	1	0	1	1	1
9	1	0	2	1	1
10	1	1	0	1	1
11	1	1	1	1	1
12	1	1	2	1	1

Tabular representations



DT2

Instance $((1, 1, 2), 1)$ – which feature matters the most for prediction 1?



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row #	x_1	x_2	x_3	$\kappa_1(\mathbf{x})$	$\kappa_2(\mathbf{x})$
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2	0	0	1	4	2
3	0	0	2	0	0
4	0	1	0	0	0
5	0	1	1	7	3
6	0	1	2	0	0
7	1	0	0	1	1
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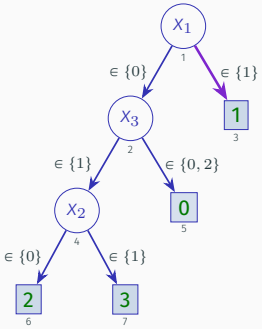
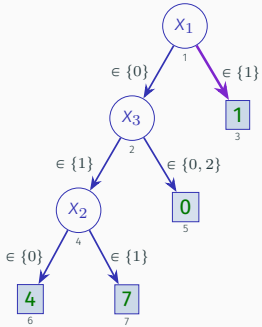


DT2

Computing XPs – make sense...



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row #	x_1	x_2	x_3	$\kappa_1(\mathbf{x})$	$\kappa_2(\mathbf{x})$
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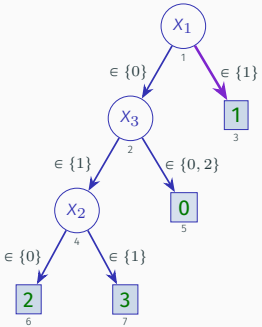
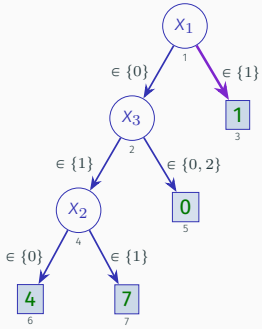
Tabular representations



DT2

XPs: AXps/CXps		
DT	AXps	CXps
DT1	$\{1\}$	$\{1\}$
DT2	$\{1\}$	$\{1\}$

Computing XPs, AEs – also make sense...



DT1

row #	x_1	x_2	x_3	$\kappa_1(\mathbf{x})$	$\kappa_2(\mathbf{x})$
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2	0	0	1	4	2
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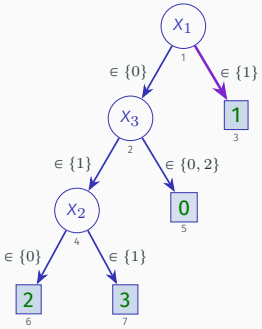

DT2

XPs: AXps/CXps		
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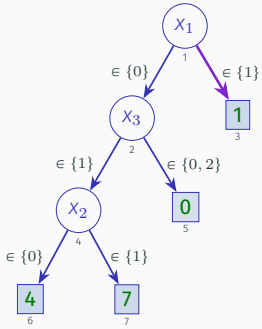
DT2	$\{1\}$	$\{1\}$

Adversarial Examples	
DT	l_0-minimal AEs
DT1	$\{1\}$
DT2	$\{1\}$

Computing XPs, AEs & Svs



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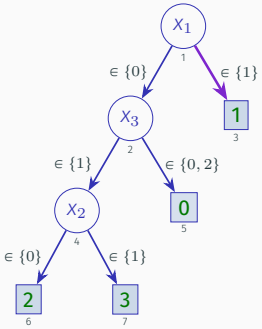

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Shapley values			
DT	Sc(1)	Sc(2)	Sc(3)
DT1	0.000	0.083	-0.500
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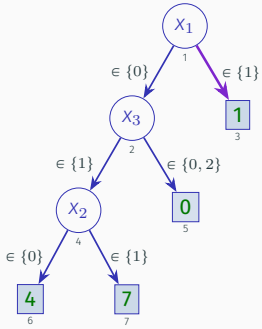
Computing XPs, AEs & Svs – what???



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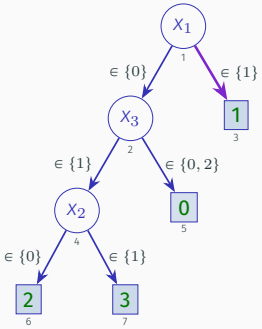

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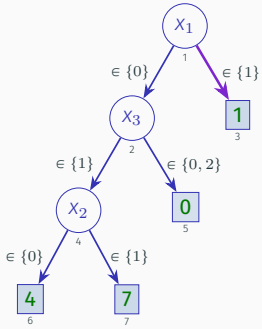

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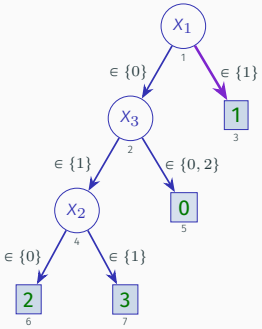

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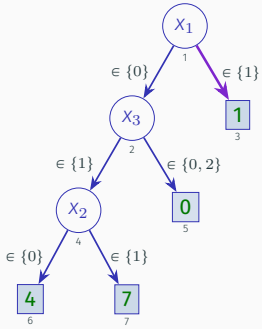
Computing XPs, AEs & Svs – what???

∴ Shapley values can mislead human decision-makers !



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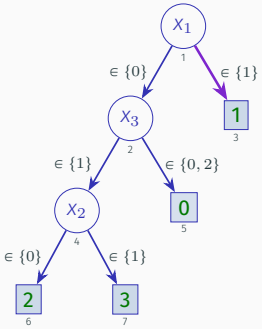
Tabular representations



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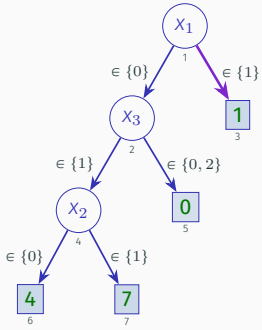
Computing XPs, AEs & Svs – what???



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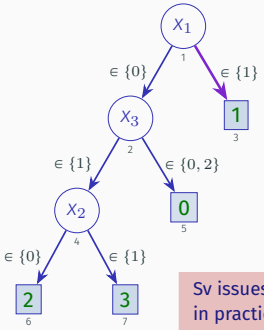
Tabular representations

$\therefore$ Shapley values can mislead human decision-makers !



Sv issues also occur in practice [HM23c]

DT2

XPs: AXps/CXps

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DT1	$\{1\}$	$\{1\}$
DT2	$\{1\}$	$\{1\}$

Adversarial Examples

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Shapley values

DT	Sc(1)	Sc(2)	Sc(3)	
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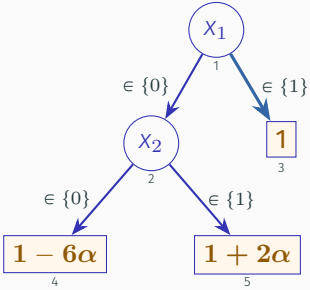
Another example – arbitrary mistakes!

