Exercícios

4.1. Use the Fourier transform analysis equation (4.9) to calculate the Fourier transforms of:

(a) $e^{-2(t-1)}u(t-1)$ **(b)** $e^{-2|t-1|}$

Sketch and label the magnitude of each Fourier transform.

$\boxed{4.1}$ a) $x(t) = e^{-2(t-1)} \cdot u(t-1)$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} \cdot dt = \int_{-\infty}^{\infty} e^{-2(t-1)} \cdot u(t-1) \cdot e^{-j\omega t} \cdot dt$$

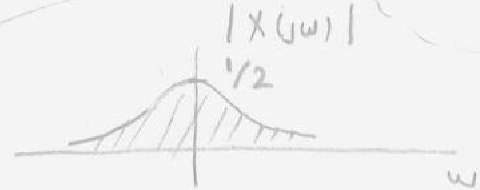
$$= \int_{1}^{\infty} e^{-2(t-1)} \cdot e^{-j\omega t} \cdot dt = \int_{1}^{\infty} e^{2} \cdot e^{-t(j\omega+2)} \cdot dt =$$

$$e^{2} \left. \frac{e^{-t(j\omega+2)}}{-(j\omega+2)} \right|_{1}^{\infty} = e^{2} \left[\frac{0 - e^{-(j\omega+2)}}{-(j\omega+2)} \right] = \boxed{\frac{e^{-j\omega}}{j\omega+2} = X(j\omega)}$$

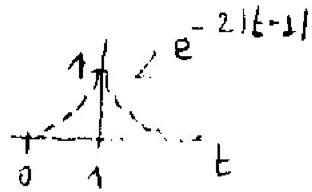
$$e^{j\theta}, \; \theta = tg^{-1}\left(\frac{\omega}{2}\right)$$

$$|X(j\omega)| = \frac{1}{\sqrt{\omega^2+4}} \qquad \measuredangle X(j\omega) = -\omega - tg^{-1}\left(\frac{\omega}{2}\right)$$

$$X(j\omega) = \frac{1}{\sqrt{\omega^2+4}} \cdot e^{-j(\omega+\theta)}$$



b) $x(t) = e^{-2|t-1|}$

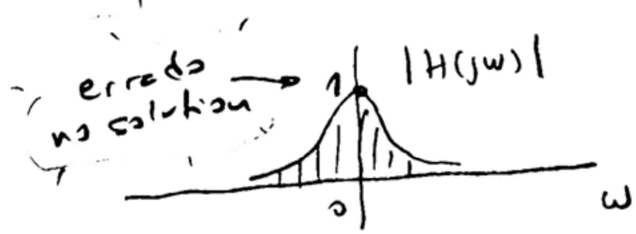


$$X(j\omega) = \int_{-\infty}^{\infty} e^{-2|t-1|} \cdot e^{-j\omega t} \cdot dt = \int_{-\infty}^{1} e^{+2(t-1)} \cdot e^{-j\omega t} \cdot dt + \int_{1}^{\infty} e^{-2(t-1)} \cdot e^{-j\omega t} \cdot dt =$$

$$= e^{-2} \cdot \int_{-\infty}^{1} e^{t(2-j\omega)} \cdot dt + e^{2} \int_{1}^{\infty} e^{-t(2+j\omega)} \cdot dt =$$

...

$$\boxed{\frac{4e^{-j\omega}}{4+\omega^2}} = X(j\omega) \quad \therefore \quad |X(j\omega)| = \frac{4}{4+\omega^2}$$



Exercícios

4.2. Use the Fourier transform analysis equation (4.9) to calculate the Fourier transforms of:

(a) $\delta(t+1)+\delta(t-1)$

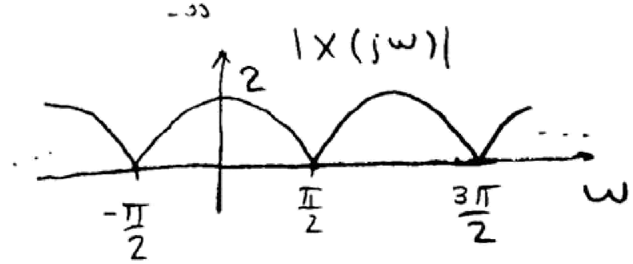
Sketch and label the magnitude of each Fourier transform.

