

# Forcing a meaningful future: Brain-spine-muscle interfaces

By Jackson Powell  
For future Jackson

Once you know the way broadly, you can see it in all things.  
– The Book of Five Rings by Miyamoto Musashi

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# Contents

0.1	Overview . . . . .	1
0.1.1	Purpose . . . . .	1
0.1.2	Content . . . . .	1
<b>I Electronics</b>		<b>2</b>
<b>1</b>	<b>Background Information</b>	<b>3</b>
1.0.1	Overview . . . . .	3
1.1	How Electricity Works . . . . .	3
1.1.1	The misconception . . . . .	3
1.1.2	Nuances in the fields . . . . .	4
1.2	Laws and Devices . . . . .	4
1.2.1	Thevenin and Ideality . . . . .	5
1.2.2	LED Circuits and PNP . . . . .	7
1.3	Resistor Lattice Digression . . . . .	8
<b>2</b>	<b>Capacitors</b>	<b>14</b>
2.1	Overview . . . . .	14
2.2	Filtration . . . . .	15
2.2.1	Integration and Differentiation . . . . .	15
2.3	Frequency Dependence . . . . .	17
2.3.1	Imaginary Numbers Digression. . . . .	17
2.4	Impedance . . . . .	18
<b>3</b>	<b>Writing Hardware</b>	<b>20</b>
3.1	Introduction . . . . .	20
3.1.1	Creating Modules . . . . .	20
3.1.2	Bus Notation . . . . .	21
3.2	Constants and Variables . . . . .	22
<b>II Math and Models</b>		<b>23</b>
<b>4</b>	<b>Moving Through States</b>	<b>24</b>
4.1	Markov-chains . . . . .	24
4.2	Diffusion . . . . .	24
4.2.1	Forward Euler's . . . . .	24
4.2.2	Backward Euler's . . . . .	25

<b>5</b>	<b>Modeling Circuits</b>	<b>27</b>
	<b>III Physiology</b>	<b>29</b>
5.1	Perspective . . . . .	30
<b>6</b>	<b>Muscles</b>	<b>31</b>
6.1	Skeletal Muscle . . . . .	31
6.1.1	A Cellular Level . . . . .	31
6.1.2	Nervous system control . . . . .	32
6.2	Smooth Muscle . . . . .	32
6.2.1	A Cellular Level . . . . .	32
6.2.2	Nervous system control . . . . .	33
6.3	Cardiac Muscle . . . . .	33
6.3.1	A Cellular Level . . . . .	33
6.3.2	Nervous system control . . . . .	33
6.3.3	Pacemakers . . . . .	33
<b>7</b>	<b>The Spinal Cord</b>	<b>34</b>
7.1	General Structure . . . . .	34
7.2	Autonomic Nervous System . . . . .	34
7.2.1	Sympathetic Nervous System . . . . .	34
7.2.2	Parasympathetic Nervous System . . . . .	35
7.3	Acetylcholine, Nicotinic Receptors, and Muscarinic Receptors . . . . .	35
7.4	Central Pattern Generation . . . . .	36
	<b>IV Spinal Cord Injury</b>	<b>37</b>
<b>8</b>	<b>The Injury Itself</b>	<b>38</b>
8.1	Cell Specific Responses . . . . .	38
8.1.1	Immune Response . . . . .	38
8.1.2	Neural Response . . . . .	39
8.2	Side comments . . . . .	40
<b>9</b>	<b>In The Clinic &amp; Therapeutic Approaches</b>	<b>42</b>
9.0.1	Electrical Stimulation . . . . .	42
9.0.2	Biomaterials . . . . .	43
9.0.3	Drug Treatment . . . . .	44
9.0.4	Surgery . . . . .	44
9.0.5	Rehabilitation . . . . .	44
9.1	Integrated Approach . . . . .	44
	<b>V Brain-Spine-Muscle Interfaces</b>	<b>45</b>
<b>10</b>	<b>Biochemistry</b>	<b>46</b>
10.1	Ion Channels . . . . .	46

<b>11 The Technology of SCI</b>	<b>47</b>
11.1 Historical Dealings . . . . .	47
11.2 Reading Thoughts . . . . .	47
11.2.1 Different Reading Devices . . . . .	48
11.3 Lorach and the Courtine Group . . . . .	49
11.3.1 Overview . . . . .	49
11.3.2 Shortcomings . . . . .	49

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## 0.1 Overview

### 0.1.1 Purpose

The purpose of this book is to aggregate all of the content and tools I will need in forging the future I desire: building better brain-spine-muscle interfaces.

### 0.1.2 Content

It is true that total recovery from spinal cord injury, in sever cases, will require both therapeutic and electronic intervention. One must therefore learn of physiology and electronics to be able to solve these problems. I will cover, in some depth, both the electronics, math, and physiology I think I'll need. While programming will likely be an essential tool in this endeavor, I will not discuss programming beyond Verilog (which is an aspect of electronics). This is because programming is something you learn by doing, and writing out C++ algorithms in a PDF would be a waste of time!

# Part I

## Electronics

Like many of the vagabonds who live in the fields, stray horses seemed to him to be good-natured things. When you're through with them, they ask for nothing; they just go off quietly somewhere by themselves.

– Musashi by Eiji Yoshikawa

# Chapter 1

## Background Information

### 1.0.1 Overview

The bulk of this information comes from the Lab Electronics course at the University of Pennsylvania, taught to me by Professor Ashmanskas<sup>1,2</sup>. As a comment on notation:  $V$  represents a variable value, while  $V$  represents a unit (or at least, that's what it should be; sometimes I forget proper notation). Graphics like this were made using CircuitTikz, whose manual can be found [here](#).

## 1.1 How Electricity Works

### 1.1.1 The misconception

One often represents electricity with the canonical metaphor of water flowing in a tube, equating the flow of water, which powers a wheel of some sort, as being the equivalent of electrons pushing through a wire. Incidentally, this simplifying schema illustrates a key misconception in the nature of electricity. This is easiest exemplified in the mode with which electricity reaches one's house from a power-plant, which before arriving will be subject to breaks in the circuit (transformers). In the traditional viewing of electricity, that which one is taught in early education, this is disconcerting as one would think that if electrons can not physically go from battery to bulb, then how can their kinetic energy be transferred and light it?

When a battery sits without wires attached, around it is an electric field. This field does not dissipate because no electrons flow from it. When wires are attached, charge accumulates on the surface of the wires. This causes a small electric field within the wires, but the drift velocity of electrons within the wire is quite slow—nowhere near the speed of light. However, the flow of electrons within the wire is sufficient to drive an electric field which exists around the wires. From this, we can determine the direction that energy will flow by taking the cross product of the electric and magnetic fields. In fact, if a lightbulb is attached, this means that energy flows from the battery to the bulb in all directions, not through the wires itself.

Let us consider now what will happen with an AC circuit, where the electromotive force flips with each cycle. In this case, both the electric and magnetic fields switch directions, meaning that their cross

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<sup>1</sup>[https://www.hep.upenn.edu/Courses/Phys364\\_spring23/](https://www.hep.upenn.edu/Courses/Phys364_spring23/)

<sup>2</sup><http://www.hep.upenn.edu/~ashmansk/>

product will remain the same and power the lightbulb. Notably, the electrons do not move much (if at all) in this setup—but this is not a surprise, as it is not the electrons that carry the energy anyway. Now, it is still essential to recognize that it is the movement of electrons within the filament of a lightbulb that creates light. This is, indeed, from kinetic energy transferred from electrons bouncing against the metals lattice, dissipating energy in the form of light. The necessary distinction is that it is not electrons that flow all the way from the battery, but rather it is vibrations of those that were, and always will be, within the bulb itself. When you consider it like this, it is straightforward—as the electric field derived from the battery is what provides the electrons with enough kinetic energy to power the bulb.

Interestingly, comparisons to the “water flow” model fail dramatically in the traditional sense, but the Venturi Effect used to describe fluid flow actually succeeds. In adding a bulb you add a resistor, which is comparable to adding a part of a pipe with a smaller diameter. As water will flow faster in this section of the tube, so too will electrons. In order to maintain the same current as is through the rest of the tube, the drift velocity,  $V_d$ , must be higher.  $V_d$  is proportional to the electromotive force,  $E$ , meaning the force is highest within the bulb.

Of course you may say: “well then why do we use wires at all?” The answer is that wires are helpful in channeling the fields, thereby making them more efficient. Knowing all of this, in this work I will almost invariably describe the flow of current as electrons moving through a wire. This is because it is much easier to think of electricity in this way, hence the ubiquitous misconception.

### 1.1.2 Nuances in the fields

It is worth explicitly highlighting that the electric field that causes the actual flow is from charges along the wires, rather than the battery. This is notable because if this were not the case, the proximity of the bulb to the battery would dictate its brightness. One may wonder how this type of charge distribution can be established so rapidly, and the reason is that the distance an electron needs to travel in order to create such a distribution is subatomic in size—meaning that with movement at the speed of light, the time it takes to establish a surface charge is effectively zero.

The whole idea is quite unintuitive, so it is appropriate to keep it smushed in the back of your mind, and to only draw it out when encountering things that are otherwise strange—such as wireless charging, which should now be much more comprehensible.

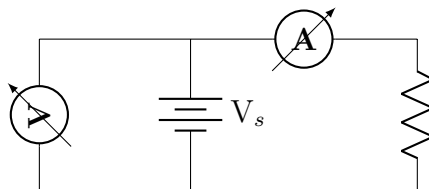
## 1.2 Laws and Devices

The currents flowing in and out of a node will always be equal. The sum of the voltages over an entire circuit will always be equal. Kirchoff’s Voltage Law (KVL) can be used to show that wires connected in parallel will have the same voltage across them. In this way, we can predict the current flowing through a circuit to be  $V = IR$ . The formula for power is given as:  $VI = P$ , which means that when one solves for voltage, and knowing that current is in units of charge/time and power in work/time, voltage is work per unit charge.

Voltage is measured using a voltmeter, a device in parallel with the load you are interested in measuring. An ammeter is used to measure current, which will be in series with the current you are trying to measure. This means that the voltmeter should have an extremely high resistance, so as to not draw



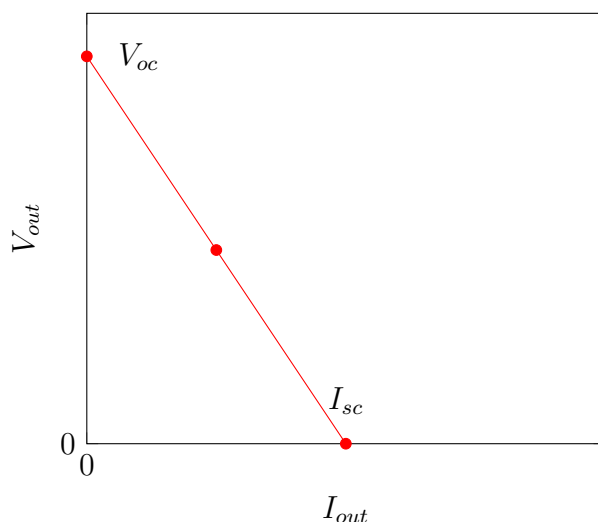
any current, and an ammeter to have a low resistance, so as to not have any voltage drop. These are important considerations, as if the resistance of the load you are measuring is large (say,  $1\text{M}\Omega$ ), it is possible that the voltmeter will have some non-negligible current flow through it. The same goes for if your circuit has very low resistance and you use an ammeter. To illustrate, one would measure the voltage and current coming from a battery as seen below:



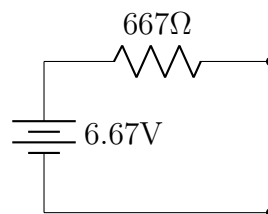
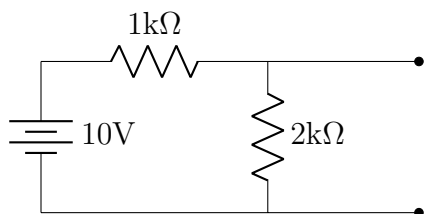
### 1.2.1 Thevenin and Ideality

A Thevenin Circuit simplifies the circuit to have a single resistance,  $R_{th}$ , and a single voltage,  $V_{th}$ .  $R_{th}$  can be calculated via replacing all of the voltage sources with a wire, and disconnecting all of the current sources. This “short circuits” your circuit and leaves you with only resistors, which can be used to calculate  $R_{th}$  using the familiar resistor rules. One can also short circuit the terminals, and determine the current flow, giving us  $R_{th} = V_{th}/I_{sc}$ .

$R_{th}$  can be measured in a circuit by varying the  $R_{load}$  added to a circuit. In this case, you will see the voltage supplied (and corresponding current) change. The slope of this change ( $\Delta V/\Delta I$ ) will equal  $R_{th}$ . If you vary the load through the two terminals and measure the voltage across it, you will get a graph that looks something like this:

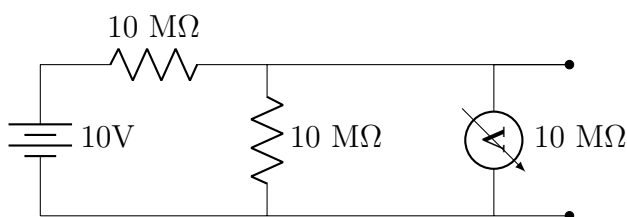


Again, the slope is what gives you  $R_{th}$ .  $I_{sc}$  is an important value which allows you to calculate  $V_{th}$ . The current that flows when you short circuit the load ( $I_{sc}$ ), multiplied by  $R_{th}$ , gives you  $V_{th}$ .



