

# Foundational Magnetic Susceptibility

## Table of Contents

<b>1. Introduction</b>	
<b>2. Simplest theoretical treatments of magnetization</b>	
a) Diamagnetism	page 5
b) Curie Paramagnetism	6
<b>3. Measuring magnetic susceptibility</b>	
a) The Gouy balance	10
b) Setting up the apparatus	14
c) Sample handling	17
d) Measurement	18
e) $B$ -calibration	23
f) Data reduction	25
g) Converting between SI and cgs unit systems	31
<b>References</b>	33

<b>Appendix A:</b>	<b>First-time use, calibration, and testing of the HRB-203 balance</b>
<b>Appendix B:</b>	<b>The standard samples provided by TeachSpin</b>
<b>Appendix C:</b>	<b>Sample preparation</b>
<b>Appendix D:</b>	<b>Magnetic surveying and linearity of magnetization</b>
<b>Appendix E:</b>	<b>Non-ideal features of the permanent-magnet's field</b>
<b>Appendix F:</b>	<b>Susceptibility of anisotropic materials</b>
<b>Appendix G:</b>	<b>Magnetic levitation of diamagnetic materials</b>
<b>Appendix H:</b>	<b>A case study in computing paramagnetic susceptibility</b>
<b>Appendix I:</b>	<b>Diamagnetic corrections to measured susceptibilities</b>
<b>Appendix J:</b>	<b>The internal field in diamagnetic water</b>
<b>Appendix K:</b>	<b>Some project ideas for Foundational Magnetic Susceptibility</b>
<b>Appendix L:</b>	<b>What is a Bohr magneton?</b>

# Foundational Magnetic Susceptibility

## 1...Introduction

Your first acquaintance with magnetism probably involved playing with permanent magnets, and that sort of play might have led you to categorize other objects into two categories: ‘magnetic’ materials (such as paper clips and steel objects in general), and ‘non-magnetic’ materials (including wood, water, plastic, and most other stuff). Similarly, in your first playing with electric circuits, you learned to make the classification of items into ‘conductors’ like wire and metal, and ‘non-conductors’ such as wood, plastic, and so on.

We can and do classify materials as conductors vs. insulators, or as magnetic vs. non-magnetic; but what are the *numerical values* underlying such a binary separation? That is to say, is there a numerical scale for ‘conductive-ness’ or ‘magnetic-ness’, more illuminating than the simple yes/no answer implied by the sorting into two categories? For classifying electrical properties, you know we can introduce the resistivity (or conductivity) of a material, which allows us to talk about *degree* of conductivity, and (for example) to introduce the intermediate category of semiconductors. Similarly, it proves valuable to take up ‘degree of magnetic-ness’, to replace that initial all-or-nothing categorization of the magnetic response of materials. We’ll see that *all* materials display a magnetic response to some degree, and we will quantify how ‘susceptible’ they are to a magnetic response by defining the ‘*magnetic susceptibility*’ of a material.

The usefulness of magnetic susceptibility is that it can be measured readily (and non-destructively), and that its value can be predicted, given hypotheses and deductions from quantum mechanics. Comparison of measured and predicted values can then *test* those hypotheses. **It is a striking fact that simple, non-invasive, tabletop, room-temperature measurements on bulk materials will give experimental values which can test quantum-mechanical predictions for the spin magnetic moments of atoms.** More striking still is to realize that the only possible prediction of classical mechanics is that no material can be magnetic at all.(!)

To get to the definition, and then to the measurement, of magnetic susceptibility, we need to start at the foundations: namely, the magnetic field  $B$ , and its consequences for materials.

### Starting points in magnetism

Everyone knows that the earth somehow controls the orientation of the needle of a compass. We describe the earth’s role in this interaction by the magnetic field  $B(r)$  it sets up in space. But in this interaction, the compass needle is special, too; it possesses a property called its magnetic moment  $\mu$ , defined via the torque  $\tau$  it experiences in response to the field:

$$\tau = \mu \times B .$$

Magnetic moment is an ‘extensive’ measure of ‘how magnetic’ a body is, one that scales with the size of a magnetized sample. Another, ‘intensive’, measure for that material is the magnetization  $\mathbf{M}$ , defined as magnetic moment per unit volume. Just as masses  $m_i$  within a volume  $\Delta V$  define a mass density

$$\rho \equiv \frac{1}{\Delta V} \sum_i m_i ,$$

so the magnetic moments  $\mu_i$  within a volume  $\Delta V$  define a ‘magnetic moment density’ called the magnetization

$$\mathbf{M} \equiv \frac{1}{\Delta V} \sum_i \boldsymbol{\mu}_i .$$

With the usual arrangement of taking  $\Delta V$  small enough (though not *too* small – we still want  $\Delta V$  to contain many atoms), we can use a limiting process to define point-values of the local magnetization  $\mathbf{M}(r)$ , a field in space, which might take on uniform (or varying) values throughout a magnetized medium, but which vanishes outside the medium.

Example:

Suppose that a single cylindrical disc of NdFeB permanent-magnet material, of diameter 1" and thickness  $\frac{1}{4}$ ", has a measured magnetic moment of  $3.1 \text{ A}\cdot\text{m}^2$ . Assuming the disc is uniformly magnetized, compute the magnetization  $\mathbf{M}$  inside the material. (Result – near  $1.0 \times 10^6 \text{ A/m}$ ; the units come from  $\text{A}\cdot\text{m}^2/\text{m}^3$ .)

Magnetization might be permanent, with  $\mathbf{M}$  persisting after its original cause is removed. Or, it might be transitory, lasting only as long as some external field causes it. The relationship between the cause  $\mathbf{B}$  and the effect  $\mathbf{M}$  is best captured via the auxiliary field  $\mathbf{H}$ , defined in SI units by

$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}) .$$

The motivation for this definition is that the field  $\mathbf{H}$  has as its source (in one sense) only the current-density of free electrical currents in wires:

$$\nabla \times \mathbf{H} = \mathbf{J}_{free} .$$

That is to say, the curl of  $\mathbf{H}$  is given by a current density that we have under our control – it does not depend on the atomic currents involved in magnetization. The field  $\mathbf{H}$ , in other words, can be a freely-chosen independent variable, even when magnetizable material is present in an experimental system.

Remarks on units:

In SI,  $\mathbf{B}$ ’s fundamental definition comes from the Lorentz force law,  $\mathbf{F} = q \mathbf{v} \times \mathbf{B}$ , and the units for  $B$  are Tesla, where  $1 \text{ Tesla} = 1 \text{ T} = 1 \text{ N}/(\text{A}\cdot\text{m})$ . By contrast, a magnetic moment has units of torque per unit field,  $\text{N}\cdot\text{m}/\text{T}$  or  $\text{J}/\text{T}$ , which is easily shown to be equal to  $\text{A}\cdot\text{m}^2$ . (In fact, the magnetic moment of a planar loop carrying current  $i$  around the periphery of an area  $A$  is given by  $\mu = i A$ , of dimensions  $\text{A}\cdot\text{m}^2$ ). Hence the magnetization field  $\mathbf{M}$  has units of (magnetic

moment)/(unit volume) or  $A \cdot m^2/m^3 = A/m$ . And in SI, the field  $H$  has the same dimensions,  $A/m$ . Show that the units of  $\mu_0$  make  $B = \mu_0 (H + M)$  a dimensionally-consistent equation.

### The motivation for studying magnetism

Now why do we *care* about the magnetic properties of materials? There are obvious technological applications of permanent magnetization, not just in the world of electric motors and generators, but also in the enormous industry of magnetic data storage. On a more fundamental level, there is deep insight to be gained in the properties of matter from its magnetic properties. How deep? Consider the Bohr-van Leeuwen theorem of statistical mechanics, which proves that any non-rotating system which is described by classical mechanics, and which is in a state of thermal equilibrium, must have magnetic moment zero. So the mere existence of a single permanent magnet demonstrates the abject failure of classical mechanics to describe matter! Thus, any explanation of any form of magnetism must appeal to quantum mechanics. To put it another way, to measure magnetism is to measure the properties of matter that only quantum mechanics is able to describe.

There are many ways to classify the magnetic properties of many kinds of matter. One familiar classification distinguishes permanent vs. temporary magnetization. Because of the relationship of elemental iron to permanent magnets, we name the familiar kind of behavior ‘ferromagnetism’. But ferromagnetism is restricted to a relatively few elements and alloys. A much broader class includes materials exhibiting no permanent magnetization, but which do respond to a magnetic field by developing a temporary magnetization  $M$ . Because the auxiliary field  $H$  is such a convenient independent variable in experiments, it is conventional to describe such materials by a (dimensionless) ‘magnetic susceptibility’  $\chi$ , defined by

$$\mathbf{M} = \chi \mathbf{H} .$$

(Notice that there is an analogous but wholly separate *electric* susceptibility; where confusion might exist, we would use  $\chi_{\text{mag}}$  and  $\chi_{\text{elec}}$  to distinguish these two responses. Notice also that we’ll have occasion to define various *forms* of the magnetic susceptibility; the one defined above is sometimes called the ‘volume susceptibility’, and it is the form which is dimensionless.)

Notice that the atomic moments making up the magnetization  $\mathbf{M}$  must in fact be responding to the local field  $\mathbf{B}$  inside the material, so the microscopic cause-effect relationship is  $\mathbf{M} = \mathbf{M}(\mathbf{B})$ . But precisely because  $\mathbf{B}$  is not, while  $\mathbf{H}$  can be, under our direct control in a suitable experimental geometry, it is traditional to regard  $\mathbf{H}$  as the independent variable for producing magnetization  $\mathbf{M}$ . Then the  $\mathbf{M} = \mathbf{M}(\mathbf{H})$  relationship might (or might not) exhibit a simple, or even a linear, dependence. [Appendix D describes a project that can test this hypothesis of linearity of magnetization.]

For materials that are not ferromagnetic, and for fields not too extreme,  $\mathbf{M}$  is found to be a linear function of the present value of  $\mathbf{H}$ . In the simplest cases of amorphous or polycrystalline matter,  $\mathbf{M}$  will have the same direction as  $\mathbf{H}$ , so  $\chi$  is a scalar as well as a constant. [Appendix F describes an experiment which confirms magnetic *anisotropy* in one of your samples, and Appendix G describes a curious demonstration in magnetic levitation that this sample allows.]

While the susceptibility  $\chi$  might depend on the material and its temperature, it will *not* depend on the material's magnetic history. Typical 'non-magnetic' materials have values of  $|\chi| \ll 1$ . Since the discovery by Faraday of the empirical distinction, we classify materials as

paramagnetic with values of  $\chi > 0$ ,

and diamagnetic with values of  $\chi < 0$ .

It is the goal of a quantum-mechanical treatment of matter to understand *why* a given material should be dia-, para-, or ferro-magnetic, and also to *predict* the value of  $\chi$  for materials. And it is the goal of this apparatus to make possible the *measurement* of the susceptibility  $\chi$  of (non-ferromagnetic) materials.

