Splitting the (Gravitational) Atom

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Abstract

Gravitational wave (GW) observations in the last decade have reinforced the Kerr Black Hole (BH) paradigm, but these conclusions rely on waveform catalogs dominated by Kerr BHs. Expanding the diversity of modeled sources is essential to uncover potential new physics. However, only bodies whose stability is known can become viable alternatives. Here, **we study the stability of Q-hairy BHs**, solutions in the Einstein Maxwell Scalar (EMS) model that evade well-known uniqueness theorems.

Model

We study the EMS model, described by the Lagrangian:

$$\mathcal{L} = \frac{R}{16\pi} - \frac{1}{4}F_{\alpha\beta}F^{\alpha\beta} - (\tilde{\nabla}_\alpha\phi)^*\tilde{\nabla}^\alpha\phi - V(|\phi|^2)\,, \tag{1}$$

where R is the Ricci scalar, $F_{\mu\nu}$ the Maxwell tensor and ϕ a complex scalar field. The scalar field is coupled to the electromagnetic field via the covariant derivative

$$\tilde{\nabla}_\mu\phi \equiv \nabla_\mu\phi + iqA_\mu\phi\,, \tag{2}$$

where q is the coupling constant and A_μ the electromagnetic 4-potential.

We consider the following potential of the scalar field

$$V(|\phi|) = \mu^2|\phi|^2(1-2\lambda|\phi|^2)^2\,, \tag{3}$$

where μ is the mass term and λ the self-interaction parameter.

Q-Hairy Black Holes

Q-hairy BHs are static, spherically symmetric solutions to the EMS model **that support scalar hair** [1]. Recent simulations in spherical symmetry show these solutions can be stable and form dynamically [2].

To generate these solutions we take the following ansatz:

$$A = U(r)dt, \qquad \phi = \Phi(r)e^{-i\omega t}, \qquad ds^2 = -e^{2\mathcal{F}_0}\frac{S_0^2}{S_1^2}dt^2 + e^{2\mathcal{F}_1}S_1^4 d\Sigma^2, \tag{4}$$

where $U(r)$ is the electric potential, ω is the scalar field frequency and $\Phi(r)$ its radial profile, $S_0 = 1 - r_h/r$ and $S_1 = 1 + r_h/r$ with r_h the horizon radius. $\mathcal{F}_0(r)$ and $\mathcal{F}_1(r)$ are unknown metric functions.

Substituting the ansatz into the field equations yields families of solutions labeled by r_h and ω that exhibit a **two-branch structure**: for each (ω, r_h) there are two possible solutions, lower and upper branch, merging at a single point. This is shown in the plot below.

