2. Speed of sound in air, and vibrations of a solid bar

A. Objectives

- Measure the speed of sound in air.
- Measure the "end correction" of a sound wave mode in an open-ended tube.
- Measure the three lowest longitudinal vibration mode frequencies of a solid metal bar, and deduce the speed of sound in the metal.
- Measure the four lowest transverse vibration mode frequencies of a solid metal bar, and deduce the speed of transverse vibrational waves at the four normal mode frequencies.

B. Equipment Required

1. Acrylic tube, stand, clamp, water jug, connecting tube, and valve
2. Tuning fork (Chose one whose frequency is different than those of your immediate neighbors.)
3. Dry erase marker
4. Meter stick
5. Breadboard
6. Mounting components for rubber bands and sound sensor (select as needed from bins)
7. Heavy rubber bands
8. Metal bar
9. Calipers (one or two shared by your section)
10. Sound sensor
11. Computer data acquisition system

C. Introduction

Young's modulus, bulk modulus, and Poisson ratio

Suppose that you take a cylinder of metal, with length ℓ and area *A*, place it into a hydraulic press, and apply a force *F* uniformly to the end faces of the cylinder. The force will cause the cylinder to deform, such that its length changes by $\Delta\ell$, as shown in Figure 2.1 (b). The *stress S* is defined as the force per unit area applied to the material; $S = F / A$. The *axial strain* ξ_a is defined as the fractional change in length $\xi_a = \Delta\ell / \ell$. Sometimes, we'll use the word "strain" by itself. This means "axial strain" unless otherwise specified.

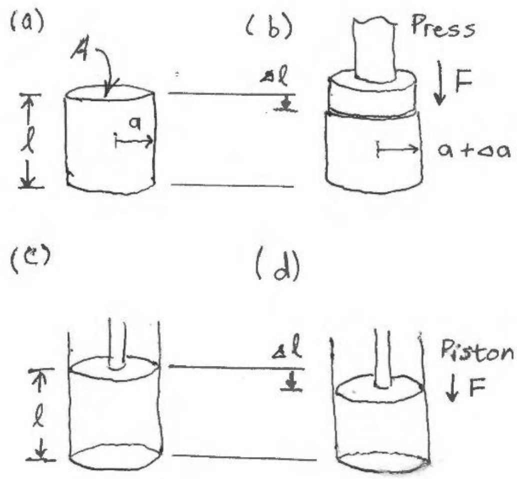

Figure 2.1. (a) Metal cylinder of length ℓ and area *A*. (b) Deformation of the cylinder under compression of its faces. (c) Piston of area *A*, filled with a fluid up to height ℓ. (d) With application of force *F*, the height of the fluid column changes by $\Delta\ell$.

When the force is removed, the material may or may not return to its initial size and shape. If it returns to its initial shape, the change is referred to as *elastic.* The maximum stress that a material can withstand without permanent change of shape is called its *elastic limit.* If the stress exceeds the elastic limit, the object will be permanently deformed. Deformations of metals are generally elastic as long as their strains are less than a few parts in 1000. In the following, we'll assume that all deformations are elastic.

For elastic deformations, the stress is proportional to the strain, according to the equation

$$S = E\xi_a \tag{2.1}$$

The constant of proportionality *E* is called *Young's modulus*, and depends on the material. For instance, for aluminum $E = 6.9\times10^{10}$ Pa. (Strictly speaking, equation (2.1) is valid only up to the *proportional limit* of a given material, which is a little smaller than the elastic limit.)

If we bond the surfaces of the press to the surfaces of the cylinder, we can consider both positive and negative stresses and strains. If we also focus on the force exerted by the material on the press, $F_e = -F$, we can rewrite equation (2.1) as

$$F_e = -\left(\frac{EA}{\ell}\right)\Delta\ell \tag{2.2}$$

This is just a special case of Hooke's law, with spring constant EA/ℓ.

Now consider a slightly different situation. A fluid is placed in a piston of area *A*, and fills the piston up to a height ℓ, as shown in Figure 2.1(c). (The following analysis also applies to a gas.) Then a force *F* is applied to the piston, which causes the pressure in the fluid to increase by

$\Delta P = F / A$. Also, the height of the fluid column will change by an amount $\Delta\ell,$ as shown in Figure 2.1(d).

We assume that the side walls of the piston are infinitely rigid, so that the change in the volume of the fluid is $\Delta V = A\Delta\ell.$ For pressures that are not too high, the fractional change in the volume of the fluid is proportional to the applied pressure change ΔP, according to the equation

$$\Delta P = -B\frac{\Delta V}{V} \tag{2.3}$$

The constant of proportionality B is called the *bulk modulus*, which depends on the material. For example, the bulk modulus of water is 2.2×10^9 Pa.

The bulk modulus and Young's modulus are not the same. In the press, the sides of the cylinder are free to expand. By contrast, the substance in the piston cannot expand laterally when compressed. Therefore, the fractional change in volume is $\Delta V / V = \Delta\ell / \ell$ in the piston, but not in the press.

There is no such thing as a Young's modulus for a liquid, since a liquid cannot hold its shape outside of a container. But we can define the bulk modulus for a solid. In order to measure the bulk modulus of a solid material, we can place an object made of that material into a fluid filled piston. When the pressure of the fluid is increased by an amount ΔP by the piston, the object will experience a uniform inward pressure increase on all of its surfaces from the fluid, and therefore will be compressed in all directions. The bulk modulus for the solid object is then also defined by equation (2.3).

When a solid cylinder is stressed along its symmetry axis as shown in Figure 2.1(b), its radius a increases to $a + \Delta a.$ The *transverse strain* ξ_t is defined as the fractional change in the dimension of the object in the direction perpendicular to the stress. In this case $\xi_t = \Delta a / a.$ The transverse strain has the opposite sign of the axial strain.

The negative of the ratio of the transverse to axial strain is a characteristic constant for a given material called the *Poisson ratio* $\nu = -\xi_t / \xi_a.$ For instance, for aluminum $\nu = 0.35.$ The following relationship between the Young's modulus, bulk modulus, and Poisson ratio will be satisfied for any material:

$$E = 3B(1-2\nu) \tag{2.4}$$

In practice, E and ν are easier to measure than B, so this equation provides a good way to determine B. The shear modulus, which we won't discuss here, can also be derived from Young's modulus and the Poisson ratio. Thus, Young's modulus and the Poisson ratio provide all the information on material properties needed to determine how a solid object deforms under applied forces, in the elastic limit.

Material	Mass density (kg/m^3)	Young's modulus (GPa = 10^9 Pa)	Poisson ratio
6061 aluminum alloy	2700	68.9	0.33
304 stainless steel	8000	193	0.29
Brass	8770	83.5	0.32

Table 2.1. Properties of metals used in this experiment.[1] The bulk modulus can be obtained from Young's modulus and the Poisson ratio with equation (2.4).

