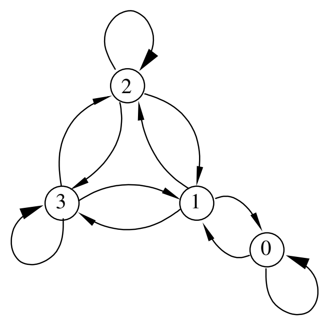
Problem 9.1

Consider a DTMC with the transition diagram of the figure.



9.1.A What condition must hold for the chain to be reversible?

9.1.B Assume

$$\begin{array}{llll} p_{00} = 1/2 & p_{01} = 1/2 & p_{02} = 0 & p_{03} = 0 \\ p_{10} = 7/12 & p_{11} = 0 & p_{12} = 2/12 & p_{13} = 3/12 \\ p_{20} = 0 & p_{21} = 2/6 & p_{22} = 1/6 & p_{23} = 3/6 \\ p_{30} = 0 & p_{31} = 1/2 & p_{32} = 1/2 & p_{33} = 0 \end{array}$$

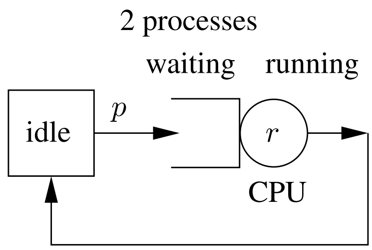
Check that the chain is reversible and compute the stationary distribution using the general solution for reversible chains.

9.1.C Solve the stationary distribution using the global balance equations (use the flux balancing method). Check the result with previous item.

9.1.D Assume that state 0 is removed and it is set $p_{11} = 7/12$. Compute the stationary distribution using the property for truncated reversible chains (that is, renormalizing the probabilities obtained before). Check the result solving the stationary distribution using the flux balancing method.

Problem 9.2

A slotted system consists of 2 processes and a CPU (see the figure). Processes can be idle, waiting in the queue or running in the CPU. Initially ($n = 0$) the 2 processes are idle. At each slot an idle process can awake with probability $p = 1/3$. Only 1 process can join the queue simultaneously. That is, if 2 processes awake simultaneously, only 1 joins the queue (chosen at random) and the other go to idle state. If a process is in the queue and the CPU is empty (no process running in the CPU) the process in the head of the queue starts running immediately. In the CPU there can be only 1 process running. After every slot a process running in the CPU can finish and go to idle state with probability $r = 1/2$. At the same time instant (i.e. if a process running in the CPU finish) an awaking process, or waiting in the queue can occupy the CPU. Define the stochastic process:



$$X(n) = \{\text{number of processes waiting or running in the CPU at time } n,\ n \geq 0\}$$

9.2.A Draw the transition diagram and derive the one step probability matrix.

9.2.B Justify that the chain is reversible, and compute the stationary distribution using the product form solution for reversible chains.

9.2.C Compute the expected number of processes in idle state.

9.2.D Compute the throughput (mean number of processes dispatched by the CPU per slot).

9.2.E Let $N \geq 1$ be the number of times an awaking process have to try before joining the queue. That is, $N = 1$ if an awaking process for the first time joins the queue; $N = 2$ if the first time the process awakes go to idle, and the same process joins the queue the second time it awakes; etc. Compute $E[N]$.