

Roll Control for LV2.3

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Introduction

The roll control system for the current generation of launch vehicle (2.3) has more or less not been functioning properly since Launch 8. **Nate: explain here some of the problems with previous incarnations of the roll control system that led up to where we are now.** In June of 2014, another attempt at designing a roll control system was initiated. The purpose of this report is to document the process of re-developing the roll control system.

Physics Models

In this section we introduce the equations for the physical phenomenon of our dynamic system.

The angular acceleration due the canards

$$\ddot{\theta} = \frac{4Lr}{I} \quad (1)$$

Where L represents Lift, and r represents the canard distance from the Z axis.

Lift is defined as

$$L = \frac{1}{2}C_L\rho v^2S \quad (2)$$

Where C_L represents the coefficient of lift, ρ is air density, v is velocity, and $S = 11.64cm^2$ is the area of a canard.

The air density ρ is defined by a basic exponential atmosphere model that depends on altitude. We use the model below:

$$1.225e^{-0.000119h} \quad (3)$$

The coefficient of lift is broken up into two categories: supersonic flow and subsonic flow. For subsonic flow it is defined as

$$C_L = K_p \cos^2(\alpha) \sin(\alpha) + K_v \cos(\alpha) \sin^2(\alpha) \quad (4)$$

And for supersonic flow it conveniently shrinks to

$$C_L = \alpha K_v \quad (5)$$

Where $K_p \approx 2.45$, $K_v \approx 3.2$, and α is the angle of the canard which is physically restricted to be between $\pm 15^\circ$.

Time domain analysis

In this section we go over the numerous experiments that have been performed using the physics models in the time domain.

to be written...

State space model

In this section we set out to develop a state space model for the roll dynamics of the rocket.

Modern control theory is founded on the state space approach. Therefore, we will review and discuss important terminology that pertains to developing our state space model.

The **state** of a dynamic system is the smallest set of variables (called *state variables*) such that knowledge of these variables at $t = t_0$, together with knowledge of the input for $t \geq t_0$, completely determines the behavior of the system for any time $t \geq t_0$. [2]

The **state variables** of a dynamic system are the variables making up the smallest set of variables that determine the state of the dynamic system. If at least n variables x_1, x_2, \dots, x_n are needed to completely

describe the behavior of a dynamic system (so that once the input is given for $t \geq t_0$ and the initial state at $t = t_0$ is specified, the future state of the system is completely determined), then such n variables are a set of state variables. [2]

If n state variables are needed to completely describe the behavior of a given system, then these n state variables can be considered the n components of a vector \mathbf{x} . Such a vector is called a **state vector**. A state vector is thus a vector that determines uniquely the system $\mathbf{x}(\mathbf{t})$ for any time $t \geq t_0$, once the state at $t = t_0$ is given and the input $\mathbf{u}(\mathbf{t})$ for $t \geq t_0$ is specified. [2]

The n -dimensional space whose coordinate axes consist of the x_1 axis, x_2 axis,..., x_n axis, where x_1, x_2, \dots, x_n are state variables, is called a **state space**. Any state can be represented by a point in the state space. [2]

The equations for the most general state-space representation of a linear system with p inputs, q outputs and n state variables below:

$$\dot{\mathbf{x}}(\mathbf{t}) = A(t)\mathbf{x}(\mathbf{t}) + B(t)\mathbf{u}(\mathbf{t}) \quad (6)$$

$$\mathbf{y}(\mathbf{t}) = C(t)\mathbf{x}(\mathbf{t}) + D(t)\mathbf{u}(\mathbf{t}) \quad (7)$$

$\mathbf{A}(\mathbf{t})$ is called the state matrix, $\dim A = n \times n$

$\mathbf{B}(\mathbf{t})$ is the input matrix, $\dim B = n \times p$

$\mathbf{C}(\mathbf{t})$ is the output matrix, $\dim C = q \times n$

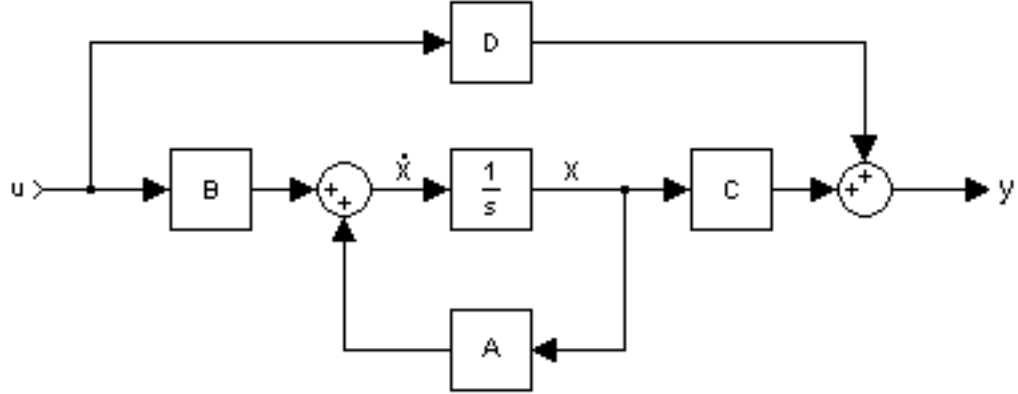
$\mathbf{D}(\mathbf{t})$ is the direct transmission matrix, $\dim D = q \times p$.

$\mathbf{x}(\mathbf{t})$ is called the state vector

$\mathbf{y}(\mathbf{t})$ is called the output vector

$\mathbf{u}(\mathbf{t})$ is called the input (or control) vector.

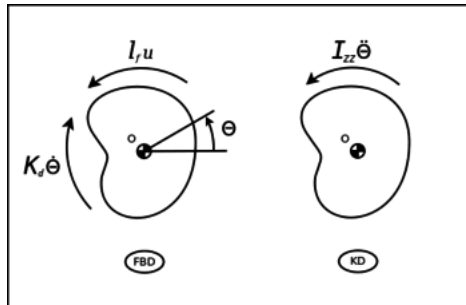
The block diagram representation of the linear state-space equations



Deriving the state space equations

Equations approximating the motion of the rocket are derived from free-body diagrams (FBD) and kinetic diagrams (KD), as taught in a mid level mechanical engineering dynamics course. Developing equations in this manner is known as developing from first principles because basic Newtonian principles $F = ma$ are used to derive them. [1]

The diagrams are shown below:



From this diagram we get the following differential equation

$$I_{zz} \ddot{\theta} = l_f u - K_d \dot{\theta} \quad [1]$$

References

- [1] Portland State Aerospace Society, Roll Control Wiki
<http://psas.pdx.edu/rollcontrol/>
- [2] Katsuhiko Ogata *Modern Control Engineering, Fifth Edition* Prentice Hall, Boston, MA, 2010