Foundations of Deep Learning

ALF

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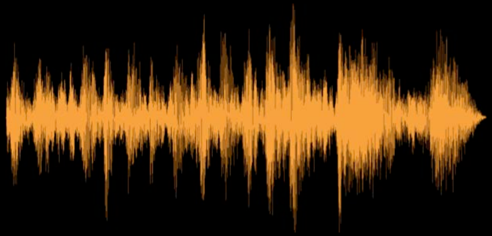
@alfcnz

Convolutional Neural Nets

Exploiting stationarity, locality, and compositionality of natural data

Signals can be represented as vectors



$$\boldsymbol{x} = [x_1 \;\; x_2 \;\; x_3 \;\; \ldots \;\; x_t \;\; \ldots]^\top$$

x_t are waveform heights

$$\boldsymbol{x} = [x_{11} \;\; x_{12} \;\; \ldots \;\; x_{1n} \;\; x_{21} \;\; x_{22} \;\; \ldots]^\top$$

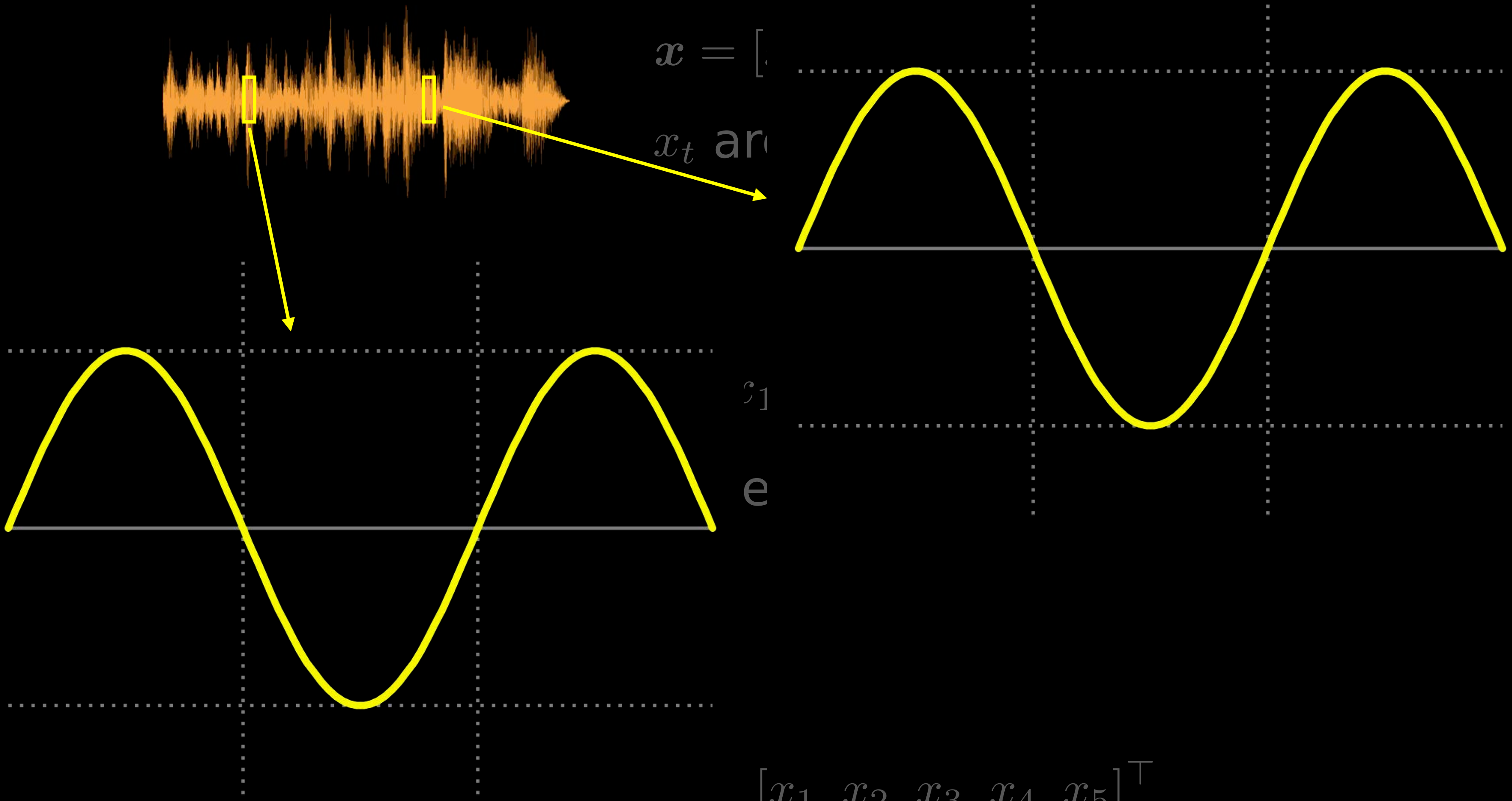
x_{ij} are pixel values

“John picked up the apple”

$$\boldsymbol{x} = [x_1 \;\; x_2 \;\; x_3 \;\; x_4 \;\; x_5]^\top$$

x_t are one-hot vectors

Signals can be represented as vectors



$\boldsymbol{x} = [$

x_t ar

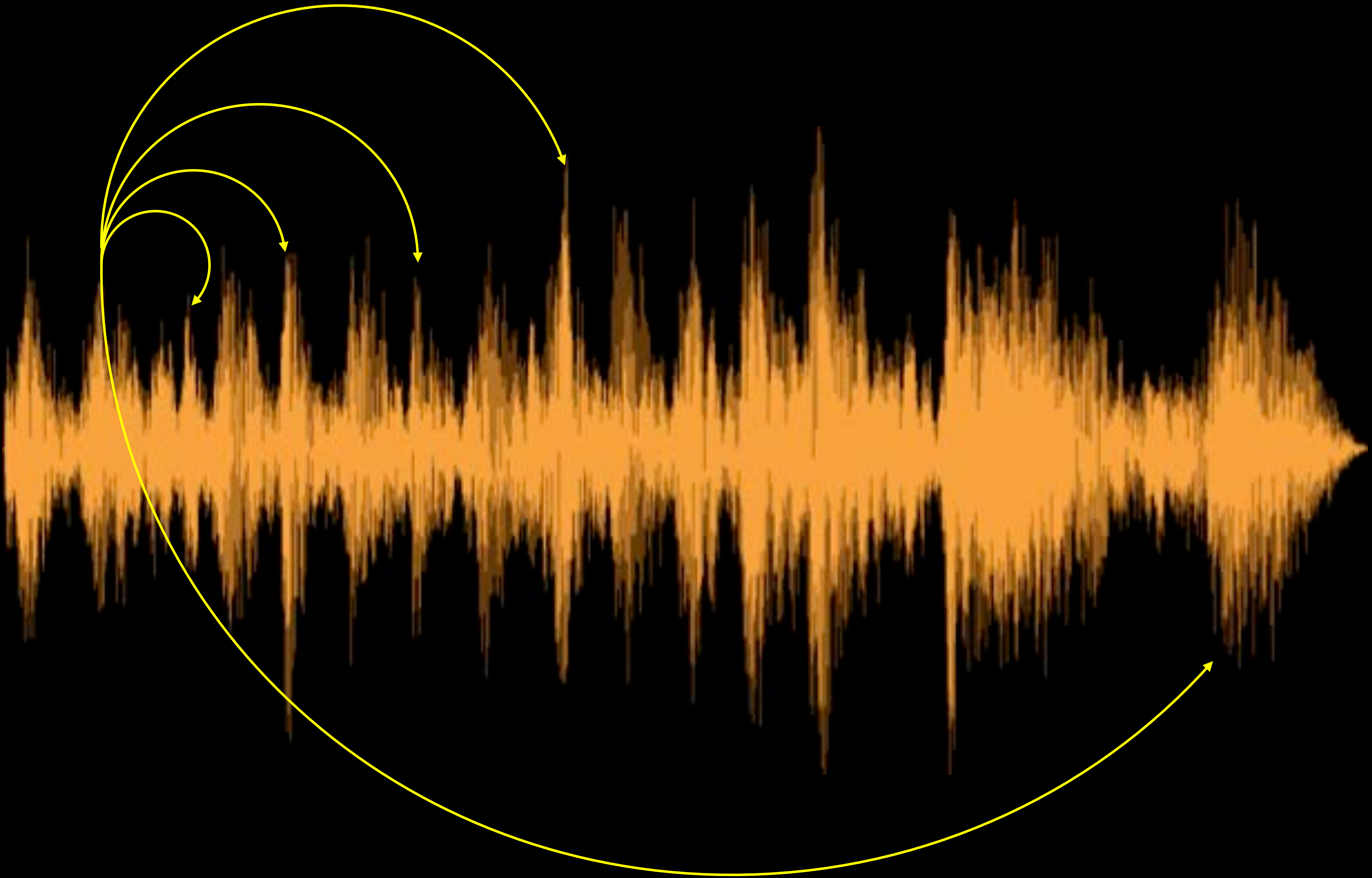
x_1

e

“John picked up the apple”

