ECE 464/564 HW-1 Solutions:

1(a) $x_c(t) = \sin(20\pi t) + \cos(40\pi t)$

is sampled with a period T, so $t = nT$

$\therefore\ x_c(nT) = \sin(20\pi nT) + \cos(40\pi nT)$ --- (1)

& $x[n] = \sin\left(\frac{n\pi}{5}\right) + \cos\left(\frac{2n\pi}{5}\right)$ --- (2)

By comparing (1) & (2), we see that

$$\frac{n\pi}{5} = 20\pi n T$$

$$\Rightarrow T = \frac{1}{100}$$

(b) This choice of T is not unique since, choosing

$$T = \frac{1}{100} + \frac{k}{10},$$ where k is any integer

can also give the same value for $x[n]$

i.e. $\sin\left(\frac{n\pi}{5}\right) + \cos\left(\frac{2n\pi}{5}\right)$

as sine/cosine are 2π periodic.

(2)

$$\frac{\sin\left(\frac{n\pi}{2}\right)}{\frac{n\pi}{2}} = \frac{\sin(10\pi nT)}{10\pi nT}$$

$$\Rightarrow \frac{n\pi}{2} = 10\pi nT$$

$$\Rightarrow T = \frac{1}{20}$$

But $x_c(t)$ is not periodic, so T is unique.

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(3) (a) $y_c(t) = \frac{d}{dt} x_c(t)$

Taking Fourier Transform on both sides, we get

$$Y_c(j\Omega) = j\Omega X_c(j\Omega)$$

$$\therefore H_c(j\Omega) = \frac{Y_c(j\Omega)}{X_c(j\Omega)} = j\Omega$$

(b) $H_c(e^{j\omega}) = j\frac{\omega}{T_s}$

(c) $$h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} j\frac{\omega}{T_s} e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi T_s} \frac{d}{dn}\left[\int_{-\pi}^{\pi} e^{j\omega n} d\omega\right]$$

$$= \frac{1}{2\pi T_s} \frac{d}{dn}\left[\left.\frac{e^{j\omega n}}{jn}\right|_{-\pi}^{\pi}\right]$$

$$= \frac{1}{2\pi T_s} \frac{d}{dn}\left[\frac{e^{j\pi n} - e^{-j\pi n}}{2j} \cdot \frac{2}{n}\right]$$

$$= \frac{1}{\pi T_s} \frac{d}{dn}\left[\frac{\sin n\pi}{n}\right]$$

$$= \frac{1}{\pi T_s} \frac{(n\pi \cos n\pi - \sin n\pi)}{n^2}$$

$$= \frac{1}{\pi n^2 T_s} (n\pi \cos n\pi) \quad (\because \sin n\pi = 0 \text{ always})$$

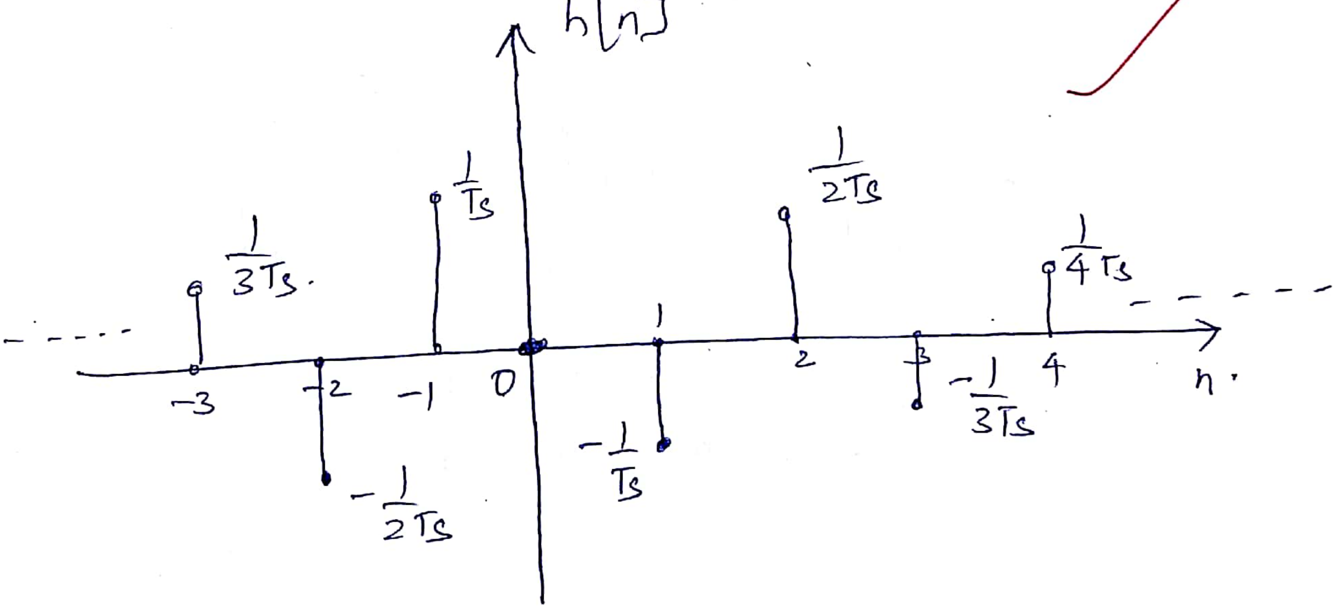
$$= \frac{\cos n\pi}{n T_s} = \frac{(-1)^n}{n T_s}, \quad n \neq 0.$$

