
Exercícios recomendados

- 6.10, 6.11, 6.12, 6.15, 6.18, 6.19
- 6.25, 6.27, 6.28 (a i...xi), 6.29a, 6.32a, 6.37(a, b, d), 6.39 (a, b, c, d, f, i), 6.42, 6.45, 6.47 (a, b, c),
- 6.48 (a, b, c, d)

corrigir livro

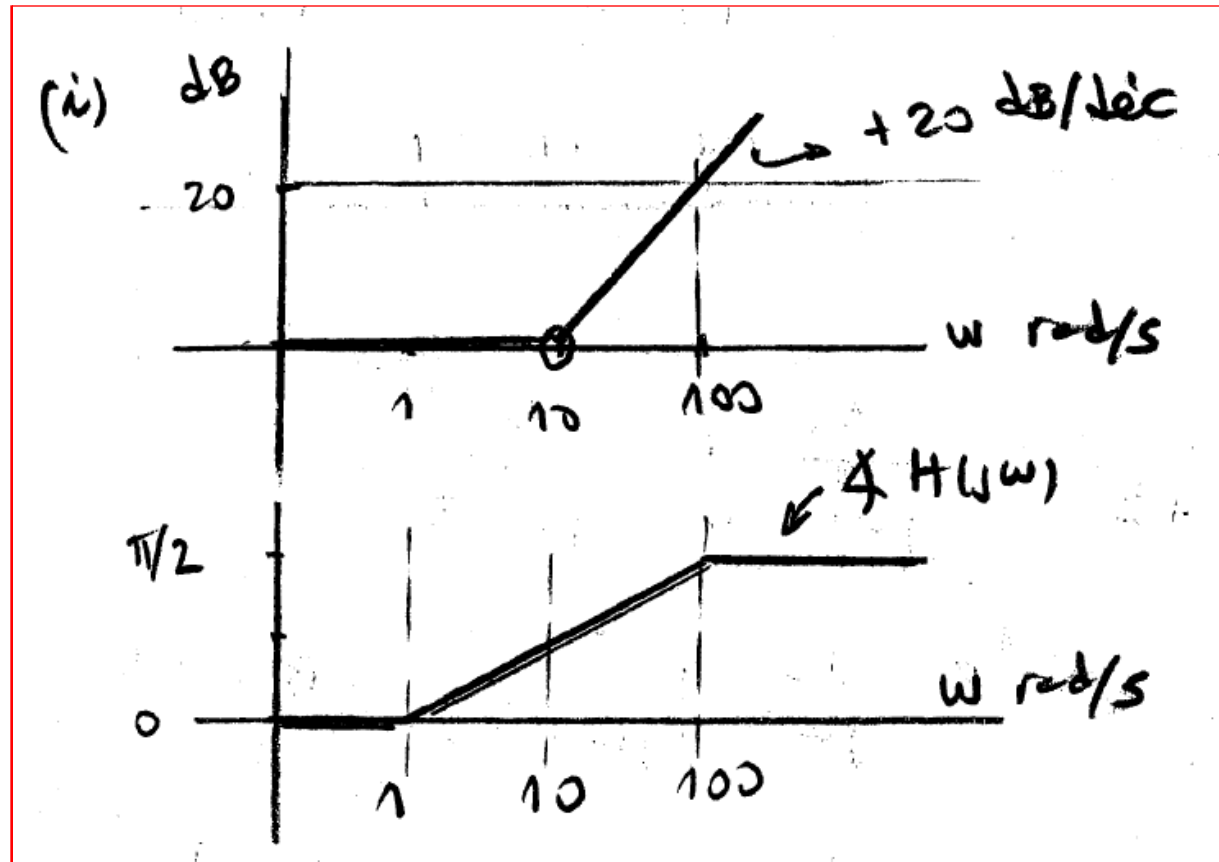
$$y[n] = b_0x[n] + b_1x[n-1] + b_2x[n-2] + b_3x[n-3]$$



6.28 (a) $H(j\omega) = 1 + \frac{j\omega}{10}$

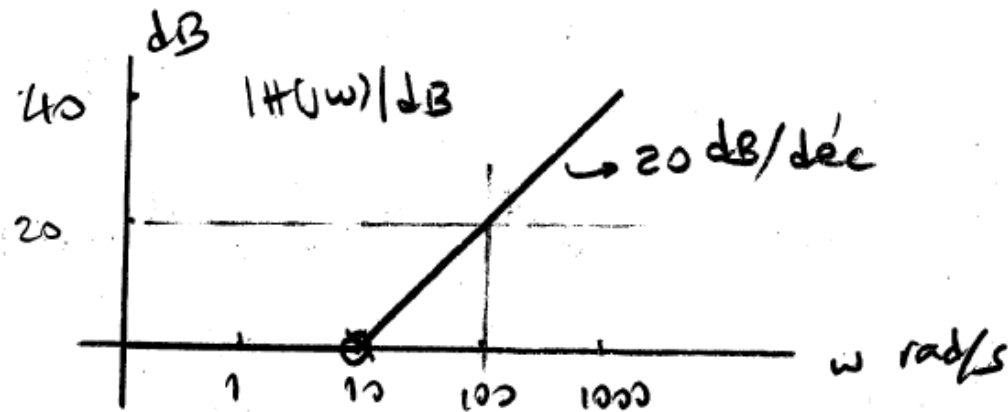
$$|H(j\omega)| = \sqrt{1 + (\omega/10)^2}$$

$$\angle H(j\omega) = \tan^{-1}(\omega/10)$$

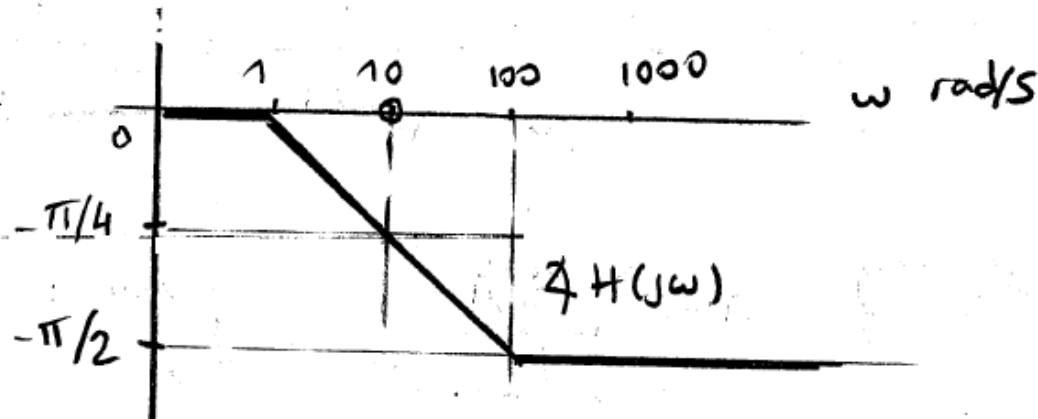


6.28 (a)

