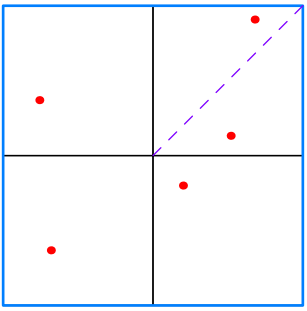

Figure 1

Example 1. Among any 5 points in a 2×2 square, show that there are two points which are at most $\sqrt{2}$ apart.

Summary — Divide the 2×2 square into suitable "boxes/pockets", so that the pigeonhole principle can be applied.

Walkthrough.

(a) Divide the 2×2 square into four unit squares.

(b) Two points among any choice of 5 points from the 2×2 square lie in one of these unit squares.

(c) The distance between any two points lying in a unit square is at most the length of any of its diagonals, that is, at most $\sqrt{2}$.