DSP ECE 464/564 HW-5 Solutions

(1a) $h[n] = 2\delta[n] + \delta[n-1] + 2\delta[n-2]$

$$H(e^{j\omega}) = 2 + e^{-j\omega} + 2e^{-2j\omega}$$

$$= e^{-j\omega}\left(2e^{j\omega} + 1 + 2e^{-j\omega}\right)$$

$$= e^{-j\omega}\left(1 + 4\cos\omega\right)$$

Compare with $\underline{e^{-j\omega\alpha + j\beta} A(e^{j\omega})}$

we get $A(e^{j\omega}) = (1 + 4\cos\omega)$

$$\alpha = 1$$

$$\beta = 0$$

Since $A(e^{j\omega})$ can be positive or negative, it is Generalized Linear Phase.

(1b) $h[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2]$

$$H(e^{j\omega}) = 1 + 2e^{-j\omega} + 3e^{-2j\omega}$$

This is not symmetric (or) anti-symmetric impulse response. So, it does not possess generalized linear phase.

(1c) $h[n] = \delta[n] + 3\delta[n-1] + \delta[n-2]$

$$H(e^{j\omega}) = 1 + 3e^{-j\omega} + e^{-j2\omega}$$

$$= e^{-j\omega}\left(e^{j\omega} + 3 + e^{-j\omega}\right) = \left(e^{-j\omega}\right)\left(2\cos\omega + 3\right)$$

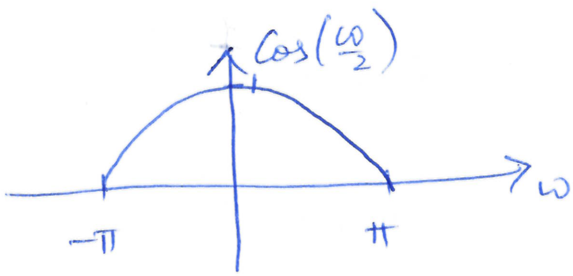
(P2)

Since $(2\cos\omega+3)>0$ always, this is a linear phase system with

$$A(e^{j\omega}) = 2\cos\omega+3$$
$$\alpha = 1$$
$$\beta = 0$$

(1d) $h[n] = \delta[n] + \delta[n-1]$

$$H(e^{j\omega}) = 1+e^{-j\omega} = e^{-j\frac{\omega}{2}}\left[e^{j\frac{\omega}{2}} + e^{-j\frac{\omega}{2}}\right]$$
$$= 2\cos\frac{\omega}{2}\, e^{-j\frac{\omega}{2}}$$



Because $2\cos(\frac{\omega}{2})$ is non-negative in the range $(-\pi,\pi)$, this is a Linear Phase System.

(1e) $h[n] = \delta[n] - \delta[n-2]$

$$H(e^{j\omega}) = 1-e^{-2j\omega}$$
$$= e^{-j\omega}\left(e^{j\omega} - e^{-j\omega}\right) = 2j\sin\omega\, e^{-j\omega}$$
$$= 2\sin\omega\, e^{-j\omega+j\frac{\pi}{2}}$$

So, here $\alpha = 1$, $\beta = \frac{\pi}{2}$, $A(e^{j\omega}) = 2\sin(\omega)$

Because $2\sin(\omega)$ can be positive or negative in the range $(-\pi,\pi)$ it is a $\boxed{\text{Generalized Linear phase system}}$

2 (a) IIR Systems: B, C, D, E

An IIR System will have H(z) of form:

$$H(z) = \frac{\prod_i (1 - c_i z^{-1})}{\prod_j (1 - d_j z^{-1})}$$

(b) A, F

The poles of an FIR system must be at $p=0$

(c) A, B, C, E, F

Because the systems are causal, the poles of stable system must be inside the unit circle

(d) E

All zeros and poles are inside unit circle

(e) A, F

All poles should be $p=0$ and zero comes in conjugate reciprocal pairs

(f) C

All pass system

(g) E (All poles and zeros should be inside unit circle).

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(h) F

The length of A is 11. The length of F is 7.

length of others is infinite

(i) E

Minimum - Phase system.

3a> This system has 5 zeros, so $M=5$

The length of the impulse response will be $M+1=6$.

b> For linear phase FIR systems group delay

is $\frac{M}{2} = \frac{5}{2} = 2.5$

c> We have zeros at $z=0.5$ & $z=e^{j\frac{\pi}{2}}$

To have symmetric impulse response for

zero at $z=0.5$, we need to have zero at $z=2$

and $\therefore c=2$

For zero at $z=e^{j\pi/2}$, we need to have zero at

$z=e^{-j\frac{\pi}{2}}$, so $a=e^{-j\frac{\pi}{2}}$

We know, $H(e^{j\omega})=0$ at $\omega=0$

$$H(z)=\left(1-e^{-j\frac{\pi}{2}}z^{-1}\right)\left(1-e^{j\frac{\pi}{2}}z^{-1}\right)\left(1-0.5z^{-1}\right)\left(1-2z^{-1}\right)\left(1-bz^{-1}\right)$$

