Exercícios recomendados

- Básicos com respostas:
  - 3.3, 3.4, 3.6, 3.7, 3.12, 3.13, 3.20
- Básicos (sem respostas)
  - 3.22, 3.23, 3.24, 3.25
- Básicos avançados
  - 3.40, 3.42, 3.46a-b, 3.54, 3.62,
- Problemas de Extensão
  - 3.65a (pares a, c, d), 3.65b, 3.65d, 3.71

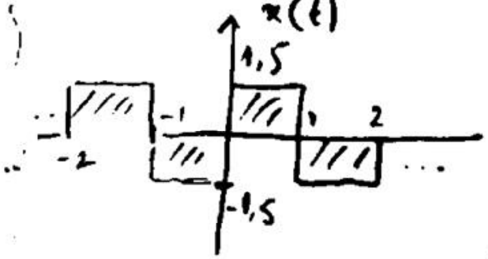
Exercícios

3.4. Use the Fourier series analysis equation (3.39) to calculate the coefficients a_k for the continuous-time periodic signal

$$x(t) = \begin{cases} 1.5, & 0 \le t < 1 \\ -1.5, & 1 \le t < 2 \end{cases}$$

Exercícios

3.4



$T = 2, \quad \omega_0 = \frac{2\pi}{T} = \pi$

(ver também Exemplos 3.5 e 3.6)

cálculo direto 1:

$$a_k = \frac{1}{T}\int_T x(t)\, e^{-jk\omega_0 t}\, dt =$$

$$= \frac{1}{T}\int x(t)\left[\cos(k\pi t) - j\,\mathrm{sen}(k\pi t)\right] dt =$$

$$= \underbrace{\frac{1}{T}\int_T x(t)\cdot\cos(k\pi t)\,dt}_{=0} - \frac{j}{T}\int x(t)\,\mathrm{sen}(k\pi t)\, dt =$$

$$-j\frac{2}{2}\int_0^1 \frac{3}{2}\,\mathrm{sen}(k\pi t)\, dt = j\frac{3}{2}\,\frac{1}{k\pi}\,\cos(k\pi t)\Big|_0^1 = \frac{+j3}{2k\pi}\left[\underbrace{\cos k\pi - 1}\right] \rightarrow$$

$$\underbrace{} = \begin{cases} 1, & k=0 \text{ e } k=\text{par} \\ -1, & k=\text{ímpar} \end{cases}$$

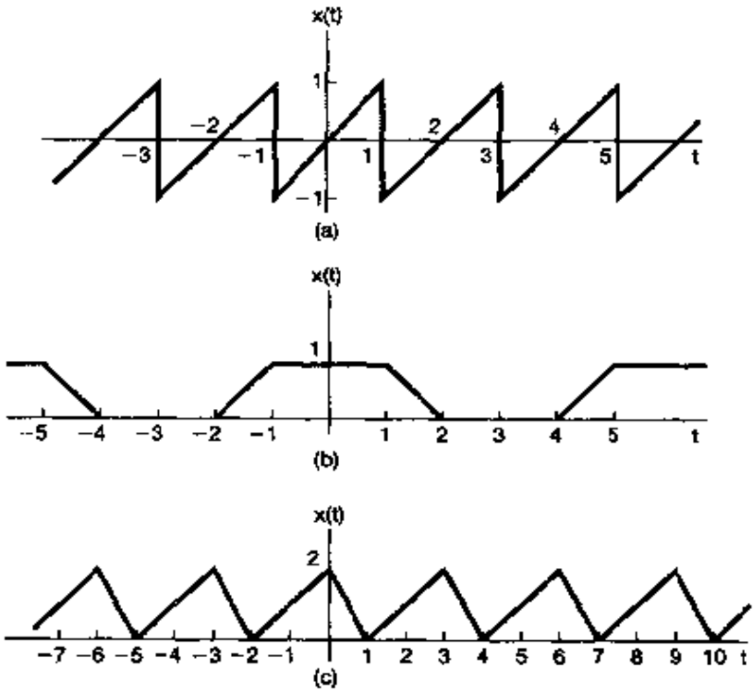
$$\rightarrow \boxed{a_k = \begin{cases} 0, & k=0 \text{ e } k=\text{par} \\ -j\,\dfrac{3}{k\pi}, & k=\text{ímpar} \end{cases}}$$

3.22. Determine the Fourier series representations for the following signals:

(a) Each $x(t)$ illustrated in Figure P3.22(a)–(f).

