# DESIGN OF EXPERIMENTS

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# Design of experiments

- Usually experimentation (i.e. simulation) is carry out as a programming exercise.
- Inaccurate statistical methods (no IID).
- Take care of the time required to collect the needed data to apply the statistical techniques, with guaranties of achieve the accomplishment of the objectives.

# Design of experiments

- How to make the **comparisons** between different configurations.
  - The comparisons must be the more homogeneous as possible.
- Study the **effect** over the answer variable of the values of the different experimental variables.
  - In a cashier: Answer variable: Queue long; factors: Number of cashiers, service time, time between arrivals.

# Principles

Principles to develop a good design of experiments:

- **Randomization:** Assignation to the random of all the factors that are not controlled by the experimentation.
- **Repetition of the experiment (replication):** Is a good method to reduce the variability between the answers.
- **Statistical homogeneity of the answers:** To compare different alternatives derived from the results, is needed that the executions of the experiments have been done under homogeny conditions. Factorial design helps to obtain this similarity between the experiments.

# Design of Experiments goals

- **Isolate** effects of each input variable.
- Determine **effects** of interactions.
- Determine **magnitude** of experimental error
- Obtain maximum **information** for given effort

# Terminology

- Response variable
  - Measured output value
    - E.g. total execution time
- Factors
  - Input variables that can be changed
    - E.g. cache size, clock rate, bytes transmitted
- Levels
  - Specific values of factors (inputs)
    - Continuous (~bytes) or discrete (type of system)

# Terminology

- Replication
  - Completely re-run experiment with same input levels
  - Used to determine impact of measurement error
- Interaction
  - Effect of one input factor depends on level of another input factor

# Tests

| Test | Dependent variables | Independent variables |
|---|---|---|
| T-test | One | One |
| ANOVA | One | One |
| Two-way ANOVA | One | Multiple |
| MANOVA | Mutiple | Multiple |

# One-Factor ANOVA

- Separates total variation observed in a set of measurements into:
  - Variation within one system
    - Due to random measurement errors
  - Variation between systems
    - Due to real differences + random error

- *One-factor experimental design*
  - *We have here two different populations?*

# Two-factor Experiments

DOE

# Two-factor Experiments

- Two factors (inputs)
  - A, B
- Separate total variation in output values into:
  - Effect due to A
  - Effect due to B
  - Effect due to interaction of A and B (AB)
  - Experimental error

# Example – User Response Time

- A = degree of multiprogramming
- B = memory size
- AB = interaction of memory size and degree of multiprogramming

| | **B (Mbytes)** | | |
|---|---|---|---|
| **A** | 32 | 64 | 128 |
| **1** | 1,125 | 1,105 | 1,075 |
| **2** | 1,26 | 1,225 | 1,18 |
| **3** | 1,405 | 1,33 | 1,25 |
| **4** | 1,75 | 1,725 | 1,35 |

# Two Factors, *n* Replications

| | | Factor A | | | | | |
|---|---|---|---|---|---|---|---|
| | | 1 | 2 | … | j | … | a |
| Factor B | 1 | … | … | … | … | … | … |
| | 2 | … | … | … | … | … | … |
| | … | … | … | … | … | … | … |
| | i | … | … | … | $y_{ijk}$ | … | … |
| | … | … | … | … | … | … | … |
| | b | … | … | … | … | … | … |

n replications

# One-factor ANOVA

- Each individual measurement is composition of
  - Overall mean
  - Effect of alternatives
  - Measurement errors

$$y_{ij} = \bar{y}_{..} + \alpha_i + e_{ij}$$

$$\bar{y}_{..} = \text{overall mean}$$

$$\alpha_i = \text{effect due to A}$$

$$e_{ij} = \text{measurement error}$$

# Two-factor ANOVA

- Factor A – *a* input levels
- Factor B – *b* input levels
- *n* measurements for each input combination
- *abn* total measurements

# Two-factor ANOVA

- Each individual measurement is composition of
  - Overall mean
  - Effects
  - Measurement errors
  - Interactions

$$y_{ijk} = \bar{y}_{...} + \alpha_i + \beta_j + \gamma_{ij} + e_{ijk}$$

$$\bar{y}_{...} = \text{overall mean}$$

$$\alpha_i = \text{effect due to A}$$

$$\beta_j = \text{effect due to B}$$

$$\gamma_{ij} = \text{effect due to interaction of A and B}$$

$$e_{ijk} = \text{measurement error}$$

# Sum-of-Squares

- As before, use sum-of-squares identity

$$\mathrm{SST} = \mathrm{SSA} + \mathrm{SSB} + \mathrm{SSAB} + \mathrm{SSE}$$

- Degrees of freedom
  - $df(\mathrm{SSA}) = a - 1$
  - $df(\mathrm{SSB}) = b - 1$
  - $df(\mathrm{SSAB}) = (a - 1)(b - 1)$
  - $df(\mathrm{SSE}) = ab(n - 1)$
  - $df(\mathrm{SST}) = abn - 1$

# Sum-of-Squares

$$\underbrace{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{r}\left(Y_{ijk}-\bar{Y}_{\dots}\right)^2}_{SS_{Total}} = \underbrace{r\cdot b\cdot\sum_{i=1}^{a}\left(\bar{Y}_{i\cdot\cdot}-\bar{Y}_{\dots}\right)^2}_{SS_A}+\underbrace{r\cdot a\cdot\sum_{j=1}^{3}\left(\bar{Y}_{\cdot j\cdot}-\bar{Y}_{\dots}\right)^2}_{SS_B}$$

$$+\underbrace{r\times\sum_{i=1}^{a}\sum_{j=1}^{b}\left(\bar{Y}_{ij\cdot}-\bar{Y}_{i\cdot\cdot}-\bar{Y}_{\cdot j\cdot}+\bar{Y}_{\dots}\right)^2}_{SS_{A\times B}}+\underbrace{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{r}\left(Y_{ijk}-\bar{Y}_{ij\cdot}\right)^2}_{SS_{within}}$$

$$MS_{within} = SS_{within}\Big/ df_{within}$$

# $n2^k$ Contrasts

- Effects of A, B and interactions.

$$w_A = y_{AB} + y_{Ab} - y_{aB} - y_{ab}$$

$$w_B = y_{AB} - y_{Ab} + y_{aB} - y_{ab}$$

$$w_{AB} = y_{AB} - y_{Ab} - y_{aB} + y_{ab}$$

# Two-Factor ANOVA table

| Source | Degrees of Freedom | SS | MS | F |
|---|---|---|---|---|
| A | a-1 | $SS_A$ | $MS_A$ | $MS_A / MS_{within}$ |
| B | b-1 | $SS_B$ | $MS_B$ | $MS_B / MS_{within}$ |
| $A \times B$ | (a-1)(b-1) | $SS_{A \times B}$ | $MS_{A \times B}$ | $MS_{A \times B} / MS_{within}$ |
| Within | ab(r-1) | $SS_{within}$ | $MS_{within}$ | |
| Total | abr-1 | $SS_{Total}$ | | |

# Replications are needed

- If no replications, n=1
  - SSE = 0, no information regarding the errors in the measurements.
- Cannot separate effect due to interactions from measurement noise
- Must *replicate* each experiment at least twice

# Replications are needed

- If n=1 (only one measurement of each configuration)
  - SSE = 0
- Since
  - SSE = SST – SSA – SSB – SSAB
- and
  - SSAB = SST – SSA – SSB

# Example

- Output = user response time (seconds)
- Want to separate effects due to
  - A = degree of multiprogramming
  - B = memory size
  - AB = interaction
  - Error
- Need replications to separate error

| | B (Mbytes) | | |
|---|---|---|---|
| **A** | 32 | 64 | 128 |
| **1** | 1,125 | 1,105 | 1,075 |
| **2** | 1,26 | 1,225 | 1,18 |
| **3** | 1,405 | 1,33 | 1,25 |
| **4** | 1,75 | 1,725 | 1,35 |

# Example

| A | B (Mbytes) | | |
|---|---|---|---|
| | 32 | 64 | 128 |
| 1 | 1,125 | 1,105 | 1,075 |
| | 1,14 | 1,095 | 1,055 |
| 2 | 1,26 | 1,225 | 1,18 |
| | 1,24 | 1,245 | 1,15 |
| 3 | 1,405 | 1,33 | 1,25 |
| | 1,38 | 1,295 | 1,305 |
| 4 | 1,75 | 1,725 | 1,35 |
| | 1,125 | 1,105 | 1,075 |

# Two-Factor ANOVA table

| | A | B | AB | Error |
|---|---|---|---|---|
| Sum of squares | $SSA$ | $SSB$ | $SSAB$ | $SSE$ |
| Deg freedom | 1 | 1 | 1 | $2^m(n-1)$ |
| Mean square | $s_a^2 = SSA/1$ | $s_b^2 = SSB/1$ | $s_{ab}^2 = SSAB/1$ | $s_e^2 = SSE/[2^m(n-1)]$ |
| Computed $F$ | $F_a = s_a^2/s_e^2$ | $F_b = s_b^2/s_e^2$ | $F_{ab} = s_{ab}^2/s_e^2$ | |
| Tabulated $F$ | $F_{[1-\alpha;1,2^m(n-1)]}$ | $F_{[1-\alpha;1,2^m(n-1)]}$ | $F_{[1-\alpha;1,2^m(n-1)]}$ | |

# Example

| ANOVA | | | | | | |
|---|---|---|---|---|---|---|
| Source of Variation | SS | df | MS | F | P-value | F crit |
| Sample | 0,842854 | 3 | 0,280951 | 460,2617 | 1,2E-12 | 3,490295 |
| Columns | 0,128802 | 2 | 0,064401 | 105,5034 | 2,43E-08 | 3,885294 |
| Interaction | 0,107915 | 6 | 0,017986 | 29,46473 | 1,65E-06 | 2,99612 |
| Within | 0,007325 | 12 | 0,00061 | | | |
| | | | | | | |
| Total | 1,086896 | 23 | | | | |

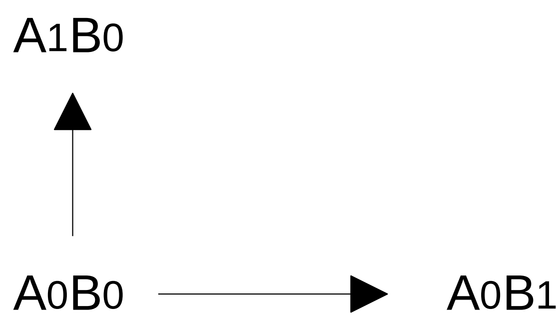
# Factorial designs

- Explore the landscape
- Assure that we are analyzing all the combinations with economy on the experimentation

# No factorial designs

- To fix two factors and modify all the levels of a third  until find a good solution. Fixing this level, start the exploration for the other factors.

- Effect A: A1B0-A0B0.
- Effect B: A0B1-A0B0



# Factorial designs

- Take in consideration the interactions.
- A effect: $$\frac{A_1B_0 - A_1B_1}{2} - \frac{A_0B_0 - A_0B_1}{2}$$
- B effect: $$\frac{A_1B_1 - A_0B_1}{2} - \frac{A_0B_0 - A_1B_0}{2}$$

$$\begin{tikzcd}
A_1B_0 \arrow[r] & A_1B_1 \\
A_0B_0 \arrow[u] \arrow[r] & A_0B_1 \arrow[u]
\end{tikzcd}$$

# Factorial designs

- Controlling "k" factors.
- "l" levels for each factor ("li" levels for the l factor).
- $l_1 \cdot l_2 \cdot \ldots \cdot l_k$ experiments
- The easiest factorial design is the $2^k$ with $li = 2 \ \forall i = 1,..,k$.

# A Problem

- *Full factorial design with replication*
  - Measure system response with all possible input combinations
  - Replicate each measurement $n$ times to determine effect of measurement error
- $k$ factors, $v$ levels, $n$ replications

  $\longrightarrow n\ v^k$ experiments
- *Example:*
  - $k = 5$ input factors, $v = 4$ levels, $n = 3$
  - $\longrightarrow 3(4^5) = 3{,}072$ experiments!

# Fractional Factorial Designs: $n2^k$ Experiments

- Special case of generalized *m*-factor experiments
- Restrict each factor to two possible values
  - High, low
  - On, off
- Find factors that have largest impact
- Full factorial design with only those factors

# Finding Sum of Squares Terms

| Sum of n measurements with (A,B) = (High, Low) | Factor A | Factor B |
| --- | --- | --- |
| **yAB** | High | High |
| **yAb** | High | Low |
| **yaB** | Low | High |
| **yab** | Low | Low |

# Contrasts for $n2^k$ with $k = 2$ factors

| **Measurements** | **Contrast** | | |
|---|---|---|---|
| | $w_a$ | $w_b$ | $w_{ab}$ |
| $y_{AB}$ | + | + | + |
| $y_{Ab}$ | + | - | - |
| $y_{aB}$ | - | + | - |
| $y_{ab}$ | - | - | + |

$$w_a = y_{AB} + y_{Ab} - y_{aB} - y_{ab}$$

$$w_b = y_{AB} - y_{Ab} + y_{aB} - y_{ab}$$

$$w_{ab} = y_{AB} - y_{Ab} - y_{aB} + y_{ab}$$

# Contrasts for $n2^k$ with $k = 3$ factors

| **Means** | **Contrast** | | | | | | |
|---|---|---|---|---|---|---|---|
| | $w_a$ | $w_b$ | $w_c$ | $w_{ab}$ | $w_{ac}$ | $w_{bc}$ | $w_{abc}$ |
| $\mathbf{y_{abc}}$ | - | - | - | + | + | + | - |
| $\mathbf{y_{Abc}}$ | + | - | - | - | - | + | + |
| $\mathbf{y_{aBc}}$ | - | + | - | - | + | - | + |
| **…** | … | … | … | … | … | … | … |

$$w_{AC} = y_{abc} - y_{Abc} + y_{aBc} - y_{abC} - y_{ABc} + y_{AbC} - y_{aBC} + y_{ABC}$$

# Important Points

- Experimental design is used to
  - Isolate the effects of each input variable.
  - Determine the effects of interactions.
  - Determine the magnitude of the error
  - Obtain maximum information for given effort
- Expand 1-factor ANOVA to *k* factors
- Use $n2^k$ design to reduce the number of experiments needed
  - But loses some information
  - Useful to underline the tendency with economy of experiments.

# Exercise 1

- We have on a factory three different Machines:
  - A, with speed from 2 to 10
  - B, with speed of 2 and 3
  - C, with speed of 2 but that can be changed for other machine with a speed of 3 for 1000€.
- Define a table for an $2^k$ experimental design that allows to analyze this.

# Solution

| | A | B | C | Answer |
|---|---|---|---|---|
| 1 | - (means 2) | -(means 2) | - (means 2) | |
| 2 | - | - | + (means 3, 1000€) | |
| 3 | - | +(means 3) | - | |
| 4 | - | + | + | |
| 5 | + (means 10) | - | - | |
| 6 | + | - | + | |
| 7 | + | + | - | |
| 8 | + | + | + | |

# Yates algorithm

Simplifying the interaction calculus on a $2^k$ factorial design

# $2^k$ factorial designs

Advantages

- Determination of the tendency with experiments economy (smoothness).
- Possibility to evolve to composite designs (local exploration).
- Basis for factorial fractional designs (rapid vision of multiple factors).
- Easy analysis and interpretation.

# $2^k$ Matrix

| Experiment | Factor 1 | Factor 2 | .... | Factor k | Answer |
|---|---|---|---|---|---|
| 1 | - | - | | - | R1 |
| 2 | + | - | | - | R2 |
| 3 | - | - | | - | R3 |
| 4 | + | - | | - | R4 |
| 5 | - | + | | - | R5 |
| 6 | + | + | | - | R6 |
| $2^k$ | + | + | | + | $R2^k$ |

# $2^k$ Matrix example

| Experiment | A | B | C | Answer |
|---|---|---|---|---|
| 1 | - | - | - | 60 |
| 2 | + | - | - | 72 |
| 3 | - | + | - | 54 |
| 4 | + | + | - | 68 |
| 5 | - | - | + | 52 |
| 6 | + | - | + | 83 |
| 7 | - | + | + | 45 |
| 8 | + | + | + | 80 |

# Effects calculus

$$Efect \quad A = \frac{A_1B_0 - A_1B_1}{2} - \frac{A_0B_0 - A_0B_1}{2}$$

$$Efect \quad B = \frac{A_1B_1 - A_0B_1}{2} - \frac{A_0B_0 - A_1B_0}{2}$$

$$\text{Main effect} = \overline{y}_+ - \overline{y}_-$$

# Effects calculus example

$$Main\ effect = \overline{y}_{+} - \overline{y}_{-}$$

$$A = \frac{72 + 68 + 83 + 80}{4} - \frac{60 + 54 + 52 + 45}{4} = 23$$

$$B = \frac{54 + 68 + 45 + 80}{4} - \frac{60 + 72 + 52 + 83}{4} = -5$$

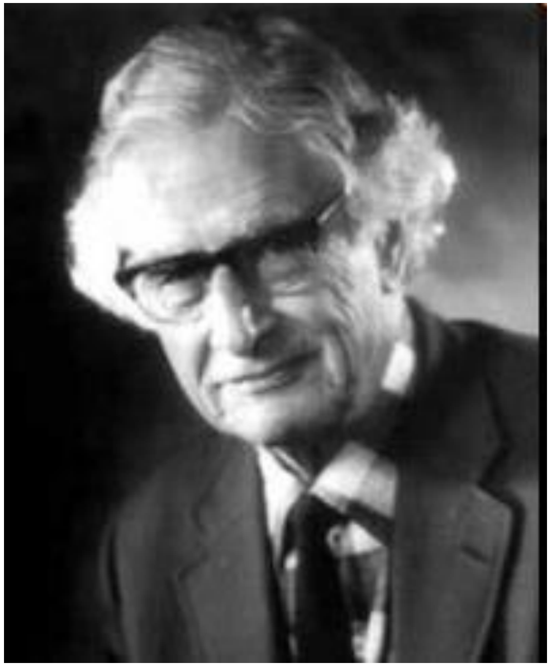
$$C = \frac{52 + 83 + 45 + 80}{4} - \frac{60 + 72 + 54 + 68}{4} = 1.5$$

# Interactions for 2 and 3 factors

$$AC = \frac{y_1 + y_3 + y_6 + y_8}{4} - \frac{y_2 + y_4 + y_5 + y_7}{4} = 10$$

$$ABC = \frac{y_{21} + y_3 + y_5 + y_8}{4} - \frac{y_1 + y_4 + y_6 + y_7}{4} = 0.5$$

# Frank Yates



- A pioneer of the Operation research of the s.XX.