$$(a) H(j\omega) = 1 - \frac{j\omega}{10}$$



$$|H(j\omega)| = \sqrt{1 + (\omega/10)^2}$$

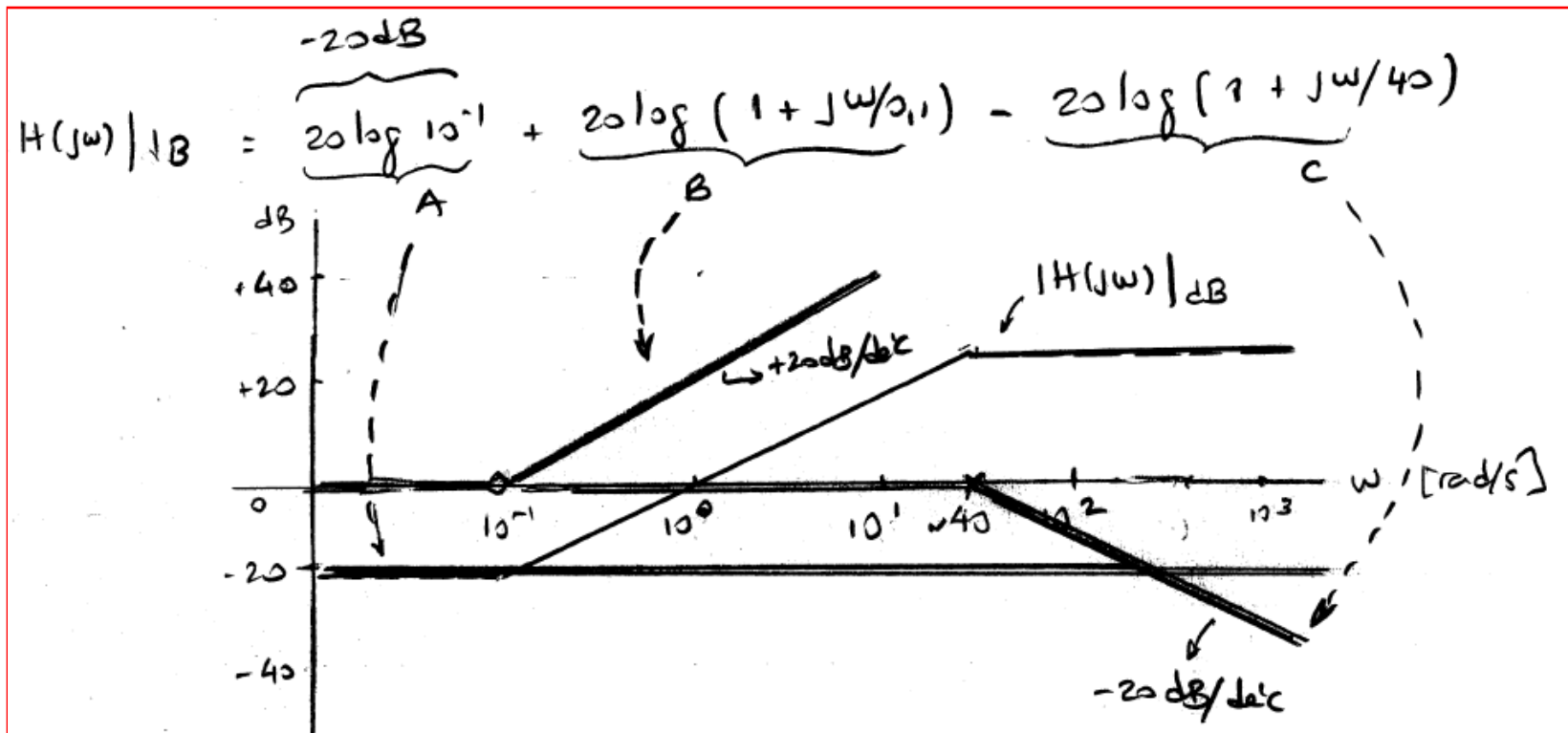


$$\angle H(j\omega) = \tan^{-1}(-\omega/10)$$

$$= -\tan^{-1}(\omega/10)$$

$$\boxed{6.10} \quad a) \quad H(j\omega) = 40 \frac{j\omega + 0,1}{j\omega + 40} = 40 (j\omega + 0,1) \times \frac{10}{10} \times \frac{1}{(j\omega + 40) \div 40}$$

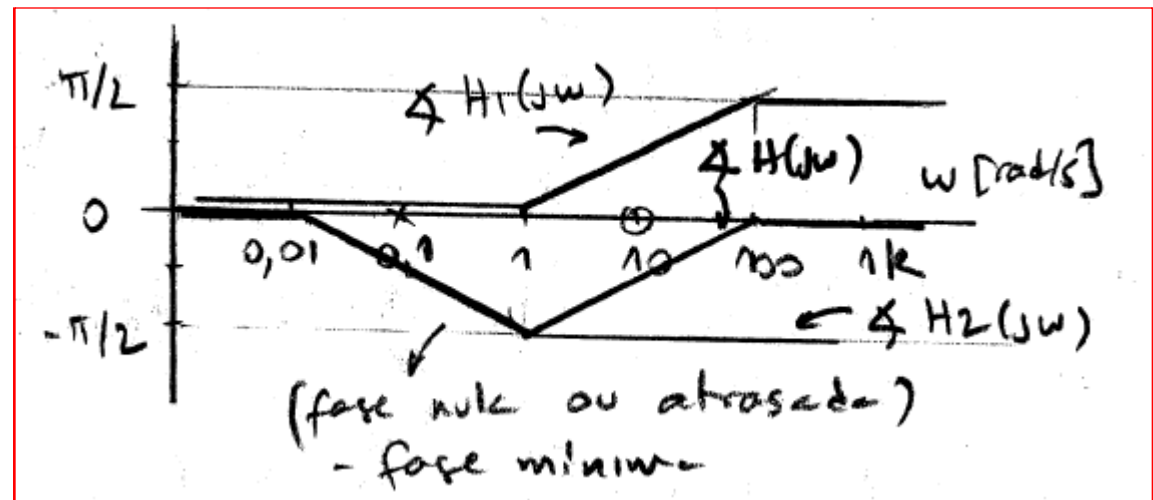
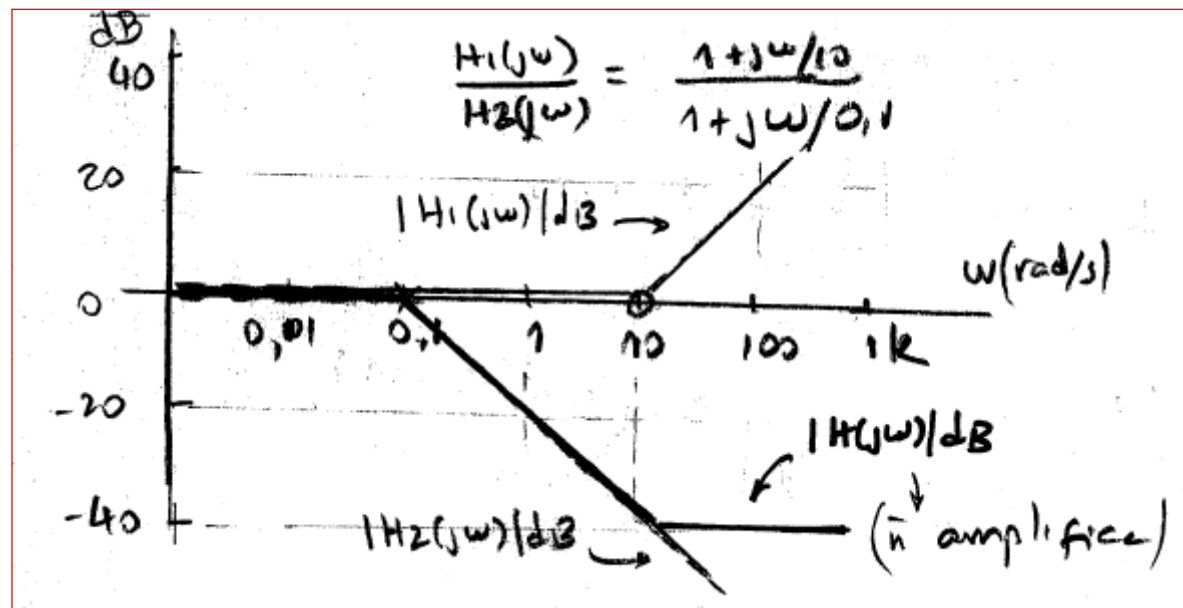
$$= \frac{1}{10} \frac{j\omega \times 10 + 1}{j\omega/40 + 1} = \frac{1}{10} \frac{j\omega/0,1 + 1}{j\omega/40 + 1}$$



