Time Series Forecasting for Antarctic Sea Ice Using SARIMA and Holt-Winters Methods: Jason Rutberg

1. Abstract

While Arctic sea ice has been rapidly declining over the last several decades, the trend of Antarctic sea ice is much less clear. The data set “Monthly Southern Hemisphere Sea Ice Area” from the National Snow and Ice Data Center (NSIDC) records the area of sea ice in the Southern Hemisphere from 1978 to 2023, but the first several years have missing data, so we only consider the time period from 1989 to 2023. Using this time series, we employ two separate forecasting methods (Seasonal ARIMA and Holt-Winters) to predict sea ice in the Southern Hemisphere in 2024 and 2025. For the seasonal ARIMA forecast, we must first choose a suitable model. Through inspection of the sample ACF and PACF plots and use of the AICc statistic to compare models, we select a SARIMA(1,1,2)x$(0,1,2)_{12}$ model. For the Holt-Winters forecast, we use an additive seasonal model. Based on the RMSE, MAE, and MAPE criteria, the Holt-Winters forecast is superior to the seasonal ARIMA forecast. We conclude that the reduction in Antarctic sea ice is expected to accelerate in the coming years, but these models may not be suitably able to deal with the zero lower bound.

2. Introduction

Climate change is altering weather patterns around the world in various ways, including causing changes in polar sea ice coverage. In Antarctica, while the area of sea ice has shown some variability in recent years, overall trends suggest a decrease in sea ice [1]. This reduction is attributed to a combination of factors, including warmer ocean temperatures and changing wind patterns [2]. The loss of sea ice in Antarctica can impact marine ecosystems and alter regional climate patterns. Data from the National Snow and Ice Data Center (NSIDC) on the area of sea ice in the Southern Hemisphere is obtained. In order to forecast sea ice coverage over the next two years, seasonal ARIMA and seasonal Holt-Winters model are formulated. These forecasts show that the pace of melting sea ice will accelerate in the next couple of years.

3. Data Set

The National Snow and Ice Data Center continuously collects data on the coverage of sea ice in the Southern Hemisphere, which is available through their “Sea Ice Index” website [3]. This particular time series records monthly data from 1978 to 2023. Due to several months with missing data, only data from 1989 to 2023 is used for this analysis for a total of 420 data points (Table 1).[1] It should be noted that sea ice area measures the total region covered by ice (in contrast to sea ice extent, which measures the region covered by at least 15% sea ice) [4]. For this data set, sea ice is measured in millions of square kilometers [5].

Table 1: Monthly Average Southern Hemisphere Sea Ice Area from 1989 to 2023

Year	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
1989	3.17	2.03	2.59	4.50	8.00	10.81	12.58	14.03	14.48	13.85	11.42	6.70
1990	3.39	2.21	2.79	5.18	7.46	10.35	12.27	14.03	14.39	13.80	11.35	6.73
…												
2022	2.44	1.38	1.86	4.25	7.29	9.56	11.91	13.67	14.07	13.34	10.84	5.23
2023	1.95	1.15	1.75	3.96	6.45	8.49	10.59	12.21	12.97	12.46	10.29	5.84

4. Results

4.1 Data Visualization

The time series is plotted below in Figure 1. The data is clearly not stationary since there is an obvious seasonal trend in the data: average monthly Antarctic sea ice peaks in August/September and bottoms out in February of each year (which corresponds to

[1] https://psl.noaa.gov/data/timeseries/monthly/data/s_icearea.mon.data

the Southern Hemisphere winter and summer, respectively). From this plot, there may be a trend over time as well (i.e. looking at changes over time in the peaks and troughs of the data).

In order to visualize a potential trend, an additive decomposition of the time series is displayed in Figure 2. An additive decomposition is selected (as opposed to a multiplicative decomposition) since the seasonal variation is roughly constant over time. The seasonal component panel shows a clear pattern as expected. The trend component shows that there is somewhat of a trend in the data: sea ice appears to slowly increase from 1990 to 2015; however, sea ice coverage begins decreasing quite noticeably after 2015.

In order to remove the trend and seasonality, a first difference (difference of lag 1) and a first seasonal difference (difference of lag 12) are applied to the data. Note that the Augmented Dickey Fuller (ADF) test had a p-value of < 0.01 for the original data, which would indicate that the data is stationary (Table A1). However, the classic ADF test is not sensitive to seasonal non-stationarity. [This analysis was initially done by only taking the seasonal difference, but the sample ACF plot suggested the data still was not stationary (i.e. linear decay instead of exponential decay), so the first difference was taken in addition to the seasonal difference despite the ADF test suggesting that this differencing was not necessary.]

Figure 1: Average Monthly Antarctic Sea Ice from 1989 to 2023

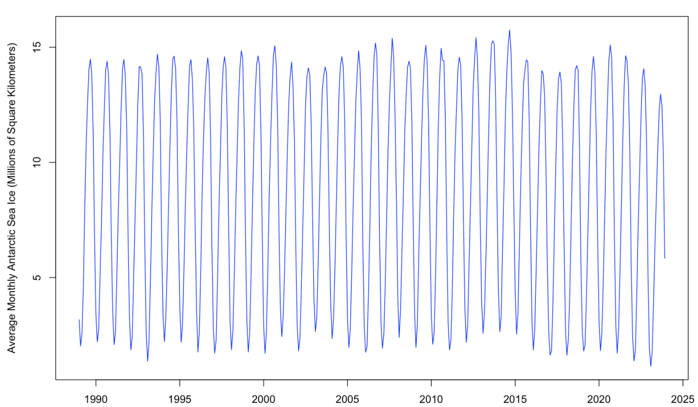
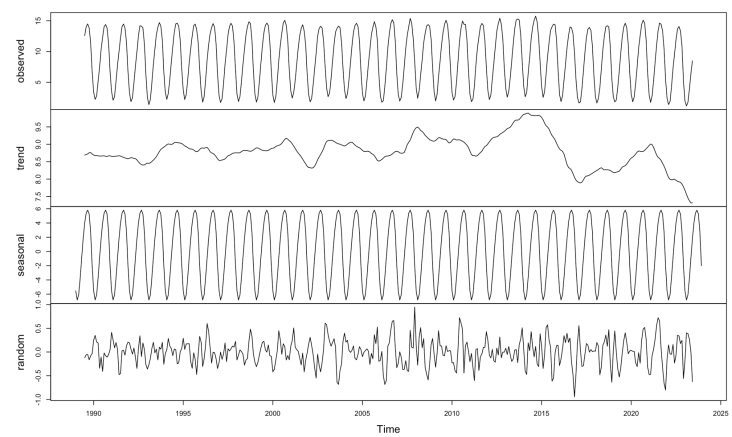


