HOMEWORK 21

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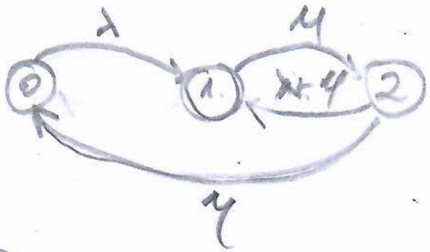
PROBLEM 21.1

21.1.A

States

in THE SYSTEM



21.1.B

$$Q = \begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & -1/2 & 1/2 & 0 \\ 1 & 0 & -1 & 1 \\ 2 & 1 & \frac{3}{2} & -\frac{5}{2} \end{array}$$

$$\pi_0 \frac{1}{2} = \pi_2$$

$$\pi_1 = \pi_0 \frac{1}{2} + \pi_2 \frac{3}{2} \Rightarrow \pi_1 = \pi_0 \frac{5}{4}$$

$$\sum \pi_i$$

$$\pi_0 = \frac{1}{1+\frac{1}{2}+\frac{5}{4}} = \boxed{\frac{4}{11}}$$

$$\boxed{\pi_1 = \frac{5}{11}}$$

$$\boxed{\pi_2 = \frac{2}{11}}$$

21.1.C

$S_2 = \lambda - L_2$ $\quad L_2$ = loss rate in 2

$$L_2 = \lambda \pi_2 = \boxed{\frac{1}{11}}$$

$$S_2 = \frac{1}{2} - \frac{1}{11} = \boxed{\frac{9}{22}}$$

21.1.D

$P_L = \pi_1 + \pi_2 = \boxed{\frac{7}{11}}$ BY PASTA

21.1.E

$$\det(\lambda I - Q) = \det \begin{pmatrix} \lambda+1/2 & 1/2 & 0 \\ 0 & \lambda+1 & 1 \\ 1 & \frac{3}{2} & \lambda+\frac{5}{2} \end{pmatrix}$$

$$\left(\lambda+\frac{1}{2}\right)\left[\left(\lambda+\frac{5}{2}\right)(\lambda+1) - \frac{3}{2}\right] + \frac{1}{2}$$

$$\left(\lambda+\frac{1}{2}\right)\left(\lambda^2+\lambda+\frac{5}{2}\lambda+1-\frac{3}{2}\right)+\frac{1}{2}$$

$$\left(\lambda+\frac{1}{2}\right)\left(\lambda^2+\frac{7}{2}\lambda-\frac{1}{2}\right)+\frac{1}{2}$$

$\lambda_0 = 0$

$\lambda_1 = -\frac{78}{25}$

$\lambda_2 = -\frac{22}{25}$

$$\pi_2(t) = 1 + a e^{-\frac{78}{25}t} + b e^{-\frac{22}{25}t}$$

$$\pi_2'(0) = 1 + a + b = 0 \qquad b = -1-a$$

$$\pi_2''(0) = -\frac{78}{25}a - \frac{22}{25}b = 0 \Rightarrow b = -\frac{39}{11}a$$

$$-1-a = -\frac{39}{11}a$$

$$\boxed{\pi_2(t) = 1 + \frac{11}{28}e^{-\frac{78}{25}t} - \frac{39}{28}e^{-\frac{22}{25}t}} \qquad \boxed{\frac{11}{28} = a} \qquad \boxed{b = -\frac{39}{28}}$$

$$T = \int_0^\infty -\frac{11}{28}e^{-\frac{78}{25}} + \frac{39}{28}e^{-\frac{22}{25}t}\,dt$$

$$= -\frac{11}{28}\frac{25}{78} + \frac{39}{28}\frac{25}{22}$$

$$= -\frac{275}{2184} + \frac{975}{616} = \boxed{\frac{625}{429}} \Rightarrow \boxed{1.46}$$

$$T = \frac{1.46}{1/2} = \boxed{2.92}$$

21.1.F

$N = 1.46 \cdot \frac{1}{2} = \boxed{0.73}$

21.1.G

Because There are 2 concatenated servers and they don't follow neither a Markovian dist nor a Deterministic

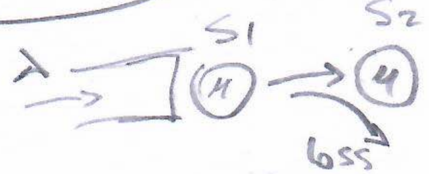
$N = \lambda T \Rightarrow N = \frac{1}{2} \, 2.92 = \boxed{1.46}$

PROBLEM 21.2

21.2.A

No Because both queues should be ∞, and if we have 1 w/ size ∅ we have loses.

21.2.B

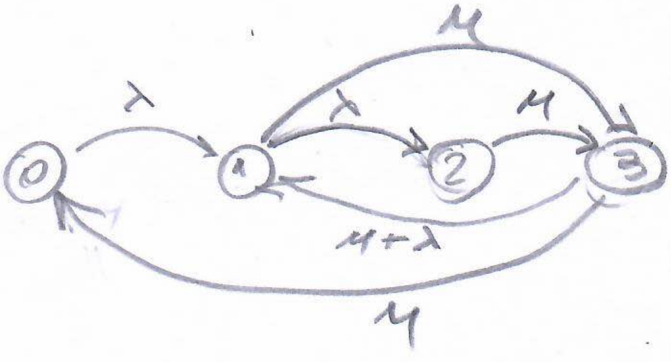


0: empty

1: S_1 busy que empty

2: " " " full

3: S_2 busy



$$\left.\begin{aligned} \pi_0 \tfrac{1}{2} &= \pi_3 \\ \pi_1 \tfrac{3}{2} &= \pi_3 \tfrac{3}{2} \\ \pi_2 &= \pi_1 \tfrac{1}{2} \end{aligned}\right\} \begin{aligned} &\Rightarrow \pi_1 \tfrac{3}{2} = \pi_0 \tfrac{3}{4} \Rightarrow \pi_1 = \pi_0 \tfrac{1}{2} \\ &\Rightarrow \pi_2 = \pi_0 \tfrac{1}{4} \end{aligned}$$

$$\pi_0 = \frac{1}{1+\frac{1}{2}+\frac{1}{4}+\frac{1}{2}} = \boxed{\frac{4}{9}}$$

$$\pi_1 = \frac{2}{9}, \quad \pi_2 = \frac{1}{9}, \quad \pi_3 = \frac{2}{9}$$

21.2.a

$$\pi_{n_1, n_2} = (1-\rho_1)\rho_1^{n} \, (1-\rho_2)\rho_2^{n}$$

$$\lambda_1 = \frac{1}{2}$$

$$\lambda_2 = \frac{1}{2}$$

$$\rho_1 = \frac{1/2}{1} = \frac{1}{2} \Rightarrow 1-\rho_1 = \frac{1}{2} = \rho_2 = 1-\rho_2$$

$$\pi_{00} = \frac{1}{2}\frac{1}{2} = \boxed{\frac{1}{4}} \neq \frac{4}{9}$$

$$\pi_{11} = \frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2} = \boxed{\frac{1}{16}}$$

PROBLEM 21.3

21.3.A

Yes because both queue are ∞

21.3.B