ECE 464/564 Hw-9 Solutions

(1) (a) $T = \frac{1}{6} \times 10^{3}$ or $f_s = \frac{1}{T} = 6 \times 10^3$ Hz

$x[n] = x_c(nT)$

Here we have $x[n] = x_c\left(\frac{n}{6000}\right)$

$$\Rightarrow x[n] = x_c\left(\frac{n}{6000}\right) = \sum_{K=-9}^{9} a_K e^{j\left(\frac{2000\pi k n}{6000}\right)}$$

$$\Rightarrow x[n] = \sum_{K=-9}^{9} a_K e^{j\left(\frac{2\pi k n}{6}\right)}$$

Here $\frac{2\pi}{6} \equiv \frac{2\pi}{N}$

$N = 6$ (Fundamental Frequency) ; Periodic.

(b) $f_{Nyquist} = 2 f_{max}$

f_{max}: the maximum frequency in the signal

Here, f_{max} is for $K = 9$, which is:

$f_{max} = 9 \times \frac{1}{10^{-3}}$ Hz $= 9$ KHz

But $f_{sampling} = 6$ KHz

We know $f_{sampling} > 2 f_{max}$ for no-aliasing

which is not the case here.

So, there will be aliasing and Nyquist is violated.

1c) $$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}$$

From Part (a), $x[n]$ is periodic with $N=6$.

Substitution yields:

$$X[k] = \sum_{n=0}^{5} \left(\sum_{m=-9}^{9} a_m e^{j\frac{2\pi}{6}mn} \right) e^{-j\frac{2\pi}{6}kn}$$

$$= \sum_{n=0}^{5} \sum_{m=-9}^{9} a_m e^{j\left(\frac{2\pi}{6}\right)(m-k)n}.$$

We reverse the order of summation, and use the orthogonality relationship,

$$X[k] = 6 \sum_{m=-9}^{9} a_m \sum_{r=-\infty}^{\infty} \delta[m-k+rN]$$

Taking the infinite summation to the outside, we recognize the convolution between a_m and shifted impulses. Thus,

$$X[k] = 6 \sum_{r=-\infty}^{\infty} a_{k-6r}$$

Note that from $X[k]$, the aliasing is apparent.

(2) (a) $$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn}, \quad 0 \le k \le N-1$$

$$= \sum_{n=0}^{N-1} \delta[n] W_N^{kn}$$

$$= 1$$

(b) $$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn}, \quad 0 \le k \le N-1$$

$$= \sum_{n=0}^{N-1} \delta[n-n_0] W_N^{kn}$$

$$= W_N^{kn_0}$$

(c) $$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn}, \quad 0 \le k \le 1$$

$N = 2M$ ($\because$ N is even)

$$\therefore X[k] = \sum_{n=0}^{M} W_N^{2kn} = \frac{1 - e^{j 2\pi k}}{1 - e^{-j4\left(\frac{\pi k}{N}\right)}}$$

$$X[k] = \begin{cases} N/2, & k = 0 \\ N/2, & k = N/2 \\ 0, & \text{otherwise} \end{cases}$$

$$\boxed{X[k] = \frac{N}{2} \delta[k] + \frac{N}{2} \delta\left[k - \frac{N}{2}\right]}$$

(d) $$X[K] = \sum_{n=0}^{N-1} x[n]\, W_N^{Kn} \quad ; \; 0 \le K \le N-1$$

$$= \sum_{n=0}^{N/2-1} W_N^{Kn}$$

$$= \frac{1-e^{-j\pi K}}{1-e^{-j\frac{2\pi K}{N}}}$$

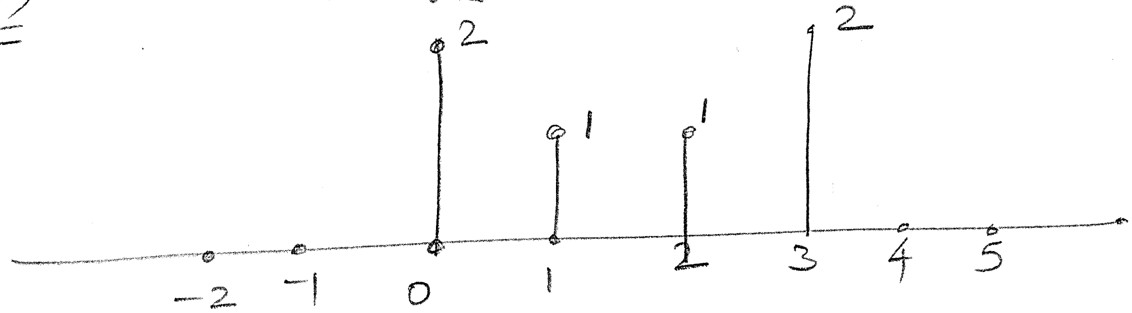
$$X[k] = \begin{cases} N/2, & k=0 \\ \dfrac{2}{1-e^{-j\frac{2\pi K}{N}}}, & K \text{ odd} \\ 0, & k \text{ even} \end{cases} \qquad 0 \le K \le N-1$$

