

Improvements in LARFT inside LAPACK

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Overview

Preliminaries

Existing Behavior

New Behavior

- Recursive LARFT

- Matrix Operation LARFT

Numerical Results

Future work

What is LAPACK

LAPACK provides interfaces for:

1. Matrix multiplication
2. Solving linear systems of equations
3. Factorizing matrices

and more!

Brief Linear Algebra Review

Householder reflectors are a way to to represent a matrix as a product of rank 1 updates of the form

$$(I - \tau_1 v_1 v_1^\top) \cdots (I - \tau_k v_k v_k^\top) = I - VTV^\top$$

Routines that use this¹

- ▶ SVD *GESVD
- ▶ Hessenberg Reduction *GEQRF
- ▶ QR Factorization *ORGQR

¹Collected by listing some functions found on the caller graph of DLARFT found [here](#)

LAPACK Implementation

The algorithm for DLARFT is given by²:

for Each column of V **do**

$$T(:, i) = -\tau_i V(:, 1 : i - 1)^\top * V(:, i)$$

$$T(:, i) = T(:, 1 : i - 1) * T(:, i)$$

$$T(i, i) = \tau_i$$

end for

²Taken from the comments of DLARFT found [here](#)

Recursive Implementation

If we collect only some of the reflectors on the first and second half, we get

$$\begin{aligned} & (I - V_1 T_1 V_1^\top)(I - V_2 T_2 V_2^\top) \\ &= I - V_1 T_1 V_1^\top - V_2 T_2 V_2^\top + V_1 T_1 V_1^\top V_2 T_2 V_2^\top \end{aligned}$$

Can be rewritten as:

$$I - VTV^\top$$

where:

$$T_3 = -T_1 V_1^\top V_2 T_2$$

$$V = [V_1 \quad V_2]$$

Matrix Operation Implementation

Based on the work done by Joffrain and Low ³ and Puglisi ⁴

$$T = V^T V \text{ (Only upper triangular part)}$$

Scale the diagonal by $\frac{1}{2}$.

$$T = T^{-1}.$$

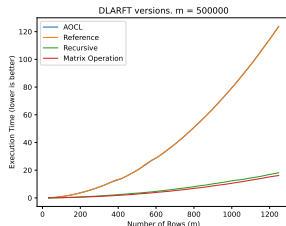
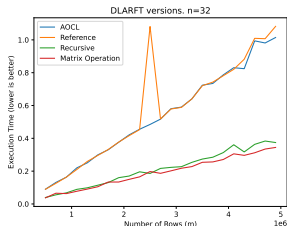
For more details about why this works, see either Theorem 2 from Joffrain and Low or the algorithm from Puglisi

³Accumulating Householder Transformations, Revisited

⁴Modification of the Householder method based on the compact wry representation

Numerical Results

We ran the following tests on the Alderaan⁵ cluster here at UC Denver



⁵Specifications for the cluster can be found [here](#)

Future work/open questions for Matrix Operation Based

- ▶ Complex arithmetic
- ▶ Stability