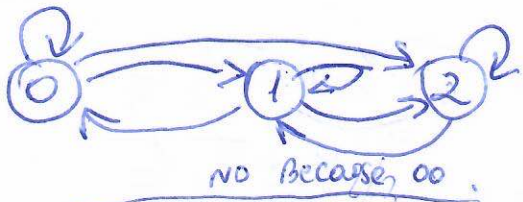
10.1.A



$$P_{00} = Q_a(0,0)(1-Q_r(1,0)) + Q_a(1,0)Q_r(0,0)$$

$$P_{00} = \frac{4}{9} + \frac{4}{9} = \boxed{\frac{8}{9}}$$

$$P_{01} = \boxed{\frac{1}{9}}$$

$$P_{0i} = Q_a(j-i,i) = \boxed{\frac{2}{9}}$$

$$P_{11} = Q_a(0,1)(1-Q_r(1,1)) + Q_a(1,1)Q_r(0,1)$$

$$P_{11} = \frac{1}{3}\cdot\left(1-\frac{1}{3}\right) + \frac{2}{3}\cdot\frac{2}{3} = \boxed{\frac{2}{3}}$$

$$P_{12} = Q_a(1,1)\left(1-Q_r(0,1)\right)$$

$$P_{12} = \frac{2}{3}\frac{1}{3} = \boxed{\frac{2}{9}}$$

$$P_{10} = Q_a(0,1)\,Q_r(1,1)$$

$$P_{10} = \frac{1}{3}\frac{1}{3} = \boxed{\frac{1}{9}}$$

$$P_{22} = Q_a(0,2)(1-Q_r(1,2)) + Q_a(1,2)Q_r(0,2)$$

$$P_{22} = 1\cdot\frac{5}{9} + = \boxed{\frac{5}{9}}$$

$$P_{21} = Q_a(0,2)\,Q_r(1,2)$$

$$P_{21} = 1\,\frac{4}{9} = \boxed{\frac{4}{9}}$$

$$Q_a(0,0) = \binom{2-0}{0}\left(\frac{2}{3}\right)^0\left(\frac{1}{3}\right)^2$$

$$Q_a(0,0) = \boxed{\frac{1}{9}}$$

$$Q_r(1,0) = \binom{0}{1}\frac{1}{3}^1\left(\frac{2}{3}\right)^{-1}$$

$$Q_r(1,0) = 0$$

$$Q_a(1,0) = \binom{2-0}{1}\frac{2}{3}^1\frac{1}{3} = \boxed{\frac{4}{9}}$$

$$Q_r(0,0) = \binom{0}{0}\frac{1}{3}^0\frac{2}{3}^0 = 1.$$

$$Q_a(2,0) = \binom{2-0}{2}\left(\frac{2}{3}\right)^2\frac{1}{3}^0 = \boxed{\frac{2}{9}}$$

$$Q_a(0,1) = \binom{2-1}{0}\frac{2}{3}^0\frac{1}{3} = \boxed{\frac{1}{3}}$$

$$Q_r(1,1) = \binom{1}{1}\frac{1}{3}^1\frac{2}{3}^0 = \boxed{\frac{1}{3}}$$

$$Q_a(1,1) = \binom{2-1}{1}\frac{2}{3}^1\left(\frac{1}{3}\right)^0 = \boxed{\frac{2}{3}}$$

$$Q_r(0,1) = \binom{1}{0}\frac{1}{3}^0\left(\frac{2}{3}\right)^1 = \boxed{\frac{2}{3}}$$

$$Q_a(0,2) = \binom{2-2}{0}\frac{2}{3}^0\frac{1}{3}^0 = \boxed{1}$$

$$Q_r(1,2) = \binom{2}{1}\frac{1}{3}^1\frac{2}{3}^1 = \boxed{\frac{4}{9}}$$

$$Q_a(1,2) = \binom{2-2}{1}$$

$$\pi_0 \frac{3}{9} + \pi_0 \frac{2}{9} = \pi_1 \frac{1}{9}$$

$$\pi_1 \frac{1}{9} + \pi_1 \frac{2}{9} = \pi_2 \frac{4}{9} + \pi_0 \frac{3}{9}$$

