John Pratt

HW #8
ECE-2026

Section L05
Rec: Prof. Zhang
Lecture: 11 AM

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HW Grader: S. Wang

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8.1 a) $y_1[n] = \frac{1}{2}x[n] - x[n-1] + \frac{1}{2}x[n-2] = \sum_{k=0}^{2} b_k \cdot x[n-k]$, $b_k = \{\frac{1}{2}, -1, \frac{1}{2}\}$

$$H_1(e^{j\omega}) = \sum_{k=0}^{2} b_k \cdot e^{-j\omega k} = \boxed{\frac{1}{2} - e^{-j\omega} + \frac{1}{2}e^{-j2\omega}}$$

b) $h_2[n] = \frac{1}{2}\delta[n-1] - \delta[n-2] + \frac{1}{2}\delta[n-3] = \sum_{k=0}^{3} b_{2k}\,\delta[n-k]$, $b_{2k} = \{0, \frac{1}{2}, -1, \frac{1}{2}\}$

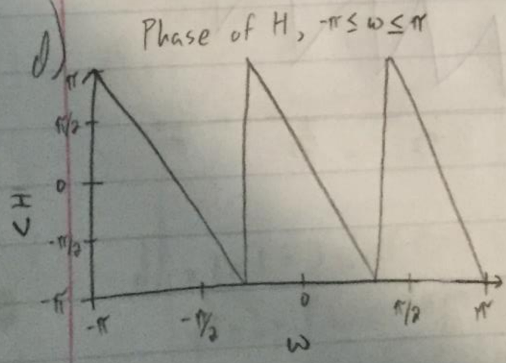
$$H_2(e^{j\omega}) = \sum_{k=0}^{2} b_{2k}\, e^{-j\omega k} = \boxed{\frac{1}{2}e^{-j\omega} - e^{-j2\omega} + \frac{1}{2}e^{-j3\omega}}$$

c) $H(e^{j\omega}) = (H_1(e^{j\omega}))\cdot(H_2(e^{j\omega})) = \frac{1}{4}(1 - 2e^{-j\omega} + e^{-j2\omega})(e^{-j\omega} - 2e^{-j2\omega} + e^{-j3\omega})$

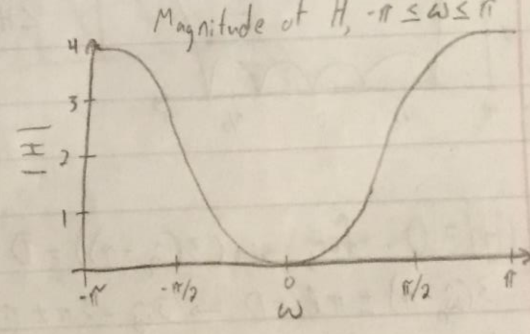
$$= \frac{1}{4}(e^{-j\omega} - 2e^{-j2\omega} + e^{-j3\omega} - 2e^{-j2\omega} + 4e^{-j3\omega} - 2e^{-j4\omega} + e^{-j3\omega} - 2e^{-j4\omega} + e^{-j5\omega})$$

$$= \frac{1}{4}e^{-j3\omega}(e^{j2\omega} + e^{-j2\omega} + -4(e^{j\omega} + e^{-j\omega}) + 6) = \boxed{\frac{1}{2}(\cos(2\omega) - 4\cos(\omega) + 6)\, e^{-j3\omega}}$$

d) Phase of H, $-\pi \le \omega \le \pi$



Magnitude of H, $-\pi \le \omega \le \pi$



e) $\omega = \frac{1}{3}\pi \rightarrow \angle H = \angle(e^{-j\omega} + -4e^{-j2\omega} + 6e^{-j3\omega} - 4e^{-j4\omega} + e^{-j5\omega})$

$= -\pi$ (this is just done substituting $\pi/3$ for ω and using $e^{j\theta} = \cos\theta + j\sin\theta$, then $\theta = \arctan\left(\frac{\sin\theta}{\cos\theta}\right)$)

$|H| = 0.25$ (same principle, $\vec{v} = \langle v_1, v_2, \ldots, v_n \rangle$, $|v| = \sqrt{v_1^2 + v_2^2 + \ldots v_n^2}$)

So: $y[n] = \frac{1}{4}(3)\cos(\frac{1}{3}\pi(n-1) - \pi) = \boxed{0.75\cos(\frac{1}{3}\pi(n-4))}$

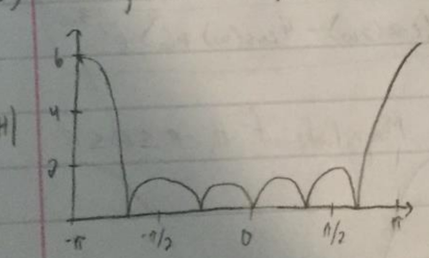
8.2 a) $H(e^{j\omega}) = \boxed{\sum_{k=0}^{5} (-1)^k e^{-j\omega k}}$

b) $H(e^{j\omega}) = \sum_{k=0}^{5} e^{-j(\hat{\omega}-\pi)k} = \frac{1 - e^{-j(\hat{\omega}-\pi)6}}{1 - e^{-j(\hat{\omega}-\pi)}}$

