# Algorithmic Game Theory
# Homework 2 - Solutions

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## 1 Problem 10

| | | P2 | |
|---|---|---|---|
| | | A | B |
| P1 | A | 6,6 | 2,7 |
| | B | 7,2 | 0,0 |

Given $\sigma^* = \{(x, 1-x), (y, 1-y)\}$

For $P1$ we know that

- If $x \neq 0 \wedge x \neq 1 \implies \text{supp}(\sigma_{p1}) = \{A, B\}$.
  Therefore $\mu_1((1,0),(y,1-y)) = \mu_1((0,1),(y,1-y))$
- If $x \neq 0 \wedge x \neq 1 \implies \text{supp}(\sigma_{p1}) = \{A, B\}$.
  Therefore $\mu_1((1,0),(y,1-y)) = \mu_1((0,1),(y,1-y))$
- If $x = 1 \implies \text{supp}(\sigma_{p1}) = \{A\}$.
  Therefore $\mu_1((1,0),(y,1-y)) > \mu_1((0,1),(y,1-y))$
- If $x = 0 \implies \text{supp}(\sigma_{p1}) = \{B\}$.
  Therefore $\mu_1((1,0),(y,1-y)) < \mu_1((0,1),(y,1-y))$

And for $P2$ we know that

- If $y \neq 0 \wedge y \neq 1 \implies \text{supp}(\sigma_{p2}) = \{A, B\}$.
  Therefore $\mu_2((x,1-x),(1,0)) = \mu_2((x,1-x),(0,1))$
- If $y \neq 0 \wedge y \neq 1 \implies \text{supp}(\sigma_{p2}) = \{A, B\}$.
  Therefore $\mu_2((x,1-x),(1,0)) = \mu_2((x,1-x),(0,1))$

- If $y = 1 \implies \text{supp}(\sigma_{p2}) = \{A\}$.
  Therefore $\mu_2((x, 1-x), (1,0)) > \mu_2((x, 1-x), (0,1))$
- If $y = 0 \implies \text{supp}(\sigma_{p2}) = \{B\}$.
  Therefore $\mu_2((x, 1-x), (1,0)) < \mu_2((x, 1-x), (0,1))$

Lets analyze case by case:

1. $P1 = A, P2 = A$

$$\mu_1((1,0),(1,0)) > \mu_1((0,1),(1,0)) \tag{1a}$$
$$6 > 7 \tag{1b}$$

   Therefore, this **IS NOT** $NE$

2. $P1 = A, P2 = B$

$$\mu_1((1,0),(0,1)) > \mu_1((0,1),(0,1)) \tag{2a}$$
$$2 > 0 \tag{2b}$$

   Therefore, this **IS** $NE$

3. $P1 = A, P2 = \{A, B\}$

$$\mu_2((1,0),(1,0)) = \mu_2((1,0),(0,1)) \tag{3a}$$
$$6 \neq 7 \tag{3b}$$

   Therefore, this **IS NOT MIXED** $NE$

4. $P1 = B, P2 = A$

$$\mu_1((0,1),(1,0)) > \mu_1((1,0),(1,0)) \tag{4a}$$
$$7 > 6 \tag{4b}$$

   Therefore, this **IS** $NE$

5. $P1 = B, P2 = B$

$$\mu_1((0,1),(0,1)) > \mu_1((1,0),(0,1)) \tag{5a}$$
$$0 > 2 \tag{5b}$$

   Therefore, this **IS NOT** $NE$

6. $P1 = B, P2 = \{A, B\}$

$$\mu_2((0,1),(1,0)) = \mu_2((0,1),(0,1)) \tag{6a}$$
$$2 \neq 0 \tag{6b}$$

Therefore, this **IS NOT MIXED** $NE$

7. $P1 = \{A, B\}, P2 = A$

$$\mu_1((1,0),(1,0)) = \mu_1((0,1),(1,0)) \tag{7a}$$
$$6 \neq 7 \tag{7b}$$

Therefore, this **IS NOT MIXED** $NE$

8. $P1 = \{A, B\}, P2 = B$

$$\mu_1((1,0),(0,1)) = \mu_1((0,1),(0,1)) \tag{8a}$$
$$2 \neq 0 \tag{8b}$$

Therefore, this **IS NOT MIXED** $NE$

9. $P1 = \{A, B\}, P2 = \{A, B\}$

When $\mu_1((1,0),(y,1-y)) = \mu_1((0,1),(y,1-y))$

$$6y + 2(1-y) = 7y + 0(1-y) \tag{9a}$$
$$4y + 2 = 7y \tag{9b}$$
$$y = \frac{2}{3} \tag{9c}$$
$$1 - y = \frac{1}{3} \tag{9d}$$

Therefore, this **IS MIXED** $NE$

When $\mu_2((x,1-x),(1,0)) = \mu_2((x,1-x),(0,1))$

$$6x + 2(1-x) = 7x + 0(1-x) \tag{10a}$$
$$x = \frac{2}{3} \tag{10b}$$
$$1 - x = \frac{1}{3} \tag{10c}$$

Therefore, this **IS MIXED** $NE$

Therefore the following are the $NE$ and MIXED $NE$ found:

$NE = \{((0,1),(1,0)),((1,0),(0,1))\}$

MIXED NE $= \{((\frac{2}{3},\frac{1}{3}),(\frac{2}{3},\frac{1}{3}))\}$

# 2 Problem 12

| | | P2 | |
|---|---|---|---|
| | | A | B |
| P1 | C | 1,1 | 4,2 |
| | D | 3,3 | 1,1 |
| | E | 2,2 | 2,3 |

Given $\sigma^* = \{(x_1, x_2, x_3), (y, 1-y)\}$

For $P1$ we know that

- If $x_1, x_2, x_3 \notin \{0,1\} \implies \text{supp}(\sigma_{p1}) = \{C, D, E\}$.
  Therefore $\mu_1((1,0,0),(y,1-y)) = \mu_1((0,1,0),(y,1-y)) = \mu_1((0,0,1),(y,1-y))$

Lets analyze the case where we can have a **fully mixed strategy**:

$P1 = \{C, D, E\}, P2 = \{A, B\}$

$$\mu_1((1,0,0),(y,1-y)) = \mu_1((0,1,0),(y,1-y)) = \mu_1((0,0,1),(y,1-y)) \tag{11a}$$

$$y + 4 - 4y = 3y + 1 - y = 2y + 2 - 2y \tag{11b}$$

$$4 - 3y = 2y + 1 = 2y + 2 - 2y \tag{11c}$$

This **IS NOT A FULLY MIXED** $NE$ because 11c leads to inequality since $2y + 1 \neq 2y + 2 - 2y$.

Therefore there is no exist a **FULLY MIXED NE**.

# 3 Problem 13

Lets analyze the utility after applying the distribution $(0.6, 0.4), (0.2, 0.4, 0.4)$

| | | P2 R | S | T |
|---|---|---|---|---|
| P1 | A | 6,6 | 2,7 | 2,6 |
| | B | 7,2 | 3,4 | 0,0 |

For Player 1

$$\mu_1((1,0),(0.2,0.4,0.4)) = \mu((0,1),(0.2,0.4,0.4)) \tag{12a}$$
$$1.2+0.8+0.8 \neq 1.4+1.2+0 \tag{12b}$$

For Player 2

$$\mu_2((0.6,0.4),(1,0,0)) = \mu_2((0.6,0.4),(0,1,0)) \quad = \mu_2((0.6,0.4),(0,0,1)) \tag{13a}$$
$$3.6+0.8 \neq 4.2+1.6 \qquad \neq 3.6 \tag{13b}$$

Therefore there is **NO NE**