When $n = 0$, $h[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{j\omega}{T_s} e^{j\omega n} d\omega = 0$

(odd symmetric)

Hence,

$$h[n] = \begin{cases} \frac{(-1)^n}{n T_s}, & n \neq 0 \\ 0, & n = 0. \end{cases}$$



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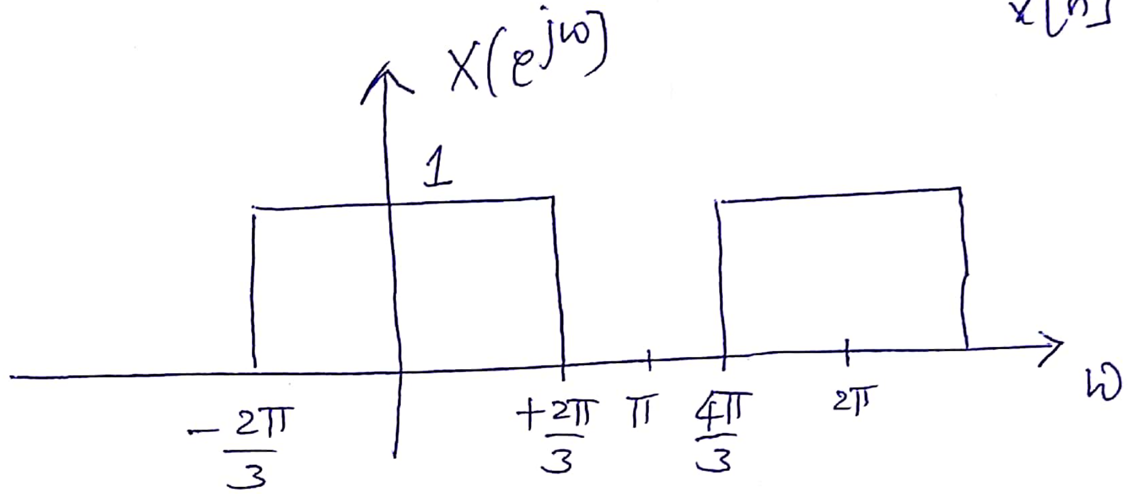
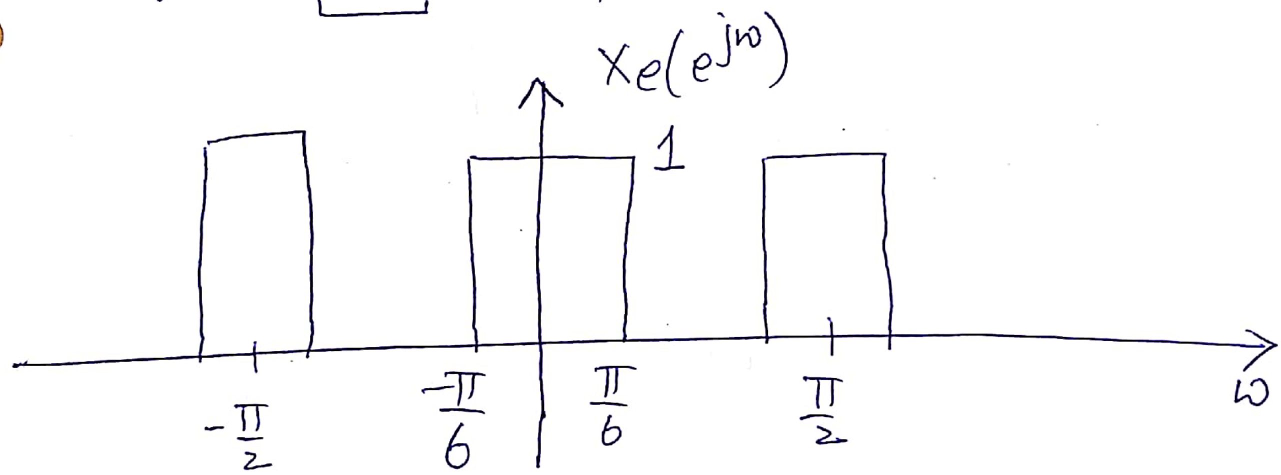
4) (a) $x[n] = \dfrac{\sin\left(\frac{2n\pi}{3}\right)}{n\pi}$ => The Fourier Transform of $x[n]$ would be:

