Improvements in LARFT inside LAPACK

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Overview

What is LAPACK

LAPACK provides interfaces for:

1. Matrix multiplication
2. Solving linear systems of equations
3. Factorizing matrices

and more!

Brief Linear Algebra Review

Householder reflectors are a way to to represent a matrix as a product of rank 1 updates of the form

$$(I - \tau_1 v_1 v_1^\top) \cdots (I - \tau_k v_k v_k^\top) = VTV^\top$$

Routines that use this[1]

- SVD *GESVD
- Hessenberg Reduction *GEQRF
- QR Factorization *ORGQR

[1] Collected by listing some functions found on the caller graph of DLARFT found here

LAPACK Implementation

The algorithm for DLARFT is given by[2]:

for Each column of V **do**

$T(:,i) = -\tau_i V(:,1:i-1)^\top * V(:,i)$

$T(:,i) = T(:,1:i-1) * T(:,i)$

$T(i,i) = \tau_i$

end for

[2]Taken from the comments of DLARFT found here

Recursive Implementation

If we collect only some of the reflectors on the first and second half, we get

$$
\begin{aligned}
&(I - V_1 T_1 V_1^\top)(I - V_2 T_2 V_2^\top) \\
&= I - V_1 T_1 V_1^\top - V_2 T_2 V_2^\top + V_1 T_1 V_1^\top V_2 T_2 V_2^\top
\end{aligned}
$$

Can be rewritten as:

$$I - VTV^\top$$

where:

$$
\begin{aligned}
T_3 &= -T_1 V_1^\top V_2 T_2 \\
V &= \begin{bmatrix} V_1 & V_2 \end{bmatrix}
\end{aligned}
$$

Matrix Operation Implementation

Based on the work done by Joffrain and Low [3] and Puglisi [4]

$T = V^\top V$ (Only upper triangular part)
Scale the diagonal by $\frac{1}{2}$.
$T = T^{-1}$.

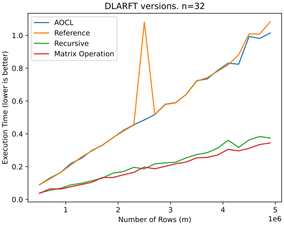
For more details about why this works, see either Theorem 2 from Joffrain and Low or the algorithm from Puglisi

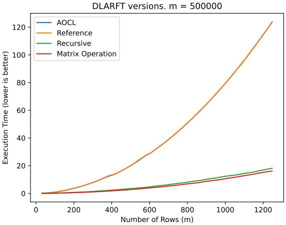
[3] Accumulating Householder Transformations, Revisited

[4] Modification of the Householder method based on the compact wy representation

Numerical Results

We ran the following tests on the Alderaan[5] cluster here at UC Denver





[5] Specifications for the cluster can be found here

Future work/open questions for Matrix Operation Based

- Complex arithmetic
- Stability