$\boxed{4.2}$ a) $x(t) = \delta(t+1) + \delta(t-1)$

$$X(j\omega) = \int_{-\infty}^{\infty} [\delta(t+1) + \delta(t-1)]\, e^{-j\omega t} \cdot dt = e^{j\omega} \int_{-\infty}^{\infty} \delta(t+1) \cdot dt + e^{-j\omega} \int \delta(t-1)\, dt$$

$$= e^{j\omega} + e^{-j\omega} = 2\cos\omega = X(j\omega)$$

zeros: $\omega = k\frac{\pi}{2}$



$$\delta(t) \xleftrightarrow{\mathcal{F}} 1$$

$$\delta(t - t_0) \xleftrightarrow{\mathcal{F}} 1 \cdot e^{-j\omega t_0}$$

Propriedade do
Deslocamento no tempo

$$\delta(t-1) \xleftrightarrow{\mathcal{F}} 1 \cdot e^{-j\omega}$$

$$\delta(t+1) \xleftrightarrow{\mathcal{F}} 1 \cdot e^{j\omega}$$

4.5. Use the Fourier transform synthesis equation (4.8) to determine the inverse Fourier transform of $X(j\omega) = |X(j\omega)|e^{j\sphericalangle X(j\omega)}$, where

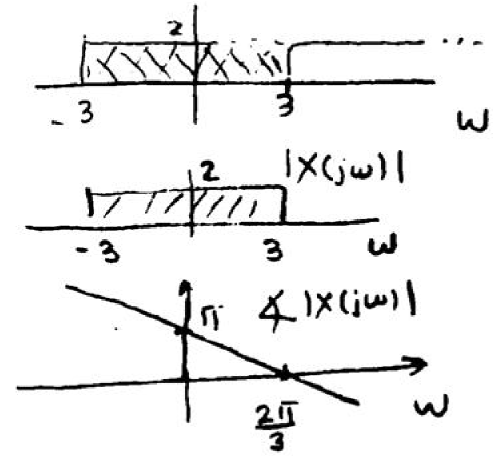
$$|X(j\omega)| = 2\{u(\omega+3) - u(\omega-3)\},$$

$$\sphericalangle X(j\omega) = -\frac{3}{2}\omega + \pi.$$

Use your answer to determine the values of t for which $x(t) = 0$.

$\boxed{4.5}$ $|X(j\omega)| = 2\left[u(\omega+3) - u(\omega-3)\right]$

$\measuredangle X(j\omega) = -\frac{3}{2}\omega + \pi = \theta(\omega)$



$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|\, e^{j\theta(\omega)} \cdot e^{j\omega t} \cdot d\omega$$

$$= \frac{1}{2\pi}\int_{-3}^{3} 2 \cdot e^{j\left(-\frac{3}{2}\omega+\pi\right)} \cdot e^{j\omega t} \cdot d\omega$$

$$= \frac{1}{\pi}\int_{-3}^{3} \underbrace{e^{j\pi}}_{=-1} \cdot e^{j\omega\left(t-\frac{3}{2}\right)} \cdot d\omega = \frac{-1}{\pi} \left.\frac{e^{j\omega(t-3/2)}}{j(t-3/2)}\right|_{\omega=-3}^{\omega=3} =$$

$$= \frac{-1}{\pi(t-3/2)} \underbrace{\frac{e^{j3(t-3/2)} - e^{-j3(t-3/2)}}{j}}_{=2\,\text{sen}\left[3(t-3/2)\right]} = \boxed{\frac{-2}{\pi(t-3/2)} \cdot \text{sen}\left[3(t-3/2)\right] = x(t)}$$

