ECE 464/564: Digital Signal Processing - Winter 2018
Homework 3
Due: Feb 6, 2018 (Tuesday)

1. A causal LTI system has the system function:

$$H(z) = \frac{(1 - 1.5z^{-1} - z^{-2})(1 + 0.9z^{-1})}{(1 - z^{-1})(1 + 0.7jz^{-1})(1 - 0.7jz^{-1})}$$

   a) Write the difference equation that is satisfied by the input and the output of the system.
   b) Plot the pole-zero diagram and indicate the ROC for the system function.
   c) Sketch $|H(e^{jw})|$
   d) Calculate and sketch $grd\{H(e^{jw})\}$

2. **Fig. 1.** shows the pole-zero plots for four different LTI systems. Based on these plots, state whether each system is an all-pass system.

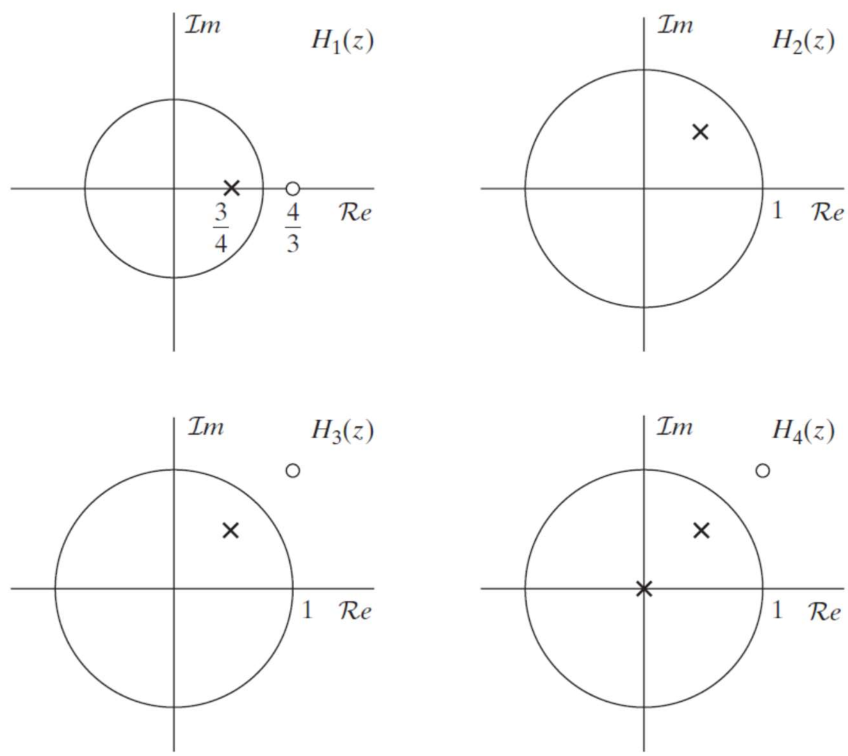

Fig. 1. Pole-Zero plots of 4 different LTI systems (for Prob. 2)

3. Consider the all-pass system described by the following z-domain transfer function:

$$H(z)=\left(\frac{(0.5-0.5j)-z^{-1}}{1-(0.5+0.5j)z^{-1}}\right)\left(\frac{(0.5+0.5j)-z^{-1}}{1-(0.5-0.5j)z^{-1}}\right)$$

   a. Sketch the zeros and poles of this system.
   b. Write and sketch the amplitude of the transfer function in the frequency domain.
   c. Write and sketch the group delay of the transfer function in the frequency domain.

4. Consider a stable LTI system whose transfer function is:

$$H(z)=\frac{(1-z^{-1})(1+2z^{-1})}{(1-0.8e^{j\pi/4}z^{-1})(1-0.8e^{-j\pi/4}z^{-1})}$$

   a. Plot the pole-zero diagram and indicate the ROC for the system function.
   b. Sketch $|H(e^{jw})|$
   c. State whether the following are true or false about the system:

      I. The system is causal.
      II. The magnitude of the frequency response has a peak at approximately $w = \pm\frac{\pi}{4}$
      III. The inverse system can be stable and causal.

5. In this problem, we demonstrate that, for a rational z-transform, a factor of the form $z-z_o$ and a factor of the form $\frac{z}{z-\frac{1}{zo^*}}$ contribute the same phase.

   (a) Let $H(z) = z - \frac{1}{a}$, where a is real and $0 < a < 1$. Sketch the poles and zeros of the system, including an indication of those at $z = \infty$. Determine $\measuredangle H(e^{jw})$, the phase of the system.

   (b) Let $G(z) = \frac{1}{1-az^{-1}}$. Sketch the pole-zero diagram of G(z). Determine $G(e^{jw})$, the phase of the system, and show that it is identical to $\measuredangle H(e^{jw})$.