## 2...Simplest theoretical treatments of magnetization

### a)...Diamagnetism

Introductory electromagnetism textbooks often give a classical explanation of diamagnetism in terms of Lenz's Law: ramping up an external field from zero to  $B$  gives a transient electric field, which can induce atomic currents constituting a magnetic moment  $\mu$  anti-parallel to  $B$ . This is a diamagnetic response; but given the Bohr-van Leeuwen theorem, this explanation cannot be complete. Yet its ‘predictions’ are supported by a correct calculation from non-relativistic quantum mechanics. For a multi-electron atom with nucleus at the origin, and electron position coordinates  $r_i$ , a calculation treating external field  $B = B \hat{z}$  as a perturbation shows that the ground-state eigen-energy of an atom depends on the field according to

$$E(B) = E(B = 0) + \frac{1}{2} \alpha B^2 + \dots,$$

where  $\alpha$  is the ‘magnetic polarizability’ of the atom; this result presumes the absence of a permanent magnetic moment of the atom. Then the magnetic moment of the perturbed atom is given by  $\mu_z = -\partial E / \partial B = -\alpha B$ . If the external field is given by  $B = \mu_0 H$ , and if  $n$  gives the number density of such atoms in space, we can compute the magnetization (magnetic moment per unit volume) as the product of (number of atoms per unit volume)  $\times$  (magnetic moment per atom), so

$$M = n \mu_z = -n \alpha B \cong -n \alpha \mu_0 H ,$$

which gives us the volume susceptibility  $\chi = -\mu_0 n \alpha$ . The point is that quantum-mechanical perturbation theory for a single atom can predict  $\alpha$ , and hence theory can predict  $\chi$  for a sample of such atoms. The result is

$$\alpha = \frac{e^2}{4m} \langle \sum_i (x_i^2 + y_i^2) \rangle$$

where  $-e$  and  $m$  are the electron charge and mass, and the sum is taken over all the electrons in one atom. For a spherically-symmetric atom, for which  $\langle x^2 \rangle = \langle y^2 \rangle = \langle z^2 \rangle = \langle r^2 \rangle / 3$ , we get

$$\chi = -\mu_0 n \cdot \frac{e^2}{4m} \sum_i \frac{2}{3} \langle r_i^2 \rangle .$$

For an atom as simple as helium, the expectation value  $\langle r^2 \rangle$  can be computed from approximate wavefunctions; the result is  $(1.19 a_0^2)$  for each of the two electrons in the ground state, where  $a_0$  is the Bohr radius. So the prediction for this two-electron atom is

$$\chi(\text{He}) = -\frac{1}{6} \mu_0 n \cdot \frac{e^2}{m} \cdot 2 (1.19 a_0^2) .$$

For He gas at STP (standard temperature and pressure, namely a pressure of 1 atmosphere =  $1.01 \times 10^5 \text{ N/m}^2$ , and a temperature of 273 K), there is one mole of He atoms in a volume of 22.4 liters, so the number density  $n = (6.02 \times 10^{23}) / (0.0224 \text{ m}^3) = 2.69 \times 10^{25} \text{ atoms/m}^3$ . Putting in the rest of the numerical values, we get the prediction

$$\chi (\text{He gas, STP}) = -1.06 \times 10^{-9} .$$

The value is dimensionless (check how this emerges!), and it's tiny, in part because the number density of atoms is rather low in gas-phase helium. Yet the principle remains: an observable magnetic response of a macroscopic volume of helium gas can be calculated, from first principles, using quantum-mechanical theory. Naturally enough, such first-principles calculations are much harder for atoms and molecules more complicated than helium – see Appendix I for some semi-empirical methods that are sometimes used in place of detailed computations.

Additional lessons of this calculation are: that diamagnetism is expected to exist in *any* atom or molecule (although it might be masked by larger paramagnetism); that it should share the temperature-dependence of the number-density  $n$ ; and that its value can give an estimate for the mean-square size of the atom or molecule.

Example:

Suppose you take as measured value for the susceptibility of liquid water, at 20 °C, the value  $-9.05 \times 10^{-6}$  (see Appendix J). What is the number density  $n$  of water molecules under these conditions? If you were to (mis) treat a water molecule as a spherical entity with 10 electrons moving about a mythical center, what value of  $\langle r_i^2 \rangle$  would you deduce for each electron's distribution about the center? Compute the 'root-mean-square' distance  $\langle r_i^2 \rangle^{1/2}$ : is this at all consistent with what you know about the size of a water molecule?

b)...Paramagnetism

Paramagnetism can be traced, quantum-mechanically, to the non-zero magnetic moments of the ground states of atoms (or molecules, or ions) having unpaired electrons. The simplest case is Curie paramagnetism, which is observed as a susceptibility that scales with absolute temperate  $T$  according to  $\chi \propto 1/T$ . This temperature dependence in Curie paramagnetism reveals a competition between the orienting effect of an external field  $B$  on the electrons spins in the atoms, and the *disorienting* effects of non-zero temperature. For a magnetic moment typical of an electron spin, of one Bohr magneton  $\mu_B$  [where  $\mu_B \equiv e \hbar / (2 m_e)$ ; see Appendix L], in a field as large as  $B = 1$  T, the energy of orientation is

$$\mu_B B \approx (9.24 \times 10^{-24} \text{ J/T}) (1 \text{ T}) \approx 10^{-23} \text{ J} ,$$

whereas the thermal energy available for *disorientation* (at room temperature) is hundreds of time larger:

$$k_B T \approx (1.38 \times 10^{-23} \text{ J/K}) (295 \text{ K}) \approx 4 \times 10^{-21} \text{ J} .$$

This suggests that paramagnetism is ordinary circumstances will be a weak effect – weak, in any case, compared to the opposite limiting case of nearly fully-aligned moments that can be expected at very large fields, or at very low temperatures, or in (single-domain) ferromagnetic materials.

The statistical mechanics of a number density  $n$  of magnetic moments  $\mu$ , each assumed to be interacting with external field  $B$  (but assumed *not* to be interacting with each other) gives a magnetization which is not as large as

$$M = n \mu , \quad (\text{the fully-aligned case})$$

but which instead takes on value

$$M = n \mu \cdot [ \coth^{-1}(a) - 1/a ] , \quad (\text{incompletely aligned case})$$

where the bracketed ‘Langevin function’ has as its argument  $a = (\mu B)/(k_B T)$ . For the typical case with  $\mu B \ll k_B T$ , the argument  $a$  is  $\ll 1$ , and a series expansion of the Langevin function gives the approximate result

$$M = n \mu \cdot [ a/3 ] ,$$

or  $M = \chi H$  with

$$\chi = \mu_0 n \mu^2 / (3 k_B T) .$$

This result illustrates the Curie-law dependence on absolute temperature. For any sample exhibiting such a  $1/T$  dependence of paramagnetism upon temperature, this law provides a simple way of measuring the magnetic moment  $\mu$  of individual atoms (or molecules, or ions). [Appendix H illustrates the first-principles numerical computation of Curie paramagnetic susceptibility in the case of a famous paramagnetic sample.]

Now the magnetic moment  $\mu$  of an atomic ground state can be *predicted*, at least in the limit where the spin-orbit effect is small (so *LS*-coupling is a good approximation). If we have a ground state with total electronic orbital angular momentum  $L$ , and total electron spin angular momentum  $S$ , then the total (angular + spin) angular momentum of all the electrons is  $J = L + S$ . And the magnetic moment of this ground state is predicted to be

$$\mu = g \mu_B \sqrt{J(J+1)} ,$$

where  $\mu_B$  is the Bohr magneton (see Appendix L), and where the Lande  $g$ -factor is also predicted by theory to be

$$g = 1 + \frac{J(J+1)+S(S+1)-L(L+1)}{2 J(J+1)} .$$

Now for an atom not isolated in space, but existing in the electric fields internal to condensed matter (as, say, an ion in water, or in a crystal lattice) it often becomes a good approximation to say that orbital motion is hindered, or ‘quenched’. In this case, the magnetic moment can be computed in the ‘spin-only approximation’ by setting  $L = 0$  and so  $J = S$ . Then knowing the net spin  $S$  of the ground state suffices for making a prediction.

But what should the spin  $S$  be? That is best computed using the number  $n_e$  of unpaired electrons (ie. those outside a closed shell, and lacking another electron of the opposite  $m_s$ -projection) and

using Hund's First Rule, which says that in a configuration of a given quantum number and orbital angular momentum, the state of lowest energy is the one of the largest possible  $J$ -value.

For example, in the  $n=2$  state of atomic helium, in the case of  $L=0$  (the s-states), the state  $1s2s\ 2^3S_1$  lies lower in energy than  $1s2s\ 2^1S_0$ , ie. the triplet state (with  $2S+1=3$ , ie.  $S=1$ ) lies lower than the singlet (with  $2S+1=1$ , ie.  $S=0$ ). Similarly, still for helium in the  $n=2$  states, but now for the case of  $L=1$  (the p-states), helium has the states  $1s2p\ 2^3P_J$  lying lower in energy than the state  $1s2p\ 2^1P_0$ .

So if there are  $n_e$  unpaired electrons, the state of maximal  $J$  that can be formed from them has  $S = n_e \cdot \frac{1}{2}$ .

Here's a concrete example worked out for the transition metal manganese, Mn in the periodic table. A chemist will tell you that its electron configuration is  $[Ar]\ 4s^2\ 3d^5$ ; that is to say, it has 18 electrons in argon's closed-shell configuration, and 7 more electrons outside that closed shell. The chemistry of Mn depends on the two 4s 'valence' electrons, so that (for example) the ion  $Mn^{2+}$  that can occur in solution, or in an ionically-bound crystal structure of a manganese compound, has the non-neutral configuration  $[Ar]\ 3d^5$ . So either alone and neutral, or in this  $Mn^{2+}$  ionic form, the atom is left with  $n_e = 5$  d-shell electrons to combine into the atomic ground state. And Hund's First Rule claims they'll combine to form a state of lowest energy that is  $S = 5/2$ . Together with the spin-only approximation, those assumptions predict a g-factor of 2, and a magnetic moment of

$$\mu = 2 \mu_B \sqrt{S(S + 1)} = 2 \mu_B \sqrt{\frac{n_e}{2} \left( \frac{n_e}{2} + 1 \right)} = \mu_B \sqrt{n_e(n_e + 2)} = \mu_B \sqrt{5 \cdot 7} .$$

Since this magnetic moment makes a quantitative prediction for the Curie paramagnetism of Mn compounds, it can actually be *tested* – you'll learn how to do the macroscopic measurement which tests this claim in Ch. 3 of this manual. And *that*, finally, is part of the evidence which backs up the theorist's claim, so glibly printed on millions of copies of the Periodic Table, that the electron configuration of manganese really is  $[Ar]\ 4s^2\ 3d^5$ .

