12. Michelson Interferometer

A. Objectives

- Observe the concentric ring interference pattern of two point sources. Observe how the pattern changes with the path length difference.
- Measure the wavelength of a helium-neon laser.
- Measure the index of refraction of air.

B. Equipment required

1. Michelson interferometer, wood platform, and foam pad (optional)

2. Helium neon laser with mounting brackets

3. Optical breadboard

4. Two dielectric mirrors in mounts

5. Two right angle post clamps, ½" posts, and post holder

6. -25 mm or -30 mm focal length lens, ½" diameter, in lens mount

7. Pedestal post mount and clamping fork for lens

8. Filter holder, mount, and white cardboard pieces to use as a screen

9. Hand vacuum pump and gas manifold

10. Gas cell

C. Introduction

Illumination of a Michelson interferometer by a plane wave

In this lab, we'll carry out experiments with a Michelson interferometer, illustrated in Figure 12.1. Let us suppose the interferometer is illuminated by an ideal plane wave of light. The beam of light enters the interferometer and hits a beamsplitter - a slab of glass that is half-silvered (partially transparent) on one side. When the beam hits the half-silvered surface of the beam splitter, it is divided into two beams. One beam is reflected from the beamsplitter and continues towards a mirror (M1 in Figure 12.1). The other beam is transmitted through the beamsplitter and continues towards another mirror (M2 in Figure 12.1). We'll call these "beam 1" and "beam 2", respectively. Beam 1 reflects from M1 and encounters the beam splitter again. It is again divided into two beams. One beam is transmitted through the beam splitter towards the observation screen. The other is reflected back towards the source of light (not shown in the figure). Similarly beam 2 reflects from M2 and encounters the beam splitter again. It is again divided into two beams. One beam is reflected from the beam splitter towards the observation screen. The other is transmitted through the beam splitter back towards the source of light (not shown in the figure).

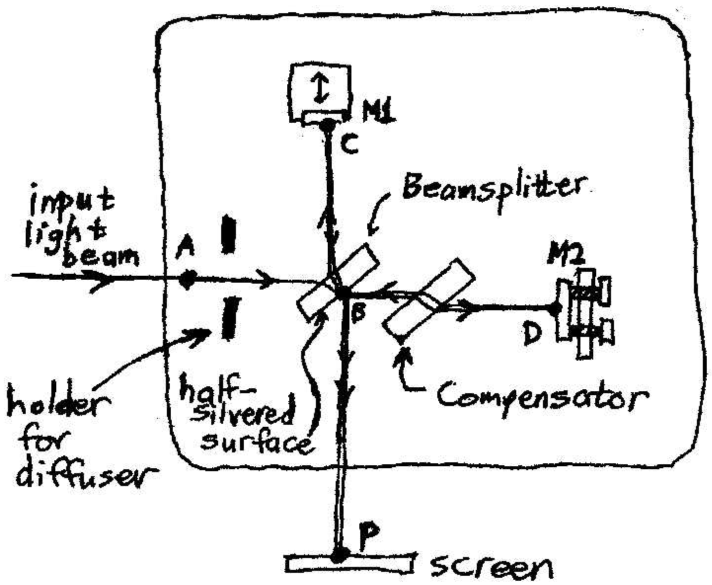

Figure 12.1. Michelson interferometer

The light that hits the screen is a superposition of two waves – one from beam 1, which follows the path A-B-C-B-P in Figure 12.1, and one from beam 2, which follows the path A-B-D-B-P in Figure 12.1. Suppose that the input beam is an ideal plane wave, and that its electric field near point A is

$$E = E_0 \cos(ks - \omega t) \tag{12.1}$$

where s is the distance measured along the beam path from point A. Also, suppose that the beam splitter produces equal intensities in the transmitted and reflected beams. It follows that the amplitude of beams after their first encounter with the beamsplitter is $E_0/\sqrt{2}$, and that the amplitude of the beams after their second encounter with the beamsplitter is $E_0/2$. Finally, suppose that the two mirrors are adjusted so that each beam is reflected exactly back in itself. Then, the electric field at point P on the observation screen is

