Jai Prasadh
Pricing Model
Stickwidit

First, we try to fit the data we have on pricing at various square-footages. I start by loading the data (estimated price points) for individual stickers, as sheet-stickers are priced differently.

In[9]:=
```
SingleStickerPricingData = {{1, 14}, {1.3`, 13.5`}, {1.6`, 13.4`}, {2, 13.2`}, {2.5`, 12.6`},
   {3, 12.1`}, {3.5`, 11.9`}, {4, 11.6`}, {4.5`, 11.3`}, {5, 11}, {6, 10.7`}, {7, 10.3`},
   {8, 10}, {9, 9.5`}, {10, 9}, {13, 8}, {16, 7}, {19, 6.8`}, {21, 6.7`}, {24, 6.3`},
   {27, 6}, {30, 5.9`}, {32, 5.7`}, {37, 5.3`}, {42, 5}, {47, 4.9`}, {53, 4.7`}, {60, 4.3`},
   {70, 4.1`}, {80, 4}, {90, 3.97`}, {106, 3.95`}, {120, 3.92`}, {140, 3.85`}, {160, 3.7`},
   {180, 3.68`}, {200, 3.63`}, {213, 3.6`}, {250, 3.5`}, {265, 3.4`}, {280, 3.3`},
   {320, 3.2`}, {360, 3.1`}, {400, 3.05`}, {420, 3}, {460, 2.85`}, {500, 2.71`},
   {524, 2.7`}, {600, 2.6`}, {700, 2.53`}, {800, 2.5`}, {900, 2.49`}, {1000, 2.48`}};
```

I guess an exponential solution, so I begin experimenting with this idea.

In[10]:=
```
Clear[A, beta, c, x, r];

model1 = NonlinearModelFit[SingleStickerPricingData, A Exp[-beta * x] + c, {A, beta, c}, x];
model1["ParameterTable"]
```

Out[12]=

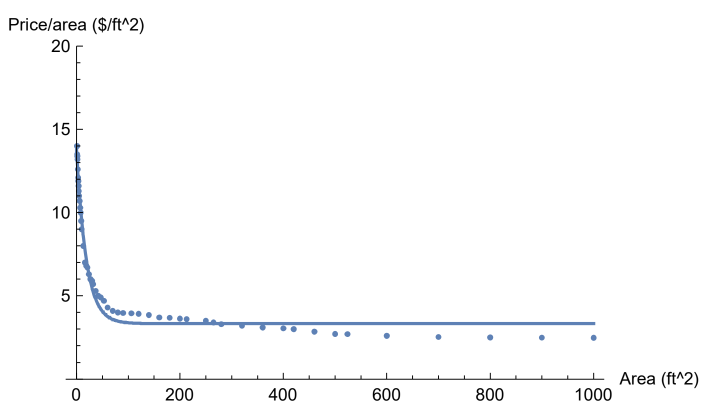
	Estimate	Standard Error	t-Statistic	P-Value
A	10.451	0.216815	48.2024	1.39441×10^{-43}
beta	0.0529966	0.00287586	18.4281	6.1349×10^{-24}
c	3.3311	0.0979234	34.0174	3.08323×10^{-36}

Set the parameters to those given by the fit, and plot the result versus the data.

In[13]:=
```
Normal[model1]
Show[Plot[model1[x], {x, 0, 1000}, PlotRange → {0, 20}],
 ListPlot[SingleStickerPricingData], AxesLabel → {"Area (ft^2)", "Price/area ($/ft^2)"}]
```

Out[13]= $3.3311 + 10.451\, e^{-0.0529966\, x}$

Out[14]=



This model does not do what we want, so try again, this getting rid of the constant c.

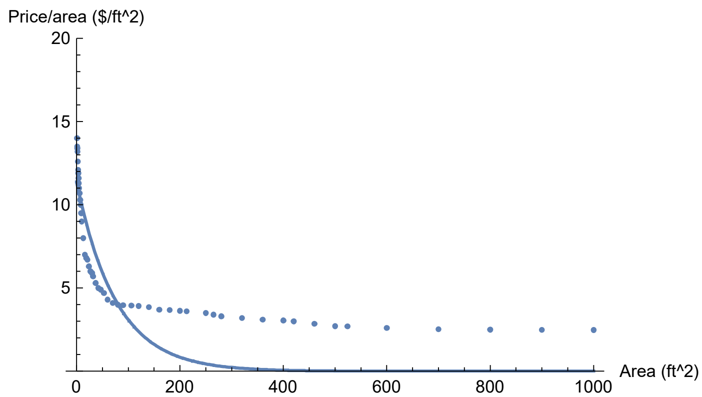
In[15]:=
```
model2 = NonlinearModelFit[SingleStickerPricingData, A Exp[-beta * x] , {A, beta}, x];
model2["ParameterTable"]
Normal[model2]
Show[Plot[model2[x], {x, 0, 1000}, PlotRange → {0, 20}],
 ListPlot[SingleStickerPricingData], AxesLabel → {"Area (ft^2)", "Price/area ($/ft^2)"}]
```