The derivation above also allows some other, easier, predictions. For covalently-bonded molecules with an even number of electrons (the vast majority of stable molecules -- though there are exceptions), we typically expect the electron spins to pair up, giving a net magnetic moment of zero, and hence no paramagnetism. (Again, there are exceptions, such as in the ground state of molecular oxygen,  $O_2$ , despite its even number of electrons.) For ionically-bonded solids, or their dissolved ionic forms, the question of paramagnetism is the question of the electronic structure of the ions. Table salt, NaCl, is in solid form  $Na^+Cl^-$ , or in dissolved form  $Na^+ + Cl^-$ ; and since both  $Na^+$  and  $Cl^-$  are even-electron, closed-shell species, with no magnetic moments, we expect ordinary salt to be diamagnetic. The same can be said for sugar, water, and a host of organic compounds. Paramagnetism becomes prominent in the case of transition-metal and rare-earth compounds and ions (in solid, or dissolved, form), since such ions lose their valence electrons but have (typically un-paired) d- or f-shell electrons contributing to a non-zero net magnetic moment of the atom or ion. Hence compounds of chromium, manganese,

copper, etc. can be *just about as paramagnetic* as the corresponding compounds of iron, nickel, or cobalt.

That completes this treatment of Curie paramagnetism. It is not the only kind of paramagnetism; in the case of metals, with a populations of electrons free to move about through a lattice, unbound to any particular atom or ion, there is another manifestation called Pauli paramagnetism. This kind of paramagnetism is of interest theoretically, because of its connection with electron states that are close in energy to the Fermi energy of electrons. And it's distinctive empirically as well, as it is predicted to have a temperature dependence weaker than the  $1/T$  behavior of Curie paramagnetism.

Note that we have treated diamagnetism and paramagnetism as separate properties; yet it's important to remember that a paramagnetic sample is still made of molecules which retain their native diamagnetism. In pursuing paramagnetism quantitatively, you might want to correct your measured results for this underlying diamagnetism, to give you a modified experimental result meant to describe 'pure paramagnetism'. One method is described in Appendix I.

### 3...Measuring magnetic susceptibility

#### Introduction

The TeachSpin apparatus you're using is designed to measure the magnetic susceptibility of dia- and para-magnetic samples, at room temperature. It does so by using the Gouy-balance technique of static weighing in a constant and rather large magnetic field; it can give *absolute* susceptibility (that is, not requiring someone else's calibration of some standard material), complete with magnitude and sign. The technique is suitable for solid, powdered, or liquid samples; and in the TeachSpin apparatus, a volume of about 3 cm<sup>3</sup> of each sample is required.

The next section describe how the susceptibility of a material,  $\chi$ , can be derived from empirically-measured observables, using this technique. You may have seen this derivation, or perhaps have worked it out in solving an end-of-chapter problem, in the treatment of the magnetic properties of matter in an electromagnetism course.

#### a)...The Gouy balance

We take from electromagnetic theory the energy density of magnetic fields in matter:

$$u_{\text{mag}} = \mathbf{B} \cdot \mathbf{H} / 2 .$$

For a material in which  $\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M})$ , this gives

$$u_{\text{mag}} = \frac{1}{2} \mathbf{B} \cdot \left( \frac{1}{\mu_0} \mathbf{B} - \mathbf{M} \right) = \frac{\mathbf{B}^2}{2\mu_0} - \frac{1}{2} \mathbf{B} \cdot \mathbf{M} .$$

This gives energy density, or energy per unit volume. We notice the first term gives the energy density of a magnetic field  $B$  in vacuum, and so the second term gives the *change* in energy due to the presence of a sample (in place of vacuum). For a sample of volume  $\Delta V$ , we get the energy attributable to the presence of the sample as

$$\Delta U_{\text{mag}} = -\frac{1}{2} \mathbf{B} \cdot \mathbf{M} \Delta V = -\frac{1}{2} \mu_0 (\mathbf{H} + \mathbf{M}) \cdot \chi \mathbf{H} \Delta V \cong -\frac{\mu_0}{2} \chi H^2 \Delta V .$$

Note that for paramagnetic materials, for which  $\chi > 0$ , the energy for a sample element goes *down* as  $H$  rises – that is to say, it is energetically *favorable* for the sample to be in a region of strong field. We conclude that paramagnetic materials are attracted into strong-field regions (but that diamagnetic materials are repelled from such regions). Now in the Gouy-balance method, we create an external  $\mathbf{H}$ -field which is horizontal and chiefly  $y$ -directed, but whose magnitude is  $z$ -dependent, so  $\mathbf{H} = \hat{\mathbf{y}} H_y(z)$ . Then a (small) sample immersed in this field has a  $z$ -dependent magnetic energy, and so it experiences a vertically-directed force,

$$\Delta F_z = -\frac{\partial}{\partial z} \Delta U_{\text{mag}} = +\frac{\mu_0}{2} \chi \frac{\partial}{\partial z} (H^2) \Delta V .$$

For the geometry we're using, this can be written as

$$\Delta F_z = +\mu_0 \chi H_y \frac{\partial H_y}{\partial z} \Delta V ,$$

which shows that we need a field ( $H_y$ ) to produce magnetic moments in the sample, and also a gradient ( $\partial H_y / \partial z$ ) to exert a force on them.

Now the motivation for creating a vertically-directed force is that ordinary weighing scales, such as one-pan electronic balances, are ideally suited to detecting such forces, which are then displayed as changes in the apparent mass of a sample. We read a mass change  $\Delta m$ , and we infer a vertical force  $\Delta F_z = \Delta m \cdot g$ .

The Gouy method eliminates the need for actually measuring the gradient  $\partial H_y / \partial z$  by making use of a sample which has a uniform horizontal cross-sectional area  $A$ , and which extends vertically, having one end in a region where  $H_y$  is maximal, and the other end located where  $H_y \approx 0$ . That is to say, a vertical stick or ‘log’ of sample is placed having one end in the central, uniform region of the field of a magnet, with the other end lying well outside that strong-field region. Using  $\Delta V = A \cdot \Delta z$ , and then integrating the  $\Delta F_z$ ’s of the various slabs of the sample, we get

$$\Delta m = \frac{1}{g} \sum \Delta F_z = \frac{1}{g} \sum \mu_0 \chi H_y \frac{\partial H_y}{\partial z} \cdot A \Delta z \rightarrow \frac{\chi A \mu_0}{g} \int H_y \frac{\partial H_y}{\partial z} dz = \frac{\chi A \mu_0}{2g} |H_y^2|_{bottom}^{top}$$

In practice, it’s easier to measure the pre-existing  $B$ -field that is present when the sample is absent, and that gives

$$\Delta m = \frac{\chi A}{2\mu_0 g} (B_{top}^2 - B_{bottom}^2) .$$

This result gives the magnetic susceptibility  $\chi$  in terms of measurable factors. The geometry involved is shown in Fig. 3.1.

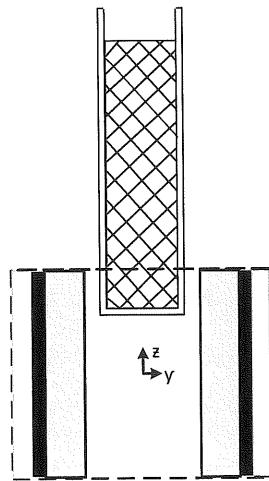


Fig. 3.1: A side view of the geometry of the Gouy method; the magnet pole-faces are shown shaded, and the sample is shown cross-hatched.

While textbooks discuss the change in apparent mass of a sample suspended in a field, it can in practice be easier to exploit Newton’s Third Law, and instead to ‘weigh the magnet’, looking for

the change in the apparent mass of a permanent-magnet structure, as a sample is fixed in turn in each of three positions:

- sample far from the magnet;
- sample high, with its bottom end centered in the magnet; and
- sample low, with its top end centered in the magnet.

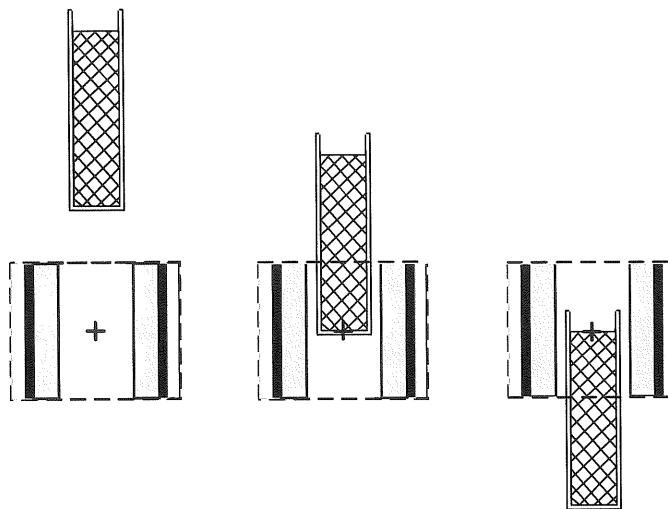


Fig. 3.2: Side view of three particular positions for the sample relative to the magnet in the Gouy method.

The Gouy method not only allows a measured  $\Delta m$  to give an absolute value for  $\chi$ , it also easily establishes the *sign* of  $\chi$  from the sign of  $\Delta m$ . We've seen above that paramagnetic materials are *attracted* into a higher-field region, while diamagnetic materials are *repelled* from a strong-field region. And it's easy to relate the experimental observations of a magnet-seeming-heavier, or a magnet-seeming-lighter, to the question of this attraction or repulsion, and hence unambiguously to the correct sign of  $\chi$ .

Notice that in the Gouy geometry, we do need to measure the strong field  $B^2$  (and we also need to show that  $B^2$  at the sample's *other* end is negligible by comparison); but we do not need to measure the actual spatial gradient of this field. Nevertheless, the magnetic force on the sample is being exerted neither at the sample's high-field end, nor at its low-field end, but instead on that intermediate portion of the sample which exists where the field has its gradient, ie. its variation from strong field to weak field. It follows that we *do* require that the sample be uniform, in area and character, over its full height.

The sensitivity of the Gouy method can be quite high, especially with the use of microbalances of high resolution. Suppose we have a sample of titanium metal with  $1\text{-cm}^2$  cross section, extending from an 'inside' region where  $B \cong 0.4\text{ T}$  to an 'outside' region where  $B \cong 0$ . If we use a handbook value [converted to SI form -- see section g) below] of  $\chi(\text{Ti}) = +182 \times 10^{-6}$ , we predict an apparent mass change

$$\Delta m = \frac{(182 \times 10^{-6})(10^{-4}\text{ m}^2)}{2(4\pi \times 10^{-7}\text{ T}\cdot\text{m}/\text{A})(9.80\text{ m/s}^2)} [(0.4\text{ T})^2 - 0] = 0.000\ 118\text{ kg} = 0.118\text{ g} .$$

(Check this calculation, and the units, too.) This numerical result might be small compared to the mass of the sample (or of the magnet, if that's being weighed), but it is adequately large compared to the resolution limit (say, 0.001 g) of relatively inexpensive electronic balances.

Finally, an absolute measure of  $\chi$  requires an absolute measure of  $B^2$ , or properly  $\langle B^2 \rangle$ , the square of the field  $B$  averaged over the cross-sectional area  $A$  occupied by the strong-field end of the sample. The preferred method for doing so is found in section e) below, and it uses the *same* balance for a weighing-the-*same*-magnet method, but using a U-shaped ribbon of electric current substituting for a vertical 'log' of sample.

