

9. Interference and Diffraction I

A. Objectives

- Measure the fringe spacing of a two-slit interference pattern, and use it to measure the laser wavelength. Study the dependence of the fringe spacing on the slit spacing.
- Measure the central spot width of a single-slit diffraction pattern, and compare it to the width of the incident laser beam and to the quantity $\lambda L / b$. See whether the pattern fits the theoretical one. Compare the pattern to a two slit-interference pattern with the same slit width. Study the dependence of the diffraction peak width on the slit width.
- Measure the diffraction pattern of multiple slits. Compare the measured diffraction angles and widths of the primary interference peaks to the theoretical angles and widths.
- Observe the Airy disk and measure its radius

B. Equipment required

1. Pasco optical rail
2. Pasco laser source, mount, and power supply, linear translator, rotary motion sensor, aperture bracket for light sensor, and slit accessory, including one single slit set and one multiple slit set (Each bin should contain one each of these items, so just take one bin.)
3. Pasco light sensor and connecting cable
4. Computer and Pasco 850 interface
5. Ruler
6. Beam blocks

C. Introduction

1. Interference of two point sources

Suppose you have two sticks A and B with a ball at each end, and that you push the two sticks into a placid body of water, a distance d apart, such that the balls are part-way immersed. Suppose also that you move the sticks vertically with a sine wave dependence in time, *i.e.* each stick's height is $h(t) = h_0 \sin(\omega t)$. Each stick will then launch a wave into the water, as shown in Figure 9.1. The displacement (height of the water relative to its undisturbed height) of the water will be

$$y_A = A(r_A) \cos\left(\frac{2\pi}{\lambda} r_A - \omega t\right) \quad (9.1)$$

$$y_B = A(r_B) \cos\left(\frac{2\pi}{\lambda} r_B - \omega t\right) \quad (9.2)$$

where y_A is the displacement due to the disturbance from stick A, r_A is the distance from stick A to the point of observation, and $A(r)$ is the amplitude of the wave (which decreases with distance r). The wavelength is $\lambda = 2\pi v / \omega$, where v is the phase velocity of the wave. Quantities with subscript B refer to the disturbance from stick B. Since the sources of the waves are points, the wavefronts (sets of points of maximum vertical displacement) are circles. These wavefronts move outward from each source at velocity v .

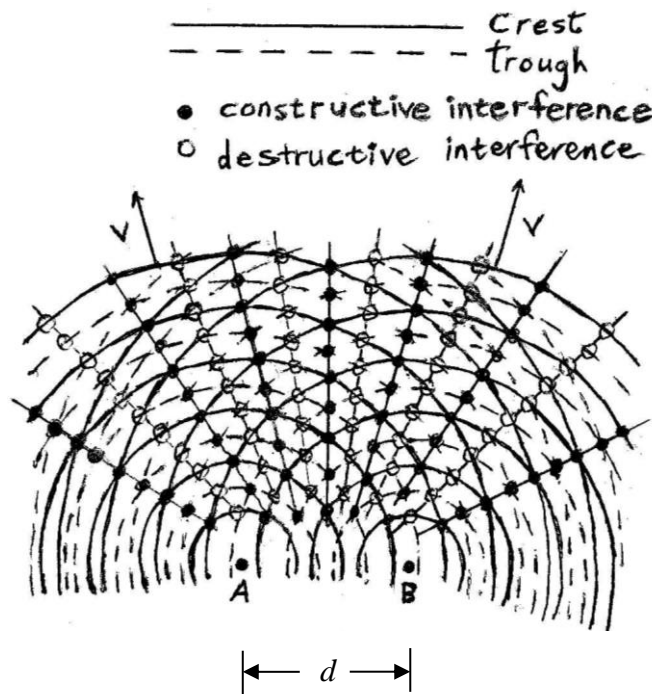


Figure 9.1. Waves generated by two point sources A and B.

Most waves obey the *superposition principle*. This means that if y_A is the wave generated by source A, and y_B the wave generated by source B, then the wave generated by the two sources combined is

$$y = y_A + y_B. \quad (9.3)$$

Waves obey the superposition principle if they are described by linear differential equations. Differential equations are linear if they contain only terms proportional to y , dy/dt , dy/dx , d^2y/dt^2 , d^2y/dx^2 , etc. An example of an equation that is not linear is one that contains a term

proportional to $(dy/dx)^2$. Waves that obey superposition include sound waves, ripples on water, electromagnetic waves in vacuum, and quantum-mechanical matter waves. All the waves we study in Physics 115L obey superposition.

Thus, the wave generated by the combination of source A and source B is just the sum of waves y_A and y_B . The crests (points of maximum positive displacement) and troughs (points of maximum negative displacement) of these waves are illustrated in Figure 9.1. If you look closely at the figure, you'll see some points where the crests of the two waves line up, or the troughs of the waves line up (shown by solid circles). At these points, the displacement of the wave is just twice that of the displacement due to one source by itself. Such positive addition of the wave displacements is referred to as *constructive interference*. Mathematically, the condition for constructive interference is that $|r_A - r_B| = n\lambda$, where n is an integer. This condition guarantees that if wave A is at a crest at that point, wave B will also be at a crest.

You can also find points where a crest of wave A aligns with a trough of wave B (shown by open circles). The displacement of the combined wave at those points is *zero* (assuming that $A(r_A) \approx A(r_B)$, which will be true for $r_A, r_B \gg d$.) Such negative addition of the two waves is referred to as *destructive interference*.

