Compact 2D FDTD Method for Modeling Photonic Crystal Fibers

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Abstract

Combining perfectly matched layer (PML) for the boundary treatment, we present an efficient compact 2-dimensional finite-difference time-domain (2D FDTD) method for modeling photonic crystal fibers. For photonic crystal fibers, if we assume that the propagation constant along the propagation direction is fixed, three-dimensional hybrid guided modes can be calculated by using only a two-dimension mesh. Because of using the real variable method, the computation time, i.e., it is of order N. Comparing with the plane wave expansion method, FDTD make the computation time and computer memory are significantly reduced. The numerical results for a triangular lattice photonic crystal fiber are in very good agreement with the results from the local basis function method. This method can easily be used for any complicated inclusions.

Keywords : Photonic crystal fiber; compact 2D FDTD method; modal field, effective refractive index.

1. Introduction

Recent years have seen an explosion of interest in the new science of photonic crystal fibers (PCFs) --- silica optical fibers consisting of a central defect region in a regular lattice of air holes. According to their light guided mechanisms, PCFs may be divided into two general categories, namely, the photonic band gap (PBG) and the total internal reflection (TIR) PCFs. Theoretical studies of guided modes in PCFs employ a wide variety of techniques, including the plane wave expansion method [1], the effective index approach [2], the localized basis function method [3], full-vector bi-orthonormal basis modal method [4], and so on. In many case, these methods employ a super-cell approach, in fact modeling a structure that extends periodically into infinity. For instance, plane-wave method (PWM) solves the full vector wave equation for the electromagnetic field and, as the name implies, is based on a plane-wave expansion of the field and the position-dependent dielectric constant. However, the periodicity of the modeled structure does not always fit well with realistic finite-sized structures.

The finite-difference time-domain (FDTD) method is a very general algorithm for calculating electromagnetic field distributions in finite structures of arbitrary geometry. It can be very accurate since it is based on a direct discretization of Maxwell's equation, making no assumptions on the kind of solution or the propagation direction of waves. Starting from a given field distribution, driven by sources at given locations, the time evolution of electromagnetic is calculated over a given spatial domain. This make it a tool that is suitable for investigating complicated wave phenomena, like multiple scattering, providing result that can be relatively easily interpreted. Another advantage of the FDTD is, that it provides results for a large range of frequencies in a single run, by applying a pulsed start field and Fourier transforming

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the response. For this reasons, it is well suitable for modeling photonic crystal structures. In the recent years, the full-wave analysis technique based on the compact 2D finite difference time domain (FDTD) method has become more and more popular due to its high flexibility. However, the compact 2D FDTD approached proposed in were based on the processing of the complex variable, such process was claimed as the disadvantage of this method. In order to overcome this drawback and further improve the efficiency of the compact 2D FDTD method, several real variable algorithms based on 2D FDTD [5, 6] were developed. The advantage of the real variable method over the complex variable method is obvious since only half of the computer memory and CPU time are required in the real algorithm. For photonic crystal fibers, if one assumes that propagation constant along z-direction (propagation direction) is fixed, three-dimensional hybrid guided modes can be calculated using only a two-dimensional mesh. Therefore the compact 2D FDTD model can be used for the analysis of photonic crystal fibers. If we just discuss the modal properties and do not consider the loss, the real variable is a suitable method for modeling PCFs.

In this paper, we present an efficient real variable compact 2D FDTD algorithm and apply a very efficient kind of absorbing boundary conditions, the perfectly Matched Layers (PML's), for modeling PCFs. In section 2, we describe the computation scheme in detail. In section 3, the method is verified numerically by comparing with the local basis function method for the case of a triangular lattice PCFs. The results are in very good agreement with the results from the local basis function method.

2. Numerical methods

2.1 FDTD time-stepping formulas in compact 2D mesh

The FDTD is a direct discretization of Maxwell's differential equations, where the differentials are replaced by finite differences. A well-known efficient implementation is based on Yee's mesh [7], where the electric and magnetic field components are evaluated at different grids having the same pitch, but which have been shifted over half a grid spacing, both in space and in time. For photonic crystal fibers, if one assumes that propagation constant along z-direction (propagation direction) is fixed, three-dimensional hybrid guided modes can be calculated using only a two-dimensional mesh that has been shown in Fig.1. Therefore the compact 2D FDTD model can be used for modeling photonic crystal fibers. Fig. 1 depicts the unit cell of the two dimensional mesh over the cross section of fiber.