Once you have measured the value of  $\langle B^2 \rangle$  by this method, the equation in boldface above gives the (volume) susceptibility  $\chi$  wholly in terms of quantities you yourself have measured. That is to say, you will have established the sign, and the magnitude, of  $\chi$  without reference to any tabulated values, and without needing a comparison to some 'standard material'. Very few workers in magnetism ever measure a susceptibility (or mass susceptibility) absolutely, as most workers use the same apparatus in succession on a 'standard' and an 'unknown'. That is to say, they depend on someone *else* to have performed, and understood, the measurement of absolute susceptibility on some standard substance, and they then report results on their unknown by comparison to this standard. Your work on the TeachSpin version of the Gouy technique will induct you into that subset of scientists who understand magnetic susceptibility, and can measure it absolutely, 'from scratch', without relying on anyone else's calibration procedures.

### b)...Setting up the apparatus

This section assumes that you have accomplished the unpacking and first operation of the NW200G balance, as described in Appendix A. It then tells you how to adapt this balance to create the Gouy-balance you'll use for measuring magnetic susceptibility.

You will notice that the balance, as shipped, includes glass panels on left and right sides, and also on top. To adapt the balance to its Gouy-balance use, you may slide out and discard the top or 'ceiling' glass panel. In its place, you need to install the plastic top panel provided by TeachSpin, the one with a cut-out region, also bearing two threaded nylon studs. Orient this panel with its studs pointing upward, and with its squarish through-hole nearest the front edge, and slide it into the slots vacated by the glass panel you've removed.

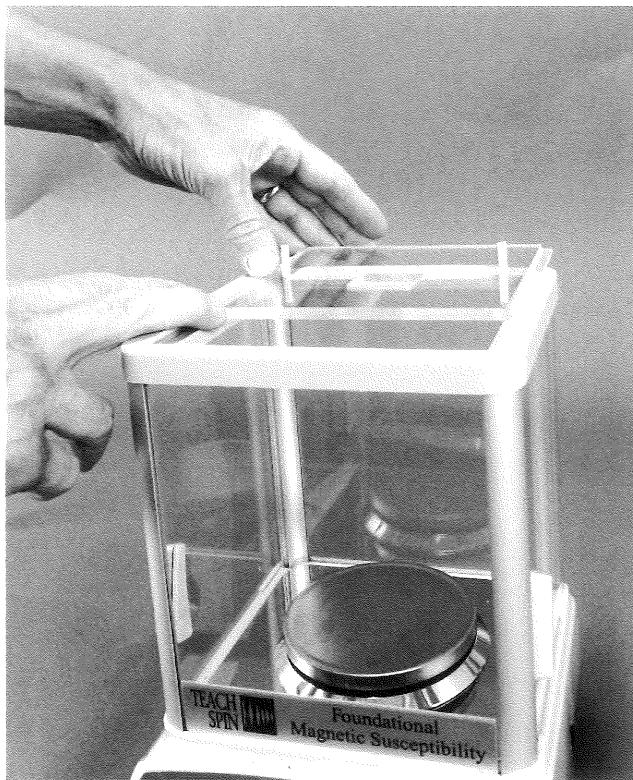


Fig. 3.3: How to install the TeachSpin top panel into the balance

You should be able to push that new top panel all the way to the front of its mounting slots. Now install onto that panel, from above, the brass and aluminum 'lifter structure' provided by TeachSpin. Orient that structure with the U-channel in its brass upright facing you, and with the double-ended shaft bearing two knobs extending from left to right, as shown below:

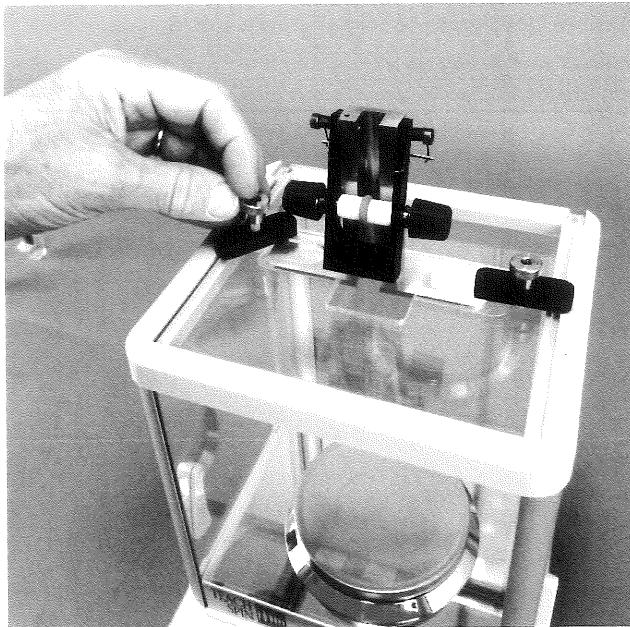


Fig. 3.4: How to install the ‘lifter structure’ onto the new top panel

Now notice the black plastic bars and the stainless-steel thumbnuts on the vertical nylon studs – these permit you to clamp together the lifter-structure’s aluminum base, the clear plastic cover, and the frame of the balance. Practice finger-tightening the thumbnuts to see how this clamping action works. Note that when you loosen the clamps, you are able to slide the clear plastic cover in the front-back direction, and that you are also able to slide the lifter structure in the left-right direction. [You will use magnetic-force measurements to optimize these adjustments in section d) below.]

Once you have the lifter structure installed in that new top panel, you are ready to load the balance. If you have successfully zeroed the balance (per Appendix A), you already have in place its stainless-steel balance pan. Place the balance in the location where you intend to do your magnetic-susceptibility work, use its bubble-indicator to level it, and turn it on. Be sure you can get a reading of 0.000 g on its display, with the stainless balance pan in its place, but empty of any load. Now slide open one of the side doors of the balance, so you can add, onto that pan, the clear plastic ‘top-hat’ structure which will hold the permanent magnet some distance above the balance pan. Install the top-hat with its wider ‘brim’ downwards, and note that it will center on the balance, and still be free to rotate about a vertical axis. Naturally, the balance’s reading will depart from zero, and take on a value near 30 g. Do *not* re-zero the balance, and do *not* ‘tare’ out this indication.

Now find the permanent-magnet structure provided by TeachSpin; it has a rectangular nickel-plated steel yoke structure, within which you can see two flat discs of rare-earth magnets. The permanent magnets are held in place both by magnetic attraction to the yoke, and by epoxy cement. Take care not to drop this magnet, and keep it away from magnetic material (including paper clips, tools, and the like).

**WARNING:** There is a high-field region ( $B \approx 0.4$  T, or 4000 gauss) largely confined to the space between the two discs of permanent-magnet material. Keep ferromagnetic objects away.

Mount that magnet onto the top surface of the plastic top-hat structure you have in place on the balance. You will see that its rectangular frame will settle into the grooves prepared for it on the top-hat's 'lid'. When you have the magnet settled and level in those grooves, you ought to see the balance stabilize to give a reading of about 180 g, the total mass of the top-hat and the entire magnet structure. Once again, do *not* zero or 'tare out' this mass. *Changes* in this mass reading will be the primary signs of magnetic interaction between the magnet, and the samples you are about to add to the system.

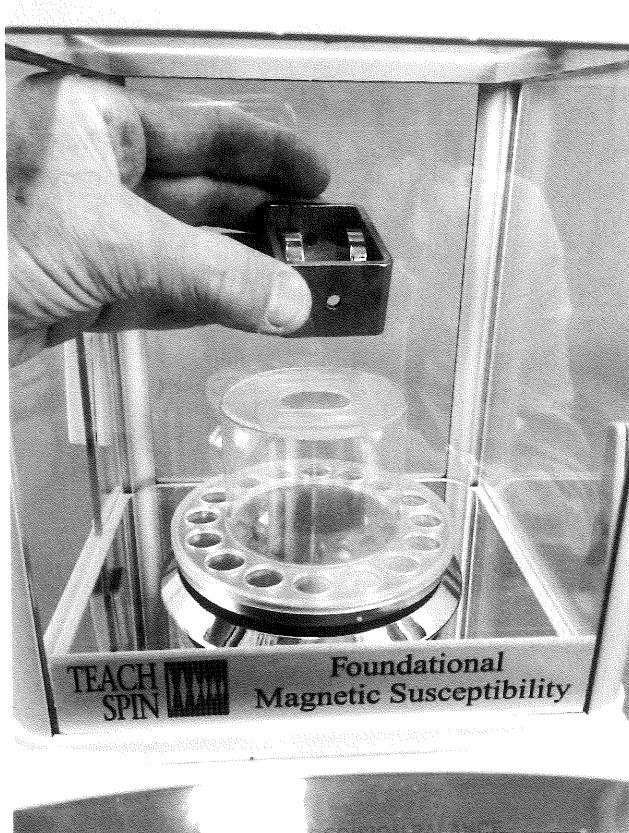


Fig. 3.5: Adding the magnet structure to the 'top hat' that's loaded onto the balance pan

When you have the balance working in this mode, slide closed the side access doors of the balance, and get an instinct for the degree to which it gives a stable reading. Only after you have had the balance located, levelled, turned on, loaded, and running for about an hour, is it worth perfecting the zero-point reading, and/or the calibration, of the balance.

To check the zero of the balance, leave it operating, and open a side access door so you can lift the whole top-hat-and-magnet structure off of the balance's weighing pan. When the balance pan is unloaded, the balance might not return to a reading of 0.000 g exactly. If it does not, wait for it to settle to a stable value, and then press the ZERO button.

Once the balance is zeroed, you may choose to calibrate it, using the procedure of Appendix A.

Once it is zeroed and calibrated to your satisfaction, you may re-install the top-hat and magnet structure onto the balance pan.

### c)...Sample handling

Sample preparation in the TeachSpin susceptibility system is simplicity itself – you may practice with one of the 16 prepared samples, or use Appendix C to fill another ‘sample vial’ or cuvette with a solid, liquid, or powdered material. Recall that the accuracy of the Gouy method depends on the sample being uniform in density over the vertical extent of the cuvette. So for a solid sample, you need a uniform cross-sectional area, as well as a density that’s uniform, along the vertical extent of the sample. Powders need to be packed, or settled, into the cuvette, to give a uniformly-dense filling over their height – this can be hard to achieve for ‘sticky’ powders.

The quantities you need to know for any sample are

Mass present,  $m$ : easy for solids, but best obtained by a cuvette-full vs. cuvette-empty weighing difference for liquids and powders;

Cross-sectional area,  $A$ : best measured by caliper dimensions for solids; may be assumed to be (10.0 mm)-squared, for liquids or powders (the cuvettes are built for spectroscopic studies on liquid samples, and are designed to offer a 10.0-mm of light path through the sample);

Height of filling,  $h$ : for powders, you do need the mass, you do *not* need the cross-sectional area, but you do need this height to which the sample is packed. Measure that by eyeball and ruler; you may want to use a ‘tamping tool’ to give a flat surface to the top of the powder sample. You will appreciate that it’s not easy to achieve high precision in the measurement of the height  $h$ .

