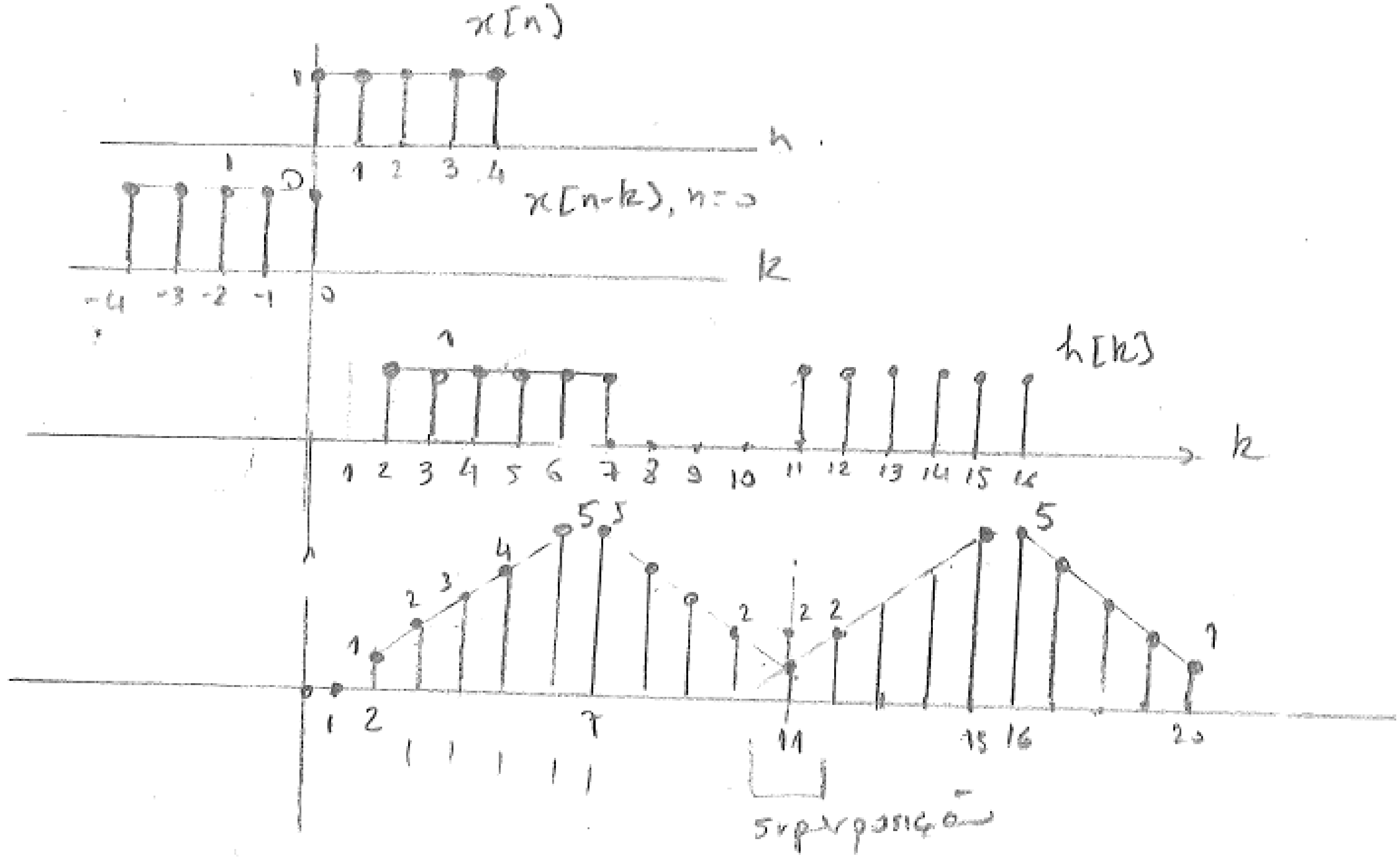
Modulo 02 - Convolução
Revisão/exercícios

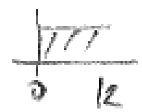
Cronograma

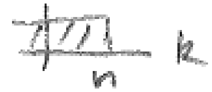
2.21 d)



$\boxed{2.21}$ a) $x[n]=\alpha^{n}\cdot u[n], \quad h[n]=\beta^{n}\cdot u[n], \quad \alpha\neq\beta$

$$y[n]=\sum_{k=-\infty}^{\infty}\alpha^{k}\cdot u[k]\cdot\beta^{(n-k)}\cdot u[n-k]=\sum_{k=0}^{n}\alpha^{k}\cdot\beta^{n}\cdot\beta^{-k}=$$





$(n\geq 0)$

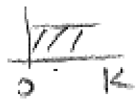
$$=\beta^{n}\sum_{k=0}^{n}\left(\frac{\alpha}{\beta}\right)^{k}=\beta^{n}\,\frac{1-\left(\frac{\alpha}{\beta}\right)^{n+1}}{1-\left(\frac{\alpha}{\beta}\right)}=\beta^{n}\,\frac{1-\alpha^{n+1}\cdot\beta^{-(n+1)}}{(1-\alpha)\cdot\beta^{-1}}=$$

$$=\frac{\beta^{n+1}-\alpha^{n+1}\cdot\beta^{-(n+1)+(n+1)}}{1-\alpha}=\boxed{\frac{\beta^{n+1}-\alpha^{n+1}}{1-\alpha}\cdot u[n]=y[n]}$$

4.21

b) $x[n] = h[n] = \alpha^n \cdot u[n]$

$$y[n] = \sum_{k=-\infty}^{\infty} \alpha^k u[k] \cdot \alpha^{n-k} \cdot u[n-k] = \sum_{k=0}^{n} \alpha^k \cdot \alpha^{n-k} \ , n>0$$



$$= \alpha^n \sum_{k=0}^{n} \alpha^k \cdot \alpha^{-k} = \alpha^n \sum_{k=0}^{n} 1 = \boxed{\alpha^n (n+1) \cdot u[n] = y[n]}$$

$\boxed{2.22}$ a) $x(t) = e^{-\alpha t} \cdot u(t)$, $h(t) = e^{-\beta t} \cdot u(t)$

$$y(t) = \int_{-\infty}^{\infty} e^{\alpha \tau} u(\tau) \cdot e^{-\beta(t-\tau)} u(t-\tau) \cdot d\tau =$$

