6. Refraction of light

A. Objectives

- Observe refraction phenomena: partial reflection from a surface, bending of light rays, total internal reflection, and dispersion
- Measure the refractive index of plexiglass.
- Find the critical angle of incidence for total internal reflection in plexiglass, and observe its effect on reflected and transmitted light rays.
- Observe total internal reflection in a 90-45-45 right angle glass prism, and explore its uses.
- Determine whether total internal reflection occurs for two flat glass surfaces pressed together, with and without a layer of water between the surfaces
- Observe the dispersive refraction of a broadband light source by an equilateral glass prism, and determine the difference in refractive index for red vs. blue light.

B. Equipment Required

1. 635 nm diode laser with power supply and mount
2. Optical breadboard with flat platform on 1" post mount
3. Laser beam blocks
4. Plexiglass block with angled surface
5. Ruler
6. Protracter
7. Paper, tape, and metal right angle blocks
8. Two 90-45-45 right angle glass prisms
9. Water bottle and washcloth
10. Equilateral glass prism
11. Two 300 mm focal length lenses in mounts
12. Showcase lamp and piece of 2x4.

C. Introduction

Ray optics and visible light

Suppose a wave is moving through space, and that its wavelength λ is much less than the size of the space through with the wave travels and the size of any obstacles in that space. In this case, the wave will look almost everywhere like the one shown in Figure 6.1. (Exceptions would occur within a few wavelengths of an obstacle, but those occur only in a tiny fraction of the space through which the wave travels.) Its *wavefronts*, which are its crests, have very little curvature over a region many wavelengths in size – they are almost planes within such a region. *Rays* are lines that are everywhere perpendicular to the wavefront, which point in the direction of the flow of energy that is transported by the wave. In the regime we are considering, the rays are almost perfectly straight lines. (Again, the only exception is within a few wavelengths of an obstacle.) In this regime, many of the common features of waves, such as their tendency to bend around corners or to diffract, are hardly noticeable. Instead, the wave seems to just transport energy along the straight lines indicated by the rays. In such cases we often let the waves fade into the background, and focus our attention only on the rays. Such a physical picture is referred to as *ray optics* or as *geometrical optics.*

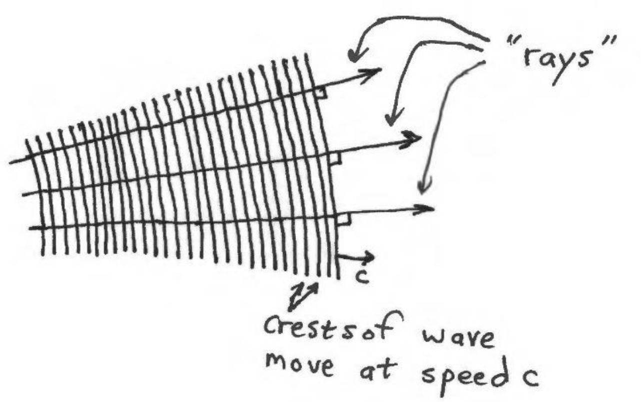

Figure 6.1. Wave with a very short wavelength moving through space. The curved lines show "wavefronts" of the wave, which are just its crests. "Rays" are lines that are everywhere perpendicular to the wavefront, and point in the direction of energy flow.

Light is an electromagnetic wave – a vibration of electric and magnetic fields. The wavelength of visible light is very short – between about 400 nm (violet) and 700 nm (red). Since this is so small compared to the millimeter or larger scale of the objects we normally deal with, light often satisfies the ray optics approximation. Then, we can understand much of what we want to know by thinking about light rays rather than electromagnetic waves. That is what we'll do in this lab.

Transparent dielectrics and refraction

To a first approximation, an electron in a material can either be strongly bound to an atom, or completely free to move through the material. Materials with essentially all electrons strongly bound are *dielectrics*. Those with many free electrons are *electrical conductors.* Again to a first approximation, a bound electron will behave as if it is harmonically bound with a resonance frequency ω_0. It sometimes happens that all of the electrons in a dielectric have resonance frequencies that are higher than the frequency of visible light, about 10^{15} Hz. If such a material is also very homogeneous, it is a *transparent dielectric,* and appears clear when viewed in normal light. Examples include many glasses, fused silica, quartz, diamond, plexiglass (acrylic), and vinyl (assuming the material has no added dyes or other impurities that absorb light). Liquids can also be transparent dielectrics. For the rest of this section, I'll use the term *dielectric* to mean a transparent dielectric.

