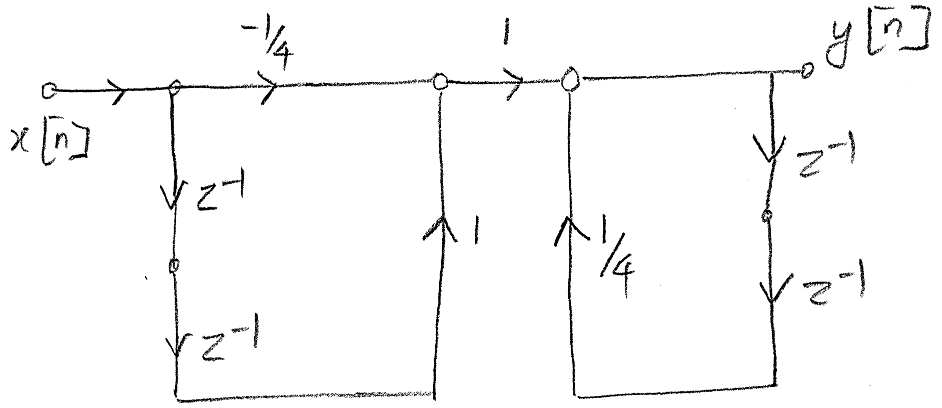
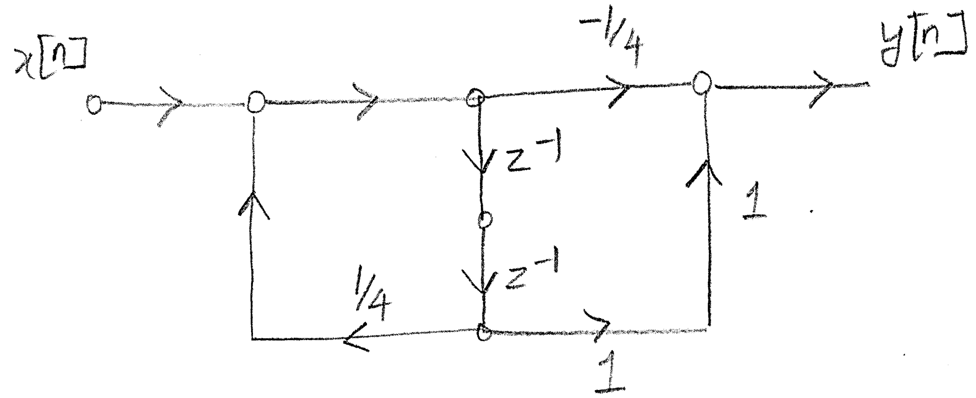
ECE 464/564 HW-6 Solutions

1) (a) Direct Form I:



(b) Direct Form II:



(c) $Y(z) - \frac{1}{4} z^{-2} Y(z) = z^{-2} X(z) - \frac{1}{4} X(z)$

$$\Rightarrow \frac{Y(z)}{X(z)} = \left(\frac{z^{-2} - \frac{1}{4}}{1 - \frac{1}{4} z^{-2}} \right) \quad (\text{or}) \quad -\frac{1}{4} \left(\frac{1 - 4z^{-2}}{1 - \frac{1}{4} z^{-2}} \right)$$

P2

2 (a) $y[n] = x[n] + v[n-1]$ ---- (1)

$v[n] = \frac{1}{2}y[n] + 2x[n] + w[n-1]$ ---- (2)

$w[n] = \frac{1}{2}y[n] + x[n]$ ------ (3)

Eliminating $w[n]$ & $w[n-1]$ terms from (2) & (3),

we get, $v[n] = \frac{1}{2}y[n] + 2x[n] + \frac{1}{2}y[n-1] + x[n-1]$

So, $v[n-1] = \frac{1}{2}y[n-1] + 2x[n-1] + \frac{1}{2}y[n-2] + x[n-2]$ ---- (4)

