Toward multilevel variance decomposition of interactions in non-linear structural equation models

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23 May 2017

Acknowledgment

This research was aided by

- Mike Neale
- Steve Boker (Univ of Virginia)
- OpenMx development team
- National Institute of Health R25-DA026119, R01-DA018673 (PI Neale)

Second order linear differential equation

$$\ddot{x}(t) = \eta x(t) + \zeta \dot{x}(t) \tag{1}$$

- x – a position (1 dimensional)
- t – time
- x(t) – position as a function of time
- η, ζ – parameters to estimate

An oscillator

$$\ddot{x}(t) = \eta x(t) + \zeta \dot{x}(t) \tag{2}$$

- When $x(t) = \dot{x}(t) = 0$ then the system is at equilibrium
- When $\eta < 0$ and $\eta + \zeta^2/4 < 0$, x will oscillate
- Otherwise, $x \to \pm \inf$ as $t \to \inf$

Resilience: Physical and psychological

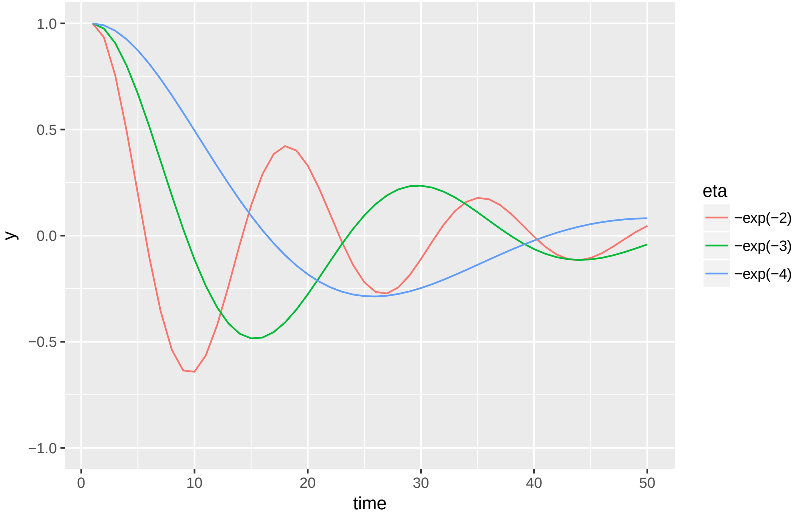
- variable thermostats
- recovery from negative (or positive) emotional shocks

As a statistical model

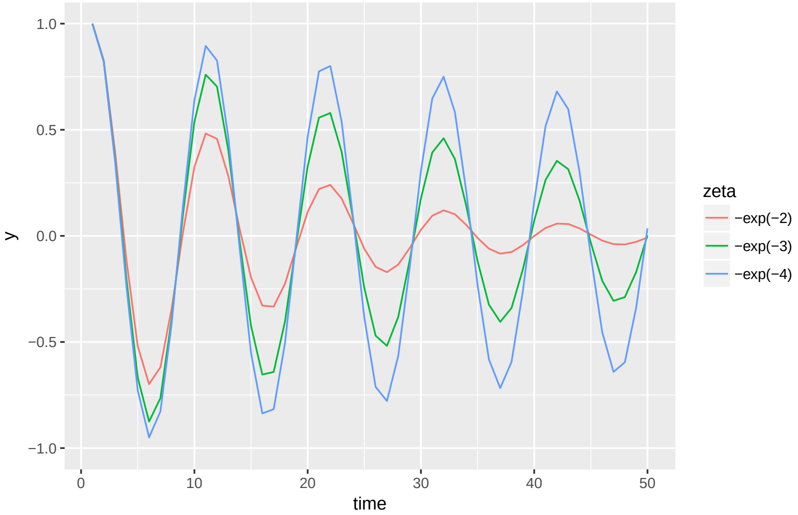
$$\ddot{x}(t) = \eta x(t) + \zeta \dot{x}(t) \tag{3}$$

- x – measured with some noise
- t – known
- η, ζ – parameters to estimate

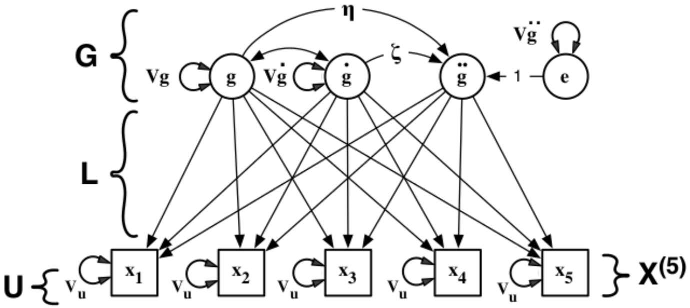
η, frequency



ζ, damping



Path diagram



Time delay embedding

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ \dots \end{pmatrix} \longrightarrow \begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \\ x_2 & x_3 & x_4 & x_5 & x_6 \\ x_3 & x_4 & x_5 & x_6 & x_7 \\ x_4 & x_5 & x_6 & x_7 & x_8 \\ x_5 & x_6 & x_7 & x_8 & x_9 \\ x_6 & x_7 & x_8 & x_9 & x_{10} \\ x_7 & x_8 & x_9 & x_{10} & x_{11} \\ x_8 & x_9 & x_{10} & x_{11} & x_{12} \\ & & \dots & & \end{pmatrix} \tag{4}$$

