A new implementation of Item Factor Analysis: Accuracy, flexibility, and speed

Joshua N. Pritikin

Department of Psychology
University of Virginia

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Why a new implementation?

- ConQuest, $750
- IRTPRO, $495
- flexMIRT, $\approx$ $100 per year

Weaknesses:

- flexibility, customization
- Windows-centric
- non-zero $ barrier to entry

Also, current open-source software is fragmented and uncompetitive.

Goal

Meet or exceed capabilities of all commercial software

Where to start?

Until recently, marginal maximum likelihood (Bock & Aitkin, 1981) with adaptive Gauss-Hermite quadrature (Schilling & Bock, 2005) was the leading algorithm (Wirth & Edwards, 2007).

The IRTPRO manual recommended

- marginal maximum likelihood (Bock & Aitkin, 1981) with analytic dimension reduction (Cai, 2010b)
- Metropolis-Hastings Robbins-Monro (Cai, 2010a)

A starting point

- marginal maximum likelihood (Bock & Aitkin, 1981) with analytic dimension reduction (Cai, 2010b)
- Metropolis-Hastings Robbins-Monro (Cai, 2010a)

Which? *Both*

- Radically different approaches
- Similar performance on a broad class of problems

An open source implementation of MH-RM exists (Chalmers, 2012).

Start with marginal maximum likelihood.

Analytic dimension reduction

Necessary restriction of the factor covariance structure

$$\begin{pmatrix} \Sigma & \\ 0 & diag(\tau) \end{pmatrix}$$

where Σ is any covariance structure. For example,

$$\begin{pmatrix} 1 & & & \\ \rho & 1 & & \\ 0 & 0 & 1 & \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

All factors have unit variance. In a multigroup model, the primary factors can be correlated. The specific factors are mutually uncorrelated and uncorrelated with the primary factors (Cai, 2010b).

Analytic dimension reduction

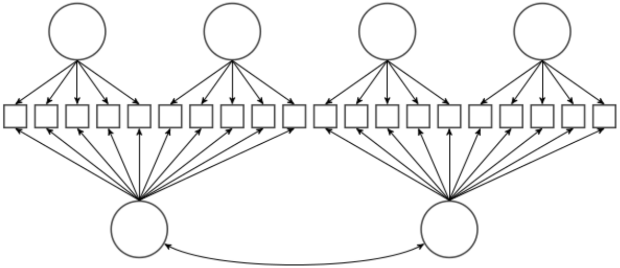


FIGURE 1.
A two-tier model for testlet-based assessments. The items measure 2 correlated primary dimensions and the test is based up of 4 testlets, creating 4 additional specific dimensions.

(Cai, 2010b, p. 583)

Analytic dimension reduction

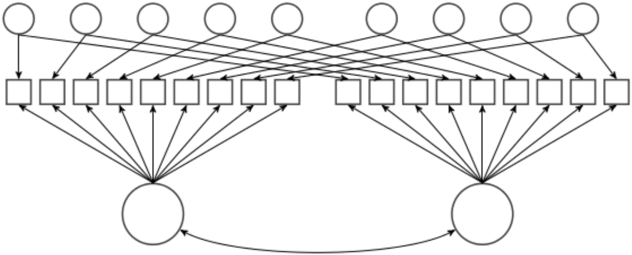


FIGURE 2.
A two-tier model for longitudinal item response data. The 9 items are administered to the same group of individuals at two time points, creating two time-specific primary dimensions and 9 item-specific doublets capturing residual correlations.

(Cai, 2010b, p. 584)

Analytic dimension reduction

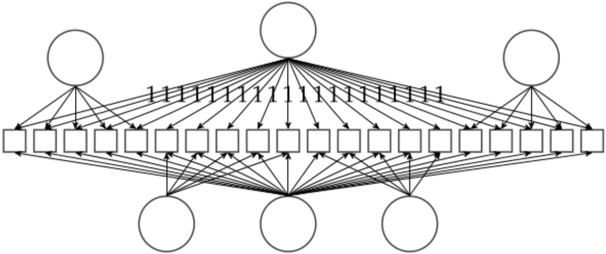


FIGURE 3.
Random intercept item bifactor model as a two-tier model. The inclusion of a random intercept that is orthogonal to the other factors is a distinguishing feature of this model. The general factor, on which all items load, and the random intercept are the primary dimensions.

