ECE 464/564 Homework-3 Solutions:

1

(a)

$$H(z) = \frac{(1-1.5z^{-1}-z^{-2})(1+0.9z^{-1})}{(1-z^{-1})(1+0.7jz^{-1})(1-0.7jz^{-1})}$$

$$\Rightarrow \frac{Y(z)}{X(z)} = \frac{1-1.5z^{-1}-z^{-2}+0.9z^{-1}-1.35z^{-2}-0.9z^{-3}}{(1-z^{-1})(1+0.49z^{-2})}$$

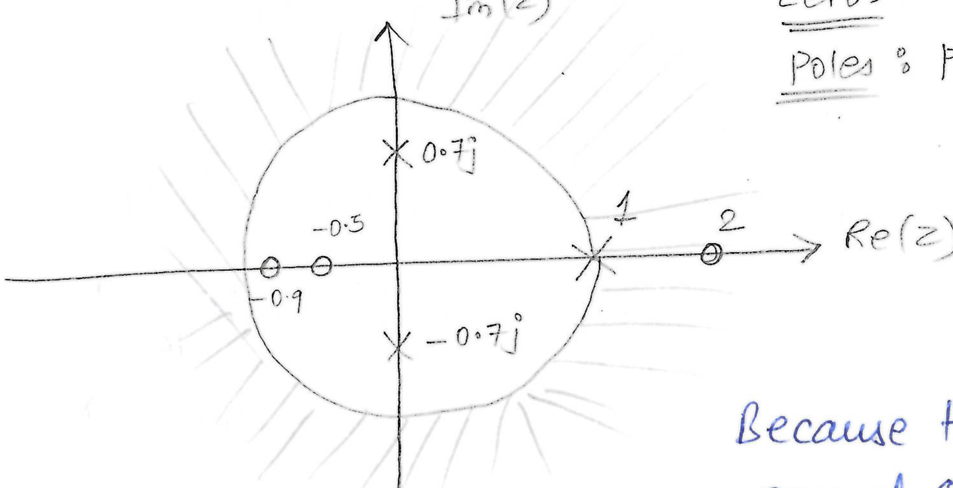
$$= \frac{1-0.6z^{-1}-2.35z^{-2}-0.9z^{-3}}{1+0.49z^{-2}-z^{-1}-0.49z^{-3}}$$

$$\Rightarrow Y(z)\left[1-z^{-1}+0.49z^{-2}-0.49z^{-3}\right] = X(z)\left[1-0.6z^{-1}-2.35z^{-2}-0.9z^{-3}\right]$$

$\Rightarrow$ By taking Inverse Z-transform on either side,

$$y[n]-y[n-1]+0.49y[n-2]-0.49y[n-3]$$
$$= x[n]-0.6x[n-1]-2.35x[n-2]-0.9x[n-3]$$

(b)



Zeros: $z_1=-0.9$, $z_2=2, -0.5$

Poles: $P_1=1$, $P_2=-0.7j$
$P_3=+0.7j$

Because H(z) is a causal system ROC is $|z|>1$;

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(c) $H(z) = \dfrac{(1-2z^{-1})(1+\frac{1}{2}z^{-1})(1+0.9z^{-1})}{(1-z^{-1})(1+0.7jz^{-1})(1-0.7jz^{-1})}$

$$\left|H(e^{j\omega})\right|_{dB} = 20\log_{10}\left|1-2e^{-j\omega}\right| + 20\log_{10}\left|1+\tfrac{1}{2}e^{-j\omega}\right| + 20\log_{10}\left|1+0.9e^{-j\omega}\right| - 20\log_{10}\left|1-e^{-j\omega}\right| - 20\log_{10}\left|1-0.7e^{-j(\omega-\frac{\pi}{2})}\right| - 20\log_{10}\left|1-0.7e^{-j(\omega+\frac{\pi}{2})}\right|$$

Zeros: $r=2,\ \theta=0:\ \omega=\theta;\ 20\log_{10}|1-r| = 0$

$\omega=\pi-\theta;\ 20\log_{10}|1+r| = 20\log_{10}3$

$r=\frac{1}{2},\ \theta=\pi;\ \omega=\theta;\ 20\log_{10}|1-r| = 20\log_{10}\frac{1}{2}$

$\omega=\pi-\theta:\ 20\log_{10}|1+r| = 20\log_{10}\frac{3}{2}$

$r=0.9,\ \theta=\pi:\ \omega=\theta:\ 20\log_{10}|1-r| = 20\log_{10}0.1$

$\omega=\pi-\theta:\ 20\log_{10}|1+r| = 20\log_{10}1.9$

Poles: $r=1,\ \theta=0;\ \omega=\theta:\ -20\log_{10}|1-r| \Rightarrow \infty$

$\omega=\pi-\theta:\ -20\log_{10}|1+r| = -20\log_{10}2$

$r=0.7,\ \theta=\frac{\pi}{2}:\ \omega=\theta:\ -20\log_{10}|1-r| = -20\log_{10}0.3$

$\omega=\pi-\theta:\ -20\log_{10}|1+r| = -20\log_{10}1.7$

$r = 0.7, \theta = \frac{-\pi}{2}$: $\omega = \theta = -20 \log_{10}|1-r| = -20\log_{10} 0.3$

$\omega = \pi - \theta = -20\log_{10}|1+r| = -20 \log_{10} 1.7$

(Plots are shown in the next page)