[LHAMS24]



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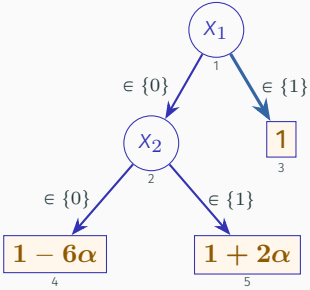

- Instance: $((1, 1), 1)$
- Obs: $\alpha \neq 0$

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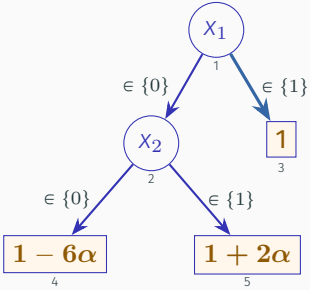

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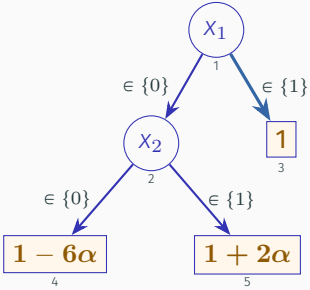

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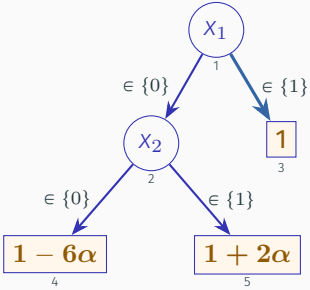

- Instance: $((1, 1), 1)$
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- $\mathrm{Sc}(1) = 0$
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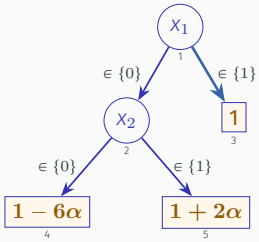
Example devised by O. Letoffe, PhD student at IRIT

More detail

row	x_1	x_2	$\rho(\mathbf{x})$	$\rho_a(\mathbf{x})$ $\alpha = 1/2$	$\rho_b(\mathbf{x})$ $\alpha = 1/4$
1	0	0	$1-6\alpha$	-2	$-1/2$
2	0	1	$1+2\alpha$	2	$3/2$
3	1	0	1	1	1
4	1	1	1	1	1



$\mathcal{S}$	rows($\mathcal{S}$)	$v_e(\mathcal{S})$
$\varnothing$	$1,2,3,4$	$1-\alpha$
$\{x_1\}$	$3,4$	1
$\{x_2\}$	$2,4$	$1+\alpha$
$\{x_1,x_2\}$	4	1

$i = 1$

$\mathcal{S}$	$v_e(\mathcal{S})$	$v_e(\mathcal{S}\cup\{1\})$	$\Delta_1(\mathcal{S})$	$\varsigma(\mathcal{S})$	$\varsigma(\mathcal{S})\times\Delta_1(\mathcal{S})$
$\varnothing$	$1-\alpha$	1	α	$1/2$	$\alpha/2$
$\{2\}$	$1+\alpha$	1	$-\alpha$	$1/2$	$-\alpha/2$
			$\mathsf{Sc}_E(1)$	$=$	0

$i = 2$

$\mathcal{S}$	$v_e(\mathcal{S})$	$v_e(\mathcal{S}\cup\{2\})$	$\Delta_2(\mathcal{S})$	$\varsigma(\mathcal{S})$	$\varsigma(\mathcal{S})\times\Delta_2(\mathcal{S})$
$\varnothing$	$1-\alpha$	$1+\alpha$	2α	$1/2$	α
$\{1\}$	1	1	0	$1/2$	0
			$\mathsf{Sc}_E(2)$	$=$	α

SHAP scores also fail with regression models

[LHM24]

- Let $\mathcal{F} = \{1, 2\}$, $\mathbb{D}_1 = \mathbb{D}_2 = \mathbb{D} = [-1/2, 3/2]$, $\mathbb{F} = \mathbb{D} \times \mathbb{D}$
  - Also, let $\mathbb{D}^+ = [1/2, 3/2]$ and $\mathbb{D}^- = \mathbb{D} \backslash \mathbb{D}^+$

- Regression model maps to real values, i.e. $\mathbb{K} = \mathbb{R}$:

$$\rho_2(x_1, x_2) = \begin{cases} x_1 & \text{if } x_1 \in \mathbb{D}^+ \\ x_2 - 2 & \text{if } x_1 \notin \mathbb{D}^+ \land x_2 \notin \mathbb{D}^+ \\ x_2 + 1 & \text{if } x_1 \notin \mathbb{D}^+ \land x_2 \in \mathbb{D}^+ \end{cases}$$

- Average values unchanged (wrt previous example), and so $\mathrm{Sc}(1) = 0$ and $\mathrm{Sc}(2) = \alpha$

SHAP scores even fail when Lipschitz continuity holds!

[LHM24]

$$\rho_3(x_1, x_2) = \begin{cases} x_1 & \text{if } x_2 \leqslant 1 \land \alpha x_1 \leqslant \alpha \\ (1 + 4|\alpha|)x_1 - 4|\alpha| & \text{if } x_2 \leqslant 1 \land \alpha x_1 \geqslant \alpha \\ 28|\alpha|x_1x_2 + (1 - 28|\alpha|)x_1 - 28|\alpha|x_2 + 28|\alpha| & \text{if } x_2 \geqslant 1 \land \alpha x_1 \leqslant \alpha \\ -4|\alpha|x_1x_2 + (1 + 8|\alpha|)x_1 + 4|\alpha|x_2 - 8|\alpha| & \text{if } x_2 \geqslant 1 \land \alpha x_1 \geqslant \alpha \end{cases}$$

- As before, average values unchanged, and so $\mathrm{Sc}(1) = 0$ and $\mathrm{Sc}(2) = \alpha$

Outline – Unit #07

Exact Shapley Values for XAI

Myth #03: Shapley Values for XAI

Corrected SHAP Scores

A Correct New SHAP – nuSHAP

Voting Power & Power Indices

Feature Importance Scores

Corrected SHAP scores & feature importance scores

[LHMS24, LHAMS24]

- Is the theory of Shapley values incorrect?

Corrected SHAP scores & feature importance scores

[LHMS24, LHAMS24]

- Is the theory of Shapley values incorrect? **No!**

Corrected SHAP scores & feature importance scores

[LHMS24, LHAMS24]

- Is the theory of Shapley values incorrect? **No!**
- What is inadequate is the **characteristic function** used in XAI [SK10, SK14, LL17]
  - In XAI: characteristic function uses the expected value
  - This defines the *marginal contribution* in SHAP lingo...