Once you have a sample ready, you ought to document it. In general, it’s worth devoting a cuvette permanently to any sample you’ve taken the trouble to acquire, prepare, weigh, fill, and document. For chemical safety and for sample integrity, we suggest you keep a cuvette’s snap-on cover permanently associated with it, and in its place atop the cuvette whenever you’re not actually using it in your Gouy balance.

#### d)...Measurement

This section assumes that you have set up the balance by the methods of b) above. When you are ready to use a chosen sample for measurement, set aside its cover, and attach the sample-cuvette to the ‘lifter rod’. You will press-fit the cuvette onto the surrogate cap that’s glued to the bottom of the lifter rod, and then sight along the structure to ensure that the cuvette is in-line with the lifter rod. For a powder or liquid sample, keep the whole structure vertical to prevent spillage. For a heavy sample, **take care** not to accelerate the lifter-rod-and-sample assembly excessively – the press-fit attachment scheme is adequate for holding the static vertical load of a sample’s weight, but not strong enough to permit you to wave the sample about.

Now you can install the lifter-rod and sample combination into the lifter structure that’s atop your balance. The photo below shows how to open out the spring-loaded lifter wheel to allow the whole sample combination to be inserted from above. Aim first to get the cuvette into place just below the top cover of the balance, and then release the spring-loaded structure to engage the lifter wheel onto the flat side of the U-channel of the lifter rod. Friction ought to hold the lifter rod in place.

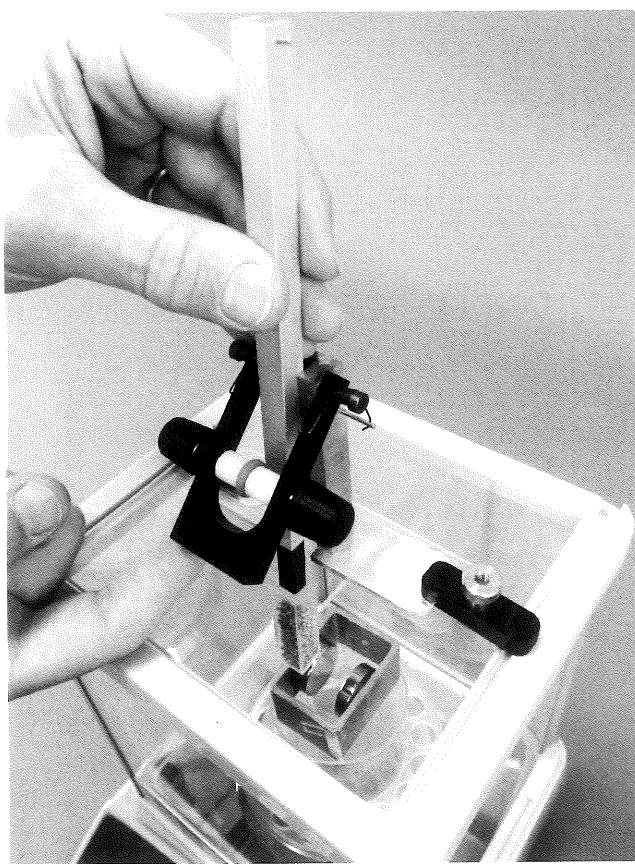


Fig. 3.6: a lifter-rod plus sample combination being installed into the lifter structure on the balance

Now use the hand-knobs to lower the lifter-rod gently down toward the magnet. You will probably find the next task – alignment – easiest if the magnet’s stand is rotated on the balance so that the magnetic-field vectors extend along the left-right direction. Once you’ve settled on

the correct orientation of the magnet structure on the weighing pan of the balance, you want to optimize three dimensions of alignment of the sample:

The left-right, or  $y$ -direction, is adjustable by sliding the lifter structure (brass and aluminum) relative to the cover of the balance. Keep the front of the aluminum base of the lifter structure aligned against the near side of the depression in the clear plastic balance cover.

The back-front, or  $x$ -direction, is adjustable by sliding the whole balance cover nearer to you, or farther from you, in the grooves in which it is mounted.

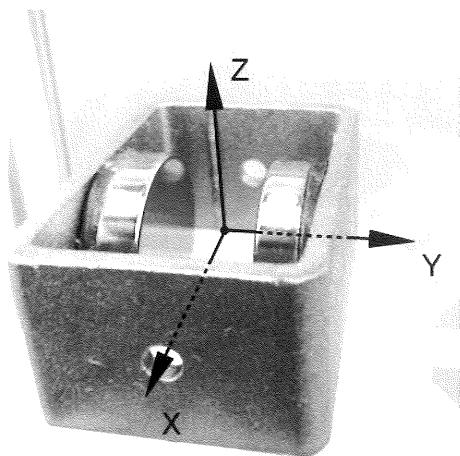


Fig. 3.7: A photo of the magnet in a chosen orientation, showing also a coordinate system used to describe alignment

Your goal in  $x$ - and  $y$ - adjustments is that, when you lower the sample cuvette toward the magnet, it will pass through the center of the magnet structure. When you have adjusted both  $y$ - and  $x$ -adjustments to your satisfaction, gently tighten the two stainless-steel thumbscrews which clamp black-plastic blocks into place. Then the whole cover and lifter structure is held steady in its chosen location.

The up-down, or  $z$ -adjustment, of the sample is the one which you achieve via the knobs on the lifter structure. There's a friction drive, via an orange O-ring, of the lifter rod that bears the sample.

Now how will you know when the  $x$ -,  $y$ -, and  $z$ -adjustments are correct? You'll use your eyeball viewing of the sample tube, relative to the magnet, to get close; and you'll use the balance's own indication of apparent mass to get precise. Roughly speaking, getting each of these adjustments to eyeball- or mm-quality correctness is good enough. To persuade yourself of that, use (as an example) a moderately-paramagnetic sample such as solid **titanium** (see Appendix B for a listing of the samples provided for you).

First aim to put the bottom of the solid sample into the 'sweet spot' in the magnet. That is, use a visual look to center the sample tube, relative to the magnet's pole-faces, in the  $x$ - and  $y$ -directions. Also use the lifter knobs to estimate when the bottom of the solid Ti sample is

centered-in- $z$  within the magnet – to check this, you may look along the line joining the two ‘sighting holes’ drilled through the magnet-structure’s steel yoke.

Now read the balance’s indication of the mass of the magnet and its support structure in two configurations. We’ll label as the ‘high position’ the case you’ve just achieved – the metal of the sample is all on the *high* side of the magnet’s centerline. And we’ll label as the ‘far position’ the case in which you use the lifter knobs to elevate the whole sample tube vertically by about 3-4 cm – *far* enough from the magnet so that the indicated mass ceases to change. Write down these two numbers.

Now do a ‘survey’ of the balance’s indication of the mass. You should see that mass-reading change, while you lower the sample from the far position to the high position. If the sample is paramagnetic, then the magnet should attract the sample as the sample approaches the high-position. That means the magnet exerts a (downward) force on the sample, so Newton’s Third Law predicts that the sample will be exerting an (upward) force on the magnet. So the magnet should ‘seem lighter’ to the balance, which thus should display a *smaller* mass reading.

How does this reading change as you use the lifter-knobs to change the  $z$ -position of the sample? Try it out, and look for a *minimum* reading of the balance. In the vicinity that minimum, you’ll see small increases, of size second-order in the departure from optimum position. Take your time in doing this sort of  $z$ -survey, noting that the balance has some finite response time before settling to its final value.

Before finalizing your choice of  $z$ -position, do a similar, though cruder, survey of sample position in the  $x$ - and  $y$ -dimensions. As you vary the  $y$ -position, you’ll be bringing the sample nearer one (or the other) of the two pole faces, and this should *increase* the forces between magnet and sample. You want to minimize-with-respect-to- $y$  the magnitude of this force; this will occur when the sample is properly centered in  $y$ .

Finally, when you vary the  $x$ -position, you’ll be bringing the sample nearer to one edge or the other of the magnet’s circular pole faces, and this should *decrease* the forces between magnet and sample. You want to maximize-with-respect-to- $x$  the magnitude of this force; this will occur when the sample is properly centered in  $x$ .

After optimizing  $x$ - and  $y$ -adjustments, which will require a slight loosening of the clamping thumbscrews, you may finalize the  $x$ - and  $y$ -positions by tightening these thumbscrews again. Now make the final adjustments in  $z$ -position, and record a final reading of the balance for the sample in the ‘high’ position. Then get a fresh ‘far’ position reading. Now supplement your data by getting a balance reading when the sample in the ‘low’ position – by this we mean *lowering* the whole sample tube until the upper surface of the sample is vertically centered in the sweet spot at the center of the magnet.

For a paramagnetic sample in the *low* position, where is the sample relative to the magnet? (Draw yourself a sketch, or look at Fig. 3.2 above.) What is the direction of the force of the magnet, on the sample? So what is the direction of the force of the sample, on the magnet? What is the direction of the force of the *earth*, on the magnet? So what’s the status of the total force on the magnet? What should the reading of the balance be: more than, or less than, the

case of the ‘far’ position? How will you fine-adjust the  $z$ -position to ensure the sample is properly set to the correct  $z$ -setting to be in the ‘low’ position? What use are the magnet yoke’s sighting holes in this adjustment? You’ll learn all this by doing it.

When you’ve understood this procedure, and written down final values for the apparent mass of the magnet structure in far-, high-, and low-positions, you may use the lifter structure to raise the sample tube clear of the magnet, and then you may hold onto the lifter stick, release the spring-loaded clamp, and remove the whole sample assembly from the balance. Do not forget to remove the sample tube from the lifter rod, to re-cap the sample tube with its own cover, and to re-store it among your collection of samples.

Now look at your three readings, for  $m_{\text{far}}$ ,  $m_{\text{high}}$ , and  $m_{\text{low}}$ . What you care about is not the ‘dead weight’ of the magnet assembly, but the two magnet-caused changes, namely

$$\Delta m_{\text{high}} \equiv m_{\text{high}} - m_{\text{far}} \quad \text{and} \quad \Delta m_{\text{low}} \equiv m_{\text{low}} - m_{\text{far}} .$$

These two changes should be equal and opposite – why? Can you think of any reasons (especially for a powdered sample) why they might *fail* to be exactly opposite each other? If the two numbers are reconcilable in magnitude, then either of them is a representation of the mass change  $\Delta m$  that arises in the Gouy formula for the susceptibility of the sample.

You might want to repeat this procedure with the **bismuth** sample that’s provided with your apparatus. (This sample is *weightier* than the previous one – take care accordingly.) Again you can do the combination of visual and balance-assisted centering in  $x$ -,  $y$ -, and  $z$ -coordinates. If you’ve been careful in sample-tube interchange, you might not need to adjust the  $x$ - and  $y$ -positions at all. But again use  $z$ -adjustments to find the two extrema that define  $m_{\text{high}}$  and  $m_{\text{low}}$ , and again subtract to find the differences  $\Delta m_{\text{high}}$  and  $\Delta m_{\text{low}}$ . Note the *sign* of these two numbers compared to the previous case of the titanium sample – what is your proof that bismuth is diamagnetic?

