- Apostila Fernando Livro Ex 9.3 9.5 9.7 9.14 9.66 9.15 9.26 9.8
- Exercicios Oppenheim 9.28 9.30
- Exemplos 9.25, 9.26 Oppenheim

Exemplo 9.25

Seja um sistema LTI com entrada x(t) e saida y(t):

$$x(t) = e^{-3t}u(t)$$

$$y(t) = [e^{-2t} - e^{-t}]u(t)$$

Determina a função de transferência e a eq diferencial do sistema

SOL

$$H(s) = Y(s) = \frac{s+3}{s^2+3s+2} \quad RE(s) > -1$$

$$X(s) = \frac{1}{s+3} \quad RE(s) > -3; \ = Y(s) = \frac{1}{s+1}\frac{1}{s+2} \quad RE(s) > -1$$

então a eq diff é

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = \frac{dx(t)}{dt} + 3x(t)$$

P20.1

Consider the signal $x(t) = 3e^{2t}u(t) + 4e^{3t}u(t)$.

(a) Does the Fourier transform of this signal converge?

(b) For which of the following values of σ does the Fourier transform of $x(t)e^{-\sigma t}$ converge?

(i) $\sigma = 1$

(ii) $\sigma = 2.5$

(iii) $\sigma = 3.5$

(c) Determine the Laplace transform $X(s)$ of $x(t)$. Sketch the location of the poles and zeros of $X(s)$ and the ROC.

S20.1

(a) The Fourier transform of the signal does not exist because of the presence of growing exponentials. In other words, $x(t)$ is not absolutely integrable.

(b) (i) For the case $\sigma = 1$, we have that

$$x(t)e^{-\sigma t} = 3e^{t}u(t) + 4e^{2t}u(t)$$

Although the growth rate has been slowed, the Fourier transform still does not converge.

(ii) For the case $\sigma = 2.5$, we have that

$$x(t)e^{-\sigma t} = 3e^{-0.5t}u(t) + 4e^{0.5t}u(t)$$

The first term has now been sufficiently weighted that it decays to 0 as t goes to infinity. However, since the second term is still growing exponentially, the Fourier transform does not converge.

(iii) For the case $\sigma = 3.5$, we have that

$$x(t)e^{-\sigma t} = 3e^{-1.5t}u(t) + 4e^{-0.5t}u(t)$$

Both terms do decay as t goes to infinity, and the Fourier transform converges. We note that for any value of $\sigma > 3.0$, the signal $x(t)e^{-\sigma t}$ decays exponentially, and the Fourier transform converges.

(c) The Laplace transform of $x(t)$ is

$$X(s) = \frac{3}{s-2} + \frac{4}{s-3} = \frac{7(s - \frac{17}{7})}{(s-2)(s-3)},$$

and its pole-zero plot and ROC are as shown in Figure S20.1.

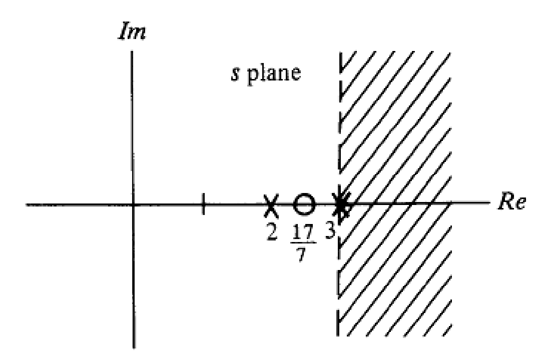


Figure S20.1

Note that if $\sigma > 3.0$, $s = \sigma + j\omega$ is in the region of convergence because, as we showed in part (b)(iii), the Fourier transform converges.

P20.4

Determine $x(t)$ for the following conditions if $X(s)$ is given by

$$X(s) = \frac{1}{(s + 1)(s + 2)}$$

(a) $x(t)$ is right-sided

(b) $x(t)$ is left-sided

(c) $x(t)$ is two-sided

S20.4

(a) For $x(t)$ right-sided, the ROC is to the right of the rightmost pole, as shown in Figure S20.4-1.

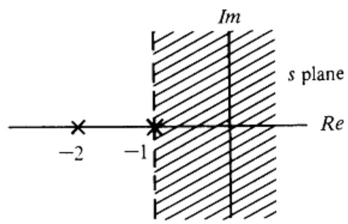


Figure S20.4-1

Using partial fractions,

$$X(s) = \frac{1}{(s+1)(s+2)} = \frac{1}{s+1} - \frac{1}{s+2},$$

so, by inspection,

$$x(t) = e^{-t}u(t) - e^{-2t}u(t)$$

(b) For $x(t)$ left-sided, the ROC is to the left of the leftmost pole, as shown in Figu S20.4-2.

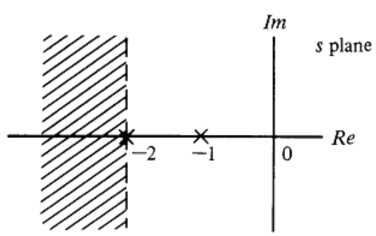


Figure S20.4-2

Since

$$X(s) = \frac{1}{s+1} - \frac{1}{s+2},$$

we conclude that

$$x(t) = -e^{-t}u(-t) - (-e^{-2t}u(-t))$$

(c) For the two-sided assumption, we know that $x(t)$ will have the form

$$f_1(t)u(-t) + f_2(t)u(t)$$

We know the inverse Laplace transforms of the following:

$$\frac{1}{s+1} = \begin{cases} e^{-t}u(t), & \text{assuming right-sided,} \\ -e^{-t}u(-t), & \text{assuming left-sided,} \end{cases}$$

$$\frac{1}{s+2} = \begin{cases} e^{-2t}u(t), & \text{assuming right-sided,} \\ -e^{-2t}u(-t), & \text{assuming left-sided} \end{cases}$$

Which of the combinations should we choose for the two-sided case? Suppose we choose

$$x(t) = e^{-t}u(t) + (-e^{-2t})u(-t)$$

We ask, For what values of σ does $x(t)e^{-\sigma t}$ have a Fourier transform? And we see that there are no values. That is, suppose we choose $\sigma > -1$, so that the first term has a Fourier transform. For $\sigma > -1$, $e^{-2t}e^{-\sigma t}$ is a growing exponential as t goes to negative infinity, so the second term does not have a Fourier transform. If we increase σ, the first term decays faster as t goes to infinity, but the second term grows faster as t goes to negative infinity. Therefore, choosing $\sigma > -1$ will not yield a Fourier transform of $x(t)e^{-\sigma t}$. If we choose $\sigma \le -1$, we note that the first term will not have a Fourier transform. Therefore, we conclude that our choice of the two-sided sequence was wrong. It corresponds to the *invalid* region of convergence shown in Figure S20.4-3.

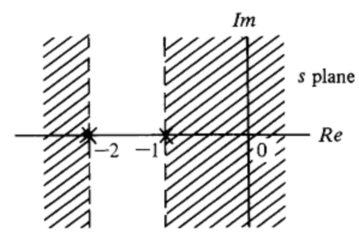


Figure S20.4-3

If we choose the other possibility,

$$x(t) = -e^{-t}u(-t) - e^{-2t}u(t),$$

we see that the valid region of convergence is as given in Figure S20.4-4.

