Vectors

  

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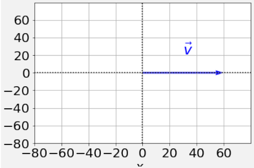
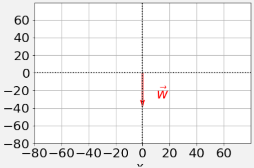
Example

A car traveling 60 miles per hour directly East.



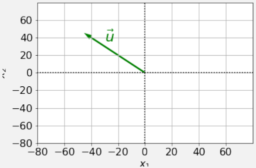
$$\mathbf{v} = \vec{v} = \begin{bmatrix} 60 \\ 0 \end{bmatrix}$$

A car traveling 40 miles per hour directly South.



$$\mathbf{w} = \vec{w} = \begin{bmatrix} 0 \\ -40 \end{bmatrix}$$

A car traveling 64 miles per hour directly North West.



$$\mathbf{u} = \vec{u} = \begin{bmatrix} -\frac{64}{\sqrt{2}} \\ \frac{64}{\sqrt{2}} \end{bmatrix}$$

Vectors in $\mathbb{R}^n$

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If the vector $\mathbf{v}$ has n different real numbers, then we say that $\mathbf{v} \in \mathbb{R}^n$, and

$$\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$

Vector Equality

Definition

Vector Equality. We say that two vectors, $\mathbf{v} = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$ $\mathbf{w} = \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix}$ are equal if and only if

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- They have the same direction and length or
- All entries of the two vectors are the same:

$$v_k = w_k \text{ for all } 1 \leq k \leq n$$

Vector Arithmetic (Addition)

Definition

We define adding two vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$ as $\mathbf{u} + \mathbf{v} = \begin{bmatrix} u_1 + v_1 \\ \vdots \\ u_n + v_n \end{bmatrix}$. Note: This means that we can only add vectors with the same dimensions together! And the result will have the same dimension!

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Adding the following vectors

$$\mathbf{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

$$\mathbf{u} = \begin{bmatrix} 0 \\ -1 \\ 5 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 7 \\ -5 \\ 10 \end{bmatrix}$$

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$$\mathbf{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

$$\mathbf{u} + \mathbf{v} = \begin{bmatrix} 1+4 \\ 2+1 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

$$\mathbf{u} = \begin{bmatrix} 0 \\ -1 \\ 5 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 7 \\ -5 \\ 10 \end{bmatrix}$$

$$\mathbf{u} + \mathbf{v} = \begin{bmatrix} 0+7 \\ -1+-5 \\ 5+10 \end{bmatrix} = \begin{bmatrix} 7 \\ -6 \\ 15 \end{bmatrix}$$

Vector Arithmetic (Scalar Multiplication)

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Computing the scalar product of the following

$c = -3, \mathbf{v} = \begin{bmatrix} 2 \\ -5 \end{bmatrix},$

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Example

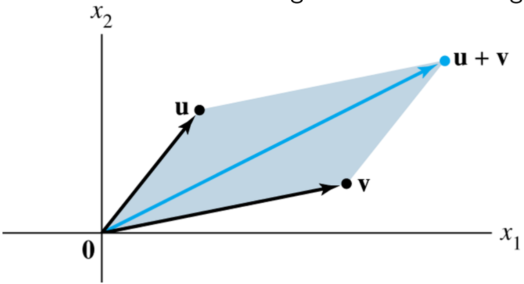
Computing the scalar product of the following

$c = -3, \mathbf{v} = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$, $\quad c\mathbf{v} = \begin{bmatrix} -3 \cdot 2 \\ -3 \cdot -5 \end{bmatrix} = \begin{bmatrix} -6 \\ 15 \end{bmatrix}$

Geometric Descriptions of Vectors in $\mathbb{R}^2$

We can think of the vector $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \in \mathbb{R}^2$ as a directed line segment (an arrow).