When light interacts with a dielectric, the vibrating magnetic field plays only a minor role and can be neglected to a first approximation. The oscillating electric field of the light exerts an oscillating force on each electron in the material. Thus, the electrons behave as driven oscillators. Since the resonance frequency of the electrons is much higher than the frequency of the light, they behave as oscillators driven far below resonance. Therefore, the electrons oscillate in phase with the driving force, and do not absorb any energy from it. (Energy absorption requires that the phase shift between the force and the oscillation $\Delta\phi \neq 0$.) That is why dielectrics are transparent.

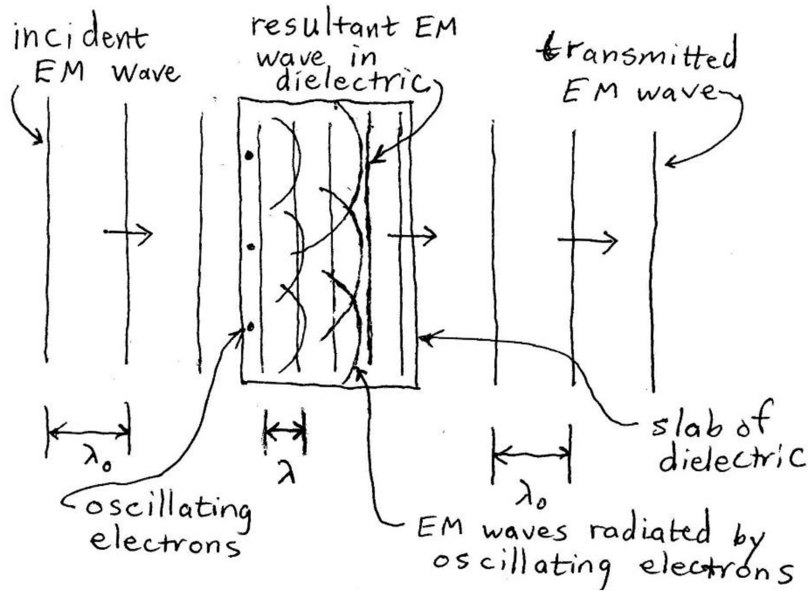

Figure 6.2. Illustration of the physical origin of refraction.

Although the electrons do not absorb light, they do have an effect on the light through a mechanism that is illustrated in Figure 6.2 for the case of an incident plane wave and a rectangular block of dielectric. Each electron in the dielectric oscillates at the frequency ω of the incident light beam, since that is the frequency of the driving electric field. An oscillating electron radiates a spherical electromagnetic wave with that same frequency. This is similar to the circular waves you can generate by pushing a stick up and down in the water. Since electromagnetic waves obey superposition,

the resultant (total) electric field everywhere in space is the sum of the field of the incident wave and the fields of the spherical waves radiated by all of the electrons.

This resultant field oscillates everywhere at the same frequency ω, because both the incident wave and all the electrons oscillate at that same frequency. It turns out that

the resultant field is a plane wave everywhere in space, the wavelength outside the dielectric is the same as it is without the dielectric, and the wavelength inside the dielectric is different.

This change in the wavelength inside the dielectric, along with related physical effects like the bending of light rays, is called *refraction.*

We won't have time to show how to derive this result in this lab course, and it's unfortunate that we often don't have time to do it in the lecture course either. If you don't cover it in the lecture this semester and want to understand it, there is a nice discussion of how this works in chapter 31 of the Feynman Lectures on Physics.

We'll denote the wavelength outside the dielectric as λ_0. According to the usual relation, $\lambda_0 = \dfrac{2\pi\omega}{c}$, where c is the speed of light in vacuum. (Strictly speaking, this is true only if the dielectric is placed in vacuum. But it's approximately true for a dielectric placed in air as discussed below.) We'll denote the modified wavelength inside the dielectric as λ. The ratio of the wavelength in vacuum to the wavelength in the material is called the *index of refraction n*:

$$n = \frac{\lambda_0}{\lambda} \tag{6.1}$$

The speed of light is also reduced in the dielectric, to a value $\mathrm{v} = c/n$. The relation between speed and wavelength in the material is $\lambda = \dfrac{2\pi\omega}{\mathrm{v}}$.

