Week 9: Linear Regression!

DSUA111: Data Science for Everyone, NYU, Fall 2020

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- This slideshow: [https://jjacobs.me/dsua111-sections/week-09 (https://jjacobs.me/dsua111-sections/week-09)](https://jjacobs.me/dsua111-sections/week-09)
- All materials: [https://github.com/jpowerj/dsua111-sections (https://github.com/jpowerj/dsua111-sections)](https://github.com/jpowerj/dsua111-sections)

Overview

1. Regression in General: What it is and what it isn't
2. Ordinary Least Squares (OLS) Regression

This is the most important topic in the course, practically speaking

- All the fancy machine learning / AI / neural net methods, they are all glorified regressions

The "best fit" line: make sure you check your intuition!

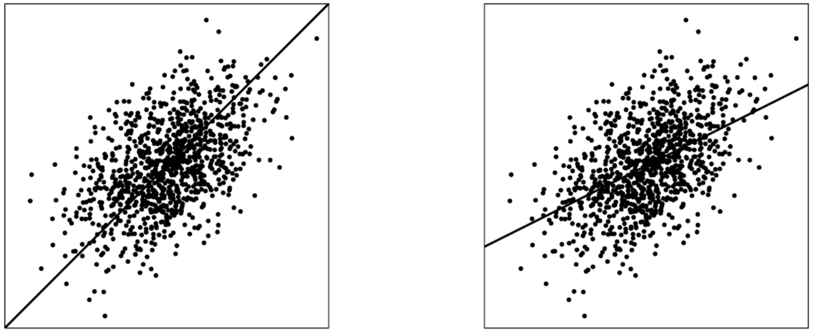


Figure 4.2 *Data simulated from a bivariate normal distribution with correlation 0.5. The regression line, which represents the best prediction of y given x, has half the slope of the principal component line, which goes closest through the cloud of points.*

- When given this sort of scatterplot (without any lines superimposed) and asked to draw the regression line of y on x, students tend to draw the principal component line shown in Figure 4.2a. However, **for the goal of predicting $\boldsymbol{y}$ from $\boldsymbol{x}$**, or for estimating the average of y for any given value of x, **the regression line is in fact better**--even if it does not appear so at first.
- The superiority of the regression line for estimating the average of y given x can be seen from a careful study of Figure 4.2.

The "best fit" line: make sure you check your intuition!

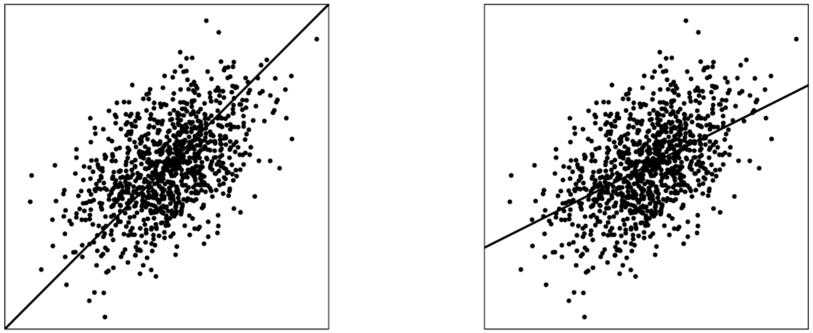


Figure 4.2 *Data simulated from a bivariate normal distribution with correlation 0.5. The regression line, which represents the best prediction of y given x, has half the slope of the principal component line, which goes closest through the cloud of points.*

