Linear Equation Review

Definition

Linear Equations:
A linear equation of n variables $x_1, \ldots, x_n$ is an equation that can be written in the form

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$$

Where b and $a_1, \ldots, a_n$ are constants

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Practice

Determine which of the following equations are linear in x_1, x_2, x_3.

1. $x_1 + 4x_2 + x_1x_3 = 3$
2. $\pi x_1 - \frac{x_2}{e^2} = 4$
3. $\cos(4)x_1 + \sin(2)x_2 + x_3 = e\pi$

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Systems of Linear Equations

Definition

A *system of linear equations* is a collection of many linear equations using the same variables

Example

$$\begin{aligned} 2x_1 + 4x_2 &= 8 \\ x_1 - 2x_2 &= 0 \end{aligned}$$

A Linear System Solution Method

There are many ways that we learned to solve systems of linear equations in other math classes, the way we will discuss is called *Elimination*. We will use this method to solve

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So, our answer is $(x_1, x_2) = (2, 1)$

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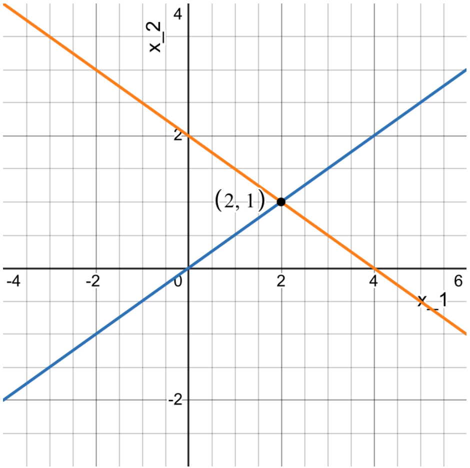
So, our answer is $(x_1, x_2) = (2, 1)$

Remark

This method is called *Elimination* because we are *eliminating* variables until we only have one per equation!

Looking At Our Solution Visually



How Many Solutions Are There?

Let's consider two different linear systems

$$\begin{aligned} 2x_1 + 4x_2 &= 4 \\ x_1 + 2x_2 &= 2 \end{aligned} \qquad\qquad \begin{aligned} 2x_1 + 4x_2 &= 4 \\ x_1 + 2x_2 &= 4 \end{aligned}$$

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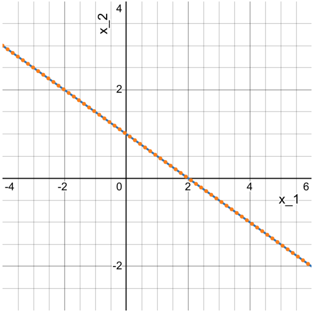

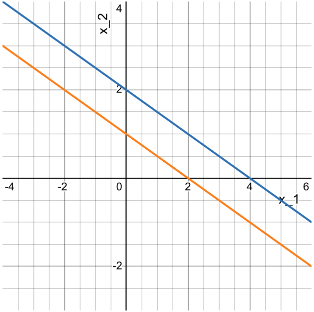

Consistency

Definition

A linear system of equations is called *Consistent* if it has at least one solution, and it is called *Inconsistent* otherwise.

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Example

From the previous slides, we would say that

$$\begin{aligned} 2x_1 + 4x_2 &= 4 \\ x_1 + 2x_2 &= 2 \end{aligned} \qquad\qquad \begin{aligned} 2x_1 + 4x_2 &= 8 \\ x_1 - 2x_2 &= 0 \end{aligned}$$

are consistent while

$$\begin{aligned} 2x_1 + 4x_2 &= 4 \\ x_1 + 2x_2 &= 4 \end{aligned}$$

is inconsistent.

Consistency Practice

Practice

Determine if the following systems are consistent or inconsistent

1. $$\begin{aligned} 2x_1 + 4x_2 &= 1 \\ 4x_1 + 8x_2 &= 2 \end{aligned}$$

2. $$\begin{aligned} 6x_1 + 3x_2 &= 10 \\ 12x_1 + 6x_2 &= 10 \end{aligned}$$

3. $$\begin{aligned} 2x_1 + 3x_2 &= 0 \\ x_1 + 5x_2 &= 0 \end{aligned}$$

4. $$\begin{aligned} x_1 + x_2 &= 1 \\ x_1 + x_2 &= 1 \end{aligned}$$

Consistency Practice Answers

Practice

Determine if the following systems are consistent or inconsistent

1. $$\begin{aligned} 2x_1 + 4x_2 &= 1 \\ 4x_1 + 8x_2 &= 2 \end{aligned}$$

   This system is consistent. It has an infinite number of solutions!