In this week's experiment, you will measure vibrations of a metal bar, which may be made of 6061 aluminum alloy, 304 stainless steel alloy, or brass. Properties of these materials are given in Table 2.1. [1]

Sound waves

Sound waves are waves of longitudinal displacement and of pressure in a material. This is illustrated in Figure 2.2 for the case of a single pulse of sound moving through a column of gas. We take z to be the coordinate along the length of the column, and focus our attention on a small volume element of gas at position z_1, in the absence of any disturbance. Figure 2.2(a) shows the state of the gas at a time t_1 before the pulse has arrived at location z_1. In the illustration, the dots represent gas molecules, and the box shows the volume element of the gas that we focus our attention on. Figure 2.2(b) shows the state of the gas at a time t_2, at which the leading edge of the pulse has reached z_1. At this time, our volume element of gas has been displaced to the right by a distance s. (Here, the volume element is defined as the boundary of a given set of gas molecules, so that it moves along with those molecules.) The element of gas has also been compressed to a smaller volume. If we generalize the displacement s to apply to a volume element of gas at arbitrary location z, then s is a function of both z and t, as illustrated in the graphs. Figure 2.2(c) shows the state of the gas at a time t_3, at which the peak of the pulse has reached z_1. At this time the displacement s of the gas element is a maximum, and the volume of the gas element has returned to its initial size. Finally, Figure 2.2(d) shows the state of the gas at

[1] Metal alloys with a numerical prefix, such as 6061 aluminum and 304 stainless steel, are alloys with precisely specified material composition, preparation, and properties. The properties given above for those two metals are the ones listed in www.matweb.com. “Brass” refers to any of a wide range of alloys of copper, zinc, and other elements, with a correspondingly wide range of properties. Since I couldn’t determine which brass alloy we have, I measured the density and Young’s modulus of the brass bars. The table gives my measured values, which are accurate to approximately 2%. I give a Poisson ratio in the middle of the range 0.30 to 0.34 that is typical for brass alloys.

a time t_4, at which the trailing edge of the pulse has reached z_1. At this time the displacement *s* has become smaller than the peak displacement, and the volume of the gas has expanded. This corresponds to *rarefaction* (opposite of compression) of the gas, and to a pressure decrease.

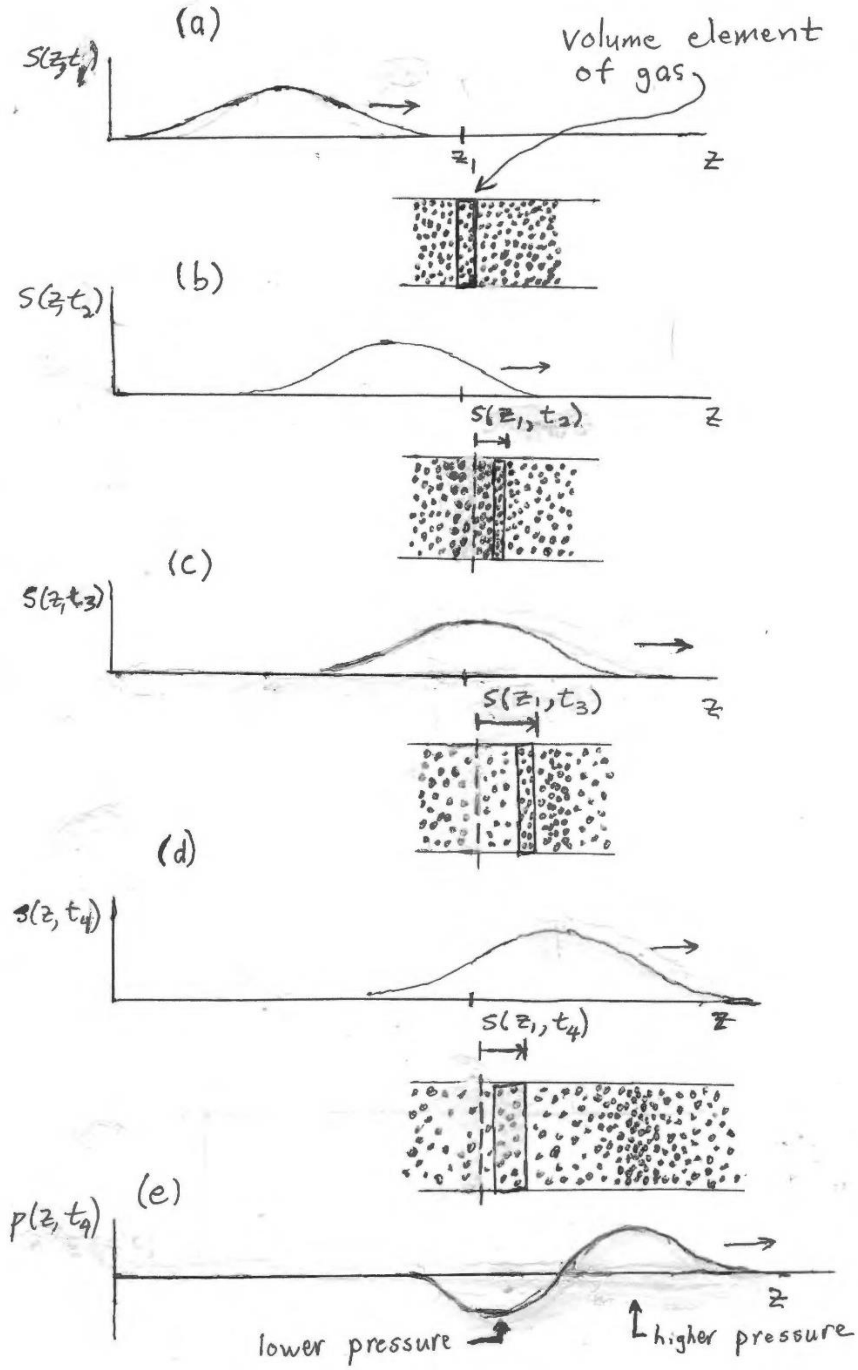

Figure 2.2. Illustration of a pulse of sound moving through a gas. This pulse consists of a single burst of positive displacement *s*. (More often, displacements in a sound wave are both positive and negative.) After the pulse passes completely by z_1, the volume element of gas returns to its equilibrium position and size as shown in (a).

The *ambient pressure* P_0 of the gas is the pressure in the absence of any disturbance (normally $1 \text{ atm} = 1.013\times 10^5 \text{ Pa for air.)}$ The total pressure P is the sum of the ambient pressure plus a change in pressure p due to the effect of the sound wave.

$$P(z,t) = P_0 + p(z,t) \tag{2.5}$$

It is the change in pressure p that exhibits wave motion. For ordinary sound waves in air, p is very tiny compared to P_0.

There is a definite relation between the pressure change and the displacement of a sound wave:

$$p = -B\frac{\partial s}{\partial z} \tag{2.6}$$

where B is the bulk modulus. Thus, it is easy to determine the pressure change p from the displacement s, and vice-versa. For instance, Figure 2.2(e) shows the pressure $p(z,t)$ at time t_4, obtained by differentiating $s(z,t)$ at time t_4. For this particular pulse, the pressure is increased in the leading edge of the pulse, and decreased in the trailing edge of the pulse.