If a point has constructive interference at some time, it will have constructive interference at all times. Thus, the points of constructive interference are stationary, even though the underlying waves are moving. The same is true for destructive interference. Also, the points of constructive or destructive interference occur everywhere on the straight lines in Figure 9.1, not just at the discrete points indicated by the circles.

The intensity of the wave is proportional to $\langle y^2 \rangle$, where the brackets indicate a time average.

Thus, the intensity at points of constructive interference will be *four times* (2^2) the intensity of the wave of a single source. And, the intensity at points of destructive interference will be *zero* (again assuming that $A(r_A) \approx A(r_B)$).

It is possible to demonstrate this effect with light, using *Young's double slit experiment*, as illustrated in Figure 9.2. In this experiment, a light wave is sent through a screen with two narrow slits a distance d apart. It turns out that each slit acts as the source of a *secondary wave*, much like the water waves illustrated in Figure 9.1. Thus, if we view the light on a screen a distance L from the slits, we'll see points of constructive interference and points of destructive interference (zero intensity). The central point on the screen will always be a point of constructive interference, since its distance from the two slits is the same. As we increase the sideways displacement x on the screen, we'll first come to a point of half wavelength of difference $|r_A - r_B|$ (destructive interference), then to a point of one wavelength difference (constructive interference again), etc. The pattern of the light on the screen will be series of

bright and dark bands called *interference fringes*. The spacing χ between the bright fringes turns out to be (for small enough displacements that $\theta \ll 1$)

$$\chi = L \frac{\lambda}{d} \quad (9.4)$$

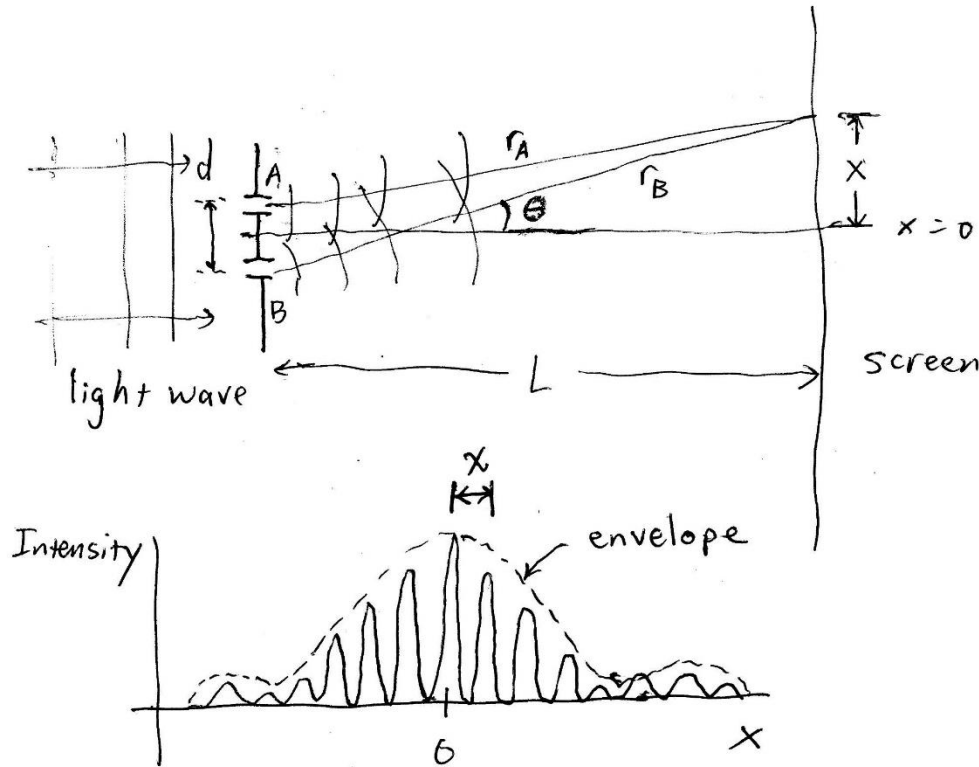


Figure 9.2. Young's double slit experiment. The “interference fringes” come from the alternation of constructive and destructive interference, as shown in Figure 9.1. These fringes are modulated by an “envelope function” that is related to the properties of light transmitted through only one slit.

2. Huygen's principle and single-slit diffraction

Huygen's principle provides a way of understanding wave propagation that does not require solution of a wave equation. Huygen's principle states that every point on a wavefront acts as a source of “secondary waves,” and that the subsequent evolution of the wave can be obtained by linear combination of these secondary waves. Each secondary wave is nearly identical to the wave that would be generated by a point source, like the stick in the water. Thus, the secondary waves have wavefronts that are circles (in 2-D), or spheres (in 3-D).

Figure 9.3(a) illustrates how to understand the propagation of a plane wave using Huygen's principle. We first choose a particular wavefront S . According to Huygen's principle, every point on S is a source of secondary waves. These secondary waves have wavefronts that are spheres, and which move outward at the wave speed c . So we imagine a set of such secondary waves

emanating from the point sources on S , as shown in the figure. (For clarity only a finite number of sources is shown; in reality the number of sources is infinite.) Anywhere to the right of S , we obtain the resultant wave by summing over the amplitudes of all of the secondary waves. The result of this sum is that new wavefronts are formed that are planes parallel to the initial wavefront S . (You can almost see the formation of these new wavefronts from the figure, and this result can be justified by carrying out the sum.) These new wavefronts move together with the secondary waves; *i.e.* they move to the right at speed c .

