Theorem Statement

Theorem

Let $T : \mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation with associated matrix $A \in \mathbb{R}^{n \times n}$.

1. *A is invertible*
2. *A has a pivot in every row*
3. *A has a pivot in every column*
4. *The system $A\mathbf{x} = \mathbf{b}$ has a unique solution for all $\mathbf{b} \in \mathbb{R}^n$*
5. *$A\mathbf{x} = \mathbf{0}$ has only the trivial solution*
6. *The columns of A are linearly independent*
7. *The columns of A span all of $\mathbb{R}^n$.*
8. *T is invertible*
9. *T is injective*
10. *T is surjective*

We have the tools to prove all these, but really, it's just a summary of all the linear transformation and matrix algebra we've done so far!

One-sided inverse Implies the Other!

Theorem

Let $A \in \mathbb{R}^{n \times n}$. If there is some $B \in \mathbb{R}^{n \times n}$ such that

$$BA = I_n \text{ or } AB = I_n$$

Then, A is invertible and $A^{-1} = B$.

$AB = I_n$ case

$BA = I_n$ case