Consider two vectors $\mathbf{u}$ and $\mathbf{v} \in \mathbb{R}^2$. The sum $\mathbf{u} + \mathbf{v}$ is the result of first moving $\mathbf{u}$ and then moving $\mathbf{v}$.



Consider the vector $\mathbf{u} \in \mathbb{R}^2$ and nonzero scalar c. The scalar product $c\mathbf{u}$ gives a vector parallel to $\mathbf{u}$ whose magnitude has been scaled by a factor of c.

- If $c > 0$, then $c\mathbf{u}$ has the same direction as $\mathbf{u}$.
- If $c < 0$, then $c\mathbf{u}$ has the opposite direction as $\mathbf{u}$.
- If $|c| < 1$, then $c\mathbf{u}$ is a compression of $\mathbf{u}$.
- If $|c| > 1$, then $c\mathbf{u}$ is a stretching of $\mathbf{u}$.

Parallelogram Rule for Addition: If $\mathbf{u}$ and $\mathbf{v}$ are in $\mathbb{R}^2$, then the sum $\mathbf{u} + \mathbf{v}$ is the diagonal of the parallelogram spanned by $\mathbf{u}$ and $\mathbf{v}$.

Properties of Vector Arithmetic

The zero vector is defined as the vector in $\mathbb{R}^n$ with no magnitude, and it is denoted by $\mathbf{0}$ or $\vec{0}$.

As a result of this definition, it follows that $\mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$.

For all $\mathbf{u}, \mathbf{v}, \mathbf{w}$ in $\mathbb{R}^n$ and all scalars c and d:

(i) $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$

(ii) $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$

(iii) $\mathbf{u} + \mathbf{0} = \mathbf{0} + \mathbf{u} = \mathbf{u}$

(iv) $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$

(v) $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$

(vi) $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$

(vii) $c(d\mathbf{u}) = (cd)\mathbf{u}$

(viii) $1\mathbf{u} = \mathbf{u}$

Linear Combinations of Vectors

Definition

Linear Combination: A linear combination of vectors $\mathbf{v}_1, \ldots, \mathbf{v}_p \in \mathbb{R}^n$ with weights $c_1, \ldots, c_p \in \mathbb{R}$ is the vector $\mathbf{y}$ given by

$$\mathbf{y} = c_1\mathbf{v}_1 + \cdots + c_p\mathbf{v}_p = \sum_{k=1}^{p} c_k\mathbf{v}_k$$

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$$2\begin{bmatrix} 2 \\ 1 \end{bmatrix} + -3\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

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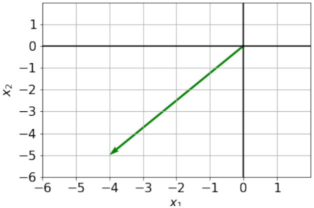
Giving Directions in a Grid



Example

Find constants x_1 and x_2 such that

$$\begin{bmatrix} -4 \\ -5 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

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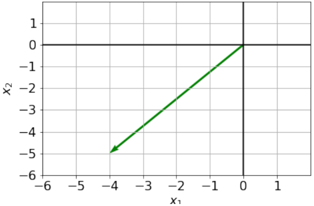

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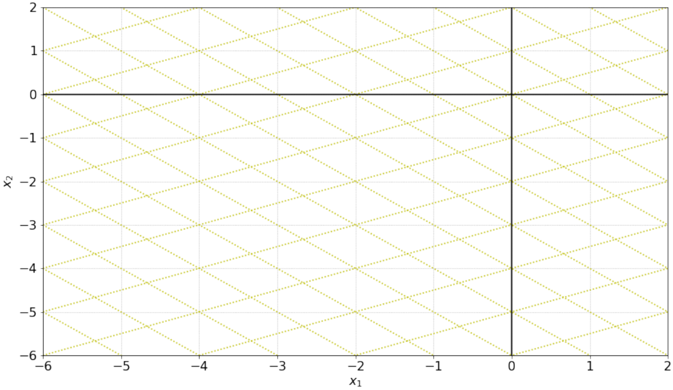
The vectors $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ are the standard column vectors.