(b) $x(t)$ periodic with period 2 and

$$x(t) = e^{-t} \quad \text{for} \quad -1 < t < 1$$



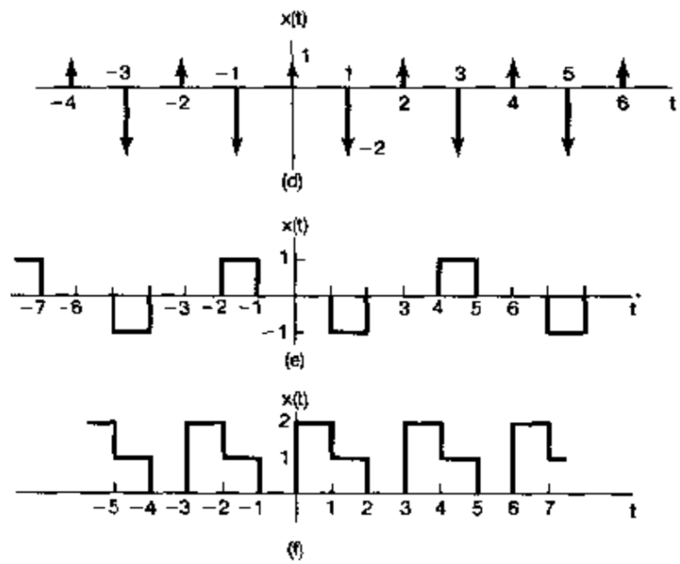
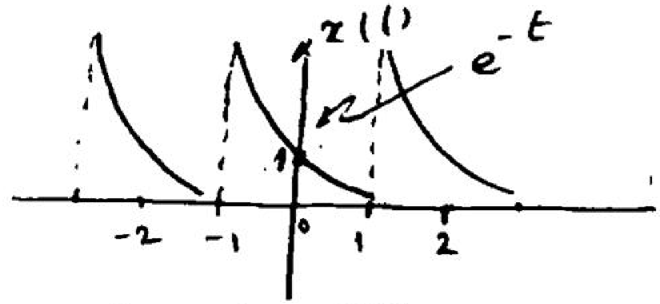


Figure P3.22 Contir

(c) $x(t)$ periodic with period 4 and

$$x(t) = \begin{cases} \sin \pi t, & 0 \le t \le 2 \\ 0, & 2 < t \le 4 \end{cases}$$

$\boxed{3.22}$ b)



$T = 2, \quad \omega_0 = \frac{2\pi}{T} = \pi$

$$a_0 = \frac{1}{T}\int_T x(t) \cdot dt =$$

$$= \frac{1}{2}\int_{-1}^{1} e^{-t} \cdot dt = -\frac{1}{2} e^{-t}\Big|_{-1}^{1} \rightarrow$$

$$a_0 = -\frac{1}{2}e^{-1} + \frac{1}{2}e^{1} = \frac{1}{2}(e^{1} - e^{-1})$$

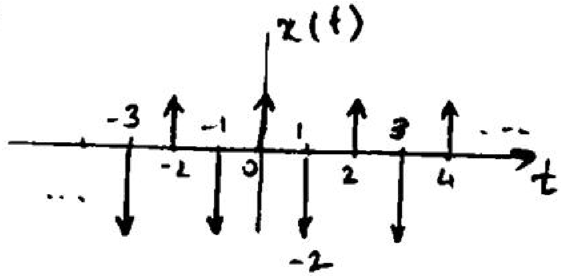
$$a_k = \frac{1}{T}\int_{-1}^{1} x(t) \cdot e^{-jk\omega_0 t} \cdot dt = \frac{1}{2}\int_{-1}^{1} e^{-t} \cdot e^{-jk\pi t} \cdot dt$$

$$= \frac{1}{2}\int_{-1}^{1} e^{-t(1+jk\pi)} \cdot dt = \frac{-e^{-t(1+jk\pi)}}{2(1+jk\pi)}\Big|_{-1}^{1} =$$

$$= \frac{-e^{jk\pi}}{2(1+jk\pi)} e^{-t}\Big|_{-1}^{1} = \frac{-(-1)^k}{2(1+jk\pi)}(e^{-1} - e) =$$

$$\boxed{a_k = \frac{(-1)^k}{2(1+jk\pi)}(e - e^{-1}) \quad \forall k}$$

$\boxed{2.22}$ (a)/d (ii)



$$T = 2, \quad \omega_0 = \frac{2\pi}{2} = \pi$$

$$a_0 = \frac{1}{T}\int_T x(t)\,dt = \frac{1}{2}\int_{-\frac{1}{2}}^{3/2}\left[\delta(t) - 2\delta(t-1)\right]dt$$