Zeros: $3\left(t-\frac{3}{2}\right) = \pm k\pi \rightarrow t = \pm\frac{k\pi}{3} + \frac{3}{2}$, $\forall k$ inteiro

4.19. Consider a causal LTI system with frequency response

$$H(j\omega) = \frac{1}{j\omega + 3}.$$

For a particular input $x(t)$ this system is observed to produce the output

$$y(t) = e^{-3t}u(t) - e^{-4t}u(t).$$

Determine $x(t)$.

$\boxed{4.19}$

$$\underset{X(j\omega)}{x(t)} \longrightarrow \boxed{H(j\omega)} \longrightarrow \underset{Y(j\omega)}{y(t)} \qquad H(j\omega) = \frac{1}{j\omega + 3} = \frac{Y(j\omega)}{X(j\omega)}$$

$$\begin{cases} y(t) = \left(e^{-3t} - e^{-4t}\right) u(t) \\ x(t) = ? \end{cases}$$

$$Y(j\omega) = \frac{1}{3+j\omega} - \frac{1}{4+j\omega} = \frac{4+j\omega-3-j\omega}{(3+j\omega)\,(4+j\omega)} = \frac{1}{(3+j\omega)(4+j\omega)}$$

$$X(j\omega) = \frac{Y(j\omega)}{H(j\omega)} = \frac{1}{(3+j\omega)(4+j\omega)} \times (3+j\omega) \Rightarrow X(j\omega) = \frac{1}{4+j\omega}$$

$$\therefore\ x(t) = \mathcal{F}^{-1}[X(j\omega)] \underset{(\text{tabela})}{=} \left| e^{-4t} \cdot u(t) = x(t) \right.$$

4.34. A causal and stable LTI system S has the frequency response

$$H(j\omega) = \frac{j\omega + 4}{6 - \omega^2 + 5j\omega}.$$

(a) Determine a differential equation relating the input $x(t)$ and output $y(t)$ of

$\boxed{4.34}$ (a) $\quad H(j\omega) = \dfrac{j\omega+4}{6-\omega^2+5(j\omega)} = \dfrac{Y(j\omega)}{X(j\omega)}$ (18)

frações parciais

$$\therefore\ 6\cdot Y(j\omega) + (j\omega)^2\cdot Y(j\omega) + 5(j\omega)\cdot Y(j\omega) = (j\omega)\,X(j\omega) + 4\cdot X(j\omega)$$

$$\Updownarrow\ \mathcal{F}^{-1}$$

$$6\,y(t) + \frac{d^2y(t)}{dt^2} + 5\,\frac{dy(t)}{dt} = \frac{dx(t)}{dt} + 4\,x(t)$$

$$\rightarrow\ \boxed{\frac{d^2y(t)}{dt^2} + 5\,y(t) + 6\,y(t) = \frac{dx(t)}{dt} + 4x(t)}$$

4.22. Determine the continuous-time signal corresponding to each of the following transforms.

(d) $X(j\omega) = 2[\delta(\omega - 1) - \delta(\omega + 1)] + 3[\delta(\omega - 2\pi) + \delta(\omega + 2\pi)]$

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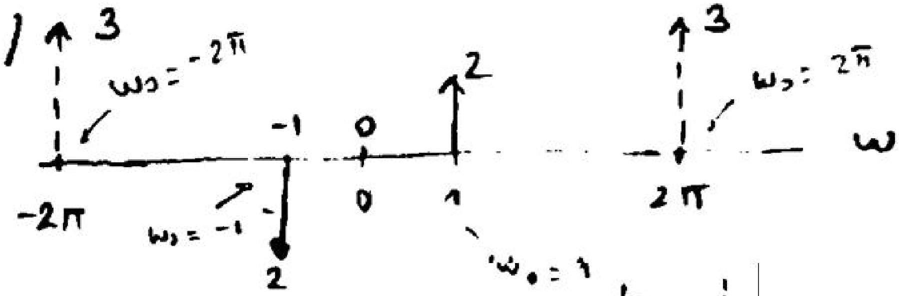
4.22

