ECE 464/564 HW-4 Solutions

1a) TRUE

Let $H_1(z)$ and $H_2(z)$ be minimum phase systems

Then the poles & zeros of both $H_1(z)$ & $H_2(z)$ are inside the unit circle.

Series connection/Cascade results in $H_1(z)H_2(z)$, the poles and zeros of this product term will also lie inside the unit circle. Therefore, it will be a minimum-phase system.

b) FALSE

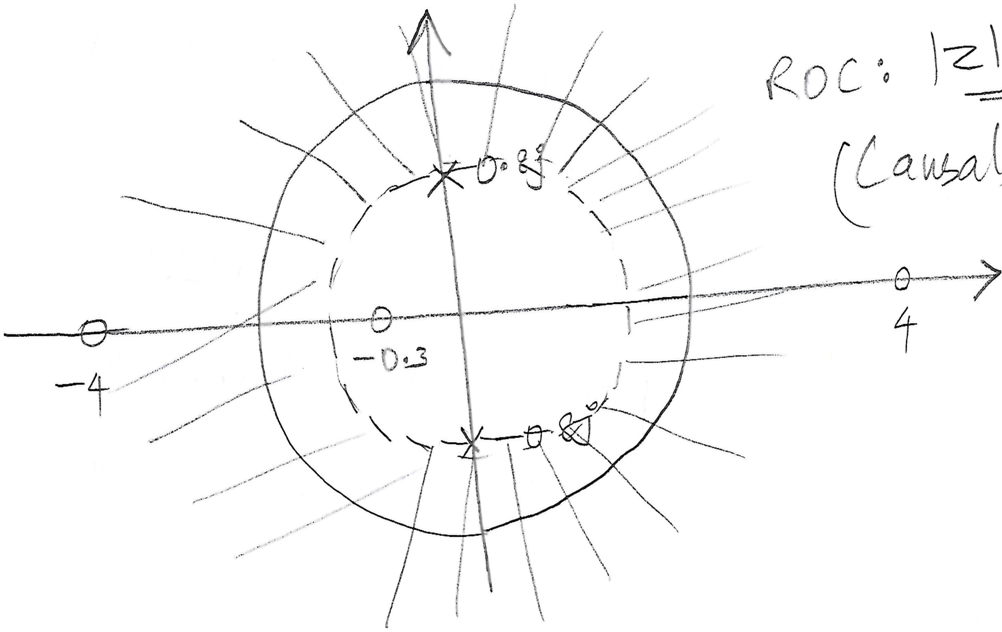
Parallel Cascade results in $H_1(z)+H_2(z)$.

The poles of this sum would lie inside the unit circle. However, we can't definitively say about the zeros.

Ex: $$\frac{1}{1-0.9z^{-1}} - 2 = -\left(\frac{1-1.8z^{-1}}{1-0.9z^{-1}}\right)$$

P2

2 (a) $$H(z) = \frac{(1+0.3z^{-1})(1-4z^{-1})(1+4z^{-1})}{(1+0.8j\,z^{-1})(1-0.8jz^{-1})}$$



ROC: $|z| > 0.8$

(Causal)

Since the ROC: $|z| > 0.8$ contains $|z| = 1$

It is a Stable system.

(b) $$H(z) = \underbrace{\frac{(1+4z^{-1})(1-4z^{-1})}{(1-\frac{1}{4}z^{-1})(1+\frac{1}{4}z^{-1})}}_{H_{ap}(z)} \cdot \underbrace{\frac{(1+0.3z^{-1})(1-\frac{1}{4}z^{-1})(1+\frac{1}{4}z^{-1})}{(1+0.8jz^{-1})(1-0.8jz^{-1})}}_{H_{min}(z)}$$

$$= \underbrace{\left(\frac{1-16z^{-2}}{1-\frac{1}{16}z^{-2}}\right)}_{H_{ap}(z)} \cdot \underbrace{\left(\frac{(1+0.3z^{-1})(1-\frac{1}{16}z^{-2})}{(1+0.64z^{-2})}\right)}_{H_{min}(z)}$$

Pg3

(3) $$H(z) = \frac{(1-4z^{-1})(1-0.6z^{-1})}{z^{-1}(1-0.25z^{-1})}$$

$$= \underbrace{\frac{(1-4z^{-1})}{(1-0.25z^{-1})}}_{H_{ap}(z)} \cdot \underbrace{\frac{(1-0.6z^{-1})}{z^{-1}}}_{H_{min}(z)}$$

There is also another way we could decompose $H(z)$:

$$H(z) = \underbrace{\frac{1}{z^{-1}}\left(\frac{1-4z^{-1}}{1-0.25z^{-1}}\right)}_{H_{ap}(z)} \underbrace{(1-0.6z^{-1})}_{H_{min}(z)}$$

$\frac{1}{z^{-1}}$ term indicates a zero at $z=0$ (origin)

This doesnot alter the amplitude response of an all-pass system.

Hence, it could be included in either the all-pass system or minimum-phase system.

P4

4a $$H(z) = \frac{z^{-2}(1-2z^{-1})}{2(1-\frac{1}{2}z^{-1})} \quad ; \quad |z| > \frac{1}{2}$$

Poles: $\frac{1}{2}$, 2 poles at the origin, $z=0$.

