Algorithmic Game Theory **Final Exam**

To be uploaded in the Racó by 23:59 December 21st, 2020

Take into account that you should justify your answers. Any answer that is not backed up by a comment, proof, algorithm, example or counterexample (whatever is adequate), will be marked with 0 points.

Exercise 1 (2.5 points) Recall that as we have seen in class computing a PNE in congestion games is PLS-complete. Contrasting with this, show that there is a polynomial time algorithm for finding a pure Nash equilibrium in *symmetric network creation games.*

Exercise 2 (2.5 points) Consider the Network creation game (Sum NCG) defined in class. Show that:

(a) When $\alpha < 1$, the complete graph K_n (Clique) is the only Nash equilibrium.

(b) The star S_n is a Nash equilibrium for $\alpha \geq 1$.

(c) There exists a Nash equilibrium that is not a star for $\alpha > 2$.

(d) When $\alpha > n^2$, all Nash equilibria are trees and the PoA is bounded by a constant.

Exercise 3 (2.5 points) For a given undirected graph $G = (V, E)$, the associated *domination game* has $N = V$ and in it a coalition wins if and only if X is a dominating set in G.

(a) Show that domination games are simple games.

(b) Are there domination games in which the core is non-empty?

(c) Analyze the computational complexity of the IsProper and IsStrong problem on domination games. (Hint. It might be useful to study the properties of the complement of a maximal independent set in G.)

(d) Analyze the computational complexity of computing a wining coalition with smallest possible size.

Exercise 4 (2.5 points)

Consider the E-Constructive Coalitional Manipulation problem (CCM). We are given a set of alternatives A, the preferences on A of a set of voters S (the nonmanipulators), another set of voters T whose preferences are still open (the manipulators), and a preferred candidate $p \in A$. We are asked whether there is a way to cast the votes in T so that p wins the election under voting protocol E.

Let us consider the following voting system: *Cup (sequential binary comparisons)*. The cup is defined by a balanced binary tree T with one leaf per candidate, and an assignment of candidates to leaves (each leaf gets one candidate). Each non-leaf node is assigned the winner of the pairwise election of the node's children; the candidate assigned to the root wins.

The cup voting protocol assumes that T and the assignment of candidates to leaves is known by the voters before they vote.

In the binary tree representing the cup, we can consider each node to be a subelection. When considering the CCM problem, we may say that the voters in T only order the candidates in that subelection since the place of the other candidates in the order is irrelevant for the subelection.

We say that a candidate can obtain a particular result in a subelection if it does so for some coalitional vote on T. This defines the set of potential winners for each subelection.

(a) Show that a candidate can win a subelection at node u in T if and only if it can win in one of u's children subelections, and it can defeat one of the potential winners of the other sibling child of u in a pairwise election.

(b) Show that, for the cup voting protocol (given the tree and the assignment of candidates to leaves), CCM can be solved in polynomial time.