Missing Land Markets, Efficiency, Insurance, and Redistribution*

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Abstract

When agricultural land is allocated to fulfill an insurance and redistribution objective, what are the implications of moving towards allocating land through markets? In this paper, I answer this question by building a model to study the efficiency, insurance and redistribution properties of communal land systems in developing countries. Under these systems, local leaders reallocate unused land on a need basis and restrict private transfers. Restrictions on private transfers ensure land remains under the purview of local leaders when households leave agriculture. However, these restrictions also prevent productive farmers from scaling up production. This implies a tradeoff whereby efficiency costs emerge while fulfilling a redistribution and insurance objective. I evaluate this tradeoff by introducing a farm sector, occupation choice and access to land through communal land into a standard model of incomplete insurance. I match the model to the Malawi economy whose micro data allow me to pin down key parameters associated with land access. The calibrated model implies that if communal land was reallocated through private markets, it would lead to an increase in welfare of 36%. The higher welfare however can be decomposed into 56% gains through higher efficiency, 12% losses through higher inequality and 8% losses through higher consumption volatility.

JEL Classification Codes: O11, E02, H11
Keywords: communal land, property rights, land misallocation, development, insurance and redistribution

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1 Introduction

Increasing evidence suggests missing land markets generate large productivity costs by preventing the reallocation of land to more productive farmers. In low income countries, this emerges under communal land systems where local leaders impose restrictions on private transfers of land. These restrictions lower agricultural productivity by keeping unproductive farmers with too much land in agriculture and preventing productive ones from scaling up production. Using detailed microdata from China and Malawi, Adamopoulos et al. (2017) and Restuccia and Santaeulalia-Llopis (2017) estimate that reallocating land efficiently would lead to gains in agricultural productivity in the order of 3.4 and 3.6 fold respectively.

Despite these productivity costs, rural institutions have long acted as a source of informal insurance in developing countries (Townsend, 1994; Udry, 1994; Morten, 2019). Under communal land systems, central governments delegate to local leaders the task of managing agricultural land. Land becomes available for reallocation as local residents move out of rural areas to pursue better economic opportunities in non-agriculture. Local leaders then reallocate unused land to poor landless households. Restrictions on private transfers are central to ensuring land stays under the purview of local leaders. In the absence of formal social safety net programs, communal land de facto operates as the only source of social insurance available. This implies communal land systems generate two key sources of welfare benefits: *redistribution* by facilitating transfers of land from rich to poor households and *insurance* by providing a productive resource to households undergoing temporary shocks.

This paper is the first to evaluate to what extent do the welfare benefits provided by communal land systems offset their productivity costs. I do this by building a model that features incomplete insurance, a farm sector, occupation choice, and land access through communal land[1]. In the baseline model in the paper, households have a common productivity in agriculture and face income risk in non-agriculture. Leaving agriculture is risky in two ways: households become exposed to non-agricultural income risk and returning to agriculture may entail long spells in low paid agricultural work while waiting for communal land to become available. I then match the model to micro data for Malawi where the panel structure and details on land tenure allow me to pin down the parameters related to communal land access.

[1]The model has the core features of a standard model of incomplete markets following a long tradition as in Imrohoroğlu (1989), Bewley (1986), Huggett (1993), Aiyagari (1994)

In the main policy experiment of the paper, I leave insurance incompleteness as is and allow farmers to trade their communal land holdings in a private market. Property rights increase efficiency as it allows landholders to allocate their labor efficiently while earning rents on their land. Without any new redistributive or social insurance policies, there is however more inequality and less insurance. I use methods developed by Benabou (2002) and Floden (2001) in order to disentangle welfare impacts through efficiency, insurance and redistribution. It reveals that while communal land has large efficiency costs, it comes with significant welfare benefits through insurance and redistribution. Under the baseline calibration, I find that privatizing land increases utilitarian welfare by 36%. The lhigher welfare however comes from a 56% gain through efficiency, a 12% loss through inequality, and an 8% loss through insurance.

The key departure in my paper is the addition of incomplete insurance for understanding the costs of allocating agricultural land through communal systems. Unlike the standard literature, this addition allows communal land to have welfare benefits through insurance and redistribution. So far, virtually all papers related to communal land abstract from incomplete insurance, and as a result can only speak to welfare costs through efficiency (Adamopoulos et al., 2017; Restuccia and Santaeulalia-Llopis, 2017; Chen et al., 2017; Gottlieb and Grobovšek, 2019; Chen et al., 2021). In Chen et al. (2017) and Chen et al. (2021) for instance, the inability to transact on communal land is at the core of the aggregate efficiency costs they find. In my paper, the inability to transact implies land becomes available for redistribution. This has significant welfare benefits as land passes down from the rich to the poorest in the economy. Ability to hold on to land also has insurance benefits implicitly by allowing households to shield themselves from risk by staying in agriculture.

I join a growing literature in macroeconomics trying to understand the implications of incomplete insurance to development (Donovan, 2020; Brooks and Donovan, 2020; Lagakos et al., 2018). Like these papers, the core of my model is a standard incomplete markets model in the tradition of Imrohoroğlu (1989), Bewley (1986), Huggett (1993), and Aiyagari (1994). In many macroeconomic applications, incomplete insurance imply households accumulate assets for self insurance. In development, lack of insurance markets interact with other means to self insure in ways that affect allocative efficiency. In Donovan (2020) for instance, farming households choose a lower amount of inputs before shocks are realized while in Lagakos et al. (2018), households can use temporary migration but have to overcome adjustment costs. In my paper, communal land systems offer ways for poor households to access income generating resources when other economic endeavors fail. However, accessing land may take time and as a result, the insurance value of communal land depends on both the length of the wait and availability of alternative means of insurance. For households already on communal land, they can self insure by staying on land. Its value to them again depends on both the ease of returning to communal land and the availability of alternative means of insurance.

2 Context

My paper focuses on Malawi whose practices regarding communal land shares many similarities with other countries in Africa where measured land misallocation is particularly severe. Table 1 illustrates the prevalence of communal land in Malawi. Households whose land were either allocated to them by traditional authorities or by their family make up about 90% of households. These represent the households living under a communal land system, which in Malawi, is described as customary tenure. Land acquired through the family then was once allocated by the chief. The other 10% of households consists of those renting illegally or through the government's leasehold sector.

Table 1: Land Tenure in Malawi

	Allocated	Inheritance/family	Rented/owned
Households (%)	16.5	73.5	10.3
Cultivated Area (%)	14.4	58.1	27.5

LSMS (2010/2011)

Formally, there are three types of land ownership delineated in the Malawian constitution: customary, public, and leasehold. Under leasehold, the government grants long term leases and collects royalties from private individuals who run what is commonly known as estate farms. Leasehold titles were originally granted to Malawi nationals from land previously owned by colonial powers. Since independence, the government issues new titles by converting customary land into leasehold land. This however happens infrequently as it requires permission from local leadership who is expected to only grant such permissions for unused land that the village no longer needs. In Malawi, estate farms are the main cultivators of the country's cash crops: tobacco and tea.

I focus on customary land which represents the primary land tenure system and where a majority of the population in Malawi live. Customary land is an official designation of indigenous land that function much in the way of other communal land systems. The village chief (or headman) manages customary land by upholding lineage based inheritance rules and allocating unused land to families in need. White (1989) sums up best the role of village chiefs regarding land in his book on village life in Malawi.

> The land belonged to the village, not to the lineage. It was the headman who was responsible for its distribution. Land which had once been allocated could not be taken back by the headman unless it had been left uncultivated, but the headman remained responsible for finding land for fresh settlers and for arbitrating boundary disputes. (p. 164)

This description illustrates the public insurance role played by village chiefs that persists to this day.

Access to agricultural land under customary tenure is achieved in two ways: either it is allocated to you through family (inter vivos or inheritance) or through the village chief. In matrilineal societies of central and southern Malawi for instance, the husband moves to the wife's village and the family is allocated land that belongs to the female lineage. In patrilineal societies of northern Malawi, the gender roles are reversed and land is passed down to sons. Access to land is obtained by virtue of membership to the village's lineage and marriage. These rules imply that searching for land is inherently an activity that follows misfortune. There are two common types of households who end up in this situation: households losing claim on land through violation of local norms (divorce, crime, disagreements, etc.) or those searching for income sources after failed ventures in non-agriculture (Peters and Kambewa, 2007).

3 Empirical Patterns

4 Model

In this section, I formalize the details of the model of communal land. In the model, households face risk in non-agriculture and have access to a risk free asset they can accumulate for self insurance (Bewley, 1986; Huggett, 1993; Aiyagari, 1994). In agriculture, households face no risks and have a common productivity. There are three key features that makes communal land valuable to households in my model. First, agricultural work functions as an investment into the possibility of acquiring land. Second, households on communal land can shield themselves from risk by staying in agriculture. And finally, leaving agriculture implies the household loses its endowment of communal land. Together, these features imply households are willing to stay in agriculture and forgo higher income in non-agriculture. The details of the the choices they face follows.

Demographics. There is a unit measure of dynastic households who discount utility at rate β and a village leader who has control over agricultural land, appropriates unused land and grants use rights to households in the economy.