Corrected SHAP scores & feature importance scores

[LHMS24, LHAMS24]

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- Replace characteristic function based on expected values by new characteristic function based on **AXps/WAXps** [LHMS24]
  - Resulting scores are (still) Shapley values & identified issues no longer observed

Corrected SHAP scores & feature importance scores

[LHMS24, LHAMS24]

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- Observed tight connection between feature attribution and power indices from a priori voting power

Corrected SHAP scores & feature importance scores

[LHMS24, LHAMS24]

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- Replace characteristic function based on expected values by new characteristic function based on **AXps/WAXps** [LHMS24]
  - Resulting scores are (still) Shapley values & identified issues no longer observed
- Observed tight connection between feature attribution and power indices from a priori voting power
  - **Feature importance scores** (more later): [LHAMS24]
    - Generalize recent axiomatic aggregations [BIL+24]
    - Adapt best known power indices
    - Devise new scores for XAI

An initial compromise

[LHAMS24]

- Replace the characteristic function used for SHAP scores:

$$v_e(\mathcal{S}) \quad := \quad \mathbf{E}[\tau(\mathbf{x}) \,|\, \mathbf{x}_{\mathcal{S}} = \mathbf{v}_{\mathcal{S}}]$$

An initial compromise

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- Recall the similarity predicate:

$$\sigma(\mathbf{x}) = \begin{cases} 1 & \text{if } (\kappa(\mathbf{x}) = \kappa(\mathbf{v})) \\ 0 & \text{otherwise} \end{cases}$$

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- Issues with non-boolean classifiers disappear; issues with boolean classifiers remain
- Developed SSHAP prototype using SHAP's code base [LHMS24]

Fixing the known issues of SHAP scores

Fixing the known issues of SHAP scores

- New characteristic function (based on WAXps):

$$v_a(\mathcal{S}) \;:=\; \begin{cases} 1 & \text{if } \mathbf{E}[\sigma(\mathbf{x}) \,|\, \mathbf{x}_{\mathcal{S}} = \mathbf{v}_{\mathcal{S}}] = 1 \\ 0 & \text{otherwise} \end{cases}$$

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- Known issues of SHAP scores guaranteed **not** to occur

- **Corrected** SHAP scores reveal tight connection between XAI by feature selection (i.e. WAXps) and feature attribution

Outline – Unit #07

Exact Shapley Values for XAI

Myth #03: Shapley Values for XAI

Corrected SHAP Scores

A Correct New SHAP – nuSHAP

Voting Power & Power Indices

Feature Importance Scores

A glimpse of NuSHAP

- Shapley values estimated with well-known algorithm – CGT [CGT09]
  - Strong theoretical guarantees on approximation of Shapley values
  - With enough sampling, given by ϵ and α:

$$\text{Prob}[|\hat{\text{Sc}}(i) - \text{Sc}(i)| \leqslant \epsilon] \geqslant 1 - \alpha$$

  - E.g. we used $\epsilon = 0.0015$ and $\alpha = 0.015$

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- Replace expected value with novel characteristic function: test for WAXp
  - New approach for computing symbolic explanations, targeting scalability for complex models
    - Replace WAXp check with sbWAXp check, i.e. polynomial-time check
    - Flexibility in the sample chosen, e.g. training data

NuSHAP vs. *SHAP

- Compute SHAP scores with NuSHAP and SHAP
- Compare features' rankings using Rank-Biased Overlap (RBO) [WMZ10]
  - Give weight to differences in first k elements of ranking
  - E.g. $\text{RBO}([0,1,2,3,4,5,6,7,8,9],[0,1,2,3,4,5,6,7,8,9],k=5)=1.0$
- Results for different datasets and ML models (using scikit, XGBoost):
  - LR: (adult, corral); DT: (iris, mux6); kNN: (connect_4, spambase, spectf); BT: (clean1, coil2000, dna); CNN: (MNIST)

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		adult	corral	iris	mux6	connect_4	spambase	spectf	clean1	coil2000	dna	MNIST
Min	nuSHAP vs. SHAP	0.08	0.17	0.31	0.32	0.0	0.01	0.0	0.0	0.0	0.0	0.0
	nuSHAP vs. |SHAP|	0.05	0.12	0.27	0.32	0.0	0.05	0.0	0.0	0.0	0.03	0.0
Max	nuSHAP vs. SHAP	0.96	0.96	0.94	0.97	0.9	0.94	0.91	0.69	0.69	0.88	0.06
	nuSHAP vs. |SHAP|	0.88	0.97	0.94	0.95	0.77	0.94	0.91	0.88	0.69	0.88	0.06
Mean	nuSHAP vs. SHAP	0.37	0.53	0.84	0.7	0.21	0.41	0.2	0.12	0.05	0.17	0.0
	nuSHAP vs. |SHAP|	0.31	0.5	0.84	0.69	0.19	0.42	0.19	0.17	0.08	0.43	0.0

NuSHAP vs. *SHAP – run times

- Compute SHAP scores with NuSHAP and SHAP
- Compare features' rankings using Rank-Biased Overlap (RBO) [WMZ10]
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Tool SHAP produces results of very poor quality!

NuSHAP vs. *SHAP – run times

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	adult	corral	iris	mux6	connect_4	spambase	spectf	clean1	coil2000	dna	MNIST
SHAP	3.4	**0.1**	**0.0**	**0.0**	21.7	**0.5**	**0.7**	6.8	28.2	23.0	281.3
nuSHAP	**1.9**	1.5	1.5	1.5	**4.5**	2.7	2.9	**2.7**	**1.7**	**4.5**	**48.9**
#Samples	68045.5	63.7	955.4	63.9	9202.5	13654.1	36459.5	3929.6	2960.8	2756.8	2929.9

Outline – Unit #07

Exact Shapley Values for XAI

Myth #03: Shapley Values for XAI

Corrected SHAP Scores

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Voting Power & Power Indices

Feature Importance Scores

Recap: weighted voting games

- General set up of weighted voting games:
  - Assembly $\mathcal{A}$ of voters, with $m = |\mathcal{A}|$
  - Each voter $i \in \mathcal{A}$ votes Yes with n_i votes; otherwise no votes are counted (and he/she votes No)
  - A coalition is a subset of voters, $\mathcal{C} \subseteq \mathcal{A}$
  - Quota q is the sum of votes required for a proposal to be approved
    - Coalitions leading to sums not less than q are **winning** coalitions
  - A weighted voting game (WVG) is a tuple $[q; n_1, \ldots, n_m]$
    - Example: $[12; 4, 4, 4, 2, 2, 1]$
  - Problem: find a measure of importance of each voter !
    - I.e. measure the a priori voting power of each voter

What are power indices?

- Power indices assign a measure of importance to each voter

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- Many power indices proposed over the years:
  - Penrose [Pen46]
  - Shapley-Shubik [SS54]
  - Banzhaf [BI65]
  - Coleman [Col71]
  - Johnston [Joh78]
  - Deegan-Packel [DP78]
  - Holler-Packel [HP83]
  - Andjiga [ACL03]
  - Responsability* [CH04, BIL+24]
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  - ...
- What characterizes power indices?
  - Account for the cases when voter is *critical* for a winning coalition
    - E.g. in previous example, Luxembourg is never critical for a winning coalition
  - Account for whether coalition is subset-minimal or cardinality-minimal

Towards defining power indices

- Understanding criticality (used at least since 1954): [SS54]

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- Understanding (subset-)minimal winning coalitions:
  - A winning coalition is subset-minimal if removing any single voter results in a losing coalition
  - A winning coalition is cardinality-minimal if it has the smallest cardinality among subset-minimal winning coalitions
  - Recall that minimal winning coalitions can be obtained by computing the AXps of a monotonically increasing boolean classifier

Example power indices I

[LHAMS24]

- Necessary definitions (using formal XAI notation...):

$$\mathbb{WA}_i(\mathcal{E}) = \{\mathcal{S} \subseteq \mathcal{F} \,|\, \mathsf{WAXp}(\mathcal{S};\mathcal{E}) \land i \in \mathcal{S}\}$$
$$\mathbb{WC}_i(\mathcal{E}) = \{\mathcal{S} \subseteq \mathcal{F} \,|\, \mathsf{WCXp}(\mathcal{S};\mathcal{E}) \land i \in \mathcal{S}\}$$
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- Power indices of Holler-Packel and Deegan-Packel: [HP83, DP78]

$$\mathsf{Sc}_H(i;\mathcal{E}) = \sum\nolimits_{\mathcal{S} \in \mathbb{A}_i(\mathcal{E})} \left(1/|\mathbb{A}(\mathcal{E})|\right)$$
$$\mathsf{Sc}_D(i;\mathcal{E}) = \sum\nolimits_{\mathcal{S} \in \mathbb{A}_i(\mathcal{E})} \left(1/(|\mathcal{S}| \times |\mathbb{A}(\mathcal{E})|)\right)$$

