Shaping the chameleon

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(Dated: November 11, 2014)

I. INTRODUCTION

The source density is $\rho(\mathbf{x})$. Different geometrical shapes of this source (such as ellipses, rectangles, triangles, etc) will yield different gravitational potentials, and will alter how the chameleon scalar responds.

The effective potential for the chameleon is

$$V_{\text{eff}} = \frac{\Lambda^5}{\phi} + \frac{\rho(\mathbf{x})}{M}\phi. \tag{1.1}$$

The equations governing the chameleon scalar and gravitational potential are

$$\nabla^2\phi = \frac{\mathrm{d}V_{\text{eff}}}{\mathrm{d}\phi}. \tag{1.2a}$$

$$\nabla^2\Phi = -\rho(\mathbf{x})/2M_{\text{pl}}^2 \tag{1.2b}$$

The force on a test particle, due to the gravitational and chameleon scalars are given by

$$\mathbf{F}_{(\phi)} = -\tfrac{1}{M}\nabla\phi, \qquad \mathbf{F}_{(\Phi)} = -\nabla\Phi. \tag{1.3}$$

The idea is to obtain the source-shape that maximises the force that a test particle will feel, that cannot be explained by forces of a purely gravitational origin.

II. EXPLANATION OF NUMERICAL METHODS

Discretize onto a grid. Using successive over relaxation and gradient flow.

The equations implemented by our numerical methods are dimensionless. By setting

$$\phi = \sqrt{M\Lambda^5}\,\tilde{\phi}, \tag{2.1a}$$

$$\Phi = \frac{M}{M_{\text{Pl}}^2}\sqrt{M\Lambda^5}\,\tilde{\Phi}, \tag{2.1b}$$

$$x^\mu = \sqrt{M}\left(M\Lambda^5\right)^{1/4}\tilde{x}^\mu, \tag{2.1c}$$

the equations become

$$\tilde{\nabla}^2\tilde{\phi} = -\frac{1}{\tilde{\phi}^2} + \rho, \tag{2.2a}$$

$$\tilde{\nabla}^2\tilde{\Phi} = -\frac{1}{2}\rho, \tag{2.2b}$$

and the ratio of the forces becomes

$$\frac{\left|\mathbf{F}_{(\phi)}\right|}{\left|\mathbf{F}_{(\Phi)}\right|} = \left(\frac{M_{\text{Pl}}}{M}\right)^2 \frac{\left|\tilde{\nabla}\tilde{\phi}\right|}{\left|\tilde{\nabla}\tilde{\Phi}\right|} \tag{2.3}$$

A. Solving for the Chameleon

B. Solving Poisson's equation

We have found conventional relaxation methods cumbersome and difficult to use for complicated shapes, mainly due to the nature of the boundary conditions. We found it much more convenient (as well as being increible computationally-cheap) to compute the gravitational force, $\mathbf{F}_{(\Phi)}$ at a given location, $\mathbf{x}$, via

$$\mathbf{F}_{(\Phi)}(\mathbf{x}) = -\frac{1}{8\pi M_{\text{Pl}}^2}\sum_i m_i \frac{\mathbf{x}-\mathbf{x}_i}{\left|\mathbf{x}-\mathbf{x}_i\right|^3}, \tag{2.4}$$

where $\{m_i, \mathbf{x}_i\}$ are the mass and location of the i^{th} constituent of the source.

III. RESULTS

IV. DISCUSSION