Giving Directions in a Different System



Example

Find constants x_1 and x_2 such that

$$\begin{bmatrix} -4 \\ -5 \end{bmatrix} = x_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Solving for the Weights

First, let's simplify our statement!

$$\begin{bmatrix} -4 \\ -5 \end{bmatrix} = x_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2x_1 - x_2 \\ x_1 + x_2 \end{bmatrix}$$

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Which becomes

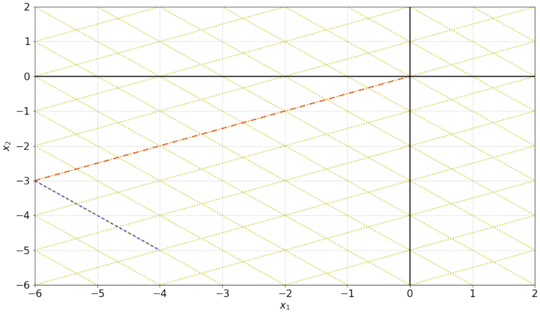
$$\begin{aligned} -4 &= 2x_1 - 1x_2 \\ -5 &= 1x_1 + 1x_2 \end{aligned}$$

Gaussian Elimination can solve this!

$$\left[\begin{array}{cc|c} 2 & -1 & -4 \\ 1 & 1 & -5 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 0 & -3 \\ 0 & 1 & -2 \end{array}\right]$$

So, $(x_1, x_2) = (-3, -2)$. Or we can say $\mathbf{x} = \begin{bmatrix} -3 \\ -2 \end{bmatrix}$

What Our Directions Look Like



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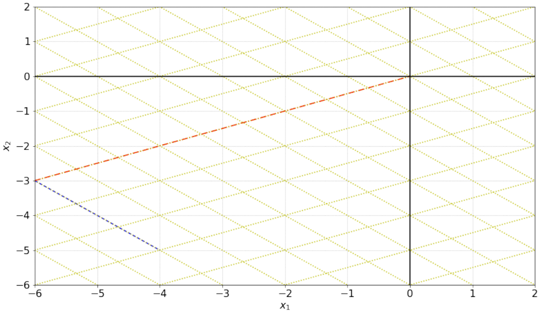

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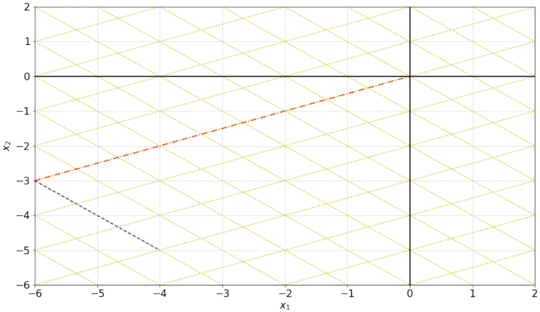

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What about an arbitrary vector $\mathbf{w} \in \mathbb{R}^2$? Can we find an $\mathbf{x}$ such that

$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = x_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Span of a Set of Vectors

Definition

Span: Let $\mathbf{v}_1, \ldots, \mathbf{v}_p \in \mathbb{R}^n$. Then we define $\text{Span}\,(\mathbf{v}_1, \ldots, \mathbf{v}_p)$ to be the subset of $\mathbb{R}^n$ containing all linear combinations of $\mathbf{v}_1, \ldots, \mathbf{v}_p$.

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$$\operatorname{Span}(\mathbf{v}_1, \ldots, \mathbf{v}_p) = \left\{ \mathbf{y} \in \mathbb{R}^n \,\middle|\, \mathbf{y} = \sum_{k=1}^{p} c_k \mathbf{v}_k \text{ for some } c_1, \ldots, c_p \in \mathbb{R} \right\}$$

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How do we test if $\mathbf{y} \in \operatorname{Span}(\mathbf{v}_1, \ldots, \mathbf{v}_p)$?

