# PROBLEM 13.1

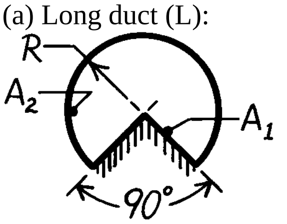
**KNOWN:** Various geometric shapes involving two areas $A_1$ and $A_2$.

**FIND:** Shape factors, $F_{12}$ and $F_{21}$, for each configuration.

**ASSUMPTIONS:** Surfaces are diffuse.

**ANALYSIS:** The analysis is not to make use of tables or charts. The approach involves use of the reciprocity relation, Eq. 13.3, and summation rule, Eq. 13.4. Recognize that reciprocity applies to two surfaces; summation applies to an enclosure. Certain shape factors will be identified by inspection. Note L is the length normal to page.

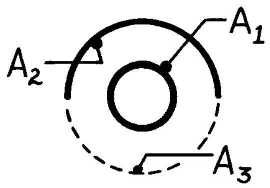
(a) Long duct (L):




By inspection, $F_{12} = 1.0$ <

By reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} = \frac{2\,RL}{(3/4)\cdot 2\pi RL} \times 1.0 = \frac{4}{3\pi} = 0.424$ <

(b) Small sphere, $A_1$, under concentric hemisphere, $A_2$, where $A_2 = 2A$

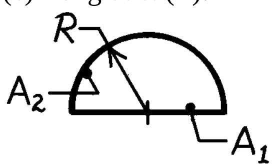



Summation rule $F_{11} + F_{12} + F_{13} = 1$

But $F_{12} = F_{13}$ by symmetry, hence $F_{12} = 0.50$ <

By reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} = \frac{A_1}{2A_1} \times 0.5 = 0.25.$ <

(c) Long duct (L):




<

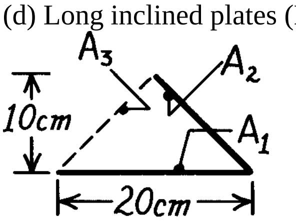
By reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} = \frac{2RL}{\pi RL} \times 1.0 = \frac{2}{\pi} = 0.637$ <

Summation rule, $F_{22} = 1 - F_{21} = 1 - 0.64 = 0.363.$ <

By inspection,

$F_{12} = 1.0$

(d) Long inclined plates (L):

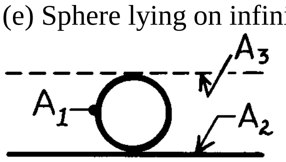



Summation rule, $F_{11} + F_{12} + F_{13} = 1$

But $F_{12} = F_{13}$ by symmetry, hence $F_{12} = 0.50$ <

By reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} = \frac{20L}{10(2)^{1/2} L} \times 0.5 = 0.707.$ <

(e) Sphere lying on infinite plane




Summation rule, $F_{11} + F_{12} + F_{13} = 1$

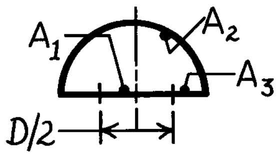
But $F_{12} = F_{13}$ by symmetry, hence $F_{12} = 0.5$ <

By reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} \to 0$ since $A_2 \to \infty$. <

Continued …..

## PROBLEM 13.1 (Cont.)

(f) Hemisphere over a disc of diameter D/2; find also $F_{22}$ and $F_{23}$.




By inspection, $F_{12} = 1.0$ <

Summation rule for surface $A_3$ is written as

$F_{31} + F_{32} + F_{33} = 1$. Hence, $F_{32} = 1.0$.

By reciprocity, $$F_{23} = \frac{A_3}{A_2} F_{32}$$

$$F_{23} = \left\{ \left[ \frac{\pi D^2}{4} - \frac{\pi (D/2)^2}{4} \right] / \frac{\pi D^2}{2} \right\} 1.0 = 0.375.$$

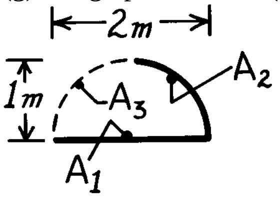
By reciprocity, $$F_{21} = \frac{A_1}{A_2} F_{12} = \left\{ \frac{\pi}{4} \left[ \frac{D}{2} \right]^2 / \frac{\pi D^2}{2} \right\} \times 1.0 = 0.125.$$ <

Summation rule for $A_2$, $$F_{21} + F_{22} + F_{23} = 1 \text{ or}$$

$$F_{22} = 1 - F_{21} - F_{23} = 1 - 0.125 - 0.375 = 0.5.$$ <

Note that by inspection you can deduce $F_{22} = 0.5$

(g) Long open channel (L):




Summation rule for $A_1$

$F_{11} + F_{12} + F_{13} = 0$

but $F_{12} = F_{13}$ by symmetry, hence $F_{12} = 0.50$. <

By reciprocity, $$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{2 \times L}{(2\pi 1)/4 \times L} = \frac{4}{\pi} \times 0.50 = 0.637.$$

**COMMENTS:** (1) Note that the summation rule is applied to an enclosure. To complete the enclosure, it was necessary in several cases to define a third surface which was shown by dashed lines.

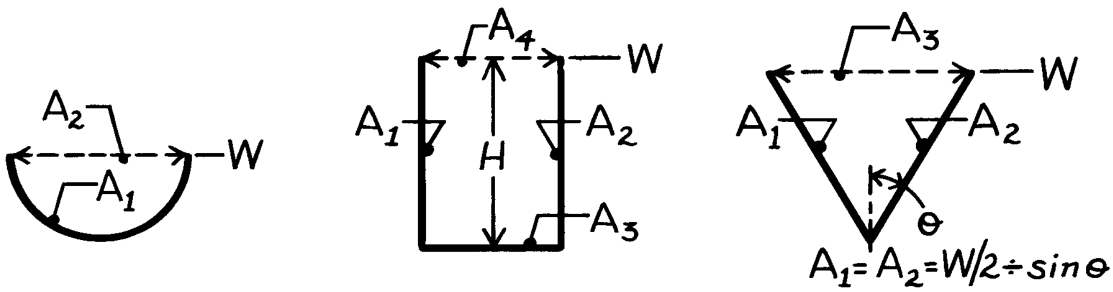
(2) Recognize that the solutions follow a systematic procedure; in many instances it is possible to deduce a shape factor by inspection.

## PROBLEM 13.2

**KNOWN:** Geometry of semi-circular, rectangular and V grooves.

**FIND:** (a) View factors of grooves with respect to surroundings, (b) View factor for sides of V groove, (c) View factor for sides of rectangular groove.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Diffuse surfaces, (2) Negligible end effects, “long grooves”.

**ANALYSIS:** (a) Consider a unit length of each groove and represent the surroundings by a hypothetical surface (dashed line).

*Semi-Circular Groove:*

$$F_{21} = 1; \qquad F_{12} = \frac{A_2}{A_1} F_{21} = \frac{W}{(\pi W/2)} \times 1$$

$$F_{12} = 2/\pi. \qquad <$$

*Rectangular Groove:*

$$F_{4(1,2,3)} = 1; \qquad F_{(1,2,3)4} = \frac{A_4}{A_1 + A_2 + A_3} F_{4(1,2,3)} = \frac{W}{H + W + H} \times 1$$

$$F_{(1,2,3)4} = W/(W + 2H). \qquad <$$

*V Groove:*

$$F_{3(1,2)} = 1; \qquad F_{(1,2)3} = \frac{A_3}{A_1 + A_2} F_{3(1,2)} = \frac{W}{\dfrac{W/2}{\sin\theta} + \dfrac{W/2}{\sin\theta}}$$

$$F_{(1,2)3} = \sin\theta.$$

(b) From Eqs. 13.3 and 13.4, $\quad F_{12} = 1 - F_{13} = 1 - \dfrac{A_3}{A_1} F_{31}.$

From Symmetry, $\quad F_{31} = 1/2.$

Hence, $\quad F_{12} = 1 - \dfrac{W}{(W/2)/\sin\theta} \times \dfrac{1}{2} \quad$ or $\quad F_{12} = 1 - \sin\theta. \qquad <$

(c) From Fig. 13.4, with X/L = H/W =2 and Y/L $\rightarrow \infty$,

$$F_{12} \approx 0.62. \qquad <$$

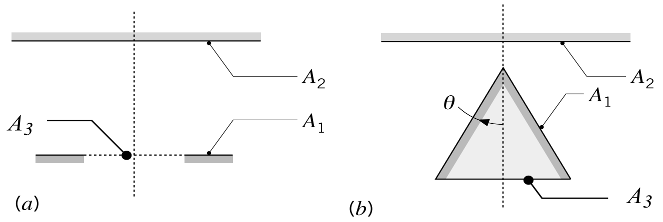
**COMMENTS:** (1) Note that for the V groove, $F_{13} = F_{23} = F_{(1,2)3} = \sin\theta$, (2) In part (c), Fig. 13.4 could also be used with Y/L = 2 and X/L = $\infty$. However, obtaining the limit of $F_{ij}$ as X/L $\rightarrow \infty$ from the figure is somewhat uncertain.

## PROBLEM 13.3

**KNOWN:** Two arrangements (a) circular disk and coaxial, ring shaped disk, and (b) circular disk and coaxial, right-circular cone.

**FIND:** Derive expressions for the view factor $F_{12}$ for the arrangements (a) and (b) in terms of the areas $A_1$ and $A_2$, and any appropriate hypothetical surface area, as well as the view factor for coaxial parallel disks (Table 13.2, Figure 13.5). For the disk-cone arrangement, sketch the variation of $F_{12}$ with θ for $0 \le \theta \le \pi/2$, and explain the key features.

**SCHEMATIC:**




**ASSUMPTIONS:** Diffuse surfaces with uniform radiosities.

**ANALYSIS:** (a) Define the hypothetical surface $A_3$, a co-planar disk inside the ring of $A_1$. Using the additive view factor relation, Eq. 13.5,

$$A_{(1,3)}\, F_{(1,3)} = A_1\, F_{12} + A_3\, F_{32}$$

$$F_{12} = \frac{1}{A_1}\left[A_{(1,3)}\, F_{(1,3)} - A_3\, F_{32}\right] \qquad <$$

where the parenthesis denote a composite surface. All the $F_{ij}$ on the right-hand side can be evaluated using Fig. 13.5.

(b) Define the hypothetical surface $A_3$, the disk at the bottom of the cone. The radiant power leaving $A_2$ that is intercepted by $A_1$ can be expressed as

$$F_{21} = F_{23} \qquad (1)$$

That is, the same power also intercepts the disk at the bottom of the cone, $A_3$. From reciprocity,

$$A_1\, F_{12} = A_2\, F_{21} \qquad (2)$$

and using Eq. (1),

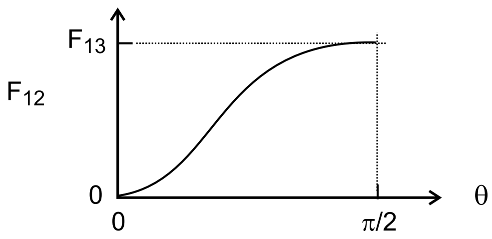
$$F_{12} = \frac{A_2}{A_1}\, F_{23} \qquad <$$

The variation of $F_{12}$ as a function of θ is shown below for the disk-cone arrangement. In the limit when $\theta \to \pi/2$, the cone approaches a disk of area $A_3$. That is,

$$F_{12}\,(\theta \to \pi/2) = F_{13}$$

When $\theta \to 0$, the cone area $A_2$ diminishes so that

$$F_{12}\,(\theta \to 0) = 0$$

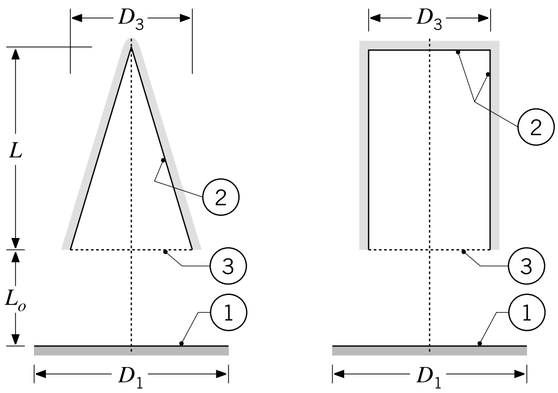


## PROBLEM 13.4

**KNOWN:** Right circular cone and right-circular cylinder of same diameter D and length L positioned coaxially a distance $L_o$ from the circular disk $A_1$; hypothetical area corresponding to the openings identified as $A_3$.

**FIND:** (a) Show that $F_{21} = (A_1/A_2) F_{13}$ and $F_{22} = 1 - (A_3/A_2)$, where $F_{13}$ is the view factor between two, coaxial parallel disks (Table 13.2), for both arrangements, (b) Calculate $F_{21}$ and $F_{22}$ for $L = L_o = 50$ mm and $D_1 = D_3 = 50$ mm; compare magnitudes and explain similarities and differences, and (c) Magnitudes of $F_{21}$ and $F_{22}$ as L increases and all other parameters remain the same; sketch and explain key features of their variation with L.

**SCHEMATIC:**



(a) Right-circular cone (b) Right-circular cylinder

**ASSUMPTIONS:** (1) Diffuse surfaces with uniform radiosities, and (2) Inner base and lateral surfaces of the cylinder treated as a single surface, $A_2$.

**ANALYSIS:** (a) For both configurations,

$$F_{13} = F_{12} \tag{1}$$

since the radiant power leaving $A_1$ that is intercepted by $A_3$ is likewise intercepted by $A_2$. Applying reciprocity between $A_1$ and $A_2$,

$$A_1 F_{12} = A_2 F_{21} \tag{2}$$

Substituting from Eq. (1), into Eq. (2), solving for $F_{21}$, find

$$F_{21} = (A_1 / A_2) F_{12} = (A_1 / A_2) F_{13}$$ <

Treating the cone and cylinder as two-surface enclosures, the summation rule for $A_2$ is

$$F_{22} + F_{23} = 1 \tag{3}$$

Apply reciprocity between $A_2$ and $A_3$, solve Eq. (3) to find

$$F_{22} = 1 - F_{23} = 1 - (A_3 / A_2) F_{32}$$

and since $F_{32} = 1$, find

$$F_{22} = 1 - A_3 / A_2$$ <

Continued …..

## PROBLEM 13.4 (Cont.)

(b) For the specified values of L, $L_o$, $D_1$ and $D_2$, the view factors are calculated and tabulated below. Relations for the areas are:

*Disk-cone:* $A_1 = \pi D_1^2/4$ $\qquad A_2 = \pi D_3/2\left(L^2 + (D_3/2)^2\right)^{1/2}$ $\qquad A_3 = \pi D_3^2/4$

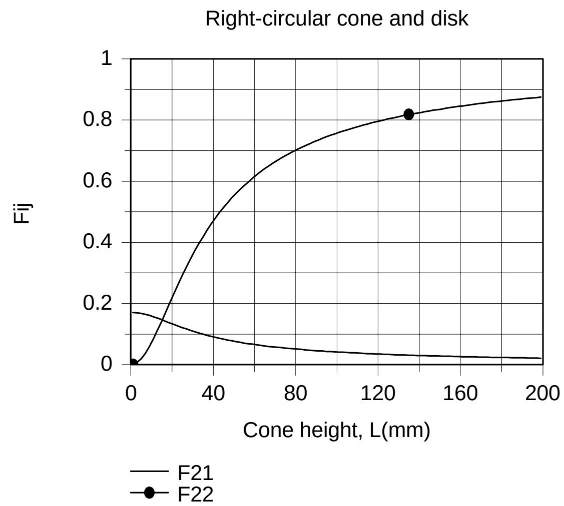
*Disk-cylinder:* $A_1 = \pi D_1^2/4$ $\qquad A_2 = \pi D_3^2/4 + \pi D_3 L$ $\qquad A_3 = \pi D_3^2/4$

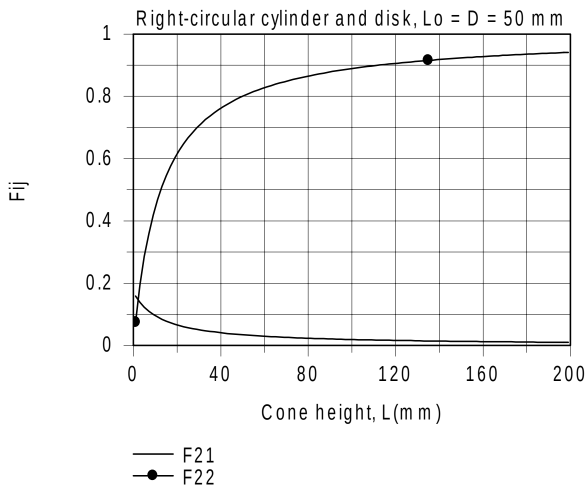
The view factor $F_{13}$ is evaluated from Table 13.2, coaxial parallel disks (Fig. 13.5); find $F_{13} = 0.1716$.

| | $F_{21}$ | $F_{22}$ |
|---|---|---|
| *Disk-cone* | 0.0767 | 0.553 |
| *Disk-cylinder* | 0.0343 | 0.800 |

It follows that $F_{21}$ is greater for the disk-cone (a) than for the cylinder-cone (b). That is, for (a), surface $A_2$ sees more of $A_1$ and less of itself than for (b). Notice that $F_{22}$ is greater for (b) than (a); this is a consequence of $A_{2,b} > A_{2,a}$.

(c) Using the foregoing equations in the IHT workspace, the variation of the view factors $F_{21}$ and $F_{22}$ with L were calculated and are graphed below.




Note that for both configurations, when L = 0, find that $F_{21} = F_{13} = 0.1716$, the value obtained for coaxial parallel disks. As L increases, find that $F_{22} \to 1$; that is, the interior of both the cone and cylinder see mostly each other. Notice that the changes in both $F_{21}$ and $F_{22}$ with increasing L are greater for the disk-cylinder; $F_{21}$ decreases while $F_{22}$ increases.

**COMMENTS:** From the results of part (b), why isn't the sum of $F_{21}$ and $F_{22}$ equal to unity?

## PROBLEM 13.5

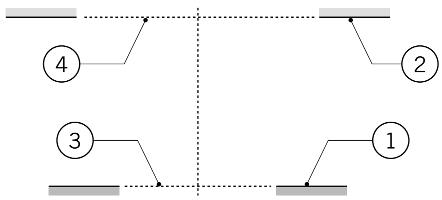
**KNOWN:** Two parallel, coaxial, ring-shaped disks.

**FIND:** Show that the view factor $F_{12}$ can be expressed as

$$F_{12} = \frac{1}{A_1}\left\{A_{(1,3)}\, F_{(1,3)(2,4)} - A_3\, F_{3(2,4)} - A_4\left(F_{4(1,3)} - F_{43}\right)\right\}$$

where all the $F_{ig}$ on the right-hand side of the equation can be evaluated from Figure 13.5 (see Table 13.2) for coaxial parallel disks.

**SCHEMATIC:**




**ASSUMPTIONS:** Diffuse surfaces with uniform radiosities.

**ANALYSIS:** Using the additive rule, Eq. 13.5, where the parenthesis denote a composite surface,

$$F_{1(2,4)} = F_{12} + F_{14}$$

$$F_{12} = F_{1(2,4)} - F_{14} \tag{1}$$

*Relation for $F_{1(2,4)}$:* Using the additive rule

$$A_{(1,3)}\, F_{(1,3)(2,4)} = A_1\, F_{1(2,4)} + A_3\, F_{3(2,4)} \tag{2}$$

where the check mark denotes a $F_{ij}$ that can be evaluated using Fig. 13.5 for coaxial parallel disks.

*Relation for $F_{14}$:* Apply reciprocity

$$A_1\, F_{14} = A_4\, F_{41} \tag{3}$$

and using the additive rule involving $F_{41}$,

$$A_1\, F_{14} = A_4\left[F_{4(1,3)} - F_{43}\right] \tag{4}$$

*Relation for $F_{12}$:* Substituting Eqs. (2) and (4) into Eq. (1),

$$F_{12} = \frac{1}{A_1}\left\{A_{(1,3)}\, F_{(1,3)(2,4)} - A_3\, F_{3(2,4)} - A_4\left(F_{4(1,3)} - F_{43}\right)\right\} \quad <$$

**COMMENTS:** (1) The $F_{ij}$ on the right-hand side can be evaluated using Fig. 13.5.

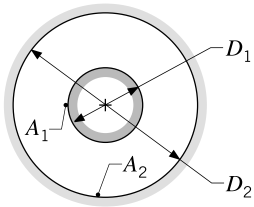
(2) To check the validity of the result, substitute numerical values and test the behavior at special limits. For example, as $A_3$, $A_4 \rightarrow 0$, the expression reduces to the identity $F_{12} \equiv F_{12}$.

# PROBLEM 13.6

**KNOWN:** Long concentric cylinders with diameters $D_1$ and $D_2$ and surface areas $A_1$ and $A_2$.

**FIND:** (a) The view factor $F_{12}$ and (b) Expressions for the view factors $F_{22}$ and $F_{21}$ in terms of the cylinder diameters.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Diffuse surfaces with uniform radiosities and (2) Cylinders are infinitely long such that $A_1$ and $A_2$ form an enclosure.

**ANALYSIS:** (a) *View factor* $F_{12}$. Since the infinitely long cylinders form an enclosure with surfaces $A_1$ and $A_2$, from the summation rule on $A_1$, Eq. 13.4,

$$F_{11} + F_{12} = 1 \qquad (1)$$

and since $A_1$ doesn't see itself, $F_{11} = 0$, giving

$$F_{12} = 1 \qquad < \ (2)$$

That is, the inner surface views only the outer surface.

(b) *View factors* $F_{22}$ *and* $F_{21}$. Applying reciprocity between $A_1$ and $A_2$, Eq. 13.3, and substituting from Eq. (2),

$$A_1\, F_{12} = A_2\, F_{21} \qquad (3)$$

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{\pi D_1 L}{\pi D_2 L} \times 1 = \frac{D_1}{D_2} \qquad < \ (4)$$

From the summation rule on $A_2$, and substituting from Eq. (4),

$$F_{21} + F_{22} = 1$$

$$F_{22} = 1 - F_{21} = 1 - \frac{D_1}{D_2} \qquad <$$

## PROBLEM 13.7

**KNOWN:** Right-circular cylinder of diameter D, length L and the areas $A_1$, $A_2$, and $A_3$ representing the base, inner lateral and top surfaces, respectively.

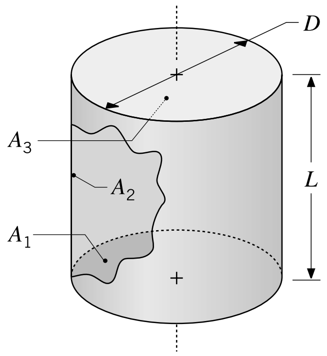
**FIND:** (a) Show that the view factor between the base of the cylinder and the inner lateral surface has the form

$$F_{12} = 2\,H\left[\left(1+H^2\right)^{1/2} - H\right]$$

where H = L/D, and (b) Show that the view factor for the inner lateral surface to itself has the form

$$F_{22} = 1 + H - \left(1+H^2\right)^{1/2}$$

**SCHEMATIC:**




**ASSUMPTIONS:** Diffuse surfaces with uniform radiosities.

**ANALYSIS:** (a) *Relation for $F_{12}$, base-to-inner lateral surface*. Apply the summation rule to $A_1$, noting that $F_{11} = 0$

$$F_{11} + F_{12} + F_{13} = 1$$

$$F_{12} = 1 - F_{13} \tag{1}$$

From Table 13.2, Fig. 13.5, with i = 1, *j* = 3,

$$F_{13} = \frac{1}{2}\left\{S - \left[S^2 - 4\left(D_3 / D_1\right)^2\right]^{1/2}\right\} \tag{2}$$

$$S = 1 + \frac{1+R_3^2}{R_1^2} = \frac{1}{R^2} + 2 = 4\,H^2 + 2 \tag{3}$$

where $R_1 = R_3 = R = D/2L$ and $H = L/D$. Combining Eqs. (2) and (3) with Eq. (1), find after some manipulation

Continued …..

## PROBLEM 13.7 (Cont.)

$$F_{12} = 1 - \frac{1}{2}\left\{4\,H^2 + 2 - \left[\left(4\,H^2 + 2\right)^2 - 4\right]^{1/2}\right\}$$

$$F_{12} = 2\,H\left[\left(1 + H^2\right)^{1/2} - H\right] \tag{4}$$

(b) *Relation for $F_{22}$, inner lateral surface*. Apply summation rule on $A_2$, recognizing that $F_{23} = F_{21}$,

$$F_{21} + F_{22} + F_{23} = 1 \qquad\qquad F_{22} = 1 - 2\,F_{21} \tag{5}$$

Apply reciprocity between $A_1$ and $A_2$,

$$F_{21} = \left(A_1 / A_2\right) F_{12} \tag{6}$$

and substituting into Eq. (5), and using area expressions

$$F_{22} = 1 - 2\,\frac{A_1}{A_2}\,F_{12} = 1 - 2\,\frac{D}{4\,L}F_{12} = 1 - \frac{1}{2\,H}F_{12} \tag{7}$$

where $A_1 = \pi D^2/4$ and $A_2 = \pi DL$.

Substituting from Eq. (4) for $F_{12}$, find

$$F_{22} = 1 - \frac{1}{2\,H}2\,H\left[\left(1 + H^2\right)^{1/2} - H\right] = 1 + H - \left(1 + H^2\right)^{1/2}$$ <

## PROBLEM 13.8

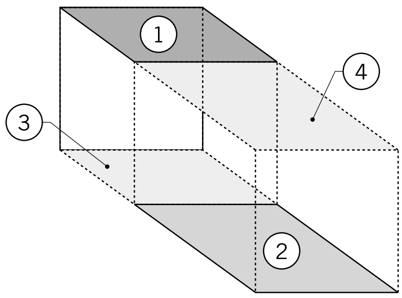
**KNOWN:** Arrangement of plane parallel rectangles.

