Stochastic Network Modeling
Homework 7 - Solutions

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Problem 7.1

7.1.1

The period of the chain is $d = 3$ and the cyclic classes are: $C_0 = \{a, d, e, f\}, C_1 = \{b, c\}, C_2 = \{g, h\}$

7.1.2

$$P = \begin{array}{cc} & \begin{array}{cccccccc} a & b & c & d & e & f & g & h \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \end{array} & \begin{bmatrix} 0 & 0.2 & 0.8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.3 & 0.7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.3 & 0.7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0.8 & 0 & 0 & 0.2 & 0 & 0 & 0 & 0 \\ 0.8 & 0 & 0 & 0 & 0 & 0.2 & 0 & 0 \end{bmatrix} \end{array}$$

$$P^7 = \begin{bmatrix} 0.00000 & 0.03200 & 0.12800 & 0.00000 & 0.00000 & 0.00000 & 0.21000 & 0.63000 \\ 0.67200 & 0.00000 & 0.00000 & 0.05480 & 0.06080 & 0.21240 & 0.00000 & 0.00000 \\ 0.67200 & 0.00000 & 0.00000 & 0.05080 & 0.06080 & 0.21640 & 0.00000 & 0.00000 \\ 0.00000 & 0.13440 & 0.53760 & 0.00000 & 0.00000 & 0.00000 & 0.08800 & 0.24000 \\ 0.00000 & 0.13440 & 0.53760 & 0.00000 & 0.00000 & 0.00000 & 0.08400 & 0.24400 \\ 0.00000 & 0.13440 & 0.53760 & 0.00000 & 0.00000 & 0.00000 & 0.08000 & 0.24800 \\ 0.26240 & 0.00000 & 0.00000 & 0.05792 & 0.25536 & 0.42432 & 0.00000 & 0.00000 \\ 0.26240 & 0.00000 & 0.00000 & 0.05632 & 0.25536 & 0.42592 & 0.00000 & 0.00000 \end{bmatrix}$$

$$P^{15} = \begin{bmatrix} 0.00000 & 0.06559 & 0.26235 & 0.00000 & 0.00000 & 0.00000 & 0.16802 & 0.50405 \\ 0.53765 & 0.00000 & 0.00000 & 0.05328 & 0.12462 & 0.28445 & 0.00000 & 0.00000 \\ 0.53765 & 0.00000 & 0.00000 & 0.05328 & 0.12462 & 0.28446 & 0.00000 & 0.00000 \\ 0.00000 & 0.10753 & 0.43012 & 0.00000 & 0.00000 & 0.00000 & 0.11560 & 0.34675 \\ 0.00000 & 0.10753 & 0.43012 & 0.00000 & 0.00000 & 0.00000 & 0.11559 & 0.34676 \\ 0.00000 & 0.10753 & 0.43012 & 0.00000 & 0.00000 & 0.00000 & 0.11558 & 0.34676 \\ 0.36988 & 0.00000 & 0.00000 & 0.05538 & 0.20431 & 0.37044 & 0.00000 & 0.00000 \\ 0.36988 & 0.00000 & 0.00000 & 0.05538 & 0.20431 & 0.37044 & 0.00000 & 0.00000 \end{bmatrix}$$

Problem 7.2

7.2.1

Period is $d = 2$ and the cycles are 2 $C_0 = \{a, c\}, C_1 = \{b\}$

7.2.2

$$P = \begin{bmatrix} & a & b & c \\ a & 0 & 1 & 0 \\ b & \alpha & 0 & 1-\alpha \\ c & 0 & 1 & 0 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} & a & b & c \\ a & 0 & 1 & 0 \\ b & \alpha^2 & 0 & (1-\alpha)^2 \\ c & 0 & 1 & 0 \end{bmatrix}$$

$$P^{20} = \begin{array}{cc} & \begin{array}{ccc} a & b & c \end{array} \\ \begin{array}{c} a \\ b \\ c \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ \alpha^{20} & 0 & (1-\alpha)^{20} \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$P^{\infty} = \begin{array}{cc} & \begin{array}{ccc} a & b & c \end{array} \\ \begin{array}{c} a \\ b \\ c \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

7.2.3

Given the chain the eigenvalues are $\lambda_1 = 1, \lambda_2 = -1, \lambda_3 = 0$

Therefore for $\pi_a(n)$ we have:

$$\begin{aligned} \pi_a(n) &= a\lambda_1^n + b\lambda_2^n && \text{(1a)} \\ \pi_a(0) &= a + b = (\pi(0)P^0)_a = 1 && \text{(1b)} \\ \pi_a(1) &= a - b = (\pi(0)P^1)_a = 0 && \text{(1c)} \\ \pi_a(2) &= a + b = (\pi(0)P^2)_a = \alpha && \text{(1d)} \\ & && \text{(1e)} \end{aligned}$$

Solving the equation system we have that $a = \frac{2-\alpha}{2}, b = \frac{\alpha}{2}$.
Therefore, $\pi_a(n) = \frac{2-\alpha}{2} + (-1)^n \frac{\alpha}{2}$

Therefore for $\pi_b(n)$ we have:

$$\begin{aligned} \pi_b(n) &= a\lambda_1^n + b\lambda_2^n && \text{(2a)} \\ \pi_b(0) &= a + b = (\pi(0)P^0)_b = 0 && \text{(2b)} \\ \pi_b(1) &= a - b = (\pi(0)P^1)_b = 1 && \text{(2c)} \\ \pi_b(2) &= a + b = (\pi(0)P^2)_b = 0 && \text{(2d)} \\ & && \text{(2e)} \end{aligned}$$

Solving the equation system we have that $a = \frac{1}{2}, b = -\frac{1}{2}$.
Therefore, $\pi_b(n) = \frac{1}{2} + (-1)^n - \frac{1}{2}$

Therefore for $\pi_c(n)$ we have:

$$\pi_c(n) = a\lambda_1^n + b\lambda_2^n \tag{3a}$$
$$\pi_c(0) = a + b = (\pi(0)P^0)_c = 0 \tag{3b}$$
$$\pi_c(1) = a - b = (\pi(0)P^1)_c = 0 \tag{3c}$$
$$\pi_c(2) = a + b = (\pi(0)P^2)_c = 1 - \alpha \tag{3d}$$
$$\tag{3e}$$

Solving the equation system we have that $a = \frac{1-\alpha}{2}, b = \frac{1-\alpha}{2}$.
Therefore, $\pi_c(n) = \frac{1-\alpha}{2} + (-1)^n \frac{1-\alpha}{2}$

Problem 7.3

I havent follow this problem. How to compute the stationary distribution without values? On the other hand i havent followed the problem properly.