ECE 464/564: Digital Signal Processing - Winter 2018
Homework 8
Due: Mar 15, 2018 (Thursday)

1. Suppose that we wish to design an FIR lowpass filter with the following specifications:

$$0.92 < H(e^{j\omega}) < 1.02 \qquad 0 \le |\omega| \le 0.63\pi,$$
$$|H(e^{j\omega})| < 0.1 \qquad 0.65\pi \le |\omega| \le \pi$$

by applying a window to the impulse response $h_d[n]$ for the ideal discrete-time lowpass filter with cutoff $\omega_c = 0.64\pi$.

   a) For each the following windows: Hamming, Hanning, and Bartlett modify the MATLAB code 'HW8-prob1_code.m' to determine the minimum value of M that satisfies the specification.
   b) To support your answer, for each window plot the frequency response of the filter you generated in part (a). Show that with M-1 the constraints are not satisfied.

2. Impulse invariance and the bilinear transformation are two methods for designing discrete time filters. Both methods transform a continuous-time system function $H_c(s)$ into a discrete time system function $H(z)$. Answer the following questions by indicating which method(s) will yield the desired result:
   a) A minimum-phase continuous-time system has all its poles and zeros in the left-half s-plane. If a minimum-phase continuous-time system is transformed into a discrete-time system, which method(s) will result in a minimum-phase discrete-time system?
   b) If the continuous-time system is an all-pass system, its poles will be at locations s_k in the left-half s-plane, and its zeros will be at corresponding locations $-s_k$ in the right-half s-plane. Which design method(s) will result in an all-pass discrete-time system?
   c) Which design method(s) will guarantee that:

$$H(e^{j\omega})\big|_{\omega=0} = H_c(j\Omega)\big|_{\Omega=0}?$$

   d) If the continuous-time system is a band stop filter, which method(s) will result in a discrete-time band stop filter?
   e) Suppose that $H_1(z)$, $H_2(z)$ and $H(z)$ are transformed versions of $H_{c1}(s)$, $H_{c2}(s)$ and $H_c(s)$ respectively. Which design method will guarantee that $H(z)= H_1(z)H_2(z)$ whenever $H_c(s)=H_{c1}(s)\, H_{c2}(s)$?
   f) Suppose that $H_1(z)$, $H_2(z)$ and $H(z)$ are transformed versions of $H_{c1}(s)$, $H_{c2}(s)$ and $H_c(s)$ respectively. Which design method will guarantee that $H(z)= H_1(z)+H_2(z)$ whenever $H_c(s)=H_{c1}(s) +H_{c2}(s)$?

3. Consider designing a discrete-time filter with system function $H(z)$ from a continuous-time filter with rational system function $H_c(s)$ by the transformation.

$$H(z) = H_c(s)\Big|_{s=\beta\left[\left(1-z^{-\alpha}\right)/\left(1+z^{-\alpha}\right)\right]}$$

Where α is a nonzero integer and β is real.

a) If $\alpha>0$, for what values of β does a stable, causal continuous-time filter with rational $H_c(s)$ always lead to a stable, causal discrete-time filter with rational $H(z)$?
b) If $\alpha< 0$, for what values of β does a stable, causal continuous-time filter with rational $H_c(s)$ always lead to a stable, causal discrete-time filter with rational $H(z)$?

4. An ideal discrete-time Hilbert transformer is a system that introduces -90º ($-\pi/2$ radians) of phase shift for 0 and +90º ($\pi/2$ radians) of phase shift for $-\pi<\omega<0$. The magnitude of the frequency response is constant (unity) for $-\pi<\omega<0$ and for $0<\omega<\pi$. Such systems are also called ideal 90º phase shifters.

$$H(e^{j\omega}) = \begin{cases} -j, & 0 < \omega < \pi, \\ j, & -\pi < \omega < 0. \end{cases}$$

a) Plot the phase response of this system for $-\pi<\omega<\pi$
b) Suppose that we wish to use the window method to design a linear-phase approximation to the ideal Hilbert transformer. Use $H(e^{j\omega})$ given above, to determine the ideal impulse response $h[n]$ if the FIR system is to be such that $h[n]=0$ for $n < 0$ and $n > M$.
c) What type(s) of FIR linear-phase systems (I, II, III, or IV) can be used to approximate the ideal Hilbert transformer in part (a)?

5. (**This is a bonus problem**)
Download the attached file and load it into your MATLAB by using 'HW8_Bonus.mat'. A piece of music is added with a high-pass noise. Please design a low-pass filter to eliminate this noise. The specification of that high-pass noise is:

f_{stop} = 10 kHz
f_{pass} = 12 kHz

The original music has a sample rate equals to f_{sample} = 44.1 kHz.

You can use command 'sound' in MATLAB to play the music. [sound (y, f_s) sends audio signal y to the speaker at sample rate f_s].

For the filtering, please use command y = filter(b,a,x). [y = filter(b,a,x) filters the input data x,

using a rational transfer function defined by the numerator and denominator coefficients b and a, respectively.] Hint: your low pass filter only need to cover the general frequency range of the sound. Choose any filter of your choice to filter the signal.

a) Please attach the MATLAB code for the problem with the information about the choice of filter, cutoff frequency and the order of the pole.
b) Plot the magnitude and frequency response for the filter of your choice in MATLAB.