# Stochastic Network Modeling
# Homework 8 - Solutions

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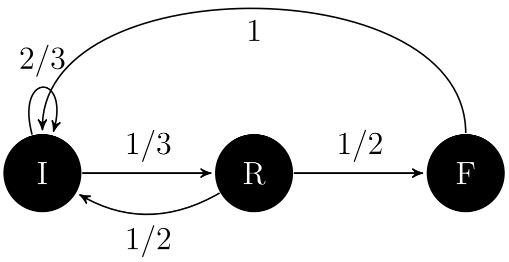
October 10, 2020

**Problem 8.1**

**8.1.1**




$$P = \begin{array}{c} \\ I \\ R \\ F \end{array} \begin{bmatrix} I & R & F \\ 2/3 & 1/3 & 0 \\ 1/2 & 0 & 1/2 \\ 1 & 0 & 0 \end{bmatrix}$$

**8.1.2**

$$\begin{cases} \pi_I \frac{1}{3} & = \pi_R \frac{1}{2} \implies \pi_I \frac{2}{3} = \pi_R \\ \pi_I \frac{1}{3} & = \pi_F \\ \sum \pi_i = 1 \end{cases} \tag{1a}$$

$$\pi_I = \frac{1}{1 + \frac{2}{3} + \frac{1}{3}} = \frac{1}{2} \tag{2a}$$
$$\tag{2b}$$

Therefore $\pi_I = \frac{1}{2}, \pi_R = \frac{1}{3}, \pi_F = \frac{1}{6}$

### 8.1.3

$S = p \times \pi_F = \frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$

### 8.1.4

$\begin{cases} 1 & \text{if it reaches to the end } \pi_F \\ 0 & \text{otherwise} \end{cases}$

$$E[T] = \sum_{i=0}^{n} \pi_F \tag{3a}$$
$$= \sum_{i=0}^{n} \frac{1}{6} \tag{3b}$$
$$= \frac{n}{6} \tag{3c}$$

## Problem 8.2

### 8.2.1

$$\begin{cases} \pi_a & = \pi_b \alpha \\ \pi_b(1-\alpha) & = \pi_c \\ \sum \pi_i = 1 \end{cases} \tag{4a}$$

$$\pi_b = \frac{1}{1 + \alpha + (1 - \alpha)} = \frac{1}{2} \tag{5a}$$
$$\tag{5b}$$

Therefore $\pi_a = \frac{\alpha}{2}, \pi_b = \frac{1}{2}, \pi_c = \frac{1-\alpha}{2}$

### 8.2.2

### 8.2.3

$S = \pi_a = \frac{\alpha}{2}$

### 8.2.4

$E[T] = \frac{1}{\pi_a} = 2$

## Problem 8.3

Lets the probabilities according to the definition be:

- $p_{ab} = \frac{1}{3}$
- $p_{ac} = \frac{1}{3}$
- $p_{ad} = \frac{1}{3}$
- $p_{ba} = 1$
- $p_{db} = 1$
- $p_{ca} = \frac{1}{2}$
- $p_{cd} = \frac{1}{2}$

Solving by flux method

$$\begin{cases} \pi_a \frac{1}{3} & = \pi_b \\ \pi_a \frac{1}{3} & = \pi_c \frac{1}{2} \implies \pi_a \frac{2}{3} = \pi_c \\ \pi_a \frac{1}{3} & = \pi_d \\ \sum \pi_i = 1 \end{cases} \tag{6a}$$

$$\pi_a = \frac{1}{1 + \frac{1}{3} + \frac{2}{3} + \frac{1}{3}} \tag{7a}$$

$$= \frac{3}{7} \tag{7b}$$

Therefore $\pi_a = \frac{3}{7}, \pi_b = \pi_d = \frac{1}{7}, \pi_c = \frac{2}{7}$