

Homework #7: 8.3 8.5 Note: 8.1\* was removed from the homework

8.3:

Assume  $z_{0,m} = 0.01$  m,  $u_* = 1$  m s<sup>-1</sup>,  $\bar{u}(z_r) = 10$  m s<sup>-1</sup> and  $\bar{v}(z_r) = 5$  m s<sup>-1</sup> at  $z_r = 10$  m. Calculate the eddy diffusion coefficient for momentum,  $K_m$ , z x. Notes:  $z_{0,m}$  is the surface roughness length for momentum, the height above the surface at which the log profile of wind speed vs altitude extrapolates to zero wind speed  $u_*$  is the friction wind speed: vertical flux of horizontal momentum in the surface layer  $\bar{u}(z_r)$  and  $\bar{v}(z_r)$  are the mean horizontal wind speed at the reference height  $z_r$  is the reference height  $v_h$  is the horizontal windspeed  $K_{mzx}$  is the eddy diffusion coefficient for momentum

$$K_{m,zx} = K_{m,zy} = \frac{u_*^2}{|v_h(z_r)|} (z_r - z_{0,m})$$

$$|v_h| = \sqrt{u^2 + v^2}$$

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u_z_r <- 10 #mean horizontal wind speed east west, m/s
v_z_r <- 5 #mean horizontal wind speed north south, m/s
u_s <- 1 #m/s, friction wind speed
z_r <- 10 #m, reference height
z_om <- .01 #m, surface roughness length for momentum

v_h_z_r <- sqrt(u_z_r^2 + v_z_r^2) #horizontal windspeed at reference height
kmzx <- (u_s^2) / (abs(v_h_z_r)) * (z_r - z_om) #Eddy diffusion coefficient
for momentum in the surface layer from similarity theory, m2/s
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The value of  $kmzx$ , 0.8935328 indicates that eddy formation is fairly small. I assume this value is accurate and reasonable given the surface roughness length being low, and the friction wind speed low as well.

8.5:

Compare vertical turbulent sensible-heat fluxes over the ocean from bulk aerodynamic formulae and Monin–Obukhov similarity theory. Assume the same conditions as in Problem 8.2, except assume  $z_{0,m} = 0.00001$  m. Assume  $\theta_v$  at the roughness length for energy equals that at the roughness length for momentum. Discuss differences in results.

$z_{om} = .000001$  m  $z_{oh} = .0005$  m  $u(z_r) = 8$  ms  $v(z_r) = 2$  m  $t(z_r) = 285$  K  $T(z_{oh}) = 286$  K  
 $p_a(z_r) = 1004$  hPa  $p_a(z_{oh}) = 1005$  hPa  $g = 9.80665$  m s<sup>-2</sup>

2.96

Bulk heat transter coefficient:

$$(w'\theta_v')_s = -C_H |v_h(z_r)| [\theta_v(z_r) - \theta_v(z_{0,h})]$$

## Monin-Obukhov Similarity theory

$$\theta_v = T_v \left( \frac{1000}{pa} \right)^k$$

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$$Ri_b = \frac{g[\theta_v(z_r) - \theta_v(z_{0,h})](z_r - z_{0,m})^2}{\theta_{a_v}(z_{0,h})[u(z_r)^2 + v(z_r)^2](z_r - z_{0,h})}$$

8.41

$$G_m, G_h = \frac{1}{(1 + 4.7 Ri_b)^2}$$

8.40

$$u_* = \frac{k |v_h(z_r)| \ln(z_r/z_{0,m})}{\sqrt{G_m}}$$

$$\theta_* = \frac{(k^2 |v_h(z_r)| [\theta_v(z_r) - \theta_v(z_{0,h})])}{u_* Pr_t \ln^2(z_r/z_{0,m})} G_h$$

Kinematic vertical turbulent sensible-heat flux in the surface layer from K-theory 8.33

$$(w'\theta_v')_s = -u_*\theta_*$$

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zr = 10 #m
zom = .000001 #m
zoh = .0005 #m
uzr = 8 #m/s
vzr = 2 #m
tzt = 285 #K
tzoh = 286 #K
pazr = 1004 #hPa
pazoh = 1005 #hPa
g = 9.80665 #m/ s-2
kk = .286
k = .4
prt = .95
Ch =.005 #assumption that the ocean is a near flat surface

ovzr <- tzr *(1000 / pazr) ^ kk
ovzoh <- tzoh *(1000 / pazoh) ^ kk
#Bulk heat transfer
wtheta = - Ch * abs(vzr) * (ovzr - ovzoh)

#Monin-Obukhov similarity theory
rib <- (g *(ovzr - ovzoh)*(zr - zom)^2 ) / (ovzoh *(uzr^2 + vzr^2) * (zr -

```

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zoh))
Gm = Gh = 1 / ((1 + 4.7*rib)^2)
ust <- (k * abs(vzr) / log2(zr/zom) ) * sqrt(Gm)
thest <- (k^2 * abs(vzr) * (ovzr - ovzoh)) / (ust * prt * log2(zr/zom))

wth <- -ust*thest

```

The bulk heat transfer method gives us 0.0091753 while the Monin-Obukhov method gives us 0.0132911. They are within an order of magnitude, but the value from the bulk heat transfer method is calculated with an assumed value of  $Ch$ , which may or may not be representative of the surface measured. In addition, the assumed  $Ch$  value is for a smooth surface, of which the ocean is not.