Calculating portfolio-level risk information with Barra-supplied text files

Converting monthly BARRA COV and RSK text file data (or *flatfiles*) into risk numbers for a portfolio can seem overwhelming. This paper reviews the necessary formulas to ease the process. Following along with the COV and RSK files can help understanding – the examples here use the USE30006.COV and USE30006.RSK files. The techniques shown can be used with any BARRA model. This paper will assist in understanding and calculating:

- Data contained in COV and RSK files
- Exposures to risk indices or industries
- Active exposures to risk indices or industries
- Historical beta and predicted beta values
- Specific risk
- Active specific risk
- Common factor risk
- Active common factor risk
- Total risk
- Total active risk

Data contained in COV and RSK files

The RSK file is a text file that has J rows, where J is the number of assets covered in the particular model. For each of the J assets, there are M columns, which contain BARRA-calculated information for each asset. Some columns contain generally available information, such as an asset's ID, historical beta, price, market capitalization, and indicator variables (also called flags) that indicate to which major indices it belongs. The other columns give BARRA-proprietary information for the asset, such as BARRA-predicted beta; specific risk; total risk; and the asset's exposure to BARRA common factors (K), for example, risk indices such as Momentum and Volatility, and industries such as Financial Services and Computer Software.

The COV file is a matrix of variances and covariances between common factors (K x K). Diagonal elements are common factor variances and off-diagonal elements are covariances between factors.

Exposures to risk indices or industries

If each of the J rows in the RSK file lists the BARRA exposures for any given asset, then how can these be aggregated to form portfolio-level exposures? For example, the RSK file may tell us that IBM has a Momentum exposure of 0.637, and that GE has a Momentum exposure of 0.432, but if we are invested in both GE and IBM, what is our portfolio's exposure to Momentum?

To find portfolio-level exposures, two vectors are necessary:

- Portfolio weight vector - A vector that sums to 1 (or 100%), in which each element corresponds to the proportion of the portfolio value that is invested in a particular asset
- Asset exposure vector - A vector made up of the exposures of each asset to the particular risk index or industry

The portfolio-level exposure to a BARRA common factor is equal to the weight vector multiplied by the individual asset exposures vector.

$$\text{Portfolio Weight Vector} \times \text{Asset Exposures Vector} = \text{Portfolio-Level Exposure}$$

For example, imagine a three-asset portfolio with the following investments:

IBM	30%
GE	45%
MSFT	25%

To find the portfolio's exposure to Momentum:

1. Find the portfolio weight vector.

   (0.30, 0.45, 0.25)

2. Find the individual asset exposures vector.

   The individual asset exposures to Momentum given by the RSK file are:

IBM	– 0.542
GE	– 0.010
MSFT	–0.482

The asset exposures vector is (-0.542, -0.010, -0.482).

3 Multiply the portfolio weight vector by the asset exposures vector. This gives you the weighted average exposure. Thus, multiplying these two vectors together is equivalent to taking the sum of the product of their elements.

$$(0.30 \times -0.542) + (0.45 \times -0.010) + (0.25 \times -0.482) = -0.2876$$

Active exposures to risk indices or industries

The appropriate way to measure how a portfolio compares to a benchmark is to look at its active exposures. Active exposure takes into consideration how much each asset is exposed to a common factor and how the managed portfolio weights compare to those in the benchmark. An active weight vector describes the difference in weights between the benchmark and managed portfolios.

The active weight vector is the portfolio weight vector minus the benchmark weight vector.

$$\text{Portfolio Weight Vector} - \text{Benchmark Weight Vector} = \text{Active Weight Vector}$$

By definition, the elements in the active weight vector will sum to 0. The elements in the managed weight vector and the benchmark weight vector each must sum to 1, so the difference between the two, the active weight vector, must be 0. An asset is represented in the active weight vector if it is either in the portfolio or the benchmark or both. If there are assets in the benchmark that are not in the portfolio, those assets will show up in the active weight vector with negative active weights.

The portfolio-level active exposure is equal to the active weight vector multiplied by the asset exposures vector:

$$\text{Active Weight Vector} \times \text{Asset Exposures Vector} = \text{Portfolio-Level Active Exposure}$$

For example, compare our previous three-asset portfolio's asset weights with those of a benchmark.

Asset	Weight in Managed Portfolio	Weight in Benchmark Portfolio
IBM	30%	25%
GE	45%	50%
MSFT	25%	25%

To find the portfolio-level active exposure to Momentum:

1 Find the active weight vector.

Managed Weight Vector	–	Benchmark Weight Vector	=	Active Weight Vector
(0.30, 0.45, 0.25)	–	(0.25, 0.50, 0.25)	=	(0.05, – 0.05, 0.0)

2 Find the asset exposure vector.

From the RSK file, the asset exposures to Momentum are:

IBM	– 0.542
GE	– 0.010
MSFT	–0.482

The asset exposure vector is (–0.542, –0.010, –0.482).