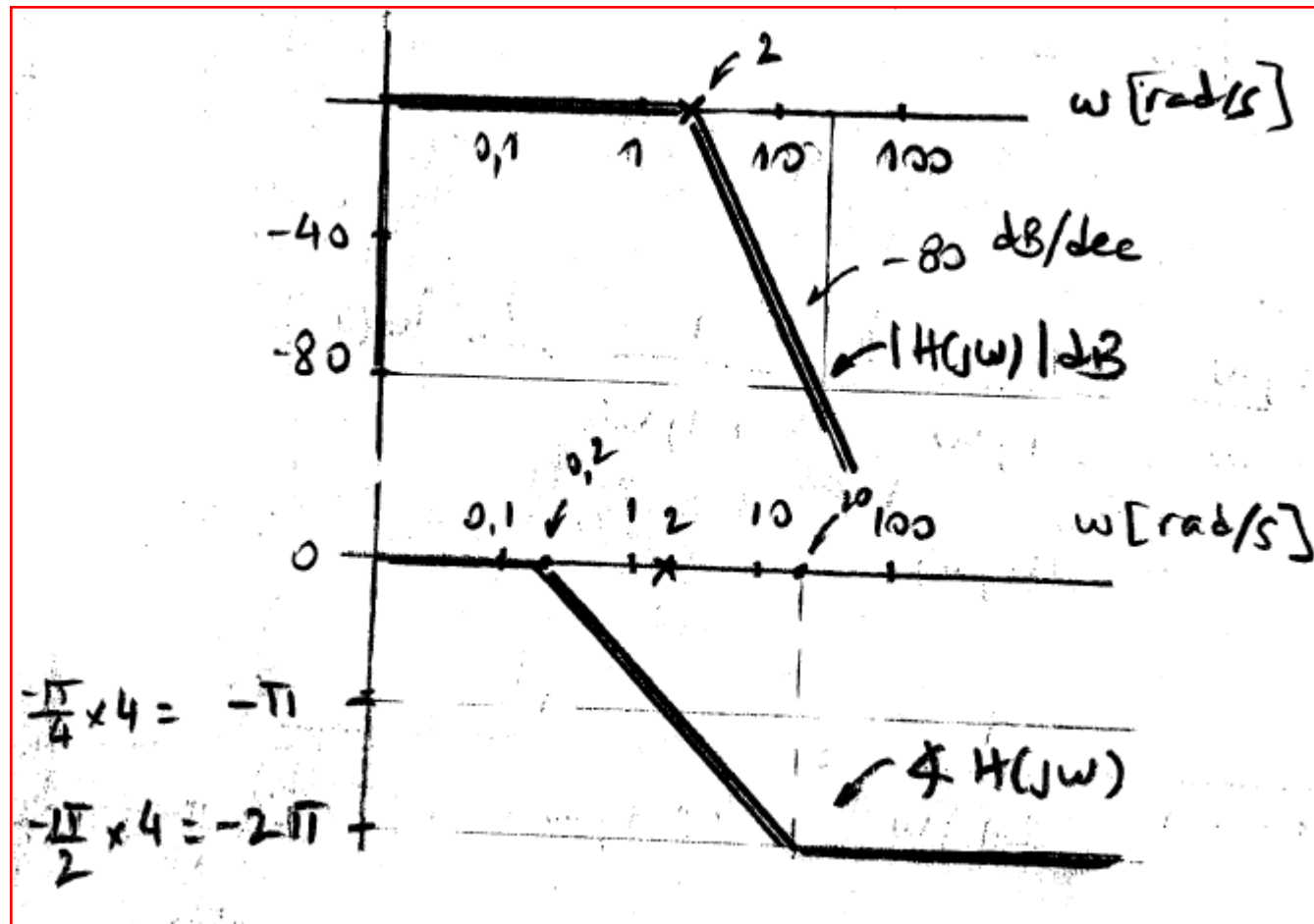
6.29 a) (i)

$$H(j\omega) = \frac{1 + j\omega/10}{1 + 10j\omega}$$

$$\frac{H_1(j\omega)}{H_2(j\omega)} = \frac{1 + j\omega/10}{1 + j\omega/0,1}$$



6.28 (a) (iii) $H(j\omega) = \frac{16}{(j\omega + 2)^4} = \frac{16/16}{(\frac{j\omega + 2}{2})^4} = \frac{1}{(1 + j\omega/2)^4}$



6.11 2ª ordem padrão: $H(j\omega) = \frac{\omega_n^2}{(j\omega)^2 + 2\zeta\omega_n(j\omega) + \omega_n^2}$

a) $H(j\omega) = \frac{250}{(j\omega)^2 + \underbrace{50,5 \cdot j\omega}_{= 2\zeta \cdot 5} + \underbrace{25}_{\omega_n^2}}$