(Cai, 2010b, p. 585)

Status

- R, $\approx$ 1300 lines
- C, $\approx$ 2350 lines

Implements marginal maximum likelihood (Bock & Aitkin, 1981) with analytic dimension reduction (Cai, 2010b)

OpenMx plugin, not yet merged

EAP person scores (Bock & Mislevy, 1982)

Item models: M3PL, GPCM, or invent your own

$S-\chi^2$ with polytomous extension (Orlando & Thissen, 2000)

Infit & outfit (Wright & Masters, 1982)

Plots: item characteristic curve (1d), data vs model, and information

$S-\chi^2$

(Explain on blackboard)

High-priority wish list

- Merge into OpenMx (Boker et al., 2011)
- Nominal model w/ analytic Newton-Raphson (Baker & Kim, 2004) (open source math exist; Chalmers, 2012)
- Item parameter standard errors (Cai, 2008)
- Multi-group (Cai, Yang, & Hansen, 2011)
- Structural latent trait model
- Hierarchical factor model

Item parameter standard errors

What do they mean?

$$\mathcal{I}(\hat{\theta}|Y_o) = \mathcal{I}_c(\hat{\theta}) - \mathcal{I}_m(\hat{\theta})$$

where the information matrix $\mathcal{I}$ at the item parameter vector estimate $\hat{\theta}$ conditional on the observed data Y_o is the complete information matrix $\mathcal{I}_c$ minus the information matrix of the missing data $\mathcal{I}_m$ (Cai, 2008).

Accuracy, performance

Let's take a look ...

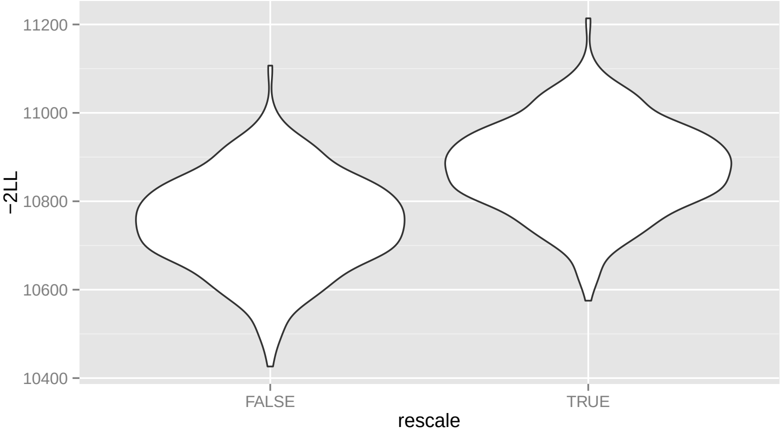
Standardize (rescale) the latent distribution or not?

- 20 2PL items
- 500 persons
- 17 point GH quadrature
- 0% missing
- 500 Monte Carlo replications M
- Rescale or not (Liu, Rubin, & Wu, 1998)

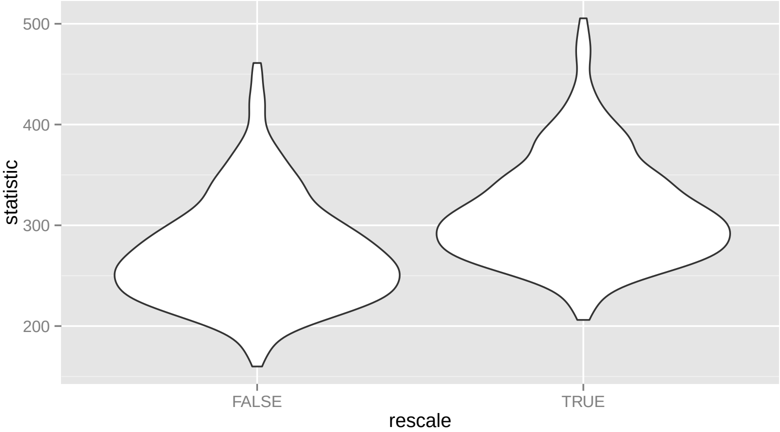
Examine -2LL, $S-\chi^2$, and bias.