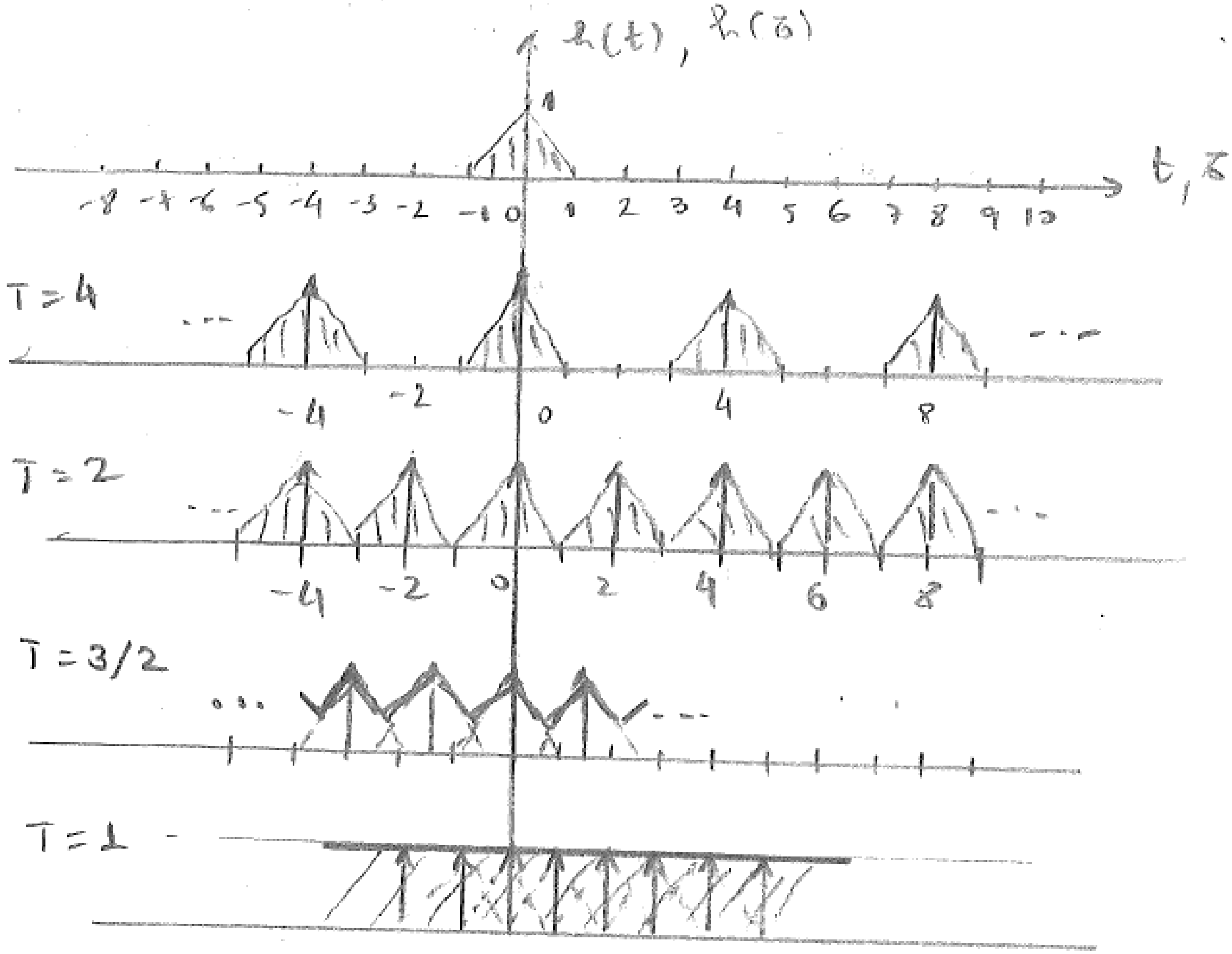
$$= \int_0^t e^{-\alpha \tau} \cdot e^{-\beta(t-\tau)} \cdot d\tau = e^{-\beta t} \int_0^t e^{-(\alpha-\beta)\tau} \cdot d\tau =$$

$$= e^{-\beta t} \left. \frac{e^{(\beta-\alpha)\tau}}{\beta-\alpha} \right|_0^t = \boxed{\frac{e^{-\beta t}}{\beta-\alpha} \left[e^{(\beta-\alpha)t} - 1 \right] \cdot u(t), \quad \beta \neq \alpha \quad = y(t)}$$

se $\alpha = \beta$, $y(t) = \int_0^t e^{-\alpha \tau} \cdot e^{-\alpha(t-\tau)} d\tau = e^{-\alpha t} \int_0^t d\tau =$

$$\rightarrow \boxed{y(t) = t \cdot e^{-\alpha t} \cdot u(t)}, \quad \alpha = \beta$$

2.23



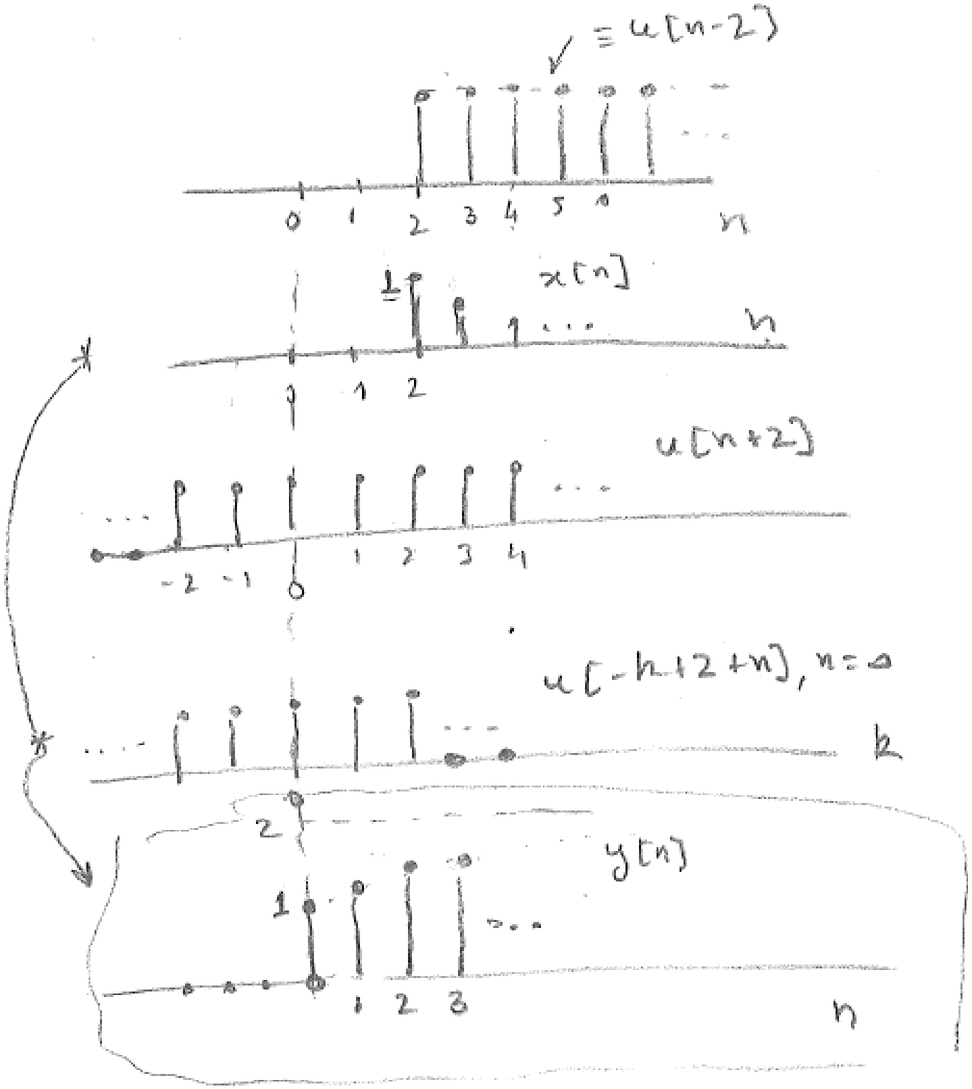
$\boxed{2.3}$ $x[n] = \left(\frac{1}{2}\right)^{n-2} \cdot u[n-2]$

$h[n] = u[n+2]$

$y[n] = x[n] * h[n]$

$= \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k] =$

$= \sum_{k=-\infty}^{\infty} \underbrace{\left(\frac{1}{2}\right)^{k-2} \cdot u[k-2]}_{x[k]} \cdot u[-k+2+n]$



$\begin{cases} k-2 = r \\ k = r+2 \end{cases}$

$= \sum_{r=-\infty}^{\infty} \left(\frac{1}{2}\right)^{r} \cdot u[r] \cdot u[-r+n]$

$= \sum_{r=0}^{n} \left(\frac{1}{2}\right)^{r} = \frac{1-\left(\frac{1}{2}\right)^{n+1}}{1-\left(\frac{1}{2}\right)} \rightarrow y[n] = 2\left[1-\left(\frac{1}{2}\right)\cdot\left(\frac{1}{2}\right)^{n}\right] = = 2-\left(\frac{1}{2}\right)^{n}, \ n \geq 0$

$(n \geq 0)$

$y[n] = \left[2-\left(\frac{1}{2}\right)^{n}\right] \cdot u[n]$

$\boxed{2.4}$ $x[n] = \begin{cases} 1, & 3 \le n \le 8 \\ 0, & cc \end{cases}$ $8-3+1=$ $h[n] = \begin{cases} 1, & 4 \le n \le 15 \\ 0, & cc \end{cases}$ $15-4+1=12$ em

