Homework 6
Network Dynamics Analysis

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Contents

1 Introduction

In this homework we have analyzed Network Dynamics (Network Dynamics) on different models proposed for this work. The main goal of this work is to identify the best model which fit on both aspects of Network Dynamics, Degree Distribution (Degree Distribution) and Growth of Vertex Degree over time (Time Growth Degree). The following sections are going to show the results obtained and after that the discussion about the methodology used for the analysis and some conclusions at the end.

All the models that we have generated for the analysis of Barabasi-Albert model (Barabasi-Albert model) over time have been done using **C++** Language and you can find the source code of this solution under `code/main.cpp`.

The solution of this work is divided as follows:

- **code**: Under this folder you are going to find *C++* code for simulating and generating the different networks using the different strategies: Growth + Preferential Attachment (G Pref Attachment), Growth + Random Attachment (G Random Attachment) and No Growth + Preferential Attachment (NG Pref Attachment), as well as the *R* scripts for generating plots and doing graph analysis.
- **code/data**: Data Generated for each strategy
- **report**: This report in Latex and PDF format.

2 Results

2.1 Degree Distribution Analysis

2.1.1 Growth + Preferential Attachment

Parameter	Value
M	200002
N	100002
MAX	1105
M/N	1.99
N/M	0.50
MP	37263.76
C	205119.83

Table 1: Data Analysis over Growth + Pref Attachment

Distribution	Estimation
Displaced Poisson	1.59
Displaced Geometric	0.50
Zeta Gamma	2.29
Zeta Truncated	103002

Table 2: Estimation of Parameters: Growth + Pref Attachment

Distribution	AIC Value
Displaced Poisson	253849.2
Displaced Geometric	33974.32
Zeta Gamma 3	17100.8
Zeta Gamma	**0**
Zeta Truncated	1.966677

Table 3: AIC Selection: Growth + Pref Attachment

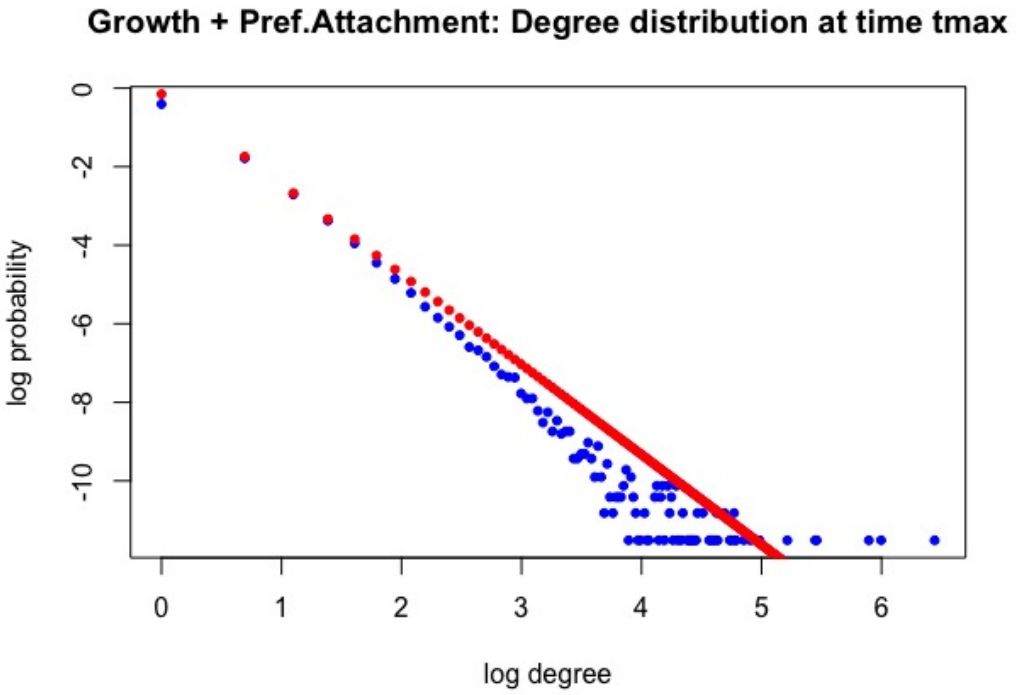

Figure 1: Degree t_{max} Growth + Pref Att with Zeta $\Gamma = 3$

2.1.2 Growth + Random Attachment

Parameter	Value
M	200002
N	100002
MAX	16
M/N	1.99
N/M	0.50
MP	50775.59
C	136242.14

