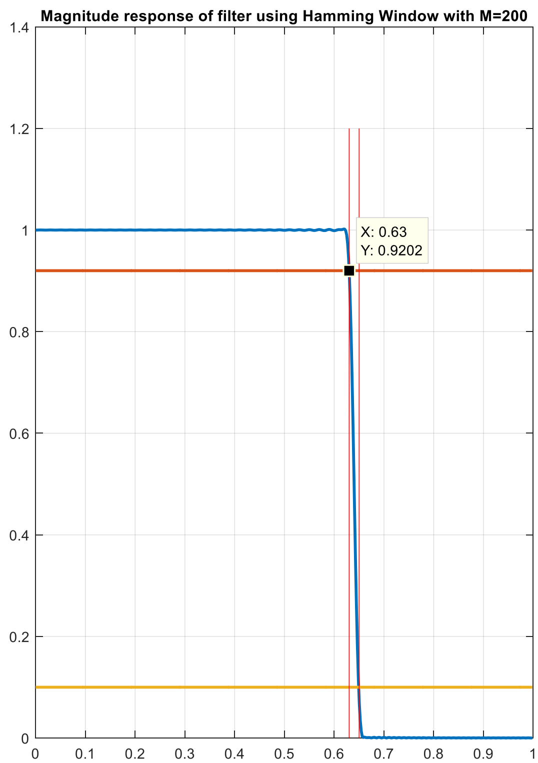
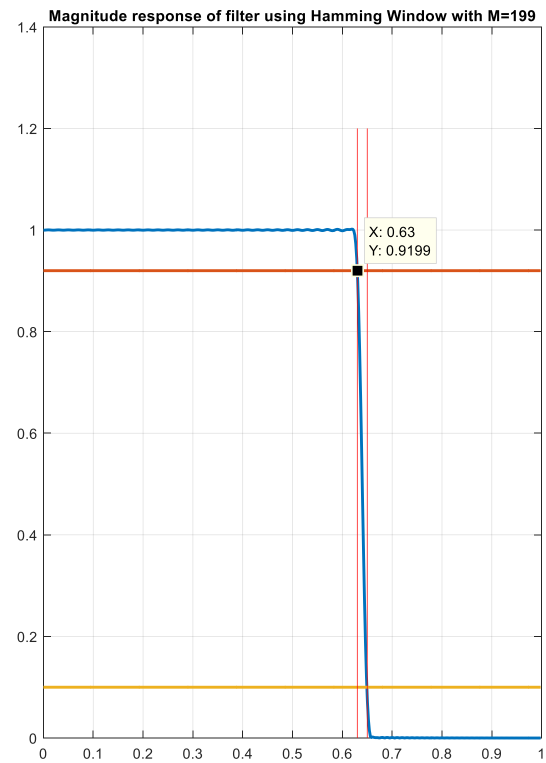
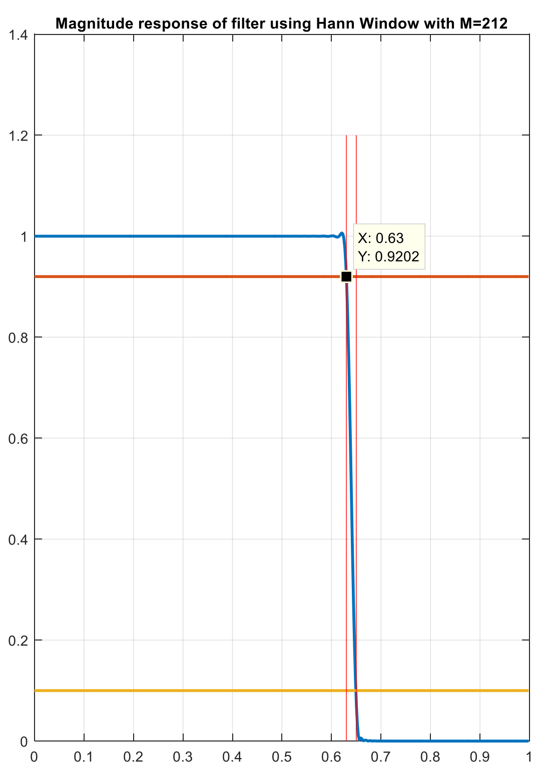
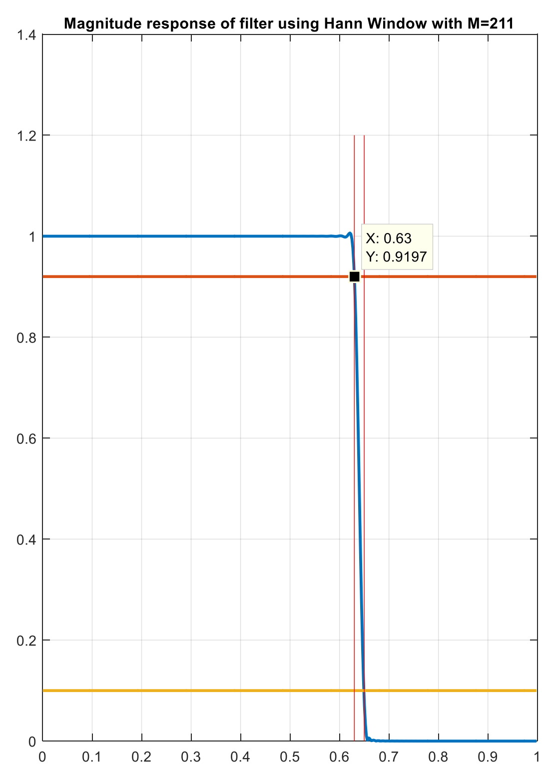
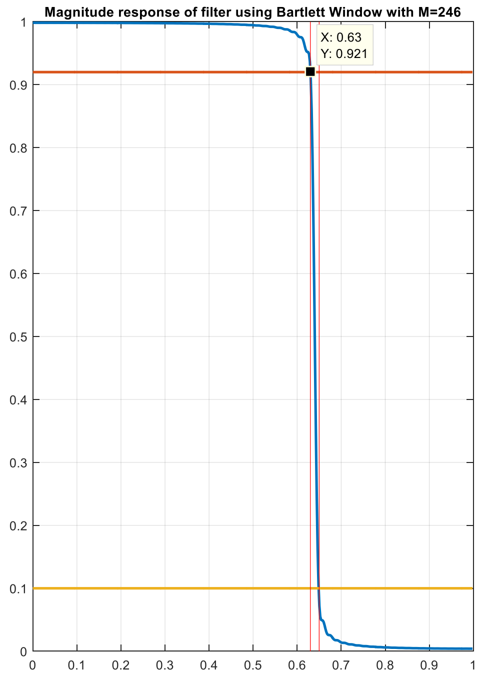
ECE 464/564 Homework-8 solutions

