
welcome
to Lecture 8
Computation and the Brain

What happened last Wednesday

Talk by Dr. Kenneth Kay

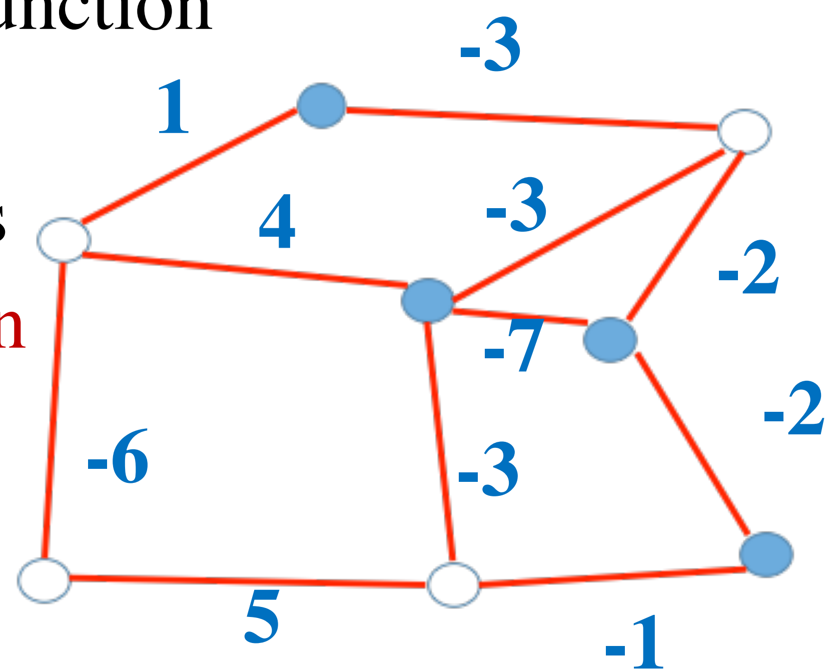
Hopfield networks

”Total happiness” is Lyapunov function

Like Ising model only easier

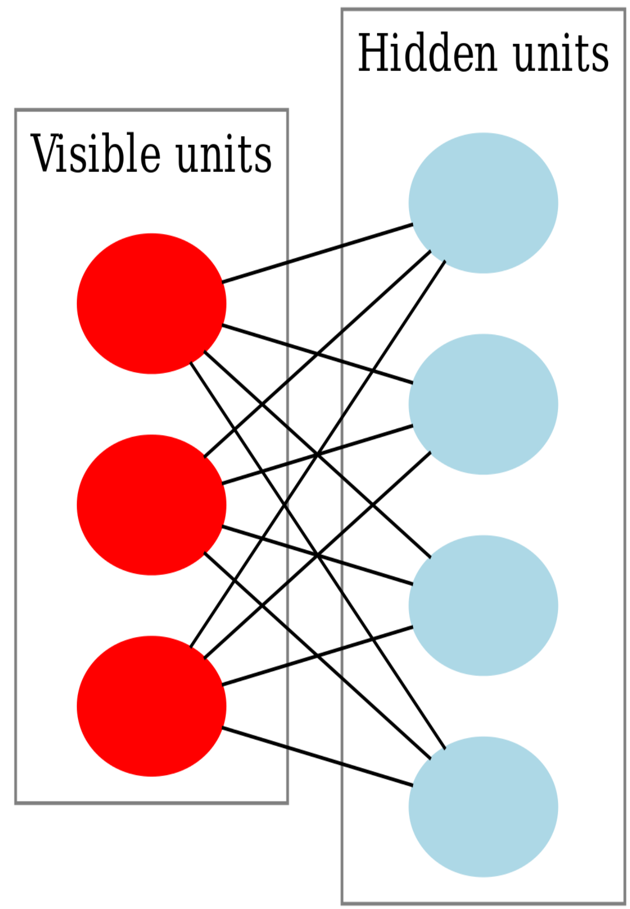
Can be trained to store memories with associative recall (~ “pattern completion”)

Capacity ~ n/7, not very robust



Restricted Boltzmann Machines

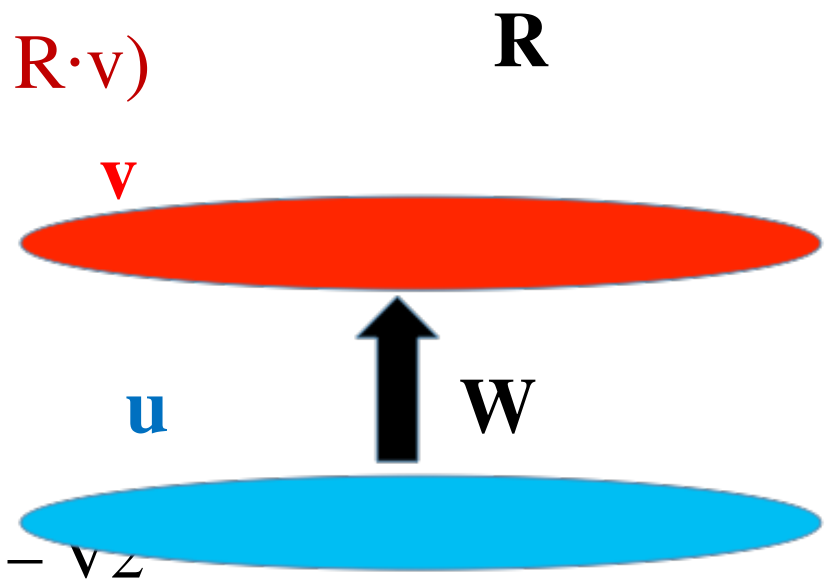
- Restricted Boltzmann machine: graph is bipartite [Hinton 2005]
- Input layer, hidden layer
- Learning happens!
- Can be stacked to deep net
- In fact, that's how the whole thing started…



The Feedforward + Recurrent network

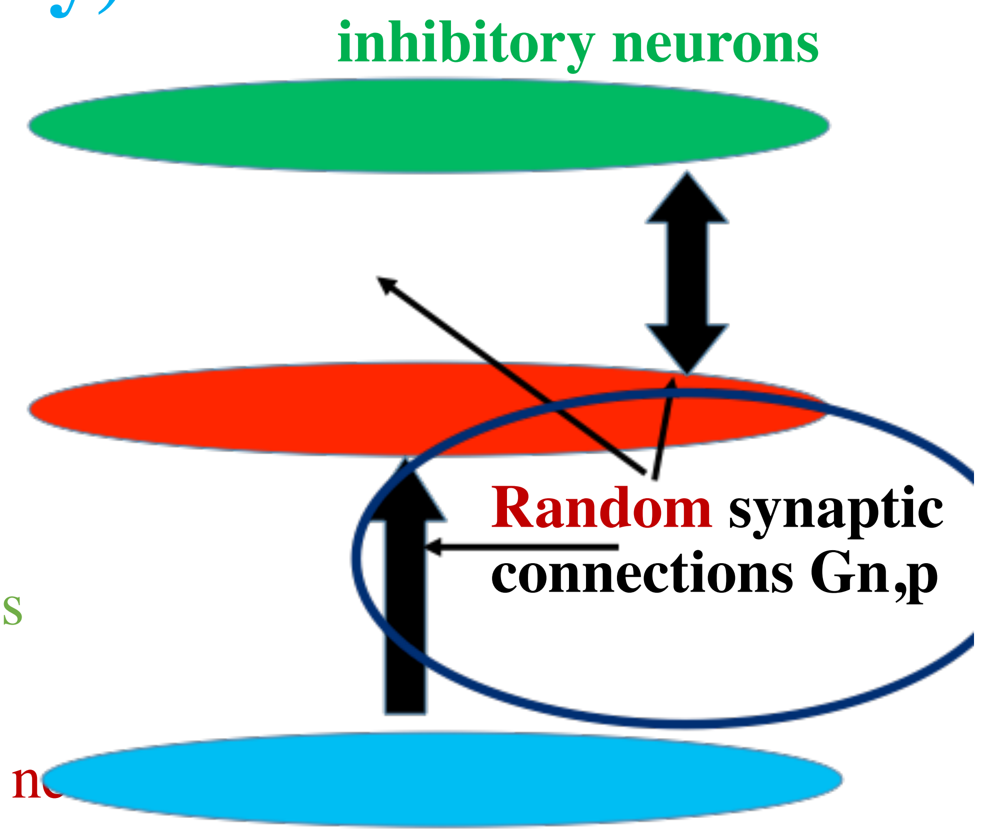
$$\tau \cdot dv/dt = -v + F(W \cdot u + R \cdot v)$$

- The linear case: Amplifies the input projection on the main eigenvector
- Analysis of nonlinear system:
- Simple and complex cells in V1 – V2



Another FFR network: Excitation-Inhibition balance (will revisit today)

- The blue cells spike
- Many red cells receive input and fire
- The green cells receive input
- They fire, and inhibit the red
- Maybe too much
- Now they receive less
  input from the red, and they inhibit less
- All these synapses are random
- By the law of large numbers, they are n



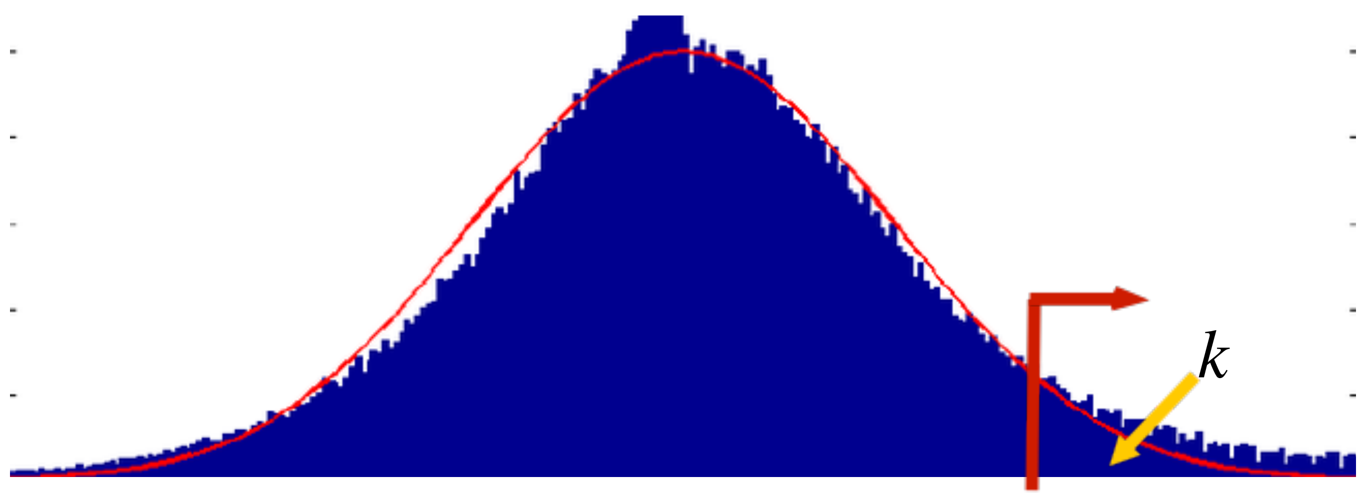
Questions? Thoughts? Feedback?

