HOMEWORK 18 | JUAN PABLO ROYO SALES.

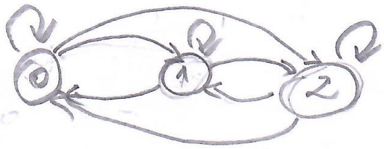
18.1.A

X: Number packet in system

0 : No packets

1 : 1 ✓

2 : 2 ✓



$P_{00} = (1-pa)^2 = \frac{1}{4}$ $\qquad P_{10} = ps(1-pa)^2 = \frac{1}{12}$

$P_{01} = pa = \frac{1}{2}$ $\qquad P_{11} = ps\,pa = \frac{1}{6}$

$P_{02} = \frac{1}{4}$

$P_{12} = \frac{3}{4}$

$P_{20} = ps^2(1-pa)^2 = \frac{1}{36}$ $\qquad P_{22} = \frac{11}{12}$

$P_{21} = ps(1-ps)(1-pa)^2 = \frac{1}{18}$

18.1.B

	0	1	2
0	1/4	1/2	1/4
1	1/12	1/6	3/4
2	1/36	1/18	11/12

18.1.C

$\pi_0 \frac{3}{4} = \pi_1 \frac{1}{12} + \pi_2 \frac{1}{36} \Rightarrow (1)\ \pi_0 \frac{3}{4} = \pi_0 \frac{1}{20} + \pi_2 \frac{1}{180} + \pi_2 \frac{1}{36}$

$\pi_1 \frac{5}{6} = \pi_0 \frac{1}{2} + \pi_2 \frac{1}{18} \Rightarrow \pi_1 = \pi_0 \frac{3}{5} + \pi_2 \frac{1}{15}$

$(1)\ \boxed{\pi_0 21 = \pi_2}$

$\pi_2 \frac{1}{12} = \pi_0 \frac{1}{4} + \pi_1 \frac{3}{4}$

$\pi_0 \frac{7}{4} = \pi_0 \frac{1}{4} + \pi_1 \frac{3}{4}$

$\sum \pi_i = 1$

$\pi_0 \frac{3}{2} = \pi_1 \frac{3}{4} \Rightarrow \boxed{\pi_0 2 = \pi_1}$

$\pi_0 = \frac{1}{1+2+21} = \boxed{\frac{1}{24}}$ $\qquad \boxed{\pi_1 = \frac{1}{12}}$ $\qquad \boxed{\pi_2 = \frac{7}{8}}$

18.1.D

$$P(\text{Loss}) = \frac{\pi_1 + \pi_2}{2} = \frac{23}{48}$$

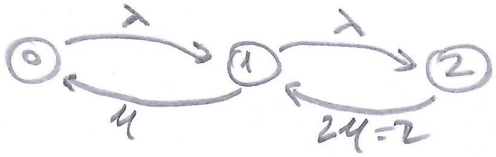
18.1.E

$$\text{Loss rate} = \pi_1 p_a^2 + \pi_2 p_a = \frac{1}{12}\frac{1}{4} + \frac{7}{8}\frac{1}{2} = \boxed{\frac{11}{24}}$$

$$S = \pi_1 p_s + \pi_2 p_s^2 = \frac{1}{12}\frac{1}{3} + \frac{7}{8}\frac{1}{9} = \boxed{\frac{1}{8}}$$

18.2

18.2 A



$$Q = \begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & -2 & 2 & 0 \\ 1 & 1 & -3 & 2 \\ 2 & 0 & 2 & -2 \end{array}$$

18.2 B

$$\pi_0 = \frac{1}{G} \qquad G = 1 + 2 + 2 = \boxed{5}$$

$$\pi_1 = \frac{1}{G}\frac{2}{1} = \frac{1}{G}2$$

$$\pi_2 = \frac{1}{G}2\frac{2}{2} = \frac{1}{G}2 \cdot 1$$

$$\pi_0 = \frac{1}{5}$$

$$\pi_1 = \frac{2}{5}$$

$$\pi_2 = \frac{2}{5}$$

18.2.C

$P(B \geq 0) = \pi_1(0)$

$$\det(\lambda I - Q) = \begin{vmatrix} \lambda+2 & 2 & 0 \\ 1 & \lambda+3 & 2 \\ 0 & 2 & \lambda+2 \end{vmatrix} = (\lambda+2)\left[(\lambda+3)(\lambda+2) - 4\right]$$

$\lambda_0 = 0, \lambda_1 = -2, \lambda_2 = -5$

$\pi_2(t) = 1 + ae^{-2t} + be^{-5t}$

$\pi_2(0) = 1 + a + b = 0 \Rightarrow a = -b-1$ $\qquad -\frac{5}{2}b = -b-1$

$\pi_2'(0) = -2a - 5b = 0 \Rightarrow -2a = 5b$ $\qquad 1 = \frac{1}{2}b$

$a = -\frac{5}{2}b$

$\boxed{2 = b}$

$\boxed{a = -3}$

$$\boxed{\pi_2(t) = 1 - 3e^{-2t} + 2e^{-5t}}$$

18.2.D

$(\pi_1 + \pi_2) = L = \boxed{\frac{4}{5}}$

18.2.F

$N_S = N - N_Q = \rho$ $\qquad N = \frac{\rho}{1-\rho} \Rightarrow \rho = \frac{\lambda}{\mu} = \frac{2}{1}$

$N = \lambda T$ $\qquad N = \frac{2}{1-2} = \boxed{-2}$ $\qquad N_Q = \frac{4}{1-2} = \boxed{-4}$

$-2 = 2T$ $\qquad \boxed{T = -1}$

$\boxed{N_S = 2}$ $\qquad -2-(-4) = \boxed{2}$