

Antarctic Sea Ice Time Series Project

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Import Data and Convert into Time Series Object

```
setwd("~/Desktop/Spring 2024/MA 585/Project")
Ice <- read.table("Sea Ice.txt")
Ice <- c(t(as.matrix(Ice)))
Ice <- ts(data = Ice, start = c(1989, 1), frequency = 12)
```

Plot Time Series (Figure 1)

```
plot(Ice, xlab = "Year",
      ylab = "Average Monthly Antarctic Sea Ice (Millions of Square Kilometers)",
      main = "Figure 1: Average Monthly Antarctic Sea Ice from 1989 to 2023",
      col = "blue", cex.main = 2)
```

Classical Decomposition (Figure 2)

```
decomp.plot <- function(x, main = NULL, ...) {
  if(is.null(main))
    main <- paste("Decomposition of", x$type, "time series")
  plot(cbind(observed = x$random + if (x$type == "additive")
    x$trend + x$seasonal
  else x$trend * x$seasonal, trend = x$trend, seasonal = x$seasonal, random = x
    $random), main = main, ...)
}

z <- decompose(Ice, type = "additive")
decomp.plot(z, main = "Figure 2: Additive Decomposition of Sea Ice Data", cex
  .main = 2)
```

ADF Test + Plot of Seasonally Differenced Data (Figure 3)

```
library(tseries)

adf.test(Ice) # Appears stationary (but we know it's NOT)

## Warning in adf.test(Ice): p-value smaller than printed p-value

##
## Augmented Dickey-Fuller Test
##
## data: Ice
```

```
## Dickey-Fuller = -9.5004, Lag order = 7, p-value = 0.01
## alternative hypothesis: stationary

Ice_diff <- diff(diff(Ice, lag = 12)) # seasonal difference + first difference
adf.test(Ice_diff) # stationary

## Warning in adf.test(Ice_diff): p-value smaller than printed p-value

##
## Augmented Dickey-Fuller Test
##
## data: Ice_diff
## Dickey-Fuller = -9.2703, Lag order = 7, p-value = 0.01
## alternative hypothesis: stationary

plot(Ice_diff, xlab = "Year",
      ylab = "Differenced Average Monthly Antarctic Sea Ice (Millions of Square Kilometers)",
      main = "Figure 3: Differenced Average Monthly Antarctic Sea Ice",
      col = "orange", cex.main = 2)
```

ACF and PACF Plots (Figure 4)

```
ACF <- acf(Ice_diff, main = "Figure 4a: Differenced Data Sample ACF", ylim =
c(-1,1), lag.max = 50)

PACF <- pacf(Ice_diff, main = "Figure 4b: Differenced Data Sample PACF", ylim =
c(-1,1), lag.max = 50)
```

Manual SARIMA Model Selection

```
fit_1 <- Arima(Ice, order = c(1,1,1), seasonal = list(order = c(0,1,1), period = 12))
fit_1

## Series: Ice
## ARIMA(1,1,1)(0,1,1)[12]
##
## Coefficients:
##          ar1          ma1          sma1
##          0.7126   -0.9024   -0.9999
## s.e.    0.0795    0.0532    0.2016
##
## sigma^2 = 0.08396: log likelihood = -93.61
## AIC=195.22   AICc=195.32   BIC=211.26

fit_2 <- Arima(Ice, order = c(1,1,2), seasonal = list(order = c(0,1,1), period = 12))
fit_2
```

```

## Series: Ice
## ARIMA(1,1,2)(0,1,1)[12]
##
## Coefficients:
##          ar1          ma1          ma2          sma1
##      0.4421  -0.5196  -0.2519  -0.9999
## s.e.  0.1089   0.1074   0.0576   0.2797
##
## sigma^2 = 0.08141: log likelihood = -86.63
## AIC=183.26   AICc=183.41   BIC=203.31

fit_3 <- Arima(Ice, order = c(3,1,0), seasonal = list(order = c(0,1,1), perio
d = 12))
fit_3

## Series: Ice
## ARIMA(3,1,0)(0,1,1)[12]
##
## Coefficients:
##          ar1          ar2          ar3          sma1
##      -0.0366  -0.2372  -0.1280  -1.0000
## s.e.   0.0492   0.0478   0.0493   0.1429
##
## sigma^2 = 0.08393: log likelihood = -92.66
## AIC=195.32   AICc=195.47   BIC=215.37

fit_4 <- Arima(Ice, order = c(0,1,3), seasonal = list(order = c(0,1,1), perio
d = 12))
fit_4

## Series: Ice
## ARIMA(0,1,3)(0,1,1)[12]
##
## Coefficients:
##          ma1          ma2          ma3          sma1
##      -0.0768  -0.3105  -0.1566  -1.0000
## s.e.   0.0492   0.0478   0.0471   0.1146
##
## sigma^2 = 0.08159: log likelihood = -87.03
## AIC=184.06   AICc=184.21   BIC=204.1

fit_5 <- Arima(Ice, order = c(1,1,1), seasonal = list(order = c(0,1,2), perio
d = 12))
fit_5

## Series: Ice
## ARIMA(1,1,1)(0,1,2)[12]
##
## Coefficients:
##          ar1          ma1          sma1          sma2
##      0.7419  -0.9300  -0.8670  -0.1329