$$\pi_0 \left(\frac{3}{9} + \frac{2}{9}\right) = \pi_1 \frac{1}{9} \Rightarrow \pi_0 \frac{5}{9} = \pi_1 \frac{1}{9}$$

$$\pi_1 \frac{3}{9} = \pi_2 \frac{4}{9} + \pi_0 \frac{3}{9} \qquad \pi_0 = \pi_1 \frac{1}{15}$$

$$\pi_1 \frac{3}{9} = \pi_2 \frac{4}{9} + \pi_1 \frac{1}{5} \frac{3}{9}$$

$$\pi_1 \frac{3}{9} = \pi_2 \frac{4}{9} + \pi_1 \frac{1}{15}$$

$$\pi_1 \frac{3}{9} - \pi_1 \frac{1}{15} = \pi_2 \frac{4}{9}$$

$$\pi_1 \frac{4}{15} = \pi_2 \frac{4}{9} \rightarrow \pi_2 = \pi_1 \frac{3}{5}$$

$$\pi_1 = \frac{1}{1 + \frac{3}{5} + \frac{1}{15}} = \boxed{\frac{3}{5}}$$

$$\pi_0 = \boxed{\frac{1}{25}}$$

$$\pi_2 = \boxed{\frac{9}{25}}$$

4.0.. 1.B

$$P_{succ}(i) = Q_a(1,i) Q_r(0,i) + Q_a(0,i) Q_a(1,i)$$

$$S = \sum_{i=0}^{2} P_{succ}(i) \pi_i$$

$$P_{succ}(0) = \frac{4}{9} 1 + \frac{1}{9} 0 = \boxed{\frac{4}{9}}$$

$$P_{succ}(1) = \frac{2}{3} \frac{2}{3} + \frac{1}{3} \frac{1}{3} = \boxed{\frac{5}{9}}$$

$$P_{succ}(2) = \boxed{\frac{4}{9}}$$

$$S = P_{succ}(0) \pi_0 + P_{succ}(1) \pi_1 + P_{succ}(2) \pi_2$$

$$S = \frac{4}{9} \frac{1}{25} + \frac{5}{9} \frac{3}{5} + \frac{4}{9} \frac{9}{25} = \boxed{\frac{23}{45}}$$

Problem 10.12

10.2.A

NOT SURE ABOUT THIS

No because no retransmitions

$$P_{00} = Q_a(0,0)\left(1-Q_r(1,0)\right) + Q_a(1,0)\,Q_r(0,0)$$

$$P_{00} = \cancel{\frac{4}{9}} + \frac{4}{9} = \frac{4}{9}$$

$$P_{02} = \frac{5}{9} \qquad P_{20} = 1$$

$$P_{10} = \frac{2}{3}$$

$$P_{01} = \frac{1}{3}$$

$$\pi_0 \frac{8}{9} = \pi_1 \frac{2}{3} + \pi_2 \cdot$$

$$\pi_1 \frac{2}{3} = \pi_0 \frac{1}{3} \Rightarrow \boxed{\pi_1 = \pi_0 \frac{1}{2}}$$

$$\boxed{\pi_2 = \pi_0 \frac{5}{9} =}$$

$$\pi_0 \frac{8}{9} = \pi_1 \frac{2}{3} + \pi_0 \frac{5}{9}$$

$$\pi_0 \frac{8}{9} - \pi_0 \frac{5}{9} = \pi_1 \frac{2}{3}$$

$$\pi_0 \frac{3}{9} = \pi_1 \frac{2}{3}$$

$$\boxed{\pi_0 \frac{1}{2} = \pi_1}$$

NOT SURE

$$\pi_0 = \frac{1}{1 + \frac{1}{2} + \frac{5}{9}} = \boxed{\frac{18}{37}} = \pi_0$$

$$\boxed{\pi_1 = \frac{9}{37}} \qquad \boxed{\pi_2 = \frac{10}{37}}$$

10.2B

$$S = \frac{4}{9}\frac{18}{37} + \frac{5}{9}\frac{9}{37} + \frac{4}{9}\frac{10}{37} = \boxed{\frac{157}{333}} = 0.47$$