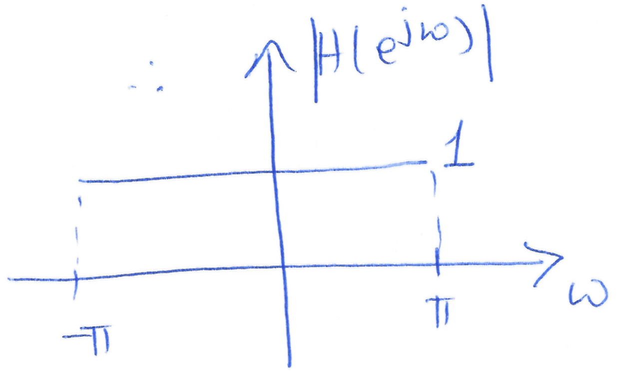
Zeros: 2

$$\Rightarrow H(z) = \frac{-z^{-2}(z^{-1} - \frac{1}{2})}{(1-\frac{1}{2}z^{-1})}$$ in canonical form.

In this form,

$$|H(e^{j\omega})| = \underbrace{|e^{-2j\omega}|}_{=1} \underbrace{\left|\frac{z^{-1}-\frac{1}{2}}{1-\frac{1}{2}z^{-1}}\right|}_{=1}$$ (All pass system in canonical form)

$\therefore$



So, this is also an all-pass system, as the amplitude is constant with frequency.

(4b) $H(z) = (-z^{-2})\dfrac{(z^{-1} - 1/2)}{(1 - \frac{1}{2}z^{-1})}$

$$H(e^{j\omega}) = -e^{-2j\omega} \cdot \frac{(e^{-j\omega} - 1/2)}{(1 - \frac{1}{2}e^{-j\omega})}$$

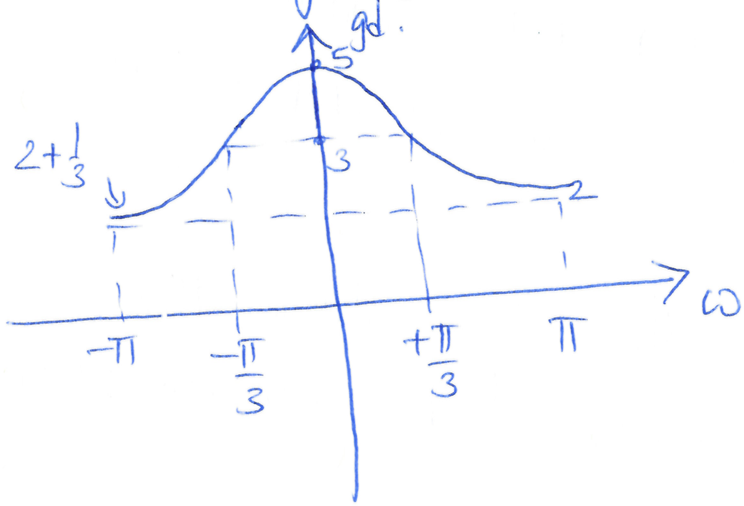
$$grd[H(e^{j\omega})] = grd(e^{-2j\omega}) + \underbrace{grd\left(\frac{e^{-j\omega} - 1/2}{1 - \frac{1}{2}e^{-j\omega}}\right)}$$

All pass in canonical form.

$$= 2 + \frac{1 - r^2}{1 + r^2 - 2r\cos(\omega - \theta)}$$

here $r = 1/2$ and $\theta = 0$ (Pole location).

$$\therefore grd[H(e^{j\omega})] = 2 + \frac{1 - \frac{1}{4}}{1 + \frac{1}{4} - \cos(\omega)} = 2 + \frac{3}{5 - 4\cos\omega}.$$



P6

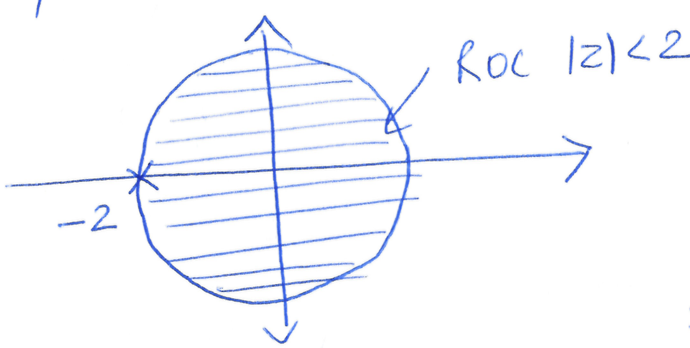
Q5 (a) $h[n] = \delta[n] + 2\delta[n]$

$H(z) = 1 + 2z^{-1}$; ROC : $|z| > 0$

Inverse system $H_{inv}(z) = \frac{1}{1+2z^{-1}}$; ROC $|z| > 2$ or $|z| < 2$.

For $H_{inv}(z)$ to be stable, ROC: $|z| < 2$ is appropriate

So, $h_{inv}[n] = -(-2)^n u[-n-1]$



Since the largest pole $z = -2$, lies outside the ROC, System can't be causal.

(b) $H(z) = 1 + \alpha z^{-1}$: ROC : $|z| > 0$

$H_{inv}(z) = \frac{1}{1+\alpha z^{-1}}$: ROC : $|z| < |\alpha|$ or $|z| > |\alpha|$

For it to be causal, $|z| > |\alpha|$ should be the ROC.

But $|z| = 1$ should be inside $|z| > |\alpha|$ for it to be stable,

So, $|\alpha| < 1$ is the only condition that makes the inverse system to be causal & stable.