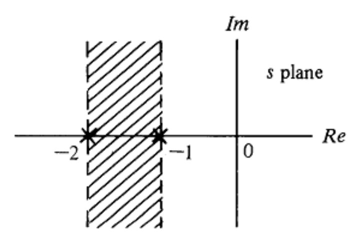


Figure S20.4-4

P20.5

An LTI system has an impulse response $h(t)$ for which the Laplace transform $H(s)$ is

$$H(s) = \int_{-\infty}^{+\infty} h(t)e^{-st}dt = \frac{1}{s+1}, \qquad Re\{s\} > -1$$

Determine the system output $y(t)$ for all t if the input $x(t)$ is given by

$$x(t) = e^{-t/2} + 2e^{-t/3} \qquad \text{for all } t.$$

We consider the solution of this problem as the superposition of the respon to two signals $x_1(t)$, $x_2(t)$, where $x_1(t)$ is the noncausal part of $x(t)$ and $x_2(t)$ is th causal part of $x(t)$. That is,

$$x_1(t) = e^{-t/2}u(-t) + 2e^{-t/3}u(-t),$$
$$x_2(t) = e^{-t/2}u(t) + 2e^{-t/3}u(t)$$

This allows us to use Laplace transforms, but we must be careful about the ROCs

Now consider $\mathcal{L}\{x_1(t)\}$, where $\mathcal{L}\{\cdot\}$ denotes the Laplace transform:

$$\mathcal{L}\{x_1(t)\} = X_1(s) = -\frac{1}{s+\frac{1}{2}} - \frac{2}{s+\frac{1}{3}}, \qquad Re\{s\} < -\frac{1}{2}$$

Now since the response to $x_1(t)$ is

$$y_1(t) = \mathcal{L}^{-1}\{X_1(s)H(s)\},$$

then

$$Y_1(s) = -\frac{1}{(s+1)(s+\frac{1}{2})} - \frac{2}{(s+\frac{1}{3})(s+1)}, \qquad -1 < Re\{s\} < -\frac{1}{2},$$
$$= \frac{2}{s+1} + \frac{-2}{s+\frac{1}{2}} + \frac{-3}{s+\frac{1}{3}} + \frac{3}{s+1},$$
$$= \frac{5}{s+1} - \frac{2}{s+\frac{1}{2}} - \frac{3}{s+\frac{1}{3}},$$

so

$$y_1(t) = 5e^{-t}u(t) + 2e^{-t/2}u(-t) + 3e^{-t/3}u(-t)$$

The pole-zero plot and associated ROC for $Y_1(s)$ is shown in Figure S20.5-1.

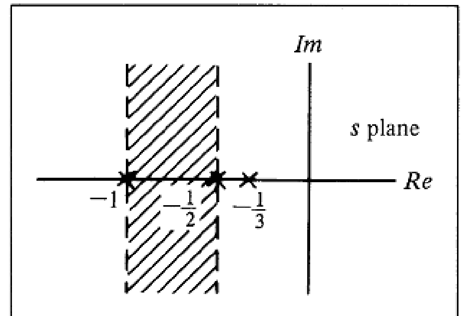


Figure S20.5-1

Next consider the response $y_2(t)$ to $x_2(t)$:

$$x_2(t) = e^{-t/2}u(t) + 2e^{-t/3}u(t),$$
$$X_2(s) = \frac{1}{s+\frac{1}{2}} + \frac{2}{s+\frac{1}{3}}, \qquad Re\{s\} > -\frac{1}{3},$$
$$Y_2(s) = X_2(s)H(s) = \frac{1}{(s+\frac{1}{2})(s+1)} + \frac{2}{(s+\frac{1}{3})(s+1)},$$
$$Y_2(s) = \frac{2}{s+\frac{1}{2}} + \frac{-2}{s+1} + \frac{3}{s+\frac{1}{3}} + \frac{-3}{s+1},$$

so

$$y_2(t) = -5e^{-t}u(t) + 2e^{-t/2}u(t) + 3e^{-t/3}u(t)$$

The pole-zero plot and associated ROC for $Y_2(s)$ is shown in Figure S20.5-2.

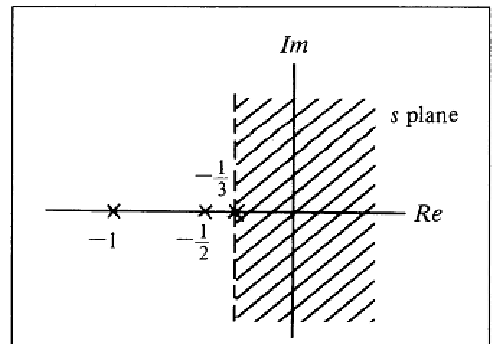


Figure S20.5-2

Since $y(t) = y_1(t) + y_2(t)$, then

$$y(t) = 2e^{-t/2} + 3e^{-t/3} \qquad \text{for all } t$$

P21.2

Consider the LTI system with input $x(t) = e^{-t}u(t)$ and impulse response $h(t) = e^{-2t}u(t)$.

(a) Determine $X(s)$ and $H(s)$.

(b) Using the convolution property of the Laplace transform, determine $Y(s)$, the Laplace transform of the output, $y(t)$.

(c) From your answer to part (b), find $y(t)$.

S21.2

(a) By definition,

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}\,dt = \int_{0}^{\infty} e^{-t}e^{-st}\,dt$$

We limit the integral to $(0, \infty)$ because of $u(t)$, so

$$X(s) = \int_{0}^{\infty} e^{-(1+s)t}\,dt = \frac{-1}{1+s}e^{-(1+s)t}\Big|_{0}^{\infty}$$

If the real part of $(1 + s)$ is positive, i.e., $Re\{s\} > -1$, then

$$\lim_{t\to\infty} e^{-(1+s)t} = 0$$

Thus

$$X(s) = \frac{0(-1)}{1+s} - \frac{1(-1)}{1+s} = \frac{1}{1+s}, \qquad Re\{s\} > -1$$

The condition on $Re\{s\}$ is the ROC and basically indicates the region for which $1/(1 + s)$ is equal to the integral defined originally. Similarly,

$$H(s) = \int_{-\infty}^{\infty} e^{-2t}u(t)e^{-st}\,dt = \int_{0}^{\infty} e^{-(2+s)t}\,dt = \frac{1}{s+2}, \qquad Re\{s\} > -2$$

b) By the convolution property of the Laplace transform, $Y(s) = H(s)X(s)$ in a manner similar to the property of the Fourier transform. Thus,

$$Y(s) = \frac{1}{(s+1)(s+2)}, \qquad Re\{s\} > -1,$$

where the ROC is the intersection of individual ROCs.

c) Here we can use partial fractions:

$$\frac{1}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2},$$

$$A = Y(s)(s+1)\Big|_{s=-1} = 1,$$

$$B = Y(s)(s+2)\Big|_{s=-2} = -1$$

Thus,

$$Y(s) = \frac{1}{s+1} - \frac{1}{s+2}, \qquad Re\{s\} > -1$$

Recognizing the individual Laplace transforms, we have

$$y(t) = e^{-t}u(t) - e^{-2t}u(t)$$

P21.4

Consider a continuous-time LTI system for which the input $x(t)$ and output $y(t)$ are related by the differential equation

$$\frac{d^2y(t)}{dt^2} - \frac{dy(t)}{dt} - 2y(t) = x(t)$$

Let $X(s)$ and $Y(s)$ denote the Laplace transforms of $x(t)$ and $y(t)$, and let $H(s)$ denote the Laplace transform of the impulse response $h(t)$ of the preceding system.