## The ideality of sources.

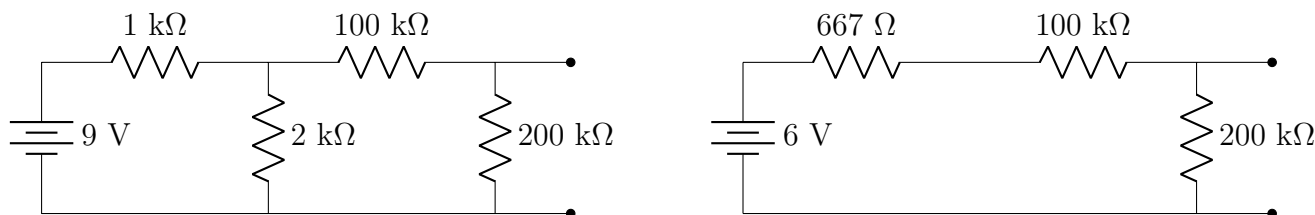
This coaxes us lightly into the topic of source ideality. Imagine all voltage or current sources as having a resistor in parallel with it, but inside of the component itself. An ideal battery, or voltage source, would be able to drive the same voltage, irrespective of the resistor/current. This would be like having a battery whose internal resistance is 0, causing the entirety of the voltage drop to occur on the circuit fragments outside of the battery. In the real world, batteries are not ideal. The canonical illustration of a batteries ideality is in trying to use a 9V battery to start your car. Naturally, the voltage dwindles as the current supplied increases. You can calculate the internal resistance of a battery by adding increasingly large loads to it, thereby giving you the batteries IV curve. It is worth considering this, as if your  $R_{load}$  is only  $\approx 10 \times R_{th}$ , then you may see drooping in the voltage supplied. Another way to state this is to make sure that the *input resistance* of your voltage source is much smaller than the *output resistance* of the upcoming circuit fragment you are attempting to drive. An example of when this fails is as follows:



Because the circuit has a non-negligible resistance relative to the voltmeter, you should expect to read something inconsistent with using an ideal voltmeter. As we are adding a  $10\text{ M}\Omega$  voltmeter in parallel to our  $10\text{ M}\Omega$  resistor, we expect that the “ $R_{load}$ ” in this case will now be  $5\text{ M}\Omega$ , so the voltage divider at  $V_{out}$  will now be  $1/3 \times 20\text{V}$ , or  $6.67\text{V}$ .

This contrasts to a current source, whose desired internal resistance is  $\infty$ , as you will want no current to flow through it, and to flow entirely through the circuit fragments outside of the component. Once again, as the real world is not ideal, your goal in this case will be to have downstream circuit components whose input resistance is much smaller than the components output resistance.

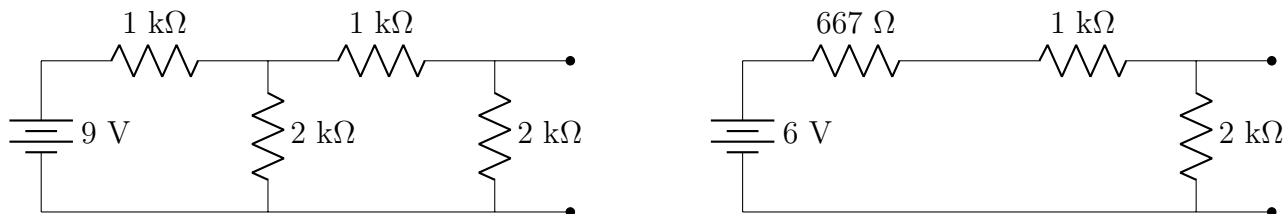
Let us consider another example of input resistance vs. output resistance:



If we draw a black box around the first voltage divider, we can convert it to the circuit schematic on the right because  $1\text{ k}\Omega || 2\text{ k}\Omega = 667\text{ k}\Omega$  just as before. We can find the  $V_{th}$  by first finding  $I_{sc}$ , when we short the black box’s load. So,  $I_{sc} = 9\text{V}/1\text{k}\Omega = 0.9\text{mA}$ . We multiply  $I_{sc}$  ( $9\text{ mA}$ ) by  $R_{th}$  ( $6.67\text{ k}\Omega$ ) to get a  $V_{th}$  of  $6\text{ V}$ . If you were to compare the output resistance of the black box,  $6.67\text{ k}\Omega$ , to the input resistance of the upcoming voltage divider,  $300\text{ k}\Omega$ , you would find that it is much smaller. This naturally means that the entirety of the  $6\text{ V}$  drop will occur over this part of the circuit.

Another way to think about this is as two successive voltage dividers, and qualitatively noting that the second’s total resistance is much higher allows us to simplify things greatly. As a reminder, a

voltage divider can be solved as  $R_A I = R_A \times V_{total} / R_{total} = V_{total} \times R_A / (R_A + R_B)$ . If you redraw as the Thevenin equivalent, the first voltage divider can effectively be ignored, because the voltage drop across this component will be minimal. Not by coincidence, since the first voltage divider is a  $2/3$  divider, and the second is a  $2/3$  divider, the voltage measured between our two terminals is  $2/3 \times 2/3 \times 9V$ , or alternatively,  $2/3 \times 6V$  ( $V_{th}$ ). This equivalency does not work when the input and output resistances of each fragment are comparable, here is an example:

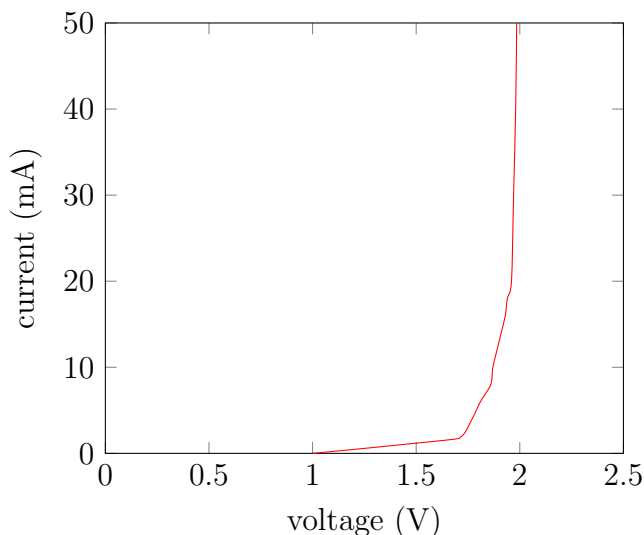


In this case,  $R_{th}$  will be the same, and  $R_{load}$  will be 3 kΩ.  $V_{th} \times R_A / (R_A + R_B) = 3,000 / 3,667 \approx 5.45V$  for the output of the first divider (i.e., between  $R_{th}$  and the 1 kΩ resistor). And then naturally, if you were to measure the voltage between the 1 kΩ and 2 kΩ resistors, it would be  $2/3 \times 5.45V \approx 3.63V$  for the voltage at the second divider.

This kind of Thevenin analysis only works when you have a linear IV curve. When might you have a non-linear IV curve?

## 1.2.2 LED Circuits and PNP

The IV curve across a light emitting diode (LED) should look something like this:



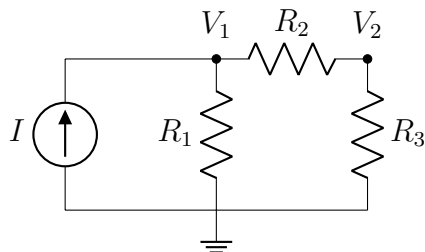
The IV curve for a diode, like an LED, is exponential in that the current slowly increases after the voltage across a diode hits some “threshold,” after which the current rises exponentially with voltage. Why is this the case? A diode is a P-N junction bridged by some depletion zone. The P side of the diode contains positively charged elements that act as “holes” (a silly way to say there is an absent electron position). The N side contains elements whose outer layers are loosely filled with electrons (i.e., low ionization energy). Effectively, the P side is devoid of electrons, while the N side has many free to give. What does this mean with regard to current and voltage? It means that the “depletion zone” between the two requires electrons to be able to bridge the gap. This really can’t happen unless

they have a certain amount of energy, so increasing the voltage helps reach the “threshold” energy requires to pass the depletion zone (think of  $P = IV$ ). Thus, as the electrons somewhat saturate the diode, you can theoretically pass an infinite current through it, as it will be effectively a short circuit.

## 1.3 Resistor Lattice Digression

One issue with modern BSIs is the usage of ECoGs, or EEGs, or other large measuring devices<sup>3</sup>. Too, they are almost universally hard electronics that require intense surgeries to implant. Therefore, if we could replace with soft electronics, we can cover considerable ground. For example, if one could drill a small hole into a patient’s skull and spread over the cortex a fabric that contained electrodes, one could achieve a similar amount of readings with a minimally invasive surgery. It is probable that there will be electrodes small enough to accomplish this, but let’s say there aren’t. Another way that this could be solved is using a lattice of resistors, with probes at either corner. These corners can exit the brain and be the points at which a computer interfaces with them.

Don’t be annoying, just go with the process.



Please, don’t be annoying—the solution is trivial but we will be talking about methods you can generalize. This example is from the SPICE method<sup>4</sup>. Considering KCL at the nodes gives us these two equations:

$$\begin{aligned} I &= \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} \\ \frac{V_1 - V_2}{R_2} &= \frac{V_2}{R_3} \end{aligned} \tag{1.1}$$

As it goes, we can reformat this so as to easily turn it into a matrix in the following way:

$$\begin{aligned} I &= \frac{1}{R_1} V_1 + \frac{1}{R_2} (V_1 - V_2) \\ I &= \frac{1}{R_1} V_1 + \frac{1}{R_2} V_1 - \frac{1}{R_2} V_2 \\ I &= \left( \frac{1}{R_1} + \frac{1}{R_2} \right) V_1 - \frac{1}{R_2} V_2 \end{aligned} \tag{1.2}$$

<sup>3</sup>This is expanded on in the later parts.

<sup>4</sup>Thank you Prof. Ashmanskas, <http://www.ecircuitcenter.com/SpiceTopics/Overview/Overview.htm>

$$\begin{aligned}
\frac{V_1 - V_2}{R_2} &= \frac{V_2}{R_3} \\
0 &= -\frac{1}{R_2}V_1 + \frac{1}{R_2}V_2 + \frac{1}{R_3}V_2 \\
0 &= -\frac{1}{R_2}V_1 + \left(\frac{1}{R_2} + \frac{1}{R_3}\right)V_2
\end{aligned} \tag{1.3}$$

These sorts of equations will usually be simplified using conductance as below:

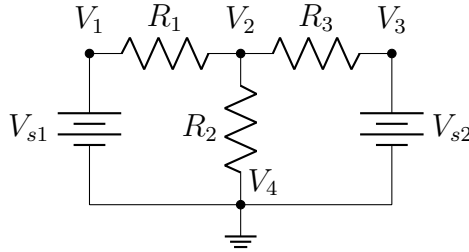
$$\begin{aligned}
(G_1 + G_2)V_1 - G_2V_2 &= I \\
-G_2V_1 + (G_2 + G_3)V_2 &= 0
\end{aligned} \tag{1.4}$$

$$\begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I \\ 0 \end{bmatrix} \tag{1.5}$$

So if  $R_{1,2,3} = 100\Omega$ , and  $I = 1\text{mA}$ , then:

$$x = A^{-1}B = \begin{bmatrix} 67\text{mV} \\ 33\text{mV} \end{bmatrix} \tag{1.6}$$

This method is called *nodal analysis*. You'll find more trouble trying to use a voltage source rather than a current source, and in fixing this, a method of *modified nodal analysis* was invented. Let us take the below example<sup>5</sup>:



In this system, we have 4 nodes, but node 4 largely functions as a reference for the other 3 nodes. A small note to make this a tad clearer is that in thinking about the current flowing into a node, such as into  $V_1$ , you will view this as current going from  $V_2$  to  $V_1$ , but because everything in electronics is backwards, this is calculated as  $(V_1 - V_2)/R_1$ , and the opposite for current flowing into  $V_2$ . Annoyance aside, using KCL, and then our definitions, we can gather these equations:

$$\begin{aligned}
I_1 + \frac{V_1 - V_2}{R_1} &= 0 \\
\frac{V_2 - V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_2 - V_3}{R_3} &= 0 \\
I_2 + \frac{V_3 - V_2}{R_3} &= 0 \\
V_1 &= V_{s1} \\
V_3 &= V_{s2}
\end{aligned} \tag{1.7}$$

<sup>5</sup><https://cheever.domains.swarthmore.edu/Ref/mna/MNA2.html>

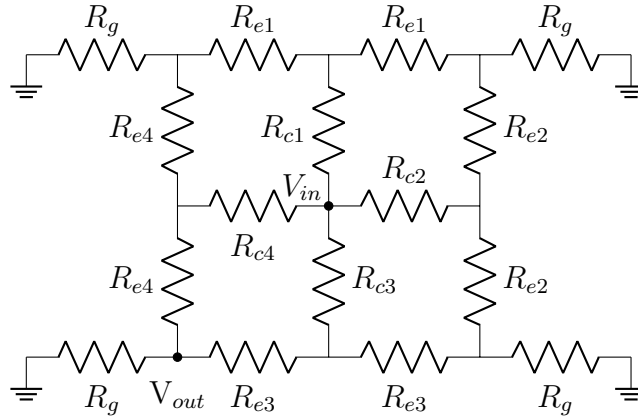
Which we convert to:

$$\begin{aligned}
I_1 + G_1 V_1 - G_1 V_2 &= 0 \\
G_1 V_1 + (-G_1 + G_2 + G_3) V_2 - G_3 V_3 &= 0 \\
I_3 - G_3 V_2 + G_3 V_3 &= 0 \\
V_1 &= V_{s1} \\
V_3 &= V_{s2}
\end{aligned} \tag{1.8}$$

