ECE 464/564: Digital Signal Processing - Winter 2018
Homework 5
Due: Feb 20, 2018 (Tuesday)

1. Consider the class of discrete-time filters whose frequency response has the form:

$$H(e^{j\omega}) = |H(e^{j\omega})| \, e^{-j\omega\alpha}$$

where $|H(e^{j\omega})|$ is a real and nonnegative function of ω and α is a real constant. These class of filters are referred to as linear-phase filters.

Consider also the class of discrete-time filters whose frequency response has the form

$$H(e^{j\omega}) = A(e^{j\omega}) \, e^{-j\omega\alpha + j\beta}$$

where $A(e^{j\omega})$ is a real function of ω (can be both positive and negative), α and β are real constants. Filters in this class are referred to as generalized *linear-phase* filters.

For each of the filters in Figure 1, determine whether it is a generalized linear phase or linear phase filter. If it is generalized linear-phase, then find $A(e^{j\omega})$, α and β.

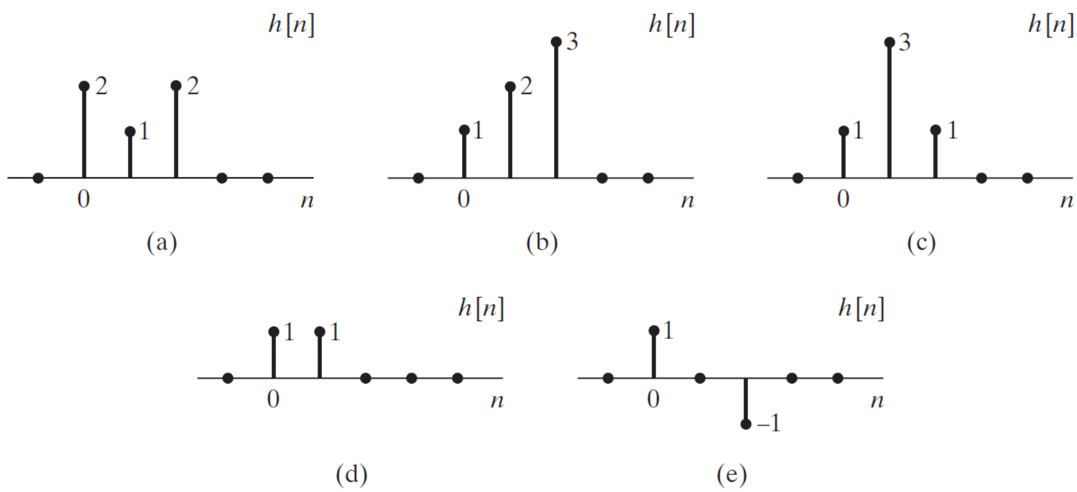

Fig. 1. Different filters (for Prob. 1)