(a) Determine $H(s)$. Sketch the pole-zero plot.

(b) Sketch the ROC for each of the following cases:

- (i) The system is stable.
- (ii) The system is causal.
- (iii) The system is neither stable nor causal.

(c) Determine $h(t)$ when the system is causal.

S21.4

(a) From the properties of the Laplace transform,

$$Y(s) = X(s)H(s)$$

A second relation occurs due to the differential equation. Since

$$\frac{d^k x(t)}{dt^k} \overset{\mathcal{L}}{\longleftrightarrow} s^k X(s)$$

and using the linearity property of the Laplace transform, we can take Laplace transform of both sides of the differential equation, yielding

$$s^2Y(s) - sY(s) - 2Y(s) = X(s).$$

Therefore,

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2 - s - 2} = \frac{1}{(s-2)(s+1)}$$

The pole-zero plot is shown in Figure S21.4-1.

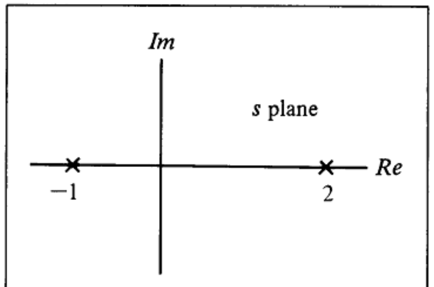


Figure S21.4-1

(b) (i) For a stable system, the ROC must include the $j\omega$ axis. Thus the ROC must be as drawn in Figure S21.4-2.

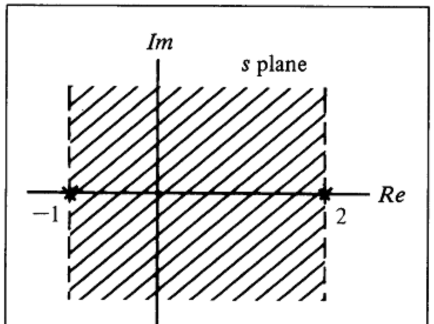


Figure S21.4-2

(ii) For a causal system, the ROC must be to the right of the rightmost pole, as shown in Figure S21.4-3.

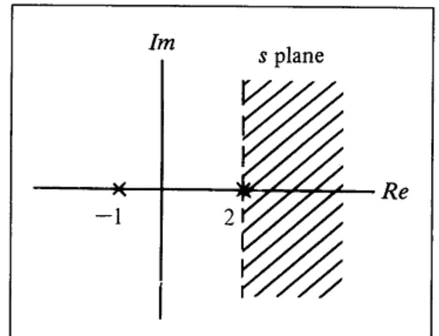


Figure S21.4-3

(iii) For a system that is not causal or stable, we are left with an ROC that is to the left of $s = -1$, as shown in Figure S21.4-4.

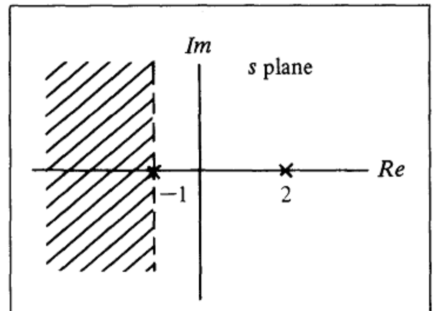


Figure S21.4-4

(c) To take the inverse Laplace transform, we use the partial fraction expansion:

$$H(s) = \frac{1}{(s+1)(s-2)} = \frac{A}{s+1} + \frac{B}{s-2} = \frac{-\frac{1}{3}}{s+1} + \frac{\frac{1}{3}}{s-2}$$

We now take the inverse Laplace transform of each term in the partial fraction expansion. Since the system is causal, we choose right-sided signals in both cases. Thus,

$$h(t) = -\tfrac{1}{3}e^{-t}u(t) + \tfrac{1}{3}e^{+2t}u(t)$$

P21.5

Consider the following system function $H(s)$ and its corresponding pole-zero plot in Figure P21.5.

$$H(s) = \frac{s}{s+1}$$

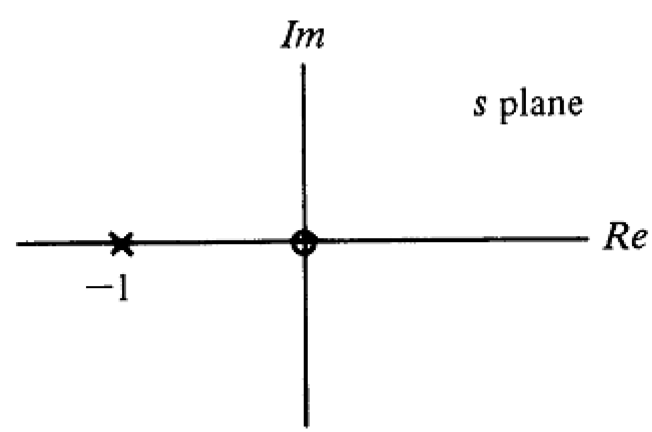


Figure P21.5

Using the graphical method discussed in the lecture, find $|H(0)|$, $\sphericalangle H(0)$, $|H(j1)|$, $\sphericalangle H(j1)$, $|H(j\infty)|$, and $\sphericalangle H(j\infty)$. Sketch the functions $|H(j\omega)|$ and $\sphericalangle H(j\omega)$.

S21.5

$\omega = 0$: Since there is a zero at $s = 0$, $|H(j0)| = 0$. You may think that the phase is also zero, but if we move slightly on the $j\omega$ axis, $\sphericalangle H(j\omega)$ becomes

$$(\text{Angle to } s = 0) - (\text{Angle to } s = -1) = \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

$\omega = 1$: The distance to $s = 0$ is 1 and the distance to $s = -1$ is $\sqrt{2}$. Thus

$$|H(j1)| = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

The phase is

$$(\text{Angle to } s = 0) - (\text{Angle to } s = -1) = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4} = \sphericalangle H(j1)$$

$\omega = \infty$: The distance to $s = 0$ and $s = -1$ is infinite; however, the ratio tends to 1 as ω increases. Thus, $|H(j\infty)| = 1$. The phase is given by

The magnitude and phase of $H(j\omega)$ are given in Figure S21.5.

