Problem 19.1

Consider an M/D/1/2 queue (deterministic services and queue size of 1 position). Service time is 1 time unit, and arrival rate is 1/2 customers/time unit.

19.1.A Compute the probability of i arrivals during a service time:

$$v_i = \int_{x=0}^{\infty} \frac{(\lambda\, x)^i}{i\,!}\, \mathrm{e}^{-\lambda\, x}\, f_S(x)\; \mathrm{d}x$$

19.1.B Draw the DTMC state transition diagram of the queue observed at departure time instants, in terms of v_i.

19.1.C Compute the stationary distribution of the previous chain, π_i^d.

Problem 19.2

Consider the semi-Markov process obtained observing the M/D/1/2 queue of problem 19.2 at transition instants of the number in the system observed at arrival and departing times.

19.2.A Build the embedded MC and compute the stationary distribution.

19.2.B Compute the stationary distribution of the continuous time process, π_i^a.
Hint: The distribution function of a random variable A exponentially distributted with rate λ, given that occurs in an interval $[0, T]$, is

$$F_A(t|T) = \frac{1 - \mathrm{e}^{-\lambda\, t}}{1 - \mathrm{e}^{-\lambda T}},\; t \in [0, T].$$

Recall that

$$\mathrm{E}[A|T] = \int_0^T (1 - F_A(t|T))\, \mathrm{d}t = \frac{1 - \alpha - \alpha\, \lambda\, T}{\lambda\, (1 - \alpha)} = \frac{1}{\lambda} - \frac{\alpha\, T}{1 - \alpha},\; \text{where } \alpha = \mathrm{e}^{-\lambda\, T}.$$

19.2.C Compute the loss probability, p_L, using the stationary distribution computed in the previous item. Check it with the loss probability formula for an M/G/1/K queue.

19.2.D Check that the stationary distributions obtained in items 19.2.C and 19.3.B satisfy:

$$\pi_i^a = \pi_i^d\, (1 - p_L),\; i = 0, 1 \tag{3}$$

where p_L is the loss probability.