Modulo 05 - SF, SFTD, TF, TFTD
Exercícios

Ex 1

A particular discrete-time system has input $x[n]$ and output $y[n]$. The Fourier transforms of these signals are related by the following equation:

$$Y(\Omega) = 2X(\Omega) + e^{-j\Omega}X(\Omega) - \frac{dX(\Omega)}{d\Omega}$$

(a) Is the system linear? Clearly justify your answer.

(b) Is the system time-invariant? Clearly justify your answer.

(c) What is $y[n]$ if $x[n] = \delta[n]$?

Ex 1

Here

$$Y(\Omega) = 2X(\Omega) + e^{-j\Omega}X(\Omega) - \frac{dX(\Omega)}{d\Omega}$$

(a) (i) The system is linear because if

$$x[n] = ax_1[n] + bx_2[n],$$

then

$$y[n] = ay_1[n] + by_2[n],$$

where $y_1[n]$ is obtained from $x_1[n]$ via the given transfer function. The similar result applies for $y_2[n]$.

(ii) The system is time-varying by the following argument.
If $x[n] \to y[n]$, does $x[n-1] \to y[n-1]$?

$$x[n-1] \overset{\mathcal{F}}{\longleftrightarrow} e^{-j\Omega}X(\Omega)$$

The corresponding $Y(\Omega)$ is

$$2e^{j\Omega}X(\Omega) + e^{-j\Omega}X(\Omega)e^{-j\Omega} + je^{-j\Omega}X(\Omega) - e^{-j\Omega}\frac{dX(\Omega)}{d\Omega}$$

$$\neq e^{-j\Omega}\left[2X(\Omega) + e^{-j\Omega}X(\Omega) - \frac{dX(\Omega)}{d\Omega}\right]$$

(iii) If $x[n] = \delta[n]$, $X(\Omega) = 1$. Then

$$Y(\Omega) = 2 + e^{-j\Omega},$$
$$y[n] = 2\delta[n] + \delta[n-1]$$

Ex 2

Suppose we have an LTI system characterized by an impulse response

$$h[n] = \frac{\sin \frac{\pi n}{3}}{\pi n}$$

(a) Sketch the magnitude of the system transfer function.

(b) Evaluate $y[n] = x[n] * h[n]$ when

$$x[n] = (-1)^n \cos \frac{3\pi}{4} n$$

Ex 2

We are given an LTI system with impulse response

$$h[n] = \frac{\sin(\pi n/3)}{\pi n}$$

(a) We know from duality that $H(\Omega)$ is a pulse sequence that is periodic with peri 2π. Suppose we assume this and adjust the parameters of the pulse so that

$$\frac{1}{2\pi}\int H(\Omega)e^{j\Omega n}\,d\Omega = h[n]$$

Let a be the pulse amplitude and let $2W$ be the pulse width. Then

$$\frac{a}{2\pi}\int_{-W}^{W} e^{j\Omega n}\,d\Omega = \frac{a}{2\pi}\left(\frac{e^{j\Omega W} - e^{-j\Omega W}}{jn}\right)$$
$$= \frac{a}{2\pi}\,\frac{2\sin Wn}{n},$$

so $a = 1$ and $W = \pi/3$, as indicated in Figure S11.9-1.

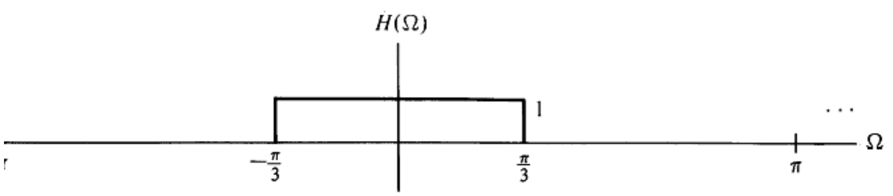


Figure S11.9-1

We know that

$$\cos\frac{3\pi}{4}n \overset{\mathcal{F}}{\longleftrightarrow} \pi\left[\delta\left(\Omega - \frac{3\pi}{4}\right) + \delta\left(\Omega + \frac{3\pi}{4}\right)\right],$$

periodically repeated, and that multiplication by $(-1)^n$ shifts the periodic spectrum by π, so the spectrum $Y(\Omega)$ is as shown in Figure S11.9-2.

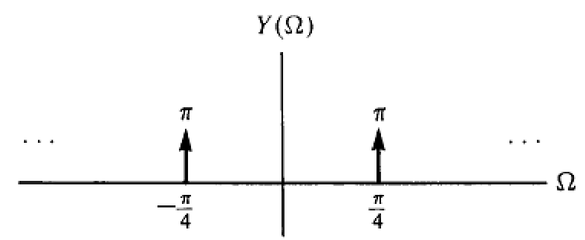


Figure S11.9-2

From Figures S11.9-1 and S11.9-2, we can see that

$$Y(\Omega) = H(\Omega)X(\Omega) = X(\Omega)$$

Therefore,

$$y[n] = x[n] = (-1)^n\cos\frac{3\pi}{4}n = \cos\frac{\pi n}{4}$$

Ex 3

Determine the Fourier series coefficients for each of the following periodic discrete-time signals. Plot the magnitude and phase of each set of coefficients a_k.

(a) $x[n] = \sin\left[\dfrac{\pi(n-1)}{4}\right]$

(b) $x[n] = \cos\left(\dfrac{2\pi n}{3}\right) + \sin\left(\dfrac{2\pi n}{7}\right)$

(c) $x[n] = \cos\left(\dfrac{11\pi n}{4} - \dfrac{\pi}{3}\right)$

Ex 3

(a) $\tilde{x}[n] = \sin\left[\frac{\pi(n-1)}{4}\right]$

To find the period, we set $\tilde{x}[n] = \tilde{x}[n+N]$. Thus,

$$\sin\left[\frac{\pi(n-1)}{4}\right] = \sin\left[\frac{\pi(n+N-1)}{4}\right] = \sin\left[\frac{\pi(n-1)}{4} + \frac{\pi N}{4}\right]$$

Let $(\pi N)/4 = 2\pi i$, when i is an integer. Then $N = 8$ and

$$\tilde{x}[n] = \frac{1}{2j} e^{j[\pi(n-1)/4]} - \frac{1}{2j} e^{-j[\pi(n-1)/4]}$$
$$= \frac{1}{2j} e^{-j(\pi/4)} e^{j(\pi n/4)} - \frac{1}{2j} e^{j(\pi/4)} e^{-j(\pi n/4)}$$

Therefore,

$$a_1 = \frac{e^{-j(\pi/4)}}{2j}, \qquad a_7 = -\frac{e^{j(\pi/4)}}{2j}$$

All other coefficients a_k are zero, in the range $0 \le k \le 7$. The magnitude phase of a_k are plotted in Figure S10.11-1.

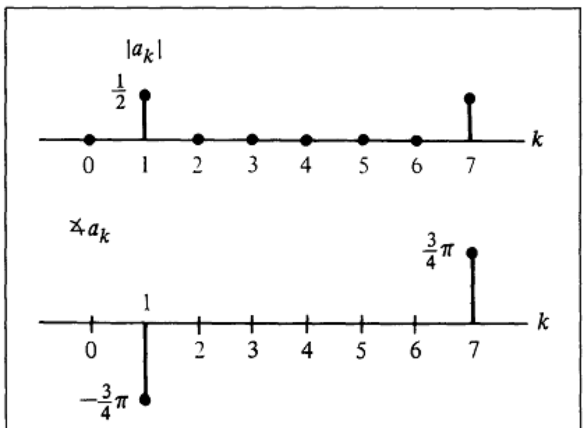


Figure S10.11-1

(b) The period $N = 21$ and the Fourier series coefficients are

$$a_7 = a_{14} = \frac{1}{2}, \qquad a_3 = a_{18}^* = \frac{1}{2j}$$

The rest of the coefficients a_k are zero. The magnitude and phase of a_k are given in Figure S10.11-2.

