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- Let $A, B \in \mathbb{R}^{m \times n}$ (same size and shape!). The sum, $C = A + B$ is defined as

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Example

Let $A = \begin{bmatrix} -1 & 2 & 1 \\ 0 & 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 & 4 \\ 0 & 4 & 2 \end{bmatrix}$. Compute $2A - B$.

Matrix Addition & Scalar Multiplication Properties

For each of the following properties, let $A, B, C \in \mathbb{R}^{m\times n}$, and r, s be scalars

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Let $A \in \mathbb{R}^{m \times n}$. The transpose of A, denoted $A^\top \in \mathbb{R}^{n \times m}$ is the matrix with columns formed from rows of A. IE:

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Composition of Linear Transformations: Let $T : \mathbb{R}^m \to \mathbb{R}^p$ and $S : \mathbb{R}^n \to \mathbb{R}^m$ be linear transformations defined by

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2. Component-wise
3. Sums of other matrices

Matrix Multiplication Method 1 (Column-wise)

Let $A \in \mathbb{R}^{m\times n}$, $B \in \mathbb{R}^{n\times p}$. Then we define the matrix product as:

$$AB = A\begin{bmatrix}\mathbf{b}_1 & \mathbf{b}_2 & \dots & \mathbf{b}_p\end{bmatrix} = \begin{bmatrix}A\mathbf{b}_1 & A\mathbf{b}_2 & \dots & A\mathbf{b}_p\end{bmatrix}$$

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Let $A = \begin{bmatrix}1 & -4\\ 2 & -5\\ 3 & -6\end{bmatrix}$ and $B = \begin{bmatrix}2 & 1\\ 1 & 4\end{bmatrix}$. Then

$$C = AB = \begin{bmatrix}\begin{bmatrix}1 & -4\\ 2 & -5\\ 3 & -6\end{bmatrix}\begin{bmatrix}2\\ 1\end{bmatrix} & \begin{bmatrix}1 & -4\\ 2 & -5\\ 3 & -6\end{bmatrix}\begin{bmatrix}1\\ 4\end{bmatrix}\end{bmatrix} = \begin{bmatrix}-2 & -15\\ -1 & -18\\ 0 & -21\end{bmatrix}$$

Matrix Multiplication Method 2 (Component-wise)

Let $A \in \mathbb{R}^{m \times n}, B \in \mathbb{R}^{n \times p}$. Then we define the matrix product as the matrix $C = AB$ where for all i, j:

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{in}b_{nj}$$

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Matrix Multiplication Method 3 (Sums of other matrices)

Let $A \in \mathbb{R}^{m\times n}, B \in \mathbb{R}^{n\times p}$. Then we define the matrix product as:

$$AB = \begin{bmatrix} \mathbf{a}_1 & \dots & \mathbf{a}_n \end{bmatrix} \begin{bmatrix} \mathbf{b}_1^\top \\ \vdots \\ \mathbf{b}_n^\top \end{bmatrix} = \mathbf{a}_1\mathbf{b}_1^\top + \cdots + \mathbf{a}_n\mathbf{b}_n^\top$$

where $\mathbf{a}_i \in \mathbb{R}^{m\times 1}$ are the columns of A and $\mathbf{b}_j^\top \in \mathbb{R}^{1\times p}$ are the rows of B (columns of $B^\top$!)

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$$C = AB = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 2 & 1 \end{bmatrix} + \begin{bmatrix} -4 \\ -5 \\ -6 \end{bmatrix} \begin{bmatrix} 1 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 2 \\ 6 & 3 \end{bmatrix} + \begin{bmatrix} -4 & -16 \\ -5 & -20 \\ -6 & -24 \end{bmatrix}$$

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Is It The Correct Shape?

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}, C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E = \begin{bmatrix} 4 & 6 \\ 0 & 3 \\ 1 & 0 \end{bmatrix}, F = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

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1. For which matrices is addition with A defined?

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1. For which matrices is addition with A defined?
2. For which matrices is addition with B defined?

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$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}, C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E = \begin{bmatrix} 4 & 6 \\ 0 & 3 \\ 1 & 0 \end{bmatrix}, F = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

1. For which matrices is addition with A defined?
2. For which matrices is addition with B defined?
3. For which matrices is addition with C defined?

