Problem 2.1
The week fuel demand x in a petrol station is distributed as

$$f(x) = \begin{cases} \alpha\, x^2, & 0 \le x \le G \\ \alpha\, \dfrac{G^4}{x^2}, & x \ge G \end{cases}$$

where G is a constant.

2.1.A Compute the value of α.
2.1.B Compute the cummulative distribution function, $F(x)$.
2.1.C Assume that the station has a fuel tank refilled weekly. Compute the size of the tank in terms of G, such that the tank will not be emptied with 95% probability.

Problem 2.2
A number $X_1 \in \{1, 2, \cdots 6\}$ is obtained rolling a die. Then, a number $X_2 \in [0, \infty]$ is obtained exponentially distributed with parameter X_1. Compute $f(x_1, x_2)$, $f(x_2)$ and $\mathrm{E}[X_2]$.
Note: X is a continous RV exponentially distributed with parameter λ if:

$$F(x) = P(X \le x) = 1 - \mathrm{e}^{-\lambda\, x},\ x \ge 0 \tag{1}$$

$$f(x) = \frac{d\, F(x)}{dx} = \lambda\, \mathrm{e}^{-\lambda\, x},\ x \ge 0 \tag{2}$$

Problem 2.3
A random variable (X, Y) has density $f(x, y) = 3\,(1-x)$ in the region $x > 0, y > 0, x+y < 1$. Compute:

2.3.A The probability that $X > Y$
2.3.B The marginal density of Y.
2.3.C The mean and variance of Y.

Problem 2.4
We choose two points X, Y randomly in the interval $[0, 1]$. Compute the cummulative distribution and density function of the random variable

$$U = \min\{X, Y\}.$$

Problem 2.5
We choose two points X, Y randomly in the interval $[0, 1]$. Compute the cummulative distribution of the random variable $V = \max\{X, Y\}$.

Problem 2.6
We choose two points X, Y randomly in the interval $[0, 1]$. Compute the joint cummulative distribution and density ($F(u, v)$ and $f(u, v)$) of the random variables $U = \min\{X, Y\}$, $V = \max\{X, Y\}$.
Hint: compute $P(U > u, V \le v)$.

Problem 2.7
Craps is a game played with a pair of dice. The player rolls once. If the dice show 7 or 11, the player wins. If the dice show 2, 3 or 12 the player loses. If the dice show another value, this is the *player's point*. Then the player keep rolling the dice until either the dice show 7 (the player loses) or the dice show the player's point, and wins. Compute the player's winning probability.