$[x_1 \;\; x_2 \;\; x_3 \;\; x_4 \;\; x_5]^\top$

x_t are one-hot vectors



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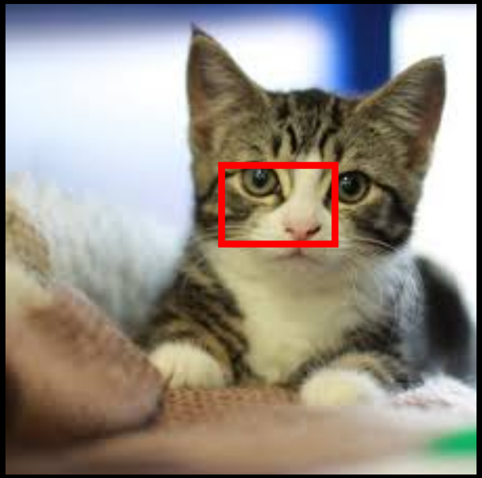

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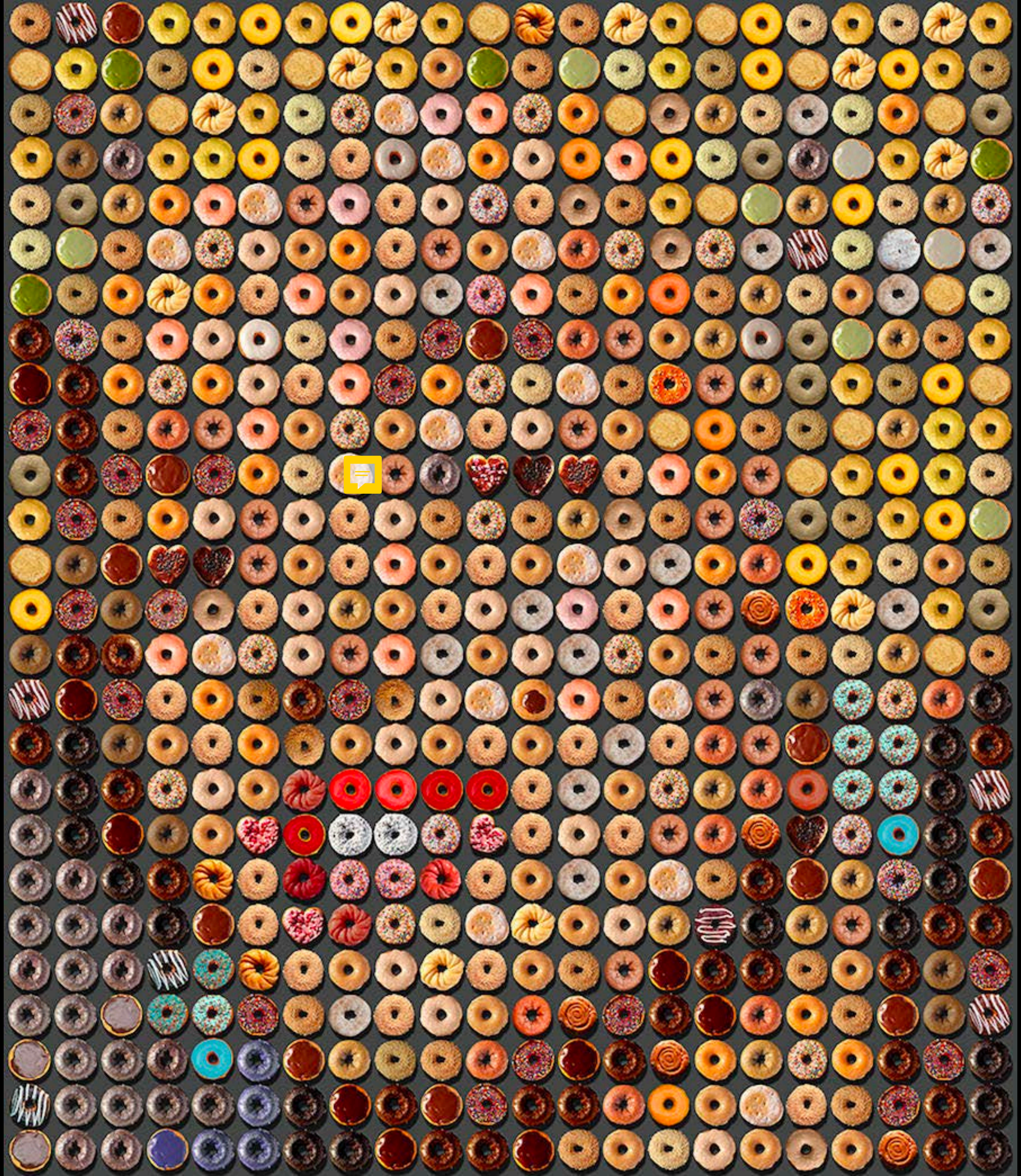
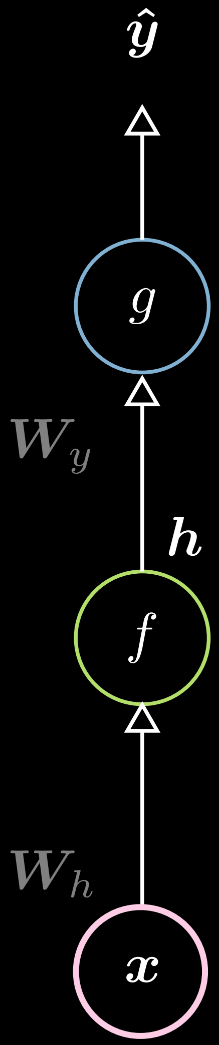


Fully connected (FC) layer

j-th row of $W^{(1)}$

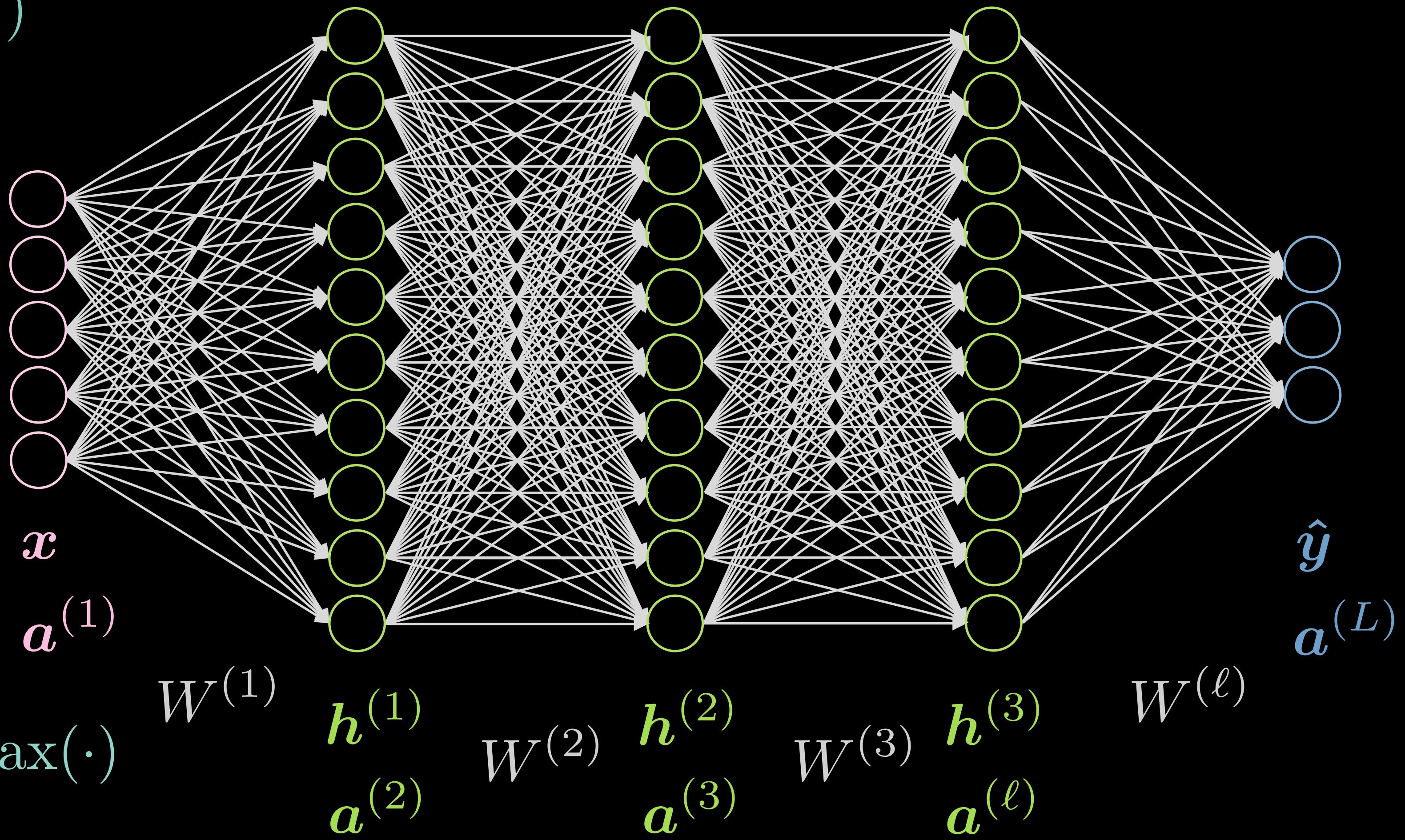
$$a_j^{(2)} = f(\boxed{\boldsymbol{w}^{(j)}}\boldsymbol{x} + b_j) = f\Big(\big(\sum_{i=1}^{n} w_i^{(j)} x_i\big) + b_j\Big)$$



