Integer Programming. Exercise Set 1

E1 Show that

$$\begin{aligned} X &= \{x \in B^4 \mid 97x_1 + 32x_2 + 25x_3 + 20x_4 \leqslant 139 \} \\ &= \{x \in B^4 \mid 2x_1 + x_2 + x_3 + x_4 \leqslant 3\} \\ &= \{x \in B^4 \mid x_1 + x_2 + x_3 \leqslant 2 \\ &\quad \wedge\, x_1 + x_2 + x_4 \leqslant 2 \\ &\quad \wedge\, x_1 + x_3 + x_4 \leqslant 2\} \end{aligned}$$

E2 Show that any instance of SAT can be transformed in polynomial time into an equivalent instance in which all clauses have more than one literal.

E3 Show that any instance of SAT can be transformed in polynomial time into an equivalent instance in which all clauses have at most three literals.

E4 Suppose that you are interested in choosing a set of investments $\{1, \ldots, 7\}$ using 0-1 variables. Model the following constraints:

1. You cannot invest in all of them.
2. You must choose at least one of them.
3. Investment 1 cannot be chosen if investment 3 is chosen.
4. Investment 4 can be chosen only if investment 2 is also chosen.
5. You must choose either both investments 1 and 5 or neither.
6. You must choose either at least one of the investments 1,2,3, or at least two investmnets from 2,4,5,6.

E5 Formulate the n-Queens Problem as a BIP: Place n queens on an $n \times n$ chessboard such that no two queens share any row, column, or diagonal.