The Determinant and the Inverse

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But what about $\det\left(A^{-1}\right)$?
We claim that $\det\left(A^{-1}\right) = \frac{1}{\det(A)}$.

Finding $\det\left(A^{-1}\right)$

Let $A \in \mathbb{R}^{n \times n}$ such that $\det(A) \neq 0$.

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$$\begin{aligned} 1 &= \det(I_n) \\ &= \det\left(AA^{-1}\right) \\ &= \det(A)\det\left(A^{-1}\right) \\ \frac{1}{\det(A)} &= \det\left(A^{-1}\right) \end{aligned}$$

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Last Content on Exam 1

The previous two slides are the last content that is fair game for the first exam