Preferences. Dynastic households maximize expected discounted utility.

$$U = \sum_{t=s}^{\infty} \beta^{t-s} u(c_t) \tag{1}$$

where we can abstract for sectoral differences in consumption goods as we assume they are perfectly substitutible. The utility function is CRRA with risk aversion parameter σ so per period utility can be written as: $u(c) = \frac{c_t^{1-\sigma}}{1-\sigma}$

Occupation and endowments. Households are endowed with one unit of labor which they supply inelastically. They have common permanent productivity denoted by z_a in agriculture and 1 in non-agriculture. When working in agriculture, landholders are endowed with ℓ_c and called smallholder farmers while those who are landless are called farm workers. When working in non-agriculture, households face idiosyncratic transitory risk denoted by $z_t \in \mathcal{Z}$.

As in Roy (1951), households choose the sector where they want to produce after observing their productivity in non-agriculture and access to communal land. Land access is stochastic and depends on the household's occupation choice. For landless households who choose to work in agriculture, with probability π_a they are allocated ℓ_c and become smallholder farmers, and with probability $1 - \pi_a$ they become farm workers. For landholders, while those who remain in agriculture keep their land as smallholder farmers, those who leave agriculture also lose their land endowment.

Technology. Smallholder farmers have access to farming technology based on their permanent productivity z_a and produce $z_a f_a(\ell_c, n_a)$ by combining ℓ_c with their own and hired labor. They earn profits and the return on their own labor w_a. At any point, the household can choose to work in non-agriculture and access a technology to produce according to $f_n(z) = z$ where z is drawn from an AR1 process that can be written as follows:

$$\log(z_{t+1}) = (1-\rho_z)\bar{z} + \rho_z \log(z_t) + \sigma_z \epsilon \tag{2}$$

Decisions. Households make dynamic decisions according to their current situation - access to land, productivity and assets - and their future possibilities. These individual states can be represented by $x \in \mathcal{Z} \times \mathcal{B} \times \mathcal{L}$ such that $\mathcal{L}$ is binary and represents access to land and the other states are continuous. Let $W : \mathcal{Z} \times \mathcal{B} \to \mathbb{R}$ represent the value of a landless household let $V : \mathcal{Z} \times \mathcal{B} \to \mathbb{R}$ represent the value of a landholder. Then we can also define $W^n : \mathcal{Z} \times \mathcal{B} \to \mathbb{R}$ as the value when running a non-farm business and $V^a : \mathcal{Z} \times \mathcal{B} \to \mathbb{R}$ and $W^a : \mathcal{Z} \times \mathcal{B} \to \mathbb{R}$ as the values when working in agriculture as a smallholder farmer and as a farm worker respectively. And finally, we can write the value of a landholder and a landless household as:

$$W(z,b) = \max\left\{\pi_a V^a(z,b) + (1-\pi_a)W^a(z,b), W^n(z,b)\right\} \tag{3}$$

$$V(z,b) = \max\left\{V^a(z,b), W^n(z,b)\right\} \tag{4}$$

where π_a is the probability of obtaining a plot of communal land and is determined in equilibrium. Hence we can write the value of working in non-agriculture as

$$W^n(b,z) = \max_{c,b'} \quad u(c) + \beta \mathbb{E}_{z'|z}[W(b',z')]$$
$$c + b' = f_n(z) + Rb, \quad b' \geq -\phi$$

My model has two types of agricultural workers: landholders who I call smallholder farmers and landless households who I call farm workers. Lanholders choosing agriculture become smallholder farmers and solve the following consumption savings problem:

$$V^a(b,z) = \max_{c,b',n_a} \quad u(c) + \beta \mathbb{E}_{z'|z}\left[V(b',z')\right]$$
$$c + b' = z_a f_a(\ell_c, n_a) - (n_a - 1)(1+\tau)w_a + Rb, \quad b' \geq -\phi$$

Those who do not get allocated ℓ_c, become farm workers and solve the following problem:

$$W^a(b,z) = \max_{c,b'} \quad u(c) + \beta \mathbb{E}_{z'|z}\left[W(b',z')\right]$$
$$c + b' = w_a + Rb, \quad b' \geq -\phi$$

This sums up the individual choices of agents in the economy, so now we can define the equilibrium concept I rely on in the analysis.

Equilibrium. I focus on a stationary equilibrium where the distribution of states in the economy are constant. A stationary equilibrium in this economy is an agricultural wage (w_a), a land allocation probability (π_a), and value functions $V(b, z), W(b, z)$ such that:

- Households solve their individual problems
- Agricultural labor markets clear:

$$
\begin{aligned}
&(n_a(w_a) - 1) \left(\int_V \mathbb{I}_{(occ=a)}(x) dF(x) + \pi_a \int_W \mathbb{I}_{(occ=a)}(x) dF(x) \right) \\
&= (1 - \pi_a) \int_W \mathbb{I}_{(occ=a)}(x) dF(x)
\end{aligned}
$$

- Communal land used in agriculture production cannot exceed supply

$$
\ell_c \int_V dF(x) = \ell_c N_c \leq L
$$

- Stationarity implies flow of new communal households equal the flow of new landless

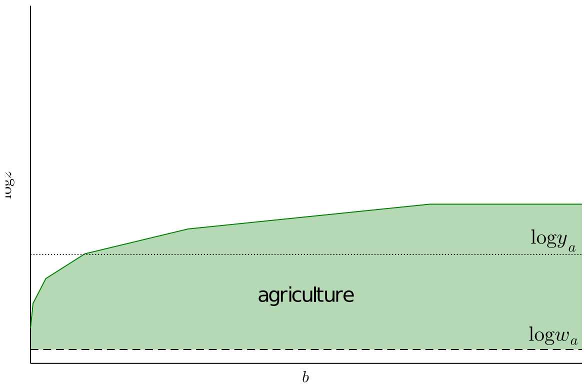
$$
\pi_a \int_W \mathbb{I}_{(occ=a)}(x) dF(x) = \int_V \mathbb{I}_{(occ=n)}(x) dF(x)
$$

5 Discussion of Model Mechanics

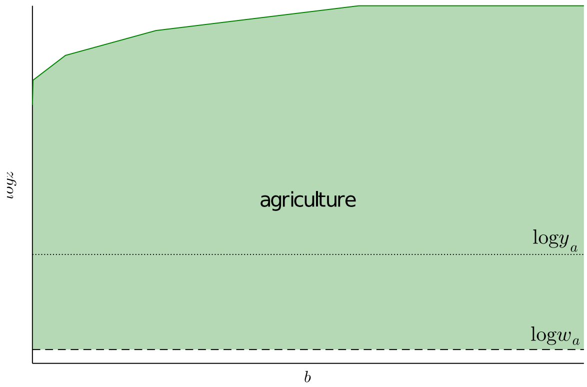
I begin by clarifying the mechanics of the model in order to highlight the ways in which communal land affects outcomes in the economy. There are two key takeawasys from this section. First, communal land implies different occupation choices based on land access. It implies landless households face the possibility of gaining access to communal land. Because of market incompleteness, whether or not they forego higher income in non-agriculture depends on their level of assets. Second, the value of communal land to smallholders depends on how long it takes households in farm work to regain access to communal land. In order to illustrate these points, I first show the indicicvidual occupation choices for both landless households and landholders. In order to clarify the role of communal land, I then vary π_a and illustrate the outcomes it delivers in the models.

Occupation Choices. Working in agriculture as a farm worker is like an investment into the ability to gain access to land and generates two types of farmers in my model: smallholders and farm workers. Inability to insure consumption implies their lifetime depends on how easy it is to access communal land, the level of income in agriculture relative to non-agriculture, the risk in non-agriculture and other means to smooth consumption. In my model, π_a plays a key role in determining these outcomes. In this section, I will show how π_a determines patterns in occupation, consumption levels, and consumption volatility faced by households.

Figure 1: Occupation policies



(a) Landless Households



(b) Landholders

In figures 1a and 1b , I plot the occupation choices of landless households and landholders respectively. The shaded region represent combinations of b and z under which households go into agriculture, otherwise, they go into non-agriculture. The dashed line represents the income from farm work while the dotted line represents income from smallholder farming. The stark contrast between the occupation patterns of landless households and landholders illustrate the different incentives these households face under communal land. I describe these contrasting incentives in detail next.

