Homework 12 – Deadline 26/11/2020

Problem 12.1

Compute the expected value and variance of a random variable **exponentially distributed** with parameter λ: $f(t) = \lambda\, e^{-\lambda t},\ t \geq 0$.

Problem 12.2

Show that the random variable equal to the minimum between two independent exponentially distributed random variables $H_1,\ H_2$ with parameters λ_1 and λ_2, is exponentially distributed with parameter $\lambda_1 + \lambda_2$. Generalize the result for the minimum of n independent exponentially distributed random variables: $\min\{H_1,\ H_2,\ \cdots H_n\}$. Hint: compute $P(\min\{H_1,\ H_2\} > t)$.

Problem 12.3

Let $\{H_1,\ H_2,\ \cdots H_n\}$ be independent exponentially distributed random variables with parameters $\lambda_1,\ \lambda_2 \cdots \lambda_n$. Using the result of problem 12.2, show that the probability that $\min\{H_1, H_2, \cdots H_n\} = H_i$, i.e. $H_i,\ i \in \{1, \cdots n\}$ has a value lower than $H_j,\ j \neq i$, is given by:

$$P\{H_i < \min_{j \neq i} H_j\} = \frac{\lambda_i}{\sum_j \lambda_j}$$

Hint: recall that $P\{A < B\} = \int_{x=0}^{\infty} P\{B > x\}\, f_A(x)\, dx$.

Problem 12.4

Assume a *Carrier Sense Multiple Access with Collision Avoidance* (CSMA/CA) MAC protocol. This protocol is similar to Aloha, but nodes listen before transmitting (CSMA). If the medium is idle the packet is transmitted, otherwise the node enters in backlogged state, and CSMA is tried again after a backoff time (CA). Assume that:

- Initially nodes are in thinking state, and enter this state after a packet transmission.
- Nodes in thinking state run CSMA in a time exponentially distributed with rate λ.
- Transmission time is exponentially distributed with rate μ.
- Nodes in backlogged state run CSMA in a backoff time exponentially distributed with rate α.
- The propagation delay is 0. Thus, transmissions are always successful (there are no collisions).

12.4.A Build the state transition diagram and the rate matrix if there are 2 nodes.

12.4.B Build the state transition diagram if there are N nodes. Hint: consider a 2-dimensional Markov chain with states (i, j), where i is the number of nodes transmitting a packet, and j is the number of nodes in backlogged state.