Out[16]=

	Estimate	Standard Error	t-Statistic	P-Value
A	11.3743	0.575321	19.7703	1.44466×10^{-25}
beta	0.013005	0.0021052	6.17757	1.07982×10^{-7}

Out[17]= $11.3743\, e^{-0.013005\, x}$

Out[18]=



This isn’t what we want, either. Try again, this time raising x to some exponent.

In[19]:=
```
model3 =
  NonlinearModelFit[SingleStickerPricingData, A Exp[-beta * (x^c)], {A, beta, c}, x];
model3["ParameterTable"]
Normal[model3]
Show[Plot[model3[x], {x, 0, 1000}, PlotRange → {0, 20}],
 ListPlot[SingleStickerPricingData], AxesLabel → {"Area (ft^2)", "Price/area ($/ft^2)"}]
```

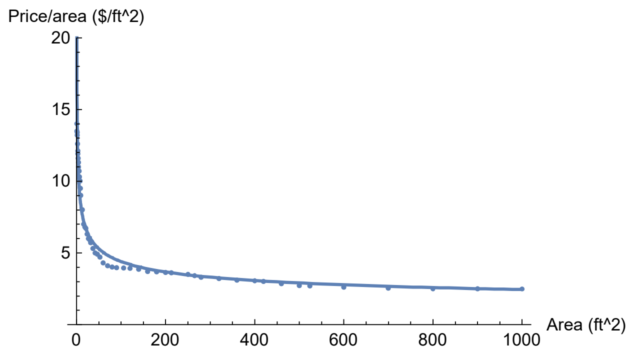
NonlinearModelFit: Failed to converge to the requested accuracy or precision within 100 iterations.

Out[20]=

	Estimate	Standard Error	t-Statistic	P-Value
A	0.000238827	0.00264976	0.0901315	0.928543
beta	−11.1107	11.079	−1.00285	0.32076
c	−0.0267728	0.0287197	−0.932208	0.355709

Out[21]= $0.000238827\, e^{\frac{11.1107}{x^{0.0267728}}}$

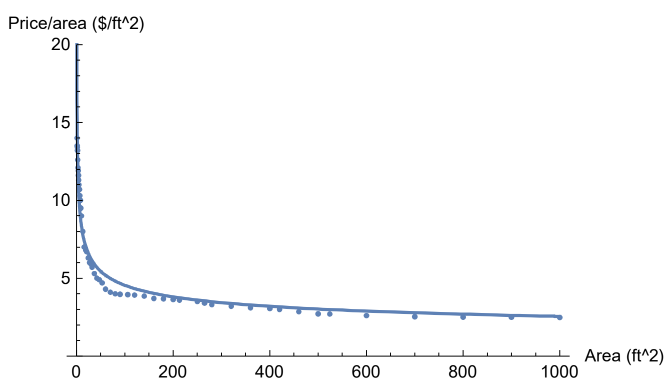
Out[22]=



This regression fails to converge, but gives us a good idea of what the parameters should be. We will try plotting these rough estimates.

In[23]:=
```
A0 = 0.00022;
beta0 = -11.2;
c0 = -0.026;
price[x_] := A0 Exp[-beta0 * (x^c0)];

Show[Plot[price[x], {x, 0, 1000}, PlotRange → {0, 20}],
 ListPlot[SingleStickerPricingData], AxesLabel → {"Area (ft^2)", "Price/area ($/ft^2)"}]
```

Out[27]=



This isn't a good model, since it is hypersensitive to each of its parameters and responds in unintuitive ways, so it isn't easy to make pricing changes. Try a model that is not exponential.