**FIND:** Show that the view factor between $A_1$ and $A_2$ can be expressed as

$$F_{12} = \frac{1}{2\,A_1}\left[A_{(1,4)}\ F_{(1,4)(2,3)} - A_1\ F_{13} - A_4\ F_{42}\right]$$

where all $F_{ij}$ on the right-hand side of the equation can be evaluated from Fig. 13.4 (see Table 13.2) for aligned parallel rectangles.

**SCHEMATIC:**




**ASSUMPTIONS:** Diffuse surfaces with uniform radiosity.

**ANALYSIS:** Using the additive rule where the parenthesis denote a composite surface,

$$A_{(1,4)}\ F^*_{(1,4)(2,3)} = A_1\ F^*_{13} + A_1\ F_{12} + A_4\ F_{43} + A_4\ F^*_{42} \quad (1)$$

where the asterisk (*) denotes that the $F_{ij}$ can be evaluated using the relation of Figure 13.4. Now, find suitable relation for $F_{43}$. By symmetry,

$$F_{43} = F_{21} \quad (2)$$

and from reciprocity between $A_1$ and $A_2$,

$$F_{21} = \frac{A_1}{A_2} F_{12} \quad (3)$$

Multiply Eq. (2) by $A_4$ and substitute Eq. (3), with $A_4 = A_2$,

$$A_4\ F_{43} = A_4\ F_{21} = A_4 \frac{A_1}{A_2} F_{12} = A_1\ F_{12} \quad (4)$$

Substituting for $A_4\ F_{43}$ from Eq. (4) into Eq. (1), and rearranging,

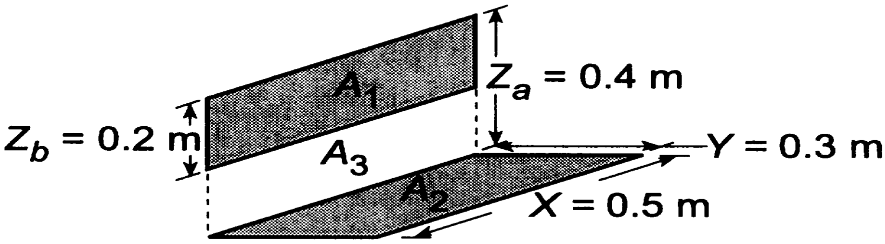
$$F_{12} = \frac{1}{2\,A_1}\left[A_{(1,4)}\ F^*_{(1,4)(2,3)} - A_1\ F^*_{13} - A_4\ F^*_{42}\right] \quad <$$

## PROBLEM 13.9

**KNOWN:** Two perpendicular rectangles not having a common edge.

**FIND:** (a) Shape factor, $F_{12}$, and (b) Compute and plot $F_{12}$ as a function of $Z_b$ for $0.05 \le Z_b \le 0.4$ m; compare results with the view factor obtained from the two-dimensional relation for perpendicular plates with a common edge, Table 13.1.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) All surfaces are diffuse, (2) Plane formed by $A_1 + A_3$ is perpendicular to plane of $A_2$.

**ANALYSIS:** (a) Introducing the hypothetical surface $A_3$, we can write

$$F_{2(3,1)} = F_{23} + F_{21}. \qquad (1)$$

Using Fig. 13.6, applicable to perpendicular rectangles with a common edge, find

$$F_{23} = 0.19: \quad \text{with } Y = 0.3, \quad X = 0.5, \quad Z = Z_a - Z_b = 0.2, \text{ and } \frac{Y}{X} = \frac{0.3}{0.5} = 0.6, \quad \frac{Z}{X} = \frac{0.2}{0.5} = 0.4$$

$$F_{2(3,1)} = 0.25: \quad \text{with } Y = 0.3,\ X = 0.5,\ Z_a = 0.4, \text{ and } \frac{Y}{X} = \frac{0.3}{0.5} = 0.6, \quad \frac{Z}{X} = \frac{0.4}{0.5} = 0.8$$

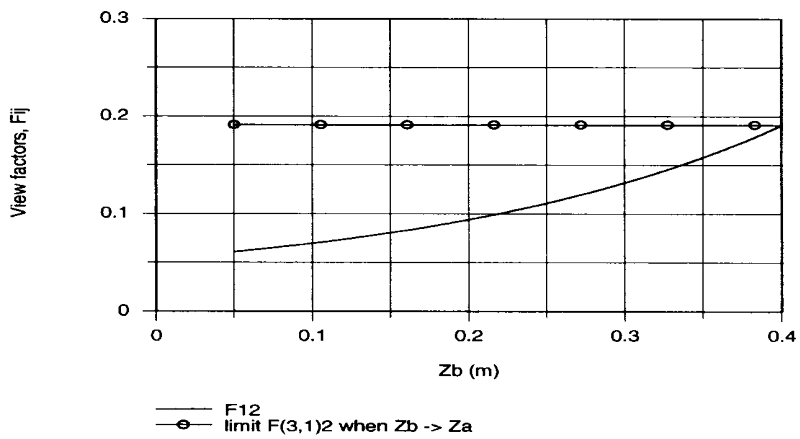
Hence from Eq. (1)

$$F_{21} = F_{2(3.1)} - F_{23} = 0.25 - 0.19 = 0.06$$

By reciprocity,

$$F_{12} = \frac{A_2}{A_1} F_{21} = \frac{0.5 \times 0.3\,\text{m}^2}{0.5 \times 0.2\,\text{m}^2} \times 0.06 = 0.09 \qquad (2) \; <$$

(b) Using the *IHT Tool – View Factors* for *Perpendicular Rectangles with a Common Edge* and Eqs. (1,2) above, $F_{12}$ was computed as a function of $Z_b$. Also shown on the plot below is the view factor $F_{(3,1)2}$ for the limiting case $Z_b \to Z_a$.

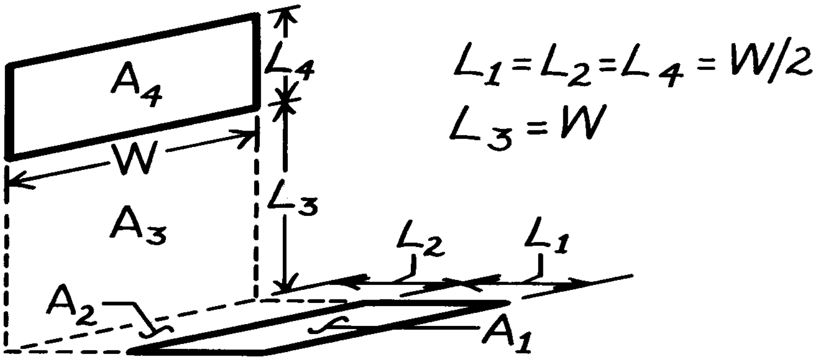


## PROBLEM 13.10

**KNOWN:** Arrangement of perpendicular surfaces without a common edge.

**FIND:** (a) A relation for the view factor $F_{14}$ and (b) The value of $F_{14}$ for prescribed dimensions.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Diffuse surfaces.

**ANALYSIS:** (a) To determine $F_{14}$, it is convenient to define the hypothetical surfaces $A_2$ and $A_3$. From Eq. 13.6,

$$(A_1 + A_2)F_{(1,2)(3,4)} = A_1\ F_{1(3,4)} + A_2\ F_{2(3,4)}$$

where $F_{(1,2)(3,4)}$ and $F_{2(3,4)}$ may be obtained from Fig. 13.6. Substituting for $A_1\ F_{1(3,4)}$ from Eq. 13.5 and combining expressions, find

$$A_1\ F_{1(3,4)} = A_1\ F_{13} + A_1\ F_{14}$$

$$F_{14} = \frac{1}{A_1}\left[(A_1 + A_2)F_{(1,2)(3,4)} - A_1\ F_{13} - A_2\ F_{2(3,4)}\right].$$

Substituting for $A_1\ F_{13}$ from Eq. 13.6, which may be expressed as

$$(A_1 + A_2)F_{(1,2)3} = A_1\ F_{13} + A_2\ F_{23}.$$

The desired relation is then

$$F_{14} = \frac{1}{A_1}\left[(A_1 + A_2)F_{(1,2)(3,4)} + A_2F_{23} - (A_1 + A_2)F_{(1,2)3} - A_2\ F_{2(3,4)}\right].$$ <

(b) For the prescribed dimensions and using Fig. 13.6, find these view factors:

*Surfaces (1,2)(3,4)* $\quad (Y/X) = \frac{L_1 + L_2}{W} = 1, \quad (Z/X) = \frac{L_3 + L_4}{W} = 1.45, \quad F_{(1,2)(3,4)} = 0.22$

*Surfaces 23* $\quad (Y/X) = \frac{L_2}{W} = 0.5, \quad (Z/X) = \frac{L_3}{W} = 1, \quad F_{23} = 0.28$

*Surfaces (1,2)3* $\quad (Y/X) = \frac{L_1 + L_2}{W} = 1, \quad (Z/X) = \frac{L_3}{W} = 1, \quad F_{(1,2)3} = 0.20$

*Surfaces 2(3,4)* $\quad (Y/X) = \frac{L_2}{W} = 0.5, \quad (Z/X) = \frac{L_3 + L_4}{W} = 1.5, \quad F_{2(3,4)} = 0.31$

Using the relation above, find

$$F_{14} = \frac{1}{(WL_1)}\left[(WL_1 + WL_2)0.22 + (WL_2)0.28 - (WL_1 + WL_2)0.20 - (WL_2)0.31\right]$$

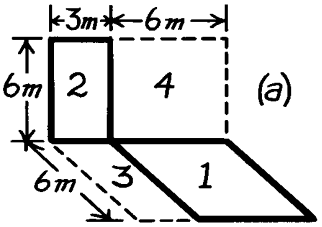
$$F_{14} = \left[2(0.22) + 1(0.28) - 2(0.20) - 1(0.31)\right] = 0.01.$$ <

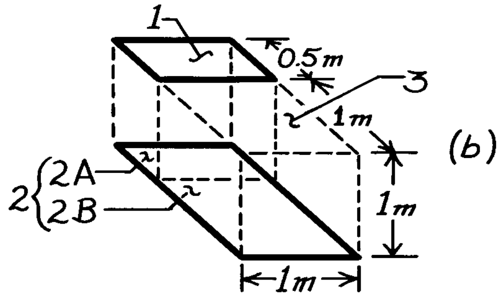
# PROBLEM 13.11

**KNOWN:** Arrangements of rectangles.

**FIND:** The shape factors, $F_{12}$.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Diffuse surface behavior.

**ANALYSIS:** (a) Define the hypothetical surfaces shown in the sketch as $A_3$ and $A_4$. From the additive view factor rule, Eq. 13.6, we can write

$$A_{(1,3)} \overset{\surd}{F}_{(1,3)(2,4)} = A_1F_{12} + A_1 \overset{\surd}{F}_{14} + A_3 \overset{\surd}{F}_{32} + A_3F_{34} \quad (1)$$

Note carefully which factors can be evaluated from Fig. 13.6 for perpendicular rectangles with a common edge. (See √). It follows from symmetry that

$$A_1F_{12} = A_4F_{43}. \quad (2)$$

Using reciprocity,

$$A_4F_{43} = A_3F_{34}, \qquad \text{then} \qquad A_1F_{12} = A_3F_{34}. \quad (3)$$

Solving Eq. (1) for $F_{12}$ and substituting Eq. (3) for $A_3F_{34}$, find that

$$F_{12} = \frac{1}{2A_1}\left[A_{(1,3)}F_{(1,3)(2,4)} - A_1F_{14} - A_3F_{32}\right]. \quad (4)$$

Evaluate the view factors from Fig. 13.6:

| $F_{ij}$ | Y/X | Z/X | $F_{ij}$ |
|---|---|---|---|
| (1,3) (2,4) | $\frac{6}{9} = 0.67$ | $\frac{6}{9} = 0.67$ | 0.23 |
| 14 | $\frac{6}{6} = 1$ | $\frac{6}{6} = 1$ | 0.20 |
| 32 | $\frac{6}{3} = 2$ | $\frac{6}{3} = 2$ | 0.14 |

Substituting numerical values into Eq. (4) yields

$$F_{12} = \frac{1}{2\times(6\times6)\text{m}^2}\left[(6\times9)\text{m}^2\times0.23 - (6\times6)\text{m}^2\times0.20 - (6\times3)\text{m}^2\times0.14\right]$$

$$F_{12} = 0.038.$$ <

Continued …..

## PROBLEM 13.11 (Cont.)

(b) Define the hypothetical surface $A_3$ and divide $A_2$ into two sections, $A_{2A}$ and $A_{2B}$. From the additive view factor rule, Eq. 13.6, we can write

$$A_{1,3}\overset{\surd}{F}_{(1,3)2} = A_1F_{12} + A_3F_{3(2A)} + A_3\overset{\surd}{F}_{3(2B)}. \tag{5}$$

Note that the view factors checked can be evaluated from Fig. 13.4 for aligned, parallel rectangles. To evaluate $F_{3(2A)}$, we first recognize a relationship involving $F_{(24)1}$ will eventually be required. Using the additive rule again,

$$A_{2A}F_{(2A)(1,3)} = A_{2A}\overset{\surd}{F}_{(2A)1} + A_{2A}F_{(2A)3}. \tag{6}$$

Note that from symmetry considerations,

$$A_{2A}F_{(2A)(1,3)} = A_1F_{12} \tag{7}$$

and using reciprocity, Eq. 13.3, note that

$$A_{2A}F_{2A3} = A_3F_{3(2A)}. \tag{8}$$

Substituting for $A_3F_{3(2A)}$ from Eq. (8), Eq. (5) becomes

$$A_{(1,3)}\overset{\surd}{F}_{(1,3)2} = A_1F_{12} + A_{2A}F_{(2A)3} + A_3\overset{\surd}{F}_{3(2B)}.$$

Substituting for $A_{2A}\,F_{(2A)3}$ from Eq. (6) using also Eq. (7) for $A_{2A}\,F_{(2A)(1,3)}$ find that

$$A_{(1,3)}\overset{\surd}{F}_{(1,3)2} = A_1F_{12} + \left(A_1F_{12} - A_{2A}\overset{\surd}{F}_{(2A)1}\right) + A_3\overset{\surd}{F}_{3(2B)} \tag{9}$$

and solving for $F_{12}$, noting that $A_1 = A_{2A}$ and $A_{(1,3)} = A_2$

$$F_{12} = \frac{1}{2A_1}\left[A_2\overset{\surd}{F}_{(1,3)2} + A_{2A}\overset{\surd}{F}_{(2A)1} - A_3\overset{\surd}{F}_{3(2B)}\right]. \tag{10}$$

Evaluate the view factors from Fig. 13.4:

| $F_{ij}$ | X/L | Y/L | $F_{ij}$ |
|---|---|---|---|
| (1,3) 2 | $\frac{1}{1}=1$ | $\frac{1.5}{1}=1.5$ | 0.25 |
| (2A)1 | $\frac{1}{1}=1$ | $\frac{0.5}{1}=0.5$ | 0.11 |
| 3(2B) | $\frac{1}{1}=1$ | $\frac{1}{1}=1$ | 0.20 |

Substituting numerical values into Eq. (10) yields

$$F_{12} = \frac{1}{2(0.5\times1)\,m^2}\left[(1.5\times1.0)\,m^2\times0.25 + (0.5\times1)\,m^2\times0.11 - (1\times1)\,m^2\times0.20\right]$$

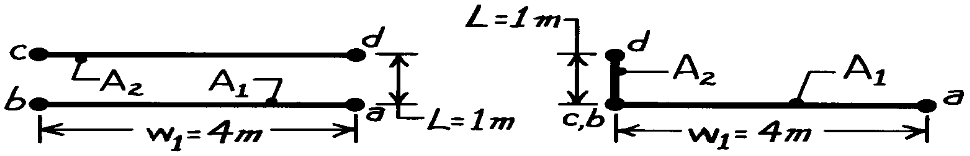
$$F_{12} = 0.23.$$ $<$

# PROBLEM 13.12

**KNOWN:** Two geometrical arrangements: (a) parallel plates and (b) perpendicular plates with a common edge.

**FIND:** View factors using "crossed-strings" method; compare with appropriate graphs and analytical expressions.

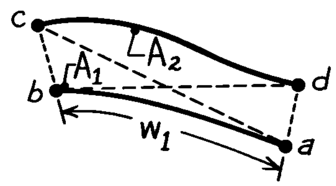
**SCHEMATIC:**




(a) Parallel plates (b) Perpendicular plates with common edge

**ASSUMPTIONS:** Plates infinite extent in direction normal to page.

**ANALYSIS:** The "crossed-strings" method is applicable
to surfaces of infinite extent in one direction having an obstructed view of one another.




$$F_{12} = (1/2w_1)[(ac + bd) - (ad + bc)].$$

(a) *Parallel plates:* From the schematic, the edge and diagonal distances are

$$ac = bd = \left(w_1^2 + L^2\right)^{1/2} \qquad bc = ad = L.$$

With $w_1$ as the width of the plate, find

$$F_{12} = \frac{1}{2w_1}\left[2\left(w_1^2 + L^2\right)^{1/2} - 2(L)\right] = \frac{1}{2\times 4\ m}\left[2\left(4^2 + 1^2\right)^{1/2} m - 2(1\ m)\right] = 0.781.$$ <

Using Fig. 13.4 with X/L = 4/1 = 4 and Y/L = ∞, find $F_{12} \approx 0.80$. Also, using the first relation of Table 13.1,

$$F_{ij} = \left\{\left[\left(W_i + W_j\right)^2 + 4\right]^{1/2} - \left[\left(W_i - W_j\right)^2 + 4\right]^{1/2}\right\} / 2\,W_i$$

where $w_i = w_j = w_1$ and W = w/L = 4/1 = 4, find

$$F_{12} = \left\{\left[(4+4)^2 + 4\right]^{1/2} - \left[(4-4)^2 + 4\right]^{1/2}\right\} / 2\times 4 = 0.781.$$

(b) *Perpendicular plates* with a common edge: From the schematic, the edge and diagonal distances are

$$ac = w_1 \qquad bd = L \qquad ad = \left(w_1^2 + L^2\right) \qquad bc = 0.$$

With $w_1$ as the width of the horizontal plates, find

$$F_{12} = (1/2w_1)\left[2(w_1 + L) - \left(\left(w_1^2 + L^2\right)^{1/2} + 0\right)\right]$$

$$F_{12} = (1/2\times 4\ m)\left[(4+1)m - \left(\left(4^2 + 1^2\right)^{1/2} m + 0\right)\right] = 0.110.$$ <

From the third relation of Table 13.1, with $w_i = w_1 = 4$ m and $w_j = L = 1$ m, find

$$F_{ij} = \left\{1 + \left(w_j / w_i\right) - \left[1 + \left(w_j / w_i\right)^2\right]^{1/2}\right\} / 2$$

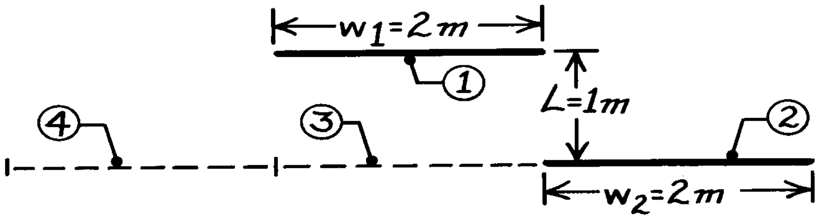
$$F_{12} = \left\{1 + (1/4) - \left[1 + (1/4)^2\right]^{1/2}\right\} / 2 = 0.110.$$

## PROBLEM 13.13

**KNOWN:** Parallel plates of infinite extent (1,2) having aligned opposite edges.

**FIND:** View factor $F_{12}$ by using (a) appropriate view factor relations and results for opposing parallel plates and (b) Hottel's string method described in Problem 13.12

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Parallel planes of infinite extent normal to page and (2) Diffuse surfaces with uniform radiosity.

**ANALYSIS:** From symmetry consideration ($F_{12} = F_{14}$) and Eq. 13.5, it follows that

$$F_{12} = (1/2)\left[F_{1(2,3,4)} - F_{13}\right]$$

where $A_3$ and $A_4$ have been defined for convenience in the analysis. Each of these view factors can be evaluated by the first relation of Table 13.1 for parallel plates with midlines connected perpendicularly.

$F_{13}$: $\quad W_1 = w_1/L = 2 \qquad W_2 = w_2/L = 2$

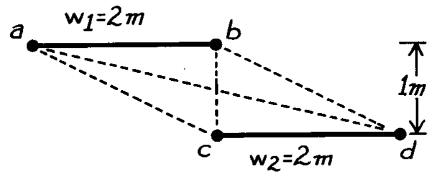
$$F_{13} = \frac{\left[(W_1+W_2)^2+4\right]^{1/2} - \left[(W_2-W_1)^2+4\right]^{1/2}}{2W_1} = \frac{\left[(2+2)^2+4\right]^{1/2} - \left[(2-2)^2+4\right]^{1/2}}{2\times 2} = 0.618$$

$F_{1(2,3,4)}$: $\quad W_1 = w_1/L = 2 \qquad W_{(2,3,4)} = 3w_2/L = 6$

$$F_{1(2,3,4)} = \frac{\left[(2+6)^2+4\right]^{1/2} - \left[(6-2)^2+4\right]^{1/2}}{2\times 2} = 0.944.$$

Hence, find $\quad F_{12} = (1/2)[0.944 - 0.618] = 0.163.$ <

(b) Using Hottel's string method,




$$F_{12} = (1/2w_1)\left[(ac+bd)-(ad+bc)\right]$$

$$ac = \left(1+4^2\right)^{1/2} = 4.123$$

$$bd = 1$$

$$ad = \left(1^2+2^2\right)^{1/2} = 2.236$$

$$bc = ad = 2.236$$

and substituting numerical values find

$$F_{12} = (1/2\times 2)\left[(4.123+1)-(2.236+2.236)\right] = 0.163.$$ <

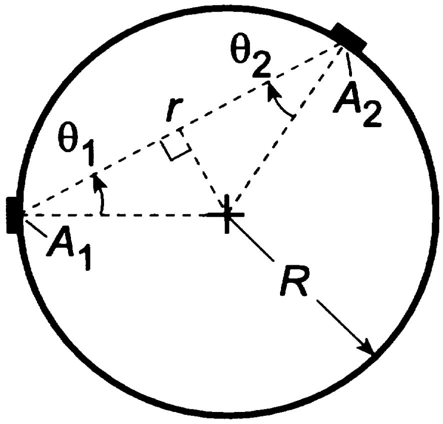
**COMMENTS:** Remember that Hottel's string method is applicable only to surfaces that are of infinite extent in one direction and have unobstructed views of one another.

## PROBLEM 13.14

**KNOWN:** Two small diffuse surfaces, $A_1$ and $A_2$, on the inside of a spherical enclosure of radius R.

**FIND:** Expression for the view factor $F_{12}$ in terms of $A_2$ and R by two methods: (a) Beginning with the expression $F_{ij} = q_{ij}/A_i J_i$ and (b) Using the view factor integral, Eq. 13.1.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Surfaces $A_1$ and $A_2$ are diffuse and (2) $A_1$ and $A_2 << R^2$.

**ANALYSIS:** (a) The view factor is defined as the fraction of radiation leaving $A_i$ which is intercepted by surface j and, from Section 13.1.1, can be expressed as

$$F_{ij} = \frac{q_{ij}}{A_i J_i} \tag{1}$$

From Eq. 12.5, the radiation leaving intercepted by $A_1$ and $A_2$ on the spherical surface is

$$q_{1\to 2} = (J_1/\pi)\cdot A_1 \cos\theta_1 \cdot \omega_{2-1} \tag{2}$$

where the solid angle $A_2$ subtends with respect to $A_1$ is

$$\omega_{2-1} = \frac{A_{2,n}}{r^2} = \frac{A_2 \cos\theta_2}{r^2} \tag{3}$$

From the schematic above,

$$\cos\theta_1 = \cos\theta_2 \qquad\qquad r = 2R\cos\theta_1 \tag{4,5}$$

Hence, the view factor is

$$F_{ij} = \frac{(J_1/\pi) A_1 \cos\theta_1 \cdot A_2 \cos\theta_2 / 4R^2 \cos\theta_1}{A_1 J_1} = \frac{A_2}{4\pi R^2} \qquad <$$

(b) The view factor integral, Eq. 13.1, for the small areas $A_1$ and $A_2$ is

$$F_{12} = \frac{1}{A_1}\int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} dA_1 dA_2 = \frac{\cos\theta_1 \cos\theta_2 A_2}{\pi r^2}$$

and from Eqs. (4,5) above,

$$F_{12} = \frac{A_2}{\pi R^2} \qquad <$$

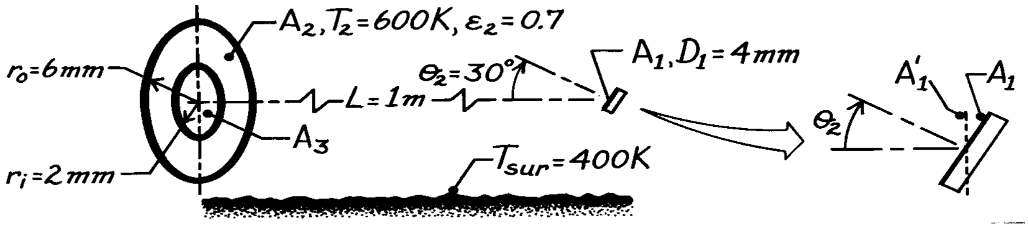
**COMMENTS:** Recognize the importance of the second assumption. We require that $A_1$, $A_2$, $<< R^2$ so that the areas can be considered as of differential extent, $A_1 = dA_1$, and $A_2 = dA_2$.

## PROBLEM 13.15

**KNOWN:** Disk $A_1$, located coaxially, but tilted 30° of the normal, from the diffuse-gray, ring-shaped disk $A_2$. Surroundings at 400 K.

**FIND:** Irradiation on $A_1$, $G_1$, due to the radiation from $A_2$.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) $A_2$ is diffuse-gray surface, (2) Uniform radiosity over $A_2$, (3) The surroundings are large with respect to $A_1$ and $A_2$.

