Randomized Algorithms Quiz 1

4-10-2018, Total=10 punts

Name:

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1. (2) There are four communication links between two servers to provide resilience to failures. Each link fails with probability 0.4 independently of the others. What is the probability that a message is successfully sent?

2. (2) Let $A[1 \ldots n]$ be an array with n different integers. Let $ran(n)$ a randomized function that outputs an integer i, $1 \leq i \leq n$, under a uniform distribution. Let us consider the following algorithm:

   Input: $A[1 \ldots n]$

   from $i = 1$ **to** n **do**

   exchange values $A[i]$ and $A[ran(n)]$

   end

   Prove if the algorithm outputs a permutation of A with uniform distribution. Justify your answer.

3. (2) I have a fair coin and a two-headed coin. I choose one of the two coins randomly with equal probability and flip it. Given that the flip was heads, what is the probability that I flipped the two-headed coin? (define the events and use conditional probability)

4. (4) A bag contains 3 red balls and 4 white balls. One ball is drawn from the bag and put to one side. A second ball is now drawn from the bag. Let R_1 be the event that the in the first draw we get a red ball, and let W_2 the event that in the second drawn the ball is white. What is the probability that the first ball is red and the second ball is white? Are the events of drawing a red ball and then a white ball independent? Suppose now that we put the first ball back into the bag after drawing it. What is the probability that the first ball is red and the second ball is white? Are the events R_1 and W_2 independents?