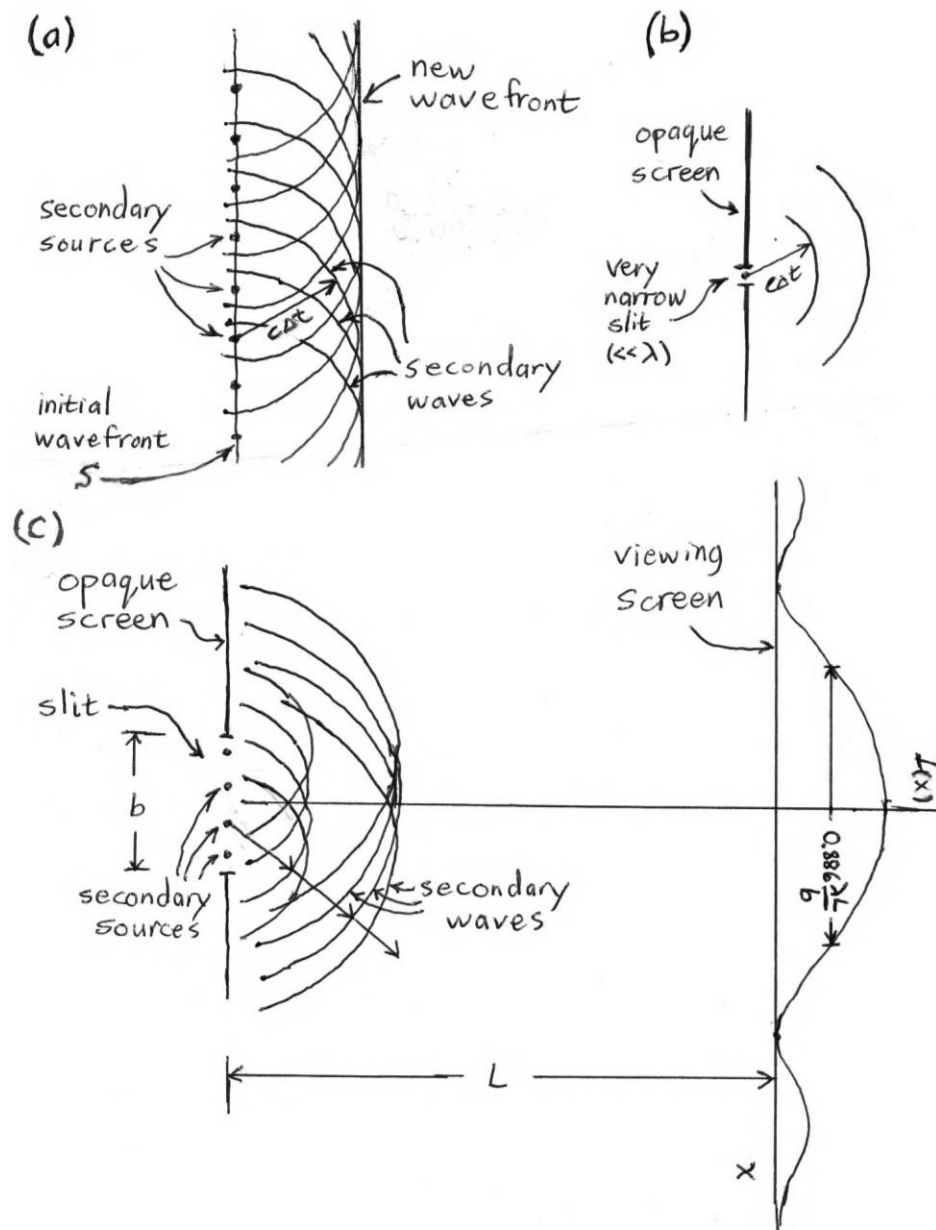


Figure 9.3. (a) Illustration of the explanation of a plane wave with Huygen's principle. (b) Huygen's principle applied to the propagation of light through a slit that is very narrow compared to a wavelength. (c) Huygen's principle applied to the propagation of light through a slit of finite width b .

Huygen's principle gives essentially exact results for wave propagation problems in free space, provided that some mathematical adjustments are made that I won't discuss here. Of course, it's not especially helpful for understanding plane waves since it's not difficult to solve the wave equation in that limit. But for problems involving the propagation of light through slits and other apertures, Huygen's principle is very helpful.

Another simple limit is the propagation of light through a slit that is narrow compared to a wavelength of light, as shown in Figure 9.3(b). We take our wavefront S to be the plane surface that includes the opaque screen with the slit. Since the screen blocks all the light except for that going through the slit, the only secondary sources are the points in the slit. Since the slit is so narrow, it is sufficient to consider a single secondary source in the center of the slit. (The problem is effectively two dimensional, since we are assuming the slit and the wave are both large in the third dimension, perpendicular to the page.) Thus, the wave leaving the slit is just that of the single secondary source – *i.e.* it is an outgoing circular wave.

