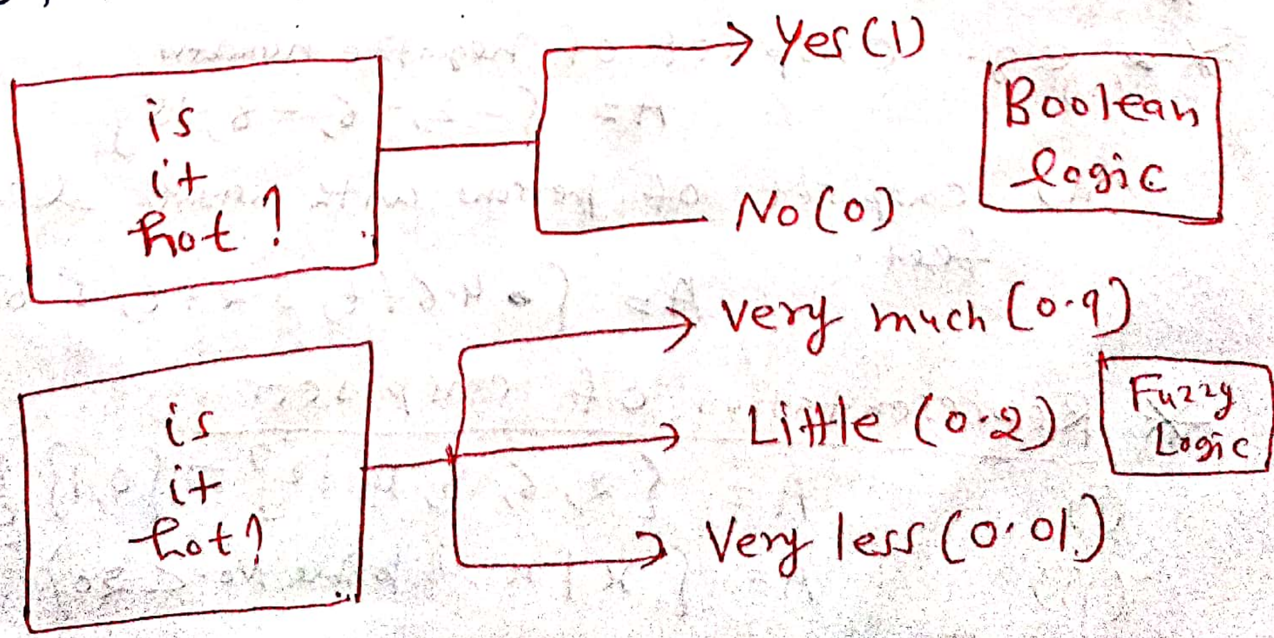
Fuzzy Logic: Fuzzy logic is an approach of computing based on "degree of truth" rather than usual "true or false (1,0) boolean logic on which the modern computer is based.

* It is an attempt to use more human like reasoning by employing "degree of truth".
* The idea of fuzzy logic was given by Dr. Lotfi Zadeh in 1965.
* Fuzzy logic includes 0 and 1 as extreme cases of truth but also includes the various states of truth in between.
* fuzzy logic seems closer to the way our brain works.
* The term "Fuzzy" refer to things which are not clear.
* In real word, some times we face a situation, where we can not determine whether the state is true or false.



Advantages and disadvantages of fuzzy logic System

Advantages	Disadvantages
(1) Fuzzy logic system can work with any type of inputs	(1) There is no systematic approach to solve a given problem through fuzzy logic
(2) Construction of fuzzy system is easy and understandable.	(2) Proof of its characteristics is difficult or impossible in most cases.
(3) It provides a very effecient solution to complex problems.	(3) since fuzzy logic works on precise as well as imprecise data, so most of the time, accuracy is compromised.
(4) It provides a very effecient solution to complex problems.	
(5) The algorithm can be described with little data.	

~~Fuzzy set~~: Classical sets (crisp sets)

* A crisp set is a collection of distinct objects
* e.g. (i) crisp set of negative numbers

$$A = \{-2, -6, -8, -15\}$$

(ii) crisp set of persons with heights less than 6 feet.

$$A = \{4.6ft, 5.2ft, 5.6ft, 5.9ft\}$$

* Representation of crisp sets

(i) $A = \{2, 6, 8, 10, 12\} = \{0, 1\}$ Binary crisp set

(ii) $A = \{x \mid x \text{ is prime No.} < 20\}$

(iii) Membership function of Classic set is represented by $\mu_A(x)$ for a set A having all values of x.

$$\mu_A(x) = \begin{cases} 1, & \text{if } x \in A \\ 0, & \text{if } x \notin A \end{cases}$$

* Operations on Classical (Crisp) Sets

(i) Union: The union between two sets gives all those elements that belongs to either set A or set B or both.

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

(ii) Intersection: The intersection between (b/w) two sets gives all those elements that is common to both A and B.

$$A \cap B = \{x \mid x \in A \text{ AND } x \in B\}$$

(iii) Complement: The complement of set A is defined as a collection of all elements in universe X that ~~does no~~ do not belongs to set A.

$$\overline{A} = \{x \mid x \notin A, x \in X\}$$

(iv) Difference: The difference of set A with respect to set B is a collection of all elements that belongs to A but do not belong to B.

$$A - B = \{x \mid x \in A \text{ AND } x \notin B\}$$

Properties of Crisp sets

1. Commutativity: $A \cup B = B \cup A$, $A \cap B = B \cap A$.

2. Associativity: $A \cup (B \cup C) = (A \cup B) \cup C$
   $A \cap (B \cap C) = (A \cap B) \cap C$

3. Distributivity: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
   $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

4. idempotency: $A \cup A = A$
   $A \cap A = A$

5. Transitivity: If $A \subseteq B \subseteq C$
   then $A \subseteq C$

6. Involution: $\overline{\overline{A}} = A$

7. Law of Contradicition $A \cap \overline{A} = \phi$

8. De Morgan's Law

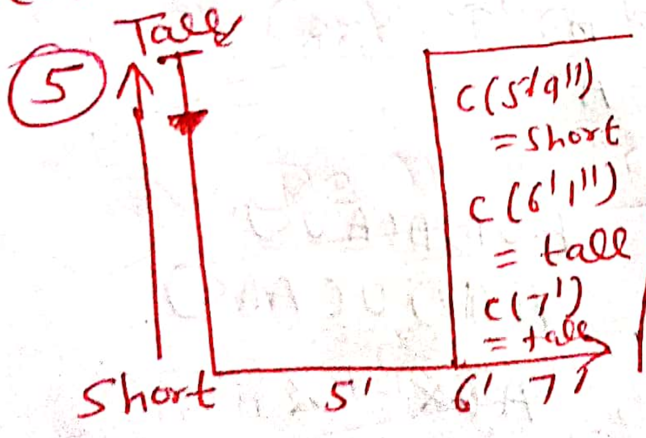
(i) $\overline{A \cap B} = \overline{A} \cup \overline{B}$