If the sound wave propagates in a liquid such as water, the bulk modulus B is the same as the bulk modulus we'd measure in a piston experiment as shown in Figure 2.1. However, if a sound wave propagates in a gas, there is one more point that needs to be considered. When a gas is compressed, its temperature tends to increase. But the gas will also tend to cool back to the ambient temperature by heat conduction. There are two possible limits for the compression. If the compression is very slow, the heat conduction keeps the gas at nearly the same temperature during the compression. Such compressions are *isothermal.* If the compression is very fast, there won't be enough time for a significant amount of heat to be conducted out of the gas, and the temperature of the gas will increase in a predictable way. Such compressions are *adiabatic.* The measured bulk modulus is different in these two limits – *isothermal bulk modulus* for slow compression, and *adiabatic bulk modulus* for fast compression. The difference is large for a gas, but very small for a liquid or solid.

So, for sound waves in a gas, we need to pay attention to the time scale of the compressions and rarefactions. Almost always, these occur too quickly for heat to be conducted, and so the compressions and rarefactions are adiabatic. Thus, for sound waves in a gas, the correct bulk modulus is the adiabatic bulk modulus, which turns out to be

$$B(\text{gas}) = \gamma P_0 \tag{2.7}$$

where P_0 is the ambient pressure of the gas, and γ is the *adiabatic index*. For air at room temperature, $\gamma = 1.40.$ (You'll learn more about this in Phys. 369.)

The wave speed for a string under tension is $\sqrt{T / \rho_L}$, where *T* is the tension of the string, and ρ_L is its mass per unit length. This illustrates a general characteristic of vibration phenomena: the speed of the process is proportional to the square root of the restoring force, and inversely proportional to square root of the inertia (or mass) of the vibrating object. *Stiffer* $\rightarrow$ *faster* and *lighter* $\rightarrow$ *faster.* For waves in solids, the restoring forces are proportional to the bulk modulus *B*, and the inertia is proportional to the mass density ρ (mass per unit volume). Thus we expect that the wave speed should be proportional to $\sqrt{B / \rho}$. A careful analysis shows that the *speed of sound c* is in fact equal to $\sqrt{B / \rho}$;

$$c = \sqrt{\frac{B}{\rho}} \qquad \begin{pmatrix} \text{sound wave speed in a gas or fluid, or in bulk} \\ \text{solid material in the short wavelength limit} \end{pmatrix} \qquad (2.8)$$

Sound wave modes of air-filled pipes.

Consider an air-filled pipe, closed on one end, and open on the other. Such pipes are important as components of musical instruments. This pipe can support *sound wave normal modes,* as illustrated in Figure 2.3. The normal mode oscillations resemble those that you studied for the clamped string in week 1. However, the oscillations are longitudinal rather than transverse, and the boundary conditions are a little different. One boundary condition is that *there must be a node in the displacement at the closed end.* That is because the closed end is an impenetrable barrier for the gas, so the displacement at that end must be zero. This implies that if we measure distance *z* from the closed end, the displacement near the closed end must be $s(z,t) = s_0 \sin(kz)\cos(\omega t)$. From equation (2.6), we then deduce that the pressure near the closed end must be $p(z,t) = -Bks_0 \cos(kz)\cos(\omega t)$. From the factor $\cos(kz)$, we see that *there must be an antinode in the pressure at the closed end.*

To a first approximation, there is a node in the pressure at the open end. This is due to the fact that there is a large volume of gas just outside the open end of the tube at ambient pressure. Going from the open tube end into this large volume, the pressure change quickly drops to zero, since there is not that much air inside the tube pushing outward, relative to the quantity of air outside the tube. Then, by the analogous argument we made for the closed end, it follows that *to a first approximation, there is an antinode in the displacement at the open end.*

This open-end boundary condition is not precise, since there is a finite transition region between the inside and outside of the tube. Thus, the *apparent* position of the pressure node is a little bit outside the tube. This difference in position between the physical end of the tube and the apparent node position is referred to as an *end correction.*

A normal mode for sound waves in a tube is shown in Figure 2.3. The condition for resonance is that there must be an odd integer multiple of quarter wavelengths between the closed end and the "apparent" position of the open end, *i.e.* that

$$L + L_E = n\frac{\lambda}{4}, \qquad n = 1,3,5,7,..... \tag{2.9}$$

where L is the physical length of the tube, and L_E is the end correction. As always, the relation between wavelength λ, frequency f (in Hz), and wave speed c is

$$c = f\lambda \tag{2.10}$$

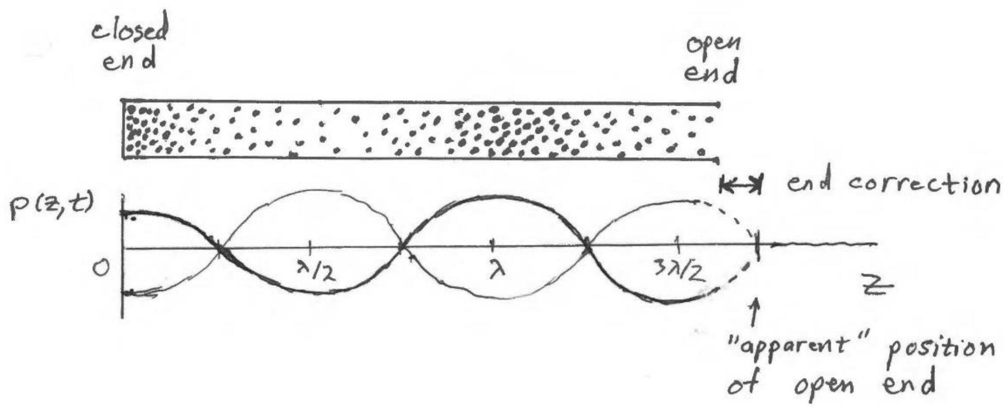

Figure 2.3. Sound wave normal mode of an air column in a tube with one end open and one end closed.

Longitudinal vibrations of a solid bar

Solids can also support sound waves with speed $c = \sqrt{B/\rho}$. However, such waves exist *only* if the size of the object is large compared to the wavelength in all directions. For instance, sound waves in the earth usually obey this condition, as do waves in an ultrasonic imaging device.

The closest analog to the air column discussed above is a longitudinal vibration of a long, thin rod or bar. Again, the wave speed is proportional to the square root of the restoring force over the inertia. Since the force on a small, solid cylinder is proportional to Young's modulus E rather than to the bulk modulus B, we'd naively expect that longitudinal vibrations for a thin solid bar would have wave speed

$$c = \sqrt{\frac{E}{\rho}} \qquad \text{(longitudinal wave, thin solid bar or rod)} \tag{2.11}$$

This turns out to be the correct longitudinal wave speed for a rod or bar, provided that the width of the rod or bar is small compared to a wavelength. In practice the difference between the limiting wave speeds $\sqrt{E/\rho}$ (bar thin compared to wavelength) and $\sqrt{B/\rho}$ (object large compared to wavelength in all directions) is of the order of 10% or less.

