Boolean Combinations of Weighted Voting Games

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Agenda

Agenda

Introduction

Basic Notions

- Based on *Boolean Combinations of Weighted Voting Games paper* ***BWVG***[1]
- It is a natural Generalization over **Weighted Voting Games**
- Intuitively is a decision making process via multiple committees
- Each committee has the authority to decide the outcome *"yes" or "no"* about an issue.
- And each committee is a WVG
- Individuals can appear in multiple committees
- Different committees can have different Threshold values

[1] Piotr Faliszewski, Edith Elkind, and Michael Wooldridge. 2009. Boolean combinations of weighted voting games. In Proceedings of The 8th International Conference on Autonomous Agents and Multiagent Systems - Volume 1 (AAMAS '09). International Foundation for Autonomous Agents and Multiagent Systems, Richland, SC, 185–192.

Introduction

Questions to be answered?

- Which coalitions might be able to bring the goal about?
- How important is a particular individual with respect to the achievement of the goal?

Introduction

Goals of the Paper

- Formal Definition of **BWVG**
- Investigate Computational Properties of **BWVG**

Agenda

Preliminary Definitions

Propositional Logic

- Let $\Phi = \{p, q, \dots\}$ be a fixed non-empty vocabulary of Boolean variables
- Let $\mathcal{L}$ denote the set of formulas of propositional logic over Φ
- If "$\vee$" and "$\wedge$" are the only operators appearing in formula φ, se say that φ is **monotone**
- If $\xi \subseteq \Phi$, we write $\xi \models \varphi$ mean that φ is true satisfied by valuation ξ

Preliminary Definitions

Simple Games

- A coalitional game is Simple if $v(C) \in \{0, 1\} \forall C \subseteq N$
- C wins if $v(C) = 1$ and C losses otherwise.
- A Simple Game is **monotone** if $v(C) = 1 \implies v(C') = 1$ for any $C \subseteq C'$.
- In this paper authors consider both *monotone* and *non-monotone* Simple Games.
- They assume games with finite numbers of players $|N| = n$, $N = \{1, \ldots, n\}$

Preliminary Definitions

Weighted Voting Games

- Given $N = \{1, \dots, n\}$ players
- A list of n weights $w = (w_1, \dots, w_n) \in \mathbb{R}^n$
- A threshold $T \in \mathbb{R}$
- When N is clear from the context $q = (T; w_1, \dots, w_n)$ to denote a *WVG* g
- $w(C)$ total weight of coalition C, $w(C) = \sum_{i \in C} w_i$
- Characteristic function given by $v(C) = 1$ if $w(C) \geq T$ and $v(C) = 0$ otherwise.
- If all Weights are non-negative the game is monotone.

Preliminary Definitions

Computational Complexity

- P, NP, $coNP$, Σ_2^p, Π_2^p
- D^p: A Language $L \in D^p$ if $L = L_1 \cap L_2$, for some language $L_1 \in NP$ and $L_2 \in coNP$
- D_2^p: A Language $L \in D_2^p$ if $L = L_1 \cap L_2$, for some language $L_1 \in \Sigma_2^p$ and $L_2 \in \Pi_2^p$
- A Language $L \in UP$ if its Characteristic Function is in $\#P$

Agenda

Boolean Weighted Voting Games

Definition

A **BWVG** is a tuple $G = \langle N, \mathcal{G}, \Phi, \varphi \rangle$, where:

- $N = \{1, \ldots, n\}$ is a set of players;
- $\mathcal{G} = \{g^1, \ldots, g^n\}$ is a Set of **WVG** over N, where jth game, g^j, is given by a vector of weights $w^j = (w_1^j, \ldots, w_n^j)$ and a Threshold T^j. $\mathcal{G}$ is called the **component games** of G;
- $\Phi = \{p^1, \ldots, p^n\}$ Set of Propositional Variables, in which each p^j correspond with the **component** g^j;
- φ is a propositional formula over Φ.

Shorthand Definition

Example:

- $g^1 \wedge g^2 \equiv \langle N, \{g^1, g^2\}, \{p^1, p^2\}, p^1 \wedge p^2 \rangle$

Boolean Weighted Voting Games

Winning Coalition

We say that C is a *wins* G if:

$$\exists \xi_1 \subseteq \Phi_C \ : \ \forall \xi_2 \subseteq (\Phi \setminus \Phi_C) \ : \ \xi_1 \cup \xi_2 \models \varphi \tag{1}$$

Intuitively 1

A coalition C wins if it is able to fix variables under its control in such a way that the goal formula φ is guaranteed to be **True**.

Notes

It is allowed ***BWVG*** to contain *negative* weights

Agenda

Representational Complexity

Preliminaries

- Any Simple Game with n players can be represented as a *K-Vector Weighted Voting Game* for $k = O(2^n)$, and therefore as a **BWVG** with $O(2^n)$ **component games** $\mathcal{G}$.
- That worst-case can be improved in **BWVG**

Representational Complexity

Proposition 1

The total number of Boolean weighted voting games with $|N| = n$ *and* $|\varphi| = s$ *is most* $2^{O(sn^2 \log(sn))}$

Proof.

- Any weighted voting game[2] can be represented using Integer weights whose absolute values do not exceed $2^{O(n \log n)}$
- *w.l.g.* we assumed that $|\mathcal{G}| = |\Phi|$ and $|\Phi| \leq |\varphi| = s$
- Given a **BWVG** G with n players and $|\varphi| = s$, we can find a equivalent representation using $O(sn^2 \log n)$ bits to represent all weights in ALL components, plus another $O(s \log s)$ bits to represent $\mathcal{G}, \Phi$ and φ.
- Therefore, the total number of **distinct games** can be represented as **BWVG** with $|N| = n$ and $|\varphi| = s$ is $2^{O(sn^2 \log(sn))}$

□

[2] S. Muroga. Threshold Logic and its Applications. Wiley, 1971.

Agenda

Shapley Value

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Agenda

The Core

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Thank you!!