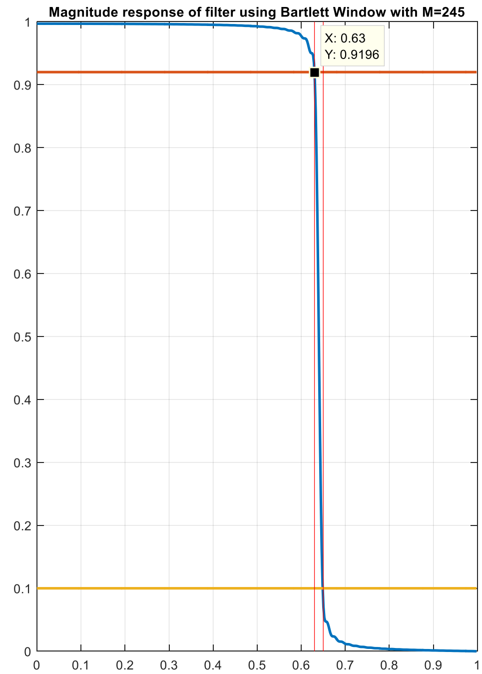













```
M=246;
n=0:1:M;
h=sin(0.64*pi*(n-M/2))./(pi*(n-M/2));
if rem(M,2)==0
    h(M/2+1)=0.64;
end

 %%% Bartlett window
subplot(1,2,1);
w=bartlett(M+1)';
h=h.*w;
[a,b]=freqz(h);
plot(b/pi,abs(a),b/pi,0*b+0.92,b/pi,0*b+0.1,'LineWidth',2);
yline=get(gca,'ylim');
line([0.63 0.63],yline,'color',[1 0 0]);
line([0.65 0.65],yline,'color',[1 0 0]);
title('Magnitude response of filter using Bartlett Window with M=246');
grid ON

subplot(1,2,2);
M=245;
n=0:1:M;
h=sin(0.64*pi*(n-M/2))./(pi*(n-M/2));
if rem(M,2)==0
    h(M/2+1)=0.64;
end
w=bartlett(M+1)';
h=h.*w;
[a,b]=freqz(h);
plot(b/pi,abs(a),b/pi,0*b+0.92,b/pi,0*b+0.1,'LineWidth',2);
yline=get(gca,'ylim');
line([0.63 0.63],yline,'color',[1 0 0]);
line([0.65 0.65],yline,'color',[1 0 0]);
title('Magnitude response of filter using Bartlett Window with M=245');
grid ON
```