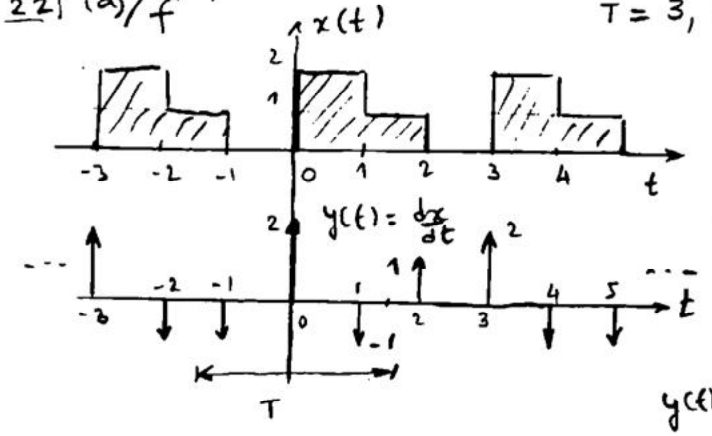
$$a_0 = 1 - 2 \rightarrow \boxed{a_0 = -1}$$

$$\underset{k\neq 0}{a_k} = \frac{1}{T}\int_T x(t)\cdot e^{-jk\omega_0 t}\,dt = \frac{1}{2}\int_{-1/2}^{3/2}\left[\delta(t) - 2\delta(t-1)\right]\cdot e^{-jk\pi t}\,dt =$$

$$= \frac{1}{2}\underbrace{\int_{-1/2}^{3/2}\delta(t)\cdot e^{0}\,dt}_{=1} - \frac{2}{2}\int_{-1/2}^{3/2}\delta(t-1)\cdot e^{-jk\pi\cdot 1}\,dt =$$

$$= \frac{1}{2} - e^{-jk\pi}\int_{-1/2}^{3/2}\delta(t-1)\,dt = \frac{1}{2} - \underbrace{e^{-jk\pi}}_{(e^{-j\pi})^k = (-1)^k} = \boxed{\frac{1}{2} - (-1)^k = \underset{k\neq 0}{a_k}}$$

3.22 (a)



$T = 3, \quad \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{3}$

$a_0 = \frac{1}{T}\int_T x(t)\cdot dt =$

$a_0 = \frac{1}{3}(1\times 2 + 1) \rightarrow \boxed{a_0 = 1}$

$x(t) \overset{SF}{\longleftrightarrow} a_k$

$y(t) = \frac{dx(t)}{dt} \overset{SF}{\longleftrightarrow} b_k = j k \omega_0 \, a_k$

Propriedade da derivada

$$b_k = \frac{1}{T}\int_T y(t)\, e^{-jk\omega_0 t}\cdot dt = \frac{1}{3}\int_{-1,5}^{1,5}\left[-\delta(t+1) - \delta(t-1) + \delta(t)\right]\cdot e^{-j\frac{2k\pi t}{3}}\cdot dt$$

$$= \frac{1}{3}\int_{-1,5}^{1,5} -\left[e^{j\frac{2k\pi}{3}}\cdot\delta(t+1) + e^{-j\frac{2k\pi}{3}}\,\delta(t-1) + 2e^{0}\,\delta(t)\right] dt$$

$$= -\frac{1}{3}\left[e^{+2j\frac{k\pi}{3}}\underbrace{\int_{-1,5}^{1,5}\delta(t+1)\cdot dt}_{=1,\ \text{etc}\dots} + e^{-j\frac{2k\pi}{3}}\right] + \frac{2}{3}$$

$$= -\frac{1}{3}\left[\frac{e^{j2k\pi/3} + e^{-j2k\pi/3}}{2}\right]\times 2 + \frac{2}{3} = -\frac{2}{3}\cos\left(\frac{2k\pi}{3}\right) + \frac{2}{3}$$

