Open Economic Model

February 1, 2025

Homework 2 problem 19 looks something like this for y'all.

> An economy has three industries, farming, building, and clothing. For every dollar of food produced, the farmers use \$0.18, the builders use \$0.04, and the tailors use \$0.14. For every dollar of building, the builders use \$0.03, the farmers use \$0.12, and the tailors use \$0.04. For every dollar of clothing produced, the tailors use \$0.05, the builders use \$0.03, and the farmers use \$0.13. If the external demand for food is \$160 million, for building is \$240 million, and for clothing is \$230 million, what should be the total production for each industry?

This problem is a little different than the type we covered in class. We assumed that we would be using all of our output internally. However, this problem requires us to have a certain amount of net production to sell (Think of this like budgeting in savings in your monthly expenses. While your food or gas bill will depend on how much you exercise and how much drive, a savings goal is static).

More information on this problem can be found with the Linear Algebra With Applications textbook on page 137. Note: This is later in the book than we currently are so some of the vocabulary and things they prove are things we won't worry about yet!

For the example above, we first set up our system as follows where p_f denotes the amount of money produced from farming, p_b denotes the amount of money produced from building, and p_c denotes the amount of money produced from clothing. Since we are given part of our data in millions, we will use millions of dollars as our units.

$$\begin{aligned} p_f &= \text{Cost of production} + \text{External demand} = (.18p_f + .12p_b + .13p_c) + 160 \\ p_b &= \text{Cost of production} + \text{External demand} = (.04p_f + .03p_b + .03p_c) + 240 \\ p_c &= \text{Cost of production} + \text{External demand} = (.14p_f + .04p_b + .05p_c) + 230 \end{aligned}$$

Which can be rearranged as below

$$\begin{aligned} .82p_f - .12p_b - .13p_c &= 160 \\ -.04p_f + .97p_b - .03p_c &= 240 \\ -.14p_f - .04p_b + .95p_c &= 230 \end{aligned}$$

Giving us the following augmented matrix

$$\left[\begin{array}{ccc|c} .82 & -.12 & -.13 & 160 \\ -.04 & .97 & -.03 & 240 \\ -.14 & -.04 & .95 & 230 \end{array}\right]$$

This is a bit of a pain to perform our row reductions due to the decimals and the size of the numbers. I won't ask you to do one like this by hand on the written homework or on an exam. If this happens again in a future homework, you can use python to reduce the matrix if it doesn't play nice with a couple steps for the MyOpenMath assignments (I'll also make this announcement in class).

The python to reduce this matrix and give us RREF is below

```
import sympy as sym
A = sym.Matrix([[.82, -.12, -.13, 160],[-.04, .97, -.03, 240],[-.14, -.04, .95, 230]])
display(A) # To double check we input the matrix correctly
[RREF, pivots] = A.rref() # Stores the RREF form in RREF and the pivot information in pivots
display(RREF[:,3]) # Prints out the right hand side
```

Which gives us the output

$$\begin{bmatrix} 281.100694254633 \\ 268.132619034603 \\ 294.820423112666 \end{bmatrix}$$

Note: Computers can't do math exactly like we can, so this might be a little different than if you were to do it by hand, so we'd round to 2 decimal points, and it'll be good enough for our uses.

$$\begin{bmatrix} 281.10 \\ 268.13 \\ 294.82 \end{bmatrix}$$