(d)



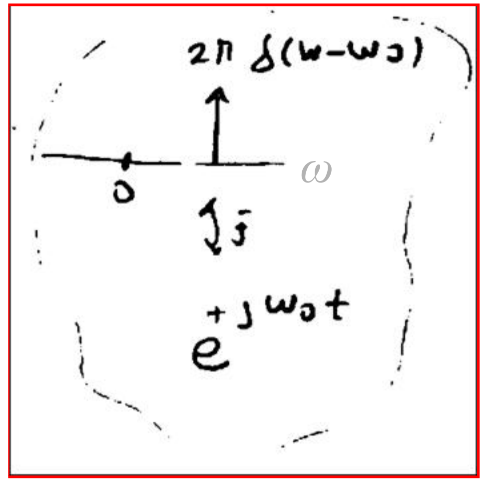
$$\frac{1}{2\pi}\left[-2e^{-j\omega_0 t} + 2e^{+j\omega_0 t}\right]$$

$$= -\frac{2j}{\pi}\left[\frac{e^{jt} - e^{-jt}}{2j}\right]$$

$$= -\frac{2j}{\pi}\,\text{sen}(t)$$

$$\frac{1}{2\pi}\left[3e^{-j2\pi t} + 3e^{j2\pi t}\right] =$$

$$\boxed{\frac{3}{\pi}\cos 2\pi t}$$



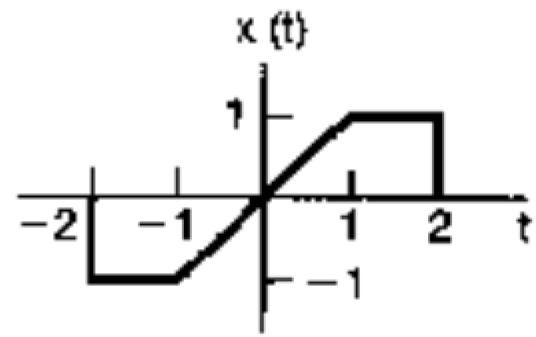
Deslocamento em freqüência

$$x(t) = -\frac{2j}{\pi}\,\text{sen}(t) + \frac{3}{\pi}\cos(2\pi t)$$

4.21. Compute the Fourier transform of each of the following signals:

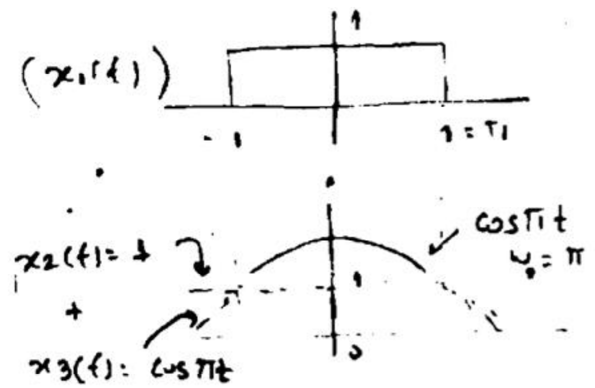
(e) $x(t) = \begin{cases} 1 + \cos \pi t, & |t| \leq 1 \\ 0, & |t| > 1 \end{cases}$

