10. Interference and Diffraction II

A. Objectives

- Measure the diffraction of light by a diffraction grating for normal incidence light and for light incident at 12 degrees from normal incidence. Use your measurement of the first order diffraction angles at normal incidence to determine your laser wavelength. Determine if your remaining measured diffraction angles match what you would expect using your measured wavelength.
- Measure the Fresnel diffraction pattern of a semi-infinite opaque screen, and compare measured quantities with diffraction theory.
- Make qualitative observations of the Fresnel diffraction pattern of a solid disk. See for yourself if the "Arago spot" exists, or not.

B. Equipment required

1. Optical breadboard
2. Thorlabs diode laser module, mount, and power supply
3. Laser beam blocks
4. Aluminum platform with post mount
5. 600 line per mm transmission diffraction grating in aluminum mount
6. 6” aluminum rail
7. Paper, tape, pencil, ruler, meter stick, and protractor
8. 4” diameter plano-convex lens in mount with 2” post and post holder
9. ½” diameter plano-concave lens, -25 mm or -30 mm focal length, in mount
10. Pasco optical rail, linear translator, rotary motion sensor, light sensor, and aperture bracket for light sensor
11. Computer and Pasco 850 interface
12. Thin aluminum plate
13. Filter holder with post and post holder
14. Brass disk on 2” diameter glass plate in mount, with 3” or 4” post and post holder

C. Introduction

1. Review of multiple slit interference

In the previous lab, you measured the diffraction pattern of a multiple slit. This occurs when a wave is incident on an opaque screen with *N* slits, equally spaced by a distance *d*, as illustrated in Figure 10.1. We observe the interference pattern on a screen at a distance *L* from the slits.

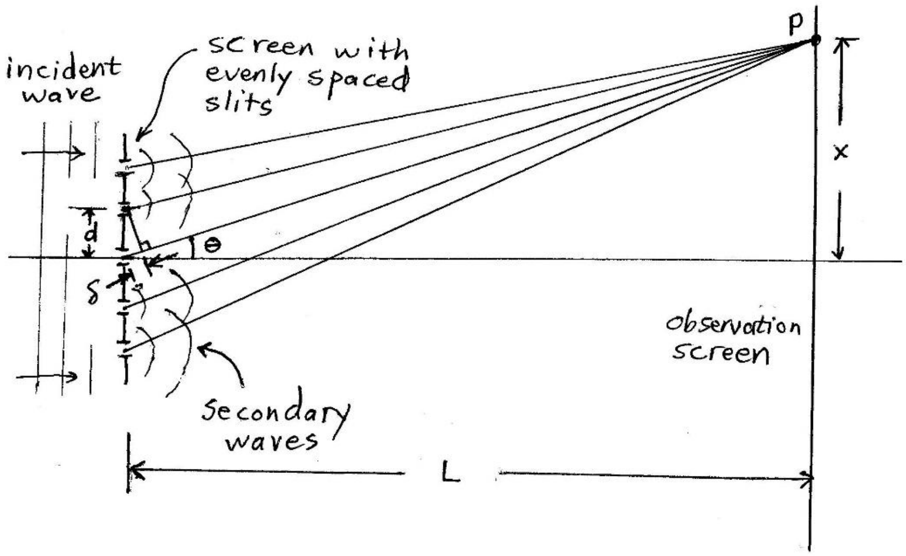

Figure 10.1. Multiple slit interference for a normally incident wave.

A calculation of the intensity on the screen, valid in the limit of large *L*, gives the result

$$I(\theta) = I_0 f(\theta) \frac{\sin^2\left(\dfrac{\pi N d \sin(\theta)}{\lambda}\right)}{\sin^2\left(\dfrac{\pi d \sin(\theta)}{\lambda}\right)} \tag{10.1}$$

where $I_0 f(\theta)$ is the intensity from one slit, $\theta = \tan^{-1}\left(\dfrac{x}{L}\right)$ is the angle between the ray from the center of the slits to the observation point *P* and the incident light ray, and *x* is the position on observation screen. In deriving equation (10.1), we have assumed that both the source of light and the observation point are very far away, so that the rays entering the slits are nearly parallel, and the rays leaving the slits are nearly parallel. An example of the diffraction pattern, for the case $N = 30,$ is shown in Figure 10.2.

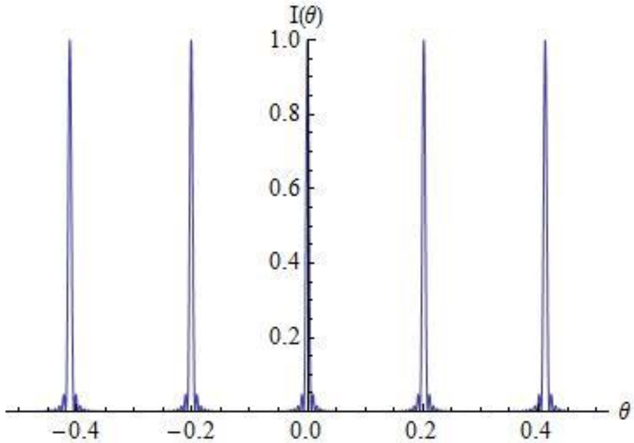

Figure 10.2. Intensity $I(\theta)$ for a normally incident multiple-slit interference pattern, in units of $N^2 I_0$, with $d/\lambda = 5$, for $N = 30$ slits. (In this figure we have set $f(\theta) = 1$.)