(ii) $\overline{A \cup B} = \overline{A} \cap \overline{B}$

Difference between Crisp set and Fuzzy set

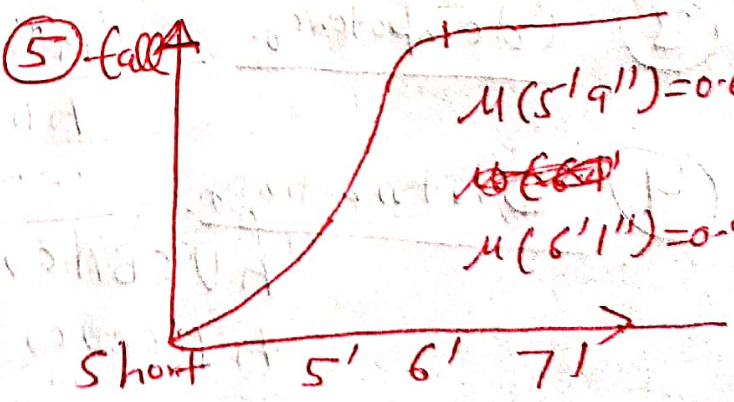
Crisp set	Fuzzy set
(1) It defines either value is 0 or 1.	(1) It defines value between 0 and 1 including both 0 and 1.
(2) It is also called Classic set.	(2) It specifies the degree to which something is true.
(3) It shows full membership Yes or No True or False 1 or 0	(3) It shows partial membership Yes ---> No True ---> False 1 ---> 0
(4) Crisp set example (i) She is 18 years old (ii) Rahul is 1.6 m tall	(4) Fuzzy set example (i) She is about 18 Yrs old (ii) Rahul is about 1.6 m tall

(5)



Crisp set

(5)



fuzzy set

Fuzzy Set:

Fuzzy set is a set that specifies the degree to which something is true. It defines the values between 0 and 1 including both 0 and 1. The elements of a fuzzy set have varying degree of membership.

A fuzzy set allows its members to have different degree of membership called membership function within interval [0,1]. It is an extension of classical set.

Properties of fuzzy set:

① Commutativity:

$A \cup B = B \cup A$

$A \cap B = B \cap A$

② Associativity:

$A \cup (B \cup C) = (A \cup B) \cup C$

$A \cap (B \cap C) = (A \cap B) \cap C$

③ idempotency:

$A \cup A = A$

$A \cap A = A$

④ Distributivity:

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

⑤ identity

$A \cup \phi = A$, $A \cup X = X$

$A \cap \phi = \phi$, $A \cap X = A$

X = Universe set

⑥ Transitivity: If $(A \subset B) \cap (B \subset C)$ then $A \subset C$

⑦ Involution: $\overline{\overline{A}} = A$

⑧ De-Morgan's Law:

$\overline{A \cap B} = \overline{A} \cup \overline{B}$

$\overline{A \cup B} = \overline{A} \cup \overline{B}$

Operations on fuzzy set:

1 Union (OR)

$X = \{a, b, c, d, e\}$

Let $A = \{(a, 0.2), (b, 0.4), (c, 0.7), (d, 0.9)\}$

$B = \{(a, 0.4), (b, 0.1), (c, 0.9), (d, 0.2)\}$

$\boxed{\text{Union} = \text{Max}\{\mu_A(X), \mu_B(X)\}, X \in U}$

$= \{(a, 0.4), (b, 0.4), (c, 0.9), (d, 0.9)\}$

2 Intersection (AND):

$\boxed{\text{Intersection} = \min\{\mu_A(X), \mu_B(X)\}, x \in U}$

$= \{(a, 0.2), (b, 0.1), (c, 0.7), (d, 0.2)\}$

3 Complement (NOT):

$\boxed{\text{Complement} = \overline{\mu_A(x)} = 1 - \mu_A(x), x \in U}$

$= \{(a, 0.8), (b, 0.6), (c, 0.3), (d, 0.1)\}$

$= \boxed{\overline{\mu_B(x)} = 1 - \mu_B(x), x \in U}$

$= \{(a, 0.6), (b, 0.9), (c, 0.1), (d, 0.8)\}$

4 Difference (A-B)

$\boxed{A - B = \text{Min}\{\mu_A(x), 1 - \mu_B(x)\}}$

$1 - \mu_B(x) = \{(a, 0.6), (b, 0.9), (c, 0.1), (d, 0.8)\}$

$\mu_A(x) = \{(a, 0.2), (b, 0.4), (c, 0.7), (d, 0.9)\}$

$\min\{\mu_A(x), 1 - \mu_B(x)\} = A - B = \{(a, 0.2), (b, 0.4), (c, 0.1), (d, 0.8)\}$

5 Algebraic Sum:

$A = \{(a, 0.2), (b, 0.3), (c, 0.4), (d, 0.5)\}$

$B = \{(a, 0.1), (b, 0.2), (c, 0.2), (d, 1)\}$

$$\boxed{\mu_{A+B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x)}$$

$= \{(a, 0.3), (b, 0.5), (c, 0.6), (d, 1.5)\}$
$- \{(a, 0.02), (b, 0.06), (c, 0.08), (d, 0.5)\}$

$\mu_{A+B}(x) = \{(a, 0.28), (b, 0.44), (c, 0.52), (d, 1.0)\}$

6 Algebraic Product:

$$\boxed{\mu_{A \cdot B}(x) = \mu_A(x) \cdot \mu_B(x)}$$

$A = \left\{\frac{0.1}{2} + \frac{0.3}{4} + \frac{0.2}{5} + \frac{0.6}{8}\right\}$

$B = \left\{\frac{0.2}{2} + \frac{0.4}{4} + \frac{0.1}{5} + \frac{0.3}{8}\right\}$

$\mu_{A \cdot B}(x) = \left\{\frac{0.02}{2} + \frac{0.12}{4} + \frac{0.02}{5} + \frac{0.18}{8}\right\}$

7 Bounded Sum (BS)

$\mu_{BS}(x) = \min\{1, \mu_A(x) + \mu_B(x)\}$

If $A = \{(\text{amit}, 0.4), (\text{Rahul}, 0.3), (\text{Ram}, 0.5), (\text{AJay}, 0.8)\}$

$B = \{(\text{amit}, 0.2), (\text{Rahul}, 0.4), (\text{Ram}, 0.7), (\text{AJay}, 0.5)\}$

where A = set of Age

B = Set of heights

$\mu_A(x) + \mu_B(x) = \{(amit, 0.6), (Rahul, 0.7), (Ram, 1.2), (Ajay, 1.3)\}$

Bounded Sum $= \min\{1, \mu_A(x) + \mu_B(x)\}$

$\min\{1, \{(amit, 0.6), (Rahul, 0.7), (Ram, 1.2), (Ajay, 1.3)\}\}$

$$\boxed{\text{Bounded sum} = \{(amit, 0.6), (Rahul, 0.7), (RAm, 1), (Ajay, 1)\}}$$