P5

$$H(z) = \left[1 - z^{-1}\left(e^{j\frac{\pi}{2}} + e^{-j\frac{\pi}{2}}\right) + z^{-2}\right]\left[1 - 2.5z^{-1} + z^{-2}\right]\left[1 - bz^{-1}\right]$$

$$= \left[1 + z^{-2}\right]\left[1 - 2.5z^{-1} + z^{-2}\right]\left[1 - bz^{-1}\right]$$

$$= \left[1 - 2.5z^{-1} + 2z^{-2} - 2.5z^{-3} + z^{-4}\right]\left[1 - bz^{-1}\right]$$

$$H(e^{j\omega}) = \left[1 - 2.5e^{-j\omega} + 2e^{-2j\omega} - 2.5e^{-3j\omega} + e^{-j4\omega}\right]\left[1 - be^{-j\omega}\right]$$

at $\omega = 0$, $H(e^{j\omega}) = 0$

$$\Rightarrow \left[1 - 2.5 + 2 - 2.5 + 1\right]\left[1 - b\right] = 0$$

$$\Rightarrow b = 1$$

$$\therefore H(z) = \left(1 - e^{-j\frac{\pi}{2}}z^{-1}\right)\left(1 + e^{j\frac{\pi}{2}}z^{-1}\right)\left(1 - 0.5z^{-1}\right)\left(1 - 2z^{-1}\right)\left(1 - z^{-1}\right)$$

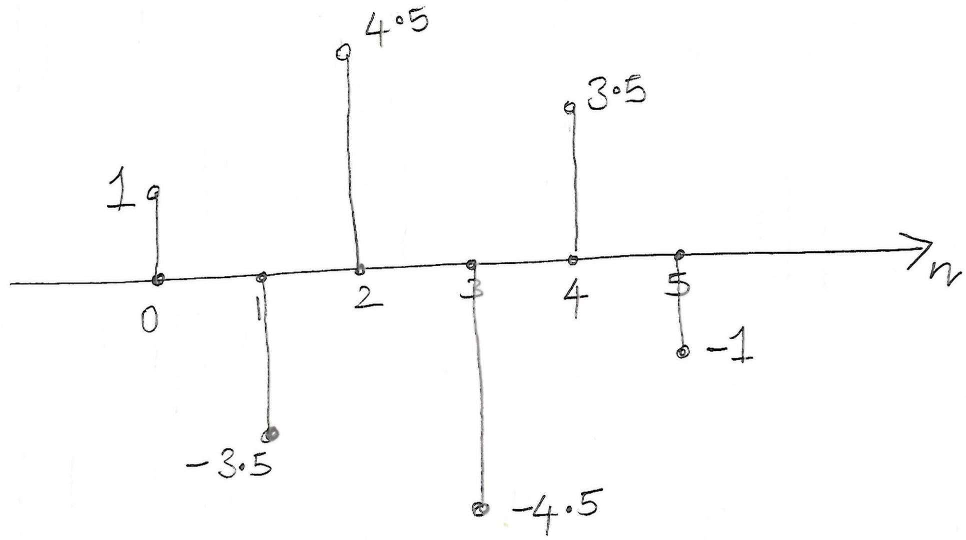
(d) $H(z) = \left[1 - 2.5z^{-1} + 2z^{-2} - 2.5z^{-3} + z^{-4}\right]\left[1 - z^{-1}\right]$

$$= 1 - 2.5z^{-1} + 2z^{-2} - 2.5z^{-3} + z^{-4} - z^{-1} + 2.5z^{-2} - 2z^{-3} + 2.5z^{-4} - z^{-5}$$

$$= 1 - 3.5z^{-1} + 4.5z^{-2} - 4.5z^{-3} + 3.5z^{-4} - z^{-5}$$

$$\therefore h[n] = \delta[n] - 3.5\delta[n-1] + 4.5\delta[n-2] - 4.5\delta[n-3] + 3.5\delta[n-4] - \delta[n-5]$$

P6



(e) Here $h[n] = -h[M-n]$, M: odd

& anti-symmetry

So it is a Type IV filter.

(4) Here, the length of filter impulse response $= M+1 = 8$

$\therefore M = 7$, (The number of zeros).

There is a zero at $z = -2$

There must be another zero at $z = -\frac{1}{2}$.

For the zero $z = 0.8e^{j\frac{\pi}{4}}$, due to the property of FIR.

Other zeros are $z = 1.25e^{j\frac{\pi}{4}}$, $z = 0.8e^{-j\frac{\pi}{4}}$, $z = 1.25e^{-j\frac{\pi}{4}}$.

From $h[n] = -h[7-n]$, Antisymmetric & M = odd

Type IV FIR filter

For type IV system, there is a zero at $z=1$.

Hence $H(z) = (1-z^{-1})(1+2z^{-1})(1+\frac{1}{2}z^{-1})(1-0.8e^{j\frac{\pi}{4}}z^{-1})(1-0.8e^{-j\frac{\pi}{4}}z^{-1})$
$(1-1.25e^{j\frac{\pi}{4}}z^{-1})(1-1.25e^{-j\frac{\pi}{4}}z^{-1})$

(5) Given $H(z) = \dfrac{(1-2z^{-1})(1-0.75z^{-1})}{z^{-1}(1-0.5z^{-1})}$

we could decompose this into:

$$\underbrace{\frac{(1-2z^{-1})(1-0.5z^{-1})}{z^{-1}}}_{H_{lin}(z)} \cdot \underbrace{\frac{(1-0.75z^{-1})}{(1-0.5z^{-1})^2}}_{H_{min}(Z)}$$

Because in a linear phase system, for every real zero at r, there must be a zero at $1/r$ as well.