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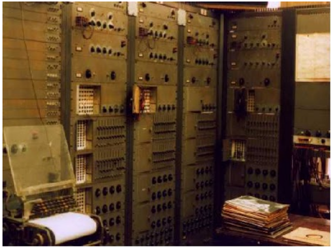

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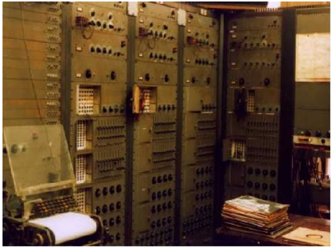

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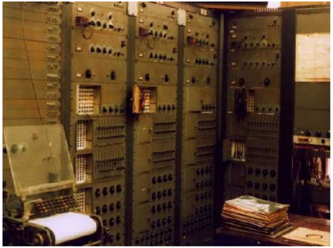

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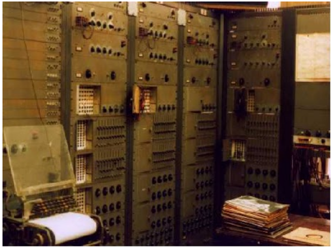

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		Amount Purchased by		
Sector	Total Output (in Billions)	Renewables	Electricity	Manufacturing
Renewable Energy	40	10 (25%)	25 (62.5%)	5 (12.5%)
Electricity	100	7 (7%)	18 (18%)	75 (75%)
Manufacturing	125	20 (16%)	50 (40%)	55 (44%)

Exchange Tables

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Proportion of Output from:			
Renewable Energy	Electricity	Manufacturing	Purchased by
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- How can we determine the equilibrium prices for each sector?

Setting Up a System of Linear Equations

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- Down each column is the proportion of that sector which is purchased by each of the other three sectors.
- Across each row we see for a given sector, what proportion of their inputs came from each sector's output.
- For the renewable energy sector, we therefore have the following linear equation:

$$\text{output from renewables} = (\text{input from renewables}) + (\text{input from electricity}) + (\text{input from r}$$

$$p_r = 0.25p_r + 0.07p_e + 0.16p_m$$

$$0.75p_r - 0.07p_e - 0.16p_m = 0$$

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Similarly deriving linear equations for the other sectors, we have the following system:

$$
\begin{aligned}
0.75p_r &- 0.07p_e - 0.16p_m = 0 \\
-0.625p_r &+ 0.82p_e - 0.4p_m = 0 \\
-0.125p_r &- -0.75p_e + 0.56p_m = 0
\end{aligned}
$$

Finding the Equilibrium for the Economy

We have the following augmented matrix associated to the system:

$$\left[\begin{array}{ccc|c} 0.75 & -0.07 & -0.16 & 0 \\ -0.625 & 0.82 & -0.4 & 0 \\ -0.125 & -0.75 & 0.56 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & -0.28 & 0 \\ 0 & 1 & -0.70 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

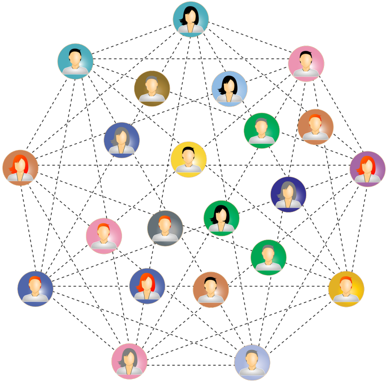
Notice p_m is a free variable, and we have equilibrium solution:

$$\mathbf{p} = \begin{bmatrix} p_r \\ p_e \\ p_m \end{bmatrix} = \begin{bmatrix} 0.28p_m \\ 0.7p_m \\ p_m \end{bmatrix} = p_m \begin{bmatrix} 0.28 \\ 0.7 \\ 1 \end{bmatrix}$$

- If this economy has $p_m = 125$ billion dollars,
- Then if we want to ensure the economy is functioning at is equilibrium level (everything produced is used by other sectors):
  - Set $p_r = (0.28)(125) = 35$ billion dollars, and
  - Set $p_e = (0.7)(125) = 87.5$ billion dollars.

Network Flow

- A network consists of a set of points, called nodes with lines, called branches connecting some or all of the nodes.



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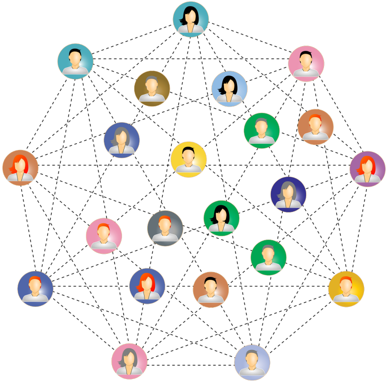

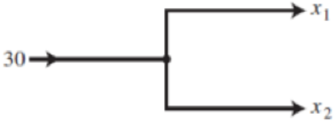

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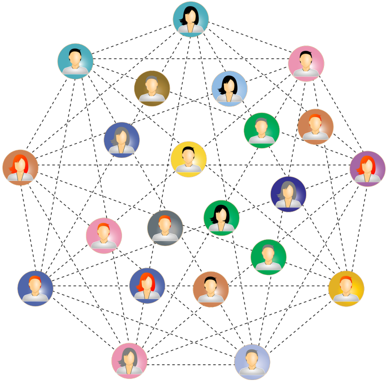

