# Improvements in LARFT inside LAPACK

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# Overview

# What is LAPACK

LAPACK provides interfaces for:

1. Matrix multiplication
2. Solving linear systems of equations
3. Factorizing matrices

and more!

# Brief Linear Algebra Review

Householder reflectors are a way to to represent a matrix as a product of rank 1 updates of the form

$$\left(I - \tau_1 v_1 v_1^\top\right) \cdots \left(I - \tau_k v_k v_k^\top\right) = V T V^\top$$

Routines that use this[1]

- ▶ SVD *GESVD
- ▶ Hessenberg Reduction *GEQRF
- ▶ QR Factorization *ORGQR

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[1]Collected by listing some functions found on the caller graph of DLARFT found here

# LAPACK Implementation

The algorithm for DLARFT is given by[2]:

**for** Each column of $V$ **do**

$T(:, i) = -\tau_i V(:, 1 : i - 1)^\top * V(:, i)$

$T(:, i) = T(:, 1 : i - 1) * T(:, i)$

$T(i, i) = \tau_i$

**end for**

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[2]Taken from the comments of DLARFT found here

# Recursive Implementation

If we collect only some of the reflectors on the first and second half, we get

$$(I - V_1 T_1 V_1^\top)(I - V_2 T_2 V_2^\top)$$
$$= I - V_1 T_1 V_1^\top - V_2 T_2 V_2^\top + V_1 T_1 V_1^\top V_2 T_2 V_2^\top$$

Can be rewritten as:

$$I - VTV^\top$$

where:

$$T_3 = -T_1 V_1^\top V_2 T_2$$
$$V = \begin{bmatrix} V_1 & V_2 \end{bmatrix}$$

# Matrix Operation Implementation

Based on the work done by Joffrain and Low [3] and Puglisi [4]

$T = V^\top V$ (Only upper triangular part)
Scale the diagonal by $\frac{1}{2}$.
$T = T^{-1}$.

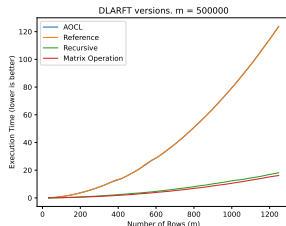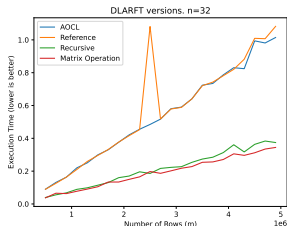For more details about why this works, see either Theorem 2 from Joffrain and Low or the algorithm from Puglisi

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[3] Accumulating Householder Transformations, Revisited
[4] Modification of the Householder method based on the compact wy representation

# Numerical Results

We ran the following tests on the Alderaan[5] cluster here at UC Denver



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[5]Specifications for the cluster can be found here

# Future work/open questions for Matrix Operation Based

- Complex arithmetic
- Stability