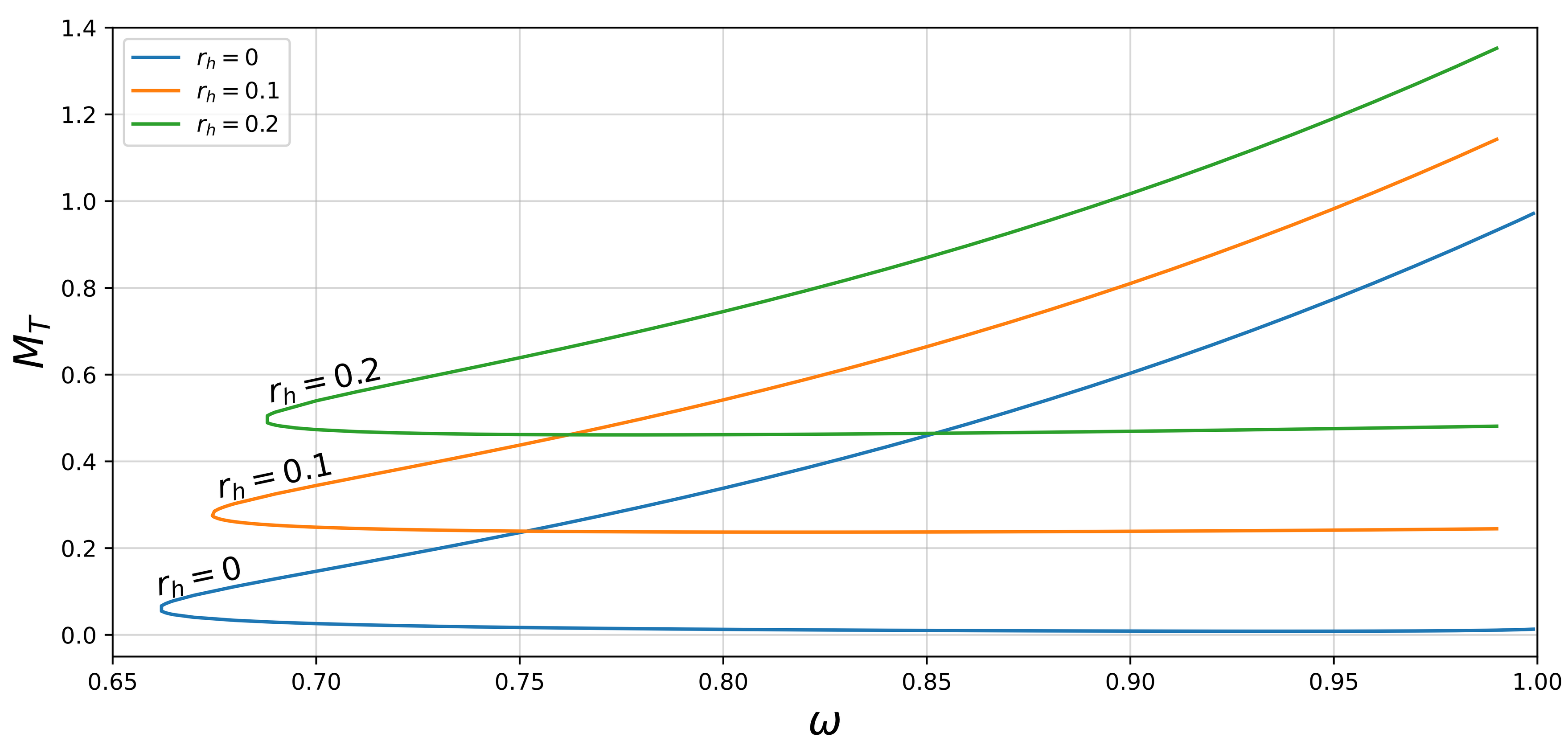

Figure 1. Total mass M_T versus the frequency of the scalar field ω for different values of horizon radius r_h.

Evolutions

We evolved Q-hairy BHs, without symmetry constraints, for different values of (ω, r_h). Our simulations show that **all solutions are unstable** and have **two distinct outcomes: Fission or Absorption**. The snapshots below illustrate the key stages of each outcome.

