ECE 464/564: Digital Signal Processing - Winter 2018
Homework 9

1. Suppose $x_c(t)$ is a periodic continuous-time signal with period 1 ms and for which the Fourier series is:

$$x_c(t) = \sum_{k=-9}^{9} a_k e^{j(2000\pi kt)}$$

The Fourier series coefficients a_k are zero for $|k| > 9$. $x_c(t)$ is sampled with a sample spacing $T = \frac{1}{6} \times 10^{-3}$ s to form *x[n]*. That is,

$$x[n] = x_c\left(\frac{n}{6000}\right)$$

   a) Is x[n] periodic and, if so, with what period?
   b) Is the sampling rate above the Nyquist rate? That is, is T sufficiently small to avoid aliasing?
   c) Find the DFS coefficients of x[n] in terms of a_k.

2. Compute the DFT of each of the following finite-length sequences considered to be of length N (where N is even):

   (a) $x[n] = \delta[n]$

   (b) $x[n] = \delta[n-n_0],\ 0 \le n_0 \le N-1$

   (c) $x[n] = \begin{cases} 1, & n\ even, \quad 0 \le n \le N-1 \\ 0, & n\ odd, \quad 0 \le n \le N-1 \end{cases}$

   (d) $x[n] = \begin{cases} 1, & 0 \le n \le N/2-1 \\ 0, & N/2 \le n \le N-1 \end{cases}$

   (e) $x[n] = \begin{cases} a^n, & 0 \le n \le N-1 \\ 0, & otherwise \end{cases}$

3. Consider the finite-length sequence *x[n]* in Fig 1. below. The five-point DFT of *x[n]* is denoted by *X [k]*. Plot the sequence *y[n]* whose DFT is

$$Y[k] = W_5^{-2k} X[k].$$

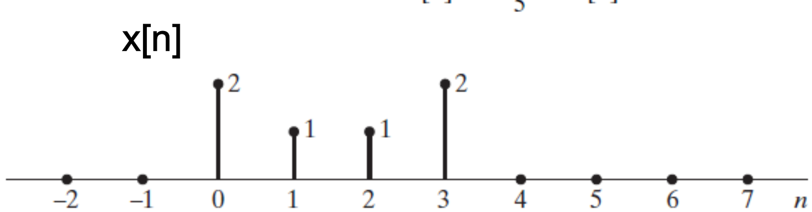

Fig. 1. Sequence x[n] for prob. 3

4. Consider the six-point sequence:

$$x[n] = 6\delta[n] + 5\delta[n-1] + 4\delta[n-2] + 3\delta[n-3] + 2\delta[n-4] + \delta[n-5]$$

shown in Figure 2.

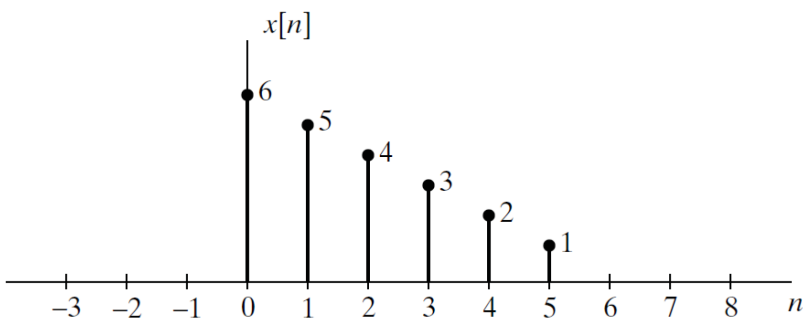

Fig. 2. Sequence x[n] for prob. 4

a) Determine X[k], the six-point DFT of x[n]. Express your answer in terms of $W_6 = e^{-j2\pi/6}$
b) Compute the DTFT of x[n].

5. The two eight-point sequences $x_1[n]$ and $x_2[n]$ are shown in Figure 3 have DFTs $X_1[k]$ *and* $X_2[k]$, respectively. Determine the relationship between $X_1[k]$ *and* $X_2[k]$.

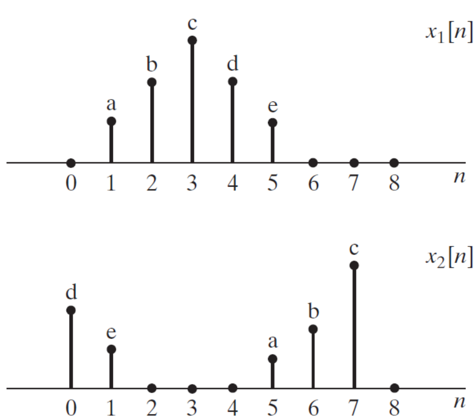

Fig. 3. Sequences $x_1[n]$ and $x_2[n]$ for prob. 5