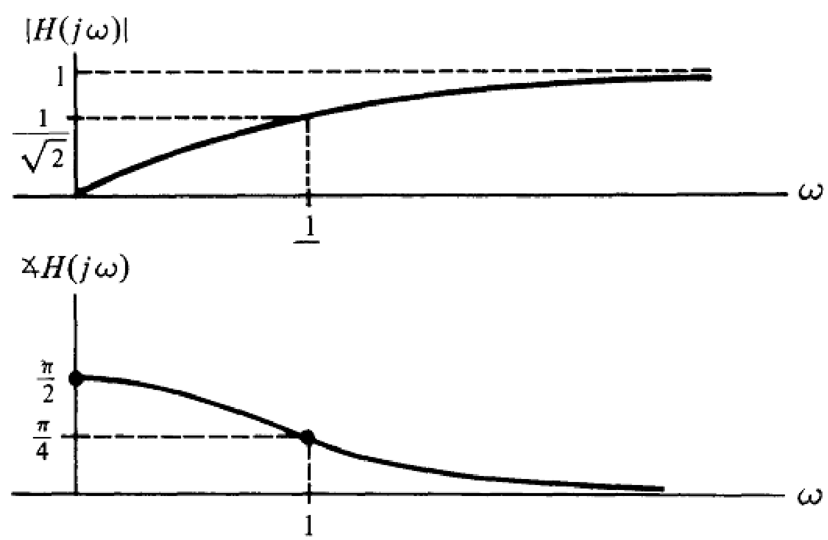


Figure S21.5

P21.7

(a) Draw the block diagram for the following second-order system in terms of integrators, coefficient multipliers, and adders.

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

(b) Sketch the pole-zero plot of $H(s)$ and plot $|H(j\omega)|$ under the following conditions.

(i) ω_n is kept constant, but ζ is varied from close to 0 to close to 1.

(ii) ζ is kept constant, but ω_n is varied from about 0 to infinity.

You don't have to be precise but show how the bandwidth and location of the peak changes for the two cases above.

S21.7

(a) Let $y(t)$ be the system response to the excitation $x(t)$. Then the differential equation relating $y(t)$ to $x(t)$ is

$$\frac{d^2y(t)}{dt^2} + 2\zeta\omega_n \frac{dy(t)}{dt} + \omega_n^2 y(t) = \omega_n^2 x(t)$$

Integrating twice, we have

$$y(t) + 2\zeta\omega_n \int_{-\infty}^{t} y(\tau)\,d\tau + \omega_n^2 \int_{-\infty}^{t}\int_{-\infty}^{\tau'} y(\tau)\,d\tau\,d\tau' = \omega_n^2 \int_{-\infty}^{t}\int_{-\infty}^{\tau'} x(\tau)\,d\tau\,d\tau',$$

or

$$y(t) = -2\zeta\omega_n \int_{-\infty}^{t} y(\tau)\,d\tau - \omega_n^2 \int_{-\infty}^{t}\int_{-\infty}^{\tau'} y(\tau)\,d\tau\,d\tau' + \omega_n^2 \int_{-\infty}^{t}\int_{-\infty}^{\tau'} x(\tau)\,d\tau\,d\tau',$$

shown in Figure S21.7-1.

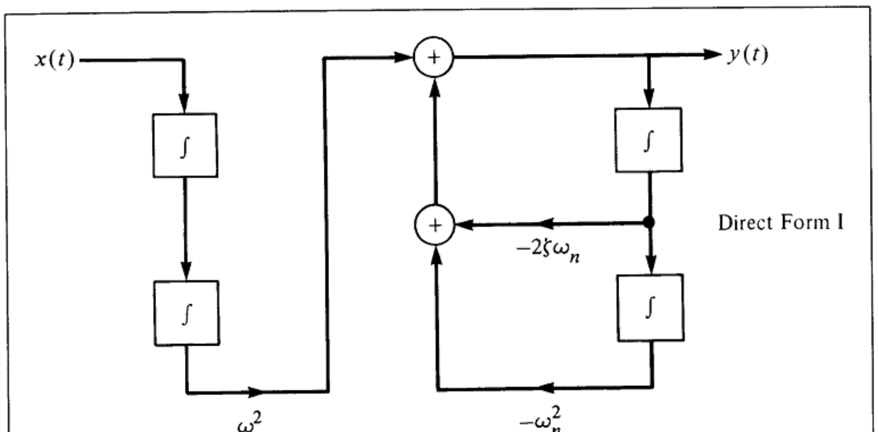


Figure S21.7-1

Recall that Figure S21.7-1 can be simplified as given in Figure S21.7-2.

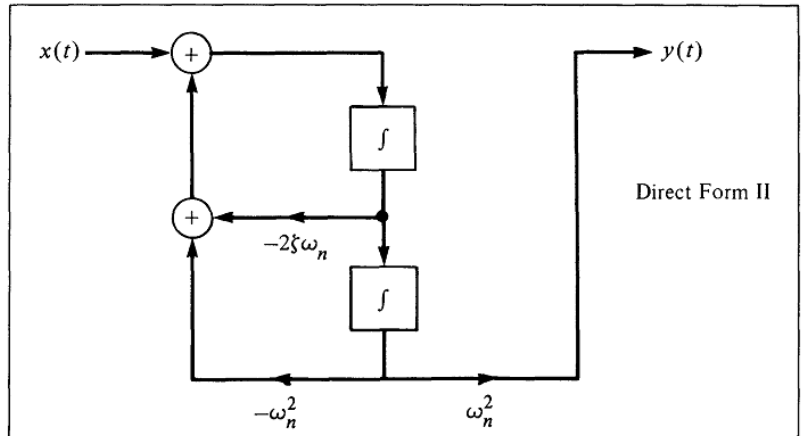


Figure S21.7-2

(b) (i) For a constant ω_n and $0 \leq \zeta < 1$, $H(s)$ has a conjugate pole pair on a circle centered at the origin of radius ω_n. As ζ changes from 0 to 1, the poles move from close to the $j\omega$ axis to $-\omega_n$, as shown in Figures S21.7-3, S21.7-4, and S21.7-5.

Figure S21.7-3 shows that for $\zeta \simeq 0$ the pole is close to the $j\omega$ axis, so $|H(j\omega)|$ has a peak very near ω_n.

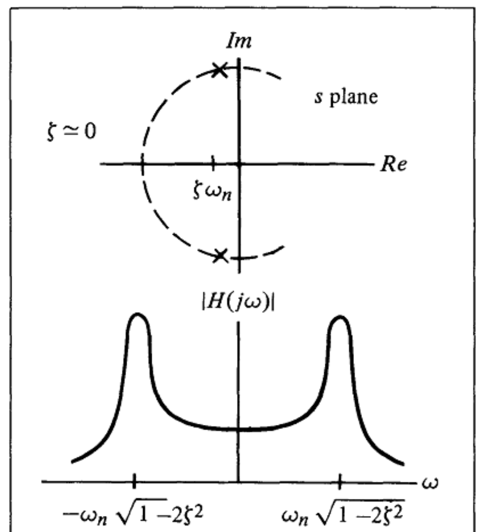


Figure S21.7-3

Figure S21.7-4 shows that the peaks are closer together and more spread out at $\zeta = 0.5$.

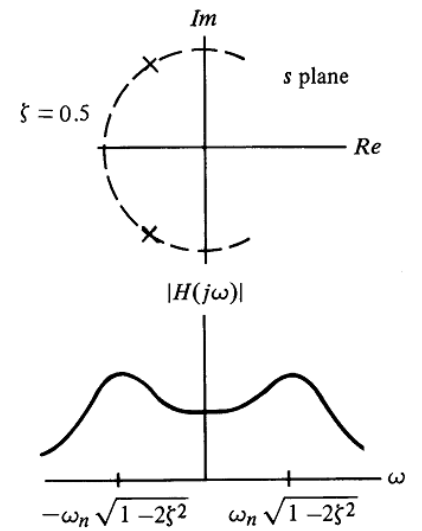


Figure S21.7-4

Figure S21.7-5 shows that at $\zeta \simeq 1$ the poles are so close together and far from the $j\omega$ axis that $|H(j\omega)|$ has a single peak.