3 Multiply the active weight vector by the asset exposures vector to find the portfolio-level active exposure to Momentum.

$$(0.05 \times -0.542) + (-0.05 \times -0.010) + (0.0 \times -0.482) = -0.0271$$

Historical beta and predicted beta values

For each asset, the RSK file has two columns, HBETA and BETA, that are calculated by BARRA but are not BARRA common factors.

The HBETA column holds the historical beta (relative to the S&P 500 for the USE3 model) for an asset – this beta is calculated from a 60-month historical regression of monthly returns for the asset against S&P 500 monthly returns. (BARRA uses an ordinary least squares regression.) If the asset does not have a long enough history, HBETA cannot be estimated for that asset. Assets without a historical beta appear as -999. These values should be treated as missing values. Options include setting the missing value to 1, which is the typical HBETA, or removing the data from your calculations.

The BETA column holds the BARRA-predicted beta (relative to the S&P 500 for the USE3 model) for that asset. One advantage of the BARRA-predicted beta is that an asset beta can be predicted even when there is less than 60 months of history.

Like asset exposures, betas have the *portfolio property*. That is, if we have the betas for the individual assets in a portfolio, and we know the weight of each asset (as proportion of total portfolio value), the weighted average of the individual asset betas gives us the portfolio beta.

$$\text{Portfolio Weight Vector} \times \text{Asset Beta Vector} = \text{Portfolio-Level Beta}$$

For example, look at our three-asset portfolio again. To find the portfolio-level historical beta:

1 Find the portfolio weight vector.

IBM	30%	**Portfolio Weight Vector**
GE	45%	(0.30, 0.45, 0.25)
MSFT	25%	

2 Find the asset historical beta vector from the HBETA column in the RSK file.

IBM	1.1	**Historical Beta Vector**
GE	1.229	(1.1, 1.229, 1.68)
MSFT	1.68	

3 Multiply the two vectors.

$$(0.30 \times 1.10) + (0.45 \times 1.229) + (0.25 \times 1.68) = 1.30305$$

Similarly, the BETA column in the RSK file provides an asset BARRA-predicted beta vector: (1.027, 0.887, 1.44). Multiplying the weight vector by the asset BARRA-predicted beta vector gives us the portfolio BARRA-predicted beta.

$$(0.30 \times 1.027) + (0.45 \times 0.887) + (0.25 \times 1.44) = 1.06725$$

Substituting the active weight vector for the portfolio weight vector in the preceding calculations yields active historical or predicted betas.

For example, consider our three-asset portfolio compared to a benchmark.

Asset	Weight in Managed Portfolio	Weight in Benchmark Portfolio	
IBM	30%	25%	**Active Weight Vector**
GE	45%	50%	(0.05, –0.05, 0.0)
MSFT	25%	25%	

$$\text{Active Historical Beta} = (0.05 \times 1.1) + (-0.05 \times 1.229) + (0.0 \times 1.68) = -0.00645$$

$$\text{Active BARRA-predicted Beta} = (0.05 \times 1.027) + (-0.05 \times 0.887) + (0.0 \times 1.44) = -0.007$$

Specific risk

The RSK file contains extensive risk information for each asset, and in particular it includes a column called SRISK%. As the name implies, this column contains the BARRA-predicted specific risk for each asset that month, in units of annualized percent (that is, this value is the standard deviation of specific return, not the variance). Using the SRISK% value for each asset in a portfolio, and the weight vector for a portfolio, we can calculate the portfolio-level specific risk.

Because the standard deviation function is not additive, in order to add up the assets' specific risk, we must first convert the standard deviations into variances, and then take the square root of that sum to find the specific risk for the portfolio.