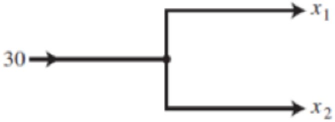

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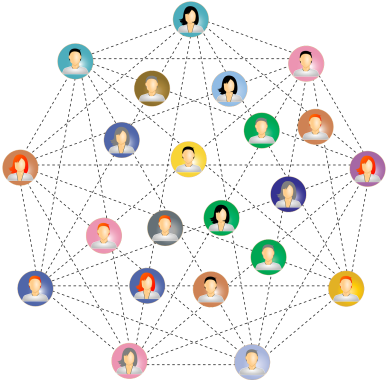

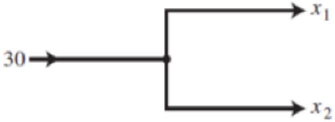

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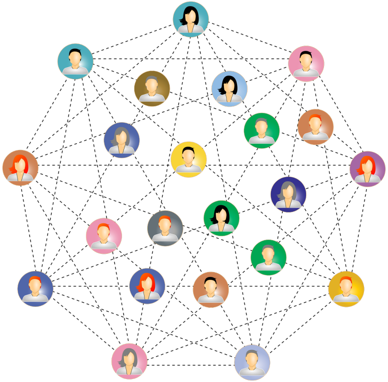

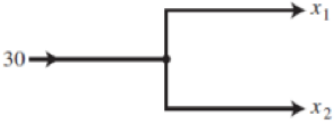

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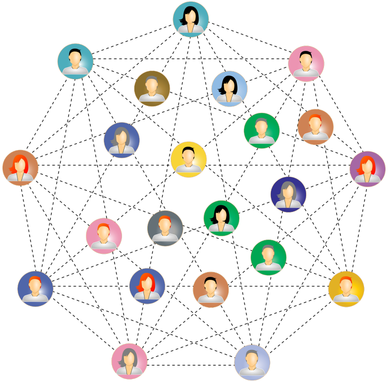
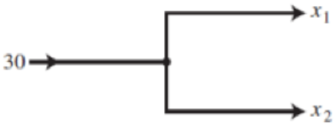
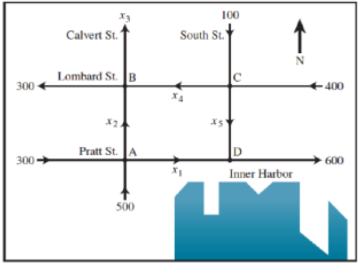
- Network flows have applications to current flow through a circuit, flow of goods through supply chains, social networks, and urban planning to name a few.





Traffic Flow in Baltimore

The network in the figure shows the flow of traffic (in vehicles per hour) over several one way streets in downtown Baltimore during a typical early afternoon. Determine the general flow pattern for the network.



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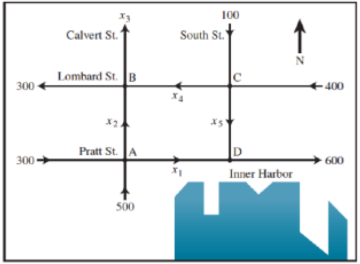

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Intersection	Flow in		Flow out
A	$300 + 500$	$=$	$x_1 + x_2$
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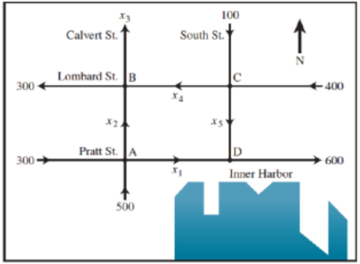

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\begin{array}{rcrcrcrcrcl}
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 & & x_2 & - & x_3 & + & x_4 & & & = & 300 \\
 & & & & & & x_4 & + & x_5 & = & 500 \\
x_1 & & & & & & & + & x_5 & = & 600 \\
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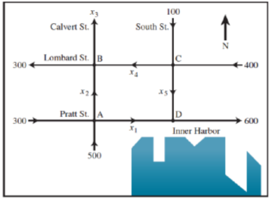
\end{array}
$$

Solving the System

We need to solve the following nonhomogeneous linear system of equations:



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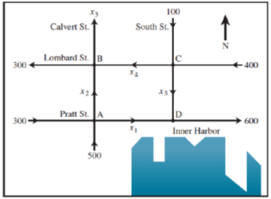

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We have an augmented matrix

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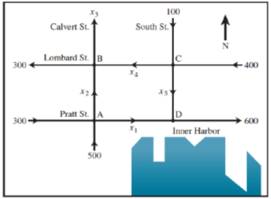

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\rightarrow
\left[\begin{array}{ccccc|c}
1 & 0 & 0 & 0 & 1 & 600 \\
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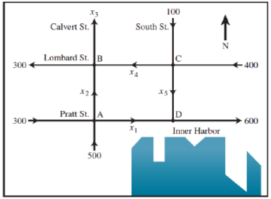

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