Most transparent dielectrics have refractive indices n in the range between 1.3 and 2.5. This implies that the wavelength of light in a material is smaller than the wavelength in vacuum. Air has a refractive index of about 1.0003. So, except for very precise work, the change in wavelength from vacuum to air is negligible. Also, for this reason the wavelength of light outside the dielectric is almost exactly equal to λ_0 even if the dielectric is placed in air.

There are important side effects of this wavelength change. One is that *there is a partial reflection, called Fresnel reflection, of light when it encounters a sudden discontinuity of refractive index.* For instance, when a light beam travelling through air hits a piece of glass, it is partly reflected from it. The reflected beam contains about 4% of the power of the incident beam;

the remaining 96% of the power is transmitted into the glass. Another effect is that the ray transmitted through a dielectric interface is *bent*, so that it propagates in a different direction.

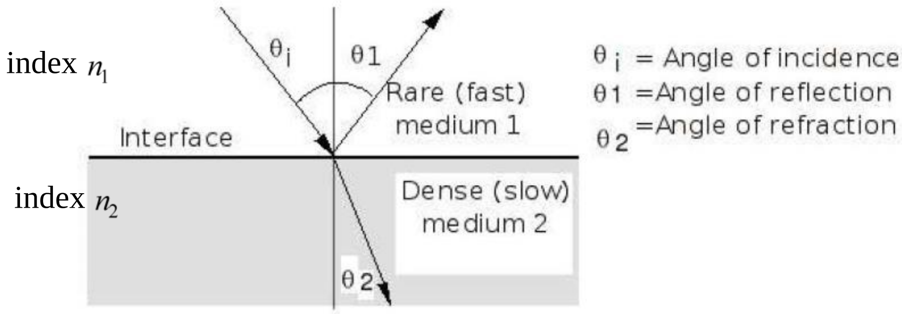

Figure 6.3. Refraction and reflection of light rays at an interface between dielectric materials of refractive index n_1 and n_2, for the case $n_2 > n_1$.

When a light beam encounters such a surface, it is conventional to define an *angle of incidence* θ_i, *angle of reflection* θ_1, and *angle of refraction* θ_2, as shown in Figure 6.3. The incident ray, reflected ray, and refracted ray all lie in the same plane. The incident ray undergoes a *specular* reflection, which means that $\theta_i = \theta_1$. The refracted (or "transmitted") ray is bent as given by *Snell's law of refraction:*

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{6.2}$$

For a ray moving from air into glass, the ray is bent towards the surface normal. For a ray moving from glass to air, the ray is bent away from the surface normal. We will not give the physical explanation of Snell's law here, but it has a nice, intuitive explanation based on Huygens' principle, which you will hopefully study in the lecture course.

Total internal reflection

We can rewrite Snell's law as follows

$$\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right) \tag{6.3}$$

Suppose that $n_2 < n_1$. Think about what happens as the angle of incidence θ_1 is increased towards $90°$. Since $n_1 / n_2 > 1$, and since $\sin\theta \to 1$ as $\theta \to 90°$, there will be some angle of incidence above which the argument of the $\sin^{-1}$ function is larger than 1. This is called the *critical angle* θ_c. Since the sine of any angle is always less than or equal to 1, equation (6.3) will have no solution for $\theta_1 > \theta_c$. We must go back to the physics of the situation to determine what happens in this case. It turns out, perhaps not surprisingly, that *there is no transmitted beam when* $\theta_1 > \theta_c$. This effect is called *total internal reflection.* It's called "internal reflection" because it only happens when $n_1 / n_2 > 1$, which implies that the beam must be incident on a material to air interface from inside the material.