$$\text{bias} = \hat{\theta} - \theta_{true} \text{ where } \hat{\theta} = \frac{1}{M}\sum_{m=1}^{M} \theta_m$$

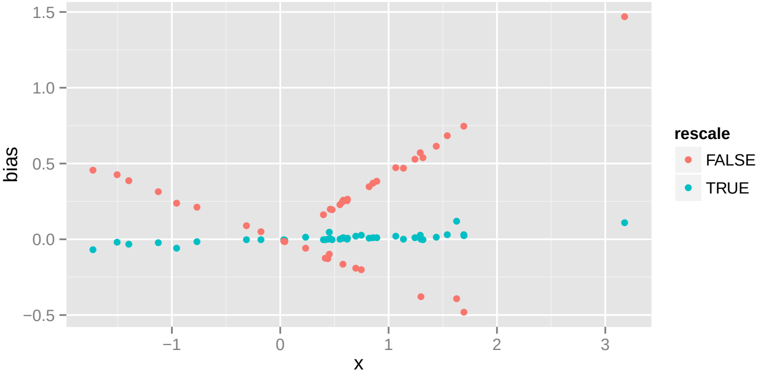
-2LL



$S-\chi^2$



Bias



Bias ranged from -0.0695 to 0.1188 with 50% of the bias between -0.0033 to 0.0174 with the median at 0.0056. For comparison, Winstep obtained bias ranging from 0.01 to 0.13 (Wang & Chen, 2005).

Which model? How many quadrature points?

- 20 2PL items
- 500 persons
- 0% missing
- GH quadrature
- This and subsequent studies are rescaled (Liu et al., 1998)