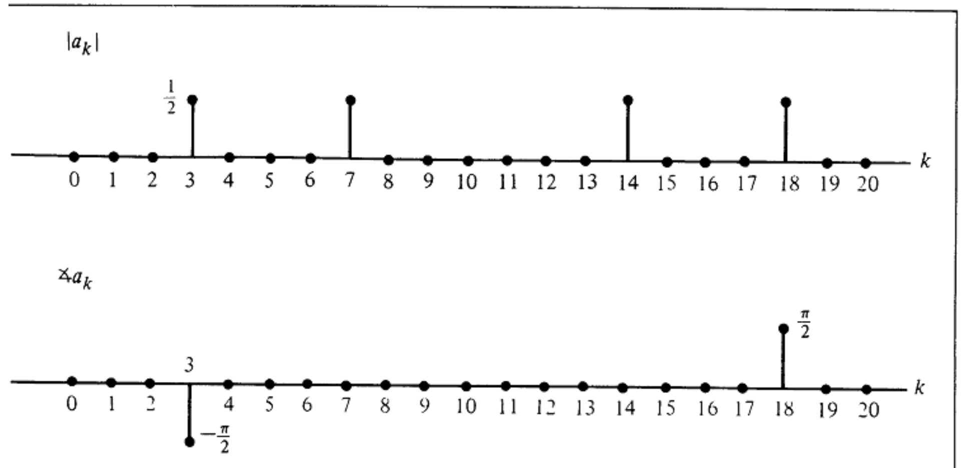


Figure S10.11-2

(c) The period $N = 8$.

$$a_3 = a_5^* = \tfrac{1}{2} e^{-j(\pi/3)}$$

The rest of the coefficients a_k are zero. The magnitude and phase of a_k are given in Figure S10.11-3.

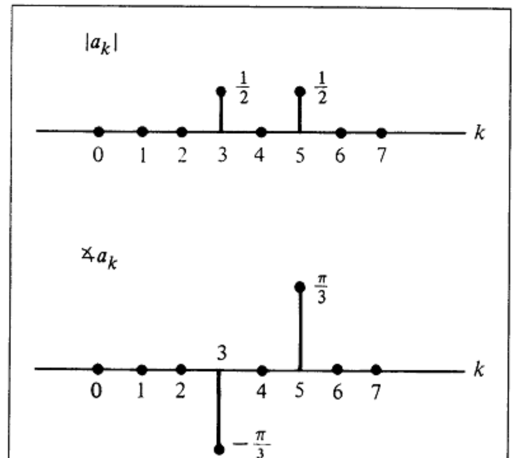


Figure S10.11-3

Ex 4

In parts (a)–(d) we specify the Fourier series coefficients of a signal that is periodic with period 8. Determine the signal $x[n]$ in each case.

(a) $a_k = \cos\left(k\frac{\pi}{4}\right) + \sin\left(3k\frac{\pi}{4}\right)$

(b) $a_k = \begin{cases} \sin\left(\frac{k\pi}{3}\right), & 0 \le k \le 6 \\ 0, & k = 7 \end{cases}$

(c) a_k as in Figure P10.7(a)

(d) a_k as in Figure P10.7(b)

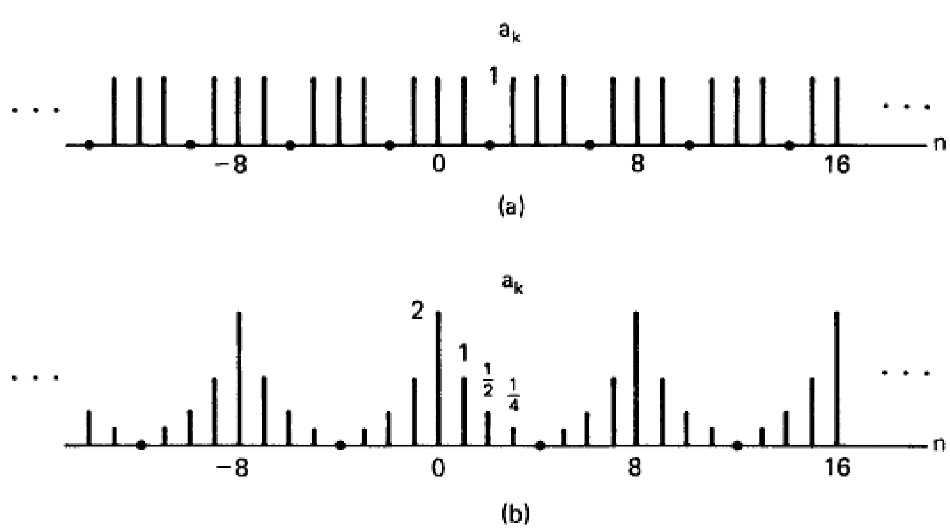


Figure P10.7

Ex 4

The Fourier series coefficients of $x[n]$, which is periodic with period N, are given by

$$a_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{-jk(2\pi/N)n}$$

For $N = 8$,

$$a_k = \frac{1}{8}\sum_{n=0}^{7} x[n]e^{-jk(\pi/4)n} \qquad \text{(S10.7-1)}$$

(a) We are given that

$$a_k = \cos\left(\frac{\pi k}{4}\right) + \sin\left(\frac{3\pi k}{4}\right), \qquad \text{(S10.7-2)}$$

$$a_k = \frac{1}{2}e^{j(\pi k/4)} + \frac{1}{2}e^{-j(\pi k/4)} + \frac{1}{2j}e^{j(3\pi k/4)} - \frac{1}{2j}e^{-j(3\pi k/4)}$$

Hence, by comparing eqs. (S10.7-1) and (S10.7-2) we can immediately write

$$x[n] = 4\delta[n-1] + 4\delta[n-7] - 4j\delta[n-3] + 4j\delta[n-5], \quad 0 \le n \le 7$$

(b)
$$\begin{aligned}
x[n] &= \sum_{k=0}^{7} a_k e^{jk(2\pi/8)} = \sum_{k=0}^{7} a_k e^{jk(\pi/4)n} \\
&= \sum_{k=0}^{6}\left[\frac{1}{2j}e^{j(k\pi/3)} - \frac{1}{2j}e^{-j(k\pi/3)}\right] e^{jk(\pi/4)n} \\
&= \frac{1}{2j}\sum_{k=0}^{6} e^{jk\pi[(1/3)+(n/4)]} - \frac{1}{2j}\sum_{k=0}^{6} e^{-jk\pi[(1/3)-(n/4)]} \\
&= \frac{1}{2j}\,\frac{1-e^{j[(7\pi n/4)+(7\pi/3)]}}{1-e^{j[(\pi n/4)+(\pi/3)]}} - \frac{1}{2j}\,\frac{1-e^{j[(7\pi n/4)-(7\pi/3)]}}{1-e^{j[(\pi n/4)-(\pi/3)]}} \\
&= \frac{1}{2j}\left[\frac{1-e^{j[(7\pi n/4)+(7\pi/3)]}}{1-e^{j[(\pi n/4)+(\pi/3)]}} - \frac{1-e^{j[(7\pi n/4)-(7\pi/3)]}}{1-e^{j[(\pi n/4)-(\pi/3)]}}\right]
\end{aligned}$$

(c)
$$\begin{aligned}
x[n] &= \sum_{k=0}^{7} a_k e^{jk(2\pi/8)n} = \sum_{k=0}^{7} a_k e^{jk(\pi/4)n} \\
&= 1 + e^{j(\pi/4)n} + e^{j(3\pi/4)n} + e^{j\pi n} + e^{j(5\pi/4)n} + e^{j(7\pi/4)n} \\
&= 1 + (-1)^n + 2\cos\left(\frac{\pi}{4}n\right) + 2\cos\left(\frac{3\pi}{4}n\right), \qquad 0 \le n \le 7
\end{aligned}$$

(d) Using an analysis similar to that in part (c), we find

$$x[n] = 2 + 2\cos\left(\frac{\pi}{4}n\right) + \cancelto{0}{\cos\left(\frac{\pi}{2}n\right)} + \frac{1}{2}\cos\left(\frac{3\pi}{4}n\right), \qquad 0 \le n \le 7$$

Ex 5

(a) Determine and sketch the discrete-time Fourier transform of the sequence in Figure P10.4-1.