From eqns (1) & (4), we get

$y[n] = x[n] + \frac{1}{2}y[n-1] + 2x[n-1] + \frac{1}{2}y[n-2] + x[n-2]$

$\Rightarrow y[n] - \frac{1}{2}y[n-1] - \frac{1}{2}y[n-2] = x[n] + 2x[n-1] + x[n-2]$

$$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{1+2z^{-1}+z^{-2}}{1-\frac{1}{2}z^{-1}-\frac{1}{2}z^{-2}}$$

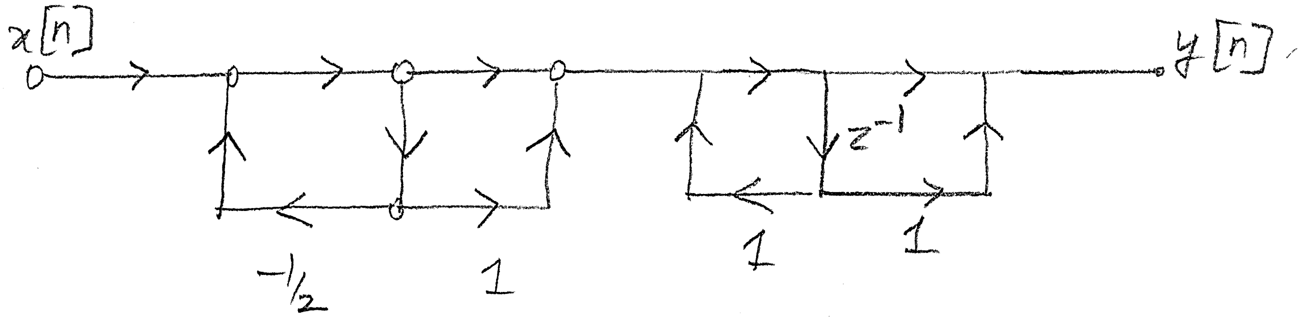
(b) $H(z)$ can be decomposed into $H_1(z)$ & $H_2(z)$ such that $H(z) = H_1(z)\cdot H_2(z)$

$$\Rightarrow H(z) = \frac{1+2z^{-1}+z^{-2}}{1-\frac{1}{2}z^{-1}-\frac{1}{2}z^{-2}} = \frac{(1+z^{-1})(1+z^{-1})}{(1+\frac{1}{2}z^{-1})(1-z^{-1})}$$

So, $H_1(z) = \left(\frac{1+z^{-1}}{1+\frac{1}{2}z^{-1}}\right)$ $H_2(z) = \left(\frac{1+z^{-1}}{1-z^{-1}}\right)$

$$\Rightarrow \frac{Y_1(z)}{X_1(z)} = \frac{1+z^{-1}}{1+\frac{1}{2}z^{-1}} \Rightarrow y_1[n] + \frac{1}{2}y_1[n-1] = x_1[n] + x_1[n-1]$$

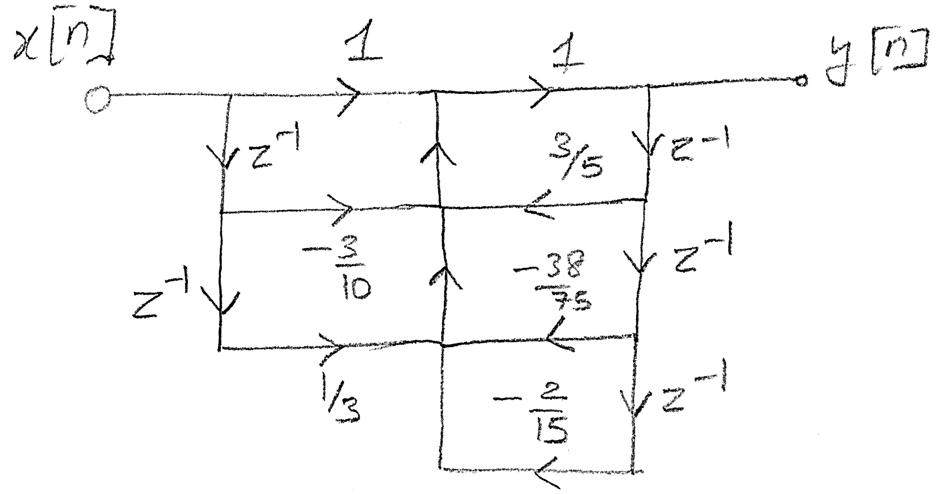
$$\Rightarrow \frac{Y_2(z)}{X_2(z)} = \frac{1+z^{-1}}{1-z^{-1}} \Rightarrow y_2[n] - y_2[n-1] = x_2[n] + x_2[n-1]$$