Dispersion

A *non-dispersive wave* is one which has a speed that doesn't depend on frequency. Electromagnetic waves in vacuum are precisely non-dispersive: they propagate at speed *c* no matter what their frequency is. Transverse vibrations of strings and longitudinal sound waves are non-dispersive to a very good approximation. A *dispersive wave* is one that has a speed that depends on frequency. We saw an example of a highly dispersive wave in Lab 2: transverse bending waves of a thin bar.

It turns out that light waves in dielectrics are slightly dispersive. This happens because the index of refraction is a function of frequency, or equivalently a function of wavelength. The index of refraction is a function of frequency because the oscillation amplitude of the electrons depends on frequency. That happens for the same reason that the driven cantilever amplitude in Experiment 3 depends on frequency. Thus, the wave speed in the material is

$$\mathrm{v} = \frac{c}{n(\lambda)} \tag{6.4}$$

Generally speaking n increases with increasing frequency (increases with decreasing wavelength). Thus, a dielectric will be a little more strongly refracting for blue light than for red light. Therefore, the angles of refraction given by Snell's law also depend on wavelength.

D. Experimental Procedure

Proper care of optical components

This lab used to be difficult to do, because the optical components (plexiglass block, lenses, and prisms) were in dreadful shape. The blocks were so scratched up that you sometimes couldn't see the laser beam after transmission through just two surfaces. The prisms were actually "pieces of prisms". Recently, we have repolished the surfaces of the blocks and purchased new prisms for this lab. This will make the lab much easier to do.

You are always expected to take good care of the equipment. That not only helps us to keep the lab in good shape, but also demonstrates that you are responsible, can follow directions, and are making an effort to understand how the equipment works. You are graded in part by how well you meet this expectation.

We ask that you take special care with the new glass prisms since they are expensive and fragile. The prisms' average cost was $130 each. To take proper care of optics, including the prisms, you should:

- Always handle an optical component by its edges or unpolished surfaces, not by the polished optical surfaces.

- Never bring any object, including your fingers, into direct contact with a polished optical surface. Hard objects may scratch or ding the surface. Fingerprints can usually be removed by cleaning but in some cases cannot be completely removed. This also means you should never set a polished optical surface directly onto the table.
- Handle the components carefully so that you don't drop them. If you do drop a prism, you probably will knock a piece of the prism off.
- Never clean an optical component with a paper towel, because that paper will scratch the surface. In research labs, only special grades of lens tissue and solvent are used for cleaning. In this lab it will be OK to use a cotton towel to clean optics. Or, ask your instructor to clean an optic for you with lens tissue and methanol.
- Store optical components their protective containers. One of the worst things you can do is to store optics in a loose pile.

We appreciate your efforts to help us keep the lab equipment in good shape.

Refraction through plexiglass

Set up the laser, breadboard, and metal platform as shown in Figure 6.4. Follow all precautions on the use of the laser given in the instructions for Lab 3. The laser beam height should be about 1" above the platform. When doing this experiment, you will end up spraying the laser beam around in various directions in a horizontal plane at the same level as the laser. For eye safety, you need to place beam blocks so that the laser beam cannot leave the work area, as shown in Figure 6.4(b). To do this, **place the large beam block in front of the breadboard, and two smaller beam blocks at the two ends of the breadboard with no gap to the large breadboard**. This is important, since in this week's experiments, laser beams will be sprayed in many different directions. Place another beam block behind the breadboard for use as a screen to observe deflected laser beams.

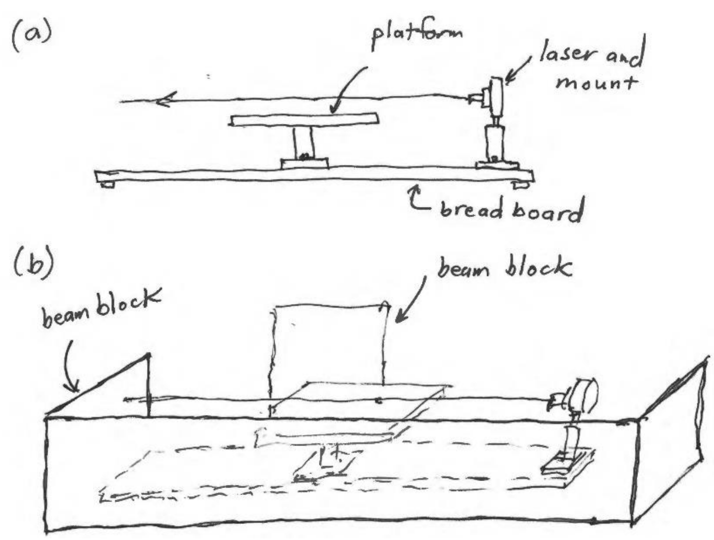

Figure 6.4. (a) Basic experimental set-up. (b) Set-up showing placement of beam blocks.