# Yates algorithm

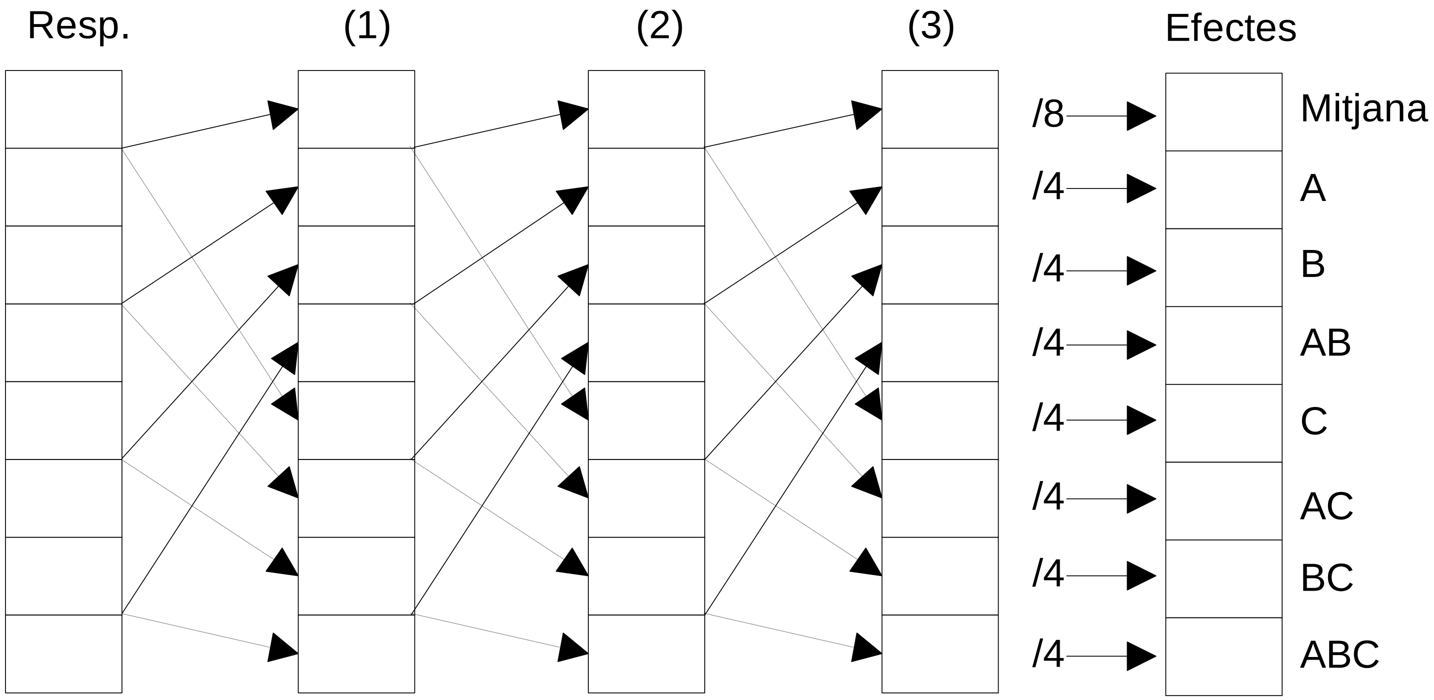
To make systematic the interactions calculus using a table.

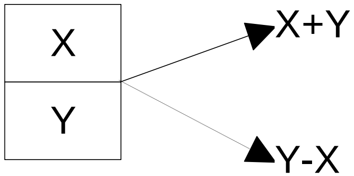
- Add the **answer** in the column "i" in the standard form of the matrix of the experimental design.
- Add **auxiliary columns** as factors exists.
- Add a new column dividing the first value of the last auxiliary column by the number of experimental conditions "E", and the others by the half of "E".

# Yates algorithm

- In the last column the first value is the mean of the answers, the last values are the effects.
- The correspondence between the values and effects is done through localize the + values in the corresponding rows of the matrix. A value with a single + in the B column is representing the principal effect of B. A row wit two + on A and C corresponds to the interaction of AC, etc.

# Yates algorithm



# Yates algorithm example

| Exp. | A | B | C | Answer | (1) | (2) | (3) | div. | effect | Id |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | - | - | - | 60 | 132 | 254 | 514 | 8 | 64.25 | Mean |
| 2 | + | - | - | 72 | 122 | 260 | 92 | 4 | 23.0 | A |
| 3 | - | + | - | 54 | 135 | 26 | -20 | 4 | -5.0 | B |
| 4 | + | + | - | 68 | 125 | 66 | 6 | 4 | 1.5 | AB |
| 5 | - | - | + | 52 | 12 | -10 | 6 | 4 | 1.5 | C |
| 6 | + | - | + | 83 | 14 | -10 | 40 | 4 | 10.0 | AC |
| 7 | - | + | + | 45 | 31 | 2 | 0 | 4 | 0.0 | BC |
| 8 | + | + | + | 80 | 35 | 4 | 2 | 4 | 0.5 | ABC |

# Wooden industry example

- Wooden industry that allows to reduce the cost.
- 4 variables to consider
  - Change the light to natural light (open the ceiling).
  - Increase the speed of the machines.
  - Increase the lubricant use.
  - Increase the working space.

# Wooden industry example

| Comb. | 1 | 2 | 3 | 4 | Description | obs. |
|---|---|---|---|---|---|---|
| (1) | - | - | - | - | | 71 |
| a | + | - | - | - | Natural light | 61 |
| b | - | + | - | - | Increase the speed of the machines | 90 |
| ab | + | + | - | - | | 82 |
| c | - | - | + | - | Increase the use of lubricant | 68 |
| ac | + | - | + | - | | 61 |
| bc | - | + | + | - | | 87 |
| abc | + | + | + | - | | 80 |
| d | - | - | - | + | Increase the working space. | 61 |
| ad | + | - | - | + | | 50 |
| bd | - | + | - | + | | 89 |
| abd | + | + | - | + | | 83 |
| cd | - | - | + | + | | 59 |
| acd | + | - | + | + | | 51 |
| bcd | - | + | + | + | | 85 |
| abcd | + | + | + | + | | 78 |

# Wooden industry example

| Comb. | obs. | 1 | 2 | 3 | 4 | Effect | Description |
|---|---|---|---|---|---|---|---|
| (1) | 71 | | | | | | |
| a | 61 | | | | | | |
| b | 90 | | | | | | |
| ab | 82 | | | | | | |
| c | 68 | | | | | | |
| ac | 61 | | | | | | |
| bc | 87 | | | | | | |
| abc | 80 | | | | | | |
| d | 61 | | | | | | |
| ad | 50 | | | | | | |
| bd | 89 | | | | | | |
| abd | 83 | | | | | | |
| cd | 59 | | | | | | |
| acd | 51 | | | | | | |
| bcd | 85 | | | | | | |
| abcd | 78 | | | | | | |

# Wooden industry example

| Comb. | obs. | 1 | 2 | 3 | 4 | Effect | Description |
|---|---|---|---|---|---|---|---|
| (1) | 71 | 132 | 304 | 600 | 1156 | 72,25 | Mean |
| a | 61 | 172 | 296 | 556 | -64 | -8 | A |
| b | 90 | 129 | 283 | -32 | 192 | 24 | B |
| ab | 82 | 167 | 273 | -32 | 8 | 1 | AB |
| c | 68 | 111 | -18 | 78 | -18 | -2,25 | C |
| ac | 61 | 172 | -14 | 114 | 6 | 0,75 | AC |
| bc | 87 | 110 | -17 | 2 | -10 | -1,25 | BC |
| abc | 80 | 163 | -15 | 6 | -6 | -0,75 | ABC |
| d | 61 | -10 | 40 | -8 | -44 | -5,5 | D |
| ad | 50 | -8 | 38 | -10 | 0 | 0 | AD |
| bd | 89 | -7 | 61 | 4 | 36 | 4,5 | BD |
| abd | 83 | -7 | 53 | 2 | 4 | 0,5 | ABD |
| cd | 59 | -11 | 2 | -2 | -2 | -0,25 | CD |
| acd | 51 | -6 | 0 | -8 | -2 | -0,25 | ACD |
| bcd | 85 | -8 | 5 | -2 | -6 | -0,75 | BCD |
| abcd | 78 | -7 | 1 | -4 | -2 | -0,25 | ABCD |

# Exercise 2: Clean industry

- We have a system that processes some kind of pieces. The time needed to process this pieces can be represented by an **exponential distribution** with a parameter μ that depends on the technology used on the process. This parameter μ can be calculated depending on several factors that affect it. Each factor adds time to the process:
  - The time needed to clean the pieces by a cleaner machine (range from 10 to 50 seconds).
  - The amount of machines that can be used for glue the different pieces (range from 1 to 5, each machine that increases the number over 2 reduces the time needed by 1 seconds).
  - The amount of workers that take the finished pieces (1 or 2), with one worker the time is 1 second, with two workers is 0,5 seconds.

# Perform a DOE for the proposed system

- Set the objectives.
- Select the process variables.
- Define an experimental design.
- Execute the design.
- Check that the data are consistent with the experimental assumptions.
- Analyze and interpret the results, detect effects of main factors and interactions.
- Remember:

$$r = 1 - e^{-\alpha \cdot x} \Rightarrow x = \frac{\ln(1-r)}{-\alpha} = \frac{\ln(r)}{-\alpha}$$

# Perform a DOE for the proposed system

- Set the objectives.
  - Detect the effects and the interactions of the three main factors
- Select the process variables.
  - Cleaner, Machines, workers.
- Define an experimental design.
  - We define a 2k experimental design.
- Execute the design.
  - Using Excel we “simulate” the behavior for each proposed model.
- Check that the data are consistent with the experimental assumptions.
  - Independence, homoscedasticity, normality, etc?.
- Analyze and interpret the results, detect effects of main factors and interactions.
  - Done on the Excel

# Answer

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | Yates | | | | |
| Cleaner | Machines | Workers | VALUES | | | µ | 1/µ | x1 | x2 | mean | | | | | |
| - | - | - | 50 | 0 | 1 | 51 | 0,019607843 | 20,8852 | 20,2611 | 20,57315 | 55,67501 | 206,082 | 246,6539 | **30,83174** | Mean |
| - | - | + | 50 | 0 | 0,5 | 50,5 | 0,01980198 | 33,83603 | 36,36768 | 35,10185 | 150,407 | 40,57198 | -8,42686 | **-2,10671** | Workers |
| - | + | - | 50 | -4 | 1 | 47 | 0,021276596 | 9,909855 | 142,0044 | 75,95712 | 25,41431 | 13,02142 | 84,47532 | **21,11883** | Machines |
| - | + | + | 50 | -4 | 0,5 | 46,5 | 0,021505376 | 17,76593 | 131,1337 | 74,44984 | 15,15767 | -21,4483 | -2,71458 | **-0,67864** | Machines*Workers |
| + | - | - | 10 | 0 | 1 | 11 | 0,090909091 | 42,759 | 0,040152 | 21,39958 | 14,5287 | 94,73195 | -165,51 | **-41,3775** | Cleaner |
| + | - | + | 10 | 0 | 0,5 | 10,5 | 0,095238095 | 5,702481 | 2,326982 | 4,014731 | -1,50728 | -10,2566 | -34,4697 | **-8,61743** | Cleaner*Workers |
| + | + | - | 10 | -4 | 1 | 7 | 0,142857143 | 10,48071 | 8,740406 | 9,610556 | -17,3848 | -16,036 | -104,989 | **-26,2471** | Cleaner*Machines |
| + | + | + | 10 | -4 | 0,5 | 6,5 | 0,153846154 | 5,977532 | 5,116704 | 5,547118 | -4,06344 | 13,32141 | 29,35739 | **7,339348** | Cleaner*Machines*Workers |

$$\boxed{r = 1 - e^{-\alpha \cdot x} \Rightarrow x = \frac{\ln(1-r)}{-\alpha} = \frac{\ln(r)}{-\alpha}}$$

# Signification of the effects

The variability is now on the effects

# Have the effects significations?

- The effects have been calculated using the answers that owns variability.
  - The variability goes to the effects.
    - $\hat{A} = \bar{y}(+) - \bar{y}(-)$

# Have signification?

- From the variance of the effects.
  - If we have replications, or if we know an estimation of the answer's variance.
- Using the normal Distribution (and its probabilistic features).
  - If we do not have replications and we do not have any estimation of the answer's variance.

# We have replications

- Calculation of the variance
  - $S_R^2 = \frac{(n_1-1)s_1^2+(n_2-1)s_2^2+..+(n_N-1)s_N^2}{n_1+n_2+..+n_N-N}$, assuming that the amount of replications in each experiment is different.
  - If not $S_R^2 = \frac{s_1^2+s_2^2+..+s_N^2}{N}$
  - Where $N=2^k$, and $Nt=v*2^k$.

# We have replications

- Variance for an effect
  - $\widehat{effect} = \overline{y_+}$-$\overline{y_-}$
  - Where $\overline{y_+}$ is the mean of the Nt/2 experiments (+ sign), and $\overline{y_-}$ is the mean of the Nt/2 experiments (- sign).
- The calculus is:
  - $v(\widehat{effect}) = v(\overline{y_+}-\overline{y_-}) = \frac{S^2}{{N_T}/{2}} + \frac{S^2}{{N_T}/{2}} = \frac{4S^2}{N_T}$

# Example

| x1 | x2 | Mean | S2 | | | |
|---|---|---|---|---|---|---|
| 60 | 64 | 62 | 8 | | | |
| 72 | 74 | 73 | 2 | | | |
| 54 | 55 | 54,5 | 0,5 | | | |
| 68 | 70 | 69 | 2 | | v(efect) | 12,1942 |
| 52 | 54 | 53 | 2 | | | 3,492019 |
| 83 | 87 | 85 | 8 | | | |
| 45 | 50 | 47,5 | 12,5 | | | |
| 80 | 85 | 82,5 | 12,5 | | | |
| | | S2= | 24,38839 | | | |