The tall peaks in the interference pattern are referred to as "primary interference peaks". These peaks stay in the same place and get narrower as the number *N* of slits increases (assuming that the slit spacing *d* remains constant). The width of the peaks (FWHM) is

$$\delta\theta \approx 2.8 \frac{\lambda}{\pi N d} \tag{10.2}$$

Thus, if the number of slits *N* becomes very large, the peaks will become very narrow. The angles at which the primary peaks occur is given by

$$d \sin\theta = m\lambda \tag{10.3}$$

where $m = 0, \pm 1, \pm 2, ...$ is the *order number*. Thus, the peak at $\theta = 0$ is the *zeroth order peak*. The first ones to either side (near $\theta = \pm 0.2$ in this example) are the *first order diffraction peaks*. Sometimes we distinguish between these two orders as the "+1 order" and the "-1 order". Similarly, the next two peaks to either side are the *second order diffraction peaks* (near $\theta = \pm 0.4$ in this example), the next two the *third order diffraction peaks*, and so on.

The angles given by equation (10.3) are the ones for which *all secondary waves interfere constructively*. To see this, denote the difference between the distance from a slit to point *P* and the distance from the next slit over to point *P* by δ, as shown in Figure 10.1. From the geometry we see that $\delta = d \sin\theta$. So, condition (10.3) just says that $\delta = m\lambda$; *i.e.* that the path length difference is an integer number of wavelengths. For instance, the first order peaks are those for which $\delta = \pm\lambda$. With equation (10.3) satisfied, the crests of all the secondary waves coincide at the observation point *P*, which gives complete constructive interference, and the largest possible intensity.

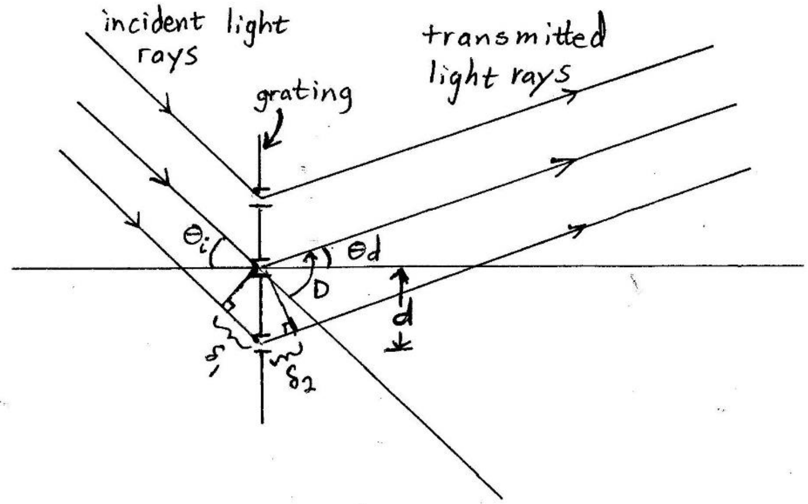

Figure 10.3. Diffraction grating used at non-normal incidence.

In multiple slit interference, we have to consider the possibility that light is not normally incident on the grating. For instance, consider light hitting the slits with angle to the normal θ_i, as illustrated in Figure 10.3. We still find sharp peaks corresponding to specific orders of diffraction, which occur for beam directions such that the difference in path length of the rays passing between adjacent slits is an integer number of wavelengths. But now there is a contribution $\delta_1 = d\sin\theta_i$ to that path length difference from the front side of the grating, as well as the contribution $\delta_2 = d\sin\theta_d$ from the back side, where θ_d is the angle that the diffracted rays make with the normal to the grating. Thus, the condition to have a diffraction peak becomes

$$d\sin\theta_i + d\sin\theta_d = m\lambda \qquad (10.4)$$

Equation (10.4) is called the *grating equation.*

2. Diffraction gratings

A *diffraction grating* is an optical component with a surface that has an array of equally spaced grooves. It is used to produce a multiple-slit interference pattern. In principle, the grooves could take the form of narrow slits as shown in Figure 10.1. However, almost no real diffraction grating is made this way. One reason is that this kind of grating is inefficient – much of the light is blocked.

Most real gratings are made of a piece of glass or plastic with grooves, but no slits, as shown in Figure 10.4. Such gratings do not block any light at all. They are nearly always produced from *master gratings,* which are one of two types: *ruled gratings,* and *holographic gratings.* Ruled master gratings are made with a machine called a *ruling engine.* It uses a sharp diamond tool to cut a groove in a piece of metal. Then it moves the tool by a distance *d* and cuts another groove. It repeats this groove by groove until the desired number of grooves has been cut.

To make a holographic master grating, you coat a surface with a photoresist, expose it to an interference pattern made with laser beams, and then chemically etch it. The etch removes photoresist in proportion to its light exposure. In this way, the bright and dark bands of the interference pattern are transferred to the material as a set of grooves.