Table 4: Data Analysis over Growth + Random Attachment

Distribution	Estimation
Displaced Poisson	1.59
Displaced Geometric	0.50
Zeta Gamma	2.07
Zeta Truncated	103002

Table 5: Estimation of Parameters: Growth + Random Attachment

Distribution	AIC Value
Displaced Poisson	82119.5
Displaced Geometric	**0**
Zeta Gamma 3	64197.51
Zeta Gamma	24921.01
Zeta Truncated	24922.54

Table 6: AIC Selection: Growth + Random Attachment

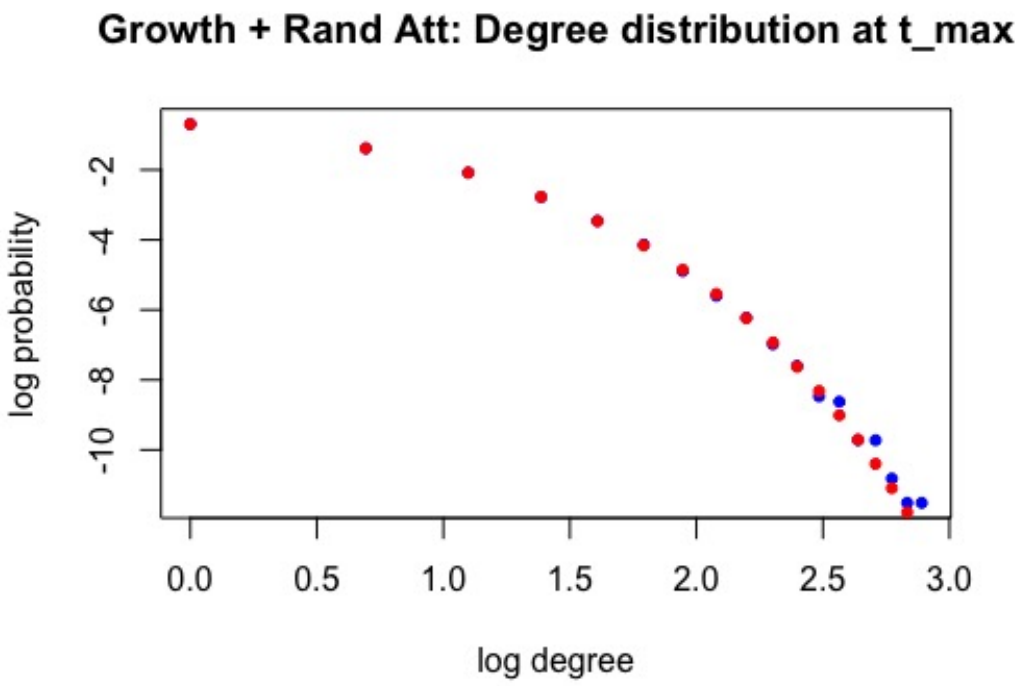

Figure 2: Degree t_{max} Growth + Pref Att with Displaced Geometric

2.1.3 No Growth + Preferential Attachment

Parameter	Value
M	219726
N	10000
MAX	60
M/N	21.97
N/M	0.04
MP	30332.60
C	495569.98

Table 7: Data Analysis over No Growth + Pref Attachment

Distribution	Estimation
Displaced Poisson	21.97
Displaced Geometric	0.04
Zeta Gamma	1.28
Zeta Truncated	13000

Table 8: Estimation of Parameters: No Growth + Preferential Attachment

Distribution	AIC Value
Displaced Poisson	**0**
Displaced Geometric	8558.90
Zeta Gamma 3	112899.5
Zeta Gamma	33428.39
Zeta Truncated	31537.34

