ECE 464/564: Digital Signal Processing - Winter 2018
Homework 7
Due: Mar 8, 2018 (Thursday)

1. The system function of a discrete-time system is:

$$H(z) = \frac{2}{1 - e^{-0.2}z^{-1}} - \frac{1}{1 - e^{-0.4}z^{-1}}$$

   a) Assume that this discrete-time filter was designed by the impulse invariance method with $T_d = 2$; i.e., $h[n]=2h_c(2n)$, where $h_c(t)$ is real. Find the system function $H_c(s)$ of a continuous-time filter that could have been the basis for the design

   b) Assume that $H(z)$ was obtained by the bilinear transform method with $T_d = 2$. Find the system function $H_c(s)$ that could have been the basis for the design

2. Consider a causal continuous-time system with impulse response $h_c(t)$ and system function:

$$H_c(s) = \frac{s + a}{(s + a)^2 + b^2}$$

   a) Use impulse invariance to determine $H_1(z)$ for a discrete-time system such that $h_1[n]=Th_c(nT)$

   b) Plot the magnitude and frequency response for both the continuous-time and discrete-time filters using MATLAB assuming a = 1, b = 1 and T=1. (Hint: Use MATLAB commands `freqs' and `freqz' to get the frequency responses).

3. A discrete-time low-pass filter is to be designed by applying the impulse invariance method to a continuous time Butterworth filter having magnitude-squared function:

$$|H_c(j\Omega)|^2 = \frac{1}{1 + (\Omega/\Omega_c)^{2N}}$$

   The specifications for the discrete-time system are:

$$0.9 \le \left|H\left(e^{jw}\right)\right| \le 1, \quad 0 \le |\omega| \le 0.2\pi$$

$$\left|H\left(e^{jw}\right)\right| \le 0.1, \quad 0.3\pi \le |\omega| \le \pi$$

   Assume that aliasing will not be a problem; i.e., design the continuous-time Butterworth filter to meet passband and stopband specifications as determined by the desired discrete-time filter.

   a) Sketch the tolerance bounds on the magnitude of the frequency response, $|H_c(j\Omega)|$ of the continuous-time Butterworth filter such that after application of the impulse invariance method (i.e., $h[n]=T_d h_c(nT_d)$), the resulting discrete-time filter will satisfy the given design specifications. Assume that $T_d = 1$.

b) Determine the integer order N and the quantity Ω_c such that the continuous-time Butterworth filter exactly meets the specifications determined in part (a) at the passband edge.
c) Determine system function *H(s)* and get *H(z)* by impulse invariance.
d) Use MATLAB to plot the magnitude and phase response of *H(z)*.

4. Determine the system function *H(z)* of the lowest-order Chebyshev Type I digital filter that meets the following specifications:
   a) 1-dB ripple in the passband $0 \leq |\omega| \leq 0.2\pi$.
   b) At least 15 dB attenuation in the stopband $0.3\pi \leq |\omega| \leq \pi$.
   c) Intuitively, what happens to N (order of the filter), if the minimum stop band attenuation required is 30 dB instead of 15 dB as in part (b)?