ECE 464/564: Digital Signal Processing - Winter 2018
Homework 4
Due: Feb 13, 2018 (Tuesday)

1. In each of the following parts, state whether the statement is always TRUE or FALSE. Justify each of your answers.
   (a) "An LTI discrete-time system consisting of the cascade connection of two minimum-phase systems is also minimum-phase."
   (b) "An LTI discrete-time system consisting of the parallel connection of two minimum-phase systems is also minimum-phase."

2. A discrete-time casual LTI system has the system function:

$$H(z) = \frac{(1+0.3z^{-1})(1-16z^{-2})}{(1+0.64z^{-2})}$$

   a. Is the system stable?
   b. Determine expressions for minimum-phase system $H_1(z)$ and all-pass system $H_{ap}(z)$ such that

$$H(z) = H_1(z)H_{ap}(z)$$

3. H(z) is the system function for a stable LTI system and is given by:

$$H(z) = \frac{(1-4z^{-1})(1-0.6z^{-1})}{z^{-1}(1-0.25z^{-1})}$$

   H(z) can be represented as a cascade of a minimum phase system $H_1(z)$ and a unity-gain all-pass system $H_{ap}(z)$, i.e.

$$H(z) = H_{\min 1}(z)H_{ap}(z)$$

   Determine a choice for $H_{min1}(z)$ and $H_{ap}(z)$ and specify whether they are unique up to a scale factor.

4. Consider an LTI system with system function:

$$H(z)=\frac{z^{-2}\left(1-2z^{-1}\right)}{2\left(1-\frac{1}{2}z^{-1}\right)},\ |z|>\frac{1}{2}$$

(a) Is H(z) an all-pass system? Explain.
(b) Find and sketch the group delay of H(z)

5. Consider the cascade of an LTI system with its inverse system shown in Figure 1 below. The impulse response of the first system is h[n] = δ[n] + 2δ[n − 1].

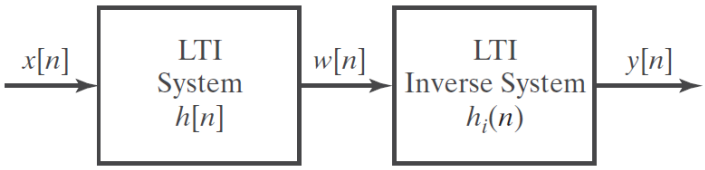

Fig. 1. LTI system with its inverse system in cascade (for Prob. 5)

a. Determine the impulse response $h_i[n]$ of a stable inverse system for h[n]. Is the inverse system causal?
b. Now consider the more general case where h[n] = δ[n] + αδ[n − 1]. Under what conditions on α will there exist an inverse system that is both stable and causal?