# MATHEMATICS

Time allowed: 3 hours | Max Marks: $80$

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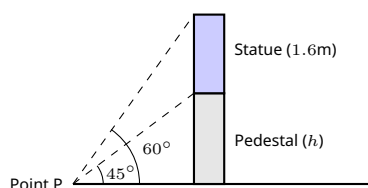**Code No. 30/1**

[1pt]

## General Instructions

1. All questions are compulsory.
2. The question paper consists of $31$ questions divided into four sections — A, B, C and D.
3. Section A contains questions $1$ to $4$ (1 mark each).
4. Section B contains questions $5$ to $10$ (2 marks each).
5. Section C contains questions $11$ to $22$ (3 marks each).
6. Section D contains questions $23$ to $31$ (4 marks each).

## SECTION A

1. If $x = 3$ is one root of the quadratic equation $x^2 - 2kx - 6 = 0$, then find the value of $k$.

2. What is the HCF of the smallest prime number and the smallest composite number?

3. Find the distance of a point $P(x, y)$ from the origin.

4. In an AP, if the common difference $d = -4$, and the seventh term $a_7$ is $4$, then find the first term $a$.

## SECTION B

5. Find the ratio in which $P(4, m)$ divides the line segment joining the points $A(2, 3)$ and $B(6, -3)$. Hence find $m$.

6. Two different dice are tossed together. Find the probability:
   (i) of getting a doublet        (ii) of getting a sum $10$, of the numbers on the two dice.

7. An integer is chosen between $70$ and $100$. Find the probability that it is:
   (a) a prime number                      (b) divisible by $7$

8. Solve for $x$: $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$.

9. Find the number of natural numbers between $101$ and $999$ which are divisible by both $2$ and $5$.

10. A statue $1.6$ m tall stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is $60°$ and from the same point the angle of elevation of the top of the pedestal is $45°$. Find the height of the pedestal.
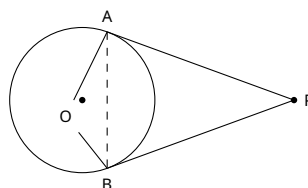
## SECTION C

11. Use Euclid's division algorithm to find the HCF of $4052$ and $12576$.

12. Find the zeroes of the quadratic polynomial $6x^2 - 3 - 7x$ and verify the relationship between the zeroes and the coefficients.

13. Solve the following pair of linear equations: $3x + 4y = 10$ and $2x - 2y = 2$.

14. Evaluate: $\int (x+3)\sqrt{3 - 4x - x^2}\, dx$.

15. Find the particular solution of the differential equation $\frac{dy}{dx} = -\frac{x + y\cos x}{1 + \sin x}$ given that $y(0) = 1$.

16. Find the particular solution of $2ye^{x/y}dx + (y - 2xe^{x/y})dy = 0$, given $x = 0$ when $y = 1$.

17. Show that the points $A(4, 5, 1)$, $B(0, -1, -1)$, $C(3, 9, 4)$, and $D(-4, 4, 4)$ are coplanar.

18. Find the foot of the perpendicular from $A(-1, 8, 4)$ to the line joining $B(0, -1, 3)$ and $C(2, -3, -1)$. Hence find the image of point $A$ in line $BC$.

19. Three numbers are chosen at random from the first six positive integers. Let $X$ be the largest number. Find the probability distribution of $X$.

20. Let $A = \mathbb{R} \times \mathbb{R}$ and define $(a, b) * (c, d) = (a + c, b + d)$. Show that $*$ is commutative and associative.

21. Prove that $y = \frac{4\sin\theta}{2 + \cos\theta} - \theta$ is increasing on $[0, \pi/2]$.

## SECTION D

22. Using integration, find the area of the triangle with vertices $(2, 2)$, $(4, 3)$, and $(1, 2)$.

23. Find the equation of the plane containing the line of intersection of the planes $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$ and $\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$ and which is perpendicular to the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 0$.

24. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.



25. The sum of the first three terms of an AP is $33$. If the product of the first and the third term exceeds the second term by $29$, find the AP.

26. A hemispherical tank full of water is emptied by a pipe at the rate of $3\frac{4}{7}$ litres per second. How much time will it take to empty half the tank, if it is $3$ m in diameter?

27. If $\sec\theta + \tan\theta = p$, then find the values of $\sin\theta$ and $\cos\theta$ in terms of $p$.

28. A bucket is in the form of a frustum of a cone with a capacity of $12308.8$ cm$^3$. The radii of the top and bottom circular ends are $20$ cm and $12$ cm respectively. Find the height of the bucket and its surface area. (Use $\pi = 3.14$)

29. If the median of the distribution given below is $28.5$, find the values of $x$ and $y$.

| Class interval | $0-10$ | $10-20$ | $20-30$ | $30-40$ | $40-50$ | $50-60$ | **Total** |
|---|---|---|---|---|---|---|---|
| **Frequency** | 5 | $x$ | 20 | 15 | $y$ | 5 | **60** |

30. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.