# Signification test

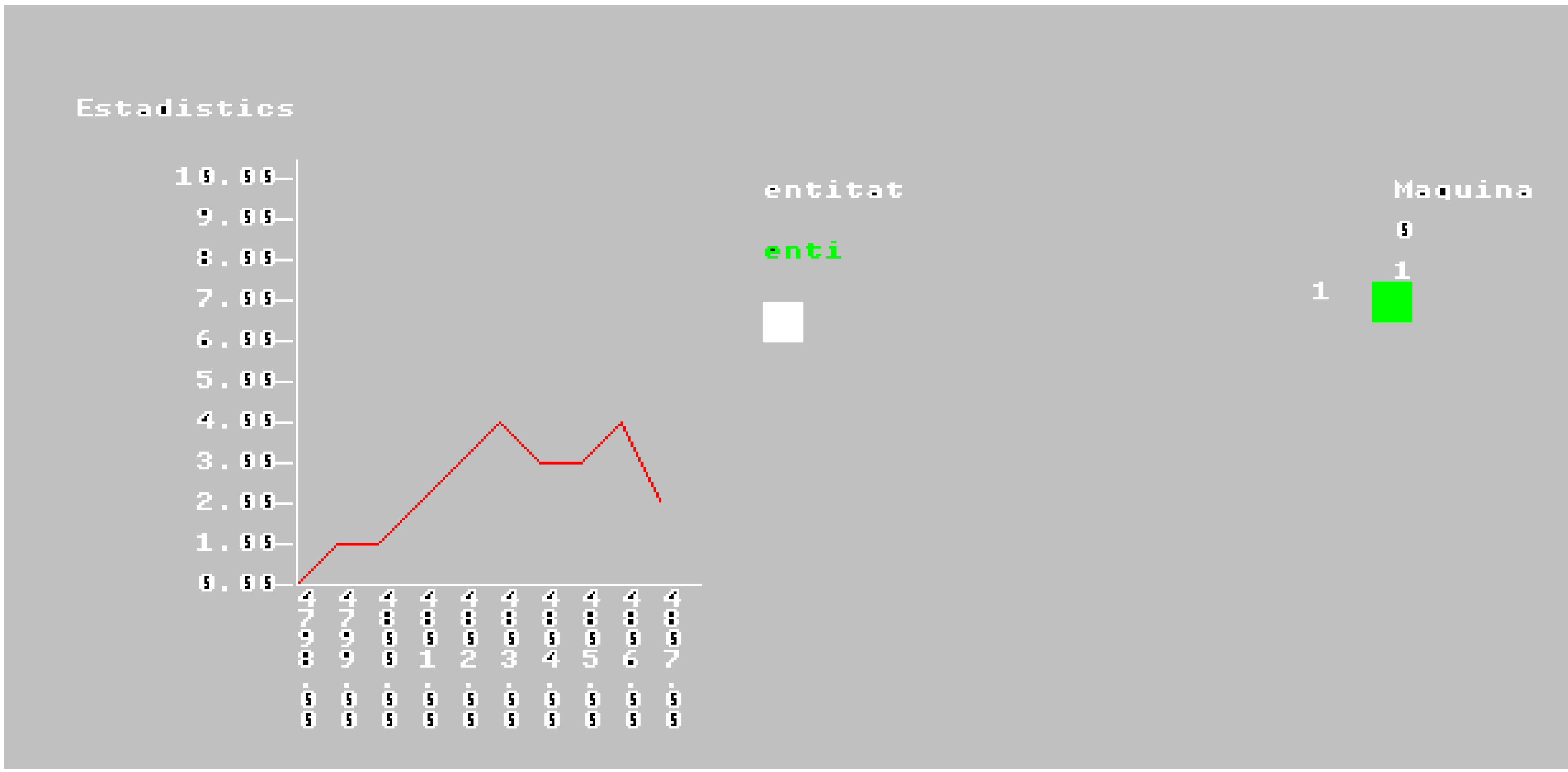
- Hypothesis testing
  - $H_0$: effect=0
  - $H_1$: effect≠0
- Statistic of the test
  - $t = \frac{\widehat{effect}-0}{S_{effect}}$
  - t-Student with $n_1+n_2+..+n_n$-N degrees of freedom ($N=2^k$)
- Calculus of the I.C.
  - $\widehat{effect} \pm x * S_{effect}$

# Replications

Number of replications calculus.

Methods to perform the replications.

# Interest variable calculus



# Experimentation

- Be x an interest variable

  x11,…x1i,…,x1m

  x21,…x2i,…,x2m

  ………………

  xn1,…xni,…,xnm

- *n* is the number of replications.
- *xi* is the value of each one of the replications.

# Sample mean and variance

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i .$$

The use of the term *n* − 1 is called Bessel's correction, for the *unbiased sample variance*

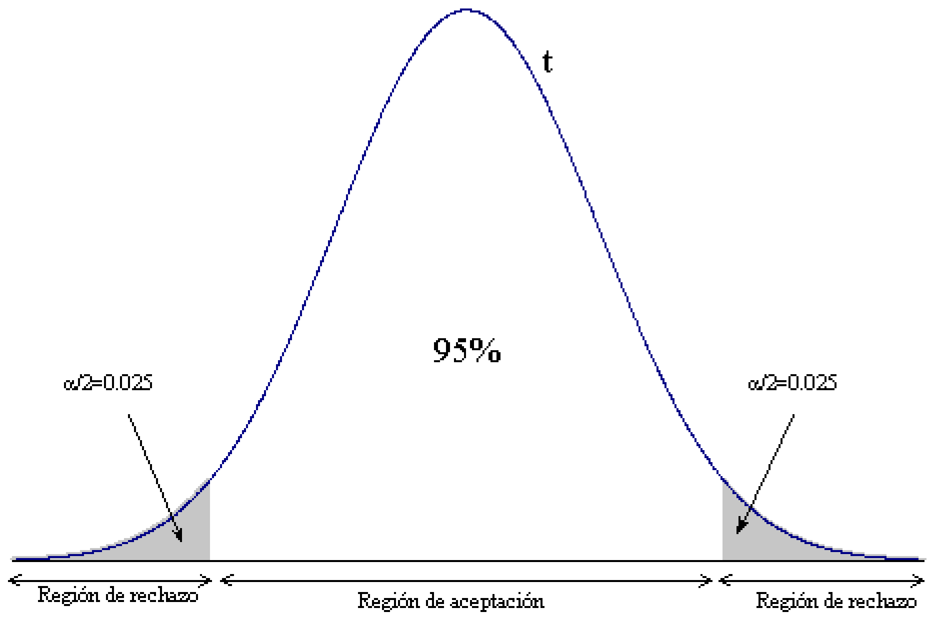
$$s^2 = \frac{n}{n-1}\sigma_y^2 = \frac{n}{n-1}\left(\frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})^2\right) = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$$

# Confidence interval

- Need to know how far is μ and $\bar{y}$ .
- Student's t-distribution of n-1 degrees of freedom.

$$\bar{y} \pm t_{1-\alpha/2,n-1}\sqrt{\frac{S^2}{n}}$$

# Student's t-distribution



# What is the correct n?

| Replication | Value from the model |
|---|---|
| 1 | 28.841 |
| 2 | 35.965 |
| 3 | 31.219 |
| 4 | 37.090 |
| 5 | 38.734 |
| 6 | 30.923 |
| 7 | 30.443 |
| 8 | 32.175 |
| 9 | 30.683 |
| 10 | 28.745 |

# Calculus of S an X

$$\bar{y} = 32.4818$$

$$S^2 = 3.5149$$

# Calculus of the self-confidence interval

$$h = t_{1-\alpha/2,n-1} \frac{S}{\sqrt{n}}$$

$t_{9,0.975} = 2{,}26$

$h = 2{,}512$

| | 1 − α | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| r | 0.75 | 0.80 | 0.85 | 0.90 | 0.95 | 0.975 | 0.99 | 0.995 |
| 1 | 1.000 | 1.376 | 1.963 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 |
| 2 | 0.816 | 1.061 | 1.386 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 |
| 3 | 0.765 | 0.978 | 1.250 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 |
| 4 | 0.741 | 0.941 | 1.190 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 |
| 5 | 0.727 | 0.920 | 1.156 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 |
| | | | | | | | | |
| 6 | 0.718 | 0.906 | 1.134 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 |
| 7 | 0.711 | 0.896 | 1.119 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 |
| 8 | 0.706 | 0.889 | 1.108 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 |
| 9 | 0.703 | 0.883 | 1.100 | 1.383 | 1.833 | **2.262** | 2.821 | 3.250 |
| 10 | 0.700 | 0.879 | 1.093 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 |

# Confidence interval:

- ( 32.4818-2.512 = 29.9698, 32.4818 + 2.512 = 34.9938 )
- The interpretation is that with a probability of 0.95, the random interval (29.9698, 34.9938) includes the real value of the mean.

# More replications needed.

- If we specify that we want an interval between a 5% of the sample mean with a confidence level of a 95%, we need more replications.
- $0.05 \cdot ( 32.4818 ) = 1.62$ but we have 2.512

# Number of needed replications

- where:
- $n$ = initial number of replications.
- $n^*$ = total replications needed.
- $h$ = half-range of the confidence interval for the initial number of replications.
- $h^*$ = half-range of the confidence interval for all the replications (the desired half-range).

$$n^* = n\left(\frac{h}{h^*}\right)^2$$

# Number of replications calculus.

$$n^* = 10\left(\frac{2.512}{1.62}\right)^2 = 24.04$$

# More replications…

| Replication | Value from the model |
|---|---|
| 11 | 33.020 |
| 12 | 29.472 |
| 13 | 27.693 |
| 14 | 31.803 |
| 15 | 30.604 |
| 16 | 33.227 |
| 17 | 28.085 |
| 18 | 35.910 |
| 19 | 30.729 |
| 20 | 30.844 |
| 21 | 32.420 |
| 22 | 39.040 |
| 23 | 32.341 |
| 24 | 34.310 |
| 25 | 28.418 |

# New mean and variance

$$\bar{y} = 32.1094$$

$$S^2 = 3.1903$$

# New self-confidence interval

- In that case is enough, but the process can be iterative.

$$h = t_{1-\alpha/2,n-1} \frac{S}{\sqrt{n}}$$

h = 1.3144 < 1.62

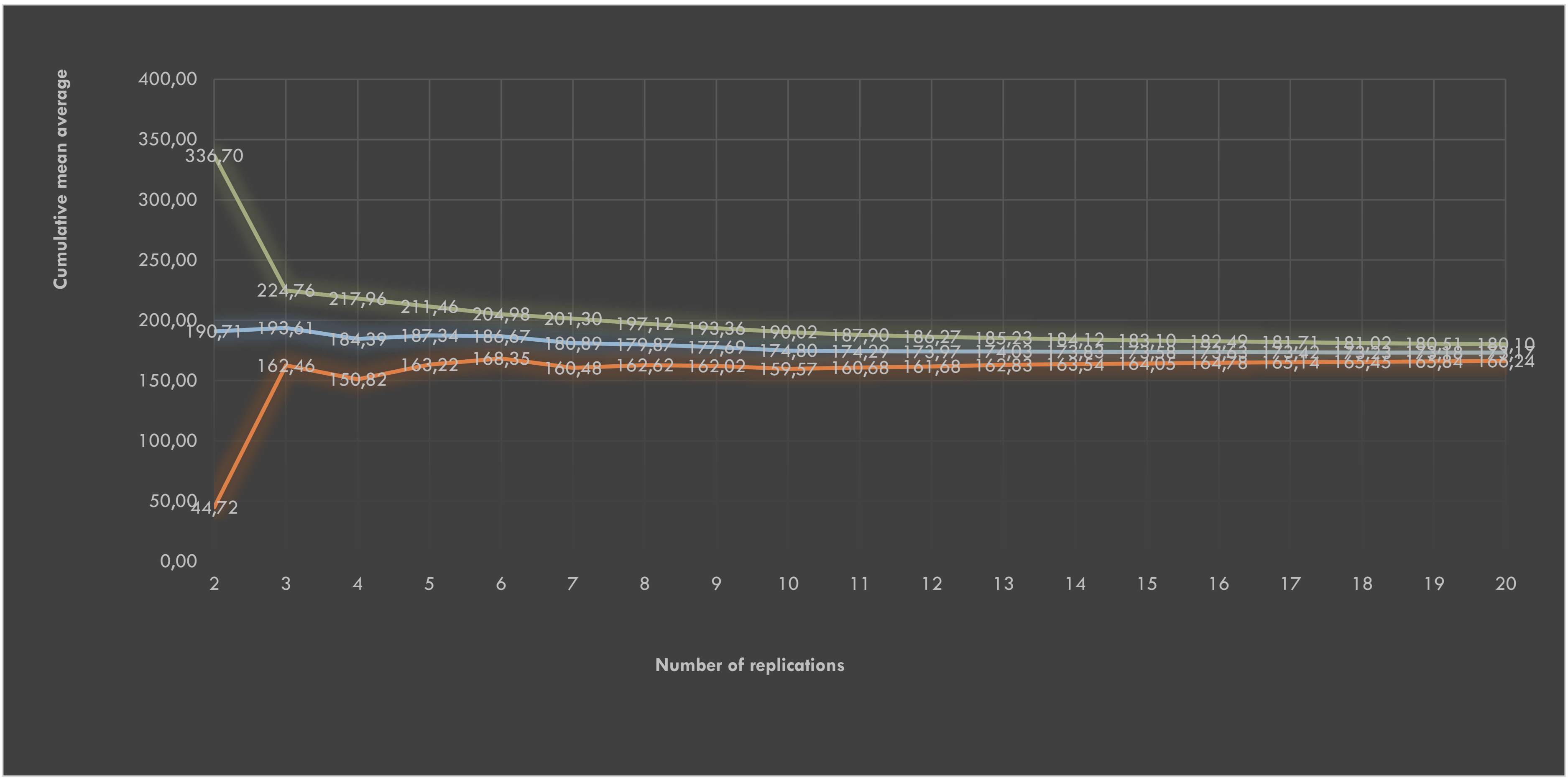
# Exercise 3

- Calculate the amount of replications needed for the Exercise 2

| Cleaner | Machines | Workers | x1 | x2 |
|---|---|---|---|---|
| - | - | - | 20,885 | 20,261 |
| - | - | + | 33,836 | 36,368 |
| - | + | - | 9,9099 | 142 |
| - | + | + | 17,766 | 131,13 |
| + | - | - | 42,759 | 0,0402 |
| + | - | + | 5,7025 | 2,327 |
| + | + | - | 10,481 | 8,7404 |
| + | + | + | 5,9775 | 5,1167 |

| | 1 − a | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| r | 0.75 | 0.80 | 0.85 | 0.90 | 0.95 | 0.975 | 0.99 | 0.995 |
| 1 | 1.000 | 1.376 | 1.963 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 |
| 2 | 0.816 | 1.061 | 1.386 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 |
| 3 | 0.765 | 0.978 | 1.250 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 |
| 4 | 0.741 | 0.941 | 1.190 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 |
| 5 | 0.727 | 0.920 | 1.156 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 |
| | | | | | | | | |
| 6 | 0.718 | 0.906 | 1.134 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 |
| 7 | 0.711 | 0.896 | 1.119 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 |
| 8 | 0.706 | 0.889 | 1.108 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 |
| 9 | 0.703 | 0.883 | 1.100 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 |
| 10 | 0.700 | 0.879 | 1.093 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 |
| | | | | | | | | |
| 11 | 0.697 | 0.876 | 1.088 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 |
| 12 | 0.695 | 0.873 | 1.083 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 |
| 13 | 0.694 | 0.870 | 1.079 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 |
| 14 | 0.692 | 0.868 | 1.076 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 |
| 15 | 0.691 | 0.866 | 1.074 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 |
| | | | | | | | | |
| 16 | 0.690 | 0.865 | 1.071 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 |
| 17 | 0.689 | 0.863 | 1.069 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 |
| 18 | 0.688 | 0.862 | 1.067 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 |
| 19 | 0.688 | 0.861 | 1.066 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 |
| 20 | 0.687 | 0.860 | 1.064 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 |
| | | | | | | | | |
| 21 | 0.686 | 0.859 | 1.063 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 |
| 22 | 0.686 | 0.858 | 1.061 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 |
| 23 | 0.685 | 0.858 | 1.060 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 |
| 24 | 0.685 | 0.857 | 1.059 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 |
| 25 | 0.684 | 0.856 | 1.058 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 |
| | | | | | | | | |
| 26 | 0.684 | 0.856 | 1.058 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 |
| 27 | 0.684 | 0.855 | 1.057 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 |
| 28 | 0.683 | 0.855 | 1.056 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 |
| 29 | 0.683 | 0.854 | 1.055 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 |
| 30 | 0.683 | 0.854 | 1.055 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 |
| | | | | | | | | |
| 40 | 0.681 | 0.851 | 1.050 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 |
| 60 | 0.679 | 0.848 | 1.046 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 |
| 120 | 0.677 | 0.845 | 1.041 | 1.289 | 1.658 | 1.980 | 2.358 | 2.617 |
| ¥ | 0.674 | 0.842 | 1.036 | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 |

# With more replications



# Replications

Methods to execute the replications.