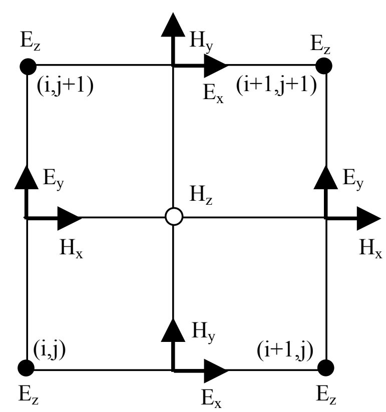


Fig.1. Unit cell of 2-D FDTD mesh

For guided modes in photonic crystal fibers, we assume that the propagation constant along the z-direction (propagation direction) is β. Thus, the field variation along the propagation direction z is of the form exp(-j βz). The z-derivatives can be replaced by -j β in Maxwell's equations. Furthermore, when computing guided modes in photonic crystal fibers, one wishes to refer only to real numbers. To ensure that Maxwell's equations to be discretized using FDTD scheme that only contains real variable, we represent the fields in the form

$$\{E,H\} = \{jE_x, jE_y, E_z, H_x, H_y, jH_z\}e^{-j\beta z} \tag{1}$$

The discrete form of the corresponding Maxwell's equation can be expressed as[6]

$$H_x\Big|_{i,j}^{n+\frac{1}{2}} = H_x\Big|_{i,j}^{n-\frac{1}{2}} - \frac{\Delta t}{\mu_{i,j}}\left(\frac{E_z\big|_{i,j+1}^{n} - E_z\big|_{i,j}^{n}}{\Delta y} - \beta E_y\big|_{i,j}^{n}\right) \tag{2}$$

$$H_y\Big|_{i,j}^{n+\frac{1}{2}} = H_y\Big|_{i,j}^{n-\frac{1}{2}} + \frac{\Delta t}{\mu_{i,j}}\left(\frac{E_z\big|_{i+1,j}^{n} - E_z\big|_{i,j}^{n}}{\Delta x} - \beta E_x\big|_{i,j}^{n}\right) \tag{3}$$

$$H_z\Big|_{i,j}^{n+\frac{1}{2}} = H_z\Big|_{i,j}^{n-\frac{1}{2}} + \frac{\Delta t}{\mu_{i,j}}\left(\frac{E_x\big|_{i,j+1}^{n} - E_x\big|_{i,j}^{n}}{\Delta y} - \frac{E_y\big|_{i+1,j}^{n} - E_y\big|_{i,j}^{n}}{\Delta x}\right) \tag{4}$$

$$E_x\Big|_{i,j}^{n+1} = \frac{\varepsilon_{i,j} - \sigma_{i,j}\Delta t/2}{\varepsilon_{i,j} + \sigma_{i,j}\Delta t/2} E_x\Big|_{i,j}^{n} + \frac{\Delta t}{\varepsilon_{i,j} + \sigma_{i,j}\Delta t/2} \times \left(\frac{H_z\big|_{i,j}^{n+\frac{1}{2}} - H_z\big|_{i,j-1}^{n+\frac{1}{2}}}{\Delta y} + \beta H_y\Big|_{i,j}^{n+\frac{1}{2}}\right) \tag{5}$$

$$E_y\Big|_{i,j}^{n+1} = \frac{\varepsilon_{i,j} - \sigma_{i,j}\Delta t/2}{\varepsilon_{i,j} + \sigma_{i,j}\Delta t/2} E_y\Big|_{i,j}^{n} + \frac{\Delta t}{\varepsilon_{i,j} + \sigma_{i,j}\Delta t/2} \times \left(\frac{H_z\big|_{i,j}^{n+\frac{1}{2}} - H_z\big|_{i-1,j}^{n+\frac{1}{2}}}{\Delta x} + \beta H_x\Big|_{i,j}^{n+\frac{1}{2}}\right) \tag{6}$$

