Stochastic Network Modeling
Homework 9 - Solutions

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Problem 9.1

9.1.1

It should fullfil that $p_{ij} = p^r_{ji}$ or filling the Komologov criteria.

9.1.2

If the chain is reversible it must fullfil the following equality

$$p_{12}p_{23}p_{31} = p_{13}p_{32}p_{21} \tag{1a}$$
$$\frac{2}{12}\frac{3}{6}\frac{1}{2} = \frac{3}{12}\frac{1}{2}\frac{2}{6} \tag{1b}$$
$$\tag{1c}$$

It is reversible. Then doing the equations for calculating the stationary distribution we get:

$$\begin{cases} \pi_0 &= \frac{1}{G} \\ \pi_1 &= \frac{1}{G}\frac{6}{7} \\ \pi_2 &= \frac{1}{G}\frac{6}{7}\frac{1}{2} \\ \pi_3 &= \frac{1}{G}\frac{6}{7}\frac{1}{2} \end{cases} \tag{2a}$$

Therefore,

$$G = 1 + \frac{6}{7} + \frac{6}{7}\frac{1}{2} + \frac{6}{7}\frac{1}{2} \tag{3a}$$
$$= \frac{19}{7} \tag{3b}$$

Therefore,

$$\begin{cases} \pi_0 & = \frac{7}{19} \\ \pi_1 & = \frac{6}{19} \\ \pi_2 & = \frac{3}{19} \\ \pi_3 & = \frac{3}{19} \end{cases} \tag{4a}$$

9.1.3

$$\begin{cases} \pi_1 \frac{7}{12} & = \pi_0 \frac{1}{2} \\ \pi_1 \frac{2}{12} & = \pi_2 \frac{2}{6} \\ \pi_1 \frac{3}{12} & = \pi_3 \frac{1}{2} \\ \sum pi_i = 1 \end{cases} \tag{5a}$$

9.1.4

$$\pi_1 \frac{7}{6} = \pi_0 \tag{6a}$$
$$\pi_1 \frac{1}{2} = \pi_2 \tag{6b}$$
$$\pi_1 \frac{1}{2} = \pi_3 \tag{6c}$$
$$\pi_1 = \frac{1}{1 + \frac{7}{6} + \frac{1}{2} + \frac{1}{2}} \tag{6d}$$
$$= \frac{6}{19} \tag{6e}$$

Therefore,

$$\begin{cases} \pi_0 & = \frac{7}{19} \\ \pi_1 & = \frac{6}{19} \\ \pi_2 & = \frac{3}{19} \\ \pi_3 & = \frac{3}{19} \end{cases} \tag{7a}$$

9.1.5

$$\begin{cases} \pi_1 &= \frac{1}{G} \\ \pi_2 &= \frac{1}{G}\frac{1}{2} \\ \pi_3 &= \frac{1}{G}\frac{1}{2} \end{cases} \tag{8a}$$

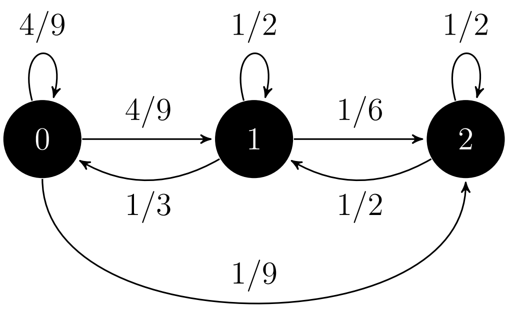
$$G = 1 + \frac{1}{2} + \frac{1}{2} \tag{9a}$$
$$= 2 \tag{9b}$$

Therefore,

$$\begin{cases} \pi_1 &= \frac{1}{2} \\ \pi_2 &= \frac{1}{4} \\ \pi_3 &= \frac{1}{4} \end{cases} \tag{10a}$$

Problem 9.2

9.2.1



$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{ccc} 0 & 1 & 2 \end{array} \begin{bmatrix} 4/9 & 4/9 & 1/9 \\ 1/3 & 1/2 & 1/6 \\ 0 & 1/2 & 1/2 \end{bmatrix}$$

9.2.2

Tree is reversible because it forms a tree

9.2.3

$I_{dl} = (\pi_0 + \pi_1)r = (\frac{9}{31} + \frac{15}{31})\frac{1}{2} = \frac{12}{31}$

9.2.4

$L = (\pi_1 + \pi_2)s = (\frac{15}{31} + \frac{7}{31})\frac{1}{2} = \frac{11}{31}$

9.2.5

$$
\begin{align}
E[N] &= \sum_{i=0}^{\infty} (ps)^{n-1} r \tag{11a}\\
&= \sum_{i=0}^{\infty} \frac{1}{6}^{n-1} \frac{1}{2} \tag{11b}\\
&= \frac{1}{2} \frac{1}{(1 - \frac{1}{6})^2} \tag{11c}\\
&= \frac{18}{25} \tag{11d}
\end{align}
$$