High quality master gratings are difficult and expensive to produce. For this reason they are almost never directly used in experiments. Instead, *replica gratings* are produced from the master gratings. To make such replicas, a thin layer of resin is applied to a substrate, pressed onto the master, cured, and then separated from the master. This provides a thin plastic grating bonded to the substrate with a surface that is the negative of the master grating. If desired, the process can be repeated to produce a replica of the replica, which will have a surface that matches the master grating. A master grating can be used to produce thousands of replica gratings. These replicas are what you'll normally see in both teaching and research labs.

Replica gratings can be further divided into *reflection gratings* and *transmission gratings.* To make a reflection grating, you apply a very thin film of metal to the replica grating. This kind of grating is opaque, but can be used as a grating that works by reflecting light. Transmission gratings are replica gratings with a glass substrate and no metal coating, so you can shine a beam of light through them as illustrated in Figure 10.4. In this lab, you'll use a transmission grating.

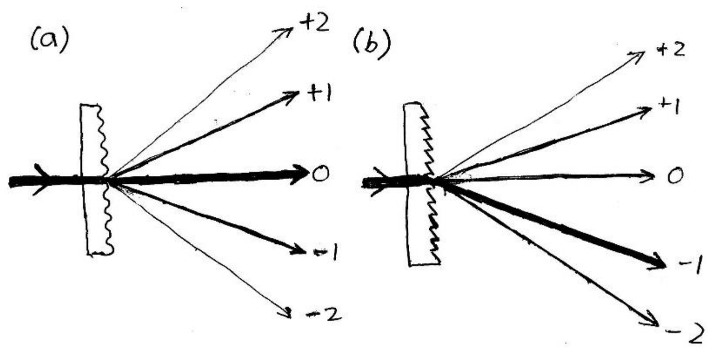

Figure 10.4. Transmission diffraction gratings. Thicker lines in the figure indicate more intense beams. (a) Transmission holographic grating. These gratings typically have a sinusoidal groove profile as shown here. (b) Transmission grating with a sawtooth groove profile. This grating is blazed to maximize the power in a first-order diffracted beam.

Transmission gratings don't cut off the beam anywhere, so this is not the situation of N nearly-point sources as depicted in Figure 10.1. Instead, each groove acts a little like a tiny lens or prism, which deflects, diffracts, and/or focuses the light in a way that depends on the shape of the groove. However, the total field transmitted by the grating is still a superposition of the outputs of the N grooves. The condition for constructive interference is the same as before, so the grating will have a set of output beams at the angles given by equation (10.3) (for normal incidence) or equation (10.4) (for non-normal incidence).

There are two important differences between a real transmission grating and the N-slit grating shown in Figure 10.1. First, the grating transmits nearly all of the power of the incident beam into the various diffracted beams. Second, the various orders of diffraction don't have relative intensities $f(\theta)$ given by a single-slit diffraction pattern. The relative intensity of the diffraction orders depends on the shape of the grooves.

Holographic gratings with a sinusoidal groove profile, as shown in Figure 10.4(a), are one of the most common types. That is because this fabrication method naturally tends to produce a sinusoidal groove shape. For these gratings, the zero-order beam will usually be the most intense. For normal incidence diffraction, the +1 and -1 order beams will have equal intensity to each other, and similarly the $+2$ and -2 orders will have the same intensity. The first order beams are usually less intense than the zeroth order beam, and the second order beams are usually less intense than the first order beams.

It is possible to design gratings with a groove shape that optimizes the single slit function $f(\theta)$ for an application. Often, this is done to maximize the power in one of the first-order diffracted beams, as shown in Figure 10.4(b). Such a grating is said to be *blazed*.

If you look again at equation (10.2), you may realize why diffraction gratings can be so useful. Equation (10.2) shows that the larger the number of lines, the sharper the diffraction peaks. Diffraction gratings typically have thousands or tens of thousands of lines. Because such gratings can provide very sharp diffraction peaks, they can be used to make a very accurate measurement of the diffraction angle, and of the wavelength.

3. Fresnel vs. Fraunhofer diffraction

So far, we've always made the assumption that the distance *L* from the diffracting object to the point of observation is "very large." This type of diffraction is called *Fraunhofer diffraction*. The quantitative condition for Fraunhofer diffraction is that the size of the diffracted beam is large compared to the size *b* of the diffracting object, as shown in Figure 10.5(a). In the figure, *b* is the width of a slit, *L* is the distance from the slit to an observation screen, λ is the wavelength of the wave, and *x* measures the position on the screen. The angular spread of the diffracted beam is $\lambda / b,$ and the size of the diffracted beam on the observation screen is $\lambda L / b.$ Thus, Fraunhofer diffraction occurs if $\lambda L / b \gg b,$ *i.e.* if $L \gg b^2 / \lambda.$ <u>All of the diffraction physics we have discussed in this lab course up to this point is Fraunhofer diffraction physics</u>.

If *L* is not large compared to $b^2 / \lambda,$ then the diffracted beam is not large compared to the diffracting object. This type of diffraction is called *Fresnel diffraction*, and is illustrated in Figure 10.5(b).