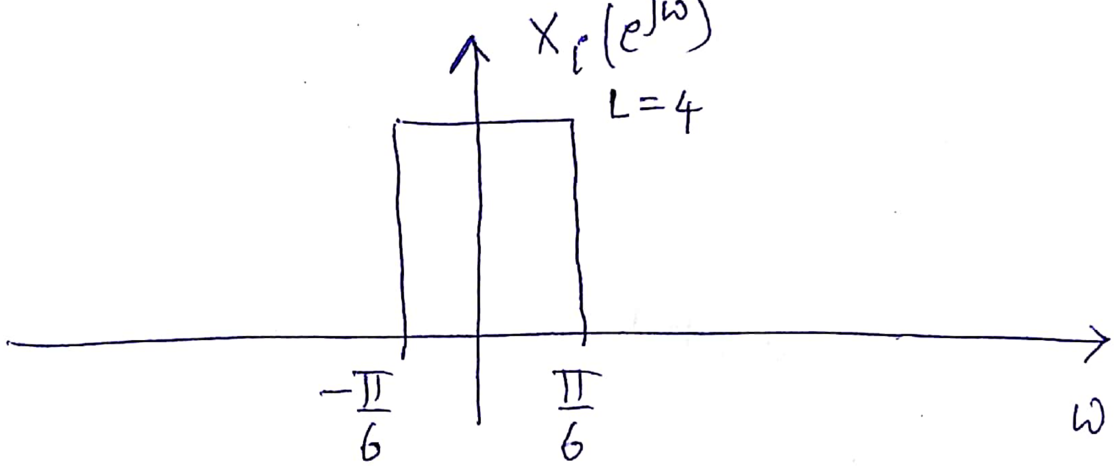


After $\boxed{\uparrow L}$, $L=4$

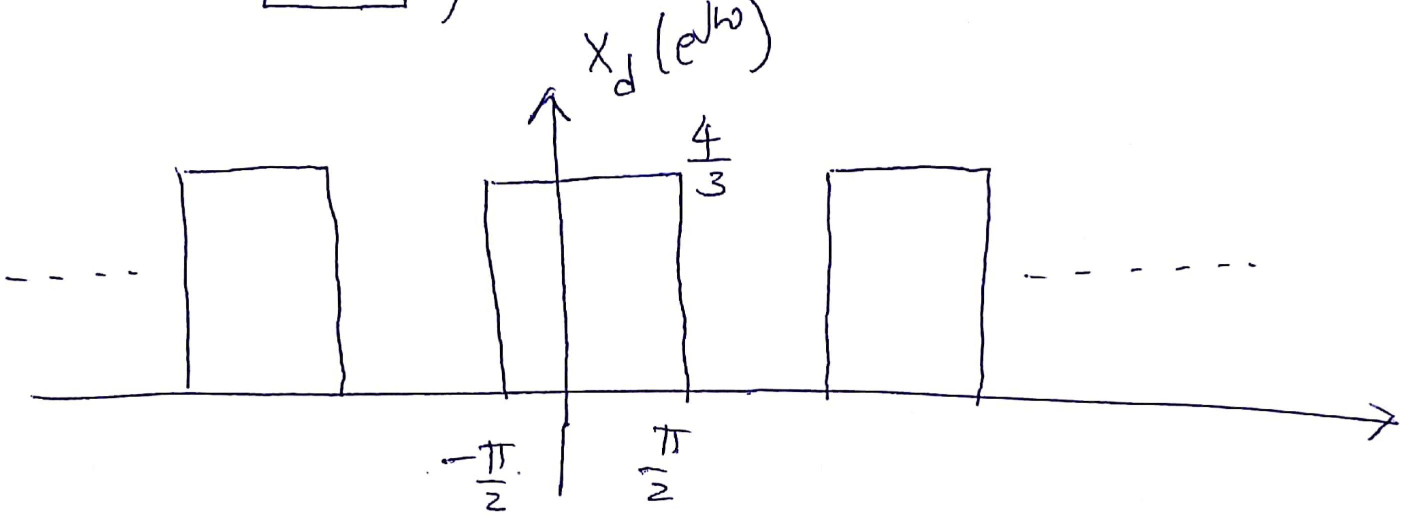


Then $X_e(e^{jw})$ is passed through a LPF $H(e^{jw})$

$$H(e^{jw}) = \begin{cases} L, & |w| \leq \min\left(\frac{\pi}{4}, \frac{\pi}{3}\right) \\ 