Finally, you might choose the powdered **Mohr’s salt** sample from your set of samples, and perform the measurements that give you the  $\Delta m_{\text{high}}$  and  $\Delta m_{\text{low}}$  for this sample. You’ve now worked on a solid, a granular, and a powdered sample, and in section f) below you’ll see that powdered or granular (as opposed to solid) samples require a slightly different calculation that takes you from  $\Delta m$ ’s to the desired value of the susceptibility  $\chi$ .

After you have learned to use the method above to form  $\Delta m_{\text{high}}$  and  $\Delta m_{\text{low}}$  by after-the-fact subtraction, you might be ready to get a more vivid and immediate presentation of the mass-changes  $\Delta m$  by using the TARE function of your balance. Assuming that the balance has been temporarily relieved of its load (the magnet structure and the stand on which it sits) and has been ZEROed, you can put the magnet and stand back into place on the balance’s pan. With the sample in the ‘far’ position, the balance should settle to display the familiar  $m_{\text{far}}$  reading. Now if you push the TARE button once, you will cause an electronic subtraction, of size  $m_{\text{far}}$ , to be applied to all the subsequent readings of the balance. So with the sample in the far position, the display should now read not some value near 180 g, but instead 0.000 g. When you now move the sample into the vicinity of the ‘high’ and ‘low’ positions, the display should now display in real time just the differences  $\Delta m_{\text{high}}$  and  $\Delta m_{\text{low}}$  you were previously getting by after-the-fact

subtraction. See if this technique gives you a better feel for finding the optimal ‘high’ and ‘low’ positions of the sample relative to the magnet. See also if it gives you a set of numerical results consistent with your previous method. See, finally, if you can find the right way to exit from the TARE mode.

### e) ... B-calibration

The Gouy-method theory laid out in section a) above shows that you can get first-principles values of magnetic susceptibility  $\chi$  for samples, provided that you know some auxiliary constants. For any sample, the balance's indication of mass change  $\Delta m$  needs to be turned into a force value – that's why you need the local free-fall acceleration  $g$ . The cross-sectional area  $A$  of the sample is also necessary. But you also need the value of  $B^2$ , the (average of the) square of the magnetic field to which the high-field end of the sample is subjected. This section discusses the technique, designed into the TeachSpin apparatus, by which you'll get this value of magnetic-field strength.

Of course  $B$ 's value could be surveyed over the relevant planar area, the cross-sectional area of the sample's high-field end, by using a Hall-effect device; but such a result would depend on someone *else*'s calibration of the Hall probe. To make a measurement of magnetic field that you can follow and understand from first principles, the TeachSpin kit includes an alternative lifter-rod structure; it's the one with red and black electrical wires emerging from its top end.

**WARNING:** This lifter-rod structure engages the lifter mechanism atop the balance just like an ordinary sample-rod combination, and it's built using another of those plastic cuvettes you've seen holding samples. But for this calibration device, you must *NOT* try to remove the cuvette from the lifter rod – you'd break the internal electrical wiring if you were to try.

The cuvette in this calibration device contains a square-U structure of bent copper ribbon, which allows a horizontal conductor of length  $L_x$  carrying an externally-driven dc current  $i$  to interact with horizontal field  $B_y$  to produce a vertical force. This (vertical) force, of the magnet on the copper ribbon, will have magnitude  $F_z = i L_x B_y$ . Again we rely on Newton's Third Law, and choose to measure this force via its reaction, the force of the copper ribbon structure on the magnet. The change it causes in the apparent mass of a supported magnet structure should be

$$\Delta m = \frac{1}{g} F_z = \frac{i L_x B_y}{g} .$$

For the TeachSpin copper-ribbon structure, the internal dimensions of the cuvette enforce the length  $L_x = (9.9 \pm 0.1)$  mm, so a current of  $i = \pm 2$  A interacting with a field  $B$  near 0.4 T will give an indicated mass change of order

$$\Delta m \cong \frac{(\pm 2 \text{ A})(0.01 \text{ m})(0.4 \text{ T})}{9.80 \text{ m/s}^2} = \pm 0.00082 \text{ kg} = \pm 0.82 \text{ g} ,$$

which can be easily read with adequate resolution. If the field existing over the area of the lower end of the copper ribbon is adequately uniform, then the  $\langle B_y \rangle$ -value deduced from  $\Delta m$  gives the desired value of  $\langle B^2 \rangle \cong \langle B_y^2 \rangle \cong \langle B_y \rangle^2$ . See Appendix E for a testable model which suggests that the desired value of  $\langle B^2 \rangle$  is connected to the measured value of  $\langle B_y \rangle$  via

$$\langle B^2 \rangle = \langle B_y \rangle^2 [1 + \left( \frac{1}{4} \pm \frac{1}{4} \right) \%] .$$

The same copper-ribbon structure can be used to measure (or to get an upper bound on) the value of  $\langle B_y \rangle$  over *another* region of area 1 cm<sup>2</sup>, but is displaced by (say) 3 cm vertically above the

center of the magnet structure. This provides the needed estimate for  $\langle B^2 \rangle$  at the ‘other’, the weak-field, end of the sample.

To use this calibration structure, you will want your magnet to have equilibrated to the temperature at which you will also use it for magnetic-susceptibility measurements. (That’s because the strength of the  $B$ -field it creates has a temperature coefficient – it drops with temperature, by a few tenths of a per cent per Kelvin.) And you will want to orient the magnet properly, relative to the copper-ribbon ‘hairpin bend’, so that you get the maximal vertical force. To do this orientation, you can use the lifter structure to raise the entire calibration structure vertically, and now you can rotate the entire top-hat-plus-magnet assembly relative to the balance’s weighing pan, about a vertical axis, to get the field direction pointing properly relative to the copper ‘hairpin’. Now lower the current-hairpin just as if it were a magnetic-susceptibility structure, and use the same sort of  $x$ -,  $y$ -, and  $z$ -adjustments as before to center the copper ribbon in the highest-field region of the magnet.

When you have the copper ribbon centered to eyeball precision, use an external, auxiliary dc current supply to send a current  $i$  of 1-2 A through the ribbon. (You will need only a trifling voltage to achieve this current.) Now take note of the balance’s reading when  $i = 0$ , and when  $i$  takes on positive (and negative) values of your choosing. Look for the balance to indicate apparent mass changes  $\Delta m$  (of both signs) relative to its  $i = 0$  reading. To ensure you have the system aligned properly, use the knobs to make small  $z$ -adjustments of the lifter structure, searching for the extreme values of  $\Delta m$  that you can achieve.

When you have the calibration structure aligned to your satisfaction, record some raw mass indications as a function of the current  $i$  that you use, and make a plot of apparent mass vs. current. Your plot ought to include points obtained at negative, as well as positive, values of  $i$ . Why should your data fall along a line in your plot? What ought to be the values of the intercept, and the slope, of this line?

When you have finished with this episode of data-taking, you could try a variation. Use the lifter’s knobs to withdraw the calibration structure vertically, and then rotate the entire top-hat-plus-magnet structure, about a vertical axis, through a  $180^\circ$  angle on the balance’s weighing pan. Now put the calibration structure back into place, align it properly, and repeat your data-taking and -plotting. You ought to get another, linear, dependence of apparent mass upon current  $i$ . But why is the slope the opposite of what you got previously? How does the magnitude of the slope compare?

When you have completed this calibration activity, you should have values for  $\langle B_y \rangle$ , and for  $\langle B^2 \rangle$ , that apply to your magnet, so long as you use it at a fixed temperature. It will take a bit more work, and some scientific judgment, to decide on the proper value of the *uncertainty* to be assigned to these values.

## f)...Data reduction

You've now seen the Gouy technique, by which you can assign, absolutely, a 'volume susceptibility'  $\chi$  to any sample that uniformly fills a cuvette. This section deals with the various *characters* (solid, powdered, liquid, solutions in liquid) that a sample might take, and the other forms of magnetic susceptibility that are best suited for them. In particular, this section will introduce and motivate the 'mass susceptibility'  $\chi_{\text{mass}}$  and the 'molar susceptibility'  $\chi_{\text{molar}}$ , and will illustrate how these forms of susceptibility are sometimes better suited to data reduction, and to comparison with theory.

### \*f.1: for uniform (solid or liquid) samples

First recall the simplest result applies to **uniform** solid or liquid samples, having cross-sectional area  $A$ . They give a predicted mass change of

$$\Delta m = \frac{\chi A}{2\mu_0 g} (B_{\text{top}}^2 - B_{\text{bottom}}^2).$$

So for solid or liquid samples, the quantity  $\chi$ , the (dimensionless, volume) susceptibility can be computed directly – in the standard geometries, we can use the form

$$\chi = \Delta m \cdot \frac{2\mu_0 g}{A B^2}.$$

### \*f.2: for powdered samples

But for **powdered** samples, the magnetizable material does not fill each horizontal plane of area  $A$ , so this calculation does not apply. Instead we compute the effective susceptibility, as would be measured in a Gouy-balance system, for a sample which has overall volume  $V$ , but which is composed of a mixture, with sub-volume  $V_1$  of material of susceptibility  $\chi_1$ , and sub-volume  $V_2$  of material of susceptibility  $\chi_2$ . We might, for example, mean that #1 = strawberry seeds, and #2 = the strawberry jelly between the seeds. Immersed in a common field  $H$ , we get, for the two sub-parts of the sample, magnetization values  $M_1 = \chi_1 H$  and  $M_2 = \chi_2 H$ , and magnetic moments  $\mu_1 = \chi_1 H V_1$  and  $\mu_2 = \chi_2 H V_2$ . Then the net magnetic moment of the whole sample comes to  $\mu = \mu_1 + \mu_2$ , so the volume-averaged magnetization of the entire two-component sample is

$$M = \frac{\mu}{V} = \frac{\mu_1 + \mu_2}{V_1 + V_2} = \frac{\chi_1 H V_1}{V_1 + V_2} + \frac{\chi_2 H V_2}{V_1 + V_2} = \chi_{\text{eff}} H \quad \text{with} \quad \chi_{\text{eff}} = \frac{V_1 \chi_1 + V_2 \chi_2}{V_1 + V_2}.$$

Clearly the effectively susceptibility is a *weighted average* of the susceptibilities of the two materials, and the 'weighting factors' that emerge are the fractional volumes occupied by the two materials (*not* their fractional masses). This result allows the susceptibility to be measured for a composite sample (such as powder-in-air), and then to be corrected to what would be measured for a fully-dense sample.