(4) Bounded difference (BD)

$$\boxed{\mu_{BD}(x) = \max\{0, \mu_A(x) - \mu_B(x)\}}$$

$A = \{(a, 0.2), (b, 0.3), (c, 0.4), (d, 0.5)\}$

~~$\mu_A(x) - \mu_B(x)$~~

$B = \{(a, 0.1), (b, 0.2), (c, 0.2), (d, 1)\}$

$\{(a, 0.1), (b, 0.1), (c, 0.2), (d, -0.5)\}$

$\mu_A(x) - \mu_B(x) = \{(a, 0.1), (b, 0.1), (c, 0.2), (d, -0.5)\}$

$\mu_{BD}(x) = \max\{0, \mu_A(x) - \mu_B(x)\}$

$= \max\{0, \{(a, 0.1), (b, 0.1), (c, 0.2), (d, -0.5)\}\}$

$= \max\{0, \{(a, 0.1), (b, 0.1), (c, 0.2), (d, 0)\}\}$

Bounded difference $= \{(a, 0.1), (b, 0.1), (c, 0.2), (d, 0)\}$

(5) Bounded Product (μ_{BP})

$\max(0, \mu_A(x) + \mu_B(x) - 1)$

$\mu_{BP} = \max(0, \mu_A(x) + \mu_B(x) - 1)$

$A = \{(a, 0.2), (b, 0.3), (c, 0.4), (d, 0.5)\}$

$B = \{(a, 0.1), (b, 0.2), (c, 0.2), (d, 1)\}$

$\mu_A(x) + \mu_B(x) = \{(a, 0.3), (b, 0.5), (c, 0.6), (d, 1.5)\}$

$\mu_A(x) + \mu_B(x) - 1 = \{(a, -0.7), (b, -0.5), (c, -0.4), (d, 0.5)\}$

$$\boxed{\begin{aligned}&\max(0, \mu_A(x) + \mu_B(x) - 1)\\ &\text{Bounded Product} = \{(a, 0), (b, 0), (c, 0), (d, 0.5)\}\end{aligned}}$$

6 Drastic Product (DP)

$$\mu_{DP} = \begin{cases} \mu_B(x), & \text{if } \mu_A(x) = 1 \\ \mu_A(x), & \text{if } \mu_B(x) = 1 \\ 0, & \text{if } \mu_A(x), \mu_B(x) < 1 \end{cases}$$

$A = \{(a, 0.2), (b, 0.3), (c, 0.4), (d, 0.5)\}$

$B = \{(a, 0.1), (b, 0.2), (c, 0.2), (d, 1)\}$

$$\boxed{\begin{aligned}&\mu_{DP} = \{(a, 0), (b, 0), (c, 0), (d, 0.5)\}\\ &\text{Drastic Product}\end{aligned}}$$

Fuzzy Relations: Fuzzy relations relate elements of one universe (X) to those of another universe (Y) through the cartesian product of the two universes.

* A fuzzy relation is a fuzzy set defined on the cartesian product of classical sets.
* A fuzzy relation between two sets X and Y is called binary fuzzy relation and is denoted by $R(X, Y)$.
* The matrix representing a fuzzy relation is called fuzzy matrix.

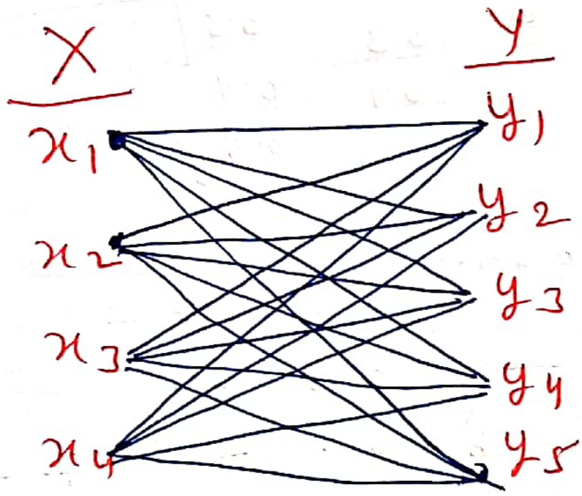
If there are two fuzzy set X and Y

Where $X = \{x_1, x_2 ---- x_n\}$

$Y = \{y_1, y_2, ---- y_m\}$

then fuzzy Relation $R = (X, Y)$

$$R(X,Y) = \begin{bmatrix} \mu_R(x_1,y_1), & \mu_R(x_1,y_2), & \cdots & \mu_R(x_1,y_m) \\ \mu_R(x_2,y_1), & \mu_R(x_2,y_2), & \cdots & \mu_R(x_2,y_m) \\ \vdots & \vdots & & \vdots \\ \mu_R(x_n,y_1) & \mu_R(x_n,y_2) & \cdots & \mu_R(x_n,y_m) \end{bmatrix}$$



If $X = \{x_1, x_2, x_3, x_4\}$

$Y = \{y_1, y_2, y_3, y_4, y_5\}$

Graphical Representation of fuzzy Relations $R(X,Y)$

example: If $A = \{(a, 0.2), (b, 0.7), (c, 0.4)\}$

$B = \{(a, 0.5), (b, 0.6)\}$

$$R(x,y) = \mu_R(x,y) = \mu_{A\times B}(x,y) = \min(\mu_A(x), \mu_B(x))$$

$$R(x,y) = \mu_R(x,y) = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{cc} a & b \end{array} \\ \begin{bmatrix} 0.2 & 0.2 \\ 0.5 & 0.6 \\ 0.4 & 0.4 \end{bmatrix} \end{array}$$

Operations on fuzzy Relations

(a) Union:

$$\boxed{\mu_{R\cup S}(x,y) = \max(\mu_R(x,y), \mu_S(x,y))}$$

Let

$$\mu_R(x,y) = \begin{array}{c|cc} & a & b \\ \hline a & 0.2 & 0.2 \\ b & 0.5 & 0.6 \\ c & 0.4 & 0.4 \end{array}$$

$$\mu_S(x,y) = \begin{array}{c|cc} & a & b \\ \hline a & 0.3 & 0.5 \\ b & 0.1 & 0.4 \\ c & 0.3 & 0.6 \end{array}$$

then fuzzy Relations

$$\mu_{R\cup S}(x,y) = \begin{array}{c|cc} & a & b \\ \hline a & 0.3 & 0.5 \\ b & 0.5 & 0.6 \\ c & 0.4 & 0.6 \end{array}$$

(b) Intersection:

$$\mu_{R\cap S}(x,y) = \min(\mu_R(x,y), \mu_S(x,y))$$

$$= \begin{array}{c|cc} & a & b \\ \hline a & 0.2 & 0.2 \\ b & 0.1 & 0.4 \\ c & 0.3 & 0.4 \end{array}$$

(c) Complement:

$$\overline{\mu_R(x,y)} = 1 - \mu_R(x,y)$$

$$= \begin{bmatrix} 0.8 & 0.8 \\ 0.5 & 0.4 \\ 0.6 & 0.6 \end{bmatrix}$$

Fuzzy Composition The operation executed on two compatible fuzzy relations to get a single fuzzy relation is called fuzzy composition.