(C) The system has a pole at $z=1$
So, the ROC cannot contain $|z|=1$
& the system is <u>not</u> stable.

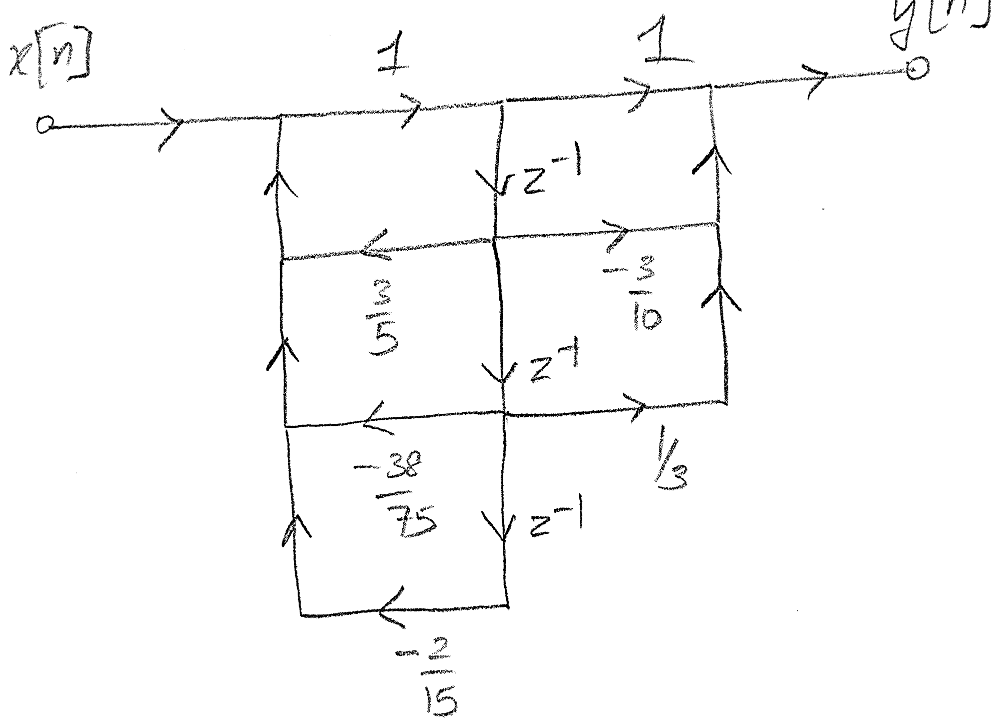
(3)
(a) $$H(z) = \frac{1 - \frac{3}{10}z^{-1} + \frac{1}{3}z^{-2}}{1 - \frac{3}{5}z^{-1} + \frac{38}{75}z^{-2} + \frac{2}{15}z^{-3}} = \frac{\frac{1}{2}}{\left(1-\frac{4}{5}z^{-1}+\frac{2}{3}z^{-2}\right)} + \frac{\frac{1}{2}}{\left(1+\frac{1}{5}z\right.}$$

