Brute force solution for vietnamese math problem

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Version 1.0

1 THE PROBLEM

In May 18, 2015, a vietnamese website posted an article [1] stating that a math book for 3rd grade had the following problem on it:





The problem also appeared in other websites such as tinmoi.vn [2]

The idea of the problem is to fill the empty squares with the numbers from 1 to 9, non repeating any and using all of them in order to get 66 for result. If you are asking ":" stands for division

The problem generated such a controversy that a local teacher, Dr. Tran Dien Show from the Hanoi National University of Education said that the problem would be a challenge even for a Ph.D not to say for a 3rd grade student [3].

Later, the problem got published on different websites, such as The Guardian [4] and Gizmodo [5].

2 Assumptions

- There's at least one array of values that solve the problem.
- No approximate solutions will be used.

3 THE SOLUTION

Based on the rules of the game, you will have to fill 9 fields with 9 different numbers from 1 to 9, non repeating them.

		I		66
+	×		-	=
13	12		11	10
×	+		+	_
:	+		×	 :

The first thing we can figure out is that there are 9! possible combinations to solve this. Why?

Consider the variables in red.

А		E	ı	F		66
+		×		-		=
13		12		11		10
×		+		+		_
В		D		G		I
:	С	+		×	Н	:

A can be any number from 1 to 9.

B can be any number from 1 to 9 but can't be the same value as A, so it can be 1 of 8.

C can be any number from 1 to 9 but can't be the same vale as A and B, so it can be 1 of 7.

See where is this going?

D can be 1 of 6.

E can be 1 of 5.

F can be 1 of 4.

G can be 1 of 3.

H can be 1 of 2.

I can be 1.

Knowing this, you will have 9! (9 * 8 * 7 * 6 * 5 * 4 * 3 * 2 * 1) or 362.880 possible combinations to evaluate.

3.1 Why brute force and not equations?

You have 9 variables and only 1 equation. The ideal scenario would be to have 9 ecuations for the 9 variables, at least for a linear equation system, but that isn't the case.

So now we know we have to test with 362.880 possible combinations, but how do we get them?

3.2 FIRST APPROACH

We will be using C++ and Microsoft Excel to solve the problem.

I chose C++ because is faster, the overall speed of C++ applications is greater than that of other languages. C++ will help us get all the possible combinations in a short amount of time.

Microsoft Excel will help us to evaluate the combinations and tell what values solve the problem.

How do we generate the combinations?

Assume for a moment you want to have all the combinations of length 3 for a binay code, 1 and 0. You will have your initial alphabet, 1 & 0. If you combine your alphabet, i.e. your length 1 words with the same length 1 words, you would get the length 2 words, like this:

0	0	00
1	1	01
		10
		11

Then, if you take your alphabet or length 1 words and combine them with the length 2 words, you would get the length 3 words:

)	00	000
1	01	001
	10	010
	11	011
		100
		101
		110
		111

If you want to get a length n word combination, you need to combine your alphabet or length 1 words with the n-1 length words, where n has to be equal or higher to 2.

So, if you want to get the length 2 words, combine your alphabet with the 2-1 (1) length words.

If you want to get the length 3 words, combine your alphabet with the 3-1 (2) length words.

•••

If you want to get the length 9 words, combine your alphabet with the 9-1 (8) length words.

Notice that we will need to have our alphabet, and the length 2, 3, 4, 5, 6, 7 and 8 words to get the length 9 words.

Something else has to be taken into account here. The words can't have repeated elements, so, from the length 2 words we will check that we are not adding words with repeated elements.