$y[n] = x[n] * h[n]$

$$y[n] = \sum_{k=-\infty}^{\infty} x[n] \cdot h[n-k]$$

i) $y[n] = 0, \; n \le 6$

ii) $7 \le n \le 12$

n	$y[n]$
7	1
8	2
⋮	⋮
12	11

$\boxed{y[n] = n-1}$

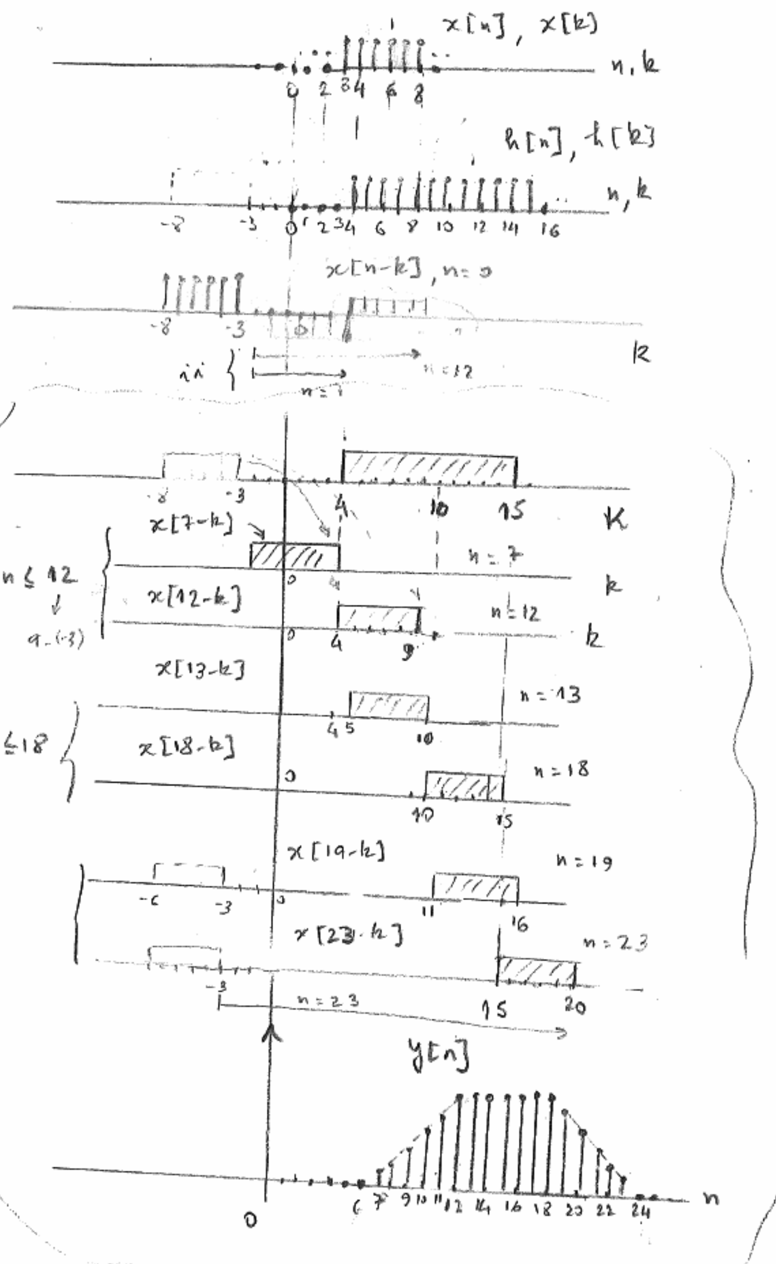
iii) $13 \le n \le 18$

$y[n] = 6$

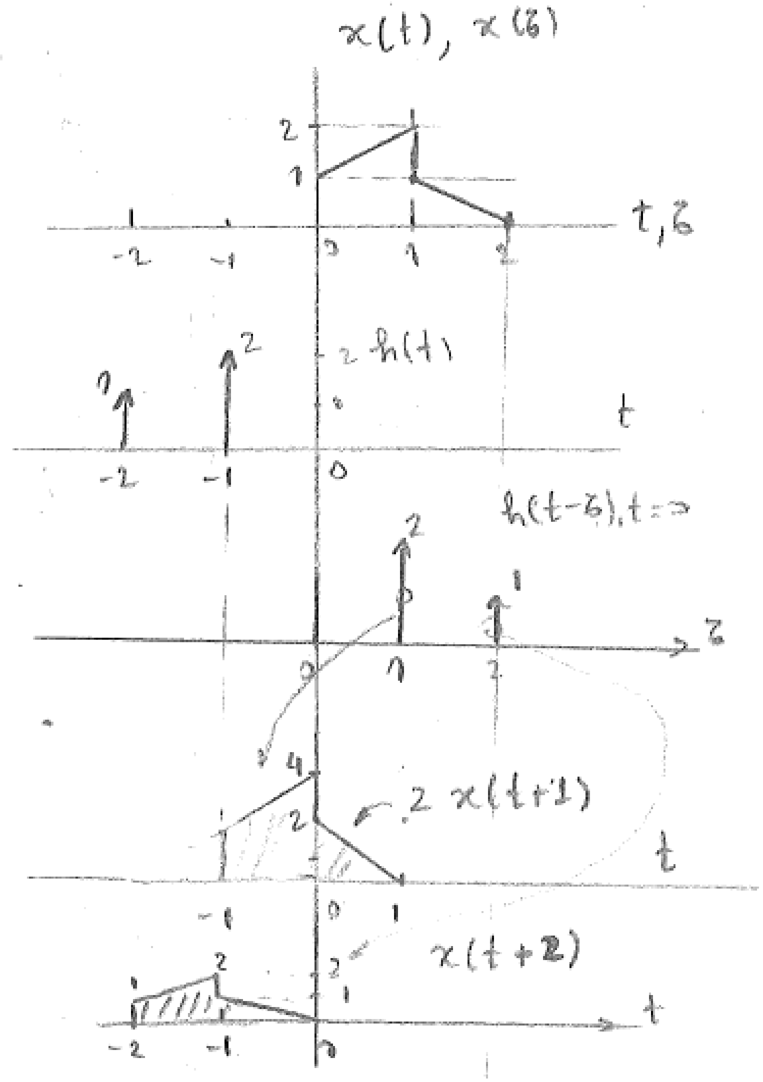
iv) $19 \le n \le 23$

n	$y[n]$
19	5
20	4
21	3
22	2
23	1

$\boxed{y[n] = 24-n}$



$\boxed{2.8}$



$$y(t) = \int_{-\infty}^{\infty} x(\tau)\cdot h(t-\tau)\cdot d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau)\cdot x(t-\tau)\, d\tau$$

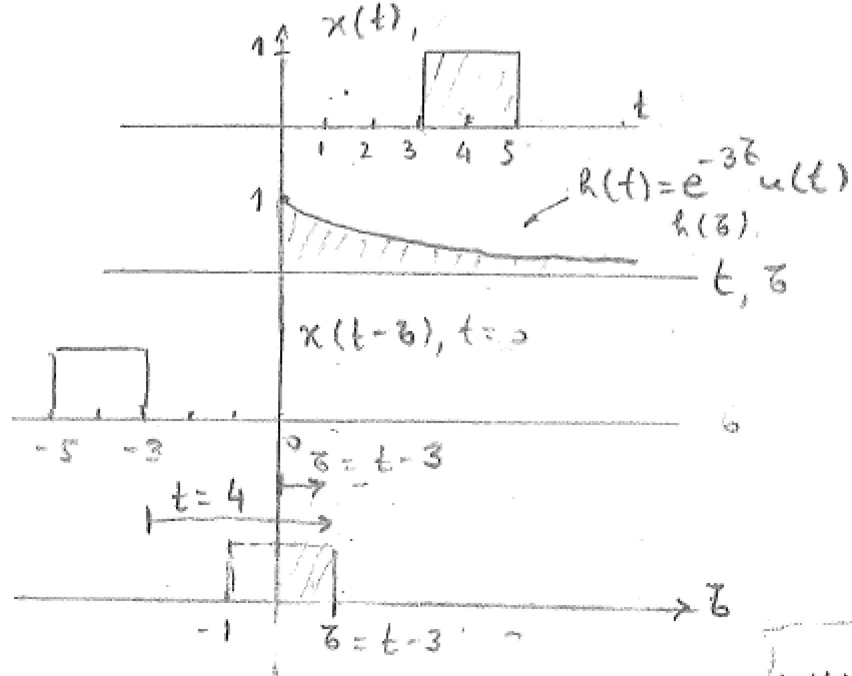
$$= \int_{-\infty}^{\infty} \left[2\cdot\delta(\tau+1) + \delta(\tau+2)\right]\cdot x(t-\tau)\cdot d\tau$$