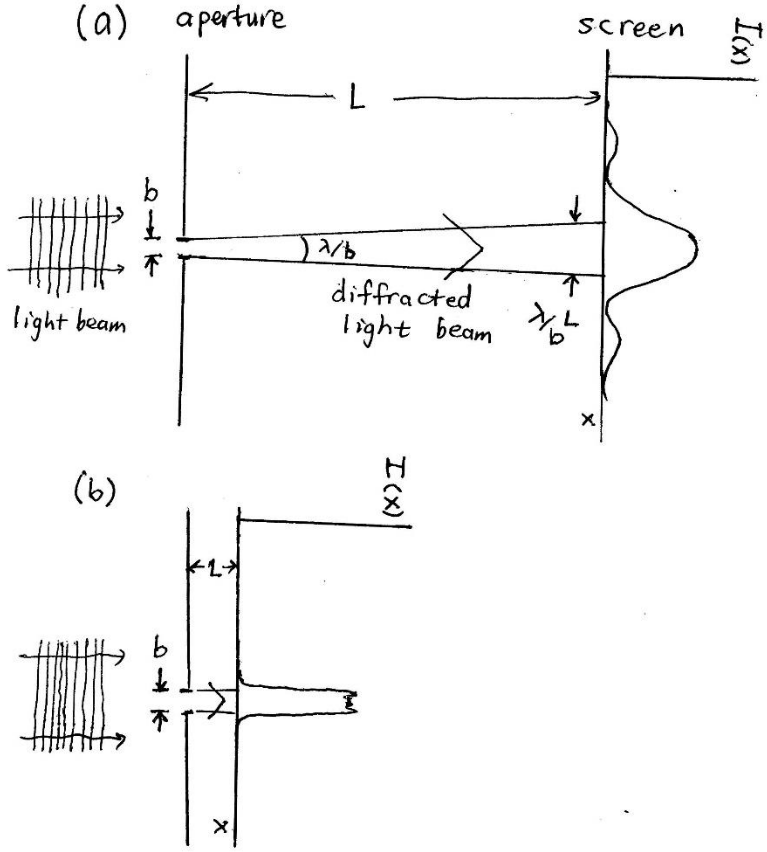

Figure 10.5 (a) Fraunhofer diffraction. The distance L to the observation screen is large enough that the diffracted beam is large compared to the diffracting object. (b) Fresnel diffraction. The distance L to the observation screen is small enough that the diffracted beam is comparable in size to the diffracting object.

To summarize,

Fraunhofer diffraction: $$L \gg \frac{b^2}{\lambda} \tag{10.5}$$

Fresnel diffraction: $$L \lesssim \frac{b^2}{\lambda} \tag{10.6}$$

4. Fresnel diffraction of the semi-infinite screen

One basic case of Fresnel diffraction is the diffraction of a wave by a semi-infinite screen, as shown in Figure 10.6. In this case, a screen with a sharp edge blocks half of the wave, and lets the other half pass. This type of diffraction is always Fresnel diffraction since the size of the diffracting object is infinite (assuming the incident wave is infinitely large).

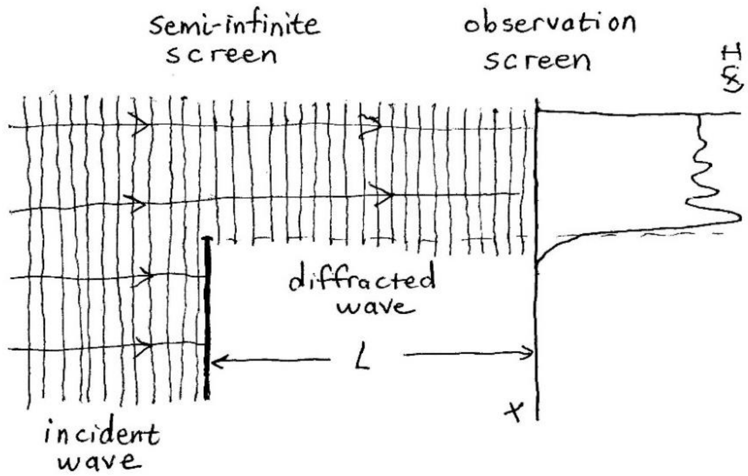

Figure 10.6. Fresnel diffraction by a semi-infinite screen.