Today:

- Invited lecture by Melina Tsitsiklis, PhD candidate in Neuroscience, Columbia

- **Then:**
  - A dynamical system for E-I balance
  - Random projection and Cap (RP&C)
  - Neuron assemblies and their operations

RP&C



Also today:

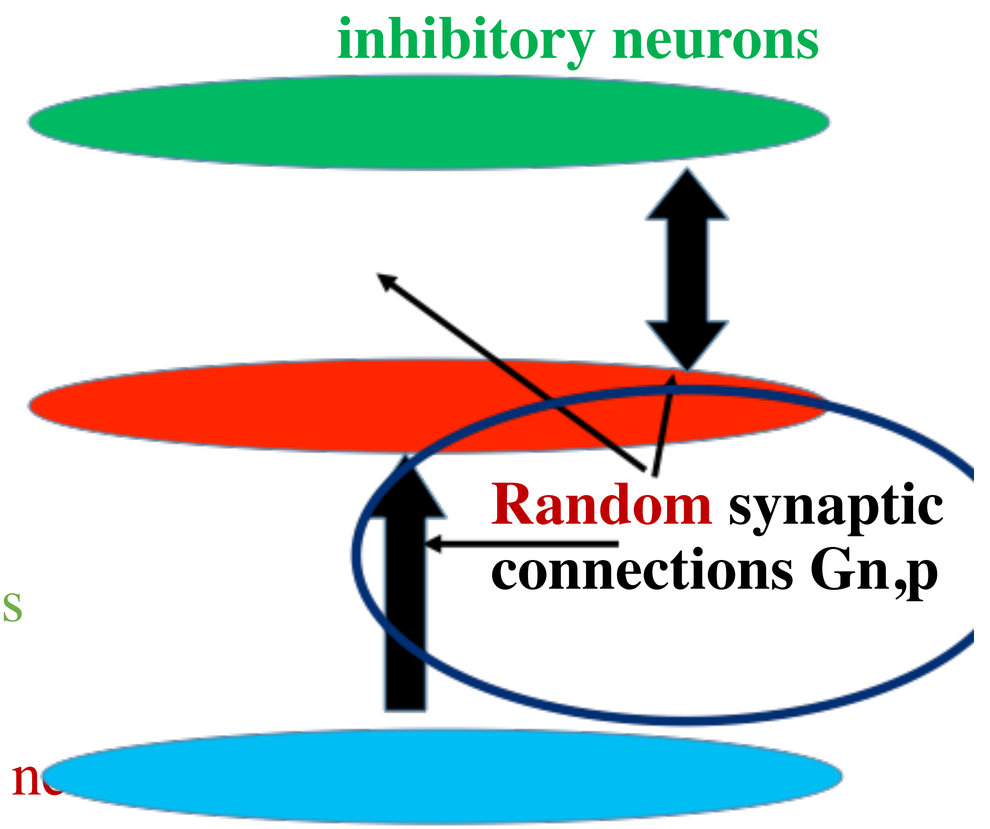
- A dynamical system for E-I balance
- Random projection and Cap (RP&C)
- Neuron assemblies and their operations
- (Work with Santosh Vempala, Wolfgang Maass, and Mike Collins)

The coming weeks:

- Locomotion and dynamical systems
- Reinforcement learning
- Language
- Anything else?
- December 6: project presentations!
- (projects due a few days later)

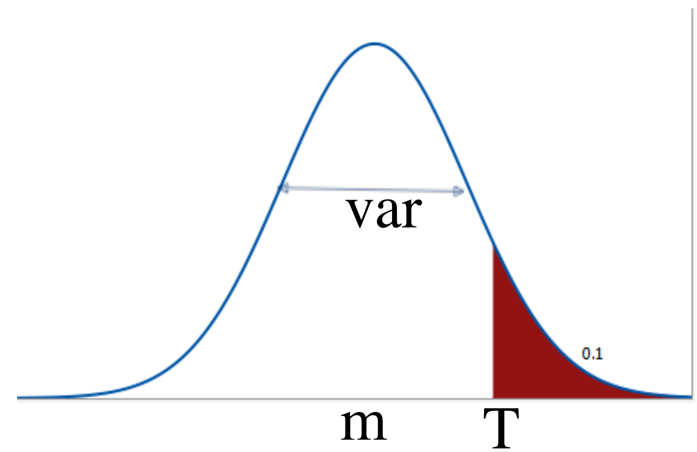
Recall: Excitation-Inhibition balance

- The blue cells spike
- Many red cells receive input and fire
- The green cells receive input
- They fire, and inhibit the red
- Maybe inhibit the red too much
- Now they receive less input from the red, and they inhibit less
- All these synapses are random
- By the law of large numbers, they are n



E – I balance

Notation: GT(T, m, var) is the Gaussian tail with parameters m and var above the threshold T



E – I balance: nonlinear ODE

- Recall lowpass filter ODE

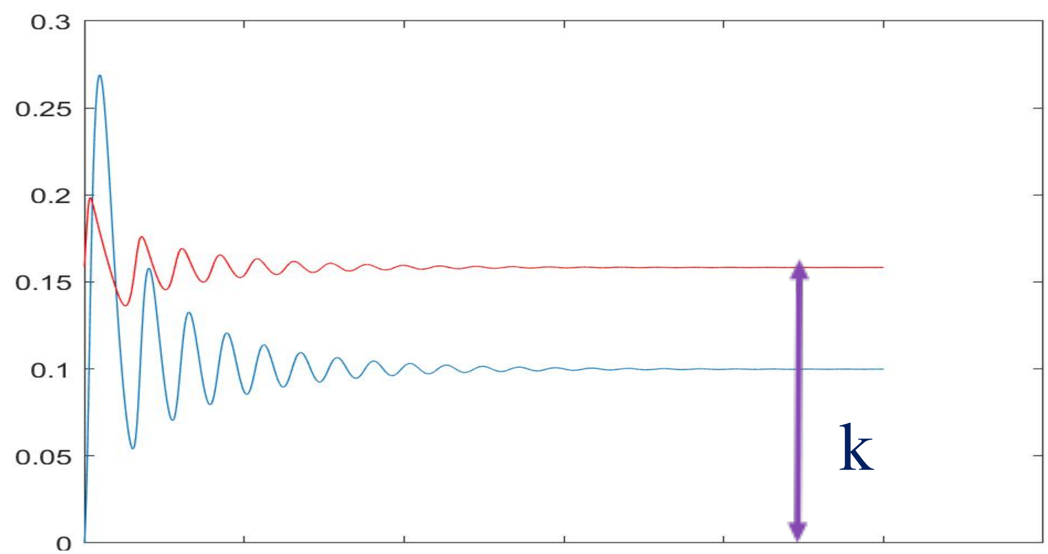
$\tau \cdot dv/dt = -v + F(W \cdot u + R \cdot v)$

$$\tau E\, \dot{E} = -E + GT(TE, np(E - I), np(1-p)(E + I))$$
$$\tau I\, \dot{I} = -I + GT(TI, npI, np(1-p)I)$$

recall notation $GT(T, m, var)$

E – I balance: Solving the ODE numerically

If **τE** is sufficiently larger than **τI**, an E – I balance will be reached after a few up and down oscillations

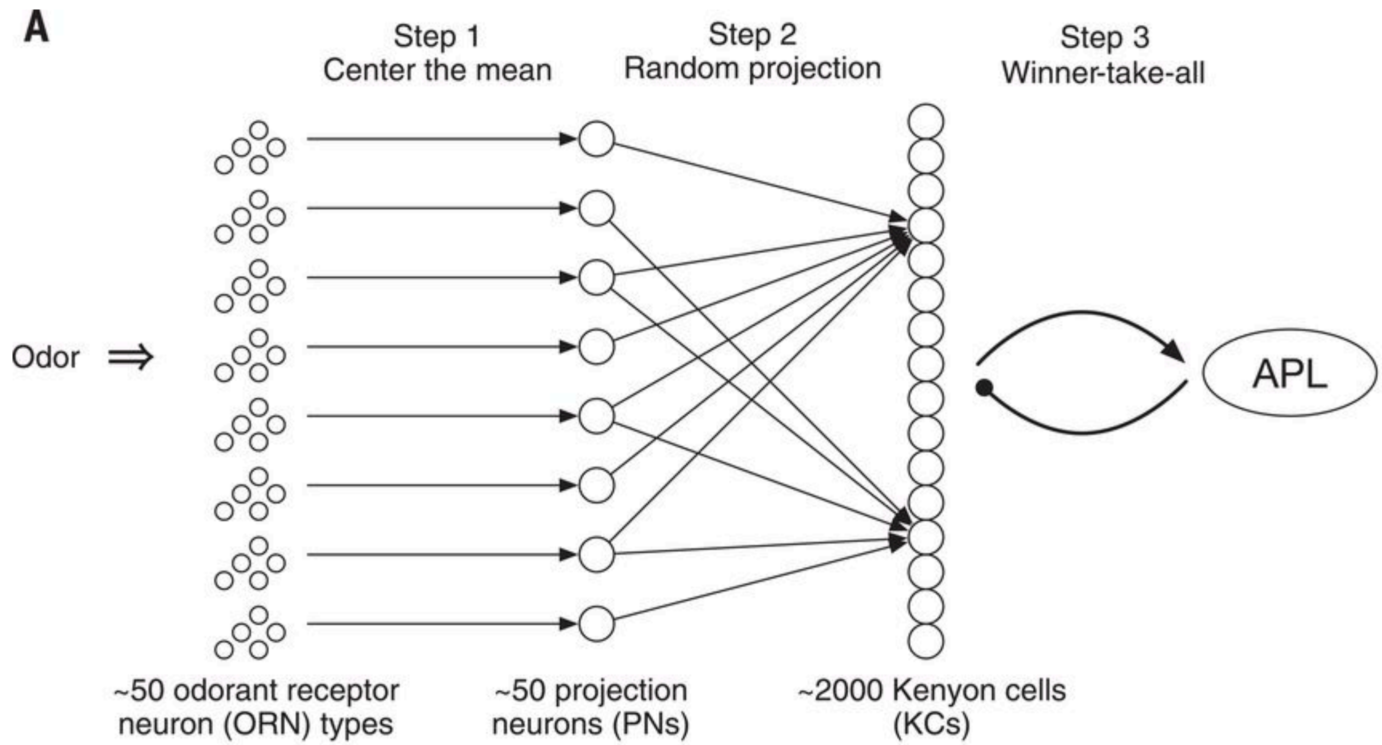


Questions? Thoughts? Feedback?