i,



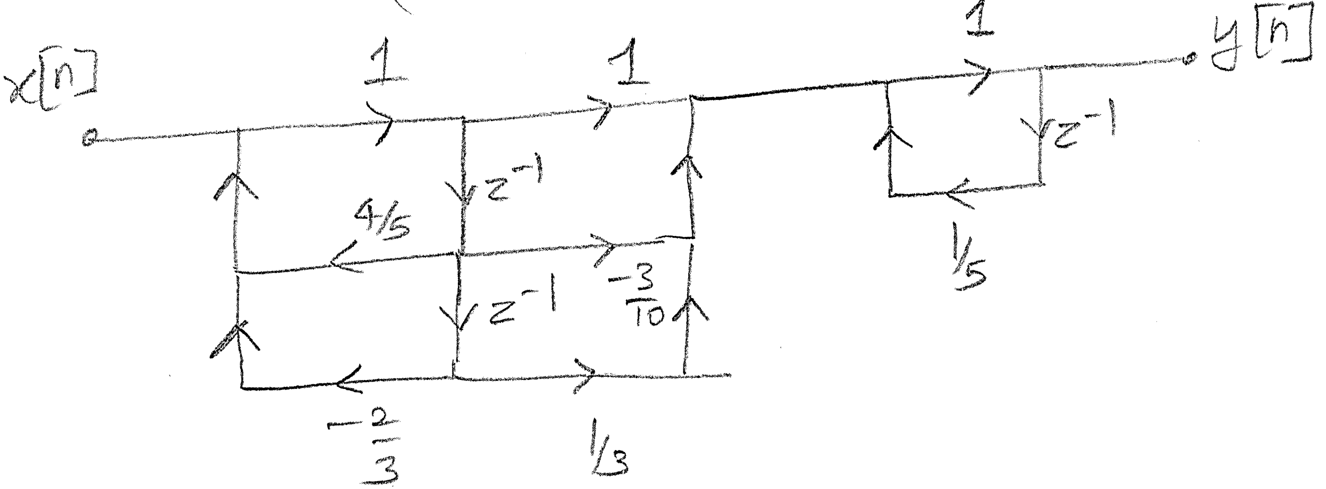
P4

i)

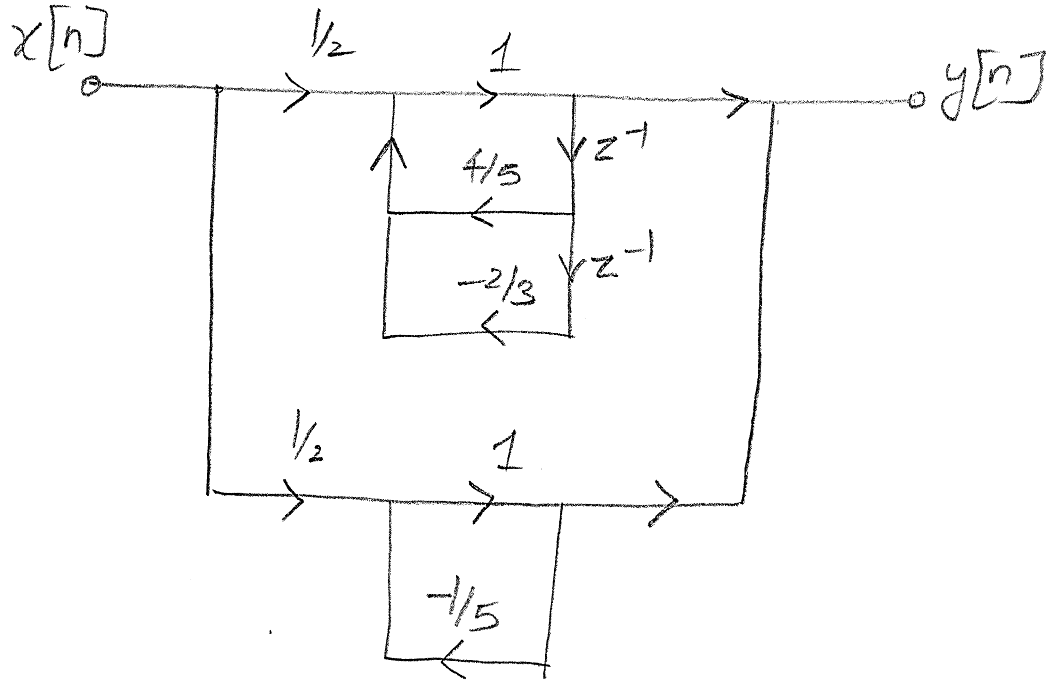


ii)