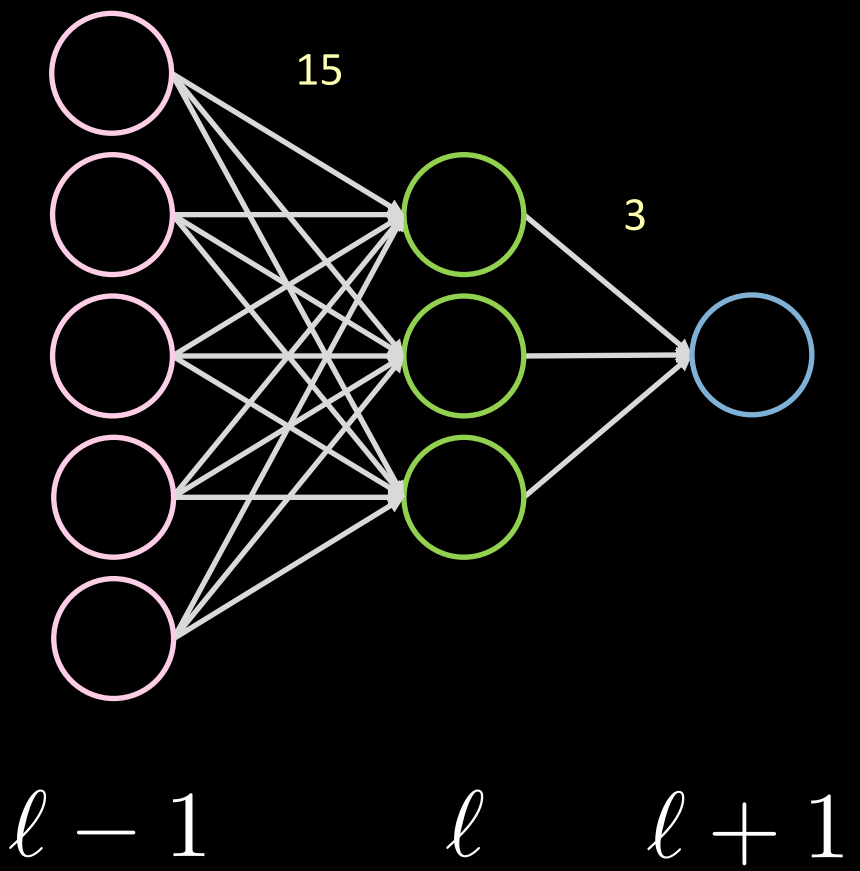
$$\boldsymbol{h} = f(\boldsymbol{W_h x} + \boldsymbol{b_h})$$

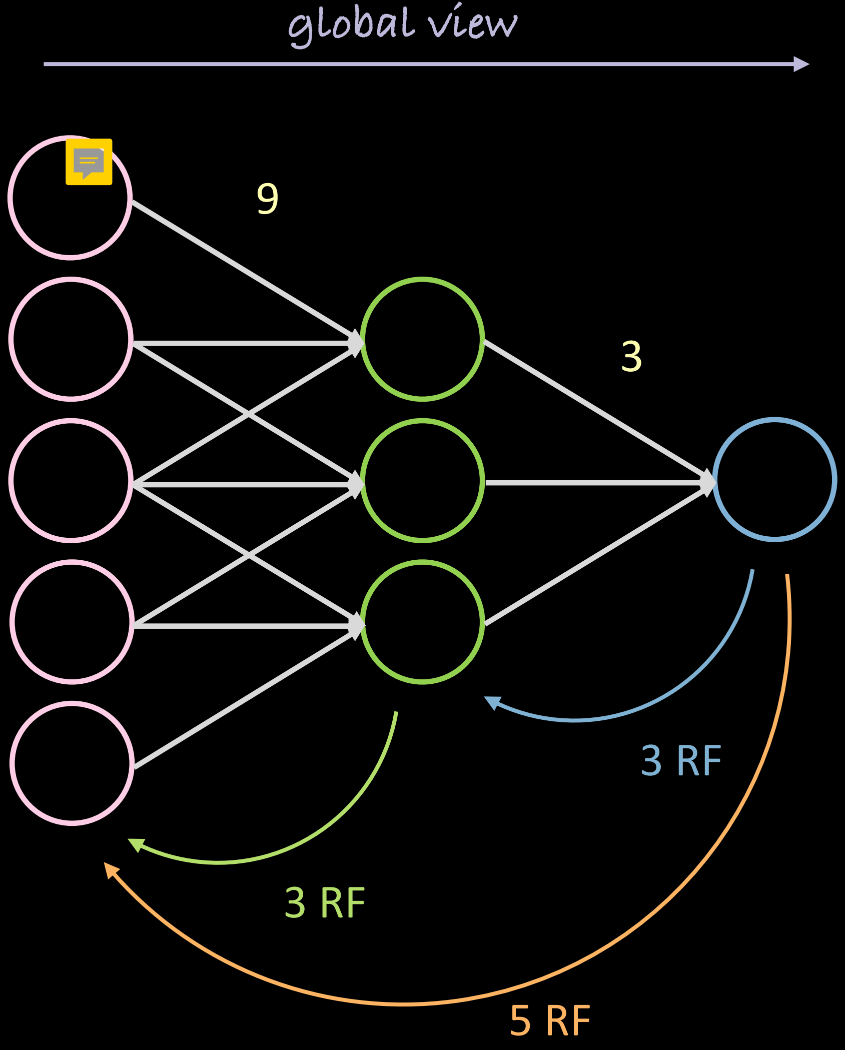
$$\hat{\boldsymbol{y}} = g(\boldsymbol{W_y h} + \boldsymbol{b_y})$$