**ANALYSIS:** The irradiation on $A_1$ is

$$G_1 = q_{21} / A_1 = (F_{21} \cdot J_2 A_2) / A_1 \qquad (1)$$

where $J_2$ is the radiosity from $A_2$ evaluated as

$$J_2 = \varepsilon_2 E_{b,2} + \rho_2 G_2 = \varepsilon_2 \sigma T_2^4 + (1-\varepsilon_2)\sigma T_{sur}^4$$

$$J_2 = 0.7 \times 5.67 \times 10^{-8} \text{W/m}^2 \cdot \text{K}^4 (600\ \text{K})^4 + (1-0.7) 5.67 \times 10^{-8} \text{W/m}^2 \cdot \text{K}^4 (400\ \text{K})^4$$

$$J_2 = 5144 + 436 = 5580\ \text{W/m}^2. \qquad (2)$$

Using the view factor relation of Eq. 13.8, evaluate view factors between $A_1'$, the normal projection of $A_1$, and $A_3$ as

$$F_{1'3} = \frac{D_i^2}{D_i^2 + 4L^2} = \frac{(0.004\ \text{m})^2}{(0.004\ \text{m})^2 + 4(1\ \text{m})^2} = 4.00 \times 10^{-6}$$

and between $A_1'$ and ($A_2$ + $A_3$) as

$$F_{1'(23)} = \frac{D_o^2}{D_o^2 + 4L^2} = \frac{(0.012)^2}{(0.012)^2 + 4(1\ \text{m})^2} = 3.60 \times 10^{-5}$$

giving $\qquad F_{1'2} = F_{1'(23)} - F_{1'3} = 3.60 \times 10^{-5} - 4.00 \times 10^{-6} = 3.20 \times 10^{-5}.$

From the reciprocity relation it follows that

$$F_{21'} = A_1' F_{1'2} / A_2 = (A_1 \cos\theta_1 / A_2) F_{1'2} = 3.20 \times 10^{-5} \cos\theta_1 (A_1 / A_2). \qquad (3)$$

By inspection we note that all the radiation striking $A_1'$ will also intercept $A_1$; that is

$$F_{21} = F_{21'}. \qquad (4)$$

Hence, substituting for Eqs. (3) and (4) for $F_{21}$ into Eq. (1), find

$$G_1 = \left(3.20 \times 10^{-5} \cos\theta_1 (A_1 / A_2) \times J_2 \times A_2\right) / A_1 = 3.20 \times 10^{-5} \cos\theta_1 \cdot J_2 \qquad (5)$$

$$G_1 = 3.20 \times 10^{-5} \cos(30°) \times 5580\ \text{W/m}^2 = 27.7\ \mu\text{W/m}^2. \qquad <$$

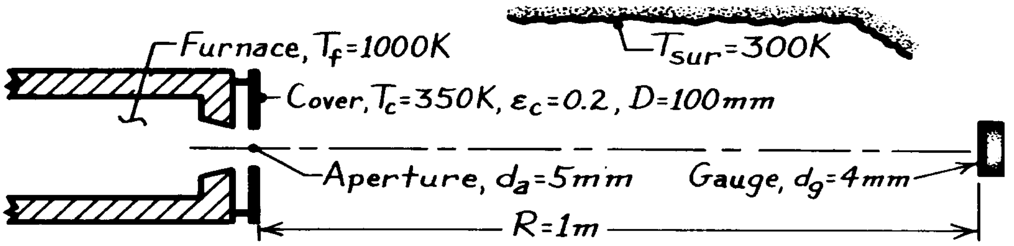
**COMMENTS:** (1) Note from Eq. (5) that $G_1 \sim \cos\theta_1$ such that $G_1$is a maximum when $A_1$ is normal to disk $A_2$.

## PROBLEM 13.16

**KNOWN:** Heat flux gauge positioned normal to a blackbody furnace. Cover of furnace is at 350 K while surroundings are at 300 K.

**FIND:** (a) Irradiation on gage, $G_g$, considering only emission from the furnace aperture and (b) Irradiation considering radiation from the cover *and* aperture.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Furnace aperture approximates blackbody, (2) Shield is opaque, diffuse and gray with uniform temperature, (3) Shield has uniform radiosity, (4) $A_g << R^2$, so that $\omega_{g-f} = A_g/R^2$, (5) Surroundings are large, uniform at 300 K.

**ANALYSIS:** (a) The irradiation on the gauge due *only* to aperture emission is

$$G_g = q_{f-g}/A_g = \left(I_{e,f} \cdot A_f \cos\theta_f \cdot \omega_{g-f}\right)/A_g = \frac{\sigma T_f^4}{\pi} \cdot A_f \cdot \frac{A_g}{R^2}/A_g$$

$$G_g = \frac{\sigma T_f^4}{\pi R^2} A_f = \frac{5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\,(1000\ \mathrm{K})^4}{\pi\,(1\ \mathrm{m})^2} \times (\pi/4)(0.005\ \mathrm{m})^2 = 354.4\ \mathrm{mW/m^2}. \qquad <$$

(b) The irradiation on the gauge due to radiation from the *aperture* (a) and *cover (c)* is

$$G_g = G_{g,a} + \frac{F_{c-g}\cdot J_c A_c}{A_g}$$

where $F_{c-g}$ and the cover radiosity are

$$F_{c-g} = F_{g-c}\left(A_g/A_c\right) \approx \frac{D_c^2}{4R^2 + D_c^2}\cdot\frac{A_g}{A_c} \qquad\qquad J_c = \varepsilon_c E_b\left(T_c\right) + \rho_c G_c$$

but $G_c = E_b\left(T_{sur}\right)$ and $\rho_c = 1-\alpha_c = 1-\varepsilon_c, J_c = \varepsilon_c\sigma T_c^4 + \left(1-\varepsilon_c\right)\sigma T_{sur}^4 = (170.2+387.4)\,\mathrm{W/m^2}$.
Hence, the irradiation is

$$G_g = G_{g,a} + \frac{1}{A_g}\left(\frac{D_c^2}{4R^2+D_c^2}\cdot\frac{A_g}{A_c}\right)\left[\varepsilon_c\sigma T_c^4 + \left(1-\varepsilon_c\right)\sigma T_{sur}^4\right]A_c$$

$$G_g = 354.4\ \mathrm{mW/m^2} + \left(\frac{0.10^2}{4\times1^2+0.10^2}\right)\left[0.2\times\sigma\,(350)^4 + (1-0.2)\times\sigma\,(300)^4\right]\mathrm{W/m^2}$$

$$G_g = 354.4\ \mathrm{mW/m^2} + 424.4\ \mathrm{mW/m^2} + 916.2\ \mathrm{mW/m^2} = 1{,}695\ \mathrm{mW/m^2}.$$

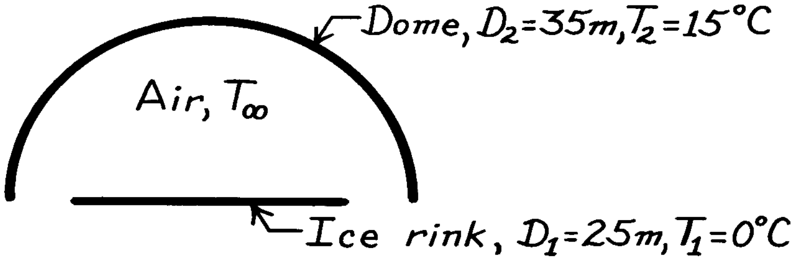
**COMMENTS:** (1) Note we have assumed $A_f << A_c$ so that effect of the aperture is negligible. (2) In part (b), the irradiation due to radiosity from the shield can be written also as $G_{g,c} = q_{c-g}/A_g = (J_c/\pi)\cdot A_c\cdot\omega_{g-c}/A_g$ where $\omega_{g-c} = A_g/R^2$. This is an excellent approximation since $A_c << R^2$.

## PROBLEM 13.17

**KNOWN:** Temperature and diameters of a circular ice rink and a hemispherical dome.

**FIND:** Net rate of heat transfer to the ice due to radiation exchange with the dome.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Blackbody behavior for dome and ice.

**ANALYSIS:** From Eq. 13.13 the net rate of energy exchange between the two blackbodies is

$$q_{21} = A_2 F_{21} \sigma \left(T_2^4 - T_1^4\right)$$

From reciprocity, $A_2\ F_{21} = A_1\ F_{12} = \left(\pi D_1^2 / 4\right)1$

$$A_2 F_{21} = (\pi / 4)(25\text{ m})^2 1 = 491\text{ m}^2.$$

Hence

$$q_{21} = 491\text{ m}^2 \left(5.67\times10^{-8}\text{W/m}^2\cdot\text{K}^4\right)\left[(288\text{ K})^4 - (273\text{ K})^4\right]$$

$$q_{21} = 3.69\times10^4\text{ W.}$$ <

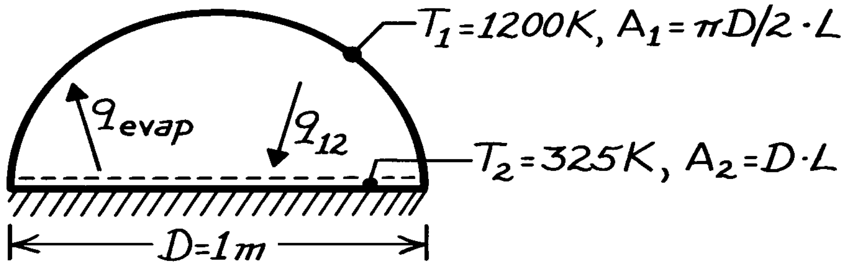
**COMMENTS:** If the air temperature, $T_\infty$, exceeds $T_1$, there will also be heat transfer by convection to the ice. The radiation and convection transfer to the ice determine the heat load which must be handled by the cooling system.

## PROBLEM 13.18

**KNOWN:** Surface temperature of a semi-circular drying oven.

**FIND:** Drying rate per unit length of oven.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Blackbody behavior for furnace wall and water, (2) Convection effects are negligible and bottom is insulated.

**PROPERTIES:** *Table A-6*, Water (325 K): $h_{fg} = 2.378 \times 10^6 \text{ J/kg}$.

**ANALYSIS:** Applying a surface energy balance,

$$q_{12} = q_{evap} = \dot{m} h_{fg}$$

where it is assumed that the net radiation heat transfer to the water is balanced by the evaporative heat loss. From Eq. 13.13

$$q_{12} = A_1 F_{12} \sigma \left(T_1^4 - T_2^4\right).$$

From inspection and the reciprocity relation, Eq. 13.3,

$$F_{12} = \frac{A_2}{A_1} F_{21} = \frac{D \cdot L}{(\pi D/2) \cdot L} \times 1 = 0.637.$$

Hence

$$\dot{m}' = \frac{\dot{m}}{L} = \frac{\pi D}{2} F_{12} \sigma \frac{\left(T_1^4 - T_2^4\right)}{h_{fg}}$$

$$\dot{m}' = \frac{\pi (1 \text{ m})}{2} \times 0.637 \times 5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \cdot \text{K}^4} \frac{(1200 \text{ K}) - (325 \text{ K})^4}{2.378 \times 10^6 \text{ J/kg}}$$

or

$$\dot{m}' = 0.0492 \text{ kg/s} \cdot \text{m}. \qquad <$$

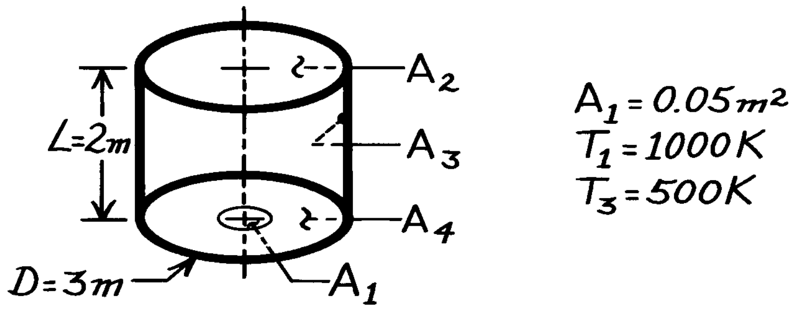
**COMMENTS:** Air flow through the oven is needed to remove the water vapor. The water surface temperature, $T_2$, is determined by a balance between radiation heat transfer to the water and the convection of latent and sensible energy from the water.

## PROBLEM 13.19

**KNOWN:** Arrangement of three black surfaces with prescribed geometries and surface temperatures.

**FIND:** (a) View factor $F_{13}$, (b) Net radiation heat transfer from $A_1$ to $A_3$.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Interior surfaces behave as blackbodies, (2) $A_2 >> A_1$.

**ANALYSIS:** (a) Define the enclosure as the interior of the cylindrical form and identify $A_4$. Applying the view factor summation rule, Eq. 13.4,

$$F_{11} + F_{12} + F_{13} + F_{14} = 1. \tag{1}$$

Note that $F_{11} = 0$ and $F_{14} = 0$. From Eq. 13.8,

$$F_{12} = \frac{D^2}{D^2 + 4L^2} = \frac{(3m)^2}{(3m)^2 + 4(2m)^2} = 0.36. \tag{2}$$

From Eqs. (1) and (2),

$$F_{13} = 1 - F_{12} = 1 - 0.36 = 0.64.$$ <

(b) The net heat transfer rate from $A_1$ to $A_3$ follows from Eq. 13.13,

$$q_{13} = A_1\ F_{13}\ \sigma\left(T_1^4 - T_3^4\right)$$

$$q_{13} = 0.05m^2 \times 0.64 \times 5.67 \times 10^{-8} W/m^2 \cdot K^4 \left(1000^4 - 500^4\right) K^4 = 1700\,W.$$ <

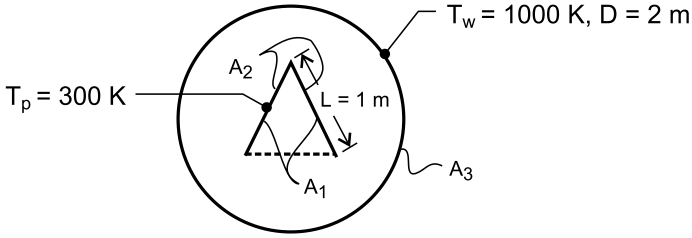
**COMMENTS:** Note that the summation rule, Eq. 13.4, applies to an enclosure; that is, the total region above the surface must be considered.

**PROBLEM 13.20**

**KNOWN:** Furnace diameter and temperature. Dimensions and temperature of suspended part.

**FIND:** Net rate of radiation transfer per unit length to the part.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) All surfaces may be approximated as blackbodies.

**ANALYSIS:** From symmetry considerations, it is convenient to treat the system as a three-surface enclosure consisting of the inner surfaces of the vee (1), the outer surfaces of the vee (2) and the furnace wall (3). The net rate of radiation heat transfer to the part is then

$$q'_{1,2} = A'_3 F_{31} \sigma \left(T_w^4 - T_p^4\right) + A'_3 F_{32} \sigma \left(T_w^4 - T_p^4\right)$$

From reciprocity,

$$A'_3 F_{31} = A'_1 F_{13} = 2L \times 0.5 = 1m$$

where surface 3 may be represented by the dashed line and, from symmetry, $F_{13} = 0.5$. Also,

$$A'_3 F_{32} = A'_2 F_{23} = 2L \times 1 = 2m$$

Hence,

$$q'_{1,2} = (1+2)\mathrm{m} \times 5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4} \left(1000^4 - 300^4\right)\mathrm{K}^4 = 1.69 \times 10^5\ \mathrm{W/m} \qquad <$$

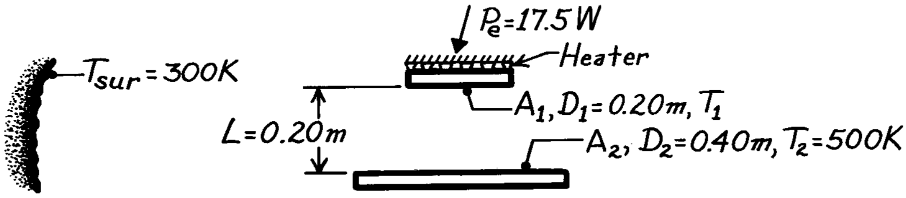
**COMMENTS:** With all surfaces approximated as blackbodies, the result is independent of the tube diameter. Note that $F_{11} = 0.5$.

## PROBLEM 13.21

**KNOWN:** Coaxial, parallel black plates with surroundings. Lower plate ($A_2$) maintained at prescribed temperature $T_2$ while electrical power is supplied to upper plate ($A_1$).

**FIND:** Temperature of the upper plate $T_1$.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Plates are black surfaces of uniform temperature, and (2) Backside of heater on $A_1$ insulated.

**ANALYSIS:** The net radiation heat rate leaving $A_i$ is

$$P_e = \sum_{j=1}^{N} q_{ij} = A_1 F_{12}\sigma\left(T_1^4 - T_2^4\right) + A_1 F_{13}\sigma\left(T_1^4 - T_3^4\right)$$

$$P_e = A_1\sigma\left[F_{12}\left(T_1^4 - T_2^4\right) + F_{13}\left(T_1^4 - T_{sur}^4\right)\right] \qquad (1)$$

From Fig. 13.5 for coaxial disks (see Table 13.2),

$$R_1 = r_1 / L = 0.10\ \text{m} / 0.20\ \text{m} = 0.5 \qquad\qquad R_2 = r_2 / L = 0.20\ \text{m} / 0.20\ \text{m} = 1.0$$

$$S = 1 + \frac{1 + R_2^2}{R_1^2} = 1 + \frac{1 + 1^2}{(0.5)^2} = 9.0$$

$$F_{12} = \frac{1}{2}\left\{S - \left[S^2 - 4(r_2 / r_1)^2\right]^{1/2}\right\} = \frac{1}{2}\left\{9 - \left[9^2 - 4(0.2/0.1)^2\right]^{1/2}\right\} = 0.469.$$

From the summation rule for the enclosure $A_1$, $A_2$ and $A_3$ where the last area represents the surroundings with $T_3 = T_{sur}$,

$$F_{12} + F_{13} = 1 \qquad\qquad F_{13} = 1 - F_{12} = 1 - 0.469 = 0.531.$$

Substituting numerical values into Eq. (1), with $A_1 = \pi D_1^2 / 4 = 3.142\times10^{-2}\,\text{m}^2$,

$$17.5\ \text{W} = 3.142\times10^{-2}\,\text{m}^2 \times 5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left[0.469\left(T_1^4 - 500^4\right)\text{K}^4 \right.$$

$$\left. +\ 0.531\left(T_1^4 - 300^4\right)\text{K}^4\right]$$

$$9.823\times10^9 = 0.469\left(T_1^4 - 500^4\right) + 0.531\left(T_1^4 - 300^4\right)$$

find by trial-and-error that $\qquad T_1 = 456\ \text{K}.$ <

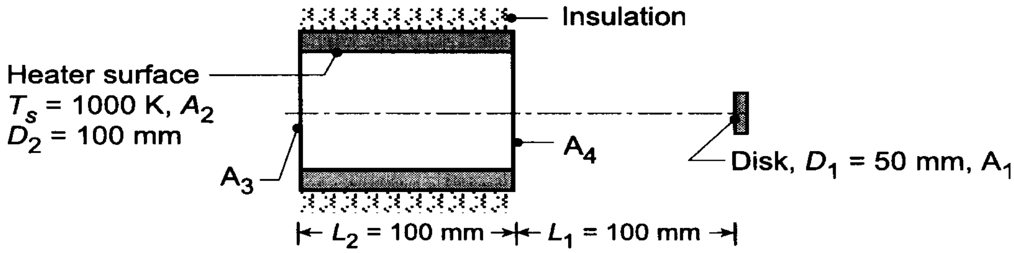
**COMMENTS:** Note that if the upper plate were adiabatic, $T_1$ = 427 K.

## PROBLEM 13.22

**KNOWN:** Tubular heater radiates like blackbody at 1000 K.

**FIND:** (a) Radiant power from the heater surface, $A_s$, intercepted by a disc, $A_1$, at a prescribed location $q_{s\to1}$; irradiation on the disk, $G_1$; and (b) Compute and plot $q_{s\to1}$ and $G_1$ as a function of the separation distance $L_1$ for the range $0 \le L_1 \le 200$ mm for disk diameters $D_1$ = 25, and 50 and 100 mm.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Heater surface behaves as blackbody with uniform temperature.

**ANALYSIS:** (a) The radiant power leaving the inner surface of the tubular heater that is intercepted by the disk is

$$q_{2\to1} = (A_2 E_{b2}) F_{21} \qquad (1)$$

where the heater is surface 2 and the disk is surface 1. It follows from the reciprocity rule, Eq. 13.3, that

$$F_{21} = \frac{A_1}{A_2} F_{12}. \qquad (2)$$

Define now the hypothetical disks, $A_3$ and $A_4$, located at the ends of the tubular heater. By inspection, it follows that

$$F_{14} = F_{12} + F_{13} \qquad \text{or} \qquad F_{12} = F_{14} - F_{13} \qquad (3)$$

where $F_{14}$ and $F_{13}$ may be determined from Fig. 13.5. Substituting numerical values, with $D_3 = D_4 = D_2$,

$$F_{13} = 0.08 \qquad \text{with} \qquad \frac{L}{r_i} = \frac{L_1 + L_2}{D_1/2} = \frac{200}{50/2} = 8 \qquad \frac{r_j}{L} = \frac{D_3/2}{L_1 + L_2} = \frac{100/2}{200} = 0.25$$

$$F_{14} = 0.20 \qquad \text{with} \qquad \frac{L}{r_i} = \frac{L_1}{D_1/2} = \frac{100}{50/2} = 4 \qquad \frac{r_j}{L} = \frac{D_4/2}{L_1} = \frac{100/2}{100} = 0.5$$

Substituting Eq. (3) into Eq. (2) and then into Eq. (1), the result is

$$q_{2\to1} = A_1 (F_{14} - F_{13}) E_{b2}$$

$$q_{2\to1} = \left[\pi \left(50\times10^{-3}\right)^2 \text{m}^2/4\right](0.20 - 0.08)\times5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\,(1000\text{ K})^4 = 13.4\text{W} \qquad <$$

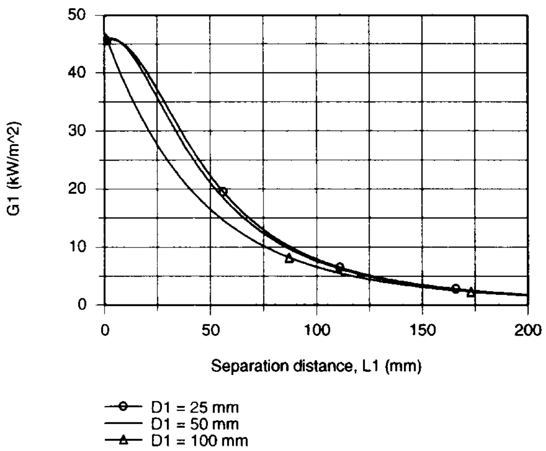
where $E_{b2} = \sigma T_s^4$. The irradiation $G_1$ originating from emission leaving the heater surface is

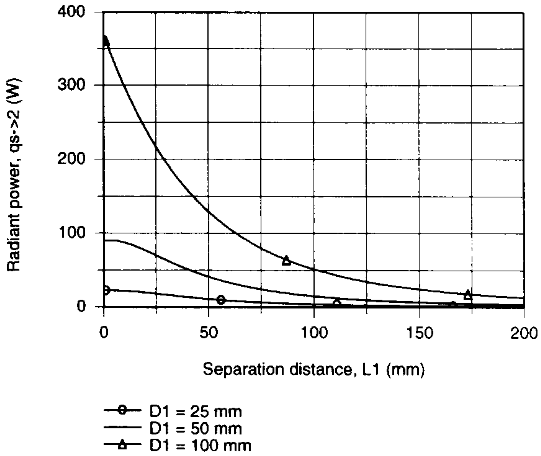
$$G_1 = \frac{q_{s\to1}}{A_1} = \frac{13.4\text{ W}}{\pi(0.050\text{ m})^2/4} = 6825\,\text{W/m}^2. \qquad (4) <$$

Continued …..

## PROBLEM 13.22 (Cont.)

(b) Using the foregoing equations in *IHT* along with the *Radiation Tool-View Factors* for *Coaxial Parallel Disks,* $G_1$ and $q_{s\to1}$ were computed as a function of $L_1$ for selected values of $D_1$. The results are plotted below.



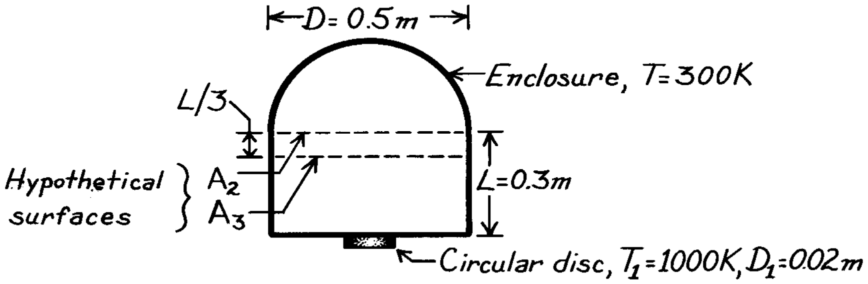

In the upper left-hand plot, $G_1$ decreases with increasing separation distance. For a given separation distance, the irradiation decreases with increasing diameter. With values of $D_1$ = 25 and 50 mm, the irradiation values are only slightly different, which diminishes as $L_1$ increases. In the upper right-hand plot, the radiant power from the heater surface reaching the disk, $q_{s\to2}$, decreases with increasing $L_1$ and decreasing $D_1$. Note that while $G_1$ is nearly the same for $D_1$ = 25 and 50 mm, their respective $q_{s\to2}$ values are quite different. Why is this so?

# PROBLEM 13.23

**KNOWN:** Dimensions and temperatures of an enclosure and a circular disc at its base.

**FIND:** Net radiation heat transfer between the disc and portions of the enclosure.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Blackbody behavior for disc and enclosure surfaces, (2) Area of disc is much less than that of the hypothetical surfaces, $(A_1/A_2) << 1$ and $(A_1/A_3) << 1$.