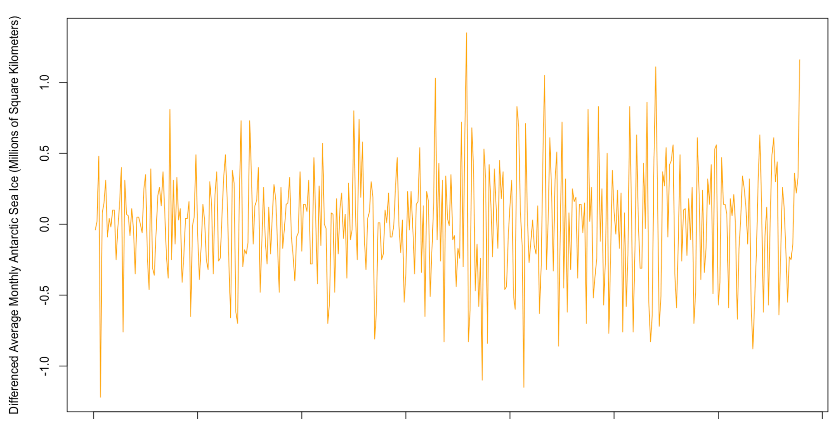
Figure 2: Additive Decomposition of Sea Ice Data



4.2 Transformations

Since the time series is relatively homoscedastic, a variance stabilizing transformation is not applied. In order to remove the seasonality and deterministic trend from the data, a seasonal difference and first difference, respectively, are applied to the data (Figure 3). Using the transformed data, the Dickey-Fuller test statistic is -9.27, and the associated p-value is < 0.01. Thus, we reject the null hypothesis at the 0.05 significance level, and we conclude that the transformed time series is stationary (Table A2).

Figure 3: Differenced Average Monthly Antarctic Sea Ice



4.3 Seasonal ARIMA Model Selection

The first step in developing a seasonal ARIMA forecast is fitting a seasonal ARIMA model, which is then used to forecast future values. Since the data is clearly seasonal (with period 12 months), a seasonal ARIMA model is fit as opposed to an ordinary ARIMA model. Using the transformed data, the sample autocorrelation (ACF) (Figure 4a) and sample partial autocorrelation (PACF) (Figure 4b) plots are used for model selection. The ACF cuts off after the third lag but is significant again at/around the first seasonal lag (and cuts off after the first seasonal lag). The PACF cuts off after lag 3 as well but is significant again at the first seasonal lag. Unlike the sample ACF plot, the sample PACF is significant at *each* seasonal lag but decays in an exponential manner.

Figure 4a: Differenced Data Sample ACF

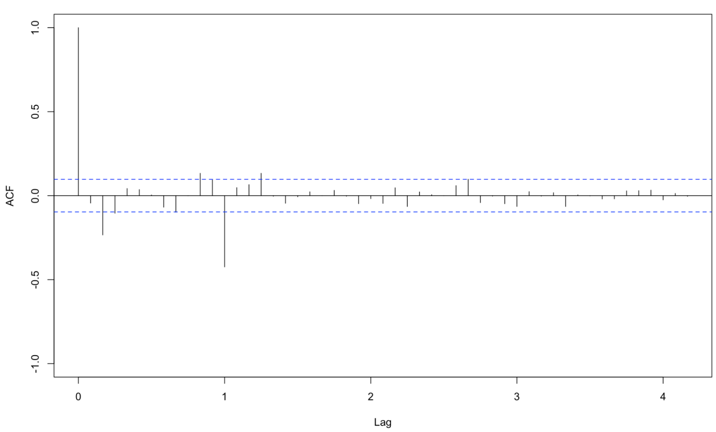
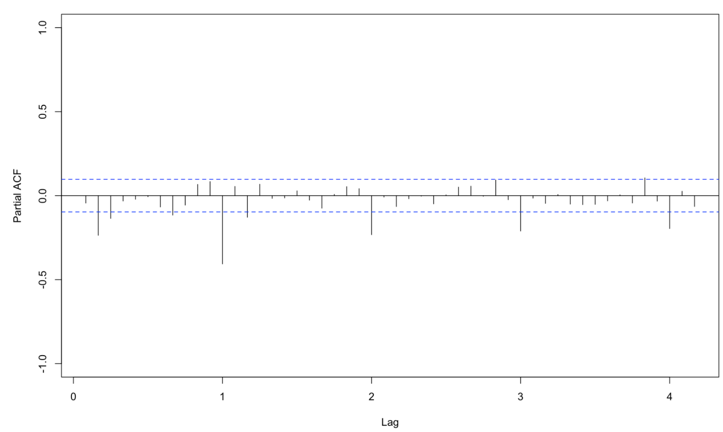


Figure 4b: Differenced Data Sample PACF



Since the sample ACF and PACF both cut off at early lags, this suggests a low-order ARMA model for the non-seasonal component (such as ARMA(1,1)). Since the sample ACF cuts off after the first seasonal lag but the PACF at seasonal lags decays exponentially to zero, this indicates an $MA(1)_{12}$ model for the seasonal component. Therefore, our initial model under consideration is $SARIMA(1,1,1)x(0,1,1)_{12}$, where d = 1 and D = 1 since we took the first difference and the first seasonal difference, respectively.

Table 2: SARIMA Model Comparison

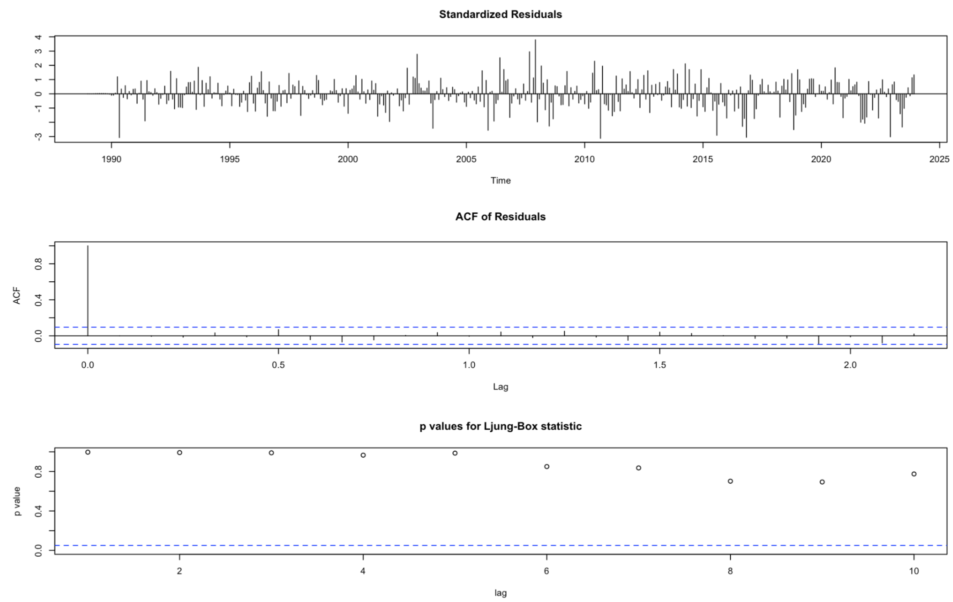
#	SARIMA Model	AICc	#	SARIMA Model	AICc
1	$(1,1,1) \times (0,1,1)_{12}$	195.32	7	$(3,1,0) \times (0,1,2)_{12}$	192.13
2	$(1,1,2) \times (0,1,1)_{12}$	183.41	8	$(0,1,3) \times (0,1,2)_{12}$	182.08
3	$(3,1,0) \times (0,1,1)_{12}$	195.47	9	$(1,1,1) \times (1,1,1)_{12}$	191.19
4	$(0,1,3) \times (0,1,1)_{12}$	184.21	10	$(1,1,2) \times (0,1,1)_{12}$	181.23
5	$(1,1,1) \times (0,1,2)_{12}$	190.94	11	$(3,1,0) \times (1,1,1)_{12}$	192.27
6	$(1,1,2) \times (0,1,2)_{12}$	181.13*	12	$(0,1,3) \times (1,1,1)_{12}$	182.23