0, & \text{otherwise} \end{cases} \Bigg\} \; 2\pi \text{ periodic}$$



After $\boxed{\downarrow M}$, $M = 3$



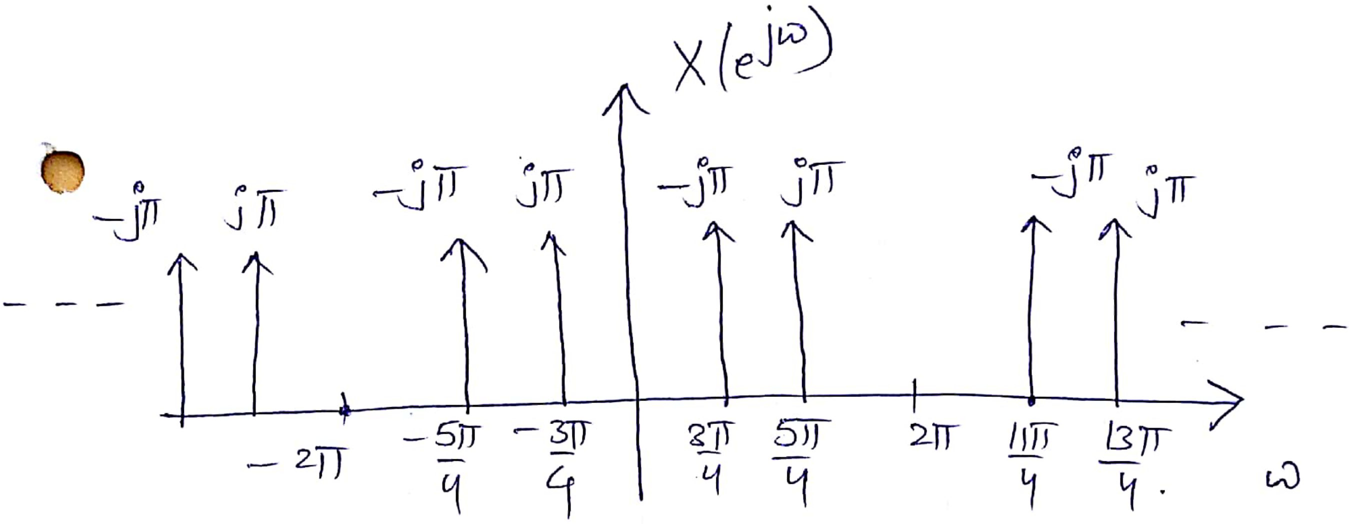
Then the Inverse DTFT would be:

$$x_d[n] = \frac{4}{3}\left[\frac{\sin\left(\frac{n\pi}{2}\right)}{n\pi}\right]$$