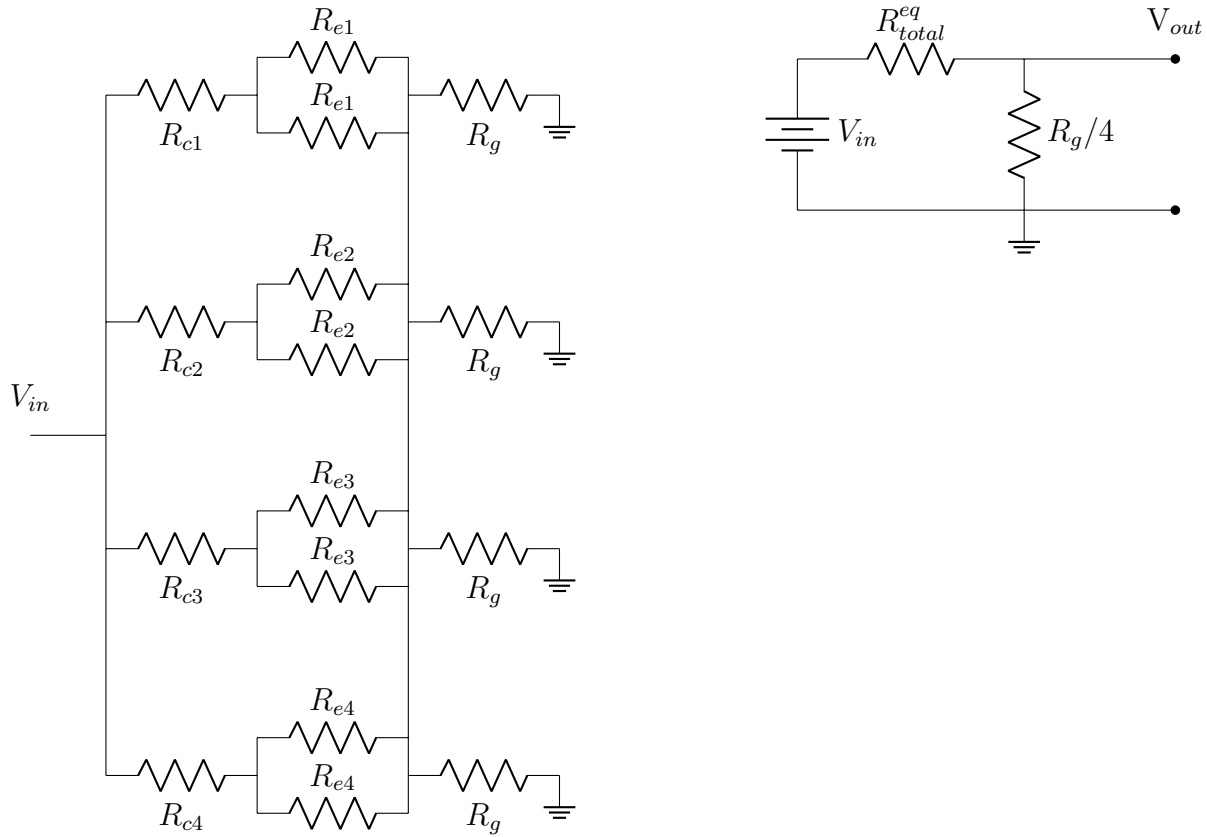
And then:

$$\begin{bmatrix} G_1 & -G_1 & 0 & 1 & 0 \\ -G_1 & G_1 + G_2 + G_3 & -G_3 & 0 & 0 \\ 0 & -G_3 & G_3 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ I_1 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ V_{s1} \\ V_{s2} \end{bmatrix} \tag{1.9}$$

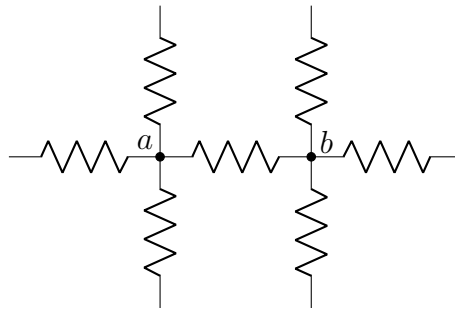
Now, I realize that one doesn't need to build a system of equations for most problems of small size, and they can be generally solved at a glance. Too, in many cases building a system is more hassle than it's worth. But, this will be important to keep in mind for the upcoming chapter **Modeling Circuits**. Let's look at some examples that are easier dealt with in the traditional way:



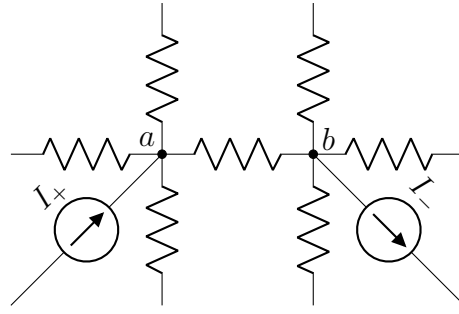
Firstly, apologies for the mess in labeling. The most difficult thing about this problem, in actuality, is probably just how you look at it. Redrawing the circuit helps a great deal. I'll guide you through how one might do this. In truth, the four grounded resistors are in parallel and can be simplified to one wire ( $R_g || R_g || R_g || R_g$ , or  $R_g/4$ ). You can probably realize that the edge resistors, for example the pair of  $R_{e1}$ , are also in parallel and are in series with  $R_{c1}$ . Therefore, we have  $(R_{e1} || R_{e1}) + R_{c1}$ . I'll call this  $R_1^{eq}$ . Then you'll realize simply that all of the equivalent  $R$  are also in parallel. This gives us  $R_1^{eq} || R_2^{eq} || R_3^{eq} || R_4^{eq}$ , which I will call  $R_{total}^{eq}$ . Now, our circuit simplifies to:



You may have had trouble at first glance because of all the “looped” sections of the circuit. It appears that current can flow any which way. Keep KVL in mind, and recall that current will only flow from high to low voltage. Therefore, you would never have current flowing from, say,  $R_{e2}$  to  $R_{e3}$ . A popular problem of the same flavor is the *infinite resistor grid*, which looks something like this:

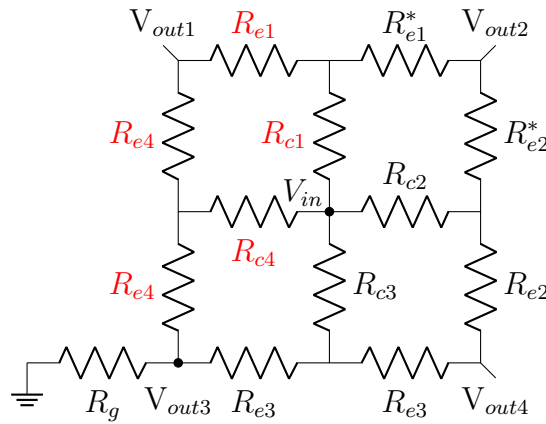


Imagine that the lattice of resistors continues out in every direction infinitely, in a grid-like fashion. The problem asks you to determine the equivalent resistance between points  $a$  and  $b$ . Considering the infinite nature, the more immediate method to solve would be to take some limit of parallel and series resistors. I actually think this is an important solution to keep in mind, as it can be generalized to other systems better. However, in this case the easiest solution is to use *superposition*. It goes as follows:



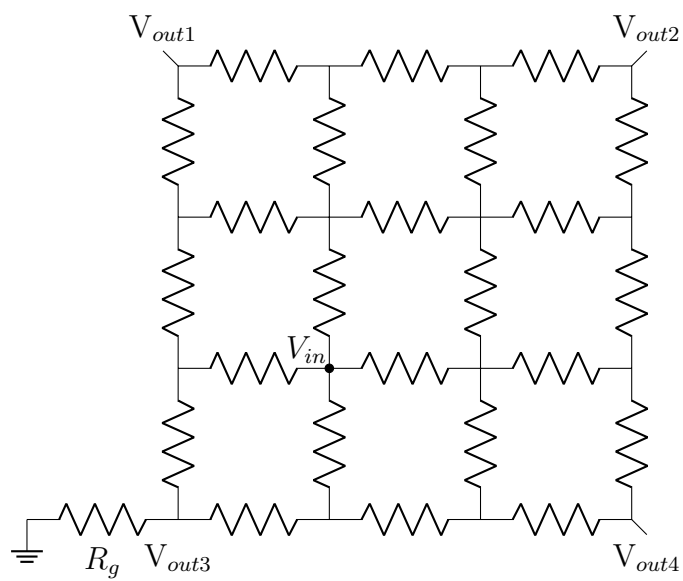
If you were to wire a positive current source to node  $a$ , you can immediately know that the current will be split equally in 4 ways, as the lattice is infinite. Then, if you apply a negative current source to node  $b$ , you can immediately know that it will draw  $1/4I$  from each resistor as well. The total current flowing between nodes  $a$  and  $b$  will then be  $1/2I$ . Given that  $V = IR$ , and by superposition, we can know that  $V_{ab} = (I/2)R$ , so  $R_{ab} = V_{ab}/I = (I/2)R/I = R/2$ . The in-line math is kind of ugly, but I think you get the gist.

Another option of a similar flavor is:



In this case, we will think about what each  $V_{out}$  will read. One thing to note immediately is that the net current at  $V_{out2}$  will be 0. Therefore,  $R_{e1}^*$  and  $R_{e2}^*$  don't really matter, and can be neglected in our calculations. This once again is easiest to solve when re-visualized. The gist being that you have two quadrants (the  $R_{c1}, R_{c4}$ , which I colored to have red labels, and the  $R_{c2}, R_{c3}$  quadrants) where  $V_{in}$  is split into. The voltage drop across these quadrants being  $3R||R$ . We then add another  $R$ , for the one that connects to  $R_g$ , and voila, we are done. Let's call this  $R_q$ , so to find the equivalent resistance from  $V_{in}$  to  $V_{out3}$  it is  $R_{q1}||R_{q2}$ . I think you get the idea. The reason for going through these simple models is to prepare ourselves for some of the upcoming, more complicated ones. Let us level up now:





# Chapter 2

## Capacitors

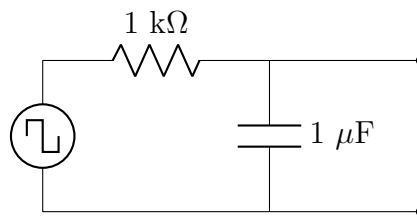
### 2.1 Overview

Getting a good intuition of capacitor's behavior is an essential part of understanding electronics. A capacitor is like a storage of charge, which can maintain a voltage for a period after the voltage supplied to the circuit is cut off. The basic structure of a capacitor is an anode and a cathode separated by a dielectric plate. Charge will be stored across it, and is released when the voltage supply dwindles.

Capacitor discharge will be exponential decay in the form:

$$\begin{aligned}V_C &= V_S \times e^{-t/RC} \\ \tau &= RC \\ V_C &= V_S \times e^{-t/\tau}\end{aligned}\tag{2.1}$$

This is quite similar to neurons, isn't it! So some examples, to build up a bit of intuition. Let's say you have the following circuit:



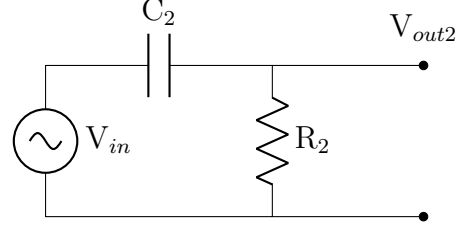
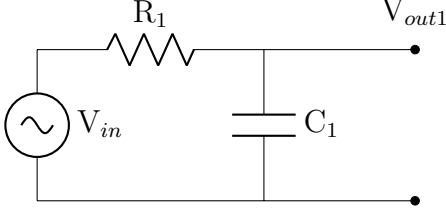
$RC$  will be  $1\text{k}\Omega \times 1\mu\text{F} = 1 \times 10^{-3}$ , or  $\tau = 1\text{ms}$ . What does this actually mean, though? It means that if you charge this capacitor up to  $10\text{V}$ , then  $V_C = V_S \times e^{-t/RC} \rightarrow 10\text{V} \times e^{-t/1\text{ms}}$ . So  $1\text{ms}$  after voltage is removed, and or, one time constant  $\tau$  after voltage is removed, the voltage at the capacitor will be  $10 \times e^{-1} \approx 10 \times 0.367 = 3.67\text{V}$ . If you were using a square wave which charged the capacitor to  $10\text{V}$ , with a very high frequency ( $100\text{kHz}$ , for example), then the period would be  $1/100\text{kHz} = 0.01\text{ms}$ . Therefore, you would not expect the capacitor to ever fully discharge, and it would maintain a constant, high voltage. This circuit happens to be what is called a *low-pass filter*, which we will discuss in-depth later. But, you can see how it might get this moniker, as this very high frequency is not allowed to pass due to the capacitor's time constant.

This was meant to serve as a basic intro to what a capacitor is. Now we can begin discussing the nuances.