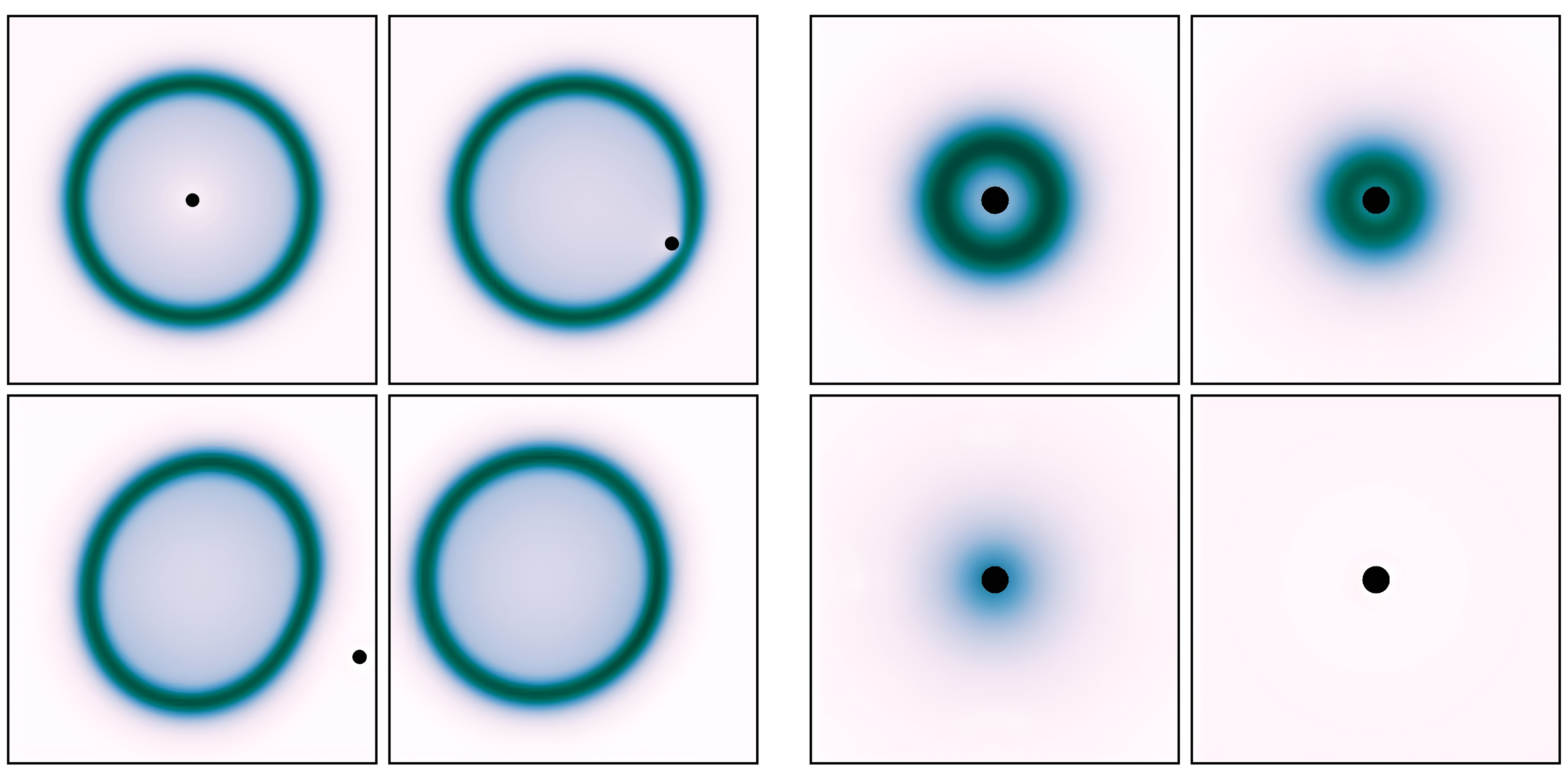

Figure 2. Key stages in the two possible outcomes: *Fission* (left), where the BH is ejected by the hair, and *Absorption* (right), where the BH absorbs the hair.

Instability Timescales

Since all solutions are unstable, **we characterize instability timescales by defining** τ_i **and** τ_f for each scenario. For *Fission*, τ_i (τ_f) marks the instant of time when the BH crosses the radius enclosing 1% (99%) of the cloud's total charge. For *Absorption*, τ_i (τ_f) correspond to the instant of time when the cloud has 99% (1%) of its initial charge.