Now we apply this to what's measured in actual experiments. In the Gouy method for a powdered sample, we can take

#1 = material under investigation,

and #2 = air interstitial between grains of powder,

and then (ignoring the susceptibility of air), we get

$$\chi_{eff} = \frac{V_1\chi_1 + V_2 \cdot 0}{V_1 + V_2} .$$

Here  $V_1 + V_2$  is the volume occupied by both materials together in the sample tube; it's given by  $A \cdot h$ , where  $A$  is the cross-sectional area of the tube, and  $h$  is the height to which it's filled. What remains is the hard-to-measure volume  $V_1$  actually occupied by material #1. What is easier to measure is  $m_1$ , the mass of material #1 actually present. These numbers are connected by the density  $\rho_1$  of (the solid form of) material #1, via

$$\rho_1 = \frac{m_1}{V_1} .$$

Then  $V_1 = m_1/\rho_1$ , and we get

$$\chi_{eff} = \frac{V_1\chi_1}{V} = \frac{(m_1/\rho_1)\chi_1}{A h} = m_1 \left( \frac{\chi_1}{\rho_1} \right) \cdot \frac{1}{A h} .$$

So the Gouy signal previously computed becomes

$$\Delta m = \frac{\chi_{eff} A}{2\mu_0 g} B^2 \rightarrow \frac{m_1 \left( \frac{\chi_1}{\rho_1} \right) \frac{1}{A h} \cdot A}{2\mu_0 g} B^2 = m_1 \left( \frac{\chi_1}{\rho_1} \right) \frac{B^2}{2\mu_0 g h} .$$

We can write this as

$$\frac{\Delta m}{m_1} = \left( \frac{\chi_1}{\rho_1} \right) \frac{B^2}{2\mu_0 g h} ,$$

and we see the cross-sectional area  $A$  has cancelled out, and that two ratios have emerged. One is  $\Delta m/m_1$ , the fractional mass change per unit mass of active material. The other is the 'mass susceptibility' of a material, defined by

$$\chi_{mass} \equiv \frac{\chi}{\rho} .$$

This development tells us that what is most directly measurable in a Gouy experiment with a powdered sample is

$$\chi_{1,mass} = \frac{\chi_1}{\rho_1} = \frac{\Delta m}{m_1} \cdot \frac{2\mu_0 g h}{B^2} ,$$

and that this  $\chi_{mass}$  can be measured, absolutely, even if the density  $\rho_1$  is *not* known. So this is the preferred form of the susceptibility to quote for materials which are available only in powdered or granular form.

Exercise:

Show that this result reduces to the previous formula for the case of a fully-dense solid sample.

We've thus seen that mass susceptibility is the number most naturally measured by the Gouy technique. To see a typical value for mass susceptibility, consider titanium metal having (mks-converted book value)  $\chi = 182 \times 10^{-6}$ , and a bulk density  $\rho = 4.50 \times 10^3 \text{ kg/m}^3$ ; this gives

$$\chi_{\text{mass}}(\text{Ti metal}) = \frac{182 \times 10^{-6}}{4.50 \times 10^3 \text{ kg/m}^3} = 40.4 \times 10^{-9} \frac{\text{m}^3}{\text{kg}}.$$

This result would apply to the grains of titanium powder, as well. You'll find that the multiplier  $10^{-9} \text{ m}^3/\text{kg}$  is a quite convenient scale on which to compare mks mass susceptibilities. These  $\text{m}^3/\text{kg}$  units also remind us that 1 kg of titanium represents a certain number of atoms, or a number of moles of atoms, of titanium. So we might also have occasion to compute or use the 'molar susceptibility' defined by

$$\chi_{\text{molar}} \equiv W \chi_{\text{mass}} = \frac{W}{\rho} \chi,$$

where  $W$  is the molar mass (the so-called 'molecular weight') of the material. In the case of titanium with  $W = 47.87 \text{ g/mole}$ , we get in SI units

$$\chi_{\text{molar}} = (0.04787 \text{ kg/mole}) \times (40.4 \times 10^{-9} \text{ m}^3/\text{kg}) = 1.94 \times 10^{-9} \text{ m}^3/\text{mole}.$$

Notice we now have a magnetic susceptibility given in terms of a certain volume associated with a mole of atoms; since we also know how many atoms are in a mole, we could even fancifully compute

$$\chi_{\text{atom}} = (1.94 \times 10^{-9} \text{ m}^3/\text{mole}) / (6.02 \times 10^{23} \text{ atoms/mole}) = 3.22 \times 10^{-33} \text{ m}^3/\text{atom}.$$

to derive a magnetic property, of dimensions volume, for one single titanium atom.

Exercise:

What is this 'volume'  $\chi_{\text{atom}}$  for a single *helium* atom? How is it related to the mean-square radius of the electron wavefunction, the value  $\langle r^2 \rangle$  for a single electron? Show that the result can be written as

$$\chi_{\text{atom}} = -\frac{4\pi}{3} r_e \langle r^2 \rangle \quad \text{where } r_e = \text{the 'classical electron radius', } \frac{e^2}{4\pi\epsilon_0 mc^2}.$$

Clearly we now have seen four related quantities, each of them giving a sort of volume:

quantity:	$\chi_{\text{atom}}$	$\chi_{\text{molar}}$	$\chi_{\text{mass}}$	$\chi$
with units:	$\text{m}^3/\text{atom}$	$\text{m}^3/\text{mole}$	$\text{m}^3/\text{kg}$	none, or $\text{m}^3/\text{m}^3$ .

Which one of these four is the 'real' susceptibility? The more productive question is to ask which one is the *useful* susceptibility, and that turns out to depend on the measurement being conducted.

- Notice that in the Gouy method using a powdered sample, the *mass* susceptibility is what emerges from the experimental results as a natural measure for the material in question. And since mass is more easily measured (than, say, the volume of material actually present in a generic sample such as a powder), the mass susceptibility is favored for this reason.

- Similarly, since a quantum-mechanical calculation can predict the susceptibility per atom or per mole (but can scarcely claim to predict the density that will be achieved in a powdered sample), the molecular or molar susceptibility is the number best suited for a comparison with theory.

{ And *why* are there these four forms? For enlightenment, consider a sample made entirely of atoms of the same kind, and filling some volume  $V$ . The fundamental definition  $M = \chi H$  tells us that  $\Sigma\langle\mu\rangle/V = \chi H$ , so the sample's net magnetic moment  $\Sigma\langle\mu\rangle$  is given by  $(\chi V)H$ . So the sample's response to a magnetic field  $H$  as 'cause' is a net magnetic moment  $\Sigma\langle\mu\rangle$  as 'effect', and the sample-information is encapsulated by the  $\chi \cdot V$  product. Now here are some alternative forms of that crucial  $\chi V$ -product:

Clearly for a sample of mass density  $\rho$ , we can write

$$\chi V = \left(\frac{\chi}{\rho}\right)(\rho V),$$

where  $\chi/\rho$  gives the previously-defined mass susceptibility  $\chi_{\text{mass}}$ , and where  $\rho V$  gives the sample mass  $m$ . So we have  $\chi V = \chi_{\text{mass}} m$ .

Similarly for a sample of density  $\rho$  and made of atoms of molar mass  $W$ , we can write

$$\chi V = \left(W \frac{\chi}{\rho}\right) \left(\frac{\rho V}{W}\right).$$

Here the first factor is the previously-defined molar susceptibility  $\chi_{\text{molar}}$ , and the second factor is the sample mass  $m$  divided by the molar mass  $W$  of its atoms. But that quotient is the number of moles present, which we will denote as  $v$ . So we also have  $\chi V = \chi_{\text{molar}} v$ .

Finally we can write

$$\chi V = \left(\frac{1}{N_A} W \frac{\chi}{\rho}\right) \left(N_A \frac{\rho V}{W}\right) = \left(\frac{1}{N_A} \chi_{\text{molar}}\right) (N_A v).$$

In this form, the first factor is the molar susceptibility divided by Avogadro's number, which we've previously called the 'atomic susceptibility'. Meanwhile the second factor is Avogadro's number times the number of moles in the sample, which is clearly  $N$ , the number of atoms in the sample. So we finally get  $\chi V = \chi_{\text{atom}} N$ .

Hence the net result is the equality of four expressions:

$$\chi V = \chi_{\text{mass}} m = \chi_{\text{molar}} v = \chi_{\text{atom}} N,$$

And now it is clear that just because there are four natural ways to quantify the amount of sample (namely volume, mass, count of moles, and count of atoms), there are also four natural ways to express the susceptibility (namely  $\chi$ ,  $\chi_{\text{mass}}$ ,  $\chi_{\text{molar}}$ , and  $\chi_{\text{atom}}$ ). Each of the four product-forms in the equation above then gives the same quantity: the factor which maps the cause (the field  $H$ ) into the effect (the sample magnetic moment  $\Sigma\langle\mu\rangle$ ). }

### \*f.3: for solutions

It turns out that the molar susceptibility is also the natural quantity to compute for a sample that's in the form of a *solution*. We might, for example, dissolve a paramagnetic salt in water, and then use some of this stock solution to fill a cuvette of cross-section area  $A$  to a depth  $h$ . If the salt is fully dissolved in the water, this procedure does give a spatially-uniform sample, so the result of \*f.1 above will give an experimental result for the volume susceptibility  $\chi$  of the sample. Now if we ignore (or correct for) the susceptibility of the water, and ascribe the whole of the resulting  $\chi$  to the paramagnetic ions in the solution, we'll see that the molar susceptibility of those ions can be extracted from the data.

We take from the section above the equality  $\chi V = \chi_{\text{molar}} v$ , so that  $\chi_{\text{molar}} = \chi(V/v)$ . This gives a form well-suited to compare with experiment. We first imagine a sample cuvette of cross-section area  $A$ , filled with solution to depth  $h$ , so the volume  $V = A h$ . We imagine that we have prepared the solution to some known concentration, so (from either the measured volume, or the measured mass) of the solution in the cuvette, we can compute  $v$ , the number of moles of magnetically-active atoms (or ions) that are present in the cuvette. Finally we use the result of section \*f.1 above, which connects the mass change  $\Delta m$  observed in the Gouy method to the effective susceptibility  $\chi$  of the uniform sample, and we get

$$\chi_{\text{molar}} = \frac{V}{v} \chi = \frac{Ah}{v} \cdot \Delta m \frac{2\mu_0 g}{AB^2} = \frac{\Delta m}{v} \cdot \frac{2\mu_0 gh}{B^2},$$

and that result gives the molar susceptibility of the magnetically-active species in the solution, in terms of measured quantities.

### \*f.4: comparing with theory

You've seen that your process of data reduction will differ, depending on the form your sample takes on. But you have also seen that for any sample, you can compute the molar susceptibility of the magnetically-active part of it, and *this* is the form most suited to be compared with theory. That molar susceptibility has the character of a volume per mole, and the units of  $\text{m}^3/\text{mole}$ ; also the multiplier  $10^{-9} \text{ m}^3/\text{mole}$  gives a convenient scale on which molar susceptibilities may be quoted. While such a result for  $\chi_{\text{molar}}$  might already be a good candidate for tabulation, there are times when it is better to transform the result into a dimensionless form. We discuss that transformation here, assuming that Curie paramagnetism characterizes your sample.

[You might first want to correct your measured whole-molecule molar susceptibility for the diamagnetic susceptibility that also present in the molecule, to leave a result attributable only to paramagnetism. The typical procedure is discussed in Appendix I.]