Let $R(x,y) = X * Y$ and
$S(y,z) = Y * Z$ are two fuzzy relations

Types of fuzzy composition:

1. fuzzy Max-Min Composition
2. fuzzy Max-product Composition

1. fuzzy max-min composition $= T_{x.z}$

$$\boxed{\mu_{R \circ S} = \max(\min(\mu_R(x,y), \mu_S(y,z)))}$$

Let
$$R(x,y) = \begin{array}{c c} & \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} 0.6 & 0.3 \\ 0.2 & 0.9 \end{bmatrix} \end{array}$$

$$S(y,z) = \begin{array}{c c} & \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{array}{c} y_1 \\ y_2 \end{array} & \begin{bmatrix} 1 & 0.5 & 0.3 \\ 0.8 & 0.4 & 0.7 \end{bmatrix} \end{array}$$

$$\mu_{R \circ S} = T_{x.z} = \begin{array}{c c} & \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} x_1 z_1 & x_1 z_2 & x_1 z_3 \\ x_2 z_1 & x_2 z_2 & x_2 z_3 \end{bmatrix} \end{array} = \begin{bmatrix} 0.6 & 0.5 & 0.3 \\ 0.8 & 0.4 & 0.7 \end{bmatrix}$$

$$(x_1,z_1) = \max[\min(x_1,y_1),(y_1,z_1)), \min(x_1,y_2),(y_2,z_1)]$$
$$= \max[\min(0.6, 1), \min(0.3, 0.8)]$$
$$= \max[0.6, 0.3] = \underline{\underline{0.6}}$$

$$(x_1,z_2) = \max[\min(0.6, 0.5), \min(0.3, 0.4)]$$
$$= \max[0.5, 0.3] = \underline{\underline{0.5}}$$

$(x_1, z_3) = \max[\min(0.6, 0.3), \min(0.3, 0.7)]$ (14)

$= \max(0.3, 0.3) = \underline{0.3}$

$(x_2, z_1) = \max[\min(0.2, 0.1), \min(0.9, 0.8)]$

$= \max[0.1, 0.8] = \underline{0.8}$

$(x_2, z_2) = \max[\min(0.2, 0.5), \min(0.9, 0.4)]$

$= \max[0.2, 0.4] = \underline{0.4}$

$(x_2, z_3) = \max[\min(0.2, 0.3), \min(0.9, 0.7)]$

$= \max[0.2, 0.7] = \underline{0.7}$

---

(2) Fuzzy Max Product Composition

If $R(x,y)$ and $S(y,z)$ are two relations, then fuzzy max-product composition

$$\boxed{\mu_{R \cdot S} = \max(\mu_R(x,y), \mu_S(y,z))}$$

Let $\mu_R(x,y) = \begin{array}{c|cc} & y_1 & y_2 \\ \hline x_1 & 0.6 & 0.3 \\ x_2 & 0.2 & 0.9 \end{array}$

$\mu_S(y,z) = \begin{array}{c|ccc} & z_1 & z_2 & z_3 \\ \hline y_1 & 1 & 0.5 & 0.3 \\ y_2 & 0.8 & 0.4 & 0.7 \end{array}$

$\mu_{R \cdot S} = \begin{array}{c|ccc} & z_1 & z_2 & z_3 \\ \hline x_1 & x_1 z_1 & x_1 z_2 & x_1 z_3 \\ x_2 & z_1 & x_2 z_2 & x_2 z_3 \end{array} = \begin{array}{c|ccc} & z_1 & z_2 & z_3 \\ \hline x_1 & 0.6 & 0.30 & 0.21 \\ x_2 & 0.72 & 0.36 & 0.63 \end{array}$

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$$\mu_{(x_1,z_1)} = \max\{(\mu_R(x_1,y_1) * \mu_S(y_1,z_1)), \mu_R(x_1,y_2) * \mu_S(y_2,z_1)\}$$
$$= \max[0.6 * 1, 0.3 \times 0.8]$$
$$= \max[0.6, 0.24] = \underline{\underline{0.6}}$$

$$\mu_{(x_1,z_2)} = \max[0.6 \times 0.5, 0.3 \times 0.4]$$
$$= \max[0.30, 0.12]$$
$$= \underline{\underline{0.30}}$$

$$\mu_{x_1,z_3} = \max[0.6 \times 0.3, 0.3 \times 0.7]$$
$$= \max[0.18, 0.21] = \underline{\underline{0.21}}$$

$$\mu_{x_2,z_1} = \max[0.2 \times 1, 0.9 \times 0.8]$$
$$= \max[0.2, 0.72] = \underline{\underline{0.72}}$$

$$\mu_{x_2,z_2} = \max[0.2 \times 0.5, 0.9 \times 0.4]$$
$$= \max[0.10, 0.36] = \underline{\underline{0.36}}$$

$$\mu_{x_2,z_3} = \max[0.2 \times 0.3, 0.9 \times 0.7]$$
$$= \max[0.06, 0.63]$$
$$= 0.63$$

$$\mu_{R \cdot S} = \begin{array}{c c} & \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} 0.6 & 0.30 & 0.21 \\ 0.72 & 0.36 & 0.63 \end{bmatrix} \end{array}$$

Fuzzy Implications (Fuzzy if-then rule)

A fuzzy implications (also known as fuzzy if-then rule / Fuzzy rule / Fuzzy conditional statement) assumes the form

(i) If x is A then y is B

(ii) If x is A then y is B else y is C

Example (i) If mango is yellow then mango is sweet.

(ii) If mango is yellow then mango is sweet else mango is sour.

If x is A then y is B, its implication relation (R) can be written as

$$R = (A \times B) \cup (\bar{A} \times Y) \quad \text{where } Y \rightarrow \text{Universe}$$

If x is A then y is B else y is C

$$R = (A \times B) \cup (\bar{A} \times C)$$

Example Let $X = \{a, b, c, d\}$, $Y = \{1, 2, 3, 4\}$

and $A' = \{(a, 0), (b, 0.8), (c, 0.6), (d, 1)\}$

$B' = \{(1, 0.2), (2, 1), (3, 0.8), (4, 0)\}$

$C' = \{(1, 0), (2, 0.4), (3, 1), (4, 0.8)\}$

Determine its ~~[illegible]~~ implication relations

(i) If X is A' then Y is B'

(ii) If X is A' then Y is B' else Y is C'

Ans (i) If X is A′ then Y is B′

$$\boxed{R = (A' \times B') \cup (\overline{A'} \times Y)}$$

$A' = \{(a, 0), (b, 0.8), (c, 0.6), (d, 1)\}$

$B' = \{(1, 0.2), (2, 1), (3, 0.8), (4, 0)\}$

$$A' \times B' = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\!\!\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.2 & 0.8 & 0.8 & 0 \\ 0.2 & 0.6 & 0.6 & 0 \\ 0.2 & 1 & 0.8 & 0 \end{bmatrix} \end{array}$$

$\overline{A'} = \{(a, 1), (b, 0.2), (c, 0.4), (d, 0)\}$

$Y = \{(1, 1), (2, 1), (3, 1), (4, 1)\}$

$$\overline{A'} \times Y = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\!\!\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0.2 & 0.2 & 0.2 & 0.2 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$(A' \times B') \cup (\overline{A'} \times Y)$

$$= \begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\!\!\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0.2 & 0.8 & 0.8 & 0.2 \\ 0.4 & 0.6 & 0.6 & 0.4 \\ 0.2 & 1 & 0.8 & 0 \end{bmatrix} \end{array}$$