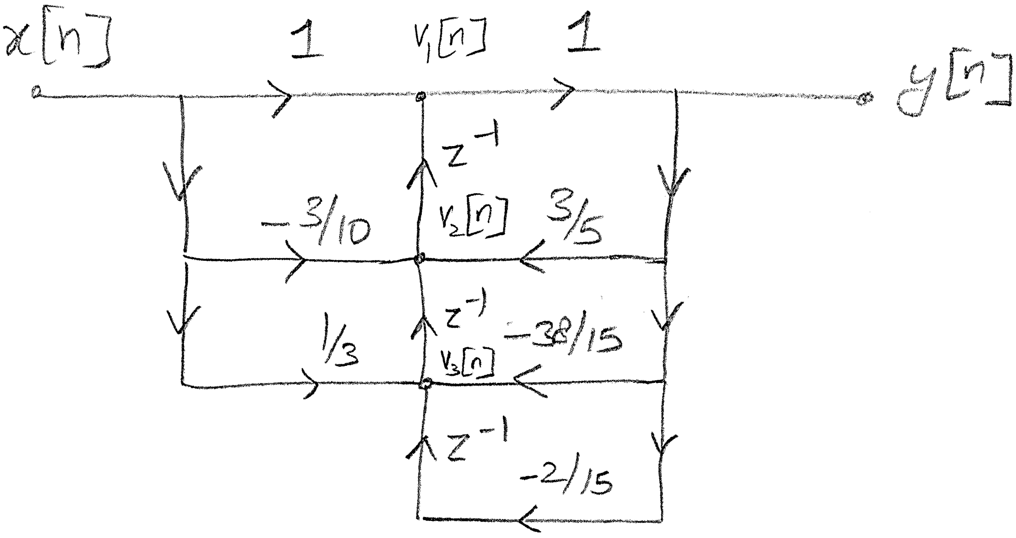
$$H(z) = \frac{1 - \frac{3}{10} z^{-1} + \frac{1}{3} z^{-2}}{\left(1 - \frac{4}{5} z^{-1} + \frac{2}{3} z^{-2}\right)\left(1 + \frac{1}{5} z^{-1}\right)} = \frac{1 - \frac{3}{10} z^{-1} + \frac{1}{3} z^{-2}}{\left(1 - \frac{4}{5} z^{-1} + \frac{2}{3} z^{-2}\right)} \cdot \frac{1}{(1+\frac{1}{5}}$$



$$H(z) = \frac{\frac{1}{2}}{1-\frac{4}{5}z^{-1}+\frac{2}{3}z^{-2}} + \frac{\frac{1}{2}}{1+\frac{1}{5}z^{-1}}$$



(V)



b. $y[n] = v_1[n] = x[n] + v_2[n-1]$

$v_2[n] = -\frac{3}{10}x[n] + v_3[n-1] + \frac{3}{5}y[n]$

$v_3[n] = \frac{1}{3}x[n] - \frac{38}{75}y[n] - \frac{2}{15}y[n-1]$

(P6)

$$V_1(z) = X(z) + z^{-1} V_2(z) \quad \text{----(1)}$$

$$V_2(z) = -\frac{3}{10} X(z) + z^{-1} V_3(z) + \frac{3}{5} Y(z) \quad \text{----(2)}$$

$$V_3(z) = \frac{1}{3} X(z) - \frac{38}{75} Y(z) - \frac{2}{15} z^{-1} Y(z) \quad \text{-----(3)}$$

