1 Semi-formal

The size n *Eternity* puzzle consists of a $n \times n$ square *grid*, and n^2 square *tiles*. Each side of the tiles has a *pattern* (out of k possibilities). The goal of the puzzle is to place each tile in one of the grid cells. Tiles can be *rotated* in their cell $0, 90, 180$ or 270 degrees. The tiles are correctly placed in the grid if every pair of adjacent tiles have the same pattern in their adjoining sides. The 0 pattern is special and must appear on the boundaries of the grid

2 Formal

In the (n, k)-*Eternity* puzzle there are n^2 tiles. A *tile* is a tuple of 4 elements, where $T[i]$ the i-th element of tile T. Each element corresponds to one of the four cardinal directions in a clockwise order (North, East, South and West, respectively). Each element of a tile is a integer number in the $[0..k]$ interval. *Shifting* a tile one position means moving each element one position towards its right except for the last element that becomes first. Note that shifting corresponds to rotating the tile 90 degrees when we consider the cardinal directions. Clearly, a tile can be shifted up to 3 position, since shifting 4 positions produces the original tile. We denote by T^d (with $0 \leq d \leq 3$) the result of T shifted d positions.

A *solution* of the (n, k)-Eternity puzzle is an assignment of each tile to a different cell in a $n \times n$ grid cell and a rotation in such a way that joined edges of adjacent tiles have the same assigned value. Formally, a solution is a function $F(T) = (i, j, d)$ satisfying *boundary* and *adjoining* conditions:

- Boundary conditions require value 0 to appear on the boundaries of the grid,
  1. (North boundary) if $i = 1$ then $T^d[1] = 0$
  2. (Soutn boundary) if $i = n$ then $T^d[3] = 0$
  3. (West boundary) if $j = 1$ then $T^d[4] = 0$
  4. (East boundary) if $j = n$ then $T^d[2] = 0$
- Adjoining conditions require adjoining sides of tiles to have the same value. Given two tiles T and S with assignment $F(T) = (i, j, d)$ and $F(S) = (i', j', d')$,
  1. if $i = i'$ and $j = j' + 1$ then $T^d[4] = S^{d'}[2]$
  2. if $i = i'$ and $j = j' - 1$ then $T^d[2] = S^{d'}[4]$
  3. if $j = j'$ and $i = i' + 1$ then $T^d[1] = S^{d'}[3]$
  4. if $j = j'$ and $i = i' - 1$ then $T^d[3] = S^{d'}[1]$