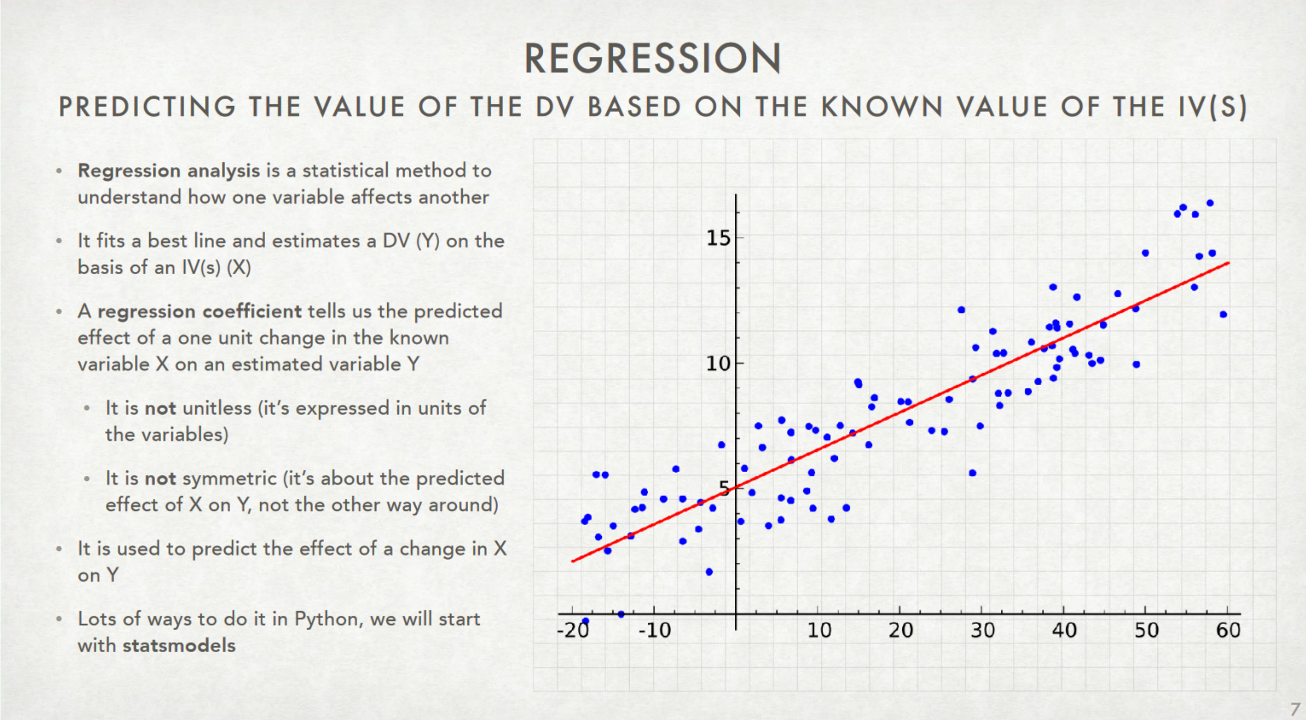
- For example, consider the points at the **extreme left** of either graph. They all lie above the principal components line but are roughly half below and half above the regression line. Thus, the principal component line **underpredicts y for low values of x**.

- Similarly, a careful study of the right side of each graph shows that the principal component line **overpredicts y for high values of x**.

- In contrast, the regression line again gives unbiased predictions, in the sense of **going through the average value of y given x**.

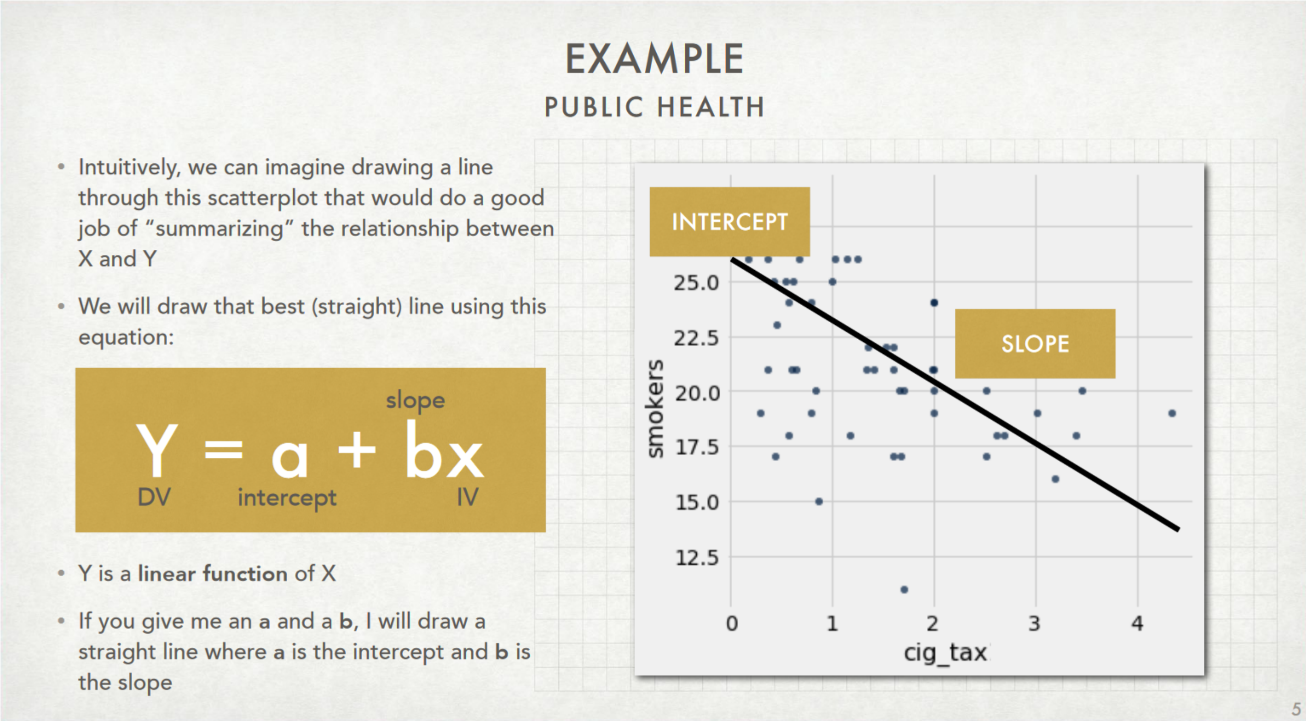
(Gelman and Hill, "Data Analysis Using Regression and Multilevel/Hierarchical Models", 58)