(e) $$X[K] = \sum_{n=0}^{N-1} a^n W_N^{kn} \qquad 0 \le k \le N-1$$

$$= \frac{1-a^N e^{-j2\pi K}}{1-ae^{-j\frac{2\pi k}{N}}}$$

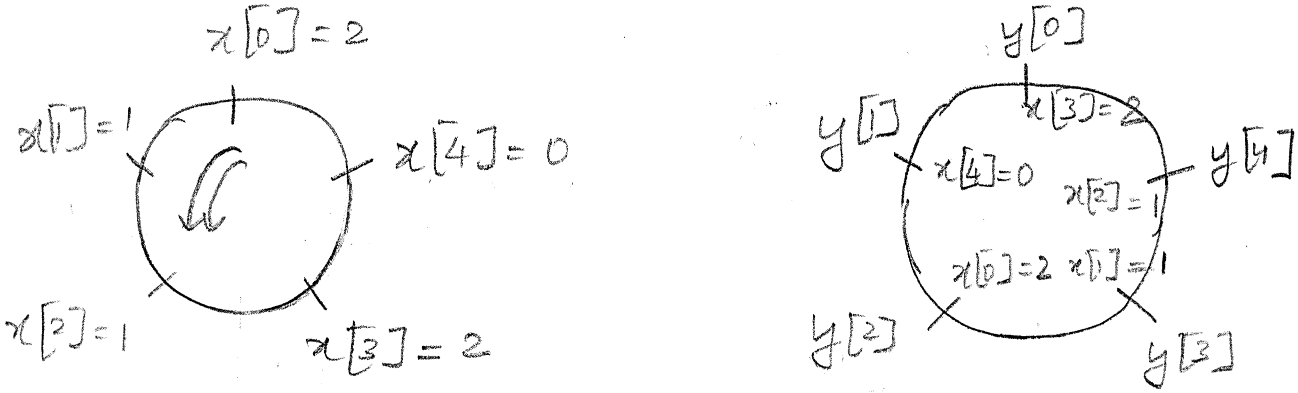
$$= \frac{1-a^N}{1-ae^{-j\frac{2\pi k}{N}}}$$

(3) Given $x[n] =$



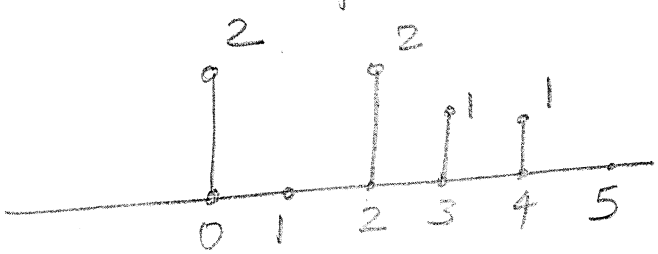
& $Y[K] = W_5^{-2k} X[K]$

From the properties of DFT, we have



$y[n] = x[((n-2))_5]$, that $y[n]$ is equal to $x[n]$ circularly shifted by 2.

$\therefore y[n] = \{ \underset{\uparrow}{2}, 0, 2, 1, 1 \}$



(4)

(a) $X[k] = \sum_{n=0}^{5} x[n] W_6^{kn} \quad ; 0 \le k \le 5$

$W_6 = e^{-j\frac{2\pi}{6}}$

$\Rightarrow X[k] = 6W_6^0 + 5W_6^{k} + 4W_6^{2k} + 3W_6^{3k} + 2W_6^{4k} + W_6^{5k}$

$X[0] = 6+5+4+3+2+1 = 21$

$X[1] = 6 + 5e^{-j\frac{\pi}{3}} + 4e^{-j\frac{2\pi}{3}} + 3e^{-j\pi} + 2e^{-j\frac{4\pi}{3}} + e^{-j\frac{5\pi}{3}}$