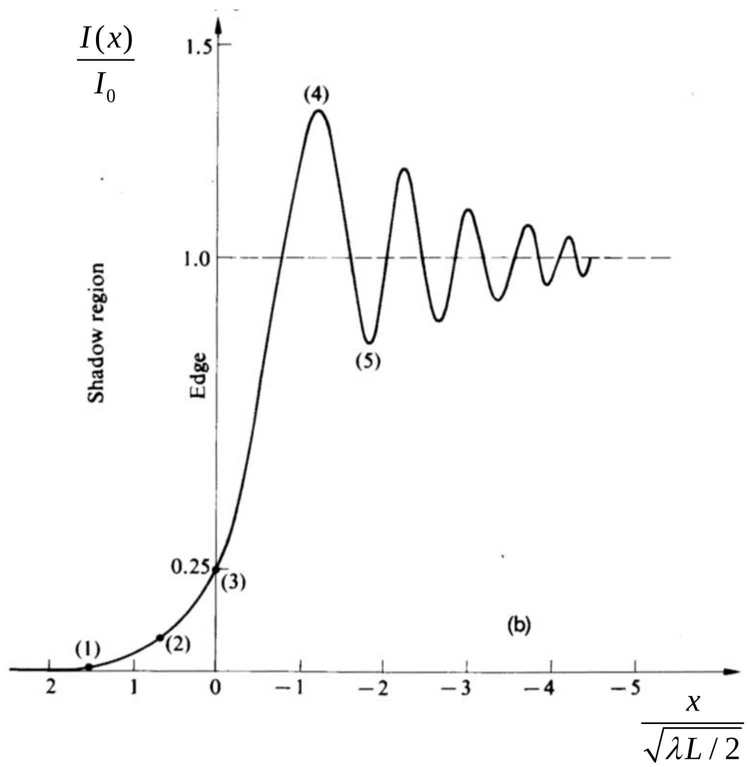

Figure 10.7. Fresnel diffraction pattern of a semi-infinite screen.

We won't cover the theory of Fresnel diffraction here, or give the formula for the diffracted intensity $I(x)$. But, Figure 10.7 shows a graph of $I(x)$. The edge in $I(x)$ is not perfectly sharp no matter how close you are to the screen. Instead, the wave intensity rises from zero up to a maximum in a distance of the order of $\sqrt{\lambda L}$. A number of interference fringes also occur near the edge. The maximum intensity of the first bright fringe is about 35% larger than I_0, where I_0 is the intensity in the absence of the screen.

5. The Arago spot

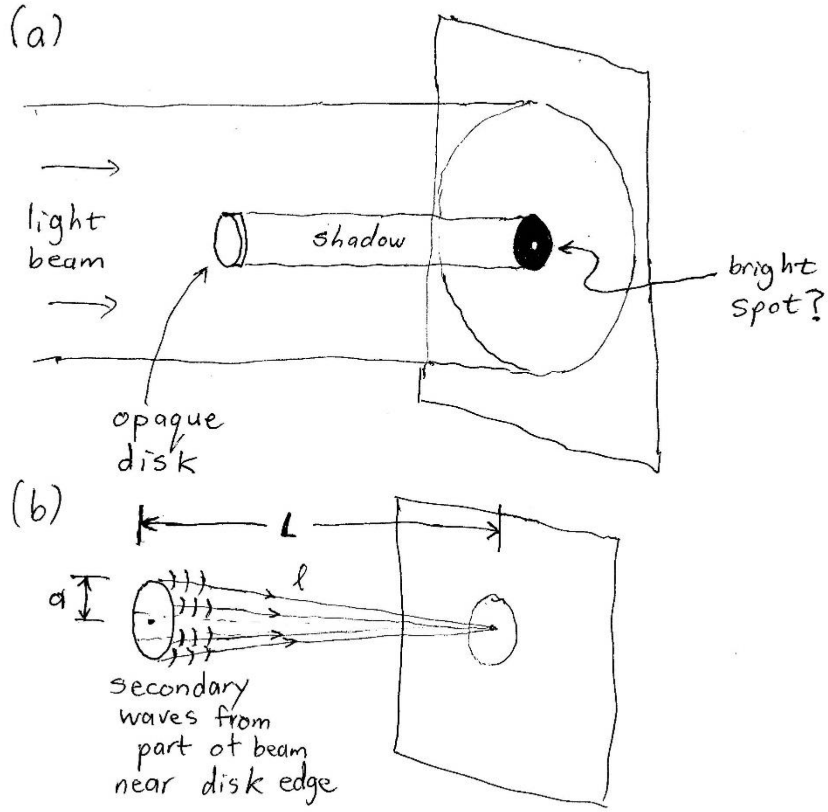

Figure 10.8. (a) Experimental arrangement for the observation of the Arago spot. (b) Explanation of the spot.

From Wikipedia:

> At the beginning of the 19th century, the idea that light does not simply propagate along straight lines gained traction…At that time, many favored Isaac Newton's corpuscular [particle] theory of light, among them the theoretician Siméon Denis Poisson. In 1818 the French Academy of Sciences launched a competition to explain the properties of light, where Poisson was one of the members of the judging committee. The civil engineer Augustin-Jean Fresnel entered this competition by submitting a new wave theory of light.
>
> Poisson studied Fresnel's theory in detail, and, being a supporter of the particle-theory of light, looked for a way to prove it wrong. Poisson thought he had found a flaw when he argued that a consequence of Fresnel's theory was that there would exist an on-axis bright spot in the shadow of a circular obstacle, where there should be complete darkness according to the particle-theory of light. Since the Arago spot is not easily

observed in every-day situations, Poisson interpreted it as an absurd result and that it should disprove Fresnel's theory.

However, the head of the committee, Dominique-François-Jean Arago—who incidentally later became Prime Minister of France—decided to perform the experiment in more detail. He molded a 2 mm metallic disk to a glass plate with wax. He succeeded in observing the predicted spot, which convinced most scientists of the wave-nature of light, and gave Fresnel the win.

Arago's experimental arrangement is illustrated in Figure 10.8(a). The figure shows the bright spot that Poisson showed was predicted by Fresnel's theory, a prediction that he considered absurd. This spot has come to be called the "Arago spot" (or sometimes the "Poisson spot").