(d) Zeros: $r = 2, \theta = 0$: $grd_1(\omega) = \dfrac{4 - 2\cos(\omega - \theta)}{1 + 4 - 4\cos(\omega - \theta)}$

$r = \frac{1}{2}, \theta = \pi$: $grd_2(\omega) = \dfrac{\frac{1}{4} - \frac{1}{2}\cos(\omega - \theta)}{1 + \frac{1}{4} - \cos(\omega - \theta)}$

$r = 0.9, \theta = \pi$: $grd_3(\omega) = \dfrac{0.81 - 0.9\cos(\omega - \theta)}{1 + 0.81 - 1.8\cos(\omega - \theta)}$

Poles: $r = 1, \theta = 0$: $grd_4(\omega) = -\left(\dfrac{1 - \cos(\omega - \theta)}{1 + 1 - 2\cos(\omega - \theta)}\right) = -0.5$

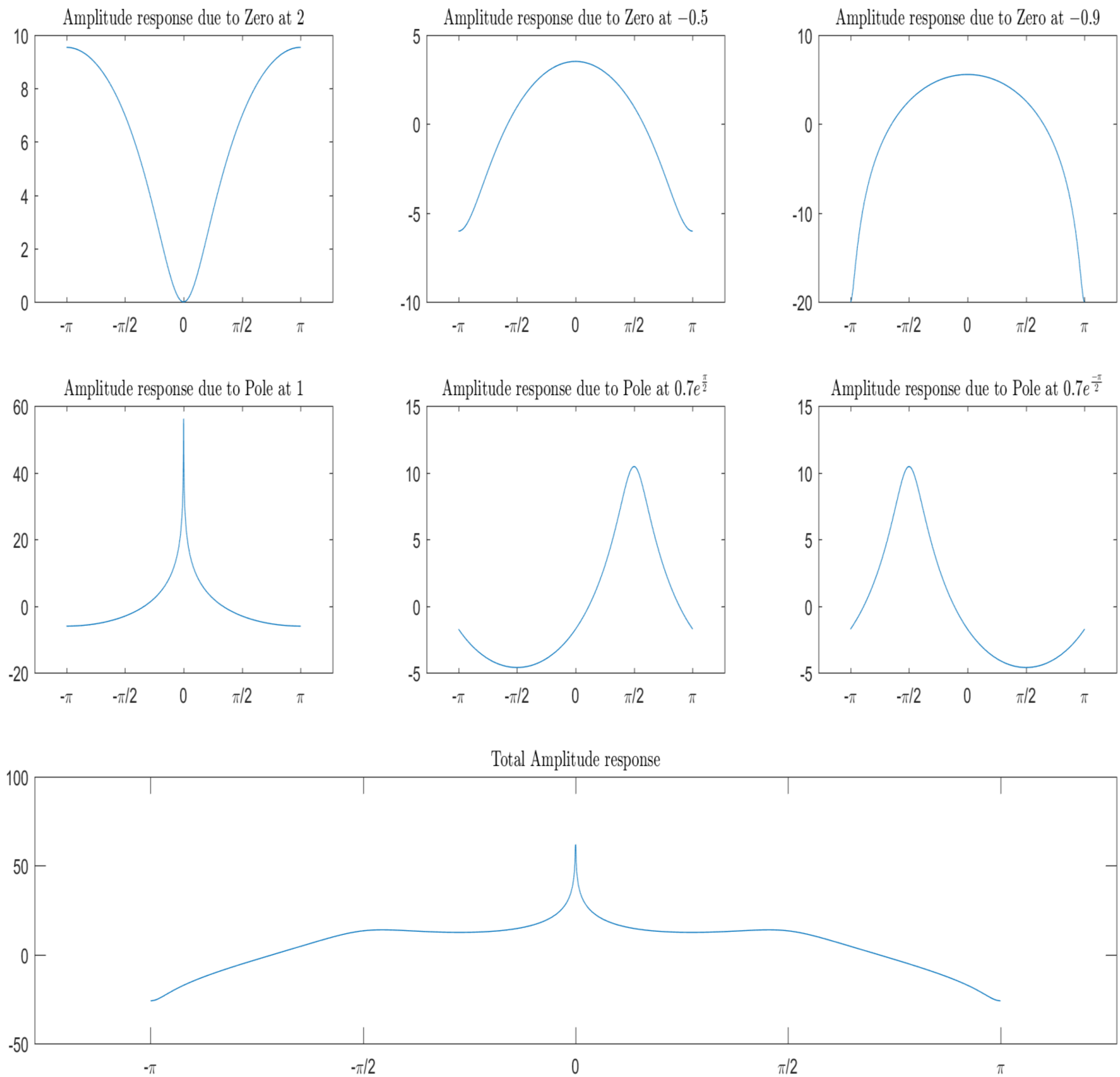
$r = 0.7, \theta = \frac{\pi}{2}$: $grd_5(\omega) = -\left(\dfrac{0.49 - 0.7\cos(\omega - \theta)}{1 + 0.49 - 1.4\cos(\omega - \theta)}\right)$

$r = 0.7, \theta = \frac{-\pi}{2}$: $grd_6(\omega) = -\left(\dfrac{0.49 - 0.7\cos(\omega - \theta)}{1 + 0.49 - 1.4\cos(\omega - \theta)}\right)$