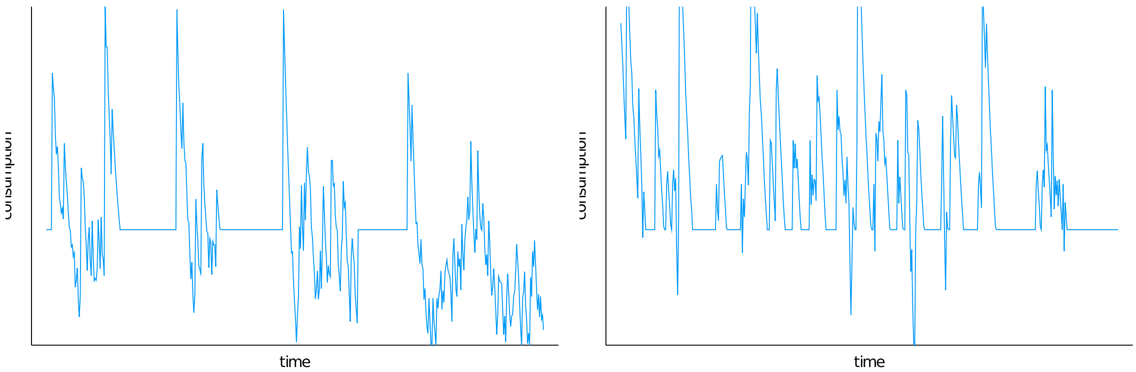
In Figure 1a, I plot the decision rules for households who enter the period with no land. They observe their productivity in non-agriculture and decide whether or not to go into agriculture. Once in agriculture, they may or may not gain access to communal land. This deters some low asset households from going into agriculture since farm work would result in a drop in consumption they could not sustain. Let us consider the situation of households who are allocated a piece of land. For these households who are also near the borrowing constraint, their income grows from z to y_a. The only households whose incomes decline are those with assets above the point in which the dotted line meets the occupation indifference curve. On the other hand, for households not accessing land, their incomes decline to w_a. This implies that households near the borrowing constraint who gain access to communal land produce more in agriculture than they would otherwise in non-agriculture. On the other hand, households who become farm workers are less productive in agriculture.

In Figure 1b, I plot the decision rules for households who enter the period as landholders on communal land. Upon observing their productivity in non-agriculture, these households decide whether or not they leave agriculture and lose their land or whether they remain smallholder farmers. The possibility of losing land prevents many of these households from leaving agriculture. The gap between y_a and the indifference curve illustrates the just how high non-agricultural productivity have to be in order to draw smallholder farmers into non-agriculture.

Communal land regimes and distributional outcomes. In order to illustrate the role of communal land in affecting economic outcomes I vary π_a to show how it changes incentives for different households. Intuitively, I higher π_a implies it takes households less time to access communal land. As illustrated in figure 1a, borrowing constraints generate two types of landless households going into agriculture. Let x^* correspond to the assset level under which the occupation indifference curve cross $\log y_a$ for landless households. Those with assets below x^* have higher productivity in agriculture if they gain access to land ($y_a > z$) and lower ($w_a < z$) if they do not. Those with assets above x^* always have a higher productivity in non-agriculture ($z > y_a > w_a$). As $\pi_a \to 1$, then $x^* \to 0$ and borrowing constraints no longer matter for productivity of labor across sector. The only indifference point becomes the productivity state in which in $z = y_a$, in which point the household would produce the same amount in the two sectors. Figures 7b and 7a in the appendix illustrate these patterns.

Before turning to cross sectional patterns, I first look at the consumption patterns of a household under different π_a regimes over their lifetime. Because households switch occupations more under a high π_a regime, this also implies higher levels and volatility. In order to show this, I first simulate a sequence of non-agricultural productivity shocks for a households under low π_a and the same shocks for a household under a high π_a. Figure 2a plots consumption over the household's lifetime under a low π_a regime. It illustrates that it takes a large shock to get the household to leave agriculture. Further, because it takes a long time to regain access to communal land, households experience low levels

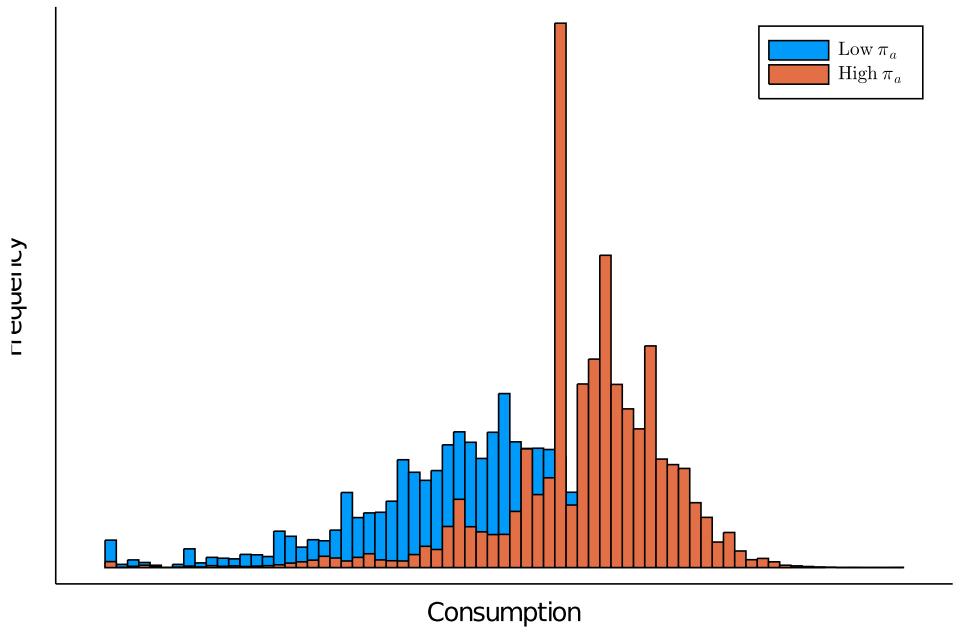
Figure 2: Consumption Path of Household under Communal Land



(a) Low π_a (b) High π_a

of consumption as it fails to regain access to communal land. On the other hand, figure 2b plots the consumption path for the same productivity shocks under a high π_a regime. This shows how the household under this regime transitions in and out of communal land many times and avoid most consumption levels below that of a households on communal land. In the appendix, figure 7b demonstrate that as π_a increases, households with land are more willing to leave non-agriculture.

Figure 3: Distribution of Log Consumption of Landless Households



Turning to cross sectional patterns, I focus on households with no land. I plot the histograms of log consumption for households under the two π_a regimes. Intuitively if shows how under the high π_a regime consumption levels rise in two ways. Low z households spend less time in low productivity states before gaining access to land and high z households switch to non-agriculture more often since returning to agriculture is easier in the event of a productivity drop. This shifts the distribution of consumption to the right as quick land access rules out long spells in farm work. Table **??** displays statistics of log consumption in the cross section. I focus on the same subsample of the population that has no land. It shows that as π_a increases, it shifts consumption pertcentiles to the right. The variance is higher under high π_a, but given the role of land in ruling out consumption at low states, we are interested in the skewness and kurtosis as well. The skewness shifts to the right households avoid low levels of consumption at low states by entering agriculture and gaining access to communal land.

Table 2: Statistics of Landless Households

Statistic	Log Consumption		
	$\pi_a = 0.03$	$\pi_a = 0.15$	$\pi_a = 0.6$
Q10	-0.44	-0.27	-0.22
Q25	-0.22	-0.22	-0.22
Q50	-0.22	-0.22	-0.06
Q75	-0.22	-0.11	0.21
Q90	-0.07	0.20	0.42
Q95	0.14	0.35	0.54
Q99	0.46	0.59	0.75
Mean	-0.25	-0.16	0.01
Mode	-0.22	-0.22	-0.22
Variance	0.07	0.08	0.09
Skewness	-1.87	-0.77	0.08
Kurtosis	10.61	6.48	2.51

In summary, there are several themes that emerged from this section. A lower π_a lowers the average lifetime consumption of households. This happens for two reasons, First landless households will experience more of the low shocks in non-agriculture when π_a is low and the wait to gain access to communal land is longer. Second, because households with land know this, they are less willing to take advantage of their higher productivity in non-agriculture as they may end up landless and poor. In terms of consumption volatility, the role of π_a here is unclear. A lower π_a doesn't necessarily lead to more volatility over the lifetime as households may gain stability by staying on land. However, this comes at the cost of higher income in non-agriculture.

6 Calibration

The discussion of the model mechanics highlight the relationship between communal land access and households' willingness to go into non-agriculture. As shown in the previous section, households may be willing to give up some income in non-agriculture in order to keep their steady income as a smallholder. This largely depends on individual risk and income levels in non-agriculture as well as the ease of access and income levels on communal land. In this section, I discuss the parameters of the model that are relevant for these patterns as well as the data that I use to discipline these parameters. I will pick parameters to match salient features of the Malawi economy, where communal land is prevalent.

In order to pick parameters so that the model best represent the data, I use simulated method of moments. This entails solving for the individual policies and the stationary distribution, simulating data, computing moments from the simulated data, and changing parameters until these these moments match analogues in the data. I calibrate the baseline model of Malawi to data from the Living Standards and Measurement Survey (LSMS) from 2010 and 2013. The appendix contains details on how I measure income and productivity from the micro data. The survey is nationally representative and includes information on both agricultural and non-agricultural production as well as wage labor. Table 3 has the list of parameters that I calibrate along with their values. Below I describe how they are picked in detail.