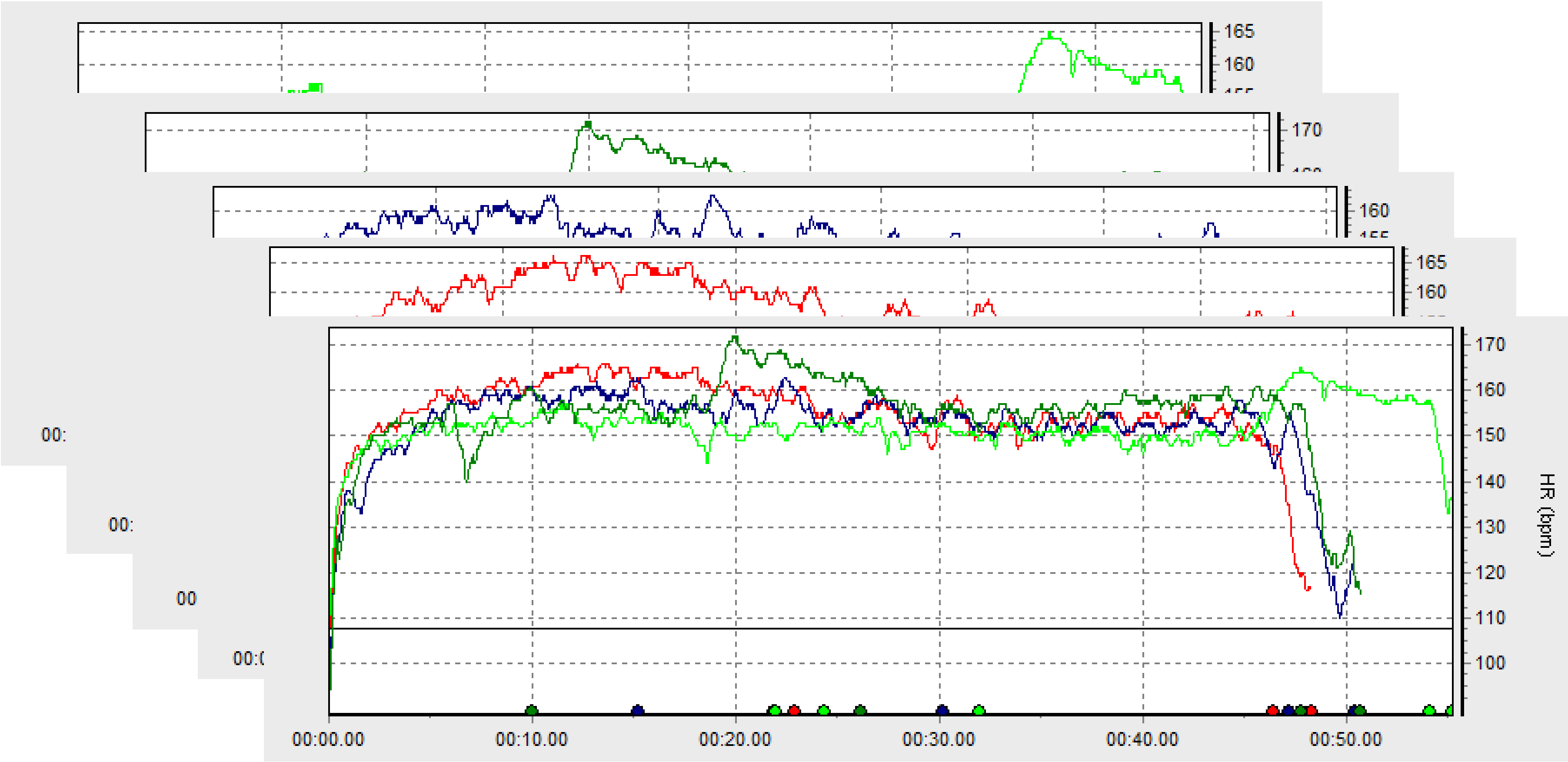
# Kind of simulations

- **Finite simulations:** Simulations where a condition defines the end of the execution. Usually time.
- **No finite simulations:** Simulations without this condition.

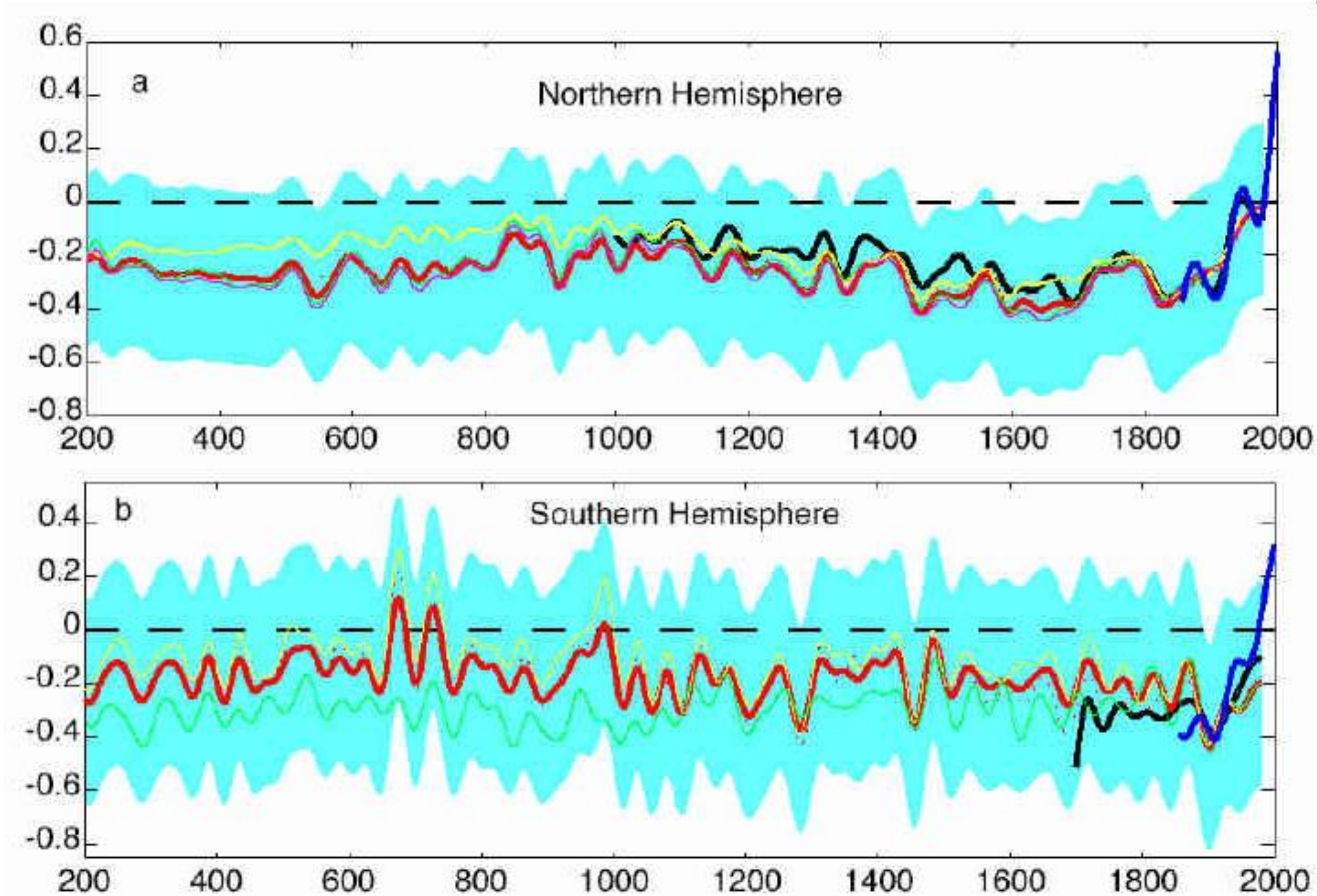
# Independent repetitions

- From the same initial state of the model, that means, with the same parameterizations and behavior, only random numbers to be used un the GAV are changed.
- This different RNG allows test again and again the new system with the different possible values of the variables that are not controlled (random variables).

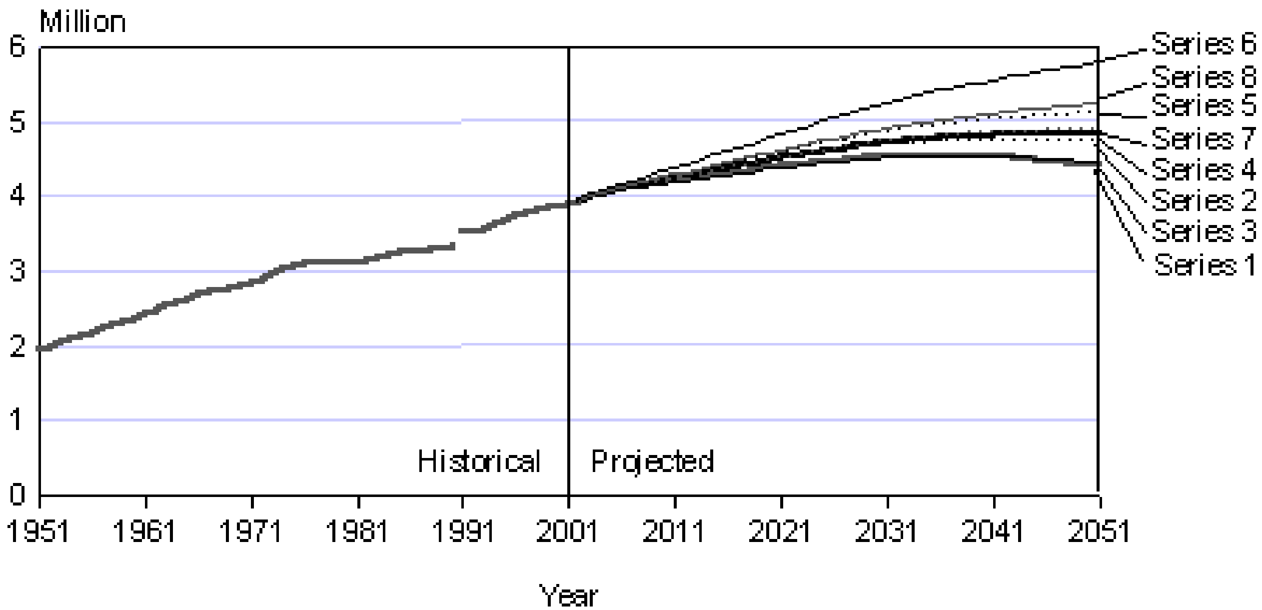
# Independent repetitions



# Independent repetitions



# Independent repetitions




**Note:** The break in series between 1990 and 1991 denotes a change from the de facto population concept to the resident population concept.

# Transient elimination

- Definitions:
  - k: number of deleted observations.
  - m: amount of observations in a single run.
  - n: amount of runs (replications).

# Transient elimination

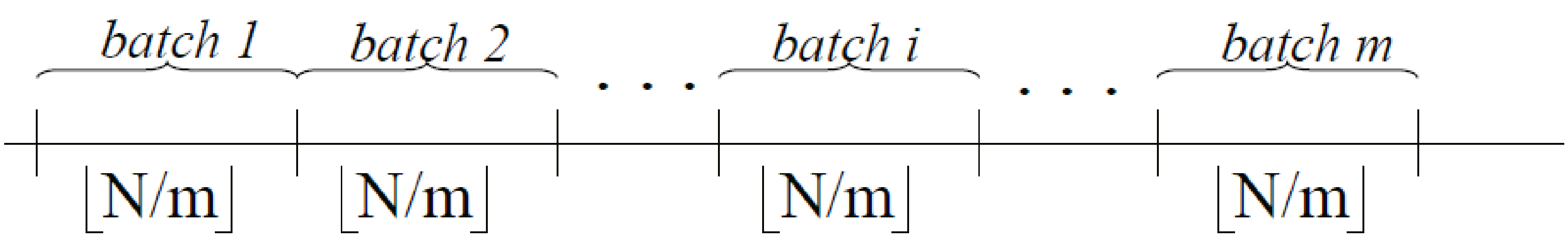
- Step 1: Compute the average of the j observation's over all runs.
  - $\bar{x}_j = \frac{1}{m}\sum_{i=1}^{m} x_{i,j}$
- Step 2: Compute the overall average.
  - $\bar{\bar{x}} = \frac{1}{n}\sum_{j=1}^{n} \bar{x}_j$
- Step 3: K=1

# Transient elimination

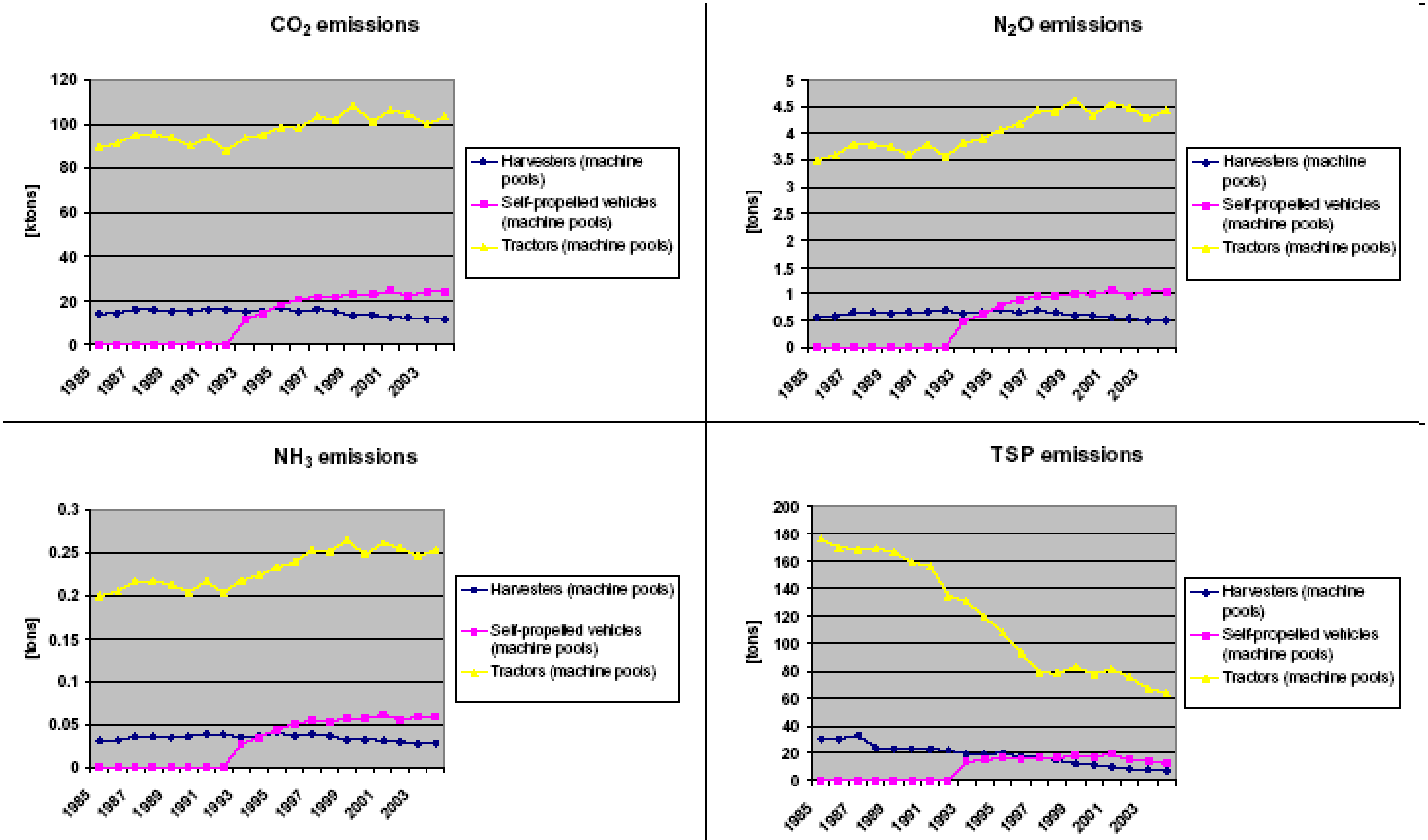
- Step 4: Compute the overall average of the j observation's over all runs without the first k observations.
  - $\overline{\overline{x_k}} = \frac{1}{n-k}\sum_{j=k+1}^{n} \bar{x}_j$
- Step 5: Calculate the relative change
  - $\Delta = \frac{\bar{\bar{x}}_k - \bar{\bar{x}}}{\bar{\bar{x}}}$
- If $|\Delta_k - \Delta_{k-1}| >$ threshold then increment k and go to step 4, else remove k observations and use

# Batch means

- Execute a long simulation and then divide it in different blocks, or execution bags “batches”.
  - We work with the mean values of these observations.
  - The size of the “batches” is $\mathrm{n} = \lfloor N/m \rfloor$
- Each one of these observations are considered as independent.
- Is desirable to determine what must be the required long of each one of these execution blocks, to assure the correctness of the experiment.