In[28]:=
```
model4 = NonlinearModelFit[SingleStickerPricingData, A (1 + x) ^r, {A, r}, x];
model4["ParameterTable"]
Normal[model4]
Show[Plot[model4[x], {x, 0, 1000}, PlotRange → {0, 20}],
 ListPlot[SingleStickerPricingData], AxesLabel → {"Area (ft^2)", "Price/area ($/ft^2)"}]
```

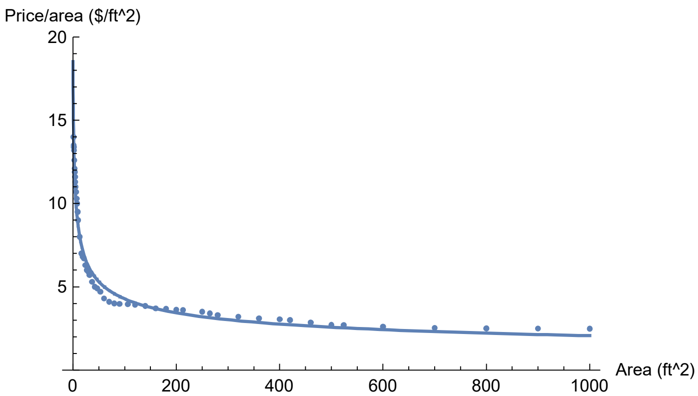
Out[29]=

	Estimate	Standard Error	t-Statistic	P-Value
A	18.5167	0.281194	65.8502	5.19518×10^{-51}
r	−0.317688	0.00598635	−53.0687	2.67483×10^{-46}

Out[30]=

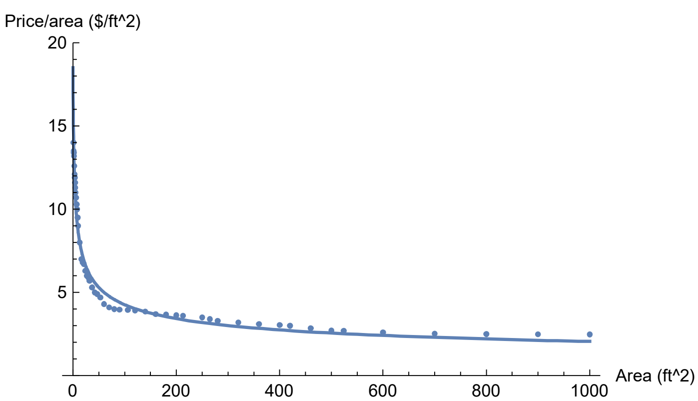
$$\frac{18.5167}{(1 + x)^{0.317688}}$$

Out[31]=



In[32]:=
```
A0 = 18.5;
r0 = -.318;
price[x_] := A0 (1 + x) ^r0;

Show[Plot[price[x], {x, 0, 1000}, PlotRange → {0, 20}],
 ListPlot[SingleStickerPricingData], AxesLabel → {"Area (ft^2)", "Price/area ($/ft^2)"}]
```

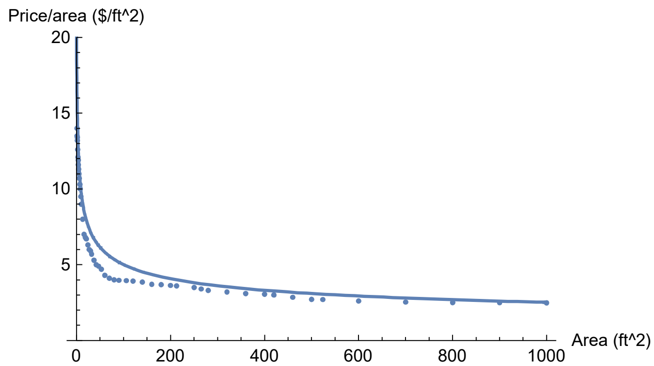
Out[35]=



In[36]:=

```
A0 = 20;
r0 = -.3;
price[x_] := A0 (1 + x)^r0;

Show[Plot[price[x], {x, 0, 1000}, PlotRange → {0, 20}],
 ListPlot[SingleStickerPricingData], AxesLabel → {"Area (ft^2)", "Price/area ($/ft^2)"}]
```

Out[39]=



This model is much better since it is simpler and responds intuitively to changes in the constant A. Some additional notes are that this model is only valid for the regime in which order area > 1 sq-ft since as order square footage becomes very small, the minimum order requirements come into play.