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  - **Obs:** One *only* needs the AXps

Example power indices II

- Additional definitions:

$$\text{Crit}(i, \mathcal{S}; \mathcal{E}) \quad := \quad \text{WAXp}(\mathcal{S}; \mathcal{E}) \land \neg\text{WAXp}(\mathcal{S} \backslash \{i\}; \mathcal{E})$$

Example power indices II

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- Power indices of Shapley-Shubik, Banzhaf and Johnston: [SS54, BI65, Joh78]

$$\begin{aligned}
\mathsf{Sc}_S(i; \mathcal{E}) &= \sum\nolimits_{\mathcal{S}\subseteq\mathcal{F}\land\mathsf{Crit}(i,\mathcal{S};\mathcal{E})} \left(1/\left(|\mathcal{F}| \times \binom{|\mathcal{F}|-1}{|\mathcal{S}|-1}\right)\right) \\
\mathsf{Sc}_B(i; \mathcal{E}) &= \sum\nolimits_{\mathcal{S}\subseteq\mathcal{F}\land\mathsf{Crit}(i,\mathcal{S};\mathcal{E})} \left(1/2^{|\mathcal{F}|-1}\right) \\
\mathsf{Sc}_J(i; \mathcal{E}) &= \sum\nolimits_{\mathcal{S}\subseteq\mathcal{F}\land\mathsf{Crit}(i,\mathcal{S};\mathcal{E})} \left(1/\Delta(\mathcal{S})\right)
\end{aligned}$$

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  - One needs the WAXps to find critical voters...

Example #01

- WVG: $[9; 9, 2, 2, 2, 2, 1, 1]$

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- AXps:

$$\begin{array}{ccccc} 1 & & & & \\ 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 4 & 5 & 7 \end{array}$$

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- WVG: $[9; 9, 2, 2, 2, 2, 1, 1]$
- AXps:

$$\begin{array}{ccccc} 1 & & & & \\ 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 4 & 5 & 7 \end{array}$$

- Holler-Packel scores: $\langle 0.333, 0.667, 0.667, 0.667, 0.667, 0.333, 0.333 \rangle$
- Banzhaf scores (normalized): $\langle 0.813, 0.040, 0.040, 0.040, 0.040, 0.013, 0.013 \rangle$
- Shapley-Shubik scores: $\langle 0.810, 0.043, 0.043, 0.043, 0.043, 0.010, 0.010 \rangle$
- Different relative orders of voter importance... which ones seem more realistic?

Example #02

- WVG: $[16; 10, 6, 4, 2, 2]$

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- WVG: $[16; 10, 6, 4, 2, 2]$
- AXps:

$$\begin{matrix} 1 & 2 & \\ 1 & 3 & 4 \\ 1 & 3 & 5 \end{matrix}$$

Example #02

- WVG: $[16; 10, 6, 4, 2, 2]$
- AXps:

$$\begin{array}{ccc} 1 & 2 & \\ 1 & 3 & 4 \\ 1 & 3 & 5 \end{array}$$

- Deegan-Packel scores: $\langle 0.389, 0.167, 0.222, 0.111, 0.111 \rangle$
- Banzhaf scores (normalized): $\langle 0.524, 0.238, 0.143, 0.048, 0.048 \rangle$
- Shapley-Shubik scores: $\langle 0.617, 0.200, 0.117, 0.033, 0.033 \rangle$
- Different relative orders of voter importance... which ones seem more realistic?

Example #03

- WVG: $[6; 4, 2, 1, 1, 1, 1]$

Example #03

- WVG: $[6; 4, 2, 1, 1, 1, 1]$
- AXps:

2	3	4	5	6
1	3	4		
1	4	5		
1	4	6		
1	3	6		
1	5	6		
1	2			
1	3	5		

Example #03

- WVG: $[6; 4, 2, 1, 1, 1, 1]$
- AXps:

$$
\begin{array}{ccccc}
2 & 3 & 4 & 5 & 6 \\
1 & 3 & 4 & & \\
1 & 4 & 5 & & \\
1 & 4 & 6 & & \\
1 & 3 & 6 & & \\
1 & 5 & 6 & & \\
1 & 2 & & & \\
1 & 3 & 5 & &
\end{array}
$$

- Deegan-Packel scores: $\langle 0.312, 0.087, 0.150, 0.150, 0.150, 0.150 \rangle$
- Banzhaf scores (normalized): $\langle 0.542, 0.125, 0.083, 0.083, 0.083, 0.083 \rangle$
- Shapley-Shubik scores: $\langle 0.533, 0.133, 0.083, 0.083, 0.083, 0.083 \rangle$
- Different relative orders of voter importance... which ones seem more realistic?

Example #04

- WVG: $[21; 12, 9, 4, 4, 1, 1, 1]$

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- WVG: $[21; 12, 9, 4, 4, 1, 1, 1]$
- AXps:

$$\begin{array}{cccc} 1 & 2 & & \\ 1 & 3 & 4 & 5 \\ 1 & 3 & 4 & 6 \\ 1 & 3 & 4 & 7 \end{array}$$

Example #04

- WVG: $[21; 12, 9, 4, 4, 1, 1, 1]$
- AXps:

$$\begin{array}{cccc} 1 & 2 & & \\ 1 & 3 & 4 & 5 \\ 1 & 3 & 4 & 6 \\ 1 & 3 & 4 & 7 \end{array}$$

- Deegan-Packel scores: $\langle 0.312, 0.125, 0.188, 0.188, 0.062, 0.062, 0.062 \rangle$
- Banzhaf scores (normalized): $\langle 0.481, 0.309, 0.086, 0.086, 0.012, 0.012, 0.012 \rangle$
- Shapley-Shubik scores: $\langle 0.574, 0.257, 0.074, 0.074, 0.007, 0.007, 0.007 \rangle$
- Different relative orders of voter importance... which ones seem more realistic?

