Randomized Algorithms Quiz 1
Solutions to the quiz

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1. (2) Failures are independent, therefore probability that all four links fail is $0.4^4 = 0.0256$. Therefore probability that at least one link is correct is $1 - 0.0256 = 0.9744$.

2. (2) No. There are $n!$ permutations of A. At each iteration the algorithm produces one of the n^n possibles outputs. If each permutation was equally possible, we should have that $\frac{n^n}{n!}$ is an integer, but if n is odd, that is not possible.

3. (2) Let A be event of having the coin C-C and B the event of having a head. We want $\mathbf{Pr}[A|B] = \frac{\mathbf{Pr}[A\cap B]}{\mathbf{Pr}[B]}$. Note $\mathbf{Pr}[B] = 3/4$ and
$A \cap B = A \Rightarrow \mathbf{Pr}[A \cap B] = \mathbf{Pr}[A]$, so $\mathbf{Pr}[A|B] = \frac{1/2}{3/4} = \frac{2}{3}$.

4. (4) There are 7 balls in the bag initially, 3 of which are red. All balls are equally likely to be drawn from the bag so the probability of a red ball is 3/7. After drawing the red ball there are now 6 balls in the bag, 2 red and 4 white. The probability of now drawing a white ball is therefore 4/6 and
$\mathbf{Pr}[\text{red then white}] = \mathbf{Pr}[\text{red first} \cap \text{white second}] = 3/7 \times 4/6 = 0.28$

$$\begin{aligned}\mathbf{Pr}[\text{white second}] &= \mathbf{Pr}[\text{white second} \,|\, \text{red first}]\,\mathbf{Pr}[\text{red first}] \\ &+ \mathbf{Pr}[\text{white second} \,|\, \text{white first}]\,\mathbf{Pr}[\text{white first}] \\ &= 4/6 \times 3/7 + 3/6 \times 4/7 = 0.57\end{aligned}$$

$\mathbf{Pr}[\text{red first}]\,\mathbf{Pr}[\text{white second}] = 3/7 \times 0.57 = 0.24.$

But $\mathbf{Pr}[\text{red first} \cap \text{white second}] = 0.28 \neq \mathbf{Pr}[\text{red first}]\,\mathbf{Pr}[\text{white second}]$, therefore the events are not independent.

When the first ball is put back into the bag, the probability of the second ball being white is now 4/7 rather than 4/6 so

$\mathbf{Pr}[\text{red then white}] = 3/7 \times 4/7 = 0.2448.$

$$\begin{aligned}\mathbf{Pr}[\text{white second}] &= \mathbf{Pr}[\text{white second} \,|\, \text{red first}]\,\mathbf{Pr}[\text{red first}] \\ &+ \mathbf{Pr}[\text{white second} \,|\, \text{white first}]\,\mathbf{Pr}[\text{white first}] \\ &= 4/7 \times 3/7 + 4/7 \times 4/7 = 4/7\end{aligned}$$

Therefore $\mathbf{Pr}[\text{red first}]\,\mathbf{Pr}[\text{white second}] + = 3/7 + 4/7 = \mathbf{Pr}[\text{red then white}]$ and the events are independent.