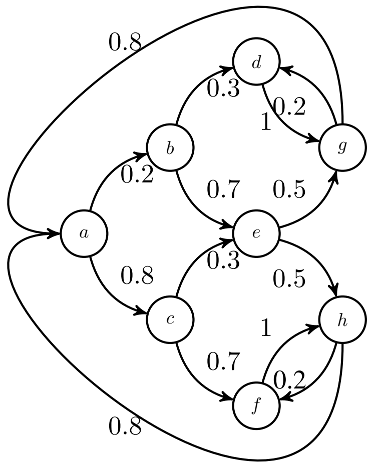
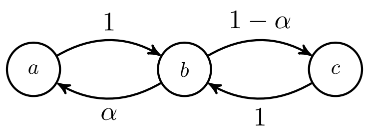
Problem 7.1
For the transition graph shown in the figure:

7.1.A Find its period and cyclic classes.
7.1.B Numerate the states and compute the transition matrix $\mathbf{P}$ such that it shows the block structure of a periodic chain.



Problem 7.2
For the transition graph shown in the figure:

7.2.A Find its period and cyclic classes.
7.2.B Numerate the states and compute the transition matrix $\mathbf{P}$ such that it shows the block structure of a periodic chain.
7.2.C Given that the chain is in state a in step $n = 0$, compute the transient probabilities $\pi_a(n)$, $\pi_b(n)$, $\pi_c(n)$ in closed form.



Problem 7.3
The system of problem 7.2 models a computer network device. Assume that when the system is in steady state, one packet arrives in every step with probability p, or no arrival occurs with probability $1-p$ (random arrivals). If the packet arrives when the system is in state a, the packet is lost. Otherwise the packet is successfully dispatched.

7.3.A Compute the stationary distribution.
7.3.B Compute the throughput, s (packets successfully dispatched per step).
7.3.C Compute the loss probability, p_l (proportion of packets that are lost).
7.3.D What relation must satisfy p, s and p_l? Check it with your previous results.

Hint: Use the *Random Arrivals See Time Averages, RASTA,* theorem: the probability that a random arrival see the system in state i is the stationary probability of the chain in state i, π_i. Note that this might not be true if arrivals are not random. E.g. periodic arrivals might find a periodic chain always in the same state.