$$E_z\Big|_{i,j}^{n+1} = \frac{\varepsilon_{i,j} - \sigma_{i,j}\Delta t/2}{\varepsilon_{i,j} + \sigma_{i,j}\Delta t/2} E_z\Big|_{i,j}^{n} + \frac{\Delta t}{\varepsilon_{i,j} + \sigma_{i,j}\Delta t/2} \times \left(\frac{H_y\big|_{i,j}^{n+\frac{1}{2}} - H_y\big|_{i-1,j}^{n+\frac{1}{2}}}{\Delta x} + \frac{H_x\big|_{i,j}^{n+\frac{1}{2}} - H_x\big|_{i-1,j}^{n+\frac{1}{2}}}{\Delta y}\right) \tag{7}$$

Where the index n denotes the discrete time step, and index i, j denote the discrete grid point in the x-y plane, respectively. Δt is the time increment, and Δx and Δy are intervals between two neighboring grid points along the x- and y-directions, respectively. ε,μ , and σ are the permittivity, permeability and conductivity of the corresponding material. Here to simplify the computation, we substitute the half grid point by the adjacent integral grid point.

If one knows all necessary information at each grid point, such as the permittivity, permeability, conductivity and the initial distribution of the fields, one can obtain the time evolution of the fields by these time-stepping formulas. From these time-stepping formulas, one can easily see that for a fixed total number of time steps the computational time is proportional to the number of discretization points in the computation domain, i.e., the FDTD algorithm is of order N. whereas the plane wave expansion method is of order N^3[1].

2.2 Numerical stability

A stability limit is determined by choosing a suitable time step Δt to ensure the solutions with purely real frequencies for all the possible wave vector **k**. To ensure the numerical stability of the above FDTD time step, the time step should satisfy [8]

$$\Delta t \le \frac{1}{C\sqrt{\Delta x^{-2} + \Delta y^{-2} + (\beta/2)^{-2}}} \tag{8}$$

where C is the speed of the light.

2.3 Boundary condition

In order to obtain a finite-sized calculation, the number of grid points should be finite. At the spatial boundaries of the calculation domain, the electromagnetic should satisfy condition such, that the space outside this domain in a desired way, e.g. a non-reflection continuation of the structure inside the calculation window. To make energy crossing the boundary not return inside the calculation window, usually a layer of absorbing material along the boundary is introduced. The best solution known is the induction of perfect Matched Layers (PML's) [9] that were introduced by Bérenger. In this paper, we use the perfectly matched layer (PML) for the boundary treatment. Fig. 2 shows the cross section of a PCF is surrounded by PML region 1 to 8, where x and y are the transverse directions, z is the propagation direction, PML regions 1,2 and 3,4 are faced with x and y direction, respectively, region 5 to 8 correspond to the four corners. The permittivity and permeability of PML medium in x and y direction are express as

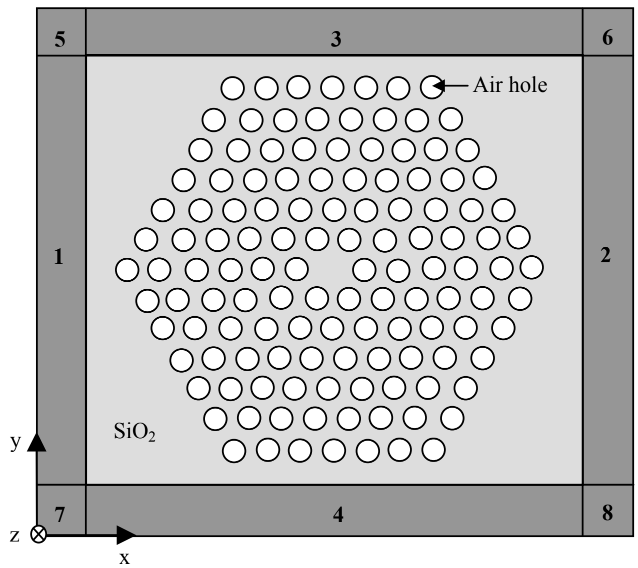


Fig.2 Calculating model of transverse cross section of photonic crystal fiber surrounded by PMLs

$$\begin{aligned} \mu &= \mu_0 \mu_r \overline{\overline{\mu}} \\ \varepsilon &= \varepsilon_0 \varepsilon_r \left[1 + \sigma/(j\omega_0 \varepsilon_0 \varepsilon_r)\right] \overline{\overline{\varepsilon}} \end{aligned} \tag{9}$$