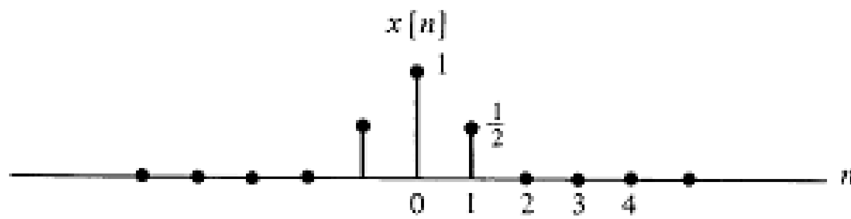


Figure P10.4-1

(b) Using your result in part (a), determine the discrete-time Fourier series of the two periodic sequences in Figure P10.4-2.

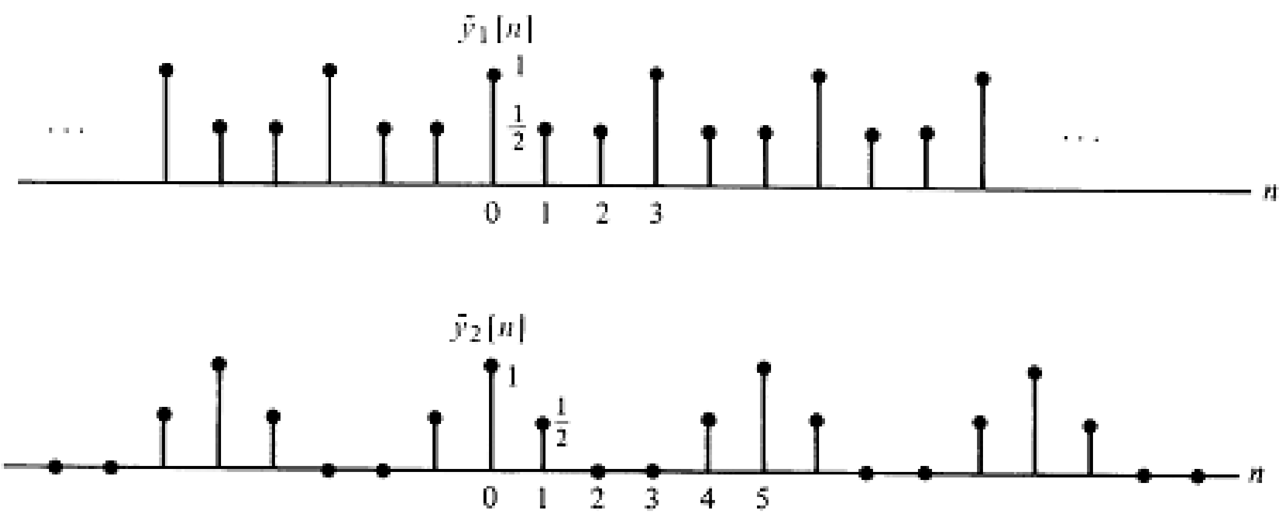


Figure P10.4-2

Ex 5

(a) The discrete-time Fourier transform of the given sequence is

$$\begin{aligned} X(\Omega) &= \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n} \\ &= \tfrac{1}{2}e^{j\Omega} + 1 + \tfrac{1}{2}e^{-j\Omega} \\ &= 1 + \cos\Omega \end{aligned}$$

$X(\Omega)$ is sketched in Figure S10.4.

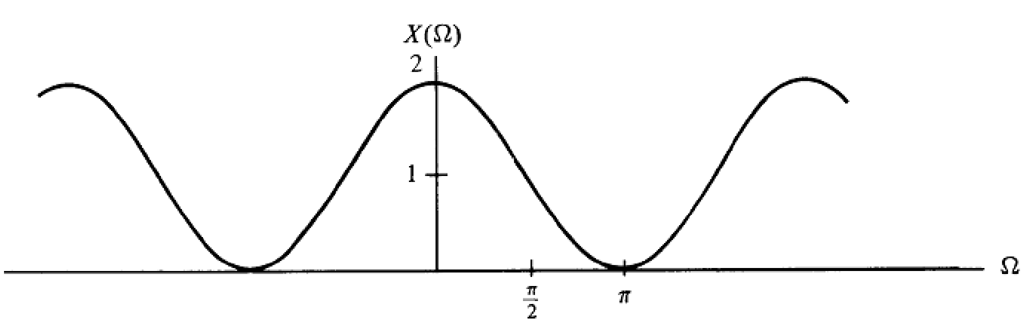


Figure S10.4

(b) The first sequence can be thought of as

$$\tilde{y}_1[n] = x[n] * \left[\sum_{k=-\infty}^{\infty} \delta[n-3k]\right]$$

Hence

$$Y_1(\Omega) = X(\Omega)\frac{2\pi}{3}\sum_{k=-\infty}^{\infty} \delta\left(\Omega - \frac{2\pi k}{3}\right)$$

Therefore, the Fourier series of $y_1[n]$ is given by

$$a_k = \frac{1}{2\pi}Y_1\left(\frac{2\pi}{3}k\right) = \frac{1}{3}\left(1 + \cos\frac{2\pi k}{3}\right), \qquad \text{for all } k$$

The second sequence is given by

$$y_2[n] = x[n] * \left[\sum_{k=-\infty}^{\infty} \delta[n-5k]\right]$$

Similarly, the Fourier series of this sequence is given by

$$a_k = \frac{1}{5}\left[1 + \cos\left(\frac{2\pi k}{5}\right)\right], \qquad \text{for all } k$$

This result can also be obtained by using the fact that the Fourier series coefficients are proportional to equally spaced samples of the discrete-time Fourier transform of one period (see Section 5.4.1 of the text, page 314).

Ex 6

Figure P9.2 shows real and imaginary parts of the Fourier transform of a signal $x(t)$.

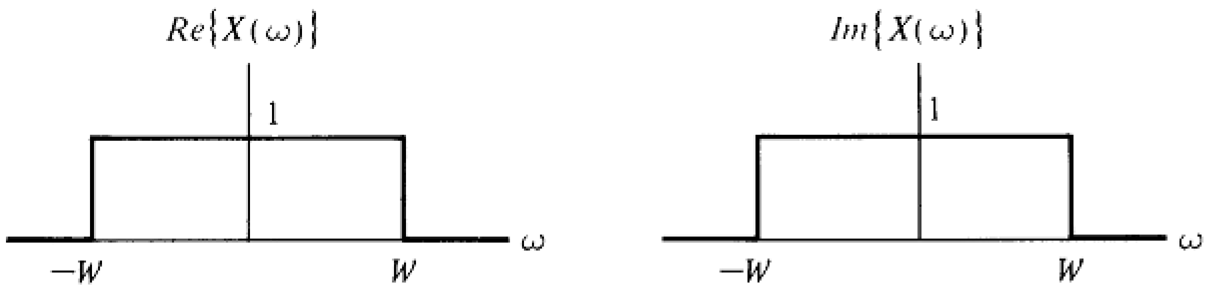


Figure P9.2

(a) Sketch the magnitude and phase of the Fourier transform $X(\omega)$.

(b) In general, if a signal $x(t)$ is real, then $X(-\omega) = X^*(\omega)$. Determine whether $x(t)$ is real for the Fourier transform sketched in Figure P9.2.