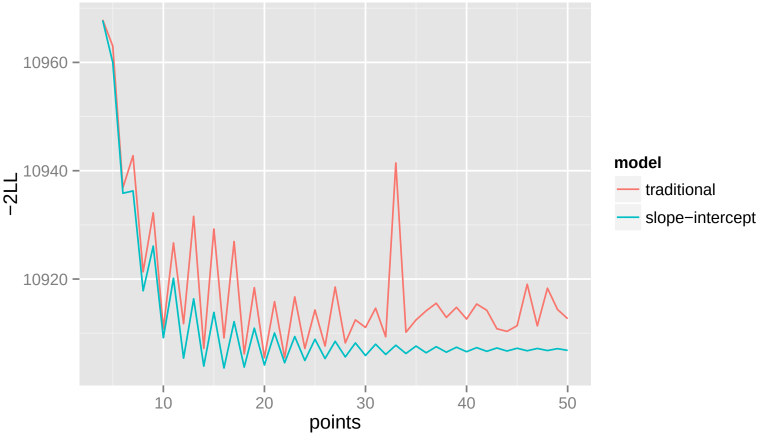
Traditional parameterization

$$\frac{1}{1+\exp(-a(\theta-b))}$$

or slope-intercept form

$$\frac{1}{1+\exp(-(a\theta+c))} \text{ where } b=\frac{c}{-a}$$

Likelihood by item model and quadrature points

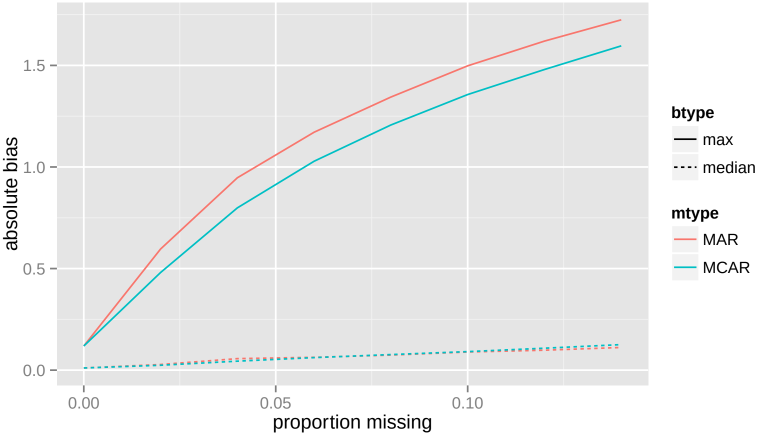


Bias by % missing

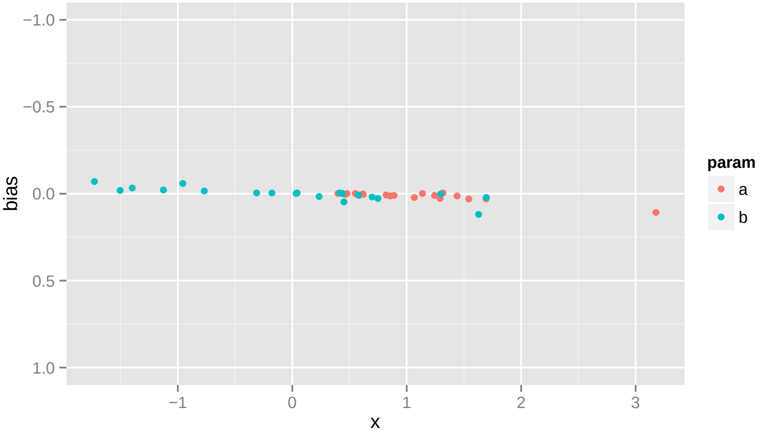
- 20 2PL items
- 500 persons
- 17 point GH quadrature
- 500 Monte Carlo replications

For the missing at random condition, data was replaced by NA depending on the first 5 items.