2(a) For a system to be minimum phase, all poles & zeros have to be inside the unit circle.

Impulse Invariance: maps left-half s-plane poles to the interior of the z-plane unit circle. However, left half s-plane zeros will not necessarily be mapped inside the z-plane unit circle.

Consider:

$$H_c(s) = \frac{s+10}{(s+1)(s+2)}$$

Using $T=1$, we could show that a minimum phase filter is transformed into a non-minimum phase discrete time filter.

Bilinear Transform:

The Bilinear transform maps a pole or zero at $s = s_0$ to a pole or zero at $z_0 = \frac{1+\frac{T}{2}s_0}{1-\frac{T}{2}s_0}$.

Thus, $$|z_0| = \left|\frac{1+\frac{T}{2}s_0}{1-\frac{T}{2}s_0}\right|$$

(P4)

Since $H_c(s)$ is a minimum phase, all the pole of $H_c(s)$ are located in the left half of s-plane.

$\therefore$ A pole $s_0 = \sigma + j\Omega$ must have $\sigma < 0$,

Using the relation for s_0, we get,

$$|z_0| = \sqrt{\frac{\left(1+\frac{T}{2}\sigma\right)^2 + \left(\frac{T}{2}\Omega\right)^2}{\left(1-\frac{T}{2}\sigma\right)^2 + \left(\frac{T}{2}\Omega\right)^2}} < 1$$

Thus, all poles & zeros will be inside the Z-plane unit circle and the discrete time filter will be minimum phase as well.

(b) Only the bilinear transform design will result in an all pass filter.

Impulse Invariance:

$$H(e^{j\omega}) = \sum_{k=-\infty}^{\infty} H_c\left(j\left(\frac{\omega}{T_d} + \frac{2\pi k}{T_d}\right)\right)$$

The aliasing terms can destroy the all pass nature of the continuous-time filter.

Bilinear Transform: This only warps the frequency axis. The magnitude response is not affected. Therefore, an all pass filter will map to an all pass filter.

(c) Only Bilinear Transformation will guarantee

$$H(e^{j\omega})\Big|_{\omega=0} = H_c(j\Omega)\Big|_{\Omega=0}.$$

Impulse Invariance:

Since impulse invariance may result in aliasing, we see that

$$H(e^{j0}) = H_c(j0)$$

if and only if

$$H(e^{j0}) = \sum_{k=-\infty}^{\infty} H_c\left(\frac{j2\pi k}{T_d}\right) = H_c(j0).$$

or equivalently

$$\sum_{k=-\infty}^{\infty} H_c\left(j\frac{2\pi k}{T_d}\right) = 0,$$ which is generally not the case.

(P6)

Bilinear Transform:

Since, under the bilinear transformation,

$\Omega = 0$ maps to $\omega = 0$

$H(e^{j0}) = H_c(j0)$ for all $H_c(s)$.

(d) Only the bilinear transform design is guaranteed to create a bandstop filter from a bandstop filter.

If $H_c(s)$ is a bandstop filter, the bilinear transform will preserve this because it just warps the frequency axis; however aliasing (in the impulse invariance technique) can fill in the stop band.

(e) This property holds under the bilinear transformation, but not under impulse invariance

Impulse Invariance: May result in aliasing.

Since the order of aliasing and multiplication are not interchangeable, the desired identity does not hold.