Ex 6

(a) The magnitude of $X(\omega)$ is given by

$$|X(\omega)| = \sqrt{X_R^2(\omega) + X_I^2(\omega)},$$

where $X_R(\omega)$ is the real part of $X(\omega)$ and $X_I(\omega)$ is the imaginary part of $X(\omega)$. It follows that

$$|X(\omega)| = \begin{cases} \sqrt{2}, & |\omega| < W, \\ 0, & |\omega| > W \end{cases}$$

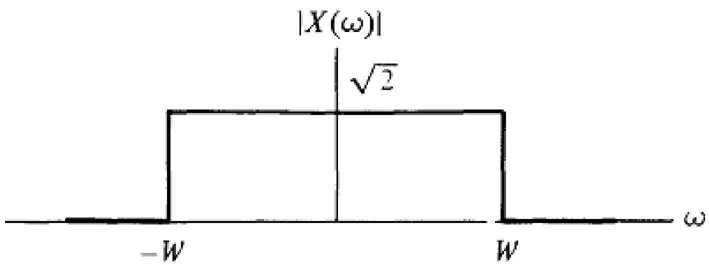


Figure S9.2-1

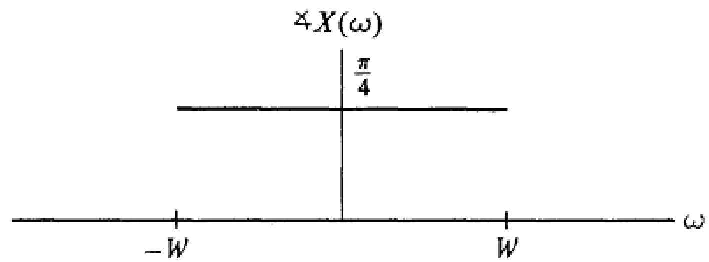


Figure S9.2-2

The phase of $X(\omega)$ is given by

$$\sphericalangle X(\omega) = \tan^{-1}\left(\frac{X_I(\omega)}{X_R(\omega)}\right) = \tan^{-1}(1), \qquad |\omega| < W$$

(b)

$$X(\omega) = \begin{cases} 1 + j, & |\omega| < W \\ 0, & \text{otherwise} \end{cases}$$

$$X(-\omega) = \begin{cases} 1 + j, & |\omega| < W \\ 0, & \text{otherwise} \end{cases}$$

$$X^*(\omega) = \begin{cases} 1 - j, & |\omega| < W \\ 0, & \text{otherwise} \end{cases}$$

Hence, the signal is not real.

Ex 7

The output of a causal LTI system is related to the input $x(t)$ by the differential equation

$$\frac{dy(t)}{dt} + 2y(t) = x(t)$$

(a) Determine the frequency response $H(\omega) = Y(\omega)/X(\omega)$ and sketch the phase and magnitude of $H(\omega)$.

(b) If $x(t) = e^{-t}u(t)$, determine $Y(\omega)$, the Fourier transform of the output.

(c) Find $y(t)$ for the input given in part (b).

Ex 7

We are given the differential equation

$$\frac{dy(t)}{dt} + 2y(t) = x(t)$$

Taking the Fourier transform of eq. (S9.7-1), we have

$$j\omega Y(\omega) + 2Y(\omega) = X(\omega)$$

Hence,

$$Y(\omega)[2 + j\omega] = X(\omega)$$

and

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{1}{2 + j\omega},$$

$$H(\omega) = \frac{1}{2 + j\omega} = \frac{1}{2 + j\omega}\left(\frac{2 - j\omega}{2 - j\omega}\right) = \frac{2 - j\omega}{4 + \omega^2}$$

$$= \frac{2}{4 + \omega^2} - j\frac{\omega}{4 + \omega^2},$$

$$|H(\omega)|^2 = \frac{4}{(4 + \omega^2)^2} + \frac{\omega^2}{(4 + \omega^2)^2} = \frac{4 + \omega^2}{(4 + \omega^2)^2},$$

$$|H(\omega)| = \frac{1}{\sqrt{4 + \omega^2}}$$

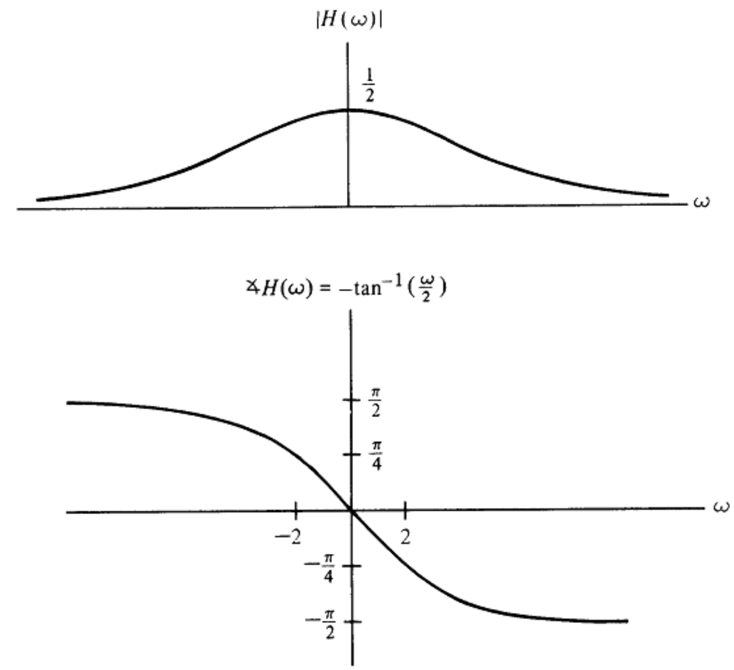


Figure S9.7

(b) We are given $x(t) = e^{-t}u(t)$. Taking the Fourier transform, we obtain

$$X(\omega) = \frac{1}{1 + j\omega}, \qquad H(\omega) = \frac{1}{2 + j\omega}$$

Hence,

$$Y(\omega) = \frac{1}{(1 + j\omega)(2 + j\omega)} = \frac{1}{1 + j\omega} - \frac{1}{2 + j\omega}$$

(c) Taking the inverse transform of $Y(\omega)$, we get

$$y(t) = e^{-t}u(t) - e^{-2t}u(t)$$

Ex 8

By first expressing the triangular signal $x(t)$ in Figure P9.8 as the convolution of a rectangular pulse with itself, determine the Fourier transform of $x(t)$.

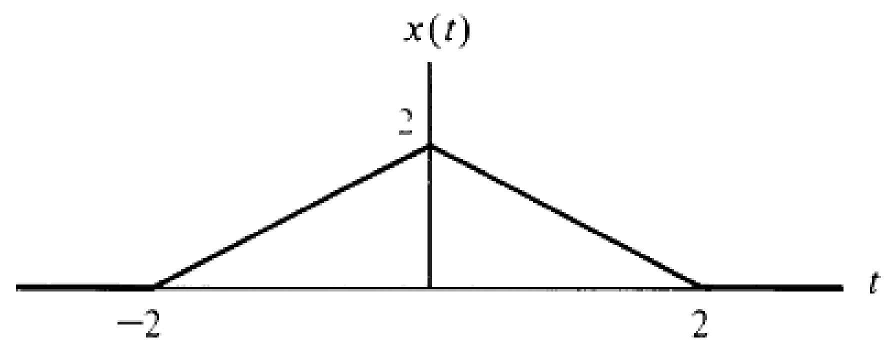


Figure P9.8

Ex 8

A triangular signal can be represented as the convolution of two rectangular pulses, as indicated in Figure S9.8.

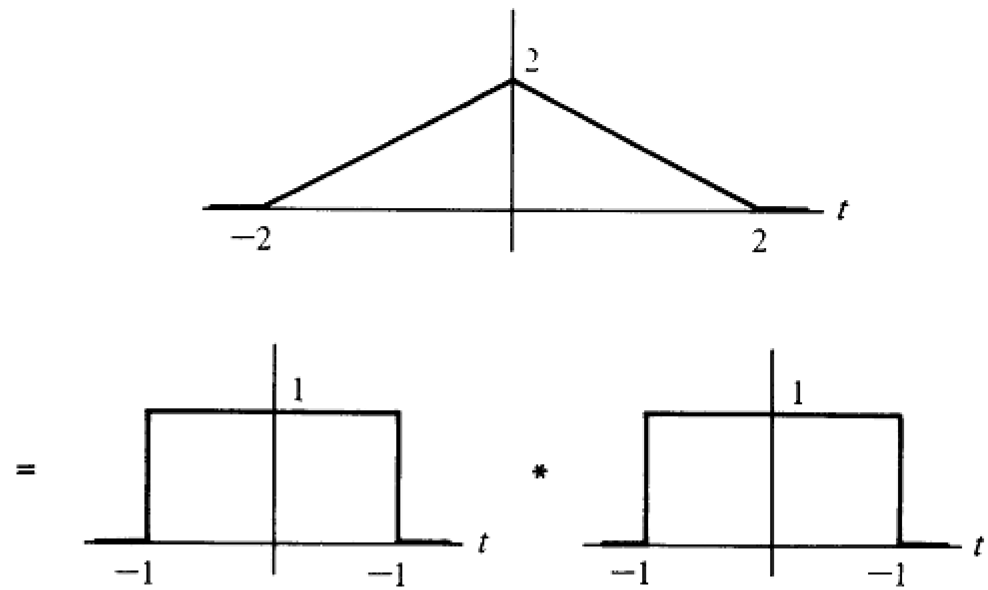


Figure S9.8

Since each of the rectangular pulses on the right has a Fourier transform given by $(2 \sin \omega)/\omega$, the convolution property tells us that the triangular function will have a Fourier transform given by the square of $(2 \sin \omega)/\omega$:

$$X(\omega) = \frac{4 \sin^2 \omega}{\omega^2}$$

Ex 9

Consider the following linear constant-coefficient differential equation (LCCDE):

$$\frac{dy(t)}{dt} + 2y(t) = A \cos \omega_0 t$$

Find the value of ω_0 such that $y(t)$ will have a maximum amplitude of $A/3$. Assume that the resulting system is linear and time-invariant.

