Homework 4 – Deadline 29/9/2020

Problem 4.1
Consider a game consisting on a coin toss. The game finish whenever 2 consecutive heads or 3 consecutive tails occur. Assume a fair coin.

- 4.1.A Define the states which allows describing the game as a DTMC. Take into account that we want to know whether the game finish because 2 consecutive heads or 3 consecutive tails occur.
- 4.1.B Draw the transition state diagram.
- 4.1.C Construct the transition matrix, $\mathbf{P}$.
- 4.1.D Identify the absorbing states.

Problem 4.2
Formulate the Craps game (see problem 2.7) as an absorbing DTMC. Try to minimize the number of states of the chain. Say what will be the transition probability matrix, $\mathbf{P}$, and the initial distribution $\boldsymbol{\pi}(0)$.

Problem 4.3
A 3 motors machine can operate properly if at least 2 of the motors are functioning during a day. At the end of each day a working motor will continue working during next day with probability $p = 2/3$, or break (and be under repair during next day) with probability $1 - p$. At the end of each day a motor under repair will be fixed and ready to work next day with probability $r = 3/4$, or continue under repair with probability $1 - r$. Initially all motors are working.

- 4.3.A Formulate the system as an absorbing DTMC, where the absorbing state is the machine failure.
- 4.3.B Compute the transition probability matrix, $\mathbf{P}$, and the initial distribution $\boldsymbol{\pi}(0)$.
- 4.3.C Let $T \geq 1$ be the random variable equal to the number of days until failure (the machine cannot operate). Explain how could you compute $\mathrm{E}[T]$ in terms of $\mathbf{P}$ and $\boldsymbol{\pi}(0)$.