Regression Overview



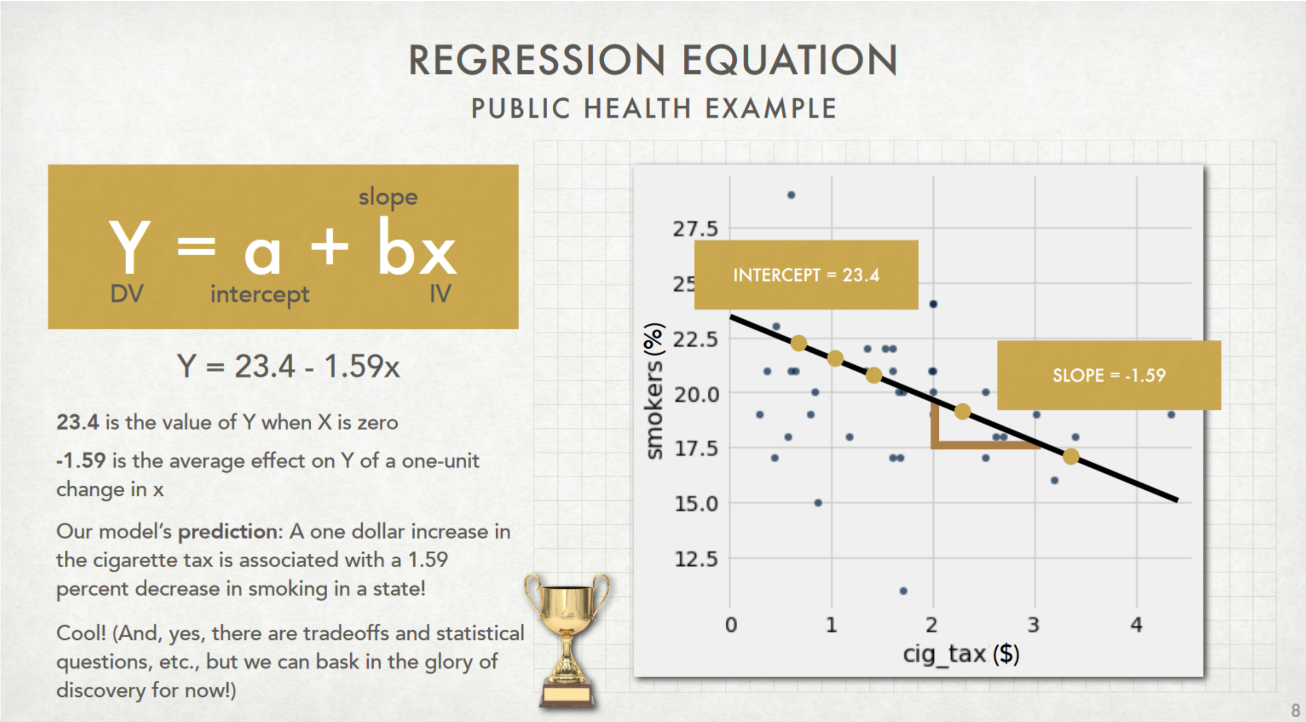
(Lecture 16.1, Slide 7)