An explanation of the Arago spot is illustrated in Figure 10.8(b). According to Huygen's principle, points near the edge of the disk should act as sources of secondary waves. The shadow of the disk is explained in wave theory as due to destructive interference of the secondary waves. However, the exact center of the shadow is a special point: the path length ℓ from the edge of the disk to the observation point is exactly equal to $\sqrt{L^2 + a^2}$ for every point on the periphery of the disk, where a is the radius of the disk. For this point only, the secondary waves in the shadow interfere constructively. Therefore there should be a bright spot at this point.

There are two reasons that the Arago spot is not normally seen in shadows in ordinary situations. One is that the disk must be round to very high accuracy. A more important reason is that the light hitting the disk must be *spatially coherent*. This means that the phase of the light wave is the same at every point near the edge of the disk. This condition is difficult to satisfy with classical light sources, but not with a laser light source.

D. Experimental Procedure

1. Measurement of light beams transmitted through a diffraction grating.

You are provided with a transmission diffraction grating that has 600 lines/mm. The grooved surface of the grating is easily damaged, and it is nearly impossible to remove fingerprints from this surface without further damaging it. For this reason, we have placed the grating in an aluminum frame, along with a glass window that covers and protects the fragile grating surface, as illustrated in Figure 10.9. You should still avoid getting any scratches or fingerprints on the exterior glass surfaces, but we can clean those surfaces if they do get dirty.

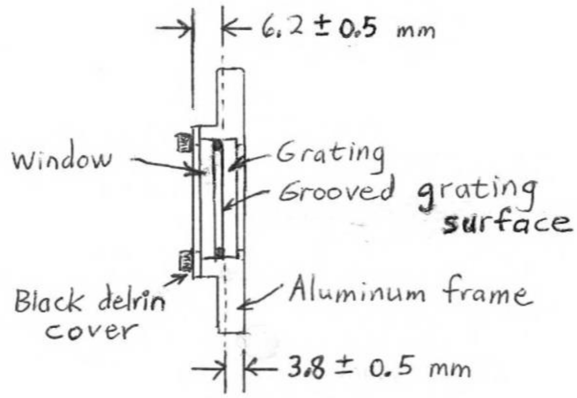

Figure 10.9. Grating and grating mount. The delicate grating surface is one of the interior surfaces in this component so that it is protected from fingerprints and scratches. The grating surface is 3.8 ± 0.5 mm from the large aluminum surface, and 6.2 ± 0.5 mm from the black delrin (plastic) surface, as shown above.

To begin this experiment, put together the experimental components as shown in Figure 10.10. You can place either side of the grating mount facing the laser. Before you turn on your laser, place beam blocks around the apparatus. As you work, check your beam blocks and adjust them as needed to prevent stray laser beams from leaving your work area.

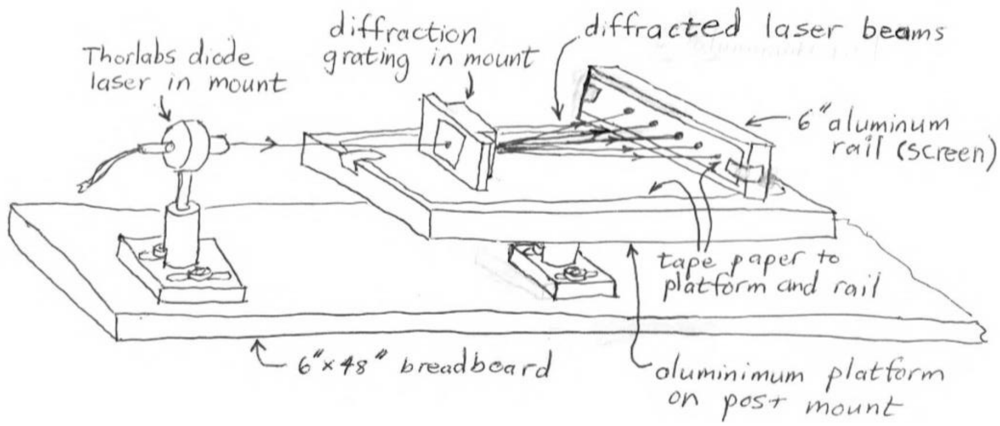

Figure 10.10. Experimental setup for the study of the diffraction of light by a diffraction grating. You must also put beam blocks around the apparatus in order to prevent stray laser beams from leaving your work area.

Connect your laser to its power supply and turn it on. Adjust the laser, diffraction grating, and screen so that the laser beam strikes the grating at normal incidence, and you can see the zeroth, first, and second order diffracted beams. (I'll refer to the aluminum rail as the "screen" from this point forward.) You should see bright spots where the diffracted beams strike the screen. You may also see some dimmer spots. These are due to beams that reflect and/or diffract two or more times from the glass surfaces in the grating assembly. Ignore these dimmer spots.

Observation: Make a qualitative estimate of the relative intensity of the +2, +1, 0, -1, and -2 diffraction orders. (The designation + *vs*. $-$ is arbitrary – just pick one.) (Aside: your eye is a poor judge of laser beam intensities – typically your eye will substantially *underestimate* the ratio of intensities.) Do the +1 and -1 order beams have equal intensities? Do the +2 and -2 order beams have equal intensities? Can you tell if this grating is blazed, or not?