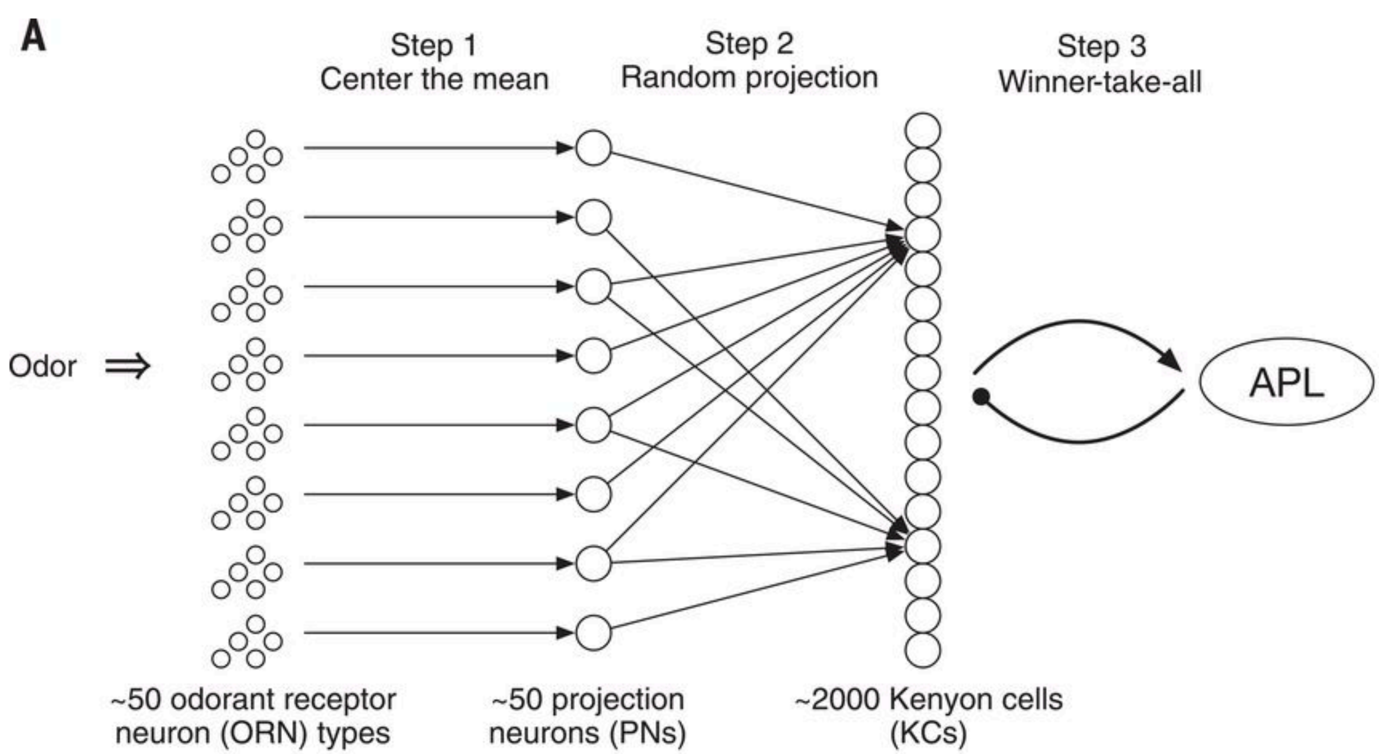
NYT last week:
Jeff Hawkins Is Finally Ready to Explain His Brain Research

More E-I balance: How fruit flies remember smells

A

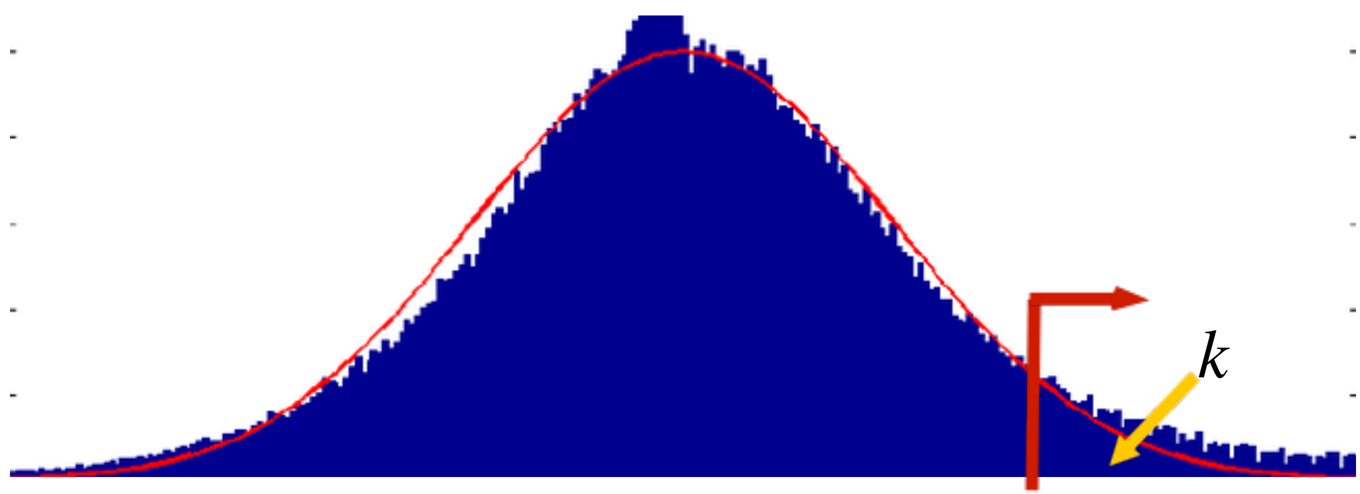


Random project and cap (RP&C)

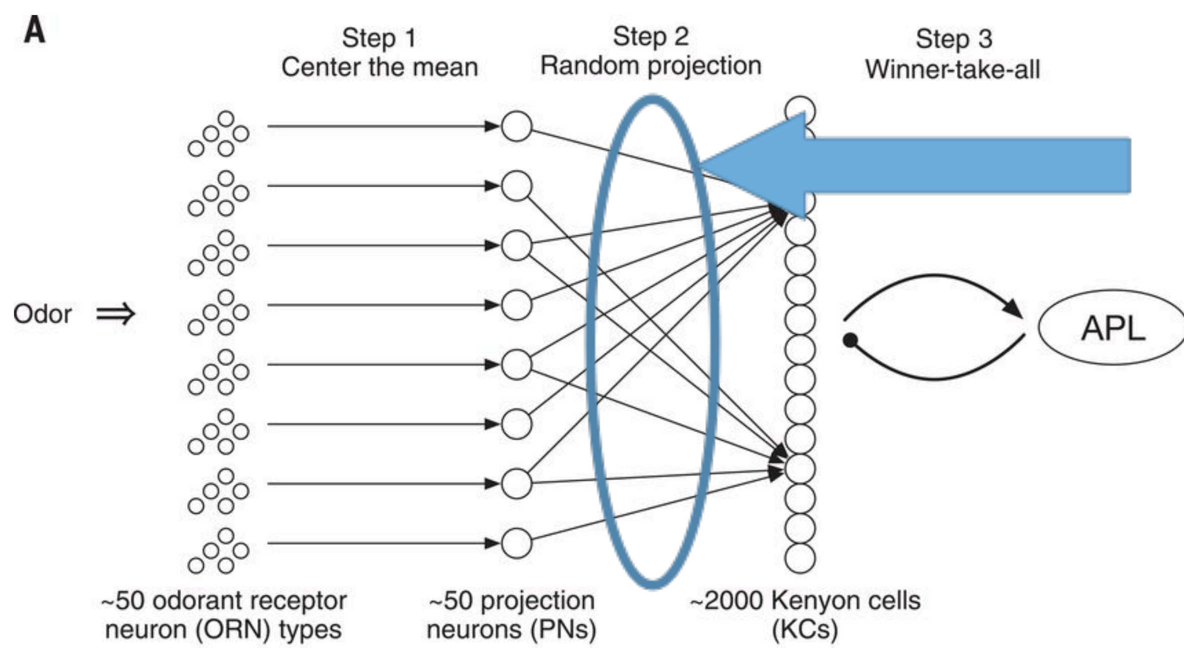


random projection followed by ***cap (RP&C):*** k ≈100 winners (out of ≈2000) take all

