① ⓐ

$$H_{eff}(j\Omega) = \begin{cases} H(e^{j\Omega T}), & |\Omega| \le \frac{\pi}{T} \\ 0, & o.w \end{cases}$$

ⓑ

$$y(t) = x(t - 0.5T) \rightarrow Y(j\Omega) = X(j\Omega)e^{-j0.5T\Omega}$$

$$H(j\Omega) = \frac{Y(j\Omega)}{X(j\Omega)} = e^{-j0.5T\Omega}$$

Want to implmnt $\boxed{H_{eff}(j\Omega) = H(j\Omega) \text{ for } |\Omega| \le \frac{\pi}{T}}$

$\Rightarrow$ For $|\Omega| \le \frac{\pi}{T}$

$$H(e^{j\Omega T}) = H_{eff}(j\Omega) = H(j\Omega) = e^{-j0.5T\Omega}$$

$\Rightarrow$ $H(e^{j\Omega T}) = e^{-j0.5 \cdot T\Omega}$ let $\omega = \Omega T$

$\Rightarrow$ for $|\omega| \le \pi$

$$\boxed{H(e^{j\omega}) = e^{-j0.5\omega}}$$

1 Ⓒ $h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{jw})\, e^{jwn}\, dw$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-j0.5w}\, e^{jwn}\, dw$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{jw(n-0.5)}\, dw = \frac{1\, e^{jw(n-0.5)}}{2\pi \cdot j(n-0.5)}\Bigg|_{-\pi}^{\pi}$$

$$= \frac{1}{2\pi j}\,\frac{e^{j\pi(n-0.5)} - e^{-j\pi(n-0.5)}}{(n-0.5)} = \frac{\sin(\pi(n-0.5))}{\pi(n-0.5)} = \boxed{\mathrm{sinc}(n-0.5)}$$

②

$$H(z) = \frac{1}{(1-100z^{-2})(1-0.9z^{-1})}$$

$$= \frac{1}{(1-10z^{-1})(1+10z^{-1})(1-0.9z^{-1})}$$

Decompose:

$$H(z) = \underbrace{\frac{1}{(1-0.9z^{-1})(1-\frac{1}{10}z^{-1})(1+\frac{1}{10}z^{-1})}}_{\text{min.}} \cdot \underbrace{\frac{(1-\frac{1}{10}z^{-1})(1+\frac{1}{10}z^{-1})}{(1-10z^{-1})(1+10z^{-1})}}_{\text{AP}}$$

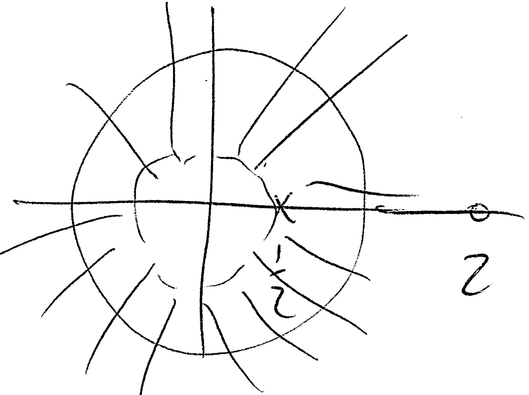
$$H_{min}(z) = \frac{1}{(1-0.9z^{-1})(1-0.1z^{-1})(1+0.z^{-1})} = \frac{1}{(1-0.9z^{-1})(1-0.01z^{-2})}$$

$$H_{AP}(z) = \frac{(1-0.1z^{-1})(1+0.1z^{-1})}{(1-10z^{-1})(1+10z^{-1})} = \frac{1-\frac{1}{100}z^{-2}}{(1+100z^{-2})}$$

$$H(z) = H_{min}(z)\, H_{AP}(z)$$

③ $H(z) = \dfrac{1-2z^{-1}}{1-\frac{1}{2}z^{-1}}$, causal

ROC $|z|>\frac{1}{2}$ ←



$H_{inv}(z) = \dfrac{1-\frac{1}{2}z^{-1}}{1-2z^{-1}}$

long div.:

