Stochastic Network Modeling
Homework 1 - Solutions

Juan Pablo Royo Sales
Universitat Politècnica de Catalunya

September 17, 2020

1 Problem 1.1

Let U be $(a, b) \mid a, b \in \{1, \dots, 6\}$.

Let $A = \{a + b \text{ is odd}\}$.

Let $B = \{a \vee b \text{ is } 6\}$.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \tag{1a}$$
$$= \frac{\frac{6}{36}}{\frac{11}{36}} \tag{1b}$$
$$= \frac{6}{11} \tag{1c}$$

2 Problem 1.2

Let U be $(a, b) \mid a, b \in \{B, G\}$ where $B = BOY$ and $G = GIRL$.

Let $A = \{$ At least 1 is a BOY$\}$.

Let $B = \{$ At least 1 is a GIRL$\}$.

$$P(B|A) = \frac{P(B \cap A)}{P(A)} \tag{2a}$$
$$= \frac{\frac{2}{4}}{\frac{3}{4}} \tag{2b}$$
$$= \frac{2}{3} \tag{2c}$$

3 Problem 1.3

Let U be $(a, b, c) \mid a, b, c \in \{1, 2, 3\}$.

Let $A = \{$ Switch door$\}$.

Let $B = \{$ Dont switch$\}$.

Let $W = \{$ Win$\}$.

Probability of winning if switch

$$P(W|A) = \sum_{k=1}^{3} P(W|A, a = k)P(a = k) \tag{3a}$$
$$= [P(W|A, a = 1) + 2P(W|A, a = 2)]\frac{1}{3} \tag{3b}$$
$$= \frac{2}{3} \tag{3c}$$

Probability of winning if we dont switch

$$P(W|B) = \sum_{k=1}^{3} P(W|B, a = k)P(a = k) \tag{4a}$$
$$= [P(W|B, a = 1) + 2P(W|B, a = 2)]\frac{1}{3} \tag{4b}$$
$$= \frac{1}{3} \tag{4c}$$

4 Problem 1.4

4.1 Problem 1.4.A

$$\begin{aligned}
E[S_n] &= E[\sum_{k=1}^{n} I_k] && (5a)\\
&= \sum_{k=1}^{n} E[I_k] && (5b)\\
&= \sum_{k=1}^{n} P(\text{k is tail}) && (5c)\\
&= \frac{n}{2} && (5d)
\end{aligned}$$

5 Problem 1.5

Let

$$A = \begin{cases} 1, & \text{if the term matches} \\ 0, & \text{otherwise} \end{cases}$$

5.1 1.5.A

In this case $P(A) = 1/10$ because at each time are equally likely.

$$\begin{aligned}
E[A] &= \sum_{i=1}^{10} P(A) && (6a)\\
&= \frac{10}{10} = 1 && (6b)
\end{aligned}$$

5.2 1.5.B

$$\begin{aligned}
E[A] &= \sum_{i=1}^{10} \frac{1}{i} && (7a)\\
&= 2.9289 && (7b)
\end{aligned}$$

6 Problem 1.6

6.1 1.6.A

Let $A =$ If the gambler win 10 euros.
Let $B =$ If the gambler win 25 euros.

Knowing that $P(A) = \frac{5}{12}$ and $P(B) = \frac{1}{18}$.

$$E[A + B] = 10 \times P(A) + 25 \times P(B) \tag{8a}$$
$$= 10 \times \frac{5}{12} + 25 \times \frac{1}{18} \tag{8b}$$
$$= \frac{50}{9} \tag{8c}$$

Expected benefit is 5.56

6.2 1.6.B

Let be $W = A + B$ the event of wining.

This is a Geometric distribuion $G(p)$.

Knowning that $E[X]$ of a Geometric distribution is $1/p$ and being $p = \frac{19}{36}$, where p is the probability of not wining.

$$E[W] = \frac{36}{19} \tag{9a}$$

Expected benefit will be the expected amount of wining in the last try $\frac{50}{9}$ minus all the 5 euros for rolling the dice and lose, $\frac{50}{9} - 5(\frac{36}{19}) = -9.44$.