Potential multilevel applications

arousal amplitude variance by age decile

- $\eta|$age decile
- $\eta|$person
- time

stressor resonance duration by group

- $\zeta|$group
- $\zeta|$person
- time

heredity of frequency and damping

What model?

Which model do we need?

Nice if we can stay in a maximum likelihood SEM framework: asymptotically unbiased and minimum variance

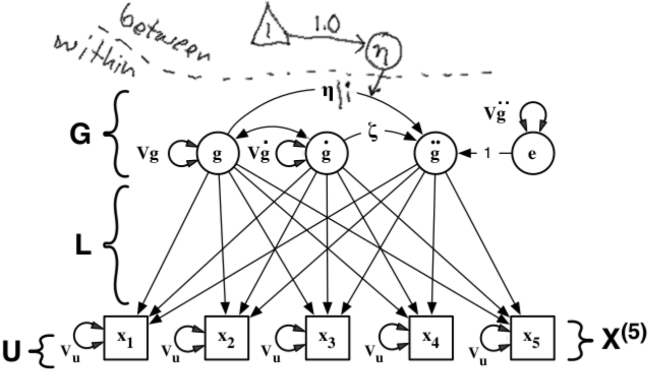
Random slopes

$$Y_{ij} = (\beta_0 + \beta_0 j) + (\beta_1 + \beta_1 j)x_{ij} \tag{5}$$

- i enumerates within groups
- j is the group
- Y is the response
- β are parameters
- x is given (e.g. measurement time)

Product is between a parameter and a given value (x)

Path diagram



Product is between two latent variables

Variance of a regression coefficient

$$Var(\eta) \equiv Var\left[\frac{Cov(\ddot{x}, x)}{Var(x)}\right] \tag{6}$$

$$Var(\zeta) \equiv Var\left[\frac{Cov(\dot{x}, x)}{Var(x)}\right] \tag{7}$$

Mean structure

In SEM, variables are assumed to be centered (mean deviation form).[1]

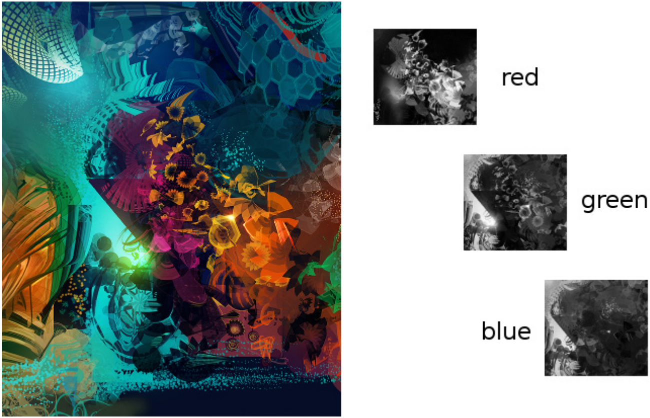
$$E(\xi_1) = E(\xi_2) = 0 \tag{8}$$

$$E(\xi_1\xi_2) = E(\xi_1)E(\xi_2) + \text{Cov}(\xi_1, \xi_2) = \text{Cov}(\xi_1, \xi_2) \tag{9}$$

$\text{Cov}(\xi_1, \xi_2)$ of Equation 9 is non-Normal

[1] Moosbrugger, Schermelleh-Engel, and Klein (1997)

What does a latent interaction look like through a Normal lens?



Mixture approaches

General approach[2]

- Components represent different outcomes of $\mathrm{Cov}(\xi_1, \xi_2)$
- Per-row component weights determined by per-row likelihood

Cannot extend to multilevel

[2] Klein and Moosbrugger (2000); Jedidi, Jagpal, and DeSarbo (1997)