2. $$\begin{aligned} 6x_1 + 3x_2 &= 10 \\ 12x_1 + 6x_2 &= 10 \end{aligned}$$

   This system is inconsistent. There cannot be a solution

3. $$\begin{aligned} 2x_1 + 3x_2 &= 0 \\ x_1 + 5x_2 &= 0 \end{aligned}$$

   This system is consistent. It has exactly 1 solution!

4. $$\begin{aligned} x_1 + x_2 &= 1 \\ x_1 + x_2 &= 1 \end{aligned}$$

   This system is consistent. It has an infinite number of solutions!

Solution Set

Definition

A *solution set* is the set of all possible solutions to a system of linear equations.

Example

The solution set for the system

$$\begin{aligned} x + y &= 1 \\ x + y &= 1 \end{aligned}$$

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The solution set for the system

$$x + y = 1$$
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would be

$$\{(x, y) | \, y = 1 - x\}$$

Solution Set Practice

Practice

Determine the solution set for each of the following systems

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Solution Set Practice Answers

Practice

Determine the solution set for each of the following systems

1.

$$\begin{aligned} 2x_1 + 4x_2 &= 1 \\ 4x_1 + 8x_2 &= 2 \end{aligned}$$

The solution set is $\left\{(x_1, x_2) \middle|\, x_2 = \frac{1-2x_1}{4}\right\}$

2.

$$\begin{aligned} 6x_1 + 3x_2 &= 10 \\ 12x_1 + 6x_2 &= 10 \end{aligned}$$

The solution set is $\emptyset$

3.

$$\begin{aligned} 2x_1 + 3x_2 &= 0 \\ x_1 + 5x_2 &= 0 \end{aligned}$$

The solution set is $\{(0,0)\}$.

Matrix Notation

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Coefficient Matrix:

$$\begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix}$$

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Augmented Matrix:

$$\left[\begin{array}{cc|c} 2 & 5 & 1 \\ 1 & 2 & 7 \end{array}\right]$$

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Augmented Matrix:

$$\left[\begin{array}{cc|c} 2 & 5 & 1 \\ 1 & 2 & 7 \end{array}\right]$$

or

$$\begin{bmatrix} 2 & 5 & 1 \\ 1 & 2 & 7 \end{bmatrix}$$

Some Properties of Matrices

How many rows and columns does the following augmented matrix have?

$$\left[\begin{array}{cccc|c} 2 & 9 & 13 & 3 & 0 \\ 1 & 0 & 2 & 1 & 1 \\ 4 & 1 & 0 & 2 & 12 \end{array}\right]$$

How many rows and columns does the following augmented matrix have?

$$3 \left\{ \left[\begin{array}{cccc|c} 2 & 9 & 13 & 3 & 0 \\ 1 & 0 & 2 & 1 & 1 \\ 4 & 1 & 0 & 2 & 12 \end{array} \right] \right.$$

- This matrix has 3 rows. This means there were 3 equations in the original system

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- This matrix has 3 rows. This means there were 3 equations in the original system
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Note: The order is very important when we use the shorthand, rows always comes first and columns always go second.