## 2.2 Filtration

### 2.2.1 Integration and Differentiation

Capacitors have this incredible ability to perform complex math, including taking derivatives or integrals of your wave form. Let's think about how this may occur using the circuit mentioned in the previous part. Firstly, recall that the current flowing through the resistor must equal the current flowing through the capacitor, and that  $Q = CV$ :



We can then solve for the voltage drop across  $R_1$  as:

$$\begin{aligned} Q &= C_1 V_{out1} \\ \frac{d}{dt} Q &= \frac{d}{dt} C_1 V_{out1} \\ I &= C_1 \frac{dV_{out1}}{dt} \end{aligned} \tag{2.2}$$

$$\begin{aligned} IR_1 &= V_{in} - V_{out1} \\ \frac{dV_{out1}}{dt} &= \frac{1}{R_1 C_1} (V_{in} - V_{out1}) \end{aligned} \tag{2.3}$$

Therefore, if  $V_{out}$  is very small compared to  $V_{in}$ , you get:

$$\begin{aligned} \frac{dV_{out1}}{dt} &= \frac{1}{R_1 C_1} V_{in} \\ \int \frac{dV_{out1}}{dt} &= \int \frac{1}{R_1 C_1} V_{in} \\ V_{out1} &= \frac{1}{R_1 C_1} \int V_{in} \end{aligned} \tag{2.4}$$

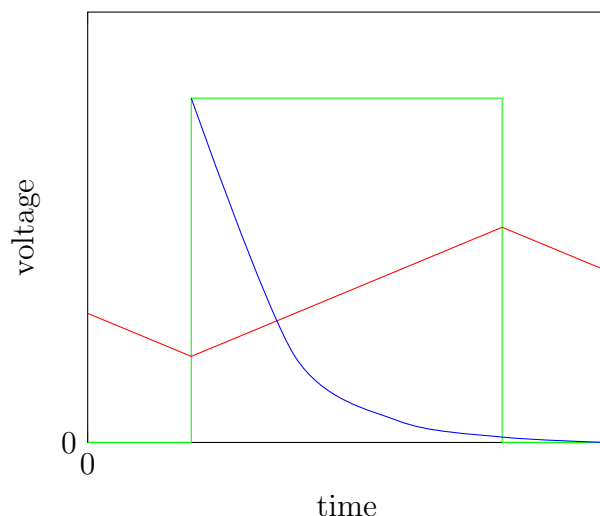
And or, that  $V_{out}$  integrates  $V_{in}$ . How will this change if we swap the positions of the resistor and capacitor in circuit 2? Once again, consider when  $V_{out}$  is much smaller than the input.

$$\begin{aligned} I &= C_2 \frac{d}{dt} (V_{in} - V_{out2}) \\ \frac{V_{out2}}{R_2} &= C_2 \frac{d}{dt} (V_{in} - V_{out2}) \\ \frac{V_{out2}}{R_2} &= C_2 \frac{dV_{in}}{dt} \\ V_{out2} &= R_2 C_2 \frac{dV_{in}}{dt} \end{aligned} \tag{2.5}$$

Thus, in this case  $V_{out}$  approximates the derivative of  $V_{in}$ .

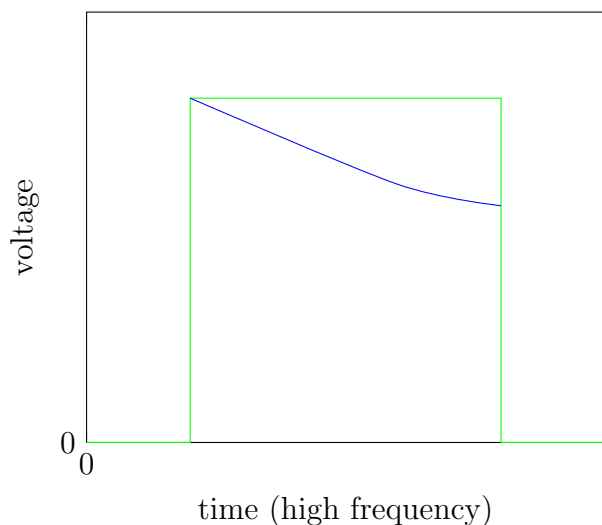
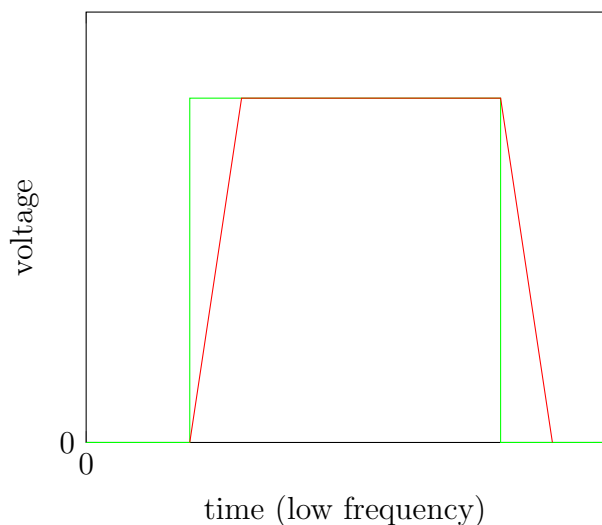
### Qualitative thinking.

The best lead in to thinking about filtration, to me, is thinking simply about how the graphs look like when something integrates or differentiates a square wave. Let us not do any math, and think only qualitatively.



If the green waveform is our  $V_{in}$  from the previous example, then our red curve is similar to how  $V_{out1}$  may look, and the blue curve is similar to how  $V_{out2}$ . That is, the blue curve *kind of* differentiates the green curve, because the relatively instant rise signifies a very positive derivative, marked by this blue spike. Similarly, the red curve *kind of* integrates the green curve, because the area under the green curve slowly accumulates, hence the constant slope of the red curve.

So if we say that the frequency is extremely slow, and the red curve is the voltage being measured at  $V_{out2}$ , how might the derivative curve look?



Similarly, if the frequency is very high, how might the integral look? Take a second to ponder the two graphs above and gather a bit of intuition on it. This should allow you to qualitatively state that

an integrating circuit will be a *low-pass filter*, and an integrating circuit will be a *high-pass filter*. As in the left graph, as the frequency gets smaller and smaller, the red plot will more and more closely match the green plot. This is the basis of capacitor filtration, an essential tool in electronics!

## 2.3 Frequency Dependence

This section will primarily be quantitative. The important bits, though, are the qualitative understandings, and the final results of this quantitative section. What you find in the interim is likely not worth understanding fully. Let's begin by supposing we apply a cos wave to a resistor. The relationship between current and voltage is as follows:

$$\begin{aligned} f &= \frac{\omega}{2\pi} \\ V(t) &= V_{pp} \cos(\omega t) \\ V(t) &= I(t)R \\ I(t) &= (V_{pp}/R) \cos(\omega t) \end{aligned} \tag{2.6}$$

Therefore, if you were to solve something like  $V_{out}/V_{in}$  your pesky cos terms will cancel. Unfortunately, this is not so for capacitor equations. We can see this below:

$$\begin{aligned} Q &= CV \\ I(t) &= C \frac{d(V_{pp} \cos(\omega t))}{dt} \\ I(t) &= -\omega CV_{pp} \sin(\omega t) \end{aligned} \tag{2.7}$$

This means that the current and the voltage will be 90° out of phase. Another way would be to write this as  $A \cos(\omega t + \phi)$ . Let us think about what will happen with our standard low-pass filter. Using KVL, we can state that:

$$\begin{aligned} V_{pp} \cos(\omega t) &= RC \frac{dv_C}{dt} + v_c \\ v_c &= A \cos(\omega t + \phi) \\ V_{pp} \cos(\omega t) &= RC \frac{d}{dt} A \cos(\omega t + \phi) + A \cos(\omega t + \phi) \end{aligned} \tag{2.8}$$

### 2.3.1 Imaginary Numbers Digression.

You probably looked at the previous equation and it is solvable (probably?) but that it would be not worth your while to do so, and that there is likely a better way to go about it. Recall Euler's Relation<sup>1</sup>:

---

<sup>1</sup>Electrical Engineers use  $j$  instead of  $i$  for imaginary numbers. The claim is that it is easier to keep track of in the math.

$$e^{j\omega t} = \cos(\omega t) + j \sin(\omega t) \quad (2.9)$$

One of the requirements of a linear circuit is superposition, and that is *kind of* the argument that allows us to use imaginary numbers. You can think that if you are using math that includes both a real and imaginary component, as long as you keep track of the real, your output will still be correct—but, don't think too hard about this. In the upcoming sections, I will use  $\mathbf{v}_c$  to denote the complex number. Let us examine:

$$\begin{aligned} V_{pp}e^{j\omega t} &= RC \frac{d\mathbf{v}_c}{dt} + \mathbf{v}_c \\ \mathbf{v}_c &= Ae^{j\omega t} \\ V_{pp}e^{j\omega t} &= RC \frac{d}{dt} Ae^{j\omega t} + Ae^{j\omega t} \end{aligned} \quad (2.10)$$

We can differentiate, and then obtain an expression for  $A$ , and solve for the voltage at the capacitor as:

$$\begin{aligned} V_{pp}e^{j\omega t} &= j\omega RC Ae^{j\omega t} + Ae^{j\omega t} \\ V_{pp} &= j\omega RC A + A \\ V_{pp} &= A(1 + j\omega RC) \\ \frac{V_{pp}}{(1 + j\omega RC)} &= A \\ \mathbf{v}_c &= \frac{V_{pp}}{1 + j\omega RC} e^{j\omega t} \end{aligned} \quad (2.11)$$

We now want to find the real component, which begins by rewriting the expression in its polar form:

$$\begin{aligned} \mathbf{v}_c &= \left( \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}} e^{j\phi} \right) V_{pp} e^{j\omega t} \\ \phi &= \tan^{-1}(-\omega RC) \\ \mathbf{v}_c &= \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}} V_{pp} e^{j(\omega t + \phi)} \end{aligned} \quad (2.12)$$

From here we can simply take the real part and be on our way:

$$v_c = \frac{V_{pp}}{\sqrt{1 + \omega^2 R^2 C^2}} \cos(\omega t + \phi) \quad (2.13)$$

This is one way to go about this problem. Another way, which will be discussed in a different section, is using impedance.

## 2.4 Impedance

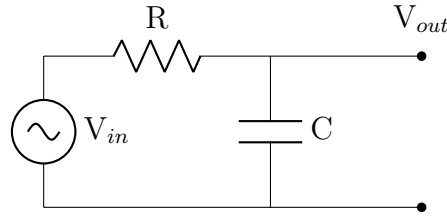
We mentioned earlier in equation (2.11) that the relationship between an input voltage and what is measured at a capacitor for a low-pass filter is:

$$V_{in} \frac{1}{1 + j\omega RC} = v_c \quad (2.14)$$

If we were to divide this fraction by  $j\omega C$ , and simplify using the representation  $Z_C$ , we would get:

$$\begin{aligned} V_{in} \frac{1/j\omega C}{1/j\omega C + R} &= v_c \\ V_{in} \frac{Z_C}{Z_C + R} &= v_c \end{aligned} \quad (2.15)$$

This looks just like a voltage divider equation! The concept of *impedance* is used to summarize other circuit fragments, like capacitors or inductors, using some resistance equivalent  $Z$ . The impedance of a resistor,  $Z_R$ , is simply  $R$ .  $Z_C$  is  $1/j\omega C$ . Let's think of how this pertains to our low-pass filter.



If we measure at  $V_{out}$ , we can think of it like a voltage divider, giving us:

$$V_{out} = \frac{Z_C}{Z_C + R} V_{in} \quad (2.16)$$

You may wonder if  $Z_C = 1/j\omega C$  has any real basis, or if it is simply an extraction from the above math. In reality, you can find it quite simply through:

$$\begin{aligned} I_c &= C \frac{dv_c}{dt} \\ I_c e^{j\omega t} &= CA \frac{d}{dt} e^{j\omega t} \\ I_c e^{j\omega t} &= j\omega CA e^{j\omega t} \\ I_c &= j\omega CA \\ I_c \frac{1}{j\omega C} &= A \end{aligned} \quad (2.17)$$

This is the equivalent of Ohm's law, where  $1/j\omega C$  is the resistance.

# Chapter 3

## Writing Hardware

### 3.1 Introduction

Verilog<sup>1</sup> is a language used to describe electronics, and allows you to avoid the physical action of wiring. This is the reason for the designation **writing hardware**. In this way, you can pick your poison: debugging code, or debugging breadboards. Importantly, though, Verilog is capable of computation and writing data files that go beyond circuit descriptions. Thus, it is not a “markdown” language and is Turing Complete<sup>2</sup>.

Tools like Field Programmable Gate Arrays (FPGAs) allow for this, as their internal composition is something of an array of transistors, which can be rewired through code in order to meet the demands of the programmer.

#### 3.1.1 Creating Modules

In Verilog, a circuit is called a `module`. Each module is defined between a `module` and `endmodule`, which can be named as shown in the example below. Different ports connect the module to things outside of the module.

```
1 module example1(o, i1, i2);
2 // example1 is the name of our module, and o, i1, and i2 are our ports
3 // it is convention to list outputs first
4
5 output o; // this defines o as an output
6 input i1, i2; // this defines i1 and i2 as inputs
7
8 endmodule
```

Gates are also initialized like modules. The way to do this is with the built in primitives for AND and OR gates (`and` and `or` respectively). For example:

```
1 module example2(o, i1, i2);
2
```

---

<sup>1</sup>A large part of the background information and general syntax comes from the YouTube channel: CompArchIllinois.

