Universitat Politècnica de Catalunya. Departament d'Arquitectura de Computadors.
Master in Innovation and Research in Informatics (MIRI). Computer Networks and Distributed Systems.

Stochastic Network Modeling (SNM). Autumn 2015.

First assessment, Discrete Time Markov Chains. 30/10/2015.

Problem 1 Assume a slotted Aloha system with 3 nodes: n_1, n_2 and n_3. n_1 transmits with probability $\sigma_1 = 1/3$ and nodes n_2 and n_3 transmit with probability $\sigma_2 = 1/2$ when they are in thinking state. Assume that n_1 is a priority node, such that if n_2 or n_3 (or both) transmit simultaneously with n_1, then n_2 or n_3 (or both) loose the packet and go into thinking state (regardless they were thinking or backlogged), while n_1 transmits successfully. Therefore, n_1 is always in thinking state. If n_1 does not transmit, and n_2 and n_3 transmit simultaneously, then n_2 and n_3 go into backlogged state. In backlogged state n_2 and n_3 transmit with probability $\nu = 1/3$.

(*a*) (1.5 points) Draw the state transition diagram of a DTMC that allows computing the throughput v_1 of the priority node n_1, and the aggregated throughput v_2 of the nodes n_2 and n_3 (in packets per slot). Say clearly what are the states and their meaning.

(*b*) (1.5 points) Compute the transition probabilities of the previous chain.

(*c*) (1.5 points) Compute the stationary distribution using the flux balancing method.

(*d*) (1.5 points) Compute the throughput v_1 and v_2.

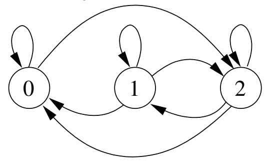
(*e*) (1 points) Compute the loss probability.

(*f*) (1.5 points) Assume that each time n_1 transmits a packet the system produces a benefit of 6 cents, each time n_2 or n_3 successfully transmit a packet the system produces a benefit of 3 cents, and that each lost packet has a cost of 9 cents. Compute the average number of slots that are necessary to produce an average benefit of 1 euro in steady state.

(*g*) (1.5 points) Assume that all nodes are in thinking state. Compute the probability that a loss occurs before both nodes n_2 and n_3 are backlogged for the first time. Explain clearly the method you use to compute this probability.

Solution

(*a*) It is enough to consider the state: $X = \{\text{number of backlogged}\}$



(*b*) Let be $X_n = 0$ and use the convention 1: transmit, 0: do not transmit. We have:

n_1	n_2	n_3	X_{n+1}
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	2
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

Define $q_1 = 1 - \sigma_1 = 2/3$, $q_2 = 1 - \sigma_2 = 1/2$, $q_\nu = 1 - \nu = 2/3$. We have:

$$p_{00} = q_1(\sigma_2^2 + 2\,q_2\,\sigma_2) + \sigma_1 = 5/6 \tag{1}$$
$$p_{02} = q_1\,\sigma_2^2 = 1/6 \tag{2}$$

Assume now n_2 blacklogged, and n_1, n_3 thinking (thus, $X_n = 1$). We have:

n_1	n_2	n_3	X_{n+1}
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	2
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

Thus,

$$p_{10} = q_1\,\nu\,q_2 + \sigma_1\,\nu = 2/9 \tag{3}$$
$$p_{11} = q_1\,q_\nu + \sigma_1\,q_\nu = 6/9 \tag{4}$$
$$p_{12} = q_1\,\nu\,\sigma_2 = 1/9 \tag{5}$$

Finally, assume n_2, n_3 blacklogged, and n_1 thinking (thus, $X_n = 2$). We have:

n_1	n_2	n_3	X_{n+1}
0	0	0	2
0	0	1	1
0	1	0	1
0	1	1	2
1	0	0	2
1	0	1	1
1	1	0	1
1	1	1	0

Thus,

$$p_{20} = \sigma_1\,\nu^2 = 1/27 \tag{6}$$
$$p_{21} = 2\,q_\nu\,\nu = 12/27 \tag{7}$$
$$p_{22} = q_\nu^2 + q_2\,\nu^2 = 14/27 \tag{8}$$

(c) We have:

$$\pi_0\,p_{02} = \pi_1\,p_{10} + \pi\,p_{20} \tag{9}$$
$$\pi_1\,(p_{10} + p_{12}) = \pi_2\,p_{21} \tag{10}$$
$$\pi_0 + \pi_1 + \pi_2 = 1 \tag{11}$$

which yields:

$$\pi_0 = 6/13 \tag{12}$$
$$\pi_1 = 3/13 \tag{13}$$
$$\pi_2 = 4/13 \tag{14}$$

(*d*) Clearly,

$$v_1 = 1/3 \tag{15}$$
$$v_2 = \pi_0\, 2\, q_1\, \sigma_2\, q_2 + \pi_1\, q_1 (\sigma_2\, q_\nu + q_2\, \nu) + \pi_2\, 2\, q_1\, \nu\, q_\nu = 38/117 \tag{16}$$

(*e*) The number of arrivals per slot is $A = A_1 + A_2$, where:

$$A_1 = 1/3 \tag{17}$$
$$A_2 = \pi_0\, (2\, \sigma_2\, q_2 + 2\, \sigma^2) + \pi_1\, \sigma_2 = 8/13 \tag{18}$$

Thus $A = 37/39$, and the loss probability:

$$P_L = 1 - \frac{v_1 + v_2}{A_1 + A_2} = 34/111 \tag{19}$$

(*f*) The benefit per slot is:

$$G = 6\, v_1 + 3\, v_2 - 9\, l = 14/39 \text{ cents} \tag{20}$$

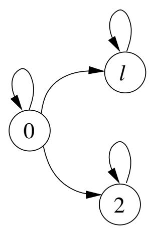
where l is the number of lost packets per slot:

$$l = A\, P_L = 34/117 \tag{21}$$

Thus, the number of slots to produce a benefit of 1 euro (100 cents) is:

$$T = \frac{100}{G} \approx 278,5 \text{ slots} \tag{22}$$

(*g*) We can compute this probability using a chain with the states loss (state l), or 2 backlogged (state 2) absorbing:



$$p_{00} = q_1\, q_2^2 + \sigma_1\, q_2^2 + 2\, q_1\, q_2\, \sigma_2 = 7/12 \tag{23}$$
$$p_{0l} = 2\, \sigma_1\, \sigma_2\, q_2 + \sigma_1\, \sigma_2^2 = 3/12 \tag{24}$$
$$p_{02} = q_1\, \sigma_2^2 = 2/12 \tag{25}$$

The first passage time to state l is the requested probability:

$$f_{0l} = \frac{p_{0l}}{1 - p_{00}} = 3/5 \tag{26}$$