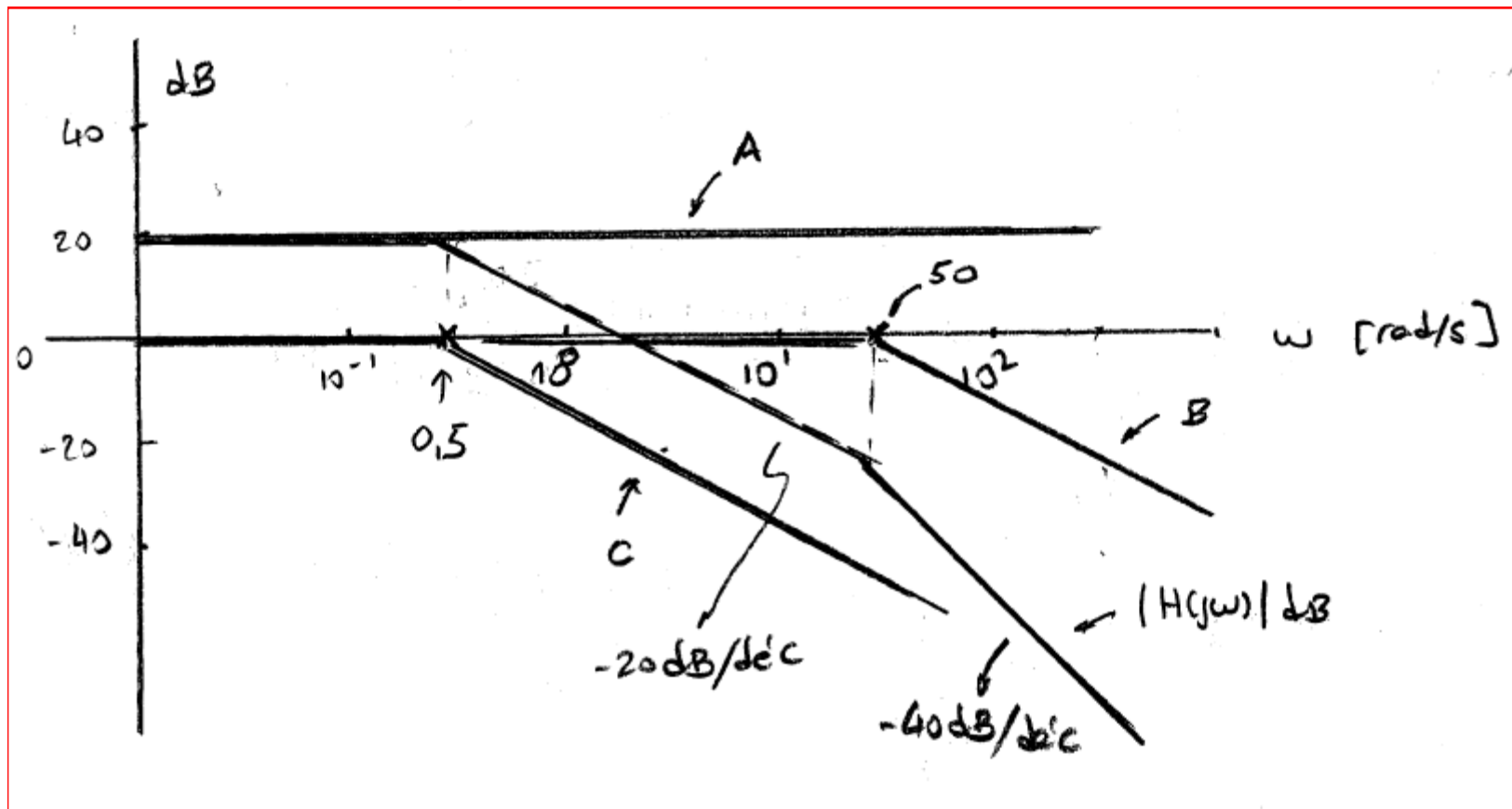
$= \frac{(5^2) \times 10}{(j\omega)^2 + 2 \times \underbrace{5,05}_{\zeta} \times \underbrace{5}_{\omega_n} \cdot j\omega + 5^2}$

(super amortecido) \rightarrow 2 pólos reais

$$H(j\omega) = \frac{250}{(j\omega + 50)(j\omega + 0,5)} = 250 \times \frac{1 \div 50}{(j\omega + 50) \div 50} \times \frac{1 \div 0,5}{(j\omega + 0,5) \div 0,5}$$

$$= \frac{\overbrace{250}^A}{\underbrace{50 \times 0,5}_{10}} \times \frac{\overbrace{1}^B}{\frac{j\omega}{50} + 1} \times \frac{\overbrace{1}^C}{\frac{j\omega}{0,5} + 1}$$

$$H(j\omega) = \frac{\overbrace{\frac{250}{50 \times 0,5}}^A}{\underbrace{10}} \cdot \frac{\overbrace{1}^B}{\frac{j\omega}{50} + 1} \cdot \frac{\overbrace{1}^C}{\frac{j\omega}{0,5} + 1}$$



$$\boxed{6.15} \quad (a) \quad \underbrace{\frac{d^2 y(t)}{dt^2}} + \underbrace{4 \frac{dy(t)}{dt}} + 4 y(t) = x(t)$$

$\Downarrow \mathcal{L}$

$$s^2 Y(s) + 4s Y(s) + 4 Y(s) = X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2 + 4s + 4}$$

