Universitat Politècnica de Catalunya. Departament d'Arquitectura de Computadors.
Master in Innovation and Research in Informatics (MIRI). Computer Networks and Distributed Systems.

Stochastic Network Modeling (SNM). Autumn 2019.

First assessment, Discrete Time Markov Chains. 4/11/2019.

Problem 1

Assume a slotted Aloha system with 2 nodes, n_1, n_2. Both nodes transmit with probability $\sigma = 1/3$ when they are thinking. When they are backlogged n_1 transmits deterministically after every 1 slot, and n_2 continues transmitting with probability $\sigma = 1/3$, as shows figure 1.

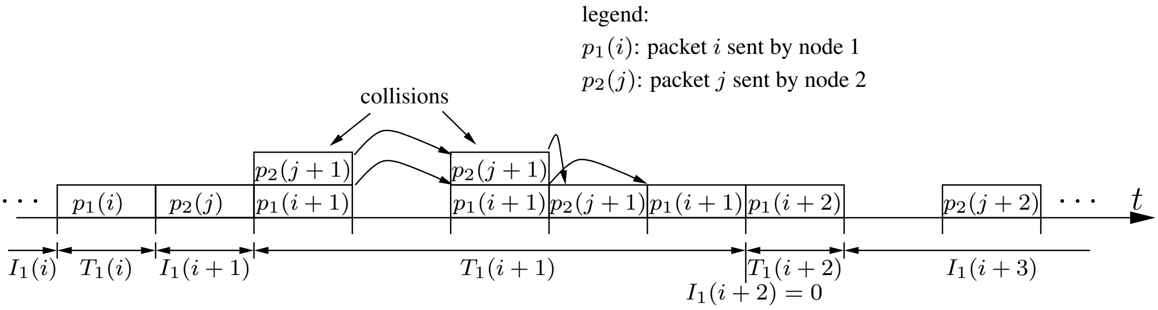

Figure 1: Time diagram of the system.

1.A (1.5 points) Draw the transition diagram and derive the one step transition probabilities of a DTMC that allows computing the throughput of each node.

1.B (1 points) Compute the stationary distribution.

1.C (1.5 point) Compute the throughput S_1, S_2, of each node (expected number of successful packets transmitted per slot).

1.D (1.5 point) Let $N_i \geq 1, i = 1, 2$ be the random variable equal to number transmissions per packet of node n_i. That is, if a new packet is successfully transmitted in the first trial, $N_i = 1$. If it collides in the first trial and it is successfully transmitted in the second trial, then $N_i = 2$, and so on. Compute $\mathrm{E}[N_i]$, $i = 1, 2$.

1.E (1 points) Let $I_1 \geq 0$ be the random variable equal to the number of slots that node n_1 is in thinking state between transmissions (idle time). That is, if n_1 transmits a new packet immediately after a n_1 successful transmission, then $I_1 = 0$, and so on (see figure 1). Compute the distribution of I_1, $P(I_1 = n)$, and its expected value $\mathrm{E}[I_1]$.

1.F (1.5 points) Let $T_1 \geq 1$ be the random variable equal to the transmission time of node n_1 (time that follows every idle time). That is, if n_1 successfully transmits a new packet in the first trial $T_1 = 1$. If it collides in the first trial and it is successfully transmitted in the second trial, then $N_1 = 3$. If it collides twice (as $p_1(i + 1)$ in figure 1), then $N_1 = 5$, and so on (see figure 1). Compute the distribution of T_1, $P(T_1 = n)$, and its expected value $\mathrm{E}[T_1]$.

1.G (1 point) Say what relation there is between the throughput of n_1, $\mathrm{E}[I_1]$ and $\mathrm{E}[T_1]$. Check it with the values obtained in the previous items.

1.H (1 point) Let A be the event $A = \{\text{both nodes are in thinking state}\}$ and $T_2 \geq 1$ be the random variable equal to the transmission time of node n_2. Compute $\mathrm{E}[T_2 \mid A]$, that is, the expected value of T_2 given that the transmission of n_2 occurs when both nodes are in thinking state. Use an absorbing DTMC, and describe clearly the meaning of each state.