$f, g = (\cdot)^+, \sigma(\cdot),$
$\tanh(\cdot), \mathrm{soft}(arg)\max(\cdot)$

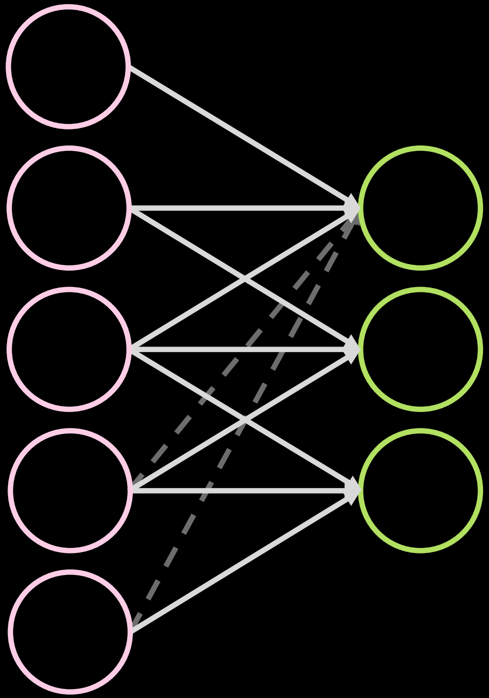


Locality ⇒ sparsity





Stationarity ⇒ parameters sharing



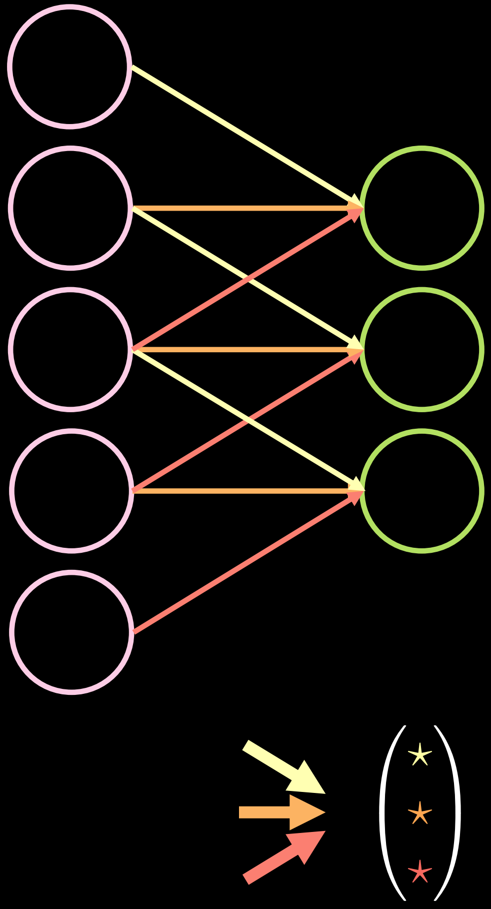
Parameters sharing

- faster convergence
- better generalisation
- not constrained to input size
- kernel independence

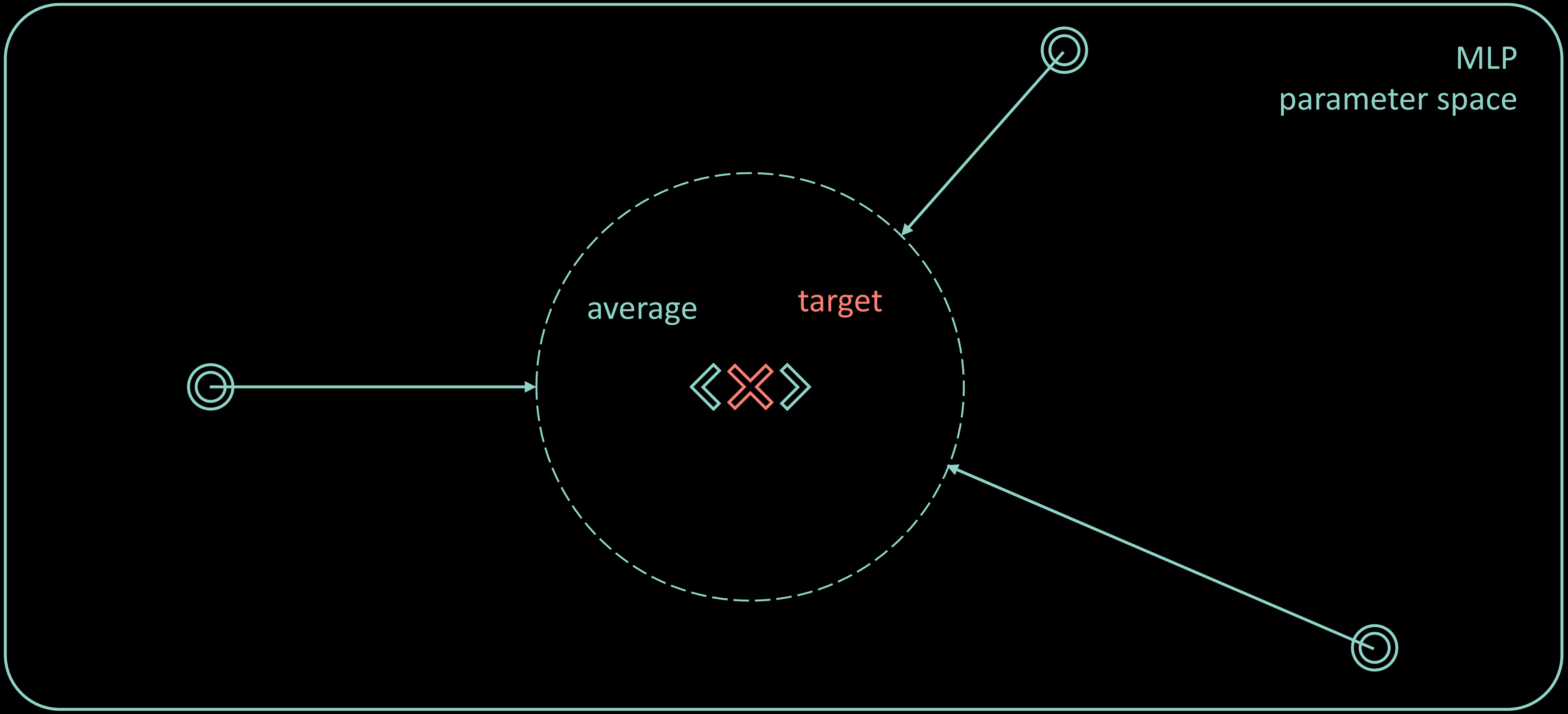
  ⇒ high parallelisation

Connection sparsity

- reduced amount of computation



Generalisation error reduction



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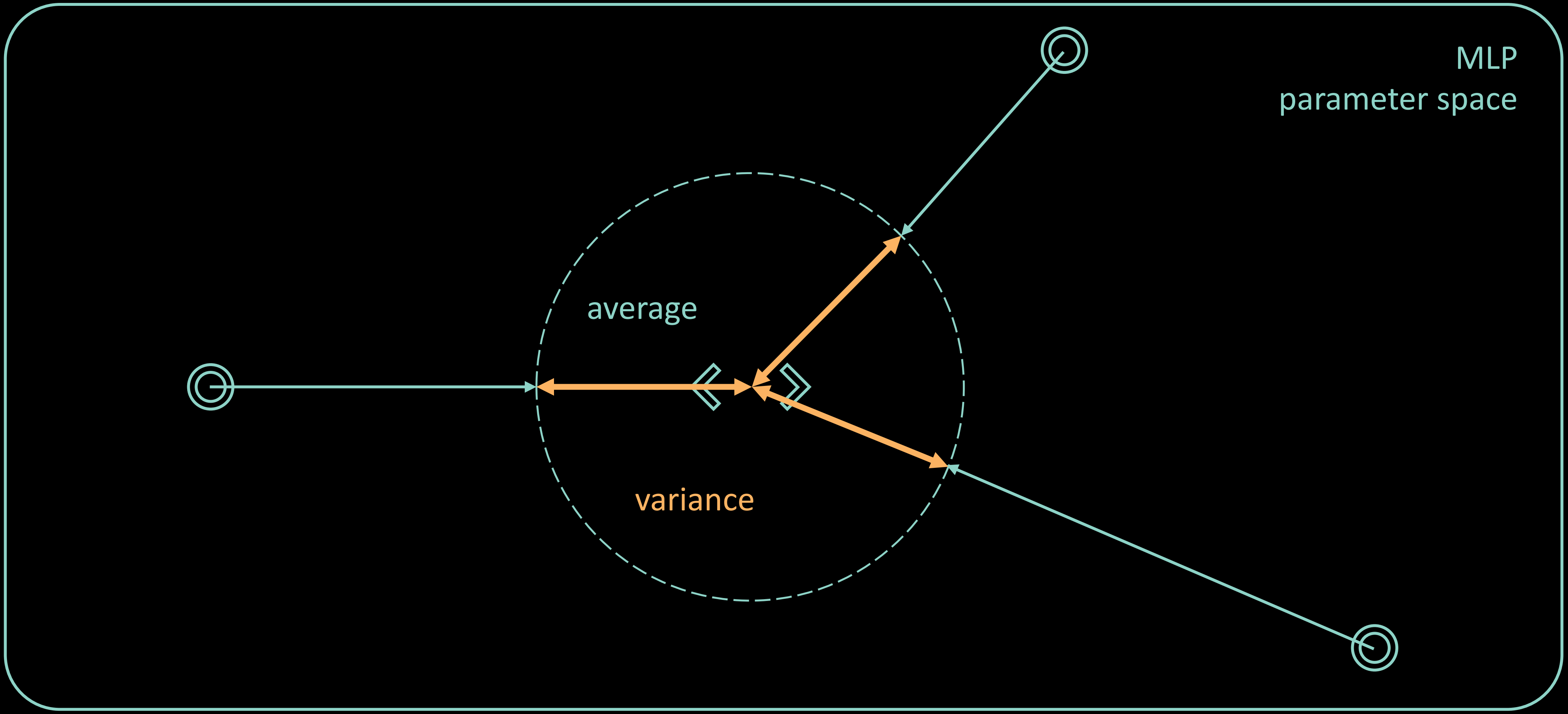

