Homework 20 – Deadline 4/12/2020

Problem 20.1

Consider an M/M/1/2 queue (queue size of 1 position). Service time is 1 time unit, and arrival rate is 1/2 customers/time unit.

20.1.A Compute the expected length of a busy period using the general formula for an M/G/1/K queue.

20.1.B Compute the distribution of a busy period. Hint: consider an absorbing chain.

20.1.C Compute the expected value of the busy period using the previous item, and check it with the value obtained in 20.1.A.

Problem 20.2

Assume an M/G/1 queue with arrival rate $\lambda = 1/2$ and services uniformly distributed with mean 1 time unit.

20.2.A Compute the expected time in the system using the P-K formula.

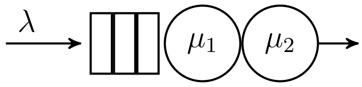
20.2.B Compute the mean busy period.

20.2.C Compute the expected number in the system.

20.2.D Compare the expected time and number in the system with those obtained for an M/M/1 queue with the same mean service and arrival rates.

Problem 20.3

Assume processes arriving to an ∞ queue with a Poisson distribution with parameter $\lambda = 1/2$ (see the figure). The server has 2 exponentially distributed stages with rates μ_1 (to be computed) and $\mu_2 = 1$. Only when the server is empty, or one process leaves stage 2, a new process can enter the stage 1 of the server.



20.3.A Let N_Q be the number of processes in the queue (i.e. not including the process in the server). Compute the minimum value of μ_1 for $\mathrm{E}[N_Q] \leq 1$.

Hint: for independent variables X_i, $\mathrm{Var}(\sum X_i) = \sum \mathrm{Var}(X_i)$. If $X \sim \exp(\mu)$, then $\mathrm{E}[X] = 1/\mu$, $\mathrm{Var}(X) = 1/\mu^2$.