Ordinary Least Squares (OLS) Regression: The *Model*



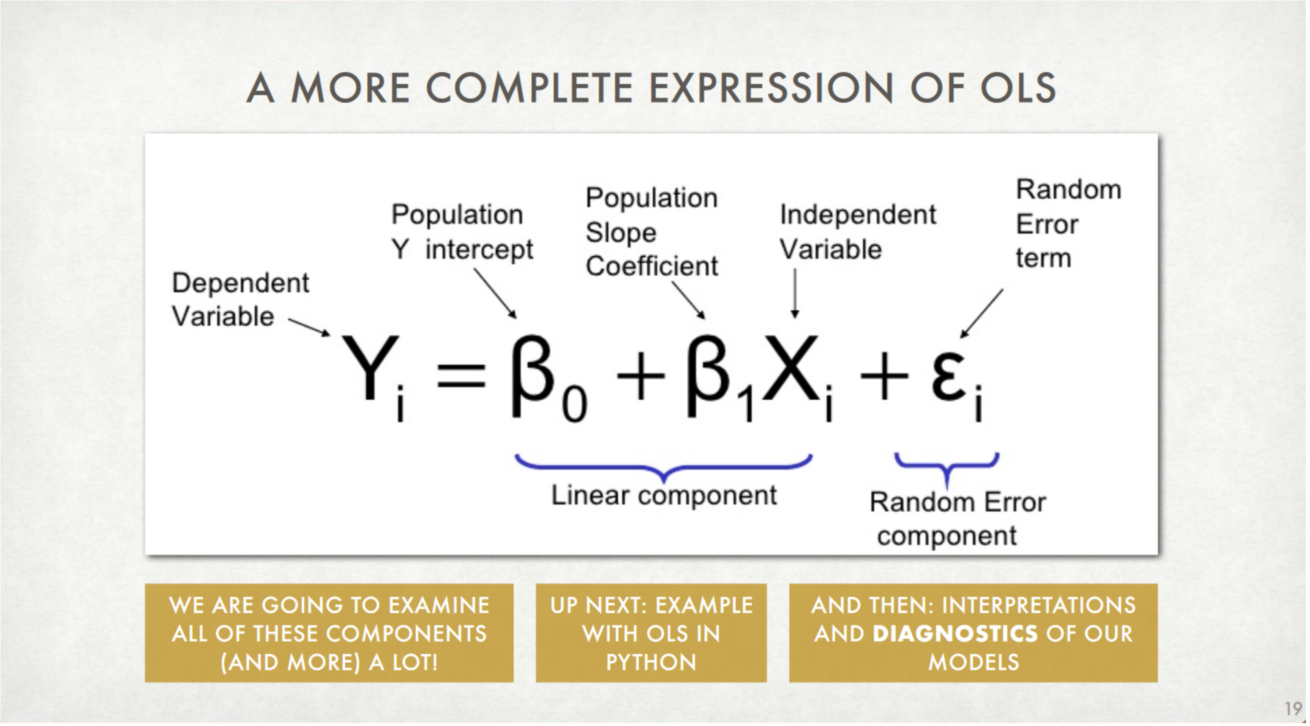
This is the non-"fitted" model, since we don't yet know the precise values of a or b

Ordinary Least Squares (OLS) Regression: The *Fitted* Model



By estimating the parameters of our model using the data in the dataset, we obtain $a = 23.4$ and $b = -1.59$

The "full" OLS model



(Lecture 16.2, Slide 19)

Regression in Python

In [2]:

```
import statsmodels.formula.api as smf
```

The Dataset: Colonial History and Life Expectancy

In [4]:
```
import pandas as pd
import numpy as np
colonial_df = pd.read_csv("colonial_life_expectancy.csv")
```

In [5]:
```
colonial_df
```

Out[5]:

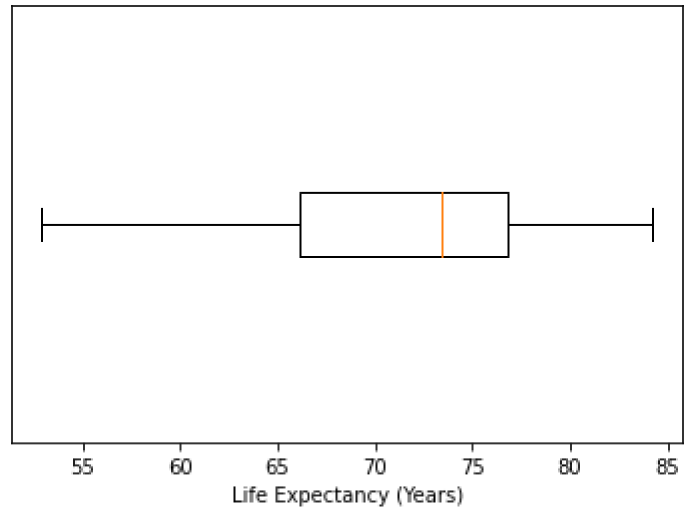
	country	ind_date	yrs_since_ind	year	life_exp
0	Afghanistan	191908	97	2016	62.7
1	Albania	191307	103	2016	76.4
2	Algeria	196207	54	2016	76.4
3	Angola	197511	41	2016	62.6
4	Antigua and Barbuda	198111	35	2016	75.0
...	...	...	...	...	...
178	Venezuela	183001	186	2016	74.1
179	Vietnam	195407	62	2016	76.3
180	Yemen	196711	49	2016	65.3
181	Zambia	196410	52	2016	62.3
182	Zimbabwe	198004	36	2016	61.4