In this lab, we'll study the longitudinal vibrations of a solid bar that is nearly free – the only contact with the bar will be rubber band supports, which have very little effect on its vibration. For a bar with such "free end" boundary conditions, *there is a node in the pressure at the bar ends.* This is due to the fact that there is essentially zero applied force to the bar ends. By an argument similar to the one we made for air columns, this means that *there is an antinode in the displacement at the bar ends.* This means that the longitudinal normal modes of the bar will look like the one shown in Figure 2.4. By inspection of the figure, we see that the condition for longitudinal resonance is that the bar must be an integer number of half-wavelengths in length,

$$L = n\frac{\lambda}{2}, \qquad n = 1, 2, 3, ... \tag{2.12}$$

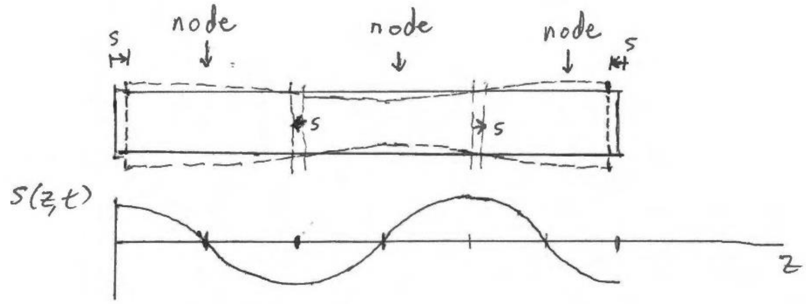

Figure 2.4. Third longitudinal vibration mode of a solid, free bar. The solid lines show the equilibrium shape of the bar, and the dashed lines show the shape of the bar when the displacement nears a maximum, as shown in the graph of $s(z,t)$.

Other vibration modes of a solid bar

A solid bar can vibrate in many other ways, some of which are shown in Figure 2.5. In fact, the vibration types shown in this figure do not exhaust the possibilities. For instance, some modes exhibit a combination of two or more of the motions shown in Figure 2.5. The full theory of these vibrations is very complicated. Generally speaking, analytical expressions for normal mode frequencies are available only in certain limiting cases. In practice, it is often necessary to resort to finite-element computation to solve the normal mode problem for vibrations of solid objects.

In this lab, we will study one more of the vibration modes – the transverse vibrations shown in Figure 2.5(b) and (c). We will not discuss the theory of these vibrations, except to state that the normal mode frequencies of the bar are given by

$$f_n = \frac{0.113h}{L^2}\sqrt{\frac{E}{\rho}}\left(2n+1\right)^2, \qquad n = 1, 2, 3, ... \tag{2.13}$$

where *L* is the length of the bar, and *h* its thickness in the direction of vibration. It turns out that the wavelengths of the modes are not simple rational multiples of the bar length *L* as we've seen before, but they can be calculated. The wavelengths of the first four transverse modes of the bar are given in Table 2.2. The derivation of equation (2.13) and the wavelengths in Table 2.2 can be found in *Laboratory observation of elastic waves in solids,* T. D. Rossing and D. A. Russell, Am. J. Phys. **58**, 1153 (1990). A copy of this paper is posted on the bulletin board near the storeroom.

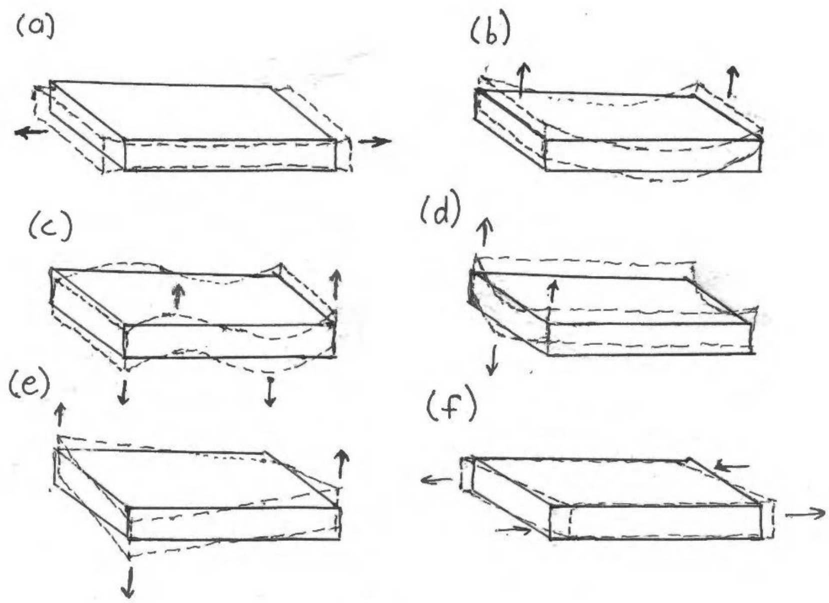

Figure 2.5. Some vibration modes of a solid, free bar. (a) Lowest longitudinal vibration mode. (b) Lowest transverse vibration mode along the length of the bar. (c) Second lowest transverse vibration mode along the length of the bar. (d) Lowest transverse vibration mode along the width of the bar. (e) A torsional vibration mode. (f) An in-plane shear vibration mode. In each drawing, the solid lines show the equilibrium shape of the bar, and the dashed lines show the deformed shape of the bar as it approaches a maximum displacement. The arrows show the direction of the displacement at that particular time.

Mode number	Wavelength
1	1.330 *L*
2	0.800 *L*
3	0.572 *L*
4	0.455 *L*

Table 2.2. Wavelengths of the first four transverse vibration modes of a long, thin bar of length *L*.

Vibrations of solid objects are important in many practical situations. They form the basis of seismology, an important tool for geology and oil exploration. Vibrating bars are components of musical instruments, such as xylophones. Resonant vibrations of piezoelectric quartz crystals are widely used to make accurate clocks. Ultrasonic vibrations can be used to make imaging devices that can "see" inside solid materials. Mechanical components always vibrate to some degree, and this can be an important limitation in laser-optical experiments or tunneling microscopy experiments in condensed matter physics. In these kinds of experiments it is crucial to have a physical understanding of the vibrations in order to know how to minimize them. In some cases, such as in gravity wave detectors, vibration of a large mass can be a desired signal. This lab and the next one provide an experimental introduction into this important topic.

D. Experimental procedure

Speed of sound in air

The concept of the first part of the experiment is shown in Figure 2.6. A tube contains an air column which sustains normal sound wave modes. We seek to measure a number of modes in the same tube, with the frequency held fixed, by varying the length of the tube. The variation of length is accomplished by filling the bottom of the tube with water, as shown in Figure 2.7. The water surface forms the "closed" end of the tube, with a height that can be changed by adding more water or draining it out. We will use a tuning fork held above the open end of the tube to excite the modes, and detect mode excitation by listening for it. Measurement of the water level differences between successive modes allows the half-wavelength to be measured. This and the known tuning fork frequency allow the speed of sound to be determined.