```

```

## s.e.  0.0933   0.0634   0.0688   0.0518
##
## sigma^2 = 0.08332:  log likelihood = -90.4
## AIC=190.79   AICc=190.94   BIC=210.84

fit_6 <- Arima(Ice, order = c(1,1,2), seasonal = list(order = c(0,1,2), perio
d = 12))
fit_6

## Series: Ice
## ARIMA(1,1,2)(0,1,2)[12]
##
## Coefficients:
##          ar1          ma1          ma2          sma1          sma2
##          0.4547  -0.5403  -0.2438  -0.8931  -0.1068
## s.e.    0.1147   0.1139   0.0598   0.0693   0.0508
##
## sigma^2 = 0.08118:  log likelihood = -84.46
## AIC=180.92   AICc=181.13   BIC=204.97

fit_7 <- Arima(Ice, order = c(3,1,0), seasonal = list(order = c(0,1,2), perio
d = 12))
fit_7

## Series: Ice
## ARIMA(3,1,0)(0,1,2)[12]
##
## Coefficients:
##          ar1          ar2          ar3          sma1          sma2
##          -0.0496  -0.2400  -0.1313  -0.8806  -0.1194
## s.e.    0.0497   0.0478   0.0494   0.0667   0.0507
##
## sigma^2 = 0.08353:  log likelihood = -89.96
## AIC=191.92   AICc=192.13   BIC=215.97

fit_8 <- Arima(Ice, order = c(0,1,3), seasonal = list(order = c(0,1,2), perio
d = 12))
fit_8

## Series: Ice
## ARIMA(0,1,3)(0,1,2)[12]
##
## Coefficients:
##          ma1          ma2          ma3          sma1          sma2
##          -0.0852  -0.3061  -0.1543  -0.8947  -0.1053
## s.e.    0.0495   0.0486   0.0483   0.0676   0.0508
##
## sigma^2 = 0.0814:  log likelihood = -84.94
## AIC=181.87   AICc=182.08   BIC=205.92

```

```

fit_9 <- Arima(Ice, order = c(1,1,1), seasonal = list(order = c(1,1,1), period = 12))
fit_9

## Series: Ice
## ARIMA(1,1,1)(1,1,1)[12]
##
## Coefficients:
##          ar1          ma1          sar1          sma1
##          0.7479  -0.9333   0.1309   -1.0000
## s.e.    0.0982   0.0672   0.0534   0.0437
##
## sigma^2 = 0.08342: log likelihood = -90.52
## AIC=191.04   AICc=191.19   BIC=211.09

fit_10 <- Arima(Ice, order = c(1,1,2), seasonal = list(order = c(1,1,1), period = 12))
fit_10

## Series: Ice
## ARIMA(1,1,2)(1,1,1)[12]
##
## Coefficients:
##          ar1          ma1          ma2          sar1          sma1
##          0.4548  -0.5384  -0.2450   0.1056  -1.0000
## s.e.    0.1147   0.1137   0.0596   0.0514   0.0456
##
## sigma^2 = 0.08124: log likelihood = -84.51
## AIC=181.02   AICc=181.23   BIC=205.08

fit_11 <- Arima(Ice, order = c(3,1,0), seasonal = list(order = c(1,1,1), period = 12))
fit_11

## Series: Ice
## ARIMA(3,1,0)(1,1,1)[12]
##
## Coefficients:
##          ar1          ar2          ar3          sar1          sma1
##          -0.0473  -0.2400  -0.1293   0.1178  -1.0000
## s.e.    0.0495   0.0478   0.0493   0.0514   0.0419
##
## sigma^2 = 0.08362: log likelihood = -90.03
## AIC=192.06   AICc=192.27   BIC=216.11

fit_12 <- Arima(Ice, order = c(0,1,3), seasonal = list(order = c(1,1,1), period = 12))
fit_12

## Series: Ice
## ARIMA(0,1,3)(1,1,1)[12]

