Integer Programming. Exercise Set 2

E1 Show that the Set Covering Problem can be written as the COP

$$\min_{S \subseteq N} \{ \sum_{j \in S} c_j \mid v(S) = v(N) \},$$

where $v(S) = \sum_{i=1}^{m} \min \{ \sum_{j \in S} a_{ij}, 1 \}$. What is the meaning of $v(S)$?

E2 Suppose that an IP or LP has n constraints and we want to ensure that at least one of them must hold. How can we use binary variables to deal with this case?

E3 Suppose now that in our IP or LP, k out of n constraints must hold. Find a way to express this fact by means of binary variables.

E4 A set of n jobs must be carried out on a single machine that can do only one job at a time. Each job j takes p_j hours to complete. Given job weights w_j for $j = 1, \ldots, n$, in what order should the jobs be carried out so as to minimize the weighted sum of their start times? Formulate this scheduling problem as a mixed integer program.

E5 Consider the following formulation of the TSP with n towns where we define $x_{ij} = 1$ if the salesman goes directly from town i to town j and $x_{ij} = 0$ otherwise:

$$\min \sum_{i=0}^{n} \sum_{j=0}^{n} c_{ij} x_{ij}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \text{ for } j = 1, \ldots, n.$$

$$\sum_{j=1}^{n} x_{ij} = 1 \text{ for } i = 1, \ldots, n.$$

$$0 \leqslant x_{ij} \leqslant 1 \quad \text{for all } i, j$$

$$x_{ij} \text{ integer for all } i, j$$

Show that we are justified to substitute the second constraint by

$$\sum_{j=1}^{n} x_{ij} = 1 \text{ for } i = 2, \ldots, n.$$