Table 3: Calibration Parameters

	Expression	Value
Predetedermined Parameters		
Interest rate	R	0.90
Discount Factor	β	0.96
Risk Aversion	σ	2.00
Elasticity of output with respect to labor	α	0.50
Calibrated Parameters		
Constant of non-agricultural productivity process	$\bar{z}$	-0.28
Probability of land allocation when ag worker	π_a	0.23
Persistence of non-agricultural productivity process	ρ_z	0.50
Variance of non-agricultural productivity process	σ_z	0.50

Parameters set exogenously. There are 5 parameters to set exogenously, 4 of which are set to standard values in the literature. I set $\beta = 0.96$ since the model is in yearly frequency and $\sigma = 2$ as in Lagakos et al. (2018) and the parameter on the production technology to $\alpha = 0.5$. Since asset markets are in partial equilibrium as in Lagakos et al. (2018), I also set the interest on assets exogenously. Since my model is set in annual frequency, I set it to $R = 0.90$.

Finally, the last parameter set exogenously is the labor distortion in agriculture. I calibrate this using the LSMS data. I use the ratio of median income from smallholder agriculture to median income from landless farm cultivation. In the appendix I describe in detail how these are calculated while table 9 has the entire distribution of returns to the different occupations in my model. The model allows me to characterize this ratio analytically if also targeting shares in the two agricultural occupations [2]. The LSMS includes information on farm wages and income. The goal is to set τ in order to match the ratio of farm income to farm wages. This captures the differences in agricultural earnings between those with land and those without. In order to compute farm income per day I compute the value added in agriculture and use the number of days spent on agricultural production[3]. I then set τ so that the ratio of smallholder income to farm cultivator income matches the data.

Parameters chosen jointly There are 4 parameters to target. I leave the supply of land L to be consistent with an equilibrium where all of the land is being used and z_a and ℓ_c to be normalized. Therefore, I am left to calibrate the probability of gaining access to communal land π_a, and the parameters associated with the non-agricultural productivity process, i.e. the constant term $\bar{z}$, the variance σ_z, and the persistence ρ_z. In order to pick these parameters, I match them to moments from the LSMS. These targeted moments include the share of the population in smallholder agriculture ($\bar{z}$), the share of the population working as a landless farm cultivator (π_a), the share of workers in agriculture in 2010 transitioning to non-agriculture in 2013 (ρ_z) and the variance of log non-agricultural income per day in 2010 (σ_z). Below I describe in turn how I calibrate each one of these parameters.

The first two parameters are chosen to pin down the share of households in the landless cultivation sector ($N_{W,a}$) and in the smallholder sector ($N_{V,a}$). Without any heterogeneity in the agricultural sector, if I match these two moments, then there is an analytical solution to the wage in agriculture. This simplifies the calibration in the sense that I don't have to find a market clearing wage, it's analytically given by these targets.

[2] In the appendix, I derive this expression

[3] Details on the construction of these measures are included in the appendix

Table 4: Data and Model Moments

Targeted Moments	Model	Data
Share of landed farming (2010)	0.83	0.83
Share of ag labor (2010)	0.06	0.06
Share of ag in 2010 in non-ag in 2013	0.043	0.04
Variance of non-ag income in 2010	0.04	1.5
Ratio of earnings from farm to ag work	3.00	5.60

Let, $n_a(w_a)$ denote the individual demand choice of labor by smallholder farmers, then the equilibrium conditions imply:

$$n_a(w_a)\,(N_{V,a}) = N_{W,a} + N_{V,a} \Rightarrow w_a = \frac{z_a(1-\alpha)}{1+\tau}\left(\frac{N_{W,a}+N_{V,a}}{N_{V,a}}\right)^{-\frac{1}{\alpha}} \tag{5}$$

So that each iteration requires finding individual policies, the stationary distribution, and updating parameters accordingly.

Next I need a measure in the data the variation in non-agricultural income. However, the data includes variation due to household size, age of manager, education, among other factors not modeled here. In order to account for this, I follow Kaboski and Townsend (2011) and others and purge the data of these factors with the regressions

$$\log(y_{i,t}) = \alpha + \beta X_{i,t} + (1+g)t + \varepsilon_{i,t} \tag{6}$$

where g is a time trend and $X_{i,t}$ is a set of controls for household i in year t that include, number of children, education, age, and gender of household head, and previous occupation. The R^2 in this regression is 0.19 so this accounts for about a fifth of the variation in non-agricultural income. I match the standard deviation of the error term $\varepsilon_{i,t}$ to the variation in active non-agricultural income in the stationary distribution.

Finally I need a measure of occupation transition between 2010 and 2013 in the LSMS. In order to measure this, I compute the share of rural households in 2010 that end up in urban areas in 2013. I match this share to an anlogous transition share in my model. In the model rural workers are either farm cultivators or smallholders. Therefore, the appropriate measure to target is the share of workers that are either a farm cultivator or a smallholder who end up in non-agriculture three years later—which can be calculated from the simulated panel. Table 4 summarizes the results of matching these moments.

7 Quantitative Analysis

Because the main goal is to quantify the insurance and redistribution value of communal land, I first need to construct a counterfactual economy where land is privatized. This is analogous to allocating labor efficiently without fixing the incomplete insurance friction. This economy is described in the appendix. I then proceed in three steps: I first compute the stationary equilibrium with the calibrated parameters. I then compute the counterfactual economy, which is analogous to giving households on communal land permanent ownership rights and allowing them to rent out. And finally I use methods developed by Benabou (2002) and Floden (2001) in order to decompose the welfare impacts of privatizing communal land into gains from efficiency, insurance and redistribution [4].

Properties of the private economy. If communal land is privatized, owners of land can earn rent on their land and supply their labor to where the return is highest. As a result, the occupation choice under a private economy is independent of land ownership or asset holdings. Rather, there is a common cutoff z^p such that households with $z < z^p$ go into agriculture while those with $z > z^p$ go into non-agriculture. Those in agriculture with land earn return on land $q\ell_c$ irrespective of their occupation choice. Anyone who chooses agriculture also earns w_a and those choosing non-agriculture earn z

Aggregate impacts of communal land. Privatizing communal land shifts employment significantly to the non-agricultural sector and increases aggregate efficiency. The share of workers in agriculture falls from 89% to 44% while the share of households in the non-farm sector rises from 11% to 56%. Now we can turn to welfare using a utilitarian social welfare function. We can decompose the welfare changes from gains through efficiency, insurance and redistribution[5]. The total welfare gains from privatizing land is 15%. However, there are losses through insurance and redistribution. About a quarter of the welfare gains through efficiency are erased due to losses through higher inequality and consumption volatility.

Table 5: Aggregate Moments

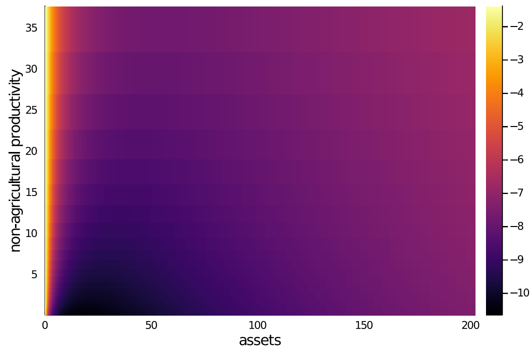
Economy	N_n	N_a	w_a	land income
Under communal land	11%	89%	0.49	1.0
Privatized communal land	56%	44%	0.70	0.36

[4] In the appendix I describe these methods in detail

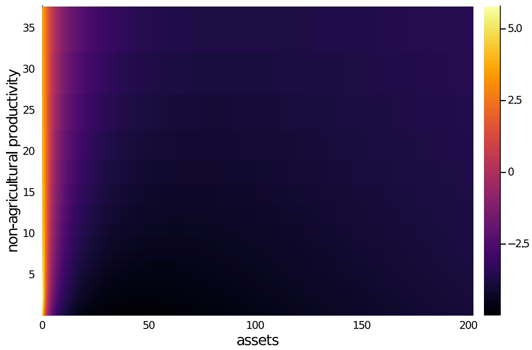
[5] Details on this decomposition are delineated in the appendix

Distributional impacts of communal land. The distributional impacts of communal land can be seen through figures 4a and 4b. In these figures, I plot the utilitarian gains from privatizing communal land along both the asset and the productivity distribution. The utilitarian losses are largest to low ability landless households who no longer have the ability to access communal land. However, high asset landowners also loose as their return on land has diminished significantly and they are exposed to more risk than before. Low asset landless households have the smallest losses while low asset landowners have the largest gains.

Figure 4: Utilitarian Gains from Privatization



(a) Landless



(b) Landowners

8 Conclusion

In this paper I study the implications of risk and incomplete insurance for privatizing communal land systems in developing countries. Since these communal systems also redistribute resources from the rich to the poor and provide means for households to shield themselves from risk, removing this function and allowing markets to allocate land also carries costs when insurance markets are incomplete. I find that privatizing land increases efficiency by allocating labor across sectors more efficiently. It increases average lifetime consumption by 20%. This however comes with a 5% welfare cost through lower insurance and higher inequality. The literature on land systems that misallocate resources has so far abstracted from this tradeoff between efficiency and social insurance that communal land provides in developing countries. This suggests that part of the difficulty in converting communal land systems to a private ownership model is the inability of formal governments in developing countries to provide the social insurance that is fulfilled through communal land. My results suggest that a policy that combines privatization with insurance and redistribution may be more amenable to households living under this context.