Ans (ii) If x is A′ then Y is B′ else Y is C′

$$R = (A' \times B') \cup (\overline{A'} \times C')$$

$C' = \{(1, 0), (2, 0.4), (3, 1), (4, 0.8)\}$

$$\overline{A'} \times C' = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\!\!\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0.4 & 1 & 0.8 \\ 0 & 0.2 & 0.2 & 0.2 \\ 0 & 0.4 & 0.4 & 0.4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$(A' \times B') \cup (\bar{A}' \times C') = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline a & & & & \\ b & & & & \\ c & & & & \\ d & & & & \end{array}$$

$$\begin{array}{c} a \\ b \\ c \\ d \end{array} \begin{bmatrix} 0 & 0.4 & 1 & 0.8 \\ 0.2 & 0.8 & 0.8 & 0.2 \\ 0.2 & 0.6 & 0.6 & 0.4 \\ 0.2 & 1 & 0.8 & 0 \end{bmatrix}$$

fuzzy Quantifier:

fuzzy quantifier are the expression allowing us to express fuzzy quantities in order to provide an approximate idea of the number of elements of a subset fullfilling a certain condition.

Types of fuzzy quantifier

① **Absolute Quantifier:** Absolute Quantifier express quantities over the total no. of elements of a particular set, stating whether the number is "much more than 10", "close to 100", "at least about 5", "round about 250", "somewhere around 20".

② **Relative Quantifier:** Relative quantifier express measurement over the total no. of elements which fulfill a certain condition as "about half", "the majority", "almost all", "Most of", "The minority" etc "Little of",

Fuzzy Modifier (Hedges)

* Fuzzy modifier are unary operators that can be employed to modify the meaning of a fuzzy set (e.g. very, somewhat, quite, slightly etc)
* A modifier may be used to further enhance the ability to describe fuzzy concepts.
* Modifiers (e.g. very, slightly) used in phrases such as very hot or slightly cold change (modify) the shape of a fuzzy set in a way that suits the meaning of the word used.
* These modifiers change the shape of a fuzzy set using mathematical operations on each point of the set.

Types of fuzzy modifiers

① **NOT Modifier:** The 'NOT' modifier returns the complement of the membership value posed as argument

$$\boxed{Y = [\mu_A(x)]'}$$

② **More or less:** The 'More or less' modifier return the $\frac{1}{2}$ of the membership value posed as argument. $\boxed{Y = [\mu_A(x)]^{1/2}}$

③ **Slightly:** The 'slightly' modifier returns the $\frac{1}{3}$ of the membership value posed as argument

$$\boxed{Y = [\mu_A(x)]^{1/3}}$$

④ **Plus:** The 'Plus' modifier returns the 1.25 of the membership value passed as argument

$$\boxed{Y = \mu_A(x)^{1.25}}$$

⑤ **Very modifier:** $\boxed{Y = \mu_A(x)^2}$

6 Extremely modifier:

$$\boxed{Y = [\mu_A(x)]^3}$$

7 Exactly modifier

$$\boxed{Y = [\mu_A(x)]^{+\infty}}$$

8 Always modifier

$$\boxed{Y = [\mu_A(x)]^0}$$

Difference between fuzzification and Defuzzification

Properties	Fuzzification	Defuzzification
Definition	Fuzzification is the process of transforming a crisp set to a fuzzy set.	Defuzzification is the process of reducing a fuzzy set into a crisp set.
Purpose	Fuzzification Converts a precise data into imprecise data.	Defuzzification Converts an imprecise data into precise data.
Example	Voltmeter	Stepper Motor, D/A Converter
methods used	Inference, Rank ordering, Angular fuzzy sets, Neural Network	Max. membership principle, Centroid method, weighted average method.
Complexity	Fuzzification is easy	Defuzzification is difficult to ~~implement~~ implement
Approach	fuzzification uses if- then rules to fuzzify the crisp value	Defuzzification uses Centre of Gravity methods.

Membership function (MF)

* Fuzzy set A of universe X is defined by function $\mu_A(x)$, Called the membership function of set A.

$\mu_A(x) : x \in [0,1]$

where $\mu_A(x) = 1$, if x is totally in A

$\mu_A(x) = 0$, if x is not in A

$0 < \mu_A(x) < 1$, if x is partially in A

$\boxed{\mu_A(x) = (X, 0.3]}$

example

* So membership function is a function that specifies the degree to which a given input belongs to a set.

Degree of membership: a value between 0 and 1, represents the degree of membership, also Called the membership Value of element x in the set A. It is the output of membership function.

* Membership function can be defined as a technique to solve practical problem by experience rather by knowledge (degree of truth).

* Membership function are used in the fuzzification and defuzzification of a fuzzy logic system.

features of membership function:

① **Core:** The core of a membership function for some fuzzy set A is defined as that region on the universe that is characterized by complete and full membership in the set

$\boxed{\mu_A(x) = 1}$

(2) **Support:** The support of a membership function for some fuzzy set A is defined as that region of the universe that is characterized by non-zero membership in the set.

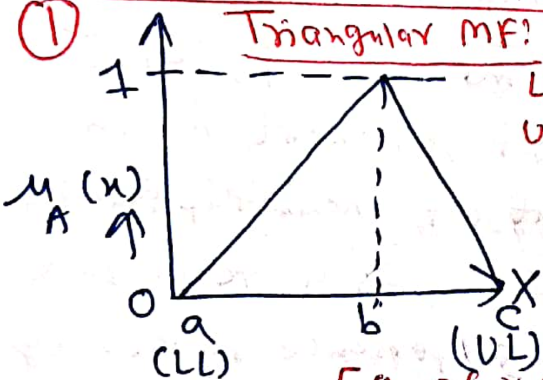
$$\boxed{\mu_A(x) > 0}$$

(3) **Boundary:** The boundary of a membership function for some fuzzy set A is defined as that region of the universe containing elements that have non-zero membership but not ~~compete~~ complete membership.

$$\boxed{0 < \mu_A(x) < 1}$$

Types of membership functions (MF)

(1) Triangular MF:

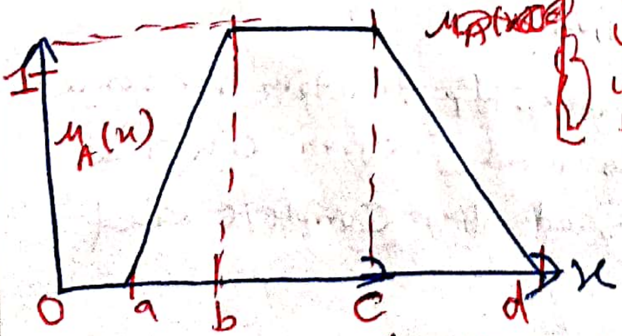


LL = lower limit
UL = upper limit

$$\mu_A(x) = \begin{cases} 0, & \text{if } x < a \\ \frac{x-a}{b-a}, & \text{if } a \le x \le b \\ c-x/c-b, & \text{if } b \le x \le c \\ 0, & \text{if } x > c \end{cases}$$