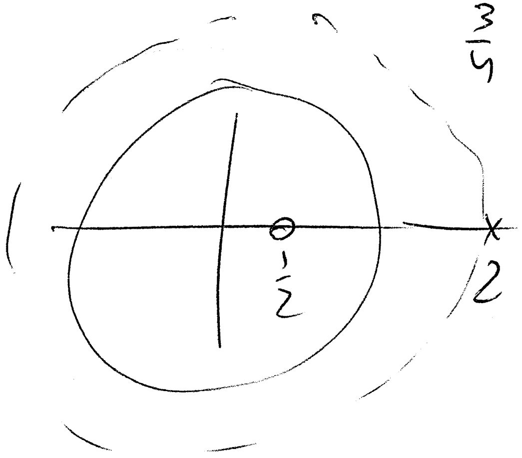
$$\begin{array}{r|l} & \frac{1}{4} \\ \hline 1-2z^{-1} & 1-\frac{1}{2}z^{-1} \\ & -\frac{1}{4}-\frac{1}{2}z^{-1} \\ \hline & \frac{3}{4} \end{array}$$

↓ (long div.)

$H_{inv}(z) = \frac{1}{4} + \dfrac{\frac{3}{4}}{1-2z^{-1}}$

2 ROC's : Ⓘ $|z| < 2$ (overlaps with $|z|>\frac{1}{2}$)

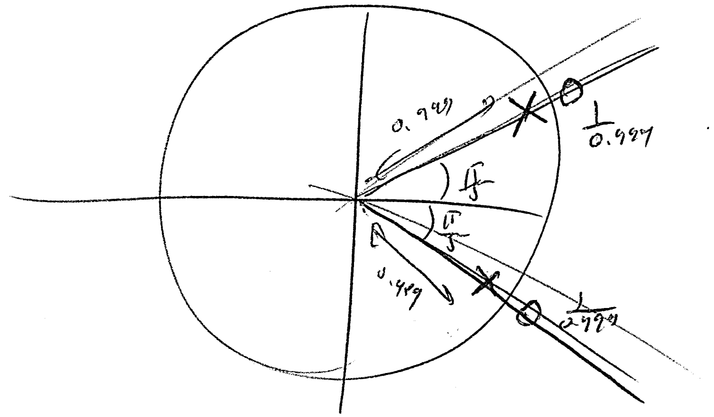
Ⓘⓘ $|z| > 2$ (overlaps with $|z|>\frac{1}{2}$)



Ⓘ $h_{inv}[n] = \frac{1}{4}\delta[n] - \frac{3}{4} 2^n u[-n-1]$ (stable, non-causal)

Ⓘⓘ $h_{inv}[n] = \frac{1}{4}\delta[n] + \frac{3}{4} 2^n u[n]$ (causal, non-stable)

④ ⓐ

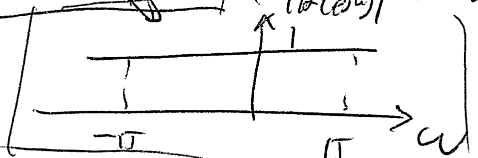


$$H = \frac{0.999e^{-j\frac{\pi}{5}} - z^{-1}}{1 - 0.999e^{j\frac{\pi}{5}}z^{-1}} \cdot \frac{0.999e^{+j\frac{\pi}{5}} - z^{-1}}{1 - 0.999e^{-j\frac{\pi}{5}}z^{-1}}$$

⑤

ⓑ All-pass (in a canonical form) $\Rightarrow \boxed{|H| = 1}$ $\left(|H(e^{j\omega})|_{dB} = 0 \right)$

$$H(z) = \frac{a^* - z^{-1}}{1 - az^{-1}} \rightarrow |H(e^{j\omega})| = 1$$



ⓒ

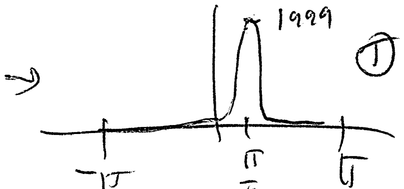
$$\boxed{gd(H(e^{j\omega})) = \underbrace{\frac{1 - 0.999^2}{1 + 0.999^2 - 2 \cdot 0.999\cos(\omega - \frac{\pi}{5})}}_{①} + \underbrace{\frac{1 - 0.999^2}{1 + 0.999^2 - 2 \cdot 0.999\cos(\omega + \frac{\pi}{5})}}_{②}}$$