```

```
##
## Coefficients:
##          ma1          ma2          ma3          sar1          sma1
##      -0.0832  -0.3066  -0.1542   0.1030  -1.0000
## s.e.   0.0494   0.0486   0.0483   0.0513   0.0436
##
## sigma^2 = 0.08146: log likelihood = -85.01
## AIC=182.02   AICc=182.23   BIC=206.07
```

SARIMA Model Diagnostics (Figure 5 and Figure A1)

```
fit <- Arima(Ice, order = c(1,1,2), seasonal = list(order = c(0,1,2), period
= 12))
```

```
tsdiag(fit)
qqnorm(residuals(fit), main = "Figure A1: Normal Q-Q Plot of SARIMA Model Res
iduals", cex.main = 2)
qqline(residuals(fit))
```

```
auto.arima(residuals(fit), max.q = 0) # Order of Minimum AICC AR Model
```

```
## Series: residuals(fit)
## ARIMA(0,0,0) with zero mean
##
## sigma^2 = 0.0777: log likelihood = -59.43
## AIC=120.86   AICc=120.87   BIC=124.9
```

SARIMA Model Forecast (Figure 6)

```
ARIMAforecast <- forecast(fit, h = 24)
ARIMAforecast
```

| ## | Point Forecast | Lo 80 | Hi 80 | Lo 95 | Hi 95 |
|-------------|----------------|------------|-----------|-------------|-----------|
| ## Jan 2024 | 2.1850082 | 1.8145861 | 2.555430 | 1.61849648 | 2.751520 |
| ## Feb 2024 | 0.8176798 | 0.3158226 | 1.319537 | 0.05015538 | 1.585204 |
| ## Mar 2024 | 1.4551682 | 0.9014767 | 2.008860 | 0.60837013 | 2.301966 |
| ## Apr 2024 | 3.8007927 | 3.2165989 | 4.384986 | 2.90734534 | 4.694240 |
| ## May 2024 | 6.5847614 | 5.9778078 | 7.191715 | 5.65650603 | 7.513017 |
| ## Jun 2024 | 9.1733101 | 8.5469250 | 9.799695 | 8.21533683 | 10.131283 |
| ## Jul 2024 | 11.3411479 | 10.6969750 | 11.985321 | 10.35597051 | 12.326325 |
| ## Aug 2024 | 12.7361920 | 12.0751678 | 13.397216 | 11.72524271 | 13.747141 |
| ## Sep 2024 | 13.2064622 | 12.5292051 | 13.883719 | 12.17068679 | 14.242238 |
| ## Oct 2024 | 12.6581979 | 11.9651721 | 13.351224 | 11.59830633 | 13.718090 |
| ## Nov 2024 | 10.1824482 | 9.4740348 | 10.890862 | 9.09902342 | 11.265873 |
| ## Dec 2024 | 5.4992322 | 4.7757643 | 6.222700 | 4.39278355 | 6.605681 |
| ## Jan 2025 | 1.9214123 | 1.1716380 | 2.671187 | 0.77473142 | 3.068093 |
| ## Feb 2025 | 0.6336378 | -0.1403631 | 1.407639 | -0.55009446 | 1.817370 |
| ## Mar 2025 | 1.3379586 | 0.5437478 | 2.132169 | 0.12331803 | 2.552599 |
| ## Apr 2025 | 3.7295454 | 2.9170056 | 4.542085 | 2.48687305 | 4.972218 |
| ## May 2025 | 6.5647628 | 5.7348961 | 7.394629 | 5.29559129 | 7.833934 |

```
## Jun 2025      9.2199698  8.3733937 10.066546  7.92524337 10.514696
## Jul 2025     11.4006831 10.5378369 12.263529 10.08107372 12.720293
## Aug 2025     12.7752296 11.8964646 13.653995 11.43127457 14.119185
## Sep 2025     13.2118722 12.3174911 14.106253 11.84403439 14.579710
## Oct 2025     12.6577101 11.7479834 13.567437 11.26640322 14.049017
## Nov 2025     10.1462139  9.2213854 11.071042  8.73181070 11.560617
## Dec 2025      5.4240693  4.4843581  6.363781  3.98690506  6.861234
```

```
plot(ARIMAforecast, main = "Figure 6: Seasonal ARIMA (1,1,2) x (0,1,2)[12] Fo
recast", xlab = "Year", ylab = "Average Monthly Antarctic Sea Ice (Millions o
f Square Kilometers)", cex.main = 2)
```