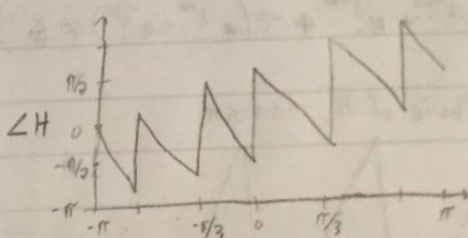
$$= \frac{e^{-j(\hat{\omega}-\pi)3}\left(e^{j(\hat{\omega}-\pi)3} - e^{-j(\hat{\omega}-\pi)3}\right)}{e^{-j(\hat{\omega}-\pi)/2}\left(e^{j(\hat{\omega}-\pi)/2} - e^{-j(\hat{\omega}-\pi)/2}\right)}$$

$$= \left(e^{-j(\omega-\pi)2.5}\right) \frac{\sin(3(\hat{\omega}-\pi))}{\sin\left(\frac{1}{2}(\hat{\omega}-\pi)\right)} \checkmark$$

c) Magnitude of H, $-\pi \le \omega \le \pi$



Phase of H, $-\pi \le \omega \le \pi$



d) $|H| = 0$ for $\sin(3(\hat{\omega}_0 - \pi)) = 0$

$3(\hat{\omega}_0 - \pi) \pm \pi l = 0 \rightarrow 3\hat{\omega}_0 - 3\pi \pm \pi l = 0.$

Given $0 < \hat{\omega}_0 < \pi$, $\hat{\omega}_0 = \{\frac{\pi}{3}, \frac{2\pi}{3}\}$

8.3) a) $H(e^{j\omega}) = \sum_{k=0}^{6} b_k e^{-j\omega k}$, $b_k = \{0,0,0,0,\frac{1}{2},1,\frac{1}{2}\}$

$$y_2[n] = \begin{cases} \frac{1}{2}c_1 + c_2 + \frac{1}{2}c_1, & n \text{ even} \\ \frac{1}{2}c_2 + c_1 + \frac{1}{2}c_2, & n \text{ even} \end{cases}$$

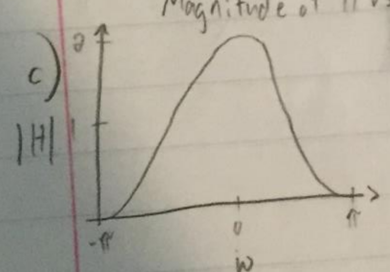
$\boxed{y_2[n] = c_1 + c_2}$ The reasoning is that any $n_{even} + n_{odd}$ is odd, and any $n_1 + n_2$ (both even or odd) is even.

b) $H(e^{j\omega}) = \frac{1}{2}e^{-j4\omega} + e^{-j5\omega} + \frac{1}{2}e^{-j6\omega}$
$= e^{-j5\omega}\left(\frac{1}{2}e^{j\omega} + \frac{1}{2}e^{-j\omega} + 1\right)$

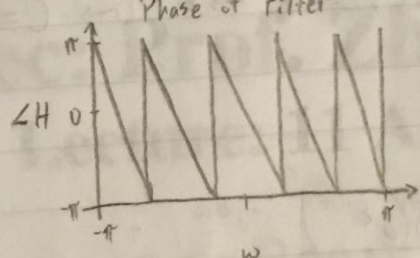
$\boxed{H(e^{j\omega}) = (1+\cos(\hat{\omega}))e^{-j5\hat{\omega}}}$

c)

Magnitude of H vs. w



Phase of Filter



d) $x_1[n] = 3\cos(\pi n) + 2\cos(\frac{5}{8}\pi n)$ LTI

$y_1[n] = \cancel{(1+\cos\pi)3\cos(\pi n)}^{0} + 2(1+\cos(\frac{5}{8}\pi))\cos(\frac{5}{8}\pi n + 5(\frac{5}{8}\pi))$

$y_1[n] = (2-\sqrt{2-\sqrt{2}})\cos(\frac{5}{8}\pi(n+5))$

8.4) a) $H(e^{j\omega}) = \frac{1}{2j}\left(e^{j\frac{3}{2}\omega} - e^{-j\frac{3}{2}\omega}\right) e^{-j\frac{7}{2}\omega} e^{-j\pi/2}$

$= \frac{1}{2\cancel{j}}\left(e^{-j2\omega} - e^{-j5\omega}\right)(-\cancel{j})$

$= -\frac{1}{2} e^{-j2\omega} + \frac{1}{2} e^{-j5\omega}$

$\boxed{h[n] = -\frac{1}{2}\delta[n-2] + \frac{1}{2}\delta[n-5]}$

8.6) a) $\frac{A_y}{A_x} = B\sin(2\hat{\omega}) \rightarrow B = \frac{A_y}{A_x \sin(2\hat{\omega})} = \frac{\frac{1}{8}}{\frac{1}{\sqrt{3}}(\sin(\frac{\pi}{3}))} = B = \frac{1}{4}$

$\phi_y - \phi_x \pm 2\pi\ell = C\hat{\omega} - \pi/2$

$-\frac{\pi}{4} - \frac{\pi}{4} \pm 2\pi\ell = C(\frac{\pi}{6}) - \pi/2 \rightarrow \ell = 1 \rightarrow C = 6$

$\boxed{B = 1/4 \qquad C = 6}$

b) $H(e^{j\omega}) = \frac{1}{4}\sin(2\omega)(e^{j\pi/2}) e^{-j6\omega}$

$= \frac{1}{8\cancel{j}}\left(e^{j2\omega} - e^{-j2\omega}\right)\cancel{j}\, e^{-6\omega}$

$= \frac{1}{8}e^{-j4\omega} - \frac{1}{8}e^{-j8\omega}$

$h[n] = \frac{1}{8}\delta[n-4] - \frac{1}{8}\delta[n-8]$