7.22. The signal $y(t)$ is generated by convolving a band-limited signal $x_1(t)$ with another band-limited signal $x_2(t)$, that is,

$$y(t) = x_1(t) * x_2(t)$$

where

$$X_1(j\omega) = 0 \qquad \text{for } |\omega| > 1000\pi$$
$$X_2(j\omega) = 0 \qquad \text{for } |\omega| > 2000\pi.$$

Impulse-train sampling is performed on $y(t)$ to obtain

$$y_p(t) = \sum_{n=-\infty}^{+\infty} y(nT)\delta(t - nT).$$

Specify the range of values for the sampling period T which ensures that $y(t)$ is recoverable from $y_p(t)$.

7.22. Using the properties of the Fourier transform, we obtain

$$Y(j\omega) = X_1(j\omega)X_2(j\omega).$$

Therefore, $Y(j\omega) = 0$ for $|\omega| > 1000\pi$. This implies that the Nyquist rate for $y(t)$ is $2 \times 1000\pi = 2000\pi$. Therefore, the sampling period T can at most be $2\pi/(2000\pi) = 10^{-3}$ sec. Therefore we have to use $T < 10^{-3}$ sec in order to be able to recover $y(t)$ from $y_p(t)$.

7.24. Shown in Figure P7.24 is a system in which the input signal is multiplied by a periodic square wave. The period of $s(t)$ is T. The input signal is band limited with $|X(j\omega)| = 0$ for $|\omega| \geq \omega_M$.

(a) For $\Delta = T/3$, determine, in terms of ω_M, the maximum value of T for which there is no aliasing among the replicas of $X(j\omega)$ in $W(j\omega)$.

(b) For $\Delta = T/4$, determine, in terms of ω_M, the maximum value of T for which there is no aliasing among the replicas of $X(j\omega)$ in $W(j\omega)$.

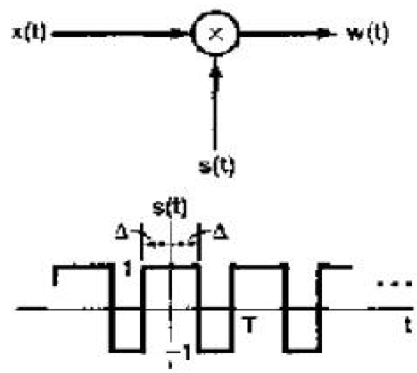

Figure P7.24

7.24. We may express $s(t)$ as $s(t) = \hat{s}(t) - 1$, where $\hat{s}(t)$ is as shown in Figure S7.24.

We may easily show that

$$\hat{S}(j\omega) = \sum_{k=-\infty}^{\infty} \frac{4\sin(2\pi k\Delta/T)}{k}\delta(\omega - k2\pi/T).$$

From this, we obtain

$$S(j\omega) = \hat{S}(j\omega) - 2\pi\delta(\omega) = \sum_{k=-\infty}^{\infty} \frac{4\sin(2\pi k\Delta/T)}{k}\delta(\omega - k2\pi/T) - 2\pi\delta(\omega).$$

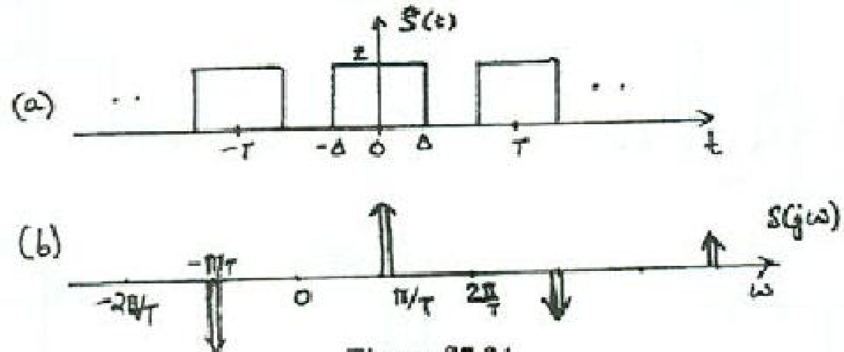

Figure S7.24

Clearly, $S(j\omega)$ consists of impulses spaced every $2\pi/T$.