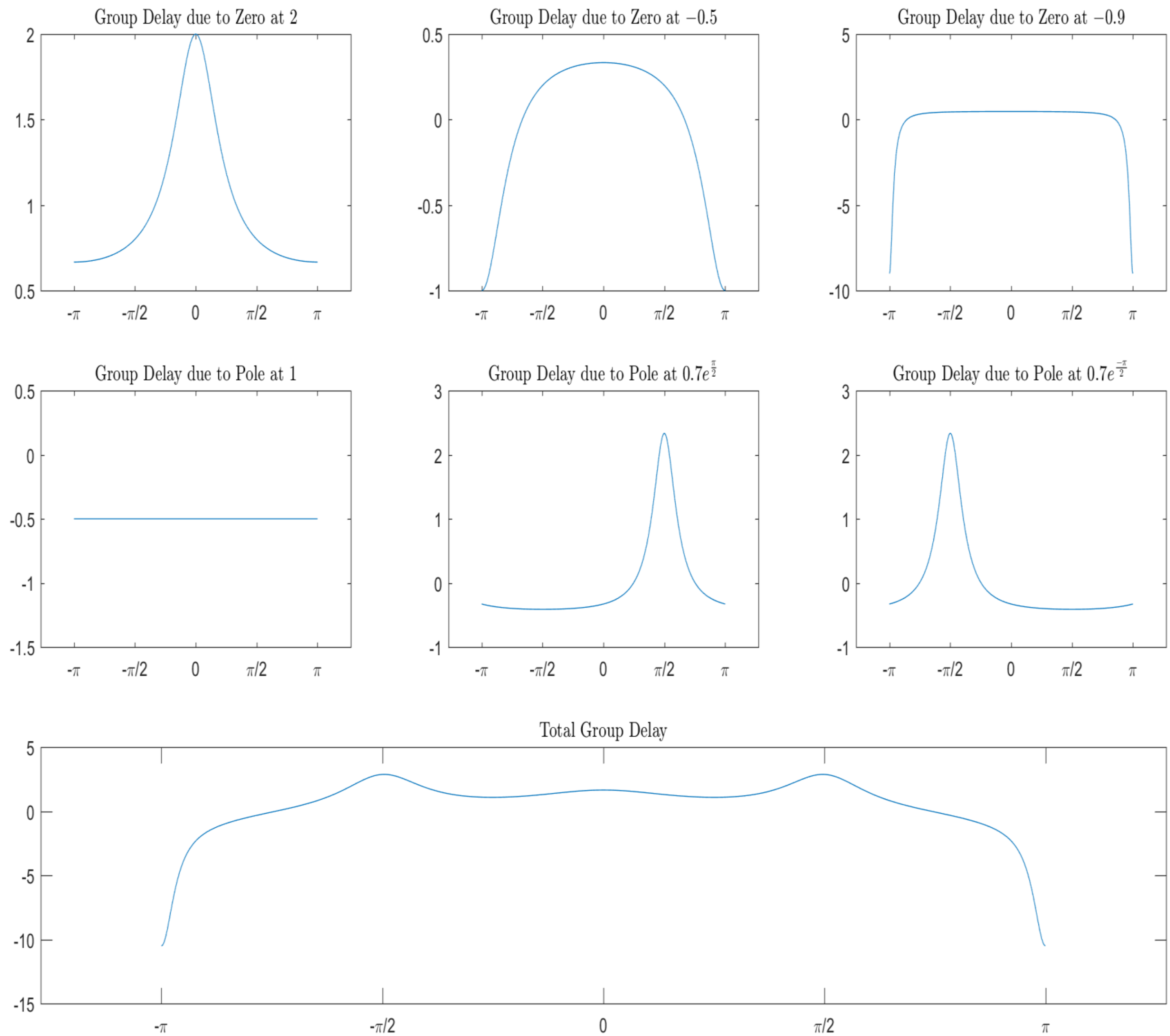
$grd(\omega) = grd_1(\omega) + grd_2(\omega) + grd_3(\omega) + grd_4(\omega) + grd_5(\omega) + grd_6(\omega).$

(Plots are shown in the next page)

1c) **Amplitude Response Plots:**



1d) **<u>Group delay Plots:</u>**



(2) For an all-pass system :

$$H_{ap}(z) = A \prod_{k=1}^{M_r} \left(\frac{z^{-1} - d_k}{1 - d_k z^{-1}} \right) \prod_{k=1}^{M_c} \left(\frac{(z^{-1} - e_k^*)(z^{-1} - e_k)}{(1 - e_k z^{-1})(1 - e_k^* z^{-1})} \right)$$

For All pass System : $|H_{ap}(e^{j\omega})| = 1$

& All the Poles & Zeros are in Conjugate Reciprocal Pairs;