Holt-Winters Forecast (Figure 7)

```
HWfit <- HoltWinters(Ice, seasonal = "additive")
HWforecast <- forecast(HWfit, h = 24)
HWforecast
```

| ## | Point Forecast | Lo 80 | Hi 80 | Lo 95 | Hi 95 |
|-------------|----------------|--------------|-----------|------------|-----------|
| ## Jan 2024 | 2.535739 | 2.112034795 | 2.959444 | 1.8877391 | 3.183740 |
| ## Feb 2024 | 1.347836 | 0.811863082 | 1.883809 | 0.5281362 | 2.167536 |
| ## Mar 2024 | 1.868988 | 1.240491431 | 2.497485 | 0.9077854 | 2.830191 |
| ## Apr 2024 | 4.185326 | 3.476278169 | 4.894374 | 3.1009308 | 5.269722 |
| ## May 2024 | 7.072052 | 6.290713122 | 7.853391 | 5.8770973 | 8.267007 |
| ## Jun 2024 | 9.813909 | 8.966423361 | 10.661394 | 8.5177917 | 11.110026 |
| ## Jul 2024 | 12.167384 | 11.258553291 | 13.076214 | 10.7774475 | 13.557320 |
| ## Aug 2024 | 13.620719 | 12.654429803 | 14.587007 | 12.1429074 | 15.098530 |
| ## Sep 2024 | 13.853213 | 12.832695525 | 14.873730 | 12.2924664 | 15.413959 |
| ## Oct 2024 | 13.142769 | 12.070763233 | 14.214775 | 11.5032777 | 14.782260 |
| ## Nov 2024 | 10.550340 | 9.429207984 | 11.671473 | 8.8357164 | 12.264964 |
| ## Dec 2024 | 5.730450 | 4.562255496 | 6.898645 | 3.9438506 | 7.517050 |
| ## Jan 2025 | 2.460862 | 1.228272975 | 3.693451 | 0.5757798 | 4.345944 |
| ## Feb 2025 | 1.272958 | -0.002587317 | 2.548504 | -0.6778204 | 3.223737 |
| ## Mar 2025 | 1.794110 | 0.477008441 | 3.111212 | -0.2202233 | 3.808444 |
| ## Apr 2025 | 4.110449 | 2.753061866 | 5.467835 | 2.0345047 | 6.186393 |
| ## May 2025 | 6.997174 | 5.600664391 | 8.393684 | 4.8613967 | 9.132952 |
| ## Jun 2025 | 9.739031 | 8.304464615 | 11.173598 | 7.5450509 | 11.933011 |
| ## Jul 2025 | 12.092506 | 10.620866741 | 13.564146 | 9.8418279 | 14.343184 |
| ## Aug 2025 | 13.545841 | 12.038040013 | 15.053642 | 11.2398584 | 15.851824 |
| ## Sep 2025 | 13.778335 | 12.235219707 | 15.321450 | 11.4183438 | 16.138326 |
| ## Oct 2025 | 13.067891 | 11.490251988 | 14.645531 | 10.6551001 | 15.480683 |
| ## Nov 2025 | 10.475463 | 8.864038650 | 12.086887 | 8.0110023 | 12.939923 |
| ## Dec 2025 | 5.655573 | 4.011057879 | 7.300087 | 3.1405044 | 8.170641 |

```
plot(HWforecast, main = "Figure 7: Holt-Winters Forecast", xlab = "Year", yla
b = "Average Monthly Antarctic Sea Ice (Millions of Square Kilometers)", cex.
main = 2)
```

