

**Definition:** When  $A$  and  $B$  are sets, we say any subset of  $A \times B$  is a **binary relation**. A relation  $R$  can also be represented as

- A function  $f_{TF} : A \times B \rightarrow \{T, F\}$  where, for  $a \in A$  and  $b \in B$ ,  $f_{TF}( (a, b) ) = \begin{cases} T & \text{when } (a, b) \in R \\ F & \text{when } (a, b) \notin R \end{cases}$
- A function  $f_{\mathcal{P}} : A \rightarrow \mathcal{P}(B)$  where, for  $a \in A$ ,  $f_{\mathcal{P}}(a) = \{b \in B \mid (a, b) \in R\}$

When  $A$  is a set, we say any subset of  $A \times A$  is a (binary) **relation** on  $A$ .