HOMEWORK 22

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PROBLEM 22.1

22.1 A

$$\lambda_1 = 1 + \lambda_2 P_{21} + \lambda_3 P_{31}$$

a) $\boxed{\lambda_1 = 1 + \lambda_2 0.5 + \lambda_3 0.2}$

$$\lambda_2 = \lambda_1 P_{12} + \lambda_3 P_{32}$$

$$\lambda_2 = \lambda_1 0.5 + \lambda_3 (1-P_3) P_1$$

b) $\boxed{\lambda_2 = \lambda_1 0.5 + \lambda_3 0.1}$ 0.85 +

$$\lambda_3 = \lambda_1 P_{13} + \lambda_2 P_{23}$$

$$\lambda_3 = \lambda_1 0.5 + \lambda_2 (1-P_2)(1-P_1)$$

c) $\boxed{\lambda_3 = \lambda_1 0.5 + \lambda_2 0.25}$

a) $\lambda_1 = 1 + (\lambda_1 0.5 + \lambda_3 0.1) 0.5 + \lambda_3 0.2$

$$\lambda_1 = 1 + \lambda_1 0.25 + \lambda_3 0.05 + \lambda_3 0.2$$

b) $$\lambda_1 = 1 + \lambda_1 0.25 + \lambda_3 0.25.$$

c) $$\lambda_3 = \lambda_1 0.5 + 0.25(\lambda_1 0.5 + \lambda_3 0.1)$$

$$\lambda_3 = \lambda_1 0.5 + \lambda_1 0.13 + \lambda_3 0.03$$

$$\lambda_3 = \lambda_1 0.63 + \lambda_3 0.03$$

$$\lambda_3 = \frac{\lambda_1 0.63}{1 - 0.03}$$

$$\boxed{\lambda_3 = \lambda_1 065}$$

a) $\lambda_1 = 1 + \lambda_1 0.25 + \lambda_1 0.16$

$\lambda_1 = 1 + \lambda_1 0.41$

$\lambda_1 = \frac{1}{1 - 0.41}$

$\boxed{\lambda_1 = 1.69}$

$\boxed{\lambda_3 = 1.10}$

$\boxed{\lambda_2 = 0.96}$

Stability $\rho_i = \frac{\lambda_i}{\mu_i} < 1$

$\rho_1 = \frac{1.69}{2} = 0.85$ ✓

$\rho_2 = \frac{1.10}{2} = 0.55$ ✓

$\rho_3 = \frac{0.96}{2} = 0.48$ ✓

$\Pi(n_1, n_2, n_3) = (1-\rho_1)\rho_1^{n_1}(1-\rho_2)\rho_2^{n_2}(1-\rho_3)\rho_3^{n_3}$

22.1.B

$\Pi_{010} = (1-\rho_1)(1-\rho_2)\rho_2^1(1-\rho_3) = 0.15\ 0.45\ 0.55\ 0.52 = 0.02$

$\Pi_{001} = (1-\rho_1)(1-\rho_2)(1-\rho_3)\rho_3 = 0.15\ 0.45\ 0.52\ 0.48 = 0.02$

$E[a] = n\Pi_{010}\rho_2 = \boxed{0.01n}$

$E[b] = n\Pi_{001}\rho_3 = \boxed{0.02n}$

22.1 C

$1 - \Pi_{000} = N_S$

$(1-P_1)(1-P_2)(1-P_3) = 0.15 \cdot 0.45 \cdot 0.2 = 0.04$

$T = 1 - 0.04 = \boxed{0.96}$

$N = \lambda T$

$N = (\lambda_1 + \lambda_2 + \lambda_3) T$

$\boxed{N = 3.60}$

Problem 22.2

22.2 A

$M = 2 \quad K = 3$

$\boxed{\lambda_1 = 1}$

$\lambda_2 = 1 + \lambda_3 \frac{1}{3} \Rightarrow \lambda_2 = \lambda + \lambda_2 \frac{1}{6}$

$\lambda_3 = \lambda_2 \frac{1}{2}$

$\lambda_2 = \frac{1}{1 - \frac{1}{6}}$

$\boxed{\lambda_2 = \frac{6}{5}}$

$\boxed{\lambda_3 = \frac{3}{5}}$

$P_1 = 1 \quad P_2 = \frac{3}{5} \quad P_3 = \frac{1}{5}$

$\Pi_{n_1 n_2 n_3} = \frac{1}{6} P_1^{n_1} P_2^{n_2} P_3^{n_3} = \frac{1}{6} P_2^{n_2} P_3^{n_3}$

$G = P_1^2 P_2^0 P_3^0 + P_1^1 P_2^2 P_3^0 + P_1^0 P_2^0 P_3^2 + P_1^1 P_2^1 P_3^0 + P_1^0 P_2^1 P_3^1 + P_1^1 P_2^0 P_3^1$

$G = 1 + \frac{9}{25} + \frac{1}{25} + \frac{3}{5} + \frac{3}{25} + \frac{1}{5} = \boxed{\frac{58}{25}}$

$$\pi_{n_1,n_2,n_3} = \frac{25}{58}\left(\frac{3}{5}\right)^{n_2}\left(\frac{1}{5}\right)^{n_3}$$

$\pi_{200} = \frac{25}{58}$ $\pi_{002} = \frac{1}{58}$ $\pi_{101} = \frac{5}{58}$

$\pi_{020} = \frac{9}{58}$ $\pi_{110} = \frac{15}{58}$ $\pi_{011} = \frac{3}{58}$

22.2.B

Because it would violate the stability cond.

22.2.C

$$S_1 = (\pi_{200} + \pi_{101} + \pi_{110})\,41$$

$S_1 = \frac{45}{58}$

22.2.D

$$T_p = \pi_{002} + \pi_{101} + \pi_{020} + \pi_{110} + \pi_{011}$$

$T_p = \frac{33}{58}$