$$= \int_{-\infty}^{\infty} \left[2\cdot\delta(\tau+1)\cdot x[t+1]\cdot d\tau\right.$$

$$+ \int_{-\infty}^{\infty} \delta(\tau+2)\cdot x(t+2)\cdot d\tau$$

$$y(t) = 2\cdot x(t+1) + x(t+2)$$

$\boxed{2.11}$



$y(t) = x(t) * h(t) =$

$\boxed{t < 3 : y(t) = 0}$

$3 \leq t \leq 5$:

$$y(t) = \int_0^{t-3} e^{-3\tau} \cdot d\tau$$

$$= \left. \frac{e^{-3\tau}}{-3} \right|_0^{t-3} = \frac{e^{-3(t-3)} - 1}{-3}$$

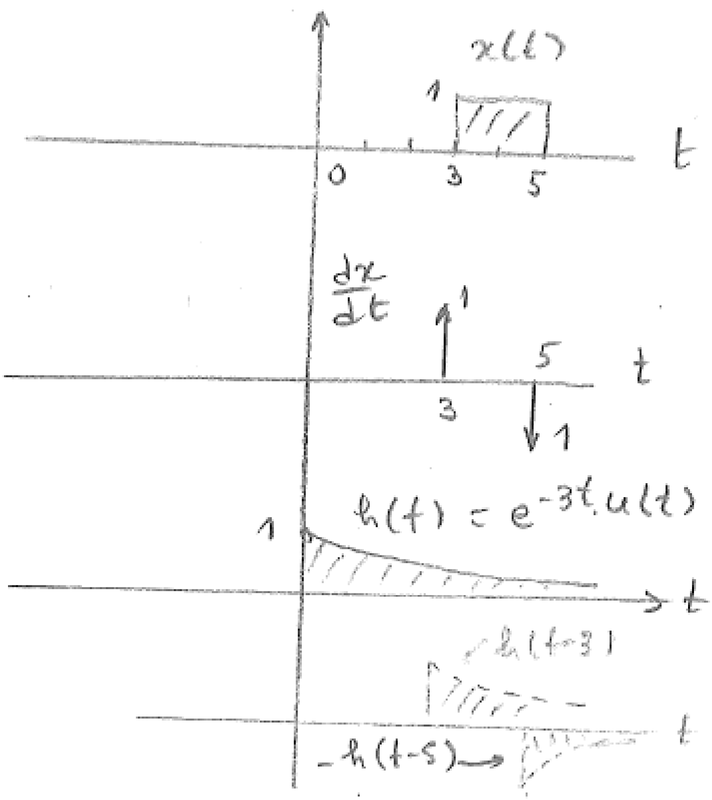
$$\boxed{y(t) = \frac{1 - e^{-3(t-3)}}{3}} \quad 3 \leq t \leq 5$$

$$t > 5 : y(t) = \int_{t-3-2}^{t-3} e^{-3\tau} \cdot d\tau = \left. \frac{e^{-3\tau}}{-3} \right|_{t-5}^{t-3} = \frac{e^{-3(t-3)} - e^{-3(t-5)}}{-3}$$

$$= \frac{1}{3}\left[e^{-3(t-5)} - e^{-3(t-3)}\right] = \frac{e^{-3t} \cdot e^{15} - e^{-3t} \cdot e^{9} \times \overbrace{(e^{15} \times e^{-15})}^{=1}}{3}$$

$$= e^{3t} \cdot e^{15} \frac{\left[1 - e^{-6}\right]}{3} \rightarrow \boxed{y(t) = e^{-3(t-5)} \cdot \frac{(1 - e^{-6})}{3}, \quad t > 5}$$

$\boxed{2.11}$ b)



$$x_2(t) = \frac{dx}{dt} = \delta(t-3) - \delta(t-5)$$

$$g(t) = \int_{-\infty}^{\infty} x_2(\tau) \cdot h(t-\tau) \, d\tau$$

$$= \int_{-\infty}^{\infty} \delta(\tau-3) \cdot h(t-\tau) \cdot d\tau - \int_{-\infty}^{\infty} \delta(t-5) \cdot \delta(t-\tau) \, d\tau$$

$$= \int_{-\infty}^{\infty} \delta(\tau-3) \cdot h(t-3) \cdot d\tau - \int_{-\infty}^{\infty} \delta(\tau-5) \cdot h(t-5) \cdot d\tau$$

$$\boxed{g(t) = h(t-3) - h(t-5)}$$

$$= e^{-3(t-3)} \cdot u(t-3) - e^{-3(t-5)} \cdot u(t-5)$$

c)

se $x(t) \xrightarrow{\text{SLIT}} y(t)$

$x'(t) \xrightarrow{\text{SLIT}} y'(t)$

$\boxed{2.14}$ a) $h_1(t) = e^{-(1-2j)t} \cdot u(t) = \overbrace{e^{-t}}^{|h_1(t)|} \cdot e^{2jt} \cdot u(t) \therefore \int_{-\infty}^{\infty} e^{-t} \cdot dt =$

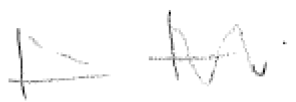
$= -e^{-t}\Big|_0^{\infty} = 0 - 1 = 1$

$\therefore$ estável

estabilidade:

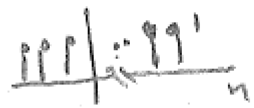
$\int_{-\infty}^{\infty} |h_1(t)| \cdot dt < \infty$?

b) $h_2(t) = e^{-t} \cdot \cos(2t) \cdot u(t) \therefore$ (vide "a") estável



$\boxed{2.15}$ a) $h_1[n] = n \cdot \cos\left(\frac{\pi}{4} n\right) \cdot u[n]$ $\therefore$ $h_1(n) \xrightarrow[n \to \infty]{} \infty$ (não estável)

b) $h_2[n] = 3^n \cdot u[-n+10]$



$\downarrow$

estável

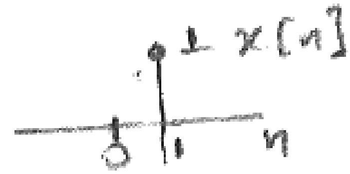
$$\begin{cases} 0 < n \le 10 : \quad \sum_{n=0}^{10} |h_2[n]|^2 < \infty \\ n < 0 : \quad \sum_{-\infty}^{-1} 3^n = \sum_{1}^{\infty} 3^{-n} \end{cases}$$

$$= \frac{3^{-1}}{1 - 3^{-1}} = \frac{1/3}{1 - 1/3} = \frac{1/3}{2/3} = \frac{1}{2}$$

ou : $\sum_{n=-\infty}^{10} 3^n = \sum_{r=10}^{\infty} 3^{-r} = \frac{3^{+10}}{1 - 1/3} =$

$= \frac{3^{10}}{2/3} = 3^{10} \times \frac{3}{2} = \frac{1}{2} \times 3^{11} < \infty$

$\boxed{2018}$ $y[n] = \underbrace{\frac{1}{4}}_{\equiv a} y[n-1] + x[n]$, $x[n] = \delta[n-1]$



$$y[0] = a \cdot \cancel{y[-1]}^{\,0} + \cancel{x[0]}^{\,0} = 0$$

$$y[1] = a \cdot y[0] + x[1] = 1$$

$$y[2] = a \cdot y[1] + \cancel{x[2]}^{\,0} = a$$

$$y[3] = a \cdot y[2] + \cancel{x[3]}_{\,0} = a^2$$

$$y[4] = a \cdot y[3] + \cancel{x[4]}_{\,0} = a^3$$

$$\vdots$$

$$\boxed{y[n] = a^{n-1} \cdot u[n-1] = \left(\frac{1}{4}\right)^{n-1} \cdot u[n-1]}$$