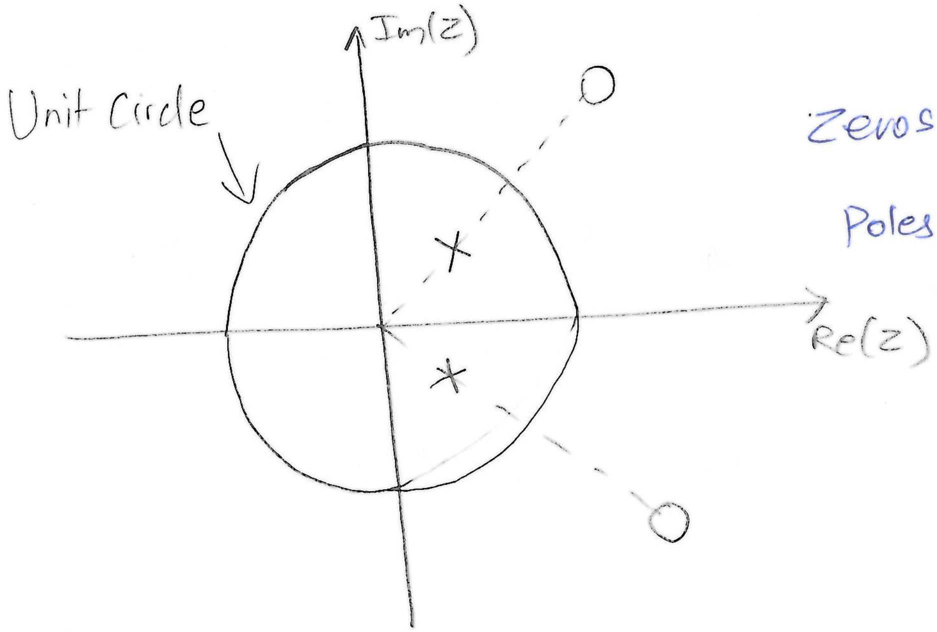
(a) $H_1(z)$ has pole at $\frac{3}{4}$ & zero at $\frac{4}{3}$ (Reciprocal)
Yes, it is an All-pass System

(b) $H_2(z)$ doesnot have Pole-zero in Conjugate reciprocal pair. No, it is not all pass

(c) $H_3(z)$ Does have PZ in Conjugate reciprocal pair. Yes, it is all pass System.

(d) Yes, $H_4(z)$ is also all pass System

3(a) $$H(z) = \left[\frac{z^{-1} - (0.5 - 0.5j)}{1 - (0.5 + 0.5j)z^{-1}}\right]\left[\frac{z^{-1} - (0.5 + 0.5j)}{1 - (0.5 - 0.5j)z^{-1}}\right]$$

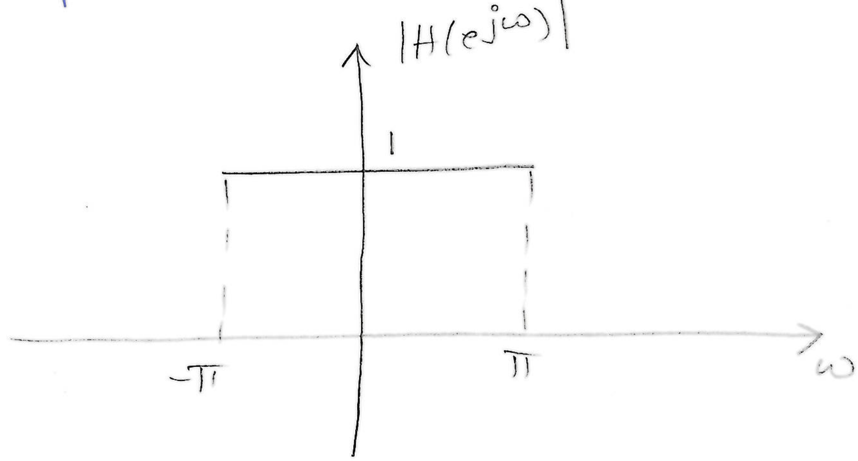


Zeros : $r = \sqrt{2}$, $\theta = \pm\frac{\pi}{4}$

Poles : $r = \frac{1}{\sqrt{2}}$, $\theta = \pm\frac{\pi}{4}$.

(b) Because $H(z)$ is an all pass system

$$|H(e^{j\omega})| = 1$$



(c) Group Delay Plot:-

① Zero : $r = \sqrt{2}$, $\theta = \frac{\pi}{4}$, $grd_1(\omega) = \dfrac{2 - \sqrt{2}\cos(\omega - \theta)}{1 + 2 - 2\sqrt{2}\cos(\omega - \theta)}$