For edge and corners, the general expressions of $\overline{\overline{\mu}}, \overline{\overline{\varepsilon}}$

$$\overline{\overline{\mu}} = \overline{\overline{\varepsilon}} = \begin{bmatrix} s_y/s_x & 0 & 0 \\ 0 & s_x/s_y & 0 \\ 0 & 0 & s_x s_y \end{bmatrix} \tag{10}$$

where $s_x = 1 + \sigma_x{}^p / j\omega\varepsilon_0$ and $s_y = 1 + \sigma_y{}^p / j\omega\varepsilon_0$. Where σ_x^P and σ_y^P are the conductivity of PML region along x- and y- direction. There are σ_y^P =0 in region 1and 2, σ_x^P =0 in region 3 and 4, σ_x^P and $\sigma_y^P \neq 0$ in the four corners.

Empirically, the PML conductivity σ_x^P and σ_y^P to vary spatially along x and y direction is chosen as follow [11]:

$$\sigma_v{}^P = \sigma_{\max} \frac{|i-i_0|^n}{N_v{}^n} \qquad (i = 0, 0.5, 1, \cdots, N_v) \text{ and } v=x, \text{ or } y \tag{11}$$

Where N_v is the number of cells in the PML region correspond to the respective region along x and y direction; n, the order of polynomial variation, is set equal to 3 or 4 in the application; i_0 denotes the initial index of the PML; and σ_{max} is an empirical constant the depends upon the size of the spatial discretization, $\Delta v(i)$ ($\Delta x(i)$ or $\Delta y(i)$), in the PML region. Here we chose σ_{max} as follow [10]

$$\sigma_{\max} \approx \frac{n+1}{15 \cdot N_v \cdot \pi \Delta v \sqrt{\varepsilon_r}} \tag{12}$$

The discrete expressions of the electromagnetic field in PML region are described in [9].

2.4 Source excitation

Although it was claimed that for the compact methods the impulse with the complex amplitude should be used in the excitation, using such an impulse as the excitation in the compact method is not an essential condition. Therefore, we use real amplitude Gaussian pulse that can be expressed as

$$E_{inc}{}^n = E^{re}(x,y) e^{-(n\Delta t - t_0)^2 / T^2} \tag{13}$$

where $E^{re}(x,y)$ is the initial field distribution of a particular guide mode, and t_0 and T are, respectively, the time delay and the width of the Gaussian pulse.

3. Numerical example

Consider a triangular lattice silica-air PCF, which is showed in Fig.2. The dielectric constant of silica is 2.1025, and the conductivity is 0. We choose the pitch Λ=2.3um that denote the distance between the nearest air holes. The diameter of the air hole d is 0.8Λ. in all of the numerical simulations present in this paper, an ten layer PML is used to truncate the FDTD. A total of 160,000(400×400) +20(PML) mesh points are used in the computation domain, and the time step Δt is chosen to be on the order of 10^{-16}s, whose actual value is dependent upon the operating frequency.

In order to validate the accuracy of the proposed algorithm, we calculate the effective index for the modes HE_{11} and TE_{01} for a PCF with FDTD and Local basis function method separately. The results are shown in Fig. 3, where the solid line are correspond to the numerical result gotten from the local basis function and the circle from the present method. This exhibits the excellent agreements of the two methods.