$X[2] = 6 + 5e^{-j\frac{2\pi}{3}} + 4e^{-j\frac{4\pi}{3}} + 3e^{-j2\pi} + 2e^{-j\frac{8\pi}{3}} + e^{-j\frac{10\pi}{3}}$

$X[3] = 6 + 5e^{-j\pi} + 4e^{-j2\pi} + 3e^{-j3\pi} + 2e^{-j4\pi} + e^{-j5\pi}$

$= 6-5+4-3+2-1 = 3$

$X[4] = 6 + 5e^{-j\frac{4\pi}{3}} + 4e^{-j\frac{8\pi}{3}} + 3e^{-j\frac{12\pi}{3}} + 2e^{-j\frac{16\pi}{3}} + e^{-j\frac{20\pi}{3}}$

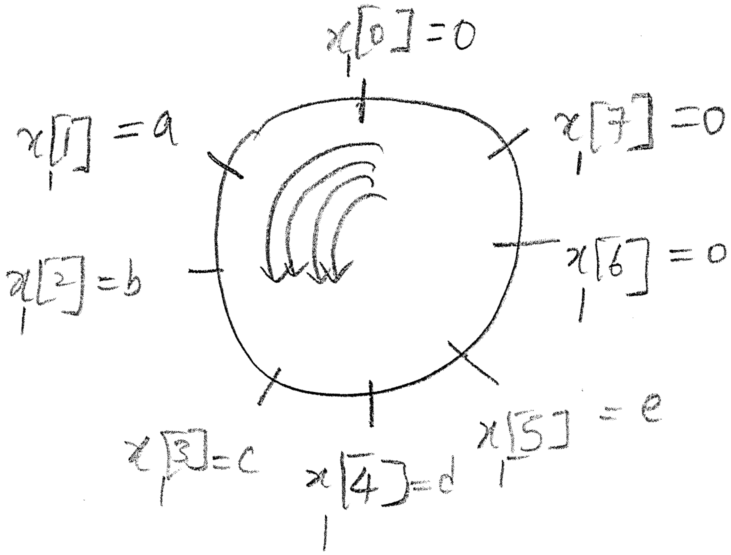
$X[5] = 6 + 5e^{-j\frac{5\pi}{3}} + 4e^{-j\frac{10\pi}{3}} + 3e^{-j\frac{15\pi}{3}} + 2e^{-j\frac{20\pi}{3}} + e^{-j\frac{25\pi}{3}}$

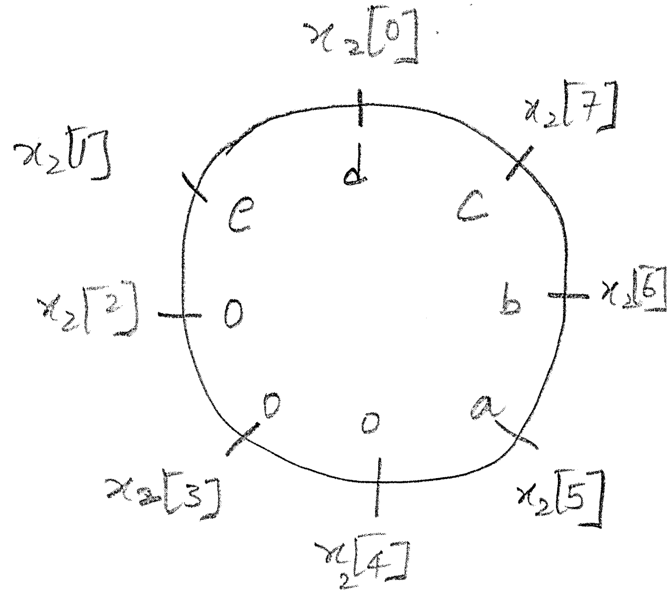
(b) $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} = \sum_{n=0}^{5} x[n] e^{-j\omega n}$

$= 6 + 5e^{-j\omega} + 4e^{-2j\omega} + 3e^{-3j\omega} + 2e^{-4j\omega} + e^{-5j\omega}$

5) From the figure, we can see the two sequences are related through a circle shift:

$$x_2[n] = x_1[((n-4))_8]$$





The DFT of $x_2[n]$ is:

$$DFT\{x_2[n]\} = W_8^{4K} X_1[K]$$

$$= e^{-j\pi K} X_1[K]$$

$$= (-1)^K X_1[K]$$