**ANALYSIS:** From Eq. 13.13 the net radiation exchange between the disc (1) and the hemispherical dome (d) is

$$q_{1d} = A_1 F_{1d}\,\sigma\left(T_1^4 - T^4\right).$$

However, since all of the radiation intercepted by the dome must pass through the hypothetical area $A_2$, it follows from Eq. 13.8 of Example 13.1,

$$F_{1d} = F_{12} \approx \frac{D^2}{4L^2 + D^2} = \frac{1}{(2L/D)^2 + 1} = \frac{1}{1.44 + 1} = 0.410.$$

Hence

$$q_{1d} = \frac{\pi}{4}(0.02\ \text{m})^2 \times 0.41 \times 5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}\left[(1000\ \text{K})^4 - (300\ \text{K})^4\right]$$

$$q_{1d} = 7.24\ \text{W}.$$ <

Similarly, the net radiation exchange between the disc (1) and the cylindrical ring (r) of length L/3 is

$$q_{1r} = A_1\ F_{1r}\ \sigma\left(T_1^4 - T^4\right)$$

where

$$F_{1r} = F_{13} - F_{12} = \frac{D^2}{4(2L/3)^2 + D^2} - 0.41 = 0.61 - 0.41 = 0.20.$$

Hence

$$q_{1r} = \frac{\pi}{4}(0.02\ \text{m})^2 \times 0.2 \times 5.67 \times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left[(1000\ \text{K})^4 - (300\ \text{K})^4\right]$$

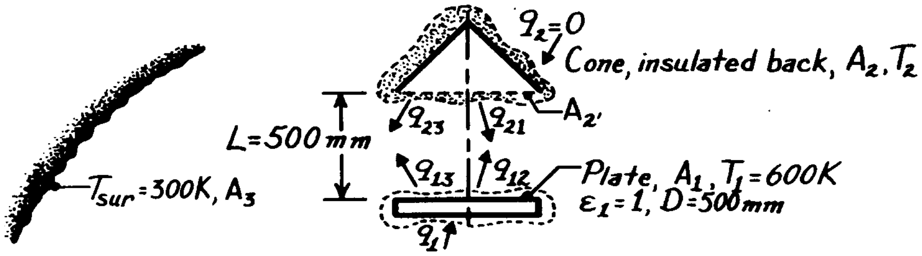
$$q_{1r} = 3.53\ \text{W}.$$ <

## PROBLEM 13.24

**KNOWN:** Circular plate ($A_1$) maintained at 600 K positioned coaxially with a conical shape ($A_2$) whose backside is insulated. Plate and cone are black surfaces and located in large, insulated enclosure at 300 K.

**FIND:** (a) Temperature of the conical surface $T_2$ and (b) Electric power required to maintain plate at 600 K.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Plate and cone are black, (3) Cone behaves as insulated, reradiating surface, (4) Surroundings are large compared to plate and cone.

**ANALYSIS:** (a) Recognizing that the plate, cone, and surroundings from a three-(black) surface enclosure, perform a radiation balance on the cone.

$$q_2 = 0 = q_{23} + q_{21} = A_2 F_{23}\, \sigma\left(T_2^4 - T_3^4\right) + A_2 F_{21}\, \sigma\left(T_2^4 - T_1^4\right)$$

where the view factor $F_{21}$ can be determined from the *coaxial parallel disks* relation (Table 13.2 or Fig. 13.5) with $R_i = r_i/L=250/500 = 0.5$, $R_j = 0.5$, $S = 1 + \left(1+R_j^2\right)/R_i^2 = 1 + (1 + 0.5^2)/0.5^2 = 6.00$, and noting $F_{2'1} = F_{21}$,

$$F_{21} = 0.5\left\{S - \left[S^2 - 4\left(r_j/r_i\right)^2\right]^{1/2}\right\} = 0.5\left\{6 - \left[6^2 - 4\left(0.5/0.5\right)^2\right]^{1/2}\right\} = 0.172.$$

For the enclosure, the summation rule provides, $F_{2'3} = 1 - F_{2'1} = 1 - 0.172 = 0.828$. Hence,

$$0.828\left(T_2^4 - 300^4\right) = 0 + 0.172\left(T_2^4 - 600^4\right)$$

$$T_2 = 413 \text{ K.}$$ <

(b) The power required to maintain the plate at $T_2$ follows from a radiation balance,

$$q_1 = q_{12} + q_{13} = A_1 F_{12} \sigma\left(T_1^4 - T_2^4\right) + A_1 F_{13} \sigma\left(T_1^4 - T_3^4\right)$$

where $F_{12} = A_{2'} F_{2'1}/A_1 = F_{21} = 0.172$ and $F_{13} = 1 - F_{12} = 0.828$,

$$q_1 = \left(\pi 0.5^2/4\right)\text{m}^2 \sigma\left[0.172\left(600^4 - 413^4\right)\text{K}^4 + 0.828\left(600^4 - 300^4\right)\text{K}^4\right]$$

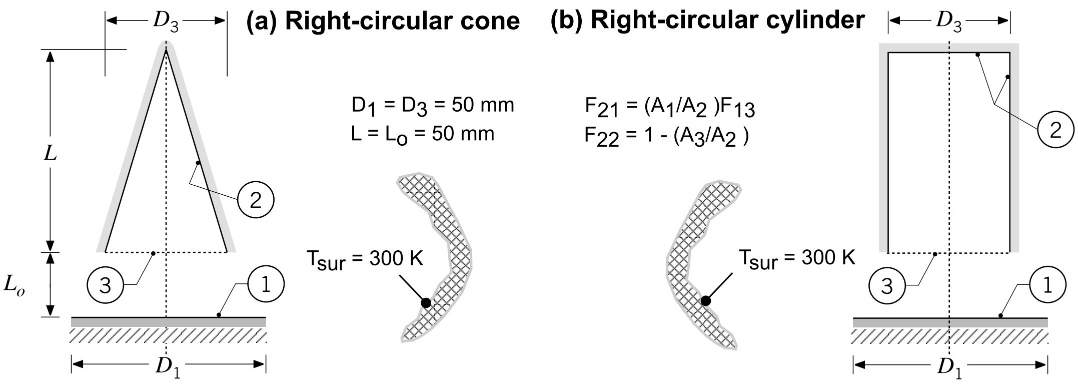
$$q_1 = 1312 \text{ W.}$$ <

## PROBLEM 13.25

**KNOWN:** Conical and cylindrical furnaces ($A_2$) as illustrated and dimensioned in Problem 13.2 (S) supplied with power of 50 W. Workpiece ($A_1$) with insulated backside located in large room at 300 K.

**FIND:** Temperature of the workpiece, $T_1$, and the temperature of the inner surfaces of the furnaces, $T_2$. Use expressions for the view factors $F_{21}$ and $F_{22}$ given in the statement for Problem 13.2 (S).

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Diffuse, black surfaces with uniform radiosities, (2) Backside of workpiece is perfectly insulated, (3) Inner base and lateral surfaces of the cylindrical furnace treated as single surface, (4) Negligible convection heat transfer, (5) Room behaves as large, isothermal surroundings.

**ANALYSIS:** Considering the furnace surface ($A_2$), the workpiece ($A_1$) and the surroundings ($A_s$) as an enclosure, the net radiation transfer from $A_1$ and $A_2$ follows from Eq. 13.14,

*Workpiece* $$q_1 = 0 = A_1 F_{12}\left(E_{b1} - E_{b2}\right) + A_1 F_{1s}\left(E_{b1} - E_{bs}\right) \qquad (1)$$

*Furnace* $$q_2 = 50\text{ W} = A_2 F_{21}\left(E_{b2} - E_{b1}\right) + A_2 F_{2s}\left(E_{b2} - E_{bs}\right) \qquad (2)$$

where $E_b = \sigma T^4$ and $\sigma = 5.67 \times 10^{-8}$ W/m$^2\cdot$K$^4$. From summation rules on $A_1$ and $A_2$, the view factors $F_{1s}$ and $F_{2s}$ can be evaluated. Using reciprocity, $F_{12}$ can be evaluated.

$$F_{1s} = 1 - F_{12} \qquad F_{2s} = 1 - F_{21} - F_{22} \qquad F_{12} = \left(A_2 / A_1\right) F_{21}$$

The expressions for $F_{21}$ and $F_{22}$ are provided in the schematic. With $A_1 = \pi D_1^2 / 4$ the $A_2$ are:

*Cone*: $A_2 = \pi D_3 / 2\left(L^2 + \left(D_3 / 2\right)^2\right)^{1/2}$ *Cylinder*: $A_2 = \pi D_3^2 / 4 + \pi D_3 L$

Examine Eqs (1) and (2) and recognize that there are two unknowns, $T_1$ and $T_2$, and the equations must be solved simultaneously. Using the foregoing equations in the *IHT* workspace, the results are

$$T_1 = 544\text{ K} \qquad T_2 = 828\text{ K}$$ <

**COMMENTS:** (1) From the *IHT* analysis, the relevant view factors are: $F_{12}$ = 0.1716; $F_{1s}$ = 0.8284; *Cone:* $F_{21}$ = 0.07673, $F_{22}$ = 0.5528; *Cylinder:* $F_{21}$ = 0.03431, $F_{22}$ =0.80.

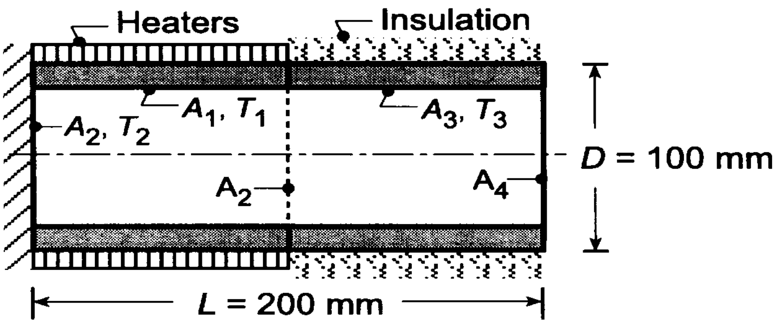
(2) That both furnace configurations provided identical results may not, at first, be intuitively obvious. Since both furnaces ($A_2$) are black, they can be represented by the hypothetical black area $A_3$ (the opening of the furnaces). As such, the analysis is for an enclosure with the workpiece ($A_1$), the furnace represented by the disk $A_3$ (at $T_2$), and the surroundings. As an exercise, perform this analysis to confirm the above results.

## PROBLEM 13.26

**KNOWN:** Furnace constructed in three sections: insulated circular (2) and cylindrical (3) sections, as well as, an intermediate cylindrical section (1) with imbedded electrical resistance heaters. Cylindrical sections (1,3) are of equal length.

**FIND:** (a) Electrical power required to maintain the heated section at $T_1$ = 1000 K if all the surfaces are black, (b) Temperatures of the insulated sections, $T_2$ and $T_3$, and (c) Compute and plot $q_1$, $T_2$ and $T_3$ as functions of the length-to-diameter ratio, with $1 \le L/D \le 5$ and D = 100 mm.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) All surfaces are black, (2) Areas (1, 2, 3) are isothermal.

**ANALYSIS:** (a) To complete the enclosure representing the furnace, define the hypothetical surface $A_4$ as the opening at 0 K with unity emissivity. For each of the enclosure surfaces 1, 2, and 3, the energy balances following Eq. 13.13 are

$$q_1 = A_1F_{12}\left(E_{b1} - E_{b2}\right) + A_1F_{13}\left(E_{b1} - E_{b3}\right) + A_1F_{14} + \left(E_{b1} - E_{b4}\right) \qquad (1)$$

$$0 = A_2F_{21}\left(E_{b2} - E_{b1}\right) + A_2F_{23}\left(E_{b2} - E_{b3}\right) + A_2F_{24}\left(E_{b2} - E_{b4}\right) \qquad (2)$$

$$0 = A_3F_{31}\left(E_{b3} - E_{b1}\right) + A_3F_{32}\left(E_{b3} - E_{b2}\right) + A_3F_{34}\left(E_{b3} - E_{b4}\right) \qquad (3)$$

where the emissive powers are

$$E_{b1} = \sigma T_1^4 \qquad E_{b2} = \sigma T_2^4 \qquad E_{b3} = \sigma T_3^4 \qquad E_{b4} = 0 \qquad (4-7)$$

For this four surface enclosure, there are $N^2$ = 16 view factors and N (N – 1)/2 = 4 × 3/2 = 6 must be directly determined (by inspection or formulas) and the remainder can be evaluated from the summation rule and reciprocity relation. By inspection,

$$F_{22} = 0 \qquad F_{44} = 0 \qquad (8,9)$$

From the coaxial parallel disk relation, Table 13.2, find $F_{24}$

$$S = 1 + \frac{1 + R_4^2}{R_2^2} = 1 + \frac{1 + (0.250)^2}{(0.250)^2} = 18.00$$

$$R_2 = r_2 / L = 0.050\,\text{m} / 0.200\,\text{m} = 0.250 \qquad R_4 = r_4 / L = 0.250$$

$$F_{24} = 0.5\left\{S - \left[S^2 - 4\left(r_4 / r_2\right)^2\right]^{1/2}\right\}$$

$$F_{24} = 0.5\left\{18.00 - \left[18.00^2 - 4(1)^2\right]^{1/2}\right\} = 0.0557 \qquad (10)$$

Consider the three-surface enclosure $1 - 2 - 2'$ and find $F_{11}$ as beginning with the summation rule,

Continued …..

**PROBLEM 13.26 (Cont.)**

$$F_{11} = 1 - F_{12} - F_{12'} \tag{11}$$

where, from symmetry, $F_{12} = F_{12'}$, and using reciprocity,

$$F_{12} = A_2 F_{21} / A_1 = \left(\pi D^2 / 4\right) F_{23} / \left(\pi DL / 2\right) = DF_{21} / 2L \tag{12}$$

and from the summation rule on $A_2$

$$F_{21} = 1 - F_{22'} = 1 - 0.172 = 0.828, \tag{13}$$

Using the coaxial parallel disk relation, Table 13.2, to find $F_{221}$,

$$S = 1 + \frac{1 + R_{2'}^2}{R_2^2} = 1 + \frac{1 + 0.50^2}{0.50^2} = 6.000$$

$$R_2 = r_2 / L = 0.050\,\text{m} / \left(0.200 / 2\,\text{m}\right) = 0.500 \qquad R_{2'} = 0.500$$

$$F_{22'} = 0.5\left\{S - \left[S^2 - 4\left(r_{2'} / r_2\right)^2\right]^{1/2}\right\}$$

$$F_{22'} = 0.5\left\{6 - \left[6^2 - 4(1)^2\right]^{1/2}\right\} = 0.1716$$

Evaluating $F_{12}$ from Eq. (12), find

$$F_{12} = 0.100\ \text{m} \times 0.828 / 2 \times 0.200\ \text{m} = 0.2071$$

and evaluating $F_{11}$ from Eq. (11), find

$$F_{11} = 1 - 2 \times F_{12} = 1 - 2 \times 0.207 = 0.586$$

From symmetry, recognize that $F_{33} = F_{11}$ and $F_{43} = F_{21}$. To this point we have directly determined six view factors (underlined in the matrix below) and the remaining $F_{ij}$ can be evaluated from the summation rules and appropriate reciprocity relations. The view factors written in matrix form, $[F_{ij}]$ are.

$$\begin{matrix} \underline{0.5858} & \underline{0.2071} & 0.1781 & 0.02896 \\ \underline{0.8284} & \underline{0} & 0.1158 & \underline{0.05573} \\ 0.1781 & 0.02896 & 0.5858 & 0.2071 \\ 0.1158 & 0.05573 & 0.8284 & \underline{0} \end{matrix}$$

Knowing all the required view factors, the energy balances and the emissive powers, Eqs. (4-6), can be solved simultaneously to obtain:

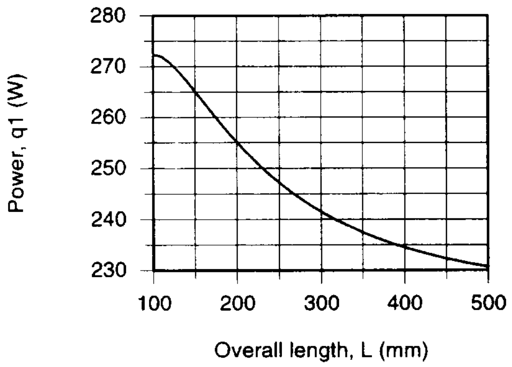
$q_1 = 255\,\text{W} \qquad E_{b2} = 5.02 \times 10^4\,\text{W/m}^2 \qquad E_{b3} = 2.79 \times 10^4\,\text{W/m}^2$ <

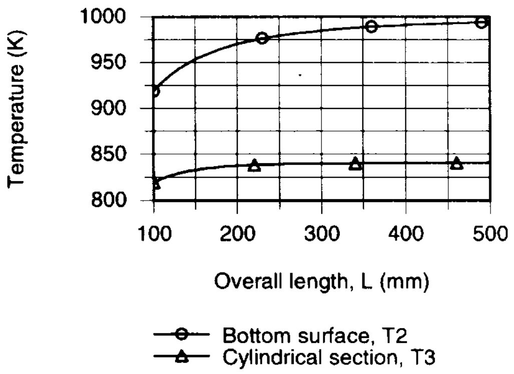
$T_2 = 970\,\text{K} \qquad T_3 = 837.5\,\text{K}$ <

Continued …..

## PROBLEM 13.26 (Cont.)

(b) Using the energy balances, Eqs. (1-3), along with the *IHT Radiation Tool, View Factors, Coaxial parallel disks,* a model was developed to calculate $q_1$, $T_2$, and $T_3$ as a function of length L for fixed diameter D = 100 m. The results are plotted below.



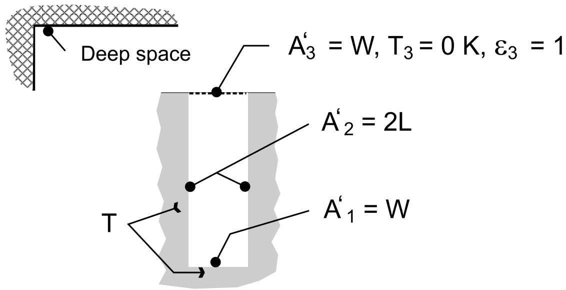

For fixed diameter, as the overall length increases, the power required to maintain the heated section at $T_1$ = 1000 K decreases. This follows since the furnace opening area is a smaller fraction of the enclosure surface area as L increases. As L increases, the bottom surface temperature $T_2$ increases as L increases and, in the limit, will approach that of the heated section, $T_1$ = 1000 K. As L increases, the temperature of the insulated cylindrical section, $T_3$, increases, but only slightly. The limiting value occurs when $E_{b3} = 0.5 \times E_{b1}$ for which $T_3 \rightarrow 840$ K. Why is that so?

**PROBLEM 13.27**

**KNOWN:** Dimensions and temperature of a rectangular fin array radiating to deep space.

**FIND:** Expression for rate of radiation transfer per unit length from a unit section of the array.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Surfaces may be approximated as blackbodies, (2) Surfaces are isothermal, (3) Length of array (normal to page) is much larger than W and L.

**ANALYSIS:** Deep space may be represented by the hypothetical surface $A_3'$, which acts as a blackbody at absolute zero temperature. The net rate of radiation heat transfer to this surface is therefore equivalent to the rate of heat rejection by a unit section of the array.

$$q_3' = A_1' F_{13} \sigma \left(T_1^4 - T_3^4\right) + A_2' F_{23} \sigma \left(T_2^4 - T_3^4\right)$$

With $\quad A_2' F_{23} = A_3' F_{32} = A_1' F_{12}, \; T_1 = T_2 = T \text{ and } T_3 = 0,$

$$q_3' = A_1' \left(F_{13} + F_{12}\right) \sigma T^4 = W \sigma T^4 \qquad <$$

Radiation from a unit section of the array corresponds to emission from the base. Hence, if blackbody behavior can, indeed, be maintained, the fins do nothing to enhance heat rejection.

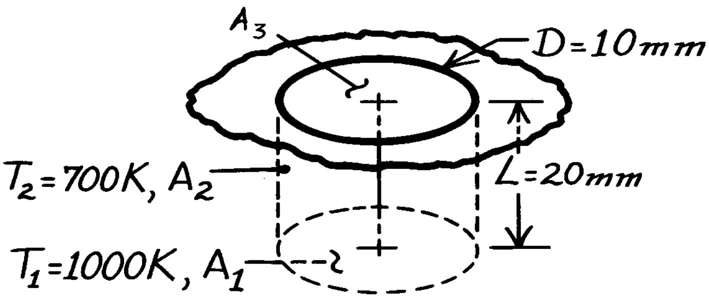
**COMMENTS:** (1) The foregoing result should come as no surprise since the surfaces of the unit section form an isothermal blackbody cavity for which emission is proportional to the area of the opening. (2) Because surfaces 1 and 2 have the same temperature, the problem could be treated as a two-surface enclosure consisting of the combined (1, 2) and 3. It follows that $q_3' = q_{(1,2)3}' = A_{(1,2)}' F_{(1,2)3} \sigma T^4 = A_3' F_{3(1,2)} \sigma T^4 = W \sigma T^4$, (3) If blackbody behavior cannot be achieved $(\varepsilon_1, \varepsilon_2 < 1)$, enhancement would be afforded by the fins.

## PROBLEM 13.28

**KNOWN:** Dimensions and temperatures of side and bottom walls in a cylindrical cavity.

**FIND:** Emissive power of the cavity.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Blackbody behavior for surfaces 1 and 2

**ANALYSIS:** The desired emissive power is defined as

$$E = q_3 / A_3$$

where

$$q_3 = A_1 \ F_{13} \, E_{b1} + A_2 \ F_{23} \, E_{b2}.$$

From symmetry, $F_{23} = F_{21}$, and from reciprocity, $F_{21} = (A_1/A_2)\ F_{12}$. With $F_{12} = 1 - F_{13}$, it follows that

$$q_3 = A_1 F_{13} E_{b1} + A_1 \left(1 - F_{13}\right) E_{b2} = A_1 E_{b2} + A_1 F_{13} \left(E_{b1} - E_{b2}\right).$$

Hence, with $A_1 = A_3$,

$$E = \frac{q_3}{A_3} = E_{b2} + F_{13} \left(E_{b1} - E_{b2}\right) = \sigma T_2^4 + F_{13}\, \sigma \left(T_1^4 - T_2^4\right).$$

From Fig. 13.15, with $(L/r_i) = 4$ and $(r_j/L) = 0.25$, $F_{13} \approx 0.05$. Hence

$$E = 5.67\times10^{-8} \mathrm{W/m^2 \cdot K^4} + 0.05\times5.67\times10^{-8} \mathrm{W/m^2 \cdot K^4} \left(1000^4 - 700^4\right) \mathrm{K^4}$$

$$E = 1.36\times10^4 \mathrm{W/m^2} + 0.22\times10^4 \mathrm{W/m^2}$$

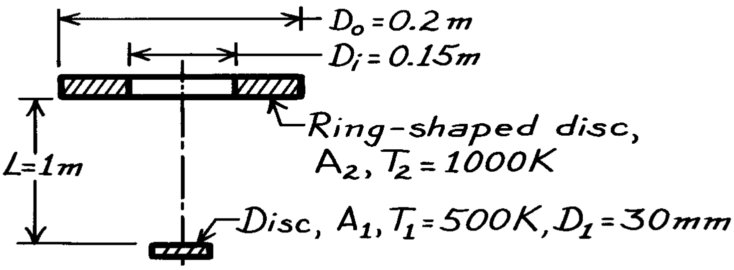
$$E = 1.58\times10^4 \mathrm{W/m^2}.$$ <

## PROBLEM 13.29

**KNOWN:** Aligned, parallel discs with prescribed geometry and orientation.

**FIND:** Net radiative heat exchange between the discs.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Surfaces behave as blackbodies, (2) $A_1 << A_2$.

**ANALYSIS:** The net radiation exchange between the two black surfaces follows from Eq. 13.13 written as

$$q_{12} = A_1 \, F_{12} \, \sigma\left(T_1^4 - T_2^4\right).$$

The view factor can be determined from Eq. 13.8 which is appropriate for a small disc, aligned and parallel to a much larger disc.

$$F_{ij} = \frac{D_j^2}{D_j^2 + 4L^2}$$

where $D_j$ is the diameter of the larger disk and L is the distance of separation. It follows that

$$F_{12} = F_{1o} - F_{1i} = 0.00990 - 0.00559 = 0.00431$$

where

$$F_{1o} = D_o^4 / \left(D_o^2 + 4L^2\right) = 0.2^2 m^2 / \left(0.2^2 m^2 + 4 \times 1 \ m^2\right) = 0.00990$$

$$F_{1i} = D_i / \left(D_i^2 + 4L^2\right) = 0.15^2 m^2 / \left(0.15^2 m^2 + 4 \times 1 \ m^2\right) = 0.00559.$$

The net radiation exchange is then

$$q_{12} = \frac{\pi\,(0.03\text{m})^2}{4} \times 0.00431 \times 5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \cdot \text{K}^4}\left(500^4 - 1000^4\right)\text{K}^4 = -0.162\,\text{W}.$$

**COMMENTS:** $F_{12}$ can be approximated using solid angle concepts if $D_o << L$. That is, the view factor for $A_1$ to $A_o$ (whose diameter is $D_o$) is

$$F_{1o} \approx \frac{\omega_{o-1}}{\pi} = \frac{A_o / L^2}{\pi} = \frac{\pi D_o^2}{4\pi L^2} = \frac{D_o^2}{4L^2}.$$

Numerically, $F_{1o} = 0.0100$ and it follows $F_{1i} \approx D_i^2 / 4L^2 = 0.00563$. This gives $F_{12} = 0.00437$. An analytical expression can be obtained from Ex. 13.1 by replacing the lower limit of integration by $D_i/2$, giving

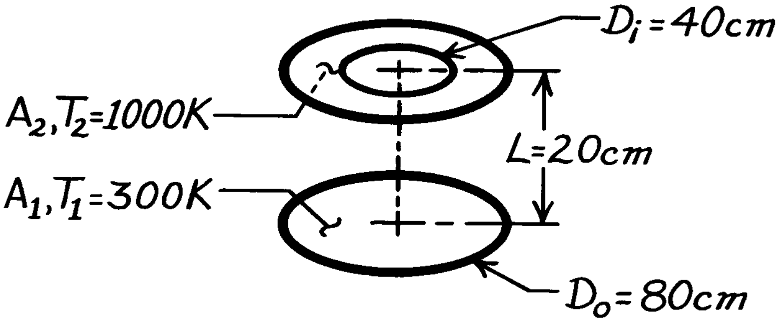
$$F_{12} = L^2\left[-1/\left(D_o^2/4 + L^2\right) + 1/\left(D_i^2/4 + L^2\right)\right] = 0.00431.$$

# PROBLEM 13.30

**KNOWN:** Two black, plane discs, one being solid, the other ring-shaped.

**FIND:** Net radiative heat exchange between the two surfaces.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Discs are parallel and coaxial, (2) Discs are black, diffuse surfaces, (3) Convection effects are not being considered.

