# Independence & Bayes Nets

Wednesday, October 19, 2016 10:02 AM

Random Variable

Full Joint Distribution – cell is a possible world $P(A, B, C, D)$

Conditional Probabilities $\underbrace{P(X \mid e)}_{\text{Query}} = \alpha P(X, e) = \alpha \sum_h P(X, e, h)$

evidence

Summing out

Conditioning $\underbrace{P(Y)}_{\text{Don't Know}} = \sum_z P(Y \mid z) P(z)$

Known

## Independence

$P(\text{CAVITY}) \neq P(\text{CAVITY} \mid \text{toothache})$

$\langle 0.2, 0.8 \rangle \neq \langle 0.6, 0.4 \rangle$

→ some variables are unobservable

→ some variables affect distribution of others

Independence: Two random variables are independent if:

$P(A \mid B) = P(A)$ OR

$P(B \mid A) = P(B)$ OR

$P(A, B) = P(A)P(B)$ ←



$P(\overset{2}{\text{CAVITY}}, \overset{2}{\text{TOOTH}}, \overset{2}{\text{CATCH}}, \overset{4 \text{ values}}{\text{WEATHER}}) = \underbrace{P(\text{CAVITY, TOOTH, CATCH})}_{8 \text{ values}} \underbrace{P(\text{WEATHER})}_{4 \text{ values}}$

32 entries
(8×4)

12 (8+4)

Product ~~Bayes~~ Rule: $P(a \wedge b) = P(a|b)P(b) = P(b|a)P(a)$

Bayes Rule: $P(a|b) = \dfrac{P(b|a)P(a)}{P(b)} = \alpha P(b|a)P(a)$

prob of symptoms given disease

Medical Diagnosis: $P(\text{cause}|\text{effect})$

NOT OBSERVABLE — OBSERVABLE

% population

$P(\text{meningitis}|\text{stiff}) = \dfrac{P(\text{stiff}|\text{meningitis})\,P(\text{meningitis})}{P(\text{stiff})}$ ← ~~% population~~ ?

if we don't know P(stiff):

$P(M|s) = \alpha \langle P(s|m)P(m),\ \overset{?}{P(s|\neg m)}P(\neg m) \rangle$

$P(s|m)P(m)\alpha + P(s|\neg m)P(\neg m)\alpha = 1$

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Combining evidence – more than 2 variables

$P(a|b,c) = \dfrac{P(c|a,b)P(b|a)P(a)}{P(b)P(c|b)} = \alpha P(c|a,b)P(b|a)P(a)$

$= P(b|a,c)P(c|a)P(a)$



$P(\text{cavity}|\text{tooth},\text{catch}) = \alpha P(\text{tooth}|\text{cavity},\text{catch})\,P(\text{catch}|\text{cavity})\,P(\text{cavity})$

→ toothache & catch are independent

$= \alpha P(\text{tooth}|\text{cavity})\,P(\text{catch}|\text{cavity})\,P(\text{cavity})$

when cause and effect, effects are generally independent

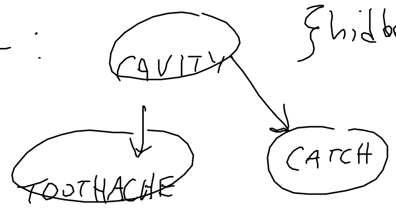
Generalized Bayes Model: when effects are independent:

$P(\text{CAUSE}|\text{EFFECT}_1, \text{EFFECT}_2, \dots, \text{EFFECT}_n) = P(\text{CAUSE}) \prod_i P(\text{EFFECT}_i|\text{CAUSE})$

... all effects are independent

Naive Bayes Assumption: Assume all effects are independent

~~Bayesian Net~~ :




} hidden / unobservable

} effects / observable

Conditional Probability Table

| CAVITY | $P(\cancel{tooth} \mid CAVITY)$ | $P(\cancel{\neg tooth} \mid CAVITY)$ |
|---|---|---|
| T | 0.6 | 0.4 |
| F | 0.1 | 0.9 |

→ Rows sum to 1