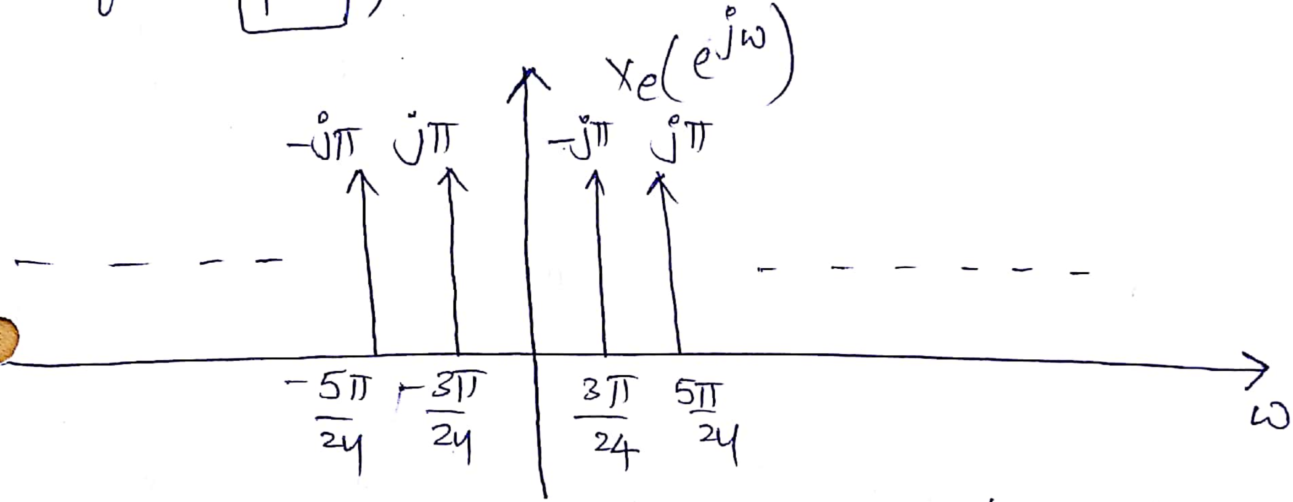
(b) $x[n] = \sin\left(\frac{3n\pi}{4}\right)$

The Fourier Transform of $x[n]$ would be

$$X(e^{j\omega}) = j\pi \sum_{k=-\infty}^{\infty}\left[-\delta\left(\omega - \frac{3\pi}{4} - 2k\pi\right) + \delta\left(\omega + \frac{3\pi}{4} - 2\pi k\right)\right]$$