Your laser should have been readjusted so that it produces a parallel beam instead of a diverging beam. If your laser beam is still diverging like it was in experiment 3, ask your instructor to readjust it for you.

Place the large plastic block on the platform so that the laser beam enters and exits through its parallel sides. Rays will reflect from the surfaces of the block due to Fresnel reflection, and rays will also be transmitted through the surfaces of the block. **In this experiment, the rays should always be confined to a horizontal plane, and be intercepted by the beam stops. If you have rays that fly up into a vertical direction, you have the block positioned incorrectly and unsafely. In that case, reposition the block so that the rays are all parallel to the table surface.**

<u>Observe the pattern of rays that you see leaving the block</u>. <u>Try moving the block around to see what happens</u>.

You should observe a pair of reflected rays coming from the front surface of the block. One of the rays comes from the front surface, and the other comes from the back surface, as shown in Figure 6.5. You might also observe weaker rays that have been reflected twice.

The separation *s* between adjacent rays is related to the quantity *h* shown in Figure 6.5 by

$$h = \frac{s}{\cos\theta_i} \tag{6.5}$$

With an application of this equation and Snell's law, you should be able to show that

$$s = \frac{2D\sin\theta_i\cos\theta_i}{\sqrt{n^2 - \sin^2\theta_i}} \tag{6.6}$$

where *n* is the index of refraction of the plexiglass.

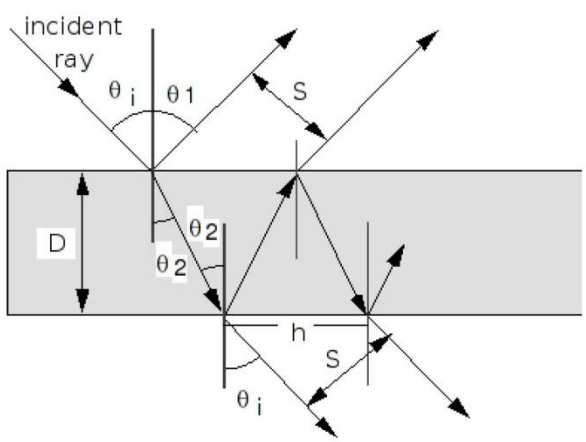

Figure 6.5: Multiple reflection and refraction of rays in a transparent block.

Calculation: From equations (6.5) and (6.6), find n in terms of the block thickness, angle of incidence, and ray spacing.

Question: How could you determine that the laser beam is normally incident on the surface?

Measurement 1 : Using a method of your choice, measure the index of refraction of plexiglass. The protractor, ruler, paper, tape, and right angle blocks may be useful for this measurement.