$$E_P = \frac{E_0}{2}\cos(ks_1 - \omega t) + \frac{E_0}{2}\cos(ks_2 - \omega t) \tag{12.2}$$

where s_1 is the total optical length of the path A-B-C-B-P, and s_2 is the total optical length of the path A-B-D-B-P. That is, the field at the screen point P is the *superposition* of the fields arising from beam 1 and beam 2. If you work it out, the total intensity at point P is

$$I_P = \varepsilon_0 c \left\langle E_P(t) \right\rangle^2 = I_0 \cos^2\left(\frac{k\left(s_1 - s_2\right)}{2}\right) \tag{12.3}$$

where the brackets denote a time average, and where $I_0 = \frac{1}{2}\varepsilon_0 c E_0{}^2$ is the intensity of the input light beam.

We see in equation (12.3) the effects of *interference* between the two beams. At point P, they have a phase difference $\Delta\varphi = k\left(s_1 - s_2\right),$ due to the fact that they have travelled a different path length. If the phase difference $k\left(s_1 - s_2\right) = 2n\pi,$ where *n* is an integer, the two beams arrive in phase (modulu 2π) at point P, and we have *constructive interference.* Their amplitudes $E_0 / 2$ add to produce a resultant amplitude $E_0,$ and the intensity of the beam at the point P is the same as the input intensity. One the other hand, if the phase difference $k\left(s_1 - s_2\right) = \left(2n+1\right)\pi,$ where *n* is an integer, the two beams arrive exactly out-of-phase (modulu 2π) at point P, and we have *destructive interference.* Their amplitudes $E_0 / 2$ add to produce a resultant amplitude zero, and the intensity of the beam at the point P is zero. For other path length differences we have a $\cos^2\left(\frac{\Delta\varphi}{2}\right)$ variation of the intensity that is characteristic of interference.

The Michelson interferometer often includes a glass plate called a *compensator*. It is identical to the beamsplitter, except that it does not have a half-silvered surface. The purpose of the compensator is to make it so that both beams travel through the same thickness of glass. This is important when broadband sources of light are used, since otherwise the phase difference between the two arms would depend on wavelength due to the dispersion of the glass. However, this is not important for the experiments we'll do today.

The Michelson interferometer usually includes one mirror that is mounted on a translation stage. This allows us to move the mirror, so that we can scan the path length difference between the two arms. At least one mirror will have tilt adjustments, so that we can align the output beams to be parallel.

The Michelson interferometer is very useful for measuring effects that cause a small phase shift of light. Michelson and Morley first used the interferometer to try to detect the motion of the earth through the "ether", which was a hypothetical medium which supported the vibrations of light beams. Their null result convincingly demonstrated that there is no such thing as "ether". Einstein's special theory of relativity was based in part on this result. We'll see another example of the measurement of a small phase shift in this lab.

Illumination of a Michelson interferometer by a point source

Let us suppose that we place a point source of light *S* near the interferometer input, as illustrated in Figure 12.2(a). As before, light from the source will be split into two by the interferometer, then split in two again, and then fall on the screen.

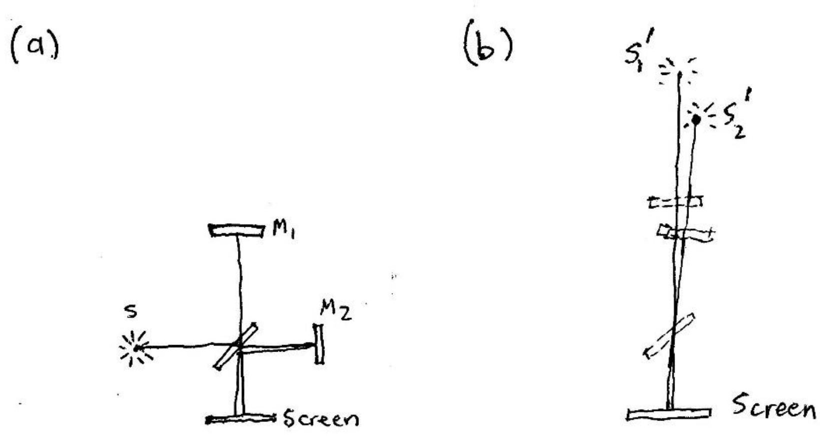

Figure 12.2. (a) Screen illuminated by Michelson interferometer and light source *S*. (b) Optically equivalent illumination of screen by sources S_1' and S_2'.

