Stochastic Network Modeling
Homework 2 - Solutions

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Problem 2.1

2.1.1

$$\begin{aligned}
1 &= \int_0^G \alpha x^2 dx + \int_G^\infty \alpha \frac{G^3}{x^2} dx && \text{(1a)}\\
&= \alpha \frac{x^3}{3} \Big|_0^G + \alpha G^3 \frac{1}{x} \Big|_G^\infty && \text{(1b)}\\
&= \alpha \frac{G^3}{3} + \left(\alpha \frac{G^3}{\infty} - \alpha \frac{G^3}{G} \right) && \text{(1c)}\\
&= \alpha \left(\frac{G^3}{3} - G^2 \right) && \text{(1d)}\\
&= \alpha \frac{G^3 - 3G^2}{3} && \text{(1e)}\\
& && \text{(1f)}
\end{aligned}$$

Therefore,

$$\alpha = \frac{3}{G^2(G-3)}$$

2.1.2

$$f(x) = \begin{cases} \frac{3x^2}{G^2(G-3)} & 0 \le x \le G \\ \frac{3G^2}{x^2(G-3)} & x \ge G \end{cases}$$

$$
\begin{aligned}
F(x) &= \int_0^x \frac{3t^2}{G^2(G-3)}dt + \int_x^{+\infty} \frac{3G^2}{t^2(G-3)}dt && \text{(2a)}\\
&= \frac{3}{G^2(G-3)}\frac{t^3}{3}\Big|_0^x && \text{(2b)}\\
&= \frac{t^3}{G^2(G-3)}\Big|_0^x && \text{(2c)}\\
&= \frac{x^3}{G^2(G-3)} && \text{(2d)}
\end{aligned}
$$

2.1.3

I am not sure but i think the idea is to put $F(x) = 0.95$. If that the case it would be

$$
\begin{aligned}
\frac{x^3}{G^2(G-3)} &= 0.95 && \text{(3a)}\\
\frac{0.95}{G^2(G-3)} &= x^3 && \text{(3b)}\\
\sqrt[3]{\frac{0.95}{G^2(G-3)}} &= x && \text{(3c)}\\
& && \text{(3d)}
\end{aligned}
$$

Problem 2.2

Not sure about this...

- $f(x_1, x_2)$

$$
\begin{aligned}
f(x_1, x_2) &= \sum_{x_1} f(x_2|\ x_1)P(x_1) && \text{(4a)}\\
&= \frac{1}{6}\sum_{x_1} x_1 e^{-x_1 x_2} && \text{(4b)}
\end{aligned}
$$

- $f(x_2)$

$$\begin{aligned}
f(x_2) &= \int_{x_1}^{+\infty} f(x_1, x_2) dx_1 && \text{(5a)}\\
&= 1 - e^{-x_1 x_2} \Big|_1^6 && \text{(5b)}\\
&= -e^{-7x_2} && \text{(5c)}
\end{aligned}$$

Problem 2.3

2.3.1

$$\begin{aligned}
F(X > Y) = P(X > Y | X + Y < 1) &= \int_{1-y}^{1} 3(1-x)dx && \text{(6a)}\\
&= 3(x - \frac{x^2}{2}) \Big|_{1-y}^{1} && \text{(6b)}\\
&= \frac{3}{2} - 3((1-y) - \frac{(1-y)^2}{2}) && \text{(6c)}\\
&= \frac{3}{2} - 3(\frac{1+y^2}{2}) && \text{(6d)}\\
&= \frac{3}{2} - \frac{3}{2} + \frac{3}{2}y^2 && \text{(6e)}\\
&= \frac{3}{2}y^2 && \text{(6f)}
\end{aligned}$$

2.3.2

$$\begin{aligned}
F(Y) &= \int_x 3(1-x)dx && \text{(7a)}\\
&= 3(x - \frac{x^2}{2}) \Big|_x^1 && \text{(7b)}\\
&= \frac{3}{2} - 3(x - \frac{x^2}{2}) && \text{(7c)}
\end{aligned}$$

2.3.3

$$\begin{aligned}
E(Y) &= \int_0^1 (1 - 3(x - \frac{x^2}{2}))dx && \text{(8a)}\\
&= \frac{x(x^2 - 3x + 2))}{2} \Big|_0^1 && \text{(8b)}\\
&= 0 && \text{(8c)}
\end{aligned}$$

Problem 2.4

$$
\begin{aligned}
F_U(u) &= P[U \leq u] && \text{(9a)}\\
&= P[min(X, Y) \leq u] && \text{(9b)}\\
&= 1 - P[min(X, Y) > u] && \text{(9c)}\\
&= 1 - P[X > u, Y > u] && \text{(9d)}\\
&= 1 - P[X > u]P[Y > u] && \text{(9e)}\\
&= 1 - (1 - F_x(u))(1 - F_y(u) && \text{(9f)}\\
&= F_x(u) + F_y(u) - F_x(u)F_y(u) && \text{(9g)}
\end{aligned}
$$

Problem 2.5

$$
\begin{aligned}
F_V(v) &= P[V \leq v] && \text{(10a)}\\
&= P[max(X, Y) \leq v] && \text{(10b)}\\
&= P[max(X, Y) \leq v] && \text{(10c)}\\
&= P[X \leq v, Y \leq v] && \text{(10d)}\\
&= P[X \leq v]P[Y \leq v] && \text{(10e)}\\
&= F_x(v)F_y(v) && \text{(10f)}
\end{aligned}
$$

Problem 2.6

$$
\begin{aligned}
P(U > u, V \leq v) &= P(U > u)P(V \leq v) && \text{(11a)}\\
&= (F_x(v)F_y(v))(F_x(u) + F_y(u) - F_x(u)F_y(u)) && \text{(11b)}\\
&= F_x(v)F_y(v)F_x(u) + F_x(v)F_y(v)F_y(u) - F_x(v)F_y(v)F_x(u)F_y(u) && \text{(11c)}
\end{aligned}
$$

Problem 2.7

Let $P[X = 7] = \frac{6}{36}$ be the probability of wining a 7 in first shot.

Let $P[X = 11] = \frac{2}{36}$ be the probability of wining a 11 in first shot.

So the $P[\text{Win in first shot}] = \frac{8}{36}$.

Let $P[X = 2] = \frac{1}{36}$ be the probability of lossing with 2 in first shot.

Let $P[X = 3] = \frac{2}{36}$ be the probability of lossing with 3 in first shot.

Let $P[X = 12] = \frac{1}{36}$ be the probability of lossing with 12 in first shot.
So the $P[\text{Lose in first shot}] = \frac{4}{36}$.
Probability of getting some number is $P(n) = \frac{n-1}{36}$

$$
\begin{aligned}
P[W] &= \frac{8}{36} + 2\sum_{3}^{5} P[W|n]P(n) && \text{(12a)}\\
&= \frac{8}{36} + 2\sum_{3}^{5} \frac{i}{36}\frac{i}{i+6} && \text{(12b)}\\
&= \frac{2}{36}(4 + \sum_{3}^{5} \frac{i^2}{i+6}) && \text{(12c)}\\
&= \frac{2}{36}(4 + \frac{9}{9} + \frac{16}{10} + \frac{25}{11}) && \text{(12d)}\\
&= 0.492929293 && \text{(12e)}
\end{aligned}
$$