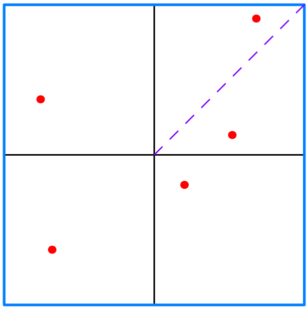

Figure 1

Example 1. Among any 5 points in a 2×2 square, show that there are two points which are at most $\sqrt{2}$ apart.

Summary — Divide the 2×2 square into suitable "boxes/pockets", so that the pigeonhole principle can be applied.

Walkthrough —

(a) Divide the 2×2 square into four unit squares.

(b) Two points among any choice of 5 points from the 2×2 square lie in one of these unit squares.

(c) Conclude!

Solution 1. Suppose we are given a set of five points in a 2×2 square. Divide the 2×2 square into four unit squares. By the pigeonhole principle, two points among those five points lie in one of these unit squares. Note that the distance between any two points lying in a unit square is at most the length of any of its diagonals. By Pythagoras' theorem, any diagonal of a unit square has length equal to $\sqrt{2}$. Consequently, two of those five points are at most $\sqrt{2}$ apart. $\blacksquare$