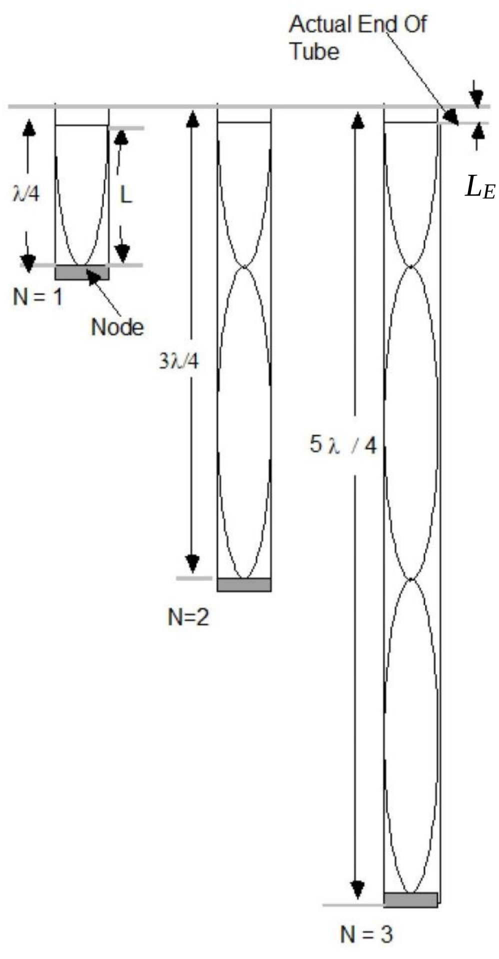

Figure 2.6. Vibration modes found as the length of the air column is increased, with the driving frequency held fixed. The curves show air displacement *vs*. position in the air column. (Note that the displacement is longitudinal – *i.e.* along the symmetry axis of the column.) N in the figure denotes the mode number, and is not the same quantity as n in equation (2.9). L_E is the end correction.

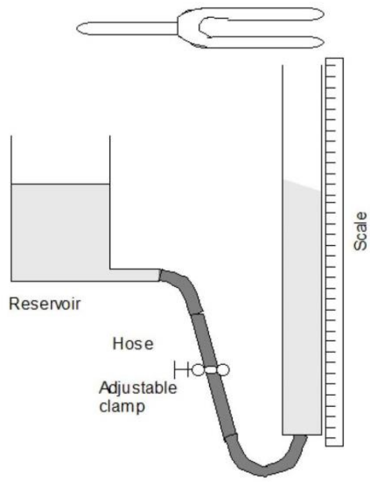

Figure 2.7. The variable length air column is obtained by regulation of the water level in a vertical acrylic tube.

Your equipment this week includes one tuning fork. The available tuning forks have different frequencies, ranging from 512 Hz to 1026 Hz. Each fork has its measured vibration frequency in Hz stamped on it. The error in the frequency (stnd. deviation) is about 1 part in 1,000, *i.e.* ± 0.5 Hz for the 512 Hz frequency forks up to ± 1 Hz for the 1026 Hz frequency forks. You should choose a tuning fork with a frequency that is different from the frequencies of your immediate neighbors'. The reason is that you will determine when a resonance occurs by listening for it, and it is easy to confuse the resonance of your tube with the resonance of your neighbor's tube if their frequencies are the same. If you see a tuning fork with a frequency of less than 512 Hz, don't use that. These lower frequency tuning forks will give too few resonances. To get the tuning fork vibrating, you'll need to strike it on a rubbery surface, such as your shoe, a mouse pad, or a rubber mallet. Don't strike it on something hard because you'll mostly excite high harmonics rather than the fundamental mode of the fork, and possibly damage the fork. Even if you strike it on a rubbery surface, the sound you hear might be dominated by the harmonics. However the fundamental will be vibrating also, and you should easily hear the fundamental tone when the tube hits resonance. You'll need to repeat the excitation of the fork every few seconds while making measurements, so find the most convenient method to do that.

To begin the experiment, secure your column to your lab bench with the C-clamp. Choose a position well away from the computer keyboard and monitor to minimize changes of damaging that equipment with a water spill. Fill your water jug partway (something like ¼ full to start). It will be most convenient to do this using the plastic pitchers near the sink to transfer water from the sink. Connect your jug via the tubing and valve to the bottom of the plexiglass tube as shown in Figure 2.7. Set the jug on the top of the mid-section of the bench. **Do not set the jug on top of any electronics or electrical outlets, and use caution that you do not spill any water onto electronics or electrical outlets.** You will fill the column by opening the valve with the jug in this elevated position. You will empty the column by setting the jug on the floor.

You are aiming to have enough water to fill the column to within about 5 cm of the top of the column. If you can't do that, add more water until you can. But, don't put in so much water that it spills over the top of the column.

Next, start with the water column empty. Strike the tuning fork as necessary to keep it vibrating, and hold it next to the tube opening. Then let water into the column and listen for the tube resonances as the water level rises. These resonances should be audible as an increased volume. It is possible that you'll start out going too fast, and miss resonances at first. You can slow the rate of change in the water level by partially closing the valve. Practice with the tuning fork and water until you can identify the resonances at different water levels. Practice holding and striking the tuning fork to give the most audible resonances, and position your ear close to the top of your tube. If you are in doubt about whether a resonance was from your tube or someone else's, go back and make sure your apparent resonance is repeatable.

Once you've got it working, one person should work the tuning fork, and the other should mark the positions of the resonances with the dry-erase marker. The marks can be erased with a paper towel if necessary. **Use the dry-erase marker, not a permanent marker,** so that you don't leave permanent marks on the tube. Record the position of each resonance (distances from the open end of the tube $L_1, L_3, ...$) in a table like Table 2.3. In this table, the subscript on L_n refers to the integer *n* in equation (2.9). Take at least two measurements of each position with the water level rising, and two with the water level falling. Take the result for each position L_n to be the average of these measurements. Averaging over the two flow directions will minimize the error associated with the observer's time lag between hearing the resonance and making the mark.

The differences between adjacent L_n values is one-half wavelength, $\lambda / 2$. You 'll want to use your measured values for L_n to determine the half wavelength to the best precision possible. A simple average of nearest-neighbor differences $\frac{(L_7 - L_5) + (L_5 - L_3) + (L_3 - L_1)}{3}$ might seem like a good way to go. But the problem with this formula is that only the measurements of the highest and lowest L_n values contribute to the result – you wouldn't be using your middle measurements at all, which would lead to lower accuracy. A better way to do your analysis is to find the best fit straight line to the points $(1, L_1), (2, L_3), (3, L_5), ...$ The slope of this line will make use of all of your measurements, and give the best estimate for the average distance between adjacent L_n.

<u>Using your measured wavelength and the known tuning fork frequency, determine the speed of sound in air. Also, from your measured wavelength and the positions L_n, determine the end correction. Express your result for the end correction both in centimeters and in units of the inner diameter of the tube.</u>

Tuning fork frequency f = ________________ (read from tuning fork)

Room temperature T = _________________ (read from thermometer near door)

Inside diameter of tube = ________________

	Moving up			Moving down			Average
	Trial 1	Trial 2	…	Trial 1	Trial 2	…	
L_1							
L_3							
L_5							
L_7							

Half wavelength $\frac{\lambda}{2}$ = ______________________

End correction L_E = _______________ End correction in units of tube diameter = ___________

Speed of sound in air at measured room temperature c = _______________________________

Table 2.3. Sample table for recording of data for the speed of sound in air. Depending on your tuning fork frequency, you might have more or fewer than the four resonances shown in the table.

