Problem 1.1
Roll a pair of fair dice once. What is the probability that the sum of the faces gives and odd number, given that at least one die shows 6?

Problem 1.2
Suppose a king from a family of two children. What is the probability that the other child is his sister (i.e. a girl)?

Problem 1.3 *Monty Hall problem*
Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. The game consists of the following

(a) First you choose a door, and keep it close.

(b) The host choose one of the other doors with a goat behind and opens it.

(c) You choose whether switch door or keep your choice, and open it.

Is it your advantage to switch your choice? Compute the probability of winning if you switch or not.[1]

Problem 1.4
A fair coin is tossed repeatedly and all tosses are independent. For each $k \geq 1$, define:

$$I_k = \begin{cases} 0, & \text{if } k \text{ toss is head} \\ 1, & \text{if } k \text{ toss is tail} \end{cases}$$

$$S_n = \sum_{k=1}^{n} I_k.$$

Using Wald's equation

1.4.A Compute $\mathrm{E}[S_n]$.
1.4.B Let N be the number of the tosses until the second head shows up. Compute $\mathrm{E}[S_N]$.

Problem 1.5
A clueless student face the following quiz: a list of 10 Roman Emperors that ruled during different terms, and the scrambled term list. The object is to match the Emperor with the term. Compute the expected number of correct matches upon the following strategies:

1.5.A At each guess choose a term at random from the list. Thus, there might be some terms never assigned.
1.5.B At each guess choose a term at random and remove it from the list. Thus, there will be a one-to-one correspondence.

Problem 1.6
A game consists of rolling 3 dice. A player has to pay 5 euros to participate (roll the dice once). The player gets: 10 euros if two equal results are obtained; 25 euros if three equal results are obtained; nothing if all three results are different. determine:

1.6.A The expected benefit in a single game (roll the dice once).
1.6.B The expected benefit if a player has decided to repeat the game until getting a prize. Check your result with Wald equation.

[1] *Monty Hall* is the name of the host of the TV show *Let's Make a Deal*, where the game was played.