Table 9: AIC Selection: No Growth + Preferential Attachment

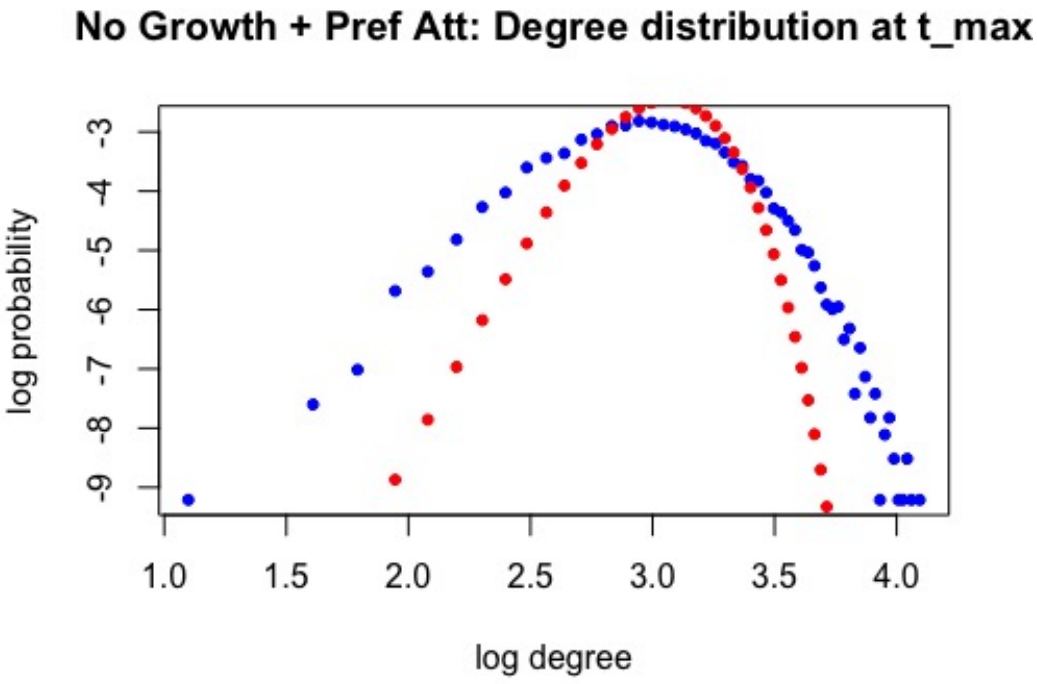

Figure 3: Degree t_{max} No Growth + Pref Att with Poisson

2.2 Time Growth Degree Analysis

2.2.1 Growth + Preferential Attachment

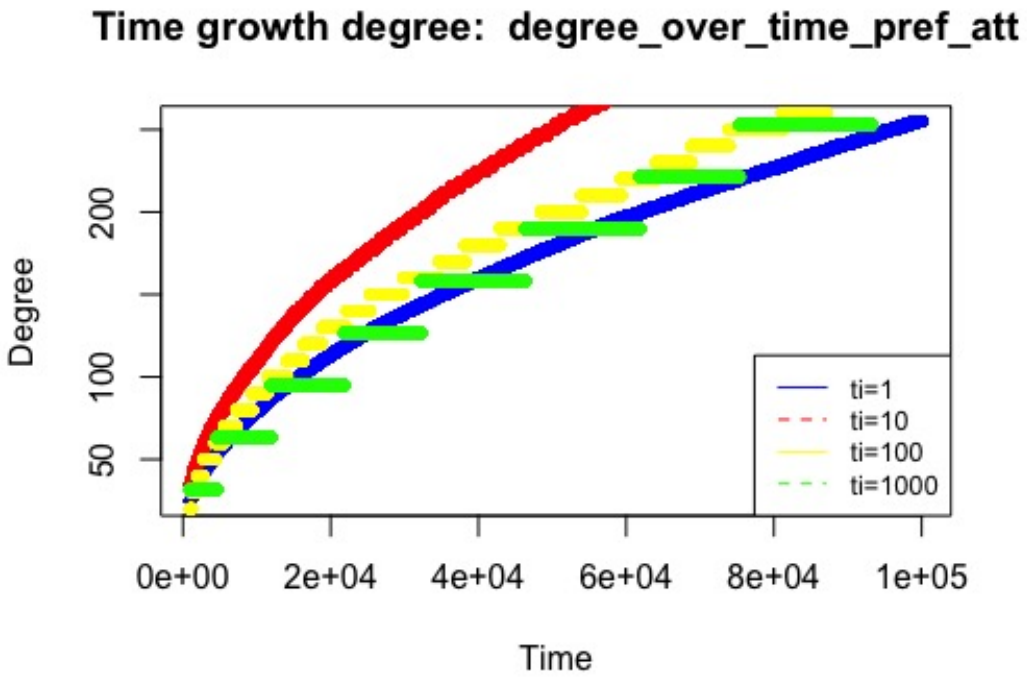

Figure 4: Time Growth Degree: Growth + Pref Att

Model	AIC Value
Model 0	827909.63
Model 1	98488.01
Model 2	98043.99
Model 3	696452.97
Model 4	869213.48
Model 0+	595689.50
Model 1+	95483.67
Model 2+	**82349.48**
Model 3+	696467.54
Model 4+	869216.84