**ANALYSIS:** The net radiative heat exchange between the solid disc, $A_1$, and the ring-shaped disc, $A_2$, follows from Eq. 13.13.

$$q_{12} = A_1 F_{12}\,\sigma\left(T_1^4 - T_2^4\right)$$

The view factor $F_{12}$ can be determined from Fig. 13.5 after some manipulation. Define these two hypothetical surfaces;

$$A_3 = \frac{\pi D_o^2}{4},\ \text{located co-planar with } A_2\text{, but a solid surface}$$

$$A_4 = \frac{\pi D_i^2}{4},\ \text{located co-planar with } A_2\text{, representing the missing center.}$$

From view factor relations and Fig. 13.5, it follows that

$$F_{12} = F_{13} - F_{14} = 0.62 - 0.20 = 0.42$$

$F_{14}$: $\quad \frac{r_j}{L} = \frac{40/2}{20} = 1, \qquad \frac{L}{r_i} = \frac{20}{80/2} = 0.5, \qquad F_{14} = 0.20$

$F_{13}$: $\quad \frac{r_j}{L} = \frac{80/2}{20} = 2, \qquad \frac{L}{r_i} = \frac{20}{80/2} = 0.5, \qquad F_{13} = 0.62.$

Hence

$$q_{12} = \left(\pi 0.80^2/4\right)\text{m}^2 \times 0.42 \times 5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(300^4 - 1000^4\right)\text{K}^4$$

$$q_{12} = -11.87\ \text{kW}. \qquad <$$

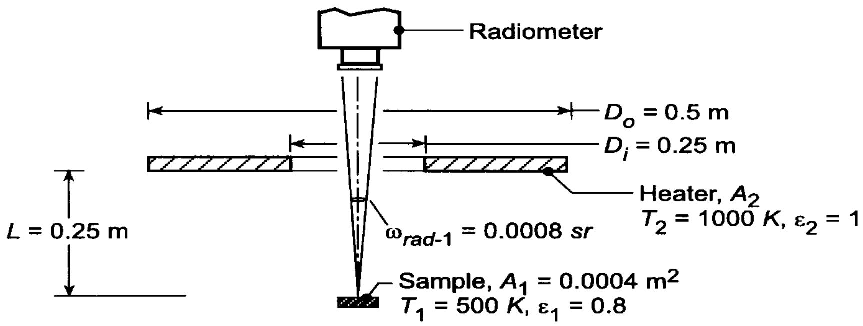
Assuming negligible radiation exchange with the surroundings, the negative sign implies that $q_1$ = -11.87 kW and $q_2$ = +11.87 kW.

**PROBLEM 13.31**

**KNOWN:** Radiometer viewing a small target area (1), $A_1$, with a solid angle $\omega = 0.0008$ sr. Target has an area $A_1 = 0.004\ m^2$ and is diffuse, gray with emissivity $\varepsilon = 0.8$. The target is heated by a ring-shaped disc heater (2) which is black and operates at $T_2 = 1000$ K.

**FIND:** (a) Expression for the radiant power leaving the target which is collected by the radiometer in terms of the target radiosity, $J_1$, and relevant geometric parameters; (b) Expression for the target radiosity in terms of its irradiation, emissive power and appropriate radiative properties; (c) Expression for the irradiation on the target, $G_1$, due to emission from the heater in terms of the heater emissive power, the heater area and an appropriate view factor; numerically evaluate $G_1$; and (d) Determine the radiant power collected by the radiometer using the foregoing expressions and results.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Target is diffuse, gray, (2) Target area is small compared to the square of the separation distance between the sample and the radiometer, and (3) Negligible irradiation from the surroundings onto the target area.

**ANALYSIS:** (a) From Eq. (12.5) with $I_1 = I_{1,e+r} = J_1/\pi$, the radiant power leaving the target collected by the radiometer is

$$q_{1\rightarrow rad} = \frac{J_1}{\pi} A_1 \cos\theta_1 \omega_{rad-1} \quad < \quad (1)$$

where $\theta_1 = 0°$ and $\omega_{rad\text{-}1}$ is the solid angle the radiometer subtends with respect to the target area.

(b) From Eq. 13.16, the radiosity is the sum of the emissive power plus the reflected irradiation.

$$J_1 = E_1 + \rho G_1 = \varepsilon E_{b,1} + (1-\varepsilon) G_1 \quad < \quad (2)$$

where $E_{b1} = \sigma T_1^4$ and $\rho = 1 - \varepsilon$ since the target is diffuse, gray.

(c) The irradiation onto $G_1$ due to emission from the heater area $A_2$ is

$$G_1 = \frac{q_{2\rightarrow 1}}{A_1}$$

where $q_{2\rightarrow 1}$ is the radiant power leaving $A_2$ which is intercepted by $A_1$ and can be written as

$$q_{2\rightarrow 1} = A_2 F_{21} E_{b2} \quad (3)$$

where $E_{b2} = \sigma T_2^4$. $F_{21}$ is the fraction of radiant power leaving $A_2$ which is intercepted by $A_1$. The view factor $F_{12}$ can be written as

Continued …..

$$F_{12} = F_{1-o} \qquad F_{1-i} = 0.5 - 0.2 = 0.3$$

where from Eq. 13.8,

$$F_{1-o} = \frac{D_o^2}{D_o^2 + 4L^2} = \frac{0.5^2}{0.5^2 + 4(0.25)^2} = 0.5 \tag{3}$$

$$F_{1-i} = \frac{D_i^2}{D_i^2 + 4L^2} = \frac{0.25^2}{0.25^2 + 4(0.25)^2} = 0.2$$

and from the reciprocity rule,

$$F_{21} = \frac{A_1 F_{12}}{A_2} = \frac{0.0004\,\mathrm{m}^2 \times 0.3}{\pi/4\left(0.5^2 - 0.25^2\right)\mathrm{m}^2} = 0.000815$$

Substituting numerical values into Eq. (3), find

$$G_1 = \frac{\pi/4\left(0.5^2 - 0.25^2\right)\mathrm{m}^2 \times 0.000815 \times 5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\,(1000\ \mathrm{K})^4}{0.0004\,\mathrm{m}^2}$$

$$G_1 = 17{,}013\,\mathrm{W/m^2}$$ <

(d) Substituting numerical values into Eq. (1), the radiant power leaving the target collected by the radiometer is

$$q_{1\to rad} = \left(6238\,\mathrm{W/m^2}/\pi\,\mathrm{sr}\right)\times 0.0004\,\mathrm{m}^2 \times 1 \times 0.0008\,\mathrm{sr} = 635\,\mu\mathrm{W}$$ <

where the radiosity, $J_1$, is evaluated using Eq. (2) and $G_1$.

$$J_1 = 0.8\times5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\times(500\ \mathrm{K})^4 + (1-0.8)\times17{,}013\,\mathrm{W/m^2}$$

$$J_1 = (2835 + 3403)\,\mathrm{W/m^2} = 6238\,\mathrm{W/m^2}$$ <

**COMMENTS:** (1) Note that the emitted and reflected irradiation components of the radiosity, $J_1$, are of the same magnitude.

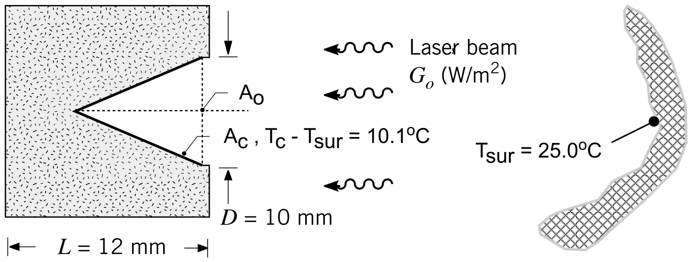
(2) Suppose the surroundings were at room temperature, $T_{sur}$ = 300 K. Would the reflected irradiation due to the surroundings contribute significantly to the radiant power collected by the radiometer? Justify your conclusion.

## PROBLEM 13.32

**KNOWN:** Thin-walled, black conical cavity with opening D = 10 mm and depth of L = 12 mm that is well insulated from its surroundings. Temperature of meter housing and surroundings is 25.0°C.

**FIND:** Optical (radiant) flux of laser beam, $G_o$ (W/m$^2$), incident on the cavity when the fine-wire thermocouple indicates a temperature rise of 10.1°C.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Cavity surface is black and perfectly insulated from its mounting material in the meter, (2) Negligible convection heat transfer from the cavity surface, and (3) Surroundings are large, isothermal.

**ANALYSIS:** Perform an energy balance on the walls of the cavity considering absorption of the laser irradiation, absorption from the surroundings and emission.

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$A_o\, G_o + A_o\, G_{sur} - A_o\, E_b\left(T_c\right) = 0$$

where $A_o = \pi\, D^2/4$ represents the opening of the cavity. All of the radiation entering or leaving the cavity passes through this hypothetical surface. Hence, we can treat the cavity as a black disk at $T_c$. Since $G_{sur} = E_b\,(T_{sur})$, and $E_b = \sigma\, T^4$ with $\sigma = 5.67 \times 10^{-8}$ W/m$^2\cdot$K$^4$, the energy balance has the form

$$G_o + \sigma(25.0 + 273)^4 K^4 - \sigma(25.0 + 10.1 + 273)^4 K^4 = 0$$

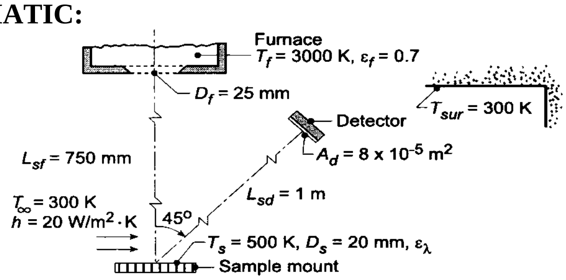
$$G_o = 63.8\ W/m^2 \qquad <$$

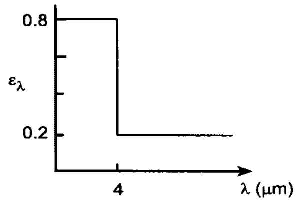
## PROBLEM 13.33

**KNOWN:** Electrically heated sample maintained at $T_s$ = 500 K with diffuse, spectrally selective coating. Sample is irradiated by a furnace located coaxial to the sample at a prescribed distance. Furnace has isothermal walls at $T_f$ = 3000 K with $\varepsilon_f$ = 0.7 and an aperture of 25 mm diameter. Sample experiences convection with ambient air at $T_\infty$ = 300 K and h = 20 W/m$^2$·K. The surroundings of the sample are large with a uniform temperature $T_{sur}$ = 300 K. A radiation detector sensitive to only power in the spectral region 3 to 5 μm is positioned at a prescribed location relative to the sample.

**FIND:** (a) Electrical power, $P_e$, required to maintain the sample at $T_s$ = 500 K, and (b) Radiant power incident on the detector within the spectral region 3 to 5 μm considering both emission and reflected irradiation from the sample.

**SCHEMATIC:**



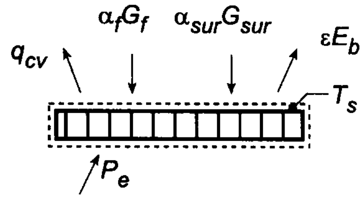




**ASSUMPTIONS:** (1) Steady-state condition, (2) Furnace is large, isothermal enclosure with small aperture and radiates as a blackbody, (3) Sample coating is diffuse, spectrally selective, (4) Sample and detector areas are small compared to their separation distance squared, (5) Surroundings are large, isothermal.

**ANALYSIS:** (a) Perform an energy balance on the sample mount, which experiences electrical power dissipation, convection with ambient air, absorbed irradiation from the furnace, absorbed irradiation from the surroundings and emission,

$$\dot{E}'_{in} - \dot{E}'_{out} = 0$$

$$P_e + \left[-h\left(T_s - T_\infty\right) + \alpha_1 G_f + \alpha_{sur} G_{sur} - \varepsilon E_b\left(T_s\right)\right] A_s = 0 \quad (1)$$




where $E_b\left(T_s\right) = \sigma T_s^4$ and $A_s = \pi D_s^2 / 4$.

*Irradiations on the sample*: The irradiation from the furnace aperture onto the sample can be written as

$$G_f = \frac{q_{f \to s}}{A_s} = \frac{A_f F_{fs} E_{b,f}}{A_s} = \frac{A_f F_{fs} \sigma T_f^4}{A_s} \quad (2)$$

where $A_f = \pi D_f^2 / 4$ and $A_s = \pi D_s^2 / 4$. The view factor between the furnace aperture and sample follows from the relation for coaxial parallel disks, Table 13.2,

$R_f = r_f / L_{sf} = 0.0125\ \text{m} / 0.750\ \text{m} = 0.01667$ $\qquad$ $R_s = r_s / L_{sf} = 0.0100\ \text{m} / 0.750\ \text{m} = 0.01333$

$$S = 1 + \frac{1 + R_s^2}{R_f^2} = 1 + \frac{1 + 0.01333^2}{0.01667^2} = 3600.2$$

Continued …..

## PROBLEM 13.33 (Cont.)

$$F_{sf} = 0.5\left\{S - \left[S^2 - 4\left(r_s / r_f\right)^2\right]^{1/2}\right\} = 0.5\left\{3600 - \left[3600^2 - 4\left(0.05 / 0.0625\right)^2\right]^{1/2}\right\} = 0.000178$$

Hence the irradiation from the furnace is

$$G_f = \frac{\pi\left(0.025\ \text{m}\right)^2 / 4 \times 0.000178 \times 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4\left(3000\ \text{K}\right)^4}{\pi\left(0.020^2\ \text{m}^2 / 4\right)} = 1277\,\text{W/m}^2$$

The irradiation from the surroundings which are large compared to the sample is

$$G_{sur} = \sigma T_{sur}^4 = 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}\left(300\text{K}\right)^4 = 459\,\text{W/m}^2$$

*Emissivity of the Sample:* The total hemispherical emissivity in terms of the spectral distribution can be written following Eq. 12.38 and Eq. 12.30,

$$\varepsilon = \int_0^\infty \varepsilon_\lambda E_{\lambda,b}\left(T_s\right) d\lambda / \sigma T^4 = \varepsilon_1 F_{\left(0-\lambda_1 T_s\right)} + \varepsilon_2\left[1 - F_{\left(0-\lambda_1 T_s\right)}\right]$$

$$\varepsilon = 0.8 \times 0.066728 + 0.2\left[1 - 0.066728\right] = 0.240$$

where, from Table 12.1, with $\lambda_1 T_s = 4\ \mu\text{m} \times 500\ \text{K} = 2000\ \mu\text{m} \cdot \text{K},\ F_{\left(0-\lambda T\right)} = 0.066728.$

*Absorptivity of the Sample:* The total hemispherical absorptivity due to irradiation from the furnace follows from Eq. 12.46,

$$\alpha_f = \varepsilon_1 F_{\left(0-\lambda_1 T_f\right)} + \varepsilon_2\left[1 - F_{\left(0-\lambda_1 T_f\right)}\right] = 0.8 \times 0.945098 + 0.2\left[1 - 0.945098\right] = 0.767$$

where, from Table 12.1, with $\lambda_1 T_f = 4\,\mu\text{m} \times 3000\ \text{K} = 12,000\ \mu\text{m} \cdot \text{K},\ F_{\left(0-\lambda T\right)} = 0.945098.$ The total hemispherical absorptivity due to irradiation from the surroundings is

$$\alpha_{sur} = \varepsilon_1 F_{\left(0-\lambda_1 T_{sur}\right)} + \varepsilon_2\left[1 - F_{\left(0-\lambda_1 T_{sur}\right)}\right] = 0.8 \times 0.00234 + 0.2\left[1 - 0.002134\right] = 0.201$$

where, from Table 12.1, with $\lambda_1 T_{sur} = 4\ \mu\text{m} \times 300\ \text{K} = 1200\ \mu\text{m} \cdot \text{K}, F_{\left(0-\lambda T\right)} = 0.002134.$

*Evaluating the Energy Balance:* Substituting numerical values into Eq. (1),

$$P_e = \left[+20\,\text{W/m}^2 \cdot \text{K}\left(500 - 300\right)\text{K} - 0.767 \times 1277\,\text{W/m}^2\right.$$

$$\left. - 0.201 \times 459\,\text{W/m}^2 + 0.240 \times 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4\left(500\ \text{K}\right)^4\right]\pi\left(0.020\ \text{m}\right)^2 / 4$$

$$P_e = 1.256\ \text{W} - 0.308\ \text{W} - 0.029\ \text{W} + 0.267\ \text{W} = 1.19\ \text{W}$$ <

(b) The radiant power leaving the sample which is incident on the detector and within the spectral region, $\Delta\lambda$ = 3 to 5μm, follows from Eq. 12.5 with Eq. 12.30,

$$q_{s-d,\Delta\lambda} = \left[E_{s,\Delta\lambda} + G_{f,ref,\Delta\lambda} + G_{sur,ref,\Delta\lambda}\right]\left(1/\pi\right) A_s \cos\theta_s \cdot A_d \cos\theta_d / L_{sd}^2$$

where $\theta_s = 45°$ and $\theta_d = 0°$. The *emitted* component is

$$E_{s,\Delta\lambda} = \int_3^{5\mu m} \varepsilon_{\lambda,b} E_{\lambda,b}\left(T_s\right)$$

$$E_{s,\Delta\lambda} = \left\{\varepsilon_1\left[F_{\left(0-4\mu\text{m},T_s\right)} - F_{\left(0-3\mu\text{m},T_s\right)}\right] + \varepsilon_2\left[F_{\left(0-5\mu\text{m},T_s\right)} - F_{\left(0-4\mu\text{m},T_s\right)}\right]\right\}\sigma T_s^4$$

Continued …..

$$E_{s,\Delta\lambda} = \{0.8[0.066728 - 0.013754] + 0.2[0.16169 - 0.066728]\}\sigma(500K)^4 = 217.5\,W/m^2$$

where, from Table 12.1, $F_{(0-3\mu m,T_s)} = 0.013754$ at $\lambda T = 3\,\mu m \times 500\text{ K} = 1500\ \mu m\cdot K$; $F_{(0-4\mu m,T_s)} = 0.066728$ at $\lambda = 4\,\mu m \times 500\text{ K} = 2000\ \mu m\cdot K$; and $F_{(0-5\mu m,T_s)} = 0.16169$ at $\lambda T = 5\mu m \times 500\text{ K} = 2500\ \mu m\cdot K$.

The *reflected irradiation from the furnace* component is

$$G_{f,ref,\Delta\lambda} = \int_3^{5\mu m}(1-\varepsilon_\lambda)G_{f,\lambda}d\lambda$$

where $G_{f,\lambda} \approx E_{\lambda,b}(T_f)$, using band emission factors,

$$G_{f,ref,\Delta\lambda} = \left\{(1-\varepsilon)\left[F_{(0-4\mu m,T_f)} - F_{(0-3\mu m,T_f)}\right] + (1-\varepsilon_2)\left[F_{(0-5\mu m,T_f)} - F_{(0-4\mu m,T_f)}\right]\right\}G_f$$

$$G_{f,ref,\Delta\lambda} = \{0.2[0.9451 - 0.8900] + 0.8[0.9700 - 0.9451]\}1277\,W/m^2 = 39.51\,W/m^2$$

where, from Table 12.1, $F_{(0-3\mu m,T_f)} = 0.8900$ at $\lambda T_f = 3\ \mu m \times 3000\text{ K} = 9000\ \mu m\cdot K$; $F_{(0-4\mu m,T_f)} = 0.9451$ at $\lambda T_f = 4\ \mu m \times 3000\text{ K} = 12{,}000\ \mu m\cdot K$; and, $F_{(0-5\mu m,T_f)} = 0.9700$ at $\lambda T_f = 5\ \mu m \times 3000\text{ K} = 15{,}000\ \mu m\cdot K$.

The *reflected irradiation from the surroundings* component is

$$G_{sur,ref,\Delta\lambda} = \int_3^{5\mu m}(1-\varepsilon_\lambda)G_{ref,\lambda}d\lambda$$

where $G_{ref,\lambda} \approx E_\lambda(T_{sur})$, using band emission factors,

$$G_{sur,ref,\Delta\lambda} = \left\{(1-\varepsilon_1)\left[F_{(0-4\mu m,T_{sur})} - F_{(0-3\mu m,T_{sur})}\right]\right.$$
$$\left. + (1-\varepsilon_2)\left[F_{(0-5\mu m,T_{sur})} - F_{(0-4\mu m,T_{sur})}\right]\right\}G_{sur}$$

$$G_{sur,ref,\Delta\lambda} = \{0.2[0.002134 - 0.0001685] - 0.8[0.013754 - 0.002134]\}459\,W/m^2 = 4.44\,W/m^2$$

where, from Table 12.1, $F_{(0-3\mu m,T_{sur})} = 0.0001685$ at $\lambda T_{sur} = 3\ \mu m \times 300\text{ K} = 900\ \mu m\cdot K$; $F_{(0-4\mu m,T_{sur})} = 0.002134$ at $\lambda T_{sur} = 4\ \mu m \times 300\text{ K} = 1200\ \mu m\cdot K$; and $F_{(0-5\mu m,T_{sur})} = 0.013754$ at $\lambda T_{sur} = 5\ \mu m \times 300\text{ K} = 1500\ \mu m\cdot K$. Returning to Eq. (3), find

$$q_{sd,\Delta\lambda} = [217.5 + 39.51 + 4.44]W/m^2(1/\pi)\left[8\pi(0.020\text{ m})^2/4\right]$$
$$\cos 45° \times 8 \times 10^{-5}m^2 \times \cos 0°/(1\text{ m})^2 = 1.48\ \mu W \qquad <$$

**COMMENTS:** (1) Note that $F_{fs}$ is small, since $A_f, A_s << L_{sf}^2$. As such, we could have evaluated $q_{f\to s}$ using Eq. 12.5 and found

$$G_f = \frac{E_{b,f}/\pi A_f\left(A_s/L_{sf}^2\right)}{A_s} = 1276\,W/m^2$$

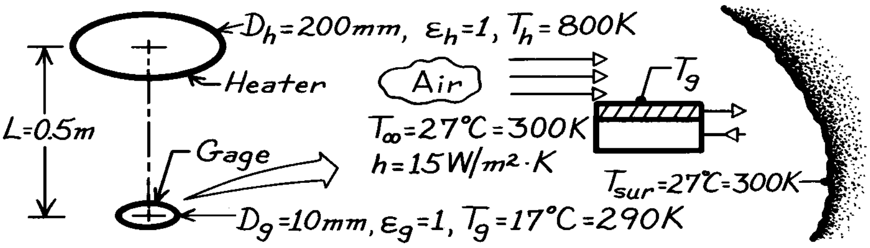
(2) Recognize in the analysis for part (b), Eq. (3), the role of the band emission factors in calculating the fraction of total radiant power for the emitted and reflected irradiation components.

# PROBLEM 13.34

**KNOWN:** Water-cooled heat flux gage exposed to radiant source, convection process and surroundings.

**FIND:** (a) Net radiation exchange between heater and gage, (b) Net transfer of radiation to the gauge per unit area of the gage, (c) Net heat transfer to the gage per unit area of gage, (d) Heat flux indicated by gage described in Problem 3.98.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Heater and gauge are parallel, coaxial discs having blackbody behavior, (2) $A_g << A_h$, (3) Surroundings are large compared to $A_h$ and $A_g$.

**ANALYSIS:** (a) The net radiation exchange between the heater and the gage, both with blackbody behavior, is given by Eq. 13.13 having the form

$$q_{h-g} = A_h F_{hg} \sigma \left(T_h^4 - T_g^4\right) = A_g F_{gh} \sigma \left(T_h^4 - T_g\right).$$

Note the use of reciprocity, Eq. 13.3, for the view factors. From Eq. 13.8,

$$F_{gh} = D_h^2 / \left(4L^2 + D_h^2\right) = (0.2m)^2 / \left(4 \times 0.5^2 m^2 + 0.2^2 m^2\right) = 0.0385.$$

$$q_{h-g} = \left(\pi 0.01^2 m^2 / 4\right) \times 0.0385 \times 5.67 \times 10^{-8} W/m^2 \cdot K^4 \left[800^4 - 290^4\right] K^4 = 69.0 \text{ mW}. \qquad <$$

(b) The net radiation *to* the gage per unit area will involve exchange with the heater and the surroundings. Using Eq. 13.14,

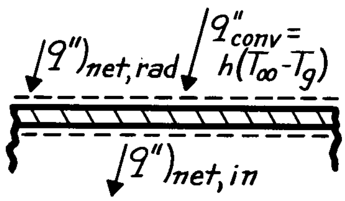
$$q''_{net,rad} = -q_g / A_g = q_{h-g} / A_g + q_{sur-g} / A_g.$$

The net exchange with the surroundings is

$$q_{sur-g} = A_{sur} F_{sur-g} \sigma \left(T_{sur}^4 - T_g 4\right) = A_g F_{g-sur} \sigma \left(T_{sur}^4 - T_g^4\right).$$

$$q''_{net,rad} = \frac{69.0 \times 10^{-3} W}{\pi (0.01m)^2 / 4} + (1 - 0.0385) 5.67 \times 10^{-8} W/m^2 \cdot K^4 \left(300^4 - 290^4\right) K^4 = 934.5 W/m^2. \qquad <$$

(c) The net heat transfer rate to the gage per unit area of the gage follows from the surface energy balance




$$q''_{net,in} = q'')_{net,rad} + q''_{conv}$$

$$q''_{net,in} = 934.5 W/m^2 + 15 W/m^2 \cdot K (300 - 290) K$$

$$q''_{net,in} = 1085 W/m^2. \qquad <$$

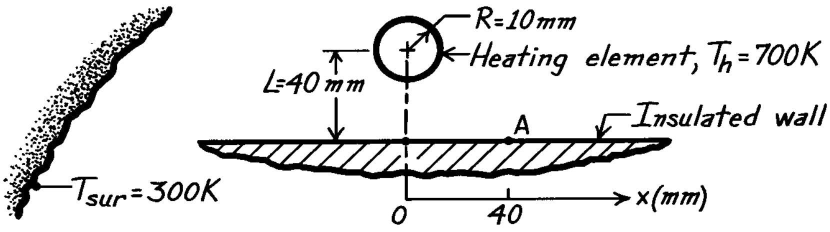
(d) The heat flux gage described in Problem 3.98 would experience a net heat flux to the surface of 1085 W/m$^2$. The irradiation to the gage from the heater is $G_g = q_{h \to g}/A_g = F_{gh} \sigma T_h^4 = 894$ W/m$^2$. Since the gage responds to net heat flux, there would be a systematic error in sensing irradiation from the heater.