Ex 9

We are given the LCCDE

$$\frac{dy(t)}{dt} + 2y(t) = A \cos \omega_0 t$$

We can view the LCCDE as

$$\frac{dy(t)}{dt} + 2y(t) = x(t),$$

the transfer function of which is given by

$$H(\omega) = \frac{1}{2 + j\omega} \qquad \text{and} \qquad x(t) = A \cos \omega_0 t$$

We have already seen that for LTI systems,

$$\begin{aligned} y(t) &= |H(\omega_0)|\, A \cos(\omega_0 t + \phi), \qquad \text{where } \phi = \measuredangle H(\omega_0) \\ &= \frac{1}{\sqrt{4 + \omega_0^2}} A \cos(\omega_0 t + \phi) \end{aligned}$$

For the maximum value of $y(t)$ to be $A/3$, we require

$$\frac{1}{4 + \omega_0^2} = \frac{1}{9}$$

Therefore, $\omega_0 = \pm\sqrt{5}$.

Ex 10

Suppose an LTI system is described by the following LCCDE:

$$\frac{d^2y(t)}{dt^2} + \frac{2dy(t)}{dt} + 3y(t) = \frac{4dx(t)}{dt} - x(t)$$

(a) Show that the left-hand side of the equation has a Fourier transform that can be expressed as

$$A(\omega)Y(\omega), \qquad \text{where } Y(\omega) = \mathcal{F}\{y(t)\}$$

Find $A(\omega)$.

(b) Similarly, show that the right-hand side of the equation has a Fourier transform that can be expressed as

$$B(\omega)X(\omega), \qquad \text{where } X(\omega) = \mathcal{F}\{x(t)\}$$

(c) Show that $Y(\omega)$ can be expressed as $Y(\omega) = H(\omega)X(\omega)$ and find $H(\omega)$.

Ex 10

$$\textbf{(a)}\quad \mathcal{F}\left\{\frac{d^2y(t)}{dt^2} + \frac{2dy(t)}{dt} + 3y(t)\right\} = -\omega^2 Y(\omega) + 2j\omega Y(\omega) + 3Y(\omega)$$

$$= (-\omega^2 + j2\omega + 3)Y(\omega),$$

$$A(\omega) = -\omega^2 + j2\omega + 3$$

$$\textbf{(b)}\quad \mathcal{F}\left\{\frac{4dx(t)}{dt} - x(t)\right\} = 4j\omega X(\omega) - X(\omega)$$

$$= (j4\omega - 1)X(\omega),$$

$$B(\omega) = j4\omega - 1,$$

$$A(\omega)Y(\omega) = B(\omega)X(\omega),$$

$$Y(\omega) = \frac{B(\omega)}{A(\omega)}X(\omega)$$

$$= H(\omega)X(\omega)$$

Therefore,

$$H(\omega) = \frac{B(\omega)}{A(\omega)} = \frac{-1 + j4\omega}{-\omega^2 + 3 + j2\omega}$$

$$= \frac{1 - j4\omega}{\omega^2 - 3 - j2\omega}$$

Ex 11

From Figure P9.13, find $y(t)$ where

$$x(t) = \frac{\sin(\omega_0 t)}{t} \qquad \text{and} \qquad h(t) = \frac{\sin(2\omega_0 t)}{t}$$

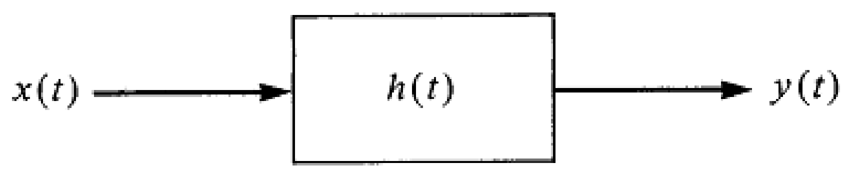


Figure P9.13

Ex 11

$$\frac{\sin Wt}{\pi t} \overset{\mathfrak{F}}{\longleftrightarrow}$$

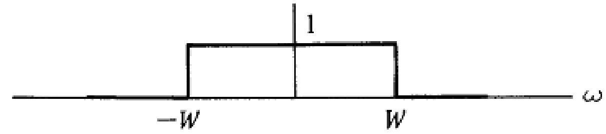


Figure S9.13-1

$$x(t) = \pi\frac{\sin \omega_0 t}{\pi t} \overset{\mathfrak{F}}{\longleftrightarrow}$$

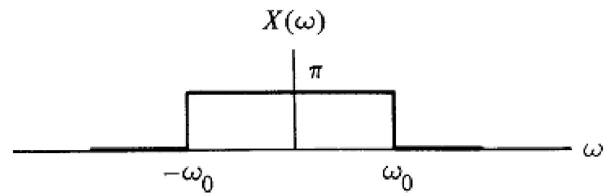


Figure S9.13-2

$$h(t) = \pi\frac{\sin (2\omega_0 t)}{\pi t} \overset{\mathfrak{F}}{\longleftrightarrow}$$

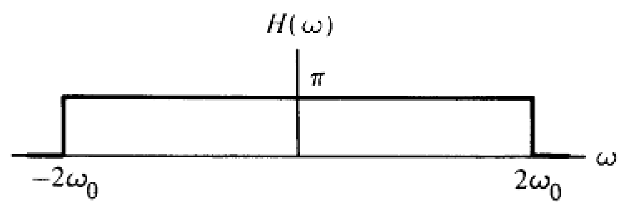


Figure S9.13-3

$$Y(\omega) = X(\omega)H(\omega) =$$

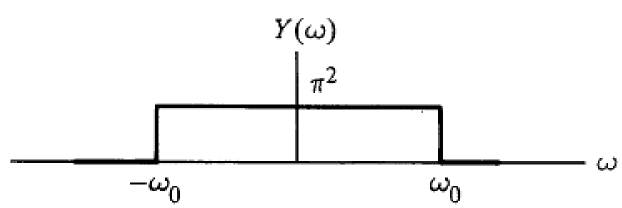


Figure S9.13-4

Therefore, $y(t) = \pi\dfrac{\sin(\omega_0 t)}{t}$.

Ex 12

(a) Determine the energy in the signal $x(t)$ for which the Fourier transform $X(\omega)$ is given by Figure P9.14.