Imagine that you place your eye at the position of the screen and look back. You'll see the point source twice – once reflected from M1, and once reflected from M2. Thus, from the perspective of the screen, there are *two* point sources of light – one being the virtual point source from M1, and the other being the virtual point source from M2. That is, illumination of the screen by the interferometer and point source *S* is equivalent to illumination of the screen by the two virtual point sources *S*1' and *S*2', as shown in Figure 12.2(b).

The two point sources are usually not exactly at the same place. For one thing, the path lengths in the two arms of the interferometer are probably not identical, so one source appears to be in front of the other. Also, if the mirrors do not reflect the beams exactly back on themselves, then there will also be a sideways displacement of the two sources as shown in Figure 12.2(b).

Let's suppose there is no sideways displacement, and focus on the case where source S_2' appears to be in front of source S_1', as shown in Figure 12.3. The field on the screen will be the linear combination of the fields from S_1' and S_2', and it will show the effects of interference between them. Now, let's focus on a particular phasefront Φ_1 of source S_1'. Since it is a point source, the phasefront will be a section of a sphere of some radius r_1. Similarly the phasefront Φ_2 of source S_2' will be a section of a sphere of some different radius r_2. Let's suppose that these two phasefronts intersect on the line joining the two sources, as shown in Figure 12.3. This assumption is equivalent to assuming that the two sources are in phase on that line. Thus, we would see a bright spot at that point due to constructive interference. As we move away from that point on surface Φ_1, the two fields will begin to show a phase difference, because phasefront Φ_2 has a different radius of curvature, and therefore begins to deviate from surface Φ_1. When the phase difference between the two fields reaches π, the fields interfere destructively, which produces a dark ring around the bright center spot. As we keep going further outward on surface Φ_1, the phase difference between the two fields eventually reaches 2π. At these points we see a bright ring formed. You can view this as a case of constructive interference due to the overlap of the phasefront Φ_1 with the phasefront Φ_2' that has radius one wavelength larger than surface Φ_2. As we continue outward, we see an alternating pattern of bright and dark rings, and the phase difference further increases through values that give constructive and destructive interference. So the bottom line is that the interference pattern of two point sources is a bull's eye pattern, as shown in Figure 12.3

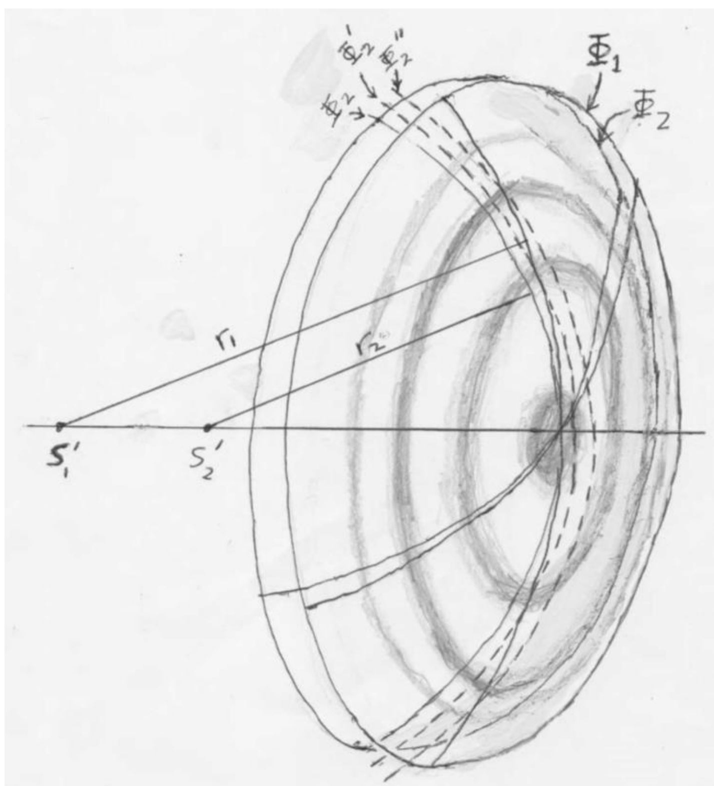

Figure 12.3. Interference of two point sources.