Finally we can use Huygen's principle to analyze the propagation of light through a slit of width b that is not small compared to a wavelength, as shown in Figure 9.3(c). This is called *single slit diffraction*. We again choose a wavefront S that lies in the plane of the opaque screen containing the slit. Again, the secondary sources consist of a set of points in that plane, but now restricted to lie in the slit, each of which radiates outward going circular secondary waves. The resultant wave to the right of the slit is the sum of these secondary waves. Since the secondary sources are restricted to a finite width b , they do not fully reconstruct a plane wave as in Figure 9.3(a). And since b is not small compared to a wavelength, they don't make a simple outgoing circular wave as in Figure 9.3(b) either – the resulting wave is intermediate between these two cases. In your lecture course, you should carry out the necessary sum over secondary waves to determine the form of the wave to the right of the slit. Here, I'll just quote the result. If we place a screen at a distance L from the slit, and if we denote the transverse coordinate on that screen by x , the intensity of the light beam on the screen is

$$I(x) \approx I_0 \text{sinc}^2\left(\frac{b\pi}{\lambda L}x\right) = I_0 \frac{\sin^2\left(\frac{b\pi}{\lambda L}x\right)}{\left(\frac{b\pi}{\lambda L}x\right)^2} \quad (9.5)$$

This formula is valid only for $x/L \ll 1$ and $L \gg b^2/\lambda$. (The latter condition specifies the so-called Fraunhofer diffraction limit.)

The single slit diffraction pattern contains a dominant central peak and a number of smaller side peaks. The width of the central peak is $0.886 \frac{\lambda}{b} L$ (full width at half maximum). In other words, the angular spread of the beam is approximately λ/b . From this, we can see that for $b \gg \lambda$, the angular spread is small – *i.e.* we have a very forward-directed beam, tending towards the plane-wave limit. The angular spread increases as b becomes small – tending toward the very narrow

slit limit as b becomes smaller than λ . In the Fraunhofer limit we're considering here ($L \gg b^2 / \lambda$), the width of the diffracted beam is much greater than the slit width b .

3. Multiple slit interference

Consider the experimental arrangement illustrated in Figure 9.4. A wave is normally incident on an opaque screen with N narrow slits, equally spaced by a distance d . We observe the interference pattern on a screen at a distance L from the slits.

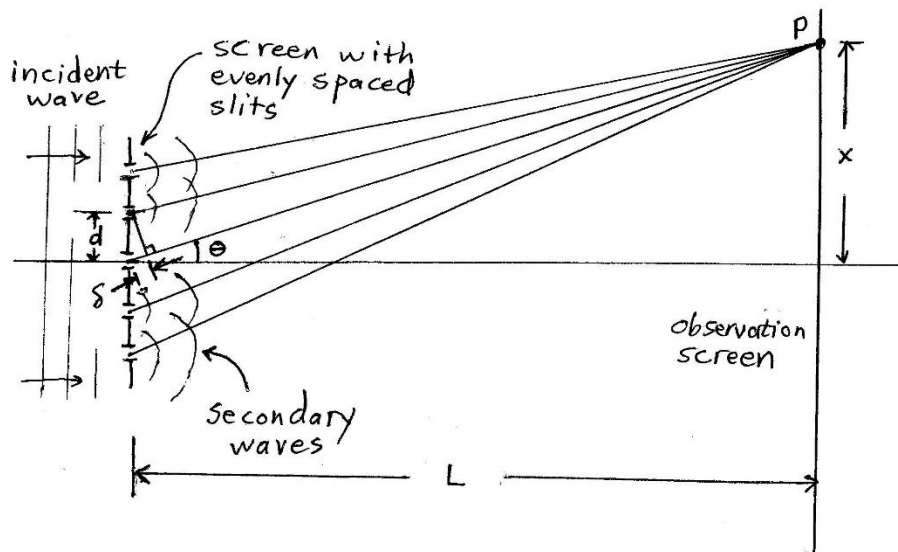


Figure 9.4. Multiple slit interference for a normally incident wave.

According to Huygen's principle, each slit acts as the source of a secondary wave with a circular wavefront. And according to the superposition principle, the wave displacement at a point P on the observation screen is the sum of the secondary waves from the slits. The intensity I of the wave is proportional to the time average of the total displacement squared. A calculation of this intensity, valid in the limit of large L , gives the result

$$I(\theta) = I_0 f(\theta) \frac{\sin^2\left(\frac{\pi N d \sin(\theta)}{\lambda}\right)}{\sin^2\left(\frac{\pi d \sin(\theta)}{\lambda}\right)} \quad (9.6)$$

where $I_0 f(\theta)$ is the intensity from one slit, $\theta = \tan^{-1}\left(\frac{x}{L}\right)$ is the angle between the ray from the center of the slits to the observation point P and the incident light ray, and x is the position on observation screen. In deriving equation (9.6), we have assumed that both the source of light and the observation point are very far away, so that the rays entering the slits and leaving the slits are nearly parallel.

$I(\theta)$ is shown in Figure 9.5 for $d/\lambda = 5$, and for (a) $N = 3$, (b) $N = 10$, and (c) $N = 30$. The diffraction pattern is no longer a cosine-squared function as it was for two slits, but has tall narrow peaks. We'll refer to these as the "primary interference peaks". These interference peaks stay in the same place and get narrower as the number N of slits increases (assuming that the slit spacing d remains constant).

The width of the primary peaks (FWHM) is

$$\delta\theta \approx 2.8 \frac{\lambda}{\pi Nd} \quad (9.7)$$

Thus, if the number of slits N becomes very large, the peaks will become very narrow. The definition of the term "FWHM" is given below.

The angles at which the tall peaks occur is given by

$$d \sin \theta = m\lambda \quad (9.8)$$

where $m = 0, \pm 1, \pm 2, \dots$ is the *order number*. Thus, the peak at $\theta = 0$ is the *zeroth order peak*.