Outline – Unit #07

Exact Shapley Values for XAI

Myth #03: Shapley Values for XAI

Corrected SHAP Scores

A Correct New SHAP – nuSHAP

Voting Power & Power Indices

Feature Importance Scores

From power indices to feature importance scores

- A Feature Importance Score (FIS) is a measure of feature importance in XAI, parameterizable on an explanation problem and a chosen characteristic function
  - Explanation problem: $(\mathcal{M}, (\mathbf{v}, q))$
  - Define characteristic function using explanation problem (more next slide)

- Obs: Can adapt (generalized) power indices as templates for feature importance scores

- Obs: Can devise new templates and/or new FISs

Some examples (1 of 2)

- More notation:

$$\Delta_i(\mathcal{S}; \mathcal{E}, v) = v(\mathcal{S}; \mathcal{E}) - v(\mathcal{S} \backslash \{i\}; \mathcal{E})$$

  - Can use **any** characteristic function, including those presented earlier in this lecture

Some examples (1 of 2)

- More notation:

$$\Delta_i(\mathcal{S};\mathcal{E},v) = v(\mathcal{S};\mathcal{E}) - v(\mathcal{S}\backslash\{i\};\mathcal{E})$$

  - Can use **any** characteristic function, including those presented earlier in this lecture

- Some templates:
  - Shapley-Shubik:

$$\text{TSc}_S(i;\mathcal{E},v) := \sum_{\mathcal{S}\in\{\mathcal{T}\subseteq\mathcal{F}\mid i\in\mathcal{T}\}} \left(\frac{\Delta_i(\mathcal{S};\mathcal{E},v)}{|\mathcal{F}| \times \binom{|\mathcal{F}|-1}{|\mathcal{S}|-1}} \right)$$

  - Banzhaf:

$$\text{TSc}_B(i;\mathcal{E},v) := \sum_{\mathcal{S}\in\{\mathcal{T}\subseteq\mathcal{F}\mid i\in\mathcal{T}\}} \left(\frac{\Delta_i(\mathcal{S};\mathcal{E},v)}{2^{|\mathcal{F}|-1}} \right)$$

Some examples (1 of 2)

- More notation:

$$\Delta_i(\mathcal{S};\mathcal{E},v) = v(\mathcal{S};\mathcal{E}) - v(\mathcal{S}\backslash\{i\};\mathcal{E})$$

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- Can use other templates

Some examples (1 of 2)

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- Can use other templates

- Can devise FISs without exploiting existing templates

Some examples (2 of 2)

- Recall WAXp-based characteristic function:

$$\upsilon_a(\mathcal{S}) \;:=\; \begin{cases} 1 & \text{if } \mathbf{E}[\sigma(\mathbf{x}) \,|\, \mathbf{x}_{\mathcal{S}} = \mathbf{v}_{\mathcal{S}}] = 1 \\ 0 & \text{otherwise} \end{cases}$$

Some examples (2 of 2)

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$$v_a(\mathcal{S}) \;:=\; \begin{cases} 1 & \text{if } \mathbf{E}[\sigma(\mathbf{x}) \,|\, \mathbf{x}_{\mathcal{S}} = \mathbf{v}_{\mathcal{S}}] = 1 \\ 0 & \text{otherwise} \end{cases}$$

- Some FISs:
  - Shapley-Shubik:

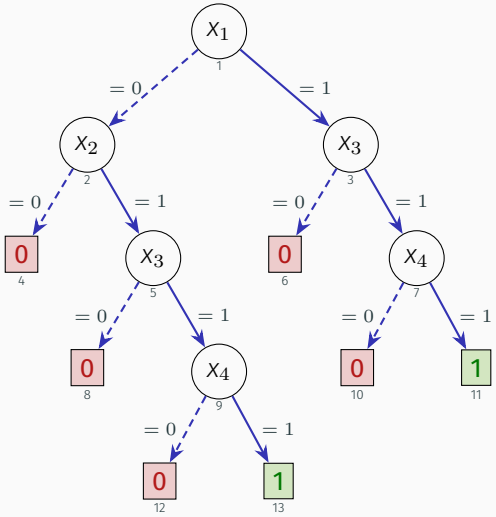
$$\mathrm{Sc}_S(i;\mathcal{E}) := \mathrm{TSc}_S(i;\mathcal{E},v_a) := \sum_{\mathcal{S}\in\{\mathcal{T}\subseteq\mathcal{F}\,|\,i\in\mathcal{T}\}} \left(\frac{\Delta_i(\mathcal{S};\mathcal{E},v_a)}{|\mathcal{F}| \times \binom{|\mathcal{F}|-1}{|\mathcal{S}|-1}} \right)$$

  - Banzhaf:

$$\mathrm{Sc}_B(i;\mathcal{E}) := \mathrm{TSc}_B(i;\mathcal{E},v_a) := \sum_{\mathcal{S}\in\{\mathcal{T}\subseteq\mathcal{F}\,|\,i\in\mathcal{T}\}} \left(\frac{\Delta_i(\mathcal{S};\mathcal{E},v_a)}{2^{|\mathcal{F}|-1}} \right)$$

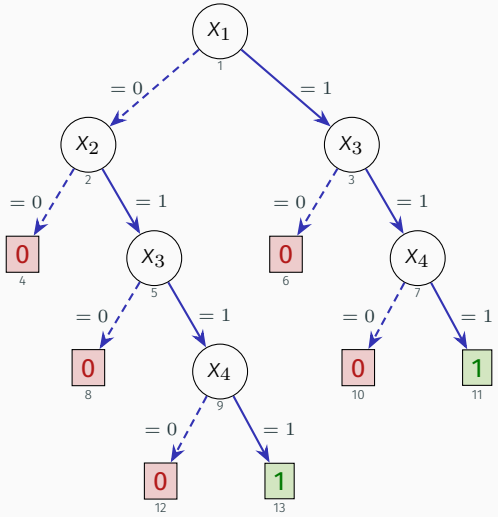
A concrete example

- AXps: $\{\{1, 3, 4\}, \{2, 3, 4\}\}$
- Feature attribution:



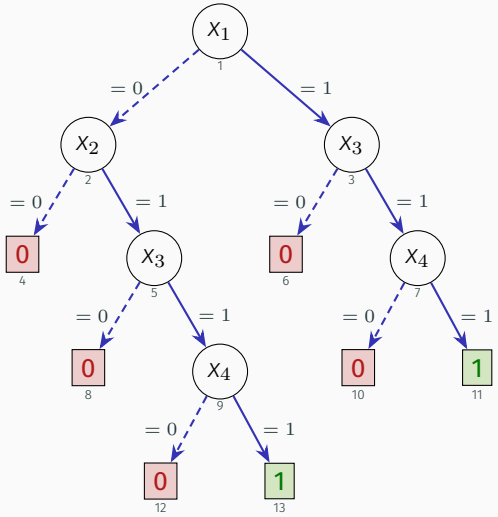
A concrete example

- AXps: $\{\{1, 3, 4\}, \{2, 3, 4\}\}$
- Feature attribution:
  - SS: $\langle 0.083, 0.083, 0.417, 0.417 \rangle$



A concrete example

- AXps: $\{\{1, 3, 4\}, \{2, 3, 4\}\}$
- Feature attribution:
  - SS: $\langle 0.083, 0.083, 0.417, 0.417 \rangle$
  - B (norm.): $\langle 0.125, 0.125, 0.375, 0.375 \rangle$



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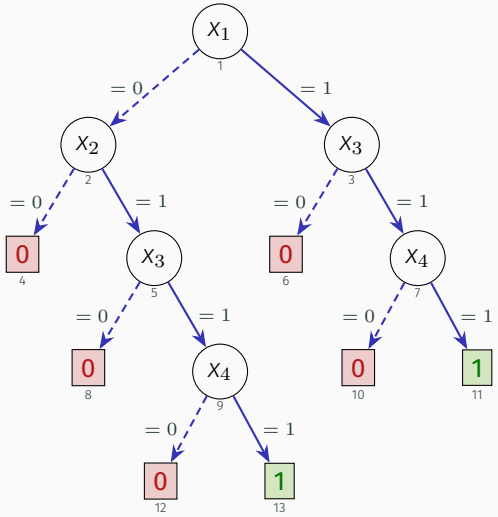
  - B (norm.): $\langle 0.125, 0.125, 0.375, 0.375 \rangle$
  - J (norm.): $\langle 0.111, 0.111, 0.389, 0.389 \rangle$



