Stochastic Network Modeling
Homework 5 - Solutions

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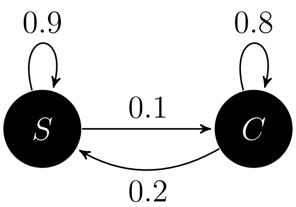
Problem 5.1

5.1.1

5.1.2



$$P = \begin{array}{c c} & \begin{array}{cc} S & C \end{array} \\ \begin{array}{c} S \\ C \end{array} & \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix} \end{array}$$

5.1.3

- (i)

$$det(\lambda I - P) = \begin{pmatrix} \lambda - 0.9 & -0.1 \\ -0.2 & \lambda - 0.8 \end{pmatrix} \tag{1a}$$

$$= \lambda^2 - \lambda 1.7 + 0.8 \tag{1b}$$

$$\tag{1c}$$

Solving applying $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$ we get $\lambda_0 = 1$ and $\lambda_1 = 0.7$.

- (ii) Trace of A $= \sum$ diagonal $A = 0.9 + 0.8 = 1.7 = \lambda_0 + \lambda_1 = 1 + 0.7$.
- $det(P) = 0.9 * 0.8 - 0.1 * 0.2 = 0.7 = \prod_i \lambda_i = 1 * 0.7$

5.1.4

For the case $\pi_s(n)$ starting in $n = 0$ with state S which is Sunny we have

$$\pi_s(0) = a + b = ([10]P^0) = P^0_{ss} = 1 \tag{2a}$$

Therefore $a = 1$

$$\pi_s(1) = a + 0.7b = ([10]P^1) = P^1_{ss} = 0.9 \tag{3a}$$

Then,

$$\begin{aligned} 0.9 &= 1 + 0.7b && (4a)\\ -0.7b &= 1 - 0.9 && (4b)\\ b &= -0.14 && (4c) \end{aligned}$$

Therefore $a = 1$ and $b = -0.14$.

Then $\pi_s(n) = 1 - 0.14 * 0.7^n, n \geq 0$

For the case cloudy $\pi_c(n)$ starting in $n = 0$ with state S which is Sunny we have

$$\pi_c(0) = a + b = ([10]P^0) = P^0_{sc} = 0 \tag{5a}$$

Therefore $a + b = 0$

$$\pi_c(1) = a + 0.7b = ([10]P^1) = P^1_{sc} = 0.1 \tag{6a}$$

Then taking that $a = -b$,

$$\begin{aligned} a &= -0.7b + 0.1 && (7a)\\ a &= -b && (7b)\\ -b &= -0.7b + 0.1 && (7c)\\ -b + 0.7b &= 0.1 && (7d)\\ -0.3b &= 0.1 && (7e)\\ b &= -0.33 && (7f) \end{aligned}$$

Therefore $a = 0.33$ and $b = -0.33$.

Then $\pi_s(n) = 0.33 - 0.33 * 0.7^n, n \geq 0$

Problem 5.2

First we need to calculate the eigen values.

$$det(\lambda I - P) = \frac{-9\lambda^3 + 23\lambda^2 - 19\lambda + 5}{9} \tag{8a}$$

And the roots are $\lambda_0 = 1, \lambda_1 = 0.55555, \lambda_2 = 0.99999$

$$\pi_2(n) = a\lambda_0^n + b\lambda_1^n + c\lambda_2^n \tag{9a}$$
$$\pi_2(0) = a + b + c = (\pi(0)P^0)_2 = 0 \tag{9b}$$
$$\pi_2(1) = a + 0.55b + 0.99c = (\pi(0)P^1)_2 = P_{12}^1 = \frac{15}{36} \tag{9c}$$
$$\pi_2(2) = a + 0.55^2 b + 0.99^2 c = (\pi(0)P^2)_2 = P_{12}^2 = \frac{20}{36}\frac{15}{36} + \frac{15}{36} = \frac{35}{54} \tag{9d}$$
$$\tag{9e}$$

Solving the equation system we have that $a = \frac{25}{27}, b = -\frac{25}{27}, c = 0$.

Therefore, $\pi_2(n) = \frac{25}{27} - \frac{25}{27} * 0.55^n$. When $n \to \infty$ Probability is $\frac{25}{27}$ because second term tends to 0.

5.2.1

If $\pi_2(\infty) = \frac{25}{27} = 0.94$. In the previous exercise we have obtained the same number but with other fraction which is $\frac{15}{16}$ but the decimal number representation is the same.