

Problem 1 (Putnam 2002). Given any five points on a sphere, show that some four of them must lie on a closed hemisphere.

Solution 1. Consider a great circle passing through at least two of the five points. Then at least one closed hemisphere contains at least two of the remaining three points. This concludes the proof. ■