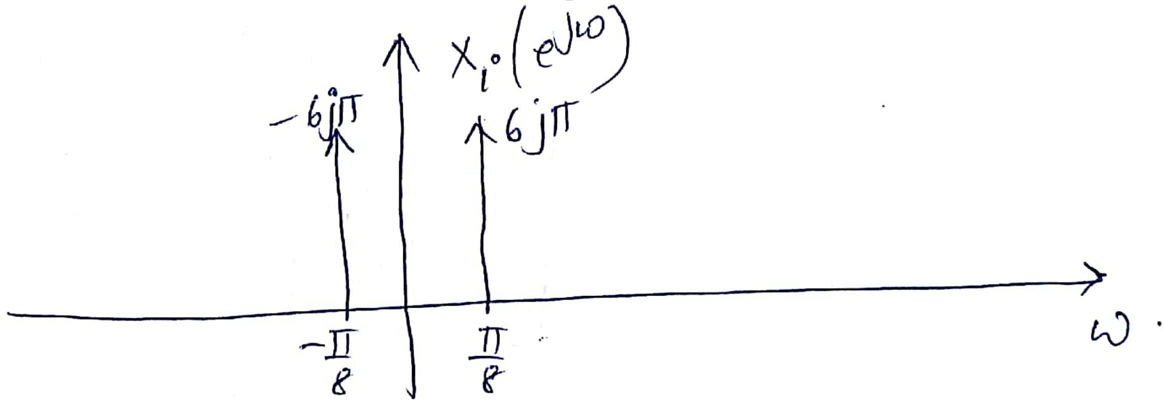


After $\boxed{\uparrow L}$, $L = 6$

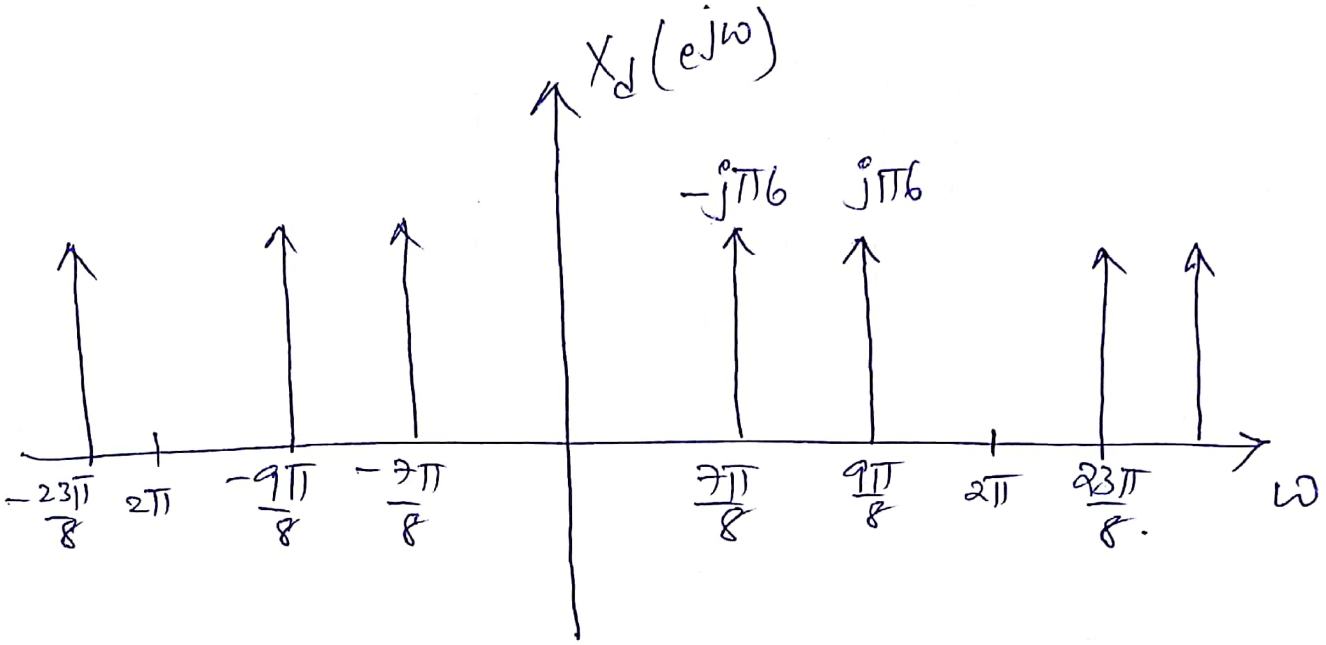


LPF $H(e^{j\omega}) = \left\{ \begin{array}{ll} L, & |\omega| \leq \min\left(\frac{\pi}{6}, \frac{\pi}{7}\right) \\ 0, & \text{0therwise} \end{array} \right\}$ 2π Periodic.

Hence after the LPF,



After $\boxed{\downarrow M}$, $M = 7$.



$$= j\pi 6 \sum_{K=-\infty}^{\infty} \left[-\delta\left(\omega - \frac{7\pi}{8} - 2\pi K\right) + \delta\left(\omega + \frac{7\pi}{8} - 2\pi K\right) \right]$$

If we take Inverse Fourier Transform, we get

$$\tilde{x}_d(n) = 6 \sin\left(\frac{7\pi n}{8}\right)$$

(or) $\frac{6}{7} \sin\left(\frac{7\pi n}{8}\right)$

If you consider that Downsampling by a factor of 7, amplitude gets scaled by the same factor.

Q5)

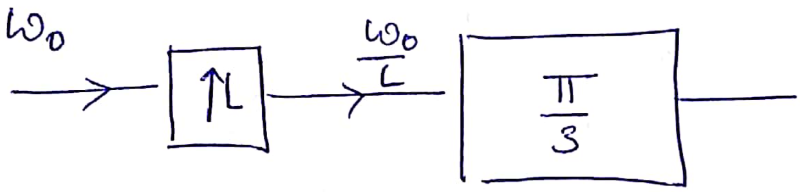
(a) Here $L=2$ & $M=3$

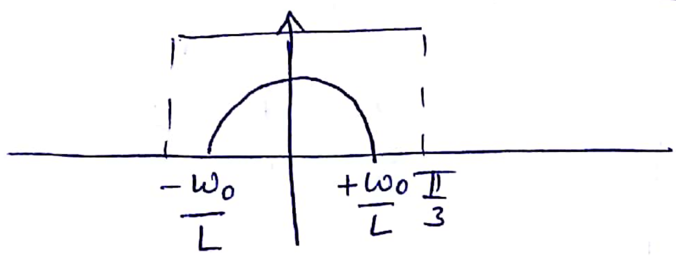
Always Upsampling tries to compress the spectrum, while Downsampling tries to expand the same