A concrete example

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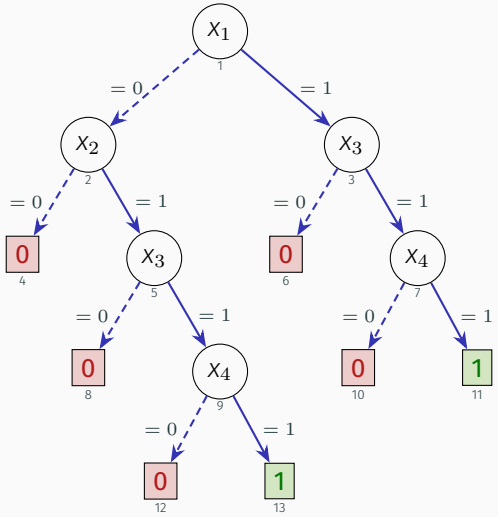
  - B (norm.): $\langle 0.125, 0.125, 0.375, 0.375 \rangle$
  - J (norm.): $\langle 0.111, 0.111, 0.389, 0.389 \rangle$
  - HP: $\langle 0.167, 0.167, 0.333, 0.333 \rangle$



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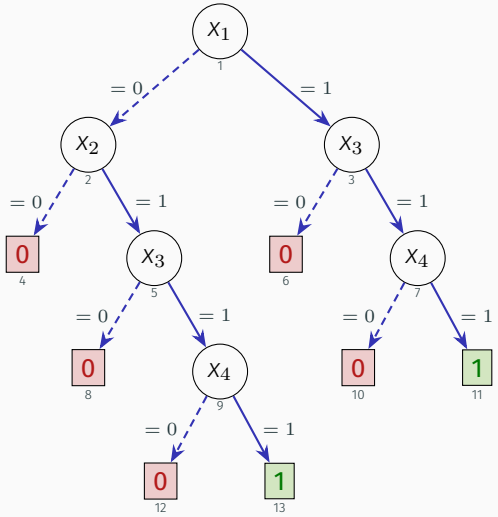
  - HP: $\langle 0.167, 0.167, 0.333, 0.333 \rangle$
  - DP: $\langle 0.167, 0.167, 0.333, 0.333 \rangle$



Questions?

Unit #08

Conclusions & Research Directions

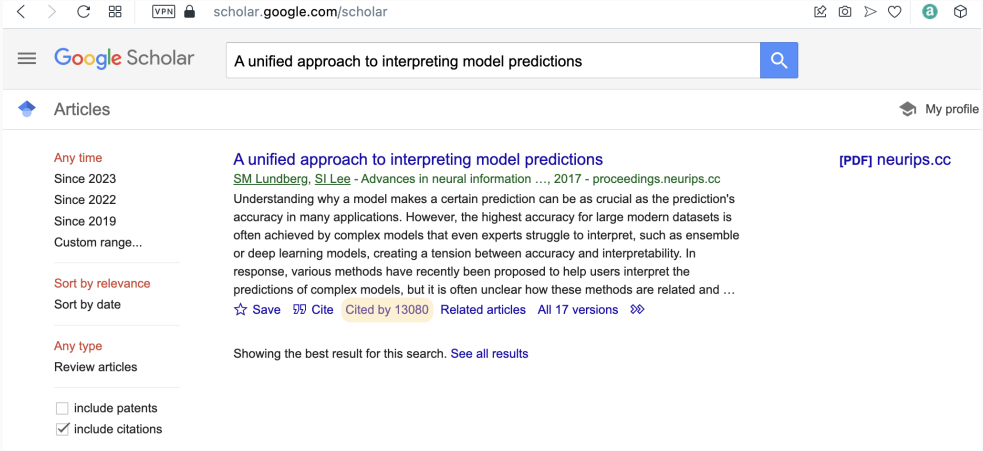
Outline – Unit #08

Some Words of Concern

Conclusions & Research Directions

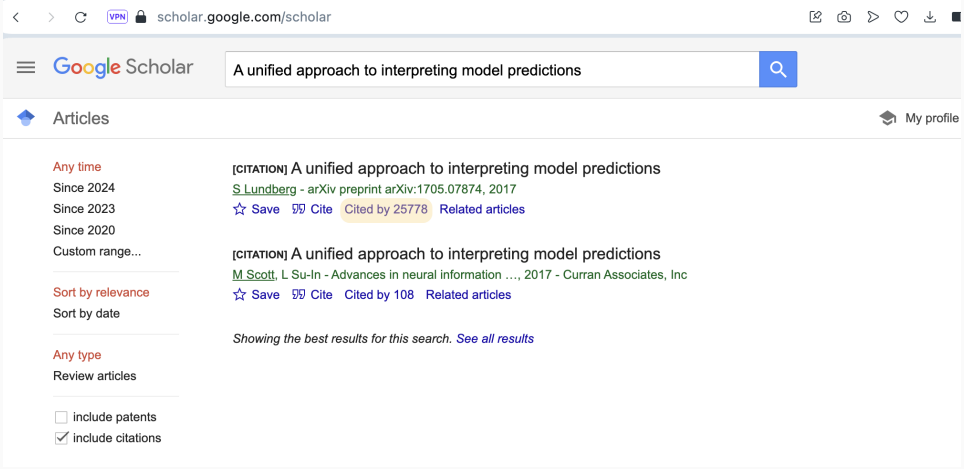
Can non-symbolic XAI's myths be stopped?

SHAP on 2023/05/31:



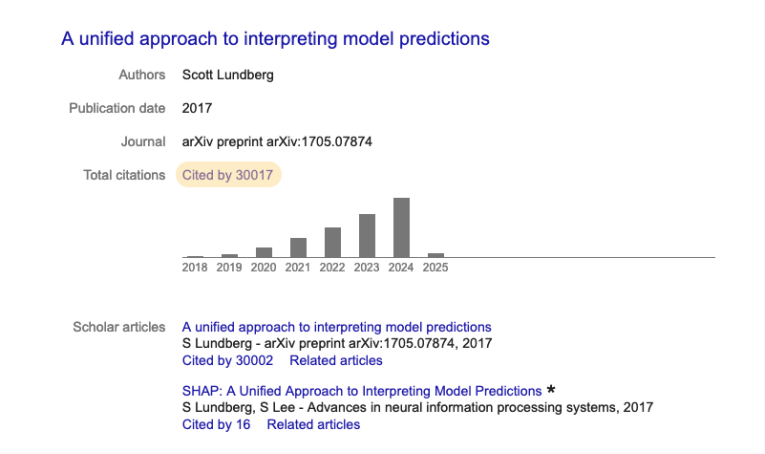
Can non-symbolic XAI's myths be stopped?

SHAP on 2024/09/15:



Can non-symbolic XAI's myths be stopped?

SHAP on 2025/01/12:

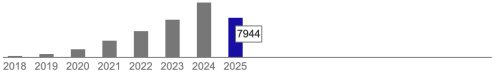


Can non-symbolic XAI's myths be stopped?

