Problem 5.1
In a certain town 90% of all sunny days are followed by sunny days and 80% of all cloudy days are followed by cloudy days. Assume that the Markov property holds.

5.1.A Build the Markov chain: transition diagram and probability matrix $\mathbf{P}$.

5.1.B Derive the characteristic polynomial, $\det(\lambda\,\mathbf{I}-\mathbf{P})$, and use it to compute the eigenvalues. Check that (i) you obtain the eigenvalue $\lambda_0 = 1$, (ii) the trace of the matrix $\mathbf{P}$ (sum of the elements of the diagonal) is equal to the sum of the eigenvalues, (iii) the determinant of $\mathbf{P}$ is equal to the product of the eigenvalues.

5.1.C If it is sunny at day $n = 0$, compute the probability of the weather to be sunny and cloudy at day $n > 0$ ($\pi_s(n)$ and $\pi_c(n)$, respectively).

Problem 5.2

5.2.A Formulate the game of problem 1.6.B (rolling 3 dice) using a DTMC, as in problem 3.4. Let $X(n)$ be the state of the chain at step n. Let 2 be the state "obtain 2 dice equal, 1 different". Compute $\pi_2(n) = P(X(n) = 2)$ in close form using the general solution for a defective transition probability matrix. Hint: when computing the unknown coefficients take into account the it must be $0 \leq \pi_2(n) \leq 1$, $n \to \infty$.

5.2.B Compute $\pi_2(\infty) = P(X(\infty) = 2)$ using the previous item and compare the result with problem 3.4.