Since $M>L$ here, we should be:

1) Careful that no-aliasing takes place &

2) Low Pass filter does not remove any important content in the signal.





$$\Rightarrow \frac{\omega_0}{L} \leq \frac{\pi}{3}$$

$$\Rightarrow \frac{\omega_0}{2} \leq \frac{\pi}{3}$$

$$\Rightarrow \omega_0 \leq \frac{2\pi}{3} \quad \text{(or)} \quad \omega_{0,max} = \frac{2\pi}{3}.$$

(b) In this case $L > M$,

So there is no chance of aliasing

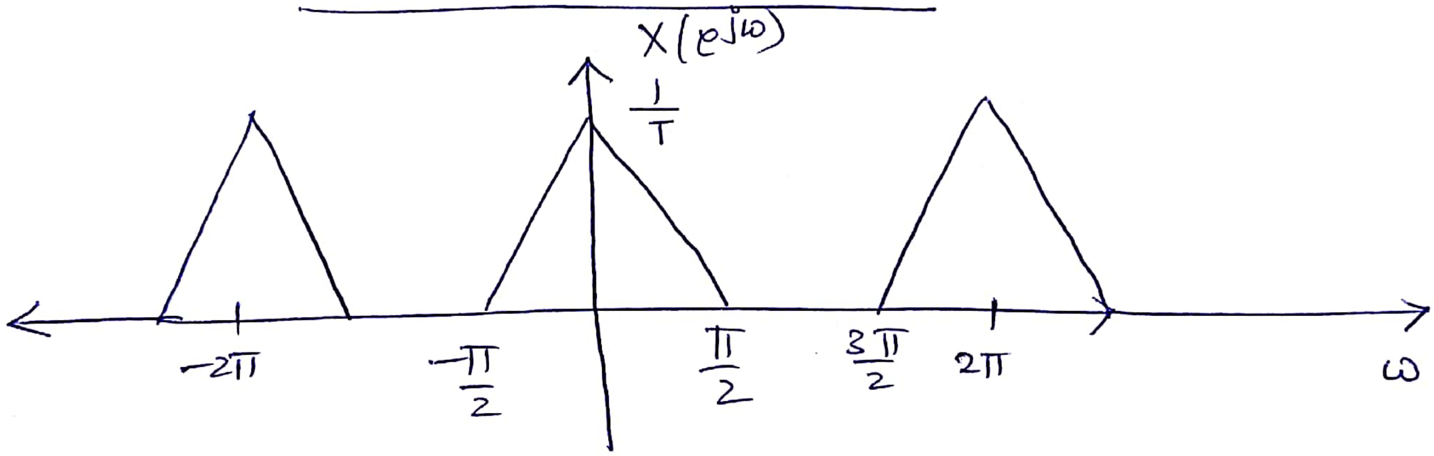
so $\frac{w_0}{L} \le \frac{\pi}{L}$

i.e. $\frac{w_0}{3} \le \frac{\pi}{3}$ or $\boxed{w_0 \le \pi}$

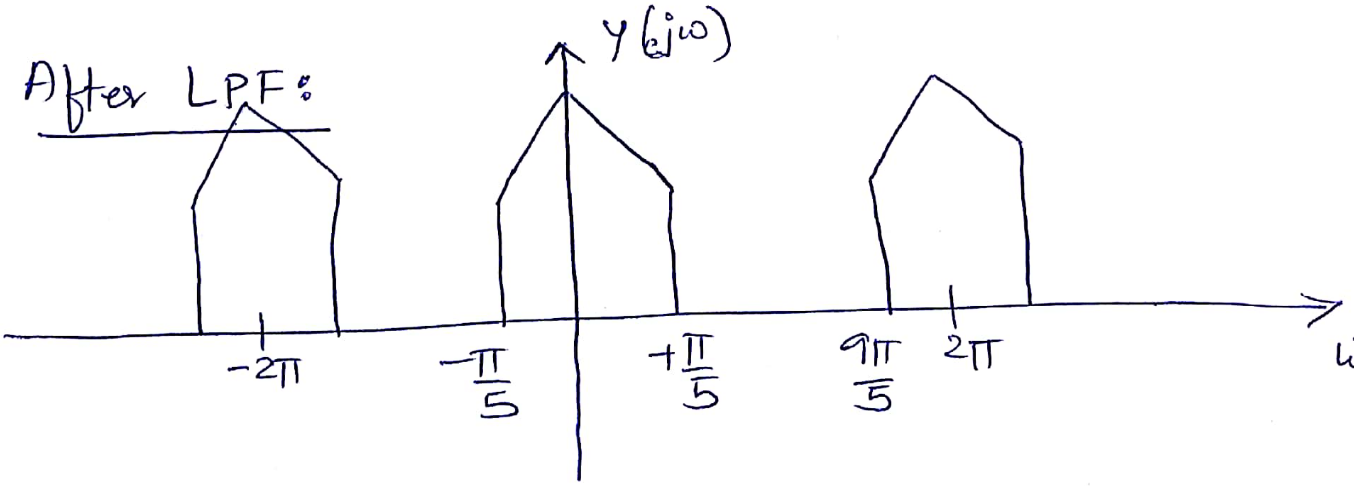
or $\boxed{w_{0,max} = \pi}$

6(a) $\omega = \Omega T = 2\pi \times 5\times10^{3} \times \frac{1}{2\times10^{4}} = 0.5\pi$