Table 10: AIC Selection: Growth + Preferential Attachment over Time Degree

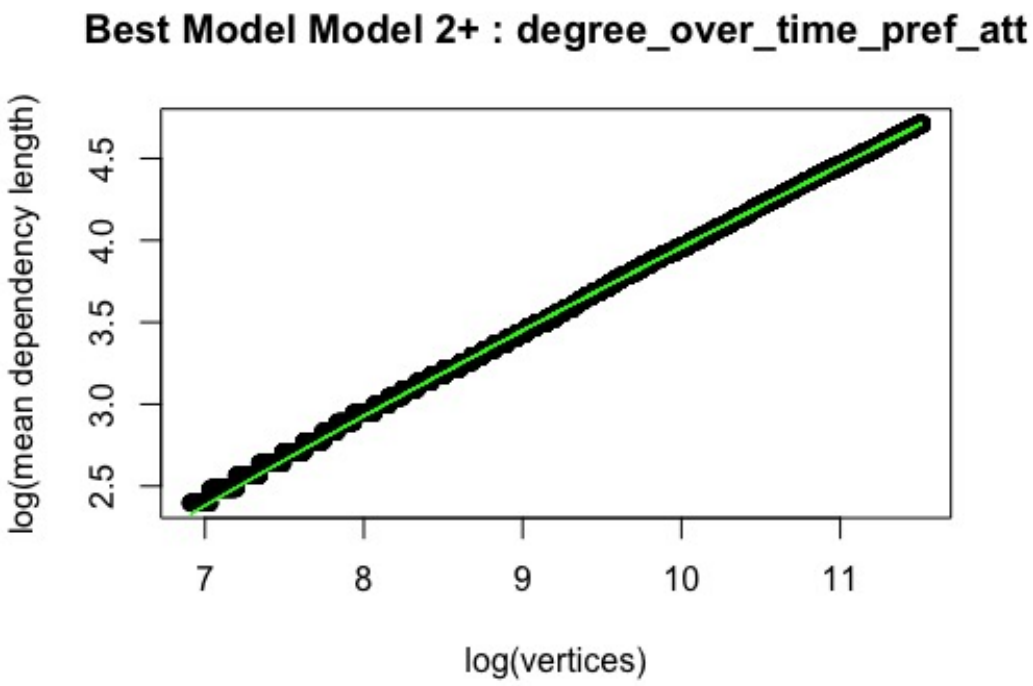

Figure 5: Best Fit Time Growth Degree: Growth + Pref Att

2.2.2 Growth + Random Attachment

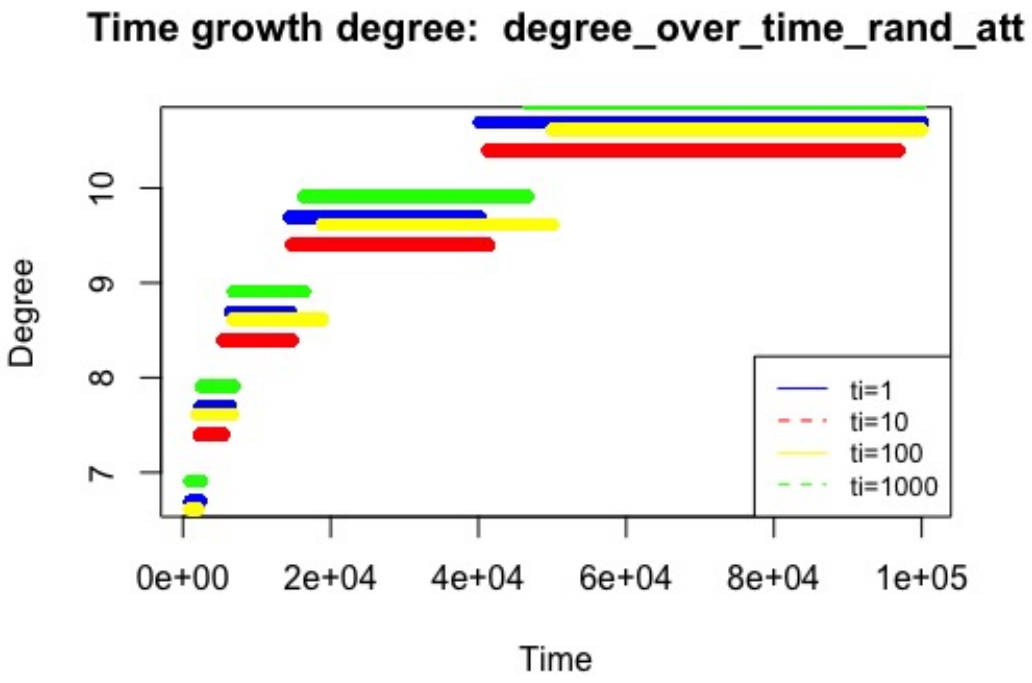

Figure 6: Time Growth Degree: Growth + Random Att

Model	AIC Value
Model 0	529661.14
Model 1	414515.14
Model 2	**40943.69**
Model 3	148894.78
Model 4	60229.36
Model 0+	140375.69
Model 1+	89613.81
Model 2+	45361.77
Model 3+	148896.86
Model 4+	60233.06