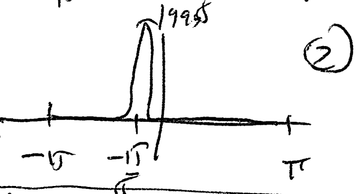
① $\omega = \frac{\pi}{5}$ $\quad \frac{1+r}{1-r} = \frac{1+0.999}{1-0.999} = 1999.$

$\omega = -\frac{4\pi}{5}$ $\quad \frac{1-r}{1+r} = \frac{1}{1999}$

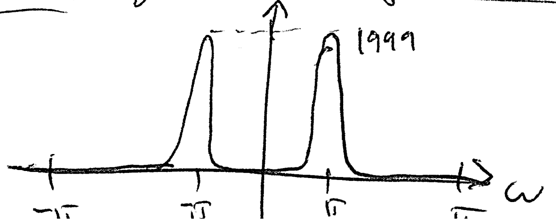
② $\omega = -\frac{\pi}{5} \rightarrow \frac{1+r}{1-r} = 1999$

$\omega = \frac{4\pi}{5} \rightarrow \frac{1-r}{1+r} = \frac{1}{1999}$





$gd(H(e^{j\omega}))$



⑤

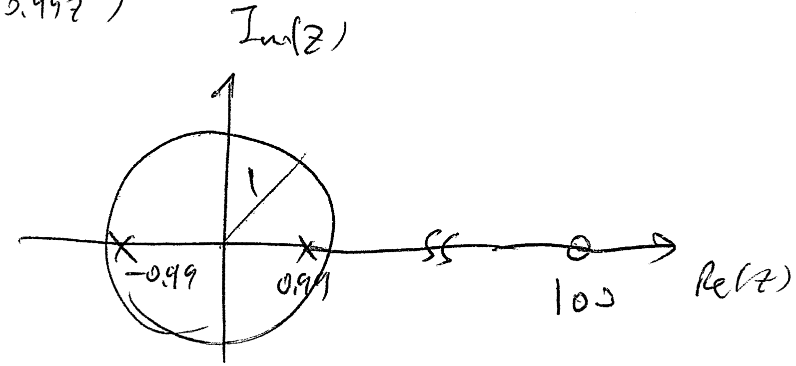
$$H(z) = \frac{1-100z^{-1}}{(1-0.99z^{-1})(1+0.99z^{-1})}$$

ⓐ

$r=0.99$, $\theta=0$ $\quad$ $r=0.99$, $\theta=\pi$

poles: $z = 0.99, -0.99$

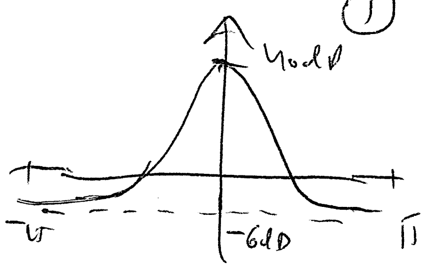
zero: $z = 100$ $\rightarrow$



ⓑ

$$\left|H(e^{j\omega})\right|_{dB} = -10\log_{10}\overset{\textcircled{1}}{(1+0.99^2-2\cdot 0.99\cos(\omega))} - 10\log_{10}\overset{\textcircled{2}}{(1+0.99^2-2\cdot 0.99\cos(\omega-\pi))}$$

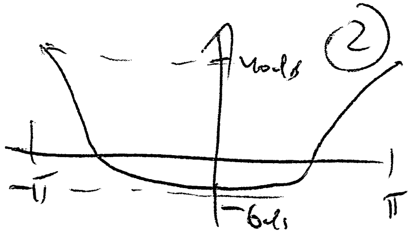
$$+ 10\log_{10}\overset{\textcircled{3}}{(1+100^2-2\cdot 100\cdot\cos(\omega))}$$



poles

$r=0.99$ $\quad$ $\omega=0$ $\quad$ $-20\log_{10}|1-r| = 40dB$

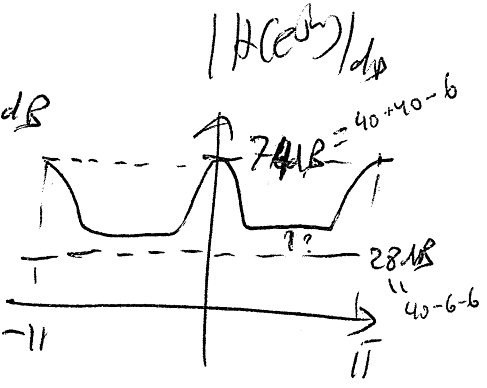
$\theta=0$ $\quad$ $\omega=\pi$ $\quad$ $-20\log_{10}|1+r| = -20\log 1.99 \approx -6dB$