For quantities measured near room temperature and atmospheric pressure, it is conventional to quote the measured result at *standard temperature and pressure* (STP). For physicists, this is a temperature of exactly $20\ °\mathrm{C}$ and pressure of exactly 1 atm (101.325 kPa). Thus, if the temperature and pressure in the lab are different than STP, you would need to make corrections, using measured or calculated temperature and pressure dependence, before reporting your results in a scientific article.

Since the departure of your pressure and temperature from STP is small, it is sufficient to use theoretical T and P dependence to make corrections. In the introduction, we saw that the speed of sound in air is $c = \sqrt{\frac{\gamma P_0}{\rho}}$. In Phys. 369, you'll learn that air is a nearly ideal gas, and that for an ideal gas $P_0 = nk_B T$, where n is the number of molecules per unit volume, k_B is Boltzmann's constant, and T is the absolute temperature. In SI units, absolute temperature is measured in Kelvin (K). The conversion between Kelvin and degrees Celsius is $T(K) = T(°\mathrm{C}) + 273.15$. The mass density ρ is also proportional to n. Thus, we have

$$c = \sqrt{\frac{\gamma P_0}{\rho}} \propto \sqrt{\frac{nT}{n}} \propto \sqrt{T} \qquad (2.14)$$

So, to a first approximation, the speed of sound in air is proportional to the square root of the absolute temperature and independent of pressure. So, to convert your result to the speed of sound at STP, you need to correct for temperature only. From equation (2.14), the formula to convert a measurement of the speed of sound $c(T)$ at temperature T (measured in degrees Celsius) to the speed of sound at STP is

$$c(20\ {}^\circ\mathrm{C}) = c(T)\sqrt{\frac{293.15}{273.15+T}} \qquad (2.15)$$

Using equation (2.15), correct your result to provide a measured speed of sound at standard temperature $c(20\ {}^\circ\mathrm{C})$.

$c(20\ {}^\circ\mathrm{C}) =$ ____________________________

Compare your result to the accepted value

$$c(20\ {}^\circ\mathrm{C}, \text{dry air, 1 atm pressure}) = 343.4\ \mathrm{m/s}. \qquad (2.16)$$

Does your result agree with the accepted value to within your error? If it doesn't, a possible reason is underestimation of experimental error. If your result were very precise, it might also be necessary to take humidity into account. The speed of sound increases with humidity, and is 344.5 m/s at 100% relative humidity and STP.

Also, compare your result to the theoretical value $\sqrt{\gamma P_0 / \rho}$. (You'll need to look up the mass density of air.)

Vibrations of a metal bar

In this part of the lab, you will study the vibrations of a metal bar. The apparatus is illustrated in Figure 2.8. All components are mounted on a 6" x 48" optical breadboard with tapped holes on 1 inch centers. The metal bar is between 26" and 40" long, between 1 and 3 inches wide, and made of brass, 304 stainless steel, or 6061 aluminum alloy. All the bars are different in some way, and are numbered. The bars are supported by heavy rubber bands stretched between posts. The rubber bands provide support for the bars without having much effect on their vibrational modes. The vibration of the bar in such a mount is a good approximation to a completely free vibration, as illustrated in Figure 2.5.

To observe the vibrations of the bar, you will position a Pasco sound sensor very close to a surface of the bar. Vibration of the bar surface is transmitted to the surrounding air, and picked up from the air by the sensor. To observe the longitudinal vibrations, this sensor should be placed very near one end of the bar (~ 1 mm gap to sensor), as shown in Figure 2.8(b). That is because only the end faces of the bar have an appreciable motion for these modes, so the sensor will pick up the largest signal in this position. The sensor would pick up very little signal from these modes if placed in other positions. To observe the transverse vibrations, this sensor should be placed very near the top surface of the bar and close to one end, as shown in Figure 2.8(c). That is because the ends of the bar vibrate in the vertical direction for these modes.

To excite the modes, you will tap the bar with a small hammer or other object. A plastic or wood-tipped object works well. In order to excite longitudinal modes, the hammer must strike the end of the bar so as to provide an impact in the longitudinal direction, as shown in Figure 2.8(b). On the other hand, if you want to excite a transverse vibration, you need to provide an impact in the transverse direction as shown in Figure 2.8(c).

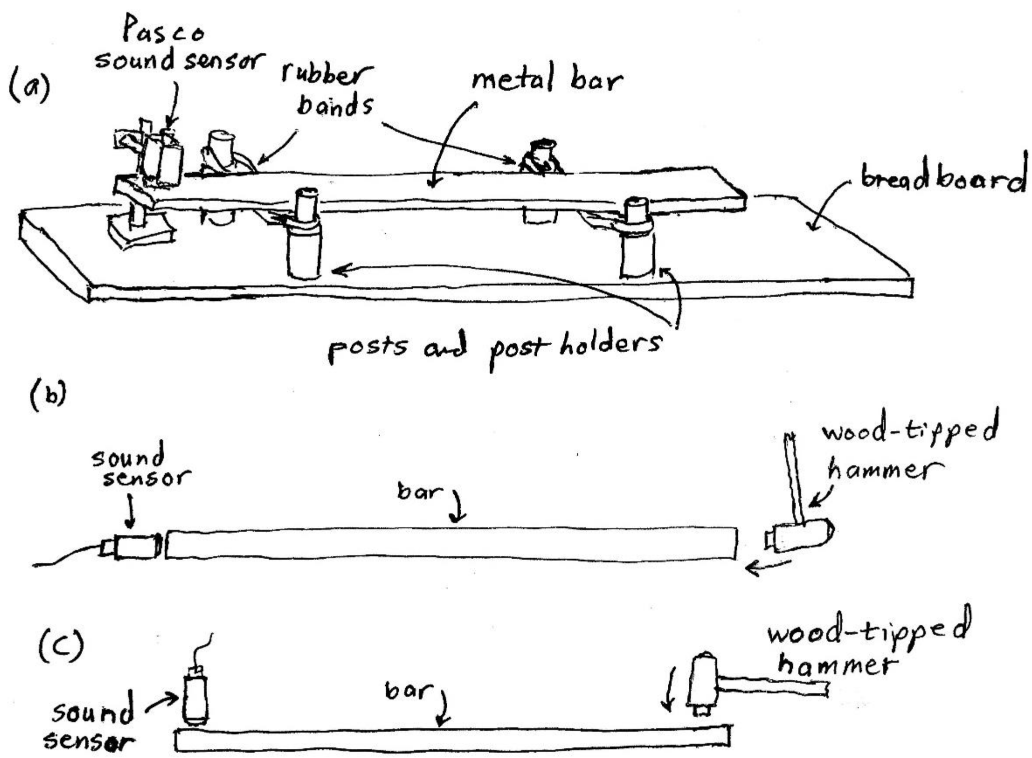

Figure 2.8. Apparatus for measuring the vibrations of a metal bar. (a) General arrangement of the components. (b) Arrangement for the study of longitudinal modes (side view). (c) Arrangement for the study of transverse modes (side view).

