Shaping the chameleon

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I. INTRODUCTION

The source density is $\rho(\mathbf{x})$. Different geometrical shapes of this source (such as ellipses, rectangles, triangles, etc) will yield different gravitational potentials, and will alter how the chameleon scalar responds.

The effective potential for the chameleon is

$$V_{\rm eff} = \frac{\Lambda^5}{\phi} + \frac{\rho(\mathbf{x})}{M}\phi + \frac{1}{2}m^2\left(\phi - \phi_\infty\right)^2. \tag{1.1}$$

The equations governing the chameleon scalar and gravitational potential are

$$\nabla^2\phi = \frac{{\rm d}V_{\rm eff}}{{\rm d}\phi}. \tag{1.2a}$$

$$\nabla^2\Phi = -\rho(\mathbf{x})/2M_{\rm pl}^2 \tag{1.2b}$$

The force on a test particle, due to the gravitational and chameleon scalars are given by

$$\mathbf{F}_{(\phi)} = -\tfrac{1}{M}\nabla\phi, \qquad \mathbf{F}_{(\Phi)} = -\nabla\Phi. \tag{1.3}$$

The idea is to obtain the source-shape that maximises the force that a test particle will feel, that cannot be explained by forces of a purely gravitational origin.

We construct the function $\rho(\mathbf{x})$ such that

$$\rho = \begin{cases} \rho_0 & \text{inside object,} \\ 0 & \text{outside object.} \end{cases} \tag{1.4}$$

II. EXPLANATION OF NUMERICAL METHODS

Discretize onto a grid. Using successive over relaxation and gradient flow.

The equations implemented by our numerical methods are dimensionless. By setting

$$\phi = \sqrt{M\Lambda^5}\,\tilde{\phi}, \tag{2.1a}$$

$$\Phi = \frac{M}{M_{\rm Pl}^2}\sqrt{M\Lambda^5}\,\tilde{\Phi}, \tag{2.1b}$$

$$x^\mu = \sqrt{M}\left(M\Lambda^5\right)^{1/4}\tilde{x}^\mu, \tag{2.1c}$$

$$\tilde{m} = mM^{1/2}\left(\Lambda^5 M\right)^{1/3}, \tag{2.1d}$$

the effective potential (1.1) becomes

$$\frac{M}{\sqrt{M\Lambda^5}}V_{\rm eff} = \frac{1}{\tilde{\phi}} + \rho\tilde{\phi} + \frac{1}{2}\tilde{m}^2\left(\tilde{\phi} - \tilde{\phi}_\infty\right)^2, \tag{2.2}$$

and the equations (1.2) become

$$\tilde{\nabla}^2\tilde{\phi} = -\frac{1}{\tilde{\phi}^2} + \rho + \tilde{m}^2\left(\tilde{\phi} - \tilde{\phi}_\infty\right), \tag{2.3a}$$

$$\tilde{\nabla}^2\tilde{\Phi} = -\frac{1}{2}\rho. \tag{2.3b}$$

The ratio of the forces becomes

$$\frac{\left|\mathbf{F}_{(\phi)}\right|}{\left|\mathbf{F}_{(\Phi)}\right|} = \left(\frac{M_{\rm Pl}}{M}\right)^2\frac{\left|\tilde{\nabla}\tilde{\phi}\right|}{\left|\tilde{\nabla}\tilde{\Phi}\right|} \tag{2.4}$$

A. Solving for the Chameleon

We obtain the profile of the chameleon scalar around a given source profile $\rho(\mathbf{x})$ using a gradient flow technique. This works by taking a initial guess for the scalar fields profile, and letting it "relax" into a profile which corresponds to a solution of the equation of motion.

B. Solving Poisson's equation

We have found conventional relaxation methods cumbersome and difficult to use for complicated shapes, mainly due to the nature of the boundary conditions (trying to put an ellipsoidal "peg" into a square "box" will always be problematic). We found it much more convenient, and incredibly computationally-cheap, to compute the gravitational force, $\mathbf{F}_{(\Phi)}$ at a given location, $\mathbf{x}$, via the simple expression

$$\mathbf{F}_{(\Phi)}(\mathbf{x}) = -\frac{1}{8\pi M_{\rm Pl}^2}\sum_i m_i\frac{\mathbf{x}-\mathbf{x}_i}{\left|\mathbf{x}-\mathbf{x}_i\right|^3}, \tag{2.5}$$

where $\{m_i, \mathbf{x}_i\}$ are the mass and location of the $i^{\rm th}$ constituent of the source. One may feel uncomfortable at the explicitly discrete nature of the assumed distribution of the gravitating source. However, in any numerical solution of a field theory some kind of discretization scheme is used, and this places any fields onto the vertices of a lattice. This manifestly generates a discrete distribution: one aims to make the lattice spacing as small as possible, in order to best model a continuous field.

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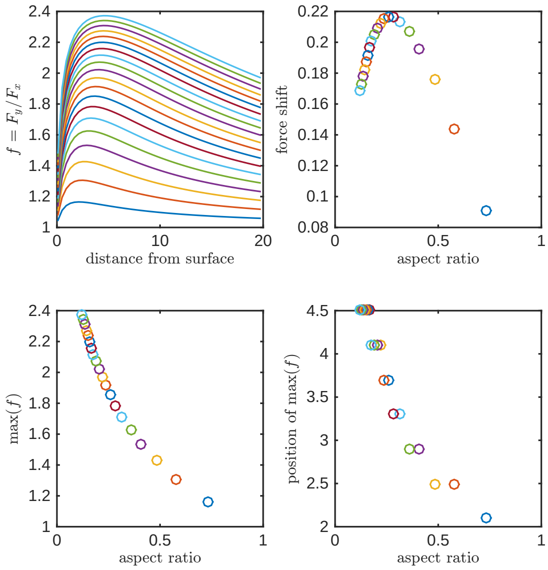

Figure 1: Interesting diagnostic quantities for the gravitational field around a squashed ellipse. The colour scheme between the panels is consistent. In the top-right panel we show the ratio f of the force down the x- and y-axes as a function of distance from the surface; each colour corresponds to a different value of the aspect ratio of the ellipse. The actual values of the aspect ratios is given in the top-left panel, which shows the "force shift". This is the fractional difference between the maximum value of the force ratio, and the force ratio at the furthest distance from the surface shown. In the bottom-left panel we show the value of the maximum force ratio, as a function of the aspect ratio; in the bottom-right panel we show the position of the maximum force ratio.

III. RESULTS

A. Properties of the gravitational force around different source geometries

The Newtonian gravitational force (2.5) is one of the simplest quantities one can compute, and its "far-field" properties are very well understood. However, we are interested in the "near-field" properties of the gravitational force: that is, how the gravitational force behaves near the source object.

Creating a shape via

$$r(\theta) = \sum_{\ell=0}^{n} a_\ell P_\ell(\cos\theta) \tag{3.1}$$

In Table I we give the values of the coefficients a_ℓ required to generate the apple shape.

IV. DISCUSSION

Coefficient	Value
a_0	0.3482169
a_1	0.0969634
a_2	-0.0450812
a_3	0.0346095
a_4	-0.0304927
a_5	0.0276200
a_6	-0.0245809
a_7	0.0209728
a_8	-0.0168116
a_9	0.0123419
a_{10}	-0.0079524
a_{11}	0.0041198
a_{12}	-0.0013419

TABLE I: Values of the coefficients in the Legendre expansion required to generate the "apple" shape.