Model Performance

```
train <- window(Ice, end = c(2019, 12))  
test <- window(Ice, start = c(2020, 1)) # Last 48 observations
```

```
# Holt-Winters Forecast
```

```
HWfit <- HoltWinters(train, seasonal = "additive")
```

```
HWforecast <- forecast(HWfit, h = 48)
```

```
HWforecast
```

| ## | Point Forecast | Lo 80 | Hi 80 | Lo 95 | Hi 95 |
|-------------|----------------|--------------|-----------|-------------|-----------|
| ## Jan 2020 | 3.133112 | 2.718804114 | 3.547420 | 2.49948265 | 3.766742 |
| ## Feb 2020 | 1.775124 | 1.259169359 | 2.291078 | 0.98603967 | 2.564208 |
| ## Mar 2020 | 2.288781 | 1.688142452 | 2.889419 | 1.37018373 | 3.207378 |
| ## Apr 2020 | 4.730717 | 4.055939771 | 5.405494 | 3.69873441 | 5.762699 |
| ## May 2020 | 7.686917 | 6.945376892 | 8.428457 | 6.55282933 | 8.821004 |
| ## Jun 2020 | 10.582066 | 9.779296522 | 11.384836 | 9.35433592 | 11.809797 |
| ## Jul 2020 | 12.767260 | 11.907610905 | 13.626909 | 11.45254010 | 14.081980 |
| ## Aug 2020 | 13.994703 | 13.081711323 | 14.907695 | 12.59840256 | 15.391004 |
| ## Sep 2020 | 14.253816 | 13.290429922 | 15.217201 | 12.78044435 | 15.727187 |
| ## Oct 2020 | 13.724182 | 12.712910982 | 14.735453 | 12.17757632 | 15.270788 |
| ## Nov 2020 | 11.062213 | 10.005223303 | 12.119203 | 9.44568678 | 12.678739 |
| ## Dec 2020 | 6.523430 | 5.422619510 | 7.624241 | 4.83988540 | 8.206976 |
| ## Jan 2021 | 3.058235 | 1.896246510 | 4.220223 | 1.28112717 | 4.835342 |
| ## Feb 2021 | 1.700246 | 0.498258516 | 2.902233 | -0.13803522 | 3.538527 |
| ## Mar 2021 | 2.213903 | 0.973205162 | 3.454601 | 0.31641932 | 4.111387 |
| ## Apr 2021 | 4.655839 | 3.377602425 | 5.934076 | 2.70094478 | 6.610733 |
| ## May 2021 | 7.612039 | 6.297335244 | 8.926743 | 5.60137297 | 9.622706 |
| ## Jun 2021 | 10.507189 | 9.157001831 | 11.857375 | 8.44225614 | 12.572121 |
| ## Jul 2021 | 12.692382 | 11.307621909 | 14.077143 | 10.57457394 | 14.810191 |
| ## Aug 2021 | 13.919826 | 12.501333713 | 15.338318 | 11.75042942 | 16.089222 |
| ## Sep 2021 | 14.178938 | 12.727498321 | 15.630378 | 11.95915258 | 16.398723 |
| ## Oct 2021 | 13.649305 | 12.165648771 | 15.132960 | 11.38024881 | 15.918360 |
| ## Nov 2021 | 10.987335 | 9.472148140 | 12.502523 | 8.67005649 | 13.304614 |
| ## Dec 2021 | 6.448553 | 4.902477205 | 7.994628 | 4.08403421 | 8.813071 |
| ## Jan 2022 | 2.983357 | 1.393142705 | 4.573571 | 0.55133412 | 5.415380 |
| ## Feb 2022 | 1.625368 | 0.005695873 | 3.245041 | -0.85170696 | 4.102444 |
| ## Mar 2022 | 2.139026 | 0.490421086 | 3.787630 | -0.38229739 | 4.660348 |
| ## Apr 2022 | 4.580962 | 2.903924187 | 6.257999 | 2.01615425 | 7.145769 |
| ## May 2022 | 7.537162 | 5.832165524 | 9.242158 | 4.92959509 | 10.144728 |
| ## Jun 2022 | 10.432311 | 8.699807174 | 12.164815 | 7.78267507 | 13.081947 |
| ## Jul 2022 | 12.617505 | 10.857923391 | 14.377086 | 9.92645722 | 15.308553 |
| ## Aug 2022 | 13.844948 | 12.058699316 | 15.631197 | 11.11311635 | 16.576780 |
| ## Sep 2022 | 14.104060 | 12.291536648 | 15.916584 | 11.33204456 | 16.876076 |
