

Bin 4 Problems

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This document holds problems that fit into the Bin 4 according to the prelim syllabus. Or are approached in a way most compatible with Bin 4

Contents

1 Prelim Problems	1
2 Axler Problems	1
3 Other Problems	2

1 Prelim Problems

- Let V be a vector space over a field \mathbb{F} . Suppose $T \in \mathcal{L}(V)$ has minimal polynomial $p(z) = 3 + 2z - z^2 + 5z^3 + z^4$
 - (5 points) Prove T is invertible
 - (15 points) Find the minimal polynomial of T .
- Answer the following
 - Is there an $n \times n$ matrix A with $A^{n-1} \neq 0$ and $A^n = 0$? Give an example to show such a matrix exists (and explain why it satisfies both conditions), or disprove it.
 - Show that an $n \times n$ upper triangular matrix with $A^n \neq 0$ and $A^{n+1} = 0$ does not exist.
- Let T be a linear map on a vector space V and $\dim V = n$
 - If for some vector v , the vectors $v, Tv, \dots, T^{n-1}v$ are linearly independent, show that every eigenvalue of T has only one corresponding eigenvector up to a scalar multiplication
 - If T has n distinct eigenvalues and vector u is a sum of n eigenvectors, corresponding to the distinct eigenvalues, show that $u, Tu, \dots, T^{n-1}u$ are linearly independent (and thus form a basis of V).
- Let $A \in \mathcal{M}_n(\mathbb{C})$ and λ be an eigenvalue of A .
 - Show that λ^r is an eigenvalue of A^r for $r \in \mathbb{N}$.
 - Provide an example showing that the multiplicity of λ^r as an eigenvalue of A^r may be strictly higher than the multiplicity of λ as an eigenvalue of A
 - Show that A^\top has the same eigenvalues as A .
 - Show that if A is orthogonal, then $\frac{1}{\lambda}$ is an eigenvalue of A .

2 Axler Problems

- Suppose V is an inner product space and $T \in \mathcal{L}(V)$ is normal. Prove that the minimal polynomial of T has no repeated zeros

3 Other Problems

These are from the Berkely Problems pdf.

1. (7.5.16) Let A and B denote real $n \times n$ symmetric matrices such that $AB = BA$. Prove that A and B share a common eigenvector in \mathbb{R}^n .