183 rows × 5 columns

Exploratory Data Analysis

- NOTE: YOU ACTUALLY DO NEED TO DO THIS IRL...

In [19]:
```
import matplotlib.pyplot as plt
plt.boxplot(colonial_df['life_exp'], vert=False)
plt.xlabel("Life Expectancy (Years)")
# https://matplotlib.org/3.1.1/api/_as_gen/matplotlib.pyplot.tick_params.html
plt.tick_params(axis='y', which='both', left=False, labelleft=False)
plt.show()
```



Outliers?

In [23]:
```
colonial_df.sort_values(by='life_exp')
```

Out[23]:

	country	ind_date	yrs_since_ind	year	life_exp
94	Lesotho	196610	50	2016	52.9
31	Central African Republic	196010	56	2016	53.0
145	Sierra Leone	196104	55	2016	53.1
32	Chad	196008	56	2016	54.3
39	Côte d'Ivoire	196008	56	2016	54.6
...	...	...	...	...	...
146	Singapore	196508	51	2016	82.9
7	Australia	190101	115	2016	82.9
154	Spain	147901	537	2016	83.1
159	Switzerland	129108	725	2016	83.3
84	Japan	159000	426	2016	84.2

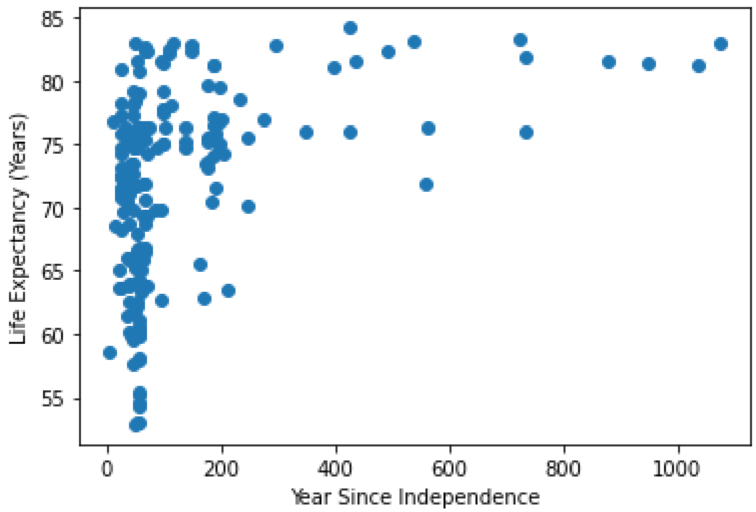
183 rows × 5 columns

(btw, "Lesotho" is pronounced "Leh-Soo-Too"... fun fact)

Scatterplottin

In [25]:

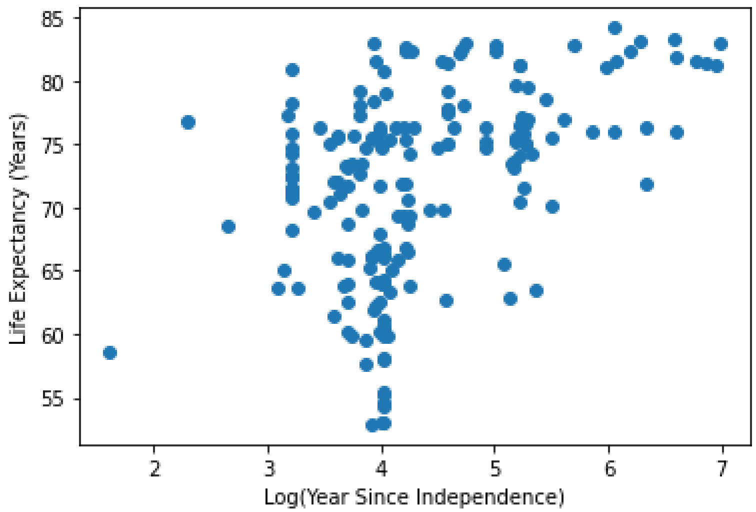
```
plt.scatter(colonial_df['yrs_since_ind'], colonial_df['life_exp'])
plt.xlabel("Year Since Independence")
plt.ylabel("Life Expectancy (Years)")
plt.show()
```



