ECE 464/564: Digital Signal Processing - Winter 2018
Homework 1
Due: Jan 23, 2018 (Tuesday)

1. The continuous-time signal

$$x_c(t) = \sin(20\pi t) + \cos(40\pi t)$$

is sampled with a sampling period T to obtain the discrete-time signal

$$x[n] = \sin\left(\frac{n\pi}{5}\right) + \cos\left(\frac{2\pi n}{5}\right)$$

(a) Determine a choice for T consistent with this information.
(b) Is your choice for T in part (a) unique? If so, explain why. If not, specify another choice of T consistent with the information given.

2. The continuous-time signal

$$x_c(t) = \frac{\sin(10\pi t)}{10\pi t}$$

is sampled with a sampling period T to obtain the discrete-time signal

$$x[n] = \frac{\sin(n\pi/2)}{(n\pi/2)}$$

(a) Determine a choice for T consistent with this information.
(b) Is your choice for T in part (a) unique? If so, explain why. If not, specify another choice of T consistent with the information given.

3. Use the system shown in Fig. 1. below to implement a differentiator:

$$y_c(t) = \frac{d}{dt}x_c(t)$$

(C/D: An ideal continuous-to-discrete time converter, D/C: An ideal discrete-to-continuous time converter)

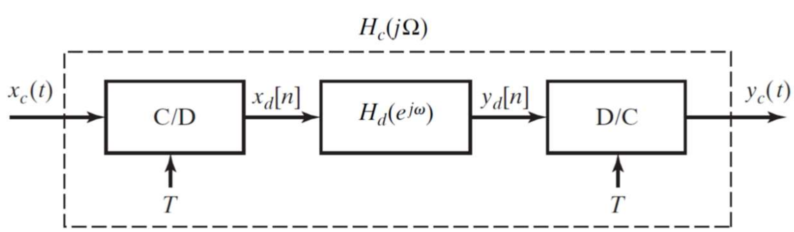

Fig. 1. For Problem 3

a. Write $H_c(j\Omega)$ for the derivative

b. Find $H_c(e^{jw})$

c. Find and plot $h[n]$

4. Each of the following parts lists an input signal x[n] and the Up-sampling and Down-sampling rates L and M for the system in Fig. 2. Determine the corresponding output $\tilde{x}_d[n]$

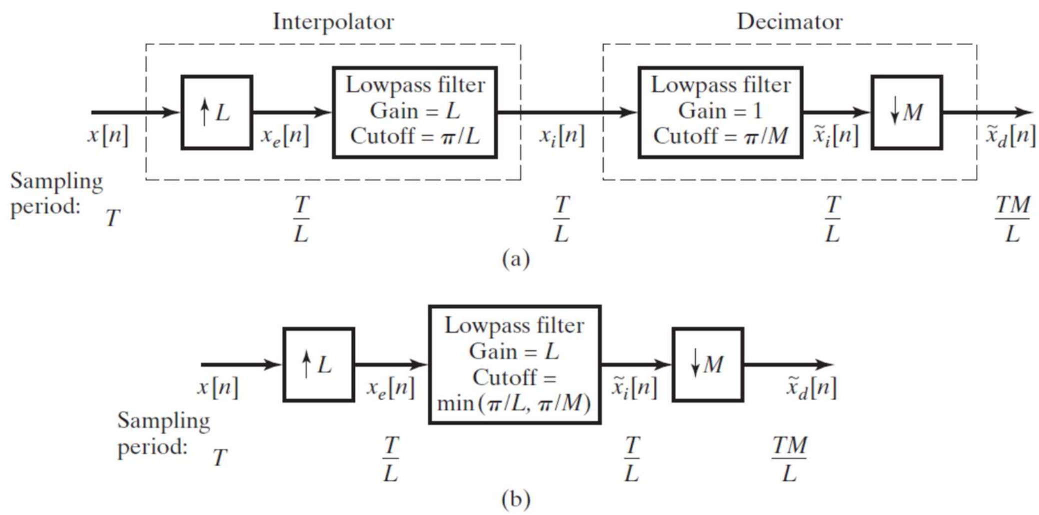

Fig. 2. For Problems 4 and 5

a. $x[n] = \sin(2\pi n/3)/\pi n$, L = 4, M = 3
b. $x[n] = \sin(3\pi n/4)$, L = 6, M = 7