Measurement: Using a method of your choice, measure the diffraction angle of each of the two first-order diffracted beams and each of the two second-order diffracted beams for *normal incidence* of the laser beam on the grating (angle in Figure 10.3 $\theta_i = 0$).

Suggestions: Bolt down your laser and platform mounts to the breadboard so they don't move. Tape pieces of paper to the platform and to the screen. This will allow you to draw lines that you can measure, and to see the spots more clearly. You can set the grating to normal incidence by observing the spot reflected from the grating and adjusting it to go back into the laser. One way to measure the angle of the diffracted beams is to place the screen exactly parallel to the grating mount, measure the screen to grating distance, measure the spot separations directly on the screen, and use trigonometry to determine the angles. For the most accurate measurement, you should take into account the exact position of the grating surface relative to the grating mount, which is shown in Figure 10.9. (For the very most accurate measurement, you would also take into account refraction by the glass in the grating assembly. However this is a very small correction, and you can skip that for this lab.)

Derived quantities:

1) Using equation (10.3), determine the laser wavelength from each of your measured first-order diffraction angles. You can determine the groove spacing *d* from the specified number of lines per mm of your grating.

2) If your measured first-order diffraction angles were not the same, an average of these two angles will provide a more accurate determination of the laser wavelength. Use this average to determine the laser wavelength, and use this wavelength value for the remainder of this lab.

3) Using equation (10.3) and your measured wavelength from step (2), predict the diffraction angle for the second order beams. Does your prediction match your measured values for this angle?

4) Using equation (10.4) and your measured wavelength from step (2), predict the diffraction angle for the first-order beams when you tilt the grating so that the laser beam strikes it at non-normal incidence with $\theta_i = 12°$ (angle illustrated in Figure 10.3).

Measurement: Using a method of your choice, measure the diffraction angle of each of the two first-order diffracted beams for *non-normal incidence* of the laser beam on the grating, with $\theta_i = 12°$.

Compare your measured angles to the values that you predicted in step (4) above.

2. Fresnel diffraction of a semi-infinite screen

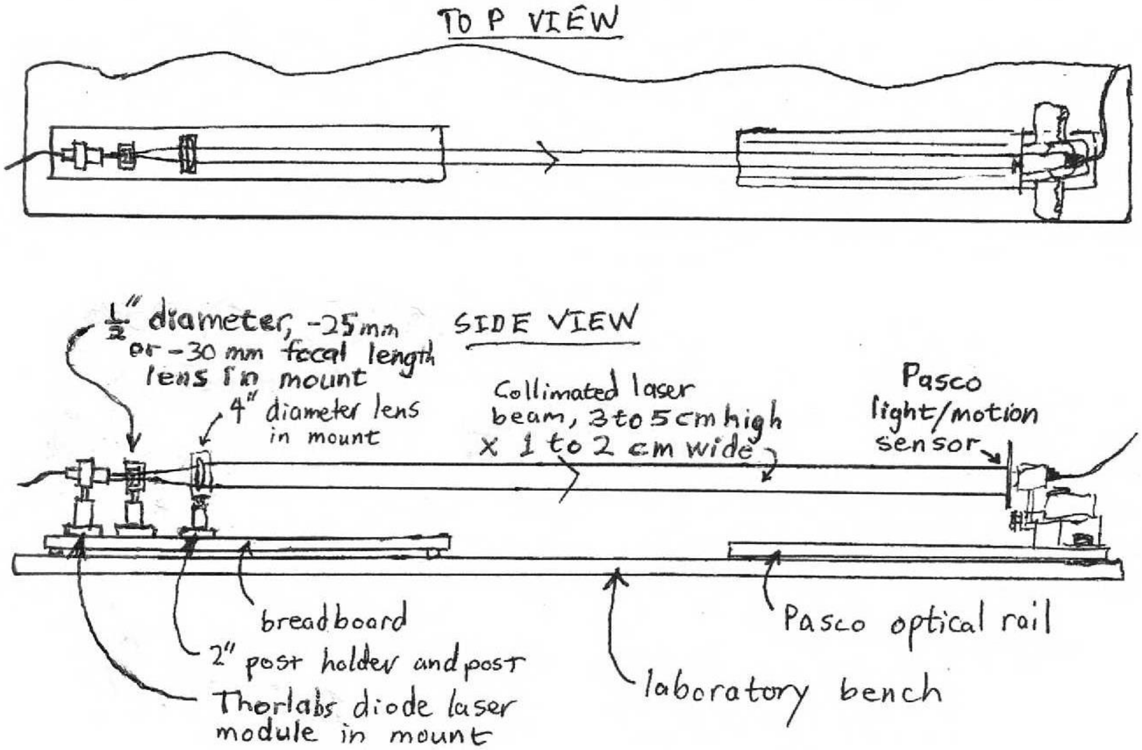

Figure 10.11. Optical bench set-up for Fresnel diffraction experiments.