10.2.c

$$P(\text{discard}) = P(\text{1 packet arrive} \mid \text{1 backlogged})$$
$$= \delta \times Q_r(0,1)$$
$$= \frac{2}{3} \times \frac{2}{3} = \boxed{\frac{4}{9}}$$

Problem 10.3

$\delta = \delta_1 \cdot \delta_2 = \frac{1}{3}$ $\quad 1 - \delta = \frac{2}{3}$

$$Q_a(0,0) = \binom{2-0}{0}\left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

$$Q_a(1,0) = \frac{4}{9}$$

$$Q_r(0,0) = 1$$

$$Q_a(0,1) = \frac{2}{3}$$

$$Q_r(1,1) = \frac{1}{3}$$

$$Q_a(1,1) = \frac{1}{3}$$

$$Q_r(0,1) = \frac{2}{3}$$

$$Q_a(0,2) = 1$$

$Q_a(1,2)$ —

$$P_{00} = Q_a(00)\left(1 - Q_r(1,0)\right) + Q_a(1,0)\,Q_r(0,0)$$

$$P_{00} = \frac{4}{9}$$

$$P_{02} = \frac{5}{9}$$

$$P_{11} = Q_a(0,1)\left(1 - Q_r(1,1)\right) + Q_a(1,1)\,Q_r(0,1)$$

$$P_{11} = \frac{2}{3}\cdot\frac{2}{3} + \frac{1}{3}\cdot\frac{2}{3} = \boxed{\frac{2}{3}}$$

$$P_{12} = Q_a(1,1)\left(1 - Q_r(0,1)\right)$$

$$P_{12} = \frac{1}{3}\cdot\frac{1}{3} = \boxed{\frac{1}{9}}$$

$$P_{10} = Q_a(0,1)\,Q_r(1,1)$$

$$P_{10} = \frac{2}{3}\cdot\frac{1}{3} = \boxed{\frac{2}{9}}$$

$$P_{22} = Q_a(0,2)\left(1 - Q_r(1,2)\right) + Q_a(1,2)\,Q_r(0,2)$$

$$P_{22} = \boxed{\frac{5}{9}}$$

$$P_{21} = Q_a(0,2)\,Q_r(1,2) = \boxed{\frac{4}{9}}$$

10.3.B

$$\begin{cases} \pi_0 \frac{5}{9} = \pi_1 \frac{2}{9} \Rightarrow \pi_0 = \pi_1 \frac{2}{5} \\ \pi_1 \frac{1}{3} = \pi_2 \frac{4}{9} \Rightarrow \pi_1 \frac{3}{4} = \pi_2 \\ \pi_2 \frac{4}{9} = \pi_1 \frac{1}{9} + \pi_0 \frac{5}{9} \Rightarrow \pi_2 \frac{4}{9} = \pi_1 \frac{1}{9} + \pi_1 \frac{2}{9} = \end{cases}$$

$$\sum \pi_i = 1 \qquad \pi_1 = \frac{1}{1 + \frac{2}{5} + \frac{3}{4}} = \boxed{\frac{20}{43}}$$

$$\boxed{\pi_2 = \frac{15}{43}} \qquad \boxed{\pi_0 = \frac{8}{43}}$$

10.3.C

$$S = \pi_1 \cdot P_{succ}(1) = \frac{20}{43} \frac{4}{9} = \boxed{\frac{80}{387}}$$

$$P_{succ}(1) = Q_a(1,1) Q_r(0,1) + Q_a(0,1) Q_r(1,1)$$

$$= \frac{1}{3} \frac{2}{3} + \frac{2}{3} \frac{1}{3} = \boxed{\frac{4}{9}}$$

10.3.D

$$m_{11} = \frac{1}{\pi_1} = \boxed{\frac{43}{20}}$$

10.3.E