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ES 292

Homework #9: 13.1, 13.4, 13.5

13.1

Using the diameter information from Table 13.1, calculate how many (a) gas molecules, (b) medium aerosol particles, and (c) fog drops make up the volume of a single medium raindrop. Use average diameters where applicable.

Volume of medium raindrops:

```
Vmr <- pi * (2000^3) / 6

print(Vmr) # Um ^3

## [1] 4188790205

dm_gs <- .0005 # typical diameter of gas molecules, in um
v_gs <- pi * (dm_gs^3) / 6 #volume of typical gas molecule, in um^3
print(Vmr/v_gs) #The number of gas molecules in a medium raindrop.

## [1] 6.4e+19

dm_map_min <- .2 #um
dm_map_max <- 1.0 #um
ave_dm_map <- mean(dm_map_min, dm_map_max)
v_map <- pi * (ave_dm_map^3) / 6
print(Vmr/v_map) #Number of average medium aerosol particles in a medium raindrop

## [1] 1e+12

dm_fg_min <- 10 #um
dm_fg_max <- 20 #um
ave_dm_fg <- mean(dm_fg_min, dm_fg_max)
v_fg <- pi * (ave_dm_fg^3) / 6
print(Vmr/v_fg) #Number of average fog droplets in a medium raindrop

## [1] 8e+06
```

The size of a medium raindrop is vastly larger than that of a gas molecule, an aerosol particle, or a fog droplet, which in turn are on a size gradient themselves. The relative diameters of the three should predict the number found in a raindrop: the fog droplet should have fewer in the raindrop than the gas molecules due their difference in sizes. That relationship is confirmed by the above results.

13.4

Calculate the number concentration of raindrops from the Marshall–Palmer distribution in the diameter ranges (a) 200–300 μm and (b) 800–900 μm, when R = 25 mm h−1. Why are the number concentrations different in the two size ranges?

EQ 13.30

$$n_i = \Delta d_i n_0 e^{-\lambda_r d_i}$$

$$\lambda_r = 4.1 * 10^{-3} R^{-.21} \mu m^{-1}$$

EQ 13.9

$$\Delta d_i = d_{i,hi} - d_{i,lo}$$

$$n_T = n_0 / \lambda_r$$

```
#diameter ranges are 200 -300 um
di_lo_a <- 200 #um
di_hi_a <- 300 #um
R <- 25 #mm h^-1
lm_r <- (4.1 * (10^-3)) * (R^-.21) #um ^-1
n0_a <- 8 * (10^-6) # particles cm^ -3 um ^-1
dd_a <- di_hi_a - di_lo_a #um, diameter width
ni_a <- dd_a * n0_a * exp(-lm_r * di_lo_a) #particles cm^-3, number concentraion for bin a


print(ni_a)

## [1] 0.0005271608

#diameter ranges are 200 -300 um
di_lo_b <- 800 #um
di_hi_b <- 900 #um

n0_b <- 8 * (10^-6) # particles cm^ -3 um ^-1
dd_b <- di_hi_b - di_lo_b #um, diameter width
ni_b <- dd_b * n0_b * exp(-lm_r * di_lo_b) #particles cm^-3, number concentraion for bin b


print(ni_b)

## [1] 0.0001508352
```

The number concentrations are constrained by the size of the droplets. In case a), ni = 5.271608510^{-4}, discrete particles are small and thus more particles will be found in a given area. In case b), ni = 1.508351710^{-4}, although the range of particle sizes is the same as case a) (100 um), the discrete particle sizes are larger. Thus, fewer droplets will be found in the same given space.

13.5

Compare the number concentrations of drops between 18 and 22 μm in diameter at the base and at the top of a nimbostratus cloud using a modified gamma distribution. Why do you think the concentrations differ in the two cases?

EQ 13.31

$$n_i = \Delta r_i A_g r_i^{\alpha_g} exp\left[-\frac{\alpha_g}{\upsilon_g}\left(\frac{\gamma_i}{\gamma_{c,g}}\right)^{\upsilon_g}\right]$$

```
r_hi <- 22/2 # radius of largest droups in bin, um
r_lo <- 18/2 # radius of smallest drops in bin, um
del_r <- r_hi - r_lo #radius width of bin, um
ri <- mean(r_hi, r_lo) # mean radius of bin

#gamma parameters for base of nimbostratus cloud, given by table 13.4
Ag_b <- .080606
al_g_b <- 5.0
up_g_b <- 1.24
r_cg_b <- 6.41 #um

#gamma parameters for top of nimbostratus cloud, given by table 13.4
Ag_t <- 1.0969
al_g_t <- 1.0
up_g_t <- 2.41
r_cg_t <- 9.67 #um

#drop number concentration in bin i for base of nimbostratus cloud

n_base <- del_r * Ag_b * (ri ^ al_g_b) * exp( -(al_g_b/up_g_b) * ( (ri / r_cg_b) ^ up
_g_b))

#drop number concentration in bin i for top of nimbostratus cloud
n_top <- del_r * Ag_t * (ri ^ al_g_t) * exp( -(al_g_t/up_g_t) * ( (ri / r_cg_t) ^ up_
g_t))

print(n_base) # particles cm ^-3

## [1] 9.847837

print(n_top) # particles cm ^-3

## [1] 13.70104
```

The drop number concentration at the base and top of a nimbostratus cloud are 9.8478374 particles cm^{-3} and 13.7010394 particles cm^{-3} respectively. I suspect that if we look at different bin widths, we will see greater number concentrations at the base in small bin widths, while at the top, as temperatures drive condensation, we will see greater number concentrations in larger bin widths. In addition, these values seem accurate. Nimbostratus clouds are heavy with precipitation, so 10~14 particles cm^-3 seem like an accurate value for this bin (micrometers).