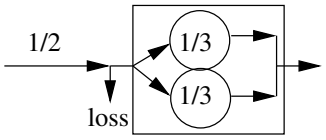
Problem 18.1

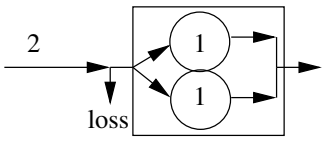
Consider the discrete time M/M/2/2 queue of the figure. We shall refer to the time units as *slots*. There are 2 servers and a queue of 0 positions. When a packet arrives, it goes immediately into service if there is a free server, or it is lost if both servers are busy. At the beginning of each slot, one packet arrives with probability $p_a = 1/2$. When a server is busy, the packet is served at the end of each slot with probability $p_s = 1/3$. Assume that arrivals of the next slot occur after departures that may occur in the current slot. So, arrivals and services can occur simultaneously. In this case, the arriving packet will occupy the server left by the departing packet. We define the stochastic process X equal to the number of packets in the system.



18.1.A Draw the transition diagram.
18.1.B Derive the one step transition probability matrix.
18.1.C Compute the stationary distribution.
18.1.D Compute the loss probability. Hint: Use RASTA theorem.
18.1.E Compute the loss rate (mean number of packets lost per slot), and throughput (mean number of packets dispatched by the servers per slot).
18.1.F Compute the average number of packets in the system, and the average number of slots the packets remain in the system, using the previous results, Check it using Little's theorem.

Problem 18.2

Consider the continuous time M/M/2/2 queue of the figure. There are 2 servers and a queue of 0 positions. When a packet arrives, it goes immediately into service if there is a free server, or it is lost otherwise. The arrivals are Poisson with rate $\lambda = 2$ packets/s. Service times are exponentially distributed with mean 1 s.



18.2.A Draw the transition diagram and derive the rate matrix.
18.2.B Solve the stationary distribution.
18.2.C Assuming that the queue is empty at time $t = 0$, compute the probability that the queue has both servers busy at time $t \geq 0$.
18.2.D Compute the loss probability. Hint: Use PASTA theorem.
18.2.E Compute the loss rate (mean number of packets lost per second), and throughput (mean number of packets dispatched by the servers per second).
18.2.F Compute average number of packets in the system, and the average time the packets remain in the system.