Problem 6

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Definition of the Problem

- Set of n players
- Partitioned into two groups.
- **Bad pairings**: *two players in such a pair do not want to be in the same group*.
- Players free to choose any group.
- Modeled as a Graph $G = (V, E)$, such that there is an edge $(i, j) \in E$ if i and j form a bad pair.
- The private objective of player i is to maximize the number of its neighbors that are in the **other group**.

Formalization

Given the problem definition

- Let $G = (V, E)$ be the graph representing with V the number of players as vertices and E the set of edges such as $\forall_{i,j \in V}\ (i, j) \in E$ if it is a bad pairing.
- $N = V$.
- $\forall_{i \in N}, A_i = \{1, 0\}$ 1 if it is in Group 1, 0 if it is in Group 2.

Formalization

Pay-off function

$$u_i(v_2, \ldots, v_n) = \begin{cases} 1 & \exists\, j \neq i, (i,j) \in E \text{ and } j \text{ is in the other group of } i \\ 0 & otherwise \end{cases}$$

Formalization

Best Response Function

$$BR_i(v_{-i}) = \begin{cases} \{1\} & \forall_{j \neq i}, (i,j) \in E \implies \sum_j 1 - v_j > \sum_j v_j \\ \{0\} & \forall_{j \neq i}, (i,j) \in E \implies \sum_j v_j > \sum_j 1 - v_j \\ \{0,1\} & otherwise \end{cases}$$

NPE Analysis

NPE Analysis

$v = (v_1, \ldots, v_n)$ *is a NPE* $\iff \forall_{i,j}, (i,j) \in E \ \sum_{i=1}^{n} 1 - v_i = \sum_{i=1}^{n} v_j$

NPE Analysis

Proof.

Part

$\Longleftarrow \forall_{i,j}, (i,j) \in E \ \sum_{i=1}^{n} 1 - v_i = \sum_{i=1}^{n} v_j \implies$

$$\implies \forall_{i,j}, (i,j) \in E \ \sum_{j \neq i} 1 - v_i > \sum_{j \neq i} v_i \implies \{1\} \in BR_i \qquad (1a)$$

$$\implies \forall_{i,j}, (i,j) \in E \ \sum_{j \neq i} 1 - v_i < \sum_{j \neq i} v_i \implies \{0\} \in BR_i \qquad (1b)$$

$\square$

NPE Analysis

Proof.

Part

$\Longrightarrow$

$\forall_{i,j}, (i,j) \in E \ \sum_{i=1}^{n} 1 - v_i \neq \sum_{i=1}^{n} v_j$ *if this holds, then*

$$\exists j, v_j = 1 \iff (i,j) \in E \ \wedge \sum_{j \neq i} 1 - v_i < \sum_{j \neq i} v_i + v_j \tag{2a}$$

$$, v_j = 0 \iff (i,j) \in E \ \wedge \sum_{j \neq i} (1 - v_i) + v_j > \sum_{j \neq i} v_i \tag{2b}$$

$$\tag{2c}$$

But if this is true i *changes to other group.*
Therefore $\forall_{i,j}, (i,j) \in E \ \sum_{i=1}^{n} 1 - v_i = \sum_{i=1}^{n} v_j$

$\square$

Complexity Analysis

- Polinomial in the size of V by the size of E
- Problem of type ***Strategy General Form***
- Therefore, $O(|V||E|)$

Thank you!!