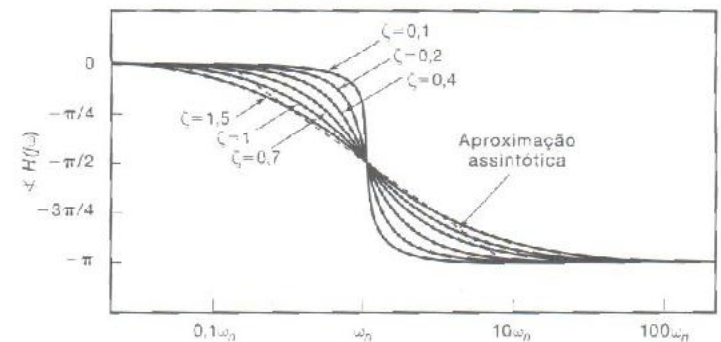
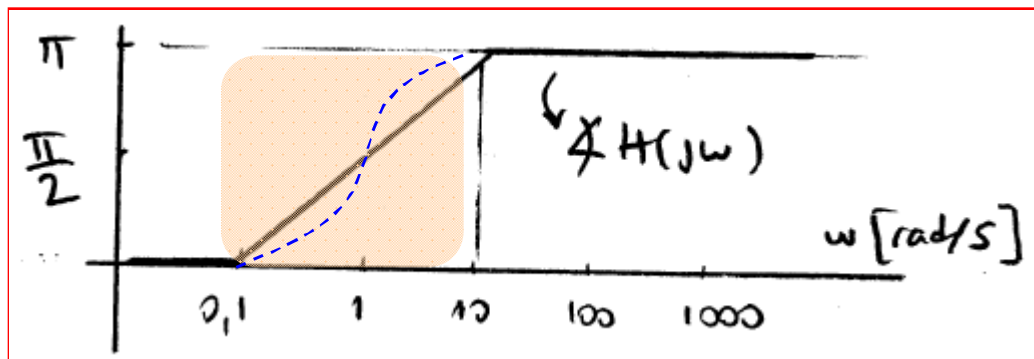
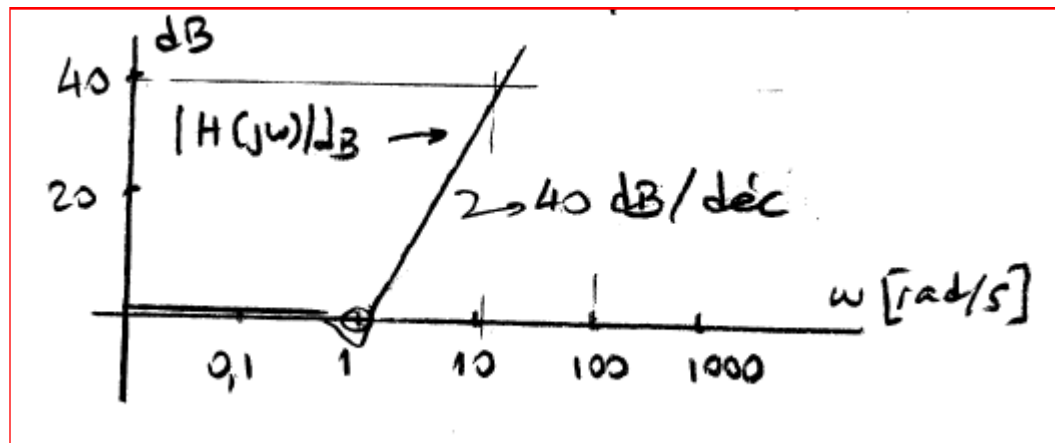
pólos: $s = \frac{-4 \pm \sqrt{4^2 - 4 \times 4}}{2} = -2$ (duplo) $\Delta = 0 \therefore$ criticamente
amortecido

$$H(s) = \frac{1}{2^2} \cdot \frac{2^2}{s^2 + 2 \cdot 2 \cdot 1 \cdot s + 2^2}$$

$\hookrightarrow \omega_n = 2$
 $\hookrightarrow \zeta = 1$
 $\hookrightarrow \omega_n$

6.28 (a) (ix) $H(j\omega) = 1 + j\omega + (j\omega)^2$

\downarrow \downarrow
 $\omega_n = 1$ $1 = 2\zeta \cdot \omega_n \rightarrow \zeta = 1/2$

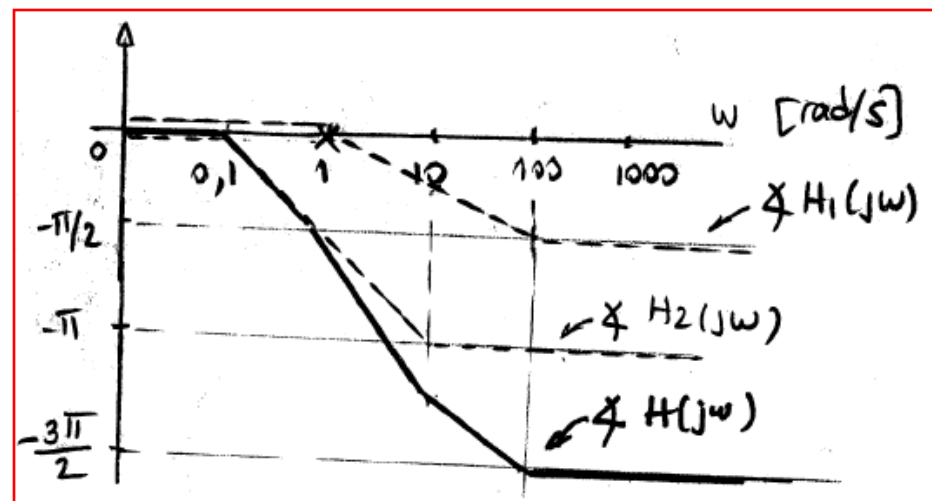
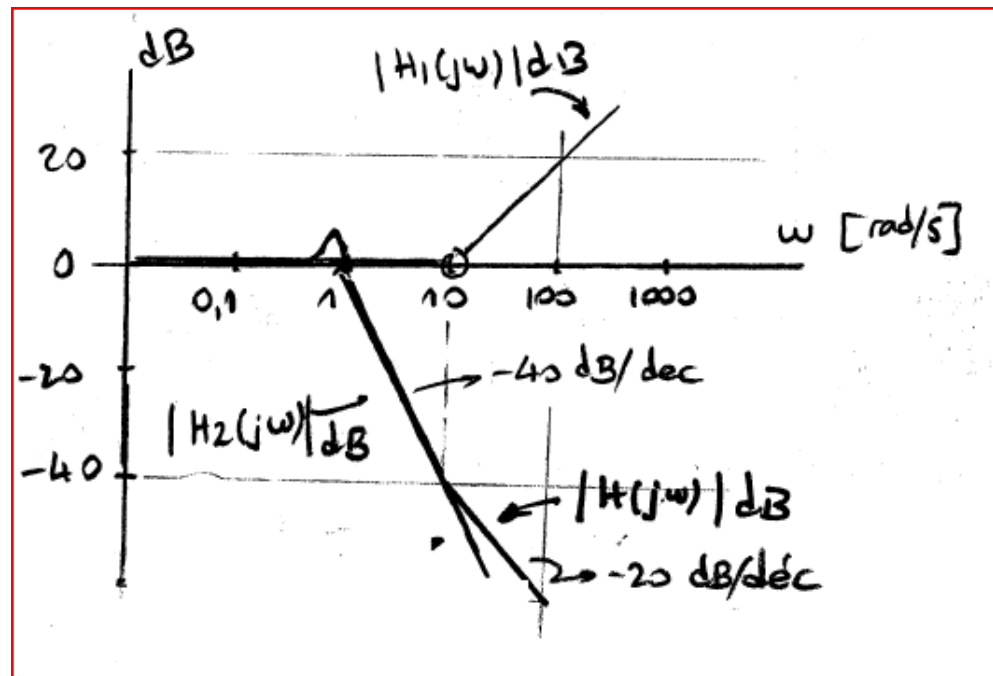


6.28

(vrai) $H(j\omega) =$

$$= \frac{1 - j\omega/10}{(j\omega)^2 + j\omega + 1} = \frac{H_1(j\omega)}{H_2(j\omega)}$$