(sidebar: for variables with skewed distributions like years since independence, you really should take the log to "de-skew" them)

In [65]:

```
plt.scatter(np.log(colonial_df['yrs_since_ind']), colonial_df['life_exp'])
plt.xlabel("Log(Year Since Independence)")
plt.ylabel("Life Expectancy (Years)")
plt.show()
```



Before we estimate the model, remember what our hypotheses are!

- H_0: Changes in the independent variable have no effect on the dependent variable
  - i.e., $\beta_1 = 0$
  - So, in our case: **number of years since independence** has **no effect** on **life expectancy**
- H_A: Changes in the independent variable have some (nonzero) effect on the dependent variable
  - i.e., $\beta_1 \neq 0$
  - In our case: **number of years since independence** has an **effect** on **life expectancy**
- (Remember our model: $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$)
  - $\text{LifeExpectancy}_i = \beta_0 + \beta_1 \text{YrsSinceIndependence}_i + \varepsilon_i$

In [28]:
```
results = smf.ols('life_exp ~ yrs_since_ind', data=colonial_df).fit()
```

(Why do we have to add `.fit()` ?)

In [79]:
```
summary = results.summary()
summary.extra_txt = None; summary
```

Out[79]:

OLS Regression Results

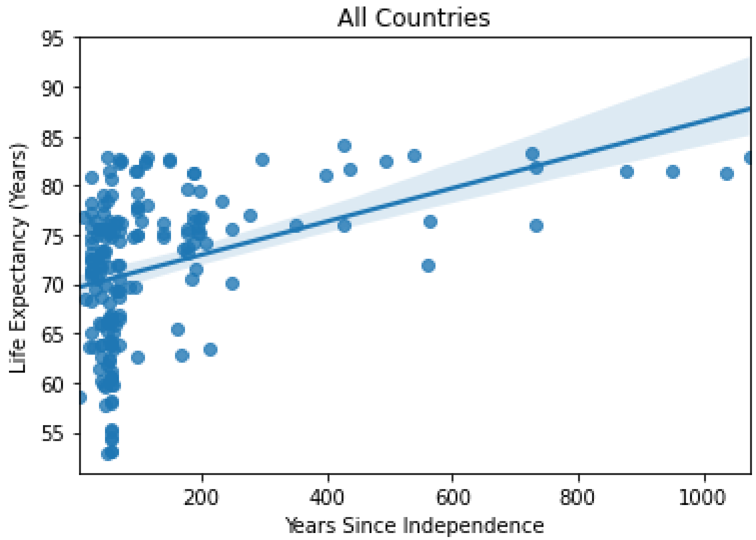
Dep. Variable:	life_exp	**R-squared:**	0.164
Model:	OLS	**Adj. R-squared:**	0.159
Method:	Least Squares	**F-statistic:**	35.52
Date:	Tue, 10 Nov 2020	**Prob (F-statistic):**	1.29e-08
Time:	00:23:41	**Log-Likelihood:**	-614.72
No. Observations:	183	**AIC:**	1233.
Df Residuals:	181	**BIC:**	1240.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	69.5870	0.636	109.483	0.000	68.333	70.841
yrs_since_ind	0.0169	0.003	5.960	0.000	0.011	0.022

Omnibus:	8.399	**Durbin-Watson:**	1.912
Prob(Omnibus):	0.015	**Jarque-Bera (JB):**	8.177
Skew:	-0.470	**Prob(JB):**	0.0168
Kurtosis:	2.564	**Cond. No.**	275.

In [58]:

```
import seaborn as sns
sns.regplot(x='yrs_since_ind', y='life_exp', data=colonial_df)
plt.title("All Countries")
plt.xlabel("Years Since Independence")
plt.ylabel("Life Expectancy (Years)")
plt.show()
```



Appendix I: Removing Outliers

- Sketchy, but in this case we have a historical reason for removing outliers: we can revise our population of interest to be *countries that achieved independence since the 1648* [Treaty of Westphalia (https://en.wikipedia.org/wiki/Peace_of_Westphalia)](https://en.wikipedia.org/wiki/Peace_of_Westphalia), which (long story short) inaugurated the era of the sovereign nation-state