Our initial C++ solution will look like this:

```
#include <algorithm>
#include <fstream>
#include <iostream>
#include <string>
#include <sstream>
using namespace std;
bool hasDuplicateCharacters(string s) {
       for(int i = 0; i < s.length(); i++) {</pre>
               for(int j = i+1; j < s.length(); j++) {</pre>
                      if(s[i] == s[j]) {
                              return true;
               }
       return false;
int stringHasUniqueElements(string strToCheck) {
       int hasUniqueElements = 0;
       if(!hasDuplicateCharacters(strToCheck)) {
               hasUniqueElements = 1;
       return hasUniqueElements;
int main(int argc, char** argv) {
       ifstream alphabet("1.txt");
       string alphabetCurrentLine;
       int iterationToGet = 9;
       /* **** */
       int checkUniqueElements = 1;
       int hasUniqueElements = 0;
       /* **** */
       int numberFileToRead = iterationToGet - 1;
       stringstream nameOfFileToRead;
       nameOfFileToRead << numberFileToRead << ".txt";</pre>
       cout << "File to read: " << nameOfFileToRead.str() << endl;</pre>
```

```
string prevIterFileName = nameOfFileToRead.str();
ifstream prevIterFile(prevIterFileName.c str());
string prevIterFileCurrentLine;
/* ***** */
stringstream nameOfFileToCreate;
nameOfFileToCreate << iterationToGet << ".txt";</pre>
string strNameOfFileToCreate = nameOfFileToCreate.str();
cout << "File to create: " << strNameOfFileToCreate << endl;</pre>
/* ***** */
ofstream outStream(strNameOfFileToCreate.c_str());
/* **** */
while(getline(alphabet, alphabetCurrentLine)) {
       while(getline(prevIterFile, prevIterFileCurrentLine)) {
               cout << alphabetCurrentLine << prevIterFileCurrentLine << endl;</pre>
               if(checkUniqueElements) {
                       hasUniqueElements = stringHasUniqueElements(alphabetCurrentLine + prevIterFileCurrentLine);
                       if(hasUniqueElements) {
                               cout << "WILL ADD " << alphabetCurrentLine << prevIterFileCurrentLine << endl;</pre>
                              outStream << alphabetCurrentLine << prevIterFileCurrentLine << endl;</pre>
                       else {
                              cout << "WILL NOT ADD " << alphabetCurrentLine << prevIterFileCurrentLine << endl;</pre>
               else {
                       outStream << alphabetCurrentLine << prevIterFileCurrentLine << endl;</pre>
       prevIterFile.close();
       prevIterFile.clear();
       prevIterFile.open(prevIterFileName.c str());
outStream.close();
return 0;
```

This is one approach of many that you could have.

This code will assume you have a local file called 1.txt with your alphabet, having each number on a single line. Then, you will need to run the code for each iteration from 1 to 9 to get the final length 9 words.

Let's see the times for each combination:

3.2.1 Length 2

```
MILL NOT ADD 88

WILL ADD 89

91

WILL ADD 91

92

WILL ADD 92

93

WILL ADD 93

94

WILL ADD 95

96

WILL ADD 96

97

WILL ADD 97

98

WILL ADD 97

98

WILL ADD 98

99

WILL ADD 99

Process exited after 0.1102 seconds with return value 0

Presione una tecla para continuar . . .
```

3.2.2 Length 3

```
MILL ADD 986
987
WILL ADD 987
989
WILL NOT ADD 989
991
WILL NOT ADD 991
992
WILL NOT ADD 992
993
WILL NOT ADD 993
994
WILL NOT ADD 994
995
WILL NOT ADD 995
996
WILL NOT ADD 996
997
WILL NOT ADD 997
998
WILL NOT ADD 998
Process exited after 0.5303 seconds with return value 0
Presione una tecla para continuar . . .
```

3.2.3 Length 4

```
MILL NOT ADD 9974
9975
WILL NOT ADD 9975
9976
WILL NOT ADD 9976
9978
WILL NOT ADD 9978
9981
WILL NOT ADD 9981
9982
WILL NOT ADD 9982
9983
WILL NOT ADD 9984
9985
WILL NOT ADD 9984
9985
WILL NOT ADD 9984
9985
WILL NOT ADD 9986
9987
WILL NOT ADD 9987

Process exited after 2.663 seconds with return value 0
Presione una tecla para continuar . . . .
```

