Brent Reasoning With Time

Monday, October 31, 2016 9:30 AM

$P(RAIN_0 = 0.5$, · uni m

$RAIN_t$

transition $RAIN_{t-1} \rightarrow RAIN_t$

R_{t-1}	$P(R_t = TRUE \mid R_{t-1})$	$P(R_t = FALSE \mid R_{t-1})$
T	0.7	0.3
F	0.3	0.7

sensor model $RAIN_t \rightarrow UMBRELLA_t$

R_t	$P(U_t = TRUE \mid R_t)$	$P(U_t = FALSE \mid R_t)$
T	0.9	0.1
F	0.2	0.8

Problems we can solve with DBN (HMM)

1) Filtering: $P(X_{t+1} \mid e_{1:t+1})$

evidence

Hidden/belief state

2) Prediction $P(X_{t+k} \mid e_{1:t})$

3) Most likely explanation

Given sequence of evidence find sequence of explanatory hidden states

~~Filtering~~

$\cancel{P(X_t \mid e_{1:t})}$

$$P(X_{t+1} \mid e_{1:t+1}) =$$

$$P(X_{t+1} \mid e_{1:t}, e_{t+1})$$ split evidence

hidden evidence

$$= \alpha P(e_{t+1} \mid X_{t+1}, e_{1:t}) P(X_{t+1} \mid e_{1:t})$$

* Bayes Rule

$$= \alpha P(e_{t+1} \mid X_{t+1}) \underbrace{P(X_{t+1} \mid e_{1:t})}_{\text{1-step prediction}}$$

* Using sensor assumption

$$= \cancel{\alpha P(e_{t+1} \mid X_{t+1})} \cdot \sum_{x_t} P(X_{t+1} \mid x_t, \cancel{e_{1:t}}) P(x_t \mid e_{1:t})$$

Sensor model little

Final

$$\overline{P(X_{t+1} \mid e_{1:t+1})} =$$

$$\alpha P(e_{t+1} \mid X_{t+1}) \sum_{x_t} P(X_{t+1} \mid x_t) P(x_t \mid e_{1:t})$$

sensor model transition model Recursive call

$RAIN_t$

transition | $RAIN_{t-1} \to RAIN_t$ 0.3

R_{t-1}	$P(R_t = TRUE \mid R_{t-1})$	$P(R_t = FALSE \mid R_{t-1})$
$r \to$ T	0.7	
$\neg r \to$ F	0.3	

R_t	$P(U_t = TRUE \mid R_t)$	$P(U_t = FALSE \mid R_t)$
T	0.9	0.1

F 0.2 0.8

sensor model RAIN$_t$ → UMBRELLA$_t$

$P(Rain_0) =$
$\langle 0.5, 0.5 \rangle$

Ex $P(RAIN_2 \mid umbrella_{1:2})$

→ Hour$_0$ → $P(Rain_0) = \langle 0.5, 5.0 \rangle$

- Hour$_1$ → Transition portion

at time 0

$\sum_{r_0} P(RAIN_1 \mid r_0) P(r_0)$

rain

rain, ¬rain

transition function

Because no evidence

$= \langle 0.7, 0.3 \rangle \times 0.5_r + \langle 0.3, 0.7 \rangle 0.5_{\neg rain}$

probability table

Prior

transition → $\langle 0.5, 0.5 \rangle$

sensor: $P(Umbrella_1 \mid RAIN_1) =$

$\langle 0.9, 0.2 \rangle$

0.182 >

Hour 1 ~~transition~~ $\langle 0.9, 0.2 \rangle \times \langle 0.5, 0.5 \rangle$

$P(RAIN_2 \mid Umbrella_{1:2}) = \alpha \langle 0.45, 0.1 \rangle \sum_{r_1} P(Rain_2 \mid r_1) P(RAIN_1 \mid u_1)$

transition funct

Comes from hour 1

$= \alpha \langle 0.7, 0.3 \rangle \times 0.818 +$

$\langle 0.3, 0.7 \rangle \times 0.182 =$

$\approx \langle 0.627, 0.373 \rangle$

$\langle 0.4, 0.2\rangle \langle 0.627, 0.373\rangle$

$\approx \langle 0.\underline{883}, \underline{0.117}\rangle$