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A Counterfactual Private Economy

This section describes the counterfactual economy that I use in order to quantify the insurance and redistribution value of communal land. This economy is analogous to having the share of the population in the stationary equilibrium on communal land with permanent rights on communal land. All functional forms are the same the bellman equations have the same interpretation, however the binary land state is now permanent.
Individual policies Therefore the occupation choices can be written as:

$$V(z,b) = \max\{V^a(z,b), V^n(z,b)\}$$
$$W(z,b) = \max\{W^a(z,b), W^n(z,b)\}$$

where these value functions for each activity $k \in \{a, n\}$ can be written as follows:

$$V^k(b,z) = \max_{c,b'} \quad u(c) + \beta \mathbb{E}_{z'|z}\left[V(b',z')\right]$$
$$c + b' = y_k(z) + q\ell_c + Rb, \quad b' \geq -\phi$$

$$W^k(b,z) = \max_{c,b'} \quad u(c) + \beta \mathbb{E}_{z'|z}\left[W(b',z')\right]$$
$$c + b' = y_k(z) + Rb, \quad b' \geq -\phi$$

where $y_k(z) = z$ if $s = n$ and $y_k(z) = w_a$ if $s = a$. Further, q is the rental rate on land.
Equilibrium. A stationary equilibrium in this economy is an agricultural wage (w_a), a rental rate on land (q), and value functions $V(b,z), W(b,z)$ such that:

- Households solve their individual problems
- Agricultural labor and land chosen optimally
- Agricultural labor markets clear
- Agricultural land markets clear

A.1 Solving for the land rental rate

Under the private economy, the representative agricultural producer solves

$$\max_{\ell, n_a} \; z_a\left(\alpha \ell^\gamma + (1-\alpha) n_a^\gamma\right)^{\frac{1}{\gamma}} - w_a n_a - q\ell \Rightarrow$$
$$z_a\left(\alpha \ell^\gamma + (1-\alpha) n_a^\gamma\right)^{\frac{1-\gamma}{\gamma}} (1-\alpha) n_a^{\gamma-1} = w_a$$
$$z_a\left(\alpha \ell^\gamma + (1-\alpha) n_a^\gamma\right)^{\frac{1-\gamma}{\gamma}} \alpha \ell^{\gamma-1} = q$$

Since demand for land is equal to the supply and the supply is fixed, we can then solve for labor demand.

$$N_a = L\alpha^{1/\gamma}\left(\left(\frac{w_a}{z_a(1-\alpha)}\right)^{\frac{\gamma}{1-\gamma}} - (1-\alpha)\right)^{-1/\gamma} \tag{7}$$

And we can write the ratio of the FOCs as follows

$$\frac{w_a}{q} = \frac{1-\alpha}{\alpha}\left(\frac{N_a}{L}\right)^{\gamma-1} \tag{8}$$

Finally combine equations 7 and 8 and write q as a function of parameters and w_a

$$q = w_a \frac{\alpha^{\frac{1}{\gamma}}}{1-\alpha}\left(\left(\frac{w_a}{z_a(1-\alpha)}\right)^{\frac{\gamma}{1-\gamma}} - (1-\alpha)\right)^{\frac{\gamma-1}{\gamma}}$$

B Welfare Decomposition

In this section I describe in detail the decomposition as described in Floden (2001). It's a slightly simplified version of what is in his paper as I don't have leisure in my economy. The goal is to devise a method that allows me to compare the welfare properties of the economy under communal land and one in which communal land is privatized. Undoubtedly, this change in policy will affect levels of consumption, distribution of income and uncertainty households face. We want a method that can capture the relative effects separately. We can start by defining lifetime utility as

Definition 1 *Lifetime utility V can be written as:*

$$V(\{c_s\}_{s=1}^{\infty}) = \sum_{s=t}^{\infty} \beta^{s-t} u(c_s) \tag{9}$$

Definition 2 *Certainty-equivalent consumption bundle* $\{\bar{c}\}$ *fulfills*

$$V(\{\bar{c}_s\}_{s=1}^{\infty}) = E_t V(\{c_s\}_{s=t}^{\infty}) \tag{10}$$

Now let C be the average consumption, $C = \int c d\lambda$ and let $\bar{C}$ denote the certainty equivalent consumption $C = \int \bar{c} d\lambda$. We are now ready to describe the utilitarian social welfare, U

Definition 3 *The utilitarian social welfare function is defined as:*

$$U = \int E_t V(\{c_s\}_{s=t}^{\infty}) d\lambda \tag{11}$$

And finally we are ready to define the welfare gain of a particular policy compared to a baseline.

Definition 4 *The utilitarian welfare gain of a policy change,* ω_U *is defined by*

$$\int E_t V(\{(1+\omega_U)c_s\}_{s=t}^{\infty}) d\lambda^A = \int E_t V(\{c_s\}_{s=t}^{\infty}) d\lambda^B \tag{12}$$

We can think of ω_U as the percent of lifetime consumption agents in economy A are prepared to give up to get the policy change. This is the classic measure of consumption equivalence from Lucas's calculations of the costs of business cycles. We can compute this for different groups. For example, I calculate this for different quintiles of the non-ag quintile distribution and of the income distribution. All one would have to do is adjust measures λ^A and λ^B so that it captures the designated groups. Now we are ready to get measures required to isolate effects through insurance versus redistribution. The idea is to first get the certainty equivalent consumption for each individual, the inequality is measured by the distribution of certainty equivalent consumption while uncertainty is measured by comparing differences in actual and certainty-equivalent consumption.

Definition 5 *The cost of uncertainty,* p_{unc} *is defined as:*

$$V(\{(1+p_{unc})C\}_{s=t}^{\infty}) = V(\{\bar{C}\}_{s=t}^{\infty}) \tag{13}$$

The cost of uncertainty can be intuitively thought of as the percent of average consumption agent is willing to give up in order to just get the certainty equivalent average every period (holding inequality fixed).

Definition 6 *The cost of inequality, p_{ine} is defined as:*

$$V(\{(1+p_{ine})\bar{C}\}_{s=t}^{\infty}) = \int V(\{\bar{c}\}_{s=t}^{\infty})d\lambda \tag{14}$$

Now we are ready to define the welfare gains from these various channels.

Definition 7 *The welfare gains from increased levels ω_{lev}:*

$$\omega_{lev} = \frac{C^B}{C^A} - 1 \tag{15}$$

Definition 8 *The welfare gains from lower uncertainty ω_{unc}:*

$$\omega_{unc} = \frac{1-p_{unc}^B}{1-p_{unc}^A} - 1 \tag{16}$$

Definition 9 *The welfare gains from lower inequality ω_{ine}:*

$$\omega_{ine} = \frac{1-p_{ine}^B}{1-p_{ine}^A} - 1 \tag{17}$$

This implies the total welfare gain ω_U approximately approaches:

$$\omega_U = (1+\omega_{lev})(1+\omega_{unc})(1+\omega_{ine}) - 1 \tag{18}$$

C Data Construction

In order to use the data to inform the parameters of my model I need to compute measures from the data with clear analogues in the model. In this section I describe these measures and how I compute them. First, I need measures of income for the three occupations in the model, i.e. agricultural income of landed farmers, income of landless agricultural cultivators, and non-agricultural income.

Agricultural Income of Smallholders For agricultural income, I only compute earnings from non-permanent crops. Fisheries and livestock income will be capital income from fishing equipment and livestock capital. I consider only income from the rain season as in Restuccia and Santaeulalia-Llopis (2017) where Value added in agriculture can be written as:

$$VA_{a,i} = Rev_{c,i} + P_{c,i}(Output_{c,i}^z - Sold_{c,i}) - Cost_{c,i} \tag{19}$$

and represents value added from product c for household i. $Rev_{c,i}$ represents household i's revenues from selling crop c, $Output^z_{c,i} - Sold_{c,i}$ represents the fraction of production of crop c that household i keeps for its own consumption and $P_{c,i,r}$ is the price received by household i for crop c (sale value of own consumption), which is replaced by the regional price when such price cannot be inferred for household i (households that report production but no sales). In order to compute this price, I proceed as follows:

1. If household i sold crop c, I use reported sales $Rev_{c,i}$ and quantity sold $Q_{c,i}$ and compute $P_{c,i,r} = Rev_{c,i}/Q_{c,i}$
2. Otherwise I attribute the median price of the crop sold by other households in the same region if available, meaning $P_{c,i} = \bar{P}_{c,j}$ where j lives in the same region as i

Finally, for the production of each crop, the household reports costs associated with various inputs and factors. I aggregate costs across inputs

$$Cost_{c,i} = \sum_v Cost^v_{c,i} \tag{20}$$

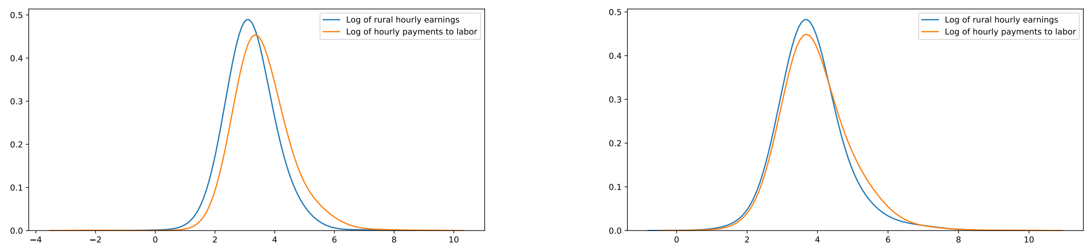
where $v \in \{intermediates, labor, capital, land, transportation\}$ represents the costs associated with production. After computing $VA_{c,i}$ for each crop and household, define household farm earnings as the sum of value added across crops

$$VA_i = \sum_c VA_{c,i} \tag{21}$$

which represents the income of farmers that get their land in Malawi through the customary tenure system.