# Batch means



$CO_2$ emissions
[ktons]
120
100
80
60
40
20
0
1985 1987 1989 1991 1993 1995 1997 1999 2001 2003
Harvesters (machine pools)
Self-propelled vehicles (machine pools)
Tractors (machine pools)
$N_2O$ emissions
[tons]
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
1985 1987 1989 1991 1993 1995 1997 1999 2001 2003
Harvesters (machine pools)
Self-propelled vehicles (machine pools)
Tractors (machine pools)
$NH_3$ emissions
[tons]
0.3
0.25
0.2
0.15
0.1
0.05
0
1985 1987 1989 1991 1993 1995 1997 1999 2001 2003
Harvesters (machine pools)
Self-propelled vehicles (machine pools)
Tractors (machine pools)
TSP emissions
[tons]
200
180
160
140
120
100
80
60
40
20
0
1985 1987 1989 1991 1993 1995 1997 1999 2001 2003
Harvesters (machine pools)
Self-propelled vehicles (machine pools)
Tractors (machine pools)

# Transient elimination

- Step 1: Set n=2
- Step 2: Compute the average of the "i" batch.
  - $\bar{x}_i = \frac{1}{n} \sum_{j=1}^{n} x_{i,j}$
- Step 3: Compute the overall average.
  - $\bar{\bar{x}} = \frac{1}{m} \sum_{i=1}^{m} \bar{x}_i$
- Step 4: Compute the variance of the batch means.
  - $Var(\bar{x}) = \frac{1}{m-1} \sum_{i=1}^{m} (\bar{x}_i - \bar{\bar{x}})^2$

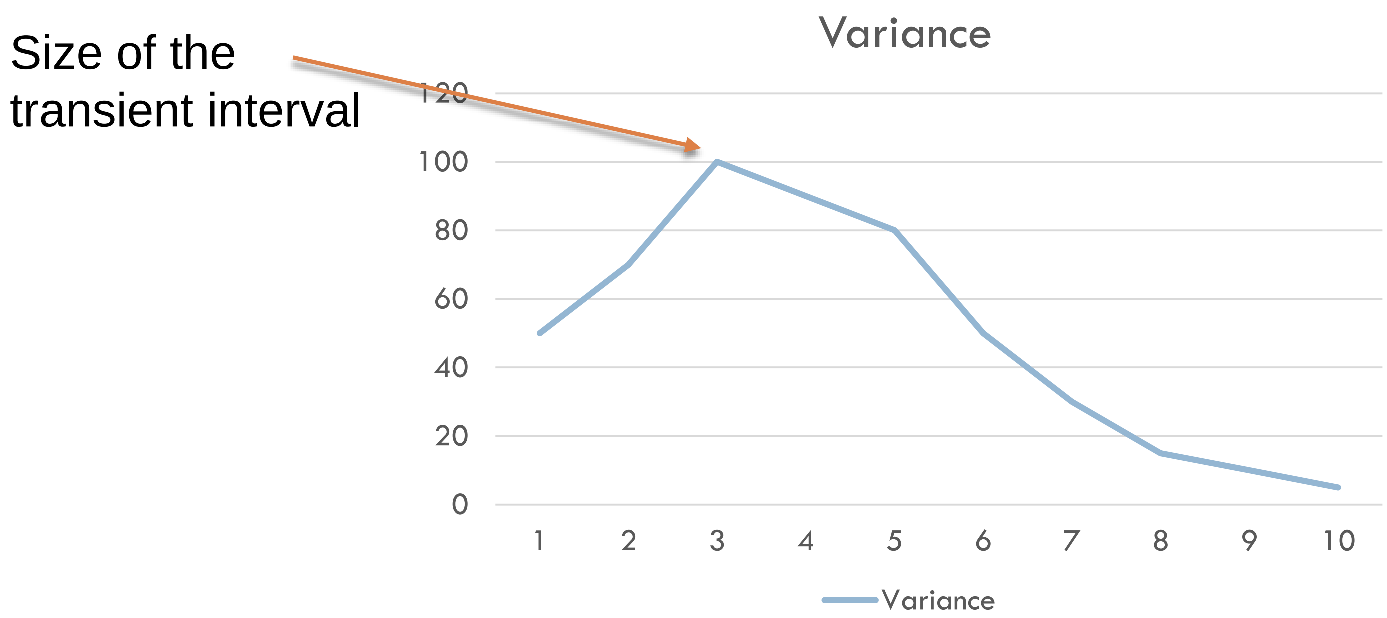
Definitions:

n: amount of observations in a single run.

m: amount of runs (replications).

# Transient elimination

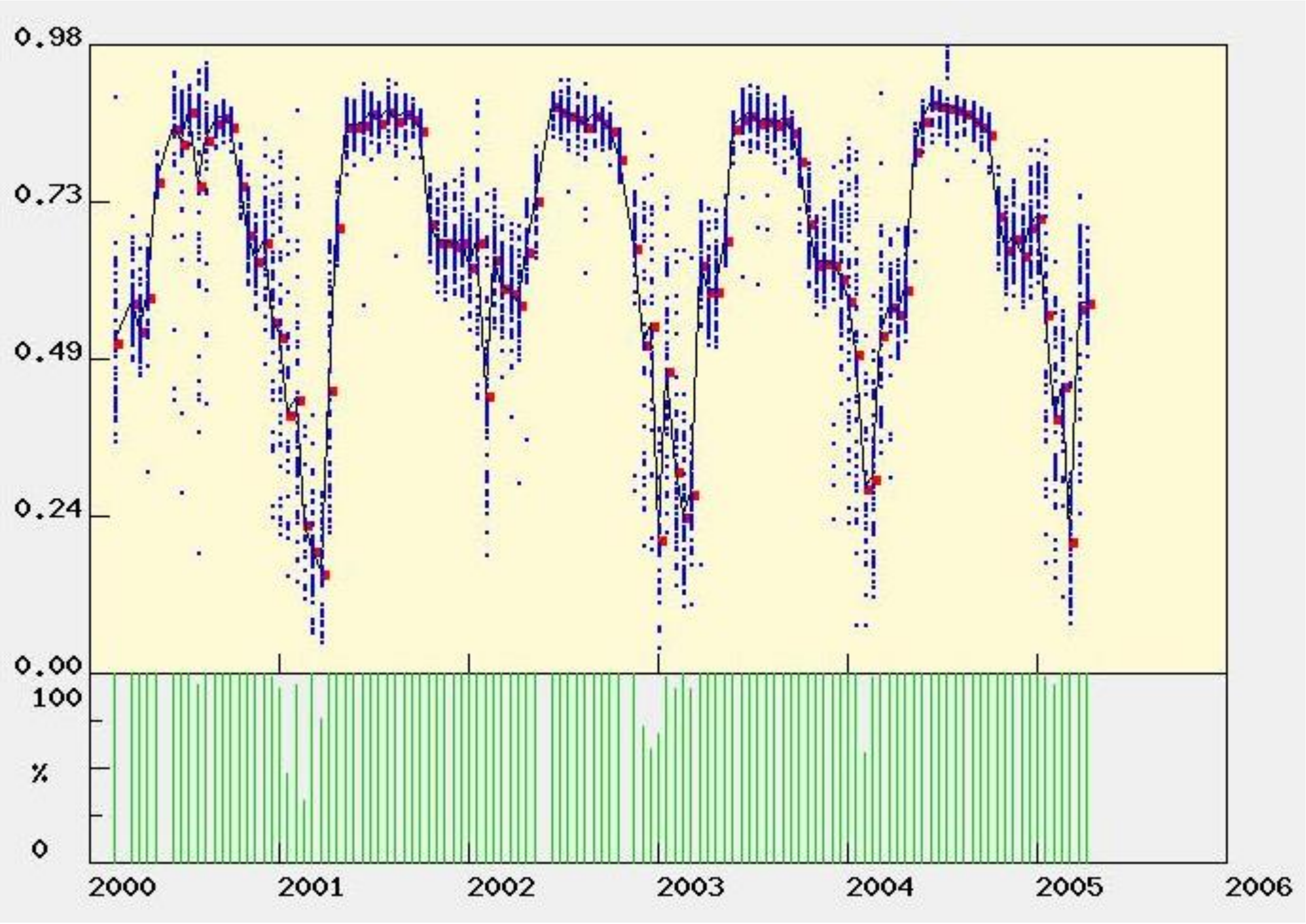
- Step 5: Increase n by 1 and go to step 2 and plot the variance as a function of n. The point at which the variance starts to decreases is the length of the transient interval.



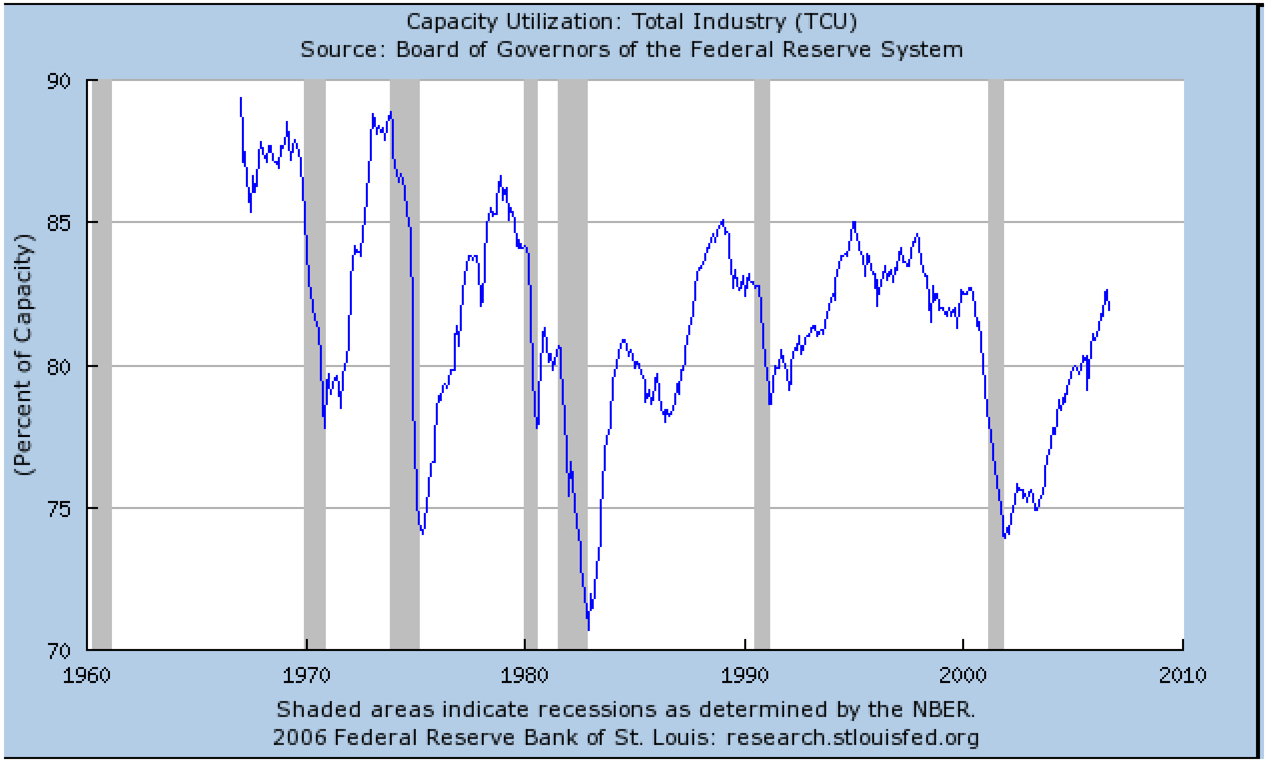
# Regenerative methods

- If the variables observed in the execution of the simulation model, represents, in some way a cyclical restart, that allows suppose the existence of cycles (in the life of the variable). Is likely to consider each one of theses cycles as a replication
- This method is not always applicable. Depends on the existence of cycles in the variables. Also the longitude of this replications must be small; if the longitude of this cycles is big we obtain a small sum of replications.

# Regenerative methods

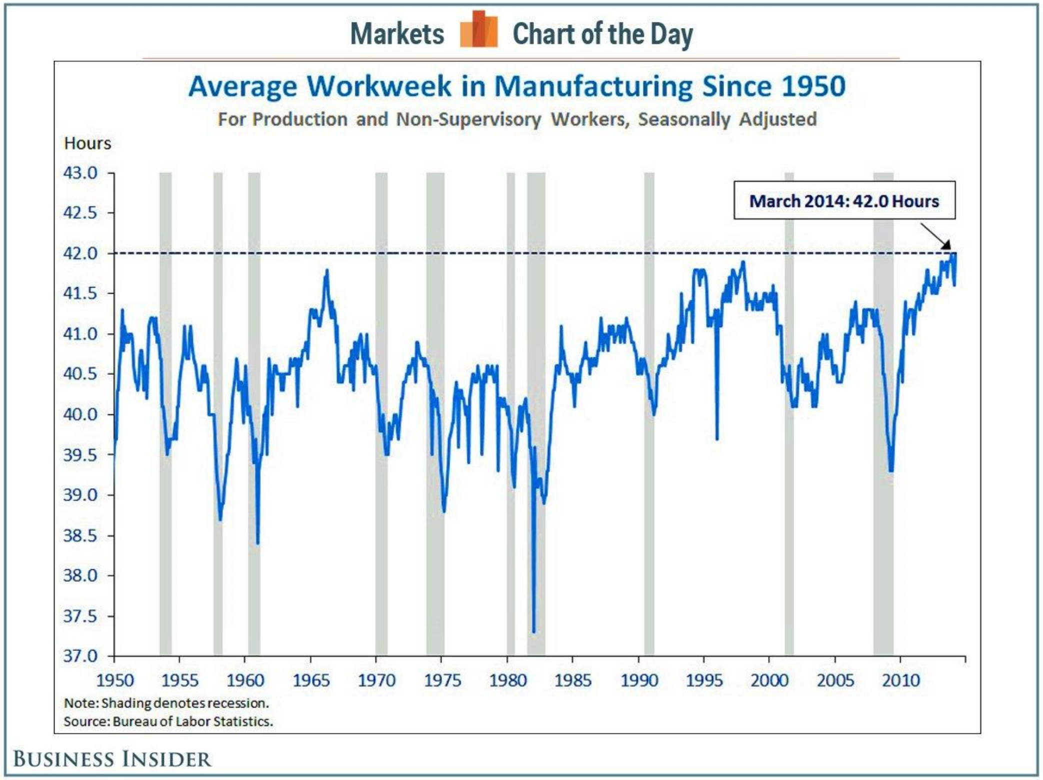


# Regenerative methods



# Regarding the chart

- Nobel laureate and Yale University economist Robert Shiller is in the camp of experts who believe the odds of a recession are very low.
  - Read more: http://www.businessinsider.com/shiller-chart-shows-why-recession-is-years-away-2014-4#ixzz30lcrlnl2



# Regenerative methods

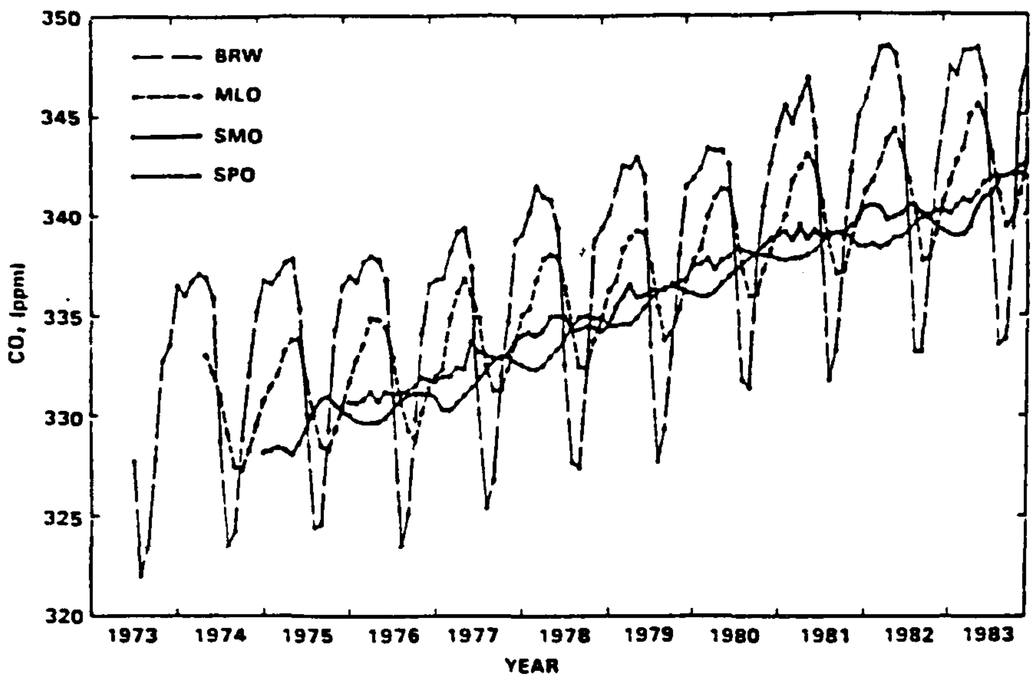



Fig. 1. Selected monthly mean carbon dioxide concentrations from continuous measurements (Barrow, Alaska (BRW); Mauna Loa, Hawaii (MLO); American Samoa (SMO); South Pole (SPO). From: WMO, 1985.

# Applicability

| | Finite simulations | No finite simulations |
|---|---|---|
| Loading period needed | Independent repetitions | Independent repetitions |
| Loading period unneeded | Independent repetitions erasing the loading period/ Batch means | Batch means |

# Variance reduction techniques

Reduce the number of replications

# Motivation

- Interest to reduce the variability introduced in the answer variable due to the use of RNG.
- The value that estimates an specific answer variable, that is represented by its confidence interval, must be adjusted (as possible).

$$(\bar{x} - k\, {}^{s}\!/\!_{\sqrt{n}},\ \bar{x} + k\, {}^{s}\!/\!_{\sqrt{n}})$$

# Motivation

- Obviously, increasing n, that is the number of observations, the standard error decreases. Variance reduction techniques try to reduce this variability without the need of increase the number of observations.

$$s/\sqrt{n}$$

# Common random numbers

- Using the same random number stream for the different configurations.
- Both streams represents “identical conditions” for both configurations.
- Is needed to establish mechanism to synchronize the streams.