Since there are several different interpretations from the sample ACF and PACF plots, eleven additional models are generated in Table 2. The bias-corrected Akaike information criterion (AICc) is used for comparisons among the models. The initial model described above (Model 1) had an AICc of 195.32. However, the $SARIMA(1,1,2)x(0,1,2)_{12}$ model (Model 6) had the lowest AICc of all the models in Table 2 at 181.13. Therefore, we proceed with this model for model diagnostics and forecasting.

4.4 Seasonal ARIMA Model Diagnostics

For the selected SARIMA(1,1,2)x(0,1,2)$_{12}$ model, diagnostics are run to determine whether this model is an appropriate fit. The middle panel of Figure 6 shows that the ACF of the model residuals are not significant at all positive lags, which indicates that the standardized model residuals are white noise as desired. The bottom panel of Figure 6 displays the associated p-values from the Ljung-Box test statistics, which are greater than alpha = 0.05 at all lags. Since we fail to reject the null hypothesis, the standardized residuals are white noise. In the appendix, Figure A1 presents the normal Q-Q plot of the standardized model residuals. Observe that there are deviations in both tails, which suggests that the residuals may deviate from a normal distribution. This same deviation from normality is observed in all of the SARIMA models tested in Table 2. The order of the minimum AICc AR model fitted to the residuals is AR(0) (Table A3). We conclude that Model 6 is good enough, and we use this model for forecasting in the next section.

Figure 6: Model Diagnostics

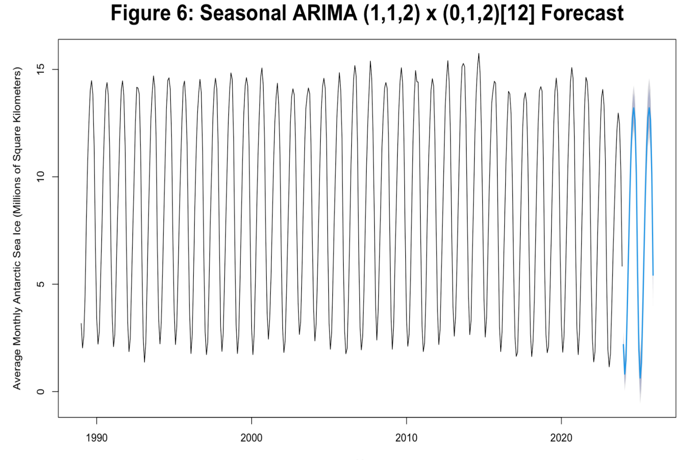


The model parameters are specified in Table A4 in the appendix. Observe that all five model parameters are statistically significant at the alpha = 0.05 significance level (i.e. the 95% confidence interval for each model parameter does not contain the value zero). Therefore, it does not make sense to remove any of the parameters from the model.

4.5 Model Based Forecast: SARIMA

Using the selected SARIMA(1,1,2)x(0,1,2)$_{12}$ model, a forecast is generated for the next two years. The forecast for Antarctic sea ice for the next two years is displayed in Figure 6 (blue line for the point estimate, dark gray shading for the 80% forecast interval, and light gray shading for the 95% forecast interval). The point estimates and the 95% forecast interval limits are displayed in Table A5 of the appendix. Observe the accelerating downward trend in the troughs of the data.

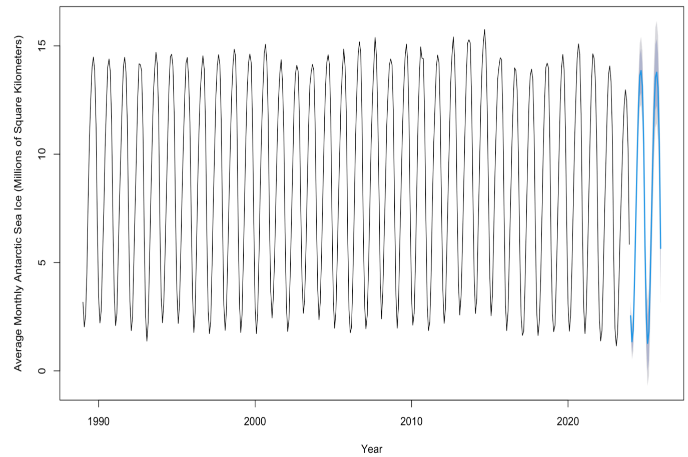
Figure 6: Seasonal ARIMA (1,1,2) x (0,1,2)[12] Forecast



4.6 Smoothing Based Forecast: Holt-Winters

An alternative forecasting method is Holt-Winters. Since the data is seasonal, this smoothing-based method uses a triple exponential smoother (level, trend, and seasonality) to model the pattern of the data and extrapolates this to the future for forecasting purposes. The additive version of the seasonal forecast is used here since the seasonal pattern stays about the same over the course of the data. The Holt-Winters forecast for Antarctic sea ice for the next two years is displayed in Figure 7 (blue line for the point estimate, dark gray shading for the 80% forecast interval, and light gray shading for the 95% forecast interval). The point estimates and the 95% forecast interval limits are displayed in Table A6 of the appendix. Observe that the forecast intervals are wider here compared with the seasonal ARIMA forecast.

Figure 7: Holt-Winters Forecast



4.7 Evaluation of Model Accuracy

To compare the accuracy of the two forecasts (SARIMA and Holt-Winters), we use an out-of-sample forecast validation technique. We treat the 48 most recent observations (4 years worth of data) in the time series as the validation sample, which represents just over 11% of the total sample size (n = 420). The remaining 372 observations are the training sample, which are used to fit the models. The measures of forecast accuracy evaluated here are root mean square error (RMSE), mean absolute error (MAE), and mean absolute percentage error (MAPE). The results are displayed in Table 3. We observe that the Holt-Winters forecast outperforms the SARIMA forecast according to all three measures. Based on this alone, we conclude that the Holt-Winters forecast is preferable.