Farm Cultivator Income. There are two ways to compute wages paid to agricultural labor. In the time use side, the only labor income that is tied to agriculture are wages paid in the formal sector. The farms paying these salaries are large and export oriented and highly regulated by the government. These workers are not the ones supplying labor to smallholder farmers who get land through customary tenure. There is another category in the time use portion of the survey that asks for the wage received for informal labor in rural areas. This is the first candidate for w_a. Another way of computing this is by looking at the production side and measuring labor payments made by landed farmers. These can be seen in figure 5 for each one of the years in our panel data. In short this tells us that in 2010, rural wages were reportedly higher than producers reported paying to agricultural labor. This is not surprising as perhaps that year, there were more non-agricultural activities in rural areas that households earned income from - and those paid more than agriculture. In 2013 however, they followed closely one another.

Figure 5: Earnings in Rural Areas Versus Labor Payments in Agriculture



(a) LSMS 2010 (b) LSMS 2013

Non-agricultural Income. There are two types of non-agricultural income. Some households run non-agricultural businesses and report in the survey the revenues and costs over the year. Therefore I can write household non-agricultural business income as:

$$VA_{b,i} = Rev_{b,i} - Cost_{b,i} \tag{22}$$

In order to have a comparable measure against other income sources, I compute the non-agricultural business income per hour of household member spent on working at the business. Further, there is also a set of formal workers who earn non-agricultural wages. These are professional workers with education who work in the formal and government sectors of the economy. I also measure their hourly wage.

Summary. In order to assign an occupation to households in our data, I assign them to the occupation (landed agriculture, agricultural labor, non-agriculture) where the household spends most of their time in. After doing this, I can characterize the distribution of income across the three occupations in our economy as seen in figure 6.

Figure 6: Earnings by Occupation

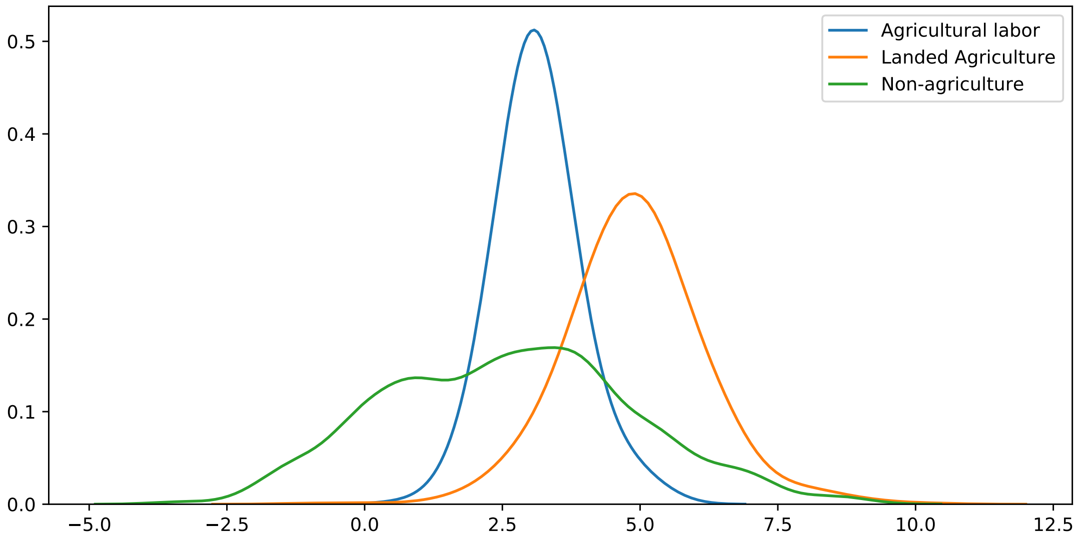


Table 6: Income (Thousands of LCU)

Survey/year	Household Income (per capita)					GDP (per capita)
	25th	50th	75th	90th	mean	
IHS3 (2010)	20.2	47.8	114.2	262.6	150.7	126.9
IHS4 (2013)	14.7	32.6	69.3	152.7	85.4	72.0

Table 7: Occupation Transitions - 2010 to 2013

	Farm cultivator	Smallholders	Non-agriculture
Farm cultivator	65	35	0
Smallholders	6	91	3
Non-agriculture	0	27	72

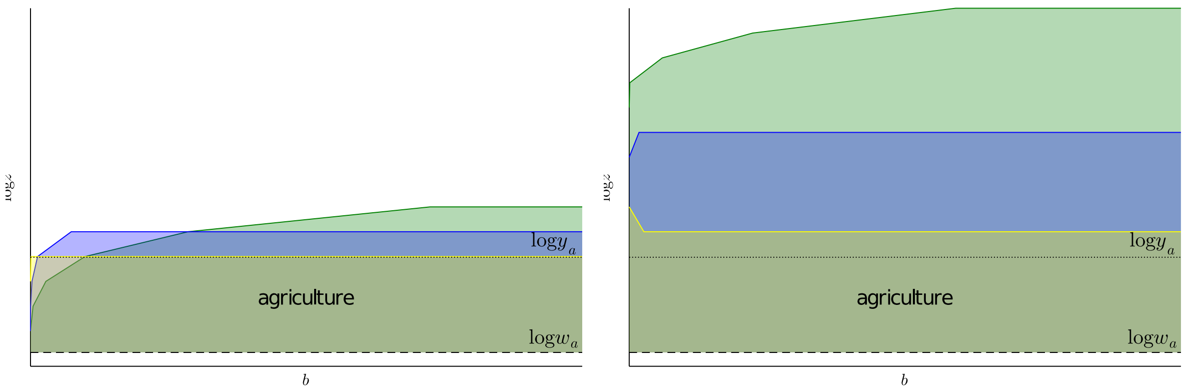
Table 8: Residence Transitions - 2010 to 2013

	Urban	Rural	Observations
Urban (%)	90	10	761
Rural (%)	*4*	96	2,343

Table 9: Hourly Earnings in 2010

Percentile	Landed farmer	Landless ag worker	Non-ag entrepreneur (worker)
25%	52	16	3 (32)
50%	*124*	*22*	21 (81)
75%	270	33	115 (192)
90%	675	55	453 (458)
95%	1200	88	960 (867)
99%	7200	166	8300 (3100)

Figure 7: Occupation policies



(a) Landless Households

(b) Landholders

Table 10: Statistics for All Households

Statistic	Consumption		
	$\pi_a = 0.03$	$\pi_a = 0.15$	$\pi_a = 0.6$
Q10	0.65	0.78	0.80
Q25	0.80	0.80	0.80
Q50	0.80	0.80	0.95
Q75	0.80	0.90	1.24
Q90	0.93	1.22	1.54
Q95	1.16	1.42	1.73
Q99	1.61	1.83	2.17
Mean	0.81	0.88	1.06
Mode	0.80	0.80	0.80
Variance	0.04	0.07	0.12
Skewness	1.74	1.75	1.42
Kurtosis	11.41	6.54	3.09

D Calibrating α

When there is no heterogeneity in agriculture, aggregation of individual decisions in agriculture lead to simple expressions of wage and profits in terms of employment shares.