Total internal reflection

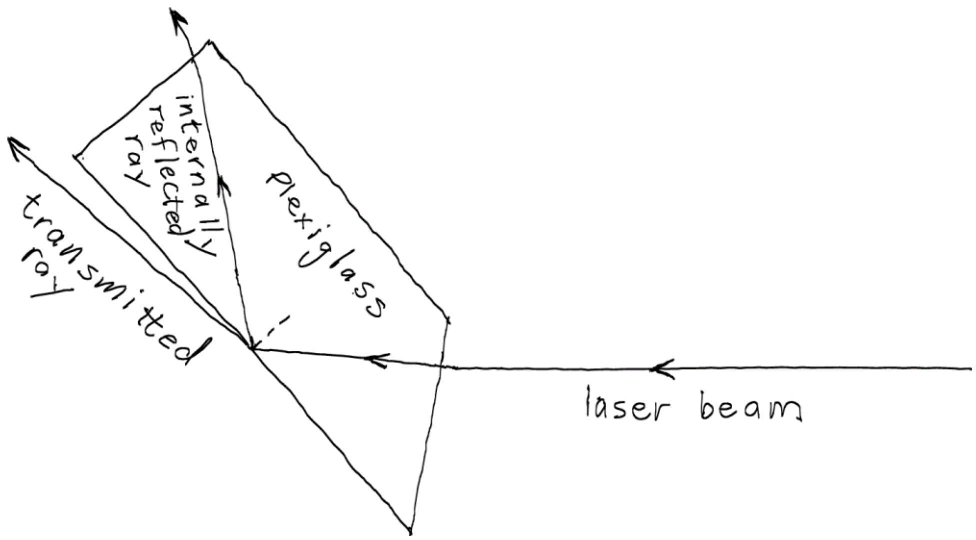

Figure 6.6. Path of laser beams for observation of total internal reflection in a block of plexiglass (view from above).

Send the laser beam through the angled surface of the plexiglass block as shown in Figure 6.6. (Make sure the plexiglass is still oriented so that the rays leaving it are horizontal and intercepted by your beam stops.) Observe the behavior of the ray as it moves through the plexiglass, especially focusing on what happens as it strikes the surface of the plexiglass internally. If you turn the room lights out, you should be able to see the laser beams inside the plexiglass. Find an orientation of the block for which the laser beam enters the angled surface, transits through the plexiglass to one of the side surfaces, and produces a transmitted beam that exits from that side surface, as shown in Figure 6.6.

Calculation: Determine the formula for the critical angle of incidence from Snell's law.

Measurement 2: Measure the critical angle at which light striking the plexiglass surface internally does not transmit back out of the plexiglass. Compare your result for the critical angle to that predicted by Snell's law and your result of Measurement 1.

Uses of total internal reflection

Total internal reflection provides a useful way to reflect light very efficiently. Optical fibers use this effect to make a guide that can transmit light over long distances with low loss – the best fibers, used for fiber-optical communications links, can transmit light for many km! Total internal reflection also provides a useful way to make an efficient mirror.

Calculation: Compute the critical angle for BK-7 optical glass, which has a refractive index of 1.515 at 635 nm. If light is incident on a 90-45-45 BK-7 glass prism as shown in Figure 6.7, what will happen to it?

Question: If we have two parallel rays R_{TOP} and R_{BOTTOM} incident on the prism as shown in Figure 6.7, what will be the relative position of the two rays when they leave the prism? What useful thing could you do with two such prisms? (Such a device is used in binoculars.)

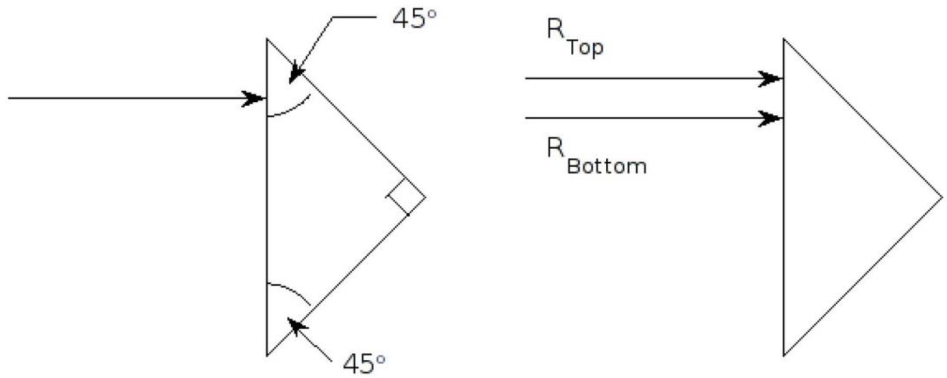

Figure 6.7. Light incident on a 90-45-45 degree prism.

Observation: You should have two 90-45-45 prisms made of BK-7 optical glass. Hold one of them so that the 90 degree corner is opposite your eye (*i.e.* look at it in the direction of the arrows in Figure 6.7.) Tilt the prism slightly from side to side. Describe briefly what you observe. Give an explanation for your observations.