Table 3: Comparison Between Model Forecasts

	SARIMA (1,1,2) x $(0,1,2)_{12}$	**Holt-Winters**
RMSE	0.7635	0.7132**
MAE	0.6143	0.5479**
MAPE	10.8042	9.4057**

5. Conclusions

Using monthly data on Antarctic sea ice from 1989 to 2023, we attempt to forecast sea ice coverage in 2024 and 2025. Two forecasting methods are analyzed: seasonal ARIMA and seasonal Holt-Winters. For the seasonal ARIMA forecasting, an appropriate seasonal ARIMA model is first selected by considering plausible models based on the sample ACF and PACF plots and then using the AICc criterion to choose the best model. To compare between the two forecasts, RMSE, MAE, and MAPE measures are used, which all show that the Holt-Winters forecast is superior to the SARIMA forecast. Both forecasts show an acceleration in the decline in Antarctic sea ice over the next couple of years. However, a few of the forecast intervals contain negative values, which is not practical since total sea ice area cannot drop below zero. Future steps could include performing an intervention analysis using somewhere around 2015 as the cut point since the trend of Antarctic sea ice seems to change considerably around this time. Other issues include dealing with the lower bound of zero Antarctic sea ice and incorporating the presence of certain global climate phenomena, such as El Niño and La Niña.

6. References

[1] Scott, Michon. "Understanding Climate: Antarctic Sea Ice Extent." NOAA Climate.gov, March 14, 2023. https://www.climate.gov/news-features/understanding-climate/understanding-climate-antarctic-sea-ice-extent.
[2] Berwyn, Bob. "Scientists Report a Dramatic Drop in the Extent of Antarctic Sea Ice." Inside Climate News, January 6, 2023. https://insideclimatenews.org/news/06012023/antarctic-sea-ice-climate-change/.
[3] "Monthly Climate Time Series: Southern Hemisphere Sea Ice Extent/Area." NOAA Physical Sciences Laboratory (PSL). Accessed April 28, 2024. https://psl.noaa.gov/data/timeseries/monthly/SHICE/.
[4] Scott, Michon. "What Is the Difference between Sea Ice Area and Extent?" National Snow and Ice Data Center, June 13, 2022. https://nsidc.org/learn/ask-scientist/what-difference-between-sea-ice-area-and-extent.
[5] Comiso, J. C., A. C. Bliss, R. Gersten, C. L. Parkinson, and T. Markus. "Current State of Sea Ice Cover." NASA Earth Sciences, 2024. https://earth.gsfc.nasa.gov/cryo/data/current-state-sea-ice-cover.

7. Appendix

Table A1: Augmented Dickey-Fuller Test on Original Data

Dickey-Fuller = –9.5004	Lag Order = 7	p-value < 0.01
Alternative Hypothesis: Stationary		

Table A2: Augmented Dickey-Fuller Test on Transformed Data

Dickey-Fuller = –9.2703	Lag Order = 7	p-value < 0.01
Alternative Hypothesis: Stationary		

Figure A1: Normal Q-Q Plot of SARIMA Model Residuals

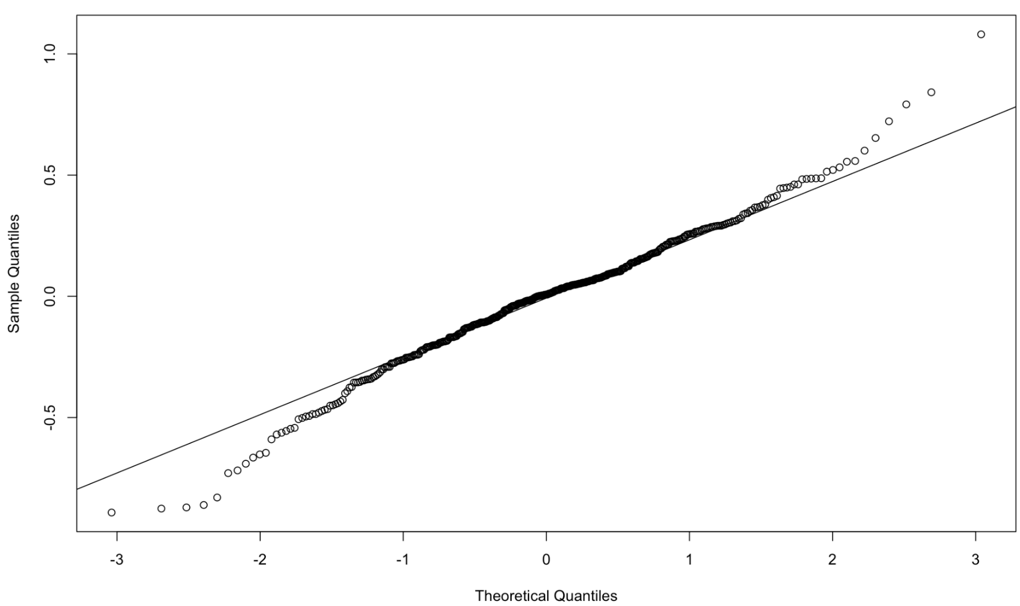


Table A3: Order of Minimum AICc AR Model for Residuals

ARIMA(0,0,0) with Zero Mean		
$\sigma^2 = 0.0777$		Log Likelihood = –59.43
AIC = 120.86	AICc = 120.87	BIC = 124.9

Table A4: SARIMA (1,1,2) x $(0,1,2)_{12}$ Model Parameters

	AR1	**MA1**	**MA2**	**SMA1**	**SMA2**
Estimate	0.4547	–0.5403	–0.2438	–0.8931	–0.1068
Std. Error	0.1147	0.1139	0.0598	0.0693	0.0508
$\sigma^2 = 0.08118$			Log Likelihood = –84.46		
AIC = 180.92		AICc = 181.13		BIC = 204.97	

Table A5: SARIMA (1,1,2) x $(0,1,2)_{12}$ Model Forecast

Month	**Point Forecast**	**95% Forecast Interval Lower Bound**	**95% Forecast Interval Upper Bound**
January 2024	2.185	1.618	2.752
February 2024	0.818	0.050	1.585
March 2024	1.455	0.608	2.302
April 2024	3.801	2.907	4.694
May 2024	6.585	5.657	7.513
June 2024	9.173	8.215	10.131
July 2024	11.341	10.356	12.326
August 2024	12.736	11.725	13.747
September 2024	13.206	12.171	14.242
October 2024	12.658	11.598	13.718
November 2024	10.182	9.099	11.266
December 2024	5.499	4.393	6.606

January 2025	1.921	0.775	3.068
February 2025	0.634	–0.550	1.817
March 2025	1.338	0.123	2.553
April 2025	3.730	2.487	4.972
May 2025	6.565	5.296	7.834
June 2025	9.220	7.925	10.515
July 2025	11.401	10.081	12.720
August 2025	12.775	11.431	14.119
September 2025	13.212	11.844	14.580
October 2025	12.658	11.266	14.049
November 2025	10.146	8.732	11.561
December 2025	5.424	3.987	6.861