SHAP on 2025/07/14:

A unified approach to interpreting model predictions

Authors	Scott M Lundberg, Su-In Lee
Publication date	2017
Journal	Advances in neural information processing systems
Volume	30
Description	Understanding why a model makes a certain prediction can be as crucial as the prediction's accuracy in many applications. However, the highest accuracy for large modern datasets is often achieved by complex models that even experts struggle to interpret, such as ensemble or deep learning models, creating a tension between accuracy and interpretability. In response, various methods have recently been proposed to help users interpret the predictions of complex models, but it is often unclear how these methods are related and when one method is preferable over another. To address this problem, we present a unified framework for interpreting predictions, SHAP (SHapley Additive exPlanations). SHAP assigns each feature an importance value for a particular prediction. Its novel components include:(1) the identification of a new class of additive feature importance measures, and (2) theoretical results showing there is a unique solution in this class with a set of desirable properties. The new class unifies six existing methods, notable because several recent methods in the class lack the proposed desirable properties. Based on insights from this unification, we present new methods that show improved computational performance and/or better consistency with human intuition than previous approaches.
Total citations	Cited by 37527



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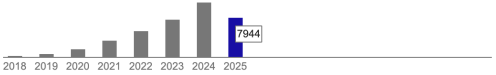
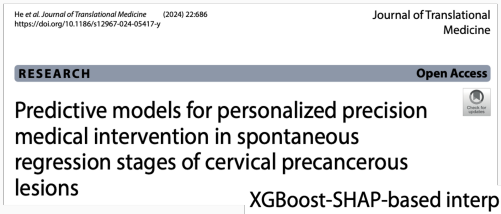


But,
Theoretical SHAP scores can mislead;
& results of tool SHAP of poor quality!

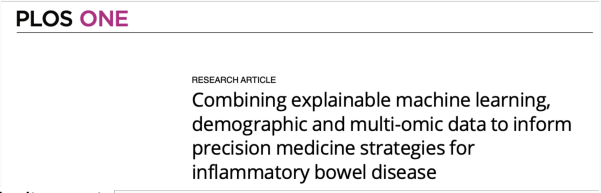
Many, many high-risk uses of SHAP & clones


He et al. Journal of Translational Medicine (2024) 22:686
https://doi.org/10.1186/s12967-024-05417-y

Journal of Translational Medicine

RESEARCH Open Access

Predictive models for personalized precision medical intervention in spontaneous regression stages of cervical precancerous lesions


PLOS ONE

RESEARCH ARTICLE

Combining explainable machine learning, demographic and multi-omic data to inform precision medicine strategies for inflammatory bowel disease

XGBoost-SHAP-based interpretable diagnostic framework for alzheimer's disease

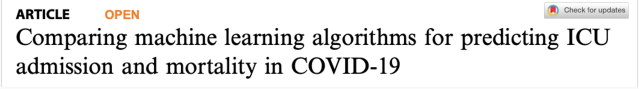

scientific reports

OPEN Explanatory predictive model for COVID-19 severity risk employing machine learning, shapley addition, and LIME

Hypothesis-free discovery of novel cancer predictors using machine learning

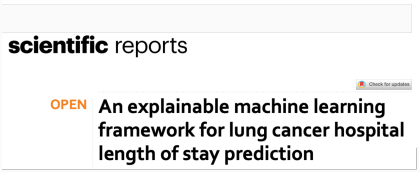
Explainable AI-based Deep-SHAP for mapping the multivariate relationships between regional neuroimaging biomarkers and cognition

npj | Digital Medicine

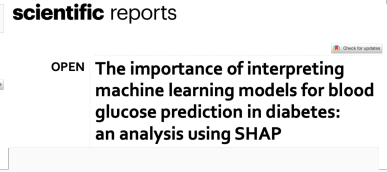

ARTICLE OPEN

Comparing machine learning algorithms for predicting ICU admission and mortality in COVID-19

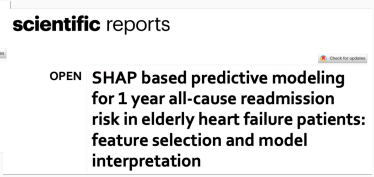
Interpretable prediction of 30-day mortality in patients with acute pancreatitis based on machine learning and SHAP


scientific reports

OPEN An explainable machine learning framework for lung cancer hospital length of stay prediction


scientific reports

OPEN The importance of interpreting machine learning models for blood glucose prediction in diabetes: an analysis using SHAP


scientific reports

OPEN SHAP based predictive modeling for 1 year all-cause readmission risk in elderly heart failure patients: feature selection and model interpretation

What next?

What next? Massive retraction (of tens of thousands) of papers??

thanks to ChatGPT...

What next? Massive retraction (of tens of thousands) of papers??

& beware of high-risk uses of SHAP! ☹

thanks to ChatGPT...

What’s the bottom line?

What's the bottom line?

- Non-symbolic XAI research experiences a persistent "*Don't Look Up*" moment...

What's the bottom line?

- Non-symbolic XAI research experiences a persistent "*Don't Look Up*" moment...

BTW, there are a multitude of proposed uses of LIME/SHAP in medicine... ⚠

Some unsettling works...

- For DTs:
  - One AXp in polynomial-time [IIM20, HIIM21, IIM22]
  - All CXps in polynomial-time [HIIM21, IIM22]

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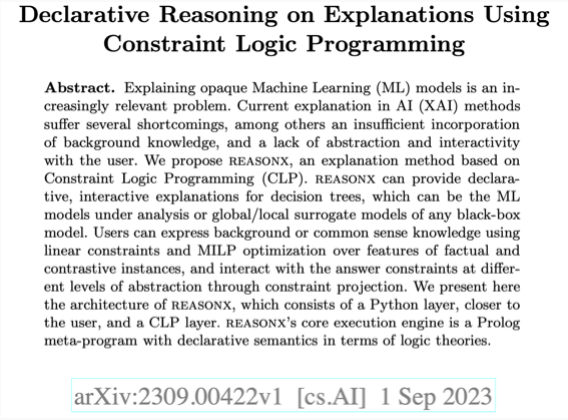

Declarative Reasoning on Explanations Using Constraint Logic Programming

Abstract. Explaining opaque Machine Learning (ML) models is an increasingly relevant problem. Current explanation in AI (XAI) methods suffer several shortcomings, among others an insufficient incorporation of background knowledge, and a lack of abstraction and interactivity with the user. We propose REASONX, an explanation method based on Constraint Logic Programming (CLP). REASONX can provide declarative, interactive explanations for decision trees, which can be the ML models under analysis or global/local surrogate models of any black-box model. Users can express background or common sense knowledge using linear constraints and MILP optimization over features of factual and contrastive instances, and interact with the answer constraints at different levels of abstraction through constraint projection. We present here the architecture of REASONX, which consists of a Python layer, closer to the user, and a CLP layer. REASONX's core execution engine is a Prolog meta-program with declarative semantics in terms of logic theories.

arXiv:2309.00422v1 [cs.AI] 1 Sep 2023

Some unsettling works...