| ## Oct 2022 | 13.574427 | 11.736003817 | 15.412850 | 10.76280138 | 16.386053 |
| ## Nov 2022 | 10.912458 | 9.048494907 | 12.776421 | 8.06177260 | 13.763143 |
| ## Dec 2022 | 6.373675 | 4.484518002 | 8.262832 | 3.48445859 | 9.262892 |
| ## Jan 2023 | 2.908479 | 0.983032253 | 4.833926 | -0.03623787 | 5.853197 |
| ## Feb 2023 | 1.550491 | -0.399356452 | 3.500338 | -1.43154321 | 4.532525 |
| ## Mar 2023 | 2.064148 | 0.090202183 | 4.038094 | -0.95474156 | 5.083037 |


```
## Apr 2023      4.506084  2.508330327  6.503837  1.45078347  7.561384
## May 2023      7.462284  5.441003024  9.483565  4.37100149 10.553567
## Jun 2023     10.357433  8.312895621 12.401971  7.23058273 13.484284
## Jul 2023     12.542627 10.475094397 14.610160  9.38060862 15.704646
## Aug 2023     13.770070 11.679795411 15.860345 10.57327065 16.966870
## Sep 2023     14.029183 11.916410312 16.141955 10.79797615 17.260389
## Oct 2023     13.499549 11.364516611 15.634582 10.23429854 16.764800
## Nov 2023     10.837580  8.680516705 12.994643  7.53863632 14.136524
## Dec 2023      6.298798  4.119926305  8.477669  2.96650152  9.631094
```

```
HWerr <- test - HWforecast$mean
HWrmse <- sqrt(mean(HWerr^2))
HWmae <- mean(abs(HWerr))
HWmape <- mean(abs((HWerr*100)/test))
```

```
HWrmse # Holt-Winters RMSE
```

```
## [1] 0.7132303
```

```
HWmae # Holt-Winters MAE
```

```
## [1] 0.5478606
```

```
HWmape # Holt-Winters MAPE
```

```
## [1] 9.405666
```

```
# SARIMA(1,1,2)x(0,1,2)[12] Forecast
```

```
arimafit <- Arima(train, order = c(1,1,2), seasonal = list(order = c(0,1,2),
period = 12))
```

```
arimaforecast <- forecast(arimafit, h = 48)
```

```
arimaforecast
```

```
##          Point Forecast      Lo 80      Hi 80      Lo 95      Hi 95
## Jan 2020      3.025450  2.6585098  3.392391  2.46426320  3.586637
## Feb 2020      1.715924  1.2299230  2.201926  0.97264932  2.459200
## Mar 2020      2.360701  1.8301858  2.891217  1.54934779  3.172055
## Apr 2020      4.729566  4.1725171  5.286616  3.87763310  5.581500
## May 2020      7.547356  6.9697964  8.124915  6.66405496  8.430657
## Jun 2020     10.239618  9.6440029 10.835234  9.32870300 11.150534
## Jul 2020     12.459174 11.8466994 13.071649 11.52247487 13.395873
## Aug 2020     13.831110 13.2024860 14.459734 12.86971243 14.792508
## Sep 2020     14.290088 13.6458177 14.934357 13.30476185 15.275413
## Oct 2020     13.733836 13.0743258 14.393346 12.72520242 14.742469
## Nov 2020     11.216815 10.5424138 11.891215 10.18540764 12.248222
## Dec 2020      6.551194  5.8622146  7.240173  5.49749099  7.604897
## Jan 2021      2.959817  2.2455474  3.674086  1.86743601  4.052198
## Feb 2021      1.660487  0.9234809  2.397494  0.53333321  2.787642
## Mar 2021      2.366973  1.6110535  3.122891  1.21089422  3.523051
## Apr 2021      4.770759  3.9975653  5.543952  3.58826155  5.953255
## May 2021      7.610913  6.8212779  8.400548  6.40327040  8.818555
## Jun 2021     10.285224  9.4796616 11.090787  9.05322240 11.517226
```