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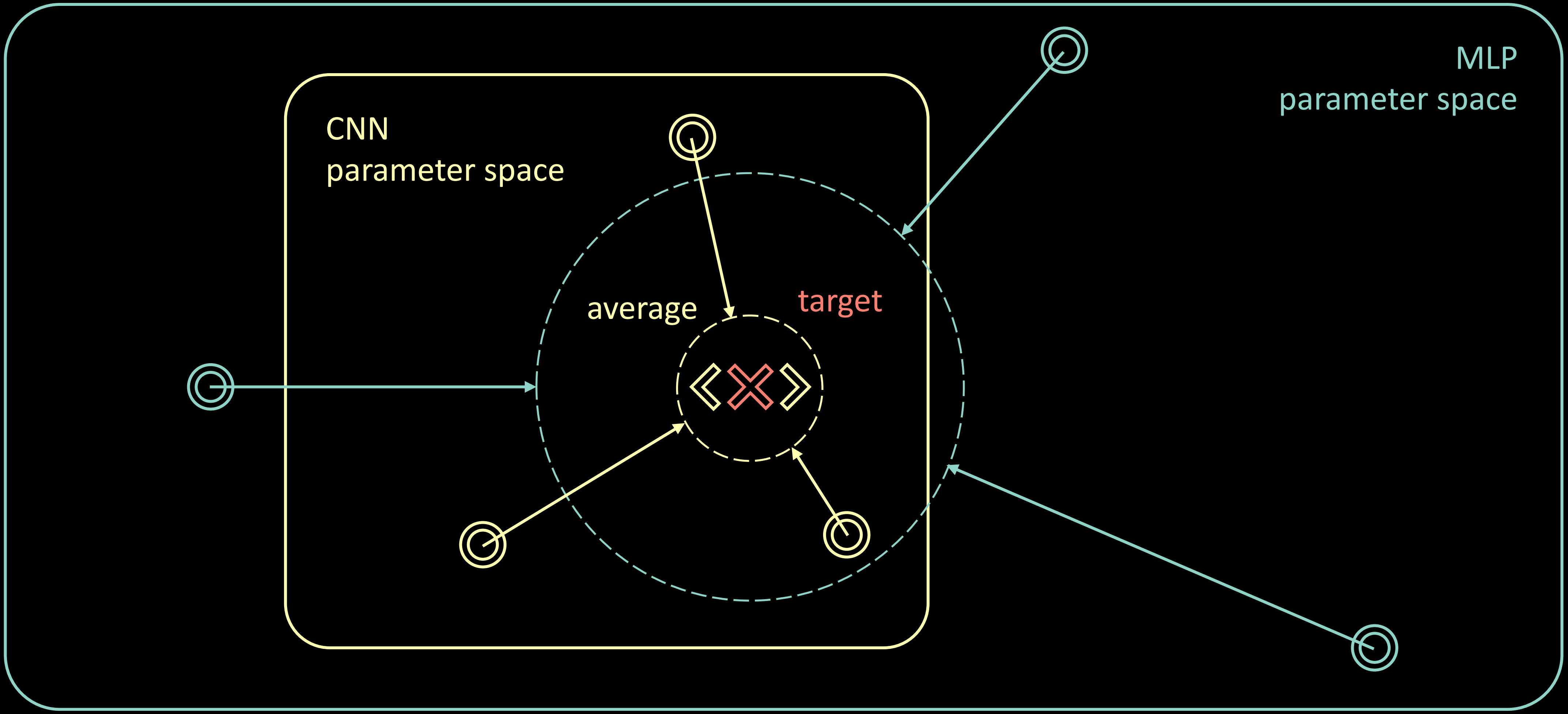