<sup>2</sup>Or at least, I think it is. I can never remember the exact definition of Turing Complete :)



```

3 output o;
4 input i1, i2;
5 wire wire1, wire2; // this initializes two wires called wire1 and wire2
6
7 or or1(wire1, i1, i2);
8 // this makes an OR gate named or1 with inputs i1 and i2, and output called
  wire1
9 and and1(o, wire1, wire2);
10 // this makes a NOT gate named not1 with input i2, and output called wire2
11 // one of the outputs of the OR gate feeds into the AND gate (via wire1) in
   this example
12
13 endmodule

```

The order in which things are initialized do not matter. It is very important to not reuse wire or other variable names, as Verilog will read these as being connected irrespective of where they are intended to be. As mentioned, the code above uses modules built into Verilog, but you could make your own module in the following way:

```

1 module andgate(output o1, input i1, input i2);
2     assign o1 = i1 & i2;
3     // for OR you would use |, and for XOR you would use ^
4 endmodule;

```

### 3.1.2 Bus Notation

Bus notation is used to simplify the pins used (in Verilog, this is called a vector). For example, a multiplexer or an adder will have many inputs, which would be inconvenient to initialize individually. Instead, we can use something like this:

```

1 module adder(c, a, b); // a, b, c are 3 bus inputs we will use
2     output [3:0] c; // initializes 4 wires within our c bus
3     input [3:0] a, b; // initializes 4 wires within our a and b buses
4 endmodule

```

Firstly, note that Verilog is 0 indexed, so `[3 : 0]` includes 4 wires. In a circuit schematic, busses are drawn as thicker wires with a slash through them and a number denoting the amount of wires in the bus. If we wanted to call individual wires from our busses into the `andgate` module we declared earlier, we could do it as:

```

1 andgate(c[0], a[0], b[0]);

```

And we can connect busses together, or wires together, using the assign command like before. For example:

```

1 wire wire3;
2 assign wire3 = c[2];
3 assign c[2:0] = a[2:0]; // wire3 will now be connected to bus a[2] through
   bus c[2]

```

## 3.2 Constants and Variables

Verilog allows us to define constants using 3 parameters, defined as their size, method of encoding, and value. For example, 8'hd7 corresponds to a size of 8 bits, hexadecimal encoding, and the value d7 (equivalently, 11010111). You can use this in `boolean` comparisons, as below:

```

1 wire wire4;
2 'define CONST1 3'b011; // CONST1 is the name of the constant
3 wire4 = (a[2:0] == CONST1);
4 // This is a tad complicated. The gist is: all of the bit values in a[2:0]
   will be compared to CONST1 in a NXOR style statement. That will then be
   compared to all of the other bits in an AND style statement. So wire4
   will be on only if all wires in correspond to CONST1.
5 // The C++ equivalent would be something like:
6 //     (a[2] == 1'b0) && (a[1] == 1'b1) && (a[0] == 1'b1)
7 // Also, this could be completely wrong. I can't check any of this without
   an actual FPGA in front of me! So who knows!

```

This does bring up a worthwhile point, which is that everything you write in Verilog has a direct circuit component underlying it (I suppose the same is true for any program, but it's more... explicit with Verilog).

# Part II

## Math and Models

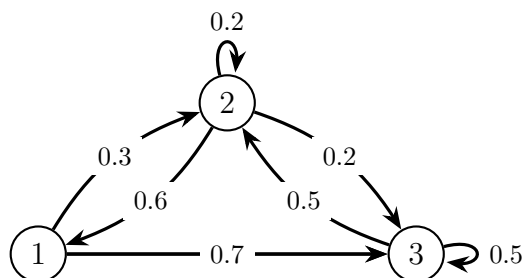
All truly strong people are kind.  
– Vagabond by Takehiko Inoue

# Chapter 4

## Moving Through States

### 4.1 Markov-chains

Markov chains are useful in predicting the next state desired.



Transition matrix:

$$A = \begin{bmatrix} 0 & 0.3 & 0.7 \\ 0.6 & 0.2 & 0.2 \\ 0 & 0.5 & 0.5 \end{bmatrix} \quad (4.1)$$

### 4.2 Diffusion

Note that in the coming sections,  $[f]$  is used to describe the concentration of some molecule  $f$ . But, this principal can be applied to anything, including the spreading of voltage across some surface.

#### 4.2.1 Forward Euler's

Diffusion is accomplished using some diffusion coefficient ( $D_c$ ) multiplied by some measure of the proportion in one compartment versus another (often  $\partial^2[f]/\partial x^2$ ).  $D_c$  can be tuned however desired. The important bit is the second derivative of  $[f]$  with respect to space. This can be done using the general form of a second derivative, as written below:

$$f'' = \frac{f_{x+1} - 2f_x + f_{x-1}}{x^2} \quad (4.2)$$

One may wonder how one would solve for an edge case, as the general form of a second derivative requires three data points. There are some nuances, but in general the solution is simply the first

derivative of the non-edge side. That is, since the second derivative is the difference in derivatives, that leaves simply the derivative of one side minus zero. This is like applying a closed end to your surface. You can ponder how to solve for an open end, if that ever arises.

This method has some slight issues in which the  $[f]$  can occasionally go negative. The way in which this occurs is stated below (note that now  $[f]$  is used instead of  $f$  to signify concentration at a value  $x$  and time  $t$ ). We can first describe a simplified version of the problem:

$$\begin{aligned}
[f]_i &= f_0 \exp(-mt) \\
\frac{d[f]}{dt} &= \frac{[f]_{i+1} - [f]_i}{\Delta t} = -m[f]_i \\
[f]_{i+1} &= (1 - m\Delta t)[f]_i = [f]_i - m\Delta t[f]_i \\
[f]_i &= (1 - m\Delta t)[f]_{i-1} = (1 - m\Delta t)^2[f]_{i-2} \\
[f]_i &= (1 - m\Delta t)^i[f]_0
\end{aligned} \tag{4.3}$$

You can see easily, from this, that if  $\Delta t$  is too big, you will abandon the characteristic decay you'd expect from  $f_0 \exp(-mt)$ , and instead get some diverging oscillatory function. How this applies to our interest in diffusion is described below:

$$\begin{aligned}
\frac{[f]_x^{t+1} - [f]_x^t}{\Delta t} &= \frac{[f]_{x+1}^t - 2[f]_x^t + [f]_{x-1}^t}{x^2} \\
[f]_x^{t+1} - [f]_x^t &= \frac{\Delta t}{x^2} ([f]_{x+1}^t - 2[f]_x^t + [f]_{x-1}^t) \\
[f]_x^{t+1} &= \frac{\Delta t}{x^2} [f]_{x+1}^t + \left(1 - 2\frac{\Delta t}{x^2}\right) [f]_x^t + \frac{\Delta t}{x^2} [f]_{x-1}^t
\end{aligned} \tag{4.4}$$

Therefore, if we want to ensure that the concentration is always positive, we are constrained by:

$$\begin{aligned}
1 - 2\frac{\Delta t}{x^2} &= 0 \\
\Delta t &< \frac{x^2}{2}
\end{aligned} \tag{4.5}$$

The relevance of this being that if one were interested in modeling on a very small  $\Delta x$ , then one would have to use a  $\Delta t$  that is not physiological, and thus waste a great deal of computing power in doing so. This can be avoided explicitly using some other methods, discussed next.

### 4.2.2 Backward Euler's

This form serves to solve the time-scale dilemma by swapping  $[f]_{x+1}$ , and can be used with any  $\Delta t$ . Let us consider the same example from above:

$$\begin{aligned}
[f]_i &= f_0 \exp(-mt) \\
\frac{d[f]}{dt} &= \frac{[f]_{i+1} - [f]_i}{\Delta t} = -m[f]_{i+1} \\
[f]_{i+1} - [f]_i &= -m\Delta t[f]_{i+1} \\
[f]_{i+1} &= \frac{1}{1 + m\Delta t} [f]_i
\end{aligned} \tag{4.6}$$

Naturally, there is no longer a concern of the size of  $\Delta t$ . Though, one immediate concern is the difficulty of solving your equation for  $[f]_{i+1}$ . Getting back to the diffusion interest, we now have:

$$\begin{aligned}\frac{[f]_x^{t+1} - [f]_x^t}{\Delta t} &= \frac{[f]_{x+1}^{t+1} - 2[f]_x^{t+1} + [f]_{x-1}^{t+1}}{x^2} \\ [f]_x^{t+1} - [f]_x^t &= \frac{\Delta t}{x^2} ([f]_{x+1}^{t+1} - 2[f]_x^{t+1} + [f]_{x-1}^{t+1})\end{aligned}\tag{4.7}$$

Which simply replaces the previous  $[f]_x^t$  with  $[f]_x^{t+1}$ . This leaves us with three unknowns (those being  $[f]^{t+1}$  at  $x-1, x, x+1$ ). We must use linear algebra to solve by first rewriting the left and right side as vectors and a matrix in the following way:

$$[\vec{f}]^{t+1} = \begin{bmatrix} [f]_0^{t+1} \\ [f]_1^{t+1} \\ \vdots \\ [f]_x^{t+1} \end{bmatrix}; [\vec{f}]^t = \begin{bmatrix} [f]_0^t \\ [f]_1^t \\ \vdots \\ [f]_x^t \end{bmatrix}\tag{4.8}$$

and

$$A = \frac{\Delta t}{x^2} \begin{bmatrix} \dots & \dots & \dots & \dots & \dots \\ 1 & -2 & 1 & & \vdots \\ \vdots & 1 & -2 & 1 & \vdots \\ \vdots & & 1 & -2 & 1 \\ \dots & \dots & \dots & \dots & \dots \end{bmatrix}\tag{4.9}$$

The corners 1, 1 and  $x, x$  were intentionally omitted, as what one desires to do with this is dependent on how they would prefer to treat their edges. As described before in the *Forward Euler's* method, one can use instead of the  $(1, -2, 1)$  pattern, simply  $(\emptyset, -1, 1)$  pattern, which signifies a closed edge. Therefore, together now we get:

$$\begin{aligned}[\vec{f}]^{t+1} - [\vec{f}]^t &= A[\vec{f}]^{t+1} \\ I[\vec{f}]^{t+1} - A[\vec{f}]^{t+1} &= [\vec{f}]^t \\ [\vec{f}]^{t+1} &= (I - A)^{-1}[\vec{f}]^t\end{aligned}\tag{4.10}$$

Where  $I$  is the identity matrix.

### Remarks.

Forward Euler's is explicit, and will be preferred whenever the differential equations are non-stiff<sup>1</sup>. It is the more accurate of the two methods, and can be less computationally intensive if your decay rates are all slow. Backward Euler's is implicit, and can not be used to solve everything, but is doable in most cases. The extra computation required to solve the system of equations more than makes up for potential limitations in your  $\Delta t$ .

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<sup>1</sup>On approximately the same time scale.

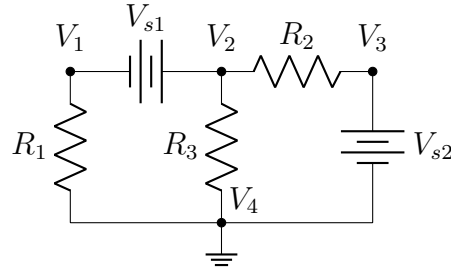
# Chapter 5

## Modeling Circuits

The idea of this algorithm comes from<sup>1</sup> and is an expansion of what was discussed earlier in the **Electronics** part of this book. Not by coincidence, both of the examples in the earlier section had symmetric matrices. The general way to extract equations from a circuit is in the following pattern:

$$\begin{bmatrix} \mathbf{G} & \mathbf{V} \\ \mathbf{V}^T & \mathbf{Z} \end{bmatrix} \quad (5.1)$$

$\mathbf{G}$  is essentially the way nodes are connected by resistors (represented as conductances). The diagonal represents the total number of resistors connected to a diagonal, and the other bits represents how nodes are connected by resistors. I will use the example provided in the footnote directly to illustrate:



There are 4 nodes, but as mentioned before, Node 4 is grounded, a reference, and thus not included in our calculations. This will mean our matrix  $\mathbf{G}$  must be  $n \times n$ , where  $n$  is  $N - 1$ . Node 1 is connected only to  $R_1$ , resolving the  $(0,0)$ <sup>2</sup> position of the matrix to be  $G_1$ . Node 2 is connected to  $R_2$  and  $R_3$ , giving us  $(1,1) = G_2 + G_3$ . Node 3 to only  $R_3$ , giving us  $(2,2) = G_3$ . Node 2 and Node 3 are connected via  $R_2$ , meaning the 1,2 and 2,1 positions will have a  $G_2$ , though notably, it will be  $-G_2$ :

$$\mathbf{G} = \begin{bmatrix} G_1 & 0 & 0 \\ 0 & G_2 + G_3 & -G_2 \\ 0 & -G_2 & G_3 \end{bmatrix} \quad (5.2)$$

There are two voltage sources ( $V = 2$ ), and 3 nodes, meaning the matrix must be  $n \times m$ , where  $n = N - 1$  and  $m = V$ . You fill the matrix as if the positive terminal of the  $j^{\text{th}}$  voltage source is connected to the  $i^{\text{th}}$  node, point  $(i,j) = 1$ , or  $(i,j) = -1$  for the negative terminal. So for the above, since the negative terminal of  $V_{s1}$  is connected to Node 1,  $(0,0) = -1$ , and as the positive terminal connects to Node 2,  $(1,0) = 1$ . Lastly, as Node 3 connects to the positive terminal of  $V_{s1}$ ,  $(2,1) = 1$ .

<sup>1</sup><https://lpsa.swarthmore.edu/Systems/Electrical/mna/MNA3.html>

<sup>2</sup>You are a computer scientist in addition to a mathematician, so of course our matrices must be 0 indexed.

$$V = \begin{bmatrix} -1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (5.3)$$

$$V^T = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (5.4)$$

And lastly, Z for zero is an  $n \times n$  matrix of zeros where  $n = V$ :

$$V^T = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (5.5)$$

Giving us an  $Ax = B$  of:

$$\begin{bmatrix} G_1 & 0 & 0 & -1 & 0 \\ 0 & G_2 + G_3 & -G_2 & 1 & 0 \\ 0 & -G_2 & G_2 & 0 & 1 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ I_1 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ V_{s1} \\ V_{s2} \end{bmatrix} \quad (5.6)$$

You may, rightfully, say “*Uh, who cares?*,” since we already know that each row corresponds to a different equation. Well, there are a few reasons. Firstly, doing this algorithmically allows us to avoid accidentally underdetermining our matrix using the ol’ eye-balling it technique. More importantly, though, the algorithmic approach allows us to solve this via code. Once I can write a bit more neatly, I’ll likely upload some snippets here.