Table 11: Statistics for All Non-ag Households (in logs)

Statistic	Consumption		
	$\pi_a = 0.03$	$\pi_a = 0.15$	$\pi_a = 0.6$
Q10	0.65	0.77	0.80
Q25	0.80	0.80	0.80
Q50	0.80	0.80	0.94
Q75	0.80	0.90	1.23
Q90	0.94	1.22	1.52
Q95	1.15	1.42	1.72
Q99	1.58	1.81	2.12
Mean	0.81	0.88	1.05
Mode	0.80	0.80	0.80
Variance	0.04	0.07	0.11
Skewness	1.69	1.76	1.40
Kurtosis	11.88	6.83	2.89

Table 12: Statistics for All Households (in logs)

Statistic	Log Consumption		
	$\pi_a = 0.03$	$\pi_a = 0.15$	$\pi_a = 0.6$
Q10	-0.43	-0.25	-0.22
Q25	-0.22	-0.22	-0.22
Q50	-0.22	-0.22	-0.05
Q75	-0.22	-0.10	0.22
Q90	-0.07	0.20	0.43
Q95	0.15	0.35	0.55
Q99	0.48	0.60	0.78
Mean	-0.24	-0.16	0.01
Mode	-0.22	-0.22	-0.22
Variance	0.07	0.08	0.09
Skewness	-1.82	-0.79	-0.01
Kurtosis	10.72	6.76	3.00

Consider the production technology of the farmer as in the main body of the paper. The goal here is to find α such that the ratio of smallholder farm income to the agricultural wages match observed patterns in the data. If we have this ratio and targets for the employment shares in agricultural cultivation and smallholders, then this parameter can be characterized analytically. First, the smallholder problem solves

$$\max \quad z_a \left(\alpha \ell_c^\gamma + (1-\alpha) n_a^\gamma\right)^{\frac{1}{\gamma}} - w_a(n_a - 1) \Rightarrow$$

$$n_a = \ell_c \alpha^{1/\gamma} \left(\left(\frac{w_a}{z_a(1-\alpha)} \right)^{\frac{\gamma}{1-\gamma}} - (1-\alpha) \right)^{-1/\gamma}$$

Since all the farmers in the baseline model are the same, we can aggregate these labor demands and use the market clearing condition. If we have a target for the number of farmers N_a^V and the number of landless workers N_a^W in agriculture, then we can write

$$N_a^V + N_a^W = N_a^V \ell_c \alpha^{1/\gamma} \left(\left(\frac{w_a}{z_a(1-\alpha)} \right)^{\frac{\gamma}{1-\gamma}} - (1-\alpha) \right)^{-1/\gamma} \Rightarrow$$

$$w_a = z_a(1-\alpha) \left(\left(\frac{N_a^V \ell_c}{N_a^V + N_a^W} \right)^{\gamma} \alpha + (1-\alpha) \right)^{\frac{1-\gamma}{\gamma}}$$

Our goal is to find an expression for farm income relative to landless agricultural income. Hence we can plug in the labor choice into the profit equation to get:

$$\pi = z_a \left(\alpha \ell_c^{\gamma} + (1-\alpha) \ell_c^{\gamma} \alpha \left(\left(\frac{w_a}{z_a(1-\alpha)} \right)^{\frac{\gamma}{1-\gamma}} - (1-\alpha) \right)^{-1} \right)^{\frac{1}{\gamma}}$$
$$-w_a \left(\ell_c \alpha^{1/\gamma} \left(\left(\frac{w_a}{z_a(1-\alpha)} \right)^{\frac{\gamma}{1-\gamma}} - (1-\alpha) \right)^{-1/\gamma} - 1 \right)$$

Note that the follwing expression can be simplified after substituting in the wage

$$\left(\frac{w_a}{z_a(1-\alpha)} \right)^{\frac{\gamma}{1-\gamma}} - (1-\alpha) = \left(\frac{N_a^V \ell_c}{N_a^V + N_a^W} \right)^{\gamma} \alpha + (1-\alpha) - (1-\alpha) = \alpha \left(\frac{N_a^V \ell_c}{N_a^V + N_a^W} \right)^{\gamma}$$

Therefore we can rewrite profits as:

$$\pi = z_a \left(\alpha \ell_c^{\gamma} + (1-\alpha) \left(\frac{N_a^V + N_a^W}{N_a^V} \right)^{\gamma} \right)^{\frac{1}{\gamma}} - w_a \left(\frac{N_a^V + N_a^W}{N_a^V} - 1 \right)$$

Note that we can rearrange the wage so that it can written as:

$$w_a = z_a(1-\alpha) \left(\frac{N_a^V + N_a^W}{N_a^V} \right)^{\gamma-1} \left(\alpha \ell_c^{\gamma} + (1-\alpha) \left(\frac{N_a^V + N_a^W}{N_a^V} \right)^{\gamma} \right)^{\frac{1-\gamma}{\gamma}}$$

This implies we can write the profit wage ratio as:

$$\frac{\pi}{w_a} = \left(\alpha \ell_c^{\gamma} + (1-\alpha) \left(\frac{N_a^V + N_a^W}{N_a^V} \right)^{\gamma} \right) \left(\frac{N_a^V + N_a^W}{N_a^V} \right)^{1-\gamma} \frac{1}{(1-\alpha)} - \left(\frac{N_a^V + N_a^W}{N_a^V} - 1 \right)$$

Finally we can solve for α as a function of the ratio of profits to wages, parameters and the employment share targets

$$\alpha = \frac{\left(\left(\frac{N_a^V}{N_a^V+N_a^W}\right)^{1-\gamma}\left(\frac{\pi}{w_a}-1\right)\frac{1}{\ell_c^\gamma}\right)}{1+\left(\left(\frac{N_a^V}{N_a^V+N_a^W}\right)^{1-\gamma}\left(\frac{\pi}{w_a}-1\right)\frac{1}{\ell_c^\gamma}\right)}$$

E Computational details

Solving models with continuous and discrete choices presents several challenges and this section details the steps I took to overcome such challenges. In order to deal with non-linearities in borrowing constraints and occupational choices as well as a large state space, I used an adaptive sparse grid method as delineated in Brumm and Scheidegger (2017). In that paper, their adaptive grid is limited to collocation with linear basis functions. I combine their adaptive grid procedure with a finite element method as in McGrattan (1996). This combination allows me to apply their adaptive grid procedure to higher order basis functions. Higher order basis functions are needed to accurately solve for policies, especially around points that are highly nonlinear. Finally, this allows me to solve for the coefficients on the value function using the Newton-Raphson algorithm which far supersedes conventional value function iteration methods in terms of both speed and accuracy. I also use extreme value shocks in order smooth out the discrete choices in the model. Below, I explain each step in detail.

E.1 Extreme value shocks

Nondiffereriabilities in the value function due to discrete choices makes calibration very challenging. I add iid extreme value shocks to the discrete choices in order to smooth out this choice and allow the use of derivatives in the calibration. Once I calibrate, I bring the role of these shocks down until it plays no role. This section describes in detail the implementation of these shocks into the finite element framework I use.

Let $\varepsilon_i^W, \varepsilon_i^V$ for $i \in \{a, n\}$ be extreme value preference shocks with common shape parameter σ_ε We can rewrite the individual choices as:

$$W(b,z,\varepsilon^W) = \max\left\{\pi_a V(b,z|a) + (1-\pi_a)W(b,z|a) + \varepsilon_a^W, W(b,z|n) + \varepsilon_n^W\right\} \tag{23}$$
$$V(b,z,\varepsilon^V) = \max\left\{V^a(b,z|a) + \varepsilon_a^V, W^n(b,z|n) + \varepsilon_n^V\right\} \tag{24}$$

Then the occupation dependent bellman equations can be written as follows:

$$V(b,z|a) = \max_{b'\in\mathcal{B}} u(c,z,b,b',c) + \beta\mathbb{E}_{z'|z}\left[\mathbb{E}_{\varepsilon^V}\left[V(b',z')\right]\right] \tag{25}$$
$$V(b,z|n) = \max_{b'\in\mathcal{B}} u(c,z,b,b',p) + \beta\mathbb{E}_{z'|z}\left[\mathbb{E}_{\varepsilon^W}\left[W(b',z')\right]\right] \tag{26}$$
$$W(b,z|a) = \pi_a\left(\max_{b'\in\mathcal{B}} u(c,z,b,b',c) + \beta\mathbb{E}_{z'|z}\left[\mathbb{E}_{\varepsilon^V}\left[V(b',z')\right]\right]\right) + \tag{27}$$
$$(1-\pi_a)\left(\max_{b'\in\mathcal{B}} u(c,z,b,b',p) + \beta\mathbb{E}_{z'|z}\left[\mathbb{E}_{\varepsilon^W}\left[W(b',z')\right]\right]\right) \tag{28}$$
$$W(b,z|n) = \max_{b'\in\mathcal{B}} u(c,z,b,b',p) + \beta\mathbb{E}_{z'|z}\left[\mathbb{E}_{\varepsilon^W}\left[W(b',z')\right]\right] \tag{29}$$

Instead of approximating the occupation specific value function, I approximate the expected value with respect to extreme value shocks. Once I have an approximation of this, I can back out the occupation specific value functions, and consequently other policies. Hence, define $\mathbb{E}_a\left[W(b,z|a)\right] = \pi_a V^a(b,z) + (1-\pi_a)W^a(b,z)$ I can write the following residual equations

$$R^W(b,z) = \mathbb{E}_{\varepsilon^W}\left[W(b,z)\right] - \left(P_a^W\mathbb{E}_a\left[W(b,z|a)\right] + P_n^W W(b,z|n)\right) \tag{30}$$
$$R^V(b,z) = \mathbb{E}_{\varepsilon^V}\left[V(b,z)\right] - \left(P_a^W V(b,z|a) + P_n^W W(b,z|n)\right) \tag{31}$$

where the probabilities follow the usual expression $P_a^V = \frac{\exp(V(b,z|a)/\sigma_\varepsilon)}{\exp(V(b,z|a)/\sigma_\varepsilon)+\exp(V(b,z|n)/\sigma_\varepsilon)}$ [6]. The goal is to approximate $\hat{W}(b,z) = \mathbb{E}_{\varepsilon^W}\left[W(b,z)\right]$ and $\hat{V}(b,z) = \mathbb{E}_{\varepsilon^V}\left[V(b,z)\right]$. In order to numerically approximate this, I use finite element methods. Now consider solving the model over the domain $\mathcal{B}$. In finite element analysis, I can define non-overlapping intervals over $\mathcal{B}$ as $\{[b_0,b_1],...,[b_i,b_i+1],...[b_{k-1},b_k]\}$. Then define lagrange basis functions of order n - $\psi^n(b,z)$ - over each interval i with associated coefficient vector $\boldsymbol{\theta}$. Then the task is