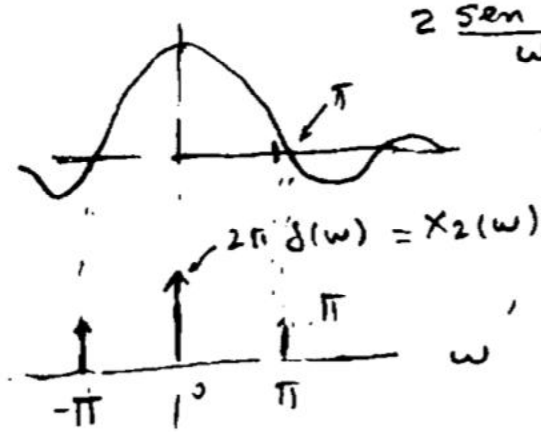
(f) $\left[\frac{\sin \pi t}{\pi t}\right]\left[\frac{\sin 2\pi(t-1)}{\pi(t-1)}\right]$



$\boxed{4.21}$ (c) $x(t)$ (10)

$$2\,\frac{\operatorname{sen}\omega}{\omega} = X_1(j\omega)$$





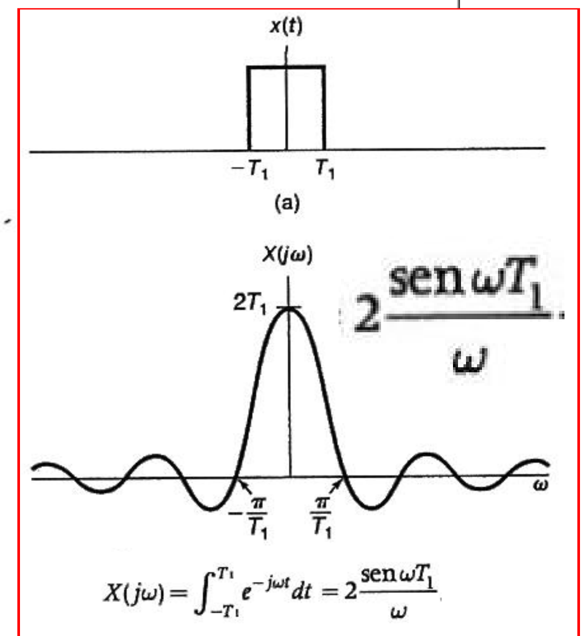
$$\therefore X_1(j\omega) = \frac{1}{2\pi}\,\underbrace{X_1(j\omega)}_{\left[2\frac{\operatorname{sen}\omega}{\omega}\right]} * \left[2\pi\,\delta(\omega) + \pi\,\delta(\omega-\pi) + \pi\,\delta(\omega+\pi)\right]$$

$$X_1(j\omega) = 2\,\frac{\operatorname{sen}\omega}{\omega} + \frac{\operatorname{sen}(\omega-\pi)}{\omega-\pi} + \frac{\operatorname{sen}(\omega+\pi)}{\omega+\pi}$$

$\operatorname{sen}(\omega \pm \pi) = -\operatorname{sen}\omega$

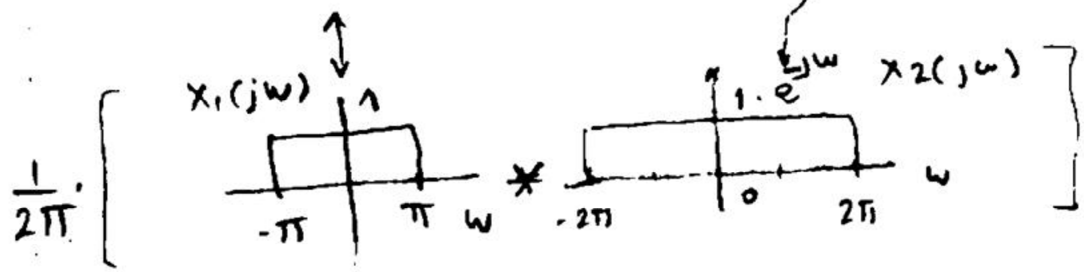
$$= 2\,\frac{\operatorname{sen}\omega}{\omega} - \frac{\operatorname{sen}\omega}{\omega-\pi} - \frac{\operatorname{sen}\omega}{\omega+\pi} =$$