$\underbrace{\quad}_{\omega_n}$
 $\underbrace{\quad}_{\zeta = 0,5}$



Temo discreto: sistema 1ª ordem

$$y[n] - ay[n-1] = x[n], \quad |a| < 1$$

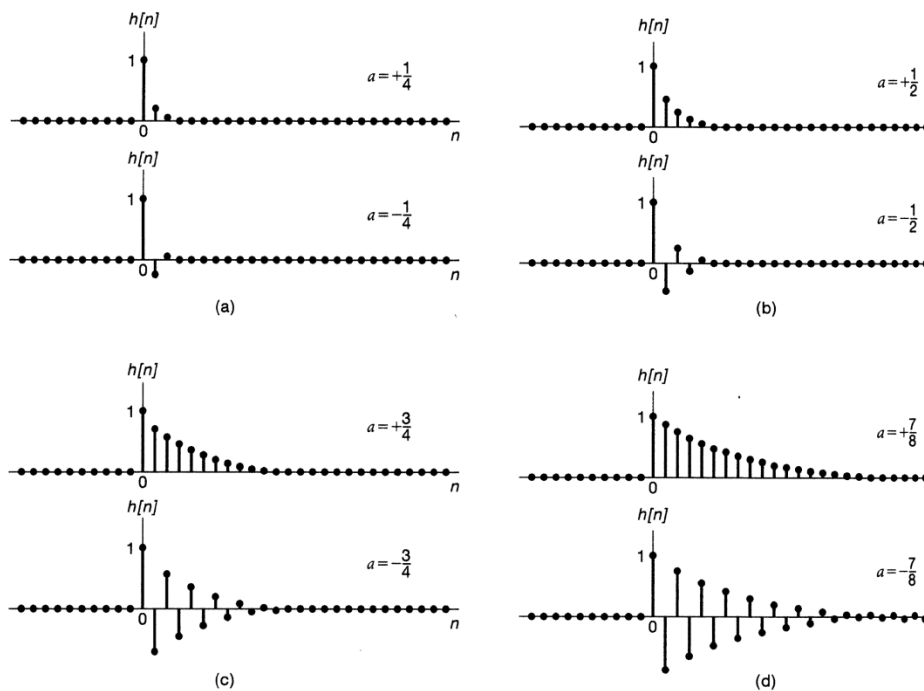
$$H(e^{j\omega}) = \frac{1}{1 - ae^{-j\omega}}$$

$$h[n] = a^n u[n]$$

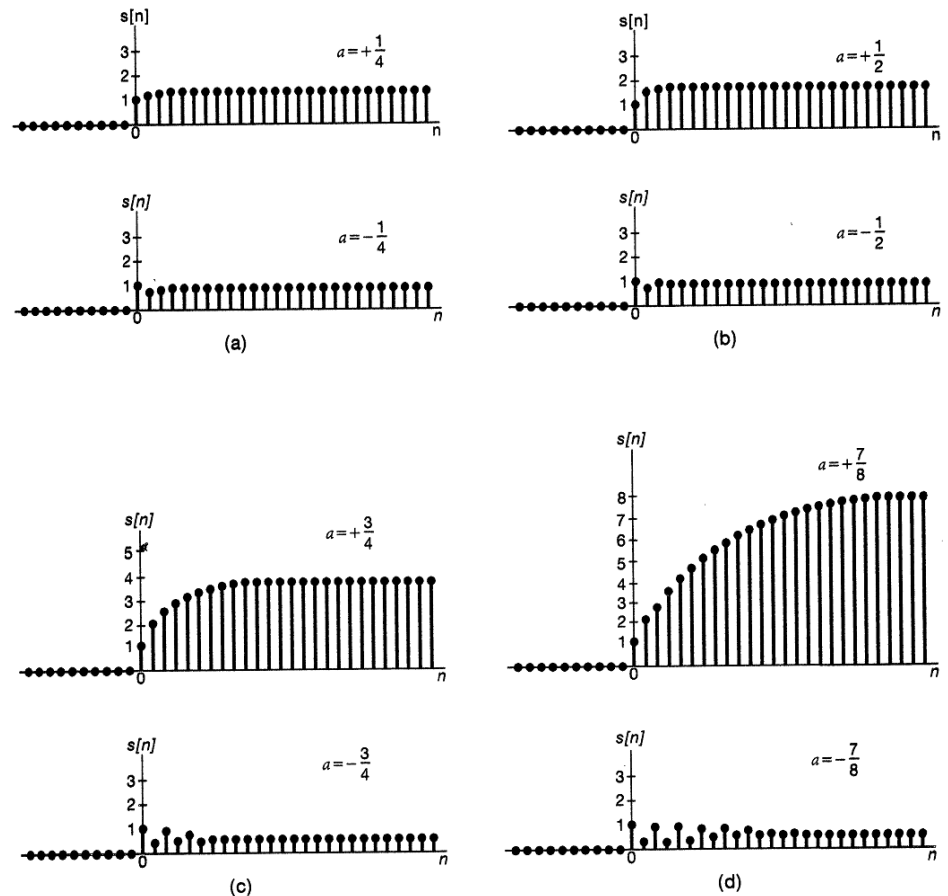
$$s[n] = h[n] * u[n] = \frac{1 - a^{n+1}}{1 - a} u[n]$$