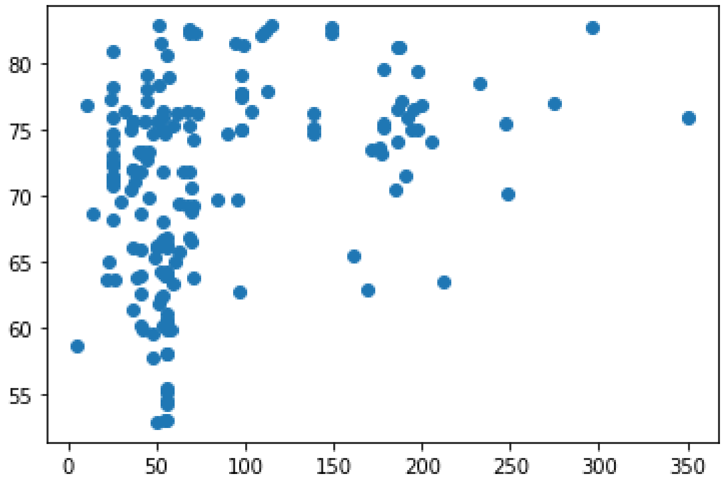
In [40]:
```
tw_df = colonial_df[colonial_df['yrs_since_ind'] < 368].copy()
```

In [43]:
```
print("Number of countries before dropping outliers: " + str(len(colonial_df)))
print("Number of countries after dropping outliers: " + str(len(tw_df)))
```

```
Number of countries before dropping outliers: 183
Number of countries after dropping outliers: 168
```

In [46]:

```
plt.scatter(tw_df['yrs_since_ind'], tw_df['life_exp'])
plt.show()
```



In [44]:
```
results_tw = smf.ols('life_exp ~ yrs_since_ind', data=tw_df).fit()
```

In [80]:
```
tw_summary = results_tw.summary()
tw_summary.extra_txt = None; tw_summary
```

Out[80]:

OLS Regression Results

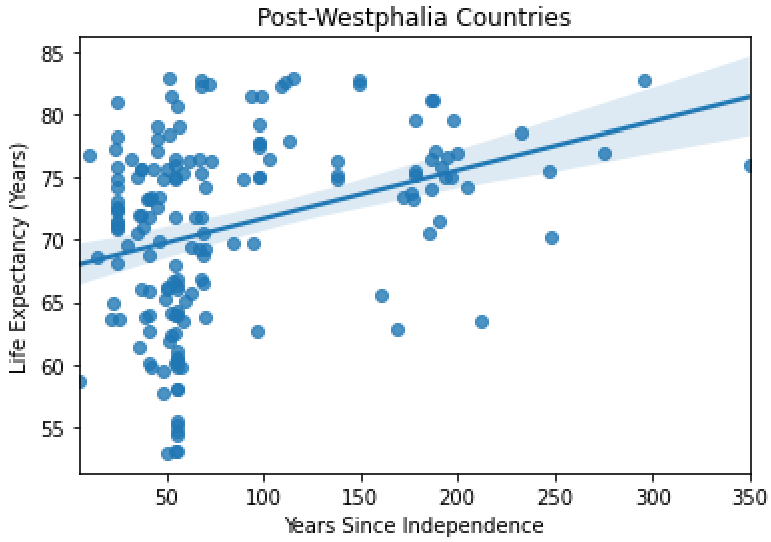
Dep. Variable:	life_exp	**R-squared:**	0.110
Model:	OLS	**Adj. R-squared:**	0.105
Method:	Least Squares	**F-statistic:**	20.61
Date:	Tue, 10 Nov 2020	**Prob (F-statistic):**	1.08e-05
Time:	00:24:07	**Log-Likelihood:**	-565.18
No. Observations:	168	**AIC:**	1134.
Df Residuals:	166	**BIC:**	1141.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	67.8297	0.889	76.285	0.000	66.074	69.585
yrs_since_ind	0.0387	0.009	4.539	0.000	0.022	0.056

Omnibus:	7.032	**Durbin-Watson:**	1.876
Prob(Omnibus):	0.030	**Jarque-Bera (JB):**	6.257
Skew:	-0.399	**Prob(JB):**	0.0438
Kurtosis:	2.493	**Cond. No.**	171.