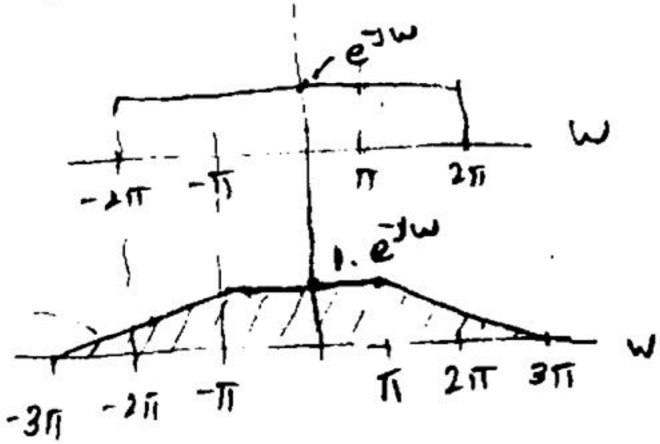
$$\boxed{X_1(j\omega) = \frac{2\operatorname{sen}\omega}{\omega} + \frac{\operatorname{sen}\omega}{\pi-\omega} - \frac{\operatorname{sen}\omega}{\pi+\omega}}$$



4.21

(f) $x(t) = \underbrace{\left[\dfrac{\operatorname{sen} \pi t}{\pi t}\right]}_{x_1(t)} \cdot \underbrace{\left[\dfrac{\operatorname{sen} 2\pi(t-1)}{\pi(t-1)}\right]}_{x_2(t)}$ deslocamento no tempo

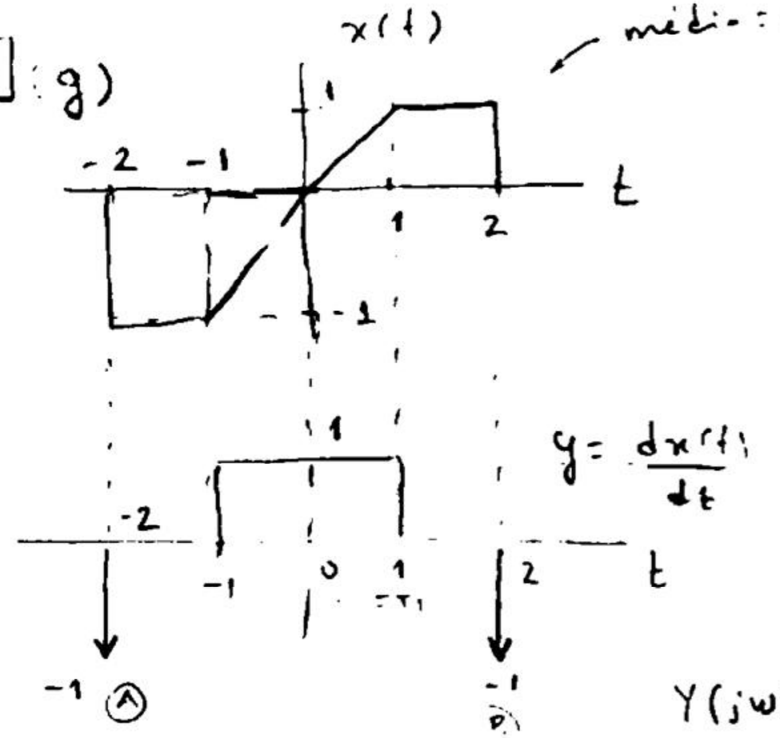




$$X(j\omega) = \frac{1}{2\pi}\left[X_1(j\omega) * X_2(j\omega)\right]$$

$$X(j\omega) = \begin{cases} 1\, e^{-j\omega}, & |\omega| \leq \pi \\ \frac{1}{2\pi}(3\pi+\omega)\, e^{-j\omega}, & -3\pi \leq \omega < -\pi \\ \frac{1}{2\pi}(3\pi-\omega)\cdot e^{-j\omega}, & \pi \leq \omega \leq 3\pi \\ 0, & \text{c.c.} \end{cases}$$