5. For the system shown in Fig. 2, $X(e^{jw})$, the Fourier transform of the input signal x[n], is shown in Fig. 3.

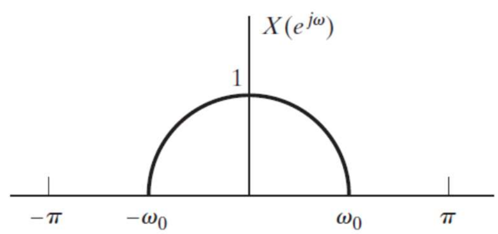

Fig. 3. For Problem 5

For each of the following choices of L and M, specify the maximum possible value of ω_0 such that $\tilde{X}_d(e^{j\omega}) = aX(e^{jM\omega/L})$ for some constant a.

(a) $M = 3, L = 2$
(b) $M = 2, L = 3$

6. Fig. 4. shows a continuous-time filter that is implemented using an LTI discrete-time filter ideal lowpass filter with frequency response over $-\pi \le \omega \le \pi$ as

$$H(e^{j\omega}) = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & \omega_c < |\omega| \le \pi. \end{cases}$$

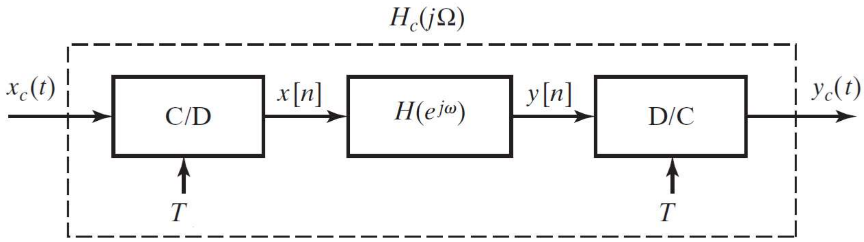

Fig. 4. Continuous-Time filter using a discrete-time LPF

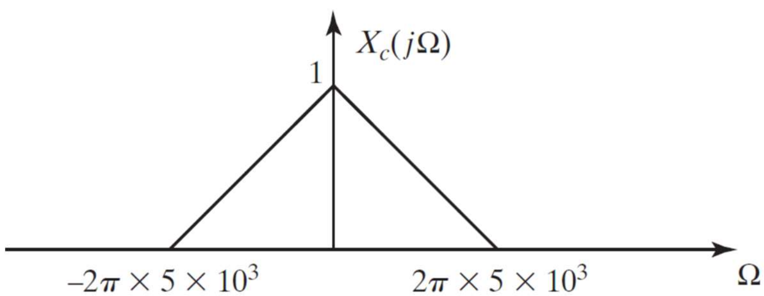

Fig. 5. Continuous-Time Fourier Transform of $x_c(t)$

(a) If the continuous-time Fourier transform of $x_c(t)$, namely $X_c(j\Omega)$, is as shown in Fig. 5. and $\omega_c = \pi/5$, sketch and label $X(e^{j\omega})$, $Y(e^{j\omega})$ and $Y_c(j\Omega)$ for $1/T = 2 \times 10^4$

(b) For $1/T = 6 \times 10^3$ and for input signals $x_c(t)$ whose spectra are bandlimited to $|\Omega| < 2\pi \times 5 \times 10^3$ (but otherwise unconstrained), what is the maximum choice of the cutoff frequency ω_c of the filter $H(e^{j\omega})$ for which no aliasing occurs. For this maximum choice of ω_c, specify $H_c(j\Omega)$.