# Control variables

- Simulation allows the observation of the system evolution during the execution of the experiment. This allows, in certain grade, to compare the values of the answer variables with the observed values.
- We can add modification to reduce the difference.

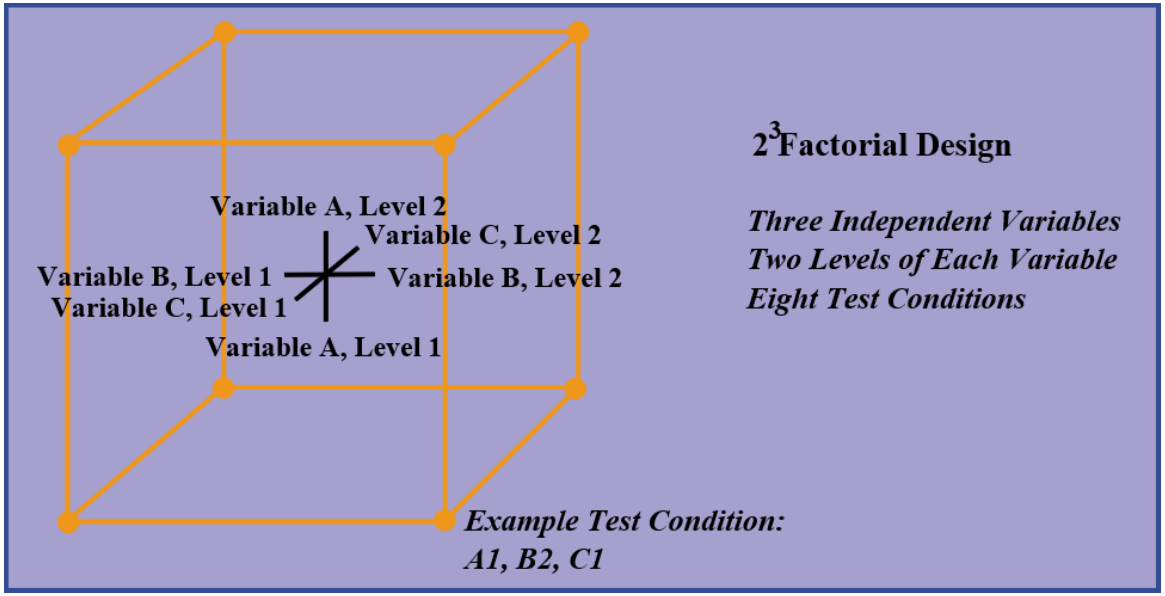
# Fractional factorial design

# Fractional factorial design

- A factorial experiment in which only an **adequately chosen fraction** of the treatment combinations required for the complete factorial experiment **is selected to be run.**

# A $2^{3-1}$ design (half of a $2^3$)

- Consider the two-level, full factorial design for three factors, namely the $2^3$ design. This implies eight runs (not counting replications or center points).




By Nicoguaro - Own work, Public Domain, https://commons.wikimedia.org/w/index.php?curid=12758102

# $2^3$ Two-level, Full Factorial Design

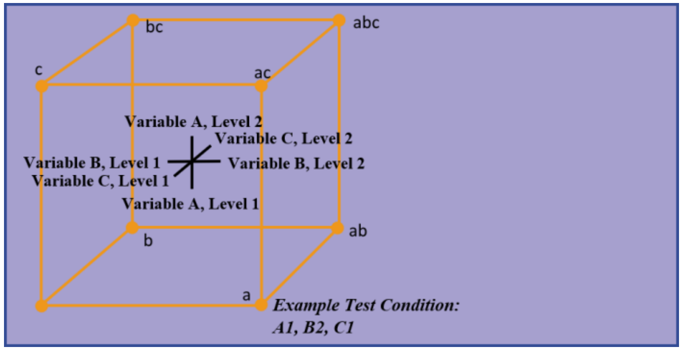
| | X1 | X2 | X3 | Y |
|---|---|---|---|---|
| 1 | -1 | -1 | -1 | $y_1 = 33$ |
| 2 | +1 | -1 | -1 | $y_2 = 63$ |
| 3 | -1 | +1 | -1 | $y_3 = 41$ |
| 4 | +1 | +1 | -1 | $Y_4 = 57$ |
| 5 | -1 | -1 | +1 | $y_5 = 57$ |
| 6 | +1 | -1 | +1 | $y_6 = 51$ |
| 7 | -1 | +1 | +1 | $y_7 = 59$ |
| 8 | +1 | +1 | +1 | $y_8 = 53$ |

# Computing the effects

- Effect of C1= (1/4)(y2 + y4 + y6 + y8) - (1/4)(y1 + y3 + y5 + y7)
- c1 = (1/4)(63+57+51+53 ) - (1/4)(33+41+57+59) = 8.5
- Suppose, however, that we only have enough resources to do four runs. Is it still possible to estimate the main effect for X1? Or any other main effect?
  - The answer is yes, and there are even different choices of the four runs that will accomplish this.

# Only 4 runs

| | C1 | C2 | C3 | Y |
|---|---|---|---|---|
| 1 | -1 | -1 | -1 | $y_1 = 33$ |
| 2 | +1 | -1 | -1 | $y_2 = 63$ |
| 3 | -1 | +1 | -1 | $y_3 = 41$ |
| 4 | +1 | +1 | -1 | $Y_4 = 57$ |
| 5 | -1 | -1 | +1 | $y_5 = 57$ |
| 6 | +1 | -1 | +1 | $y_6 = 51$ |
| 7 | -1 | +1 | +1 | $y_7 = 59$ |
| 8 | +1 | +1 | +1 | $y_8 = 53$ |



# Main effects

- C1 main effect:
  - $c1 = (1/2)(y4 + y6) - (1/2)(y1 + y7)$
  - $c1 = (1/2)(57+51) - (1/2)(33+59) = 8$
- C2 main effect
  - $c2 = (1/2)(y4 + y7) - (1/2)(y1 + y6)$
  - $c2 = (1/2)(57+59) - (1/2)(33+51) = 16$
- C3 main effect
  - $c3 = (1/2)(y6 + y7) - (1/2)(y1 + y4)$
  - $c3 = (1/2)(51+59) - (1/2)(33+57) = 10$

# Selecting the experiments to execute

- Note that, mathematically, $2^{3-1} = 2^2$

| | X1 | X2 |
|---|---|---|
| 1 | - | - |
| 2 | + | - |
| 3 | - | + |
| 4 | + | + |

# Adding the column of the interactions

- We add a new column that represents the interactions between X1 and X2

| | X1 | X2 | X1*X2 |
|---|---|---|---|
| 1 | - | - | + |
| 2 | + | - | - |
| 3 | - | + | - |
| 4 | + | + | + |

# Adding the column for the new factor

- Now we can substitute this new column for X3

| | X1 | X2 | X3 |
|---|---|---|---|
| 1 | - | - | + |
| 2 | + | - | - |
| 3 | - | + | - |
| 4 | + | + | + |

# Example

- We have 4 factors P, T, D, E.
- $2^4$.
- We want to perform at maximum 8 experiments.

# Example

| | P | T | D | E=P*T |
|---|---|---|---|---|
| 1 | - | - | - | + |
| 2 | + | - | - | - |
| 3 | - | + | - | - |
| 4 | + | + | - | + |
| 5 | - | - | + | + |
| 6 | + | - | + | - |
| 7 | - | + | + | - |
| 8 | + | + | + | + |

# Confounding

- A confounding design is one where some treatment effects (main or interactions) are estimated by the same linear combination of the experimental observations as some blocking effects.

# The price

- One price we pay for using the design table column X1*X2 to obtain column X3 is, clearly, our **inability** to obtain an **estimate of the interaction effect** for X1*X2 (i.e., c12) that is separate from an estimate of the main effect for X3.
- We have **confounded** the **main effect** estimate for factor X3 (i.e., c3) with the **estimate of the interaction effect** for X1 and X2 (i.e., with c12)

# Notation

- X3 = X1*X2 can be represented by:
  - 3=12
- Playing with this
- Multiplying with 3
  - 33=123, and 33=I (identity)
  - I=123, 2I=2**12**3, 2I=2, I=22

    3

  - I=123 is the design generator
  - 1=23, 2=13, 3=12, I=123 aliases

# Principal fraction

- We can replace any design generator by its negative counterpart and have an equivalent, but different fractional design.
- The fraction generated by positive design generators is sometimes called the principal fraction.

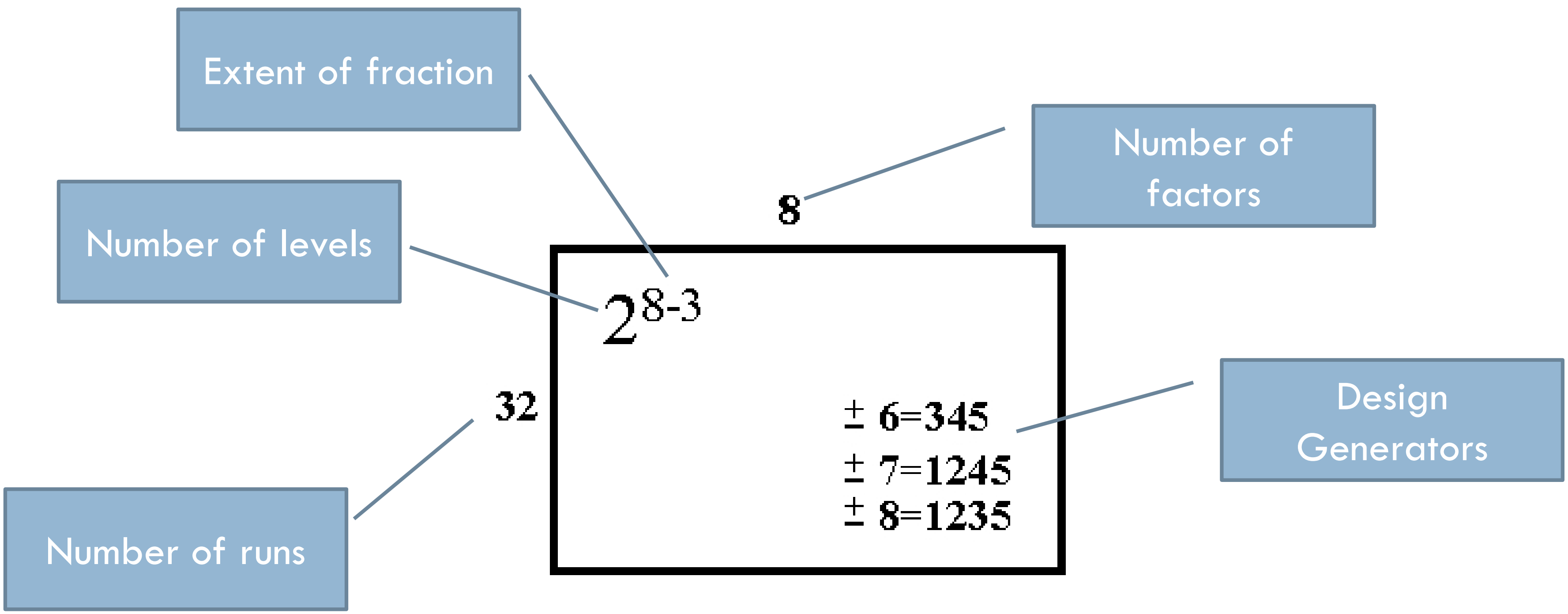
# Confounding pattern

- The confounding pattern described by 1=23, 2=13, and 3=12 tells us that all the main effects of $2^{3-1}$ design are confounded with two-factor interactions.
  - That is the price we pay for using this fractional design.

# Confounding pattern

- In the typical quarter-fraction of a $2^6$ design, i.e., in a $2^{6-2}$ design, main effects are confounded with three-factor interactions (e.g., 5=123) and so on.
- In the case of 5=123, we can also readily see that 15=23 (etc.), which alerts us to the fact that certain two-factor interactions of a $2^{6-2}$ are confounded with other two-factor interactions.

# Definition of the experiment



# How to construct this experiment?

8

32

$2^{8-3}$

$\pm 6=345$

$\pm 7=1245$

$\pm 8=1235$

# Construct a Fractional Factorial Design From the Specification

- **Write down a full factorial design** in standard order for *k*-*p* factors (8-3 = 5 factors for the example above). In the specification above we start with a $2^5$ full factorial design. Such a design has $2^5 = 32$ rows.
- **Add a sixth column** to the design table for factor 6, using 6 = 345 (or 6 = -345) to manufacture it (i.e., create the new column by multiplying the indicated old columns together).
- **Do likewise** for factor 7 and for factor 8, using the appropriate design generators.
- The resultant design matrix gives the **32 trial runs** for an 8-factor fractional factorial design.

# Example: $2^{8-3}$

8

32

$2^{8-3}$

$\pm$ 6=345
$\pm$ 7=1245
$\pm$ 8=1235

| X1 | X2 | X3 | X4 | X5 | X6 | X7 | X8 |
|---|---|---|---|---|---|---|---|
| -1 | -1 | -1 | -1 | -1 | | | |
| 1 | -1 | -1 | -1 | -1 | | | |
| -1 | 1 | -1 | -1 | -1 | | | |
| 1 | 1 | -1 | -1 | -1 | | | |
| -1 | -1 | 1 | -1 | -1 | | | |
| 1 | -1 | 1 | -1 | -1 | | | |
| -1 | 1 | 1 | -1 | -1 | | | |
| 1 | 1 | 1 | -1 | -1 | | | |
| -1 | -1 | -1 | 1 | -1 | | | |
| 1 | -1 | -1 | 1 | -1 | | | |
| -1 | 1 | -1 | 1 | -1 | | | |
| 1 | 1 | -1 | 1 | -1 | | | |
| -1 | -1 | 1 | 1 | -1 | | | |
| 1 | -1 | 1 | 1 | -1 | | | |
| -1 | 1 | 1 | 1 | -1 | | | |
| 1 | 1 | 1 | 1 | -1 | | | |
| -1 | -1 | -1 | -1 | 1 | | | |
| 1 | -1 | -1 | -1 | 1 | | | |
| -1 | 1 | -1 | -1 | 1 | | | |
| 1 | 1 | -1 | -1 | 1 | | | |
| -1 | -1 | 1 | -1 | 1 | | | |
| 1 | -1 | 1 | -1 | 1 | | | |
| -1 | 1 | 1 | -1 | 1 | | | |
| 1 | 1 | 1 | -1 | 1 | | | |
| -1 | -1 | -1 | 1 | 1 | | | |
| 1 | -1 | -1 | 1 | 1 | | | |
| -1 | 1 | -1 | 1 | 1 | | | |
| 1 | 1 | -1 | 1 | 1 | | | |
| -1 | -1 | 1 | 1 | 1 | | | |
| 1 | -1 | 1 | 1 | 1 | | | |
| -1 | 1 | 1 | 1 | 1 | | | |
| 1 | 1 | 1 | 1 | 1 | | | |

# $2^{8-3}$

8

32

$2^{8-3}$

$\pm\ 6=345$

$\pm\ 7=1245$

$\pm\ 8=1235$

| X1 | X2 | X3 | X4 | X5 | X6 | X7 | X8 |
|---|---|---|---|---|---|---|---|
| -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 |
| 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 |
| 1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 |
| -1 | -1 | 1 | -1 | -1 | 1 | 1 | -1 |
| 1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 |
| -1 | 1 | 1 | -1 | -1 | 1 | -1 | 1 |
| 1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 |
| -1 | -1 | -1 | 1 | -1 | 1 | -1 | 1 |
| 1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 |
| -1 | 1 | -1 | 1 | -1 | 1 | 1 | -1 |
| 1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 |
| -1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 |
| 1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 |
| -1 | 1 | 1 | 1 | -1 | -1 | 1 | 1 |
| 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 |
| -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 |
| 1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 |
| -1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 |
| 1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 |
| -1 | -1 | 1 | -1 | 1 | -1 | -1 | 1 |
| 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 |
| -1 | 1 | 1 | -1 | 1 | -1 | 1 | -1 |
| 1 | 1 | 1 | -1 | 1 | -1 | -1 | 1 |
| -1 | -1 | -1 | 1 | 1 | -1 | 1 | -1 |
| 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 |
| -1 | 1 | -1 | 1 | 1 | -1 | -1 | 1 |
| 1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 |
| -1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | -1 | 1 | 1 | 1 | 1 | -1 | -1 |
| -1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

# Words

- There are seven "*words*", or strings of numbers, in the defining relation for the $2^{8-3}$ design, starting with the original three generators and adding all the new "words" that can be formed by multiplying together any two or three of these original three words. These seven turn out to be I = 3456 = 12457 = 12358 = 12367 = 12468 = 3478 = 5678.
- In general, there will be $(2^p - 1)$ words in the defining relation for a $2^{k-p}$ fractional factorial.

# Resolution

- The length of the shortest word in the defining relation is called the resolution of the design.
- Resolution describes the degree to which estimated main effects are confounded (or aliased) with estimated 2-level interactions, 3-level interactions, etc.
- Resolution is added as a Roman numeral to the experiment definition.