4.21 : g)



$x(t) \longleftrightarrow X(j\omega) = ?$ (12)

$y(t) = \frac{dx(t)}{dt} \longleftrightarrow Y(j\omega) = j\omega \cdot X(j\omega)$

$y = \frac{dx(t)}{dt}$

$$\longleftrightarrow Y(j\omega) = \frac{2\,\text{sen}\,\omega}{\omega} - \overset{A}{e^{+j2\omega}} - \overset{B}{e^{-j2\omega}}$$

$$Y(j\omega) = \frac{2\,\text{sen}\,\omega}{\omega} - 2\cos(2\omega) = j\omega \cdot X(j\omega)$$

$$Y(j\omega) = 2\left[\frac{\text{sen}\,\omega}{\omega} - \cos(2\omega)\right]$$

$$\boxed{\frac{2j}{\omega}\left[\cos(2\omega) - \frac{\text{sen}\,\omega}{\omega}\right] = X(j\omega)}$$

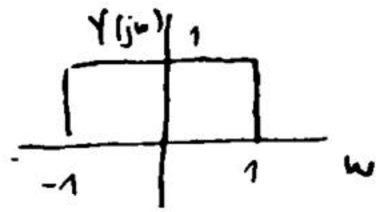
4.10. **(a)** Use Tables 4.1 and 4.2 to help determine the Fourier transform of the following signal:

$$x(t) = t\left(\frac{\sin t}{\pi t}\right)^2$$

6

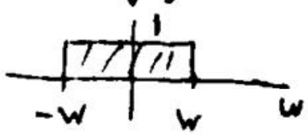
$\boxed{4.10}$ a) $x(t) = t\left(\dfrac{\text{sen}\, t}{\pi t}\right)^2 \xleftrightarrow{\mathcal{F}} X(j\omega) = j\dfrac{d}{d\omega}\underbrace{\mathcal{F}\left[\dfrac{\text{sen}\, t}{\pi t}\right]^2}_{Z(j\omega) = \frac{1}{2\pi}\left[Y(j\omega) * Y(j\omega)\right]} =$

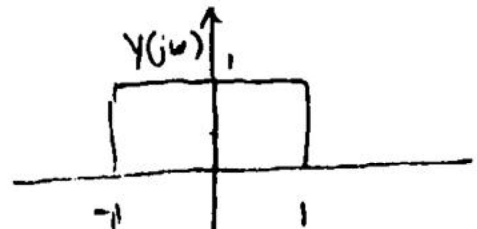
$y(t) = \dfrac{\text{sen}\, t}{\pi t} \xleftrightarrow{\mathcal{F}} Y(j\omega) =$