# Part III

## Physiology

Preoccupied with a single leaf, you won't see the tree. Preoccupied with a single tree, you'll miss the entire forest.

– Vagabond by Takehiko Inoue

## 5.1 Perspective

It is very important to avoid taking a quantized approach to studying physiology. You'll notice neuroscience majors may know some superb nuances regarding the role of HCN channels in overexcitable neural disorders, like epilepsy. However, they may be totally unaware of their canonical role as pace-makers in the heart. So too may they be unaware of the rich history of modeling  $I_h$  currents by computational biologists, preferring to look only at a channel as it functions in neurons and is testable in a cell culture. Hence, one must not become preoccupied with a single leaf.

Another bit of wisdom comes from Yoshikawa's Musashi. There is a chapter in which vagabonds are tilling a marsh's untenable land. Each time a storm came, all work was reset as flooding returned it to mud. One realized that they had been tilling the field in the shape of a square, unconsciously aiming for a uniform, symmetrical crop that is typical of any farmer. In doing so, they intended to bend nature from its gruff state into something it was not. Nature, not allowing this, cyclically reclaimed the land. But, if they tilled the land in an oblong fashion, asymmetrical and with the natural bends of the marsh, along the path that water would flow in a storm, perhaps the fields would survive the storm. Rather than trying to oppose the universe, one must first know its way, and be guided by it.

Surgical intervention is most often an opposition to the way of nature. Indeed, the most clear evidence of this is in today's BSIs which attempt to escape endogenous circuits all together. Perhaps one should focus more on enhancing nature's way first, before opposing it. And to enhance nature's way, one must understand it fully.

# Chapter 6

## Muscles

### 6.1 Skeletal Muscle

#### 6.1.1 A Cellular Level

##### **Structure.**

Skeletal muscles are composed of muscle fibers, which are composed of myofibrils. Muscles use a silly nomenclature, in which “sarco” is added to words. For example, the whole structure sits within the sarcoplasm (cytoplasm), fibers are intertwined with the the sarcoplasmic reticulum (equivalent to endoplasmic reticulum) and surrounded by the sarcolemma (equivalent to the cell membrane). An individual myofibril is made up of overlapping actin and myosin segments, held together, to some degree, by titin. The myosin heads are the canonical structure you imagine, which bud off of the end of a chain like leaves on a branch and bind to actin. Undeniably, the most uninteresting part of muscle physiology is as follows: The “centerline” of the myosin networks is called the M line, while the “centerline” of the actin network is called a Z disk. Titin branches off from the Z disks to bind to the ends of myosin chains, providing both some elasticity and support for myosin. Actin chains are centered around a line of nebulin, which too provides structural support and organization. The I band is considered to be the unbound part of the actin structure, centered around the Z disks. The H zone is the unbound part of the myosin structure, centered around the M line. The A band is the entire length of the myosin fibers, thereby encapsulating both the bound part of the actin-myosin complex and the H zone, meaning it too is centered around the M line. Therefore, when contraction occurs, the size of the H zone and I band decreases, while the size of the A band stays the same.

##### **Contraction-relaxation.**

Myosin heads desire to bind to actin, but are blocked by tropomyosin.  $\text{Ca}^{2+}$  can bind to troponin, bound to tropomyosin, to cause conformational changes resulting in the exposure of the actin to the myosin heads. In doing so, binding can occur, followed by myosin’s power stroke. The energy for the stroke comes from hydrolyzing ATP, which had already occurred by the time myosin attached to the actin. The powerstroke allows the release of the ADP and  $\text{P}_i$ . ATP then can bind to the empty active site of myosin, which causes the release of the head and prepares it for another cycle. Notably, when ATP does not bind to myosin, the muscle will be stuck in the rigor state.

In skeletal muscles, the source of  $\text{Ca}^{2+}$  is a combination of the extracellular  $\text{Ca}^{2+}$  flowing inward, and further release from the sarcoplasmic reticulum. The story goes as follows: a motor neuron releases

acetylcholine onto the motor end plate (an area which has a high density of sodium channels). This causes a depolarization, which propagates down the muscle fiber. Structures called T-tubules sink lower into the tissue, allowing for more direct access to the inner processes (visually, these look similar to gyri in the brain or the crypts of the intestinal wall). The T-tubules are lined with dihydropyridine (DHP) channels, an L-type VGCC (specifically  $\text{Ca}_v1.1$ ). DHP and ryanodine receptors (RyR) can be mechanically coupled, which influx of  $\text{Ca}^{2+}$  in through DHP mechanically opens RyR channels of the sarcoplasmic reticulum (a large store of  $\text{Ca}^{2+}$ ). The free calcium is lowered through things like active pumping out of the sarcoplasm.

A steady supply of ATP is needed to maintain pumping, and it is said that at any given time, there is 8 or so twitches worth of ATP within the muscle fiber. Therefore, frequent production and alternate stores are required for continuous movement. One such storage is phosphocreatine, whose phosphate group can be quickly transferred to ADP through creatine kinase. Muscles therefore contain high levels of this enzyme, and **testing for it in the bloodstream can be a good proxy for muscle damage**.

Notably, it is very difficult to fully deplete a muscle of its ATP. Other forms of fatigue begin before this can possibly occur, which include CNS or PNS feedback. An example of this may be that acetylcholine is not synthesized fast enough to continually stimulate muscle fibers. Continual stimulation of muscle fibers, beyond what is allowable under normal conditions, **will fully deplete ATP levels and therefore cause damage to muscles**. Another consideration is the continuous use of ATP may result in  $\text{P}_i$  buildup in the sarcoplasm, making release of  $\text{ADP} + \text{P}_i$  from myosin less likely to occur. Too, this opens the possibility of calcium phosphate forming, which can be quite damaging if it crystallizes further. There are also ion concentration changes to consider, and continued stimulation can result in tetanus.

### 6.1.2 Nervous system control

## 6.2 Smooth Muscle

Smooth muscle is much more variable than skeletal, differing by location, contraction pattern, required inputs, and structure.

### 6.2.1 A Cellular Level

#### Structure.

Smooth muscle is not considered to have sarcomeres, despite it having the same basic structural components of skeletal muscles. Smooth muscle contains much more actin than does skeletal muscle, and notably does not contain troponin like skeletal muscle does. Smooth muscle networks are connected through intermediate filaments, which usually attach to dense bodies within the cytoplasm. Actin also attaches to dense bodies, maintaining the actin-myosin network within the cell as well. Smooth muscles do not have T-tubules like skeletal muscles. A comparable structure may be calveolae, which do indent into the membrane and seem spatially associated with the sarcoplasmic reticulum. The autonomic nervous system stimulates fibers through neurotransmitter release from varicosities, or bulbous stores of the chemical. The neurons may innervate the muscle fibers, allowing multiple muscle cells to be stimulated at once, or they may be released to a few fibers, which are connected through gap junctions and stimulate the nearby ones, causing a propagating wave to stimulate others. The first case describes a multi-subunit muscle, and the second a single subunit.

## **Contraction-Relaxation.**

Initiation of contraction begins the same as skeletal muscle, in that calcium enters and the concentration is further driven up by sarcoplasmic calcium release. Though, in this case  $\text{Ca}^{2+}$  may enter either through gap junctions or membrane ion channels. As such, there are many more modes of entry than in skeletal muscle. For example, voltage-gated  $\text{Ca}^{2+}$  channels may open, but there are also ligand gated channels or stretch-activated channels, adding extra layers of possible regulation. Intracellular differences arise beginning from release from the SR. Firstly, it is no longer a mechanically gated RyR channel which allows its release. The release mechanism is now  $\text{Ca}^{2+}$  activated RyR release (commonly called  $\text{Ca}^{2+}$ -induced- $\text{Ca}^{2+}$ -release (CICR)), and the  $\text{IP}_3$  path. GPCRs activate phospholipase C, driving  $\text{IP}_3$  production which binds to SR channels and causes them to open. The  $\text{IP}_3$  path is usually considered the greatest way to drive up intracellular  $\text{Ca}^{2+}$  (or at least, that is what computational biologists seem to think). When  $\text{Ca}^{2+}$  is available, it binds to calmodulin (CaM), which then binds to the myosin light chain kinase (MLCK). MLCK phosphorylates myosin to increase myosin ATPase activity. Importantly, once contraction occurs, it stays stiff until released by a different mechanism. Because after contraction, no work is being done in the stiff state, smooth muscle is able to stay contracted for long periods. This explains why sphincters in the body are able to stay closed all the time, while one's bicep fatigues after carrying groceries for just a little while. Relaxation begins when  $\text{Ca}^{2+}$  is either pumped out of the cell through a  $\text{Ca}^{2+}$ ATPase pump, or sodium transporter. This causes CaM to unbind, myosin light chain phosphatase (MLCP) to dephosphorylate myosin, and the myosin heads to release from actin. Interestingly, diacylglycerol (DAG), another product of the  $\text{IP}_3$  path, inhibits MLCP and thereby enhances muscle contraction.

The calcium stored in the SR is maintained in a number of ways. One example being the protein STIM1 responding to lower  $\text{Ca}^{2+}$  levels within it, moving toward the cell membrane, and activating store operated  $\text{Ca}^{2+}$  channels, such as Orai1.

## **6.2.2 Nervous system control**

## **6.3 Cardiac Muscle**

### **6.3.1 A Cellular Level**

**Structure.**

**Contraction-Relaxation.**

### **6.3.2 Nervous system control**

### **6.3.3 Pacemakers**

# Chapter 7

## The Spinal Cord

### 7.1 General Structure

A cross section of the spinal cord would reveal meninges just like the brain and skull; an outer layer of dura, then arachnoid, and finally pia mater. Too, there is an outer layer of white matter followed by a grey matter interior, which is centered around a “central canal.” The spinal cord has 33 vertebrae, beginning with cervical ( $C_1$  to  $C_7$ ), then thoracic ( $T_1$  to  $T_{12}$ ), then lumbar ( $L_1$  to  $L_5$ ), then sacral ( $S_1$  to  $S_5$ ), and finally one coccyx ( $Co_1$  to  $Co_4$ ). The sacral, and especially the coccygeal, vertebrae are fused, so you may see them depicted as a single unit. Adding slightly to the confusion is that there are only 31 pairs of spinal nerves. These are  $C_1 - C_8$ ,  $T_1 - T_{12}$ ,  $L_1 - L_5$ ,  $S_1 - S_5$ , and  $Co_1$ . The spinal nerves exit the spinal cord on either side, i.e., either dorsal or ventral roots. Dorsal entry neurons carry sensory information to the CNS, while ventral exit zones carry information from the CNS to the muscles. Though, importantly, their axons can not necessarily be found on either side, this will depend on where crossing over occurs. The section of grey matter which connects to the dorsal root is called the dorsal horn, and the same is true for the ventral horn. The lateral horn is in between the dorsal and ventral horns.

Note to self: It might be fun to add a more detailed breakdown at some point, such as the general arrangement of all the nuclei in the spinal cord.

Another note to self: It would be good to integrate more clinical outcomes into this. For example, what happens in the case of SCI at various vertebrae, or in vagotomy, etc.

### 7.2 Autonomic Nervous System

Pathways of the autonomic nervous system require two neurons, one that originates in the CNS and terminates at a ganglion, and a post-ganglionic neuron which terminates at the tissue of interest. While the circuit is often considered to have only two neurons, in fact many pre-ganglionic neurons synapse onto many post-ganglionic neurons, which means one neuron can affect many target tissues.

#### 7.2.1 Sympathetic Nervous System

Pre-ganglionic neurons of the sympathetic nervous system originate in the hypothalamus or reticular formation and extend down to the mid-sections of the spinal cord, from neuron pairs  $T_1$  to  $L_2$ . Sympathetic ganglia are directly beside the spinal cord in a long chain (often called the sympathetic

chain). Generally speaking, neurons closer to the T<sub>1</sub> section correspond to organs higher in the body, such as the heart or lungs, while neurons closer to the L<sub>2</sub> section corresponds to those lower in the body, like one's reproductive organs<sup>1</sup>. Pre-ganglionic neurons release acetylcholine onto nicotinic receptors, and post-ganglionic neurons release norepinephrine onto adrenergic receptors.

## 7.2.2 Parasympathetic Nervous System

Parasympathetic neurons originate from the uppermost and lowermost neurons, either leaving directly from the cranial nerves or from the S<sub>2</sub> to S<sub>4</sub> nerves. Ganglia in the parasympathetic nervous system are close to the target tissue, meaning post-ganglionic neurons are very short compared to the pre-ganglionic ones. One of the most paths being from the vagus nerve (a cranial nerve) that carries the majority of parasympathetic signals to organs including the heart, lungs, liver, stomach, intestines, and pancreas. Like the sympathetic nervous system, pre-ganglionic neurons release acetylcholine onto nicotinic receptors. Notably, parasympathetic post-ganglionic neurons also secrete acetylcholine, but onto muscarinic receptors instead.

## 7.3 Acetylcholine, Nicotinic Receptors, and Muscarinic Receptors

Funnily enough, nicotinic receptors (nAChRs) have been described as the most well understood membrane receptor<sup>2</sup>. Nicotinic receptors are fast opening non-specific cationic channels. They are expressed throughout much of the major structures in the brain, peripheral nervous system, and skeletal muscle. As such, drugs targeting these receptors are of considerable interest in many neurological disorders. The binding site for ACh is composed of aromatic amino acids, namely W and Y. This paper describes continual exposure to drugs treating nAChRs as causing their eventual desensitization, while exposure to nicotine increases their expression greatly.

Both of these are of key interest in treating Alzheimer's disease, as loss of cholinergic synapses is one of its features (including significant reduction of both muscarinic and nicotinic receptors). Interestingly, Alzheimer's shows varying loss of nicotinic receptor subtypes across the brain. There is no shortage of drugs currently in development for reversing this.

