JUAN PABLO ROYO SALES

1

HOMEWORK 12

12.1

$$E[X] = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= \int_{-\infty}^{\infty} x \lambda e^{-\lambda x}\, dx$$

$$= \lambda \int_{-\infty}^{\infty} x e^{-\lambda x}\, dx$$ by parts

$$= \lambda\left[\frac{-x e^{-\lambda x}}{\lambda} - \int -\frac{e^{-\lambda x}}{\lambda}\, dx \right] \Rightarrow$$

$-\lambda x = z$ $\quad -\frac{1}{\lambda} = dz$

$\frac{dz}{dx} = -\lambda$

$\frac{1}{\lambda^2} \int e^z dz = \frac{e^z}{\lambda^2}$

$= \frac{e^{-\lambda x}}{-\lambda^2}$

$$\Rightarrow \left(\frac{-x e^{-\lambda x}}{\lambda} - \frac{e^{-\lambda x}}{\lambda^2} \right)$$

$$= -x e^{-\lambda x} - \frac{e^{-\lambda x}}{\lambda}$$

$$= \boxed{\frac{-(\lambda x + 1) e^{-\lambda x}}{\lambda}}$$

12.2

- $P(\min\{H_1, H_2\} > t)$

LET X be a RV s.t $P(\min\{H_1 H_2\} = X)$

$\min\{H_1, H_2\} > t \mid H_1 > t \wedge H_2 > t$

- For making

$P(\min\{H_1, H_2\} > t) = P(H_1 > t)\, P(H_2 > t)$

$= 1 - (1 - e^{-\lambda_1 t}) * 1 - (1 - e^{-\lambda_2 t})$

$$= e^{-\lambda_1 t} e^{-\lambda_2 t}$$

$$= \boxed{e^{-(\lambda_1+\lambda_2)t}}$$

$$P(\min\{H_1, H_2\} = X) = e^{-(\lambda_1+\lambda_2)t}$$

For $\min\{H_1, H_2, \dots\dots H_n\}$

$$P(\min\{H_1 \dots\dots H_n\} > t) = P(H_1 > t)\, P(H_2 > t) \dots P(H_n > t)$$

$$= 1-(1-e^{-\lambda_1 t})\, 1-(1-e^{-\lambda_2 t}) \dots 1-(1-e^{-\lambda_n t})$$

$$= e^{-\lambda_1 t}\, e^{-\lambda_2 t} \dots e^{-\lambda_n t}$$

$$= \boxed{e^{-(\lambda_1+\lambda_2 \dots \lambda_n)t}}$$

12 3

$$P(\min\{H_1 \dots H_n\} = H_1) \Rightarrow P(H_i < \min_{i \neq j} H_i) \quad \frac{\lambda_i}{\sum_j \lambda_j}$$

by 12.2 we knew that

$$P(\min H_i > t) = e^{-\sum_i \lambda_i t}$$

$$P(\min_{i \neq j} H_j > t) = e^{-\sum_{j \neq i} \lambda_j t}$$

$1-e$

$1-e$

$$P(H_i > t) = e^{-\lambda_i t}$$

by Hint $P(A<B) = \int P(B > x)\, f_A(x)\, dx$

$$P(H_i < \min_{i \neq j} H_j) = \int P(\min_{i \neq j} H_j > t)\, H_i(t)\, dt$$

$$= \int e^{-\sum_{j \neq i} \lambda_j t} (1 - e^{-\lambda_i t})\, dt$$

$$= \int \frac{e^{-\sum \lambda_j t}}{\sum \lambda_j} - \int e^{-\lambda_i t - (\sum \lambda_j t)}$$

$$= \frac{e^{-\Sigma \lambda_j t}}{\Sigma \lambda_j} - \frac{e^{-\lambda_1 t - (\Sigma \lambda_j t)}}{\Sigma \lambda_j t}$$

2

$$= \boxed{\frac{\lambda_1 t}{\Sigma \lambda_j}}$$

12.4

States

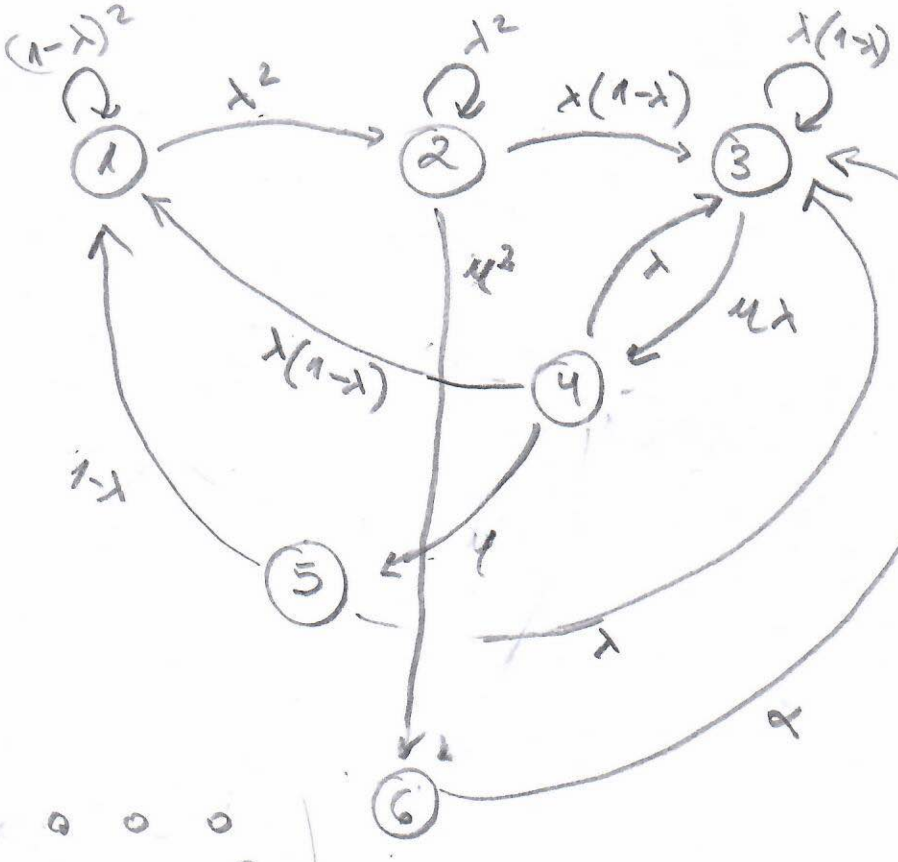
1	TT
2	LL
3	LT
4	XL
5	XT
6	BX

L T

$X \longrightarrow \mu$

$B \longrightarrow \alpha$

$L \longrightarrow \lambda$



$$\begin{matrix} (1-\lambda)^2 & \lambda^2 & 0 & 0 & 0 & 0 \\ 0 & \lambda^2 & \lambda(1-\lambda) & 0 & 0 & \mu^2 \\ 0 & 0 & \lambda(1-\lambda) & \mu\lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 & \mu & 0 \\ 1-\lambda & 0 & \lambda & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & 0 \end{matrix}$$