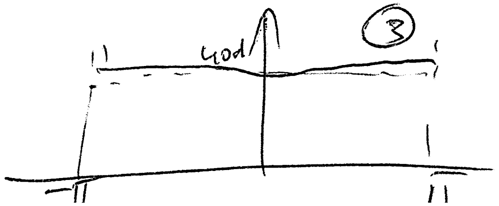


$r=0.99$ $\quad$ $\omega=\pi$ $\quad$ $-20\log|1-r| = 40dB$

$\theta=\pi$ $\quad$ $\omega=0$ $\quad$ $-20\log|1+r| = -6dB$

$\Rightarrow$





zero

$r=100$ $\quad$ $\omega=0$ $\quad$ $+20\log|1-r| = 20\log 99 \approx 20\log 100 = 40dB$

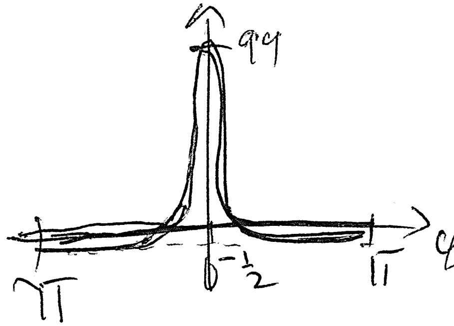
θ $\quad$ $\omega=\pi$ $\quad$ $+20\log|1+r| = 20\log 101 \approx 20\log 100 = 40dB$

⑤ (c)

$gd(H(e^{j\omega}))$

$$= -\frac{0.99^2-0.99\cos(\omega)}{1+0.99^2-2\cdot0.99\cos(\omega)} + \frac{0.99^2-0.99\cos(\omega-\pi)}{1+0.99^2-2\cdot0.99\cos(\omega-\pi)} + \frac{100^2-100\cos(\omega)}{1+100^2-2\cdot100\cos(\omega)}$$

① ② ③



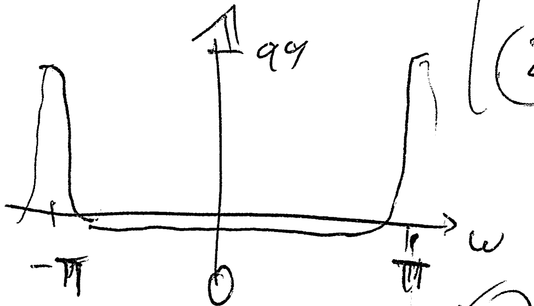
①

$r=0.99$ $\omega=0$ $-\frac{r^2-r}{(1-r)^2} = -\frac{(r-1)r}{(1-r)^2} = \frac{+r}{1-r} = \frac{0.99}{\underbrace{1-0.99}_{0.01}} = 99$

$\theta=0$

$\omega=\pi$ $-\frac{r}{r+1} = \frac{-0.99}{1.99} \simeq -\frac{1}{2}$

poles

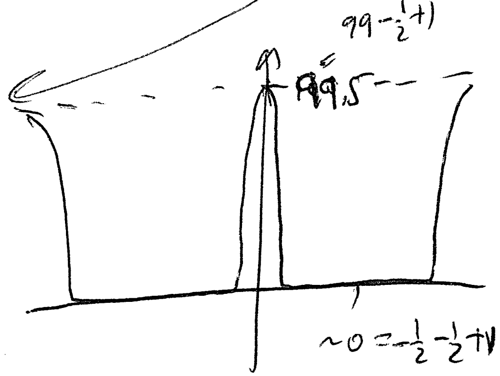


②

$r=0.99$ $\omega=\pi$ 99

$\theta=\pi$ $\omega=0$ $-\frac{1}{2}$

$\Rightarrow$



③

$r=100$ $\omega=0$ $-\frac{r}{1-r} = -\frac{100}{-99} = \frac{100}{99} \simeq 1$

$\theta=0$

$\omega=\pi$ $+\frac{r}{1+r} = \frac{100}{101} = \frac{100}{101} \simeq 1$

zero