Table 11: AIC Selection: Growth + Random Attachment over Time Degree

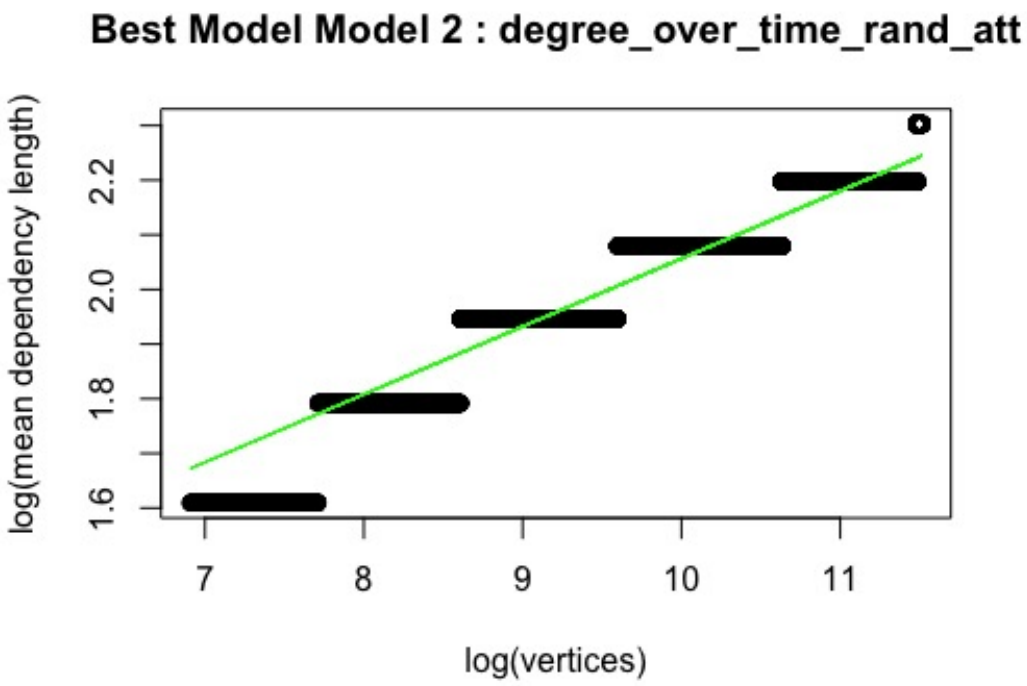

Figure 7: Best Fit Time Growth Degree: Growth + Random Att

2.2.3 No Growth + Preferential Attachment

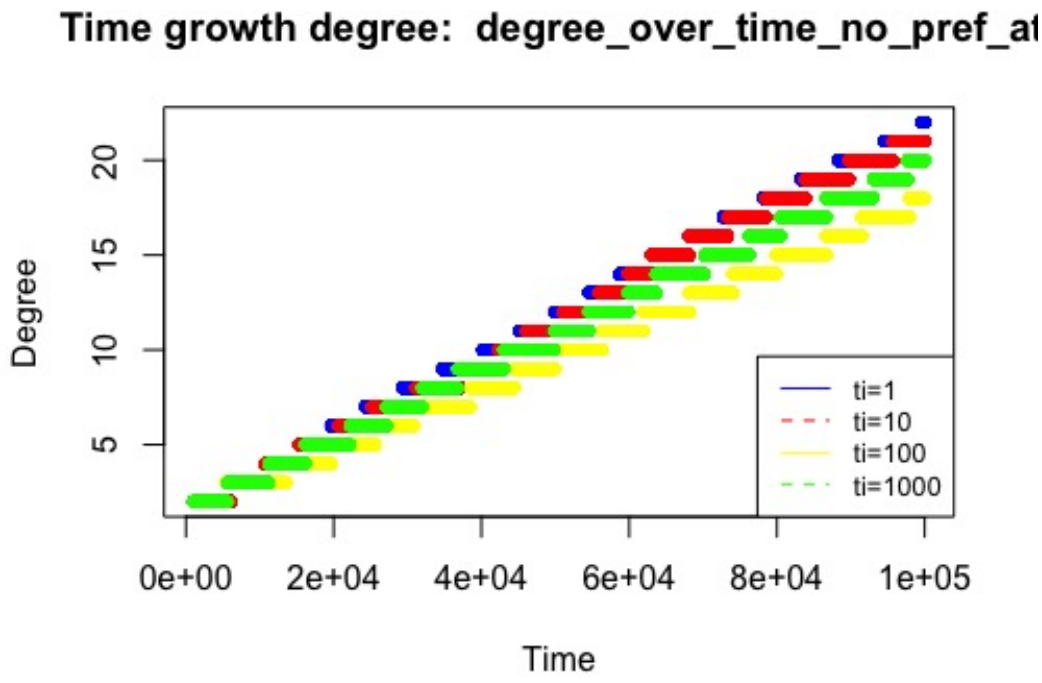

Figure 8: Time Growth Degree: No Growth + Preferential Att

Model	AIC Value
Model 0	222892.08
Model 1	419104.67
Model 2	102823.02
Model 3	338659.87
Model 4	599022.32
Model 0+	**57388.89**
Model 1+	299613.65