### Myasthenia Gravis Digression.

Myasthenia Gravis, an autoimmune disorder, is an example of an acetylcholine dysregulation caused muscle disorder. As a digression within a digression, autoimmune disorders most often target endocrine organs, and the belief is that they are to protect against mutations causing hypersecretion<sup>3</sup>. Upon first glance, you may think that Myasthenia Gravis evades this generality, but in fact it does not, as it is well associated with patients that dually have a thymoma (tumor of the thymus). Immune cells begin attacking cells with ACh receptors, tagging them with antibodies. Therefore, Myasthenia Gravis presents itself usually in the weaker muscles, such as those that control the eyes, as they have less ACh receptors and therefore a diminished response to signals. Thus, patients may have

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<sup>1</sup>Memorizing which nerves respond to which segments is likely not a good use of your time at this stage..., although it might have some clinical use to know which organs may be affected after SCI.

<sup>2</sup><https://www.nature.com/articles/nrd2927>

<sup>3</sup>[https://www.cell.com/immunity/pdf/S1074-7613\(20\)30180-1.pdf](https://www.cell.com/immunity/pdf/S1074-7613(20)30180-1.pdf)

drooped eyelids (ptosis), double vision (diplopia), or trouble following moving objects with their eyes. The disease worsens with increased activity, as this causes more antibodies to be released onto ACh receptors—but improves on rest. Fascinatingly, men and women “get” the disease at different times (women typically under 40, while men over 60).

Antibodies may also target other genes (such as MUSK or LRP4) which are involved in ACh receptor localization, or other forms of regulation.

## 7.4 Central Pattern Generation

Defining the central pattern generator is still somewhat contested. Computational models have been used to explore its existence<sup>4</sup>, and for now we should take a few unifying assumptions: **(1)** the central pattern generator exists and is used to generate rhythmic moving, such as walking, **(2)** in many cases it is initiated by the CNS, but aside from that is largely devoid from CNS input, and **(3)** the CNS can work in tandem to compute integrated moving that requires coordination and balance.

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<sup>4</sup><https://www.nature.com/articles/s41598-021-91714-1>



## Part IV

### Spinal Cord Injury

Maybe it sounds like I'm just spouting moral platitudes. But from a vagabond like me, it's not that. I can't begin to tell you how lonely I feel when I come across a beautiful view, then suddenly realize there's no one to enjoy it with me.

– Musashi by Eiji Yoshikawa

# Chapter 8

## The Injury Itself

Spinal cord injury is composed of the primary injury, prototypically, but not restrictively, due to some kind of high impact action. This is usually unpredictable and highly variable. Secondary injury, resulting from inflammation, oxidative stress, and other biological responses is much more predictable and potentially lends itself better to therapeutic intervention.

The lesion's composition is categorized in three ways: **(1)** the non-neural core, **(2)** the astrocytic scar around the core, and **(3)** the spare reactive neural tissue. In the mix of immune cell influx and scar formation, no neural cells can survive at the center of the lesion. On a neuronal level, the rostral end retracts in a process of Wallerian degeneration. The caudal end dies away. Growth from the cell body is limited both by the damaged cell's biochemistry and by the physical barriers which now present themselves in front of the axons. The physical barriers that immediately succeed injury are often called damaged axon-glia complexes (AGCs). Discussed further later, immune cell influx causes astrocytes to form a scar, meant to save the spare surrounding neural tissue, which is composed of both glia and neurons.

The traditional aim of treatment is to bridge the corticospinal tract with distant neurons through a therapeutic combination of inhibiting anti-regenerative and promoting regenerative factors. As I have commented many times, getting neurons to regenerate alone is insufficient in many cases, as reformation of the correct synapses will not necessarily follow. Forcing axon regeneration alone is, incidentally, not too hard—one can pump neurons full of metabolites or simply implant stem cells. The issue being that they do not know where to grow to. One possible route to solving this is remodeling neural circuits using interneurons to bridge these connections. There are also attempts to use biomaterials to simulate a pro-regenerative environment, hopefully enhancing plasticity of the circuits.

### 8.1 Cell Specific Responses

The discussion, for the moment, will mostly use information gathered from Hu *et al.* 2023.

#### 8.1.1 Immune Response

As SCI breaks the spine-blood barrier, influx of immune cells can cause further damage. Evidently, the nature of the immune response being helpful or harmful is still largely contested.

## Neutrophils.

Neutrophils compose part of the immediate response to injury, which are recruited by cytokines and chemokines secreted by cells damaged in the primary injury. They essentially initiate the secondary injury, and reach their peak around 1 DPI. Like most cells, the role of neutrophils cannot be characterized as solely pro- or anti-regenerative. While a high influx of neutrophils is associated with poor patient outcomes, so too are neutrophils associated with guiding macrophages to damaged tissue, suggestive of better recovery.

## Microglia.

In mouse SCI models, it seems that there are two peaks of microglial activity. The time course is remarkably long and disparate, reported 7 DPI and 60 DPI. Microglia can either promote inflammation, thereby worsening the secondary injury (called the M1 phenotype) or decrease inflammation, and promote repair (called M2). It is likely that this response depends on the subtype of microglia, which varies depending on the environment. Regardless, it is true that the earlier one treats SCI, the more likely one is to avoid negative microglial effects. Fascinatingly, in a neonatal setting microglia are able to heal SCI almost entirely through their role secreting fibrinogen, which is able to connect damaged axons back together.

## Macrophages.

Macrophages are considered to be the dominant immune cell located around the injury site. Microglia, conversely, are scattered around the borders of the injury. Depending on the type of glial scar that is formed, different types of macrophages have been found. Macrophages mediate the corraling<sup>1</sup> of cells around the injury site. The phagocytotic abilities of macrophages are of key importance, as loose fragments of cells must be removed, and microglia are incapable of keeping up such a high demand for removal. As a large part of this includes the destroyed myelin of oligodendrocytes, macrophages uptake great amounts of lipids. This can result in the formation of lipid droplets, which causes macrophages to become “foamy.” This foamy phenotype impairs further repair.

## Lymphocytes.

The adaptive immunity is fairly universally regarded as harmful to regeneration (with some exceptions, of course). T cells further break down the spine-blood barrier and increase immune cell invasion. Evidently, T cell entry is also a major source of neuropathic pain in SCI patients. In general, it seems established that the overall immune response after injury impairs further regeneration, and a good example of this can be found here<sup>2</sup>.

## 8.1.2 Neural Response

Apoptosis of neurons initiates at around 4 HAI, but peaks at only 8 HAI<sup>3</sup>.

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<sup>1</sup>Corraling is a term used to describe the formation of a barrier around the injury, preventing further injury. It is composed astrocytes, and other cells, and is important in repair.

<sup>2</sup><https://www.science.org/doi/full/10.1126/science.abd5926>

<sup>3</sup>It is worth noting that these times are likely quite inaccurate, or very injury-type specific. But, they do give a good indication of the approximate timeframe—such as that the majority of this apoptosis occurs within the first day or so after injury.

## Interneurons.

Fascinatingly, it was shown that the ability of neonatal mice to fully recover from SCI was due, in part, to interneurons maintaining excitatory conditions. In adult mice, these interneurons switch to inhibitory after SCI, which dampens signals to motor neurons. A paper investigating this can be found here<sup>4</sup>. One must wonder, can you electronically mimic the excitatory interneurons in fully grown mice? Similar approaches have been done therapeutically, such as with potassium-chloride cotransporter-2 (KCC2) agonist CLP290<sup>5</sup> which seems to dampen the overexcited, inhibitory interneurons. Perhaps one could simply use something like DBS (or in this case, DSS?) on these interneurons.

## Astrocytes.

Astrocytes, being the dominant supportive cell, plays an essential role in SCI. After injury, astrocytes form a physical barrier that is supposedly intended to limit the secondary injury. This occurs after astrocytes become activated, and are helpful in the initial stages but later form a glial scar, impairing regeneration. Astrocytes may either be activated by inflammation, causing them to be neurotoxic (called A1 cells) or by ischemia, causing them to be neuroprotective (A2 cells). The first transformation occurring through the NF- $\kappa$ B path, and the second through STAT3. Microglia are said to be the greatest contributor to activating astrocytes through release of signaling molecules. Another important factor is type 1 collagen upregulation, which results in astrocytic adhesion through cadherin, causing activation and eventual scar formation.

The astrocyte scar is surprisingly thin, only a few layers of cells. Though, its importance is not to be underestimated. When ablated, mice with SCI were worse off by almost every metric. A cornerstone paper on the topic seems to be here<sup>6</sup>. While scar tissue is primarily astrocytic in origin, it is worth mentioning that pericyte derived scar tissue (sometimes called the fibrotic scar) too play a role. Their positive roles include boosting tissue integrity, but so too do they seem to block axon regeneration as a physical barrier.

## Oligodendrocytes.

Oligodendrocytes reportedly begin apoptosis around 1 DPI and it peaks around 8 DPI. Oligodendrocyte precursor cells (OPCs) are may differentiate into oligodendrocytes or Schwann cells after SCI. OPCs have been show to remyelinate neurons after SCI, but fascinatingly, it has been shown that locomotor recovery after SCI does not necessarily require remyelination by oligodendrocytes<sup>7</sup>. Though, plenty of other research suggests it is required—so it is likely context dependent.

## 8.2 Side comments

This section serves to allow me to comment random things until I find a place for them later.

1. Non-neuronal cells secrete a collagen protein (Cthrc1), which is pro-regenerative and affects the ECM<sup>8</sup>.

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<sup>4</sup><https://www.nature.com/articles/s41593-022-01067-9>

<sup>5</sup><https://www.sciencedirect.com/science/article/pii/S009286741830730X>

<sup>6</sup><https://www.nature.com/articles/nature17623>

<sup>7</sup><https://www.nature.com/articles/s41467-018-05473-1>

<sup>8</sup>[https://www.cell.com/developmental-cell/pdf/S1534-5807\(20\)30984-9.pdf](https://www.cell.com/developmental-cell/pdf/S1534-5807(20)30984-9.pdf)

2. The ependyma is a source of stem cells, which contributes to glial scar formation after SCI. Connexin (gap junction) signaling between ependymal cells is supposedly a factor involved in their differentiation.

# Chapter 9

## In The Clinic & Therapeutic Approaches

Death rates from SCI are still as high as 20% in some countries. There is a strong age dependence to this, as the probability of walking again after SCI in those older than 50 is much lower than those under.

For this section, I will largely ignore the uninteresting, obvious approaches to axon/neuron regeneration. This includes the often successful implantation of stem cells and or treatment with various neurotrophic factors. While these represent important measures in the field, they are largely intuitive and still face the broad issue of improper circuit reformation.

### Classifying Injury

One main method of classifying injury and tracking progress of patients is called the American Spinal Injury Association (ASIA) impairment scale. ASIA scores are broken into the following categories:

1. Grade A: Complete impairment, where there is no motor or sensory information being transmitted below the injury site<sup>1</sup>
2. Grade B: Incomplete impairment, where there is no motor information being transmitted, but some sensory information is preserved.
3. Grade C: There is some motor activity preserved, but more than half of the key muscles are too weak to move against gravity (Grade 3 muscles).
4. Grade D: A fair amount of motor activity is preserved, where at least half of the key muscles are above muscle Grade 3.
5. Grade E: There is no impairment at all!

### 9.0.1 Electrical Stimulation

Fascinatingly, electrical stimulation has been used in conjunction with physical therapy in the past with good results<sup>2</sup>. The reasons may be that this promotes stem cell differentiation<sup>3</sup>, or disrupts inhibitory interneuron signaling. The optimal electrical application for differentiation has been explored extensively<sup>4</sup>. One may wonder if the benefits of BSI are in the interface itself, or simply the stimula-

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<sup>1</sup>It is not clear to me if this is measured by EMG or movement.

<sup>2</sup><https://www.nejm.org/doi/pdf/10.1056/NEJMoa1803588?articleTools=true>

<sup>3</sup><https://www.mdpi.com/2073-4409/11/5/846>

<sup>4</sup><https://www.frontiersin.org/articles/10.3389/fbioe.2021.591838/full>

tion. Combining stem cell implantation and electronics is, likely, the future.

A slightly different rose by the same name is functional electrical stimulation (FES). Many trials have shown improvement in patients treated with either external stimulation or internal stimulation.

### **Light.**

Light stimulation feels like a footnote in the electrical modulation story, to me. Though, if one wanted to control different neurons or enzymes on an alternate time course, optogenetic activation may be an option. The obvious issue being that one does not have genetic access to patients, and therefore would need to design (likely very complicated) targeted therapeutics.

### **Sound.**

Another footnote is ultrasound simulation. In this case, it will be low intensity focused ultrasound. Some approaches have seen altered gene expression, but perhaps a more promising one is modulating mechanosensitive channels as was shown here<sup>5</sup>. Notably, this paper found that many mechanically activated channels are affected in ultrasound, including Piezo, and many of the Trp family proteins.

### **Magnetics.**

I would be extremely curious to know if the magnetic field itself has any unique properties beyond its manipulation of the electric field. Still, too, the story is the same. Some seem to enhance channel activation, while others expression. Interestingly, transcranial magnetic stimulation (TMS) has been used as a treatment with some success. Incomplete spinal cord injury has seen improvements from TMS.

## **9.0.2 Biomaterials**

The overall goal in the use of biomaterials is to block a worsened immune response, scar formation, and promote neuron activity. Adding promise to stem cell implantation is the use of biomaterials that enhance proper network reformation<sup>6</sup>. Theoretically, a perfect biomaterial could be a substrate preferable for neuron growth, contain molecules that inhibit the immune response, neurotrophic factors that enhance stem cell differentiation and recruitment, and ion channel agonists. Notably, to date there have been no major publications where a “cocktail” like this has been successful. These sorts of things are usually made from hydrogels, collagens, or select inorganic fibers.