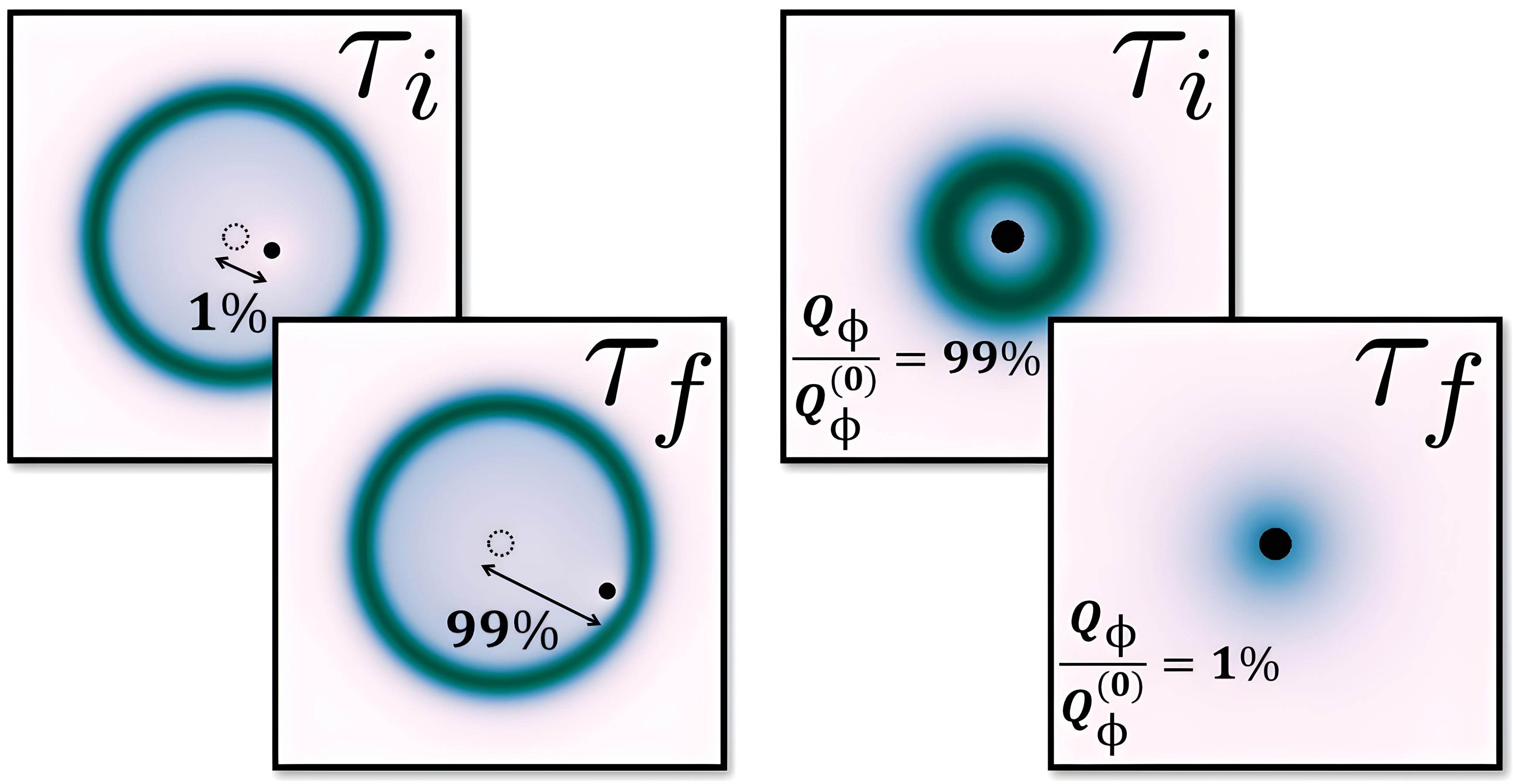

Figure 3. Definition of τ_i and τ_f for the *Fission* scenario (left) and the *Absorption* scenario (right).

In the figures below we can see the timescales for the *Fission* scenario. We can see that τ_i does not vary with ω for solutions in the upper branch and for solutions in the lower branch it decreases with ω. As for $\Delta\tau$, it increases with ω in the lower branch and has a non-monotonic pattern in the upper branch.