Temo discreto: sistema 1ª ordem

Resp ao impulso



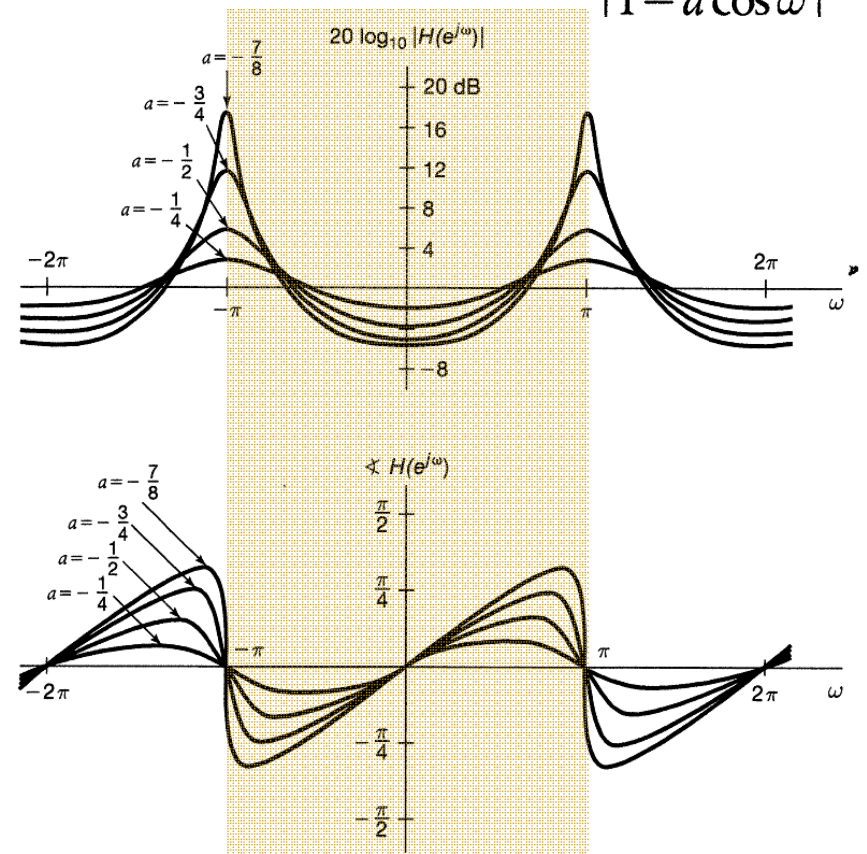
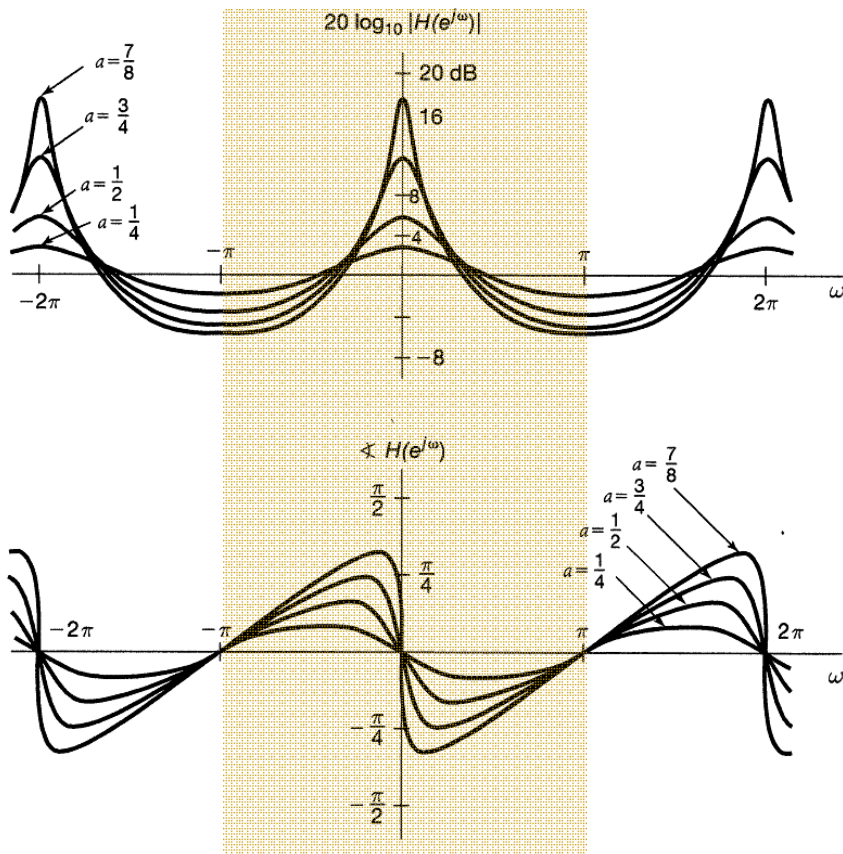
Resp ao degrau



Temo discreto: sistema 1ª ordem resposta em frequência diagrama de Bode

$$|H(e^{j\omega})| = \frac{1}{(1 + a^2 - 2a \cos \omega)^{1/2}}$$

$$\angle H(e^{j\omega}) = -\operatorname{tg}^{-1} \left[\frac{a \operatorname{sen} \omega}{1 - a \cos \omega} \right].$$



$$\boxed{6.39} \quad (a) \quad H(e^{j\omega}) = 1 + \frac{1}{2} e^{-j\omega} = \underbrace{1 + \frac{1}{2} \cos \omega}_{\text{re}} - j \underbrace{\frac{1}{2} \sin \omega}_{\text{im}}$$

$$|H(e^{j\omega})|^2 = \left(1 + \frac{1}{2} \cos \omega\right)^2 + \left(\frac{1}{2} \sin \omega\right)^2 =$$

$$= 1 + \cos \omega + \frac{1}{4} \cos^2 \omega + \frac{1}{4} \sin^2 \omega = 1 + \cos \omega + \frac{1}{4} (\underbrace{\sin^2 \omega + \cos^2 \omega}_{=1})$$

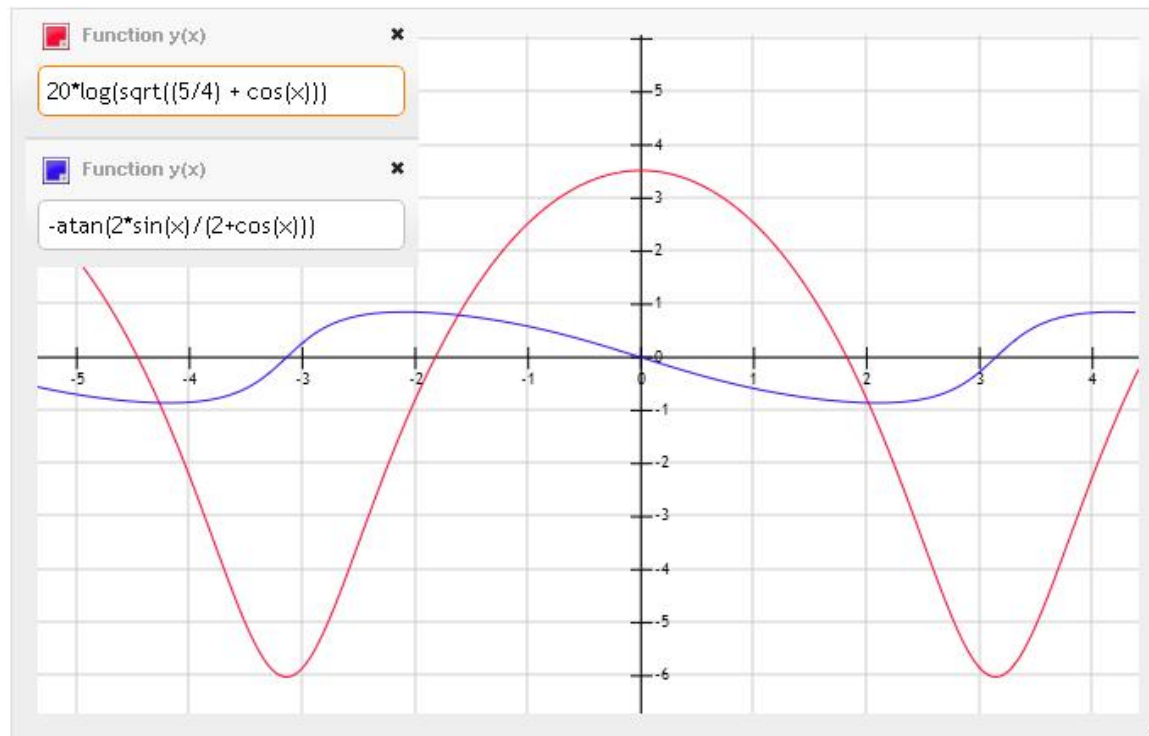
$$\boxed{|H(e^{j\omega})| = \sqrt{\frac{5}{4} + \cos \omega}}$$