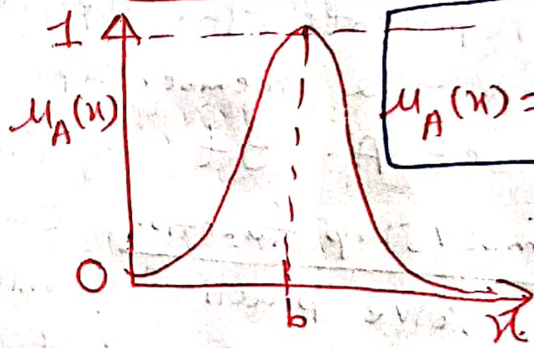
(2) Trapezoidal MF:

LSL = lower support limit
USL = upper support limit



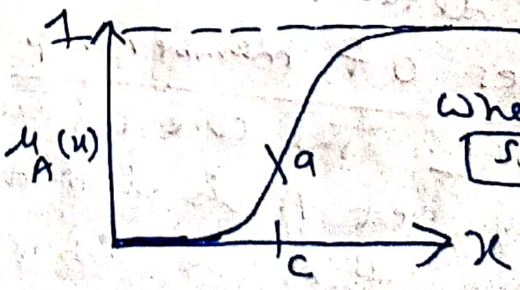
$$\mu_A(x) = \begin{cases} 0, & (x<a) \text{ OR } (x>d) \\ \frac{x-a}{b-a}, & a \le x \le b \\ 1, & b \le x \le c \\ \frac{d-x}{d-c}, & c \le x \le d \end{cases}$$

(3) Gaussian MF:



$$\boxed{\mu_A(x) = e^{-\frac{(x-b)^2}{2k^2}}}$$

(4) Sigmoidal MF



where $\boxed{\text{slope} = a}$

$$\boxed{\mu_A(x) = \frac{1}{1+e^{-a(x-c)}}}$$

Defuzzification and its methods

Defuzzification: It is a process of converting a fuzzy quantity to crisp quantity.

* Calculation of a crisp value from a fuzzy value is called Defuzzification.
* Defuzzification is realized by a decision making

Defuzzification methods

(1) Lambda-cut Method (Alpha cut Method)

If A is a fuzzy set then A_λ is the Lambda-cut set of A.

$A_\lambda : \{x \mid \mu_A(x) \geq \lambda\}$ where $0 \leq \lambda \leq 1$]

where A_λ is the crisp set.

example:

$$R = \begin{bmatrix} 1 & 0.8 & 0 \\ 0.8 & 1 & 0.4 \\ 0 & 0.4 & 1 \end{bmatrix}$$

If $\lambda = 1$ then

$$R_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

If $\lambda = 0.25$ then

$$R_{0.25} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

If $\lambda = 0.5$ then

$$R_{0.5} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

If $\lambda = 0$

$$R_0 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

If $A = \left\{ \frac{1}{a} + \frac{0.9}{b} + \frac{0.6}{c} + \frac{0.3}{d} + \frac{0.01}{e} + \frac{0}{f} \right\}$

for $\lambda = 1$, $A_1 = \{a\} = \left[\frac{1}{a}\right] \frac{1}{a}$

$\lambda = 0.8$ $A_{0.8} = \{a, b\} = \frac{1}{a} + \frac{1}{b}$

$\lambda = 0.6$, $A_{0.6} = \{a, b, c\} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$

$\lambda = 0.25$, $A_{0.25} = \{a, b, c, d\} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}$

$\lambda = 0$, $A_0 = (a, b, c, d, e, f) = \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} + \frac{1}{e} + \frac{1}{f}$

---

(2) Maxima method

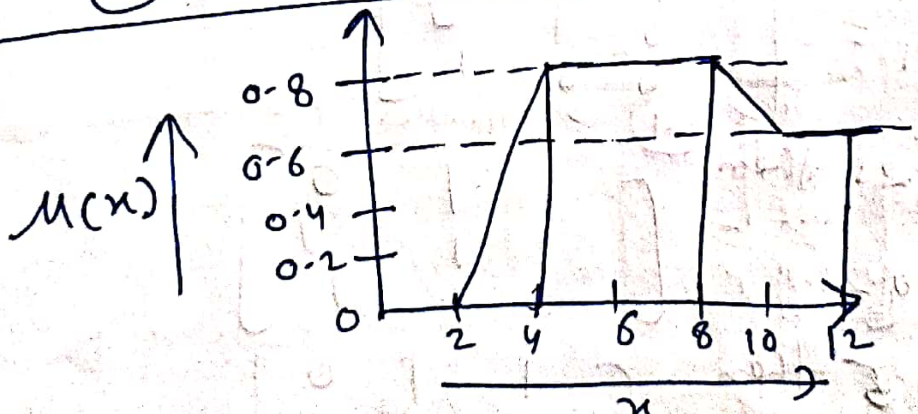
: This method deals with the maximum membership value to convert fuzzy values into crisp values.

It is of three types:

(1) First of maxima (2) Last of maxima (3) Mean of maxima

---

(1) First of Maxima Method (FOM)



FOM method gives the smallest value of domain x with maximum membership value

$$\boxed{X_{FOM} = 4}$$

Last of Maxima (LOM) Method

LOM method gives the largest value of domain x with maximum membership value.

$$\boxed{X_{LOM} = 8}$$

Mean of Maxima Method (MOM Method)

MOM method gives the mean of the values of domain x with maximum membership value.

$$X_{MOM} = \frac{4+6+8}{3} = \frac{18}{3} = 6$$

$$\boxed{X_{MOM} = 6}$$

Ex: 2 A fuzzy set 'Young' is defined as follows:
Young = {(15, 0.5), (20, 0.8), (25, 0.8), (30, 0.5), (35, 0.3)}
find the crisp value of 'Young' using MOM method.