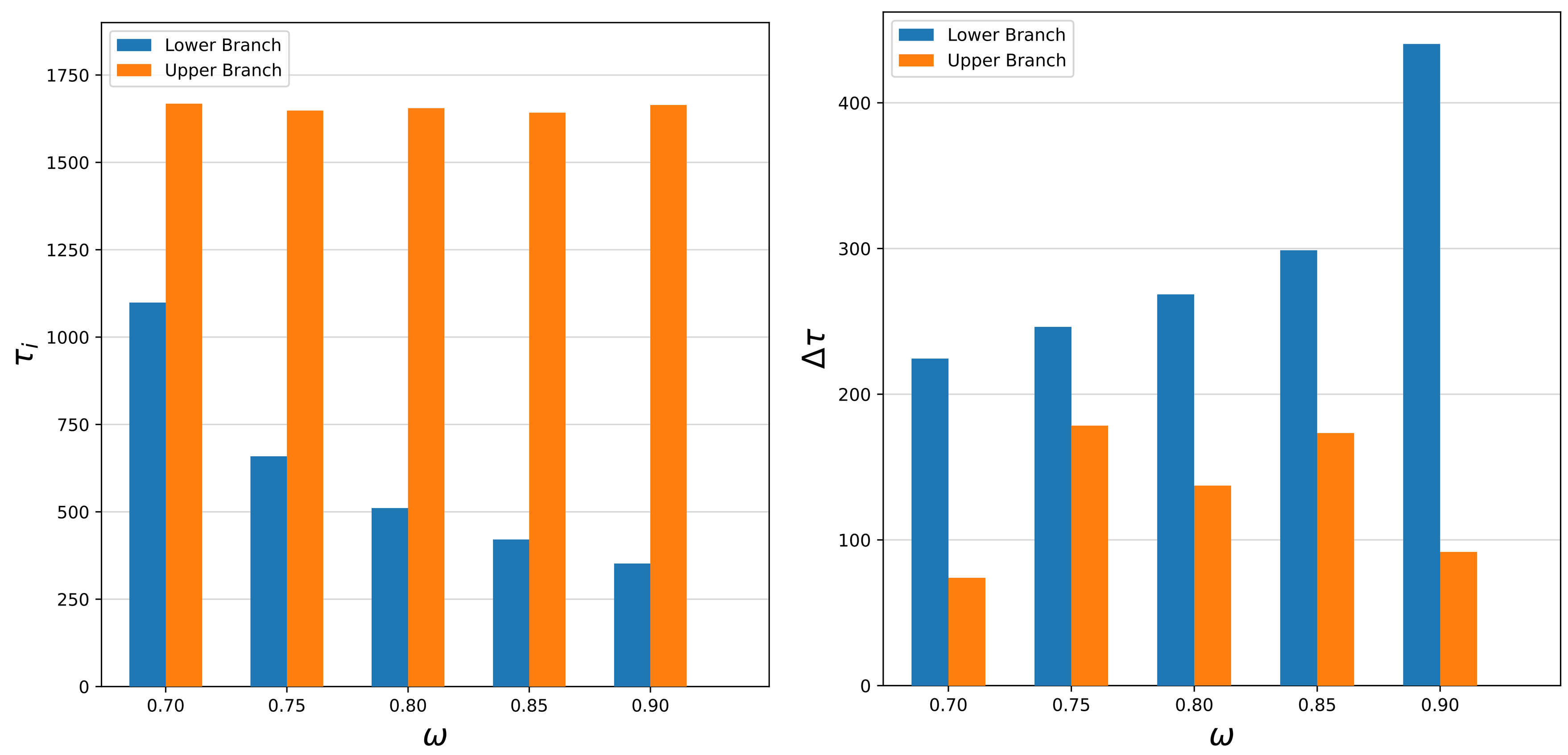

Figure 4. τ_i (left) and $\Delta\tau$ (right) vs ω for $r_h = 0.2$ in the *Fission* scenario.

Absorption only takes place in solutions which have more compact hair, which are all located in the lower branch. By fixing ω and varying r_h, we can see that both τ_i and $\Delta\tau$ decrease with r_h.