Table A6: Holt-Winters Forecast

Month	Point Forecast	95% Forecast Interval Lower Bound	95% Forecast Interval Upper Bound
January 2024	2.536	1.888	3.184
February 2024	1.348	0.528	2.168
March 2024	1.869	0.908	2.830
April 2024	4.185	3.101	5.270
May 2024	7.072	5.877	8.267
June 2024	9.814	8.518	11.110
July 2024	12.167	10.777	13.557
August 2024	13.621	12.143	15.099
September 2024	13.853	12.292	15.414
October 2024	13.143	11.503	14.782
November 2024	10.550	8.836	12.265
December 2024	5.730	3.944	7.517

January 2025	2.461	0.576	4.346
February 2025	1.273	–0.678	3.224
March 2025	1.794	–0.220	3.808
April 2025	4.110	2.035	6.186
May 2025	6.997	4.861	9.133
June 2025	9.739	7.545	11.933
July 2025	12.093	9.842	14.343
August 2025	13.546	11.240	15.852
September 2025	13.778	11.418	16.138
October 2025	13.068	10.655	15.481
November 2025	10.475	8.011	12.940
December 2025	5.656	3.141	8.171

Final Project: R Markdown

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Import Data and Convert into Time Series Object

```
setwd("~/Desktop/Spring 2024/MA 585/Project")
Ice <- read.table("Sea Ice.txt")
Ice <- c(t(as.matrix(Ice)))
Ice <- ts(data = Ice, start = c(1989, 1), frequency = 12)
```

Plot Time Series (Figure 1)

```
plot(Ice, xlab = "Year",
     ylab = "Average Monthly Antarctic Sea Ice (Millions of Square Kilometers
)",
     main = "Figure 1: Average Monthly Antarctic Sea Ice from 1989 to 2023",
     col = "blue", cex.main = 2)
```

Classical Decomposition (Figure 2)

```
decomp.plot <- function(x, main = NULL, ...) {
if(is.null(main))
main <- paste("Decomposition of", x$type, "time series")
plot(cbind(observed = x$random + if (x$type == "additive")
x$trend + x$seasonal
else x$trend * x$seasonal, trend = x$trend, seasonal = x$seasonal, random = x
$random), main = main, ...)
}

z <- decompose(Ice, type = "additive")
decomp.plot(z, main = "Figure 2: Additive Decomposition of Sea Ice Data", cex
.main = 2)
```

ADF Test + Plot of Seasonally Differenced Data (Figure 3)

```
library(tseries)

adf.test(Ice) # Appears stationary (but we know it's NOT)

## Warning in adf.test(Ice): p-value smaller than printed p-value

##
##  Augmented Dickey-Fuller Test
```

```
##
## data:  Ice
## Dickey-Fuller = -9.5004, Lag order = 7, p-value = 0.01
## alternative hypothesis: stationary
```

```
Ice_diff <- diff(diff(Ice, lag = 12)) # seasonal difference + first differenc
e
adf.test(Ice_diff) # stationary
```

```
## Warning in adf.test(Ice_diff): p-value smaller than printed p-value
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  Ice_diff
## Dickey-Fuller = -9.2703, Lag order = 7, p-value = 0.01
## alternative hypothesis: stationary
```

```
plot(Ice_diff, xlab = "Year",
     ylab = "Differenced Average Monthly Antarctic Sea Ice (Millions of Squar
e Kilometers)",
     main = "Figure 3: Differenced Average Monthly Antarctic Sea Ice",
     col = "orange", cex.main = 2)
```

ACF and PACF Plots (Figure 4)

```
ACF <- acf(Ice_diff, main = "Figure 4a: Differenced Data Sample ACF", ylim =
c(-1,1), lag.max = 50)
```

```
PACF <- pacf(Ice_diff, main = "Figure 4b: Differenced Data Sample PACF", ylim
= c(-1,1), lag.max = 50)
```