Looking at our three-asset portfolio, we can find the portfolio-level specific risk:

1 Weight each SRISK% value by that asset's weight in the portfolio.

Asset	Weight		SRISK%		Weighted SRISK
IBM	0.3	X	28.969	=	8.6907
GE	0.45	X	20.334	=	9.1503
MSFT	0.25	X	40.398	=	10.0995

2 Square the weighted SRISK% value for each asset.

Weighted SRISK		Weighted Specific Variance
8.6907^2	=	75.528
9.1503^2	=	83.728
10.0995^2	=	102.00

3 Add all the squared values to find the portfolio specific variance.

$$\begin{array}{rr} & 75.528 \\ + & 83.728 \\ + & \underline{102.00} \\ & 261.256 \end{array}$$

4 Take the square root of the portfolio specific variance to find portfolio specific risk.

$$\sqrt{261.256} = 16.16\%$$

‣ Note: In some portfolios, there is reason to believe that certain assets may have similarities in their asset-specific returns (for example, common and preferred shares of the same company). The RSK files do not provide any indication of "linked" specific risk values for related assets, and thus calculate without covariance between assets. BARRA software products do account for the cases where there are "linked" specific risk values, and thus may provide slightly different specific risk forecasts for certain portfolios.

Active specific risk

Active specific risk for a portfolio is a measure of how much specific risk a portfolio has relative to the specific risk in a chosen benchmark. Calculating active specific risk is similar to calculating portfolio specific risk except that we use active weights, instead of portfolio weights.

To find the active specific risk of our three-asset portfolio:

1 Find the active weight vector.

Asset	Weight in Managed Portfolio	Weight in Benchmark Portfolio	
IBM	30%	25%	**Active Weight Vector**
GE	45%	50%	(0.05, –0.05, 0.0)
MSFT	25%	25%	

2 Find the weighted SRISK% values and square them to find the weighted specific variance.

Asset	Active Weight		SRISK %		Weighted SRISK	Weighted Specific Variance
IBM	0.05	X	28.969	=	1.4485	2.0981
GE	–0.05	X	20.334	=	–1.0167	1.0337
MSFT	0.0	X	40.398	=	0	0

3 Add all the squared values to find the portfolio specific variance.

$$
\begin{array}{rl}
 & 2.0981 \\
+ & 1.0337 \\
+ & \underline{0} \\
 & 3.1318
\end{array}
$$

4 Take the square root of the portfolio specific variance to find portfolio specific risk.

$$\sqrt{3.1318} = 1.770\%$$

Common factor risk

To calculate a portfolio's common factor risk, we use the RSK file as well as the COV file to create a series of matrices that describe the assets, weights, and exposures. This calculation will presume some basic knowledge of linear algebra.

$$\text{Common Factor Risk} = X_p F X_p^T$$

Where do these matrices come from? X_p is built from various parts of the RSK file, and F is the COV file. To find common factor risk:

1. Build the matrix X_a, which describes common factor exposures for each asset in the portfolio. X_a has J rows and K columns, where J is the number of assets and K is the number of common risk factors (risk indices and industries) in the appropriate model.

   For our three-asset portfolio, the matrix is 3 x 67, with the exposures pulled from the RSK file. (Note that only the risk indices are shown here, not the industries, in order to conserve space.)

Ticker	VOLATILITY	MOMENTU	SIZE	SIZENONL	TRADEACT	GROWTH	EARNYLD	VALUE	EARNVAR	LEVERAGE	CURRSEN	YIELD	NONESTU
IBM	–0.64	–0.542	1.093	0.163	0.18	–0.751	0.151	–0.22	–0.538	–0.007	–0.12	–0.217	0
GE	–0.782	–0.01	1.775	0.163	–0.748	–0.585	–0.116	–0.279	–0.626	–0.259	0.045	0.158	0
MSFT	–0.016	–0.482	–1.625	0.163	–0.1	0.484	–0.192	–0.318	–0.349	–0.963	–0.428	–0.526	0

2. Find the matrix h, which describes the weights of each asset in a portfolio. The vector h has dimensions 1 x J, where J is the number of assets. For our three-asset portfolio, h has dimensions 1 x 3.

IBM	0.3
GE	0.45
MSFT	0.25

3. From X_a and h, we build the matrix X_p, which describes the portfolio exposure for each of the common factors. Finding X_p is the same as finding the portfolio-level exposure to an individual risk index or industry, except that we find it for all the common factors.

$X_p = hX_a$, and has dimensions 1 x K, where K is the number of common factors in the model. In other words, for each factor, multiply each asset's weight by its exposure to find the weighted asset factor exposure. Then, sum the weighted asset factor exposures to find the portfolio exposure to each factor.

Ticker	VOLATILITY	MOMENTU	SIZE	SIZENONL	TRADEACT	GROWTH	EARNYLD	VALUE	EARNVAR	LEVERAGE	CURRSEN	YIELD	NONESTU
Portfolio	–0.548	–0.288	1.533	0.163	–0.308	–0.368	–0.055	–0.271	–0.530	–0.359	–0.123	–0.126	0

4 Let F describe the variance and covariance between all the common factors. It is found in the COV file. F has dimensions K x K, where K is the number of common risk factors. In the USE3 Model, F is currently 67 x 67. The elements in the COV file are in units of annualized variance.

