BIOC 455/555
Fall 2016
Homework # 9
Due at the beginning of class on Thursday, November 17th.

Consider a constitutively produced protein that decays exponentially, so that the protein number obeys the following stochastic differential equation (SDE):

$$\dot{x} = \beta - \gamma x + \tilde{\eta}(t), \tag{1}$$

where β and γ are positive constants, and $\tilde{\eta}(t)$ is a Gaussian noise term with $\langle \tilde{\eta}(t) \rangle = 0$ and $\langle \tilde{\eta}(t)\tilde{\eta}(t') \rangle = 2D\delta(t - t')$.

> (1) First, find the fixed point in the absence of noise. Next, find the SDE describing the first order dynamics of small perturbations, $\epsilon(t)$, about the fixed point of Eq. 1. Assume that the noise term is small, of order $O(\epsilon)$.
>
> (2) Find the mean and variance of $\epsilon(t)$.

Now consider the case where the protein above negatively down-regulates its own production, such that the protein number now obeys the SDE:

$$\dot{x} = \frac{\alpha}{1 + x/c} - \gamma x + \tilde{\eta}(t), \tag{2}$$

where α, γ, and c are positive constants, and $\tilde{\eta}(t)$ is a Gaussian noise term with $\langle \tilde{\eta}(t) \rangle = 0$ and $\langle \tilde{\eta}(t)\tilde{\eta}(t') \rangle = 2D\delta(t - t')$.

> (3) First, find the fixed point in the absence of noise. Next, find the SDE describing the first order dynamics of small perturbations, $\epsilon(t)$, about the fixed point of Eq. 2. Assume that the noise term is small, of order $O(\epsilon)$.
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> (4) Find the mean and variance of $\epsilon(t)$.