The first ones to either side (near $\theta = \pm 0.2$ in this example) are the *first order diffraction peaks*. Sometimes we distinguish between these two orders as the "+1 order" and the "-1 order".

Similarly, the next two peaks to either side are the *second order diffraction peaks* (near $\theta = \pm 0.4$ in this example), the next two the *third order diffraction peaks*, and so on.

The angles given by equation (9.8) are the ones for which *all secondary waves interfere constructively*. To see this, denote the difference between the distance from a slit to point P and the distance from the next slit over to point P by δ , as shown in Figure 9.4. From the geometry we see that $\delta = d \sin \theta$. So, condition (9.8) just says that $\delta = m\lambda$; *i.e.* that the path length difference is an integer number of wavelengths. For instance, the first order peaks are those for which $\delta = \pm\lambda$. With equation (9.8) satisfied, the crests of all the secondary waves coincide at the observation point P , which gives complete constructive interference, and the largest possible intensity.

For other angles that do not satisfy equation (9.8), there is much less constructive interference of the secondary waves, or even destructive interference, so the intensity is much less. In between the tall peaks there are some small *secondary maxima*, but these maxima become smaller and closer together as the number N of slits increases. For large N , the secondary maxima are barely noticeable. The function $f(\theta)$ gives the angular dependence of the light diffracted from a single slit. In Figure 9.5 we have taken this function to be equal to 1. In reality this function will have a dependence on θ like that shown in Figure 9.6. The function $f(\theta)$ will "modulate" the intensity of the interference peaks so that they're not equal in intensity.

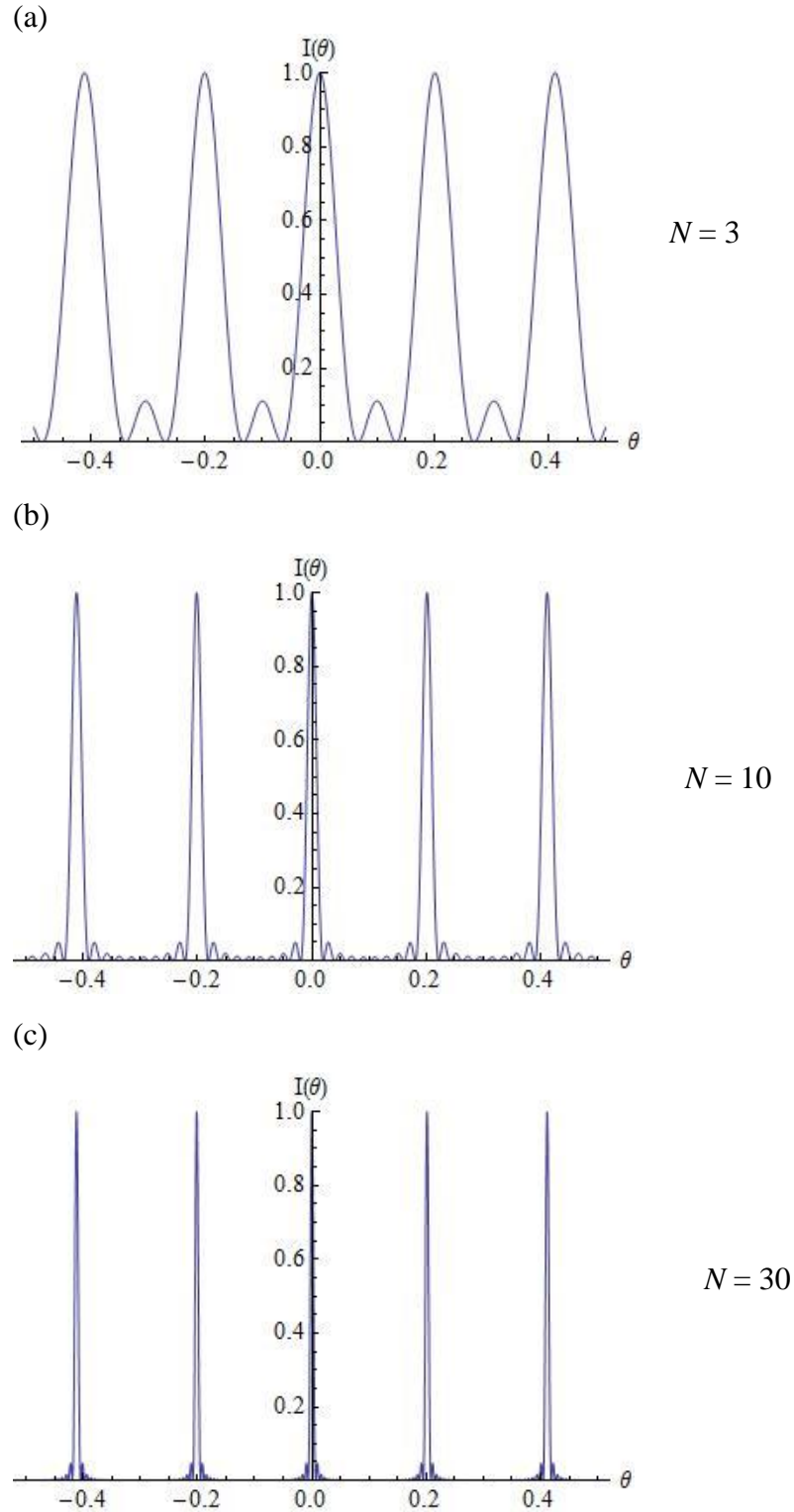


Figure 9.5. Intensity $I(\theta)$ for a normally incident multiple-slit interference pattern, in units of $N^2 I_0$, with $d/\lambda = 5$, for (a) 3 slits, (b) 10 slits, and (c) 30 slits. (In this figure we have set $f(\theta) = 1$.)