Solution

Solve the system

$$\left[\begin{array}{ccc|c} \mathbf{v}_1 & \cdots & \mathbf{v}_p & \mathbf{y} \end{array}\right]$$

Span Example

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Describe the span of the vectors $\mathbf{v}_1, \mathbf{v}_2$ given by

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \qquad \mathbf{v}_2 = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

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Solution

$$c_1\mathbf{v}_1 + c_2\mathbf{v}_2 = c_1 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} c_1 \\ 2c_2 \\ c_1 + c_2 \end{bmatrix}$$

So,

$$Span\left(\mathbf{v}_1, \mathbf{v}_2\right) = \left\{ \mathbf{y} \in \mathbb{R}^3 \,\middle|\, \mathbf{y} = \begin{bmatrix} c_1 \\ 2c_2 \\ c_1 + c_2 \end{bmatrix} \text{ for some } c_1, c_2 \in \mathbb{R} \right\}$$

Span of a Set of Vectors Practice

Describe the span of the following vectors

1. $\mathbf{v}_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

2. $\mathbf{v}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -4 \\ -2 \end{bmatrix}$

3. $\mathbf{v}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

4. $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

Span of a Set of Vectors Practice Solutions

Describe the span of the following vectors

1. $\mathbf{v}_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$.

   $$\text{Span}\left(\mathbf{v}_1\right) = \left\{ \mathbf{y} \in \mathbb{R}^3 \,\middle|\, \mathbf{y} = \begin{bmatrix} 0 \\ c_1 \\ c_1 \end{bmatrix} \text{ for some } c_1 \in \mathbb{R} \right\}$$

2. $\mathbf{v}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -4 \\ -2 \end{bmatrix}$

   $$\text{Span}\left(\mathbf{v}_1, \mathbf{v}_2\right) = \left\{ \mathbf{y} \in \mathbb{R}^2 \,\middle|\, \mathbf{y} = \begin{bmatrix} 2c_1 - 4c_2 \\ c_1 - 2c_2 \end{bmatrix} \text{ for some } c_1, c_2 \in \mathbb{R} \right\}$$

3. $\mathbf{v}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

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   $$\text{Span}\left(\mathbf{v}_1, \mathbf{v}_2\right) = \left\{ \mathbf{y} \in \mathbb{R}^3 \,\middle|\, \mathbf{y} = \begin{bmatrix} c_1 \\ c_2 \\ -2c_1 - c_2 \end{bmatrix} \text{ for some } c_1, c_2 \in \mathbb{R} \right\}$$

More Span Practice

$$\text{Determine if } y = \begin{bmatrix} 4 \\ 5 \\ -1 \end{bmatrix} \text{ is in Span} \left(\begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right)$$

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Perform Gaussian Elimination on the augmented matrix

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$$\left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 5 \\ -2 & -1 & -1 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 5 \\ 0 & 0 & 12 \end{array}\right]$$

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$$\text{Determine if } y = \begin{bmatrix} 4 \\ 5 \\ -1 \end{bmatrix} \text{ is in Span} \left(\begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right)$$

Perform Gaussian Elimination on the augmented matrix

$$\left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 5 \\ -2 & -1 & -1 \end{array} \right] \to \left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 5 \\ 0 & 0 & 12 \end{array} \right]$$

No!

Systems with Infinite Solutions

Recall for the system,

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But writing this can be cumbersome!

Using Vectors!

We recall from earlier that we can write out our answer in terms of vectors!

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Span of a Set of Vectors Conceptual Practice

Let $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{w}$ be vectors in $\mathbb{R}^n$. Prove that if $\mathbf{w} \in \operatorname{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$, then $\operatorname{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{w}) = \operatorname{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$