RP&C



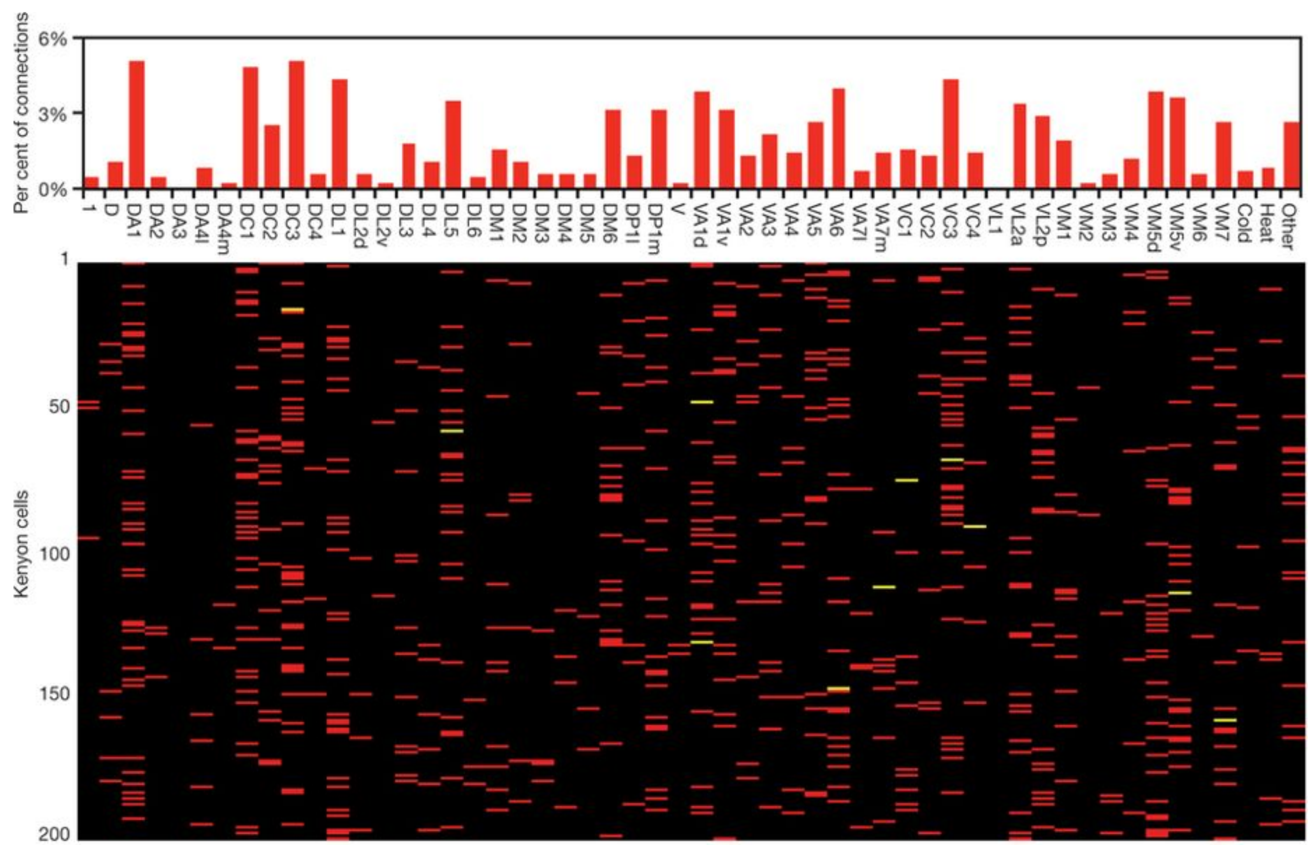
How fruit flies remember smells

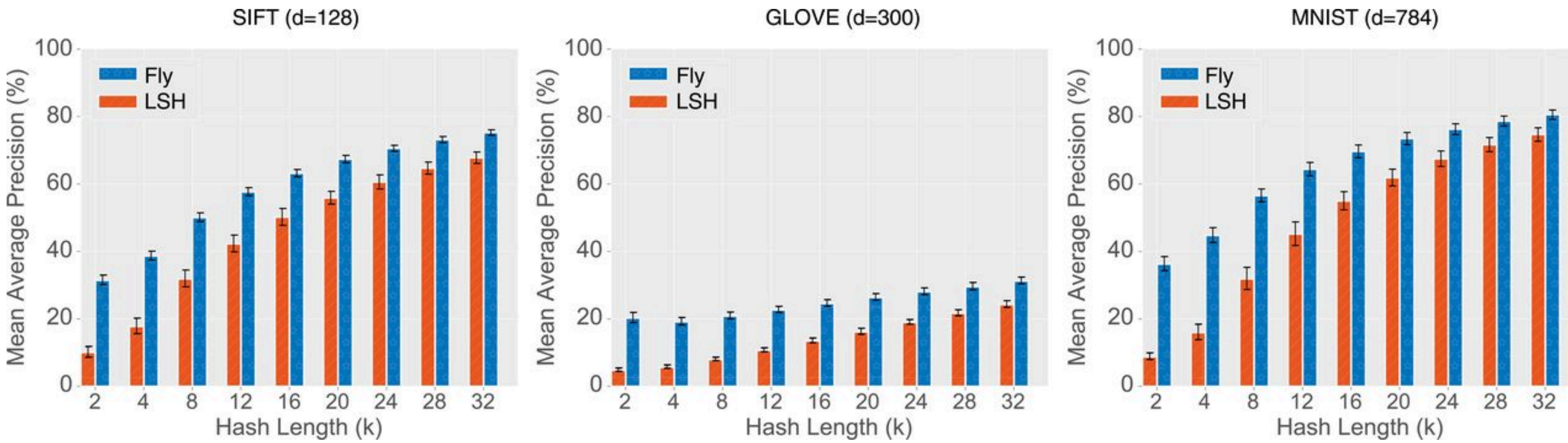


Q: but wait a minute! Is this a random bipartite graph?

A: ***Random convergence of olfactory input in the Drosophila mushroom body*** by S. Caron, V. Ruta, L. Abbott, R. Axel,2013

Bottom line: looks like a random bipartite graph, except that the degree distribution of the LHS is not uniform





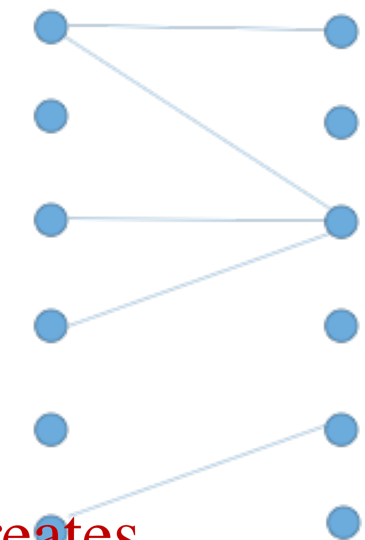
Surprise! The fly's algorithm (RP&C) preserves similarity in standard datasets to a degree that is competitive with the best similarity preserving algorithms* [Dasgupta et al. Nov 2017]

*Alex Andoni 2018: but not with the latest versions…

cf: Generalization-selectivity trade-off [Barak et al Feb. 2013]

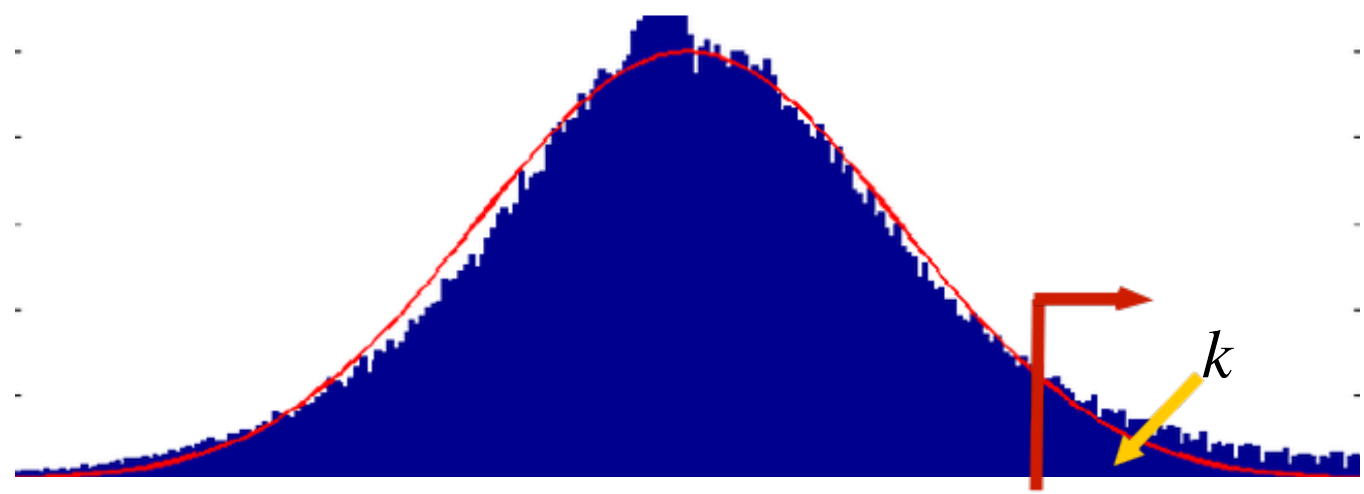
The underlying mathematical reason?

Btw: why am I taking the two sides to be *symmetric* (n nodes, k-sparse sets)?



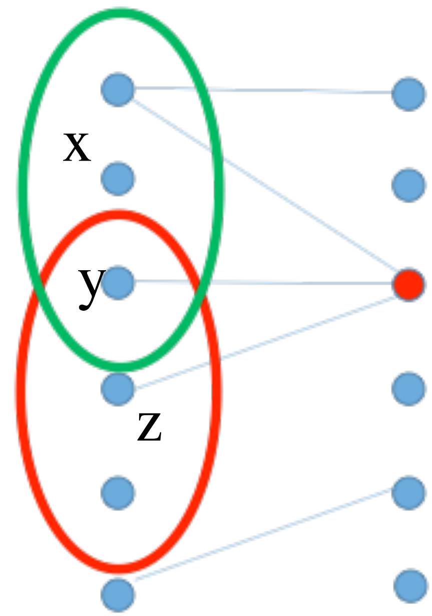
1. It will be helpful in what follows
2. It doesn't matter, anything on the LHS creates a "Bernoulli shower" on the RHS

“Bernoulli shower,” then k-cap



Calculemus…

- What is the distribution of the input
  node receives from x + y? y + z?
- What is the chance that it is in the cap of both?
- Nontrivial probabilistic calculation…

