Homework for Computational Complexity | Created: 21/02/2020
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Instructor: Albert Atserias

Homework 1

Exercise: Composition of logspace computable functions A function $f : \{0,1\}^* \to \{0,1\}^*$ is called *moderate* if there exists a polynomial $p(n)$ such that $|f(x)| \leq p(|x|)$ for every $x \in \{0,1\}^*$. The *bit-graph* of f is the following language:

$$\mathrm{BIT}_f := \{\ \langle x, i\rangle :\ 1 \leq i \leq |f(x)| \text{ and the } i\text{-th bit of } f(x) \text{ is } 1\ \}$$

We say that f is *logspace computable* if BIT_f is decidable in space $O(\log n)$; i.e., it is in L. Show that if $f : \{0,1\}^* \to \{0,1\}^*$ and $g : \{0,1\}^* \to \{0,1\}^*$ are both moderate and logspace computable, then the composition $f \circ g$ (defined by $(f \circ g)(x) = f(g(x))$) is also moderate and logspace computable.

Exercise: Logspace verifiers Recall that NP has been characterized as the class of languages A for which there exists a polynomial $p(n)$ and a language B in P such that for every string x we have

$$x \in A \Leftrightarrow \exists y \in \{0,1\}^* \text{ s.t. } |y| \leq p(|x|) \text{ and } \langle x, y\rangle \in B.$$

Show that P can be replaced by L and we still get a characterization of NP; i.e., the verifier can be restricted to running not only in polynomial time but even logarithmic space (since L $\subseteq$ P, one half of this statement is obvious; you are asked to prove the other half without proving P $\subseteq$ L which, fyi, is an open problem).