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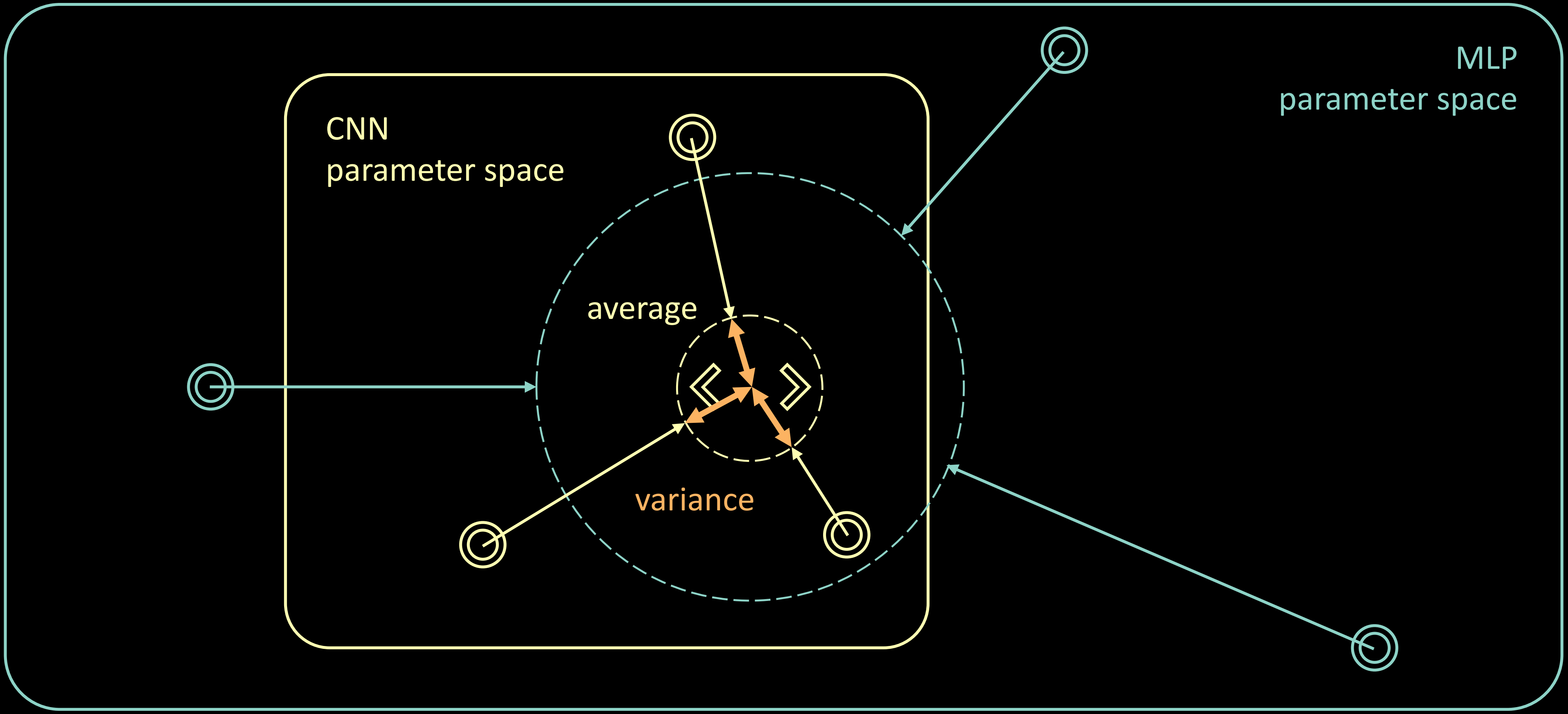
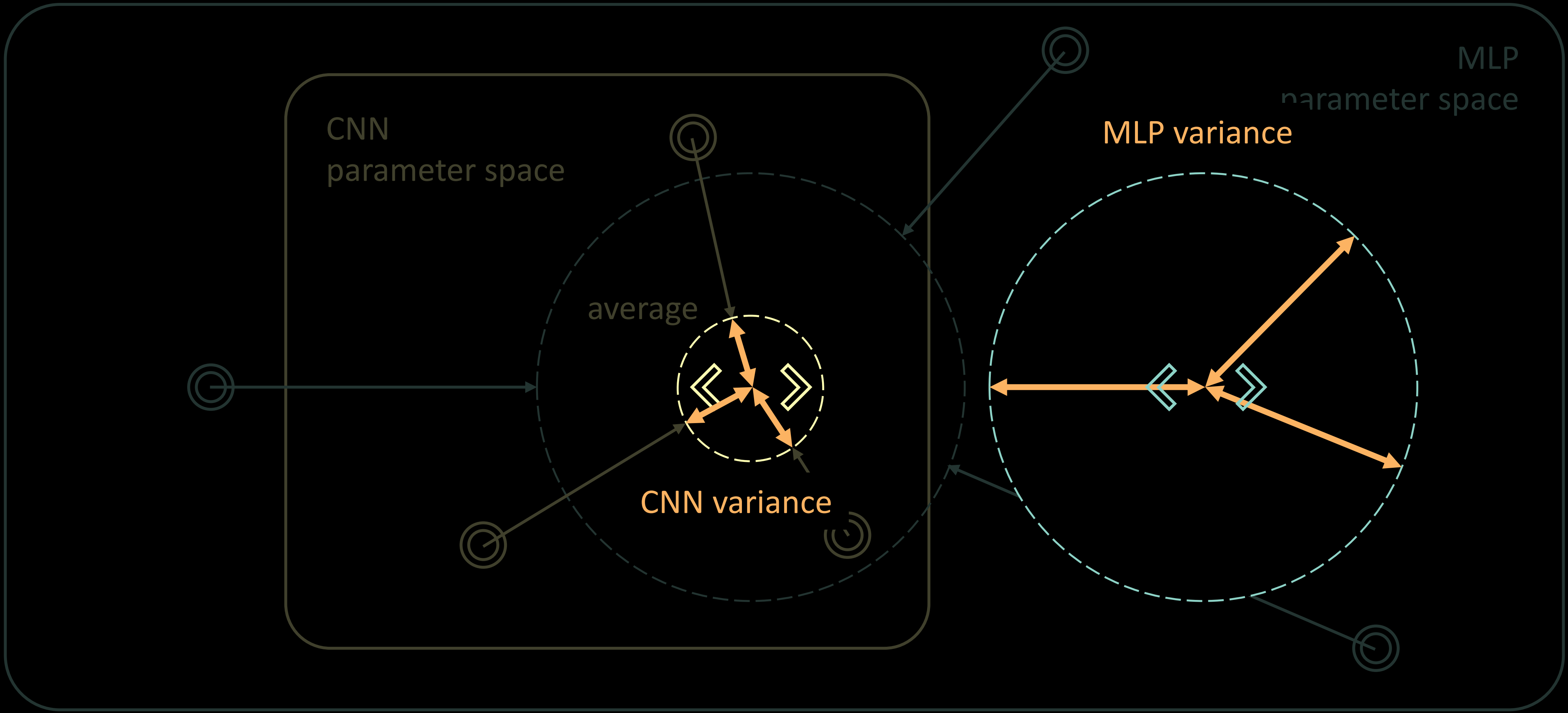
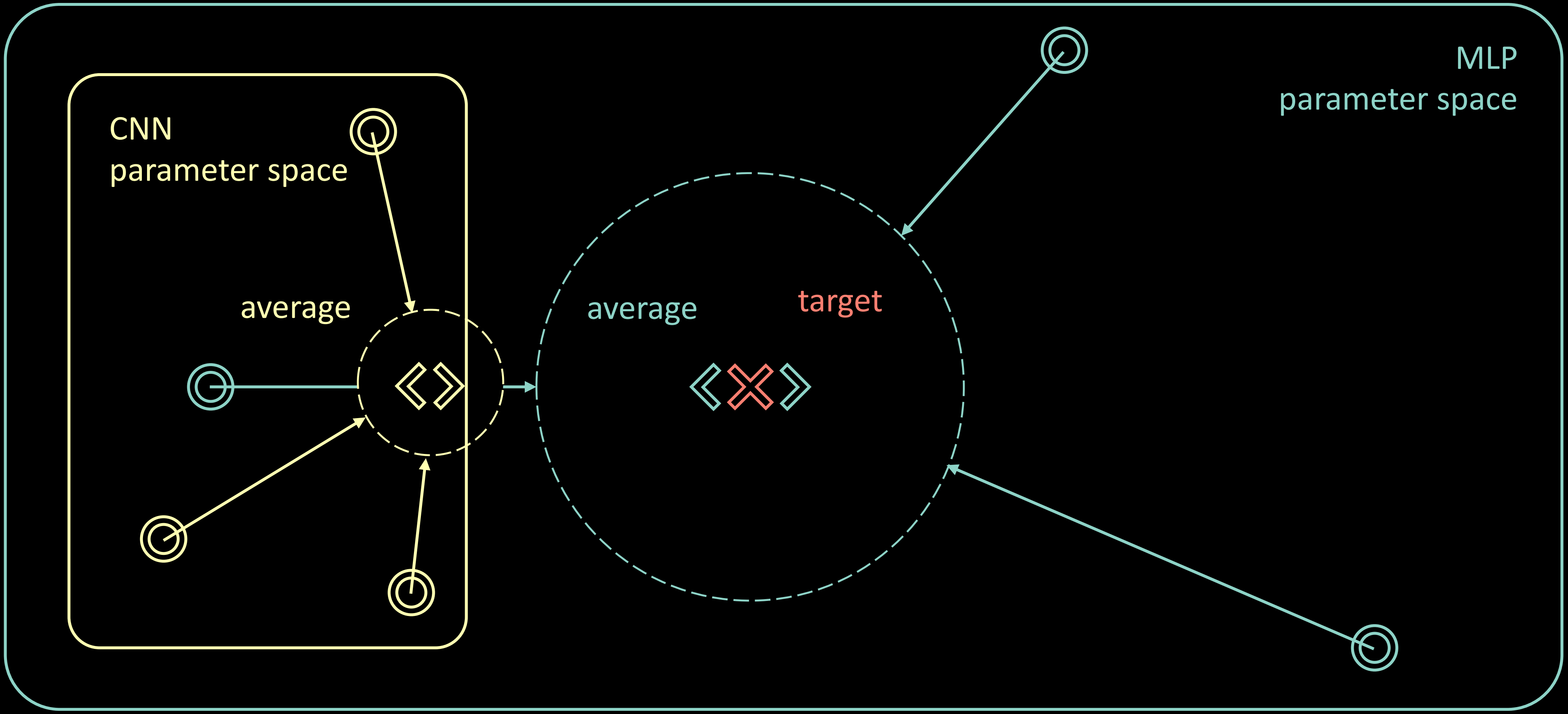


Generalisation error reduction



Misspecification of model constraints



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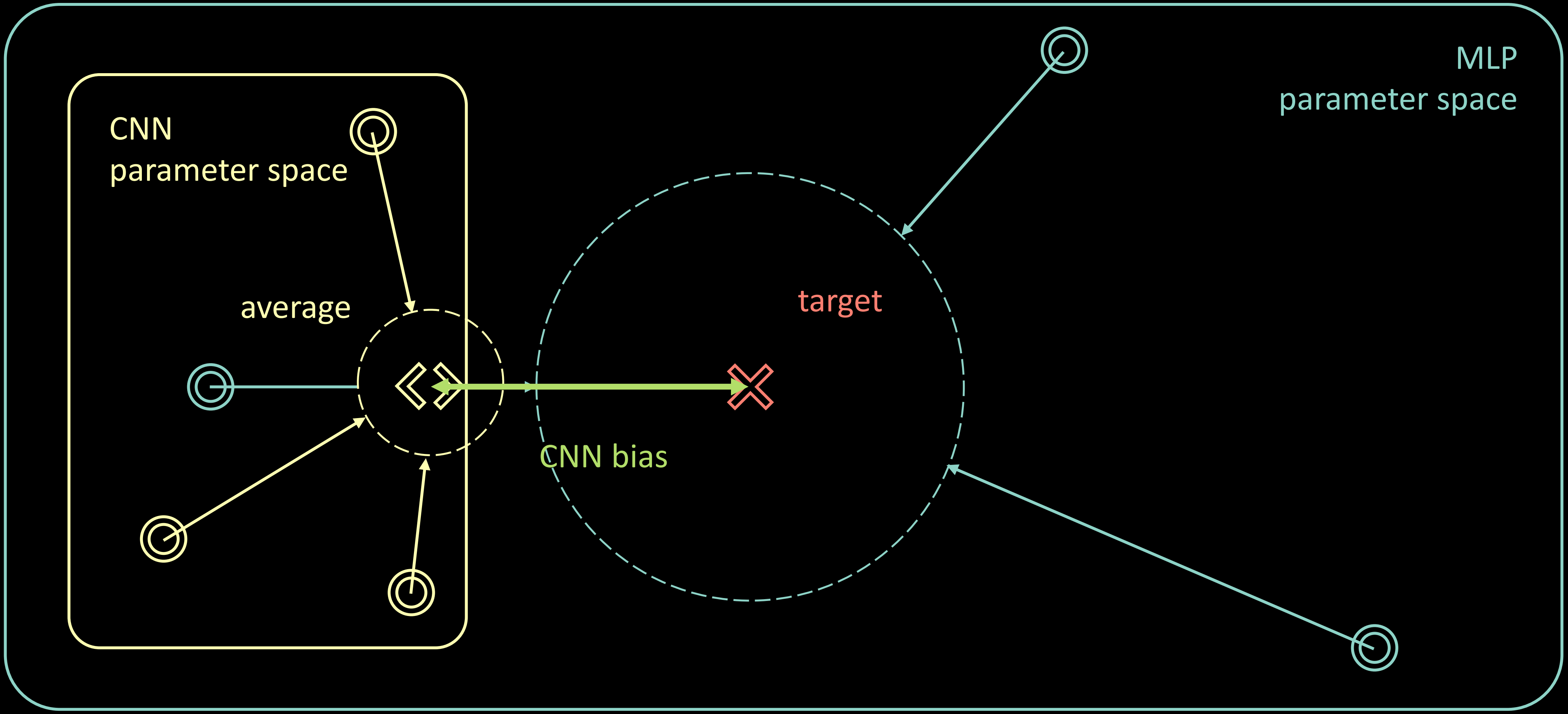