Now, we must fit the data for the sheet stickers. I again begin by loading that data.

In[40]:=

```
SheetStickerPricingData =
  {{0, 20}, {1, 18}, {1.3`, 17.7`}, {1.6`, 17.3`}, {2, 17}, {2.5`, 16.8`}, {3, 16.2`},
   {3.5`, 15.6`}, {4, 15}, {4.5`, 13}, {5, 12}, {6, 11}, {7, 10.5`}, {8, 10}, {9, 9.8`},
   {10, 9.5`}, {13, 9.2`}, {16, 9}, {19, 8.5`}, {21, 8}, {24, 7.8`}, {27, 7.5`},
   {30, 7.4`}, {32, 7.3`}, {37, 7.1`}, {42, 7}, {47, 6.9`}, {53, 6.7`}, {60, 6.4`},
   {70, 6.1`}, {80, 6}, {90, 5.5`}, {106, 5}, {120, 4.8`}, {140, 4.55`}, {160, 4.5`},
   {180, 4.45`}, {200, 4.4`}, {213, 4.2`}, {250, 4.1`}, {265, 4}, {280, 3.7`},
   {320, 3.6`}, {360, 3.58`}, {400, 3.54`}, {420, 3.5`}, {460, 3.3`}, {500, 3.25`},
   {524, 3.2`}, {600, 3.18`}, {700, 3.09`}, {800, 3}, {900, 2.97`}, {1000, 2.95`}};
```

I will guess a model of the similar form to the last.

In[41]:=
```
model5 = NonlinearModelFit[SheetStickerPricingData, A (1 + x) ^r, {A, r}, x];
model5["ParameterTable"]
Normal[model5]
Show[Plot[model5[x], {x, 0, 1000}, PlotRange → {0, 20}],
 ListPlot[SheetStickerPricingData], AxesLabel → {"Area (ft^2)", "Price/area ($/ft^2)"}]
```

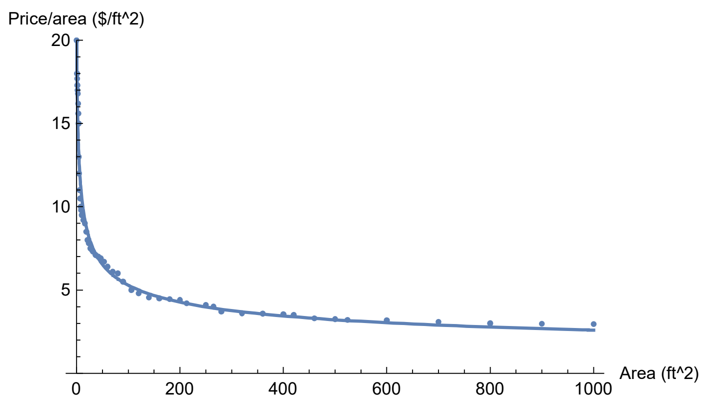
Out[42]=

	Estimate	Standard Error	t-Statistic	P-Value
A	22.2496	0.393215	56.5837	2.18485×10^{-48}
r	−0.311377	0.00729174	−42.7027	3.65309×10^{-42}

Out[43]=

$$\frac{22.2496}{(1 + x)^{0.311377}}$$

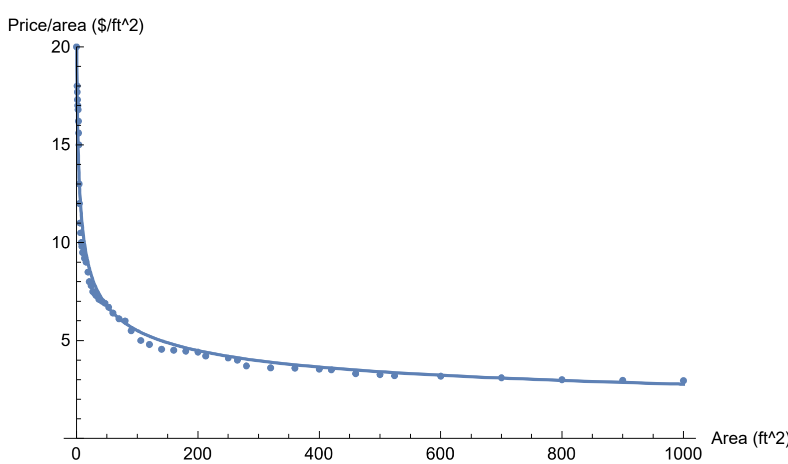
Out[44]=



This guess ended up working quite well. Let’s tweak the parameters now.