| | | | | | |
|-------------|-----------|------------|-----------|-------------|-----------|
| ## Jul 2021 | 12.467095 | 11.6459833 | 13.288207 | 11.21131299 | 13.722877 |
| ## Aug 2021 | 13.810145 | 12.9737999 | 14.646491 | 12.53106543 | 15.089225 |
| ## Sep 2021 | 14.250438 | 13.3991391 | 15.101736 | 12.94848885 | 15.552386 |
| ## Oct 2021 | 13.713014 | 12.8470163 | 14.579011 | 12.38858507 | 15.037442 |
| ## Nov 2021 | 11.209059 | 10.3285925 | 12.089526 | 9.86250163 | 12.555617 |
| ## Dec 2021 | 6.531375 | 5.6366408 | 7.426110 | 5.16299693 | 7.899754 |
| ## Jan 2022 | 2.944009 | 2.0328733 | 3.855144 | 1.55054740 | 4.337470 |
| ## Feb 2022 | 1.650218 | 0.7234528 | 2.576982 | 0.23285312 | 3.067582 |
| ## Mar 2022 | 2.358942 | 1.4174530 | 3.300430 | 0.91905902 | 3.798824 |
| ## Apr 2022 | 4.763633 | 3.8079006 | 5.719365 | 3.30196663 | 6.225299 |
| ## May 2022 | 7.604153 | 6.6344868 | 8.573819 | 6.12117664 | 9.087129 |
| ## Jun 2022 | 10.278612 | 9.2952495 | 11.261975 | 8.77468859 | 11.782536 |
| ## Jul 2022 | 12.460543 | 11.4636866 | 13.457399 | 10.93598268 | 13.985103 |
| ## Aug 2022 | 13.803617 | 12.7934522 | 14.813782 | 12.25870319 | 15.348531 |
| ## Sep 2022 | 14.243919 | 13.2206174 | 15.267221 | 12.67891411 | 15.808925 |
| ## Oct 2022 | 13.706499 | 12.6702183 | 14.742780 | 12.12164429 | 15.291354 |
| ## Nov 2022 | 11.202546 | 10.1534254 | 12.251667 | 9.59805434 | 12.807039 |
| ## Dec 2022 | 6.524863 | 5.4630125 | 7.586714 | 4.90090267 | 8.148824 |
| ## Jan 2023 | 2.937497 | 1.8610040 | 4.013990 | 1.29114309 | 4.583850 |
| ## Feb 2023 | 1.643706 | 0.5532442 | 2.734167 | -0.02401125 | 3.311423 |
| ## Mar 2023 | 2.352430 | 1.2487442 | 3.456116 | 0.66448826 | 4.040371 |
| ## Apr 2023 | 4.757121 | 3.6405914 | 5.873651 | 3.04953638 | 6.464706 |
| ## May 2023 | 7.597641 | 6.4685026 | 8.726780 | 5.87077270 | 9.324510 |
| ## Jun 2023 | 10.272101 | 9.1305275 | 11.413674 | 8.52621512 | 12.017986 |
| ## Jul 2023 | 12.454031 | 11.3001708 | 13.607892 | 10.68935403 | 14.218708 |
| ## Aug 2023 | 13.797105 | 12.6310911 | 14.963120 | 12.01384040 | 15.580370 |
| ## Sep 2023 | 14.237408 | 13.0593625 | 15.415453 | 12.43574309 | 16.039072 |
| ## Oct 2023 | 13.699988 | 12.5100233 | 14.889952 | 11.88009428 | 15.519881 |
| ## Nov 2023 | 11.196035 | 9.9942446 | 12.397825 | 9.35805539 | 13.034014 |
| ## Dec 2023 | 6.518352 | 5.3047964 | 7.731907 | 4.66237919 | 8.374324 |

```

arimaerr <- test - arimaforecast$mean
arimarmse <- sqrt(mean(arimaerr^2))
arimamae <- mean(abs(arimaerr))
arimamape <- mean(abs((arimaerr*100)/test))

```

```
arimarmse # ARIMA RMSE
```

```
## [1] 0.7635105
```

```
arimamae # ARIMA MAE
```

```
## [1] 0.6142785
```

```
arimamape # ARIMA MAPE
```

```
## [1] 10.80418
```

```

Measure <- c("SARIMA(1,1,2)x(0,1,2)[12]", "Holt-Winters")
RMSE <- round(c(arimarmse, HWrmse), 4)
MAE <- round(c(arimamae, HWmae), 4)

```

```
MAPE <- round(c(arimamape, HWmape), 4)
rbind(Measure, RMSE, MAE, MAPE)

##           [,1]           [,2]
## Measure "SARIMA(1,1,2)x(0,1,2)[12]" "Holt-Winters"
## RMSE     "0.7635"           "0.7132"
## MAE      "0.6143"           "0.5479"
## MAPE     "10.8042"          "9.4057"
```