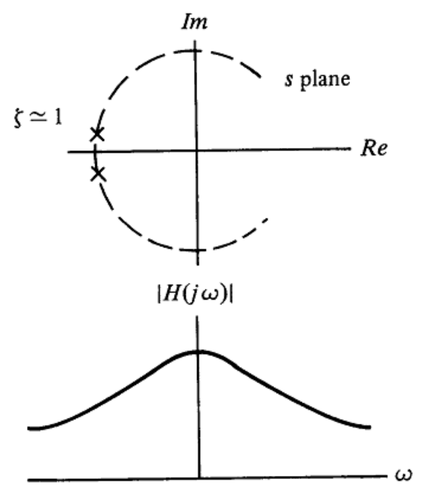


Figure S21.7-5

(ii) For constant ζ between 0 and 1, the poles are located on two straight lines. As ω_n increases, the peak frequency increases as well as the bandwidth, as indicated in Figures S21.7-6 and S21.7-7.

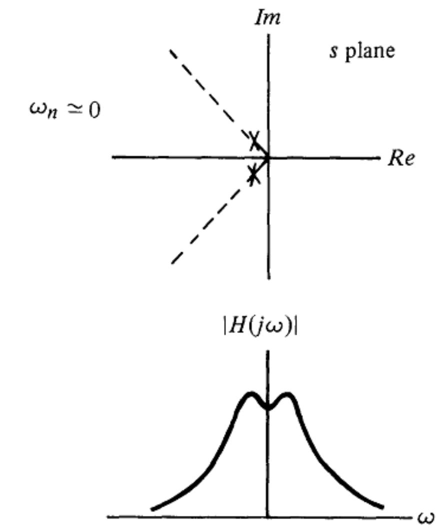


Figure S21.7-6

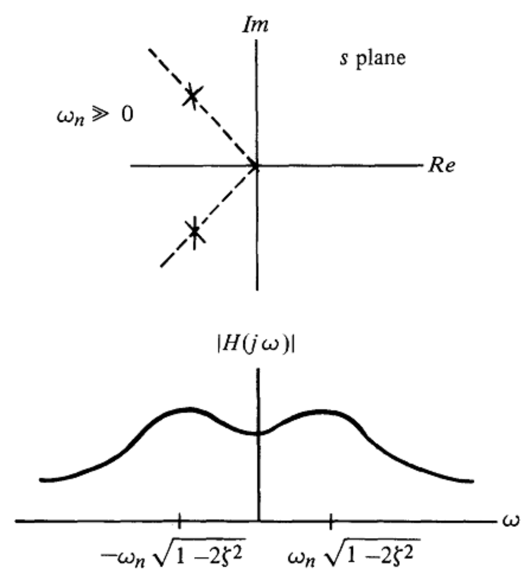


Figure S21.7-7

P21.8

(a) Consider the following system function $H(s)$.

$$H(s) = \frac{s}{s^2 + s + 1} + \frac{1}{s^2 + 2s + 2}$$

Draw the block diagram for $H(s)$ implemented as

(i) a parallel combination of second-order systems,

(ii) a cascade combination of second-order systems.

(b) Is implementation (ii) unique?

S21.8

(a) (i) The parallel implementation of $H(s)$, shown in Figure S21.8-1, can be drawn directly from the form for $H(s)$ given in the problem statement. The corresponding differential equations for each section are as follows:

$$\frac{d^2y_1(t)}{dt^2} + \frac{dy_1(t)}{dt} + y_1(t) = \frac{dx(t)}{dt},$$

$$\frac{d^2y_2(t)}{dt^2} + \frac{2dy_2(t)}{dt} + 2y(t) = x(t),$$

$$y(t) = y_1(t) + y_2(t)$$

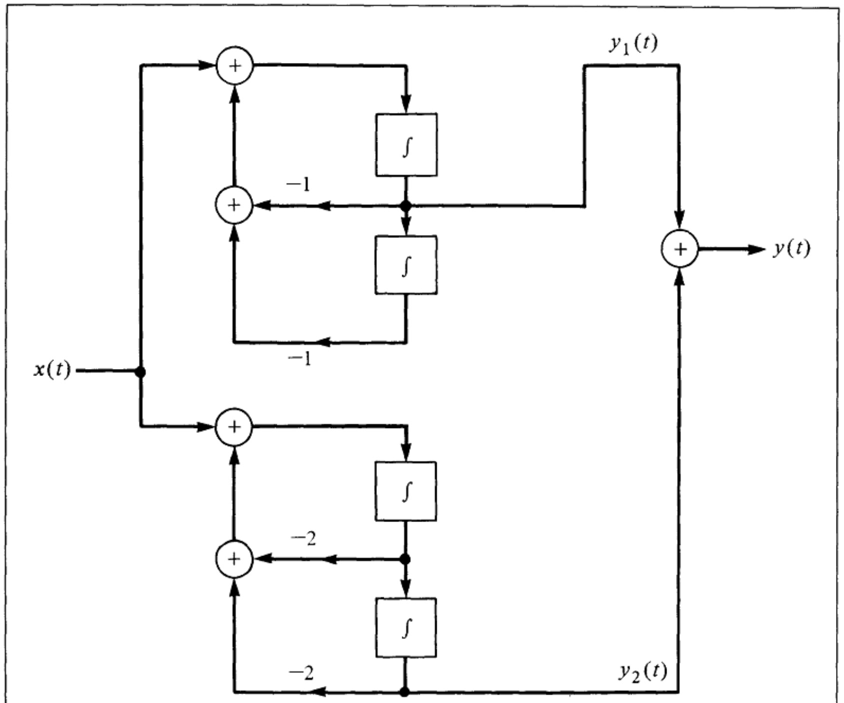


Figure S21.8-1

(ii) To generate the cascade implementation, shown in Figure S21.8-2, we first express $H(s)$ as a product of second-order sections. Thus,

$$H(s) = \frac{s(s^2 + 2s + 2) + (s^2 + s + 1)}{(s^2 + s + 1)(s^2 + 2s + 2)} = \frac{s^3 + 3s^2 + 3s + 1}{(s^2 + s + 1)(s^2 + 2s + 2)}$$

Now we need to separate the numerator into two sections. In this case, the numerator equals $(s + 1)^3$, so an obvious choice is

$$(s + 1)(s^2 + 2s + 1)$$

Thus,

$$H(s) = \left(\frac{s + 1}{s^2 + s + 1}\right)\left(\frac{s^2 + 2s + 1}{s^2 + 2s + 2}\right)$$

The corresponding differential equations are as follows:

$$\frac{d^2r(t)}{dt^2} + \frac{dr(t)}{dt} + r(t) = x(t) + \frac{dx(t)}{dt},$$

$$\frac{d^2y(t)}{dt^2} + \frac{2dy(t)}{dt} + 2y(t) = \frac{d^2r(t)}{dt^2} + \frac{2dr(t)}{dt} + r(t)$$