Now *if* your sample is a Curie paramagnet, then you can use the result

$$\chi_{\text{molar}} \equiv W \frac{\chi}{\rho} = \frac{\mu_0 N_A \mu^2}{3k_B T}$$

which is derived in Appendix H, to transform the empirically-measured value of  $\chi_{molar}$  into a microscopic result, the magnitude of the magnetic moment  $\mu$  assumed to exist in each molecule. From that, you can extract a most useful (and conveniently dimensionless) end-product  $\mu/\mu_B$ , the magnetic moment per molecule in units of the Bohr magneton. The equation above can be solved for  $\mu^2$ , or better for  $(\mu/\mu_B)^2$ :

$$\mu^2 = \chi_{molar} \cdot \frac{3k_B T}{\mu_0 N_A} , \text{ or } \left(\frac{\mu}{\mu_B}\right)^2 = \chi_{molar} \cdot T \cdot \frac{3k_B}{\mu_0 N_A \mu_B^2} .$$

To turn this into a more useful form, it is convenient to form dimensionless versions of the factors:

$$\left(\frac{\mu}{\mu_B}\right)^2 = \frac{\chi_{molar}}{10^{-9} \text{ m}^3/\text{mole}} \cdot \frac{T}{1 \text{ K}} \cdot \frac{3k_B \times 10^{-9} \text{ m}^3 \text{ K}/\text{mole}}{\mu_0 N_A \mu_B^2} .$$

Since the left-hand side, and the first two factors on the right, are all dimensionless, so also must be the last factor on the right. You can show that its purely-numerical value is  $6.3637 \times 10^{-4}$ , so upon taking the square root we can finally write

$$\frac{\mu}{\mu_B} = 0.02523 \sqrt{\frac{\chi_{molar}}{10^{-9} \text{ m}^3/\text{mole}} \cdot \frac{T}{1 \text{ K}}} .$$

[This result replaces the form that chemists often quote,  $\mu/\mu_B = "2.84(\chi T)^{1/2}"$ , which is **only** valid if the susceptibility quoted is the molar susceptibility, *in cgs form*, and is further understood to be in units of  $\text{cm}^3/\text{mole}$ , and the temperature is understood to be in Kelvin.]

## g)... Converting between SI and cgs units

Magnetic measurements have been historically conducted using the cgs or cm-g-s set of units, and using ‘gaussian electromagnetic units’ for the field quantities. In this system,  $B$  has units of Gauss,  $H$  has units of Oersteds, and  $M$  is typically quoted in ‘emu’ or ‘electromagnetic units’, meaning (in this case) ergs/(Gauss·cm<sup>3</sup>). Converting tabulated values into or out of cgs presents two distinct difficulties:

- 1) In the cgs presentation of electromagnetism, one lacks the constant  $\mu_0$ , and one defines the fields via  $B = H + 4\pi \cdot M$  [notice the exotic dimensional consistency? of this equation!], while retaining  $M^{(\text{cgs})} = \chi^{(\text{cgs})} H^{(\text{cgs})}$ , so it turns out that

$$\chi^{(\text{mks})} = 4\pi \cdot \chi^{(\text{cgs})},$$

even though both the mks and cgs  $\chi$ -numbers are dimensionless.(!)

- 2) In practice, the cgs values of  $\chi$ ,  $\chi_{\text{mass}}$ , and  $\chi_{\text{molar}}$  are not always quoted with properly-indicative subscripts, nor with their (rather mysterious) units, and considerable care is needed to decide *which of them* is being quoted.

So, for example, a standard reference might quote, for the susceptibility of helium gas at STP, the result “ $-1.88 \times 10^{-6}$  cgs” per “gram formula weight”, which is really a *mass* susceptibility of

$$\chi_{\text{mass}}^{(\text{cgs})} = \frac{-1.88 \times 10^{-6} \text{ cm}^3}{4.00 \text{ g}}.$$

The dimensionless (or volume) susceptibility is then

$$\chi^{(\text{cgs})} = \chi_{\text{mass}}^{(\text{cgs})} \cdot \rho = \left( \frac{-1.88 \times 10^{-6} \text{ cm}^3}{4.00 \text{ g}} \right) \cdot \left( \frac{4.00 \text{ g}}{22.4 \times 10^3 \text{ cm}^3} \right) = -0.0839 \times 10^{-9} \frac{\text{cm}^3}{\text{cm}^3},$$

and this finally gives the SI value

$$\chi^{(\text{mks})}(\text{He, at STP}) = 4\pi \times (-0.0839 \times 10^{-9}) = -1.054 \times 10^{-9}.$$

Notice that this number is in fine agreement with the result ( $-1.06 \times 10^{-9}$ ) predicted by quantum-mechanical theory, directly from the Schrödinger equation.

Similarly, the same handbook might give, for titanium metal at 293 K, a value of “ $+153 \times 10^{-6}$ ”, which is to be interpreted and converted as

$$\chi^{(\text{mks})}(\text{Ti, } 20^\circ\text{C}) = 4\pi \left( \frac{153 \times 10^{-6} \text{ cm}^3}{47.87 \text{ g}} \right) \left( \frac{4.50 \text{ g}}{\text{cm}^3} \right) = 181 \times 10^{-6}$$

To illustrate the wealth of confusion that is possible, consider that the susceptibility for ordinary water at 37 °C can be variously quoted as

$$\chi^{(\text{mks})} = -9.05 \times 10^{-6}$$

$$\chi^{(\text{cgs})} = -0.720 \times 10^{-6}$$

$$\chi_{\text{mass}}^{(\text{mks})} = -9.09 \times 10^{-9} \text{ m}^3/\text{kg}$$

$$\chi_{\text{mass}}^{(\text{cgs})} = -0.723 \times 10^{-6} \text{ cm}^3/\text{g}$$

$$\chi_{\text{molar}}^{(\text{mks})} = -0.164 \times 10^{-9} \text{ m}^3/\text{mole}$$

$$\chi_{\text{molar}}^{(\text{cgs})} = -1.30 \times 10^{-5} \text{ cm}^3/\text{mole}$$

Each of these six (distinct!) numbers is *correct and with its proper units -- within its system --* but the difficulty is that the units might *fail to be quoted*, or might be quoted *improperly*, in the reference or paper in which you find them. For example, even the dimensionless (volume) susceptibility  $\chi^{(\text{cgs})}$  might appear as the number  $-0.720 \times 10^{-6}$ , accompanied by no units, or by emu/cm<sup>3</sup>, or by gauss/Oersted, erg/cm<sup>3</sup>·Oe<sup>2</sup>, erg/cc·gauss<sup>2</sup>, or emu/cm<sup>3</sup>·Oe. Clearly, care is needed before any reference can be used safely! See John F. Schenk, "The role of magnetic susceptibility in magnetic resonance imaging", Med. Phys. 23, 815 (1996) for a thorough discussion of this welter of units.

For just such reasons, in the rest of this Manual we use SI units exclusively, and we also use the notations  $\chi$ ,  $\chi_{\text{mass}}$ , and  $\chi_{\text{molar}}$  consistently to distinguish these three forms of the susceptibility. There is no particular moral advantage of one unit system over another, so long as you are working only within the small sandbox of comparing the  $\chi$ -value of one material with another's. But when it comes time to ask *other* and larger physics questions, it is necessary to know and understand the unit system you're using. Here's an

Example:

Given the susceptibility(ies) quoted above for water, and given a permanent magnet whose 'sweet spot' delivers a field of  $B = 0.4 \text{ T} = 4000 \text{ gauss}$ , what is the magnetization  $M$  which develops inside a sample of water placed in this field? How does it compare to the magnetization typical within the material of a NdFeB rare-earth magnet? Hint: compute the value of  $H$  in the sweet spot, and use the definition of  $\chi$ .

## References

This annotated list of references is not meant to be complete, but it offers guidance at some points left uncovered in the Manual proper.

H. A. Bethe and E. E. Salpeter, Quantum Mechanics of One- and Two-Electron Atoms, (Springer-Verlag, 1957)

--at section 50, gives the quantum-mechanical calculation for the diamagnetism of helium, deriving the needed  $\langle r^2 \rangle = 1.19 a_0^2$  from a Hartree approximation to the two-electron wavefunction.

John F. Schenk, "The role of magnetic susceptibility in magnetic resonance imaging: MRI magnetic compatibility of the first and second kinds", Med. Phys. **23**, 815-850 (1996)

--motivated by the requirements of magnetic resonance imaging, but discusses in detail the issue of dia- and para-magnetic objects in distorting imaging fields. In particular, Schenk is careful and complete in discussing SI and cgs unit systems.

Charles Kittel, Introduction to Solid State Physics, 2<sup>nd</sup> edition (Wiley, 1956)

--contains two valuable appendices: Appendix D treats the diamagnetism of atoms by quantum-mechanical perturbation theory, and Appendix G justifies the 'spin-only approximation', the so-called 'quenching' of the orbital angular momentum by crystal fields, in forming the effective magnetic moment of atoms in paramagnetism.

Arthur Thorpe and Frank E. Senftle, "Absolute Method of Measuring Magnetic Susceptibility", Rev. Sci. Instrum. **30**, 1006-1008 (1959)

--introduces a variant on the Gouy method, which has come to be called the Thorpe-Senftle method, for measuring susceptibility. It requires a much smaller volume of sample than the Gouy method, but it does require a field 'survey' and an integration; it connects to Appendix D in this Manual.

G. A. Candela and R. E. Mundy, "Absolute Magnetic Susceptibilities by the Thorpe-Senftle Method", Rev. Sci. Instrum. **32**, 1056-1057 (1961)

--applies the Thorpe-Senftle method to measure the absolute susceptibility of Mohr's salt, and compares the results with those of the Gouy method in the same apparatus.

David J. Griffiths, Introduction to Electrodynamics, 3rd edition (Prentice Hall, 1999)

--contains in Ch. 6 a modern view of the ‘auxiliary field’  $\mathbf{H}$ , and also addresses the question of the best *names* to use for  $\mathbf{B}$  and  $\mathbf{H}$ . It also concurs with the customary definition of magnetic susceptibility, via  $\mathbf{M} = \chi \mathbf{H}$ .

John R. Reitz and Frederick J. Milford, Foundations of Electromagnetic Theory, 2<sup>nd</sup> edition (Addison-Wesley, 1967)

--contains in Ch. 12 a discussion of magnetic energy; and, at Problem 12-7, illustrates the physical basis of the Gouy method.

Alan Earnshaw, Introduction to Magnetochemistry (Elsevier, 2013)

--treats the ‘Pascal constants’ for estimating molecular diamagnetism in his introductory chapter, giving them in cgs form, and in units of ‘ $10^{-6}$  / g atom’, meaning  $10^{-6}$  cm<sup>3</sup>/mole.

G. A. Bain and J. F. Berry, “Diamagnetic Corrections and Pascal’s Constants”, J. Chem. Ed. **85**, 532-536 (2008)

--has a very complete treatment of Pascal constants, and extensive tables as well as examples of their use.