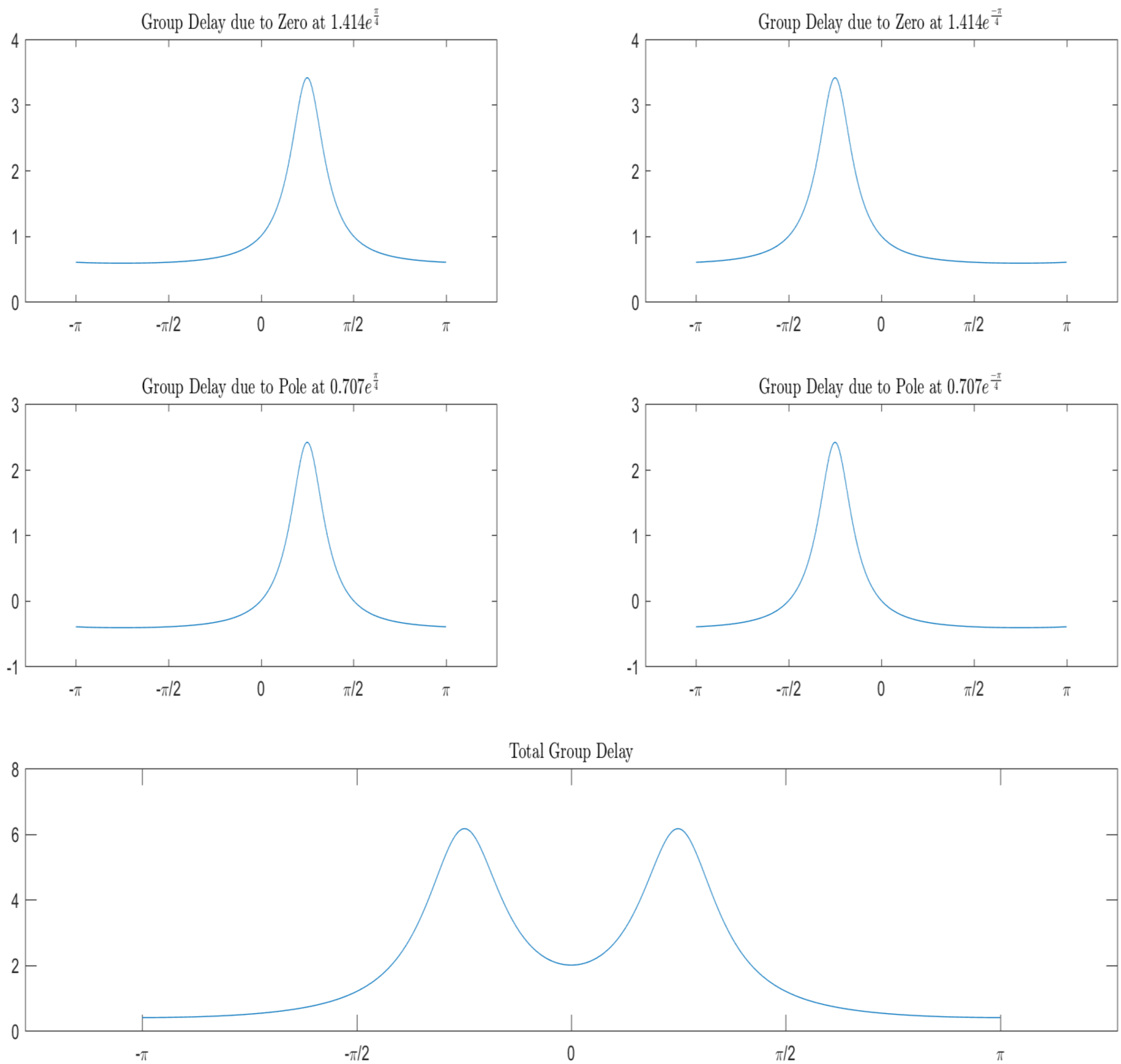
Zero: $r = \sqrt{2}$, $\theta = \frac{-\pi}{4}$, $grd_2(\omega) = \dfrac{2 - \sqrt{2}\cos(\omega - \theta)}{1 + 2 - 2\sqrt{2}\cos(\omega - \theta)}$

(2) Pole: $r = \frac{1}{\sqrt{2}}$, $\theta = \frac{\pi}{4}$, $grd_3(\omega) = -\left[\dfrac{\frac{1}{2} - \frac{1}{\sqrt{2}}\cos(\omega - \theta)}{1 + \frac{1}{2} - \sqrt{2}\cos(\omega - \theta)}\right]$

$r = \frac{1}{\sqrt{2}}$, $\theta = \frac{-\pi}{4}$, $grd_4(\omega) = -\left[\dfrac{\frac{1}{2} - \frac{1}{\sqrt{2}}\cos(\omega - \theta)}{1 + \frac{1}{2} - \sqrt{2}\cos(\omega - \theta).}\right]$

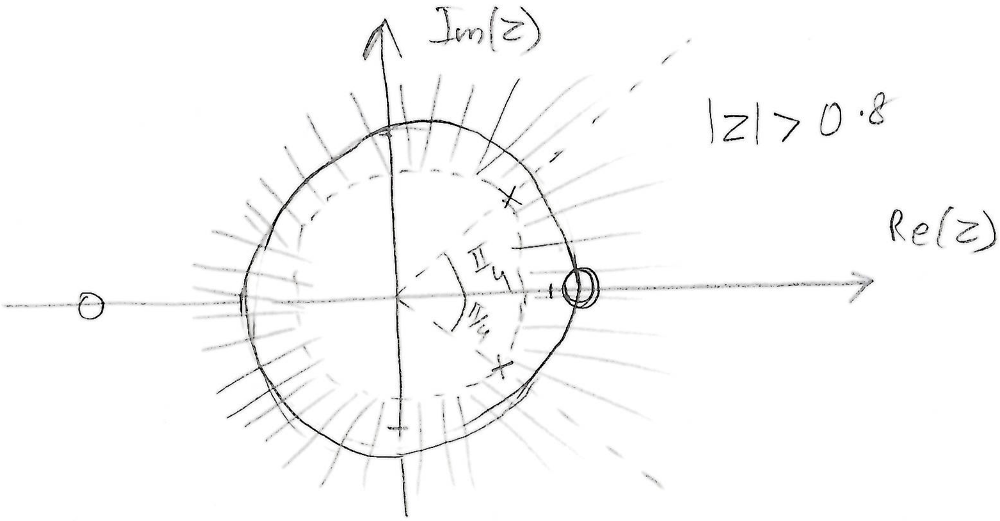
$grd(\omega) = grd_1(\omega) + grd_2(\omega) + grd_3(\omega) + grd_4(\omega)$

3c) **Group delay Plots:**



4) (a) $$H(z) = \frac{(1-z^{-1})(1+2z^{-1})}{(1-0.8e^{j\frac{\pi}{4}}z^{-1})(1-0.8e^{-j\frac{\pi}{4}}z^{-1})}$$

Since the system is stable, it must have $|z|=1$ included in the ROC.



