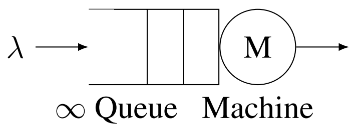
Problem 23.1



arrival rate	$\lambda = 1/10$
service rate	$\mu = 1$
failure rate	$\mu_1 = 1/3$
type 1 failure prob. (repair)	$p_1 = 3/4$
type 2 failure prob. (fatal)	$p_2 = 1/4$
repair rate	$\mu_2 = 3/4$

A system consists of a machine M dispatching Poisson arrivals, as shown in the figure. When the machine is working it dispatches items at rate μ, and fails with rate μ_1, both exponentially distributed. Failures can be of type 1 and type 2, which occurs with probabilities p_1 and p_2, respectively:

- type :1 the machine is repaired during a time exponentially distributed with rate μ_2. During repair no items are dispached.
- type 2: the machine is replaced by a new one.

23.1.A Use an absorbing chain to compute the distribution of the service time, S: $f_s(t),\ t \geq 0$.
23.1.B Compute $\mathrm{E}[S]$ and $\mathrm{E}[S^2]$.
23.1.C Compute the expected time in the system using the P-K formula.
23.1.D Compute the expected number in the system.
23.1.E Assume the process with states $(n, i),\ n \geq 0,\ i \in \{1, 2\}$, where n is the number in the system, and i the machine state: 1: woring, 2: repair. Draw the chain state transition diagram.
23.1.F Derive the rate matrix, Q, ordering the states lexicographically. Identify the states that form the initial and repetitive part. Identify the submatrices that would be used for a matrix geometric solution: $\mathbf{B}_0$, $\mathbf{L}_0$, $\mathbf{F}_0$, $\mathbf{B}$, $\mathbf{L}$, $\mathbf{F}$.
23.1.G Compute the matrix $\mathbf{R}$.
23.1.H Compute $\boldsymbol{\pi}_0$ and $\boldsymbol{\pi}_1$.
23.1.I Use the previous results to compute the expected number in the system. Compare it with the one obtained before.
23.1.J Compute the expected number of machines repaired per time unit.
23.1.K Compute the expected number of new machines bought per time unit.