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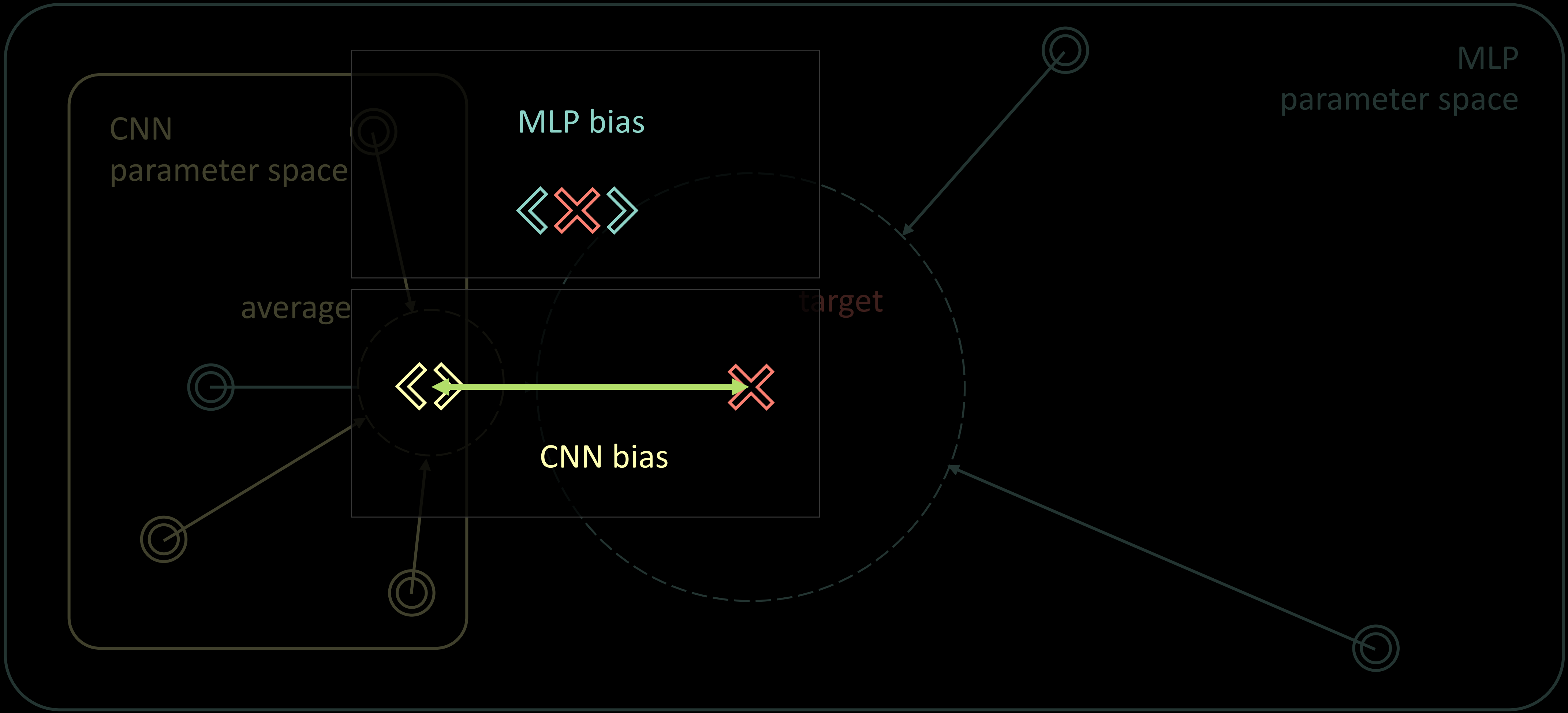
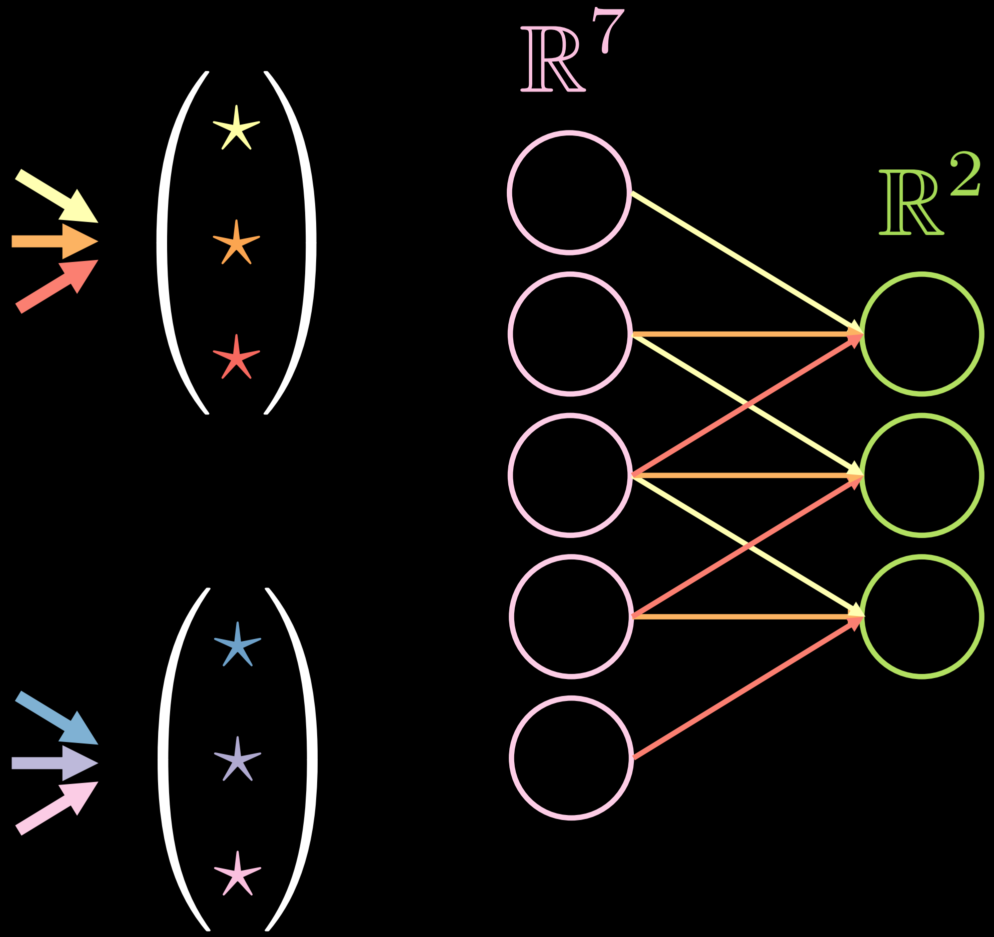


