

Shaping the chameleon

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In conversation with C. Burrage

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Introduction

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I. INTRODUCTION

$$\phi = \begin{cases} \phi_c & 0 < r < S, \\ \frac{\rho r^2}{6m} + \frac{c}{r} + d & S < r < R, \\ \frac{\rho \alpha}{r} + \phi_\infty & R < r \end{cases} \quad (1.1a)$$

Matching value and derivative at $r = S$:

$$\phi_c = \frac{\rho S^2}{6m} + \frac{c}{S} + d, \quad 0 = \frac{\rho S}{3m} - \frac{c}{S^2} \quad (1.2)$$

Matching value and derivative at $r = R$:

$$\frac{\rho R^2}{6m} + \frac{c}{R} + d = \frac{\alpha}{R} + \phi_\infty, \quad \frac{\rho R}{3m} - \frac{c}{R^2} = -\frac{\alpha}{R^2}. \quad (1.3)$$

We thus combine to obtain

$$c = \frac{\rho S^3}{3m}, \quad \alpha = \frac{\rho}{3m} (S^3 - R^3), \quad d = \phi_c - \frac{\rho S^2}{2m}. \quad (1.4)$$

Now, let all quantities acquire an angular perturbation,

$$X \rightarrow X_0 + \varepsilon X_1(\theta). \quad (1.5)$$

Collecting like powers of ε yields

$$\frac{\alpha_1}{\alpha_0} = \frac{\rho_1}{\rho_0} + \frac{3}{1 - (S_0/R_0)^3} \left[\frac{R_1}{R_0} - \left(\frac{S_0}{R_0} \right)^2 \frac{S_1}{R_0} \right]. \quad (1.6)$$

Note that α will be the magnitude of the radial component of the chameleon force outside the source. Note that if $S_0 \approx R_0$ then the denominator above will become very small, meaning that α_1/α_0 could become rather large.

The volume of the thin shell is

$$\mathcal{V} = \frac{4}{3}\pi (R^3 - S^3). \quad (1.7)$$

Hence,

$$\mathcal{V} = \mathcal{V}_0 + 4\pi R_0^3 \left[\frac{R_1}{R_0} - \left(\frac{S_0}{R_0} \right)^2 \frac{S_1}{R_0} \right] \quad (1.8)$$

Does this make sense? volume is an integrated quantity, and here we have angular-dependent terms on the RHS

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