Now, let's suppose that we have both a longitudinal and a lateral shift in the position of two sources, as illustrated in Figure 12.2(b) What would the interference pattern look like then? It is easy to determine this, if you notice that you can still draw a line between the two sources, and that there will still be a bull's eye pattern centered on that line. So, what you'd be doing is looking off to the side of that bull's eye pattern. Possibly, you'd be so far off to the side that you wouldn't see the central spot, only the rings further out.

Experimental Procedure

Observation of the interference of two point sources

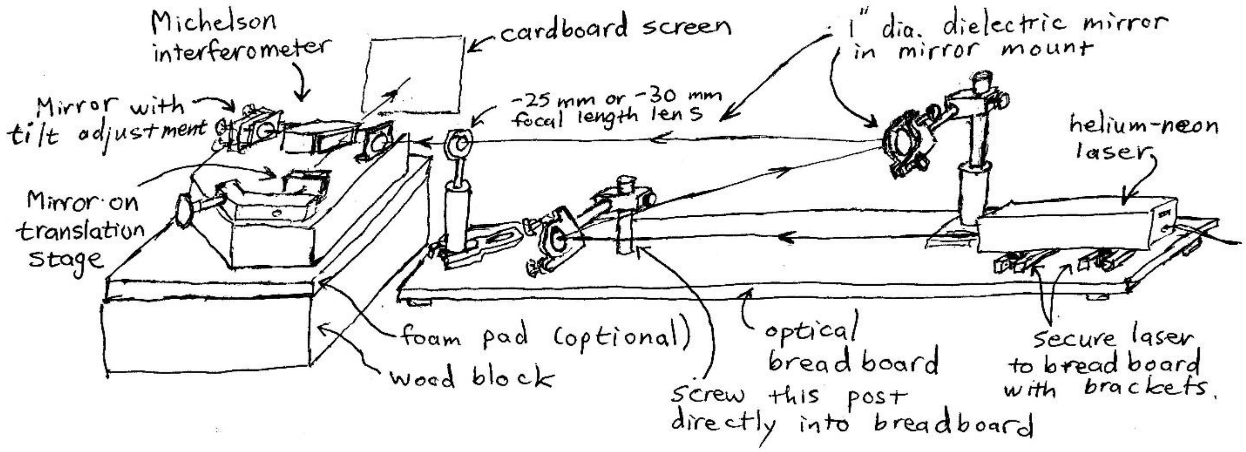

Figure 12.4. Experimental set-up.

We have two models of interferometer, a Pasco model, and an older Ealing (grey) model. They are similar enough that I won't give separate instructions, except for a few points below.

The interferometer optics have high optical quality. The same is true of the windows on the gas cell. **For this reason, please take extra care with the interferometer optics and the gas cell windows. Do not touch the surface of the optics, or bring any hard objects into contact with them. When you're done with the gas cell, disconnect it from the gas manifold with the quick disconnect fitting, and set it in its protective plastic box.** We appreciate your efforts to help us maintain the equipment in this lab.

The experimental set-up is shown in Figure 12.4. To get started, set the interferometer on the wood block, and position it so that the interferometer output is directed away from you. If you want more beam height, or think you are having problems with vibration, you can also put a foam pad between the wood block and the interferometer. The interferometer component (diffuser) holder will be left empty throughout this experiment. Place a filter holder with a piece of white cardboard for use as a screen so that you can see the output beams. Alternatively, you may use one of the beam blocks for this.

Place the breadboard near the wood block, and secure the helium neon laser to the breadboard near the end opposite to the interferometer. Leave the lens out for now.

Throughout this experiment, follow all laser safety instructions that we've discussed previously. For the following step, leave the laser off.