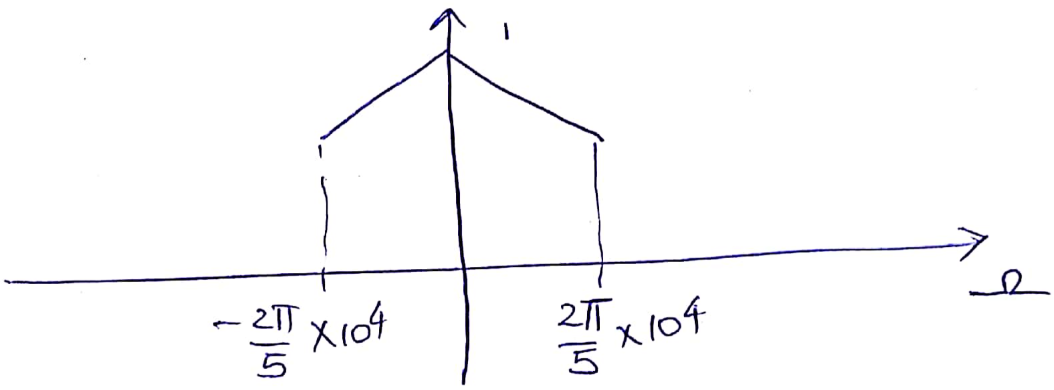
After the C/D Converter:



After LPF:



After D/C Converter:



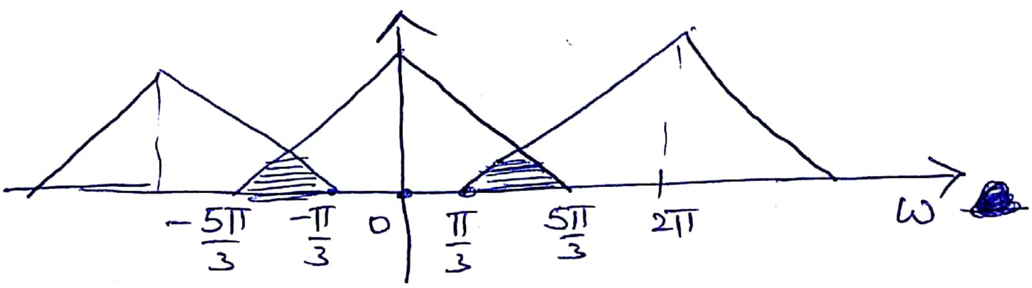
(b) $\omega = \Omega T = 2\pi \times 5\times10^3 \times \frac{1}{6\times10^3} = \frac{5\pi}{3} > \pi$

There would be aliasing here, (as shown in fig below)

the maximum cutoff frequency should therefore be $\frac{\pi}{3}$

$$\Omega_c = \frac{\omega}{T} = \frac{\pi}{3}\times 6\times10^3 = 2\pi\times10^3$$

$$H_c(j\Omega) = \begin{cases} 1, & |\Omega| \leq 2\pi\times10^3 \\ 0 & 2\pi\times10^3 < |\Omega| \leq 6\pi\times10^3 \end{cases}$$



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