$$= \frac{2}{3}\left[1 - \cos 2k\pi/3\right] = b_k = j\frac{2k\pi}{3}\cdot a_k$$

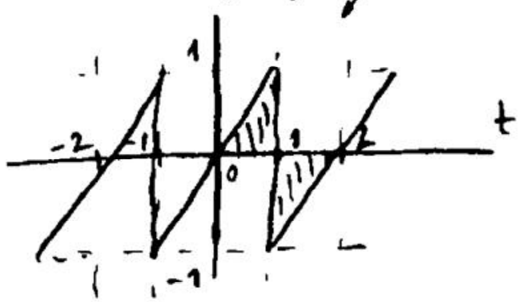
a_k complexo ⟷ $x(t)$ ímpar

$$a_k = \frac{1}{j2k\pi/3}\times\frac{2}{3}\left[1 - \cos 2k\pi/3\right] = \frac{1}{jk\pi}\left[1 - \cos 2k\pi/3\right] = a_k, \quad k \neq 0$$

$a_0 = 1$

$\boxed{3.22}$ a) / $a^{(i)}$ $\qquad \boxed{a_0 = 0}$

$x(t)$



$T = 2, \quad w_0 = \frac{2\pi}{T} = \pi$

$x(t) = t, \quad -\frac{T}{2} \leq t \leq \frac{T}{2}$

$d(u \cdot v) = u\,dv + v\,du \quad \therefore$

$\int u\,dv = u \cdot v - \int v\,du$

(método 1)

$$a_k = \frac{1}{T}\int_T x(t)\, e^{-jkw_0 t}\, dt \qquad k \neq 0$$

$$v = -\frac{\cos k\pi t}{k\pi}$$

$$= \frac{2}{T}\int_0^{T/2} (-j)\, x(t) \cdot \text{sen}(kw_0 t)\,dt = -j\,\frac{2}{2}\int_0^1 \underbrace{t}_{u} \cdot \overbrace{\text{sen}(k\pi t)\,dt}^{dv}$$

$$= -j\left[-t \cdot \frac{\cos k\pi t}{k\pi}\bigg|_0^1 - \int_0^1 -\frac{\cos k\pi t}{k\pi}\,dt\right] =$$

$$= j\left[\frac{\cos k\pi t}{k\pi} + \frac{\text{sen}\, k\pi t}{k\pi}\right]_0^1 =$$

$$= \frac{j}{k\pi}\left[\underbrace{\cos k\pi}_{(-1)^k} + \underbrace{\text{sen}\, k\pi}_{=0}\right] = \frac{j(-1)^k}{k\pi} = a_k \quad k \neq 0$$

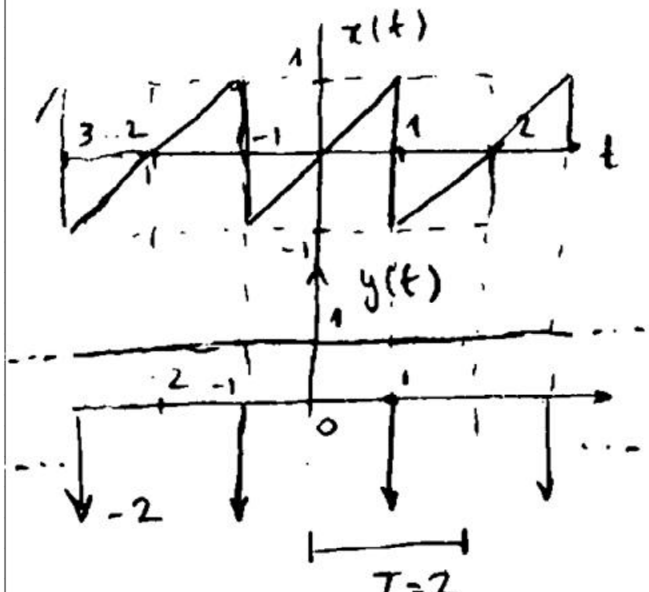
$$a_0 = 0$$

Há forma mais elegante de resolver o problema com a propriedade da derivada...