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Modeling frameworks

Two-stage maximum likelihood

Bayesian using Monte Carlo sampling

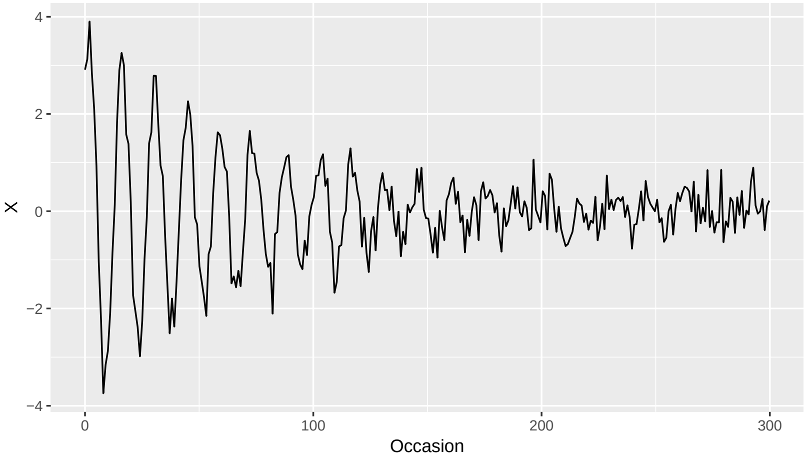
Two-stage parameter recovery simulation

Fully crossed design:

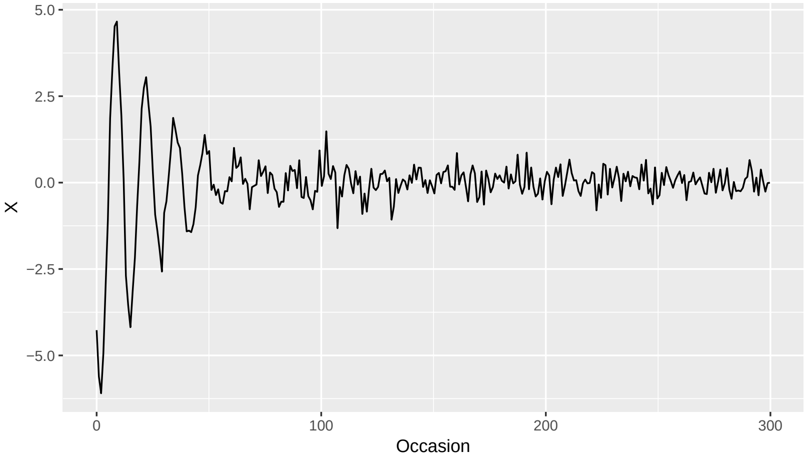
- number of twins = 100, 200, 400, 800
- additive genetic variance = 0, 0.25, 0.5, 0.75
- 300 time points
- 200 Monte Carlo replications

$$\log(-\eta) \sim \mathcal{N}(-1.6, 0.6) \tag{10}$$
$$\log(-\zeta) \sim \mathcal{N}(-3.0, 0.6) \tag{11}$$

Example data



Example data



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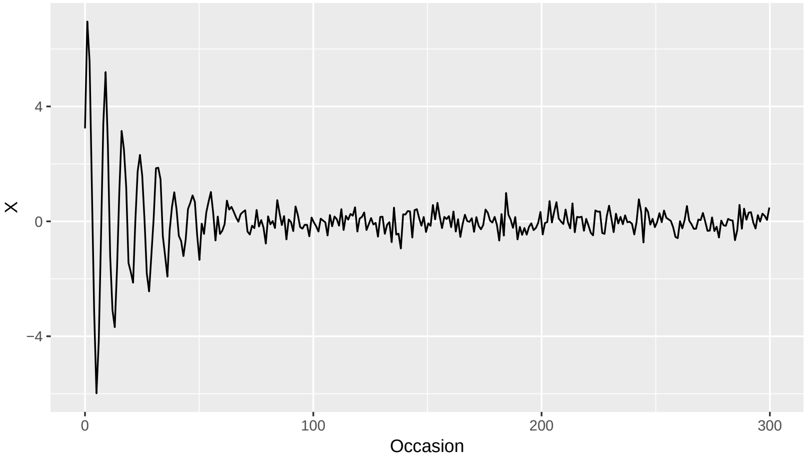

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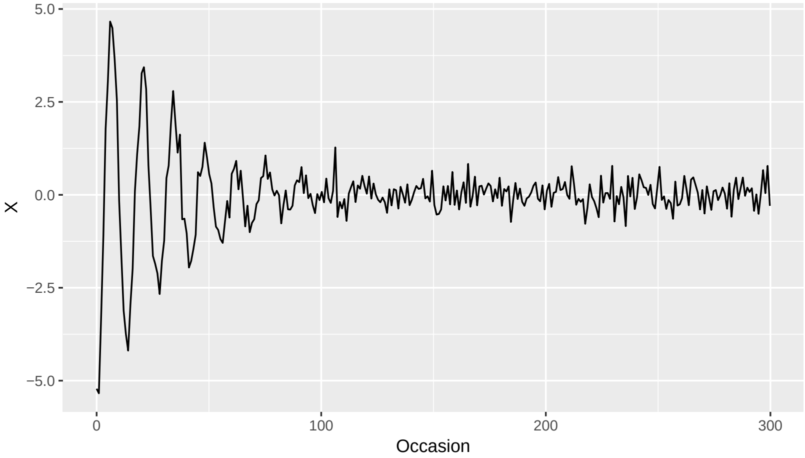