(b) $$\left|H(e^{j\omega})\right|_{dB} = 20\log_{10}\left|1-e^{-j\omega}\right| + 20\log_{10}\left|1+2e^{-j\omega}\right|$$
$$-20\log_{10}\left|1-0.8e^{-j(\omega-\frac{\pi}{4})}\right|$$
$$-20\log_{10}\left|1-0.8e^{-j(\omega+\frac{\pi}{4})}\right|$$

① Zeros: $r=1,\ \theta=0:\ \omega=\theta = 20\log_{10}|1-r| = -\infty$

$\omega=\pm\pi = 20\log_{10}|1+r| = 20\log_{10}2$

$r=2,\ \theta=\pi:\ \omega=\pi:\ 20\log_{10}|1-r| = -\infty$

$\omega=0:\ 20\log_{10}|1+r| = 20\log_{10}2$

(2) Poles : $r = 0.8,\ \theta = \frac{\pi}{4}$: $w = \theta$: $-20\log_{10}|1-r| = -20\log_{10} 0.8$

: $w = \pi - \theta$: $-20\log_{10}|1+r| = -20\log_{10} 1.8$

$r = 0.8,\ \theta = -\frac{\pi}{4}$: $w = \theta$: $-20\log_{10}|1-r| = -20\log_{10} 0.8$

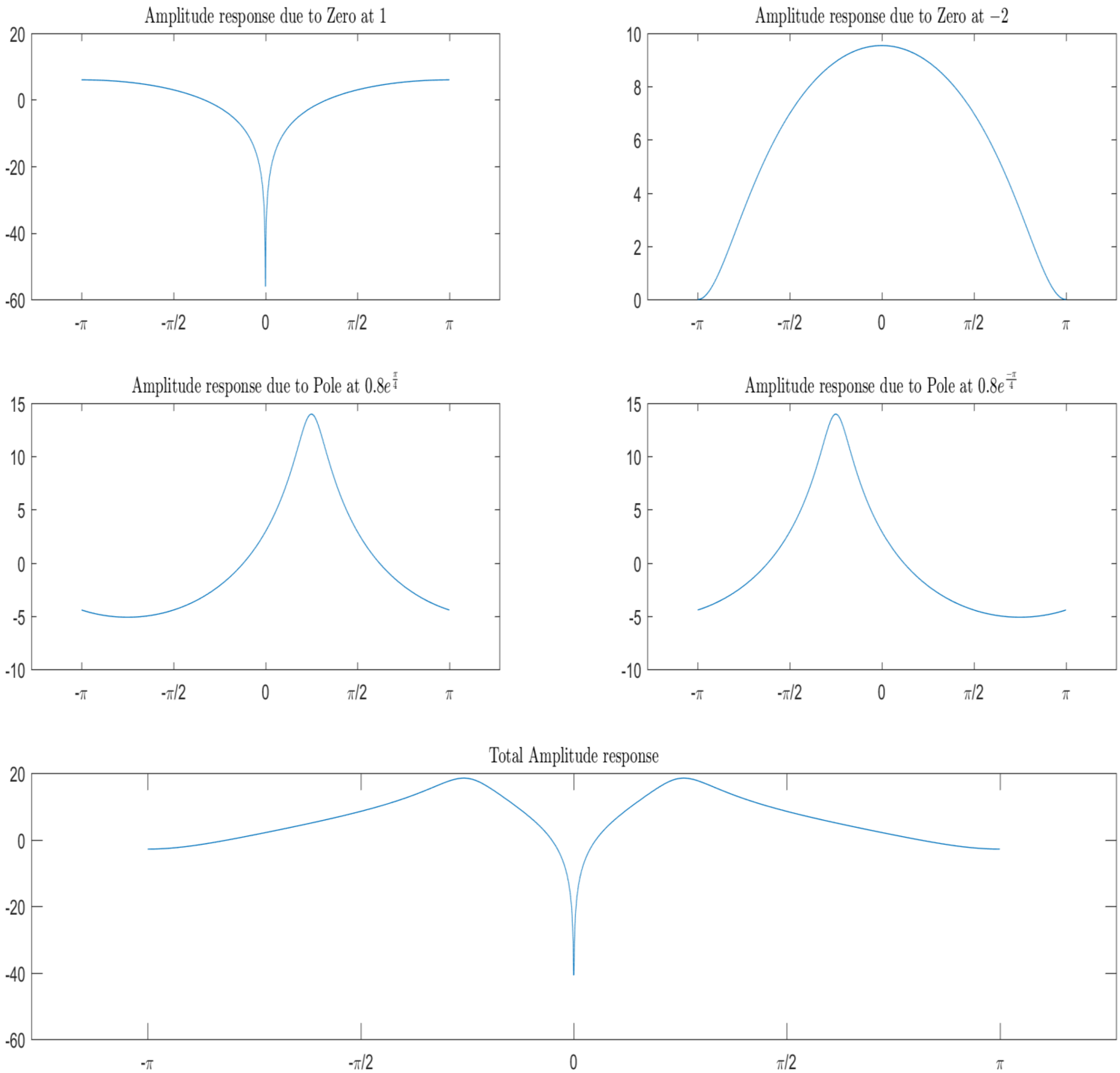
$w = \pi - \theta$: $-20\log_{10}|1+r| = -20\log_{10} 1.8$