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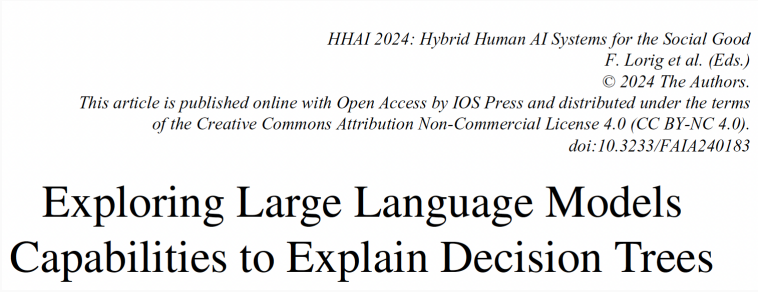

HHAI 2024: Hybrid Human AI Systems for the Social Good
F. Lorig et al. (Eds.)

doi:10.3233/FAIA240183

Exploring Large Language Models Capabilities to Explain Decision Trees

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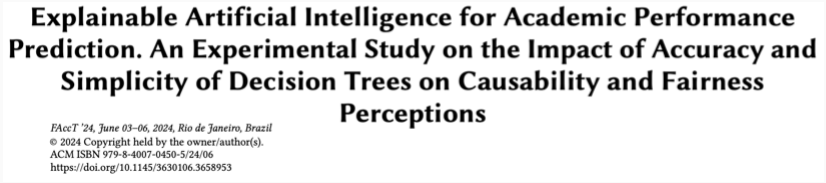

Explainable Artificial Intelligence for Academic Performance Prediction. An Experimental Study on the Impact of Accuracy and Simplicity of Decision Trees on Causability and Fairness Perceptions

FAccT '24, June 03–06, 2024, Rio de Janeiro, Brazil

ACM ISBN 979-8-4007-0450-5/24/06
https://doi.org/10.1145/3630106.3658953

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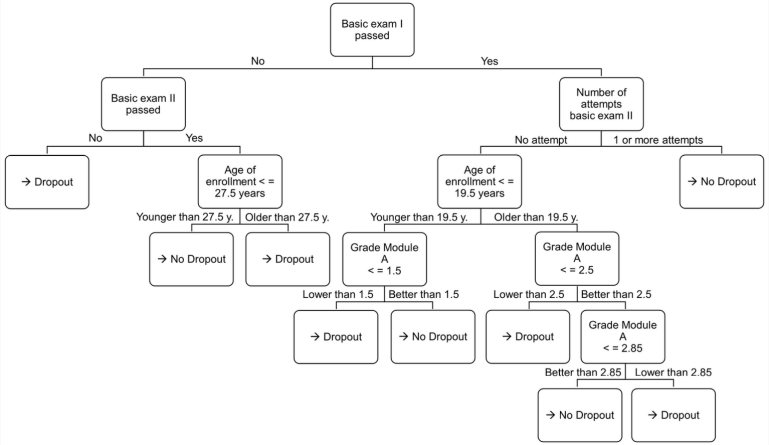

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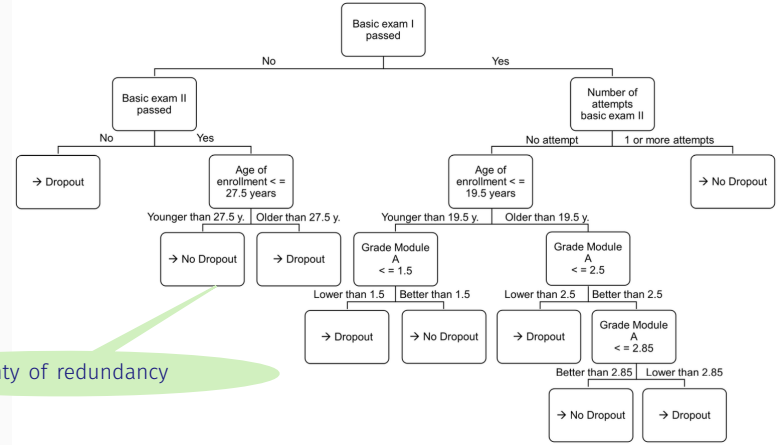

Plenty of redundancy

Outline – Unit #08

Some Words of Concern

Conclusions & Research Directions

Conclusions

- Covered logic-based (aka symbolic, aka formal) XAI & its recent progress:
  - Abductive & contrastive explanations
  - Reviewed their computation in practice
  - Duality & enumeration
  - Other explainability queries – feature necessity & relevancy

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  - Explainability using intrinsic interpretability is a **myth**
  - The rigor of model-agnostic explanations is a **myth**
  - The rigor of SHAP scores as a measure of relative feature importance is a **myth**

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  - The rigor of SHAP scores as a measure of relative feature importance is a **myth**
- Demonstrated tight connection between (rigorous) feature selection and (rigorous) feature attribution in XAI

Conclusions

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- Showed that formal XAI **disproves** some myths of (heuristic) XAI:
  - Explainability using intrinsic interpretability is a **myth**
  - The rigor of model-agnostic explanations is a **myth**
  - The rigor of SHAP scores as a measure of relative feature importance is a **myth**
- Demonstrated tight connection between (rigorous) feature selection and (rigorous) feature attribution in XAI
- Symbolic XAI exhibits links with many fields of research:
  machine learning, artificial intelligence, formal methods, automated reasoning, optimization, computational social choice (& game theory), etc.

Research directions

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- Scalabilitty, scalability, and scalability

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- Scalabilitty, scalability, and scalability
- Sample-based explanations & coherency

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- Rigorous feature attribution

Research directions

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- Rigorous feature attribution
- Checking & certification of XAI tools

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- Probabilitistic explanations

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- Any ideas from you?

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- Probabilitistic explanations
- Any ideas from you?
- … And trying to curb the massive momentum of (heuristic) XAI myths!

What this course covered

- Lecture 01 – unit(s):
  - #01: Foundations
- Lecture 02 – unit(s):
  - #02: Principles of symbolic XAI – feature selection
  - #03: Tractability in symbolic XAI (& myth of interpretability)
- Lecture 03 – unit(s):
  - #04: Intractability in symbolic XAI (& myth of model-agnostic XAI)
  - #05: Explainability queries
- Lecture 04 – unit(s):
  - #06: Recent, emerging & advanced topics
- Lecture 05 – unit(s):
  - #07: Principles of symbolic XAI – feature attribution (& myth of Shapley values in XAI)
  - #08: Corrected feature attribution – nuSHAP
  - #09: Conclusions & research directions

Some food for thought...

"All truths are easy to understand once they are discovered; the point is to discover them."
(G. Galilei)

"Beware of false knowledge; it is more dangerous than ignorance."
(G. B. Shaw)

"He who loves practice without theory is like the sailor who boards ship without a rudder and compass and never knows where he may cast."
(L. da Vinci)

Q & A

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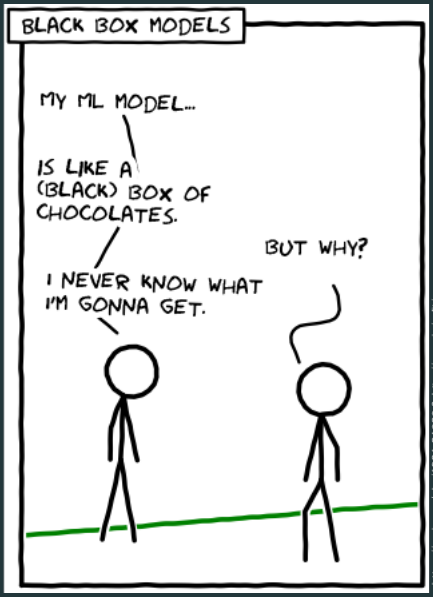
BLACK BOX MODELS
MY ML MODEL...
IS LIKE A (BLACK) BOX OF CHOCOLATES.
I NEVER KNOW WHAT I'M GONNA GET.
BUT WHY?
https://arxiv.org/abs/1901.01686 & http://cmx.io/edit/

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