Manual SARIMA Model Selection

```
fit_1 <- Arima(Ice, order = c(1,1,1), seasonal = list(order = c(0,1,1), perio
d = 12))
fit_1
```

```
## Series: Ice
## ARIMA(1,1,1)(0,1,1)[12]
##
## Coefficients:
##          ar1      ma1     sma1
##       0.7126  -0.9024  -0.9999
## s.e.  0.0795   0.0532   0.2016
##
## sigma^2 = 0.08396:  log likelihood = -93.61
## AIC=195.22   AICc=195.32   BIC=211.26
```

```r
fit_2 <- Arima(Ice, order = c(1,1,2), seasonal = list(order = c(0,1,1), perio
d = 12))
fit_2
```

```
## Series: Ice
## ARIMA(1,1,2)(0,1,1)[12]
##
## Coefficients:
##          ar1      ma1      ma2     sma1
##       0.4421  -0.5196  -0.2519  -0.9999
## s.e.  0.1089   0.1074   0.0576   0.2797
##
## sigma^2 = 0.08141:  log likelihood = -86.63
## AIC=183.26   AICc=183.41   BIC=203.31
```

```r
fit_3 <- Arima(Ice, order = c(3,1,0), seasonal = list(order = c(0,1,1), perio
d = 12))
fit_3
```

```
## Series: Ice
## ARIMA(3,1,0)(0,1,1)[12]
##
## Coefficients:
##           ar1      ar2      ar3     sma1
##       -0.0366  -0.2372  -0.1280  -1.0000
## s.e.   0.0492   0.0478   0.0493   0.1429
##
## sigma^2 = 0.08393:  log likelihood = -92.66
## AIC=195.32   AICc=195.47   BIC=215.37
```

```r
fit_4 <- Arima(Ice, order = c(0,1,3), seasonal = list(order = c(0,1,1), perio
d = 12))
fit_4
```

```
## Series: Ice
## ARIMA(0,1,3)(0,1,1)[12]
##
## Coefficients:
##           ma1      ma2      ma3     sma1
##       -0.0768  -0.3105  -0.1566  -1.0000
## s.e.   0.0492   0.0478   0.0471   0.1146
##
## sigma^2 = 0.08159:  log likelihood = -87.03
## AIC=184.06   AICc=184.21   BIC=204.1
```

```r
fit_5 <- Arima(Ice, order = c(1,1,1), seasonal = list(order = c(0,1,2), perio
d = 12))
fit_5
```

```
## Series: Ice
## ARIMA(1,1,1)(0,1,2)[12]
```

```
##
## Coefficients:
##          ar1      ma1     sma1     sma2
##       0.7419  -0.9300  -0.8670  -0.1329
## s.e.  0.0933   0.0634   0.0688   0.0518
##
## sigma^2 = 0.08332:  log likelihood = -90.4
## AIC=190.79   AICc=190.94   BIC=210.84
```

```
fit_6 <- Arima(Ice, order = c(1,1,2), seasonal = list(order = c(0,1,2), perio
d = 12))
fit_6
```

```
## Series: Ice
## ARIMA(1,1,2)(0,1,2)[12]
##
## Coefficients:
##          ar1      ma1      ma2     sma1     sma2
##       0.4547  -0.5403  -0.2438  -0.8931  -0.1068
## s.e.  0.1147   0.1139   0.0598   0.0693   0.0508
##
## sigma^2 = 0.08118:  log likelihood = -84.46
## AIC=180.92   AICc=181.13   BIC=204.97
```

```
fit_7 <- Arima(Ice, order = c(3,1,0), seasonal = list(order = c(0,1,2), perio
d = 12))
fit_7
```

```
## Series: Ice
## ARIMA(3,1,0)(0,1,2)[12]
##
## Coefficients:
##           ar1      ar2      ar3     sma1     sma2
##       -0.0496  -0.2400  -0.1313  -0.8806  -0.1194
## s.e.   0.0497   0.0478   0.0494   0.0667   0.0507
##
## sigma^2 = 0.08353:  log likelihood = -89.96
## AIC=191.92   AICc=192.13   BIC=215.97
```

```
fit_8 <- Arima(Ice, order = c(0,1,3), seasonal = list(order = c(0,1,2), perio
d = 12))
fit_8
```

```
## Series: Ice
## ARIMA(0,1,3)(0,1,2)[12]
##
## Coefficients:
##           ma1      ma2      ma3     sma1     sma2
##       -0.0852  -0.3061  -0.1543  -0.8947  -0.1053
## s.e.   0.0495   0.0486   0.0483   0.0676   0.0508
##
```

```
## sigma^2 = 0.0814:  log likelihood = -84.94
## AIC=181.87   AICc=182.08   BIC=205.92
```

```
fit_9 <- Arima(Ice, order = c(1,1,1), seasonal = list(order = c(1,1,1), perio
d = 12))
fit_9
```

```
## Series: Ice
## ARIMA(1,1,1)(1,1,1)[12]
##
## Coefficients:
##          ar1      ma1    sar1     sma1
##       0.7479  -0.9333  0.1309  -1.0000
## s.e.  0.0982   0.0672  0.0534   0.0437
##
## sigma^2 = 0.08342:  log likelihood = -90.52
## AIC=191.04   AICc=191.19   BIC=211.09
```

```
fit_10 <- Arima(Ice, order = c(1,1,2), seasonal = list(order = c(1,1,1), peri
od = 12))
fit_10
```

```
## Series: Ice
## ARIMA(1,1,2)(1,1,1)[12]
##
## Coefficients:
##          ar1      ma1      ma2    sar1     sma1
##       0.4548  -0.5384  -0.2450  0.1056  -1.0000
## s.e.  0.1147   0.1137   0.0596  0.0514   0.0456
##
## sigma^2 = 0.08124:  log likelihood = -84.51
## AIC=181.02   AICc=181.23   BIC=205.08
```

```
fit_11 <- Arima(Ice, order = c(3,1,0), seasonal = list(order = c(1,1,1), peri
od = 12))
fit_11
```

```
## Series: Ice
## ARIMA(3,1,0)(1,1,1)[12]
##
## Coefficients:
##           ar1      ar2      ar3    sar1     sma1
##       -0.0473  -0.2400  -0.1293  0.1178  -1.0000
## s.e.   0.0495   0.0478   0.0493  0.0514   0.0419
##
## sigma^2 = 0.08362:  log likelihood = -90.03
## AIC=192.06   AICc=192.27   BIC=216.11
```

```
fit_12 <- Arima(Ice, order = c(0,1,3), seasonal = list(order = c(1,1,1), peri
od = 12))
fit_12
```

```
## Series: Ice
## ARIMA(0,1,3)(1,1,1)[12]
##
## Coefficients:
##           ma1      ma2      ma3     sar1     sma1
##       -0.0832  -0.3066  -0.1542   0.1030  -1.0000
## s.e.   0.0494   0.0486   0.0483   0.0513   0.0436
##
## sigma^2 = 0.08146:  log likelihood = -85.01
## AIC=182.02   AICc=182.23   BIC=206.07
```

SARIMA Model Diagnostics (Figure 5 and Figure A1)

```
fit <- Arima(Ice, order = c(1,1,2), seasonal = list(order = c(0,1,2), period
= 12))

tsdiag(fit)
qqnorm(residuals(fit), main = "Figure A1: Normal Q-Q Plot of SARIMA Model Res
iduals", cex.main = 2)
qqline(residuals(fit))

auto.arima(residuals(fit), max.q = 0) # Order of Minimum AICC AR Model

## Series: residuals(fit)
## ARIMA(0,0,0) with zero mean
##
## sigma^2 = 0.0777:  log likelihood = -59.43
## AIC=120.86   AICc=120.87   BIC=124.9
```

SARIMA Model Forecast (Figure 6)

```
ARIMAforecast <- forecast(fit, h = 24)
ARIMAforecast

##          Point Forecast      Lo 80     Hi 80       Lo 95     Hi 95
## Jan 2024      2.1850082  1.8145861  2.555430  1.61849648  2.751520
## Feb 2024      0.8176798  0.3158226  1.319537  0.05015538  1.585204
## Mar 2024      1.4551682  0.9014767  2.008860  0.60837013  2.301966
## Apr 2024      3.8007927  3.2165989  4.384986  2.90734534  4.694240
## May 2024      6.5847614  5.9778078  7.191715  5.65650603  7.513017
## Jun 2024      9.1733101  8.5469250  9.799695  8.21533683 10.131283
## Jul 2024     11.3411479 10.6969750 11.985321 10.35597051 12.326325
## Aug 2024     12.7361920 12.0751678 13.397216 11.72524271 13.747141
## Sep 2024     13.2064622 12.5292051 13.883719 12.17068679 14.242238
## Oct 2024     12.6581979 11.9651721 13.351224 11.59830633 13.718090
## Nov 2024     10.1824482  9.4740348 10.890862  9.09902342 11.265873
## Dec 2024      5.4992322  4.7757643  6.222700  4.39278355  6.605681
## Jan 2025      1.9214123  1.1716380  2.671187  0.77473142  3.068093
## Feb 2025      0.6336378 -0.1403631  1.407639 -0.55009446  1.817370
## Mar 2025      1.3379586  0.5437478  2.132169  0.12331803  2.552599
```

```
## Apr 2025      3.7295454  2.9170056  4.542085  2.48687305  4.972218
## May 2025      6.5647628  5.7348961  7.394629  5.29559129  7.833934
## Jun 2025      9.2199698  8.3733937 10.066546  7.92524337 10.514696
## Jul 2025     11.4006831 10.5378369 12.263529 10.08107372 12.720293
## Aug 2025     12.7752296 11.8964646 13.653995 11.43127457 14.119185
## Sep 2025     13.2118722 12.3174911 14.106253 11.84403439 14.579710
## Oct 2025     12.6577101 11.7479834 13.567437 11.26640322 14.049017
## Nov 2025     10.1462139  9.2213854 11.071042  8.73181070 11.560617
## Dec 2025      5.4240693  4.4843581  6.363781  3.98690506  6.861234
```

```
plot(ARIMAforecast, main = "Figure 6: Seasonal ARIMA (1,1,2) x (0,1,2)[12] Fo
recast", xlab = "Year", ylab = "Average Monthly Antarctic Sea Ice (Millions o
f Square Kilometers)", cex.main = 2)
```