3.2.4 Length 5

```
WILL NOT ADD 99862
99863
WILL NOT ADD 99863
99864
WILL NOT ADD 99864
99865
WILL NOT ADD 99865
99867
WILL NOT ADD 99867
99871
WILL NOT ADD 99871
99872
WILL NOT ADD 99872
99873
WILL NOT ADD 99873
99874
WILL NOT ADD 99875
99876
WILL NOT ADD 99876
Process exited after 13.22 seconds with return value 0
Presione una tecla para continuar . . .
```

3.2.5 Length 6

```
■ C:\Development\Personal\github\cpp-tests\DevC... — □ ×
WILL NOT ADD 998746
998751
WILL NOT ADD 998751
998752
WILL NOT ADD 998752
998753
WILL NOT ADD 998753
998754
WILL NOT ADD 998754
998756
WILL NOT ADD 998756
998761
WILL NOT ADD 998761
998762
WILL NOT ADD 998762
998763
WILL NOT ADD 998763
998764
WILL NOT ADD 998764
998765
WILL NOT ADD 998765
Process exited after 68.98 seconds with return value 0
Presione una tecla para continuar . . . _
```

3.2.6 Length 7

```
■ C:\Development\Personal\github\cpp-tests\DevC... — □ ×
WILL NOT ADD 9987632
9987634
WILL NOT ADD 9987634
9987635
WILL NOT ADD 9987635
9987641
WILL NOT ADD 9987641
9987642
WILL NOT ADD 9987642
9987643
WILL NOT ADD 9987643
9987645
WILL NOT ADD 9987645
9987651
WILL NOT ADD 9987651
9987652
WILL NOT ADD 9987652
9987653
WILL NOT ADD 9987653
9987654
WILL NOT ADD 9987654
Process exited after 364.1 seconds with return value 0
Presione una tecla para continuar . . . _
```

3.2.7 Length 8

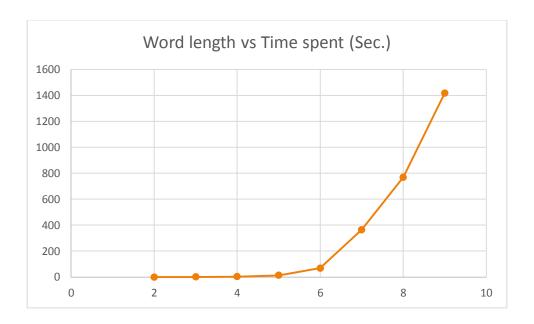
```
■ C:\Development\Personal\github\cpp-tests\DevC... - □ ×
WILL NOT ADD 99876513
99876514
WILL NOT ADD 99876514
99876521
WILL NOT ADD 99876521
99876523
WILL NOT ADD 99876523
99876524
WILL NOT ADD 99876524
99876531
WILL NOT ADD 99876531
99876532
WILL NOT ADD 99876532
99876534
WILL NOT ADD 99876534
99876541
WILL NOT ADD 99876541
99876542
WILL NOT ADD 99876542
99876543
WILL NOT ADD 99876543
Process exited after 769.4 seconds with return value 0
Presione una tecla para continuar . . . _
```

3.2.8 Length 9

```
■ C:\Development\Personal\github\cpp-tests\DevC... — □
WILL NOT ADD 998765314
998765321
WILL NOT ADD 998765321
998765324
WILL NOT ADD 998765324
998765341
WILL NOT ADD 998765341
998765342
WILL NOT ADD 998765342
998765412
WILL NOT ADD 998765412
998765413
WILL NOT ADD 998765413
998765421
WILL NOT ADD 998765421
998765423
WILL NOT ADD 998765423
998765431
WILL NOT ADD 998765431
998765432
WILL NOT ADD 998765432
Process exited after 1416 seconds with return value 0
Presione una tecla para continuar . . .
```

Now we have the length 9 words. With this we can go to Microsoft Excel, copy them, and see which one(s) solve the problem.

Length	Time (Sec.)
2	0,1102
3	0,5303
4	2,663
5	13,22
6	68,98
7	364,1
8	769,4
9	1416



Since this C++ implementation generates the words with a recursive algorithm, the execution time grows exponentially.