(a) If $\Delta = T/3$, then

$$S(j\omega) = \sum_{k=-\infty}^{\infty} \frac{4\sin(2\pi k/3)}{k}\delta(\omega - k2\pi/T) - 2\pi\delta(\omega).$$

Now, since $w(t) = s(t)x(t)$,

$$W(j\omega) = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty} \frac{4\sin(2\pi k/3)}{k}X(j(\omega - k2\pi/T)) - 2\pi X(j\omega).$$

Therefore, $W(j\omega)$ consists of replicas of $X(j\omega)$ which are spaced $2\pi/T$ apart. In order to avoid aliasing, ω_M should be less that π/T. Therefore, $T_{max} = \pi/\omega_M$.

(b) If $\Delta = T/3$, then

$$S(j\omega) = \sum_{k=-\infty}^{\infty} \frac{4\sin(2\pi k/4)}{k}\delta(\omega - k2\pi/T) - 2\pi\delta(\omega).$$

We note that $S(j\omega) = 0$ for $k = 0, \pm 2, \pm 4, \cdots$. This is as sketched in Figure S7.24

Therefore, the replicas of $X(j\omega)$ in $W(j\omega)$ are now spaced $4\pi/T$ apart. In order to avoid aliasing, ω_M should be less that $2\pi/T$. Therefore, $T_{max} = 2\pi/\omega_M$.

7.25. In Figure P7.25 is a sampler, followed by an ideal lowpass filter, for reconstruction of $x(t)$ from its samples $x_p(t)$. From the sampling theorem, we know that if $\omega_s = 2\pi/T$ is greater than twice the highest frequency present in $x(t)$ and $\omega_c = \omega_s/2$, then the reconstructed signal $x_r(t)$ will exactly equal $x(t)$. If this condition on the bandwidth of $x(t)$ is violated, then $x_r(t)$ will *not* equal $x(t)$. We seek to show in this problem that if $\omega_c = \omega_s/2$, then for any choice of T, $x_r(t)$ and $x(t)$ will always be equal at the sampling instants; that is,

$$x_r(kT) = x(kT),\ k = 0, \pm 1, \pm 2, \ldots$$

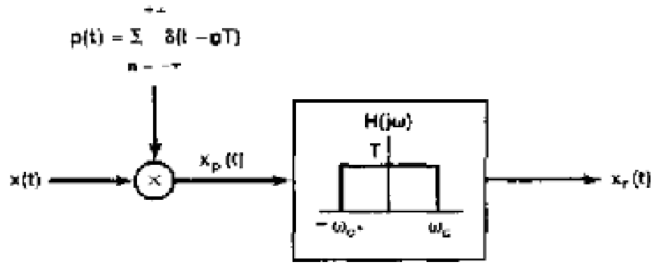

Figure P7.25

To obtain this result, consider eq. (7.11), which expresses $x_r(t)$ in terms of the samples of $x(t)$:

$$x_r(t) = \sum_{n=-\infty}^{\infty} x(nT)T\frac{\omega_c}{\pi}\frac{\sin[\omega_c(t - nT)]}{\omega_c(t - nT)}.$$

With $\omega_c = \omega_s/2$, this becomes

$$x_r(t) = \sum_{n=-\infty}^{\infty} x(nT)\frac{\sin\left[\frac{\pi}{T}(t - nT)\right]}{\frac{\pi}{T}(t - nT)}. \qquad \text{(P7.25–1)}$$

By considering the values of α for which $[\sin(\alpha)]/\alpha = 0$, show from eq. (P7.25–1) that, without any restrictions on $x(t)$, $x_r(kT) = x(kT)$ for any integer value of k.