Model 2+	248667.38
Model 3+	338696.35
Model 4+	599024.32

Table 12: AIC Selection: No Growth + Preferential Attachment over Time Degree

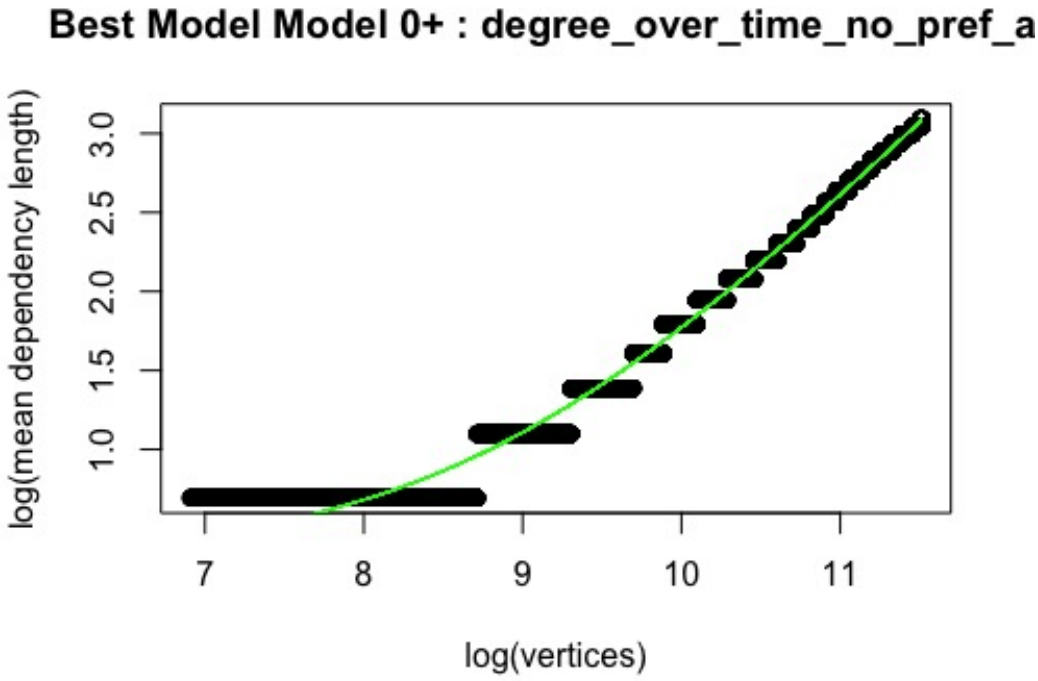

Figure 9: Best Fit Time No Growth Degree: Growth + Pref Att

3 Discussion and Methods

3.1 Methods

As we have pointed out in the introduction we have use **C++** code for generating the different kinds of Growth in the Barabasi-Albert model. This can be found in the source code file `code/main.cpp`.