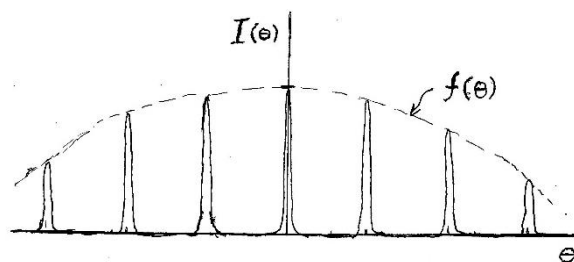


Figure 9.6. Modulation of the intensity of the multiple-slit interference peaks by the single slit diffraction pattern $f(\theta)$.

D. Experimental Procedure

1. Set-up of the Pasco interference and diffraction experiment

Next, we will quantitatively study interference and diffraction with the Pasco experimental apparatus illustrated in Figure 9.7. This includes an optical bench, diode laser and mount, and slit accessories and mounts. The diode laser and slit accessory mounts both snap into the bench from above. At this point, snap the laser mount into the bench near one end, as shown in Figure 9.7. Leave the slit accessory off the bench for now.

In this experiment, we will use an aperture disk bracket on the light sensor. To mount this bracket onto the detector, place the bracket so that its mounting hole aligns with the mounting hole in the sensor, and screw the post into the sensor as shown in Figure 9.8. Finally, place the post in its mounting hole on the translator, lower the detector until it stops, and tighten the post clamp. When the translator, detector, and aperture bracket are properly mounted on the rail, they should appear as shown in Figure 9.8.

The apparatus also includes a light sensor and aperture disk on a linear translator with a motion sensor. The linear translator has a nut and thumbscrew at its bottom center. To mount the translator to the bench, feed the nut into the slot just below the center of the bench from the end of the rail opposite the detector. Slide the translator with its nut along the bench until it is positioned near that end, and tighten the thumbscrew.

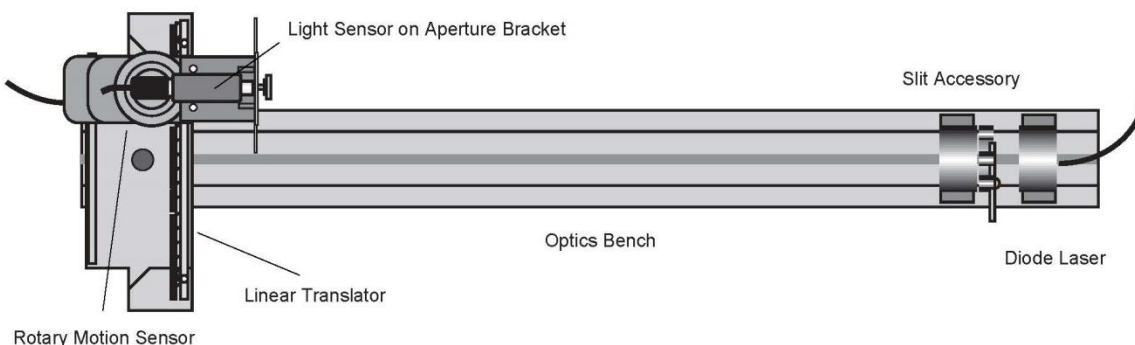


Figure 9.7. Experimental arrangement for the Pasco interference and diffraction experiment.

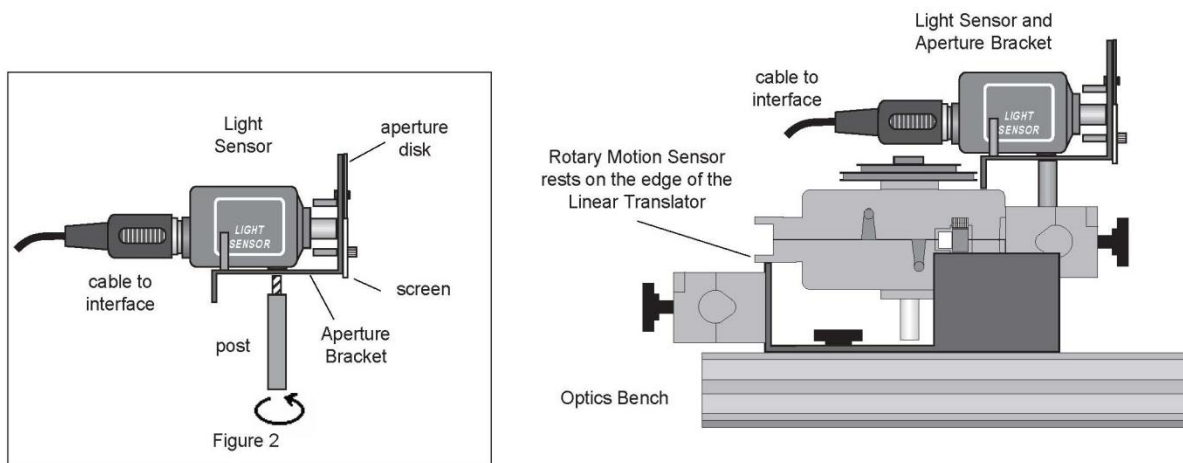


Figure 9.8. Proper set-up of the Pasco linear translator, light sensor, and light sensor aperture bracket.