Install the two mirrors with their mounts as shown in Figure 12.4. One mirror should be placed near the interferometer end, in a position that will intercept the beam. <u>Leave enough room so that you can fit the lens mount at the end of the board</u>, as shown in the figure. The beam height from the laser will be quite low. In order to get the mirror this low, you'll need to use a right angle post clamp mounted onto a post screwed directly into the breadboard (using a ¼-20 set screw), as shown in the figure. The other mirror should be placed so that it is at the same height as the mirrors in the interferometer.

You should be especially careful with the laser beam when first aligning the mirrors. Don't allow the laser beam to leave your work area. <u>Keep your face out of the vertical plane that contains the laser beams</u>. View the apparatus from the front of the apparatus, not from above or from the ends, until you're sure the laser beams are not leaving the work area. Place beam stops on each end of the set up, and try to set the initial positions of the mirrors so that the laser beam will not go above the beam stops. When you put the laser beam onto a mirror, check immediately where that beam is going. If it is leaving the work area, move the mirror until the beam is not leaving the work area before proceeding further.

Being careful to observe the above precautions, turn the laser on. If you do not see a laser beam, you may need to open the shutter at the front of the laser. Position the first mirror so that the laser beam strikes the mirror near its center. Then move that mirror so that the beam strikes the second mirror somewhere near the center. Finally, direct the beam from the second mirror into the interferometer. You can make fine adjustments of the laser beam direction with the mirror mount screws. Again, the lens should be out at this point.

Position the laser, breadboard, and interferometer so that the beam enters the middle of the front interferometer component holder, and strikes each interferometer mirror near its center. You should now see two bright spots on the screen, which are the reflections from the two mirrors. One of the two interferometer mirrors will have tilt adjustments. Use these adjustments to make it so that the two spots overlap.

Next, put the negative lens into the beam, and position it so that beam enters and exits the interferometer at about the same place as before. The lens will produce a diverging beam, and your goal is to get a beam which is ½" to 1" diameter at the output. A lens placement about 15 to 20 cm in front of the beamsplitter should be about right.

At this point, you should see an interference pattern on the screen that is some part of a concentric ring pattern. This is the interference pattern of two points sources, as discussed in the introduction. The source looks like a point source, because the negative lens produces a virtual point source as illustrated in Figure 12.5.

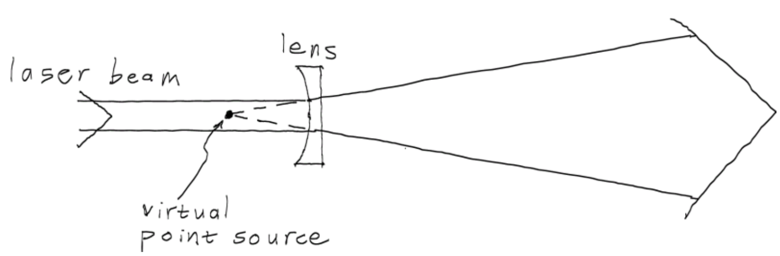

Figure 12.5. Formation of a virtual point source by the negative lens.

If you see closely spaced curved fringes, that means you're not seeing the center of the pattern, and that you should adjust the mirror tilt. When you move the mirror tilt, you move one virtual point source up and down or sideways relative to the other point source. You want to adjust it until the bull's eye pattern is centered, which means that you have adjusted the line through the two sources to hit the center of the screen.

Your interferometer has one mirror that can be translated with a micrometer screw adjustment. Turn the adjustment slowly. You should be able to see the fringes move toward or away from the center. Think about why that is happening until you understand it.

Next, turn the screw through a much larger adjustment. You should be able to see the spacing of the fringes get larger or smaller. Why is this happening? See if you can make the fringe spacing so large that you see only one large central spot- i.e. the entire interference pattern goes bright or dark. (This may not be possible for some of the interferometers. If you are using the Pasco interferometer, you may need to unbolt the mirror from the translation stage, move it, and bolt it back down.) What would be the relative location of virtual sources S_1' when this condition is realized?

Measurement of the laser wavelength

The scanning Michelson interferometer is a standard wavelength measurement device. (For instance, you could buy this one.) To do this, you simply move the mirror by a calibrated amount and count fringes. Each new fringe corresponds to a change in the path length difference of one wavelength. Since the light beam transverses the distance between the beamsplitter and the mirror twice (once each way), you get a new fringe for each ½ wavelength of motion of the mirror.