Observation: Throughout this observation, maintain a prism orientation such that the laser beam is incident normally on the first prism surface. Set one of the prisms on the platform so that the laser beam enters it as depicted in Figure 6.8 (*i.e.* with the prism on the right missing). Record whether you have a beam reflected from the angled surface, and/or a beam transmitted through the angled surface. Next, place the second prism up against the first one, as depicted in Figure 6.8, and again record whether you have reflected and/or transmitted beams. Next, drop some water onto the interface between the prisms, and wait a few seconds. Again, record whether you have reflected and transmitted beams. When you are done with this part of the experiment, remove the water from prisms and the platform with the washcloth.

Question: Explain the observed behavior. Do the side-by-side prisms behave as one solid block of glass, or two separate ones? Why do you think that is? If the water has an effect, explain why.

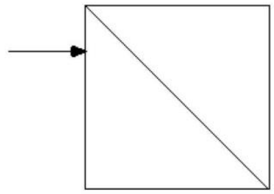

Figure 6.8. Experimental arrangement for light incident on a pair of prisms.

Dispersion

We will start this part of the lab by measuring the index of refraction of the equilateral glass prism for red light using the method of minimum deflection. (You should not expect to get the index of BK-7 since the equilaterial prism is made from a different kind of glass.) To do this, send the laser beam through the prism as shown in Figure 6.9. We'll define the deflection angle δ as the angle between the laser beam input to the prism, and the laser beam exiting the prism. Rotate the prism, and watch what happens to the deflected beam. You should see that there is an orientation of the prism for which the angle δ is a minimum. It turns out that when the prism is oriented for minimum deflection angle δ, the rays inside the prism are parallel to its base, as shown in Figure 6.9. Also, the relation between deflection angle δ prism apex angle α and the prism index of refraction with this condition is

$$n = \frac{\sin\left(\frac{1}{2}(\alpha+\delta)\right)}{\sin\left(\frac{1}{2}\alpha\right)} \qquad \text{(with prism adjusted minimum deflection)} \qquad (6.7)$$

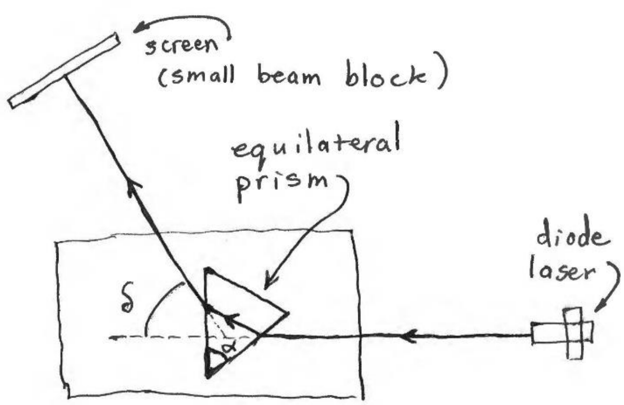

Figure 6.9. Laser beam refracted by an equilateral prism.

Measurement: Measure the minimum deflection angle δ, and use this measurement to determine the index of refraction of the prism glass for red light using equation (6.7). (α is determined from the fact that the prism shape is an equilateral triangle.)

This method to determine an index of refraction is a good one, since only two quantities are needed (α and δ), and they can be determined pretty accurately. Since the deflection is near a minimum point, it is insensitive to the prism orientation, which is more difficult to measure accurately.

Next we will study dispersion. To begin, set up the arrangement shown in Figure 6.10. To do this, switch the diode laser off and unplug its power supply. Place a piece of 2x4 on the breadboard in front of the laser, place the showcase lamp on top of the 2x4, and plug the lamp in and turn it on. Orient the lamp so that the filament is closer to the platform than the filament support. Next, place one of the 300 mm focal length lenses on top of the platform. Adjust the spacing between the lamp and this lens so that a nearly parallel beam of light comes out of the lens. (This will happen with a lamp-to-lens distance approximately equal to the lens focal length.) This will make it so that all the rays pass through the prisms travelling in the same direction, as was the case for the laser. Next, set the equilateral prism behind the lens so that it intercepts a large fraction of the rays. Rotate the prism so that you are able to observe the rays refracted by the prism on the small beam block, used as a screen. Once you have this set, put the second lens between the prism and the screen so that it intercepts most of the refracted rays. You should see the beginnings of a rainbow color pattern on the screen. Adjust the distance between the screen and the lens to give the clearest rainbow. Again, this will occur with a lens-to-screen distance approximately equal to the focal length. With this lens position, all rays entering the lens at the same angle will be focused to the same position on the screen.