# Complete definition of the DOE

8

32

$$2^{8-3}_{IV}$$

$\pm\ 6=345$

$\pm\ 7=1245$

$\pm\ 8=1235$

# Example

- We have a limited budget to analyze the different factors to consider on our model. Each individual experiment costs 100€ and we have a total budget of 20.000€ to be destined to experimentation. Define an experiment design, with this constraint, considering that we need at least 3 replications for each experiment.

| GORE | BUSH | BUCHANAN | NADER | BROWNE | HAGELIN | HARRIS | MCREYNOLDS | MOOREHEAD | PHILLIPS |
|---|---|---|---|---|---|---|---|---|---|
| ALACHUA | 47300 | 34062 | 262 | 3215 | 658 | 42 | 4 | 658 | 21 |
| BAKER | 2392 | 5610 | 73 | 53 | 17 | 3 | 0 | 0 | 3 |
| BAY | 18850 | 38637 | 248 | 828 | 171 | 18 | 5 | 3 | 37 |
| BRADFORD | 3072 | 5413 | 65 | 84 | 28 | 2 | 0 | 0 | 3 |
| BREVARD | 97318 | 115185 | 570 | 4470 | 643 | 39 | 11 | 11 | 76 |
| | | | | | | | | | More than 100 cases... |

# Answer

- We consider that GORE categorical variable are not going to be used on our analysis.
- Assuming that we can spend all the budget in an initial experimentation without considering the possible inconvenient that can appear due to possible high variability of certain variables (that lead us to increase the number of needed replications that is initial considered by 3), the maximum amount of experiment is 64:
  - $64\ experiment * 3\ replications * 100€ = 19200€$

# Answer

- This design don't allow a complete experimentation considering all the factors we have on our model, we have 9 factors that implies a 2^9 experiments meaning 512 experiments with 3 replications each one of them.
- This implies that we need to reduce the amount of variables to be considered using a fractional factorial design.

# Answer

- To do this it is needed to define a confounding pattern and select those factors that are going to be confounded.
- The design will be defined as:
  - $2^{9-3}$
- Hence 3 confounding patterns must be defined.
- We select in our case the last four variables to be confounded using the first 6 variables.

# Answer

9

64

$2^{9-3}$

VI

$\pm 7 = 123456$

$\pm 8 = 12345$

$\pm 9 = 12346$

# Plackett-Burman designs

# Plackett-Burman designs

- In 1946, R.L. Plackett and J.P. Burman published their now famous paper "The Design of Optimal Multifactorial Experiments" in *Biometrika* (vol. 33). This paper described the construction of very economical designs with the run number a multiple of four (rather than a power of 2).
- Plackett-Burman designs are very efficient screening designs when only main effects are of interest.

# Plackett and Burman designs (1946)

- Effects of main factors only
  - Logically minimal number of experiments to estimate effects of m input parameters (factors)
  - Ignores interactions
- Requires O(m) experiments
  - Instead of O(2m) or O(vm)

# Plackett and Burman Designs

- PB designs exist only in sizes that are multiples of 4
- Requires X experiments for m parameters
  - X = next multiple of 4 $\geq$ m
- PB design matrix
  - Rows = configurations
  - Columns = factor's values in each configuration
    - High/low = +1/ -1
  - First row = from P&B paper
  - Subsequent rows = circular right shift of preceding row
  - Last row = all (-1)

# Plackett and Burman Designs

- PB designs also exist for 20-run, 24-run, and 28-run (and higher) designs.
- With a 20-run design you can run a screening experiment for up to 19 factors, up to 23 factors in a 24-run design, and up to 27 factors in a 28-run design.

# PB Design Matrix

| Config | Input Parameters (factors) | | | | | | | Response |
|---|---|---|---|---|---|---|---|---|
| | **A** | **B** | **C** | **D** | **E** | **F** | **G** | |
| 1 | +1 | +1 | +1 | -1 | +1 | -1 | -1 | |
| 2 | -1 | +1 | +1 | +1 | -1 | +1 | -1 | |
| 3 | -1 | -1 | +1 | +1 | +1 | -1 | +1 | |
| 4 | +1 | -1 | -1 | +1 | +1 | +1 | -1 | |
| 5 | -1 | +1 | -1 | -1 | +1 | +1 | +1 | |
| 6 | +1 | -1 | +1 | -1 | -1 | +1 | +1 | |
| 7 | +1 | +1 | -1 | +1 | -1 | -1 | +1 | |
| 8 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | |
| **Effect** | | | | | | | | |

# PB Design Matrix

| Config | Input Parameters (factors) | | | | | | | Response |
|---|---|---|---|---|---|---|---|---|
| | A | B | C | D | E | F | G | |
| 1 | +1 | +1 | +1 | -1 | +1 | -1 | -1 | 10 |
| 2 | -1 | +1 | +1 | +1 | -1 | +1 | -1 | 12 |
| 3 | -1 | -1 | +1 | +1 | +1 | -1 | +1 | 3 |
| 4 | +1 | -1 | -1 | +1 | +1 | +1 | -1 | 5 |
| 5 | -1 | +1 | -1 | -1 | +1 | +1 | +1 | 6 |
| 6 | +1 | -1 | +1 | -1 | -1 | +1 | +1 | 5 |
| 7 | +1 | +1 | -1 | +1 | -1 | -1 | +1 | 8 |
| 8 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 9 |
| **Effect** | | | | | | | | |

# PB Design Matrix

| Config | Input Parameters (factors) | | | | | | | Response |
|---|---|---|---|---|---|---|---|---|
| | **A** | **B** | **C** | **D** | **E** | **F** | **G** | |
| 1 | +1 | +1 | +1 | -1 | +1 | -1 | -1 | 10 |
| 2 | -1 | +1 | +1 | +1 | -1 | +1 | -1 | 12 |
| 3 | -1 | -1 | +1 | +1 | +1 | -1 | +1 | 3 |
| 4 | +1 | -1 | -1 | +1 | +1 | +1 | -1 | 5 |
| 5 | -1 | +1 | -1 | -1 | +1 | +1 | +1 | 6 |
| 6 | +1 | -1 | +1 | -1 | -1 | +1 | +1 | 5 |
| 7 | +1 | +1 | -1 | +1 | -1 | -1 | +1 | 8 |
| 8 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 9 |
| **Effect** | -0,5 | | | | | | | |

$$-0.5 = (+1(10) + 1(5) + 1(5) + 1(8))/4 - (1(12) + 1(3) + 1(6) + 1(6))/4$$

# PB Design Matrix

| Config | Input Parameters (factors) | | | | | | | Response |
|---|---|---|---|---|---|---|---|---|
| | A | B | C | D | E | F | G | |
| 1 | +1 | +1 | +1 | -1 | +1 | -1 | -1 | 10 |
| 2 | -1 | +1 | +1 | +1 | -1 | +1 | -1 | 12 |
| 3 | -1 | -1 | +1 | +1 | +1 | -1 | +1 | 3 |
| 4 | +1 | -1 | -1 | +1 | +1 | +1 | -1 | 5 |
| 5 | -1 | +1 | -1 | -1 | +1 | +1 | +1 | 6 |
| 6 | +1 | -1 | +1 | -1 | -1 | +1 | +1 | 5 |
| 7 | +1 | +1 | -1 | +1 | -1 | -1 | +1 | 8 |
| 8 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 9 |
| **Effect** | -0,5 | 3,5 | 0,5 | -0,5 | -2,5 | -0,5 | -3,5 | |

# PB Design

- Magnitude of effect is important, sign is meaningless.
- In the previous example (from most important to least important effects): C, D, E, F, G, A and B.

# Example

- A statistical approach to the experimental design of the sulfuric acid leaching of gold-copper ore.
- http://www.scielo.br/scielo.php?script=sci_arttext&pid=S0104-66322003000300010

# Our factors

**Table 4: Mineralogical analysis of transition ore sample and sulfuric acid leach residue**

| Minerals | Molecular formulae | Assay (%) | |
|---|---|---|---|
| | | Transition ore | Leach residue |
| Native Cu | $Cu$ | 0.39 | 0.30 |
| Chalcopyrite | $CuFeS_2$ | r | r |
| Bornite | $Cu_5FeS_4$ | r | r |
| Chalcosite | $Cu_2S$ | rr | rr |
| Covelite | $CuS$ | rr | rr |
| Cuprite | $Cu_2O$ | t | - |
| Malachite | $Cu_2(CO_3)(OH)_2$ | t | - |
| Goethite/Limonite | $HFeO_2$ /$Fe_2O_3.H_2O$ | 26 | 21 |
| Iron oxide | $Fe_2O_3$, $Fe_3O_4$ | 8 | 7 |
| Clorite | $(Mg,Al,Fe)_{12}[(Si,Al)_8O_{20}](OH)_{16}$ | 33 | 35 |
| Quartz | $SiO_2$ | 26 | 33 |

Notations -: not detected rr: very rare (some cristals) r: rare (~0.2%) t: trace (~0.5%) <1: ~0.8%

**Table 5: Matrix for twelve replicated experiments based on the Plackett-Burman method and their respective copper extraction responses ($R_1$, $R_2$)**

| Test | Variable | | | | | | | | | | | Response (%) | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | A | X1 | B | C | D | X2 | E | F | G | X3 | X4 | $R_1$ | $R_2$ |
| 1 | + | + | - | + | + | + | - | - | - | + | - | 70.5 | 71.8 |
| 2 | + | - | + | + | + | - | - | - | + | - | + | 77.6 | 77.4 |
| 3 | - | + | + | + | - | - | - | + | - | + | + | 67.2 | 68.2 |
| 4 | + | + | + | - | - | - | + | - | + | + | - | 73.2 | 74.4 |
| 5 | + | + | - | - | - | + | - | + | + | - | + | 77.0 | 76.8 |
| 6 | + | - | - | - | + | - | + | + | - | + | + | 76.5 | 77.9 |
| 7 | - | - | - | + | - | + | + | - | + | + | + | 74.0 | 73.5 |
| 8 | - | - | + | - | + | + | - | + | + | + | - | 59.9 | 59.9 |
| 9 | - | + | - | + | + | - | + | + | + | - | - | 73. 9 | 73.9 |
| 10 | + | - | + | + | - | + | + | + | - | - | - | 80.2 | 78.0 |
| 11 | - | + | + | - | + | + | + | - | - | - | + | 66.5 | 64.9 |
| 12 | - | - | - | - | - | - | - | - | - | - | - | 69.3 | 68.0 |

# Latin squares

More than two levels.


Fisher window
Anders Sandberg

# When to apply?

- Latin square designs are used when the factors of interest have more than two levels (k).
- We have tree factors.
- There are no (or only negligible) interactions between factors.

# Pros and cons

- PROS:
  - They handle the case when we have several nuisance factors and we either cannot combine them into a single factor or we wish to keep them separate.
  - They allow experiments with a relatively small number of runs.

- CONS:
  - The number of levels of each blocking variable must equal the number of levels of the treatment factor.
  - The Latin square model assumes that there are no interactions between the blocking variables or between the treatment variable and the blocking variable.

# Example

- What is the best (fast) browser depending on the OS and the computer?
- Browsers are labeled by A,B, C and D.

| | OS | | | |
|---|---|---|---|---|
| Computer | 1 | 2 | 3 | 4 |
| I3 | A | B | C | D |
| I5 | B | C | D | A |
| I7 | C | D | A | B |
| XEON | D | A | B | C |

# Terminology

- A Latin Square is a **Standard Latin Square** when the letters of the first row and first column are arranged in alphabetical order (the previous example).
- Exists an unique standard $3 \times 3$ Latin Square, however there are four standard $4 \times 4$ Latin Square.

| Square 1 | | | | Square 2 | | | | Square 3 | | | | Square 4 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | B | C | D | A | B | C | D | A | B | C | D | A | B | C | D |
| B | C | D | A | B | A | D | C | B | A | D | C | B | D | A | C |
| C | D | A | B | C | D | A | B | C | D | B | A | C | A | D | B |
| D | A | B | C | D | C | B | A | D | C | A | B | D | C | B | A |

# Creating a Latin Square design

- Write down any Latin square of the required size (it could be a standard Latin square).
- And randomize:
  - Randomize the order of the rows.
  - Randomize the order of the columns.
  - Randomize the allocation of treatments to the letters of the square.

# The model

- $Y_{ijt}$=μ+$R_i$+$C_j$+$T_k$+$\varepsilon_{ijk}$
- Where
- $Y_{ijt}$ denoting any observation for which
  - $X_1$ = i, $X_2$ = j, $X_3$ = k
  - $X_1$ and $X_2$ are blocking factors
  - $X_3$ is the primary factor (treatment).
- μ denoting the general location parameter
- $R_i$ denoting the effect for block "i"
- $C_j$ denoting the effect for block "j"
- $T_k$ denoting the effect for treatment "k"

# Calculating the model

- Estimate for μ: $\bar{Y}$ = the average of all the data
- Estimate for Ri: $\bar{Y_i} - \bar{Y}$
  - $\bar{Y_i}$ = average of all Y for which X1 = i
- Estimate for Cj: $\bar{Y_j} - \bar{Y}$
  - $\bar{Y_j}$ = average of all Y for which X2 = j
- Estimate for Tk: $\bar{Y_k} - \bar{Y}$
  - $\bar{Y_k}$ = average of all Y for which X3 = k
- Estimate for $\varepsilon_{ijk}$
  - $\varepsilon_{ijk} = (y_{ijk} - \bar{y}_i - \bar{y}_j - \bar{y}_k + 2\bar{y})$

# The ANOVA table

| Source | SS | df (N-1)=$k^2$-1 | MS | F |
|---|---|---|---|---|
| Row | SSROW | k-1 | MSROW =SSROW /df | MSROW/MSE |
| Column | SSCOL | k-1 | MSCOL =SSCOL/df | MSCOL/MSE |
| Treatments | SSTR | k-1 | MSTR =SSTR/df | MSTR/MSE |
| Error | SSE | (k-1)(k-2) | MSE =SEE/df | |

$$\underbrace{\sum_{j=1}^{k}\sum_{i=1}^{k}(y_{ijt}-\bar{y})^2}_{\text{TSS}} = \underbrace{k\sum_{i=1}^{k}(\bar{y}_{i..}-\bar{y})^2}_{\text{SSROW}} + \underbrace{k\sum_{j=1}^{k}(\bar{y}_{.j.}-\bar{y})^2}_{\text{SSCOL}} + \underbrace{k\sum_{t=1}^{k}(\bar{y}_{.t.}-\bar{y})^2}_{\text{SSTR}} +$$

$$+\underbrace{\sum_{i=1}^{k}\sum_{j=1}^{k}\sum_{t=1}^{k}(y_{ijt}-\bar{y}_{i..}-\bar{y}_{.j.}-\bar{y}_{..t}+2\bar{y})^2}_{\text{SSE}}$$

# Example

- A courier company is interested in deciding between five brands (D,P,F,C and R) of car for its next purchase of fleet cars.
  - The brands are all comparable in purchase price.
  - The company wants to carry out a study that will enable them to compare the brands with respect to operating costs.
  - For this purpose they select five drivers (Rows).
  - In addition the study will be carried out over a five week period (Columns = weeks).