Let's call the displacement coordinate along the translator axis x . The goal of this experiment is to measure the intensity of the diffracted laser as a function of x . If we were to use the sensor alone for this, the experiment would not work very well, because the sensor is several mm in diameter, and therefore would measure the intensity averaged over an area several mm in size. The purpose of the sensor aperture disk is to restrict the light entering the sensor as shown in Figure 9.9. In this way, only a narrow sliver of light enters the detector at any given time, so that the sensor measures the light intensity only over that narrow range in x . We can then measure light intensity vs. x by translating the position of the detector.

The translator contains a linear rack gear that couples to a pinion gear on the rotary sensor. With this mechanism, the rotation sensor rotates through an angle proportional to the translator displacement. The rotation sensor has a digital encoder that transmits pulses to the Pasco 850 interface for given increments of rotation angle, which correspond to given increments of displacement. The computer can record the translation displacement by measurement of these pulses. The arrangement is such that the computer can determine both the magnitude and sign of the displacement.

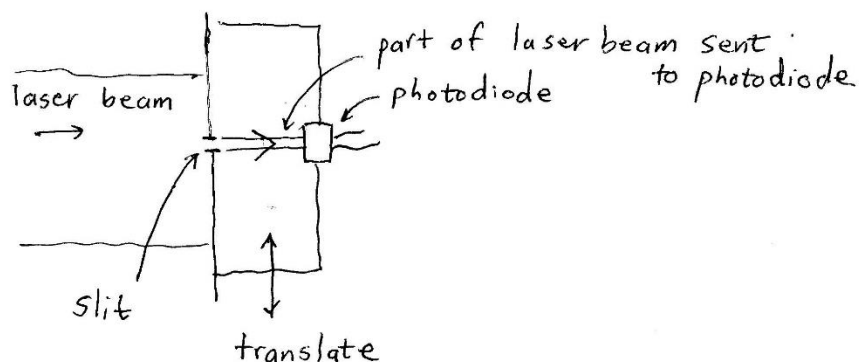


Figure 9.9. Restriction of light entering the light sensor by the light sensor aperture (slit).

Rotate the aperture disk so that the large open hole lies in front of the light sensor, and slide the sensor so that it is centered on the bench. Plug in the laser and turn it on. Using the alignment screws on the back of the laser, adjust the laser beam so that it runs down the centerline of the bench, and has a height that matches the light sensor at the other end of the bench.

The first measurement will be to determine the width of the laser beam. To do this, rotate the detector aperture disk so that the narrowest slit lies in front of the detector.

IMPORTANT: Leave the aperture disk with the narrow slit in front of the light sensor throughout the remainder of the experiment.

Again, the slit accessory should not be mounted to the rail yet.

Turn on the computer and Pasco 850 interface, start Capstone, and plug the light sensor into analog channel A. Using Hardware Setup, select "light sensor" for this input. Plug the two microphone connectors from the rotation sensor into digital inputs 1 and 2. It doesn't matter which connector goes into input 1, and which goes into input 2. Using Hardware Setup, select "Rotary Motion Sensor" for these two inputs. Then, click the tools button at the bottom right of the hardware setup window. For "Resolution", select "High: 1440 counts per revolution". For "Linear accessory", select "Rack and Pinion". Click "OK" and close "Properties" and "Hardware Setup."

Next, open a "Graph" window, select "Light intensity" for the graph's y-axis, and "Position (m)" for the graph's x-axis. Next, position the detector a little to one side of the laser beam, press "Record", translate the detector by hand across the laser beam, and press "Stop." Examine the data you have recorded, magnifying the axis scales if necessary. Note that Capstone records a certain number of samples per second. Each of these samples consists of one light intensity value and one position value. Thus, if you are stopped at a certain position, Capstone will record many data points at that position. But whatever points you record will be displayed as light intensity vs. position. At first, your traces may be a little ragged, and contain spurts of widely spaced points. Practice with the translator until you can get data with smoothly spaced points.

In this experiment, you'll need to adjust the light sensor sensitivity switch (1, 10, 100). If the maximum light signal is saturated at 100%, then set the sensitivity to a lower value. If the maximum light intensity is a very small number (say less than 5%), then switch the sensitivity to a higher value.

When you have recorded a good trace, you'll have the laser intensity vs. position. Save a copy of this data. *Question:* what is the width of the laser beam at the light sensor?

Now, we are ready to study diffraction and interference with this apparatus. To get started, shift the detector so that the laser beam falls on the white screen area. Then push the single slit accessory into the optical bench, close to the laser as shown in Figure 9.7. Orient the slits within their mount so that the slits are vertical. You want to do this so that the diffraction patterns are oriented horizontally, and the detector can measure them with its horizontal displacement.

Various slits will click into place as you rotate the disk, and the laser should be automatically aligned to hit the slit that is selected with each click. Click through the different slits, and observe the patterns that you see on the screen. Repeat this step with the multiple slit accessory.