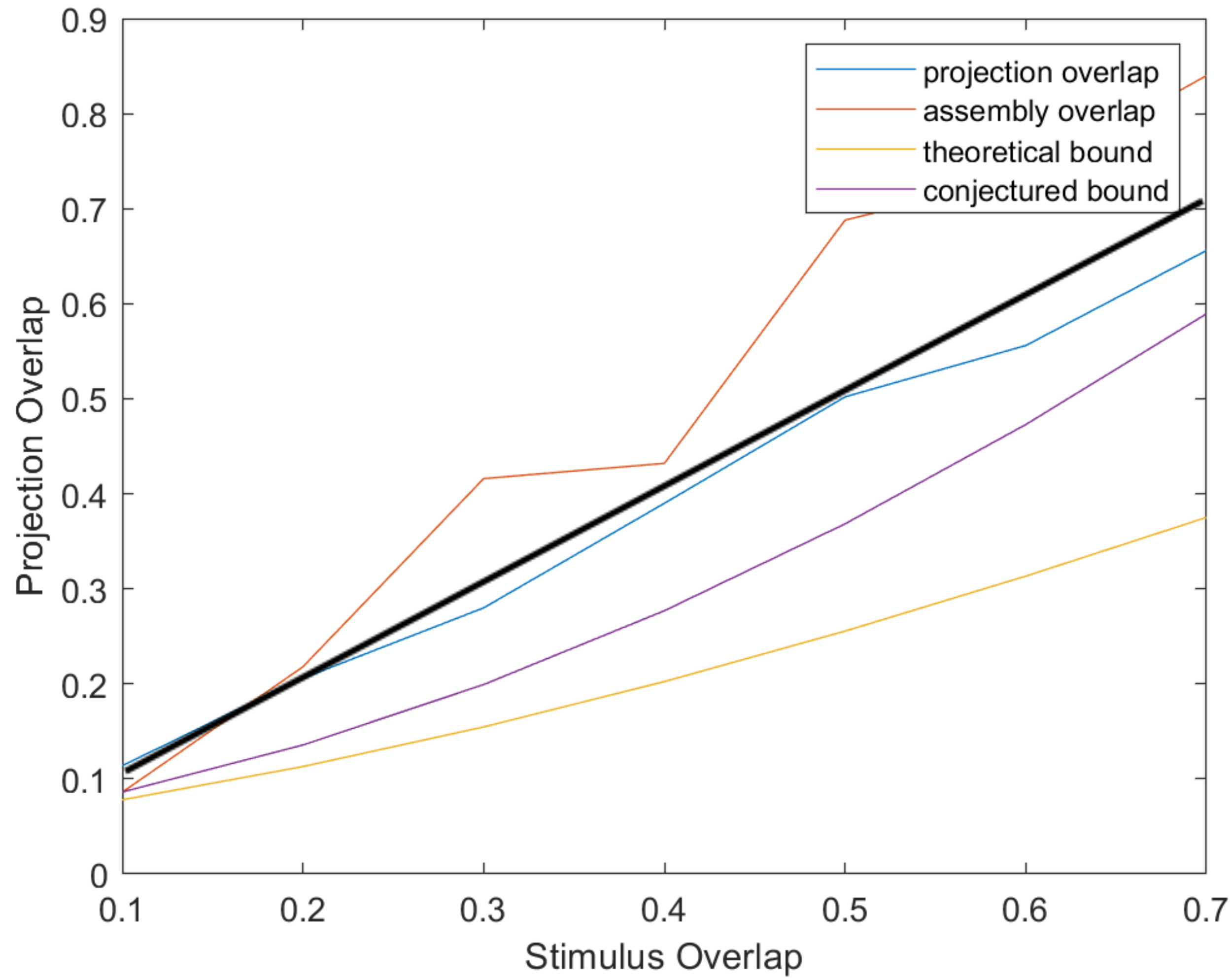


The underlying mathematical reason:

- **Theorem** [P., Vempala, 2018
  The intersection of **cap(A)** ar
  probability, at least

Conjecture:
no denominator

The underlying mathematical reason: compare with simulations

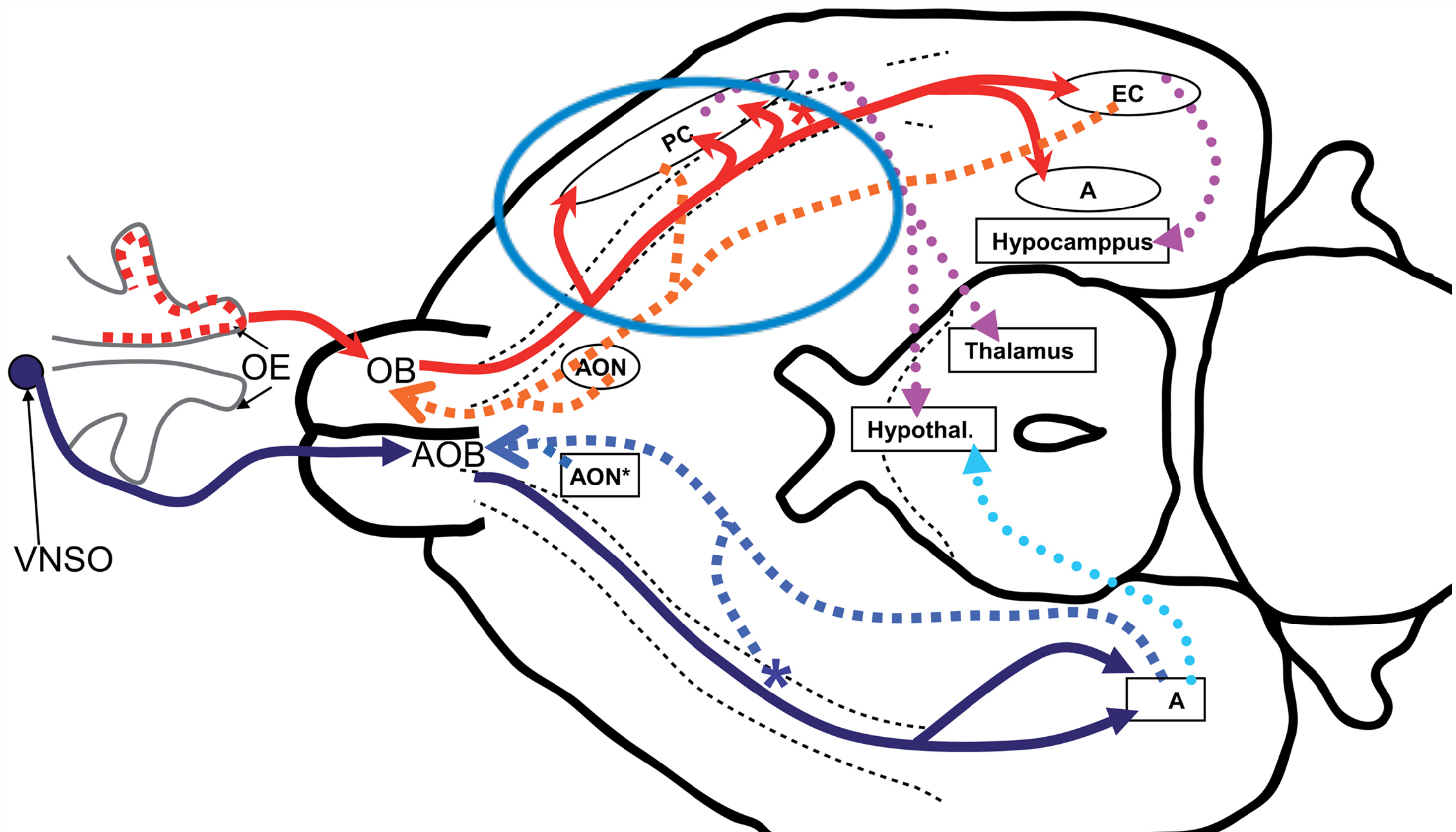


So much for the fruit fly…

- Q: Does something homologous happen in mammals?

(read: in us?)

Yes!



K. Franks, M. Russo, S. Sosulki, A. Mulligan, S. Siegelbaum, R. Axel

"Recurrent Circuitry Dynamically Shapes the Activation of Piriform Cortex." Neuron October 2011

From the *Discussion* section of Franks *et al*.

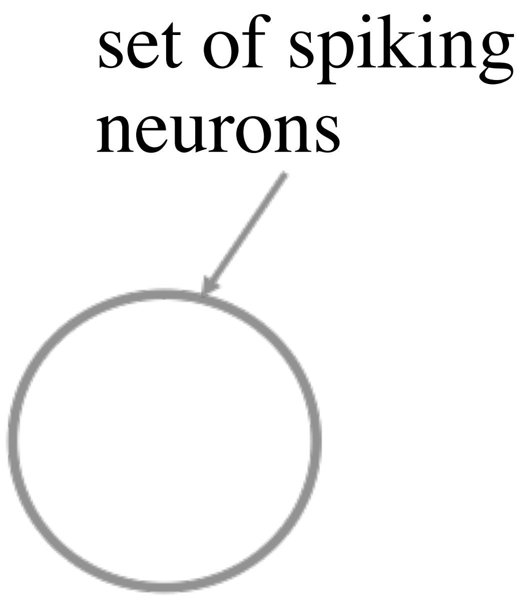
An odorant may [cause] a small subset of [PC] neurons [to fire].

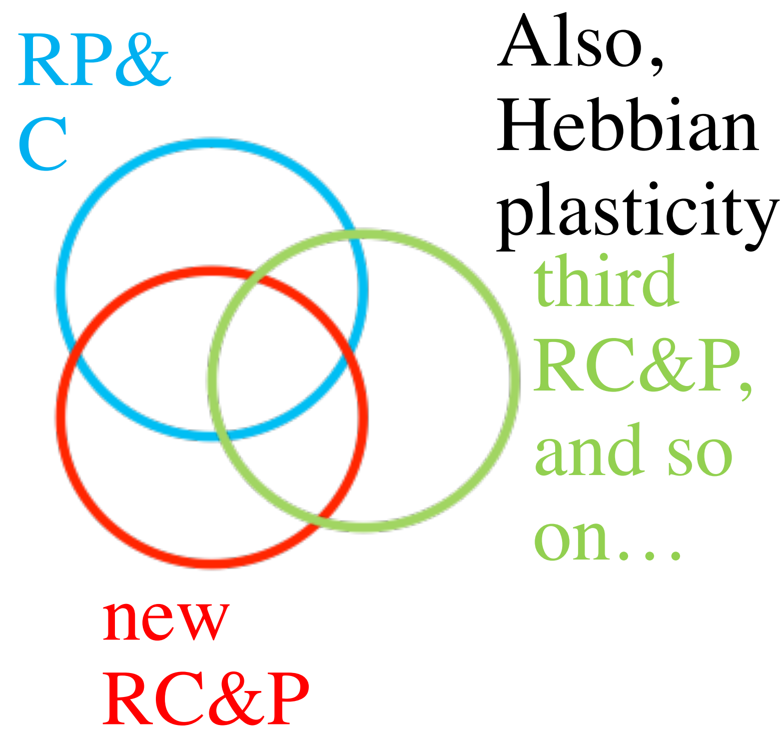
Inhibition triggered by this activity will prevent further firing

This small fraction of ... cells would then generate sufficient recurrent excitation to recruit a larger population of neurons.