Mounting components

In this lab, you'll select mounting components from the bins on the side tables. These are the same components that are used in many research labs throughout RLM, and you'll use them for a number of labs this semester. For the first few of these experiments, we'll give a suggested arrangement for each application. However, you're not required to use our suggestion, and can use any arrangement you prefer provided that

- your arrangement holds the experimental components securely (except for the bar),
- your arrangement provides the flexibility to adjust the positions of your components as needed, and
- your arrangement does not damage any of the components.

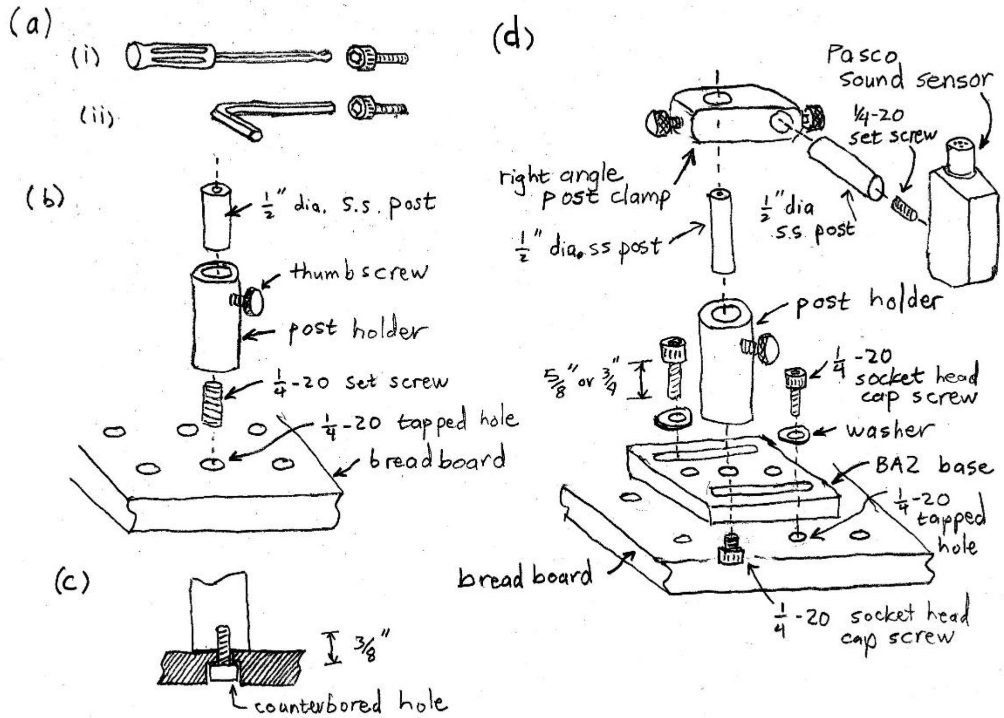

Figure 2.9. (a) (i) Ball driver and socket head cap screw. (ii) Hex key wrench (also called an Allen wrench) and socket head cap screw. (b) Possible arrangement to mount post for rubber band. (c) Cross-sectional view of a component held in place with a socket head cap screw in a counterbored hole. (d) Possible arrangement for mounting the Pasco sound sensor in this experiment. In this arrangement, the post holder is secured to a countersunk hole in the BA2 base as illustrated in Fig. (c).

One component we'll use is the *socket-head cap screw*, depicted in Figure 2.9(a). These screws have a head with a hexagonal socket. You can turn these screws with either the *ball driver*, or one of the *hex key wrenches* that is provided with your kit, as illustrated in Figure 2.9(a). You'll mostly use "1/4-20" screws. The ¼ indicates that the screw threads will just fit through a ¼" diameter hole, and the 20 means that the screw has 20 threads per inch. The other screw size you'll often use this semester is an "8-32" screw. The number 8 indicates the thread diameter (not in any particular units) and 32 means that the screw has 32 threads per inch. A *set screw* is a screw that has the threaded part only, without any head. These have a hexagonal hole on one end that can be turned with a hex key wrench. A *tapped hole* is a hole that has been cut with threads that match a particular screw. For instance a "¼ -20 tapped hole" has the threads that match the ¼ - 20 screw.

Your kit also contains ½" diameter stainless steel posts, from 2" to 6" in length, and post holders. The posts will slide into the post holders as illustrated in Figure 2.9(b). You may need to unscrew the thumbscrew to allow the post to slide in. You can secure the post in the holder by tightening the thumbscrew. **Do not replace the thumbscrews in the post mounts with the steel cap screws**, since the steel screws can damage the posts, and the posts will then get stuck into the holders.

The posts have a tapped 8-32 hole in one end and a tapped ¼-20 hole in the other ends. This will allow you to secure components to the end of the post. For example, in this lab you can secure the Pasco sound sensor to a post by screwing one end of a ¼-20 set screw into the tapped hole in the sensor, and the other end into a post, as illustrated in Figure 2.9(c).

One method of securing a post-holder to a breadboard is shown in Figure 2.9(b). You first screw one end of a ¼-20 set screw into the threaded hole in the bottom of the post-holder. Then, you screw the other end of the set-screw into the ¼-20 threaded hole in the breadboard, and turn the post-holder by hand until it is tight. This is the suggested method to provide a mount for your rubber bands in this lab. Just secure a post into the post-holder, and pull your rubber band over the post.

Sometimes, it is desirable to have some adjustment in the position of post. One way to do this is with a Thorlabs model BA-2 base, as illustrated in Figure 2.9(d). The three holes down the center of this base are "counterbored." This means that the holes have a bigger diameter on one side than on the other, such that it is possible for the head of a screw to drop into the larger diameter side, as illustrated in Figure 2.9(c). To use these, you put a ¼-20 socket head cap screw into the larger diameter side of one hole, and then screw that into the tapped hole at the bottom of a post holder and tighten it. (A screw with a 3/8" length of thread works well for this purpose.) This secures the post-holder to the base. Then you can position one of the slots in the base above a tapped hole in the breadboard, and secure the base to the breadboard with another ¼-20 cap screw. (For this purpose, screws with 5/8" or ¾" of thread length work well.) The advantage of this method of mounting is that you can slide the base back and forth along the slots. Whenever you use a screw like this, it is good practice to use a washer as shown in Figure 2.9(d). This distributes the mechanical load of the screw over the load-bearing surface, so that you don't dent or mar that surface. If you need maximum stability of the mount, you should use one screw in each slot as shown in Figure 2.9(d). For less critical applications a single screw in one slot is adequate.

To complete the mounting of the Pasco sound sensor, we suggest that you place a post in the post mount, then secure a right angle clamp to that post, and finally to secure the post on the sound sensor into the other hole in the right angle clamp as shown in Figure 2.9(d). This provides a very flexible mount. For instance, you can move the mount sideways in both directions, or up and down. You can also position the sound sensor to point in the vertical or horizontal direction. When you have the mount in the position you want, you can tighten the thumbscrews by hand to keep it there.