Regarding *R* scripts we have the following scripts:

- `code/FromHomework2.R`: Which is the same script we have used on the Homework 2 for Fitting model analysis but we have change Zeta in order to fix $\Gamma = 3$
- `code/Task1.R`: We have developed here the analysis and plotting of the first task about analysis of Degree Distribution
- `code/Task2.R`: We have developed here the analysis and plotting of the first task about analysis of Time Growth Degree

3.1.1 Experiments

As we are dealing with randomness, we have realized that we needed different number of experiments in order to be able to empirically test the theoretical the expected behavior.

As it can bee seen in the `code/main.cpp` code, we have conducted 100 experiments on each case to reach this mean values.

3.1.2 Parameters Selection

We have 2 cases in parameter selection, on the one hand we have chosen parameters and **Initial Graph Configuration** for G Pref Attachment and Growth + Random Attachment (G Random Attachment) and on the other hand we have chosen a different set of parameters and **Initial Graph Configuration** of initial graph for NG Pref Attachment.

- **Grow + Preferential and Random Attachment**: Regarding the selection of parameters we have selected $m_0 = 1$, $n_0 = 2$, $t_{max} = 100.000$. This selection is based on the fact that we have tried different kind of alternatives, but the only one which has given the best fit in 2.1 is this set of parameters. Other option that we have tested are:
  - $m_0 = 2$, $n_0 = 3$, $t_{max} = 100.000$
  - $m_0 = 3$, $n_0 = 5$, $t_{max} = 100.000$
  - $m_0 = 5$, $n_0 = 5$, $t_{max} = 100.000$

  Another important thing to point out is the **Initial Graph Configuration** for these 2 cases have been **Complete Graph** selection.
- **No Grow + Preferential Attachment**: The selected parameters have been $m_0 = 1$, $n_0 = 10.000$, $t_{max} = 100.000$. We have chosen a big value for n_0 in order to approximate to the binomial and we have reached to that number after trying out different combinations of n_0 and t_{max} On the other hand the **Initial Graph Configuration** for this case was a **Ring**.

3.2 Discussions

3.2.1 Degree Distribution Analysis

- Regarding the Degree Distribution analysis we can appreciate that for G Pref Attachment we have checked that the best model that fits is the

Zeta Distribution as it is expected according to its Akaike value and the plot obtained here 3 and here 1.

- In the case of G Random Attachment the best fit is also given as expected by Displaced Geometric as we can appreicate by its Akaike value and the plot obtained here 6 and here 2.
- At the same time we can see that NG Pref Attachment best fit with Displaced Poisson as it is expected according to tis Akaike value. This can be seen in the table and plot here 9 and here 3.

3.2.2 Vertex Growth Degree over Time Analysis

In the case of the Time Growth Degree analysis we have detected that once we apply the re-escalation function either on G Pref Attachment or G Random Attachment, the **time** where the vertex was inserted does not affect the distribution and all the different moments are similar. In the case of the model NG Pref Attachment we didn't have the need to re-escalate because all nodes are inserted in t_0 and there is **No Growth** as its name indicates.

On the other hand all the correspondent models fits as expected, for example in the case of G Pref Attachment the best model which fits is **Model 2+** as we can see here 10 and here 5, and the same we can appreciate for NG Pref Attachment. Although this, we couldn't fit properly strategy G Random Attachment, because in the exercise it is said that **Model 4** should be the best, but in our experiments according to our results the best model which fits is the **Model 0**. This can be seen here 11 and here 7.

4 Conclusions

In our humble opinion we think it is better to try out different parameters rather than spend so much time coding and having the different parts to work together. When we refer to *trying out different parameters*, we are talking about experiment with different sets of m_0, n_0, t_{max}, the **Initial Graph Configuration**, etc. Although we have tried some of them because of lack of time and because we have been focusing on having a piece of working code, we think that perhaps in other circumstances a better and deeper analysis could be conducted.

On very interested thing of this Homework was the possibility of integrating different parts of homeworks that we have conducted before into a single

work: the possibility of using different fitting methods as **Maximum likelihood estimation**, **NLS**, etc.