In[45]:=
```
A0 = 22;
r0 = -0.3;
price[x_] := A0 (1 + x) ^r0;

Show[Plot[price[x], {x, 0, 1000}, PlotRange → {0, 20}],
 ListPlot[SheetStickerPricingData], AxesLabel → {"Area (ft^2)", "Price/area ($/ft^2)"}]
```

Out[48]=



So with that, we have a function for price depending on area ordered for both kinds of stickers.

Edit: The single sticker pricing model didn't behave quite as we wanted. We want to enforce the end behavior to asymptotically approach $2.75 $/ft^2 and for the mid-range pricing to be lower (making the function more convex). I will add an asymptote to the model and also allow the convexity to vary with x in a linear fashion.

In[49]:=
```
Clear[B, H, r, x, gamma];
```

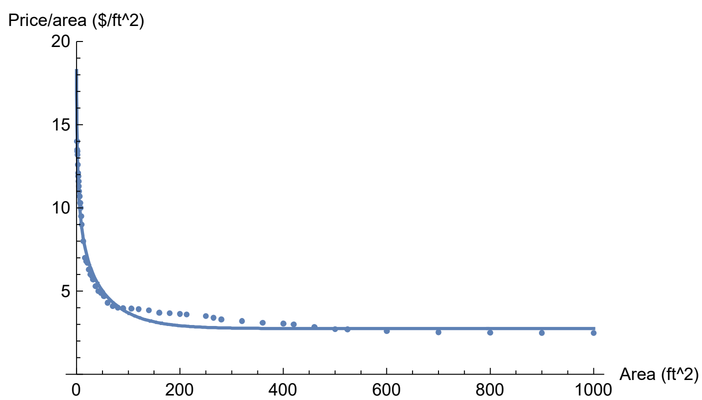
In[50]:=
```
model6 = NonlinearModelFit[SingleStickerPricingData,
   2.75 + B (1 + x) ^ (r - gamma x), {B, r, gamma}, x];
model6["ParameterTable"]
Normal[model6]
Show[Plot[model6[x], {x, 0, 1000}, PlotRange → {0, 20}],
 ListPlot[SingleStickerPricingData], AxesLabel → {"Area (ft^2)", "Price/area ($/ft^2)"}]
```

Out[51]=

	Estimate	Standard Error	t-Statistic	P-Value
B	15.4921	0.506684	30.5755	5.02369×10^{-34}
r	−0.367434	0.0221012	−16.6251	5.19541×10^{-22}
gamma	0.00239659	0.000407891	5.87556	3.4145×10^{-7}

