Exam 2
BIOC 455/555
Fall 2016

Instructions:

1. Please print your name on each page.
2. Show all work.
3. Circle your final answer for each question.
4. You may NOT work together, but you may use your class notes.
5. You have a maximum of 4 hours to work on this exam.
6. This take-home exam is due Thursday, Nov. 10^{th} at the beginning of class.

Print name:

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Question #1:	________	/ 30
Question #2:	________	/ 30
Question #3:	________	/ 20
Question #4:	________	/ 20
Total:	________	**/ 100**

1. Consider the probability density function

$$P(x) = \begin{cases} a(1 - \frac{x}{c}) & 0 \leq x \leq c \\ 0 & x > c, \end{cases}$$

where a and c are positive real constants and $x \in [0, \infty)$.

(a) Find a in terms of c. (*10 points*)

(b) Find $\langle x \rangle$ and $\langle x^2 \rangle$ in terms of c for this distribution. (*20 points*)

2. Consider a stationary phase cell (*i.e.* it is not growing or dividing) in which a certain protein is produced with an average rate of 90 proteins/min. Meanwhile, a diffusion-limited protease degrades this protein so that it has a half-life of 20 mins. Assume that at time $t = 0$ there are 100 proteins inside the cell.

(a) Assume you are simulating this process with the Gillespie algorithm. What is the PDF that describes the distribution of waiting times until the next reaction (of either type)? (*10 points*)

(b) On average, at what time will the number of proteins next change? (*10 points*)

2. (*cont.*)

(c) What is the probability that, when it does change, the number will decrease? (*10 points*)

3. Consider the exponential distribution, $P(x) = \mu e^{-\mu x}$, where $\mu > 1$ and $x \geq 0$.

(a) Find the moment generating function for the exponential distribution. In other words, find $\langle e^{\lambda x} \rangle$, where λ is the moment generating parameter. (*10 points*)

3. (*cont.*)

(b) Use the moment generating function to find $\langle x^0 \rangle$, $\langle x^2 \rangle$, and $\langle x^4 \rangle$ for the exponential distribution given above. (*10 points*)

4. In class we showed that the constitutive production and exponential decay of mRNA could be modeled by a master equation of the form:

$$\begin{aligned} \frac{dP_0}{dt} &= -\lambda P_0 + \gamma P_1 \\ \frac{dP_n}{dt} &= \lambda\left(P_{n-1} - P_n\right) + \gamma\left[(n+1)P_{n+1} - nP_n\right] \qquad n \geq 1, \end{aligned}$$

where $P_n(t)$ is the probability of having n mRNA molecules at time t, λ is the constant rate of production, and $\ln(2)/\gamma$ is the cell cycle time. Starting from this master equation, show that:

$$\frac{d\langle n\rangle}{dt} = \lambda - \gamma\langle n\rangle,$$

where $\langle n\rangle$ is the average number of mRNA molecules. (*20 points*)

HINT: Recall that

$$\langle n\rangle = \sum_{n=0}^{\infty} nP_n.$$

4. (*cont.*) extra workspace…