$\boxed{2.19}$

$$x[n] \to \boxed{S1} \xrightarrow{w[n]} \boxed{S2} \to y[n]$$

$$w[n] = \frac{1}{2} w[n-1] + x[n]$$

$$y[n] = \alpha\, y[n-1] + \beta \cdot w[n]$$

$$w[n-1] = \frac{y[n-1] - \alpha\, y[n-2]}{\beta}$$

a) $y[n] = \alpha \cdot y[n-1] + \beta \Big\{ \underbrace{\frac{1}{2} w[n-1] + x[n]}_{w[n]} \Big\} =$

(eliminar "w")

$$y[n] = \alpha\, y[n-1] + \frac{\beta}{2} \cdot w[n-1] + \beta \cdot x[n]$$

$$y[n] = \alpha \cdot y[n-1] + \frac{\beta}{2} \times \Big\{ \frac{1}{\beta} y[n-1] - \frac{\alpha}{\beta} y[n-2] \Big\} + \beta \cdot x[n]$$

$$y[n] = \alpha \cdot y[n-1] + \frac{1}{2} y[n-1] - \frac{\alpha}{2} \cdot y[n-2] + \beta\, x[n]$$

$$y[n] = -\frac{\alpha}{2} \cdot y[n-2] + \left(\alpha + \frac{1}{2}\right) \cdot y[n-1] + \beta \cdot x[n]$$

$$\equiv \frac{1}{8} y[n-2] + \frac{3}{4} y[n-1] + x[n] \quad \therefore \boxed{\beta = 1}$$

$$\alpha + \frac{1}{2} = \frac{3}{4} \to \boxed{\alpha = \frac{1}{4}}$$

$\boxed{2.19}$

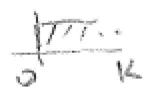
b) $x[n] \longrightarrow \boxed{S_1 * S_2} \longrightarrow y[n]$

$h[n] = h_1[n] * h_2[n]$

$h_1[n] = \left(\frac{1}{2}\right)^n u[n]$

$y[n] = \frac{1}{4} y[n-1] + w[n] \therefore h_2(n) = \left(\frac{1}{4}\right)^n u[n]$

$$h[n] = \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^k u[n] \cdot \left(\frac{1}{4}\right)^{n-k} u[n-k] = \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k \left(\frac{1}{4}\right)^{n-k} = \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k \left(\frac{1}{2}\right)^{2(n-k)}$$



$$= \left(\frac{1}{2}\right)^{2n} \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{k-2k} = \left(\frac{1}{2}\right)^{2n} \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{-k} = \left(\frac{1}{2}\right)^{2n} \frac{1-\left(\frac{1}{2}\right)^{-(n+1)}}{1-\left(\frac{1}{2}\right)^{-1}} = \boxed{*}$$

$$\boxed{*} = \left(\frac{1}{2}\right)^{2n} \frac{1-\left(\frac{1}{2}\right)^{-n} \cdot 2}{1-2} =$$

$$\left(\frac{1}{2}\right)^{2n} \left[2\left(\frac{1}{2}\right)^{-n} - 1\right] =$$

$$2\left(\frac{1}{2}\right)^{n} - \left(\frac{1}{2}\right)^{2n} =$$

$$\left[2\left(\frac{1}{2}\right)^{n} - \left(\frac{1}{4}\right)^{n}\right] \cdot u[n]$$

$\boxed{2.28}$ a) $h[n] = \left(\frac{1}{5}\right)^n \cdot u[n]$ ∴ causal $h[n] = 0 \ \forall n < 0$

$$\sum_{-\infty}^{\infty} |h[n]| = \sum_{n=0}^{\infty} \left(\frac{1}{5}\right)^n = \frac{1}{1-\left(\frac{1}{5}\right)} = \frac{1}{4/5} = 5/4 < \infty$$ ∴ estável

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b) $h[n] = (0{,}8)^n \cdot u[n+2]$ ∴ não causal

$$\sum_{n=-\infty}^{\infty} h[n] = \sum_{n=-2}^{\infty} (0{,}8)^n = \frac{(8/10)^{-2}}{1-8/10} < \infty$$ ∴ estável

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c) $h[n] = \left(\frac{1}{2}\right)^n \cdot u[-n]$ ∴ $\underset{n<0:}{}$ ∴ $h[-5] = \left(\frac{1}{2}\right)^{-5} \cdot u[+5]$ → não causal

$$\sum_{-\infty}^{\infty} h[n] = \sum_{n=-\infty}^{0} \left(\frac{1}{2}\right)^n = \sum_{r=0}^{\infty} \left(\frac{1}{2}\right)^{-r} = \sum_{r=0}^{\infty} 2^r \to \infty$$ ∴ não estável

$\boxed{2.29}$ a) causal pois $h(t)=0 \ \forall\, t<0$

$$\int_{-\infty}^{\infty}|h(t)|\cdot dt = \int_{2}^{\infty} e^{-4t}\cdot dt = -\frac{1}{4}\cdot e^{-4t}\Big|_{2}^{\infty} = -\frac{1}{4}\left(0-e^{-8}\right) = \frac{e^{-8}}{4} < \infty$$

$\therefore$ estável

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b) não causal : $h(t) \neq 0 \ \forall\, t<0$

$$\int_{-\infty}^{\infty}|h(t)|\cdot dt = \int_{-\infty}^{\infty} e^{-6t}\cdot u(3-t)\cdot dt = \int_{-\infty}^{3} e^{-6t}\cdot dt = -\frac{1}{6}\, e^{-6t}\Big|_{-\infty}^{3} =$$

$$-\frac{1}{6}\left(e^{-18}-e^{\infty}\right) \to \infty \ \therefore$$

não estável

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c) $h(t) = e^{-2t}\cdot u(t+50)$ $\therefore$ não causal

$$\int_{-\infty}^{\infty} e^{-2t}\cdot u(t+50)\cdot dt = \int_{-50}^{\infty} e^{-2t}\, dt = -\frac{1}{2}\, e^{-2t}\Big|_{-50}^{\infty} = \frac{1}{2}\, e^{100} < \infty$$

estável

$\boxed{2.30}$ $y[n] + 2y[n-1] = x[n] \rightarrow$ $x[n] = \delta[n]$ e repouso inicial $y[n] = 0,\ n < 0$

$y[n] = -2 \cdot y[n-1] + x[n]$;

$y[0] = -2 \cdot y[-1] + 1 = 1$

$y[1] = -2\, y[0] + x[1] = 1$

$y[2] = -2\, y[1] + x[2] = -2$

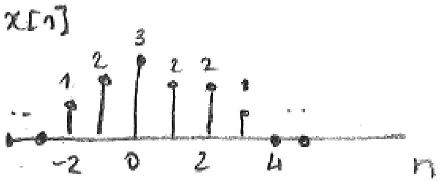
$y[3] = -2\, y[2] + x[3] = -2 \cdot -2 = (-2)^2$

...

$y[n] = (-2)^n \cdot u[n]$

$n \geq 0$

$\boxed{2.31}$



$$y[n] = -2 \cdot y[n-1] + x[n] + 2 \cdot x[n-2]$$

$$y[n] = 0, \quad n < 2$$

$$y[-2] = -2 \cdot \underbrace{y[-3]}_{=0} + \underbrace{x[-2]}_{1} + 2 \cdot \underbrace{x[-4]}_{0} \rightarrow y[-2] = 1$$

$$y[-1] = -2 \cdot \underbrace{y[-2]}_{1} + \underbrace{x[-1]}_{2} + 2 \cdot \underbrace{x[-3]}_{0} \rightarrow y[-1] = 0$$

$$y[0] = -2\,\underbrace{y[-1]}_{0} + \underbrace{x[0]}_{3} + 2\,\underbrace{x[-2]}_{1} \rightarrow y[0] = 5$$

$$-10 + 2 + 4$$

$$y[1] = -2\,\underbrace{y[0]}_{5} + \underbrace{x[1]}_{2} + 2\,\underbrace{x[-1]}_{2} \rightarrow y[1] = -4$$

$$8 + 2 + 6$$

$$y[2] = -2\,\underbrace{y[1]}_{-4} + \underbrace{x[2]}_{2} + 2 \cdot \underbrace{x[0]}_{3} \rightarrow y[2] = 16$$

$$-32 + 1 + 4$$

$$y[3] = -2\,\underbrace{y[2]}_{16} + \underbrace{x[3]}_{1} + 2\,\underbrace{x[1]}_{2} \rightarrow y[3] = -27$$

$$y[4] = -2 \cdot \underbrace{y[3]}_{-27} + \underbrace{x[4]}_{0} + 2\,\underbrace{x[2]}_{2} \rightarrow y[4] = 56$$