(C) i) True. Because ROC outside all poles

ii) True

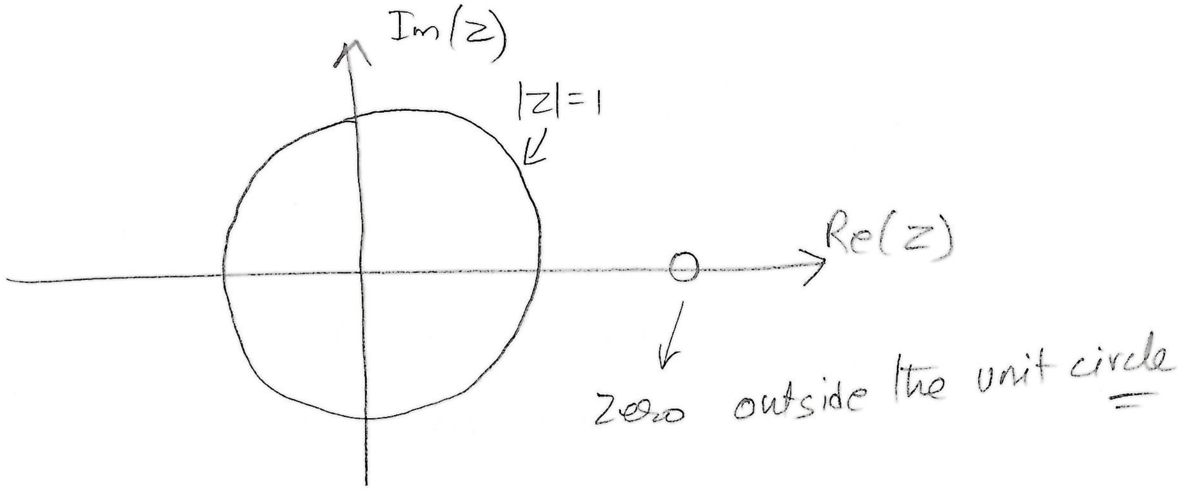
iii) False. Because the zeros of the original system are the poles of inverse system. It's greater than 1, the inverse system can't be stable and causal

4b) **Amplitude Response Plots:**



5(a) $H(z) = z - \frac{1}{a}$

The system has a zero at $z = \frac{1}{a}$ and pole at $z = \infty$



$H(e^{j\omega}) = e^{j\omega} - \frac{1}{a}$, Replacing $z = e^{j\omega}$

$\measuredangle H(e^{j\omega})$ needs to be calculated

$H(e^{j\omega}) = \cos\omega + j\sin\omega - \frac{1}{a} = \left(\cos\omega - \frac{1}{a}\right) + j\sin\omega$

$\measuredangle H(e^{j\omega}) = \tan^{-1}\left(\frac{\sin\omega}{\cos\omega - \frac{1}{a}}\right)$

(b) $G(z) = \frac{1}{1 - az^{-1}}$

$G(e^{j\omega}) = \frac{1}{1 - ae^{-j\omega}} = \frac{e^{j\omega}}{e^{j\omega} - a}$

(10)

$$\measuredangle G(e^{j\omega}) = \frac{1}{1-ae^{-j\omega}} \; ; \; \frac{1}{G(e^{j\omega})} = 1 - ae^{-j\omega}$$

$$= 1 - a\cos\omega + ja\sin\omega$$

$$\measuredangle \frac{1}{G(e^{j\omega})} = \tan^{-1}\left(\frac{a\sin\omega}{1-a\cos\omega}\right) = \tan^{-1}\left(\frac{\sin\omega}{\frac{1}{a} - \cos\omega}\right)$$

$$\measuredangle G(e^{j\omega}) = -\tan^{-1}\left(\frac{\sin\omega}{\frac{1}{a} - \cos\omega}\right) = \tan^{-1}\left(\frac{\sin\omega}{\cos\omega - \frac{1}{a}}\right)$$