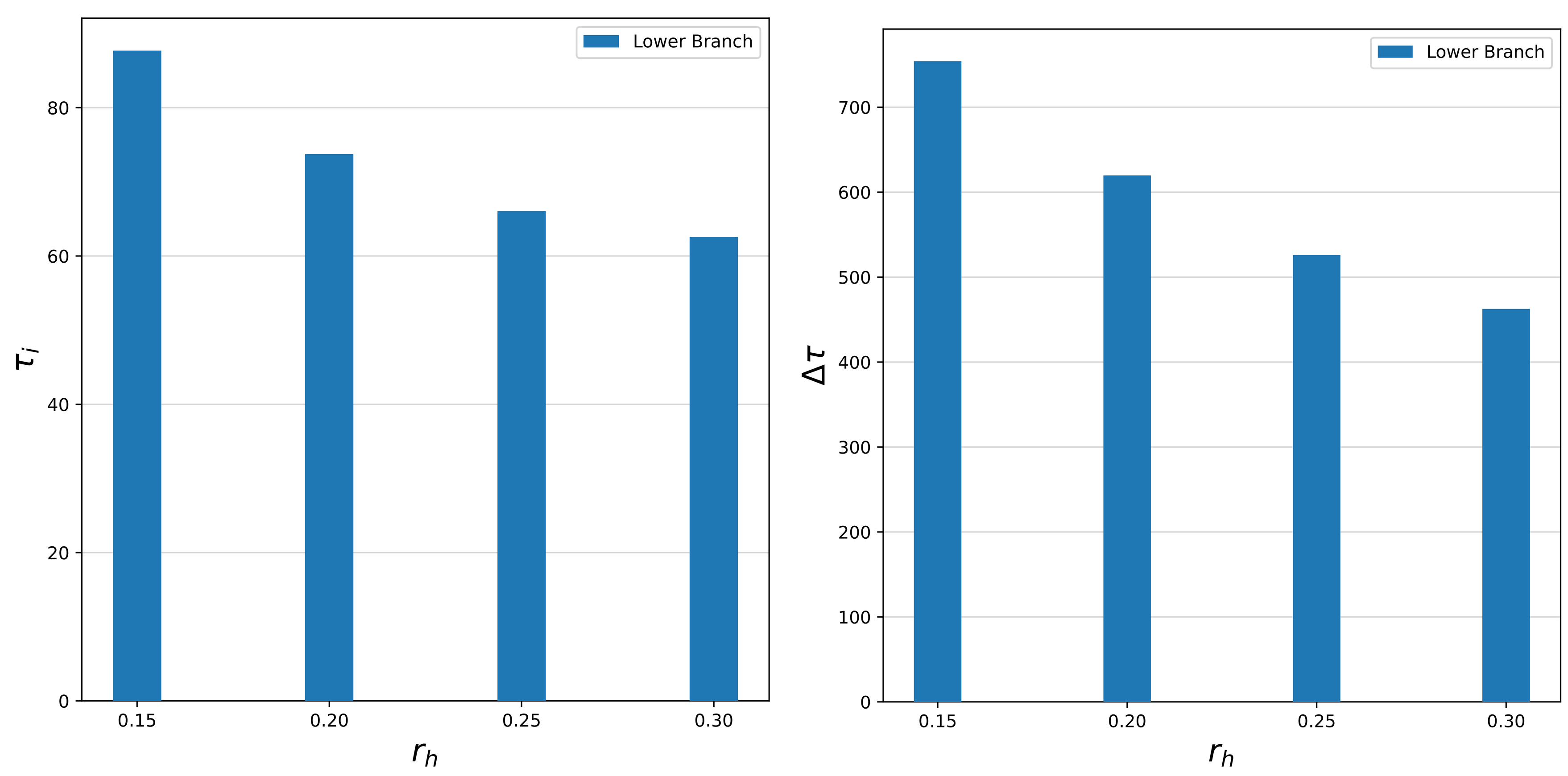

Figure 5. τ_i (left) and $\Delta\tau$ (right) vs r_h for $\omega = 0.95$ in the *Absorption* scenario.

Conclusions

We evolved Q-Hairy BHs without imposing any symmetries. **All configurations exhibited an instability**, some driven by a non-spherically symmetric mode. The instabilities arise due to the delicate balance between gravitational attraction and electromagnetic repulsion. We correlated the timescales of the instabilities with the parameters of the solution.

References

[1] Carlos A. R. Herdeiro and Eugen Radu. "Spherical electro-vacuum black holes with resonant, scalar Q-hair". In: *The European Physical Journal C* 80.5 (Apr. 1, 2020). ISSN: 1434-6052.

[2] Cheng-Yong Zhang et al. "Nonlinear self-interaction induced black hole bomb". In: *Phys. Rev. D* 110 (4 Aug. 2024), p. L041505.

The authors would like to acknowledge financial support from HORIZON-MSCA2021-SE-01 Grant No. NewFunFiCO-101086251 and from CIDMA through FCT under the Multi-Annual Financing Program for R&D Units, PTDC/FIS-AST/3041/2020, 2022.04560.PTDC and 2024.05617.CERN