$$\nabla H(e^{j\omega}) = +g^{-1} \frac{-\frac{1}{2} \sin \omega}{1 + \frac{1}{2} \cos \omega}$$

$$\boxed{\nabla H(e^{j\omega}) = -g^{-1} \frac{\sin \omega}{2 + \cos \omega}}$$

$$|H(e^{j\omega})| = \sqrt{\frac{5}{4} + \cos \omega}$$

$$\angle H(e^{j\omega}) = -\tan^{-1} \frac{\sin \omega}{2 + \cos \omega}$$



6.45 (i) $H_1(e^{j\omega}) = \frac{1}{(1 - \frac{1}{2}e^{-j\omega})(1 - \frac{1}{3}e^{-j\omega})(1 - \frac{1}{4}e^{-j\omega})}$

obs: $a^n u[n]$
 \updownarrow
 $\frac{1}{1 - ae^{-j\omega}}$

\downarrow
 $a = +\frac{1}{2}$

\downarrow
 $a = +\frac{1}{3}$

\downarrow
 $a = +\frac{1}{4}$

$\therefore h[n]$ não tem termo oscilatório

(ii) $H_2(e^{j\omega}) = \frac{1}{(1 + \frac{1}{2}e^{-j\omega})(1 - \frac{1}{3}e^{-j\omega})(1 - \frac{1}{4}e^{-j\omega})}$

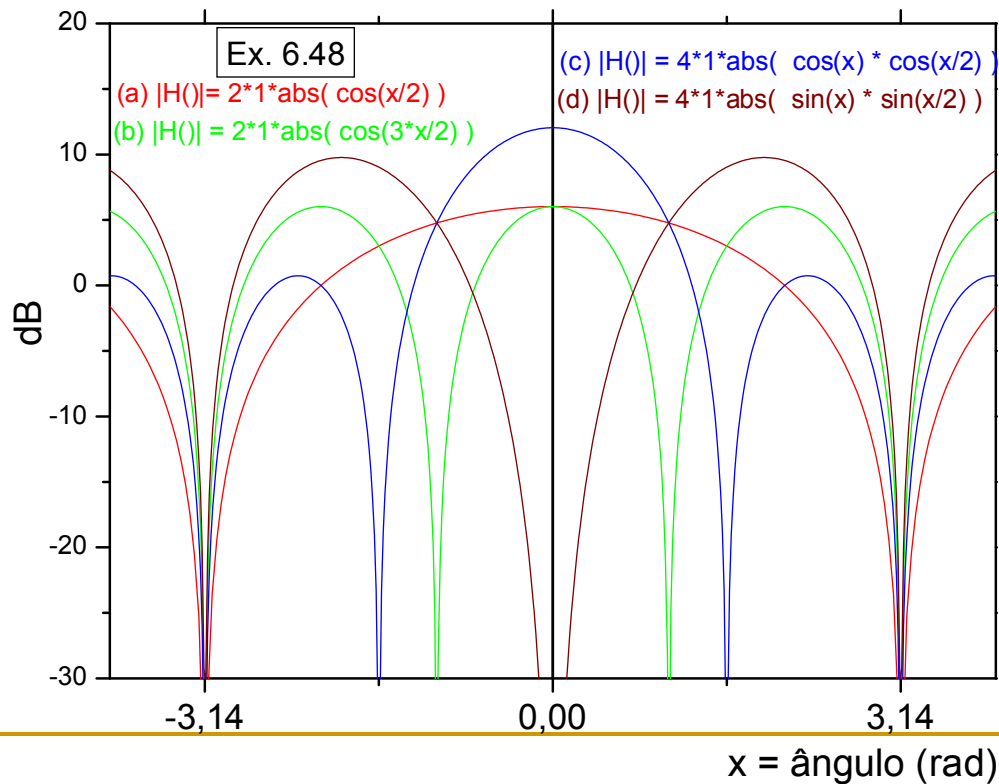
\downarrow
 $a = -\frac{1}{2}$ \therefore termo $(-\frac{1}{2})^n \cdot u[n]$ é oscilatório

6.48 $y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + b_3 x[n-3]$

a) $b_0 = b_3 = 0, b_1 = b_2$

$y[n] = b_1 \{x[n-1] + x[n-2]\} \xrightarrow{\mathcal{F}} Y(e^{j\omega}) = b_1 [e^{-j\omega} + e^{-2j\omega}] \cdot X(e^{j\omega})$

$H_1(e^{j\omega}) = b_1 \cdot e^{-j\frac{3\omega}{2}} [e^{j\omega/2} + e^{-j\omega/2}] = 2b_1 \cdot \cos\left(\frac{\omega}{2}\right) \cdot e^{-j\frac{3\omega}{2}}$
phase linear



6.48 (c) $b_0 = b_1 = b_2 = b_3$

$$y[n] = b_0 \{ x[n] + x[n-1] + x[n-2] + x[n-3] \}$$

$$Y(e^{j\omega}) = b_0 \underbrace{\left\{ 1 + e^{-j\omega} + e^{-2j\omega} + e^{-3j\omega} \right\}}_{\text{WF}} x(e^{j\omega})$$

$$H_3(e^{j\omega}) = b_0 \left\{ \underbrace{e^{-j\frac{3\omega}{2}} [e^{j\frac{3\omega}{2}} + e^{-j\frac{3\omega}{2}}]}_A + \underbrace{e^{-j\frac{3\omega}{2}} [e^{+j\frac{\omega}{2}} + e^{-j\frac{\omega}{2}}]}_B \right\} \times \frac{1}{2}$$

$$H_3(e^{j\omega}) = 2b_0 \cdot e^{-j\frac{3\omega}{2}} \left[\underbrace{\cos\left(\frac{3\omega}{2}\right)}_{a+b} + \underbrace{\cos\left(\frac{\omega}{2}\right)}_{a-b} \right]$$

$$= 2b_0 e^{-j\frac{3\omega}{2}} \times 2 \cos\left(\frac{\omega}{2}\right) \cdot \cos\left(\frac{\omega}{2}\right)$$

$$H_3(e^{j\omega}) = 4b_0 \cos(\omega) \cdot \cos\left(\frac{\omega}{2}\right) e^{-j\frac{3\omega}{2}}$$

(phase linear)

$$\cos(a+b) + \cos(a-b) =$$

$$2 \cos(a) \cdot \cos(b)$$

$$a+b = 3\omega/2$$

$$a-b = \omega/2$$

$$2a = 2\omega \Rightarrow a = \omega$$

$$b = \omega/2$$