Ejercitación Unidad 4

1. $\Omega = \{1C, 1X, 2C, 2X, 3C, 3X, 4C, 4X, 5C, 5X, 6C, 6X\}$
$R_x = \{2, 1, 3, 2, 4, 3, 5, 4, 6, 5, 7, 6\}$

a. Obtenga la función de probabilidad $p(x)$

$$p(x) = \begin{cases} 0 & 7 < x < 0 \\ \frac{1}{12} & x = 1 \\ \frac{1}{12} & x = 7 \\ \frac{2}{12} & 2 \leq x \leq 6 \end{cases}$$

b. Obtenga la función de distribución acumulada $F_X(x)$
$F(x) = \sum_{i=1}^{7} p(x_i) = 1$

c. Halle $P(X > 3)$
$P(X > 3) = P(4) + P(5) + P(6) + P(7) = \frac{7}{12}$

d. Halle la probabilidad de que el puntaje obtenido sea un número impar.
C = {x es impar}
C = {1,3,5,7}
$P(x) = \frac{6}{12}$

e. Halle $E(X), V(X)$ y $E(2X + 3)$
$E(x) =$
$V(x) =$
$E(2X + 3) =$

2. **a.** Determinar el valor de la constante k
$\int_0^2 \int_0^2 k(x + y)dxdy = 1$

$$\begin{aligned} \int_0^2 k(x + y)dx &= \int_0^2 kxdx + \int_0^2 kydx \\ &= \frac{kx^2}{2}\Big|_0^2 + kxy\Big|_0^2 \\ &= \frac{4k}{2} + 2ky \\ &= 2k(y + 1) \end{aligned}$$

$$\begin{aligned}\int_0^2 2k(y+1)dy &= \int_0^2 2kydy + \int_0^2 2kdy \\ &= ky^2\Big|_0^2 + 2ky\Big|_0^2 \\ &= 4k + 4k \\ &= 8k\end{aligned}$$

$$\int_0^2 \int_0^2 k(x+y)dxdy = 8k = 1$$

$$\boxed{k = \frac{1}{8}}$$

b. Obtener las funciones de densidad marginales.

$$\begin{aligned}f_x(X) &= \int_0^2 \frac{1}{8}(x+y)dy \\ &= \int_0^2 \frac{1}{8}xdy + \int_0^2 \frac{1}{8}ydy \\ &= \frac{1}{8}xy\Big|_0^2 + \frac{1}{16}y^2\Big|_0^2 = \\ &= \frac{1}{4}x + \frac{1}{4} \\ &= \boxed{\frac{1}{4}(x+1)}\end{aligned}$$

$$\begin{aligned}f_y(Y) &= \int_0^2 \frac{1}{8}(x+y)dx \\ &= \int_0^2 \frac{1}{8}xdx + \int_0^2 \frac{1}{8}ydx \\ &= \frac{1}{16}x^2\Big|_0^2 + \frac{1}{8}yx^2\Big|_0^2 \\ &= \frac{1}{4} + \frac{1}{4}y \\ &= \boxed{\frac{1}{4}(y+1)}\end{aligned}$$

c. Calcular la $P(X < 1, Y \leq 1{,}5)$, $P(X \leq 1)$, $P(Y \leq 1{,}5|X \leq 1)$

$$\begin{aligned} F_x(x,y) &= \int_0^x \int_0^y f(x,y)dydx \\ &= \int_0^x \int_0^y \frac{1}{8}(x+y)dydx \end{aligned}$$

$$\begin{aligned} \int_0^y \frac{1}{8}xdy + \int_0^y \frac{1}{8}ydy &= \frac{1}{8}xy\Big|_0^y + \frac{1}{16}y^2\Big|_0^y \\ &= \frac{1}{8}xy + \frac{1}{16}y^2 \\ &= \frac{1}{16}y(2x+y) \end{aligned}$$

$$\begin{aligned} \int_0^x \frac{1}{16}y(2x+y)dx = \int_0^x \frac{1}{8}yxdx + \int_0^x \frac{1}{16}y^2dx &= \frac{1}{16}yx^2\Big|_0^x + \frac{1}{16}y^2\Big|_0^x \\ &= \frac{1}{16}xy(x+y) \end{aligned}$$

$$\begin{aligned} P(X<1, Y \leq 1{,}5) &= \frac{1}{16}xy(x+y) \\ &= \boxed{0{,}234375} \end{aligned}$$

$$\begin{aligned} P(X \leq 1) &= \frac{1}{4}(x+y) \\ &= \boxed{0{,}5} \end{aligned}$$

$$\begin{aligned} P(Y \leq 1{,}5 | X \leq 1) &= \frac{f(x,y)}{f_x(X)} \\ &= \frac{\frac{1}{16}yx(x+y)}{\frac{1}{4}(x+1)} \\ &= \boxed{0{,}46835} \end{aligned}$$

d. ¿Son independientes X e Y?

$$F_x(x,y) = \frac{1}{16}xy(x+y)$$

$$
\begin{aligned}
F_x(x) \cdot F_y(y) &= \frac{1}{4}(x+1) \cdot \frac{1}{4}(y+1) \\
&= \frac{1}{16}(xy + x + y + 1)
\end{aligned}
$$

$$\boxed{\begin{array}{l} F_x(x,y) \neq F_x(x) \cdot F_y(y) \\ \text{No son independientes} \end{array}}$$

3. $P(Y > X) = \dfrac{\lambda_1}{\lambda_1 + \lambda_2}$