$$\begin{bmatrix} A_{11} & C_1 \\ A_{21} & C_2 \end{bmatrix}$$

Figure 1: Decomposition of A used in DORGQR

Contents

1 What does Reference DORGQR do?

The algorithm for the current Reference DORGQR is as follows. We will assume for simplicity's sake that n is a perfect multiple of k and that we start blocking exactly at k. This is not exactly how DORQR is currently written, but it will allow us to talk about the algorithm and ignore some technical difficulties in the implimentations

Algorithm 1 Reference DORGQR

```
Require: $A \in \mathbb{R}^{m \times n}$ output of DEQRF. IE the $i^{\text{th}}$ column of $A$ is the vector defining the $i^{\text{th}}$ elementary reflector ($H_i$ for the starting matrix for $i = 1, \ldots, k$, and $m \geq n$
Ensure: $A = Q \in \mathbb{R}^{m \times n}$ such that $Q = H_1 H_2 \cdots H_k$
  if blocking then
    determine blocking parameter $nb$
  else
    $Q \leftarrow$ DORG2R($A$)
    return $Q$
  end if
  Break $A$ down according to Figure 1 where $C_1 \in \mathbb{R}^{k \times n-k}$ and $C_2 \in \mathbb{R}^{m-k \times n-k}$ are the last $k$ columns of $A$.
  $C_1 \leftarrow \mathbf{0}$
  $C_2 \leftarrow$ DORG2R($C_2$)
  for $i = k - nb, k - 2nb, \cdots 1$ do
    Construct $T$ such that $H = I - VTV^\top$ where $H = H_i H_{i+1} \cdots H_{i+nb-1}$
    Apply
  end for
```

2 What did we aim to do?/Deliverables

We aim to increase the performance of the reference DORGQR in both time and memory used

3 Why do we care?

4 Hardware Used

We ran the following tests using a Lenovo Thinkpad E430 running Arch Linux with the following system specifications

- Kernel: 6.5.8-arch1-1
- CPU: Intel i3-3120M (4 cores, 8 threads) @ 2.500GHz

5 Version 1

The file that contains just the changes mentioned here is: `my_dorgqr_v1.f`

5.1 Changes

For the first version, we aimed to take advantage of the fact that in our first step, we have an identity matrix in the slot of C_2, and 0 inside the slot of C_1 on every iteration.

5.2 Numerical Performance

We compare this version against two baselines.

- Reference DORGQR
- MKL DORGQR

6 Version 2

The file that contains the changes mentioned here and the ones described in Section 5 is: `my_dorgqr_v2.f`

6.1 Changes

6.2 Numerical Performance

7 Summary