Ans

$$\boxed{X_{MOM} = \frac{20+25}{2} = 22.5}$$

$$\boxed{X_{FOM} = 20}, \quad \boxed{X_{LOM} = 25}$$

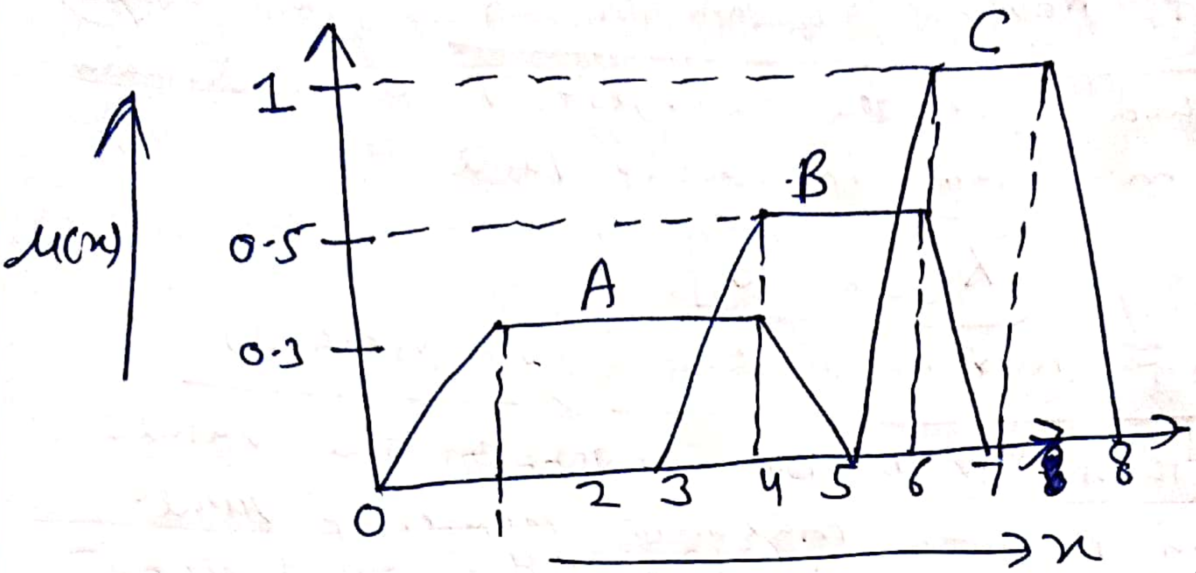
(3) Weighted Average Method:

This is also one of the defuzzification method to convert fuzzy set into crisp set or value.

Let us consider

fuzzy set A = {(0, 0), (1, 0.3), (4, 0.3), (5, 0)}
fuzzy set B = {(1, 0), (4, 0.5), (6, 0.5), (7, 0)}
fuzzy set C = {(5, 0), (6, 1), (7, 1), 8, 0}

→



Crisp value using weighted average method

$$x = \frac{\left(\frac{1+4}{2}\right)\times 0.3 + \left(\frac{4+6}{2}\right)\times 0.5 + \left(\frac{6+7}{2}\right)\times 1}{0.3+0.5+1}$$

$$\boxed{x = 5.146}$$

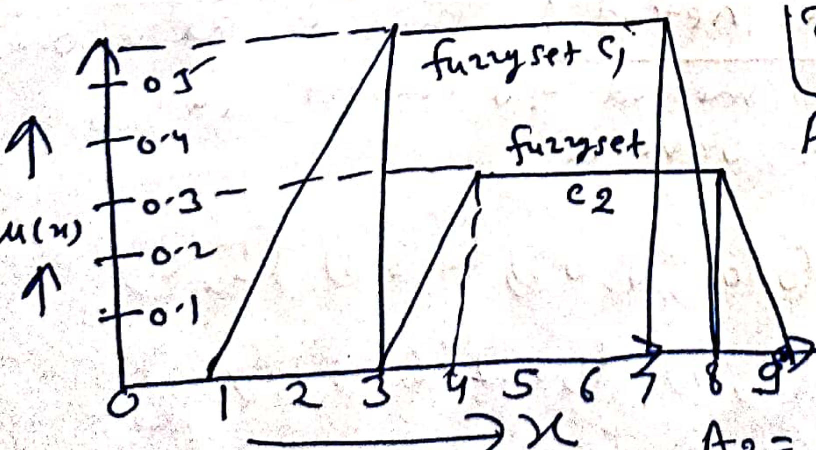
(4) Centroid method

(i) Centre of Sum (COS) method In this method, we calculate the sum of areas of individual fuzzy sets and then find the centre of their area.

Let us consider

$c_1 = \{(1,0), (3,0.5), (7,0.5), (8,0)\}$

$c_2 = \{(3,0), (4,0.3), (8,0.3), (9,0)\}$



$$\boxed{\text{Crisp value } (x) = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2}}$$

$A_1 = \frac{1}{2}\times[(3-1)\times 0.5] + (7-3)\times 0.5 + \frac{1}{2}\times(8-7)\times 0.5$

$A_1 = \underline{2.75}$

x_1 = centre of area $= \frac{7+3}{2} = \underline{5}$

$A_2 = \frac{1}{2}\times(4-3)\times 0.3 + (8-4)\times 0.3 + \frac{1}{2}\times(8-7)\times 0.3$

$A_2 = 1.5$

x_2 = centre of area $= \frac{8+4}{2} = 6$

So $x = \frac{2.75\times 5 + 1.5\times 6}{2.75+1.5} = \underline{5.35}$

Centre of largest Area

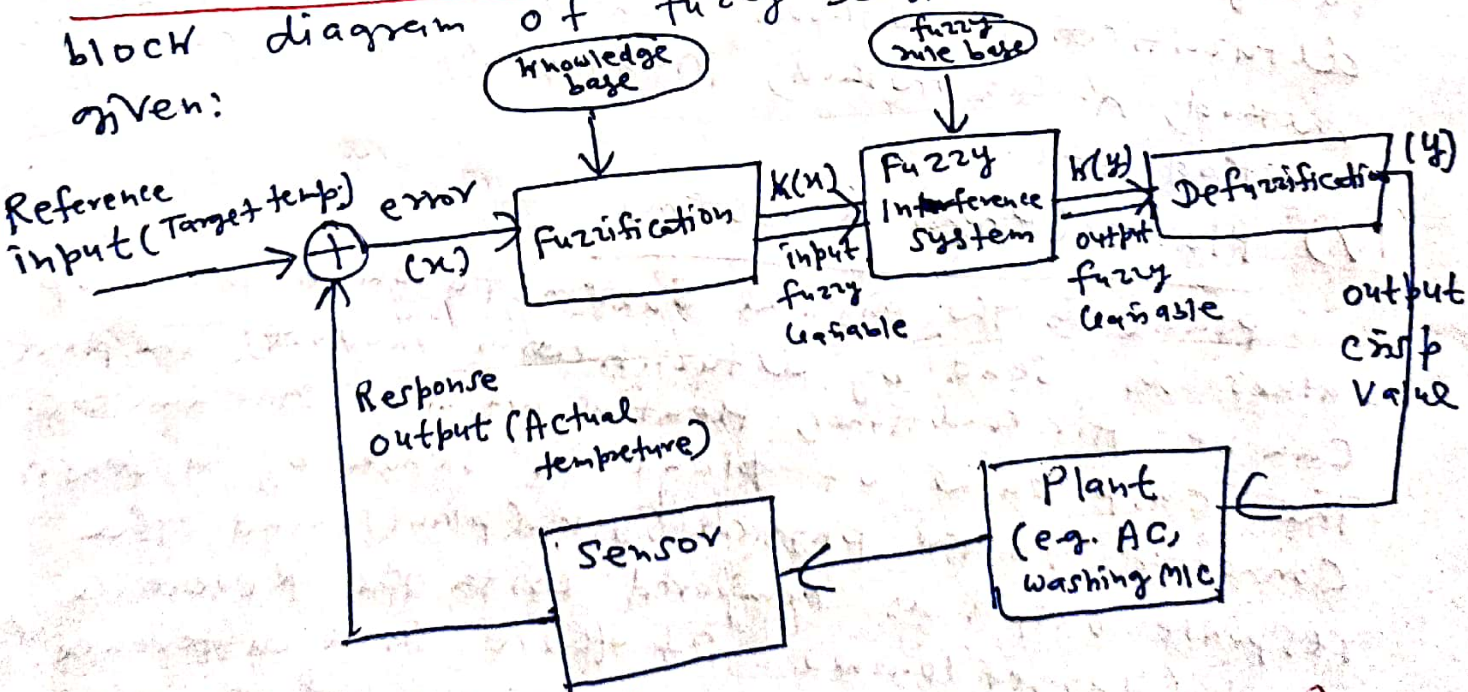
In this method, the defuzzified value is the mean value of the fuzzy set having the maximum or the largest area

$$x = \frac{7+3}{2.75} = \frac{10}{2.75} = 3.64$$

Fuzzy Logic Controller (FLC)

Fuzzy Logic Controller (FLC): is a system which is used to control the working ~~of~~ of a physical system (plant) with the help of fuzzy logic. Fuzzy logic controller is used to control washing machine, air conditioner, heater system, fan regulator, traffic control and braking system controller etc.