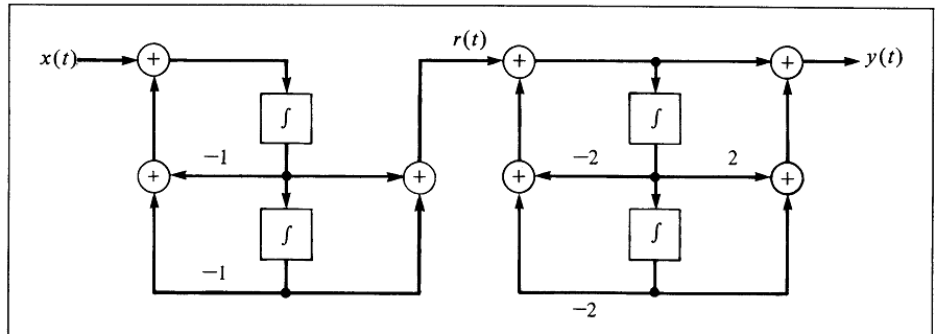


Figure S21.8-2

(b) We see that we could have decomposed $H(s)$ as

$$H(s) = \left(\frac{s^2 + 2s + 1}{s^2 + s + 1}\right)\left(\frac{s + 1}{s^2 + 2s + 2}\right)$$

Thus, the cascade implementation is not unique.

9.28. (a) The possible ROCs are

(i) $\mathcal{R}e\{s\} < -2$.

(ii) $-2 < \mathcal{R}e\{s\} < -1$.

(iii) $-1 < \mathcal{R}e\{s\} < 1$.

(iv) $\mathcal{R}e\{s\} > 1$.

(b) (i) Unstable and anticausal.

(ii) Unstable and non causal.

(iii) Stable and non causal.

(iv) Unstable and causal.

9.28. Consider an LTI system for which the system function $H(s)$ has the pole-zero pattern shown in Figure P9.28.

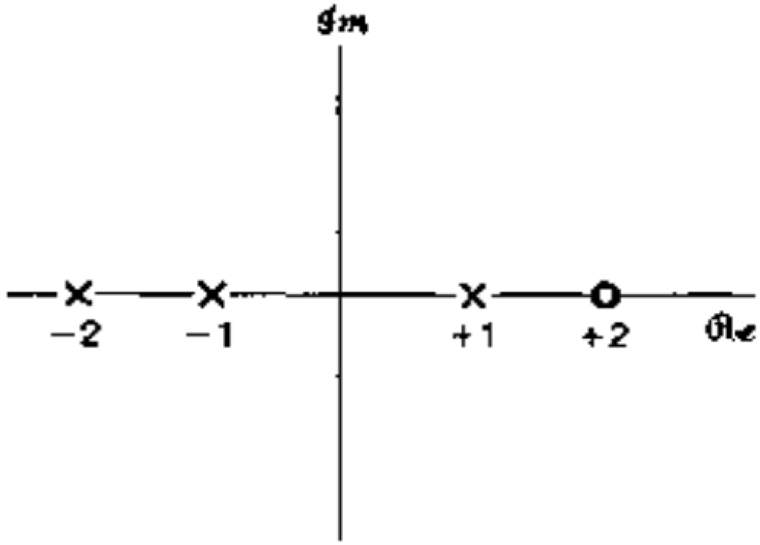


Figure P9.28

(a) Indicate all possible ROCs that can be associated with this pole-zero pattern.

(b) For each ROC identified in part (a), specify whether the associated system is stable and/or causal.

9.30. A pressure gauge that can be modeled as an LTI system has a time response to a unit step input given by $(1 - e^{-t} - te^{-t})u(t)$. For a certain input $x(t)$, the output is observed to be $(2 - 3e^{-t} + e^{-3t})u(t)$.

For this observed measurement, determine the true pressure input to the gauge as a function of time.

9.30. For the input $x(t) = u(t)$, the Laplace transform is

$$X(s) = \frac{1}{s}, \qquad \mathcal{R}e\{s\} > 0.$$

The corresponding output $y(t) = [1 - e^{-t} - te^{-t}]u(t)$ has the Laplace transform

$$Y(s) = \frac{1}{s} - \frac{1}{s+1} - \frac{1}{(s+1)^2} = \frac{1}{s(s+1)^2}, \qquad \mathcal{R}e\{s\} > 0.$$

Therefore,

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{(s+1)^2}, \qquad \mathcal{R}e\{s\} > 0.$$

Now, the output $y_1(t) = [2 - 3e^{-t} + e^{-3t}]u(t)$ has the Laplace transform

$$Y_1(s) = \frac{2}{s} - \frac{3}{s+1} + \frac{1}{s+3} = \frac{6}{s(s+1)(s+3)}, \qquad \mathcal{R}e\{s\} > 0.$$

Therefore, the Laplace transform of the corresponding input will be

$$X_1(s) = \frac{Y_1(s)}{H(s)} = \frac{6(s+1)}{s(s+3)}, \qquad \mathcal{R}e\{s\} > 0.$$

Taking the inverse Laplace transform of the partial fraction expansion of $X_1(s)$, we obtain

$$x_1(t) = 2u(t) + 4e^{-3t}u(t).$$

Considere o sinal

$$x(t) = e^{-5t}u(t) + e^{-\beta t}u(t)$$

e que a sua Transformada de Laplace é $X(s)$. Quais são as restrições a serem impostas sobre a parte real e imaginária de β para que a região de convergência de $X(s)$ seja $\Re(s) > -3$?

1) (9.3)

$x(t) = e^{-5t}u(t) + e^{-\beta t}u(t)$

$X(s) = \mathcal{L}\{x(t)\}$

$= \dfrac{1}{s+5} + \dfrac{1}{(s+\beta)} = \dfrac{[2s+(\beta+5)]}{(s+5)(s+\beta)}$

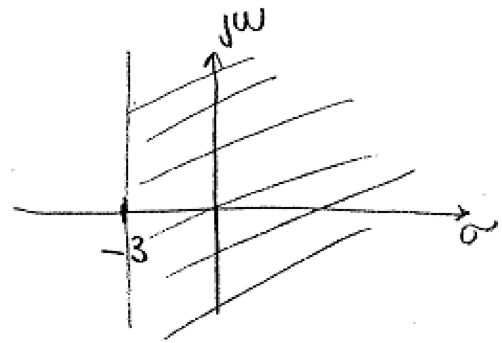
RDC : $Re\{s\} > \max(-5, Re\{\beta\})$

se $Re\{s\} > -3$

$\Rightarrow Re\{\beta\} = -3$

$Im\{\beta\} = bj, \quad \forall b \in \mathbb{R}$

$x(t) = e^{-at}u(t) \overset{\mathcal{L}}{\longleftrightarrow} X(s) = \dfrac{1}{s+a}$



⇐ sinais abertos à direita

• EXERCÍCIO **0.3** (Op 9.7) região de convergência

Determine quantos sinais tem Tranformada de Laplace expressa por

$$\frac{s-1}{(s+2)(s+3)(s^2+s+1)}$$

considerando suas regiões de convergência.

