# Manipulation

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- Assume $\succ$ is a preference profile so that $\succ_i$ is the true preferences of voter $i$.
- A voting rule $F$ is strategy-proof if for every preference profile $\succ' = (\succ_{-i}, \succ'_i)$, it is not the case that $F(\succ') \succ_i F(\succ)$

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- Yes, but not very satisfactory!

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- Onto: Every alternative can win under some preference profile.
- Non-dictatorial: There is no voter $i$ such that $F(\succ)$ is always the top alternative for voter $i$.

# Gibbard-Satterthwaite

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- *dictatorial: there exists a single fixed voter whose most-preferred alternative is chosen for every profile;*
- *imposing: there is at least one alternative that does not win under any profile;*
- *manipulable (i.e., not strategyproof).*

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- *manipulable (i.e., not strategyproof).*

The first two properties are not acceptable in most voting settings. So, we need to assume that the voters have an incentive to misreport true preferences.

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- Maybe the rule is manipulable, but it is NP-hard to find a successful manipulation.
  For once NP-hardness can be good!!

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- The problem belongs to NP provided $F$ is computable in polynomial time.
- For plurality, this problem is computationally trivial:
- The only sensible manipulation is to put $a$ as your most preferred candidate!

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  - Responsive: the candidate with the largest score wins (in the voting under the joint profile)
  - Monotone: for any two preference orders $P$ and $P'$ and for any candidate $a$, if for each voter $i$, $\{b \mid a \ P \ b\} \subseteq \{b \mid a \ P' \ b\}$, then $S(P, a) \leq S(P', a)$.

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  If so, place $b$ in the next position, otherwise terminate claiming that order does not exists.

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**Theorem**

*For any voting rule $F$ satisfying the BTT conditions, G-Man solves the $F$-$\textsc{Manipulation}$ problem.*

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- Assume that such an order exists and that Greedy-Manipulation terminates without providing an ordering. Let us reach a contradiction.

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- Consider any completion $P$ of the preference order started by G-Man that places $u$ in the first unassigned place.

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- By initialization $S(P, c) \geq S(P', c)$.
- So, $S(P, c) \geq S(P, u)$.
- But G-Man did not assign $u$, so $S(P, c) < S(P, u)$ and we get the contradiction.

# Manipulable rules by Greedy

**Corollary**

*For any voting rule $F$ satisfying the BTT conditions, and for which the scoring rule can be computed in polynomial time G-Man solves the $F$-$\textsc{Manipulation}$ problem in polynomial time.*

By monotonicity, it should be possible to computer the score to of the alternative ranked "first" among a set of unranked alternatives

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- Both, Copeland vote and the score can be computed in polynomial time.
- Copeland is polynomial time manipulable.

# Maximin

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  - $Score(x) = min_y n_{x \succ y}$
  - elect $x^*$ with the maximum score
- Working in a similar way, Maximin is polynomial time manipulable.

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# STV

STV-Manipulation is NP-hard (Bartholdi III and Orlin, Social Choice and Welfare, 1991)

- The NP-hardness follows by a reduction from the 3-cover problems which is NP-complete problem (3-Cover).
- The basic idea is to build a large election instance introducing all sorts of constraints on the ballot of the manipulator, such that finding a ballot meeting those constraints solves a given instance of 3-Cover.