Solution

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Problem 1

Assume a slotted Aloha system with 2 nodes. Node 1 and node 2 transmit with probability $\sigma_1 = 2/4$ and $\sigma_2 = 1/4$, respectively, regardless whether they are thinking or backlogged. The transmission time of node 1 is 1 slot, and the transmission time of node 2 is 2 slots (see figure 1). If the transmission of node 2 collide in any of the two slots, the packet is destroyed and both nodes become backlogged. Note that in the middle of a transmission of node 2 (when node 2 has transmitted the first of the 2 slots), node 2 cannot start the transmission of another packet.

1.A (2 points) Draw the transition diagram and derive the one step transition probabilities of a DTMC that allows computing the throughput. Indicate clearly the meaning of each state.

1.B (0,5 points) Discuss whether the chain is reversible.

1.C (1 points) Compute the stationary distribution.

1.D (1 point) Compute node 1 and node 2 throughput, S_1 and S_2 respectively (expected number of successful packets transmitted per slot).

1.E (1 point) Compute node 1 and node 2 loads, L_1 and L_2 respectively (expected number of packet arrivals per slot).

1.F (1 point) Compute node 1 and node 2 collision probabilities, p_1 and p_2 respectively (proportion of colliding packets).

1.G (1 point) Assume slots of 1 ms and a line bitrate of 10 Mbps. What is the average transmission time of 1 Mbyte (10^6 bytes), transmitted by node 1, and by node 2, T_1 and T_2 respectively? (in seconds)

1.H (2.5 points) Let B be the random variable equal to the number of consecutive busy slots, regardless the packets collide or not (see figure 1). Compute E$[B]$ using a DTMC. Indicate clearly the meaning of the states of the chain.

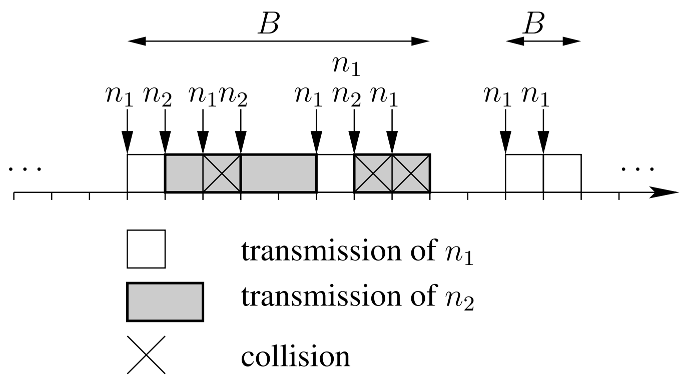

Figure 1: Time diagram of the Aloha system.

Solution

Problem 1

1.A The nodes are indistinguishable of being in thinking or backlogged state. Thus, it is enought to remember wheter node 2 is transmitting its first slot (since in the end of first slot, node 2 cannot initiate a new transmission). Thus, the states are (see figure 2):

0 node 2 is not transmitting its first slot
1 node 2 is transmitting its first slot

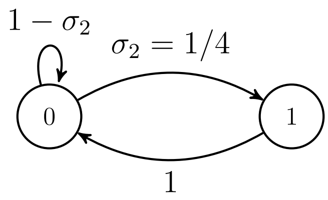

Figure 2: DTMC.

1.B The DTMC is an irreducible tree, and thus, reversible.

1.C Using the general solution for reversible chains we have:

$$\pi_0 = \frac{1}{G}$$
$$\pi_1 = \frac{1}{G}\frac{1}{4}$$

which yields: $G = 5/4$ and

$$\pi_0 = 4/5$$
$$\pi_1 = 1/5$$

1.D

$$S_1 = \pi_0\,\sigma_1\,(1-\sigma_2) = 3/10 \quad \text{packets/slot}$$
$$S_2 = \pi_0\,\sigma_2\,(1-\sigma_1)^2 = 1/20 \quad \text{packets/slot}$$

1.E

$$L_1 = \sigma_1 = 1/2 \quad \text{packets/slot}$$
$$L_2 = \pi_0\,\sigma_2 = 1/5 \quad \text{packets/slot}$$

1.F

$$p_1 = \frac{\pi_0\,\sigma_1\,\sigma_2 + \pi_1\,\sigma_1}{\sigma_1} = 2/5$$
$$p_2 = \frac{\pi_0\,\sigma_2\,(2\,\sigma_1\,(1-\sigma_1)+\sigma_1^2)}{\pi_0\,\sigma_2} = 3/4$$

Check:

$$p_1 = 1 - S_1/L_1 = 2/5$$
$$p_2 = 1 - S_2/L_2 = 3/4$$

1.G We have:

$$S_1 = \frac{3}{10}\,\frac{\text{packets}}{\text{slot}}\,\frac{1\text{ slot}}{10\text{ ms}}\,\frac{10\text{ Mbps}\,10\text{ ms}}{1\text{ packet}} = 3\text{ Mbps}$$
$$S_2 = \frac{1}{20}\,\frac{\text{packets}}{\text{slot}}\,\frac{1\text{ slot}}{10\text{ ms}}\,\frac{10\text{ Mbps}\,20\text{ ms}}{1\text{ packet}} = 1\text{ Mbps}.$$

Thus,

$$T_1 = \frac{10^6\text{ bytes}\,\frac{8\text{ bits}}{1\text{ byte}}}{3\text{ Mbps}} = 8/3\text{ seconds}$$
$$T_2 = \frac{10^6\text{ bytes}\,\frac{8\text{ bits}}{1\text{ byte}}}{1\text{ Mbps}} = 8\text{ seconds}$$

1.H Consider a chain with states (see figure 3):

0 node 2 is not transmitting its first slot
1 node 2 is transmitting its first slot
2 node 1 and node 2 are not transmitting

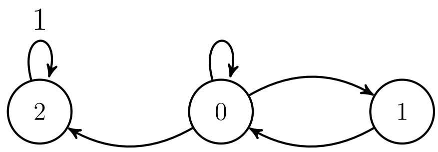

Figure 3: Absorbing DTMC.

where:

$$\begin{aligned} p_{00} &= \sigma_1\,(1-\sigma_2) = 3/8 \\ p_{01} &= \sigma_2 = 1/4 \\ p_{02} &= (1-\sigma_1)\,(1-\sigma_2) = 3/8. \end{aligned}$$

We have:

$$\mathrm{E}[B] = \pi_0(0)\,m_{02} + \pi_1(0)\,m_{12} \tag{1}$$

where:

$$\pi_0(0) = \boldsymbol{P}\{\text{first state is } 0 \mid \text{some node transmit}\} = \frac{\sigma_1\,(1-\sigma_2)}{1-(1-\sigma_1)(1-\sigma_2)} = \frac{3}{5}$$
$$\pi_1(0) = \boldsymbol{P}\{\text{first state is } 1 \mid \text{some node transmit}\} = \frac{\sigma_2}{1-(1-\sigma_1)(1-\sigma_2)} = \frac{2}{5}$$

$$\begin{aligned} m_{02} &= p_{02} + p_{00}\,(1+m_{02}) + p_{01}\,(1+m_{12}) = 1 + p_{00}\,m_{02} + p_{01}\,m_{12} \\ m_{12} &= 1 + m_{02} \end{aligned}$$

which yields $m_{02} = 10/3$, $m_{12} = 13/3$, and substituting in (1):

$$\mathrm{E}[B] = \pi_0(0)\,m_{02} + \pi_1(0)\,m_{12} = \frac{3}{5}\,\frac{10}{3} + \frac{2}{5}\,\frac{13}{3} = \frac{56}{15} \approx 3,73$$