Measurement of lowest longitudinal modes (sound wave in metal)

<u>Measure the length of your bar with a meter stick, and measure the width and thickness of the bar with a calipers</u>. (The class will share one or two calipers, and you can borrow one from your instructor.) Check the width and thickness in several places, and use the average of those measurements if they are different. **Each bar is numbered. Record the bar number you are using, and include that number in your lab report.**

Set up the bar and sound sensor as shown in Figure 2.8(b). Start Pasco Capstone, and set it up to display the sound sensor signal in an FFT window. (As before, it is best to use this in "Fast Monitor Mode".) **Set the window to display frequencies from 0 to 10 kHz**. Tap one end of the bar, and observe the signal. You should see peaks belonging to various vibration modes.

See if you can identify the longitudinal mode peaks. It is easy in this lab to be confused about which modes are transverse and which are longitudinal. You will be able to see both longitudinal and transverse modes in your spectrum, even though you are set up to preferentially observe longitudinal modes as in Figure 2.8(b). Some ways to tell the difference:

(i) The longitudinal modes will be somewhat more pronounced when you are set up as in Figure 2.8(b), and the transverse will be somewhat more pronounced when you are set up as in Figure 2.8(c).

(ii) The longitudinal modes will occur at significantly higher frequencies than the transverse modes. This is the reason for the advice to start with a 0 to 10 kHz frequency scale.

(iii) The longitudinal modes will have evenly spaced frequencies $f_n = nf_1$, with f_1 the fundamental mode frequency. The transverse modes won't have evenly spaced frequencies.

If you're still unsure which modes are the longitudinal modes, estimate the fundamental mode frequency with equation (2.10), equation (2.12), and a rough estimate that the speed of sound $\left(\sqrt{E/\rho}\right)$ should lie between about 3,000 and 4,000 m/s for brass, and between about 4,000 and 6,000 m/s for stainless steel and aluminum.

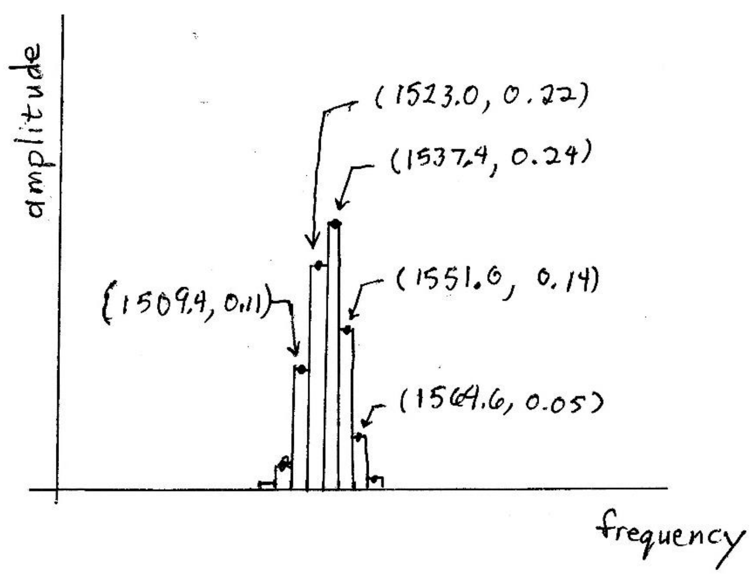

Figure 2.10. Example of a peak you might observe in the FFT window in Capstone.

You will use the FFT to measure the mode frequencies. You may find it convenient to use the measurement tool in the FFT window. To obtain the best accuracy, you will want to get the peaks as narrow as you can. Also, you should not assume that the frequency of the data point with the highest amplitude is the true mode frequency. For example, suppose one of your peaks is the one shown in Figure 2.10. The point with the largest amplitude occurs at a frequency of 1537.4 Hz. However, it would be wrong to state that you measured a frequency of 1537.4 Hz. That is because this statement incorrectly implies an uncertainty of the order of 0.1 Hz. This uncertainty is much too small, because the data points in this example are 14.4 Hz apart, and the true frequency is very unlikely to occur within 0.1 Hz of any given point. In this example, it's clear that the true peak lies in between 1537.4 Hz and the next lower frequency point at 1523.0 Hz, since that one has a larger amplitude than the next higher frequency point. But the frequency is clearly closer to 1537.4 Hz than to 1523.0 Hz. A reasonable estimate for this example would be to say that it's somewhere between 60% and 80% of the way between 1523.0 and 1537.4, *i.e.* at $1533 \pm 2\text{ Hz}$. An even better method would be to find the average frequency of the data points within one peak, with weight factors given by the intensities (*i.e.* the square of the amplitude).

<u>Using the FFT display, measure the frequency of the fundamental mode and at least the two lowest harmonic longitudinal modes of vibration. From your measured frequencies and bar length, and the equations in the introduction, calculate the speed of sound at each frequency</u>.

Address the following questions:

- Are the speeds of sound the same, to within experimental error? If not, can you think of reasons why they would not be the same?

- How does your measured speed compare to the expected value $\sqrt{E/\rho}$? To the speed of sound in bulk metal $\sqrt{B/\rho}$? You can also compare your measured sound speed to those given in the table tacked on to the bulletin board near the class storeroom.

Observation of lowest transverse modes

Set up the experiment to study transverse modes as shown in Figure 2.8(c). Again, find the modes by striking the bar and observing the FFT window. The lowest transverse mode will be at a much lower frequency than the lowest longitudinal mode, and the transverse modes will not be equally spaced. (See equation (2.13).) If you have trouble figuring out which modes are the longitudinal modes, you can use (i) an estimate of the frequency the modes should have according to equation (2.13), (ii) the uneven spacing of the modes as predicted by equation (2.13) , and (iii) the fact that the transverse modes should stand out more as you detect and strike the bar vertically at the ends. (In other words, try striking at an angle, off center, *etc.* – the non-transverse modes should become more pronounced relative to the transverse modes. For very clean vertical excitation and detection, as shown in Figure 2.8(c), the transverse modes should stand out above the others.)

Measure the frequencies of the four lowest transverse vibration modes. Compare your measured frequencies to those predicted by equation (2.13). Do they agree within experimental error?

Finally, calculate the speed of sound *c* for each mode. Since it is difficult to measure the wavelength in this experiment, you should use the theoretical wavelengths given in Table 2.2 for this calculation. You should find a substantial dependence of this wave speed *c* on frequency *f*. A wave with a speed that depends on frequency is called a *dispersive* wave, so this is the first example of a (substantially) dispersive wave in this class. Waves with a speed that does not depend on frequency are called *non-dispersive* waves. The vibrations of a string and sound waves are non-dispersive, to a good approximation. (Here, I am supposing that the longitudinal vibration of a solid is a "sound" wave, whereas the transverse vibration is not.)

What does the functional dependence of this wave speed on frequency seem to be? Show how a function of this form fits your data for $c(f)$.

In your lab report, please remember to state the number of the bar that you used in your measurements. This will allow your instructor to determine whether your results were correct for that particular bar.