9. Assume that a WVG is described by $\Gamma = (q; w_1, \ldots, w_n)$. Analyze the computational complexity of the problemS
   - Compute the smallest number of players that can form a wining coalition in Γ.
   - Compute the biggest number of players that can form a losing coalition in Γ.

10. **The diameter game.** Consider a cooperative game which is defined on an undirected connected graph $G = (V, E)$. The players are the edges in the graph. For $X \subseteq E$, let $G_X = (V, X)$ be the graph formed by V and the edges in X. The valuation function is the following

$$v(X) = \begin{cases} 2|X| - diam(G_X) & \text{if } G_X \text{ is connected} \\ \frac{|X|}{2} & \text{otherwise,} \end{cases}$$

    where $diam(H)$ is the diameter of the graph H.

    (a) Is the valuation function monotone? supperadditive? supermodular?

    (b) Are there connected graphs such that the core of the associated diameter game is non-empty?

11. **Games on social networks** One of the criticism to simple games is the fact of assuming that any coalition can be formed. In the context in which the players participate in a social networks, a natural restriction on a coalition to take effect is that the members of a should at least be able to establish some level of communication among themselves.

    For simplicity you can assume that a simple game $\Gamma = (N, \mathcal{W})$ is defined and that the social network is an undirected graph $H = (N, E)$.

    On top of that we can came out with different combinations for defining winning coalitions in an associated *social game*, Γ_s on N. Consider the following options:

    (a) A coalition X is winning in Γ_s iff X wins in Γ and $H[X]$ has no isolated vertices.

    (b) A coalition X is winning in Γ_s iff X wins in Γ and $H[X]$ is connected.

    (c) A coalition X is winning in Γ_s iff there is $Y \subseteq X$, so that Y wins in Γ and $H[Y]$ is connected.

    Under which of the options (a), (b) or (c) is Γ_s a simple game?

    For those cases in which a simple game is defined, assuming that you have access to a polynomial time algorithm that given X decides whether $X \in \mathcal{W}$, analyze the computational complexity of the problem of deciding whether Γ_s has an empty core.

12. **Vertex cover games** For a given undirected graph $G = (V, E)$, the associated *vertex cover game* has $N = V$ and in it a coalition wins iff and only if X is a vertex cover in G.

    (a) Show that vertex cover games are simple games.

    (b) Are there games in which the core is non-empty?

    (c) Analyze the computational complexity of the IsProper and IsStrong problem on vertex cover games