Block diagram of Fuzzy logic Controller The block diagram of fuzzy logic controller is as given:



Fuzzy Logic Controller (FLC)

Components of fuzzy Logic Controller (FLC)

(i) Fuzzification Module:

It Converts crisp values into fuzzy values with the help of knowledge base. Knowledge base uses membership functions to define the input variables into fuzzy variables.

(ii) Fuzzy Inference system (FIS)

Fuzzy Inference system consists of fuzzy rule base which takes fuzzy variables as inputs and generate possible fuzzy outputs, which is given as input to defuzzification module.

(iii) Defuzzification (Module)

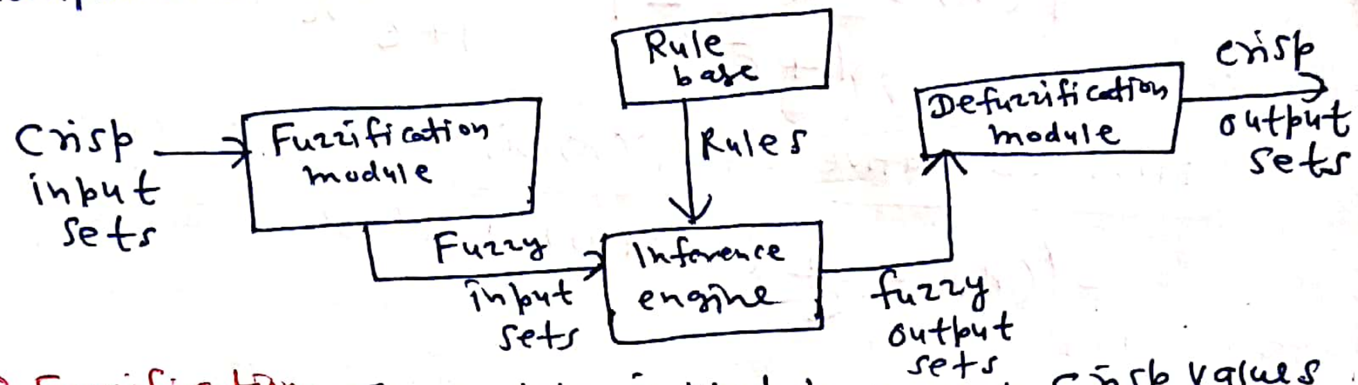
Defuzzification module is used to convert fuzzy values into crisp values using various defuzzification methods such as weighted average method, λ-cut method, Maxima method, centroid method etc.

(iv) Plant (Physical system)

Plant is the physical system to which the crisp values is send by defuzzification module. The plant can be Air conditioner, Heater, washing machine etc. The crisp value of plant is sensed by a sensor connected to the physical system (plant) and the response output is compared with the Reference input (Target value). If there is a difference between the target value & response output, then an error is generated and again sent to the fuzzification module & defuzzification module to minimize the difference. The process continue till the error is eliminated or becomes negligible.

Fuzzy Logic System (FLS) Fuzzy Logic System is a rule based system that rely on experience rather than knowledge.

Architecture of fuzzy Logic System: The various Components of FLS are as given:



① **Fuzzification Module.** This module is used to convert crisp values into fuzzy values using membership functions. This process of conversion is called fuzzification. It divides the crisp input sets into following 5 states in any fuzzy logic system (i) Large Positive (very cold) (ii) medium Positive (cold) (iii) Small (warm) (iv) Medium Negative (hot) (v) Large Negative (very hot).

② **Rule base:** This Component is used to store the set of rules and the if-then Conditions given by the experts are used for Controlling the decision making system
e.g. If (tempreture is cold OR very Cold) AND (target is WARM) then Command is heat)

③ **Inference Engine:** Processing is done in the Inference engine. It allow users to find matching degree between fuzzy inputs and rules. After matching degree, this system determines which rule is to be added according to given input. When all rules are fired, then they are Combined for developing the control action

④ **Defuzzification module:** This module takes the fuzzy output generated by inference engine as input and then transform them as crisp output set

This fuzzy logic system is used with Physical system (AC, washing machine, heater etc) to make Fuzzy Logic Controller

$$\boxed{\text{Fuzzy Logic System} + \text{Physical system (Plant)} = \text{Fuzzy Logic Controller}}$$

Applications of fuzzy systems

① Aerospace: In Aerospace, fuzzy logic is used in the following areas:

(a) Altitude Control of aircraft
(b) Satellite altitude Control
(c) flow and mixture regulation in aircraft vehicles.

② Automotive:

(a) Trainable fuzzy systems for idle speed control.
(b) Shift Scheduling method for automatic transmission
(c) Intelligent highway systems
(d) Traffic Control.
(e) Improving effeciency of automatic transmissions.

③ Business:

(a) Decision making support systems
(b) Personnel evaluation in a large Company.

④ Defence:

(a) Underwater target recognition
(b) Automatic target recognition of thermal Infrared images.
(c) Naval decision support aids.
(d) Control of a hypervelocity interceptor.
(e) fuzzy set modeling of NATO decision making.

⑤ Electronics:

(a) Control of automatic exposure in video Cameras
(b) Humidity in a Clean room.
(c) Air conditioning system
(d) Washing Machine timing
(e) Microwave ovens.
(f) Vacuum Cleaners

6 Finance:

(a) Banknote transfer Control
(b) Fund management
(c) Stock market Predictions

7 Industrial Sector:

(a) Cement kiln Control heat exchanger Control
(b) Activated sludge wastewater treatment process Control.
(c) Water Purification plant Control.
(d) Quantitative pattern analysis for Industrial quality assurance.
(e) Control of Constraint Satisfaction problem in Structural design.
(f) Control of water purification plants.

8 Manufacturing:

(a) optimization of cheese production
(b) optimization of milk production

9 Medical

(a) medical diagnostic support system
(b) Control of arterial pressure during anestesia
(c) multivariable Control of anesthesia.
(d) Radiology diagnosis
(e) Fuzzy inference diagnosis of diabetes and prostate cancer

10 Pattern Recognition and Classification.

(a) Fuzzy Logic based speech recognition
(b) Handwriting recognition
(c) facial Characterstic analysis
(d) Command analysis
(e) image search.