7.25. Here, $x_r(kT)$ can be written as

$$x_r(kT) = \sum_{n=-\infty}^{\infty} x(nT)\frac{\sin[\pi(k-n)]}{\pi(k-n)}.$$

Note that when $n \neq k$,

$$\frac{\sin[\pi(k-n)]}{\pi(k-n)} = 0$$

and when $n = k$,

$$\frac{\sin[\pi(k-n)]}{\pi(k-n)} = 1.$$

Therefore,

$$x_r(kT) = x(kT).$$

values of L and M.

7.35. Consider a discrete-time sequence $x[n]$ from which we form two new sequences, $x_p[n]$ and $x_d[n]$, where $x_p[n]$ corresponds to sampling $x[n]$ with a sampling period of 2 and $x_d[n]$ corresponds to decimating $x[n]$ by a factor of 2, so that

$$x_p[n] = \begin{cases} x[n], & n = 0, \pm 2, \pm 4, \ldots \\ 0, & n = \pm 1, \pm 3, \ldots \end{cases}$$

and

$$x_d[n] = x[2n].$$

(a) If $x[n]$ is as illustrated in Figure P7.35(a), sketch the sequences $x_p[n]$ and $x_d[n]$.
(b) If $X(e^{j\omega})$ is as shown in Figure P7.35(b), sketch $X_p(e^{j\omega})$ and $X_d(e^{j\omega})$.

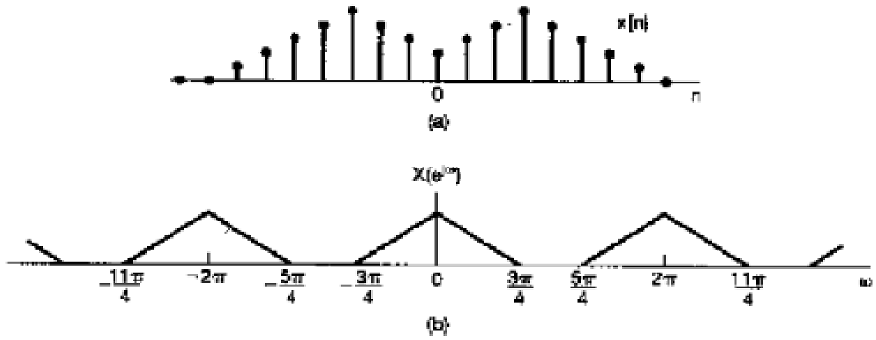

Figure P7.35

7.35. (a) The signals $x_p[n]$ and $x_d[n]$ are sketched in Figure S7.35.

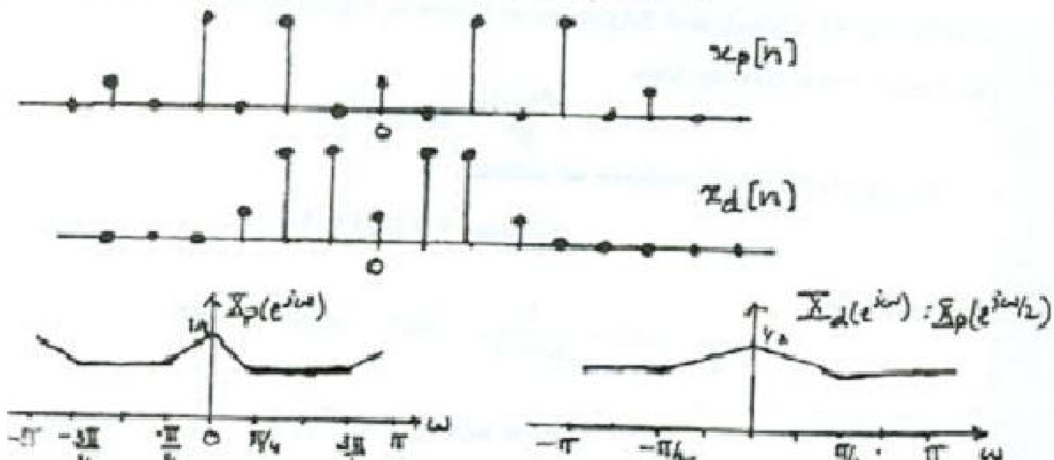

Figure S7.35