propriedade da derivada

3.22 a)

$$y(t) = \frac{dx(t)}{dt} \longleftrightarrow b_k = (jk\omega_0)\,a_k = jk\pi \cdot a_k$$



$$b_k = \frac{1}{T}\int_T y(t)\, e^{-jk\omega_0 t}\, dt =$$

$$= \frac{1}{2}\int_0^2 \left[1 - 2\,\delta(t-1)\right] e^{-jk\pi t}\, dt$$

$$= \frac{1}{2}\int_0^2 e^{-jk\pi t}\, dt - \int_0^2 \underbrace{e^{-jk\pi(1)}\,\delta(t-1)}_{\text{propriedade da amostragem do impulso}}\, dt \qquad t=1$$

$$= \frac{1}{2}\, \frac{e^{-jk\pi t}}{-jk\pi}\Bigg|_0^2 - e^{-jk\pi}\underbrace{\int_0^2 \delta(t-1)\, dt}_{=1} =$$

$$= \frac{1}{2}\,\frac{1}{-jk\pi}\Big(\underbrace{e^{-j2k\pi}}_{=1} - 1\Big) - \underbrace{e^{-jk\pi}}_{=(-1)^{-k}}$$

$$\to b_k = -(-1)^k = jk\pi \cdot a_k \to \underset{(k\neq 0)}{a_k} = \frac{-(-1)^k}{jk\pi} = \boxed{\frac{j(-1)^k}{k\pi} = \underset{k\neq 0}{a_k}}$$

3.24. Let

$$x(t) = \begin{cases} t, & 0 \le t \le 1 \\ 2 - t, & 1 \le t \le 2 \end{cases}$$

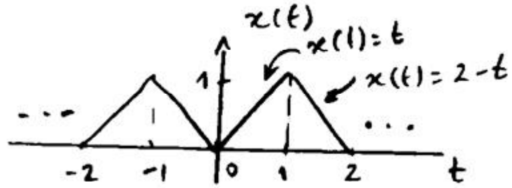
be a periodic signal with fundamental period $T = 2$ and Fourier coefficients a_k.

(a) Determine the value of a_0.

(b) Determine the Fourier series representation of $dx(t)/dt$.

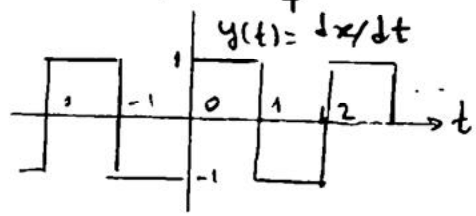
(c) Use the result of part (b) and the differentiation property of the continuous-time Fourier series to help determine the Fourier series coefficients of $x(t)$.

3.24

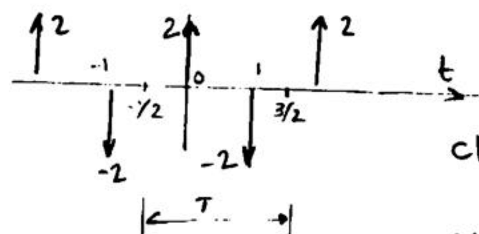


a) $a_0 = \frac{1}{T}\int_T x(t)\,dt = \frac{1}{2}\cdot\frac{2\times 1}{2} \rightarrow a_0 = \frac{1}{2}$

$T = 2,\ \omega_0 = \frac{2\pi}{T} = \pi$



$y(t) \xleftrightarrow{SF} b_k = jk\omega_0 \cdot a_k$



$\frac{dy}{dt} = g(t) \xleftrightarrow{SF} c_k = j\omega_0 k \cdot b_k = (j\omega_0 k)^2 \cdot a_k$

$c_k = \frac{1}{T}\int_T g(t)\cdot e^{-jk\omega_0 t}\,dt =$

$= \frac{1}{2}\int_{-1/2}^{3/2} 2\left[\delta(t) - \delta(t-1)\right]\cdot e^{-jk\pi t}\,dt$

$c_k = \int_{-1/2}^{3/2}\left[\delta(t)\cdot e^{0} - \delta(t-1)\,e^{-jk\pi}\right]dt =$

$= \int_{-1/2}^{3/2}\delta(t)\,dt - e^{-jk\pi}\int_{-1/2}^{3/2}\delta(t-1)\cdot dt = 1 - e^{-jk\pi} = 1 - (-1)^k = c_k$

$\therefore\ c_k = (jk\omega_0)^2\cdot a_k \rightarrow a_k = \frac{1-(-1)^k}{(jk\pi)^2} \Rightarrow \boxed{a_k = \frac{(-1)^k - 1}{(k\pi)^2},\quad a_0 = 1/2}$

$k \neq 0$

$\frac{dx}{dt} \xleftrightarrow{SF} b_k$

$c_k = (jk\omega_0)\cdot b_k \rightarrow \boxed{b_k = \frac{1-(-1)^k}{jk\pi}} \xleftrightarrow{SF} \frac{dx(t)}{dt}$