Holt-Winters Forecast (Figure 7)

```
HWfit <- HoltWinters(Ice, seasonal = "additive")
HWforecast <- forecast(HWfit, h = 24)
HWforecast
```

```
##          Point Forecast        Lo 80     Hi 80      Lo 95     Hi 95
## Jan 2024       2.535739  2.112034795  2.959444  1.8877391  3.183740
## Feb 2024       1.347836  0.811863082  1.883809  0.5281362  2.167536
## Mar 2024       1.868988  1.240491431  2.497485  0.9077854  2.830191
## Apr 2024       4.185326  3.476278169  4.894374  3.1009308  5.269722
## May 2024       7.072052  6.290713122  7.853391  5.8770973  8.267007
## Jun 2024       9.813909  8.966423361 10.661394  8.5177917 11.110026
## Jul 2024      12.167384 11.258553291 13.076214 10.7774475 13.557320
## Aug 2024      13.620719 12.654429803 14.587007 12.1429074 15.098530
## Sep 2024      13.853213 12.832695525 14.873730 12.2924664 15.413959
## Oct 2024      13.142769 12.070763233 14.214775 11.5032777 14.782260
## Nov 2024      10.550340  9.429207984 11.671473  8.8357164 12.264964
## Dec 2024       5.730450  4.562255496  6.898645  3.9438506  7.517050
## Jan 2025       2.460862  1.228272975  3.693451  0.5757798  4.345944
## Feb 2025       1.272958 -0.002587317  2.548504 -0.6778204  3.223737
## Mar 2025       1.794110  0.477008441  3.111212 -0.2202233  3.808444
## Apr 2025       4.110449  2.753061866  5.467835  2.0345047  6.186393
## May 2025       6.997174  5.600664391  8.393684  4.8613967  9.132952
## Jun 2025       9.739031  8.304464615 11.173598  7.5450509 11.933011
## Jul 2025      12.092506 10.620866741 13.564146  9.8418279 14.343184
## Aug 2025      13.545841 12.038040013 15.053642 11.2398584 15.851824
## Sep 2025      13.778335 12.235219707 15.321450 11.4183438 16.138326
## Oct 2025      13.067891 11.490251988 14.645531 10.6551001 15.480683
## Nov 2025      10.475463  8.864038650 12.086887  8.0110023 12.939923
## Dec 2025       5.655573  4.011057879  7.300087  3.1405044  8.170641
```

```
plot(HWforecast, main = "Figure 7: Holt-Winters Forecast", xlab = "Year", yla
b = "Average Monthly Antarctic Sea Ice (Millions of Square Kilometers)", cex.
main = 2)
```