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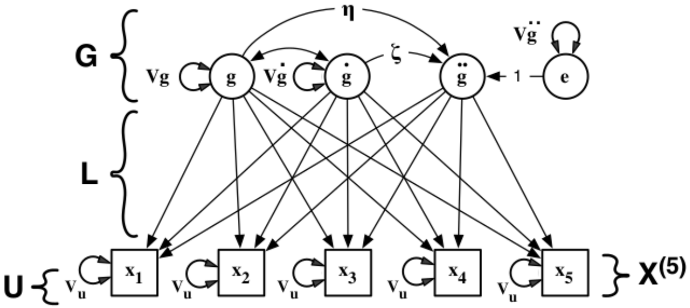


Variance decomposition

$$U = \begin{pmatrix} \eta_1 & \eta\zeta_1 & 0 & 0 \\ \eta\zeta_1 & \zeta_1 & 0 & 0 \\ 0 & 0 & \eta_2 & \eta\zeta_2 \\ 0 & 0 & \eta\zeta_2 & \zeta_2 \end{pmatrix} \tag{12}$$

$$AE = \begin{pmatrix} A + E & kA \\ kA & A + E \end{pmatrix} \tag{13}$$

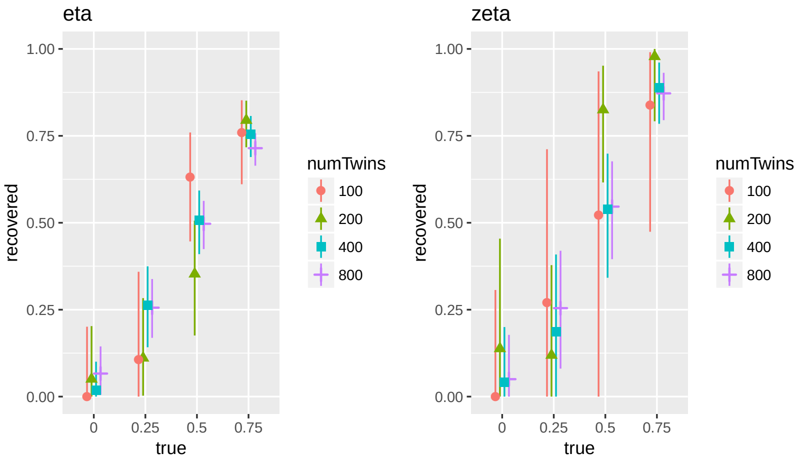
$$\Sigma = U + AE \tag{14}$$

- U is populated with the inverse Hessian
- A and E are 2-by-2 covariance matrices
- k is 1.0 for MZ and 0.5 for DZ
- means are freely estimated

95% interval coverage

numTwins	agv	eta	zeta
100	0.00	0.97	0.96
100	0.25	0.97	0.99
100	0.50	0.94	0.97
100	0.75	0.98	0.99
200	0.00	0.98	0.97
200	0.25	0.96	0.98
200	0.50	0.96	1.00
200	0.75	0.96	0.99
400	0.00	0.96	0.98
400	0.25	0.96	0.99
400	0.50	0.94	0.99
400	0.75	0.95	1.00
800	0.00	0.92	0.94
800	0.25	0.91	0.97
800	0.50	0.92	1.00
800	0.75	0.94	1.00

One replication



Future Directions

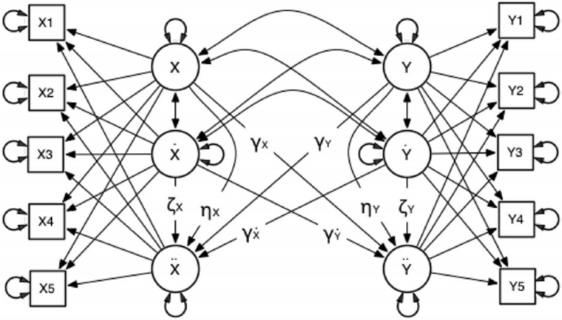


Figure 3. Coupled latent differential equation model (coupled LDE).

Accounting for a common environment?[3]

[3] Hu, Boker, Neale, and Klump (2014)

Future Directions

Full Bayesian

Bisconti, T. L., Bergeman, C. S., & Boker, S. M. (2006). Social support as a predictor of variability: An examination of the adjustment trajectories of recent widows. *Psychology and Aging*, *21*(3), 590.

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Klein, A., & Moosbrugger, H. (2000). Maximum likelihood estimation of latent interaction effects with the lms method. *Psychometrika*, *65*(4), 457–474.

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