In the extreme, some cells could receive enough recurrent input to fire … without receiving [initial] input…

In pictures…

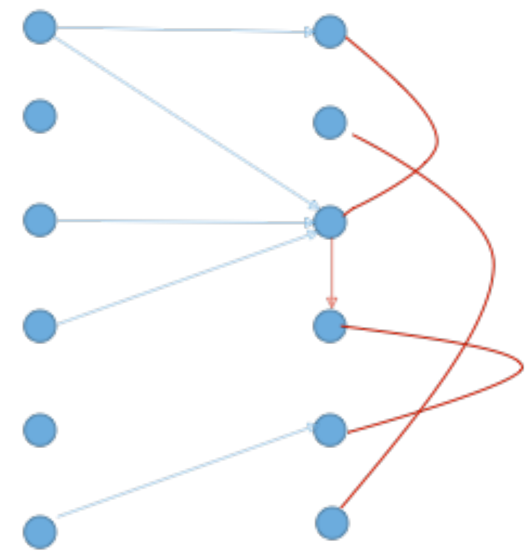

set of spiking
neurons


RP&
C
new
RC&P
third
RC&P,
and so
on…
Also,
Hebbian
plasticity

Does this process converge?

And does it preserve similarity?

Upgrade the model: the GNP model



- Fruit fly, plus:
- Recurrent synapses
- All random connections with prob. p
- Discrete time
- Hebbian plasticity: i-j synaptic weight increased by β – or multiplied by $(1 + \beta)$
- A fixed number of brain areas, each with n excitatory neurons and recurrent connectivity, plus

Main parameters, intended values

- $n \sim 107$
- $k \sim 103 - 4$
- $p \sim 0.001$
- $\beta \sim 0.20$

So, can this model predict what happens in the piriform cortex?

I stem

“probability” of activation

Input from stimulus

$$x_j(t+1) = s_j + \Sigma_{i \rightarrow j} \; x_i(t)\, w_{ij}(t)$$

synaptic weigh

(additive) plasticity

$$w_{ij}(t+1) = w_{ij}(t) + \beta\, x_i(t)\, x_j(t+1)$$

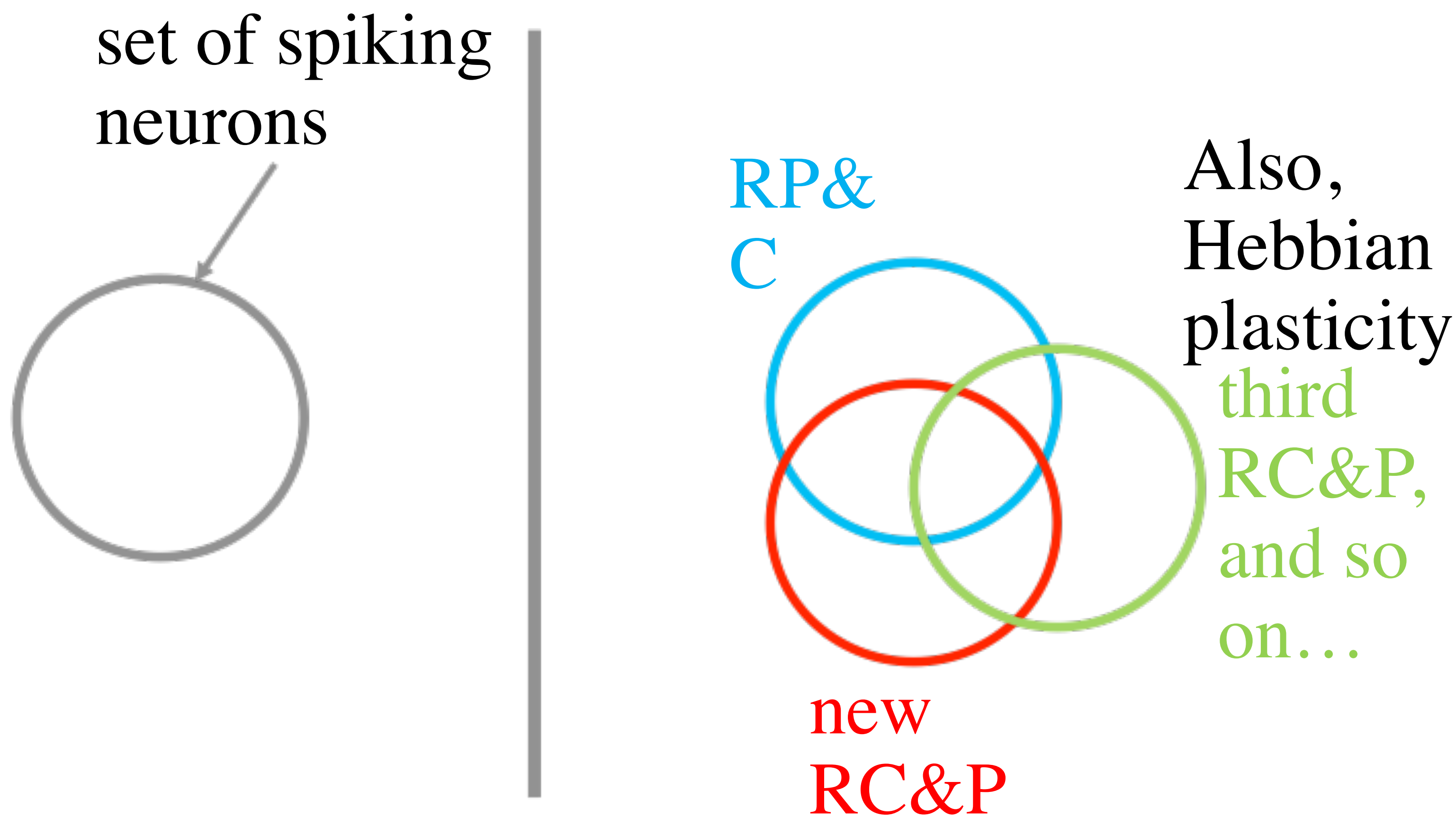
Linear system: solution

Theorem (P., Vempala, Maass, ITCS 2018]: The linearized dynamics converges geometrically and with high probability to

OK, how about the real, nonlinear system?

- After the first RP&C: the recurrent network kicks in
- And plasticity X $(1 + \beta)$

In pictures…



set of spiking
neurons
RP&
C
new
RC&P
Also,
Hebbian
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third
RC&P,
and so
on…

OK, how about the real, nonlinear system?

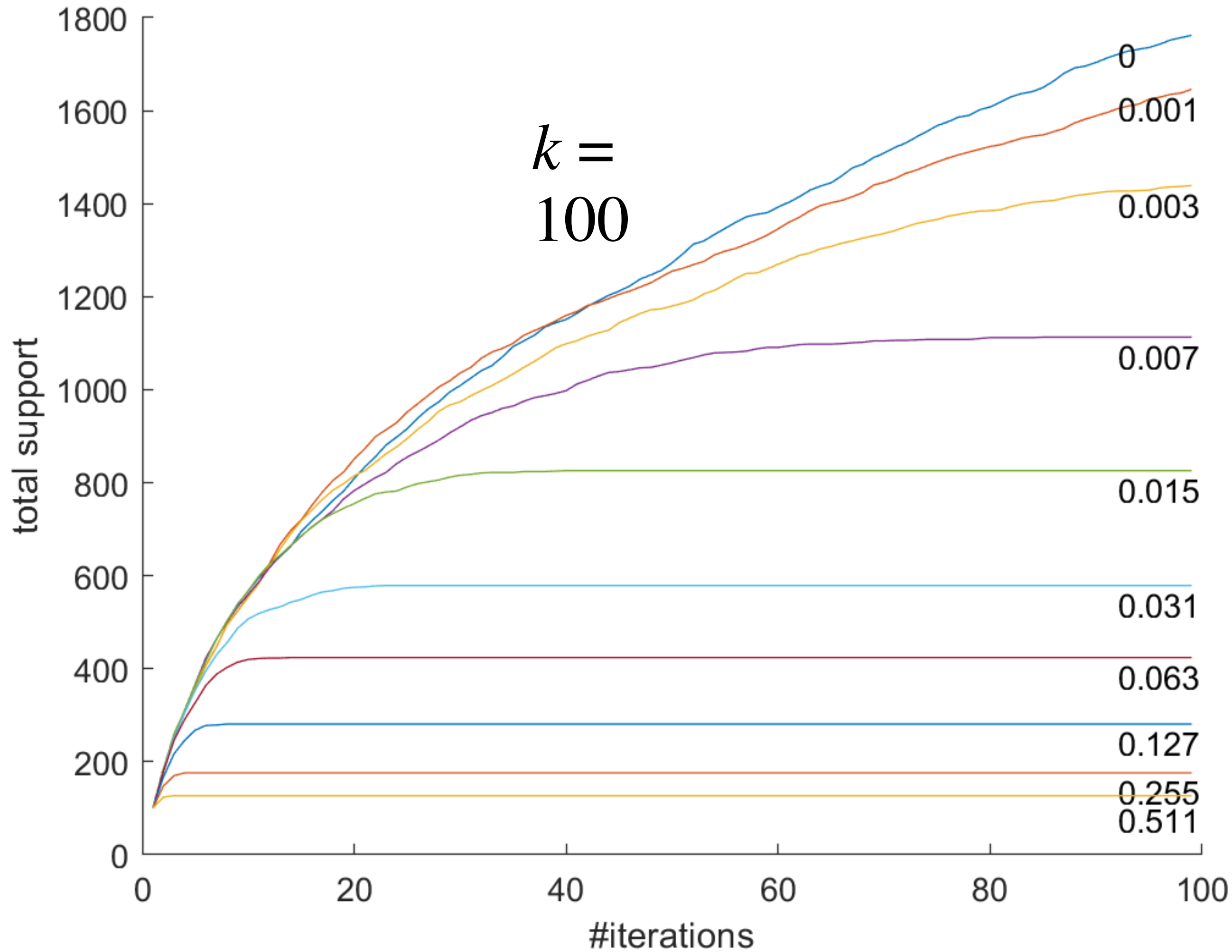
- After the first RP&C (call it A1): the recurrent network kicks in…
- …and multiplicative plasticity $(1 + \beta)$
- The new RP&C, A2 , has substantial intersection (about half) with A1
- Then again, A3 etc.

OK, how about the real, nonlinear system?