In [81]:

```
import seaborn as sns
sns.regplot(x='yrs_since_ind', y='life_exp', data=tw_df)
plt.title("Post-Westphalia Countries")
plt.xlabel("Years Since Independence")
plt.ylabel("Life Expectancy (Years)")
plt.show()
```



Appendix II: ...You really should log the skewed variables

In [82]:
```
colonial_df['log_yrs_since_ind'] = colonial_df['yrs_since_ind'].apply(np.log)
```

In [83]:
```
results_log = smf.ols('life_exp ~ log_yrs_since_ind', data=colonial_df).fit()
```

In [84]:
```
summary_log = results_log.summary()
summary_log.extra_txt = None; summary_log
```

Out[84]:

OLS Regression Results

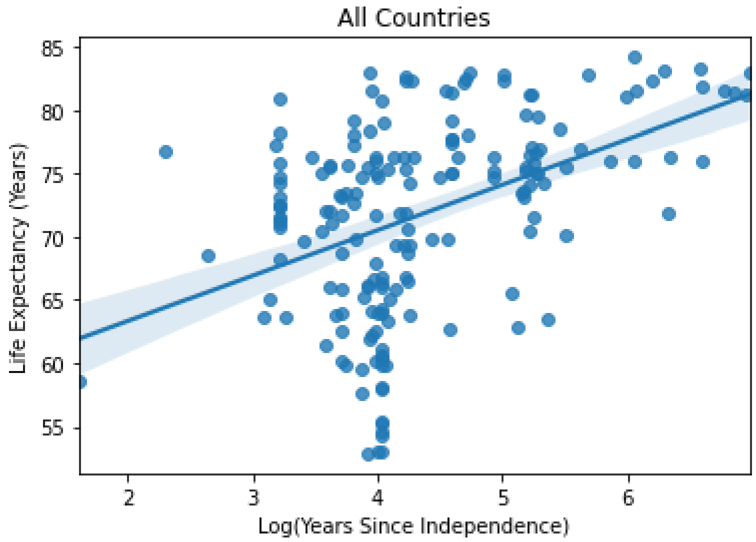
Dep. Variable:	life_exp	**R-squared:**	0.190
Model:	OLS	**Adj. R-squared:**	0.186
Method:	Least Squares	**F-statistic:**	42.53
Date:	Tue, 10 Nov 2020	**Prob (F-statistic):**	6.74e-10
Time:	00:24:32	**Log-Likelihood:**	-611.80
No. Observations:	183	**AIC:**	1228.
Df Residuals:	181	**BIC:**	1234.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	56.1455	2.452	22.897	0.000	51.307	60.984
log_yrs_since_ind	3.5956	0.551	6.521	0.000	2.508	4.684

Omnibus:	8.079	**Durbin-Watson:**	1.850
Prob(Omnibus):	0.018	**Jarque-Bera (JB):**	8.422
Skew:	-0.505	**Prob(JB):**	0.0148
Kurtosis:	2.712	**Cond. No.**	22.5

In [85]:

```
import seaborn as sns
sns.regplot(x='log_yrs_since_ind', y='life_exp', data=colonial_df)
plt.title("All Countries")
plt.xlabel("Log(Years Since Independence)")
plt.ylabel("Life Expectancy (Years)")
plt.show()
```



(and now with just the post-Westphalia countries)

In [86]:
```
tw_df['log_yrs_since_ind'] = tw_df['yrs_since_ind'].apply(np.log)
```

In [87]:
```
results_tw_log = smf.ols('life_exp ~ log_yrs_since_ind', data=tw_df).fit()
```

In [88]:
```
results_summary = results_tw_log.summary()
results_summary.extra_txt = None; results_summary
```

Out[88]:

OLS Regression Results

Dep. Variable:	life_exp	**R-squared:**	0.093
Model:	OLS	**Adj. R-squared:**	0.088
Method:	Least Squares	**F-statistic:**	17.02
Date:	Tue, 10 Nov 2020	**Prob (F-statistic):**	5.84e-05
Time:	00:24:42	**Log-Likelihood:**	-566.81
No. Observations:	168	**AIC:**	1138.
Df Residuals:	166	**BIC:**	1144.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	57.6155	3.297	17.477	0.000	51.107	64.124
log_yrs_since_ind	3.2210	0.781	4.125	0.000	1.680	4.763

Omnibus:	7.992	**Durbin-Watson:**	1.857
Prob(Omnibus):	0.018	**Jarque-Bera (JB):**	7.861
Skew:	-0.484	**Prob(JB):**	0.0196
Kurtosis:	2.570	**Cond. No.**	26.8

In [89]:

```
import seaborn as sns
sns.regplot(x='log_yrs_since_ind', y='life_exp', data=tw_df)
plt.title("Post-Westphalia Countries")
plt.xlabel("Log(Years Since Independence)")
plt.ylabel("Life Expectancy (Years)")
plt.show()
```