Now we can get the content from 9.txt and copy it to an empty Microsoft Excel sheet.

In my case, I added some columns names from B2 to M2.

Combination	Α	В	С	D	Ε	F	G	Н	I	Result	Bingo?
-------------	---	---	---	---	---	---	---	---	---	--------	--------

From B3 to B362883 I pasted the words.

In order to get each number, from column C to column K I added the following:

Column	Variable	Value
С	A	=EXTRAE(B3;1;1)
D	В	=EXTRAE(B3;2;1)
E	С	=EXTRAE(B3;3;1)
F	D	=EXTRAE(B3;4;1)
G	E	=EXTRAE(B3;5;1)
Н	F	=EXTRAE(B3;6;1)
I	G	=EXTRAE(B3;7;1)
J	Н	=EXTRAE(B3;8;1)
K	I	=EXTRAE(B3;9;1)

EXTRAE is the MID function in Spanish [6].

This step is completely unnecessary if you get the combinations separated by a tabulation or by a comma and let Excel process them. I just used an old code to generate word combinations.

The full Microsoft Excel file I used can be seen here:

 $\underline{https://github.com/jpruiz114/cpp-tests/blob/master/DevCppTest02/Solution.xlsx?raw=true}$

In the column L I just added the equation, like this:

=C3+13*D3/E3+F3+12*G3-H3-11+I3*J3/K3-10

You might be asking. What about the parentheses? Operators have their priority and you should not need to add any parenthesis so Microsoft Excel know what elements should evaluate first.

The equation has division and multiplication (With the highest priority) and subtraction and addition (With the lowest priority).

The last step is to add a formula on column M like this: =SI(L3=66; "Bingo"; "") – "SI" is the equivalent for the IF function of Microsoft Excel in Spanish [6].

In addition, to make things easy, a filter that shows only the records where column M equals "Bingo" would give you the 136 answers

In the end, you get the following solutions:

1 2 6 4 7 8 3 5 9	1 5 2 8 4 7 9 3 6	2 8 6 9 4 1 5 7 3	3 9 2 8 1 5 7 6 4
1 2 6 4 7 8 5 3 9	1 5 3 9 4 2 7 8 6	2 8 6 9 4 1 7 5 3	3 9 6 2 5 1 4 7 8
1 3 2 4 5 8 7 9 6	1 5 3 9 4 2 8 7 6	2 9 6 3 5 1 4 7 8	3 9 6 2 5 1 7 4 8
1 3 2 4 5 8 9 7 6	1 8 3 7 4 5 2 6 9	2 9 6 3 5 1 7 4 8	4 2 6 1 7 8 3 5 9
1 3 2 9 5 6 4 7 8	1 8 3 7 4 5 6 2 9	3 1 4 2 7 9 5 6 8	4 2 6 1 7 8 5 3 9
1 3 2 9 5 6 7 4 8	1 9 6 4 5 8 3 7 2	3 1 4 2 7 9 6 5 8	4 3 2 1 5 8 7 9 6
1 3 4 7 6 5 2 9 8	1 9 6 4 5 8 7 3 2	3 2 1 5 4 7 8 9 6	4 3 2 1 5 8 9 7 6
1 3 4 7 6 5 9 2 8	1 9 6 7 5 2 3 4 8	3 2 1 5 4 7 9 8 6	4 3 9 1 7 8 2 5 6
1 3 6 2 7 9 4 5 8	1 9 6 7 5 2 4 3 8	3 2 4 8 5 1 7 9 6	4 3 9 1 7 8 5 2 6
1 3 6 2 7 9 5 4 8	2 1 4 3 7 9 5 6 8	3 2 4 8 5 1 9 7 6	4 9 6 1 5 8 3 7 2
1 3 9 4 7 8 2 5 6	2 1 4 3 7 9 6 5 8	3 2 8 6 5 1 7 9 4	4 9 6 1 5 8 7 3 2
1 3 9 4 7 8 5 2 6	2 3 6 1 7 9 4 5 8	3 2 8 6 5 1 9 7 4	5 1 2 9 6 7 3 4 8
1 4 8 2 7 9 3 5 6	2 3 6 1 7 9 5 4 8	3 5 2 1 4 8 7 9 6	5 1 2 9 6 7 4 3 8
1 4 8 2 7 9 5 3 6	2 4 8 1 7 9 3 5 6	3 5 2 1 4 8 9 7 6	5 2 1 3 4 7 8 9 6
1 5 2 3 4 8 7 9 6	2 4 8 1 7 9 5 3 6	3 6 4 9 5 8 1 7 2	5 2 1 3 4 7 9 8 6
1 5 2 3 4 8 9 7 6	2 6 9 8 5 1 4 7 3	3 6 4 9 5 8 7 1 2	5 3 1 7 2 6 8 9 4
1 5 2 8 4 7 3 9 6	2 6 9 8 5 1 7 4 3	3 9 2 8 1 5 6 7 4	5 3 1 7 2 6 9 8 4
[[4 4 0 2 7 2 0 C	720065244	0 2 2 7 5 0 1 6 4	
5 4 1 9 2 7 3 8 6	7 2 8 9 6 5 3 1 4	8 3 2 7 5 9 1 6 4	9 1 4 7 6 5 3 2 8
5 4 1 9 2 7 8 3 6	7 3 1 5 2 6 8 9 4	8 3 2 7 5 9 6 1 4	9 2 8 7 6 5 1 3 4
5 4 8 9 6 7 1 3 2	7 3 1 5 2 6 9 8 4	8 5 2 1 4 7 3 9 6	9 2 8 7 6 5 3 1 4
5 4 8 9 6 7 3 1 2	7 3 2 8 5 9 1 6 4	8 5 2 1 4 7 9 3 6	9 3 1 6 2 5 7 8 4
5 7 2 8 3 9 1 6 4	7 3 2 8 5 9 6 1 4	8 5 2 7 4 9 1 3 6	9 3 1 6 2 5 8 7 4
5 7 2 8 3 9 6 1 4	7 3 4 1 6 5 2 9 8	8 5 2 7 4 9 3 1 6	9 3 2 1 5 6 4 7 8
5 9 3 6 2 1 7 8 4	7 3 4 1 6 5 9 2 8	8 6 4 7 5 9 1 3 2	9 3 2 1 5 6 7 4 8
5 9 3 6 2 1 8 7 4	7 5 2 8 4 9 1 3 6	8 6 4 7 5 9 3 1 2	9 4 1 5 2 7 3 8 6
6 2 8 3 5 1 7 9 4	7 5 2 8 4 9 3 1 6	8 6 9 2 5 1 4 7 3	9 4 1 5 2 7 8 3 6

6	2	8	3	5	1	9	7	4
6	3	1	9	2	5	7	8	4
6	3	1	9	2	5	8	7	4
6	9	3	5	2	1	7	8	4
6	9	3	5	2	1	8	7	4
7	1	4	9	6	5	2	3	8
7	1	4	9	6	5	3	2	8
7	2	8	9	6	5	1	3	4

_								
7	6	4	8	5	9	1	3	2
7	6	4	8	5	9	3	1	2
7	8	3	1	4	5	2	6	9
7	8	3	1	4	5	6	2	9
7	9	6	1	5	2	3	4	8
7	9	6	1	5	2	4	3	8
8	2	4	3	5	1	7	9	6
8	2	4	3	5	1	9	7	6

8	6	9	2	5	1	7	4	3
8	7	2	5	3	9	1	6	4
8	7	2	5	3	9	6	1	4
8	9	2	3	1	5	6	7	4
8	9	2	3	1	5	7	6	4
9	1	2	5	6	7	3	4	8
9	1	2	5	6	7	4	3	8
9	1	4	7	6	5	2	3	8