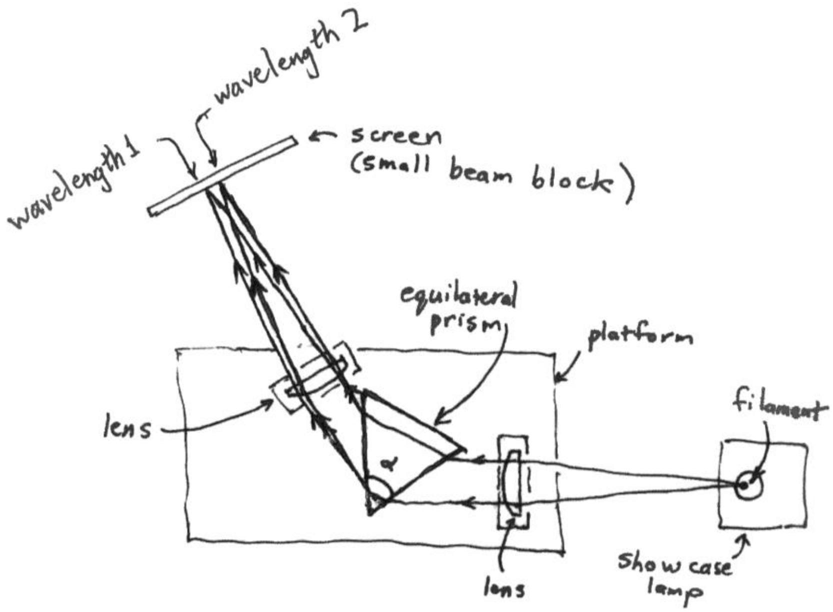

Figure 6.10. Experimental arrangement to study the dispersion of glass.

The rainbow color pattern occurs because different wavelengths of light (different colors) are refracted at different angles by the prism, because the index of refraction depends on wavelength. This causes the rays of one wavelength to appear at a different position on the screen than the rays of another wavelength, as shown in Figure 6.10.

Question : From the pattern you observe, which wavelength has the larger index of refraction in the glass – blue light or red light?

Measurement: <u>Determine the difference in the index of refraction of the prism glass for blue light and the index of refraction of the prism glass for red light</u>.

The best way to do this is to use the method of minimum deflection again. To do that, orient the prism so that the red light is deflected by the minimum angle. Readjust the lens positions if necessary. Tape a piece of paper to the screen, and mark the position of the red light. Tweak the prism again to see if you can deflect the red light by an even smaller angle, and if you can, move the screen so that the mark aligns with the new position of the red light. Repeat until you are sure that the mark coincides with the red light position for the minimum possible deflection.

Next, tweak the prism a bit, and see if you can reduce the deflection angle for the blue light. (If you're like me, you'll find that you can't.) So, the prism orientation that results in minimum deflection angle for the red light also results in minimum deflection angle for the blue light, for practical purposes.

Since we already measured the minimum deflection angle for red light in the previous step, we do not need to do it again. Therefore we just want to know the minimum deflection angle for the blue light. That angle is equal to the angle for the red light plus a small change in angle. With the experimental arrangement, it is easy to get that small change in angle. You just make another mark where the blue light is for minimum deflection. With some thought, you should be able to figure out how to use the distance between your two marks to give you the change in minimum deflection angle from red to blue. From that and equation (6.7), you should be able to get the index of refraction for blue light. This completes the measurements for this week's experiment.

The arrangement shown in Figure 6.10 forms the basis of a prism spectrometer – a device to measure the power contained in a light beam as a function of wavelength. These days, a spectrometer would have a CCD camera in place of the screen in order to take quantitative spectral measurements. A working spectrometer would also have a slit at the position of the filament, and the light to be analyzed would enter through that slit. This would be done so that the angle of incidence of the light on the prism is very well-defined.