Stochastic Network Modeling
Homework 6 - Solutions

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Problem 6.1

6.1.1

$$\begin{aligned} f_{ss} &= p_{ss} + p_{sc} f_{ss} && \text{(1a)} \\ &= 0.9 + 0.1 f_{ss} && \text{(1b)} \\ &= \frac{0.9}{1 - 0.1} && \text{(1c)} \\ &= 1 && \text{(1d)} \end{aligned}$$

$$\begin{aligned} f_{cc} &= p_{cc} + p_{cs} f_{cc} && \text{(2a)} \\ &= 0.8 + 0.2 f_{cc} && \text{(2b)} \\ &= \frac{0.8}{1 - 0.2} && \text{(2c)} \\ &= 1 && \text{(2d)} \end{aligned}$$

6.1.2

$$\begin{aligned} m_{ss} &= p_{ss} + p_{sc}(1 + m_{ss}) && \text{(3a)} \\ &= 0.9 + 0.1 + 0.1 m_{ss} && \text{(3b)} \\ &= \frac{1}{1 - 0.1} && \text{(3c)} \\ &= 1.11 && \text{(3d)} \end{aligned}$$

$$\begin{aligned}
m_{cc} &= p_{cc} + p_{cs}(1 + m_{cc}) && \text{(4a)}\\
&= 0.8 + 0.2 + 0.2m_{cc} && \text{(4b)}\\
&= \frac{1}{1 - 0.2} && \text{(4c)}\\
&= 1.25 && \text{(4d)}
\end{aligned}$$

Problem 6.2

$$\begin{aligned}
f_{4w} &= p_{4w} + p_{44}f_{4w} + p_{45}f_{4w} + p_{46}f_{4w} && \text{(5a)}\\
&= \frac{3}{36} + \frac{27}{36}f_{4w} + 0 + 0 && \text{(5b)}\\
&= \frac{1}{3} && \text{(5c)}
\end{aligned}$$

$$\begin{aligned}
f_{5w} &= p_{5w} + p_{55}f_{5w} + p_{54}f_{5w} + p_{56}f_{5w} && \text{(6a)}\\
&= \frac{4}{36} + \frac{26}{36}f_{5w} + 0 + 0 && \text{(6b)}\\
&= \frac{2}{5} && \text{(6c)}
\end{aligned}$$

$$\begin{aligned}
f_{6w} &= p_{6w} + p_{66}f_{6w} + p_{65}f_{6w} + p_{64}f_{6w} && \text{(7a)}\\
&= \frac{5}{36} + \frac{25}{36}f_{6w} + 0 + 0 && \text{(7b)}\\
&= \frac{5}{11} && \text{(7c)}
\end{aligned}$$

Probability of wining is

$$\begin{aligned}
P(\text{wining}) &= (1 - f_{ll}) * f_{ww} * f_{4w} * f_{5w} * f_{6w} && \text{(8a)}\\
&= (1 - \frac{4}{36}) * \frac{8}{36} * \frac{1}{3} * \frac{2}{5} * \frac{5}{11} \qquad = \frac{32}{2673} && \text{(8b)}\\
&= 0.01 && \text{(8c)}
\end{aligned}$$

Problem 6.3

6.3.1

First lets do the $f_{i(HH)}$

$$\begin{aligned} f_{H(HH)} &= p_{H(HH)} + p_{H(T)} f_{T(HH)} && \text{(9a)} \\ &= \frac{1}{2} + \frac{1}{2} f_{T(HH)} && \text{(9b)} \end{aligned}$$

$$\begin{aligned} f_{(TT)(HH)} &= p_{(TT)(HH)} + p_{(TT)(TTT)} f_{(TTT)(HH)} + p_{(TT)(H)} f_{H(HH)} && \text{(10a)} \\ &= 0 + 0 + \frac{1}{2}(\frac{1}{2} + \frac{1}{2} f_{T(HH)}) && \text{(10b)} \\ &= \frac{1}{4} + \frac{1}{4} f_{T(HH)} && \text{(10c)} \end{aligned}$$

$$\begin{aligned} f_{T(HH)} &= p_{T(HH)} + p_{TH} f_{H(HH)} + p_{T(TT)} f_{(TT)(HH)} && \text{(11a)} \\ &= 0 + 0 + \frac{1}{2}(\frac{1}{2} + \frac{1}{2} f_{T(HH)}) + \frac{1}{2} f_{(TT)(HH)} && \text{(11b)} \\ &= \frac{1}{4} + \frac{1}{4} f_{T(HH)} + \frac{1}{2} f_{(TT)(HH)} && \text{(11c)} \\ &= \frac{1}{4} + \frac{1}{4} f_{T(HH)} + \frac{1}{2}(\frac{1}{4} + \frac{1}{4} f_{T(HH)}) && \text{(11d)} \\ &= \frac{1}{4} + \frac{1}{4} f_{T(HH)} + \frac{1}{8} + \frac{1}{8} f_{T(HH)} && \text{(11e)} \\ &= \frac{3}{8} + \frac{3}{8} f_{T(HH)} && \text{(11f)} \\ &= \frac{3}{5} && \text{(11g)} \end{aligned}$$

Here 11d plugin 10c

Now applying 11g into 10c we have that $f_{(TT)(HH)} = \frac{2}{5}$.