Remove the platform from your breadboard and position the breadboard near the left hand-side of your workbench. Your goal at this point is to make a collimated laser beam that is a few cm in size and shines toward the right end of your workbench. In order to do this, place the 4" diameter lens about 30 cm in front of your laser. Use the 2" post and post mount for the lens; otherwise the lens will probably be too high compared to the laser. Place a beam stop near the end of the breadboard opposite the laser, and adjust it as necessary to prevent the laser beam from leaving your work area as you proceed. Plug your laser into its power supply and turn it on. Adjust the lens and laser mounts so that the laser beam shines through the middle of the lens. Next, position your -25 mm or -30 mm focal length lens in the laser beam just in front of the laser. (Your lenses will have lower aberration if you place the curved sides toward the parrallel input and output beams, as shown in the figure.) Adjust the laser and two lens positions so that the laser beam goes through the middle of both lenses, and then heads straight down towards the opposite end of the workbench, and so that the laser beam is collimated (doesn't change size or shape much with distance from the 4" lens.) In this condition, you set-up should look like the left half of Figure 10.11, the two lenses should be about 22 cm apart, and the beam shape should be approximately rectangular, about 3-5 cm high and 1-2 cm wide, depending on which laser and lenses that you have.

You will study the intensity of this laser beam as a function of position using the same Pasco apparatus that you used in Experiment 9. To do this, set up the Pasco optics rail with the light/motion sensor as you did last week. You will not be using the Pasco laser or slit accessory this week. Position the Pasco optical rail so that the light/motion sensor is near the opposite end of your workbench from the laser, as shown in Figure 10.11. Adjust the position of your components so that the collimated laser beam strikes the light sensor.

Next, place the thin aluminum plate in the filter holder and mount, and position it just after your 4" lens, so that it blocks ½ of the laser beam as shown in Figure 10.12. Just after the plate, the beam should have a sharp, vertical edge due to obstruction by the plate, and this edge should be in the center of the unobstructed beam. The part of the beam that is near the edge will then provide an experimental realization of the diffraction of light by a semi-infinite screen depicted in Figure 10.6 (top view).

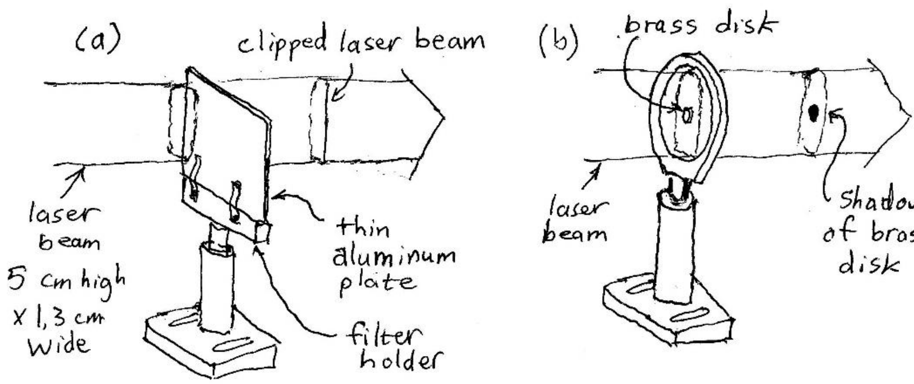

Figure 10.12. (a) Placement of the aluminum plate into the laser beam, for study of the Fresnel diffraction of a semi-infinite screen. (b) Placement of the brass disk in the laser beam.

Using a beam block as an observation screen, observe the beam at various distances from the plate. The edge should look very sharp near the plate. See if you can observe evidence of diffraction further away from the plate.

Next, make measurements of the Fresnel diffraction pattern $I(x)$ of the plate edge using the Pasco light/motion sensor. You will take measurements exactly as you did last week for the single and multiple slit experiments. For best results, you should tweak the plate so that the edge is parallel to the slit in front of the light sensor. Take measurements for the following distances between the plate and the detector: 1 cm, 30 cm, and 2.5 m. (For the first two, you'll place the plate on the Pasco rail, not on the breadboard.)

<u>Analysis of this data</u>

(i) For any plate positions where you can see one or more fringes, measure the ratio of the intensity of the light at the first maximum to the intensity of the light at the same position with the plate removed. See if you get the same ratio of maximum intensity to I_0 that is shown in the theory curve of Figure 10.7.

(ii) For any plate positions where you can see two or more fringes, measure the distance between the first two fringe maxima. See if you get the same spacing that is shown in the theory curve of Figure 10.7.

Arago's spot, or not?

Remove the aluminum plate from the beam path. Replace it with the 6.32 mm diameter brass disk as illustrated in Figure 10.12(b). Place the disk just after the lens. Your goal is to realize Arago's experiment, as shown in Figure 10.8(a).

Observe the shadow of the disk. Make qualitatively correct drawings of the appearance of the shadow of the disk for a few distances from the disk. Alternatively, you may use cell phone pictures of the spot, if you’re able to get good ones. You might choose a set of distances such as 4 cm, 16 cm, 60 cm, and 240 cm. If you are careful, you may remove the Pasco optics rail and observe the spot all the way to the wall of the room. It may help to turn the room lights off for this part of the experiment.

Comment on your results. Do you observe the Arago spot, or not? If so, is it there for all distances from the disk, or only for specific distances? Do you see any other structure in the light beam, either just outside the disk, or in the shadow region? If so, can you give any explanation for it? Do you think there would be any way to explain your results with the particle theory of light that was initially favored by Poisson, and many others?

- END OF EXPERIMENT 10 -