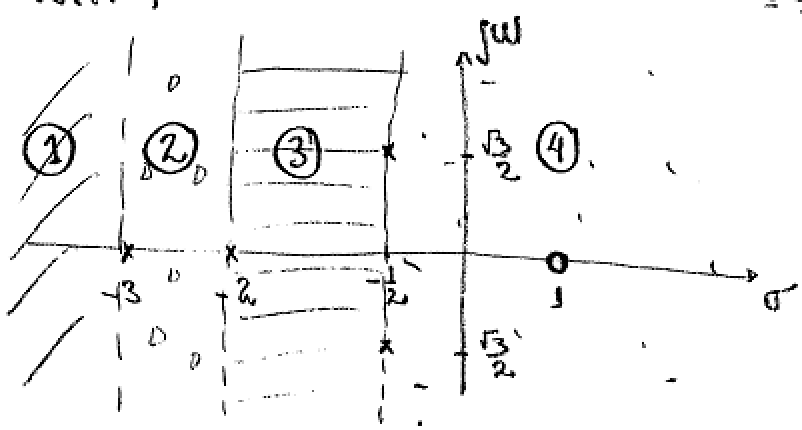
3) (9.7)

$$x(t) \xleftrightarrow{\mathcal{L}} X(s) = \frac{(s-1)}{(s+2)(s+3)\underbrace{(s^2+s+1)}}$$

$\sqrt{1-4}$

$x(t)$?

$= \frac{-1}{2} \pm j\frac{\sqrt{3}}{2} \Rightarrow \left[s-\left(-\frac{1}{2}-j\frac{\sqrt{3}}{2}\right)\right]\left[s-\left(-\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)\right]$



$\Rightarrow$ Considerando que existem 4 possíveis RDC

$\Downarrow$

existem 4 possíveis sinais bilaterais associados à esta transformada de Laplace

$\Rightarrow$ Observação

$x_1(t) = g(t)\,u(t)$ $\quad \overset{\mathcal{L}}{\Rightarrow} X(s)$ $\quad$ RDC : $Re\{s\} > \alpha$,

$x_2(t) = -g(t).\,u(-t)$ $\quad\quad\quad\quad$ $Re\{s\} < \alpha$, em que $\alpha \in \mathbb{R}^+$

exemplo: $x_1(t) = e^{at}\,u(t)$

$x_2(t) = -e^{at}\,u(-t)$

- EXERCÍCIO **0.4** (Op 9.14)

Suponha que os seguintes fatos sobre o sinal $x(t)$ com Transformada de Laplace $X(s)$ foram dados:

1. $x(t)$ é real e par
2. $X(s)$ tem quatro pólos finitos mas nenhum zero finito
3. $X(s)$ tem um pólo em $s = \frac{e^{j\pi/4}}{2}$
4. $\int_{-\infty}^{\infty} x(t)dt = 4$

4) (9.14)

$x(t) \xleftrightarrow{\mathcal{L}} X(s)$

i) $x(t) \in \mathbb{R}$, $x(t) = x(-t)$

ii) $X(s)$ tem 4 pólos finitos, n

iii) $X(s)$ tem um pólo em $s =$

iv) $\int_{-\infty}^{\infty} x(t)\,dt = 4$

i) Se $x(t)$ é par $\Rightarrow$ $x(t)$ é bil

ii) $X(s) = \dfrac{K}{(s+a)(s+b)(s+c)(s+}$

iii) $s = \dfrac{e^{j\pi/4}}{2} = \dfrac{\cos \pi/4}{2} + j\,\dfrac{\sin \pi/4}{2} = \dfrac{\sqrt{2}}{4} + j\dfrac{\sqrt{2}}{4}$

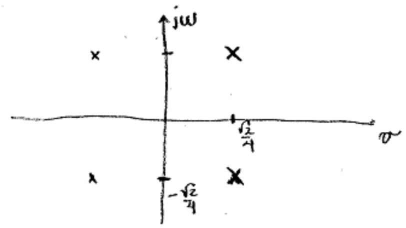
i) $x(t) \in \mathbb{R}$

$a = \dfrac{\sqrt{2}}{4} + j\dfrac{\sqrt{2}}{4}$, $b = \dfrac{\sqrt{2}}{4} - j\dfrac{\sqrt{2}}{4}$

i) e iii)

se $x(t)$ é par $\Rightarrow$ apresenta simetria de pólos

$x(t)$ é par $\Rightarrow X(s) = X(-s)$



$\Rightarrow$ $c = -\dfrac{\sqrt{2}}{4} + j\dfrac{\sqrt{2}}{4}$, $d = -\dfrac{\sqrt{2}}{4} - j\dfrac{\sqrt{2}}{4}$

$\left(s - \frac{\sqrt{2}}{4} - j\frac{\sqrt{2}}{4}\right)\left(s - \frac{\sqrt{2}}{4} + j\frac{\sqrt{2}}{4}\right)$

$s^2 - \frac{\sqrt{2}}{2}s + \frac{2}{16} + \frac{2}{16}$

iv)

Se $X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\,dt$

$s = 0 \Rightarrow X(0) = \int_{-\infty}^{\infty} x(t)\,dt$

$\therefore X(s) = \dfrac{K}{\left(s^2 - \frac{\sqrt{2}}{2}s + \frac{1}{4}\right)\left(s^2 + \frac{\sqrt{2}}{2}s + \frac{1}{4}\right)}$

$X(0) = \dfrac{K}{\frac{1}{4}\cdot\frac{1}{4}} = 4 \Rightarrow K = 1/4$

$\Rightarrow X(s) = \dfrac{1/4}{\left(s^2 - \frac{\sqrt{2}}{2}s + \frac{1}{4}\right)\left(s^2 + \frac{\sqrt{2}}{2}s + \frac{1}{4}\right)}$

$X(s) = \int_{-\infty}^{\infty} x(-t+a)\, e^{-st}\,dt$

$\tau = -t + a \Rightarrow t = -\tau + a$ $\quad -\infty \to +\infty$

$d\tau = -dt$ $\quad \infty \to -\infty$

$X(s) = -\int_{\infty}^{-\infty} x(\tau)\, e^{-s(-\tau+a)}\,d\tau$

$= \int_{-\infty}^{\infty} x(\tau)\, e^{-s(-\tau)}\, e^{-sa}\,d\tau$

$= e^{-as} \int_{-\infty}^{\infty} x(\tau)\, e^{-(-s)\tau}\,d\tau$

$= e^{-as}\, X(-s)$

• EXERCÍCIO 0.9 (Op. 9.66)

Considere o circuito RL mostrado abaixo. Assuma que a corrente $i(t)$ tenha alcançado o regime permamente com a chave na posição A. No tempo $t = 0$, a chave é movida da posição A para B.

(a) Encontre a equação diferencial relacionando $i(t)$ e $v_2(t)$ para $t < 0^-$. Determine a condição inicial, $i(0^-)$, para a equação diferencial em termos de $v_1(t)$.