## PROBLEM 13.35

**KNOWN:** Long cylindrical heating element located a given distance above an insulated wall exposed to cool surroundings.

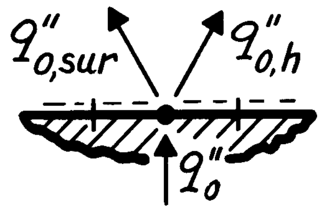
**FIND:** Maximum temperature attained by the wall and temperature at location A.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Insulated wall, (3) Negligible conduction in wall, (4) All surfaces are black.

**ANALYSIS:** Consider an elemental area at point x = 0; this is the location that will attain the maximum temperature. Since the wall is insulated and conduction is negligible, the net radiation leaving $dA_o$ is zero. From Eq. 13.13,




$$q''_o = q''_{o,h} + q''_{o,sur} = F_{o,h}\,\sigma\left(T_o^2 - T_h^4\right) + F_{o,sur}\left(T_o^5 - T_{sur}^4\right) = 0 \tag{1}$$

where $F_{o,sur} = 1 - F_{o,h}$ and $F_{o,h}$ can be found from the relation for a cylinder and parallel rectangle, Table 13.1, with $s_1$ = 2 mm, $s_2$ = 0 mm, L = 40 mm, and r = R = 10 mm.

$$F_{o,h} = \frac{r}{s_1 - s_2}\left[\tan^{-1}\frac{s_1}{L} - \tan^{-1}\frac{s_2}{L}\right] = \frac{10\,\text{mm}}{2\,\text{mm} - 0}\left[\tan^{-1}\frac{2}{40} - \tan^{-1}0\right] = 0.25 \tag{2}$$

Rearranging Eq. (1) and substituting numerical values, find

$$T_o^4 = \left[T_h^4 + \frac{\left(1 - F_{o,h}\right)}{F_{o,h}}T_{sur}^4\right] \Big/ \left[1 + \frac{1 - F_{o,h}}{F_{o,h}}\right] \tag{3}$$

$$T_o^4 = \left[(700\text{ K})^4 + \frac{1 - 0.25}{0.25}(300\text{ K})^4\right] \Big/ \left[1 + \frac{1 - 0.25}{0.25}\right] \qquad T_o = 507\text{ K.}$$ <

For the point A located at x = 40 mm, use the same relation of Table 13.1 to find $F_{A,h}$ (for this point, $s_1$ = 41 mm, $s_2$ = 39 mm, r = R = 10 mm, L = 40 mm),

$$F_{A,h} = \frac{10\text{ mm}}{(41 - 39)\text{mm}}\left[\tan^{-1}\frac{41}{40} - \tan^{-1}\frac{39}{40}\right] = 0.125.$$

Substituting numerical values into Eq. (3), find

$$T_A^4 = \left[(700\text{ K})^4 + \frac{1 - 0.125}{0.125}(300\text{ K})^4\right] \Big/ \left[1 + \frac{1 - 0.125}{0.125}\right] \quad T_A = 439\text{ K.}$$ <

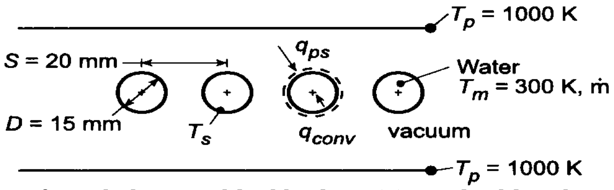
**COMMENTS:** Note the importance of the assumptions that the wall is insulated and conduction is negligible. In calculating $F_{o,h}$ and $F_{A,h}$ we are finding the view factor for a small area or point. Hence, we need only specify that $s_1 - s_2$ is very small compared to L.

## PROBLEM 13.36

**KNOWN:** Diameter and pitch of in-line tubes occupying evacuated space between parallel plates of prescribed temperature. Temperature and flowrate $\dot{m}$ of water through the tubes.

**FIND:** (a) Tube surface temperature $T_s$ for $\dot{m}$ = 0.20 kg/s, (b) Effect of $\dot{m}$ on $T_s$.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Surfaces behave as blackbodies, (2) Negligible tube wall conduction resistance, (3) Fully-developed tube flow.

**PROPERTIES:** *Table A-6*, water ($T_m$ = 300 K): $\mu = 855 \times 10^{-6}$ N·s/m$^2$, k = 0.613 W/m·K, Pr = 5.83.

**ANALYSIS:** (a) Performing an energy balance on a single tube, it follows that $q_{ps} = q_{conv}$, or

$$A_p F_{ps} \sigma \left(T_p^4 - T_s^4\right) = h A_s \left(T_s - T_m\right)$$

From Table 13.1 and D/S = 0.75, the view factor is

$$F_{ps} = 1 - \left[1 - \left(\frac{D}{S}\right)^2\right]^{1/2} + \left(\frac{D}{S}\right) \tan^{-1}\left(\frac{S^2 - D^2}{D^2}\right)^{1/2} = 0.881$$

With $Re_D = 4\dot{m}/\pi D \mu = 4\left(0.20 \text{ kg/s}\right)/\pi\left(0.015\,\text{m}\right)855 \times 10^{-6}\,\text{N}\cdot\text{s/m}^2 = 19{,}856$, fully-developed turbulent flow may be assumed, in which case Eq. 8.60 yields

$$h = \frac{k}{D}\left(0.023\,Re_D^{4/5}\,Pr^{0.4}\right) = \frac{0.613\ \text{W/m}\cdot\text{K}}{0.015\,\text{m}}\left(0.023\right)\left(19{,}856\right)^{4/5}\left(5.83\right)^{0.4} = 5220\,\text{W/m}^2\cdot\text{K}$$

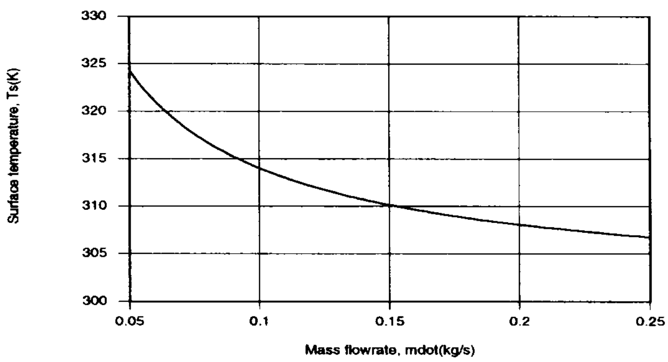
Hence, with $(A_p/A_s) = 2S/\pi D = 0.849$,

$$T_s - T_m = \frac{F_{ps}\sigma}{h}\frac{A_p}{A_s}\left(T_p^4 - T_s^4\right) = \frac{0.881 \times 5.67 \times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4}{5220\ \text{W/m}^2\cdot\text{K}}\left(0.849\right)\left(T_p^4 - T_s^4\right)$$

With $T_m$ = 300 K and $T_p$ = 1000 K, a trial-and-error solution yields

$$T_s = 308\ \text{K}$$ <

(b) Using the *Correlations and Radiation* Toolpads of *IHT* to evaluate the convection coefficient and view factor, respectively, the following results were obtained.




The decrease in $T_s$ with increasing $\dot{m}$ is due to an increase in h and hence a reduction in the convection resistance.

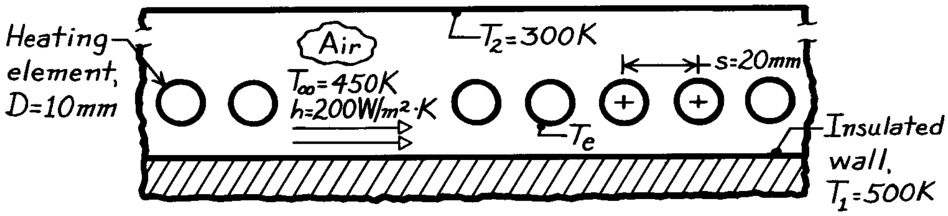
**COMMENTS:** Due to the large value of h, $T_s << T_p$.

## PROBLEM 13.37

**KNOWN:** Insulated wall exposed to a row of regularly spaced cylindrical heating elements.

**FIND:** Required operating temperature of the heating elements for the prescribed conditions.

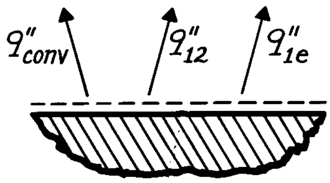
**SCHEMATIC:**




**ASSUMPTIONS:** (1) Upper and lower walls are isothermal and infinite, (2) Lower wall is insulated, (3) All surfaces are black, (4) Steady-state conditions.

**ANALYSIS:** Perform an energy balance on the insulated wall considering convection and radiation.

$$\dot{E}''_{in} - \dot{E}''_{out} = -q''_1 - q''_{conv} = 0$$




where $q''_1$ is the net radiation leaving the insulated wall per unit area. From Eq. 13.13,

$$q''_1 = q''_{1e} + q''_{12} = F_{1e}\sigma\left(T_1^4 - T_e^4\right) + F_{12}\sigma\left(T_1^4 - T_2^4\right)$$

where $F_{12} = 1 - F_{1e}$. Using Newton's law of cooling for $q''_{conv}$ solve for $T_e$,

$$T_e^4 = \left[T_1^4 + \frac{(1-F_{1e})}{F_{1e}}\left(T_1^4 - T_2^4\right)\right] + \frac{h}{\sigma}\frac{1}{F_{1e}}\left(T_1 - T_\infty\right).$$

The view factor between the insulated wall and the tube row follows from the relation for an infinite plane and row of cylinders, Table 13.1,

$$F_{1e} = 1 - \left[1 - \left(\frac{D}{S}\right)^2\right]^{1/2} + \left(\frac{D}{S}\right)\tan^{-1}\left(\frac{s^2 - D^2}{D^2}\right)^{1/2}$$

$$F_{1e} = 1 - \left[1 - \left(\frac{10}{20}\right)^2\right]^{1/2} + \left(\frac{10}{20}\right)\tan^{-1}\left(\frac{20^2 - 10^2}{10^2}\right)^{1/2} = 0.658.$$

Substituting numerical values, find

$$T_e^4 = \left[(500\ \text{K})^4 + \frac{1-0.658}{0.658}\left(500^4 - 300^4\right)\text{K}^4\right] + \frac{200\ \text{W/m}^2\cdot\text{K}}{5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4}\times\frac{1}{0.658}\left(500-450\right)\text{K}$$

$$T_e = 774\ \text{K}.$$ <

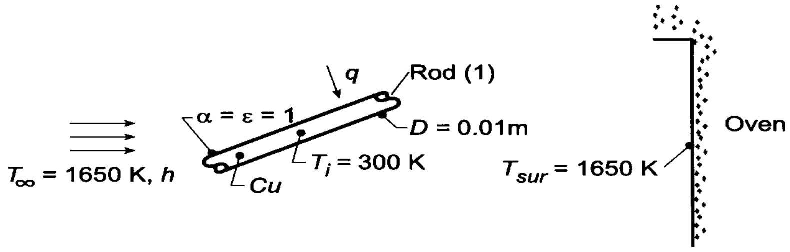
**COMMENTS:** Always express temperatures in kelvins when considering convection and radiation terms in an energy balance. Why is $F_{1e}$ independent of the distance between the row of tubes and the wall:

## PROBLEM 13.38

**KNOWN:** Surface radiative properties, diameter and initial temperature of a copper rod placed in an evacuated oven of prescribed surface temperature.

**FIND:** (a) Initial heating rate, (b) Time $t_h$ required to heat rod to 1000 K, (c) Effect of convection on heating time.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Copper may be treated as a lumped capacitance, (b) Radiation exchange between rod and oven may be approximated as blackbody exchange.

**PROPERTIES:** *Table A-1*, Copper (300 K): $\rho$ = 8933 kg/m$^3$, $c_p$ = 385 J/kg·K, k = 401 W/m·K.

**ANALYSIS:** (a) Performing an energy balance on a unit length of the rod, $\dot{E}_{in} = \dot{E}_{st}$, or

$$q = Mc_p \frac{dT}{dt} = \rho\left(\frac{\pi D^2}{4}\times 1\right)c_p \frac{dT}{dt}$$

Neglecting convection, $q = q_{rad} = A_2\, F_{21}\, \sigma\left(T_{sur}^4 - T^4\right) = A_1\, F_{12}\, \sigma\left(T_{sur}^4 - T^4\right)$, where $A_1 = \pi D \times 1$ and $F_{12} = 1$. It follows that

$$\frac{dT}{dt} = \frac{\sigma\pi D\left(T_{sur}^4 - T^4\right)}{\rho\left(\pi D^2/4\right)c_p} = \frac{4\sigma\left(T_{sur}^4 - T^4\right)}{\rho D c_p} \tag{1}$$

$$\left.\frac{dT}{dt}\right)_i = \frac{4\left[(1650\ \text{K})^4 - (300\ \text{K})^4\right]5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4}{8933\ \text{kg/m}^3\,(0.01\,\text{m})\,385\,\text{J/kg}\cdot\text{K}} = 48.8\ \text{K/s}. \qquad <$$

(b) Using the *IHT Lumped Capacitance Model* to numerically integrate Eq. (2), we obtain

$$t_s = 15.0\ \text{s} \qquad <$$

(c) With convection, $q = q_{rad} + q_{conv} = A_1\, F_{12}\, \sigma\left(T_{sur}^4 - T^4\right) + hA_1\,(T_\infty - T)$, and the energy balance becomes

$$\frac{dT}{dt} = \frac{4\sigma\left(T_{sur}^4 - T^4\right)}{\rho D c_p} + \frac{4h\left(T_\infty - T\right)}{\rho D c_p}$$

Performing the numerical integration for the three values of h, we obtain

| | | | |
|---|---|---|---|
| h (W/m$^2$·K): | 10 | 100 | 500 |
| $t_h$ (s): | 14.6 | 12.0 | 6.8 |

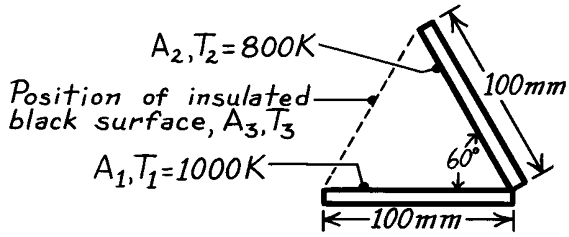
**COMMENTS:** With an initial value of $h_{rad,i} = \sigma\left(T_{sur}^4 - T^4\right)/(T_{sur} - T)$ = 311 W/m$^2$·K, Bi = $h_{rad}$ (D/4)/k = 0.002 and the lumped capacitance assumption is justified for parts (a) and (b). With h = 500 W/m$^2$·K and h + $h_{r,i}$ = 811 W/m·K in part (c), Bi = 0.005 and the lumped capacitance approximation is also valid.

## PROBLEM 13.39

**KNOWN:** Long, inclined black surfaces maintained at prescribed temperatures.

**FIND:** (a) Net radiation exchange between the two surfaces per unit length, (b) Net radiation transfer to surface $A_2$ with black, insulated surface positioned as shown below; determine temperature of this surface.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Surfaces behave as blackbodies, (2) Surfaces are very long in direction normal to page.

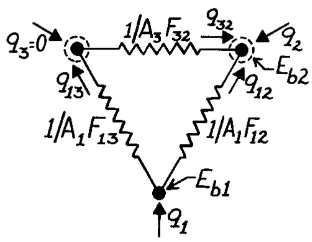
**ANALYSIS:** (a) The net radiation exchange between two black surfaces is given by Eq. 13.13,

$$q_{12} = A_1F_{12}\sigma\left(T_1^4 - T_2^4\right)$$

Noting that $A_1$ = width×length $(\ell)$ and that from symmetry, $F_{12} = 0.5$, find

$$q'_{12} = \frac{q_{12}}{\ell} = 0.1\text{ m}\times 0.5\times 5.67\times 10^{-8}\text{W/m}^2\cdot\text{K}^4\left(1000^4 - 800^4\right)\text{K}^4 = 1680\text{ W/m}. \qquad <$$

(b) With the insulated, black surface $A_3$ positioned as shown above, a three-surface enclosure is formed. From an energy balance on the node representing $A_2$, find




$$-q'_2 = q'_{32} + q'_{12}$$

$$-q_2 = A_3F_{32}\left[E_{b3} - E_{b2}\right] + A_1F_{12}\left[E_{b1} - E_{b2}\right].$$

To find $E_{b3}$, which at present is not known, perform an energy balance on the node representing $A_3$. Note that $A_3$ is adiabatic and, hence $q_3 = 0$, $q_{13} = q_{32}$.

$$A_1F_{13}\left[E_{b1} - E_{b3}\right] = A_3F_{32}\left[E_{b3} - E_{b2}\right]$$

Since $F_{13} = F_{23} = 0.5$ and $A_1 = A_3$, it follows that

$$E_{b3} = (1/2)\left[E_{b1} + E_{b2}\right]$$

and

$$-q'_2 = (A_3/\ell)F_{32}\left[(E_{b1} + E_{b2})/2 - E_{b2}\right] + q'_{12}$$

$$-q'_2 = 0.1\text{ m}\times 0.5\times 5.67\times 10^{-8}\text{W/m}^2\cdot\text{K}^4\left[\left(1000^4 + 800^4\right)/2 - 800^4\right]\text{K}^4$$

$$+1680\text{ W/m} = 2517\text{ W/m} \qquad <$$

Noting that $E_{b3} = \sigma T_3^4 = (1/2)\,[E_{b1} + E_{b2}]$, it follows that

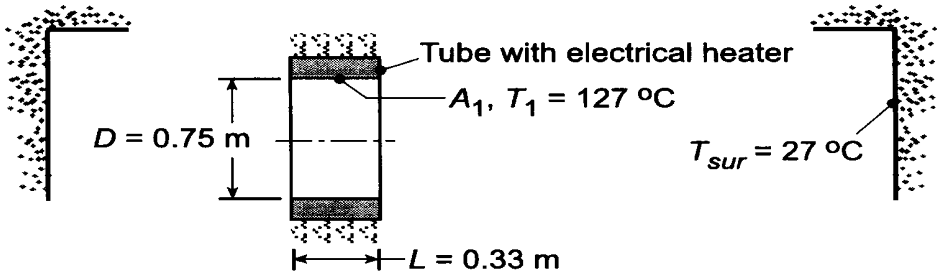
$$T_3 = \left[\left(T_1^4 + T_2^4\right)/2\right]^{1/4} = \left[\left(1000^4 + 800^4\right)/2\right]^{1/4}\text{ K} = 916\text{ K}. \qquad <$$

## PROBLEM 13.40

**KNOWN:** Electrically heated tube suspended in a large vacuum chamber.

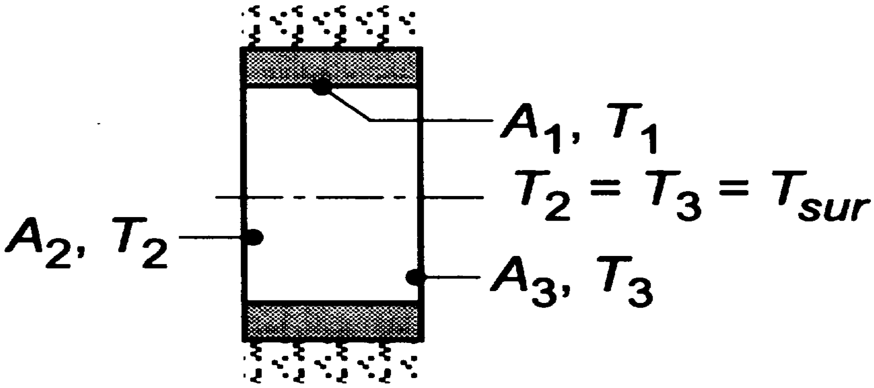
**FIND:** (a) Electrical power supplied to the heater, $P_e$, to maintain it at $T_1$ = 127°C, and (b) Compute and plot $P_e$ as a function of tube length L for the range 25 ≤ L ≤ 250 mm for tube temperatures of $T_1$ = 127, 177 and 227°C.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) All surfaces are blackbodies, (2) Tube of area $A_1$ is isothermal, (3) The surroundings are very large compared to the tube.

**ANALYSIS:** (a) Recognize that the surroundings can be represented by the surfaces $A_2$ and $A_3$, which are blackbodies at $T_{sur}$. This situation then permits calculation of necessary shape factors.




The net radiative heat rate from the heater, surface $A_1$, follows from Eq. 13.14, with $T_2 = T_3 = T_{sur}$ as

$$P_e = q_1 = A_1F_{12}\sigma\left(T_1^4 - T_2^4\right) + A_1F_{13}\sigma\left(T_1^4 - T_3^4\right). \quad (1)$$

Note that $F_{12} = F_{13}$ from symmetry considerations. Write now the summation rule for surface $A_2$

$$F_{21} + F_{22} + F_{23} = 1 \qquad \text{or} \qquad F_{21} = 1 - F_{23}$$

where $F_{23}$ is determined from Fig. 13.5 using

$$\frac{r_j}{L} = \frac{0.75/2}{0.33} = 1.14 \qquad\qquad \frac{L}{r_i} = \frac{0.33}{0.75/2} = 0.88$$

giving $F_{23}$ = 0.37. Hence $F_{21}$ = 1 – 0.37 = 0.63. Using reciprocity, find now $F_{12}$

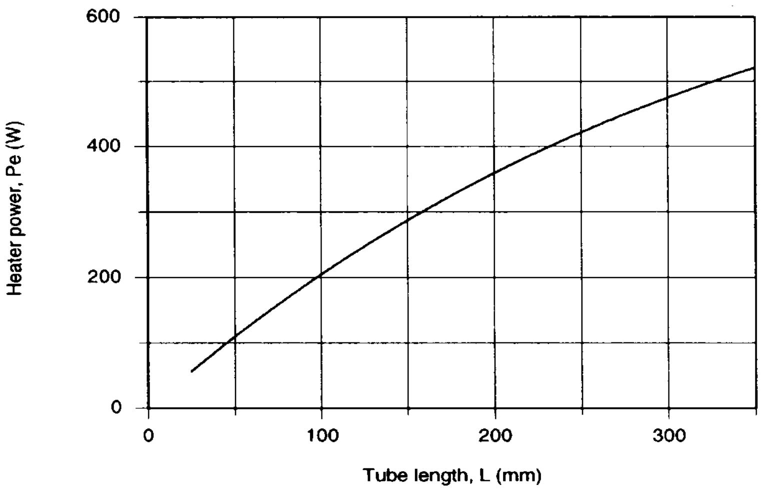
$$F_{12} = \frac{A_2}{A_1}F_{21} = \frac{\pi D^2/4}{\pi DL} \times F_{21} = \frac{\pi\left(75^2/4\right)}{\pi\left(75\times 33\right)} \times 0.63 = 0.36.$$

Noting that $F_{12} = F_{13}$ and that $T_2 = T_3$, the electrical power using Eq. (1) with numerical values can be written as,

Continued …..

$$P_e = 2\left[\pi \times 0.75\text{m} \times 0.33\text{m} \times 0.36 \times 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4 \left[(127+273)^4 - (27+273)^4\right]\text{K}^4 = 555\,\text{W}.\right.$$ <

(b) Using the energy balance Eq. (1) of the foregoing analysis with the *IHT Radiation Tool-View Factors, Coaxial parallel disks*, $P_e$ was computed as a function of L for selected tube temperatures.



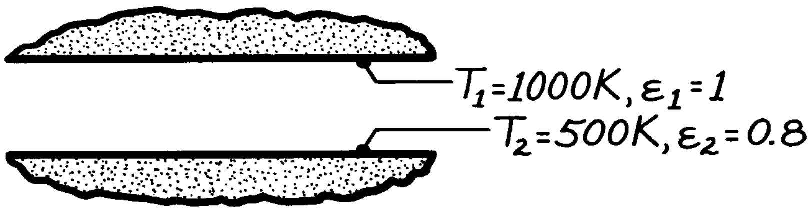

As the tube length L increases, the heater power $P_e$ required to maintain the tube at $T_1$ increases. Note that for small values of L, say L < 100 mm, P is linear with L. For larger values of L, $P_e$ is not linear with L. Why is this so? What is the relationship between $P_e$ and L for L >> 300 mm?