Out[52]= $2.75 + 15.4921\,(1 + x)^{-0.367434-0.00239659\,x}$

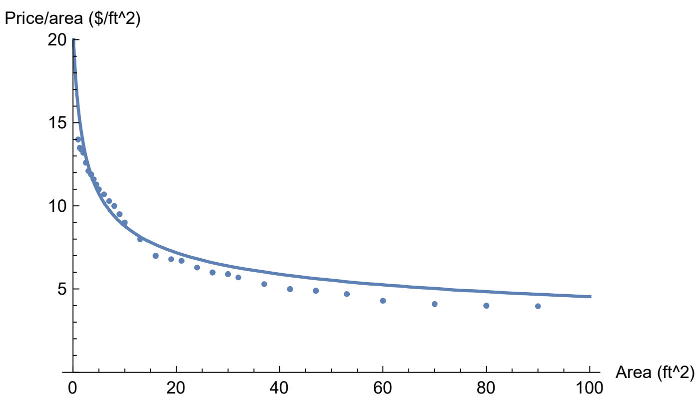
Out[53]=



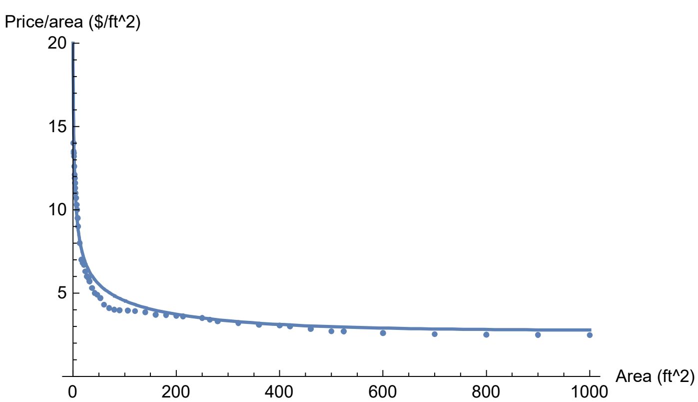
In[54]:=
```
B = 18;
H = 2.75;
r = -0.45;
gamma = 0.0005;
price[x_] := H + B (1 + x) ^ (r - gamma x);

Normal[price[x]]
Show[Plot[price[x], {x, 0, 100}, PlotRange → {0, 20}],
 ListPlot[SingleStickerPricingData], AxesLabel → {"Area (ft^2)", "Price/area ($/ft^2)"}]
Show[Plot[price[x], {x, 0, 1000}, PlotRange → {0, 20}], ListPlot[SingleStickerPricingData],
 AxesLabel → {"Area (ft^2)", "Price/area ($/ft^2)"}]
```

Out[59]= $2.75 + 18 (1 + x)^{-0.45-0.0005 x}$

Out[60]=



Out[61]=



Let's test some points in this new model.

In[62]:=
```
price[0]
price[200]
price[1000]
price[10 000]
```

Out[62]= 20.75

Out[63]= 3.7239

Out[64]= 2.7754

Out[65]= 2.75

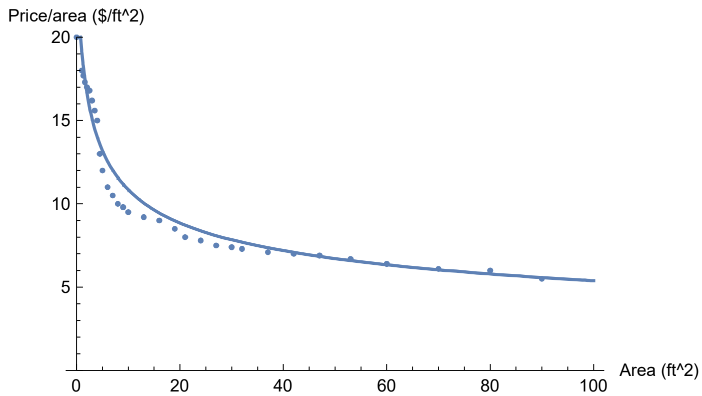
This looks about right, so let’s try to extend this model to the sheet sticker pricing.

In[226]:=
```
B = 22;
H = 2.5;
r = -0.4;
gamma = 0.0004;
price[x_] := H + B (1 + x) ^ (r - gamma x);
```

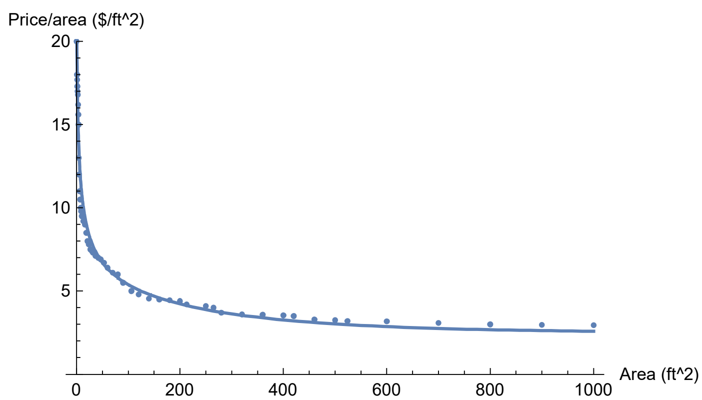
In[231]:=
```
Normal[price[x]]
Show[Plot[price[x], {x, 0, 100}, PlotRange → {0, 20}],
 ListPlot[SheetStickerPricingData], AxesLabel → {"Area (ft^2)", "Price/area ($/ft^2)"}]
Show[Plot[price[x], {x, 0, 1000}, PlotRange → {0, 20}],
 ListPlot[SheetStickerPricingData], AxesLabel → {"Area (ft^2)", "Price/area ($/ft^2)"}]
```

Out[231]= $2.5 + 22\,(1 + x)^{-0.4-0.0004\,x}$

Out[232]=



Out[233]=



In[234]:=
```
price[0]
price[20]
price[100]
```

Out[234]= 24.5

Out[235]= 8.85268

Out[236]= 5.3875