From ①, ②, ③ we get

$$Y(z) = X(z) + z^{-1} \left[-\frac{3}{10} X(z) + z^{-1}\left(\frac{1}{3} X(z) - \frac{38}{75} Y(z) - \frac{2}{15} \cdot Y(z) z^{-1}\right) + \frac{3}{5} Y(z) \right]$$

$$\Rightarrow Y(z) = X(z) - \frac{3}{10} z^{-1} X(z) + \frac{1}{3} z^{-2} X(z) - \frac{38}{75} z^{-2} Y(z) - \frac{2}{15} z^{-3} Y(z) + \frac{3}{5} z^{-1} Y(z)$$

$$\Rightarrow Y(z) - \frac{3}{5} Y(z) z^{-1} + \frac{38}{75} z^{-2} Y(z) + \frac{2}{15} z^{-3} Y(z) = X(z) - \frac{3}{10} z^{-1} X(z) + \frac{1}{3} z^{-2} X(z).$$

$$\Rightarrow \frac{Y(z)}{X(z)} = \frac{1 - \frac{3}{10} z^{-1} + \frac{1}{3} z^{-2}}{1 - \frac{3}{5} z^{-1} + \frac{38}{75} z^{-2} + \frac{2}{15} z^{-3}}$$

4a) The equations obtained from the flow graph are:

$$y[n] = ab\,w[n] + ab\,y[n] + x[n] + b\,w[n-1]$$

$$w[n] = -y[n]$$

$$y[n] = -ab\,y[n] + ab\,y[n] + x[n] + b\,w[n-1]$$

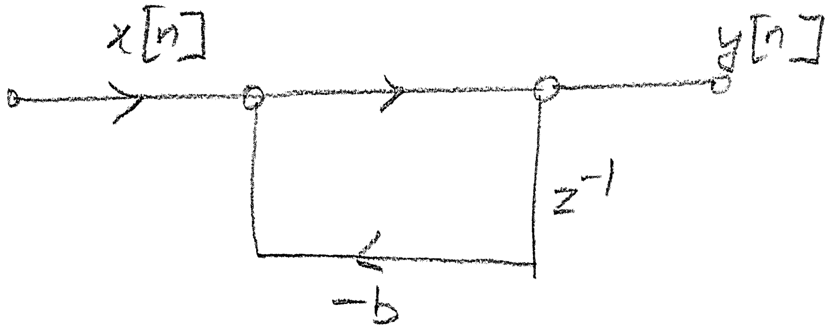
$$\Rightarrow \boxed{y[n] = x[n] - b\,y[n-1]}$$