Source: http://www.slideshare.net/prkhadka19/latin-square-design?next_slideshow=1

# Example

- Each week a driver is assigned to a car using randomization and a Latin Square Design.
- The average cost per mile is recorded at the end of each week and is tabulated below:

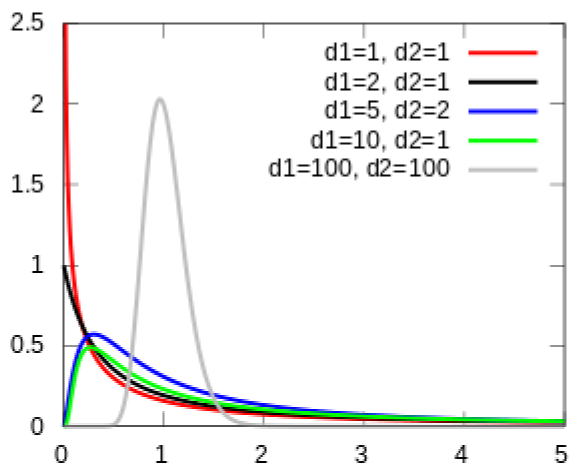
| | | Week | | | | |
|---|---|---|---|---|---|---|
| Drivers | | 1 | 2 | 3 | 4 | 5 |
| | 1 | 5.83 | 6.22 | 7.67 | 9.43 | 6.57 |
| | | D | P | F | C | R |
| | 2 | 4.80 | 7.56 | 10.34 | 5.82 | 9.86 |
| | | P | D | C | R | F |
| | 3 | 7.43 | 11.29 | 7.01 | 10.48 | 9.27 |
| | | F | C | R | D | P |
| | 4 | 6.60 | 9.54 | 11.11 | 10.84 | 15.05 |
| | | R | F | D | P | C |
| | 5 | 11.24 | 6.34 | 11.30 | 12.58 | 16.04 |
| | | C | R | P | F | D |

# Example

- The Anova Table for Example

| Source | S.S. | d.f. | M.S. | F ratio | F (table $\alpha$=0.05%) |
|---|---|---|---|---|---|
| **Week** | 51.17887 | 4 | 12.79472 | 16.06 | F(4,12)=3.25 |
| **Driver** | 69.44663 | 4 | 17.36166 | 21.79 | |
| **Car** | 70.90402 | 4 | 17.72601 | 22.24 | |
| **Error** | 9.56315 | 12 | 0.79693 | | |
| **Total** | 201.09267 | 24 | | | |

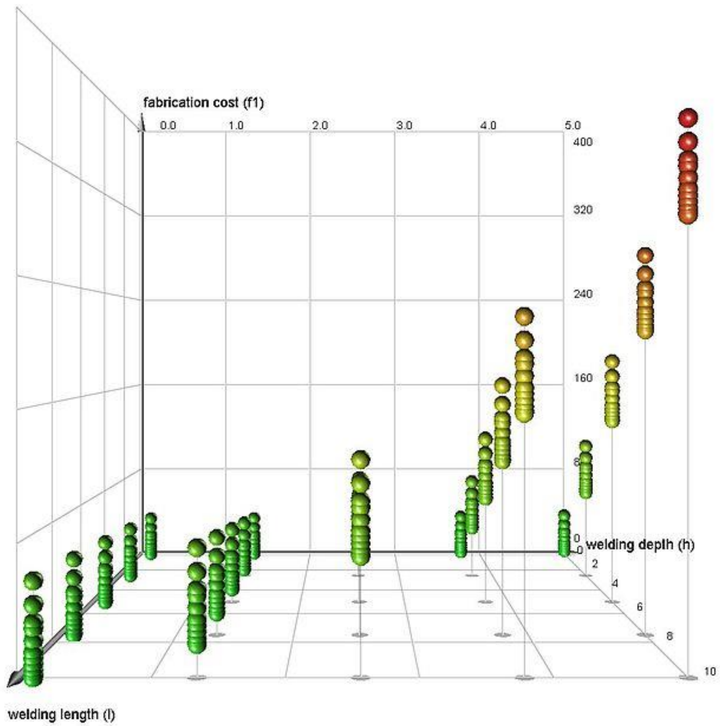
# F-table

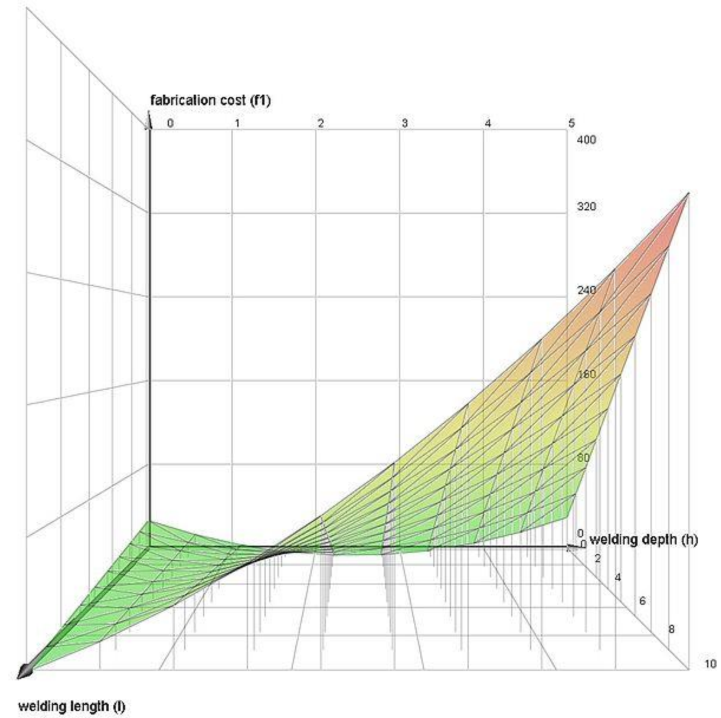



| | Numerator df: $df_B$ | | | | |
|---|---|---|---|---|---|
| $df_W$ | 1 | 2 | 3 | 4 | 5 |
| 5 5% | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 |
| 1% | 16.3 | 13.3 | 12.1 | 11.4 | 11.0 |
| 10 5% | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 |
| 1% | 10.0 | 7.56 | 6.55 | 5.99 | 5.64 |
| 12 5% | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 |
| 1% | 9.33 | 6.94 | 5.95 | 5.41 | 5.06 |
| 14 5% | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 |
| 1% | 8.86 | 6.51 | 5.56 | 5.04 | 4.70 |

# Response surface methods

# From full factorial design to the surface



# Steps

1. Set objectives
2. Select process variables
3. Select an experimental design
4. Execute the design
5. Check that the data are consistent with the experimental assumptions
6. Analyze and interpret the results
7. Use/present the results (may lead to further runs or DOE's).

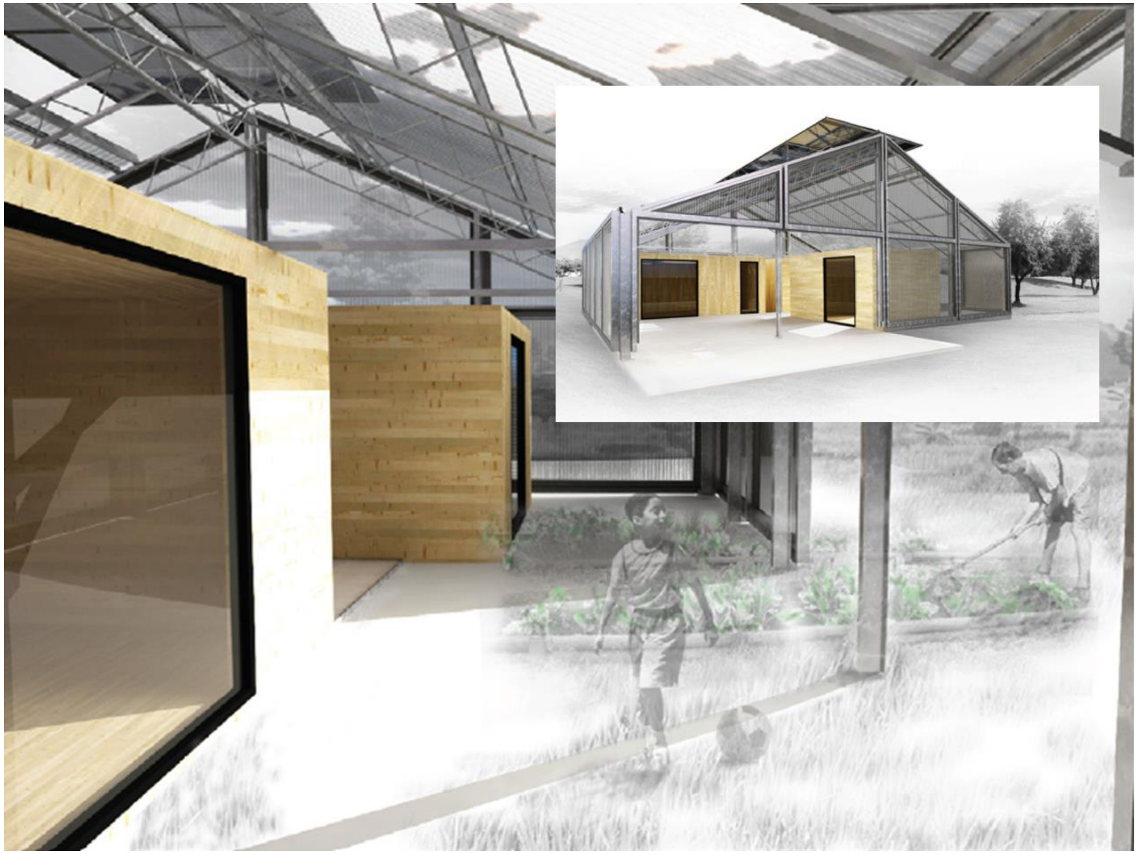
# Planning and running DOE

- Check performance of gauges/measurement devices first.
- Keep the experiment as simple as possible.
- Check that all planned runs are feasible.
- Watch out for process drifts and shifts during the run.
- Avoid unplanned changes (e.g., swap operators at halfway point).
- Allow some time (and back-up material) for unexpected events.
- Obtain buy-in from all parties involved.
- Maintain effective ownership of each step in the experimental plan.
- Preserve all the raw data--do not keep only summary averages!
- Record everything that happens.
- Reset equipment to its original state after the experiment.

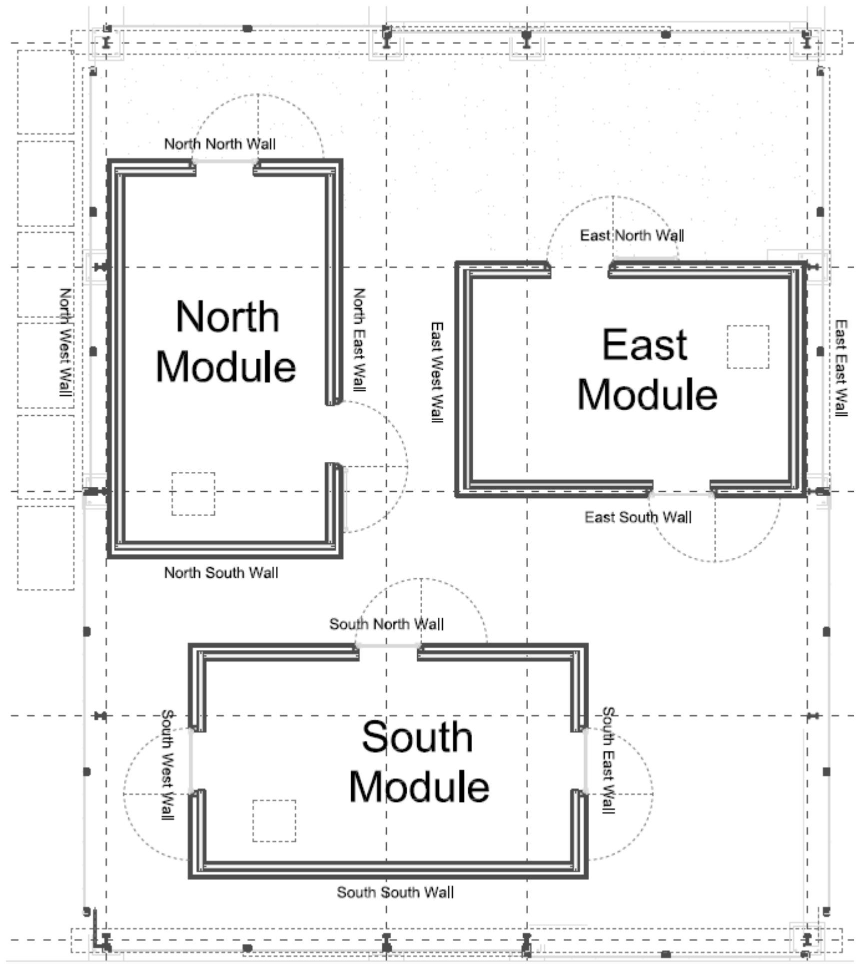
# Case study

DOE applied to construction problems

# Solar Decathlon Europe, LOW3



# Solar Decathlon Europe, LOW3

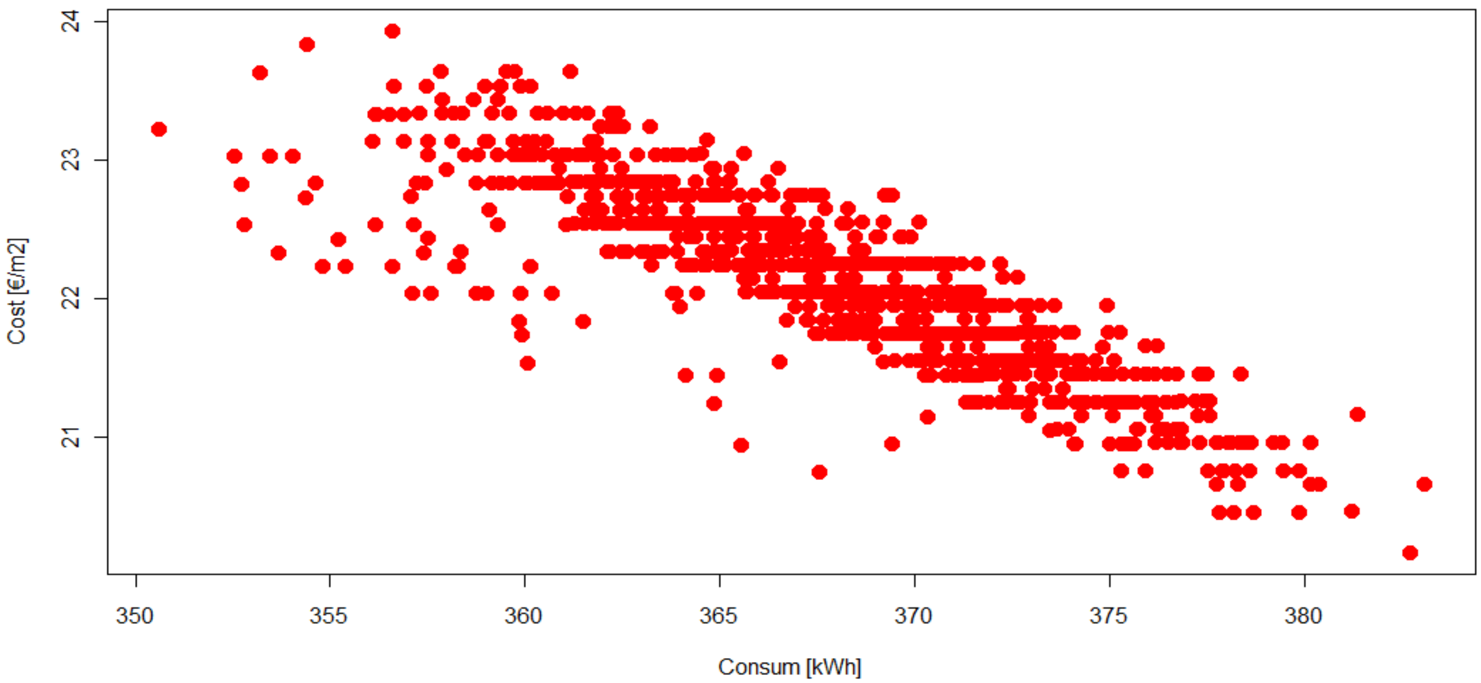


# Solar Decathlon Europe, LOW3

| | North MODULE | | | | South MODULE | | | | East MODULE | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **U** (W/m2 K) | North Wall | West Wall | South Wall | East Wall | North Wall | West Wall | South Wall | East Wall | North Wall | West Wall | South Wall | East Wall |
| **s1** | 0,212 | 0,212 | 0,212 | 0,212 | 0,212 | 0,212 | 0,212 | 0,212 | 0,212 | 0,212 | 0,212 | 0,212 |
| **s2** | 0,212 | 0,212 | 0,212 | 0,212 | 0,212 | 0,212 | 0,212 | 0,212 | 0,212 | 0,212 | 0,212 | 0,174 |
| **s3** | 0,212 | 0,212 | 0,212 | 0,212 | 0,212 | 0,212 | 0,212 | 0,212 | 0,212 | 0,212 | 0,174 | 0,212 |
| **s4** | 0,212 | 0,212 | 0,212 | 0,212 | 0,212 | 0,212 | 0,212 | 0,212 | 0,212 | 0,174 | 0,212 | 0,212 |
| **...** | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| **S4096** | 0,174 | 0,174 | 0,174 | 0,174 | 0,174 | 0,174 | 0,174 | 0,174 | 0,174 | 0,174 | 0,174 | 0,174 |

| | North MODULE | | | | South MODULE | | | | East MODULE | | | | Results |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| U (W/m2ºK) | North Wall | West Wall | South Wall | East Wall | North Wall | West Wall | South Wall | East Wall | North Wall | West Wall | South Wall | East Wall | KWh |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 386,99 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 383,30 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 383,71 |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 378,86 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 381,82 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 380,42 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 379,96 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 375,42 |

# Solar Decathlon Europe, LOW3



# Solar Decathlon Europe, LOW3

- Using Yates

| Module | Wall | Effects | Mean |
|---|---|---|---|
| East | East Wall | 4,35 | 3,34 |
| | South Wall | 3,09 | |
| | West Wall | 3,26 | |
| | North Wall | 2,66 | |
| South | East Wall | 2,23 | 2,54 |
| | South Wall | 2,16 | |
| | West Wall | 3,44 | |
| | North Wall | 2,35 | |
| North | East Wall | 2,74 | 3,46 |
| | South Wall | 4,65 | |
| | West Wall | 1,60 | |
| | North Wall | 4,86 | |

# Solar Decathlon Europe, LOW3

- To know more:
  - Fonseca i Casas, P., Fonseca i Casas, A., Garrido-Soriano, N., & Casanovas, J. (2014). Formal simulation model to optimize building sustainability. *Advances in Engineering Software*, *69*, 62–74. doi:10.1016/j.advengsoft.2013.12.009