Set your translator back to a position where you see at least two or three concentric fringes. Then, measure the displacement of the mirror needed to produce 50 fringes. From your measurement, determine the wavelength of the laser.

Notes:

(i) The screw has a mechanical advantage over the translation stage. For the Ealing interferometer, the mechanical advantage is a factor of 5, *i.e.* the translation stage moves 1/5 of the distance moved by the micrometer. For the Pasco interferometer, the specified stage motion is 25 microns per micrometer revolution, or 1 micron of stage motion for each small division of the micrometer.

(ii) The screw will have some backlash if you change direction of motion. For this reason, the measurement should be taken as part of a continuous motion of the screw in one direction only.

(iii) Accuracy will be improved if you make a pencil mark at a position two or three fringes out from the center, and count each fringe when it moves past that mark. You can also average several measurements, or count more fringes for greater accuracy.

Measurement of the index of refraction of air

The Michelson interferometer is very sensitive to any effect that causes the phase of light in one arm to change relative to the phase in the other arm. In this part of the lab, we'll put this sensitivity to use to measure the index of refraction of air.

Recall that the wavelength of light in a medium with index of refraction *n* is $\lambda = \lambda_0 / n$, where λ_0 is the wavelength in vacuum. The phase change due to propagation through a length ℓ of the medium is $\frac{2\pi}{\lambda}\ell = \frac{2\pi}{\lambda_0}n\ell$. Thus, if we change the index by an amount Δn over a length ℓ in one arm of the interferometer, we change the relative phase of the two arms by

$$\Delta\phi = 2 \cdot \frac{2\pi}{\lambda_0}\Delta n\,\ell, \tag{12.4}$$

where the factor of two accounts for the fact that we pass through the medium twice. Since the interferometer is very sensitive to phase, it can measure a very small Δn.

You're supplied with a gas cell and a hand vacuum pump with a gas manifold. The internal length of the cell is $\ell = 38.5$ mm. The manifold includes a vacuum gauge and a needle valve that can be used to vent the cell to atmosphere. The needle valve can regulate air flow down to very low levels. It is delicate and will be damaged if you crank down hard to close it. For this reason **do not overtighten the needle valve. Close it with a modest force only.** The manifold also includes a quick-connect for the vacuum connection to the cell.

Secure the cell into the interferometer using the two tapped 8-32 holes that match the base of the cell. Set up the gas manifold near the interferometer, and connect the cell to the manifold with the quick-connect. Close the needle value. Try pumping air out of the cell with the hand pump, and watch the fringes as you do so. You should see the fringes move as you remove the air. Then, try to slowly open the needle value. You should be able to vent the cell very slowly, so that it takes many seconds to reach atmospheric pressure, and so that you can easily count fringes.

The gauge measures pressure relative to atmospheric pressure, which is 1 atm = 101.3 kPa (kiloPascal). Thus, a gauge reading of -72 kPa means that the pressure in the cell is $101 - 72 = 29$ kPa. The gauge that is included with the hand pump is probably not very accurate. The gauge that is included with the manifold measures pressure differences to an accuracy of 1%. The hand pump can only reach pressures of 20% to 30% of atmospheric pressure.

To make your measurement, remove as much air from the cell as you can with the hand pump, record your pressure, slowly vent the cell to air with the needle valve, and count fringes as the pressure climbs. To obtain the most accurate measurement, stop the change a little short of atmospheric pressure and take a second gauge reading. Then, you'll have a measured number *N* of fringes, for a certain measured pressure change ΔP. The measured number of fringes can be converted to a phase change $\Delta\phi,$ since each fringe corresponds to 2π of phase. Include fractional fringes in your measurement if you're able to.

From your measurements and equation (12.4), obtain $\Delta n\left(\Delta P\right)$. To find the index of refraction of air, use linear extrapolation of your result to a pressure change of 1 atmosphere. Also note that you're measuring a change in the index from that of the vacuum, $n_{vacuum} = 1$.

- THE END! -