Problem 11.1

Assume a slotted Aloha system with 2 nodes and parameters $\sigma = 2/3$ and $\nu = 1/3$. Consider the random variable T equal to the number of consecutive slots that one node remains in backlogged state.

11.1.A Derive an absorbing DTMC that allows computing $\mathrm{E}[T]$.
11.1.B Compute the fundamental matrix $\mathbf{N} = (\mathbf{I} - \mathbf{Q})^{-1}$ using the cofactors formula.
11.1.C Compute $\mathrm{E}[T]$ using $\mathbf{N}$.

Problem 11.2

Let n_1 be one of the nodes in the previous Aloha system. Define the event:

$$A = \left\{ \begin{array}{c} \text{Upon } n_1 \text{ and } n_2 \text{ become backlogged, } n_2 \text{ can transmit 2 packets} \\ \text{successsfully before } n_1 \text{ can successfully transmitt the backlogged packet} \end{array} \right\}$$

11.2.A Derive an absorbing DTMC that allows computing $P(A)$.
11.2.B Compute the fundamental matrix $\mathbf{N} = (\mathbf{I} - \mathbf{Q})^{-1}$ using Gaussian elimination.
11.2.C Compute $P(A)$ using $\mathbf{N}$.