Solving a Linear System With Matrices

Let's look at how we can use this augmented matrix by solving a system our old way and the new way simultaneously

Solving a Linear System With Matrices

$$\begin{aligned} 2x_1 + 4x_2 &= 8 \\ 1x_1 - 2x_2 &= 0 \end{aligned}$$

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$$\begin{aligned} 2x_1 + 4x_2 &= 8 \\ 1x_1 - 2x_2 &= 0 \end{aligned}$$

$$\left[\begin{array}{cc|c} 2 & 4 & 8 \\ 1 & -2 & 0 \end{array}\right]$$

Solving a Linear System With Matrices

$$\begin{aligned} 2x_1 + 4x_2 &= 8 \\ 1x_1 - 2x_2 &= 0 \end{aligned}$$

$$\begin{aligned} 2x_1 + 4x_2 &= 8 \\ 0x_1 - 4x_2 &= -4 \end{aligned}$$

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$$R_2 = R_2 - \tfrac{1}{2}R_1$$

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$$R_1 = R_1 + R_2$$

$$\begin{aligned} 2x_1 + 0x_2 &= 4 \\ 0x_1 - 4x_2 &= -4 \end{aligned}$$

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$$R_1 = R_1 + R_2$$

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$$R_1 = \tfrac{1}{2}R_1,\ R_2 = -\tfrac{1}{4}R_2$$

$$\begin{aligned} 1x_1 + 0x_2 &= 2 \\ 0x_1 + 1x_2 &= 1 \end{aligned}$$

$$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 1 \end{array}\right]$$

A Larger System!

Practice

Solve the following system of equations for x_1, x_2, x_3

$$\begin{aligned} x_1 + x_2 + x_3 &= 7 \\ x_1 - x_2 + 2x_3 &= 7 \\ 5x_1 + x_2 + x_3 &= 11 \end{aligned}$$

A Larger System Solution

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 1 & -1 & 2 & 7 \\ 5 & 1 & 1 & 11 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 0 & -2 & 1 & 0 \\ 5 & 1 & 1 & 11 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 0 & -2 & 1 & 0 \\ 0 & -4 & -4 & -24 \end{array}\right] \to$$

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$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & -6 & -24 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & 1 & 4 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 0 & -2 & 0 & -4 \\ 0 & 0 & 1 & 4 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 4 \end{array}\right] \rightarrow$$

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Elementary Row Operations

We have a name for the kinds of changes we are making to these matrices, we call them "Elementary Row Operations"

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$$\left[\begin{array}{ccc|c} 1 & -8 & 0 & 6 \\ 0 & 0 & 2 & 8 \\ 0 & 1 & 0 & -3 \end{array}\right]$$

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$$\left[\begin{array}{ccc|c} 1 & -8 & 0 & 6 \\ 0 & 0 & 2 & 8 \\ 0 & 1 & 0 & -3 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & -18 \\ 0 & 0 & 2 & 8 \\ 0 & 1 & 0 & -3 \end{array}\right]$$

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Example

$$\left[\begin{array}{ccc|c} 1 & -8 & 0 & 6 \\ 0 & 0 & 2 & 8 \\ 0 & 1 & 0 & -3 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & -18 \\ 0 & 0 & 2 & 8 \\ 0 & 1 & 0 & -3 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & -18 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 2 & 8 \end{array}\right]$$

Elementary Row Operations

We have a name for the kinds of changes we are making to these matrices, we call them "Elementary Row Operations"

Definition

An operation on a row of a matrix called is an Elementary Row Operation if it is one of

1. Replacement: Replace one row by itself plus a multiple of another row
2. Interchange: Swap two rows
3. Scaling: Multiply an entire row by the same constant

Example

$$\left[\begin{array}{ccc|c} 1 & -8 & 0 & 6 \\ 0 & 0 & 2 & 8 \\ 0 & 1 & 0 & -3 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & -18 \\ 0 & 0 & 2 & 8 \\ 0 & 1 & 0 & -3 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & -18 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 2 & 8 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & -18 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

Equivalent Systems

Definition

Two matrices (augmented or not augmented) are Row Equivalent if we can transform them into each other using only Elementary Row Operations

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are Row Equivalent augmented matrices.

Remark

If two augmented matrices are row equivalent, then the solutions to their respective systems are the same!

Equivalent Systems

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Two linear systems are called Equivalent if they have the same solution.

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Example

$$\begin{aligned} 2x_1 + 4x_2 &= 2 \\ 1x_1 + 3x_2 &= 5 \end{aligned} \qquad\qquad \begin{aligned} 4x_1 + 8x_2 &= 4 \\ 2x_1 + 6x_2 &= 10 \end{aligned}$$

Are equivalent systems because the right one is just the left with both rows multiplied by 2.