Example 1 Part 1

Let

$$A = \begin{bmatrix} 4 & 14 & -4 \\ -12 & 28 & -8 \\ 12 & -13 & 18 \end{bmatrix}, \qquad B = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 20 & 10 \\ 0 & 0 & 20 \end{bmatrix}, \qquad C = \begin{bmatrix} 2 & 2 & -3 \\ 0 & 2 & -2 \\ -3 & -1 & 0 \end{bmatrix}$$

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Which is the same as AB.

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Let A, B be given below, then compute a value of k such that A and B are similar.

$$A = \begin{bmatrix} 5 & 2 \\ 4 & k \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 0 \\ 0 & 7 \end{bmatrix}$$

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We can solve this however we want, but the first equation gives us $k = 3$ and plugging this into the second gives us what we need, so k must be 3.

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Now we find the eigenvectors of A and then we would be done!