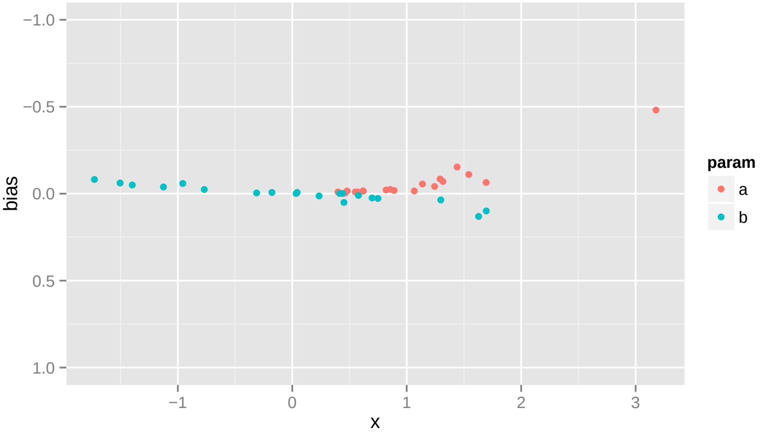
Bias by % missing



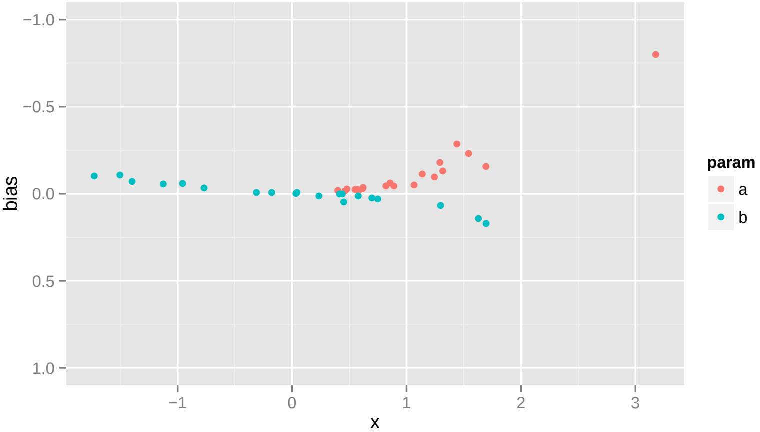
Bias by % missing, MCAR 0%



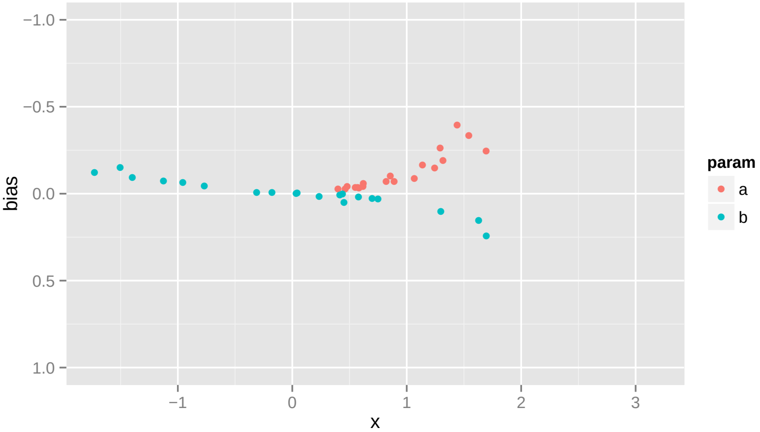
Bias by % missing, MCAR 2%



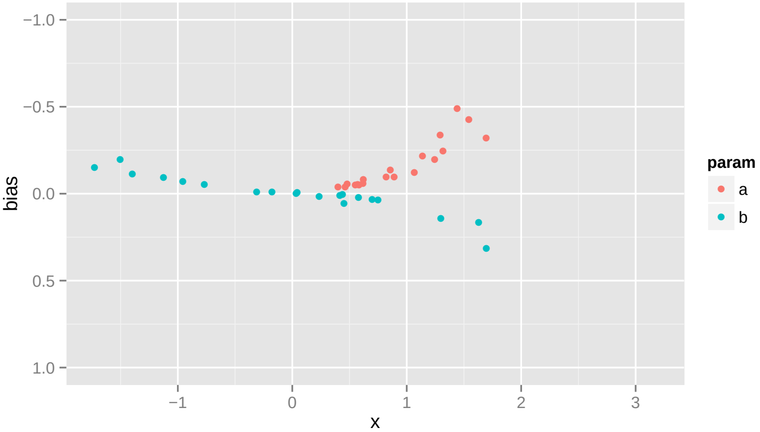
Bias by % missing, MCAR 4%



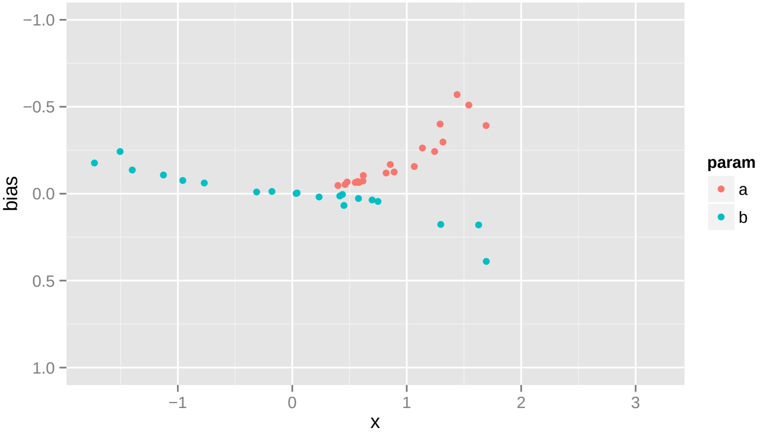
Bias by % missing, MCAR 6%



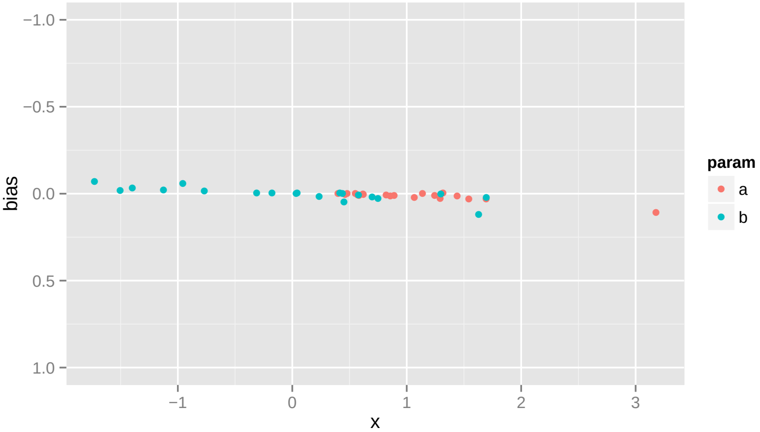
Bias by % missing, MCAR 8%



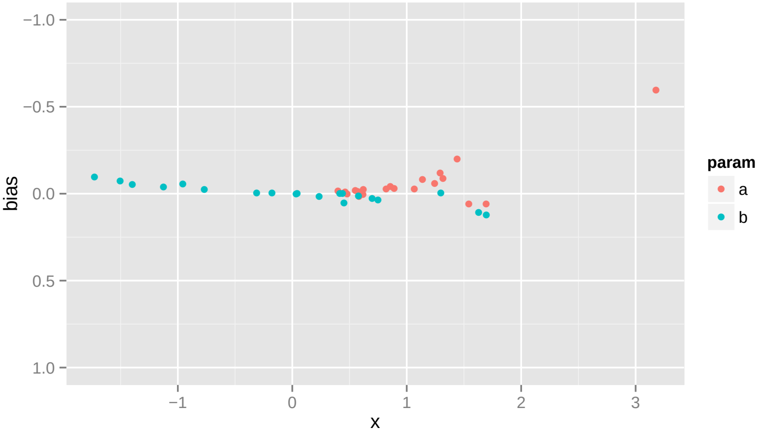
Bias by % missing, MCAR 10%