$$= \measuredangle H(e^{j\omega})$$

```
%%To Plot amplitude response

clear all;
clc;
%Amplitude response plots
w= linspace(-pi,pi,2000);

%Zeros
r_z1 =2;
theta_z1 = 0;

r_z2 =0.5;
theta_z2 = pi;

r_z3 =0.9;
theta_z3 = pi;


%Poles
r_p1 =1;
theta_p1 = 0;

r_p2 =0.7;
theta_p2 = pi/2;

r_p3 =0.7;
theta_p3 = -pi/2;

%Amplitude response due to each zero
ampZero1 = 10*log10(1+r_z1^2-2*r_z1*cos(w-theta_z1));
ampZero2 = 10*log10(1+r_z2^2-2*r_z2*cos(w-theta_z2));
ampZero3 = 10*log10(1+r_z3^2-2*r_z3*cos(w-theta_z3));


%Amplitude response due to each pole
ampPole1 = -(10*log10(1+r_p1^2-2*r_p1*cos(w-theta_p1)));
ampPole2 = -(10*log10(1+r_p2^2-2*r_p2*cos(w-theta_p2)));
ampPole3 = -(10*log10(1+r_p3^2-2*r_p3*cos(w-theta_p3)));

ampTotal = ampZero1+ampZero2+ampZero3+ampPole1+ampPole2+ampPole3;

subplot(3,3,1);
plot(w,ampZero1);
set(gca,'XTick',-pi:pi/2:pi)
set(gca,'XTickLabel',{'-\pi','-\pi/2','0','\pi/2','\pi'})
title('Amplitude response due to Zero at $$2$$','interpreter','latex')

subplot(3,3,2);
plot(w,ampZero2);
set(gca,'XTick',-pi:pi/2:pi)
set(gca,'XTickLabel',{'-\pi','-\pi/2','0','\pi/2','\pi'})
```

```
title('Amplitude response due to Zero at $$-
0.5$$','interpreter','latex')

subplot(3,3,3);
plot(w,ampZero3);
set(gca,'XTick',-pi:pi/2:pi)
set(gca,'XTickLabel',{'-\pi','-\pi/2','0','\pi/2','\pi'})
title('Amplitude response due to Zero at $$-
0.9$$','interpreter','latex')

subplot(3,3,4);
plot(w,ampPole1);
set(gca,'XTick',-pi:pi/2:pi)
set(gca,'XTickLabel',{'-\pi','-\pi/2','0','\pi/2','\pi'})
title('Amplitude response due to Pole at $$1$$','interpreter','latex')

subplot(3,3,5);
plot(w,ampPole2);
set(gca,'XTick',-pi:pi/2:pi)
set(gca,'XTickLabel',{'-\pi','-\pi/2','0','\pi/2','\pi'})
title('Amplitude response due to Pole at
$$0.7e^\frac{\pi}{2}$$','interpreter','latex')

subplot(3,3,6);
plot(w,ampPole3);
set(gca,'XTick',-pi:pi/2:pi)
set(gca,'XTickLabel',{'-\pi','-\pi/2','0','\pi/2','\pi'})
title('Amplitude response due to Pole at $$0.7e^\frac{-
\pi}{2}$$','interpreter','latex')

subplot(3,3,[7 8 9]);
plot(w,ampTotal);
set(gca,'XTick',-pi:pi/2:pi)
set(gca,'XTickLabel',{'-\pi','-\pi/2','0','\pi/2','\pi'})
title('Total Amplitude response','interpreter','latex')
```

```
%%To Plot group delay
clear all;
clc;
%Group delay plots
w= linspace(-pi,pi,2000);

%Zeros
r_z1 =2;
theta_z1 = 0;

r_z2 =0.5;
theta_z2 = pi;

r_z3 =0.9;
theta_z3 = pi;


%Poles
r_p1 =1;
theta_p1 = 0;

r_p2 =0.7;
theta_p2 = pi/2;

r_p3 =0.7;
theta_p3 = -pi/2;

%Group delay due to each zero
grdZero1 = ((r_z1^2-r_z1*cos(w-theta_z1))./(1+r_z1^2-2*r_z1*cos(w-
theta_z1)));
grdZero2 = ((r_z2^2-r_z2*cos(w-theta_z2))./(1+r_z2^2-2*r_z2*cos(w-
theta_z2)));
grdZero3 = ((r_z3^2-r_z3*cos(w-theta_z3))./(1+r_z3^2-2*r_z3*cos(w-
theta_z3)));


%Group delay due to each pole
grdPole1 = -((r_p1^2-r_p1*cos(w-theta_p1))./(1+r_p1^2-2*r_p1*cos(w-
theta_p1)));
grdPole2 = -((r_p2^2-r_p2*cos(w-theta_p2))./(1+r_p2^2-2*r_p2*cos(w-
theta_p2)));
grdPole3 = -((r_p3^2-r_p3*cos(w-theta_p3))./(1+r_p3^2-2*r_p3*cos(w-
theta_p3)));

grdTotal = grdZero1+grdZero2+grdZero3+grdPole1+grdPole2+grdPole3;

subplot(3,3,1);
plot(w,grdZero1);
set(gca,'XTick',-pi:pi/2:pi)
set(gca,'XTickLabel',{'-\pi','-\pi/2','0','\pi/2','\pi'})
title('Group Delay due to Zero at $$2$$','interpreter','latex')
```

```
subplot(3,3,2);
plot(w,grdZero2);
set(gca,'XTick',-pi:pi/2:pi)
set(gca,'XTickLabel',{'-\pi','-\pi/2','0','\pi/2','\pi'})
title('Group Delay due to Zero at $$-0.5$$','interpreter','latex')

subplot(3,3,3);
plot(w,grdZero3);
set(gca,'XTick',-pi:pi/2:pi)
set(gca,'XTickLabel',{'-\pi','-\pi/2','0','\pi/2','\pi'})
title('Group Delay due to Zero at $$-0.9$$','interpreter','latex')

subplot(3,3,4);
plot(w,grdPole1);
set(gca,'XTick',-pi:pi/2:pi)
set(gca,'XTickLabel',{'-\pi','-\pi/2','0','\pi/2','\pi'})
title('Group Delay due to Pole at $$1$$','interpreter','latex')

subplot(3,3,5);
plot(w,grdPole2);
set(gca,'XTick',-pi:pi/2:pi)
set(gca,'XTickLabel',{'-\pi','-\pi/2','0','\pi/2','\pi'})
title('Group Delay due to Pole at
$$0.7e^\frac{\pi}{2}$$','interpreter','latex')

subplot(3,3,6);
plot(w,grdPole3);
set(gca,'XTick',-pi:pi/2:pi)
set(gca,'XTickLabel',{'-\pi','-\pi/2','0','\pi/2','\pi'})
title('Group Delay due to Pole at $$0.7e^\frac{-
\pi}{2}$$','interpreter','latex')

subplot(3,3,[7 8 9]);
plot(w,grdTotal);
set(gca,'XTick',-pi:pi/2:pi)
set(gca,'XTickLabel',{'-\pi','-\pi/2','0','\pi/2','\pi'})
title('Total Group Delay','interpreter','latex')
```