Model Performance

```r
train <- window(Ice, end = c(2019, 12))
test <- window(Ice, start = c(2020, 1)) # last 48 observations

# Holt-Winters Forecast
HWfit <- HoltWinters(train, seasonal = "additive")
HWforecast <- forecast(HWfit, h = 48)
HWforecast
```

```
##          Point Forecast         Lo 80     Hi 80       Lo 95     Hi 95
## Jan 2020       3.133112  2.718804114  3.547420  2.49948265  3.766742
## Feb 2020       1.775124  1.259169359  2.291078  0.98603967  2.564208
## Mar 2020       2.288781  1.688142452  2.889419  1.37018373  3.207378
## Apr 2020       4.730717  4.055939771  5.405494  3.69873441  5.762699
## May 2020       7.686917  6.945376892  8.428457  6.55282933  8.821004
## Jun 2020      10.582066  9.779296522 11.384836  9.35433592 11.809797
## Jul 2020      12.767260 11.907610905 13.626909 11.45254010 14.081980
## Aug 2020      13.994703 13.081711323 14.907695 12.59840256 15.391004
## Sep 2020      14.253816 13.290429922 15.217201 12.78044435 15.727187
## Oct 2020      13.724182 12.712910982 14.735453 12.17757632 15.270788
## Nov 2020      11.062213 10.005223303 12.119203  9.44568678 12.678739
## Dec 2020       6.523430  5.422619510  7.624241  4.83988540  8.206976
## Jan 2021       3.058235  1.896246510  4.220223  1.28112717  4.835342
## Feb 2021       1.700246  0.498258516  2.902233 -0.13803522  3.538527
## Mar 2021       2.213903  0.973205162  3.454601  0.31641932  4.111387
## Apr 2021       4.655839  3.377602425  5.934076  2.70094478  6.610733
## May 2021       7.612039  6.297335244  8.926743  5.60137297  9.622706
## Jun 2021      10.507189  9.157001831 11.857375  8.44225614 12.572121
## Jul 2021      12.692382 11.307621909 14.077143 10.57457394 14.810191
## Aug 2021      13.919826 12.501333713 15.338318 11.75042942 16.089222
## Sep 2021      14.178938 12.727498321 15.630378 11.95915258 16.398723
## Oct 2021      13.649305 12.165648771 15.132960 11.38024881 15.918360
## Nov 2021      10.987335  9.472148140 12.502523  8.67005649 13.304614
## Dec 2021       6.448553  4.902477205  7.994628  4.08403421  8.813071
## Jan 2022       2.983357  1.393142705  4.573571  0.55133412  5.415380
## Feb 2022       1.625368  0.005695873  3.245041 -0.85170696  4.102444
## Mar 2022       2.139026  0.490421086  3.787630 -0.38229739  4.660348
## Apr 2022       4.580962  2.903924187  6.257999  2.01615425  7.145769
## May 2022       7.537162  5.832165524  9.242158  4.92959509 10.144728
## Jun 2022      10.432311  8.699807174 12.164815  7.78267507 13.081947
## Jul 2022      12.617505 10.857923391 14.377086  9.92645722 15.308553
## Aug 2022      13.844948 12.058699316 15.631197 11.11311635 16.576780
## Sep 2022      14.104060 12.291536648 15.916584 11.33204456 16.876076
## Oct 2022      13.574427 11.736003817 15.412850 10.76280138 16.386053
## Nov 2022      10.912458  9.048494907 12.776421  8.06177260 13.763143
## Dec 2022       6.373675  4.484518002  8.262832  3.48445859  9.262892
## Jan 2023       2.908479  0.983032253  4.833926 -0.03623787  5.853197
## Feb 2023       1.550491 -0.399356452  3.500338 -1.43154321  4.532525
## Mar 2023       2.064148  0.090202183  4.038094 -0.95474156  5.083037
```

```
## Apr 2023       4.506084  2.508330327  6.503837  1.45078347  7.561384
## May 2023       7.462284  5.441003024  9.483565  4.37100149 10.553567
## Jun 2023      10.357433  8.312895621 12.401971  7.23058273 13.484284
## Jul 2023      12.542627 10.475094397 14.610160  9.38060862 15.704646
## Aug 2023      13.770070 11.679795411 15.860345 10.57327065 16.966870
## Sep 2023      14.029183 11.916410312 16.141955 10.79797615 17.260389
## Oct 2023      13.499549 11.364516611 15.634582 10.23429854 16.764800
## Nov 2023      10.837580  8.680516705 12.994643  7.53863632 14.136524
## Dec 2023       6.298798  4.119926305  8.477669  2.96650152  9.631094
```

```
HWerr <- test - HWforecast$mean
HWrmse <- sqrt(mean(HWerr^2))
HWmae <- mean(abs(HWerr))
HWmape <- mean(abs((HWerr*100)/test))

HWrmse # Holt-Winters RMSE
```

```
## [1] 0.7132303
```

```
HWmae # Holt-Winters MAE
```

```
## [1] 0.5478606
```

```
HWmape # Holt-Winters MAPE
```

```
## [1] 9.405666
```

```
# SARIMA(1,1,2)x(0,1,2)[12] Forecast
arimafit <- Arima(train, order = c(1,1,2), seasonal = list(order = c(0,1,2),
period = 12))
arimafcast <- forecast(arimafit, h = 48)
arimafcast
```

```
##          Point Forecast      Lo 80     Hi 80       Lo 95     Hi 95
## Jan 2020       3.025450  2.6585098  3.392391  2.46426320  3.586637
## Feb 2020       1.715924  1.2299230  2.201926  0.97264932  2.459200
## Mar 2020       2.360701  1.8301858  2.891217  1.54934779  3.172055
## Apr 2020       4.729566  4.1725171  5.286616  3.87763310  5.581500
## May 2020       7.547356  6.9697964  8.124915  6.66405496  8.430657
## Jun 2020      10.239618  9.6440029 10.835234  9.32870300 11.150534
## Jul 2020      12.459174 11.8466994 13.071649 11.52247487 13.395873
## Aug 2020      13.831110 13.2024860 14.459734 12.86971243 14.792508
## Sep 2020      14.290088 13.6458177 14.934357 13.30476185 15.275413
## Oct 2020      13.733836 13.0743258 14.393346 12.72520242 14.742469
## Nov 2020      11.216815 10.5424138 11.891215 10.18540764 12.248222
## Dec 2020       6.551194  5.8622146  7.240173  5.49749099  7.604897
## Jan 2021       2.959817  2.2455474  3.674086  1.86743601  4.052198
## Feb 2021       1.660487  0.9234809  2.397494  0.53333321  2.787642
## Mar 2021       2.366973  1.6110535  3.122891  1.21089422  3.523051
## Apr 2021       4.770759  3.9975653  5.543952  3.58826155  5.953255
## May 2021       7.610913  6.8212779  8.400548  6.40327040  8.818555
## Jun 2021      10.285224  9.4796616 11.090787  9.05322240 11.517226
```

```
## Jul 2021      12.467095 11.6459833 13.288207 11.21131299 13.722877
## Aug 2021      13.810145 12.9737999 14.646491 12.53106543 15.089225
## Sep 2021      14.250438 13.3991391 15.101736 12.94848885 15.552386
## Oct 2021      13.713014 12.8470163 14.579011 12.38858507 15.037442
## Nov 2021      11.209059 10.3285925 12.089526  9.86250163 12.555617
## Dec 2021       6.531375  5.6366408  7.426110  5.16299693  7.899754
## Jan 2022       2.944009  2.0328733  3.855144  1.55054740  4.337470
## Feb 2022       1.650218  0.7234528  2.576982  0.23285312  3.067582
## Mar 2022       2.358942  1.4174530  3.300430  0.91905902  3.798824
## Apr 2022       4.763633  3.8079006  5.719365  3.30196663  6.225299
## May 2022       7.604153  6.6344868  8.573819  6.12117664  9.087129
## Jun 2022      10.278612  9.2952495 11.261975  8.77468859 11.782536
## Jul 2022      12.460543 11.4636866 13.457399 10.93598268 13.985103
## Aug 2022      13.803617 12.7934522 14.813782 12.25870319 15.348531
## Sep 2022      14.243919 13.2206174 15.267221 12.67891411 15.808925
## Oct 2022      13.706499 12.6702183 14.742780 12.12164429 15.291354
## Nov 2022      11.202546 10.1534254 12.251667  9.59805434 12.807039
## Dec 2022       6.524863  5.4630125  7.586714  4.90090267  8.148824
## Jan 2023       2.937497  1.8610040  4.013990  1.29114309  4.583850
## Feb 2023       1.643706  0.5532442  2.734167 -0.02401125  3.311423
## Mar 2023       2.352430  1.2487442  3.456116  0.66448826  4.040371
## Apr 2023       4.757121  3.6405914  5.873651  3.04953638  6.464706
## May 2023       7.597641  6.4685026  8.726780  5.87077270  9.324510
## Jun 2023      10.272101  9.1305275 11.413674  8.52621512 12.017986
## Jul 2023      12.454031 11.3001708 13.607892 10.68935403 14.218708
## Aug 2023      13.797105 12.6310911 14.963120 12.01384040 15.580370
## Sep 2023      14.237408 13.0593625 15.415453 12.43574309 16.039072
## Oct 2023      13.699988 12.5100233 14.889952 11.88009428 15.519881
## Nov 2023      11.196035  9.9942446 12.397825  9.35805539 13.034014
## Dec 2023       6.518352  5.3047964  7.731907  4.66237919  8.374324
```

```r
arimaerr <- test - arimafcast$mean
arimarmse <- sqrt(mean(arimaerr^2))
arimamae <- mean(abs(arimaerr))
arimamape <- mean(abs((arimaerr*100)/test))

arimarmse # ARIMA RMSE

## [1] 0.7635105

arimamae # ARIMA MAE

## [1] 0.6142785

arimamape # ARIMA MAPE

## [1] 10.80418

Measure <- c("SARIMA(1,1,2)x(0,1,2)[12]", "Holt-Winters")
RMSE <- round(c(arimarmse, HWrmse), 4)
MAE <- round(c(arimamae, HWmae), 4)
```

```r
MAPE <- round(c(arimamape, HWmape), 4)
rbind(Measure, RMSE, MAE, MAPE)
```

```
##         [,1]                          [,2]
## Measure "SARIMA(1,1,2)x(0,1,2)[12]" "Holt-Winters"
## RMSE    "0.7635"                      "0.7132"
## MAE     "0.6143"                      "0.5479"
## MAPE    "10.8042"                     "9.4057"
```