Lab 06 - Higher Order Mappings

Numerical Solution of PDEs Using the Finite Element Method

MHPC P2.13_seed

Martin Kronbichler kronbichler@lnm.mw.tum.de and **Luca Heltai** luca.heltai@sissa.it

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1. The topic of this lab session is a modified version of step-4 made available for you https://www.dealii.org/8.5.0/doxygen/deal.II/step_4.html
2. For more information in higher order mappings see step-10 https://www.dealii.org/8.5.0/doxygen/deal.II/step_10.html
3. Learn how deal.II passes the mapping to the functions that involve integrals and evaluation of positions, namely `FEValues` and `VectorTools::interpolate_boundary_values`.
4. Run the program and check the graphical and text output.
5. Adjust the right-hand side and solution to get the manufactured solution

   $$u(x,y) = r^2 \sin(3\theta)\cos(\frac{1}{2}\pi r)$$

   and apply zero boundary conditions. You can use wolframalpha.com to compute $-\triangle u$. Make sure the L2 errors are converging. Also look at the error field (in addition to the numerical solution called `solution` and the analytical solution `analytic_solution`) in ParaView.
6. What mapping degree is required to get optimal convergence of the L2 error based on the polynomial degree of the finite element space?
7. Try getting H1 convergence to work correctly too.
8. Change the right hand side and the solution to

   $$u(x) = \sin(\pi x) \cdot \cos(\pi y)$$

   from lab05, but now solved on the circle. Record the convergence rates as you increase the polynomial degree. Look at the error field in the ParaView output. Where is the error largest?
9. In the asymptotic regime, the highest possible convergence rate on the cirlce appears to be 3.5 irrespective of the degree of the polynomial. Change the mesh to `GridGenerator::hyper_shell` and set the manifold of all cells to the spherical manifold, not just the faces on the boundary. Ensure that you get optimal convergence rates

   $$h^{p+1}$$

   in the L2 norm for pth-degree polynomials.