Stochastic Network Modeling
Homework 4 - Solutions

Juan Pablo Royo Sales
Universitat Politècnica de Catalunya

September 27, 2020

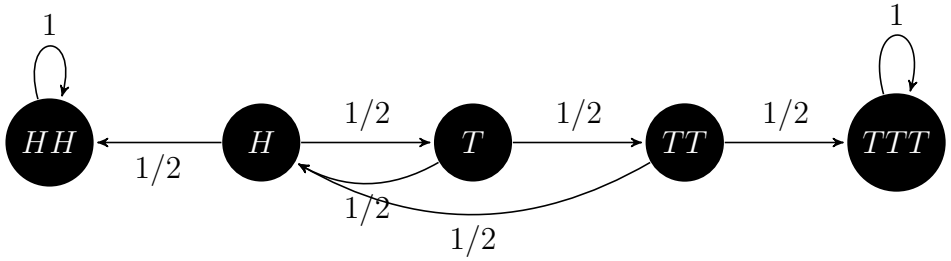
Problem 4.1

4.1.1

States are:

- H When a head shows up
- T When a tail shows up
- HH When a head shows up after H
- TT When a tail shows up after T
- TTT When a tail shows up after TT

4.1.2



4.1.3

$$P = \begin{bmatrix} & H & T & HH & TT & TTT \\ H & 0 & 1/2 & 1/2 & 0 & 0 \\ T & 1/2 & 0 & 0 & 1/2 & 0 \\ HH & 0 & 0 & 1 & 0 & 0 \\ TT & 1/2 & 0 & 0 & 0 & 1/2 \\ TTT & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

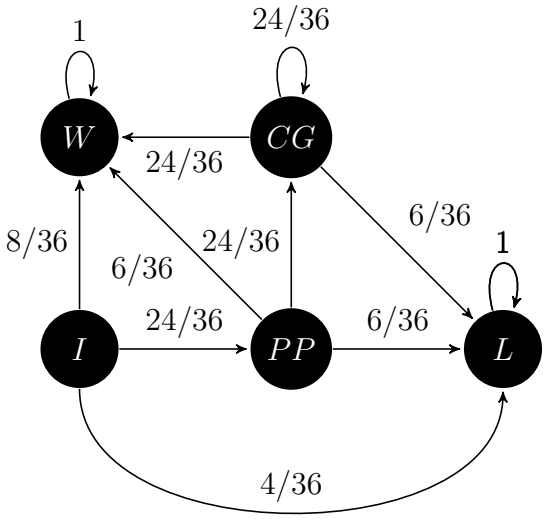
4.1.4

Absorbing states are TTT and HH.

Problem 4.2

States are:

- I: Initial state
- W: Win $\{7, 11\}$ or after second point.
- L: Loses $\{2, 3, 12\}$ or after 7 after point.
- PP: Player Point.
- CG: Continue Gambling.



$$P = \begin{bmatrix} & I & W & L & PP & CG \\ I & 0 & 8/36 & 4/36 & 24/36 & 0 \\ W & 0 & 1 & 0 & 0 & 0 \\ L & 0 & 0 & 1 & 0 & 0 \\ PP & 0 & 6/36 & 6/36 & 0 & 24/36 \\ CG & 0 & 6/36 & 6/36 & 0 & 24/36 \end{bmatrix}$$

$$\pi(0) = (1, 0, 0, 0, 0) \tag{1a}$$

Problem 4.3

4.3.1

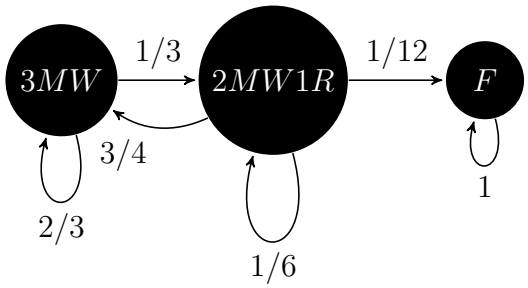
States are:

- F: Machine Failure
- $3MW$: 3 Motors working
- $2MW1R$: 2 Motors working 1 Repair

Probabilities are:

- $P(1R|\ 2MW1R) = P(1R)P(2W) = \frac{2}{3}\frac{1}{4}$: Probability that 1 continue in repair when 2 is working.
- $P(1B|\ 2MW1R) = P(1B)P(2W) = \frac{1}{3}\frac{1}{4}$: Probability that 1 fail when 2 is working.

- The rest are known by the statement.



4.3.2

$$P = \begin{bmatrix} & 3MW & 2MW1R & F \\ 3MW & \frac{2}{3} & \frac{1}{3} & 0 \\ 2MW1R & \frac{3}{4} & \frac{1}{6} & \frac{1}{12} \\ F & 0 & 0 & 1 \end{bmatrix}$$

$$\pi(0) = (1, 0, 0) \tag{2a}$$

4.3.3

$$E[T] = T \times \pi(0)P^T \tag{3a}$$