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ES 292

**Homework #10: 16.4**

**16.4**

Calculate the critical radius and critical supersaturation when a 0.1-μm radius particle contains:   
(a) 0.05 mol kg−1 of an organic compound of molecular weight 120 g mol−1 , and  
(b) 0.05 mol kg−1 of sodium chloride of molecular weight 58.44 g mol−1.  
Assume the organic compound does not dissociate, sodium chloride dissociates completely, and T = 298.15 K. Which of the two solutes enhances CCN (Cloud Condensation Nuclei) activation the most? Explain.

The Critical Radius the radius at which the sum of the saturation ratio and the surface tension is at its maximum. Critical saturation is the equilibrium staturation found at this radius. Supersaturation is this value minus one. Cloud concensation nuclei are particles that water can heterogeneously nucleate on. Particles smaller than the critcal radius cannot activate to cloud drop size unless ambient saturation ratio exceeds the critical saturation ratio.

**Equation 16.43**

um (%)

Where mp is the average particle molecular weight (g mol -1), mv is the molecular weight of water (g mol-1), Ms is the mass of solute in the particle(g), ms is the molecular weight of the solute (g mol -1), iv is the degree of dissociation of the solute (NaCl, iv = 2; organic, iv = 1), o\_p is the surface tension of water containing ions, R\* is the universal gas constant(8.31451 × 107 g cm2 s−2 mol−1 K−1), T is absolute temperature, and rhop,i is the average particle density (g cm-3)

Unfortunately, I must make assumptions regarding either the density of the compounds or the partial pressures of the water/gases. I'm going to use 1.0 g cm-3 for the gasses density and 2.0 g cm-3 for the water density.

univ <- 8.31451 \* (10^7) #g cm2 s−2 mol−1 K−1, universal gas constant  
  
afunc <- function(sigp, Temp, mp, rhopi){  
   
 answ <- (2 \* sigp \* mp) / (univ \* Temp \* rhopi)  
 print(answ)  
   
}  
  
bfunc <- function(mv, iv, Ms, rhow, ms){  
   
 answ <- (3 \* mv \* iv \* Ms) / (4 \* pi \* rhow \*ms)  
 print(answ)  
}  
  
  
Massp <- function(radius, pv){  
 answ <- (4/3) \* pi \* radius^3 \* pv  
 print(answ)  
}  
  
critr <- function(a, b){  
 answ <- sqrt((3 \* b)/a)  
 print(answ)  
}  
  
critss <- function(a, b){  
 answ <- sqrt((4 \* a^3) / (27 \*b))  
 print(answ)  
}

1. Surface tension of water containing dissolved organics is

dyn cm-1 (g s-2)

Where sigmaw/a is the surface tension of liquid water and mc is the molality of carbon dissolved in water (mol C k-1)

When 0 < T-273.15 < 40

**Part A**

sigpfunc1 <- function(tempe, mc){  
 # dyn cm -1, g s-2  
 sigwa <- 76.1 - 1.55 \* (tempe - 273.15)  
 answ <- sigwa - .0187 \* tempe \* log(1 + 628.14\*mc)  
 print(answ)  
}

ra1 <- .1 \* .0001 #converting radius to cm (cm)  
tm <- 298.15 #k  
ptsz <- .1 #um particle radius  
molC <- .05 #mol kg -1  
molW <- 18.015 # g mol-1 molecular weight of water  
molv1 <- 120 #g mol -1, molecular weight of compound  
den\_assumed <- 1 #g / cm^3 molecular density of compound  
den\_water <- 2 #g/ cm^3 molecular density of water  
ivv <- 1 #degree of dissociation, 1 for nondissociated organic compound  
  
mss <- Massp(radius = ra1, pv = den\_assumed)

## [1] 4.18879e-15

siga <- sigpfunc1(tempe = tm, mc = molC)

## [1] 17.95665

ap1 <- afunc(sigp = siga, Temp = tm, mp = molW, rhopi = den\_assumed) #cm

## [1] 2.609866e-08

bp1 <- bfunc(mv = molW, iv = ivv, Ms = mss, rhow = den\_water, ms = molv1) #cm^3

## [1] 7.50625e-17

rcrit\_a <- critr(a = ap1, b = bp1)

## [1] 9.288873e-05

scrit\_a <- critss(a = ap1, b = bp1)

## [1] 0.0001873113

The critical r for compound a is **9.2888734 \* 10^{-5}** cm, while the supersaturation ratio is **1.8731126 \* 10^{-4}.** It must be noted that water and compound density were assumed to be 2 and 1 g/cm^3.

**Part B**

ra1 <- .1 \* .0001 #converting radius to cm (cm)  
tm <- 298.15 #k  
ptsz <- .1 #um particle radius  
molC <- .05 #mol kg -1  
molW <- 18.015 #g mol -1  
molv2 <- 58.44 #g mol -1, molecular density of NaCl  
den\_assumed <- 1 #g / cm^3  
den\_water <- 2 #g/ cm^3  
ivv2 <- 2 #degree of dissociation, 2 for dissociated NaCl  
  
mss2 <- Massp(radius = ra1, pv = den\_assumed)

## [1] 4.18879e-15

sigb <- sigpfunc1(tempe = tm, mc = molC)

## [1] 17.95665

ap2 <- afunc(sigp = sigb, Temp = tm, mp = molW, rhopi = den\_assumed) #cm

## [1] 2.609866e-08

bp2 <- bfunc(mv = molW, iv = ivv2, Ms = mss, rhow = den\_water, ms = molv2) #cm^3

## [1] 3.082649e-16

rcrit\_b <- critr(a = ap2, b = bp2)

## [1] 0.0001882407

scrit\_b <- critss(a = ap2, b = bp2)

## [1] 9.243009e-05

The critical r for compound b is **1.8824071 \* 10^{-4} cm**, while the supersaturation ratio is **9.243009 \* 10^{-5}.** It must be noted that water and compound density were assumed to be 2 and 1 g/cm^3.

The critical radii are less than the particle radius, so they will not grow to the CCN unless the critical supersaturation exceeds the ambient saturation ratio. I suspect that the difference between the ambient and the super saturation ratio will play a greater role in determining which particle reaches CCN first. Particle a has a lower supersaturation ratio, making it more likely to reach CCN. The critical radius of particle a also assures us that ri < r\*, a necessary condition for CCN activation if Sq > S\*.