**PROBLEM 13.41**

**KNOWN:** Two horizontal, very large parallel plates with prescribed surface conditions and temperatures.

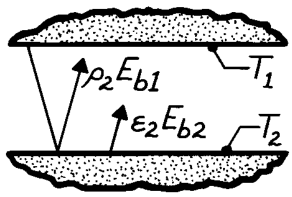
**FIND:** (a) Irradiation to the top plate, $G_1$, (b) Radiosity of the top plate, $J_1$, (c) Radiosity of the lower plate, $J_2$, (d) Net radiative exchange between the plates per unit area of the plates.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Plates are sufficiently large to form a two surface enclosure and (2) Surfaces are diffuse-gray.

**ANALYSIS:** (a) The irradiation to the upper plate is defined as the radiant flux incident on that surface. The irradiation to the upper plate $G_1$ is comprised of flux emitted by surface 2 and reflected flux emitted by surface 1.




$$G_1 = \varepsilon_2 E_{b2} + \rho_2 E_{b1} = \varepsilon_2 \sigma T_2^4 + (1-\varepsilon_2)\sigma T_1^4$$

$$G_1 = 0.8 \times 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4 (1000\ \text{K})^4 + (1-0.8) \times 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4 (500\ \text{K})^4$$

$$G_1 = 2835\,\text{W/m}^2 + 11{,}340\,\text{W/m}^2 = 14{,}175\,\text{W/m}^2.$$ <

(b) The radiosity is defined as the radiant flux leaving the surface by emission and reflection. For the blackbody surface 1, it follows that

$$J_1 = E_{b1} = \sigma T_1^4 = 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4 (1000\ \text{K})^4 = 56{,}700\,\text{W/m}^2.$$ <

(c) The radiosity of surface 2 is then,

$$J_2 = \varepsilon_2 E_{b2} + \rho_2 G_2.$$

Since the upper plate is a blackbody, it follows that $G_2 = E_{b1}$ and

$$J_2 = \varepsilon_2 E_{b1} + \rho_2 E_{b1} = \varepsilon_2 \sigma T_2^4 + 1(1-\varepsilon_2)\sigma T_1^4 = 14{,}175\,\text{W/m}^2.$$ <

Note that $J_2 = G_1$. That is, the radiant flux leaving surface 2 ($J_2$) is incident upon surface 1 ($G_1$).

(d) The net radiation heat exchange per unit area can be found by three relations.

$$q_1'' = J_1 - G_1 = (56{,}700 - 14{,}175)\,\text{W/m}^2 = 42{,}525\,\text{W/m}^2$$

$$q_{12}'' = J_1 - J_2 = (56{,}700 - 14{,}175)\,\text{W/m}^2 = 42{,}525\,\text{W/m}^2$$ <

The exchange relation, Eq. 13.24, is also appropriate with $\varepsilon_1 = 1$,

$$q_1'' = -q_2'' = q_{12}''$$

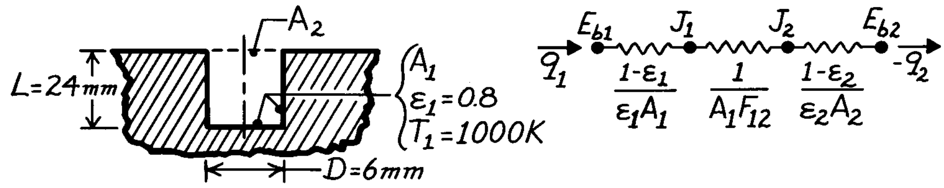
$$q_1'' = \varepsilon_2 \sigma \left(T_1^4 - T_2^4\right) = 0.8 \times 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4 \left(1000^4 - 500^4\right)\text{K}^4 = 42{,}525\,\text{W/m}^2.$$

## PROBLEM 13.42

**KNOWN:** Dimensions and temperature of a flat-bottomed hole.

**FIND:** (a) Radiant power leaving the opening, (b) Effective emissivity of the cavity, $\varepsilon_e$, (c) Limit of $\varepsilon_e$ as depth of hole increases.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Hypothetical surface $A_2$ is a blackbody at 0 K, (2) Cavity surface is isothermal, opaque and diffuse-gray.

**ANALYSIS:** Approximating $A_2$ as a blackbody at 0 K implies that all of the radiation incident on $A_2$ from the cavity results (directly or indirectly) from emission by the walls and escapes to the surroundings. It follows that for $A_2$, $\varepsilon_2 = 1$ and $J_2 = E_{b2} = 0$.

(a) From the thermal circuit, the rate of radiation loss through the hole ($A_2$) is

$$q_1 = (E_{b1} - E_{b2}) / \left[\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}\right]. \qquad (1)$$

Noting that $F_{21} = 1$ and $A_1 F_{12} = A_2 F_{21}$, also that

$$A_1 = \pi D^2/4 + \pi DL = \pi D(D/4 + L) = \pi(0.006\,m)(0.006\,m/4 + 0.024\,m) = 4.807 \times 10^{-4}\,m^2$$

$$A_2 = \pi D^2/4 = \pi(0.006\,m)^2/4 = 2.827 \times 10^{-5}\,m^2.$$

Substituting numerical values with $E_b = \sigma T^4$, find

$$q_1 = 5.67 \times 10^{-8}\,W/m^2 \cdot K^4 \left(1000^4 - 0\right) K^4 / \left[\frac{1-0.8}{0.8 \times 4.807 \times 10^{-4}\,m^2} + \frac{1}{2.827 \times 10^{-5}\,m^2} + 0\right]$$

$$q_1 = 1.580\ W. \qquad <$$

(b) The effective emissivity, $\varepsilon_e$, of the cavity is defined as the ratio of the radiant power leaving the cavity to that from a blackbody having the same area of the cavity opening and at the temperature of the inner surfaces of the cavity. For the cavity above,

$$\varepsilon_e = \frac{q_1}{A_2 \sigma T_1^4}$$

$$\varepsilon_e = 1.580\,W / 2.827 \times 10^{-5}\,m^2 \left(5.67 \times 10^{-8}\,W/m^2 \cdot K^4\right)(1000\ K)^4 = 0.986. \qquad <$$

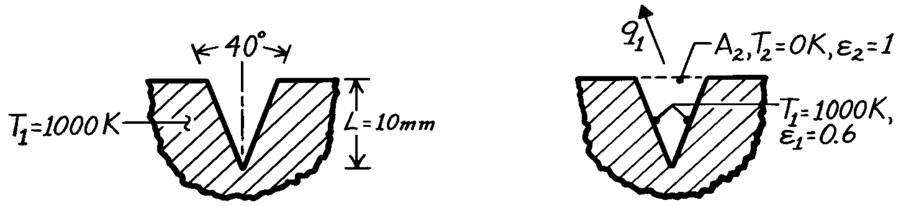
(c) As the depth of the hole increases, the term $(1 - \varepsilon_1)/\varepsilon_1 A_1$ goes to zero such that the remaining term in the denominator of Eq. (1) is $1/A_1 F_{12} = 1/A_2 F_{21}$. That is, as L increases, $q_1 \to A_2 F_{21} E_{b1}$. This implies that $\varepsilon_e \to 1$ as L increases. For L/D = 10, one would expect $\varepsilon_e = 0.999$ or better.

## PROBLEM 13.43

**KNOWN:** Long V-groove machined in an isothermal block.

**FIND:** Radiant flux leaving the groove to the surroundings and effective emissivity.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Groove surface is diffuse-gray, (2) Groove is infinitely long, (3) Block is isothermal.

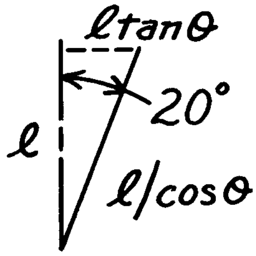
**ANALYSIS:** Define the hypothetical surface $A_2$ with $T_2 = 0$ K. The net radiation leaving $A_1$, $q_1$, will pass to the surroundings. From the two surface enclosure analysis, Eq. 13.23,

$$q_1 = -q_2 = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1 F_{12}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

Recognize that $\varepsilon_2 = 1$ and that from reciprocity, $A_1 F_{12} = A_2 F_{21}$ where $F_{21} = 1$. Hence,

$$\frac{q_1}{A_2} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1-\varepsilon_1}{\varepsilon_1}\dfrac{A_2}{A_1} + 1}$$

With $A_2/A_1 = 2\ell\ \tan 20° / \left(2\ell / \cos 20°\right) = \sin 20°$, find




$$q_1'' = \frac{5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\left(1000^4 - 0\right)\mathrm{K^4}}{\dfrac{(1-0.6)}{0.6}\times\sin 20° + 1} = 46.17\ \mathrm{kW/m^2}. \quad <$$

The effective emissivity of the groove follows from the definition given in Problem 13.42 as the ratio of the radiant power leaving the cavity to that from a blackbody having the area of the cavity opening and at the same temperature as the cavity surface. For the present situation,

$$\varepsilon_e = \frac{q_1''}{E_b\left(T_1\right)} = \frac{q_1''}{\sigma T_1^4} = \frac{46.17\times10^{+3}\,\mathrm{W/m^2}}{5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\left(1000\ \mathrm{K}\right)^4} = 0.814. \quad <$$

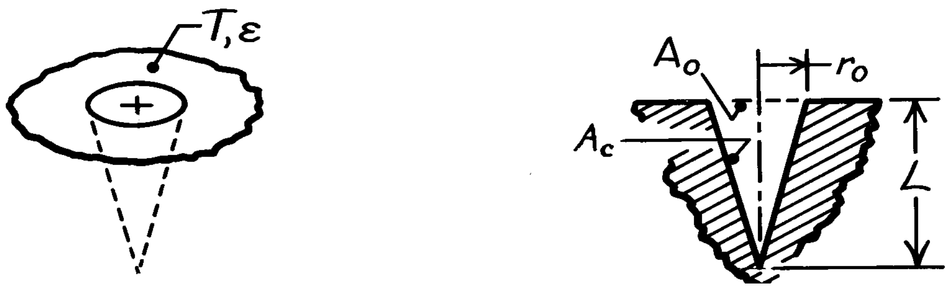
**COMMENTS:** Note the use of the hypothetical surface defined as black at 0 K. This surface does not emit and absorbs all radiation on it; hence, is the radiant power to the surroundings.

## PROBLEM 13.44

**KNOWN:** Conical cavity formed in an isothermal, opaque, diffuse-gray material of emissivity ε and temperature T.

**FIND:** Radiant power leaving the opening of the cavity in terms of T, ε, $r_o$, and L.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Material is opaque, diffuse-gray, and isothermal, (2) Cavity opening is hypothetical black surface at 0 K.

**ANALYSIS:** Define $A_o$, the opening of the cavity, as a black surface at $T_o$ = 0 K. Considering $A_o$ and $A_c$ as a two surface, diffuse-gray enclosure, the radiant power leaving the cavity opening is

$$q_{cavity} = -q_o = \left[E_b(T) - E_b(T_o)\right] / \left[\frac{1-\varepsilon}{\varepsilon A_c} + \frac{1}{A_c F_{co}} + \frac{1-\varepsilon_o}{\varepsilon_o A_o}\right]$$

Recognizing that $E_b(T_o) = 0$ and $\varepsilon_o = 1$ and also, using reciprocity,

$$A_c F_{co} = A_o F_{oc}$$

and from the enclosure, note $F_{oc} = 1$. Hence,

$$q_{cavity} = \frac{E_b(T)}{\dfrac{1-\varepsilon}{\varepsilon A_c} + \dfrac{1}{A_c F_{co}}} = \frac{\sigma T^4}{\dfrac{1-\varepsilon}{\varepsilon A_c} + \dfrac{1}{A_o F_{oc}}} = \frac{A_o \sigma T^4}{\dfrac{1-\varepsilon}{\varepsilon} \cdot \dfrac{A_o}{A_c} + 1}. \qquad (1)$$

Noting that $A_o = \pi r_o^2$ and $A_c = \pi r_o \left(L^2 + r_o^2\right)^{1/2}$, find

$$q_{cavity} = \frac{\pi r_o^2 \cdot \sigma T^4}{\dfrac{1-\varepsilon}{\varepsilon} \cdot \dfrac{1}{\left[(L/r_o)^2 + 1\right]^{1/2}} + 1}. \qquad <$$

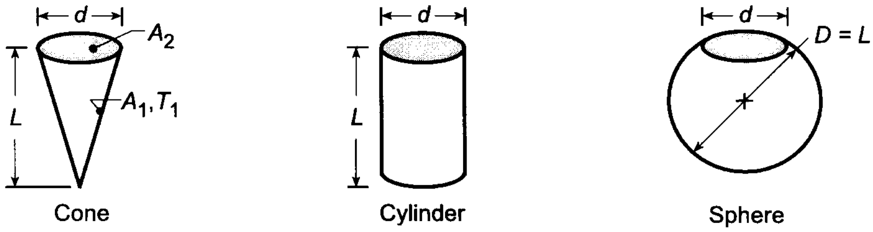
**COMMENTS:** When L increases or $A_o/A_c << 1$, the radiant power approaches that of a blackbody according to Eq. (1).

# PROBLEM 13.45

**KNOWN:** Cavities formed by a cone, cylinder, and sphere having the same opening size (d) and major dimension (L) with prescribed wall emissivity.

**FIND:** (a) View factor between the inner surface of each cavity and the opening of the cavity; (b) Effective emissivity of each cavity as defined in Problem 13.42, if the walls are diffuse-gray with $\varepsilon_w$; and (c) Compute and plot $\varepsilon_e$ as a function of the major dimension-to-opening size ratio, L/d, over the range from 1 to 10 for wall emissivities of $\varepsilon_w$ = 0.5, 0.7, and 0.9.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Diffuse-gray surfaces, (2) Uniform radiosity over the surfaces.

**ANALYSIS:** (a) Using the summation rule and reciprocity, determine the view factor $F_{12}$ for each of the cavities considered as a two-surface enclosure.

*Cone*: $F_{21} + F_{22} = F_{21} + 0 = 1 \qquad F_{21} = 1$

$$F_{12} = A_2\,F_{21}/A_1 = \left(\pi d^2/4\right)/(\pi d/2)\left[L^2 + (d/2)^2\right]^{1/2} = (1/2)\left[(L/d)^2 + 1/4\right]^{-1/2} \quad <$$

*Cylinder*: $F_{21} = 1$

$$F_{12} = A_2\,F_{21}/A_1 = A_2/A_1 = \left(\pi d^2/4\right)/\left[\pi dL + \pi d^2/4\right] = (1 + 4L/d)^{-1} \quad <$$

*Sphere*: $F_{21} = 1$

$$F_{12} = A_2\,F_{21}/A_1 = A_2/A_1 = \left(\pi d^2/4\right)/\left[\pi D^2 - \pi d^2/4\right] = \left(4D^2/d^2 - 1\right)^{-1}. \quad <$$

(b) The effective emissivity of the cavity is defined as

$$\varepsilon_{eff} = q_{12}/q_c$$

where $q_c = A_2\sigma T_1^4$ which presumes the opening is a black surface at $T_1$ and for the two-surface enclosure,

$$q_{12} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{(1-\varepsilon_1)/\varepsilon_1 A_1 + 1/A_1 F_{12} + (1-\varepsilon_2)/\varepsilon_2 A_2} = \frac{A_1\sigma T_1^4}{(1-\varepsilon_1)/\varepsilon_1 + 1/F_{12}}$$

since $T_2$ = 0K and $\varepsilon_2 = 1$. Hence, since $A_2/A_1 = F_{12}$ for all the cavities, with $\varepsilon_1 = \varepsilon_w$

$$\varepsilon_{eff} = \frac{1/F_{12}}{(1-\varepsilon_w)/\varepsilon_w + 1/F_{12}} = \frac{1}{F_{12}(1-\varepsilon_w)/\varepsilon_w + 1}$$

*Cone*: $$\varepsilon_{eff} = 1\Big/\left\{(1/2)\left[(L/d)^2 + 1/4\right]^{-1/2}(1-\varepsilon_w)/\varepsilon_w + 1\right\} \qquad (1) \quad <$$

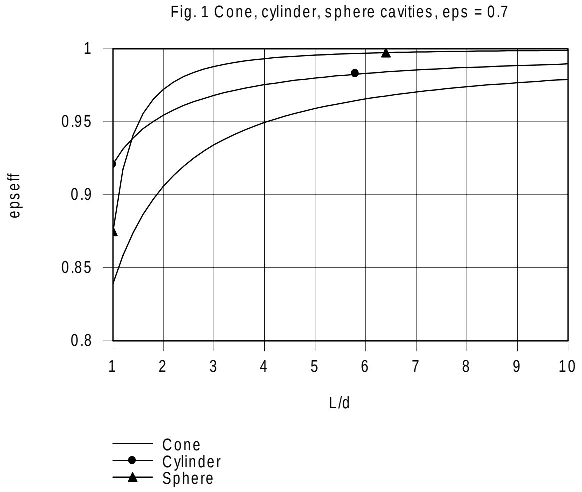
Continued …..

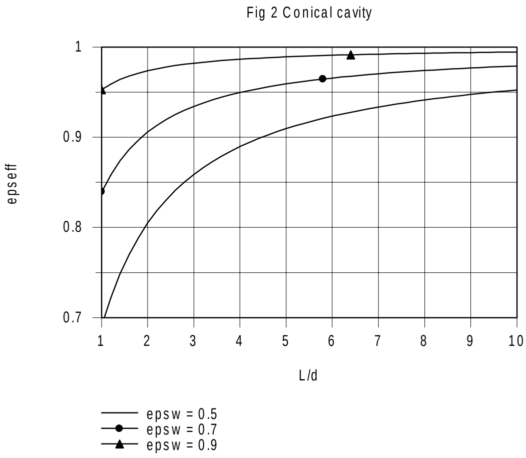
# PROBLEM 13.45 (Cont.)

*Cylinder*: $\varepsilon_{eff} = 1 \Big/ \left\{ \left[1 + 4L/d\right]^{-1} \left(1 - \varepsilon_w\right)/\varepsilon_w + 1 \right\}$ (2) <

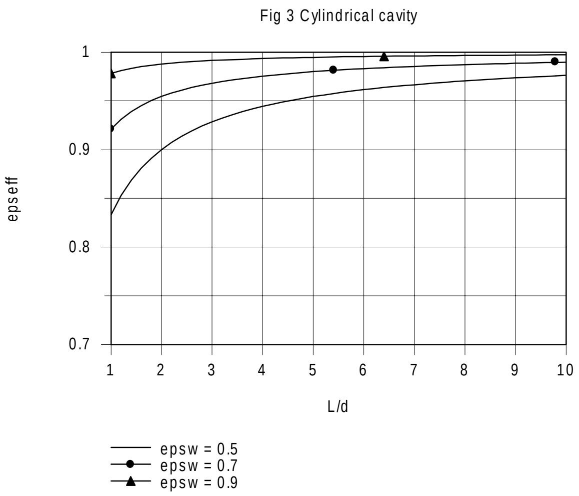
*Sphere*: $\varepsilon_{eff} = 1 / \left\{ \left[4D^2/d^2 - 1\right]^{-1} \left(1 - \varepsilon_w\right)/\varepsilon_w + 1 \right\}$ (3) <

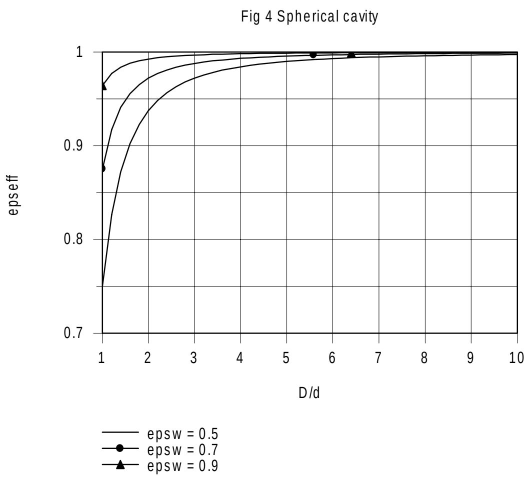
(c) Using the *IHT* Workspace with eqs. (1,2,3), the effective emissivity was computed as a function of L/d (cone, cylinder and sphere) for selected wall emissivities. The results are plotted below.




In Fig. 1, $\varepsilon_{eff}$ is shown as a function of L/d for $\varepsilon_w = 0.7$. For larger L/d, the sphere has the highest $\varepsilon_{eff}$ and the cone the lowest. Figures 2, 3 and 4 illustrate the $\varepsilon_{eff}$ vs. L/d for each of the cavity types. As expected, $\varepsilon_{eff}$ increases with increasing wall emissivity.




Note that for the spherical cavity, with L/d ≥ 5, $\varepsilon_{eff} > 0.98$ even with $\varepsilon_w$ as low as 0.5. This feature makes the use of spherical cavities for high performance radiometry applications attractive since $\varepsilon_{eff}$ is not very sensitive to $\varepsilon_w$.

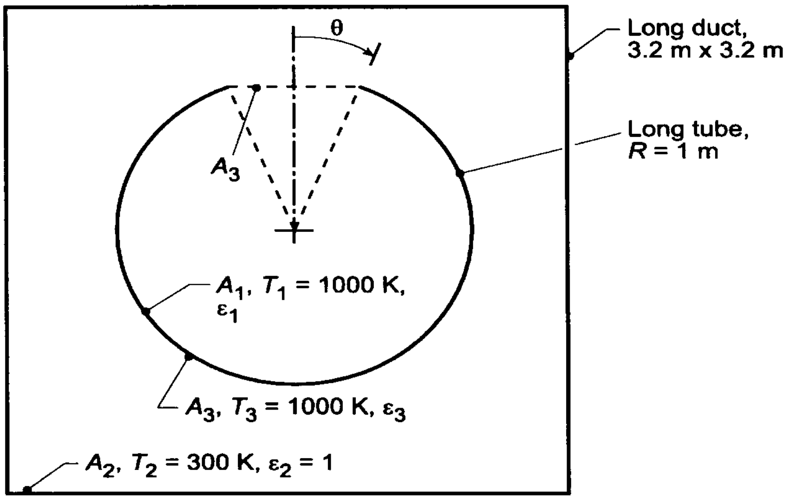
**COMMENTS:** In Fig. 1, intercomparing $\varepsilon_{eff}$ for the three cavity types, can you give a physical explanation for the results?

## PROBLEM 13.46

**KNOWN:** Very long diffuse, gray, thin-walled tube of 1-m radius contained inside a long black duct of square cross-section, 3.2 m × 3.2 m. Top portion is open as shown schematically.

**FIND:** (a) Net radiant heat transfer rate per unit length of the tube from the opening, $q_1' = q_1/L$, and the effective emissivity of the opening, $\varepsilon_{eff}$, for the condition when θ = 45° and (b) Compute and plot $q_1'$ and $\varepsilon_{eff}$ as a function of θ for the range 0 ≤ θ ≤ 180°.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Tube is very long compared to its radius R and duct dimension, (2) Interior of cylinder is diffuse, gray, (3) Interior of the duct is black.

**ANALYSIS:** (a) Consider the two-surface enclosure formed by the inner surface of the tube, $A_1$, and the hypothetical surface formed by the opening, $A_3$. The surface $A_3$ behaves as a blackbody ($\varepsilon_3 = 1$) at a temperature $T_3 = T_2 = 300$ K. The net heat rate leaving the opening follows from Eq. 13.23.

$$q_1 = -q_1 = \frac{\sigma\left(T_1^4 - T_2^4\right)}{(1-\varepsilon_1)/\varepsilon_1 A_1 + 1/A_1F_{13} + (1-\varepsilon_3)/\varepsilon_3 A_3} \qquad (1)$$

The view factor $F_{13}$ can be determined from the reciprocity relation recognizing that $F_{31} = 1$.

$$F_{13} = A_3F_{31}/A_1 = 1.414L \times 1/4.712L = 0.300 \qquad (2)$$

where the areas $A_1$ and $A_3$ are, with θ = 45°,

$$A_1 = 2R(\pi - \theta)L = 2\times 1\,\text{m}\left[\pi - 45\times\pi/180\right]\times L = 4.712L \qquad (3)$$

$$A_3 = 2R\sin\theta\, L = 2\times 1\,\text{m}\times\sin 45°\times L = 1.414L \qquad (4)$$

Substituting numerical values, find

$$q_1 = -q_3 = \frac{5.67\times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(1000^4 - 300^4\right)\text{K}^4}{(1-0.1)/(0.1\times 4.712L) + 1/(4.712L\times 0.300) + 0}$$

$$q_1/L = -q_3/L = 21{,}487\ \text{W/m}$$ <

Continued …..

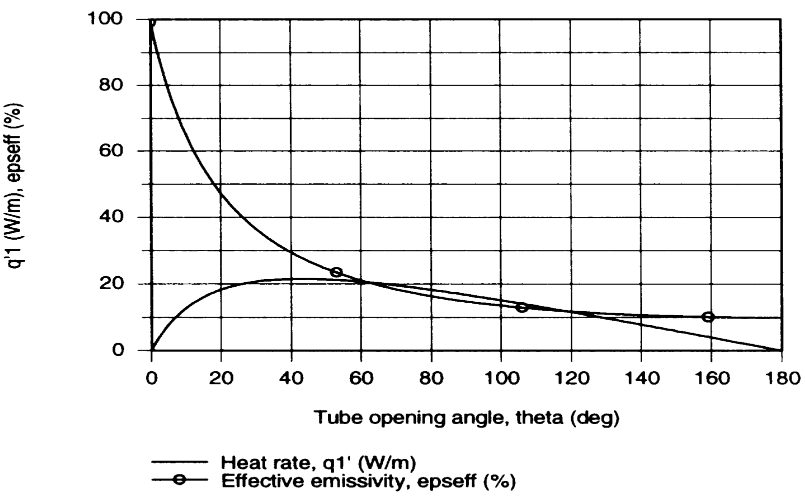
## PROBLEM 13.46 (Cont.)

The effective emissivity of the opening is the ratio of the radiant power leaving the opening to that of a blackbody having the area of the opening ($A_3$) and a temperature of the inner surface of the cavity ($T_1$).

$$\varepsilon_{eff} = \frac{q_1}{A_3 \sigma T_1^4} = \frac{q_1 / L}{(A_3 / L)\sigma T_1^4} \tag{5}$$

$$\varepsilon_{eff} = \frac{21,487\ W/m}{1.414\,m \times 5.67 \times 10^{-8}\ W/m^2 \cdot K^4 (1000\ K)^4} = 0.268$$ <

(b) Using the foregoing equations, Eqs. (1-5), in the *IHT* workspace, $q_1' = q_1 / L$ and $\varepsilon_{eff}$ as a function of $\theta$ were computed and are plotted below.