5 Using the COV file, multiply $X_pFX_p^T$ to find the common factor variance. In our portfolio, X_p is a 1 x 67 matrix, so we multiply it by F (which is 67 x 67) and multiply that result by X_p^T (which is 67 x 1). The common factor variance is 231.1.

6 Take the square root to find the portfolio-level common factor risk.

$$\sqrt{231.1} = 15.2\%$$

You can practice this calculation using the USE30006.COV file, the three rows from the USE30006.RSK file for IBM, GE and MSFT, and the active asset weights.

Active common factor risk

The calculation for portfolio-level active common factor risk is an extension of the process described in the Common Factor Risk section. The h vector is now the *active* weight vector, not the *portfolio* weight vector. So if our benchmark weight vector is (0.25, 0.5, 0.25) as above, then our active weight vector (the new h vector) is (.05, -.05, 0.0). Using the methodology in the Common Factor Risk section, with the new active h vector, the active portfolio common factor risk is 1.77%. You can practice this calculation using the USE30006.COV file, the three rows from the USE30006.RSK file for IBM, GE, and MSFT, and the active asset weights.

Total risk

In order to determine the total risk of a portfolio, both the portfolio specific risk, denoted by Δ, and the portfolio common factor risk are necessary.

$$\text{Risk} = XFX^T + \Delta$$

At first glance, it might seem that we can add the common factor risk to the specific risk to get total risk. However, this is not possible, because standard deviations are not additive. Summing two values and then taking their standard deviation is *not* the same as taking the individual standard deviations first, and then summing. What is possible, however, is to sum up the specific and common factor *variance* to get the total *variance* of the portfolio, because variances are additive.

So, since we have just seen how to calculate the common factor risk and specific risk (which are standard deviations) we can calculate total risk by converting them to variances, and then taking the sum. Taking the square root of the total variance gives the total portfolio risk.

In our three-asset portfolio of IBM, GE, and MSFT, the weights are (0.30, 0.45, 0.25). To find the portfolio's total risk:

1. Find the common factor variance. In the Common Factor Risk section, we found that using the USE30006.* files gave us a portfolio common factor risk of 15.20%. Squaring this gives a common factor variance of 231.1.

   ‣ Note: If we had not already calculated common factor risk, we would follow the steps described in that section, but stop before taking the square root, thus giving us the common factor variance.

2. Find the specific variance of the portfolio. In the Specific Risk section, we found that using the USE30006.* files gave us a portfolio specific risk of 16.16%. Squaring this gives a specific variance of 261.2.

   ‣ Note: If we had not already calculated specific risk, we would follow the steps described in that section, but stop before taking the square root, thus giving us the specific variance.

3. Add together the common factor and specific variance. Our three-asset portfolio's total variance is 492.3 (492.3 = 231.1 + 261.2).

4. Take the square root of the total variance to transform it into a standard deviation value. In this case it turns out that the portfolio total risk is 22.19% (22.19 = sqrt of 492.3).

Total active risk

Calculating the total active risk for a portfolio is just like calculating the total risk – the only difference is that we use the *active* specific risk and the *active* common factor risk as the building blocks. As in the total risk case, we must first transform the active specific risk and active common factor risk into variances, then add those variances together, and take the square root of that sum to get the total active risk.

Our three-asset portfolio of IBM, GE, and MSFT, has weights of (0.30, 0.45, 0.25), where the benchmark has the same assets in weights of (0.25, 0.50, 0.25). This gives an active weight vector of (.05, -.05, 0.0). To find total active risk:

1. Find the active common factor variance. In the Active Common Factor Risk section, we found that using the USE30006.* files gave us an active common factor risk of 0.68%. Squaring this gives an active common factor variance = 0.46.

   ‣ Note: If we had not already calculated active common factor risk, we would follow the steps described in that section, but stop before taking the square root, thus giving us the active common factor variance.

2. Find the active specific variance. In the Active Specific Risk section, we found that using the USE30006.* files gave us an active specific risk of 1.77%. Squaring this gives an active specific variance = 3.13.

   ‣ Note: If we had not already calculated active specific risk, we would follow the steps described in that section, but stop before taking the square root, thus giving us the active specific variance.

3. Add together the active common factor and active specific variance. Our three-asset portfolio has a portfolio active variance of 3.59 (3.59 = 0.46 + 3.13).

4. Take the square root of the portfolio active variance to transform it into a standard deviation value – in this case it turns out that the portfolio active risk is 1.90% (1.90 = sqrt of 3.59).

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