Kernels – 1D data

kernel size: $2 \times 7 \times 3$

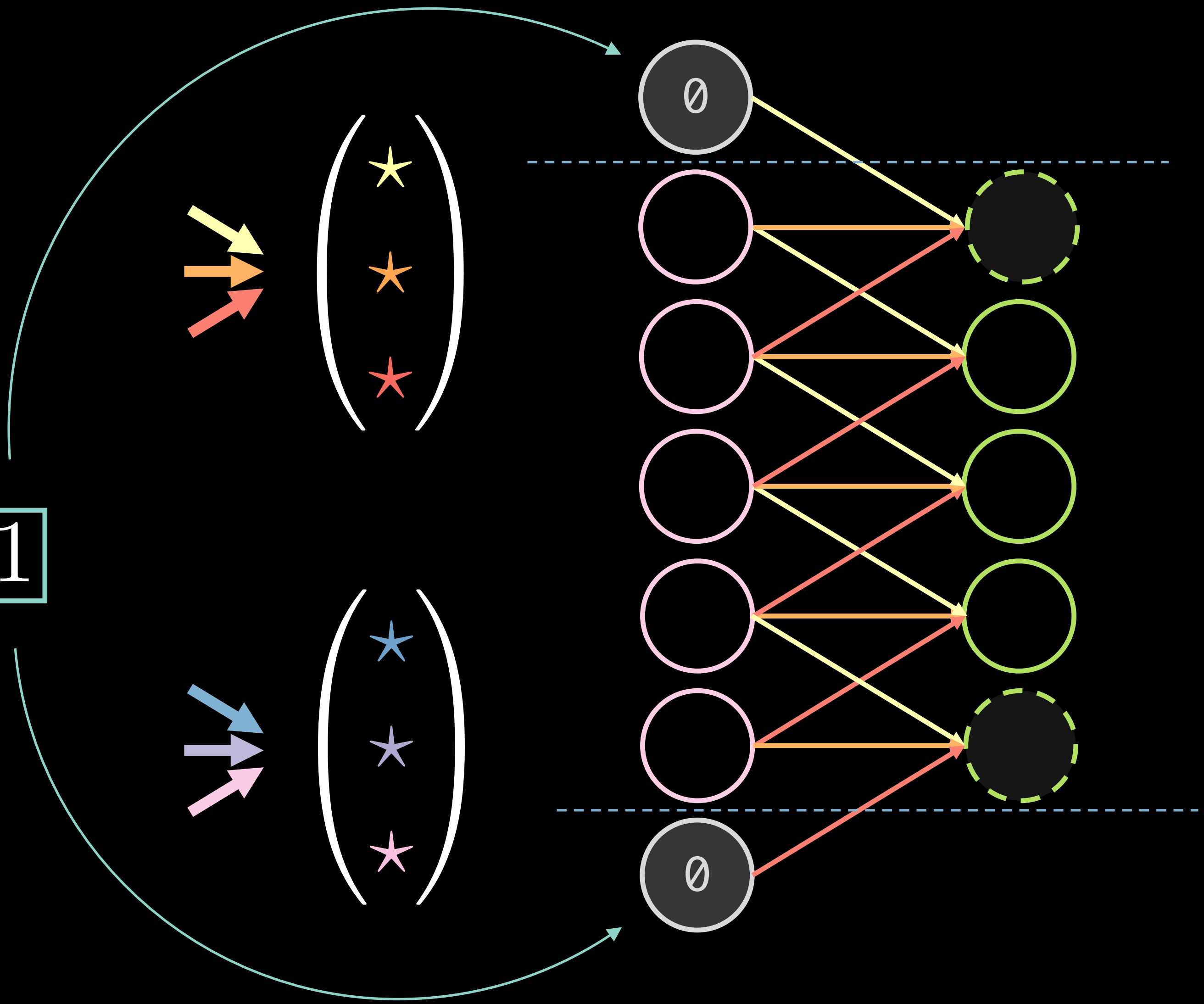
1D data uses 3D kernels-collection!



Padding – 1D data

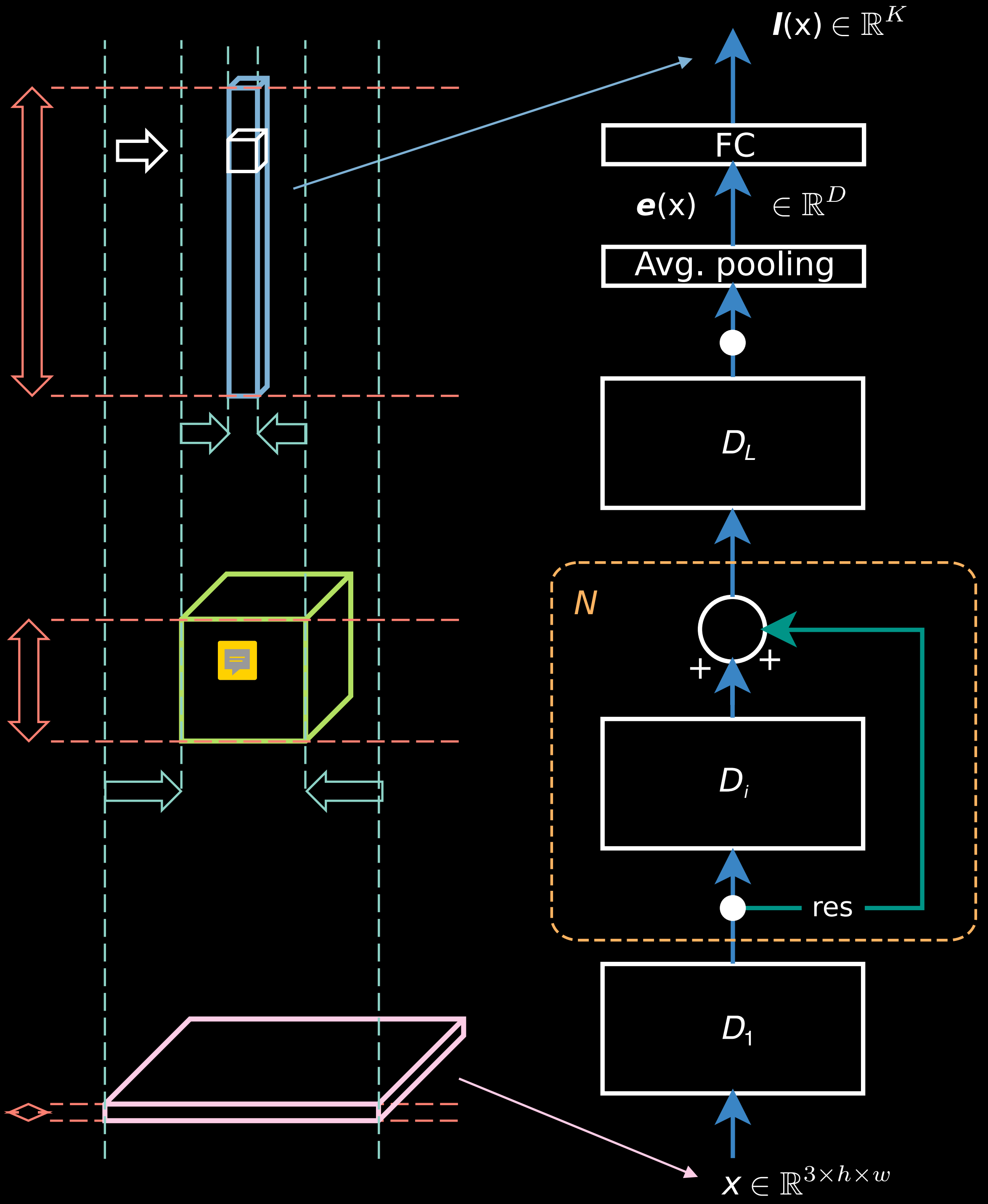
kernel size: $2 \times 7 \times \boxed{3}$

zero padding: $(\boxed{3} - 1)/2 = \boxed{1}$



Standard spatial CNN

- Multiple layers
  - Convolution
  - Non-linearity (ReLU and Leaky)
  - Pooling
  - Batch normalisation
- Residual bypass connection


$\boldsymbol{l}(x) \in \mathbb{R}^K$
FC
$\boldsymbol{e}(x)$
$\in \mathbb{R}^D$
Avg. pooling
D_L
N
+
+
D_i
res
D_1
$x \in \mathbb{R}^{3\times h\times w}$

Pooling

$$\|\underline{x}\|_p := \left(\sum_i |x_i|^p \right)^{1/p}$$

$$\|\underline{x}\|_p \to \max(\underline{x}), \; p \to +\infty$$