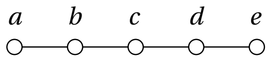
Problem 3.1

Suppose a particle moving in a straight line over a set of 5 points (see the figure). The particle starts in the middle ($X(0) = c$), and in each step it moves to the right with probability p and to the left with probability $q = 1 - p$.

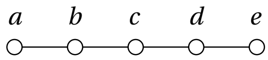


3.1.A Build a tree with the possible outcomes of $X(0)$, $X(1)$, $X(2)$, and compute the joint probabilities $P(X(0), X(1), X(2))$.

3.1.B Compute the probabilities $P(X(2) = i)$, $i = \{a, b, c, d, e\}$.

Problem 3.2

Suppose a particle moving in a straight line over a set of 5 points. In each step it moves to the right with probability p and to the left with probability q.



Assume the following behavior in the boundary points (a and e):

(a) In a returns to b with probability p or remains with probability $(1 - p)$. In e returns to d with probability $(1 - p)$ or remains with probability p.

(b) Return with probability 1 to the previous state.

(c) Remain with probability 1.

For the previous chains:

3.2.A Draw the state transition diagram.

3.2.B Construct the transition matrix, $\mathbf{P}$.

Problem 3.3

Modify the MC of the problem 3.2 such that the particle remains two steps in the boundary points and returns to the previous point in the 3-th step. That is, in the boundary points, the particle returns to the previous point in 3 steps with probability 1.

3.3.A Say why this chain with 5 states cannot be a Markov chain.

3.3.B Show that it is possible to find a Markov chain if more that 5 states are allowed. Construct the transition matrix of the Markov chain.

Problem 3.4

3.4.A Formulate the game of problem 1.6.B using a DTMC: define the states, transition diagram, compute the one step transition probabilities, transition probability matrix $\mathbf{P}$, and the initial distribution $\boldsymbol{\pi}(0)$.

3.4.B Let $X(n)$ be the state of the chain at step n in the previous item. Let 2 be the state "obtain 2 dice equal, 1 different". Compute $\pi_2(\infty) = P(X(\infty) = 2)$ using the law of total probability.

3.4.C Compute the expected benefit of the player using the DTMC.