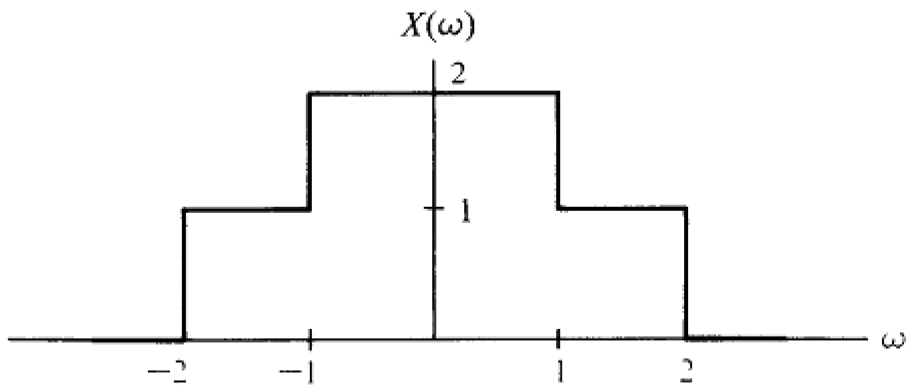


Figure P9.14

Ex 12

(a) Energy $= \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty} |X(\omega)|^2\, d\omega$

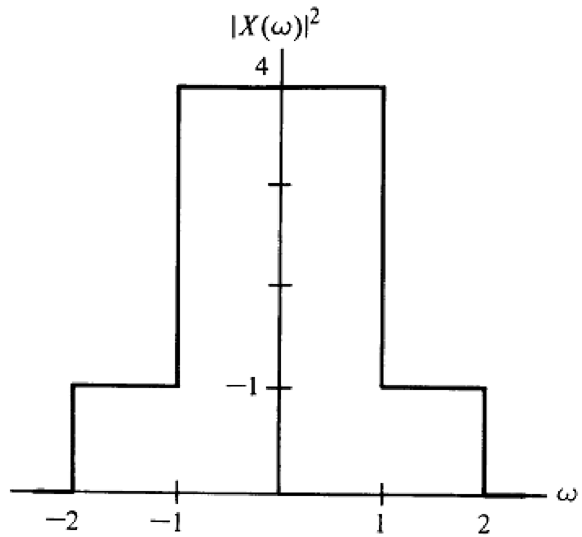


Figure S9.14-1

$$\begin{aligned}\text{Area} &= (4)(2) + (2)(1)(1)\\ &= 10\\ \text{Energy} &= \frac{5}{\pi}\end{aligned}$$

Ex 13

P8.6

Find the signal corresponding to the following Fourier transforms.

(a) $X_a(\omega) = \dfrac{1}{7 + j\omega}$

(b)

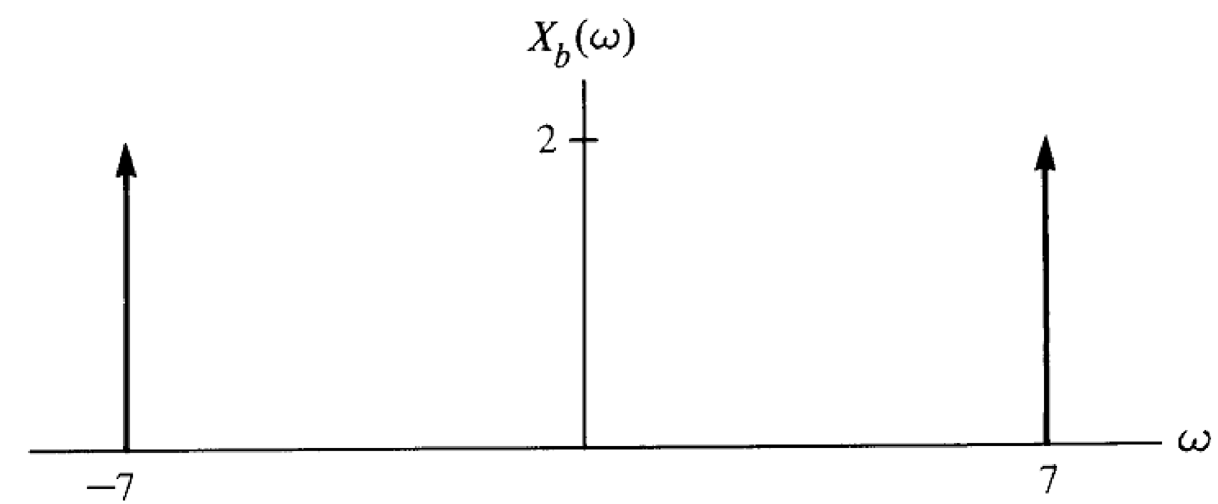


Figure P8.6-1

(c) $X_c(\omega) = \dfrac{1}{9 + \omega^2}$

See Example 4.8 in the text (page 191).

S8.6

Ex 20

(a) By inspection,

$$e^{-at}u(t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{a + j\omega}$$

Thus,

$$e^{-7t}u(t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{7 + j\omega}$$

Direct inversion using the inverse Fourier transform formula is very difficu

(b) $X_b(\omega) = 2\delta(\omega + 7) + 2\delta(\omega - 7)$,

$$x_b(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_b(\omega)e^{j\omega t}\,d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} 2[\delta(\omega + 7) + \delta(\omega - 7)]e^{j\omega t}\,d\omega$$

$$= \frac{1}{\pi}e^{-j7t} + \frac{1}{\pi}e^{j7t} = \frac{2}{\pi}\cos 7t$$

(c) From Example 4.8 of the text (page 191), we see that

$$e^{-a|t|} \overset{\mathcal{F}}{\longleftrightarrow} \frac{2a}{a^2 + \omega^2}$$

However, note that

$$\alpha x(t) \overset{\mathcal{F}}{\longleftrightarrow} \alpha X(\omega)$$

since

$$\int_{-\infty}^{\infty} \alpha x(t)e^{-j\omega t}\,dt = \alpha\int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt = \alpha X(\omega)$$

Thus,

$$\frac{1}{2a}e^{-a|t|} \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{a^2 + \omega^2} \qquad \text{or} \qquad \frac{1}{9 + \omega^2} \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{6}e^{-3|t|}$$

Ex 14

Consider the impulse train

$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$$

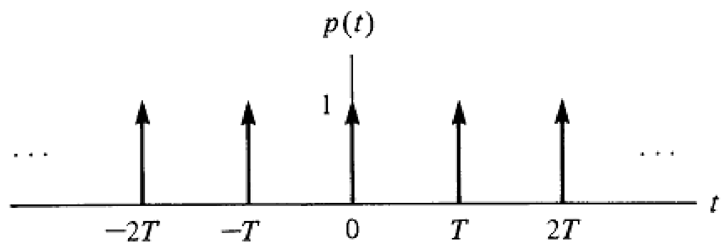


Figure P8.8-1

(a) Find the Fourier series of $p(t)$.

(b) Find the Fourier transform of $p(t)$.

(c) Consider the signal $x(t)$ shown in Figure P8.8-2, where $T_1 < T$.

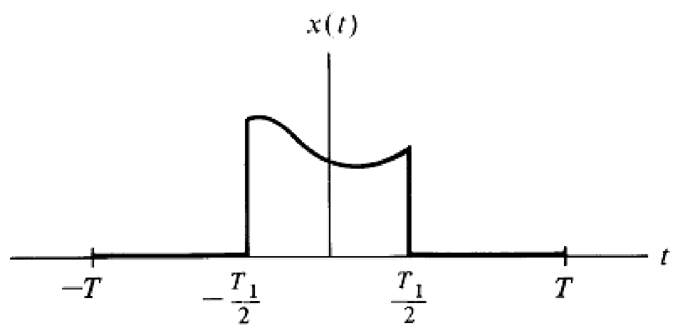


Figure P8.8-2

Show that the periodic signal $\tilde{x}(t)$, formed by periodically repeating $x(t)$, satisfies

$$\tilde{x}(t) = x(t) * p(t)$$

Ex 14

(a) Using the analysis equation, we obtain

$$a_k = \frac{1}{T}\int_{-T/2}^{T/2} \delta(t) e^{-jk(2\pi/T)t}\,dt = \frac{1}{T}$$

Thus all the Fourier series coefficients are equal to $1/T$.

(b) For periodic signals, the Fourier transform can be calculated from a_k as

$$X(\omega) = 2\pi \sum_{k=-\infty}^{\infty} a_k \delta\left(\omega - \frac{2\pi k}{T}\right)$$

In this case,

$$P(\omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{T}\right)$$

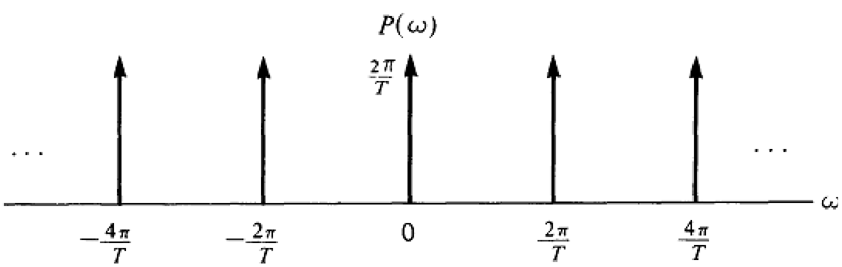


Figure S8.8

(c) We are required to show that

$$\tilde{x}(t) = x(t) * p(t)$$

Substituting for $p(t)$, we have

$$x(t) * p(t) = x(t) * \left[\sum_{k=-\infty}^{\infty} \delta(t - kT)\right]$$

Using the associative property of convolution, we obtain

$$x(t) * p(t) = \sum_{k=-\infty}^{\infty} [x(t) * \delta(t - kT)]$$

From the sifting property of $\delta(t)$, it follows that

$$x(t) * p(t) = \sum_{k=-\infty}^{\infty} x(t - kT) = \tilde{x}(t)$$

Thus, $x(t) * p(t)$ is a periodic repetition of $x(t)$ with period T.