9 4 8 5 6 7 1 3 2 9 4 8 5 6 7 3 1 2 9 5 3 1 4 2 7 8 6 9 5 3 1 4 2 8 7 6 9 6 4 3 5 8 1 7 2 9 8 6 2 4 1 5 7 3 9 8 6 2 4 1 5 7 3									
9 5 3 1 4 2 7 8 6 9 5 3 1 4 2 8 7 6 9 6 4 3 5 8 1 7 2 9 6 4 3 5 8 7 1 2 9 8 6 2 4 1 5 7 3	9	4	8	5	6	7	1	3	2
9 5 3 1 4 2 8 7 6 9 6 4 3 5 8 1 7 2 9 6 4 3 5 8 7 1 2 9 8 6 2 4 1 5 7 3	9	4	8	5	6	7	3	1	2
9 6 4 3 5 8 7 1 2 9 8 6 2 4 1 5 7 3	9	5	3	1	4	2	7	8	6
9 6 4 3 5 8 7 1 2 9 8 6 2 4 1 5 7 3	9	5	3	1	4	2	8	7	6
9 8 6 2 4 1 5 7 3	9	6	4	3	5	8	1	7	2
	9	6	4	3	5	8	7	1	2
9 8 6 2 4 1 7 5 3	9	8	6	2	4	1	5	7	3
	9	8	6	2	4	1	7	5	3

3.3 SECOND APPROACH

Programming languages like Python provide ways to solve this problem with less code and faster than the way seen on the first approach. The following code, found on a Stack Exchange post [7] answering the same problem uses a different and efficient approach:

```
import itertools

p = itertools.permutations([1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, 9.0])

def is_66(a):
    result = (a[0] + 13 * a[1] / a[2] + a[3] + 12 * a[4] - a[5] - 11 + a[6] * a[7] / a[8] - 10)

    # handle the floats correctly, i.e. result == 66.00 will exclude some solutions
    return (result > 65.99) and (result < 66.01)

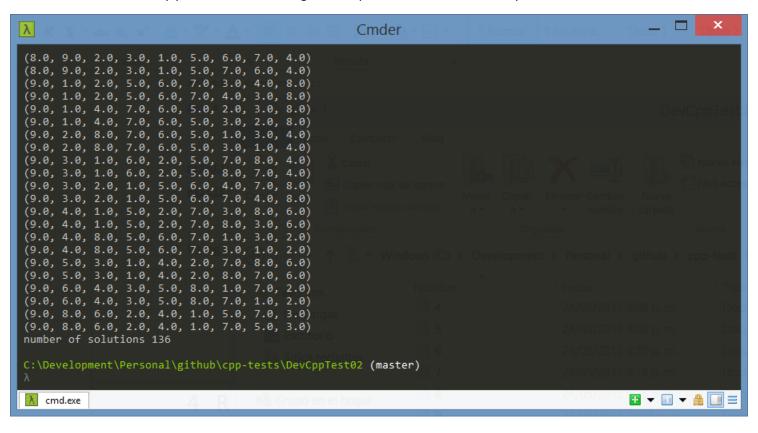
solution_counter = 0;

for a in p:
    if(is_66(a)):
        print(a);
        solution_counter += 1;

print("number of solutions %d" % solution_counter);

exit();</pre>
```

Using python and the **itertools** library you can iterate through all the possible combinations very fast.



4 Conclusions

- Using a recursive algorithm will increase the execution time of the process, and might not be a good idea if time is a problem. It might be helpful if memory consumption is an issue.
- Brute force attacks can help you find the solution of a system in a short amount of time with a high accuracy.

5 Work to do

- Create an alternate C++ non recursive solution.
- Compare the time spent and the memory consumption with the first C++ approach and the pending C++ approach.
- Create a more detailed version of this document contemplating both the old approaches and the new ones.

6 Sources

https://github.com/jpruiz114/cpp-tests/tree/master/DevCppTest02

7 REFERENCES

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- [4] A. Bellos, "The Guardian," 5 20 2015. [Online]. Available: http://www.theguardian.com/science/alexs-adventures-in-numberland/2015/may/20/can-you-do-the-maths-puzzle-for-vietnamese-eight-year-olds-that-has-stumped-parents-and-teachers. [Accessed 5 22 2015].
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