2. Measurement of the laser wavelength and double slit interference

Select one of the four double slits from the multiple slit accessory. The two numbers on the slit give the width of the slits and the distance d between their centers, in units of mm. You should see a two-slit interference pattern on the screen. Record this interference pattern with Capstone, by recording the laser intensity vs. translator displacement. Examine the data closely. You should find at least 5 to 10 interference fringes. Measure the spacing χ between the fringes. (Think about the most accurate way to do this.) Measure the distance L from the slit accessory to the sensor aperture disk. Finally, using your measurement along with equation (9.4), determine the wavelength of the laser.

There are three more double slits on the accessory. Using your result from the first double slit, predict the fringe spacing χ that you expect for the other three. Then, see if measurements of χ for those other three double slits agree with your prediction. Make and save a measurement of the intensity vs. translator displacement for the 0.08/0.50 double slit, because you'll need that for the next section.

3. Measurement of single-slit diffraction.

Replace the multiple slit accessory with the single-slit accessory, and select the slit with a width of 0.08 mm. You should see the single slit diffraction pattern on the screen. Measure this pattern with Capstone.

Question: What is the width of the central peak of this pattern? How does this compare to the width of the laser beam that you measured earlier? If the pattern is wider than the laser beam, how can you explain that? Also, the rule of thumb for diffraction is that when a wave of wavelength λ passes through a slit of width b , the angular spread of the diffracted beam is about λ/b . How does your measured angular spread of the beam compare to λ/b ?

Predict the width of the single slit diffraction peak you should see for the other single slits on the accessory. Then, measure those widths and see if they agree with your prediction.

Save your diffraction pattern for the 0.08 mm slit. In your report, show a fit of this pattern to the

function $I(x) = a \frac{\sin^2(p(x-q))}{(p(x-q))^2}$, with fit parameters a , p , and q . You can use Capstone's "fit

user defined function" for this, or you can use other programs like Excel or Mathematica.

Compare the value you got for p with the one you would derive from the theoretical function of equation (9.5).

In your report, plot out the interference pattern for the 0.08 mm single slit on the same scale as the 0.08/0.50 double slit pattern you recorded in the previous part, so that you can clearly see how they compare with each other.

Question: The double slit interference fringes have both fine-scale interference fringes and a wider-scale "envelope" of intensity vs. position. What determines the structure of the wider-scale envelope?

4. Measurement of multiple slit interference patterns

Next, place the multiple slit wheel in front of the laser, and align it so that you can see multiple slit diffraction patterns at the detector. There is a series of multiple slits on this wheel containing $N = 2, 3, 4$, and 5 slits, with a constant slit spacing of 0.125 mm, and a constant slit width of 0.04 mm. Put each of these multiple slits into the laser, and make qualitative observations of the diffraction patterns by eye. See if you can observe the main features of the diffraction pattern that are described in the introduction.

Next, place the $N = 2$ multiple slit into the beam, and record its intensity pattern $I(x)$ with Capstone. Repeat with the $N = 3, 4$, and 5 multiple slits. Print out your graphs for these data on paper. Take note of your length L since that will be important for analysis.

Analysis of these data:

For the following, you don't have to fit a function to the data. It will be fine, and probably easiest, to just make measurements of the data on paper with a ruler (along with measurements of the scale with a ruler to get the results in correct length units).

- (i) Measure the diffraction angles for the first and second orders of each of the four patterns. Check if your measured angles agree with the prediction of equation (9.8). You may use the fact that the Pasco laser wavelength is 650 nm.
- (ii) Measure the height of the highest *secondary* maximum in the $N = 3, 4$, and 5 patterns, as a fraction of the height of the primary maxima. (Remember that the secondary maxima are the small peaks in between the primary maxima.) How quickly are these dropping with N ?
- (iii) Measure the widths of the +1 order diffraction peak and the -1 order diffraction peak for each of the four patterns. For a definition of "width", use the Full-Width-at-Half-Maximum (FWHM), as illustrated in Figure 9.10. For each N , find the average of these two first-order peak widths. Plot this average width as a function of N . According to equation (9.7), these widths should be given by $\delta\theta L \approx 2.8 \frac{\lambda L}{\pi N d}$. Plot this function on top of your data to see how well your data agree with the theory.

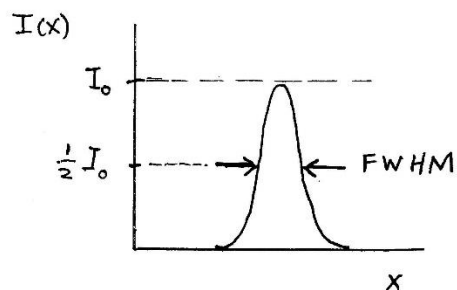


Figure 9.10. Illustration of the Full-Width-at-Half-Maximum (FWHM). If the peak signal level is I_0 , the FWHM is the full-width of the peak at a signal level equal to $I_0 / 2$.

5. The Airy disk

Place your single slit accessory in the bench, dial to one of the two circular apertures, and observe the diffraction pattern on the screen. You will see the diffraction pattern of a circular aperture called the *Airy disk*. It contains a bright central spot surrounded by a number of dimmer rings. Repeat for the other circular aperture.

Pick one of the two circular apertures. Using a ruler, measure the radius r_1 of the first dark ring in the pattern. According to diffraction theory, the angle r_1 / L should be equal to $1.22\lambda / b$, where b is the aperture diameter. How close does your measurement come to this theoretical value? (According to the Pasco manual, the diameters of the two circular apertures are 0.2 mm and 0.4 mm.)

The Airy disk is important because it determines the ultimate limit on the resolution of optical instruments such as microscopes and telescopes.

- End of experiment 9 -