Problem 13.1

Assume a server where clients arrive exponentially distributed with rate 2 clients per time unit. The server takes a time exponentially distributed with mean 1 time unit to dispatch a client. If a client arrives and finds the server busy (serving another client), the arriving client is lost. The server is empty at time $t = 0$.

13.1.A Build the Markov chain: transition diagram and rate matrix.

13.1.B Compute the probability that the server is busy at time $t \geq 0$.

13.1.C Compute the probability that no client is lost in $[0, t]$. Hint: add an absorbing state reached when a client is lost.

13.1.D Use the previous result to compute the average time until a packet is lost.

13.1.E Assume now that the system in steady state. Compute the average number of clients dispatched by the server per time unit. Hint: use the sojourn time in each state.

Problem 13.2

Assume the CSMA/CA protocol of problem 12.4 with 2 nodes and parameters $\mu = 1$, $\lambda = 1/4$ and $\alpha = 3/4$.

13.2.A Let X be the random variable equal to the time since a node enters backlogged state until it leaves the backlogged state (it starts transmitting the packet). Compute the distribution of X. Hint: consider the CTMC with one absorbing state.

13.2.B Compute the expected value of X.