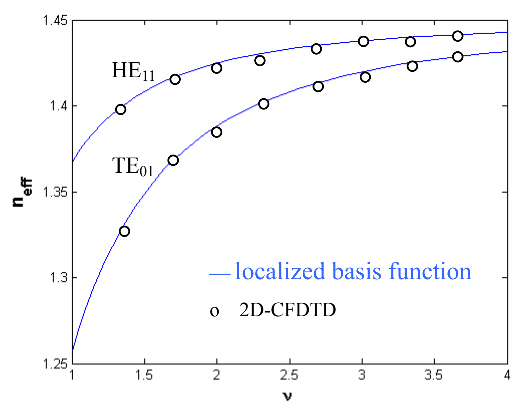
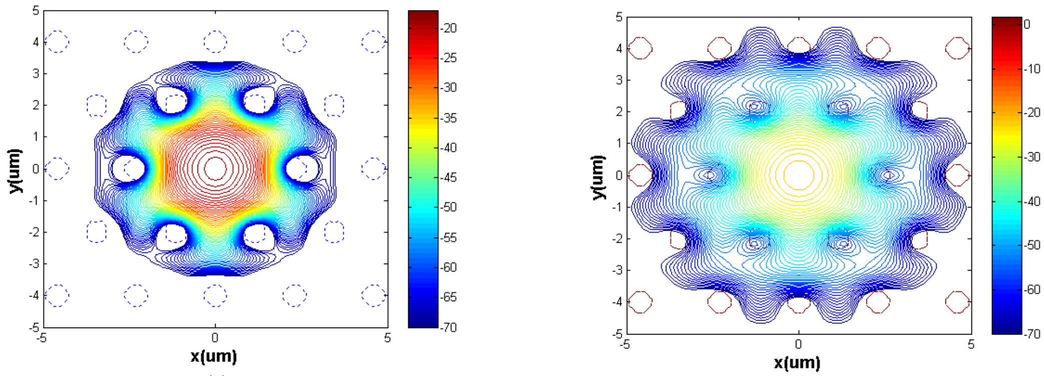


Fig. 3 Propagation characteristics of HE_{11} and TE_{01} for a triangular lattice PCF with Λ=2.3um and d=0.8Λ



(a) (b)

Fig. 4 Electric field distribution of the fundamental mode in a PCF with Λ=2.3um, d=0.3Λ at different wavelength (a) λ=633nm (b) λ=1550nm

Using the present FDTD method, one can also compute the electromagnetic fields of the guided mode. Fig 4 shows the intensity pattern of the fundamental modal field for a fiber with Λ=2.3um and d /Λ=0.3 at different wavelength 633nm and 1550nm. Fig. 5 shows the intensity of electromagnetic field for the fundament mode computed at 1550nm and 633nm along x axis (y=0). The contour maps are at 1dB interval. Fig.4 and Fig.5 clearly show that at the shorter wavelength the electromagnetic field concentrates in the core part of the PCF. This also implies that the PCF will have better wave guide feature at a shorter wavelength than at longer one for the same PCFs. The modal field distribution exhibits the six-fold symmetry of the cladding region in this PCF. The result are in very good agreement with those in [11] and [12].

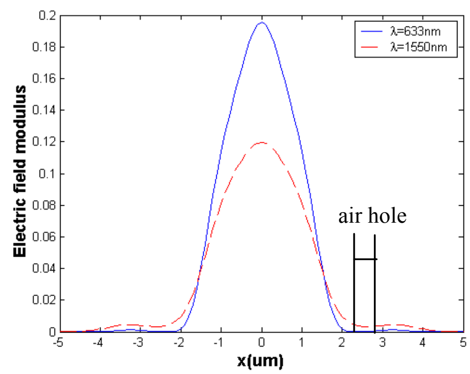


Fig. 5 Field intensity along x direction at y=0 at different wavelength

4. Conclusions

When the propagation constant along z-direction for the PCFs is fixed, three dimensional hybrid modes can be calculated with a two-dimensional mesh. In this paper, we have applied a highly efficient and flexible compact 2D-FDTD approach for modeling the triangular lattice PCF. We presented the field distribution of the fundamental modes and the effective index for modes HE_{11} and TE_{01}. The results for the triangular silica-air PCFs accord with the results from the other analysis method. Moreover, the FDTD method is an order-N method, whereas the plane expansion method is an order-N^3. Therefore, the computation time and computer memory are significantly reduced. In addition, this method can also be widely used to analyze the other properties such as the band structure [13], the chromatic dispersion, waveguide coupling, transmission, reflection and so on.

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