(b) Usando as propriedades da Transformada de Laplace Unilateral, determine e esboce a corrente $i(t)$ para cada um dos seguintes valores de $v_1(t)$ e $v_2(t)$:

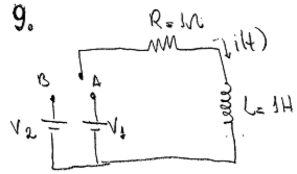
(i) $v_1 = 0\ V,\ v_2 = 2\ V$

(ii) $v_1 = 4V,\ v_2 = 0\ V$

(iii) $v_1 = 4V,\ v_2 = 2\ V$. O que pode ser dito desta condição em relação às outras?

ap. 5.44

9.



a) $v_1(t) = RI(t) + L\dot{i}(t)$, em que $t < 0^-$ significa o tempo quando a chave A está aberta

$V_1(s) = RI(s) + LsI(s)$

$I(s) = \dfrac{V_1(s)}{Ls + R}$

$V_2(s) = RI(s) + L(sI(s) - i(0^-))$

$V_2(s) + Li(0^-) = I(s) \cdot (Ls + R)$

$I(s) = \dfrac{V_2(s)}{Ls+R} + \dfrac{Li(0^-)}{Ls+R}$

se $V_1(s) = \dfrac{V_1}{s}$ $\Rightarrow$ $i(0^-) = \lim_{t\to\infty} i(t) = \lim_{s\to 0} sI(s) = \lim_{s\to 0} \cancel{s} \dfrac{V_1}{\cancel{s}} \cdot \dfrac{1}{L\cancel{s}+R}$

$= \dfrac{V_1}{R} = V_1$

b) $i(t) = ?$

1) $v_1 = 0$, $\underline{v_2 = 2V}$ $\Rightarrow$ CI = 0

$V_2(s) = \dfrac{2}{s}$

$I(s) = \dfrac{2/s}{s+1} = \dfrac{2}{(s+1)s} = \dfrac{A}{s} + \dfrac{B}{(s+1)}$

$A(s+1) + B(s) = 2$

$(A+B) = 0$

$A = 2 \Rightarrow B = -2$

$= \dfrac{2}{s} - \dfrac{2}{s+1}$

$i(t) = 2u(t) - 2e^{-t}u(t)$

2) $v_1 = 4$ $v_2 = 0$

$I(s) = \dfrac{\cancel{V_2(s)}^{\,0}}{Ls+R} + \dfrac{L \cdot V_1/R}{Ls+R} = \dfrac{1 \cdot 4/1}{s+1} = \dfrac{4}{s+1}$

$i(t) = 4e^{-t}u(t)$

3) $v_1 = 4$ $v_2 = 2$

$i(t) = i_1(t) + i_2(t)$ $\Rightarrow$ princípio da superposição

9.15. Consider two right-sided signals $x(t)$ and $y(t)$ related through the differential equations

$$\frac{dx(t)}{dt} = -2y(t) + \delta(t)$$

and

$$\frac{dy(t)}{dt} = 2x(t).$$

Determine $Y(s)$ and $X(s)$, along with their regions of convergence.

5) 0-9.15

$x(t)$, $y(t)$ são sinais à direita

$$\begin{cases} \dot{x} = -2y + \delta(t) \\ \dot{y} = 2x \end{cases}$$

$X(s)$ e $Y(s)$ e RDC?

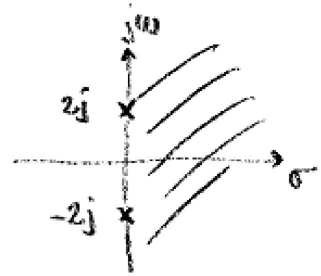
$$\xrightarrow{\mathcal{L}} \begin{cases} s X(s) = -2 Y(s) + 1 \\ s Y(s) = 2 X(s) \end{cases} \Rightarrow s \cdot \left(s \frac{Y(s)}{2} \right) + 2 Y(s) = 1$$

$$(s^2 + 4) Y(s) = 2$$

$$\therefore Y(s) = \frac{2}{s^2+4}$$

$$X(s) = \frac{s}{s^2+4}$$

$\Rightarrow$ pólos em $s = \pm 2j$



RDC: $\mathcal{R}\{s\} > 0$ pois $x(t)$ e $y(t)$ são lateral à direita

9.26. Consider a signal $y(t)$ which is related to two signals $x_1(t)$ and $x_2(t)$ by

$$y(t) = x_1(t-2) * x_2(-t+3)$$

where

$$x_1(t) = e^{-2t}u(t) \qquad \text{and} \qquad x_2(t) = e^{-3t}u(t).$$

Given that

$$e^{-at}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+a}, \quad \Re e\{s\} > a,$$

use properties of the Laplace transform to determine the Laplace transform $Y(s)$ of $y(t)$.

6) (9.26)

$y(t) = x_1(t-2) * x_2(-t+3)$

em que $x_1(t) = e^{-2t}u(t)$; $e^{-at}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+a}$, $Re\{s\} > -a$.

$x_2(t) = e^{-3t}u(t)$

$Y(s) = ?$

$\Rightarrow x_1(t) \overset{\mathcal{L}}{\longleftrightarrow} X_1(s) = \frac{1}{s+2}$, $Re\{s\} > -2$

$x_2(t) \overset{\mathcal{L}}{\longleftrightarrow} X_2(s) = \frac{1}{s+3}$, $Re\{s\} > -3$

$\Rightarrow$

$x_1(t-2) \overset{\mathcal{L}}{\longleftrightarrow} X_1(s) = \frac{e^{-2s}}{s+2}$, $Re\{s\} > -2$

$\boxed{\begin{array}{c} x(t-\tau) \\ \updownarrow \\ e^{-s\tau}X(s) \end{array}}$

$\underbrace{x_2(-t+3)}_{x_2(-(t-3))} \overset{\mathcal{L}}{\longrightarrow} X_2(s) = \underbrace{\frac{+e^{-3s}}{-s+3}}_{-\frac{e^{-3s}}{(s-3)}}$, $Re\{s\} > -3$

$\boxed{\begin{array}{c} x(-t+\tau) \\ \downarrow \\ e^{-s\tau}X(-s) \end{array}}$

$Y(s) = X_1(s) \cdot X_2(s)$

$= \frac{-e^{-5s}}{(s+2)(s-3)}$, $Re\{s\} > -2$

9.8. Let $x(t)$ be a signal that has a rational Laplace transform with exactly two poles, located at $s = -1$ and $s = -3$. If $g(t) = e^{2t}x(t)$ and $G(j\omega)$ [the Fourier transform of $g(t)$] converges, determine whether $x(t)$ is left sided, right sided, or two sided.

Ex. 9.8

$x(t) \overset{\mathcal{L}}{\longleftrightarrow} X(s)$ é racional

tem pólos em $s = -1$ e $s = -3$

$g(t) = e^{2t} x(t)$ e $G(j\omega)$ converge

$x(t)$ é aberto à esquerda, à direita ou é bilateral?

$\Rightarrow g(t) = e^{2t} x(t) \overset{\mathcal{L}}{\longleftrightarrow} G(s) = X(s-2)$

Logo $G(s)$ tem pólos em $\underbrace{s = 1 \text{ e } s = -1}_{\text{deslocados de 2 para a direita}}$.

Como $G(j\omega)$ existe, a ROC inclui o eixo $j\omega$.

Assim $g(t)$ é um sinal bilateral.

Como $x(t) = e^{-2t} g(t)$, tem-se que $x(t)$ também é bilateral.