[6]Numerically, this blows up easily, hence we have to make the following transformation in order to make the algorithm more stable. First let $\bar{V}(a,b) = \max\{V(b,z|a), V(b,z|n)\}$. Then we can multiply P_a^V by $\frac{\exp(\bar{V}(a,b)/\sigma_\varepsilon)}{\exp(\bar{V}(a,b)/\sigma_\varepsilon)}$ and get that $P_a^V = \frac{\exp\left((V(b,z|a)-\bar{V}(a,b))/\sigma_\varepsilon\right)}{\exp\left((V(b,z|a)-\bar{V}(a,b))/\sigma_\varepsilon\right)+\exp\left((V(b,z|n)-\bar{V}(a,b))/\sigma_\varepsilon\right)}$

to find $\boldsymbol{\theta}$ such that

$$\int_{b_i}^{b_{i+1}} \psi_i(b,z)R^J(b,z;\theta)db = 0, \quad i = 0,1,...,k, \quad z \in \mathcal{Z}, \quad J \in \{V,W\} \tag{32}$$

and the approximated value function can be written as:

$$\hat{V}(b,z) = \psi_i(b,z)\theta_i + ... + \psi_{i+n-1}(b,z)\theta_{i+n-1} \quad b \in [b_i, b_{i+1}] \tag{33}$$

So for cubic basis functions, $n = 4$. The ability to evaluate the equations in 32 over each element independently makes finite element analysis very amenable to paralellization. I use shared memory in order to solve each individual problem. This allows me to compute equations 32 for different elements concurrently. In the model with permanent heterogeneity, I use distributed memory in order to solve independent individual problems for different levels of agricultural productivity concurrently.

E.2 Solving for Coefficients

Note that solving for the value function requires solving a root finding problem for equations in 32. This implies that having analytical expressions for the jacobian of 32 [7] is an essential step. Unlike the model without extreme value shocks (when the residual equations are linear in $\boldsymbol{\theta}$ like other residual methods like collocation for instance), with extreme value shocks, it takes a bit more work to derive the jacobian since the residual is now nonlinear in the coefficient vector. This is due to the value function appearing in the choice probabilities. This section goes through this process in detail. Let 30 and 31 be the two residual equations that we'll use in order to approximate W and V. Then, these are solely functions of our coefficients and we need the jacobian with respect to coefficients. First, for the current value, it's simple. For any point $b_i \in \mathcal{B}$, if the grid for communal and landless households coincide, then we can write

$$\frac{\partial V(b_i,z)}{\partial \theta_{b_i}} = \frac{\partial W(b_i,z)}{\partial \theta_{b_i}} = \psi(b_i,z)$$

Now note that the maximand on $V^a(b,z), W^a(b,z)$ and $W^n(b,z)$ are $b'_{W,a}, b'_{V,a}$ and $b'_{W,n}$ respectively. Hence, if the order of approximation is k, there will be $k-1$ non-zero values for the expected bellman for each possible $z' \in \mathcal{Z}$ as well. Hence, at each point

[7] This is the most error prone step in solving for coefficients. In practice, I construct test functions that compute both the numerical and analytical derivatives until they coincide.

$(b,z) \in \mathcal{B} \times \mathcal{Z}$, we have derivatives around optimal decisions in ag and non-ag. This says that in the event that I choose agriculture, there is also some probability that I would have chosen non-agriculture. Either way, if I end up in agriculture, I chose savings as if I was in agriculture.

Finally this implies we are ready to compute all derivatives of the RHS with respect to coefficients. There are two sets of non-zero points in the jacobian for each occupation choice. In agriculture, there is a non-zero point associated with choosing agriculture and another associated with choosing agricultural asset choice. Suppose we have land and want the derivative with respect to unknowns surrounding the agricultural savings choice. Then we can write the derivative as:

$$\frac{\partial RHS_V(b,z)}{\partial \theta_{b',z,z',a}} = P_a^V \beta \pi(z,z') \frac{\partial \hat{V}(b',z')}{\partial \theta_{b',z,z',a}} + \frac{\partial P_a^V}{\partial \theta_{b',z,z',a}} V(b,z|a) + \frac{\partial P_n^V}{\partial \theta_{b',z,z',a}} V(b,z|n) \Rightarrow$$

Since the value function of the landless households is a bit different, we'll also derive them here. Suppose we want the derivatives around the optimal asset choice when the agent gets the land (denoted by f). Let $\tilde{W}^a(b,z) = \pi_a V^a(b,z) + (1-\pi_a) W^a(b,z)$

$$\begin{aligned}
\frac{\partial RHS_W(b,z)}{\partial \theta_{b',z,z',a}} =& P_a^W \pi_a \beta \pi(z,z') \frac{\partial \hat{V}(b',z')}{\partial \theta_{b',z,z',a}} + \frac{\partial P_a^W}{\partial \theta_{b',z,z',a,V}} \pi_a V^a(b,z) + \frac{\partial P_n^V}{\partial \theta_{b',z,z',a,V}} W(b,z|n) \\
\frac{\partial RHS_W(b,z)}{\partial \theta_{b',z,z',a}} =& P_a^W (1-\pi_a) \beta \pi(z,z') \frac{\partial \hat{V}(b',z')}{\partial \theta_{b',z,z',a,W}} + \frac{\partial P_a^W}{\partial \theta_{b',z,z',a,W}} (1-\pi_a) W^a(b,z) + \\
& \frac{\partial P_n^V}{\partial \theta_{b',z,z',a,W}} W(b,z|n)
\end{aligned}$$

and an analogous scheme can be used in order to solve for other derivatives. We can use this to build the weighted residual and analytically solve for the jacobian as described above. This enables us to use a newton in order to solve the individual problem. Now we can move towards getting the partial derivatives from the choice probabilities.

$$
\begin{aligned}
\frac{\partial P_a^V}{\partial \theta_{b',a}} =& \frac{\exp\left(V(b,z|a)/\sigma_\varepsilon\right)}{\exp\left(V(b,z|a)/\sigma_\varepsilon\right) + \exp\left(V(b,z|n)/\sigma_\varepsilon\right)} \frac{\partial V(b,z|a)}{\partial \theta_{b',a}} / \sigma_\varepsilon - \\
& \frac{\exp\left(V(b,z|a)/\sigma_\varepsilon\right) \exp\left(V(b,z|a)/\sigma_\varepsilon\right)}{\left(\exp\left(V(b,z|a)/\sigma_\varepsilon\right) + \exp\left(V(b,z|n)/\sigma_\varepsilon\right)\right)^2} \frac{\partial V(b,z|a)}{\partial \theta_{b',a}} / \sigma_\varepsilon \\
&= \frac{\partial V(b,z|a)}{\partial \theta_{b',a}} \left(P_a^V/\sigma_\varepsilon - (P_a^V)^2/\sigma_\varepsilon\right) \\
\frac{\partial P_n^V}{\partial \theta_{b',a}} =& - \frac{\exp\left(V(b,z|n)/\sigma_\varepsilon\right) \exp\left(V(b,z|a)/\sigma_\varepsilon\right)}{\left(\exp\left(V(b,z|a)/\sigma_\varepsilon\right) + \exp\left(V(b,z|n)/\sigma_\varepsilon\right)\right)^2} \frac{\partial V(b,z|a)}{\partial \theta_{b',a}} / \sigma_\varepsilon \\
&= - \frac{\partial V(b,z|a)}{\partial \theta_{b',a}} P_n^V P_a^V / \sigma_\varepsilon
\end{aligned}
$$

Further we know that for each z, $\frac{\partial V(b,z|a)}{\partial \theta_{b',a}}$ relates to the future bellman through its local impacts from z' and occupation dependent savings. Consequently, we can expand the expressions above as follows

$$
\begin{aligned}
\frac{\partial P_a^V}{\partial \theta_{b',z,z',a}} &= \beta \pi(z,z') \frac{\partial \hat{V}(b',z')}{\partial \theta_{b',z,z',a}} \left(P_a^V/\sigma_\varepsilon - (P_a^V)^2/\sigma_\varepsilon\right) \\
\frac{\partial P_n^V}{\partial \theta_{b'_{V,a,z,z'}}} &= -P_n^V P_a^V \beta \pi(z,z') \frac{\partial \hat{V}(b',z')}{\partial \theta_{b',z,z',a}} / \sigma_\varepsilon
\end{aligned}
$$

This complete the derivation of the residual equations 32 and the jacobian with respect to coefficient vector $\boldsymbol{\theta}$ and allows me to solve the individual problem.