Problem 6.1

6.1.A Compute the first transition probabilities in n steps of problem 5.1 ($f_{ss}(n)$ and $f_{cc}(n)$, $n \geq 1$). Check that both states are recurrent ($\sum_{n=1}^{\infty} f_{ii}(n) = 1$).

6.1.B Compute the mean recurrence times (m_{ss} and m_{cc}) using $f_{ss}(n)$ and $f_{cc}(n)$.

6.1.C Compute the mean recurrence times (m_{ss} and m_{cc}) using the recursive equations. Check with previous item.

Problem 6.2

Formulate the Craps game (see problem 2.7) as an absorbing DTMC. Let w be the winning state. Compute the first transition probabilities f_{iw} for all other states i of the chain. Use the probabilities f_{iw} to compute the player's winning probability.

Problem 6.3

Compute the probability of reaching each absorbing state of problem 4.1 using the recursive equations of the first transition probabilities.

Problem 6.4

Assume that the game of problem 4.1 finishes because 2 consecutive heads occurs. Denote this state as state 1.

6.4.A Derive the transition diagram of this new DTMC.

6.4.B Let $X(n)$ be the state in the original chain. Compute the one step probabilities of the new chain:

$$p'_{ij} = P(X(n) = j \mid X(n-1) = i, X(\infty) = 1),$$

in terms of the probabilities of the original chain:

$$p_{ij} = P(X(n) = j \mid X(n-1) = i)$$

and the first transition probabilities f_{ij} of the original chain (obtained in problem 6.3).