Consider $H_{a1}(s) = H_{a2}(s) = e^{-sT/2}$

Bilinear Transform:

$$H(z) = H_c\left(\frac{2}{T_d}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right)$$

$$= H_{c1}\left(\frac{2}{T_d}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right) H_{c2}\left(\frac{2}{T_d}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right)$$

$$= H_1(z) \cdot H_2(z)$$

(d) The property holds for both impulse invariance and the bilinear transform.

Impulse Invariance:

$$H(e^{j\omega}) = \sum_{k=-\infty}^{\infty} H_c\left(j\left(\frac{\omega}{T_d} + \frac{2\pi k}{T_d}\right)\right)$$

$$= \sum_{k=-\infty}^{\infty} H_{c1}\left(j\left(\frac{\omega}{T_d} + \frac{2\pi k}{T_d}\right)\right) + \sum_{k=-\infty}^{\infty} H_{c2}\left(j\left(\frac{\omega}{T_d} + \frac{2\pi k}{T_d}\right)\right)$$

$$= H_1(e^{j\omega}) + H_2(e^{j\omega}).$$

Bilinear Transform:

$$H(z) = H_c\left(\frac{2}{T_d}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right)$$

$$= H_{c1}\left(\frac{2}{T_d}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right) + H_{c2}\left(\frac{2}{T_d}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right)$$

$$= H_1(z) + H_2(z)$$

(3)(a)

$$s = \beta\left(\frac{1-z^{-\alpha}}{1+z^{-\alpha}}\right)$$

$$s + s z^{-\alpha} = \beta - \beta z^{-\alpha}$$

$$(s+\beta)\, z^{-\alpha} = \beta - s$$

$$z^{-\alpha} = \left(\frac{\beta - s}{\beta + s}\right)$$

$$z^{\alpha} = \frac{\beta + s}{\beta - s}$$

If $H_c(s)$ is stable, causal continuous time filter means the poles are on the Left half plane, which means $Re\{s\} < 0$

If $H(z)$ is a stable & causal, all poles must lie within $|z|=1$; i.e, $|z|<1$

$$|z|<1 \Rightarrow |z^{\alpha}| < 1 \quad (\because \alpha > 0)$$

$$\Rightarrow \left|\frac{\beta+s}{\beta-s}\right| < 1$$

$$|\beta + s| < |\beta - s|, \quad s = j\Omega + \sigma$$

$$|\beta + \sigma + j\Omega| < |\beta - \sigma - j\Omega|$$

$$(\beta + \sigma)^2 + \Omega^2 < (\beta - \sigma)^2 + \Omega^2$$

$$4\beta\sigma < 0.$$

Since $\sigma < 0$, $\underline{\beta > 0}$

(b) $|z| < 1 \Rightarrow |z^{\alpha}| > 1$ (Because $\alpha < 0$)

$$\left|\frac{\beta + s}{\beta - s}\right| > 1$$

$$\Rightarrow |\beta + s| > |\beta - s|$$

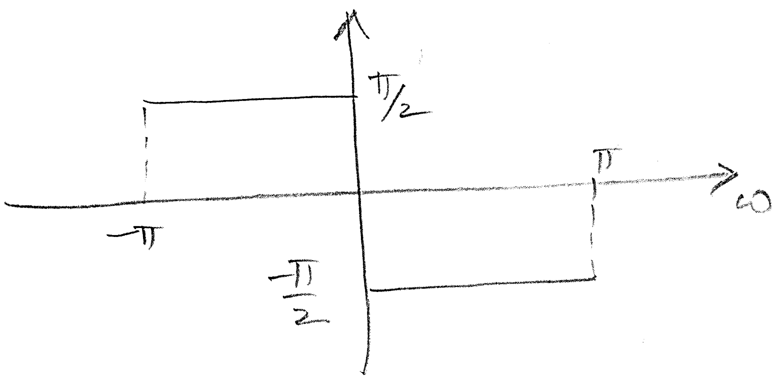
$$\Rightarrow 4\beta\sigma > 0$$

Since $\sigma < 0$, $\underline{\beta < 0}$

(4a)

$$H_d(e^{j\omega}) = \begin{cases} j, & -\pi < \omega < 0 \\ -j, & 0 < \omega < \pi. \end{cases}$$

$$\angle H_d(e^{j\omega}) = \begin{cases} \frac{\pi}{2}, & -\pi < \omega < 0 \\ -\frac{\pi}{2}, & 0 < \omega < \pi \end{cases}$$