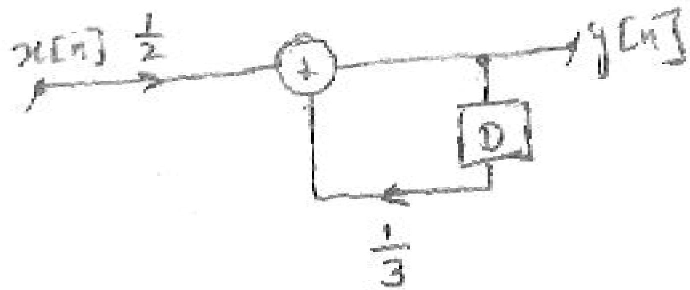
$$y[5] = -2\,\underbrace{y[4]}_{56} + \underbrace{x[5]}_{0} + 2\,\underbrace{x[3]}_{1} \rightarrow y[5] = -110$$

$$y[6] = -2\,\underbrace{y[5]}_{-110} + \underbrace{x[6]}_{0} + 2\,\underbrace{x[4]}_{0} \rightarrow y[6] = -2^{1} \times -110$$

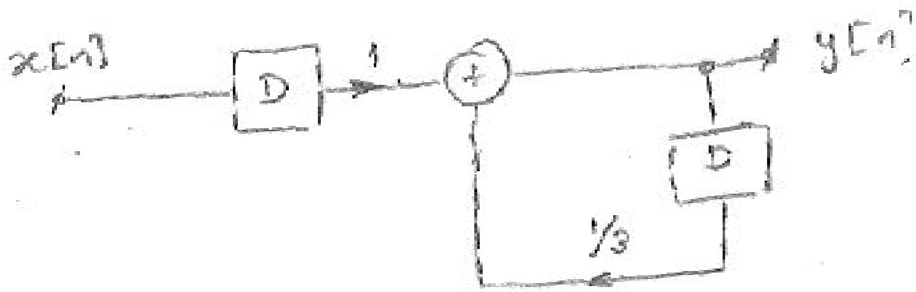
$$\underline{n \geq 5}: \quad \boxed{y[n] = (-110) \times (-2)^{n-5}}$$

$\boxed{2.38}$

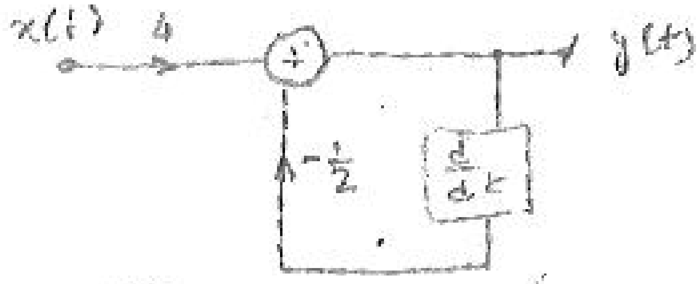
a) $y[n] = \frac{1}{3} \cdot y[n-1] + \frac{1}{2} x[n]$



b) $y[n] = \frac{1}{3} \cdot y[n-1] + x[n-1]$

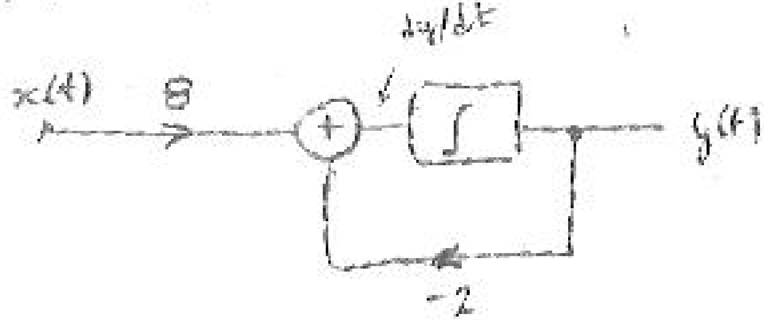


$\boxed{2.39}$ a) $y(t) = -\left(\frac{1}{2}\right)\frac{dy}{dt} + 4x(t)$

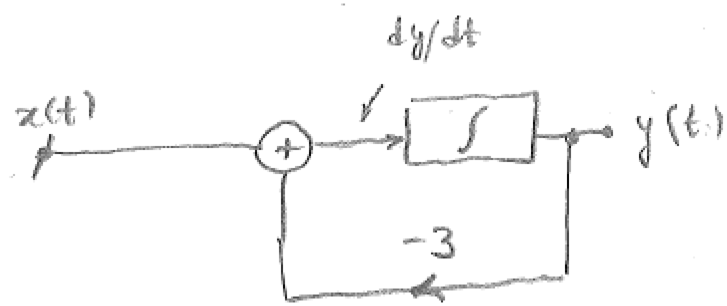


$\Downarrow$

$$\frac{dy}{dt} = -2y(t) + 8x(t)$$

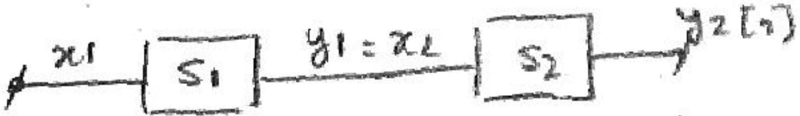


$\boxed{2.39}$ b) $\frac{dy}{dt} + 3y(t) = x(t)$



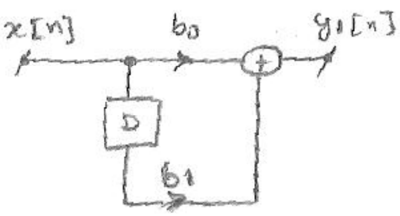
$$\frac{dy}{dt} = -3y(t) + x(t)$$

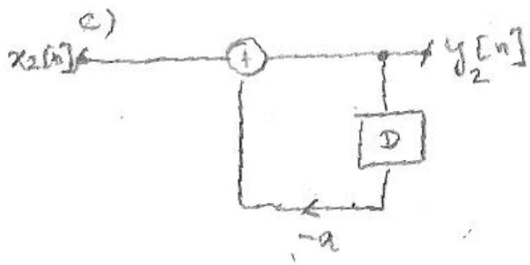
$\boxed{2.57}$: $y[n] = -a\,y[n-1] + b_0 \cdot x[n] + b_1 \cdot x[n-1]$

a) 