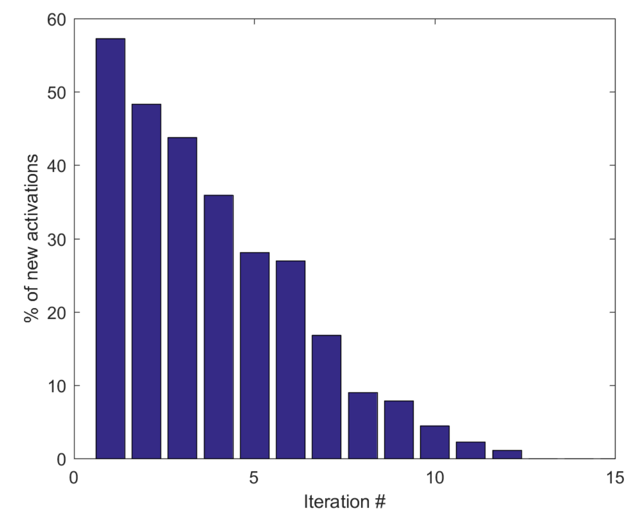
Theorem (P., Vempala 2016-18): The process converges exponentially fast, with high probability, and the *total number of cells involved* is at most:

- If $\beta \geq \beta^*$: $k + o(k)$
- If $0 < \beta < \beta^*$: $k \cdot \exp(0.17 \cdot \ln(n/k) / \beta)$

- NB: $\beta^* = (\sqrt{2} - 1)/(1 + \sqrt{pk/\ln n})$



k = 100
total support
#iterations
0
0.001
0.003
0.007
0.015
0.031
0.063
0.127
0.255
0.511



% of new activations
Iteration #

The result of such projection: an ***Assembly***

- Set of $\approx k$ neurons in a brain area whose firing (in a pattern) is tantamount to our thinking of a particular memory, concept, name, word, episode, etc.
- [Hebb 1949, Harris 2003, 2005; Buzsaki 2008, 2010]
- Also, simulations of a far more biologically accurate STDP model [Pokorny et al 2018]
- Presumably highly connected

The Big Picture

Computation in the brain: What is the right level?

- Whole brain?

?

- Spiking neurons and synapses?
- Dendrites?
- Molecules?

"…we do not have a logic for the transformation of neural activity into thought and action. I view discerning [this] logic as the most important future direction of neuroscience…"

R. Axel *Neuron*, Sep 2018

The assembly hypothesis

- There is an intermediate level of brain computation
- Implicated in carrying out higher cognitive functions such as reasoning, planning, language, story-telling, math, music…
- Assemblies are its basic representation – its main “data structure”
- What are its fundamental operations?
- NB: an operation must be useful and plausible

Useful and plausible?

- Useful: it helps us understand an experiment or two – ultimately the brain
- Plausible: no magic involved; the operation must be compiled down to neurons and synapses

The model

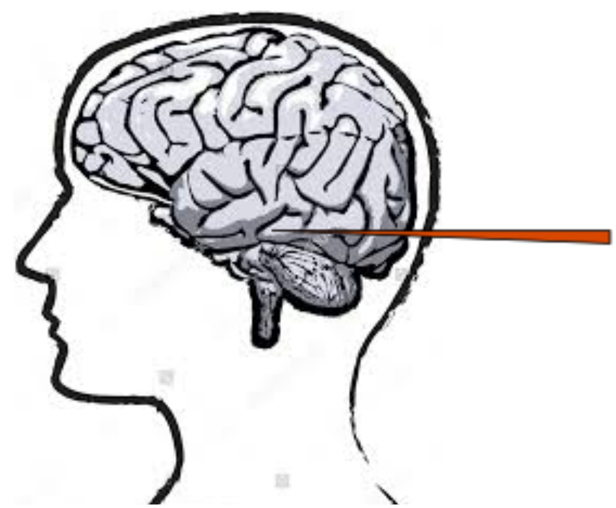
- A finite number of “brain areas” A, B, …
- Each with n (excitatory) neurons
- E – I balance: assemblies have k neurns each
- $k < \sqrt{n}$ (so assemblies barely intersect)

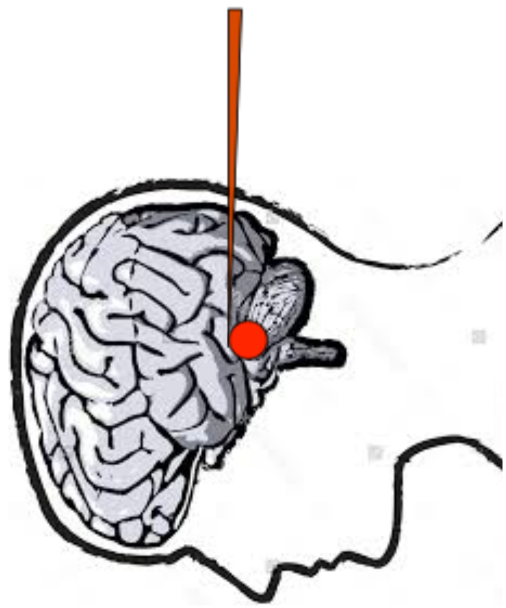
The assembly hypothesis: operations

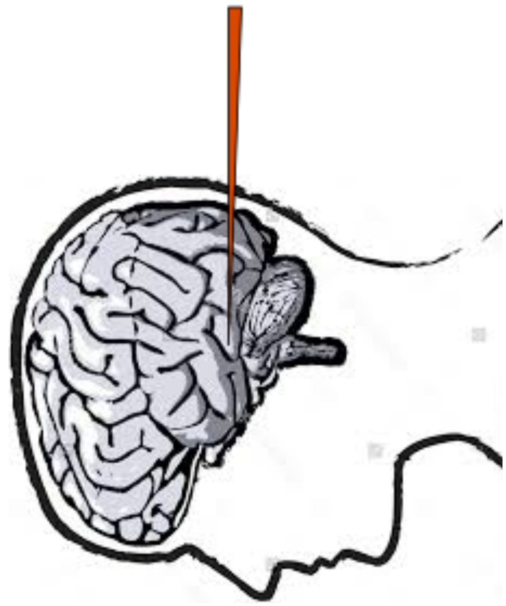
- Project(x, A, y)
- A = area(y), x = parent(y)
- (Plus, this is how an assembly is created)

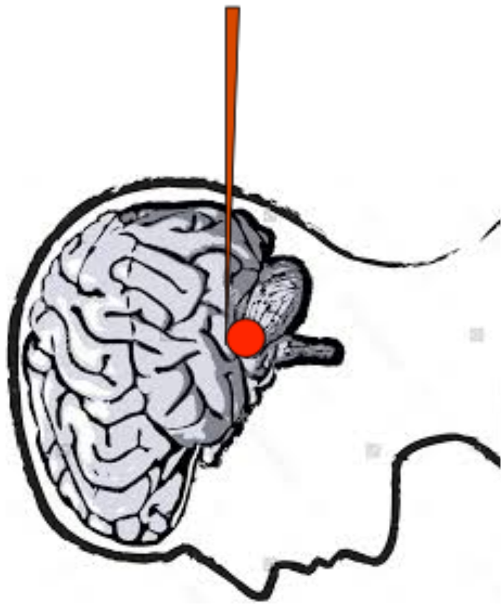
- Q: *Other operations?*
- A: Two assemblies may be associated by sharing cells
- Association encodes "affinity", similarity, co-occurrence