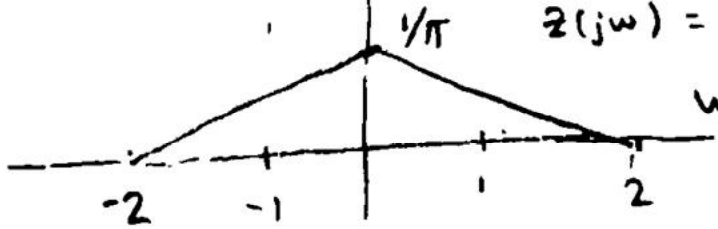
$\dfrac{\text{sen}(Wt)}{\pi t}$

$\updownarrow \mathcal{F}$

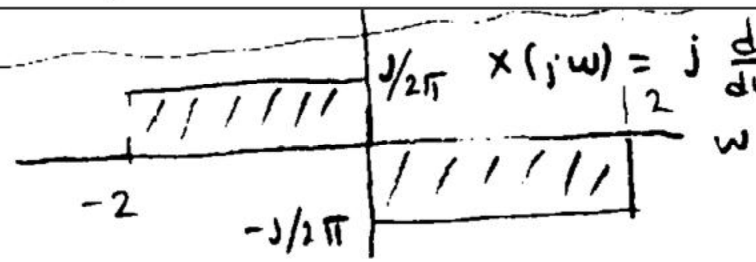




$Z(j\omega) = \dfrac{1}{2\pi}\left[Y(j\omega) * Y(j\omega)\right]$

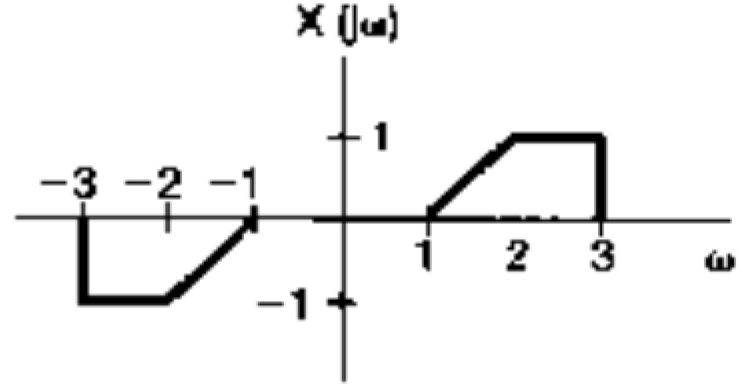


$X(j\omega) = j\dfrac{d}{d\omega}\left[Z(j\omega)\right]$



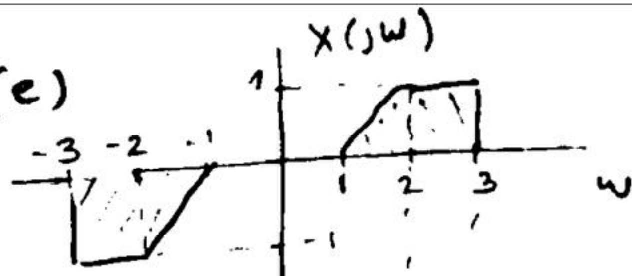
4.22. Determine the continuous-time signal corresponding to each of the following transforms.

(a)



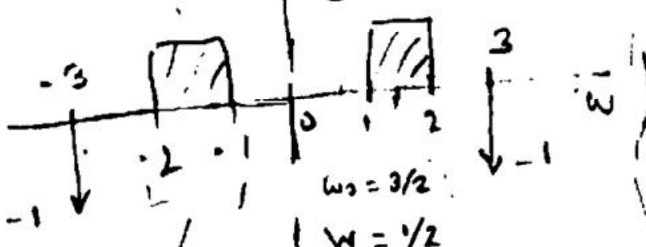
(b)

$\boxed{4.22}$ (e)



$\xleftarrow{F}$ $x(t) = ?$

$$\frac{d}{d\omega}X(\omega) = Y(\omega) \xleftarrow{F} y(t) = -jt\,x(t)$$



$$\frac{\text{sen}\,Wt}{\pi t} \cdot e^{-j\frac{3}{2}t} \qquad \frac{\text{sen}(t/2)}{\pi t} \cdot e^{+j\frac{3}{2}t}$$

$$y(t) = \frac{-1}{2\pi}\left[e^{-j3t} + e^{+j3t}\right] + \frac{\text{sen}(t/2)}{\pi t}\left[e^{j\frac{3}{2}t} + e^{-j\frac{3}{2}t}\right]$$

$$y(t) = -jt \cdot x(t)$$

$$x(t) = \frac{1}{\pi t}\cos(3t) - \frac{2}{j\pi t^2}\,\text{sen}(t/2) \cdot \cos(3t/2)$$

$(2\,\text{sen}(a) \cdot \text{sen}(b) = \text{sen}(a+b) + \text{sen}(a-b))$

$$x(t) = \frac{1}{\pi t}\cos(3t) + \frac{1}{j\pi t^2}\left[\text{sen}(t) - \text{sen}(2t)\right]$$