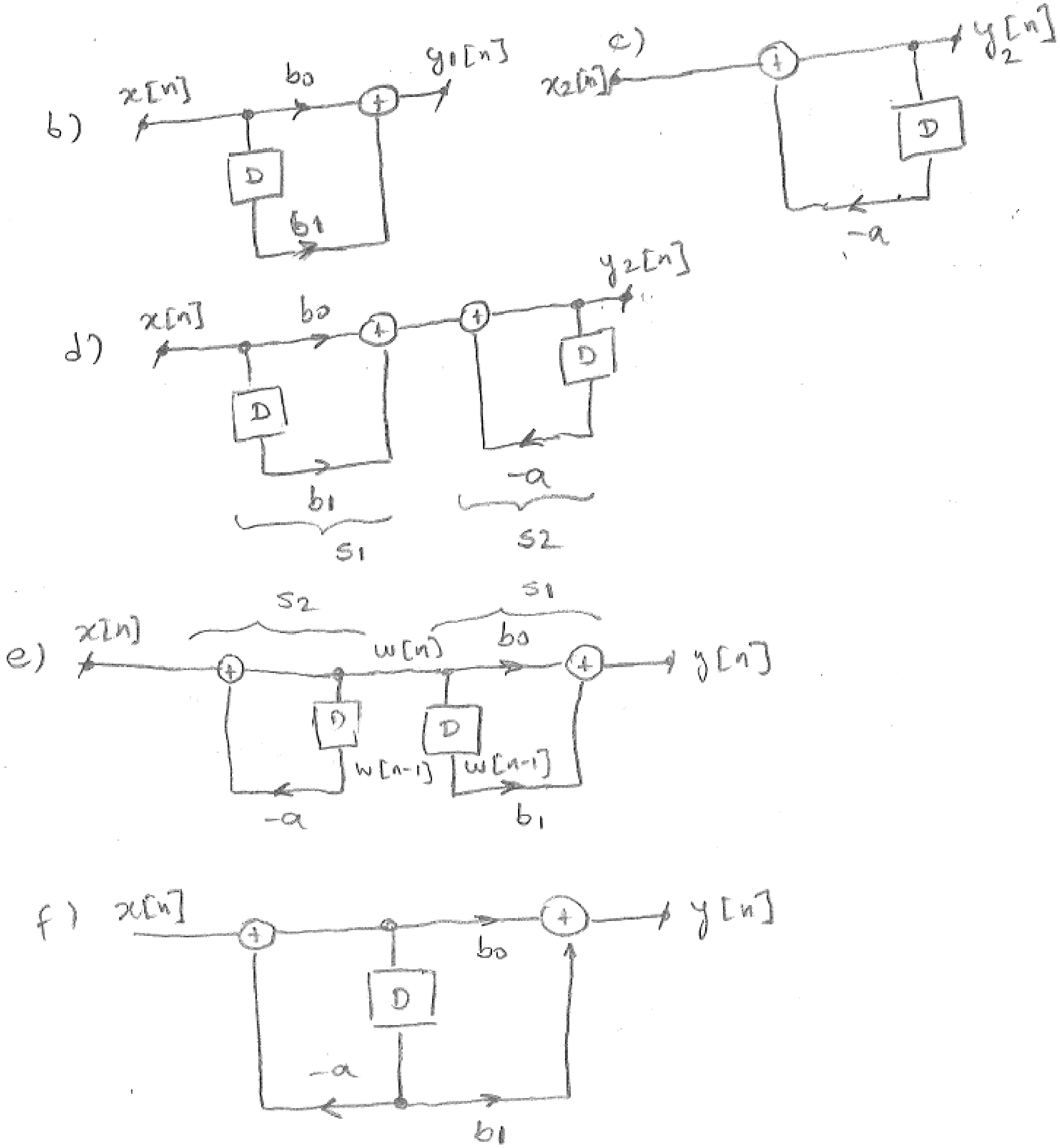
$$\begin{cases} y_1[n] = b_0\, x_1[n] + b_1\, x_1[n-1] & : S_1 \\ y_2[n] = -a\, y_2[n-1] + x_2[n] & : S_2 \end{cases}$$

$$y_2[n] = -a\, y_2[n-1] + b_0 \cdot x_1[n] + b_1\, x_1[n-1]$$

b) 

c) 

$\boxed{2.57}$



b)
x[n]
b0
y1[n]
D
b1
c)
x2[n]
y2[n]
D
-a
d)
x[n]
b0
y2[n]
D
D
b1
-a
S1
S2
e)
x[n]
S2
S1
w[n]
b0
y[n]
D
D
w[n-1]
w[n-1]
-a
b1
f)
x[n]
y[n]
b0
D
-a
b1

$\boxed{2{,}62}$ a)

$K = 2\ N/m$

$m = \frac{1}{2}\ kg$

$\downarrow y(t)$ = deslocamento = saída

força: $x(t)$ = entrada

$$x(t) = m\frac{d^2y(t)}{dt^2} + Ky(t) = \frac{1}{2}\frac{d^2y(t)}{dt^2} + 2y(t)$$

Resposta Natural

$x(t) = 0$

$$m\frac{d^2y}{dt^2} + Ky(t) = 0$$

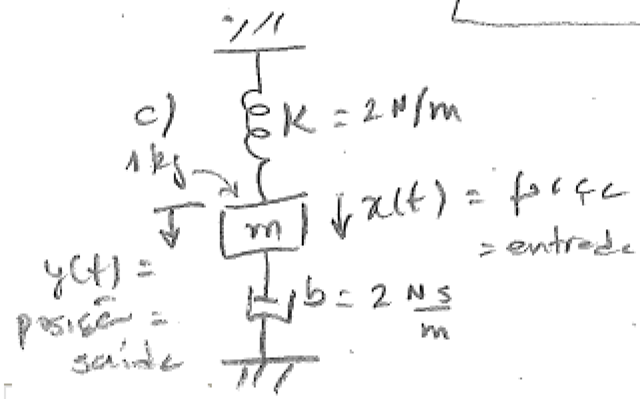
$y_h = Ae^{st}$, s = complexo

$$\therefore\ m\,A\underbrace{s^2e^{st}}_{\frac{d^2y_h}{dt^2}} + K\underbrace{Ae^{st}}_{y_h} = 0 \rightarrow ms^2 = -K \rightarrow s = \sqrt{-K/m}$$

$$s = \sqrt{\frac{-2}{1/2}} = \sqrt{-4} = \pm j2$$

$$y_{h(t)} = A\left(e^{j2t} + e^{-j2t}\right) \therefore \underset{\text{(deslocamento)}}{Re[y_h(t)]} = 2A\cos(2t)$$

$\boxed{2{,}62}$



$$x(t) = m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + K\cdot y = 0 \text{ (resposta natural)}$$

$$\therefore\ y_h(t) = Ae^{st},\ s = \text{complexo} \rightarrow$$

$$m\,s^2\cancel{Ae^{st}} + bs\,\cancel{Ae^{st}} + K\,\cancel{Ae^{st}} = 0 \rightarrow m s^2 + bs + k = 0$$

$$\rightarrow s = \frac{-b \pm \sqrt{b^2 - 4mk}}{2m} = \ldots \boxed{s = -1 \pm j}$$

P5.12

Find the combined impulse response of the LTI system in Figure P5.12. Recall that $x(t) * h(t) * h^{-1}(t) = x(t)$.

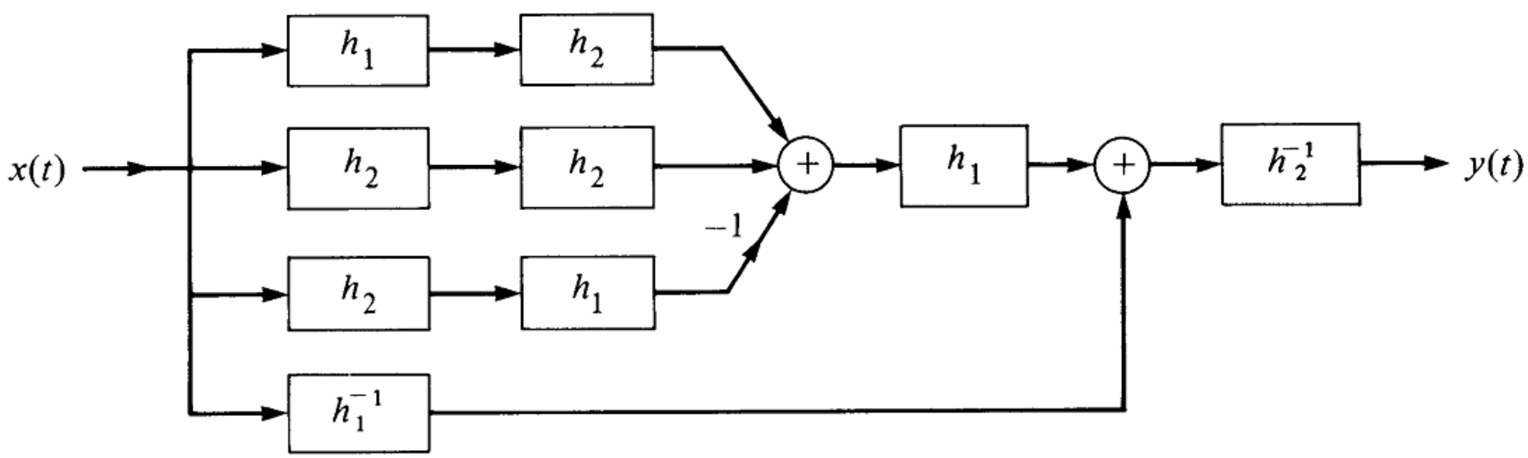


Figure P5.12

S5.12

We have a total system response of

$$h = \{[(h_1 * h_2) + (h_2 * h_2) - (h_2 * h_1)] * h_1 + h_1^{-1}\} * h_2^{-1}$$

$$h = (h_2 * h_1) + (h_1^{-1} * h_2^{-1})$$

P6.7

Consider the block diagram in Figure P6.7. The system is causal and is initially at rest.