(b) $$h_d[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{0} j e^{j\omega n}\, d\omega - \int_{0}^{\pi} j e^{j\omega n}\, d\omega\right]$$

$$= \left(\frac{j}{2\pi}\right) \frac{e^{j\omega n}}{jn}\Bigg|_{-\pi}^{0} - \frac{j}{2\pi}\frac{e^{j\omega n}}{jn}\Bigg|_{0}^{\pi}$$

$$= \frac{1}{2\pi n}\left[\left(1 - e^{-jn\pi}\right) - \left(e^{jn\pi} - 1\right)\right]$$

$$= \frac{1}{2\pi n}\left[2 - e^{-jn\pi} - e^{jn\pi}\right]$$

$$= \frac{1}{2\pi n}\left[2 - 2\cos(n\pi)\right] = \frac{1}{n\pi}\left[1 - \cos(n\pi)\right]$$

P11

$$= \frac{2\sin^2\left(\frac{n\pi}{2}\right)}{n\pi}$$

So, $h_d[n] = \begin{cases} \dfrac{\sin^2\left(\frac{n\pi}{2}\right)}{\left(\frac{n\pi}{2}\right)}, & n \neq 0 \\ 0, & n = 0 \end{cases}$ $\rightarrow$

further
when n is even,
$h_d[n] = 0$
when n is odd,
$h_d[n] = \frac{2}{n\pi}$

c) From (b), we know that

$$h_d[n] = -h_d[-n]$$

It is antisymmetric, so it can be type III or type IV.

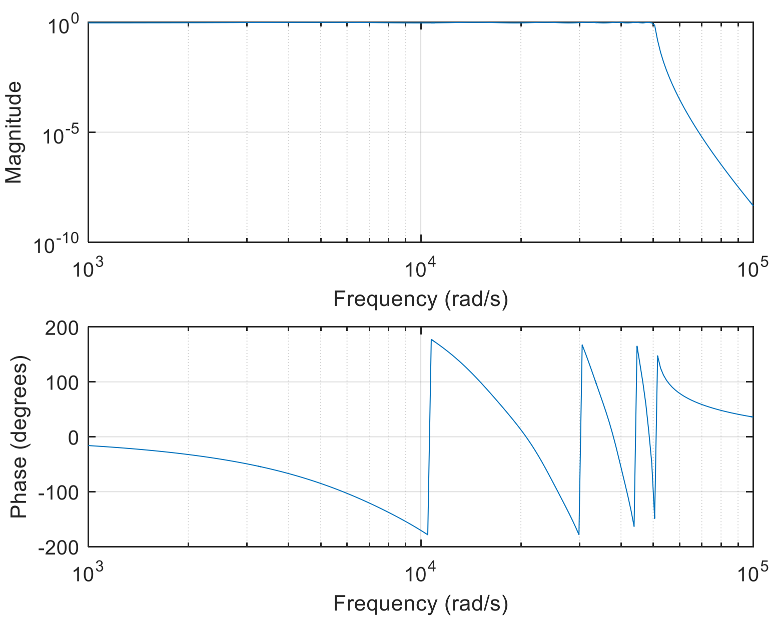
```
load('HW8_Bonus.mat');
% sound(noised_audio,fs)
Wp=2*pi*6600;
Ws=2*pi*8800;
Rp=0.5;
Rs=-50;

[N,Wp]=cheb1ord(Wp,Ws,Rp,Rs,'s');
[b,a]=cheby1(N,Rp,Wp,'low','s');

[bz,az]=bilinear(b,a,fs);
filtered_sound=filter(bz,az,noised_audio);

sound(filtered_sound,fs);
figure(1)
freqs(b,a,1000);
figure(2)
freqz(bz,az,1000);
figure(3)
subplot(121)
plot(noised_audio)
title('noisy audio')
subplot(122)
plot(filtered_sound)
title('noise free data')
```

Response of the Continuous Time Filter



Response of the Discrete Time Filter