Simetrias

3.42 $x(t)$: real, período $= T$, $x(t) \xrightarrow{SF} a_k$ [provar: $= a^*_{-k}$, $a_0 =$ real]

a) $a_k = \frac{1}{T}\int_T x(t)\, e^{-jk\omega_0 t}\, dt$, $a_0 = \frac{1}{T}\int_T x(t)\, dt =$ real

$a_k^* = \frac{1}{T}\int_T x(t)\, e^{+jk\omega_0 t}\, dt$

$\boxed{a_{-k}^* = \frac{1}{T}\int_T x(t)\, e^{-jk\omega_0 t}\, dt = a_k}$

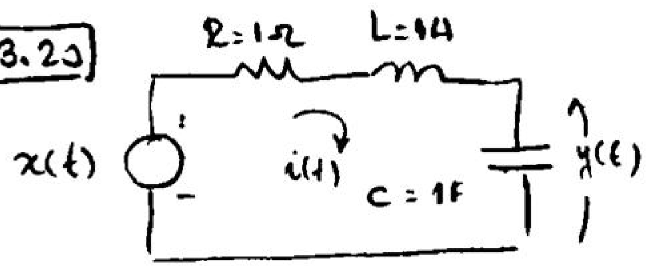
b) $x(t) =$ real e par, período $= T$, $x(t) \xrightarrow{SF} a_k$ [provar: $=$ reais e pares]

$a_k = \frac{1}{T}\int_T x(t) \cdot e^{-jk\omega_0 t}\, dt = \frac{1}{T}\int_T x(t)\left[\cos(k\omega_0 t) - j\,\mathrm{sen}(k\omega_0 t)\right] dt$

$a_k = \frac{1}{T}\underbrace{\int \underset{\text{par}}{x(t)} \cdot \underset{\text{par}}{\cos(k\omega_0 t)}\, dt}_{\text{real}} - j\frac{1}{T}\underbrace{\int_T \underset{\text{par}}{x(t)} \cdot \underset{\text{impar}}{\mathrm{sen}(k\omega_0 t)}\, dt}_{\text{imaginário} = 0}$

$\boxed{a_k = \text{real}}$, * como $\cos(-k\omega_0 t) = \cos(k\omega_0 t)$, $a_{-k} = a_k$ (par);

3.23



a) $x(t) = R\,i(t) + L\dfrac{di(t)}{dt} + y(t)$

$i(t) = C\dfrac{dy(t)}{dt}$

$$x(t) = RC\frac{dy(t)}{dt} + LC\frac{d^2y(t)}{dt^2} + y(t)$$

$$y(t) = -LC\frac{d^2y(t)}{dt^2} - RC\frac{dy(t)}{dt} + x(t)$$

autofunção

b) $x(t) = e^{j\omega t} \therefore H(j\omega)\cdot \cancel{e^{j\omega t}} = -LC(j\omega)^2 \cancel{e^{j\omega t}}\cdot H(j\omega) - RC(j\omega)\cdot\cancel{e^{j\omega t}}\cdot H(j\omega) + \cancel{e^{j\omega t}}\,1$

$\rightarrow H(j\omega)\left[1 - \omega^2 LC + j\omega RC\right] = 1 \therefore H(j\omega) = \dfrac{1}{1-\omega^2 LC + j\omega RC} = \dfrac{1}{1-\omega^2 + j\omega}$

Resposta em frequência

c) se $x(t) = \text{sen}(t)$, $y(t) = ?$ $\omega_0 = 1$

$x(t) = \dfrac{1}{2j}e^{jt} - \dfrac{1}{2j}e^{-jt} \therefore y(t) = H(j1)\cdot a_1\cdot e^{j1t} + H(-j1)\cdot a_{-1}e^{j(-1)t} =$

$\dfrac{1}{2j} \rightarrow a_1$, $\dfrac{1}{2j} \rightarrow a_{-1}$

$$= \frac{e^{jt}}{2j}\,\frac{1}{1-1^2+j\cdot 1} - \frac{e^{-jt}}{2j}\,\frac{1}{1-(-1)^2+j(-1)} =$$

$$= \frac{e^{jt}}{2j}\,\frac{1}{j} - \frac{e^{-jt}}{2j}\,\frac{1}{-j} = -\frac{e^{jt}}{2} + \frac{e^{-jt}}{2} = \boxed{-\cos(t) = y(t)}$$