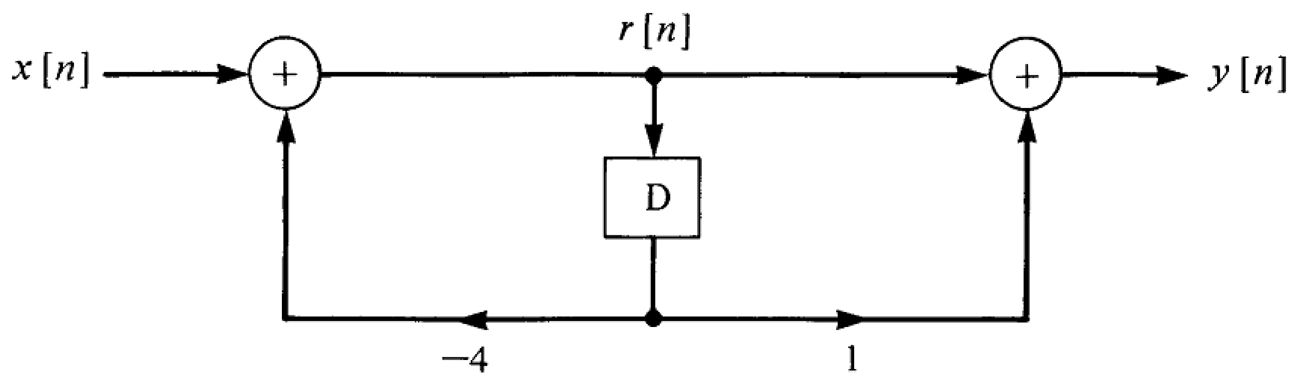


Figure P6.7

(a) Find the difference equation relating $x[n]$ and $y[n]$.

(b) For $x[n] = \delta[n]$, find $r[n]$ for all n.

(c) Find the system impulse response.

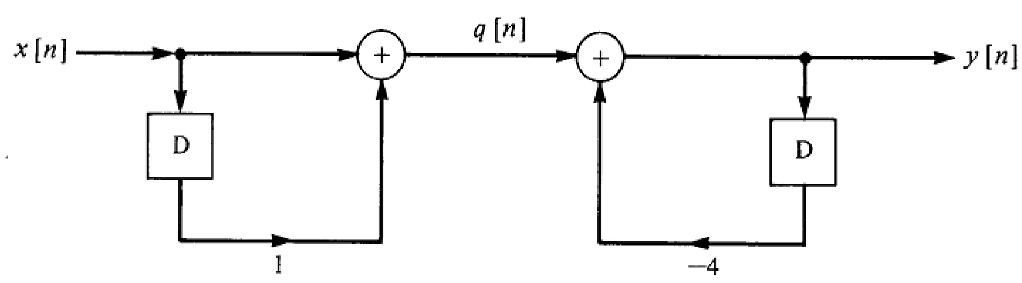


Figure S6.7

(b) The relation between $x[n]$ and $r[n]$ is $r[n] = -4r[n-1] + x[n]$. For such a simple equation, we solve it recursively when $\delta[n] = x[n]$.

n	$\delta[n]$	$r[n-1]$	$r[n]$
<0	0	0	0
0	1	0	1
1	0	1	−4
2	0	−4	16
3	0	16	−64

We see that $r[n] = (-4)^n u[n]$.

(c) $y[n]$ is related to $r[n]$ by

$$y[n] = r[n] + r[n-1]$$

Now $y[n] = h[n]$, the impulse response, when $x[n] = \delta[n]$, and

$$h[n] = (-4)^n u[n] + (-4)^{n-1} u[n-1]$$

This expression for $h[n]$ can be further simplified:

$$h[n] = (-4)^n u[n] + (-4)^{n-1} u[n-1]$$

or

$$h[n] = \begin{cases} 0, & n < 0, \\ 1, & n = 0 \end{cases}$$

For $n > 0$,

$$\begin{aligned} h[n] &= (-4)^n + (-4)^{n-1} \\ &= -3(-4)^{n-1} \end{aligned}$$

Thus,

$$h[n] = \delta[n] - 3(-4)^{n-1} u[n-1]$$

P6.8

Consider the system shown in Figure P6.8. Find the differential equation relating $x(t)$ and $y(t)$.

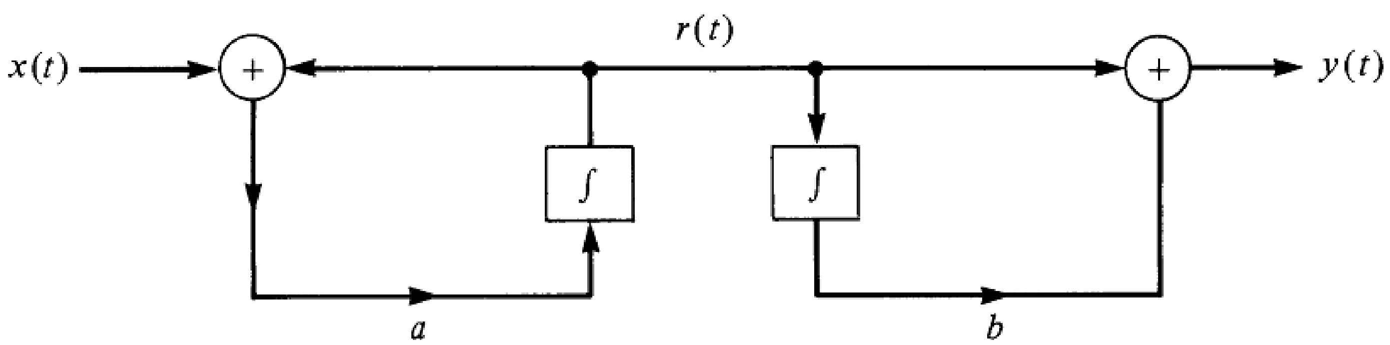


Figure P6.8

Note that the system in Figure P6.8 is not in any standard form. Relating $r(t)$ to $x(t)$ first, we have

$$\int a[x(t) + r(t)]\, dt = r(t), \qquad \text{or} \tag{S6.8-1}$$

$$\frac{dr(t)}{dt} - ar(t) = ax(t),$$

represented in the system shown in Figure S6.8.

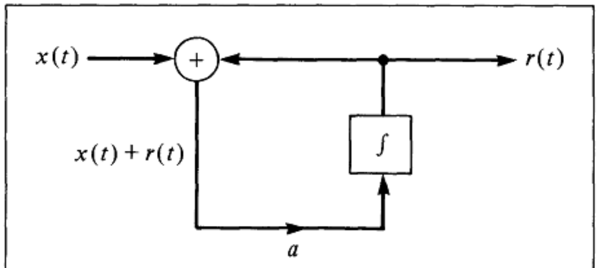


Figure S6.8

The signal $y(t)$ is related to $r(t)$ as follows:

$$r(t) + b\int r(t)\, dt = y(t), \qquad \text{or} \tag{S6.8-2}$$

$$\frac{dr(t)}{dt} + br(t) = \frac{dy(t)}{dt}$$

Solving for $dr(t)/dt$ in eqs. (S6.8-1) and (S6.8-2) and equating, we obtain

$$ar(t) + ax(t) = -br(t) + \frac{dy(t)}{dt}$$

Therefore,

$$r(t) = \frac{-a}{a+b}x(t) + \frac{1}{a+b}\frac{dy(t)}{dt} \tag{S6.8-3}$$

We now substitute eq. (S6.8-3) into eq. (S6.8-1) (or eq. S6.8-2), which, after simplification, yields

$$\frac{dy^2(t)}{dt^2} - a\frac{dy(t)}{dt} = a\frac{dx(t)}{dt} + abx(t)$$