JUAN PABLO ROYO SALES

2ND ASSESMENT

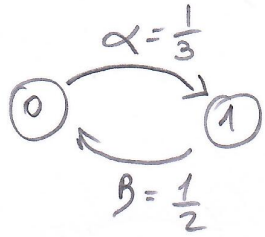
PROBLEM 1

1.1

States

0: WORKING

1: Repairing



1.2

$$Q = \begin{array}{c|cc} & 0 & 1 \\ \hline 0 & -1/3 & 1/3 \\ 1 & 1/2 & -1/2 \end{array}$$

$\pi_0 = \frac{1}{G}$ $\quad G = 1 + \frac{\alpha}{\beta} = \frac{5}{3}$

$\pi_1 = \frac{1}{G}\frac{\alpha}{\beta}$

$$\boxed{\pi_0 = \frac{3}{5}, \quad \pi_1 = \frac{2}{5}}$$

1.3

$E[B] = \pi_0 500 - \pi_1 100$

$\int_0^1 C(t)dt = 200t^2/2 \Big|_0^1 = 100$

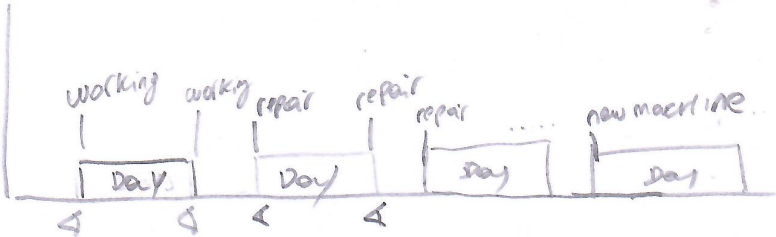
$E[B] = 300 - 40$

$$\boxed{E[B] = 260 \text{ €/day}}$$

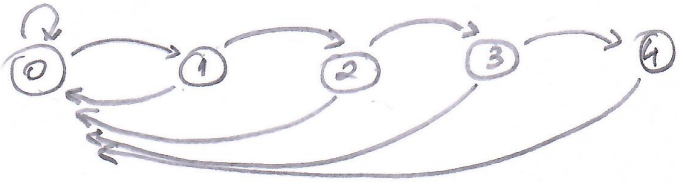
Problem 2

2.1

I observe at the beginning and end of each day.



0 : working
1 : 1 day repair
2 : 2 " "
3 : 3 " "
4 : new machine



$P_{01} = 1 - P_{00} = 0.28$

$P_{00} = e^{-\alpha} = 0.72$

$P_{12} = P_{23} = P_{34} = 1 - P_{10} = 0.39$

$P_{10} = P_{20} = P_{30} = e^{-\beta} = 0.61$

$P_{40} = 1$

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① $\pi_0^e\, 0.28 = \pi_1^e\, 0.61 + \pi_2^e\, 0.61 + \pi_3^e\, 0.61 + \pi_4^e$

② $\pi_1^e = \pi_0^e\, 0.28$

③ $\pi_2^e = \pi_1^e\, 0.39 \Rightarrow \pi_2^e = \pi_0^e\, 0.11$

④ $\pi_3^e = \pi_2^e\, 0.39 \Rightarrow \pi_3^e = \pi_0^e\, 0.04$

⑤ $\pi_4^e = \pi_3^e\, 0.39 \Rightarrow \pi_4^e = \pi_0^e\, 0.02$

$\sum \pi_i^e = 1$

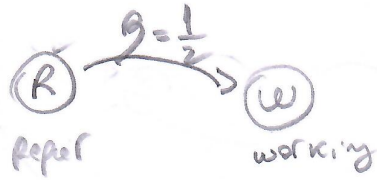
$$\pi_0^e = \frac{1}{1 + 0.28 + 0.11 + 0.04 + 0.02} = \boxed{0.69}$$

$\pi_0^e = 0.69$, $\pi_1^e = 0.19$, $\pi_2^e = 0.08$, $\pi_3^e = 0.03$, $\pi_4^e = 0.01$

2.2

$$P(Y > t) = 1 - P(Y \leq t) = 1 - \pi_R^e(t)$$

CONSIDER AN ABSORBING CHAIN



$$Q = \begin{array}{c|cc} & R & W \\ \hline & -1/2 & 1/2 \\ & 0 & 0 \end{array}$$

$\lambda_0 = 0$

$\lambda_1 = -\frac{1}{2}$

$\pi_R^e(t) = 1 + a e^{-\beta t}$

$\pi_R^e(0) = 1 + a = 0 \Rightarrow a = -1$

$$\pi_R^e(t) = 1 - e^{-t/2}$$

$$P(Y > t) = e^{-t/2}$$

2.3

$E[H_0] = \frac{1}{\alpha} = 3$

$E[H_1] = E[H_2] = E[H_3] = E[H_4] = 2$

(2.4)

$$T = \sum \pi^e E[H_i] = 2.69$$

$$\pi_0 = \frac{\pi_0^e E[H_0]}{T} = 0.77$$

$$\pi_1 = \frac{\pi_1^e E[H_1]}{T} = 0.14$$

$$\pi_2 = \frac{\pi_2^e E[H_2]}{T} = 0.06$$

$$\pi_3 = \frac{\pi_3^e E[H_3]}{T} = 0.02$$

$$\pi_4 = \frac{\pi_4^e E[H_4]}{T} = 0.01$$

(2.5)

$$E[N] = \pi_4 = \boxed{0.01}$$

(2.6)

$$E[C] = \int_0^\infty P(Y > t) = \int_0^\infty e^{-t/2} * 100 = 2 * 100 = \boxed{200}$$

(2.7)

$$E[B] = \pi_0 500 - (\pi_1 + \pi_2 + \pi_3) 100 - \pi_4 4500$$

$$E[B] = 385 - 22 - 15$$

$$\boxed{E[B] = 348}$$