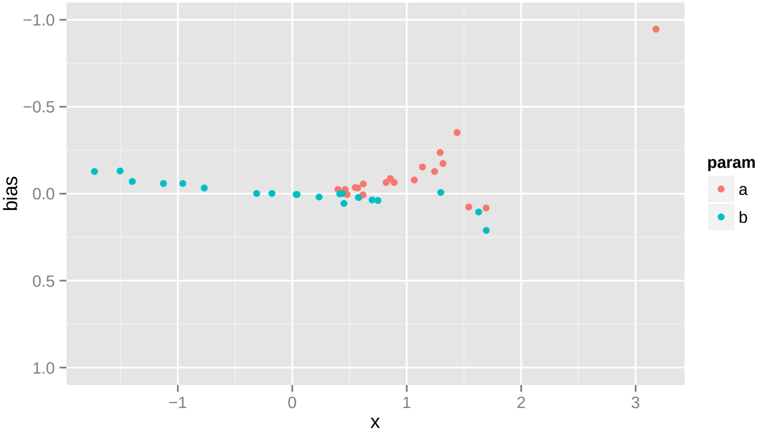
Bias by % missing, MAR 0%



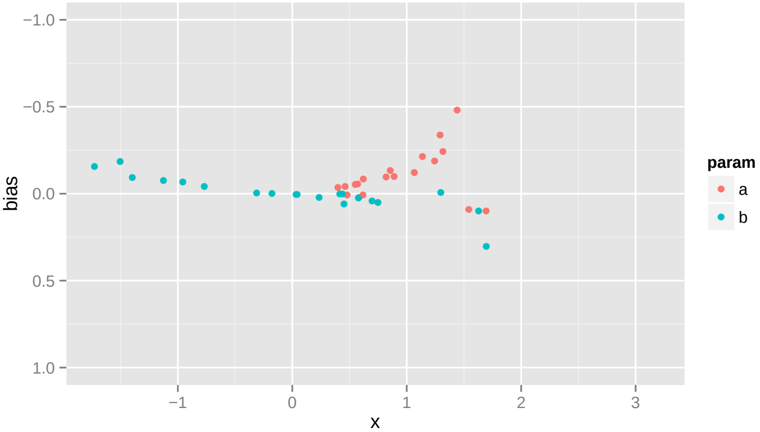
Bias by % missing, MAR 2%



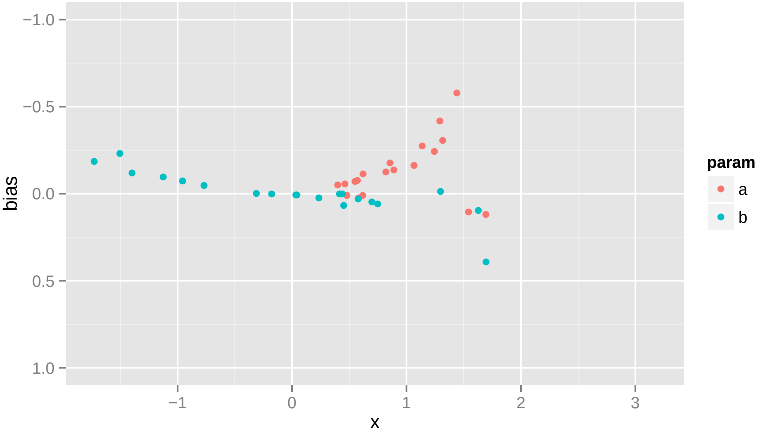
Bias by % missing, MAR 4%



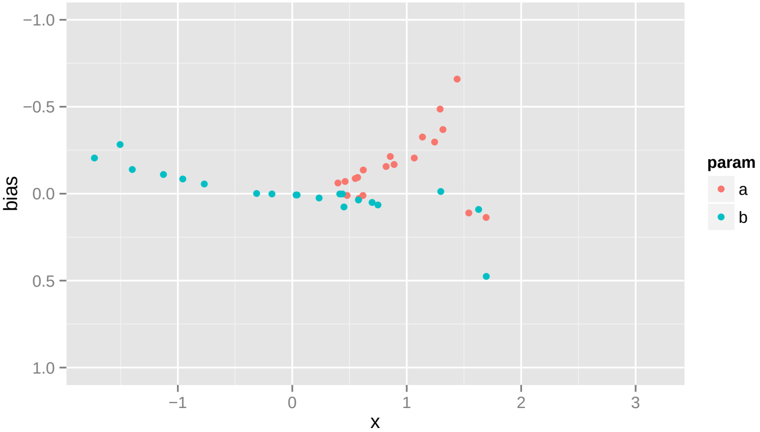
Bias by % missing, MAR 6%



Bias by % missing, MAR 8%



Bias by % missing, MAR 10%

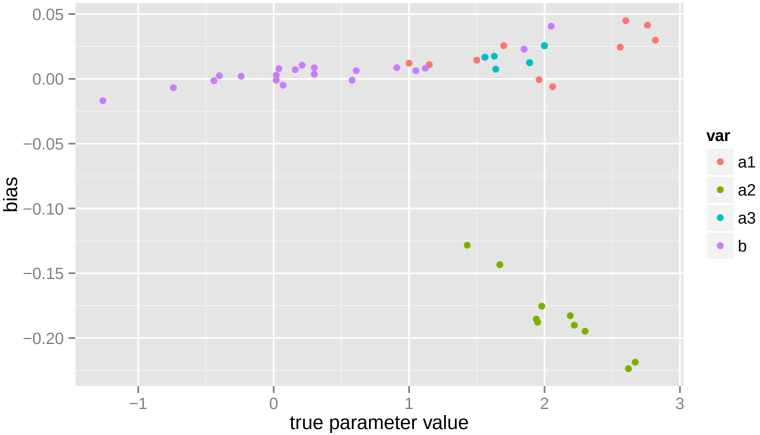


Cai (2010b) parameter recovery simulation

- 20 M2PL items
- 2 primary dimensions
- 4 specific dimensions formed by 4 pairs of item doublets
- $N = 500$ per replication
- 13 point GH quadrature[1]
- 500 Monte Carlo replications

[1] IRTPRO uses a 21 point equally spaced quadrature by default.

Cai (2010b) parameter recovery simulation



Note: slight pos bias; comparable to Cai (2010b) except for green; *very* slow

Acknowledgment

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- OpenMx development team
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Questions?

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