Stochastic Network Modeling
Homework 3 - Solutions

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Problem 3.1

3.1.1

$$
\begin{aligned}
P(X(0), X(1), X(2)) &= P(X(1) \cap X(2)) && (1a)\\
&= P(X(1))P(X(2)) && (1b)\\
&= (p+q)(p+q) && (1c)\\
&= p^2 + 2pq + q^2 && (1d)
\end{aligned}
$$

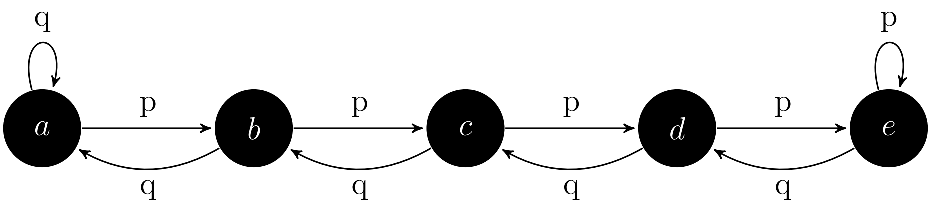
3.1.2

$$
\begin{aligned}
P(X(2) = a) &= qq && (2a)\\
P(X(2) = b) &= 0 && (2b)\\
P(X(2) = c) &= qp + pq = 2pq && (2c)\\
P(X(2) = d) &= 0 && (2d)\\
P(X(2) = e) &= pp && (2e)\\
& && (2f)
\end{aligned}
$$

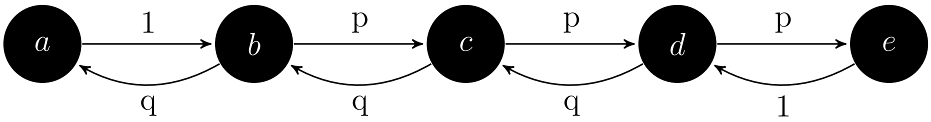
Problem 3.2

3.2.1

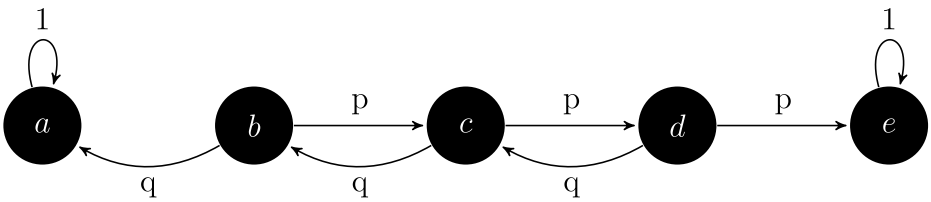
3.2.1(a)



3.2.1(b)



3.2.1(c)



3.2.2

3.2.2(a)

$$P = \begin{bmatrix} q & p & 0 & 0 & 0 \\ q & 0 & p & 0 & 0 \\ 0 & q & 0 & p & 0 \\ 0 & 0 & q & 0 & p \\ 0 & 0 & 0 & q & p \end{bmatrix}$$

3.2.2(b)

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ q & 0 & p & 0 & 0 \\ 0 & q & 0 & p & 0 \\ 0 & 0 & q & 0 & p \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

3.2.2(c)

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ q & 0 & p & 0 & 0 \\ 0 & q & 0 & p & 0 \\ 0 & 0 & q & 0 & p \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Problem 3.3

3.3.1

It is not a MC because the $\sum_j p_{ij} > 1$.

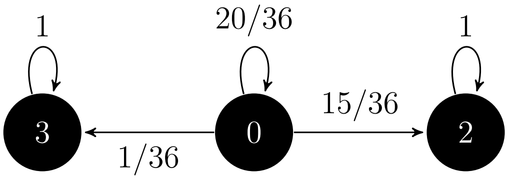
3.3.2

I think it is possible because we use that extra state to go back from a to a', then from a' to a again and after to b. The same for the case of e.

Problem 3.4

3.4.1

Let 0 be the Initial State or Loosing.
Let 2 be the state to get 2 equal dice.
Let 3 be the state to get 3 equal dice.



$$\pi(0) = \begin{bmatrix} 20/36 & 1/36 & 15/36 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} \pi(0) &= (\pi_0(0), \pi_2(0), \pi_3(0)) && \text{(3a)} \\ &= (20/36, 1/36, 15/36) && \text{(3b)} \end{aligned}$$

3.4.2

$$\begin{aligned} \pi_2(\infty) &= P(X(\infty) = 2) && \text{(4a)} \\ &= P(X(\infty - 1) = 0) P(X(\infty) = 2 | X(\infty - 1) = 0) && \text{(4b)} \\ &= 15/36 * 20/36 && \text{(4c)} \\ &= 25/108 && \text{(4d)} \end{aligned}$$

3.4.3

I am not sure how to calculate this. I think it should be summing up over all $\pi_k(0)p_{ki}(n)$ but i am not sure.