Exercício: Propriedades

Ex 15

Calcule a DTFT do sinal

$$x[n] = ne^{j\frac{\pi}{8}n}\alpha^{n-3}u[n-3]$$

Exercício: Propriedades

Ex 15

$$x[n] = (j)(-jn)e^{j\frac{\pi}{8}n}\alpha^{n-3}u[n-3]$$

- Propriedades:

$$x[n-n_0] \overset{DTFT}{\longleftrightarrow} e^{-j\omega n_0}X\left(e^{j\omega}\right)$$

$$e^{j\omega_0 n}x[n] \overset{DTFT}{\longleftrightarrow} X\left(e^{j(\omega-\omega_0)}\right)$$

$$-jnx[n] \overset{DTFT}{\longleftrightarrow} \frac{dX\left(e^{j\omega}\right)}{d\omega}$$

Exercício: Propriedades

Ex 15

- Propriedades:

$$x[n-n_0] \overset{DTFT}{\longleftrightarrow} e^{-j\omega n_0} X\left(e^{j\omega}\right)$$

$$e^{j\omega_0 n} x[n] \overset{DTFT}{\longleftrightarrow} X\left(e^{j(\omega-\omega_0)}\right)$$

$$-jnx[n] \overset{DTFT}{\longleftrightarrow} \frac{dX\left(e^{j\omega}\right)}{d\omega}$$

- Propriedades são aplicadas em:

$$x_0[n] = \alpha^n u[n] \longleftrightarrow X_0\left(e^{j\omega}\right) = \frac{1}{1-\alpha e^{-j\omega}}$$

Exercício: Propriedades

Ex 15

$$x_0[n] = \alpha^n u[n] \longleftrightarrow X_0\left(e^{j\omega}\right) = \frac{1}{1-\alpha e^{-j\omega}}$$

- Portanto

$$\begin{aligned}
X_1(e^{j\omega}) &= e^{-j3\omega} X_0\left(e^{j\omega}\right) = \frac{e^{-j3\omega}}{1-\alpha e^{-j\omega}} \\
X_2\left(e^{j\omega}\right) &= X_1(e^{j(\omega-\frac{\pi}{8})}) = \frac{e^{-j3(\omega-\pi/8)}}{1-\alpha e^{-j(\omega-\pi/8)}} \\
X_3\left(e^{j\omega}\right) &= \frac{dX_2\left(e^{j\omega}\right)}{d\omega} = j\frac{d}{d\omega}\left(\frac{e^{-j3(\omega-\pi/8)}}{1-\alpha e^{-j(\omega-\pi/8)}}\right) \\
X\left(e^{j\omega}\right) &= -\frac{j\left(3\,(-1)^{\frac{7}{8}}\,e^{-3\,j\omega}+2\,e^{-4\,j\omega}\alpha\right)}{\left(-1+\alpha e^{-1/8\,j(8\,\omega-\pi)}\right)^2}
\end{aligned}$$

Exercício: Propriedades

Ex 16

Determine a saída $y(t)$ de um sistema LIT, cuja resposta ao impulso é

$$h(t) = 2e^{-2t}u(t)$$

quando uma entrada

$$x(t) = 3e^{-t}u(t)$$

foi aplicada.

Exercício: Propriedades

Ex 16

- Pela propriedade da Convolução, temos:

$$Y(j\omega) = H(j\omega)X(j\omega)$$

- Calculando a FT dos sinais $h(t)$ e $x(t)$:

$$h(t) = 2e^{-2t}u(t) \quad \overset{FT}{\longleftrightarrow} \quad \frac{2}{j\omega + 2}$$

$$x(t) = 3e^{-t}u(t) \quad \overset{FT}{\longleftrightarrow} \quad \frac{3}{j\omega + 1}$$

- Portanto:

$$Y(j\omega) = \frac{6}{(j\omega + 2)(j\omega + 1)} = \frac{A}{j\omega + 2} + \frac{B}{j\omega + 1}$$

Exercício: *Propriedades*

Ex 16

- Calculando A e B

$$(A+B)(j\omega)+(2B+A)=6 \rightarrow \begin{cases} A+B=0 & \rightarrow \quad A=-B \\ 2B+A=6 & \rightarrow \quad 2B-B=6 \\ & \rightarrow \quad B=6 \text{ e } A=-6 \end{cases}$$

- Finalmente

$$Y(j\omega)=6\left(\frac{-1}{j\omega+2}+\frac{1}{j\omega+1}\right)$$

- $Y(j\omega)\xleftrightarrow{FT}y(t)=-6e^{-2t}u(t)+6e^{-t}u(t)$

Exercício: Propriedades

Ex 17

Seja a resposta ao impulso de um sistema LIT

$$h[n] = \frac{1}{\pi n}\text{sen}\left(\frac{\pi}{4}n\right).$$

Encontre a saída $y[n]$ em resposta às entradas:

$$\begin{cases} x_1[n] = \frac{1}{\pi n}\text{sen}\left(\frac{\pi}{8}n\right) \\ x_2[n] = \frac{1}{\pi n}\text{sen}\left(\frac{\pi}{2}n\right) \end{cases}$$

Exercício: Propriedades

Ex 17

- Notando que o sinal $h[n]$, $x_1[n]$ e $x_2[n]$ são funções **sinc**:

$$h[n] = \frac{1}{\pi n}\text{sen}\left(\frac{\pi}{4}n\right) \Longleftrightarrow H(e^{j\omega}) = \begin{cases} 1, & |\omega| \leq \frac{\pi}{4} \\ 0, & \frac{\pi}{4} < |\omega| < \pi \end{cases}$$

$$x_1[n] = \frac{1}{\pi n}\text{sen}\left(\frac{\pi}{8}n\right) \Longleftrightarrow X_1(e^{j\omega}) = \begin{cases} 1, & |\omega| \leq \frac{\pi}{8} \\ 0, & \frac{\pi}{8} < |\omega| < \pi \end{cases}$$

$$x_2[n] = \frac{1}{\pi n}\text{sen}\left(\frac{\pi}{2}n\right) \Longleftrightarrow X_2(e^{j\omega}) = \begin{cases} 1, & |\omega| \leq \frac{\pi}{2} \\ 0, & \frac{\pi}{2} < |\omega| < \pi \end{cases}$$

Exercício: Propriedades

Ex 17

- Para a entrada $x_1[n]$, na frequência, temos:

$$\begin{aligned} Y_1(e^{j\omega}) &= H(e^{j\omega})X_1(e^{j\omega}) \\ &= X_1(e^{j\omega}) \end{aligned}$$

- Logo:

$$y[n] = y_1[n] = x_1[n] = \frac{1}{\pi n}\text{sen}\left(\frac{\pi}{8}n\right)$$

Exercício: Propriedades

Ex 17

- Para a entrada $x_2[n]$, na frequência, temos:

$$\begin{aligned} Y_2(e^{j\omega}) &= H(e^{j\omega})X_2(e^{j\omega}) \\ &= H(e^{j\omega}) \end{aligned}$$

- Logo:

$$y[n] = y_2[n] = h[n] = \frac{1}{\pi n}\text{sen}\left(\frac{\pi}{4}n\right)$$

Ex 18

P11.2

(a) Consider the linear constant coefficient difference equation

$$y[n] - \tfrac{1}{2}y[n-1] = x[n],$$

which describes a linear, time-invariant system initially at rest. What is the system function that describes $Y(\Omega)$ in terms of $X(\Omega)$?

