A Report of *Type Theory and Formal Proof*

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February 9, 2021

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1 Introduction

This report is going to provide a summary over the book [NG14]. Alongside the different chapters of the book I am going to describe briefly the most important parts of each chapter and, at the same time, I am going to solve 1 or 2 of the exercises proposed by the authors.

The organization of the report is going to be the same as the chapters of the book.

2 Untyped lambda calculus

In this first chapter the authors define and describe Lambda Calculus (λ-*calculus*) system which encapsulates the formalization of basic aspects of mathematical functions, in particular construction and use. In λ-*calculus* formalization system there are *typed* and *untyped* formalization of the same system. In this first case authors introduced the first basic and simple formalization which is *untyped*.

2.1 Definition

There are *two constructions principles* and *one evaluation rule*

Construction principles:

- *Abstraction:* Given an expression M and a variable x we can construct the expression: $\lambda x.M$. This is abstraction of x over M Example: $\lambda y.(\lambda x.x - y)$ Abstraction of y over $\lambda x.x - y$
- *Application:* Given 2 expressions M and N we can construct the expression: $M\ N$. This is the application of M to N. Example: $(\lambda x.x^2+1)(3)$ Application of 3 over $\lambda x.x^2 + 1$

Evaluation Rule: Formalization of this process is called Beta Reduction (*β-reduction*). *β-reduction*: An expression $(\lambda x.M)N$ can be rewritten to $M[x := N]$, which means every x should be replaced by N in M. This process is called *β-reduction* of $(\lambda x.M)N$ to $M[x := N]$.

Example: $(\lambda x.x^2 + 1)(3)$ reduces to $(x^2 + 1)[x := 3]$, which is $3^2 + 1$.

In this book, functions on λ-*calculus* notation are *Curried*.

2.1.1 Lambda-terms

Expressions in λ-*calculus* are called Lambda Terms (λ-*term*)

Definition 1. *The set Λ of all λ-term*

1. (Variable) If $u \in V$, then $u \in \Lambda$
2. (Application) If M and $N \in \Lambda$, then $(MN) \in \Lambda$
3. (Abstraction) If $u \in V$ and $M \in \Lambda$, then $(\lambda u.M) \in \Lambda$

References

[NG14] Rob Nederpelt and Herman Geuvers. *Type Theory and Formal Proof.* Cambridge University Press, Cambridge CB2 8BS, United Kindom, 2014.