And also applying 11g into 9b we have that $f_{H(HH)} = \frac{4}{5}$.

6.3.2

Now the other absorbing state $f_{i(TTT)}$

$$\begin{aligned}
f_{H(TTT)} &= p_{H(TTT)} + p_{H(T)} f_{T(TTT)} + p_{H(HH)} f_{(HH)(TTT)} && \text{(12a)}\\
&= 0 + \frac{1}{2} f_{T(TTT)} + 0 && \text{(12b)}\\
&= \frac{1}{2} f_{T(TTT)} && \text{(12c)}
\end{aligned}$$

$$\begin{aligned}
f_{TT(TTT)} &= p_{TT(TTT)} + p_{TT(H)} f_{H(TTT)} && \text{(13a)}\\
&= \frac{1}{2} + \frac{1}{2}(\frac{1}{2} f_{T(TTT)}) && \text{(13b)}\\
&= \frac{1}{2} + \frac{1}{4} f_{T(TTT)} && \text{(13c)}
\end{aligned}$$

Here 13c is obtained applying 12c.

$$\begin{aligned}
f_{T(TTT)} &= p_{T(TTT)} + p_{T(TT)} f_{TT(TTT)} + p_{TH} f_{H(TTT)} && \text{(14a)}\\
&= 0 + \frac{1}{2} f_{(TT)(TTT)} + \frac{1}{2}(\frac{1}{2} f_{T(TTT)}) && \text{(14b)}\\
&= \frac{1}{2} f_{(TT)(TTT)} + \frac{1}{4} f_{T(TTT)} && \text{(14c)}\\
&= \frac{1}{2}(\frac{1}{2} + \frac{1}{4} f_{(T)(TTT)}) + \frac{1}{4} f_{T(TTT)} && \text{(14d)}\\
&= \frac{1}{4} + \frac{3}{8} f_{T(TTT)} && \text{(14e)}\\
&= \frac{2}{5} && \text{(14f)}
\end{aligned}$$

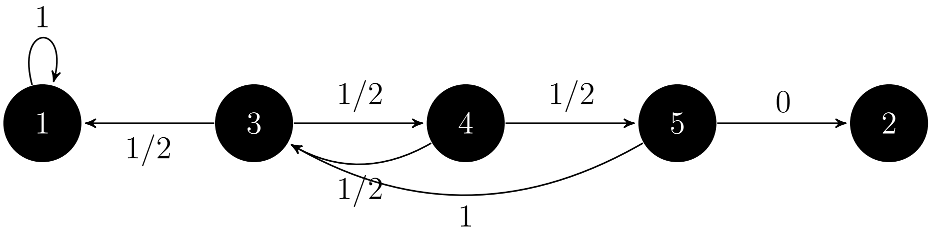
Here 14d is obtained applying 13c.

Now applying 14f to 13c we have that $f_{TT(TTT)} = \frac{3}{5}$.

Also applying 14f to 12c we have that $f_{H(TTT)} = \frac{1}{5}$.

Problem 6.4

6.4.1



6.4.2

$$
\begin{aligned}
p'_{34} &= P(X(n) = 4 | X(n-1) = 3, X(\infty) = 1) && \text{(15a)} \\
&= \frac{1}{2}\frac{4}{5} && \text{(15b)} \\
&= \frac{2}{5} && \text{(15c)}
\end{aligned}
$$

$$
\begin{aligned}
p'_{35} &= P(X(n) = 5 | X(n-1) = 3, X(\infty) = 1) && \text{(16a)} \\
&= \frac{1}{4}\frac{2}{5} && \text{(16b)} \\
&= \frac{1}{10} && \text{(16c)}
\end{aligned}
$$

$$
\begin{aligned}
p'_{35} &= P(X(n) = 5 | X(n-1) = 3, X(\infty) = 1) && \text{(17a)} \\
&= \frac{1}{4}\frac{2}{5} && \text{(17b)} \\
&= \frac{1}{10} && \text{(17c)}
\end{aligned}
$$

$$
\begin{aligned}
p'_{43} &= P(X(n) = 3 | X(n-1) = 4, X(\infty) = 1) && \text{(18a)} \\
&= \frac{1}{2}\frac{3}{5} && \text{(18b)} \\
&= \frac{3}{10} && \text{(18c)}
\end{aligned}
$$

$$\begin{aligned}
p'_{45} &= P(X(n) = 5 | X(n-1) = 4, X(\infty) = 1) && \text{(19a)} \\
&= \frac{1}{2}\frac{2}{5} && \text{(19b)} \\
&= \frac{1}{10} && \text{(19c)}
\end{aligned}$$

$$p'_{i2} = P(X(n) = 2 | X(n-1) = i, X(\infty) = 1) = 0 \qquad \text{(20a)}$$