1. The *cooperation* game is defined as follows. There is a group N of n people and a task to be performed. To perform correctly the task requires that exactly k persons cooperate. Each player can decide whether to cooperate (1) or not (0). The utility of a strategy profile $x \in \{1,0\}^i$ for player i is defined as

$$u_i(x) = \begin{cases} 1 & \text{the task is performed and } x_i = 1. \\ 0 & \text{otherwise} \end{cases}$$

   - Provide a formal characterization of the best response sets, for player $i \in N$ and strategy profile x.
   - Provide a formal characterization of the strategy profiles that are pure Nash equilibrium of the cooperation game.
   - Analyze the computational complexity of the problems related to pure Nash equilibria for this family of games.

2. The *weak cooperation* game is defined as follows. There is a group N of n people and a task to be performed. To perform correctly the task requires that at least k persons cooperate. Each player can decide whether to cooperate (1) or not (0). The utility of a strategy profile $x \in \{1,0\}^i$ for player i is defined as

$$u_i(x) = \begin{cases} 1 & \text{the task is performed and } x_i = 1. \\ 0 & \text{otherwise} \end{cases}$$

   - Provide a formal characterization of the best response sets for player $i \in N$ and strategy profile x.
   - Provide a formal characterization of the strategy profiles that are pure Nash equilibrium of this game.
   - Analyze the computational complexity of the problems related to pure Nash equilibria for this family of games.

3. The *split cooperation* game is defined as follows. There is a group N of n people and a task to be performed. To perform correctly the task requires that at least k persons cooperate. Each player can decide whether to cooperate (1) or not (0). The utility of a strategy profile $x \in \{1,0\}^i$ for player i is defined as

$$u_i(x) = \begin{cases} \frac{k}{|x|_1} & \text{the task is performed and } x_i = 1. \\ 0 & \text{otherwise} \end{cases}$$

   where $|x|_1 = |\{i \mid x_i = 1\}|$.

- Provide a formal characterization of the best response sets for player $i \in N$ and strategy profile x.
- Provide a formal characterization of the strategy profiles that are pure Nash equilibrium of this cooperation game.
- Analyze the computational complexity of the problems related to pure Nash equilibria for this family of games.

4. The *matching* game is played in a bipartite graph $G = (V_1, V_2, E)$ in which edges are connect only vertices V_1 to vertices in V_2. The players are the vertices in the graph that is $V_1 \cup V_2$. Each player has to select one of its neighbors. Player i gets utility 1 when the selection is mutual (player i selects j and player j selects i) otherwise he gets 0.

   Provide a formal characterization of the strategy profiles that are pure Nash equilibrium of the matching game. Analyze the complexity of the problems related to pure Nash equilibria for this family of games.

5. In the **cover game** the players are the vertices in an undirected graph $G = (V, E)$ on a set of n vertices. The goal of the game is to select a set of vertices X that covers a lot of edges. An edge is covered by a set X if at least one of its ends points belongs to X.

   Formally, the set of actions allowed to player i is $A_i = \{0, 1\}$. Those players playing 1 will form the set. Let $s = (s_1, \dots, s_n)$, $s_i \in \{0, 1\}$, be an strategy profile, and let $X(s) = \{i \mid s_i = 1\}$.

   The cost function for player $i \in V$ is defined as follows

   $$c_i(s) = s_i + |\{(i, j) \in E \mid i, j \notin X(s)\}|.$$

   - Provide a formal characterization of the best response set for player $i \in V$.
   - Does this game have always a pure Nash equilibrium? If not, provide an example of a game in the family with no PNE.
   - Analyze the computational complexity of the problems related to pure Nash equilibria for this family of games.

6. Consider a set of n players that must be partitioned into two groups. However, there is a set of bad pairings and the two players in such a pair do not want to be in the same group. Moreover, each player is free to choose which of the two groups to be in. We can model this by a graph $G = (V, E)$ where each player i is a vertex. There is an edge (i, j) if i and j form a bad pair. The private objective of player i is to maximize the number of its neighbors that are in the other group.

   Provide a formal characterization of the strategy profiles that are pure Nash equilibrium of this game. Analyze the complexity of the problems related to pure Nash equilibria for this family of games.

7. The Max 2SAT game is defined by a weighted 2-CNF formula on n variables. In a weighted formula each clause has a weight. The game has n players. Player i controls the i-th variable and can decide the value assigned to this variable. A strategy profile is a truth assignment $x \in \{0, 1\}^n$. Player i gets $1/3$ of the weight of the clauses that are satisfied due to its bit selection.

Provide a formal characterization of the strategy profiles that are pure Nash equilibrium of the Max 2SAT game. Analyze the complexity of the problems related to pure Nash equilibria for this family of games.

8. Assume that we have fixed a finite set K of k colors. Consider a graph $G = (V, E)$ with a labeling function $\ell : V \to 2^K$ and define an associated *coloring game* $\Gamma(G, \ell)$ as follows

   - the players are $V(G)$,
   - the set of strategies for player v is $\ell(v)$,
   - the payoff function of player v is $u_v(s) = |\{u \in N(v) \mid s_u = s_v\}|$.

   Provide a formal characterization of the strategy profiles that are pure Nash equilibrium of the coloring game game. Analyze the complexity of the problems related to pure Nash equilibria for this family of games.

9. Consider a network formation game where players are interested only in creating a connected network. In such a game we are given a connected graph G in which each edge e has a cost $c(e)$. Consider a game with one player per vertex in G. Player u can select any subset s_u of the edges incident with u in G. The cost for each player is ∞ if the subgraph resulting from the union of the selected edges is not connected, otherwise the cost for player u is the sum of the costs of the edges in s_u.

   (a) Can a best response be computed in polynomial time?

   (b) Provide a characterization of the PNE of the game. Can a PNE (if any) be computed in polynomial time?

10. Compute the Nash equilibria (pure and mixed) of the following game:

		Player 2 A	Player 2 B
Player 1	A	6,6	2,7
	B	7,2	0,0

11. We have two printers and three users. Both printers are equal and work under the same conditions. A printer is able to print a job in time proportional to the size of the job without unnecessary delays between jobs. Each user has a job to print and has to select a printer machine. The users have jobs of sizes 4, 6 and 10 respectively. Once the selection of printers is done both printers start at the same time processing their corresponding assignment and the printing is made available to the user just when the printer finishes the allocated work.

    - Assuming that the speed factor of the printers is one, provide a table with the utility for each player and pure strategy profile.
    - Find all the Nash equilibria of the game (pure and mixed).

12. A *fully mixed strategy* is a mixed strategy that assigns positive probability to all the possible actions. Consider the two player's game described by the following bi-matrix.

		Player 2	
		A	B
Player 1	C	1,1	4,2
	D	3,3	1,1
	E	2,2	2,3

Find all the NE for the game having a fully mixed strategy for each player or show that such Nash equilibria do not exists.

13. Consider the following strategic game:

		Player 2		
		R	S	T
Player 1	A	6,6	2,7	2,6
	B	7,2	3,4	0,0

Determine whether the strategy profile

$$(0.6, 0.4), (0.2, 0.4, 0.4)$$

is a Nash equilibrium.

14. Consider the cover game defined in exercise **??**. Is a fully mixed strategy profile a NE for the game?

15. Show that in a singleton congestion games on a game with n players accesing to a set of m resourses, all best response sequences have length at most $n^2 m$.