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Are we done? No! Need to show $E_2E_2^{-1} = I_3$!

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Let $A, B \in \mathbb{R}^{n\times n}$ such that both are invertible. Then we know that

- A^{-1} is invertible and $\left(A^{-1}\right)^{-1} = A$
- AB is invertible and $(AB)^{-1} = B^{-1}A^{-1}$

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Product of Elementary Matrices

Let $A = \begin{bmatrix} 0 & 1 & -2 \\ 1 & 0 & 0 \\ 0 & 0 & 5 \end{bmatrix}$. Write A as a product of elementary matrices.

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$$\to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & \frac{2}{5} \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{5} \end{array}\right]$$

How to Compute A^{-1} Practice 3×3

If possible, find the inverse of the given matrices.

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & -2 & -2 \\ 1 & -3 & 1 \end{bmatrix} \qquad \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 3 \\ -4 & 0 & -2 \end{bmatrix}$$

A^{-1} Solution 1

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & -2 & -2 \\ 1 & -3 & 1 \end{bmatrix}$$

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$$A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & -2 & -2 \\ 1 & -3 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 1 & -3 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3=R_3-R1} \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & -3 & -2 & -1 & 0 & 1 \end{array}\right] \xrightarrow{R_2=-\frac{R_2}{2}} \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & -\frac{1}{2} & 0 \\ 0 & -3 & -2 & -1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_3=R_3+3R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & -1 & -\frac{3}{2} & 1 \end{array}\right]$$

A^{-1} Solution 1

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & -2 & -2 \\ 1 & -3 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 1 & -3 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3 = R_3 - R1} \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & -3 & -2 & -1 & 0 & 1 \end{array}\right] \xrightarrow{R_2 = -\frac{R_2}{2}} \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & -\frac{1}{2} & 0 \\ 0 & -3 & -2 & -1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_3 = R_3 + 3R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & -1 & -\frac{3}{2} & 1 \end{array}\right] \xrightarrow{R_2 = R_2 - R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & -1 & -\frac{3}{2} & 1 \end{array}\right]$$

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$$\xrightarrow{R_1 - 3R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 4 & \frac{9}{2} & -3 \\ 0 & 1 & 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & -1 & -\frac{3}{2} & 1 \end{array}\right]$$

A^{-1} Solution 2

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 3 \\ -4 & 0 & -2 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 2 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & 1 & 0 \\ -4 & 0 & -2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3 = R_3 + 2R_1} \left[\begin{array}{ccc|ccc} 2 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

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Can't find the inverse!

If possible, find the inverse of the given matrices.

$$\begin{bmatrix} 3 & 6 \\ 1 & 2 \end{bmatrix} \qquad \begin{bmatrix} -2 & 4 \\ -3 & 1 \end{bmatrix}$$

A^{-1} solution 3

$$A = \begin{bmatrix} 3 & 6 \\ 1 & 2 \end{bmatrix}$$

$$\left[\begin{array}{cc|cc} 3 & 6 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{array}\right] \xrightarrow{R_2 = R_2 - \frac{R_1}{3}} \left[\begin{array}{cc|cc} 3 & 6 & 1 & 0 \\ 0 & 0 & -\frac{1}{3} & 1 \end{array}\right]$$

A^{-1} solution 3

$$A = \begin{bmatrix} 3 & 6 \\ 1 & 2 \end{bmatrix}$$

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Can't find the inverse!

A^{-1} solution 4

$$A = \begin{bmatrix} -2 & 4 \\ -3 & 1 \end{bmatrix}$$

$$\left[\begin{array}{cc|cc} -2 & 4 & 1 & 0 \\ -3 & 1 & 0 & 1 \end{array}\right] \xrightarrow{R_2 = R_2 - \frac{3R_1}{2}} \left[\begin{array}{cc|cc} -2 & 4 & 1 & 0 \\ 0 & -5 & -\frac{3}{2} & 1 \end{array}\right]$$

A^{-1} solution 4

$$A = \begin{bmatrix} -2 & 4 \\ -3 & 1 \end{bmatrix}$$

$$\left[\begin{array}{cc|cc} -2 & 4 & 1 & 0 \\ -3 & 1 & 0 & 1 \end{array}\right] \xrightarrow{R_2 = R_2 - \frac{3R_1}{2}} \left[\begin{array}{cc|cc} -2 & 4 & 1 & 0 \\ 0 & -5 & -\frac{3}{2} & 1 \end{array}\right] \xrightarrow[R_2 = \frac{-R_2}{5}]{R_1 = \frac{-R_1}{2}} \left[\begin{array}{cc|cc} 1 & -2 & -\frac{1}{2} & 0 \\ 0 & 1 & \frac{3}{10} & -\frac{1}{5} \end{array}\right]$$

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$$\xrightarrow{R_1 = R_1 + 2R_2} \left[\begin{array}{cc|cc} 1 & 0 & \frac{1}{10} & -\frac{2}{5} \\ 0 & 1 & \frac{3}{10} & -\frac{1}{5} \end{array}\right]$$

Solving a System With A^{-1}

Let's say we know A^{-1}. How can we solve linear systems like $A\mathbf{x} = \mathbf{b}$?

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   2.2 If we can, then continue
3. Multiply both sides of (1) by A^{-1}.
4. Solution is $\mathbf{x} = A^{-1}\mathbf{b}$

Note: If A is **NOT** square, then we cannot find an inverse!