Problem 16.1

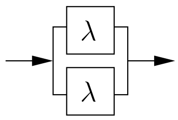
Assume the CSMA/CA protocol of problem 12.4 with 2 nodes and parameters $\lambda = 1/4$ and $\alpha = 3/4$. Assume however that the transmission time of the packets is deterministic with duration $\mu = 1$ time unit. Assume the process obtained when the system is observed at start and end transmission time of the packets.

16.1.A Discuss why such system is a semi-Markov process.
16.1.B Define the states of the chain and derive the transmission diagram of the embedded MC.
16.1.C Compute the stationary distribution of the embedded MC using the flux balancing method.
16.1.D Compute the stationary distribution of the semi-Markov process using the previous item.
16.1.E Assume that the bitrate when a packet is transmitted is 10 Mbps. Compute the average transmission time of a file of 10 Mbytes by one of the nodes. Compare the result with the expected value obtained in item 15.2.

Problem 16.2

Assume the system with redundancy of figure 2. The failure time of each device is exponentially distributed with rate $\lambda = 1$ failures/year. Upon a device failure, it is repaired, independently of the other, within a deterministic time equal to 0.1 years. If both devices fail, the system is stopped until both devices are repaired. Otherwise the system is in working state.

16.2.A Compute the expected time until the system is stopped for the first time.
16.2.B Compute how long the system is working and stopped during 1 year.
16.2.C Compute the expected number of device repairs in a year.



Hints:

- Use a semi Markov process obtained at device failure instants, and device repairs that leave the system in working state.
- In item 16.2.A consider an absorbing state when the system stops.
- The distribution function of a random variable R exponentially distributted with rate λ, given that occurs in an interval $[0, T]$, is

$$F_R(t|T) = \frac{P(R \leq t,\, R \leq T)}{P(R \leq T)} = \frac{P(R \leq t)_{t \in [0,T]}}{P(R \leq T)} = \frac{1 - \mathrm{e}^{-\lambda t}}{1 - \mathrm{e}^{-\lambda T}},\ t \in [0, T].$$

Recall that

$$\mathrm{E}[R|T] = \int_0^T (1 - F_R(t|T))\, \mathrm{d}t = \frac{1 - \alpha - \alpha\, \lambda\, T}{\lambda\, (1 - \alpha)} = \frac{1}{\lambda} - \frac{\alpha\, T}{1 - \alpha},\ \text{where } \alpha = \mathrm{e}^{-\lambda T}.$$