Shaping the chameleon

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In conversation with C. Burrage

Contents

I. INTRODUCTION

$$\phi = \begin{cases} \phi_{\rm c} & 0 < r < S, \\ \frac{\rho r^2}{6m} + \frac{c}{r} + d & S < r < R, \\ \frac{\alpha}{r} + \phi_\infty & R < r \end{cases} \tag{1.1a}$$

Matching value and derivative at $r = S$:

$$\phi_{\rm c} = \frac{\rho S^2}{6m} + \frac{c}{S} + d, \qquad 0 = \frac{\rho S}{3m} - \frac{c}{S^2} \tag{1.2}$$

Matching value and derivative at $r = R$:

$$\frac{\rho R^2}{6m} + \frac{c}{R} + d = \frac{\alpha}{R} + \phi_\infty, \qquad \frac{\rho R}{3m} - \frac{c}{R^2} = -\frac{\alpha}{R^2}. \tag{1.3}$$

We thus combine to obtain

$$c = \frac{\rho S^3}{3m}, \qquad \alpha = \frac{\rho}{3m}\left(S^3 - R^3\right), \qquad d = \phi_{\rm c} - \frac{\rho S^2}{2m}. \tag{1.4}$$

Now, let all quantities aquire an angular perturbation,

$$X \to X_0 + \varepsilon X_1(\theta). \tag{1.5}$$

Collecting like powers of ε yields

$$\frac{\alpha_1}{\alpha_0} = \frac{\rho_1}{\rho_0} + \frac{3}{1 - (S_0/R_0)^3}\left[\frac{R_1}{R_0} - \left(\frac{S_0}{R_0}\right)^2 \frac{S_1}{R_0}\right]. \tag{1.6}$$

Note that α will be the magnitude of the radial component of the chameleon force outside the source. Note that if $S_0 \approx R_0$ then the denominator above will become very small, meaning that α_1/α_0 could become rather large.

In spherical polar coordinates $x^i = (r, \theta, \chi)$, the force is

$$\mathbf{F} = -\nabla\phi = -\left(\frac{\partial\phi}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial\phi}{\partial\theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial\phi}{\partial\chi}\hat{\boldsymbol{\chi}}\right). \tag{1.7}$$

Write $F = |\mathbf{F}|$. Then,

$$F^2 = F_r^2 + F_\theta^2 + F_\chi^2, \qquad F_r \equiv \frac{\partial\phi}{\partial r}, \qquad F_\theta \equiv \frac{1}{r}\frac{\partial\phi}{\partial\theta}, \qquad F_\chi \equiv \frac{1}{r\sin\theta}\frac{\partial\phi}{\partial\chi}. \tag{1.8}$$

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Since we have $\alpha = \alpha_0 + \varepsilon\alpha_1$, we have the corresponding decomposition of ϕ:

$$\phi = \phi_\infty + \frac{\alpha_0 + \varepsilon\alpha_1(\theta,\chi)}{r}. \tag{1.9}$$

Hence, writing $\phi = \phi_0 + \varepsilon\phi_1$, we identify

$$\phi_0 = \phi_\infty + \frac{\alpha_0}{r}, \qquad \phi_1 = \frac{\alpha_1(\theta,\chi)}{r}. \tag{1.10}$$

The only non-zero contribution to the force at zeroth-order in ε is

$$F_r^{(0)} = \frac{\partial\phi_0}{\partial r} = -\frac{\alpha_0}{r^2}. \tag{1.11}$$

All contributions to the force are non-zero at first order in ε; they are

$$F_r^{(1)} = \frac{\partial\phi_1}{\partial r} = -\frac{\alpha_1}{r^2}, \qquad F_\theta^{(1)} = \frac{1}{r^2}\frac{\partial\alpha_1}{\partial\theta}, \qquad F_\chi^{(1)} = \frac{1}{r^2\sin\theta}\frac{\partial\alpha_1}{\partial\chi}. \tag{1.12}$$

Now,

$$\frac{\partial\alpha_1}{\partial\psi} = \alpha_0\left(\frac{1}{\rho_0}\frac{\partial\rho_1}{\partial\psi} + \frac{3}{1-\left(S_0/R_0\right)^3}\left[\frac{1}{R_0}\frac{\partial R_1}{\partial\psi} - \left(\frac{S_0}{R_0}\right)^2\frac{1}{R_0}\frac{\partial S_1}{\partial\psi}\right]\right), \qquad \psi\in\{\theta,\chi\}. \tag{1.13}$$