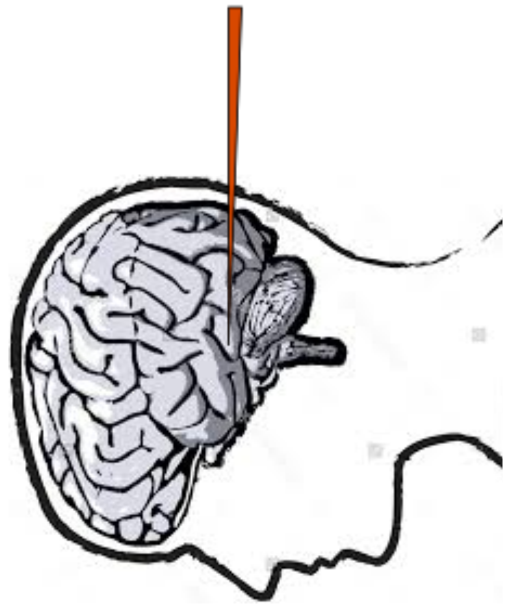
The [Ison et al. 2016] experiment

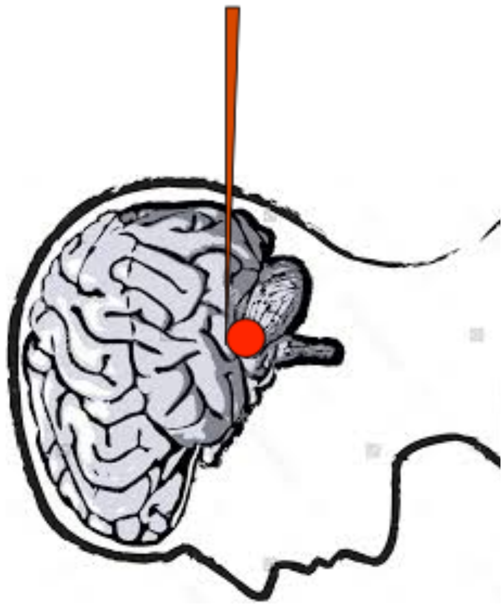


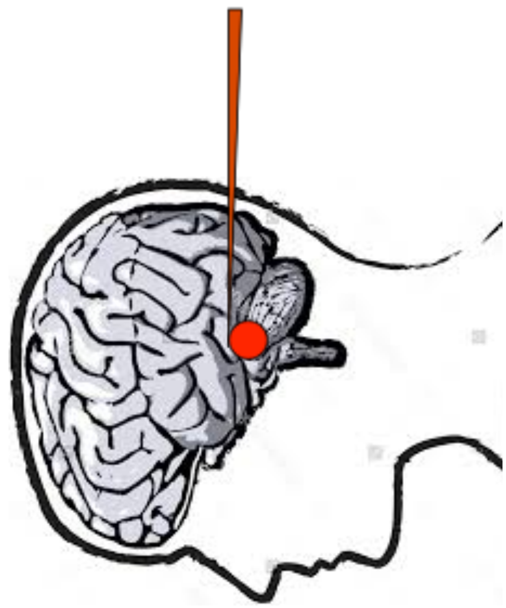


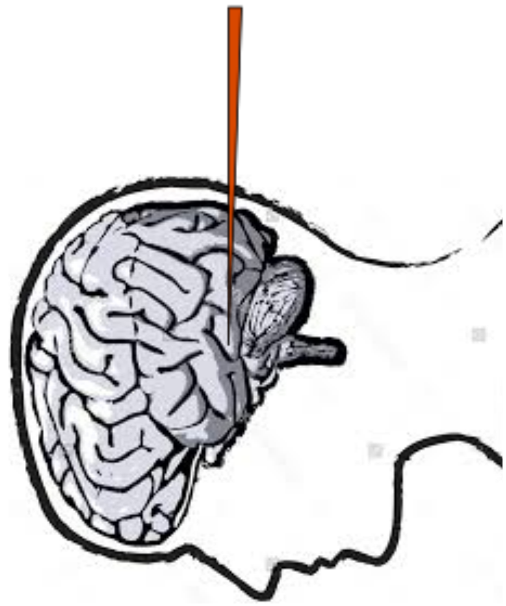


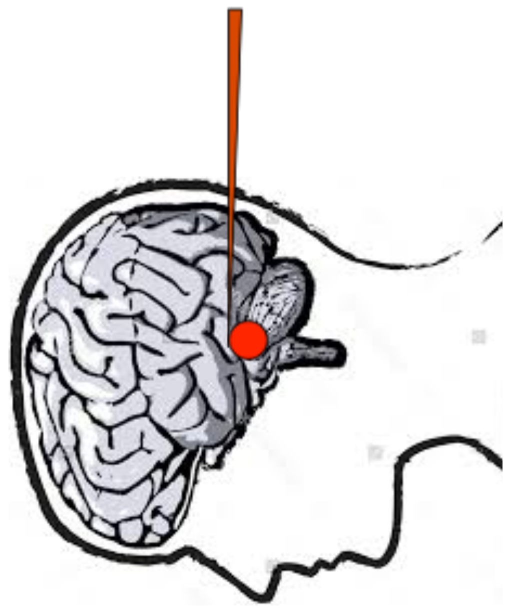


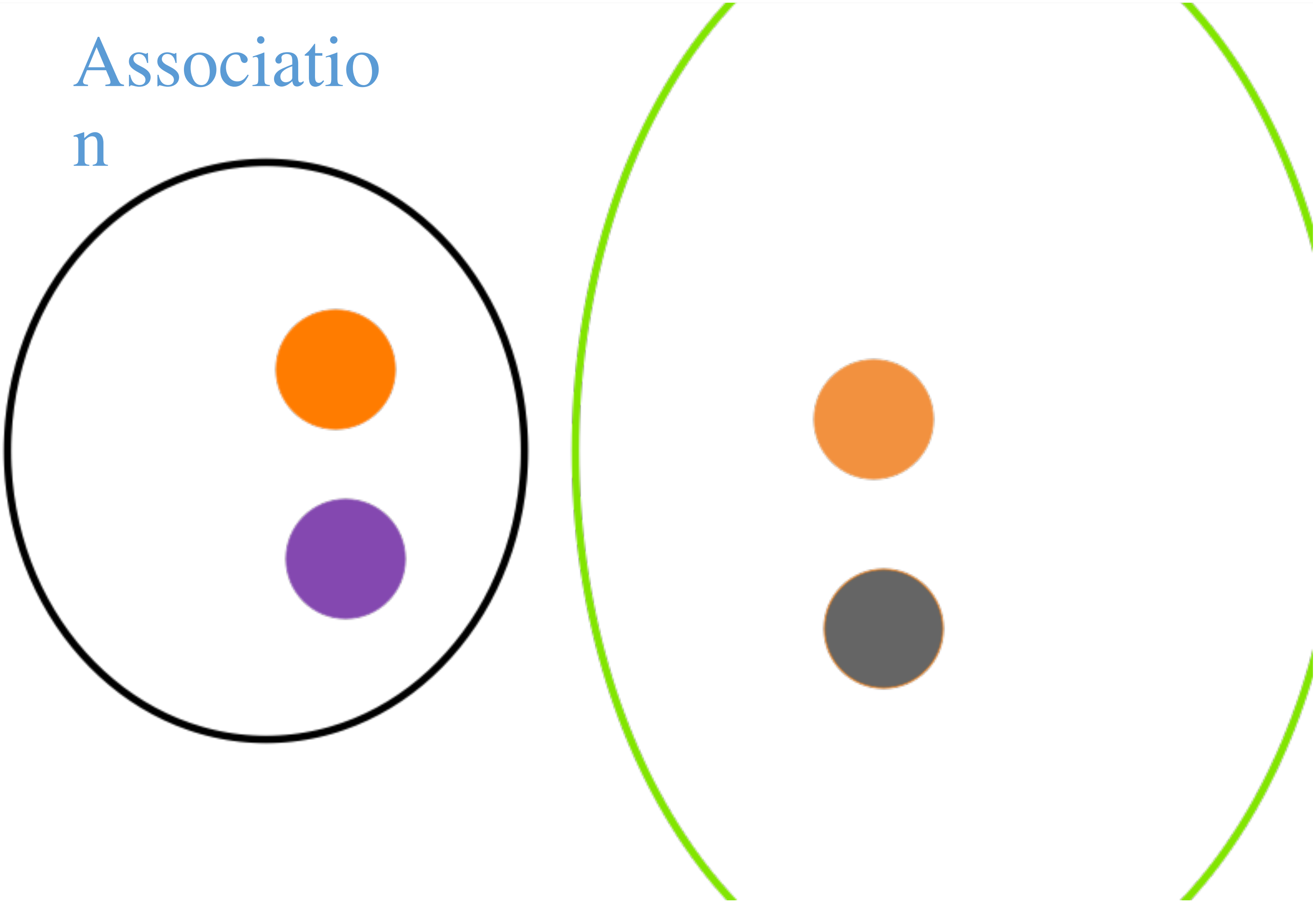











Associatio
n

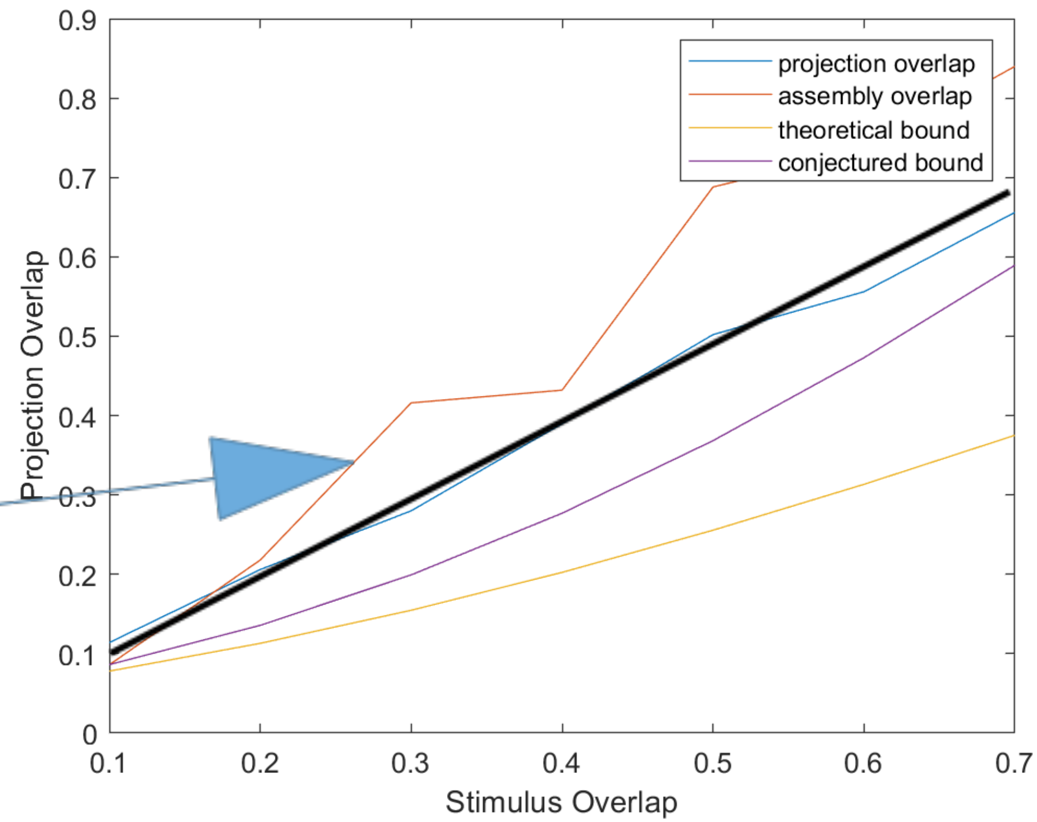
So, association is useful, but is it plausible?

- Yes, through simulations [Pokorny et al 2018] under submission
- Also, provably, in the GNP model: upon stimulus projection sequence E, O, E + O, a small fraction of E cells move to the O assembly, and vice versa
- NB: In simulations [Pokorny et al. 2018], under different parameters, new cells respond ***only*** to E + O
- Q: But is association preserved under projection?

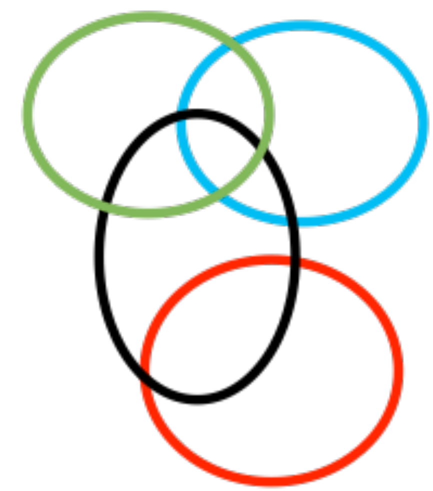
Q: is association preserved under projection?

Recall the fly,
and similarity
preservation:

association of assemblies seems to be *very well preserved* under projection



Also: assembly
association also means
associative memory through
pattern completion



Association graph: which assemblies/concepts
are similar?

Assembly expressive power: m-ary relations

e.g. *subject – verb – object*

[Frankland and Greene PNAS 2015]

time	area A	area B	…
	assembly z	assembly w	…
	assembly z	assembly y	…

Other operations?

- reciprocal_project(x, A, y)
- (now also y can activate x)

- merge(x, y, A, z)
- (assemblies x, y project to create one assembly in area A, call it z)
- Creates hierarchies
- Valuable for implementing language

Assembly Operations recap

- project(**y, B, x**)
- associate(**x, y**)
- reciprocal_project(y, B, x)
- merge(x, y, B, z)
- Plus: activate(x), area-read(), assembly-read(), disinhibit(**A**)..

Q: ***How powerful is this system?***

A: ***Turing-complete***

Turing complete:

- **Theorem**: The system can simulate with high probability arbitrary $\sqrt{n}$ space Turing machine computations
- with four brain areas, plus one brain area for every symbol and state
- Meaning?

Ultimately: Language

- An envir… *us* a few thousand…
- A "last-m…
- Hypothes… s to exploit the Brain…
- Invaluab… g the Brain
- ***A deluge … ments!***