An open question is how one could leverage biomaterials to help clear damaged parts of neurons/cells that would normally be cleared by phagocytosis. Perhaps, one could add materials that are easily oxygenated to dampen the blow of ROS.

To date, implantation of biomaterials have been relatively lackluster in treating patients. While some regeneration scaffolds have proven to improve some neurological function, no patient has regained motor function.

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<sup>5</sup><https://www.nature.com/articles/s41467-022-28040-1>

<sup>6</sup><https://pubs.rsc.org/en/content/articlepdf/2022/bm/d1bm01744f>

### 9.0.3 Drug Treatment

Drug treatment primarily follows the same paths, being reduction of inflammation and neuroprotection. Methylprednisolone (MP) is the only drug approved to treat SCI and works through reducing inflammation. Notably, some side effects have been observed and therefore MP has fallen out of favor for treatment.

### 9.0.4 Surgery

I think you'll find that there is a disappointing lack of options—signaling the primitive nature of neurosurgery! Anyway, surgical intervention aims to restabilize the spinal cord as quickly as possible, particularly through decompression. Early surgery seems indicative of shorter ICU stays and better neurological recovery. The first day or so post SCI is the critical time. Interestingly, even after decompression, the pressure within the spine remains high due to fluid build-up within the dura matter. This makes blood reperfusion more difficult, leading to more problems. While durotomy is often a complication of surgery due to progressive CSF leakage after operation, in this case it can be helpful to lessen spinal pressure, which there evidently is a long tradition of<sup>7</sup>. Duroplasty is a more modern and sophisticated alternative, and can allow opening of the dura matter without as much risk<sup>8</sup>.

Incision directly into the spinal cord itself, myelotomy, has also been done with some success. The belief is that it helps drain some of the harmful dying tissue. There seems to be time dependence in this, where if performed too late after injury it will simply reinvigorate inflammation.

### 9.0.5 Rehabilitation

As you would intuit, exercise is the most common technique, as it preserves muscle mass and promotes circuit reorganization. Another rehabilitative technique is pumping in a significant amount of oxygen, as ischemia occurs after injury. BSIs have also become more popular. Fascinatingly, decoding of handwriting has been used to generate text<sup>9</sup>. Though, these seem to require deep access to the brain.

## 9.1 Integrated Approach

So it seems to me that a good course of treatment would be something like:

1. Immediately after injury, implant a hydrogen with a good restorative milieu, and immunosuppressants.
2. As microglia peak around a week after injury, a few days later the hydrogel can be removed and stem cell implantation can begin.
3. Throughout this time, electrical stimulation can be applied.
4. Some time after this, rehabilitation through physical therapy can begin.

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<sup>7</sup><https://www.sciencedirect.com/science/article/pii/S0020138388901325>

<sup>8</sup><https://www.liebertpub.com/doi/full/10.1089/neu.2014.3668>

<sup>9</sup><https://www.nature.com/articles/s41586-021-03506-2>



## Part V

# Brain-Spine Interfaces

When we stand at the edge of the ocean, we can not understand its vastness. We know only that it is big. Ittō Ittosai is big. And what about Kojirō?

– Reworded from Vagabond by Takehiko Inoue

# Chapter 10

## Biochemistry

While one may find it odd to begin an electronics discussion with a chapter on biochemistry, the fact of the matter is an integrated approach is the only way. As such, to be able to interface with the electrophysiology of neurons, one must fully understand it.

### 10.1 Ion Channels

# Chapter 11

## The Technology of SCI

### 11.1 Historical Dealings

Using electronics to study neural circuits has a long and storied history. One can think of Hodgkin and Huxley, whose ubiquitous nature avoids a need for citation, using an operational amplifier to clamp the voltage of giant squid axons to learn of their properties. In the case of solve spinal cord circuits, so too have electronics long been used<sup>1</sup>.

Restoration of the signals which pervade severance in spinal cord injury through therapeutics has been equally explored. For example, drugs such as clonidine,  $\alpha_2$  receptor agonist, can promote walking in cats with spinal cord injury<sup>2</sup>. This effect can be blocked completely by yohimbine, an  $\alpha_2$  receptor antagonist<sup>3</sup>.

The ability to generate walking-like movements from electrical stimulation of the lower spine, the T10-L1 region, has been long known<sup>4,5</sup>. In these instances, “tonic” stimulation was able to generate step-like movements. Although the term “tonic” is dubious because other papers by the same lab identified frequency dependence in their stimulation<sup>6</sup>. While one can generate step-like movements at once frequency, continued extension can be generated with another.

### 11.2 Reading Thoughts

My personal opinion is that the limiting factor in bridging from the stimulation applied by Tator and Minassian is a reliable way to read ones objective through their brain waves. Lorach and the Courtine group solved this using the WIMAGINE system<sup>7</sup>. Let’s begin by discussing the options.

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<sup>1</sup><https://journals.physiology.org/doi/epdf/10.1152/jn.1943.6.2.111>

<sup>2</sup><https://journals.physiology.org/doi/full/10.1152/jn.1998.79.6.2941>

<sup>3</sup><https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2278596>

<sup>4</sup><https://www.sciencedirect.com/science/article/pii/B9780444521378000188>

<sup>5</sup><https://journals.sagepub.com/doi/pdf/10.1177/1073858417699790>

<sup>6</sup><https://link.springer.com/article/10.1007/s00422-004-0511-5>

<sup>7</sup><https://www.frontiersin.org/articles/10.3389/fnins.2019.00847/full>

### 11.2.1 Different Reading Devices

As you would imagine, there is a tradeoff between the resolution and the damage you will confer upon one's brain. This is described well by Schwartz in 2006, so notably technology may have advanced since then<sup>8</sup>. The four classes of devices are:

1. Electroencephalography (EEG)
2. Electrocorticography (ECoG)
3. Local field potentials (LFPs)
4. Single Unit AP

EEGs, naturally, are non-invasive and have a reading range on the order of a few centimeters. ECoG is on the tenths of centimeters level, while LPS is millimeters level. Single unit AP is as the name describes, and reads individual action potentials. EEGs sit far above the cortex, outside of the skull, and ECoG come in contact with it. Both LPS and single unit require the electrodes to be buried within the cortex itself, thus causing damage to the brain tissue. Therefore, EEG is typically preferable, as it does not require surgery. It is highlighted by Schwartz that many negatives can occur long term from invasive procedures, such as degeneration, volume displacement, or glial encapsulation. Therefore, it is for this reason that I presume the ability to read ones objective was the limiting factor in progressing the field.

For the moment, I will avoid describing individual technologies between the four groups, as it is likely that the tech progresses greatly each year, and it would be pointless to attempt to describe the reading ranges of any without being totally up-to-date. Instead, we will discuss the WIMAGINE in depth here.

#### **WIMAGINE ECoG.**

As the name suggests, the device requires the skull to be torn away and must sit directly upon the cortex. It features 64 electrodes, and a human patient would need to have two implanted in order to read both sides of one's motor cortex.

The device was tested longest in sheep, which was a 10 month trial. Surprisingly, at the end of the 10 month trial, through GFAP staining, they still found a great deal of glial migration/build-up around the site. They did not quantify this, but in my personal opinion, this does not bode well for our patient—as they will have to spend a lifetime with the device. Secondly, calcification of the dura matter over the device had begun. This is not inherently surprising, and perhaps is a good sign that the bone was not irreversibly damaged. However, as the device is wirelessly charged, and wirelessly transmits brain information, one must wonder how years of calcium buildup may impact the ability to send or receive information. Too, if the technology needs a dust-up, the surgeons will have to re-destroy these calcified layers. There is quite a lot of fuss made about the fact that the device is wireless, and how it was a decision made to best serve patient comfort. But, in my opinion, it may spell out more long-term harm than good. It is not clear to me yet whether the  $8 \times 8$  resolution of the electrodes is sufficient to delineate all necessary information. Though, this is the trade-off one must make in choosing between EEG, ECoG, and LPS.

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<sup>8</sup><https://www.sciencedirect.com/science/article/pii/S0896627306007264>

The WIMAGINE system digitizes information with 12 bits of resolution. Data is processed through pwelch spectral analysis. From what I can gather, this is simply another implementation of the FFT.

## 11.3 Lorach and the Courtine Group

This is the most recent iteration of an ongoing project by the Courtine group.

### 11.3.1 Overview

The group used surgical implantation into the skull, over the motor cortex, to record patient movements and wirelessly transmit this to the spinal cord. The technology used to capture these electrocorticographic (ECoG) signals is the WIMAGINE. The design features two antennas, and the second of which transmits motor signals that are to be decoded and sent to a pulse generator. The pulse generator is ACTIVA RC, the same that is used in deep brain stimulation (DBS) or pacemakers in the heart. There is no mention of having the patient replace this in future years, but my assumption is that a new battery will require a new surgery in years to come.

### 11.3.2 Shortcomings

The three shortcomings explicitly mentioned in the **Introduction** are that **(1)** one using this BSI must have motion sensors on in order to compensate (a sort of PID controller, perhaps), **(2)** that the patient's movement was not perceived as natural, and **(3)** that there was still considerable hurdles in traversing variable terrain. Many more shortcomings exist than this, and we will go through them below.

#### Motion Sensors

I suppose it is obvious why one would use a motion sensor rather than an EMG—as it would be too difficult to discern whether the motion itself, or strength of contraction, is too high using an EMG alone. Still, though, I wonder if this is a feasible alternative. Perhaps one can simply predict motion based on EMG patterns.

The largest drawback, in my opinion, is their desire to make it quickly programmed/calibrated. As with DBS, one would actually prefer longer calibration time. That is, one would be happy to wait a few extra hours or days if it makes their ability to walk improve.

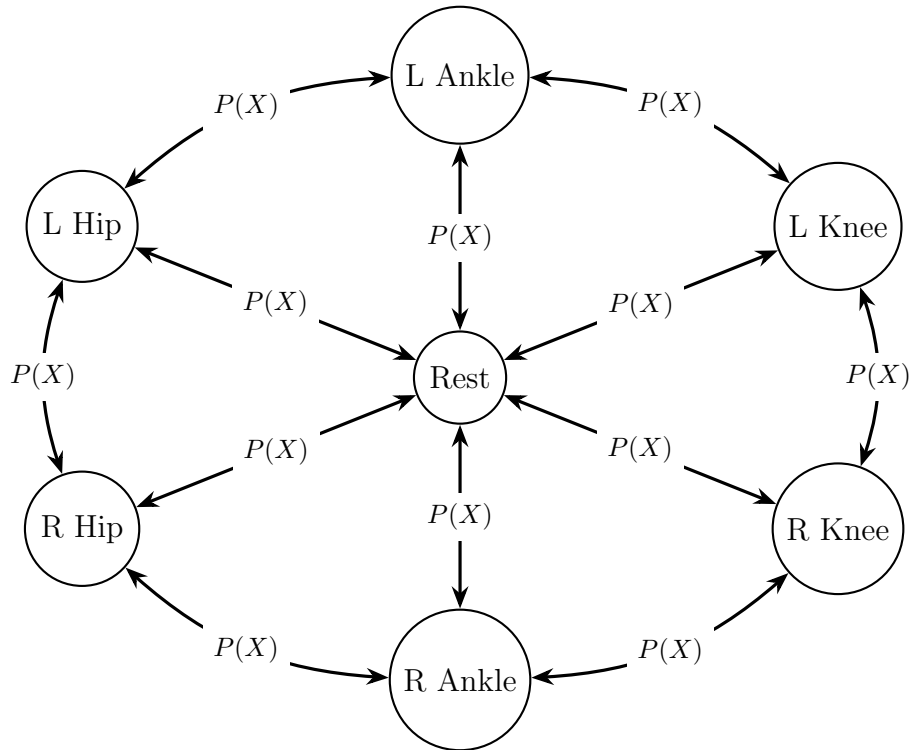
Similarly, they commented that the patient tested was able to walk without the BSI turned on (using crutches) after some time. This signals three things to me, **(1)** this patient's injury was not, per se, insurmountably severe, **(2)** the plastic abilities of a damaged spinal cord may be better than we had thought—especially since the patient's injury predated this treatment by 10 years, **(3)** that there was likely some important tuning being done by the brain-spine endogenous connection (i.e., their BSI was not doing all of the work), and **(4)** in some cases, no BSI is likely needed at all, and electrical stimulation may be sufficient. This in itself is a quite interesting, alternative patient state. For example, one can imagine a scenario where a patient's brain-spine communication is deeply dampened, but not entirely ceased by damage. Therefore, one would only need amplification of signals as opposed to complete generation of new ones. There may be some of that here. Another comment to make is that

their signals are incredibly un-sophisticated, as presented in this paper. That is, they use 16 random electrodes, which do not have connections to synapses themselves. Perhaps this design is to hijack a more reflex-based approach to walking—hence why one can not traverse variable terrain that goes beyond reflexive movement.

A comment regarding the surgery itself: The patient's are having two large, 50mm arrays placed into their skull. Naturally, most would agree to this in order to restore their ability to walk. However, it surprises me that they chose to make the device wireless, given that the surgery is already quite invasive.

## Seven States

Data is processed and used to edit the probability distribution of a Markov chain, which progresses the electrode paddle through seven defined states defined below.



Notably, they did not explicitly mention that one of the progressions could be from state to self state, but one can only assume this is true. So too, only seven states seems to limit the mobility of a patient.

Thank you.

Man takes up the sword in order to shield the wound  
in his heart sustained in a far-off time beyond re-  
membrance. Man wields the sword so that he may  
die smiling in some far-off time beyond perception.  
– Berserk by Kentaro Miura