$$\Rightarrow y[n] + b\,y[n-1] = x[n]$$

$$\Rightarrow Y(z)\left[1 + bz^{-1}\right] = X(z)$$

$$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1+bz^{-1}}$$

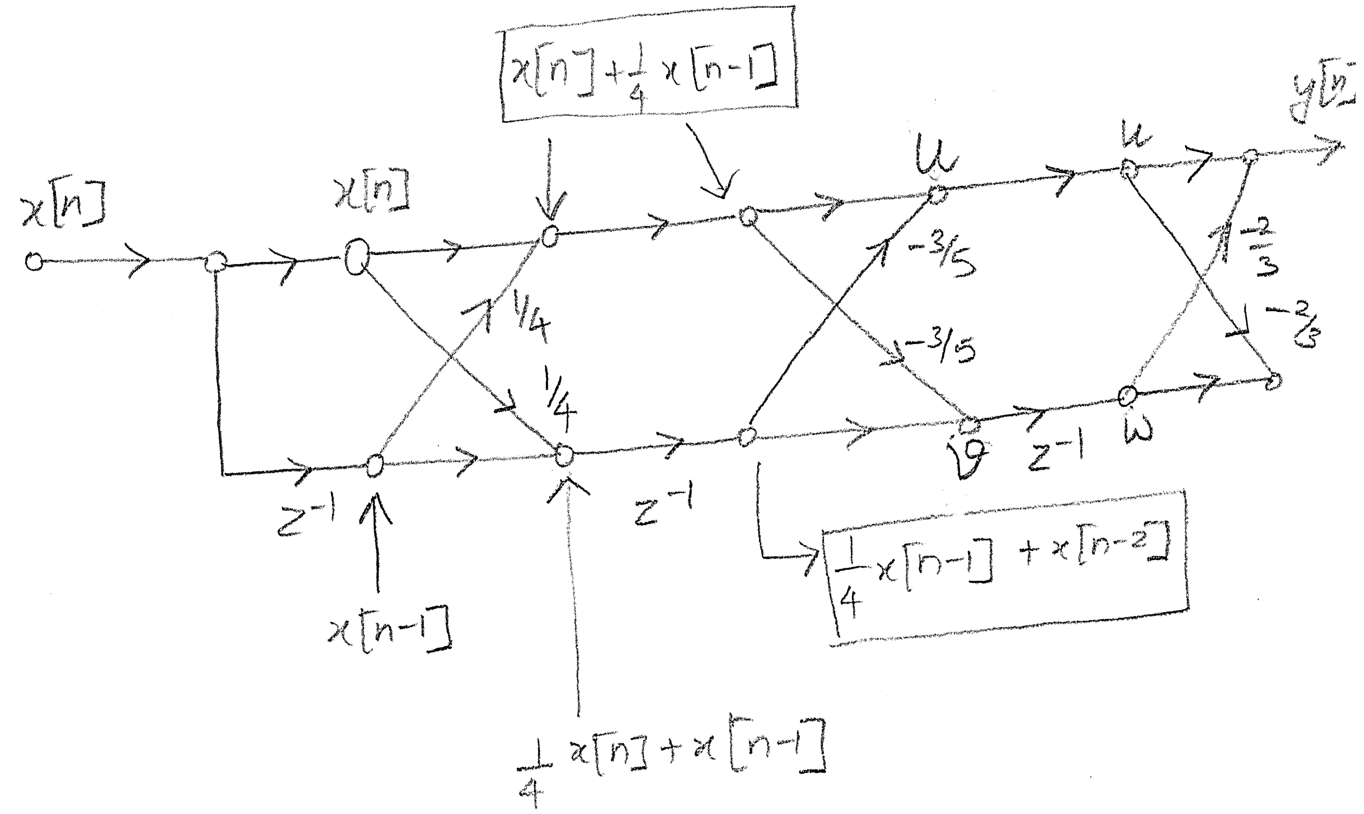
b)



Direct Form - I

P8

(5).



From the fig. above, we see that,

$$u[n] = x[n] + \frac{1}{4}x[n-1] - \frac{3}{5}\left(\frac{1}{4}x[n-1] + x[n-2]\right)$$

$$v[n] = \frac{1}{4}x[n-1] + x[n-2] - \frac{3}{5}\left(x[n] + \frac{1}{4}x[n-1]\right)$$

$$w[n] = v[n-1]$$

$$y[n] = u[n] - \frac{2}{3}w[n] = u[n] - \frac{2}{3}v[n-1]$$

$$\Rightarrow y[n] = x[n] + \frac{1}{4}x[n-1] - \frac{3}{5}\left(\frac{1}{4}x[n-1] + x[n-2]\right)$$

$$- \frac{2}{3}\left[\frac{1}{4}x[n-2] + x[n-3] - \frac{3}{5}\left(x[n-1] + \frac{1}{4}x[n-2]\right)\right]$$

$$\Rightarrow y[n] = x[n] + \frac{1}{4}x[n-1] - \frac{3}{20}x[n-1] - \frac{3}{5}x[n-2]$$

$$- \frac{2}{12}x[n-2] - \frac{2}{3}x[n-3] + \frac{2}{5}\left(x[n-1] + \frac{1}{4}x[n-2]\right)$$

$$= x[n] + \frac{1}{2}x[n-1] - \frac{2}{3}x[n-2] - \frac{2}{3}x[n-3]$$