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# Bin 1 Problems

December 5, 2023

This document holds problems that fit into the Bin 1 according to the prelim syllabus.

## Contents

<b>1 Prelim Problems</b>	<b>1</b>
<b>2 Axler Problems</b>	<b>1</b>
<b>3 Other Problems</b>	<b>1</b>

## 1 Prelim Problems

The problems are listed in most recent first. IE the first question is from August 2023, second is from January 2023, etc. In addition, the proofs provided are my own to complement the ones posted by the committees of previous years.

1. Let  $T$  be a linear map  $T : U \rightarrow V$  and  $S$  be a linear map  $S : V \rightarrow W$ .  
Prove that  $\dim U - \dim V \leq \dim \text{null } ST - \dim \text{null } S$ .
2. Suppose  $U, W$  are subspaces of a finite-dimensional vector space  $V$ .
  - (a) Show that  $\dim (U \cap W) = \dim U + \dim W - \dim (U + W)$
  - (b) Let  $n = \dim V$ . Show that if  $k < n$ , then an intersection of  $k$  subspaces of dimension  $n - 1$  always has dimension at least  $n - k$ .
3. Prelim Aug 22 Part 1.
  - (a) Let  $v_1, v_2, v_3$  be linearly dependent, and  $v_2, v_3, v_4$  are linearly independent
    - i. Show that  $v_1$  is a linear combination of  $v_2$  and  $v_3$ .
    - ii. Show that  $v_4$  is not a linear combination of  $v_1, v_2$ , and  $v_3$ .
4. Let  $V$  be a vector space of dimension  $n$  over a field  $F$ . Let  $v_1, v_2, \dots, v_n$  be a basis of  $V$  and  $T$  be an operator on  $V$ .  
Prove:  $T$  is invertible if and only if  $Tv_1, Tv_2, \dots, Tv_n$  are linearly independent.

## 2 Axler Problems

## 3 Other Problems