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$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}, C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E = \begin{bmatrix} 4 & 6 \\ 0 & 3 \\ 1 & 0 \end{bmatrix}, F = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

1. For which matrices is addition with A defined?
2. For which matrices is addition with B defined?
3. For which matrices is addition with C defined?
4. For which matrices is multiplication with A defined?

Is It The Correct Shape?

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}, C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E = \begin{bmatrix} 4 & 6 \\ 0 & 3 \\ 1 & 0 \end{bmatrix}, F = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

1. For which matrices is addition with A defined?
2. For which matrices is addition with B defined?
3. For which matrices is addition with C defined?
4. For which matrices is multiplication with A defined?
5. For which matrices is multiplication with B defined?

Is It The Correct Shape?

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}, C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E = \begin{bmatrix} 4 & 6 \\ 0 & 3 \\ 1 & 0 \end{bmatrix}, F = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

1. For which matrices is addition with A defined?
2. For which matrices is addition with B defined?
3. For which matrices is addition with C defined?
4. For which matrices is multiplication with A defined?
5. For which matrices is multiplication with B defined?
6. For which matrices is multiplication with C defined?

Example

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}, C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E = \begin{bmatrix} 4 & 6 \\ 0 & 3 \\ 1 & 0 \end{bmatrix}, F = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

Compute (if possible):

1. AF
2. FA

Example

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}, C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E = \begin{bmatrix} 4 & 6 \\ 0 & 3 \\ 1 & 0 \end{bmatrix}, F = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

Compute (if possible):

1. AF

This is defined!

2. FA

Example

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}, C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E = \begin{bmatrix} 4 & 6 \\ 0 & 3 \\ 1 & 0 \end{bmatrix}, F = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

Compute (if possible):

1. AF

This is defined!

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Not defined!

Example

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}, C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E = \begin{bmatrix} 4 & 6 \\ 0 & 3 \\ 1 & 0 \end{bmatrix}, F = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

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$$AF = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

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Not defined!

Example

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}, C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E = \begin{bmatrix} 4 & 6 \\ 0 & 3 \\ 1 & 0 \end{bmatrix}, F = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

Compute (if possible):

1. AF

This is defined!

$$AF = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \end{bmatrix}$$

2. FA

Not defined!

Example

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}, C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E = \begin{bmatrix} 4 & 6 \\ 0 & 3 \\ 1 & 0 \end{bmatrix}, F = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

Compute (if possible):

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$$AF = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 12 \\ -1 & -4 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 4 & 6 \\ 2 & 3 \end{bmatrix}$$

2. FA

Not defined!

Example

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}, C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E = \begin{bmatrix} 4 & 6 \\ 0 & 3 \\ 1 & 0 \end{bmatrix}, F = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

Compute (if possible):

1. AF

This is defined!

$$AF = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 12 \\ -1 & -4 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 4 & 6 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 12 \\ 3 & 2 \\ 3 & 7 \end{bmatrix}$$

2. FA

Not defined!

Now You Try!

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}, C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E = \begin{bmatrix} 4 & 6 \\ 0 & 3 \\ 1 & 0 \end{bmatrix}, F = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

1. BF
2. FB

Now You Try!

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}, C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E = \begin{bmatrix} 4 & 6 \\ 0 & 3 \\ 1 & 0 \end{bmatrix}, F = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

1. BF

$$BF = \begin{bmatrix} 2 & 13 \\ 4 & 6 \end{bmatrix}$$

2. FB

Now You Try!

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}, C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E = \begin{bmatrix} 4 & 6 \\ 0 & 3 \\ 1 & 0 \end{bmatrix}, F = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

1. BF

$$BF = \begin{bmatrix} 2 & 13 \\ 4 & 6 \end{bmatrix}$$

2. FB

$$FB = \begin{bmatrix} 4 & 7 \\ 8 & 4 \end{bmatrix}$$

Multiplication and Transpose Properties

Let $A, B \in \mathbb{R}^{m \times n}$, $C \in \mathbb{R}^{n \times p}$, and r be a scalar.

1. $\left(A^\top\right)^\top = A$

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4. $(AB)^\top = B^\top A^\top$