(b) Using Fourier transforms, evaluate $y[n]$ if $x[n]$ is

(i) $\delta[n]$

(ii) $\delta[n - n_0]$

(iii) $(\tfrac{3}{4})^n u[n]$

S11.2

Ex 18

(a) The difference equation $y[n] - \frac{1}{2}y[n-1] = x[n]$, which is initially at rest, has a system transfer function that can be obtained by taking the Fourier transform of both sides of the equation. This yields

$$Y(\Omega)(1 - \tfrac{1}{2}e^{-j\Omega}) = X(\Omega),$$

so

$$H(\Omega) = \frac{Y(\Omega)}{X(\Omega)} = \frac{1}{1 - (\frac{1}{2})^{-j\Omega}}$$

(b) (i) If $x[n] = \delta[n]$, then $X(\Omega) = 1$ and

$$Y(\Omega) = H(\Omega)X(\Omega) = \frac{1}{1 - \frac{1}{2}e^{-j\Omega}},$$

so

$$y[n] = (\tfrac{1}{2})^n u[n]$$

(ii) $X(\Omega) = e^{-j\Omega n_0}$, so

$$Y(\Omega) = \frac{e^{-j\Omega n_0}}{1 - \frac{1}{2}e^{-j\Omega}}$$

and, using the delay property of the Fourier transform,

$$y[n] = (\tfrac{1}{2})^{n-n_0}u[n - n_0]$$

(iii) If $x[n] = (\frac{3}{4})^n u[n]$, then

$$X(\Omega) = \frac{1}{1 - \frac{3}{4}e^{-j\Omega}},$$

$$Y(\Omega) = \left(\frac{1}{1 - \frac{1}{2}e^{-j\Omega}}\right)\left(\frac{1}{1 - \frac{3}{4}e^{-j\Omega}}\right) = \frac{-2}{1 - \frac{1}{2}e^{-j\Omega}} + \frac{3}{1 - \frac{3}{4}e^{-j\Omega}},$$

so

$$y[n] = -2(\tfrac{1}{2})^n u[n] + 3(\tfrac{3}{4})^n u[n]$$

Ex 19

P11.4

A particular LTI system is described by the difference equation

$$y[n] + \tfrac{1}{4}y[n-1] - \tfrac{1}{8}y[n-2] = x[n] - x[n-1]$$

(a) Find the impulse response of the system.

(b) Evaluate the magnitude and phase of the system frequency response at $\Omega = 0$, $\Omega = \pi/4$, $\Omega = -\pi/4$, and $\Omega = 9\pi/4$.

Ex 19

S11.4

(a) The use of the Fourier transform simplifies the analysis of the difference equation.

$$y[n] + \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = x[n] - x[n-1],$$

$$Y(\Omega)(1 + \frac{1}{4}e^{-j\Omega} - \frac{1}{8}e^{-j2\Omega}) = X(\Omega)(1 - e^{-j\Omega}),$$

$$\frac{Y(\Omega)}{X(\Omega)} = H(\Omega) = \frac{1 - e^{-j\Omega}}{(1 + \frac{1}{2}e^{-j\Omega})(1 - \frac{1}{4}e^{-j\Omega})}$$

We want to put this in a form that is easily invertible to get the impulse response $h[n]$. Using a partial fraction expansion, we see that

$$H(\Omega) = \frac{2}{1 + \frac{1}{2}e^{-j\Omega}} + \frac{-1}{1 - \frac{1}{4}e^{-j\Omega}},$$

so

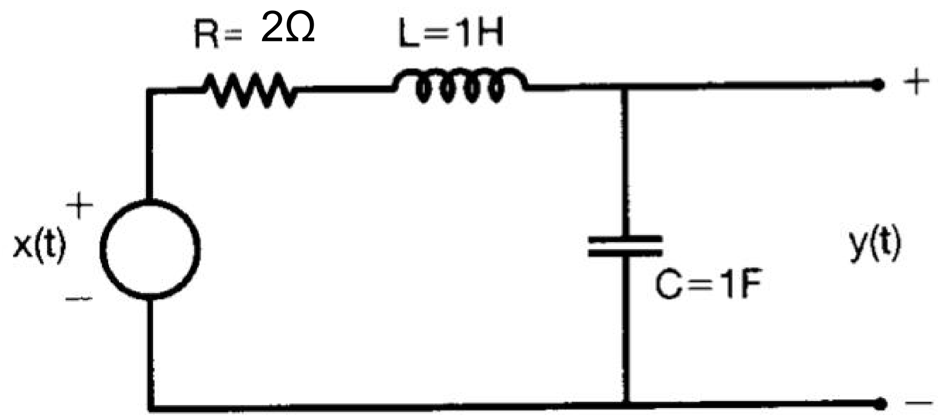
$$h[n] = 2(-\tfrac{1}{2})^n u[n] - (\tfrac{1}{4})^n u[n]$$

(b) At $\Omega = 0$, $H(\Omega) = 0$. At $\Omega = \pi/4$, $H(\Omega) = 0.65e^{j(1.22)}$. Since $h[n]$ is real, $H(\Omega) = H^*(-\Omega)$, so $H(-\Omega) = H^*(\Omega)$ and $H(-\pi/4) = 0.65e^{-j(1.22)}$. Since $H(\Omega)$ is periodic in 2π,

$$H\left(\frac{9\pi}{4}\right) = H\left(\frac{\pi}{4}\right) = 0.65e^{j(1.22)}$$

EX 20

Encontre a resposta ao impulso do sistema LTI causal representado pelo circuito RLC.



a) Encontrar a resposta ao impulso
b) Desenhar o diagrama de blocos
c) Calcular e esboçar módulo e fase da resposta em freq

2) $i = C\dot{y}$

$x = RC\dot{y} + LC\ddot{y} + y \Rightarrow \ddot{y} + \frac{R}{L}\dot{y} + \frac{1}{LC}y = \frac{1}{LC}x$

$$\Downarrow FT$$

$$(j\omega)^2 Y(j\omega) + \frac{R}{L}(j\omega)Y(j\omega) + \frac{1}{LC}Y(j\omega) = \frac{1}{LC}X(j\omega)$$

$$\Downarrow$$

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{\frac{1}{LC}}{(j\omega)^2 + \frac{R}{L}j\omega + \frac{1}{LC}}$$

sendo $L = C = 1$ e $R = 2$

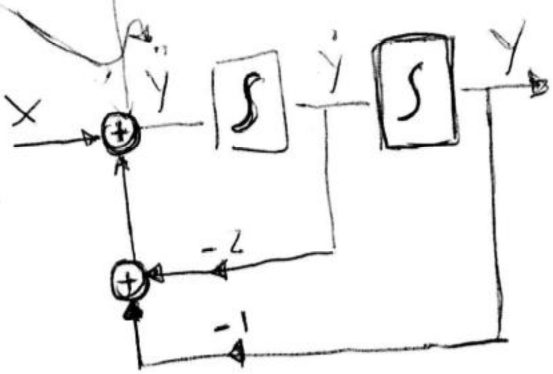
$$H(j\omega) = \frac{1}{(j\omega)^2 + 2j\omega + 1} = \frac{1}{(j\omega+1)^2}$$

$$\Downarrow FT^{-1} \text{ (tabela)}$$

$$h(t) = te^{-t}u(t)$$

b) diagrama de blocos

$$\ddot{y} = -\frac{R}{L}\dot{y} - \frac{1}{LC}y + \frac{1}{LC}x = -2\dot{y} - y + x$$



c) resp em freq

$$|H(j\omega)| = \left|\frac{1}{(j\omega+1)^2}\right| = \frac{1}{|j\omega+1|^2} = \frac{1}{1+\omega^2}$$

$$H(j\omega) = \frac{1}{(j\omega+1)^2} = \frac{1}{(j\omega)^2+2j\omega+1} = \frac{1}{\underbrace{(1-\omega^2)}_{\alpha}+j\underbrace{2\omega}_{\beta}}$$

$$\Rightarrow \angle H(j\omega) = -tg^{-1}\frac{2\omega}{1-\omega^2}$$

Note

se $c = \frac{1}{\alpha+j\beta} \left(= \frac{\alpha-j\beta}{\alpha^2+\beta^2}\right)$

$\Rightarrow |c| = \frac{1}{\sqrt{\alpha^2+\beta^2}}$

$\angle c = -tg^{-1}\frac{\beta}{\alpha}$