2. The pole-zero plots in Figure 2 describe six different causal LTI systems.

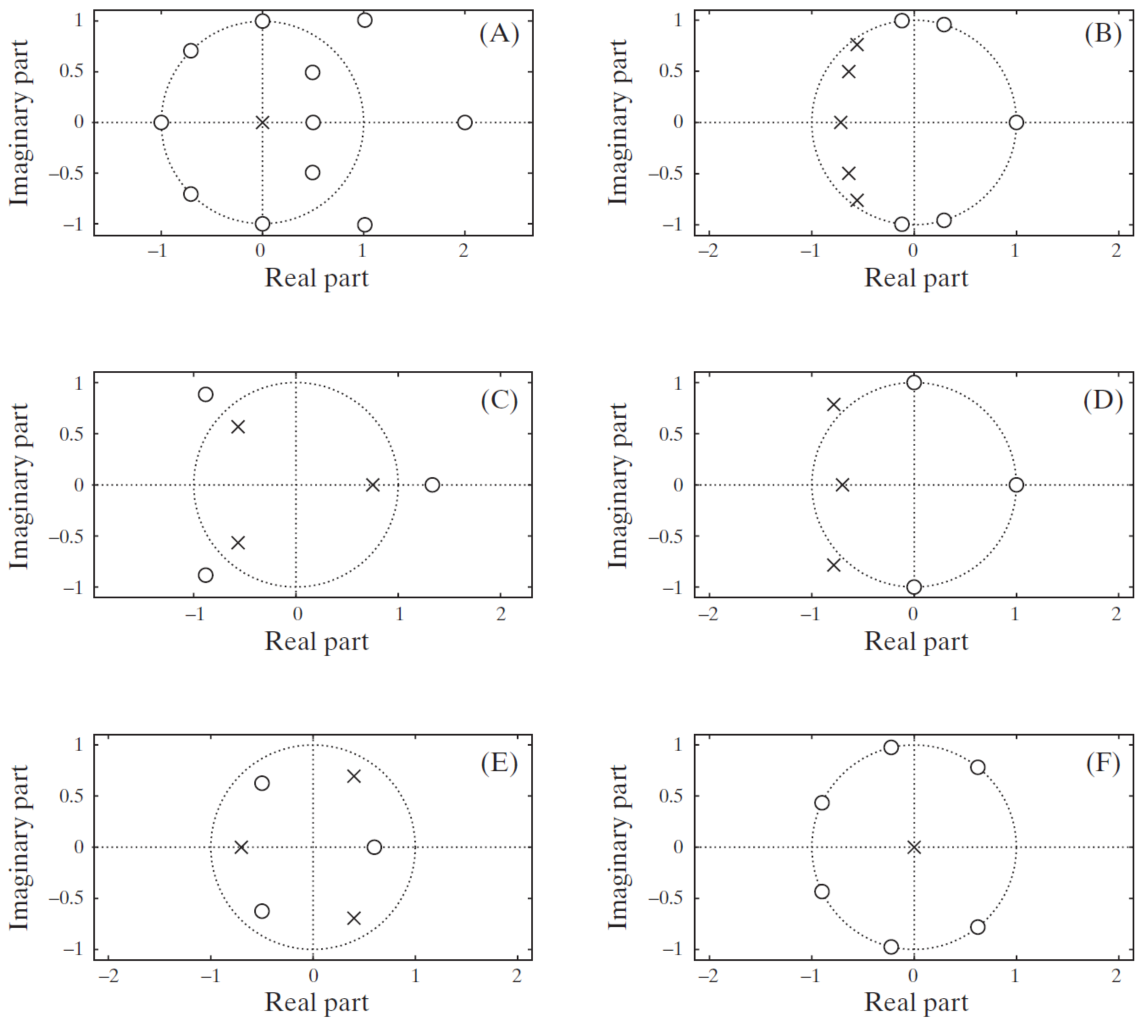

Fig. 2. Pole-zero plots for six different LTI systems (for Prob. 2)

Answer the following questions about the systems having the above pole-zero plots. In each case, an acceptable answer could be none or all.

a. Which systems are IIR systems?
b. Which systems are FIR systems?
c. Which systems are stable systems?
d. Which systems are minimum-phase systems?
e. Which systems are generalized linear-phase systems?
f. Which systems have $|H(e^{j\omega})|$ = constant for all ω?
g. Which systems have corresponding stable and causal inverse systems?
h. Which system has the shortest (least number of non-zero samples) impulse response?
i. Which systems have minimum group delay?

3. A linear phase FIR system has a real impulse response h[n] whose z-transform is known to have form:

$$H(z) = (1 - az^{-1})\left(1 - e^{j\Pi/2}z^{-1}\right)(1 - 0.5z^{-1})(1 - cz^{-1})(1 - bz^{-1})$$

where a, b and c are zeros of H(z) that you are to find. It is also known that $H(e^{j\omega}) = 0$ for $\omega = 0$. This information and knowledge of the properties of linear-phase systems are sufficient to completely determine the system function (and therefore the impulse response) and to answer the following questions:

   a. Determine the length of the impulse response (i.e. the number of nonzero samples).
   b. Determine the group delay of the system in samples.
   c. Determine the unknown zeros a, b and c ($a \neq 0$, $b \neq 0$, $c \neq 0$).
   d. Determine the values of the impulse response and sketch it as a stem plot.
   e. Is this a Type I, Type II, Type III or Type IV system?

4. A generalized linear-phase FIR system has an impulse response with real values and h[n]=0 for $n < 0$ and for $n \geq 8$, and h[n] = -h[7-n]. The system function of this system has a zero at $z = 0.8e^{j\Pi/4}$ and another zero at z = -2. What is H(z)?

5. H(z) is the transfer function of a stable LTI system and is given by:

$$H(z) = \frac{(1 - 2z^{-1})(1 - 0.75z^{-1})}{z^{-1}(1 - 0.5z^{-1})}$$

H(z) can also be expressed as $H_{min}(z)H_{lin}(z) = H(z)$ where $H_{min}(z)$ is a minimum-phase system and $H_{lin}(z)$ is a generalized linear-phase system. Determine a choice for $H_{min}(z)$ and $H_{lin}(z)$.