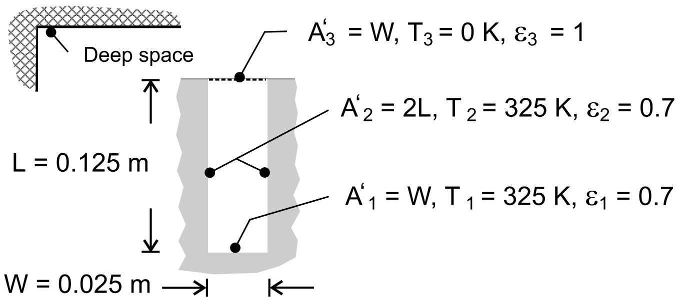

Note that $q_1' = 0$ when $\theta = 0°$ since the tube is closed and no power leaves the tube. At $\theta = 180°$, the area of the tube has been reduced to zero and hence, $q_1' = 0$. For small values of $\theta$, $\varepsilon_{eff}$ is highest and decreases as $\theta$ increases, to the limit $\varepsilon_{eff} = \varepsilon_1 = 0.1$.

# PROBLEM 13.47

**KNOWN:** Temperature, emissivity and dimensions of a rectangular fin array radiating to deep space.

**FIND:** (a) Rate of radiation transfer per unit length from a unit section to space, (b) Effect of emissivity on heat rejection.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Diffuse/gray surface behavior, (2) Length of array (normal to page) is much larger than W and L, (3) Isothermal surfaces.

**ANALYSIS:** (a) Since the sides and base of the U-section have the same temperature and emissivity, they can be treated as a single surface and the U-section becomes a two-surface enclosure. Deep space may be represented by the hypothetical surface $A'_3$, which acts as a blackbody at absolute zero temperature. From Eq. (13.23), with $T_1 = T_2 = T$ and $\varepsilon_1 = \varepsilon_2 = \varepsilon$,

$$q'_{(1,2)3} = \frac{\sigma\left(T^4 - T_3^4\right)}{\dfrac{1-\varepsilon}{\varepsilon A'_{(1,2)}} + \dfrac{1}{A'_{(1,2)} F_{(1,2)3}} + \dfrac{1-\varepsilon}{\varepsilon A'_3}}$$

where $A'_{(1,2)} = 2L + W$, $A'_3 = W$, $A'_{(1,2)} F_{(1,2)3} = A'_3 F_{3(1,2)} = W$. Hence,

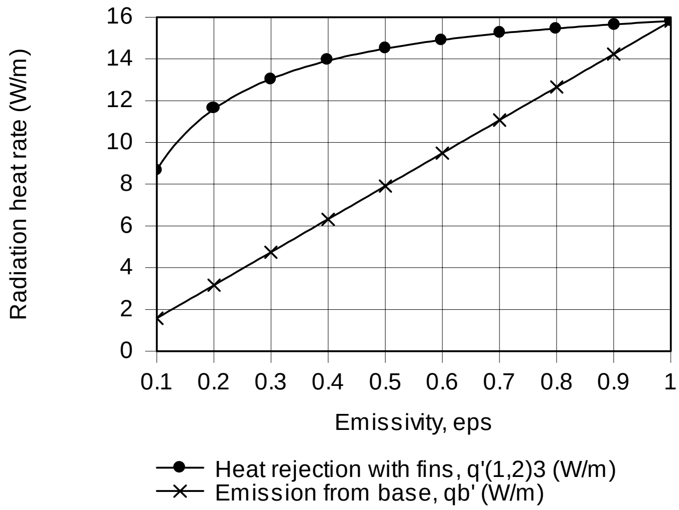
$$q'_{(1,2)3} = \frac{\sigma T^4}{\dfrac{1-\varepsilon}{\varepsilon(2L+W)} + \dfrac{1}{W} + \dfrac{1-\varepsilon}{\varepsilon W}}$$

$$q'_{(1,2)3} = \frac{5,67\times10^{-8}\ W/m^2\cdot K^4 (325\ K)^4}{\dfrac{1-0.70}{0.70(0.275m)} + \dfrac{1}{0.025m} + 0} = 15.2\ W/m$$ <

(b) For ε = 0.7 emission from the base of the U-section is $q'_b = \varepsilon A'_1 \sigma T^4 = 0.7\times0.025m \times5.67\times10^{-8}\ W/m^2\cdot K^4 (325\ K)^4 = 11.1\ W/m$. The effect of $\varepsilon$ on $q'_{(1,2)3}$ and $q'_b$ is shown as follows.

Continued …..

## PROBLEM 13.47 (Cont.)




The effect of the fins on heat transfer enhancement increases with decreasing emissivity.

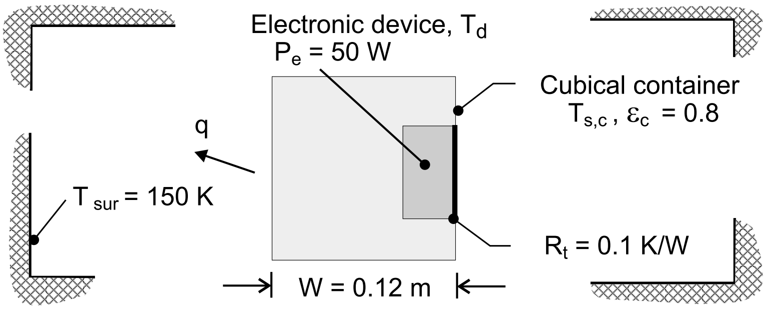
**COMMENTS:** Note that, if the surfaces behaved as blackbodies ($\varepsilon_1 = \varepsilon_2 = 1.0$), the U-section becomes a blackbody cavity for which heat rejection is simply $A'_3\ E_b(T) = q'_b$. Hence, it is no surprise that the $q'_b \rightarrow q'_{(1,2)3}$ as $\varepsilon \rightarrow 1$ in the foregoing figure. For $\varepsilon = 1$, no enhancement is provided by the fins.

## PROBLEM 13.48

**KNOWN:** Power dissipation of electronic device and thermal resistance associated with attachment to inner wall of a cubical container. Emissivity of outer surface of container and wall temperature of service bay.

**FIND:** Temperatures of cubical container and device.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state, (2) Device and container are isothermal, (3) Heat transfer from the container is exclusively by radiation exchange with bay (small surface in a large enclosure), (4) Container surface may be approximated as diffuse/gray.

**ANALYSIS:** From Eq. (13.27)

$$P_e = q = \sigma\left(6W^2\right)\varepsilon_c\left(T_{s,c}^4 - T_{sur}^4\right)$$

$$T_{s,c} = \left[\frac{q}{\sigma\left(6W^2\right)\varepsilon_c} + T_{sur}^4\right]^{1/4}$$

$$T_{s,c} = \left[\frac{50\,\text{W}}{5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\times6(0.12\text{m})^2\times0.8} + (150\,\text{K})^4\right]^{1/4} = 339.4\,\text{K} = 66.4°\text{C} \qquad <$$

With $\quad q = \left(T_d - T_{s,c}\right)/R_t$,

$$T_d = q\,R_t + T_{s,c} = 50\,\text{W}\times0.1\,\text{K/W} + 66.4°\text{C} = 71.4°\text{C} \qquad <$$

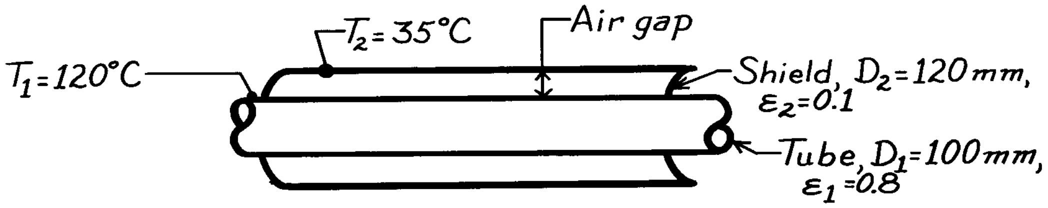
**COMMENTS:** If the temperature of the device is too large to insure reliable operation, it may be reduced by increasing $\varepsilon_c$ or W.

## PROBLEM 13.49

**KNOWN:** Long, thin-walled horizontal tube with radiation shield having an air gap of 10 mm. Emissivities and temperatures of surfaces are prescribed.

**FIND:** Radiant heat transfer from the tube per unit length.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Tube and shield are very long, (2) Surfaces at uniform temperatures, (3) Surfaces are diffuse-gray.

**ANALYSIS:** The long tube and shield form a two surface enclosure, and since the surfaces are diffuse-gray, the radiant heat transfer from the tube, according to Eq. 13.23, is

$$q_{12} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1 F_{12}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}} \tag{1}$$

By inspection, $F_{12} = 1$. Note that

$$A_1 = \pi D_1 \ell \qquad \text{and} \qquad A_2 = \pi D_2 \ell$$

where $\ell$ is the length of the tube and shield. Dividing Eq. (1) by $\ell$, find the heat rate per unit length,

$$q'_{12} = \frac{q_{12}}{\ell} = \frac{5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}\left[(273+120)^4 - (273+35)^4\right]\text{K}^4}{\dfrac{1-0.8}{0.8\pi\left(100\times10^{-3}\,\text{m}\right)} + \dfrac{1}{\pi\left(100\times10^{-3}\,\text{m}\right)\times1} + \dfrac{1-0.1}{0.1\pi\left(120\times10^{-3}\,\text{m}\right)}}$$

$$q'_{12} = \frac{842.3\ \text{W/m}^2}{(0.7958+3.183+23.87)\,\text{m}^{-1}} = 30.2\ \text{W/m}. \qquad <$$

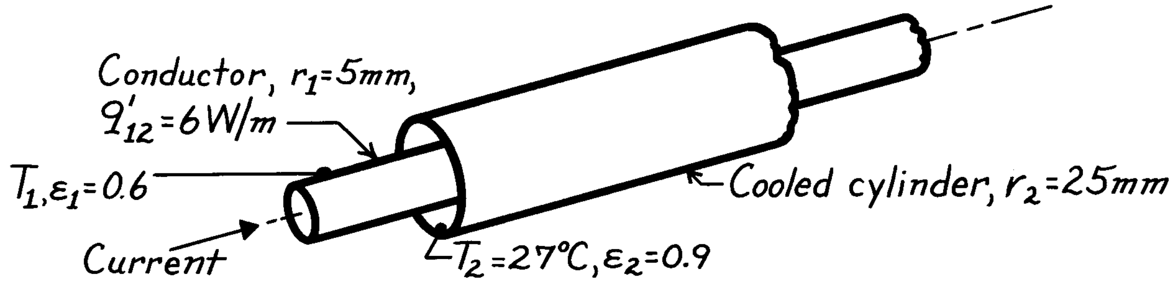
**COMMENTS:** Recognize that convective heat transfer would be important in this annular air gap. Suitable correlations to estimate the heat transfer coefficient are given in Chapter 9.

## PROBLEM 13.50

**KNOWN:** Long electrical conductor with known heat dissipation is cooled by a concentric tube arrangement.

**FIND:** Surface temperature of the conductor.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Surfaces are diffuse-gray, (2) Conductor and cooling tube are concentric and very long, (3) Space between surfaces is evacuated.

**ANALYSIS:** The heat transfer by radiation exchange between the conductor and the concentric, cooled cylinder is given by Eq. 13.25. For a unit length,

$$q'_{12} = \frac{q_{12}}{\ell} = \sigma \cdot 2\pi r_1 \left(T_1^4 - T_2^4\right) / \left[\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{r_1}{r_2}\right)\right] \quad (1)$$

where $A_1 = 2\pi r_1 \cdot \ell$. Solving for $T_1$ and substituting numerical values, find

$$T_1 = \left\{T_2^4 + \frac{q'_{12}}{\sigma \cdot 2\pi r_1}\left[\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{r_1}{r_2}\right)\right]\right\}^{1/4}$$

$$T_1 = \left\{(27+273)^4 \text{ K}^4 + \frac{6 \text{ W/m}}{5.67\times10^{-8} \text{W/m}^2 \cdot \text{K}^4 \times 2\pi\,(0.005\text{m})}\left[\frac{1}{0.6} + \frac{1-0.9}{0.9}\left(\frac{5}{25}\right)\right]\right\}^{1/4}$$

$$T_1 = \left\{(300 \text{ K})^4 + 3.368\times10^9 \text{K}^4 \left[1.667 + 0.00222\right]\right\}^{1/4} \quad (2)$$

$$T_1 = 342.3 \text{ K} = 69°\text{C}.$$ <

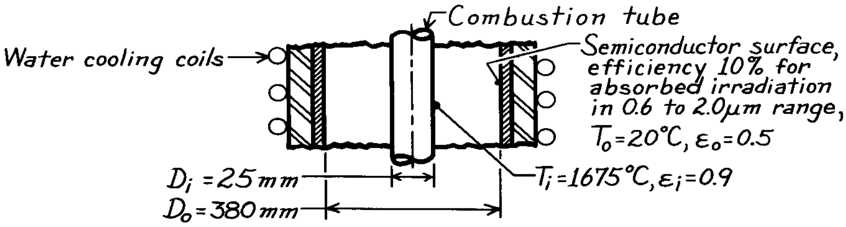
**COMMENTS:** (1) Note that Eq. (1) implies that $F_{12} = 1$. From Eq. (2) by comparison of the second term in the brackets involving $\varepsilon_2$, note that the influence of $\varepsilon_2$ is small. This follows since $r_1 \ll r_2$.

## PROBLEM 13.51

**KNOWN:** Arrangement for direct thermophotovoitaic conversion of thermal energy to electrical power.

**FIND:** (a) Radiant heat transfer between the inner and outer surface per unit area of the outer surface, (b) Power generation per unit outer surface area if semiconductor has 10% conversion efficiency for radiant power in the 0.6 to 2.0 μm range.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Surfaces are diffuse-gray, (2) Surfaces approximate long, concentric cylinder, two-surface enclosure with negligible end effects.

**ANALYSIS:** (a) For this two-surface enclosure, the net radiation exchange per unit area of the outer surface is,

$$\frac{q_{io}}{A_o} = \frac{A_i}{A_o} \cdot \frac{\sigma\left(T_i^4 - T_o^4\right)}{\frac{1}{\varepsilon_i} + \frac{1-\varepsilon_o}{\varepsilon_o}\left(\frac{r_i}{r_o}\right)} \tag{1}$$

and since $A_i/A_o = 2\pi r_i \ell / 2\pi r_o \ell = r_1 / r_o$, the heat flux at surface $A_o$ is

$$\frac{q_{io}}{A_o} = \left(\frac{0.0125}{0.190}\right)\frac{\left(5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\right)\left(1948^4 - 293^4\right)\mathrm{K}^4}{\frac{1}{0.9} + \frac{1-0.5}{0.5}\left(\frac{0.125}{0.190}\right)} = 45.62\ \mathrm{kW/m^2}. \tag{2}$$ <

(b) The power generation per unit area of surface $A_o$ can be expressed as

$$P_e'' = \eta_e \cdot G_{abs}\left(0.6 \to 2.0\mu\mathrm{m}\right) \tag{3}$$

where $\eta_e$ is the semiconductor conversion efficiency and $G_{abs}$ (0.6 → 2.0μm) represents the absorbed irradiation on $A_o$ in the prescribed wavelength interval. The *total* absorbed irradiation is $G_{abs,t} = q_{io}/A_o$ and has the spectral distribution of a blackbody at $T_i$ since $T_o^4 \ll T_i^4$ and $A_i$ is gray. Hence, we can write Eq. (3) as

$$P_e'' = \eta_e \cdot \left(q_{io}/A_o\right)\left[F_{(0\to 2\mu\mathrm{m})} - F_{(0\to 0.6\mu\mathrm{m})}\right]. \tag{4}$$

From Table 12.2: $\lambda T = 2 \times 1948 = 3896\ \mu\mathrm{m\cdot K}$, $F_{(0-\lambda T)} = 0.461$; $\lambda T = 0.6 \times 1948 = 1169\ \mu\mathrm{m\cdot K}$, $F_{(0-\lambda T)} = 0.0019$. Hence

$$P_e'' = 0.1\left(45.62\ \mathrm{kW/m^2}\right)\left[0.461 - 0.0019\right] = 2.09\ \mathrm{kW/m^2}.$$ <

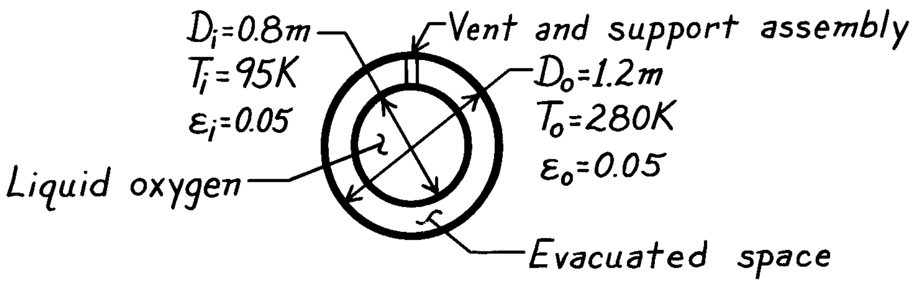
That is, the unit produces 2.09 kW per unit area of the outer surface.

**PROBLEM 13.52**

**KNOWN:** Temperatures and emissivities of spherical surfaces which form an enclosure.

**FIND:** Evaporation rate of oxygen stored in inner container.

**SCHEMATIC:**




**PROPERTIES:** Oxygen (given): $h_{fg} = 2.13 \times 10^5$ J/kg.

**ASSUMPTIONS:** (1) Opaque, diffuse-gray surfaces, (2) Evacuated space between surfaces, (3) Negligible heat transfer along vent and support assembly.

**ANALYSIS:** From an energy balance on the inner container, the net radiation heat transfer to the container may be equated to the evaporative heat loss

$$q_{oi} = \dot{m} h_{fg}.$$

Substituting from Eq. (13.26), where $q_{oi} = -q_{io}$ and $F_{io} = 1$

$$\dot{m} = \frac{-\sigma\left(\pi D_i^2\right)\left(T_i^4 - T_o^4\right)}{h_{fg}\left[\frac{1}{\varepsilon_i} + \frac{1-\varepsilon_o}{\varepsilon_o}\left(\frac{r_i}{r_o}\right)^2\right]}$$

$$\dot{m} = \frac{-5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\times\pi\,(0.8\mathrm{m})^2\left(95^4 - 280^4\right)\mathrm{K}^4}{2.13\times10^5\,\mathrm{J/kg}\left[\frac{1}{0.05} + \frac{0.95}{0.05}\left(\frac{0.4}{0.6}\right)^2\right]}$$

$$\dot{m} = 1.14\times10^{-4}\,\mathrm{kg/s}.$$ <

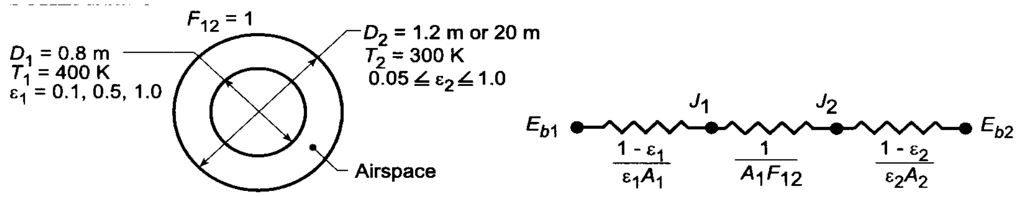
**COMMENTS:** This loss could be reduced by insulating the outer surface of the outer container and/or by inserting a radiation shield between the containers.

## PROBLEM 13.53

**KNOWN:** Emissivities, diameters and temperatures of concentric spheres.

**FIND:** (a) Radiation transfer rate for black surfaces. (b) Radiation transfer rate for diffuse-gray surfaces, (c) Effects of increasing the diameter and assuming blackbody behavior for the outer sphere. (d) Effect of emissivities on net radiation exchange.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Blackbody or diffuse-gray surface behavior.

**ANALYSIS:** (a) Assuming blackbody behavior, it follows from Eq. 13.13

$$q_{12} = A_1 F_{12} \sigma \left(T_1^2 - T_2^4\right) = \pi\,(0.8\,\text{m})^2\,(1) 5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left[(400\text{ K})^4 - (300\text{ K})^4\right] = 1995\text{ W.} \quad <$$

(b) For diffuse-gray surface behavior, it follows from Eq. 13.26

$$q_{12} = \frac{\sigma A_1\left(T_1^4 - T_2^4\right)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)^2} = \frac{5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4 \pi\,(0.8\,\text{m})^2\left[400^4 - 300^4\right]\text{K}^4}{\dfrac{1}{0.5} + \dfrac{1-0.05}{0.05}\left(\dfrac{0.4}{0.6}\right)^2} = 191\text{ W.} \quad <$$

(c) With $D_2 = 20$ m, it follows from Eq. 13.26

$$q_{12} = \frac{5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}\pi\,(0.8\,\text{m})^2\left[(400\text{ K})^4 - (300\text{ K})^4\right]}{\dfrac{1}{0.5} + \dfrac{1-0.05}{0.05}\left(\dfrac{0.4}{10}\right)^2} = 983\text{ W.} \quad <$$

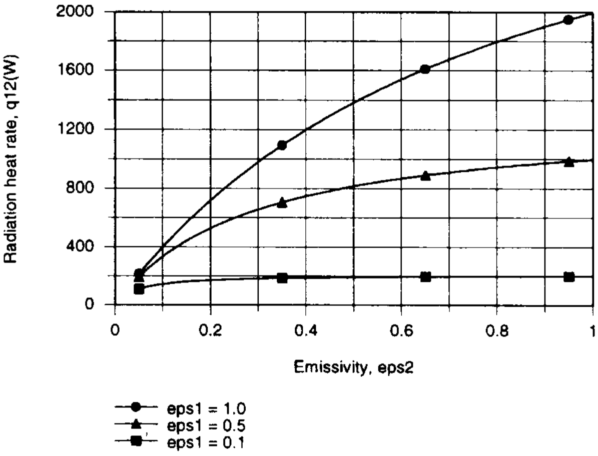
With $\varepsilon_2 = 1$, instead of 0.05, Eq. 13.26 reduces to Eq. 13.27 and

$$q_{12} = \sigma A_1 \varepsilon_1\left(T_1^4 - T_2^4\right) = 5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\pi\,(0.8\,\text{m})^2\,0.5\left[(400\text{ K})^4 - (300\text{ K})^4\right] = 998\text{ W.} \quad <$$

Continued …..

## PROBLEM 13.53 (Cont.)

(d) Using the *IHT Radiation* Tool Pad, the following results were obtained




Net radiation exchange increases with $\varepsilon_1$ and $\varepsilon_2$, and the trends are due to increases in emission from and absorption by surfaces 1 and 2, respectively.

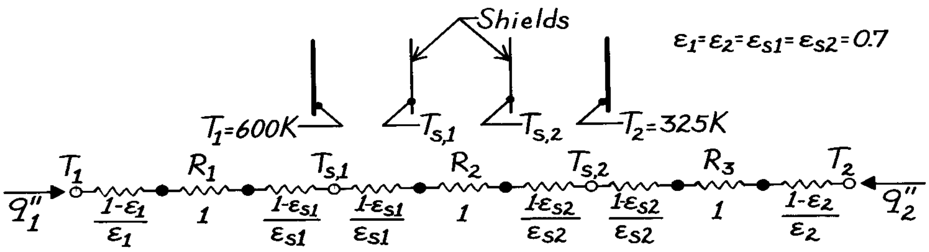
**COMMENTS:** From part (c) it is evident that the actual surface emissivity of a *large* enclosure has a small effect on radiation exchange with small surfaces in the enclosure. Working with $\varepsilon_2 = 1.0$ instead of $\varepsilon_2 = 0.05$, the value of $q_{12}$ is increased by only (998 – 983)/983 = 1.5%. In contrast, from the results of (d) it is evident that the surface emissivity $\varepsilon_2$ of a *small* enclosure has a large effect on radiation exchange with interior objects, which increases with increasing $\varepsilon_1$.

## PROBLEM 13.54

**KNOWN:** Two radiation shields positioned in the evacuated space between two infinite, parallel planes.

**FIND:** Steady-state temperature of the shields.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) All surfaces are diffuse-gray and (2) All surfaces are parallel and of infinite extent.

**ANALYSIS:** The planes and shields can be represented by a thermal circuit from which it follows that

$$q_1'' = -q_2'' = \frac{\sigma\left(T_1^4 - T_2^4\right)}{R_1'' + R_2'' + R_3''} = \frac{\sigma\left(T_1^4 - T_{s1}^4\right)}{R_1''} = \frac{\sigma\left(T_{s1}^4 - T_{s2}^4\right)}{R_2''} = \frac{\sigma\left(T_{s2}^4 - T_2^4\right)}{R_3''}.$$

Since all the emissivities involved are equal, $R_1'' = \dfrac{A_1}{A_1 F_{12}} = 1 = R_2'' = R_3''$, so that

$$T_{s1}^4 = T_1^4 - \frac{R_1''}{R_1'' + R_2'' + R_3''}\left(T_1^4 - T_2^4\right) = T_1^4 - (1/3)\left(T_1^4 - T_2^4\right)$$

$$T_{s1}^4 = (600\ \text{K})^4 - (1/3)\left(600^4 - 325^4\right)\text{K}^4 \qquad T_{1s} = 548\ \text{K}$$ <

$$T_{s2}^4 = T_2^4 + \frac{R_3''}{R_1'' + R_2'' + R_3''}\left(T_1^4 - T_2^4\right) = T_2^4 + (1/3)\left(T_1^4 - T_2^4\right)$$

$$T_{s2}^4 = (325\ \text{K})^4 + (1/3)\left(600^4 - 325^4\right)\text{K}^4 \qquad T_{s2} = 474\ \text{K}.$$ <

## PROBLEM 13.55

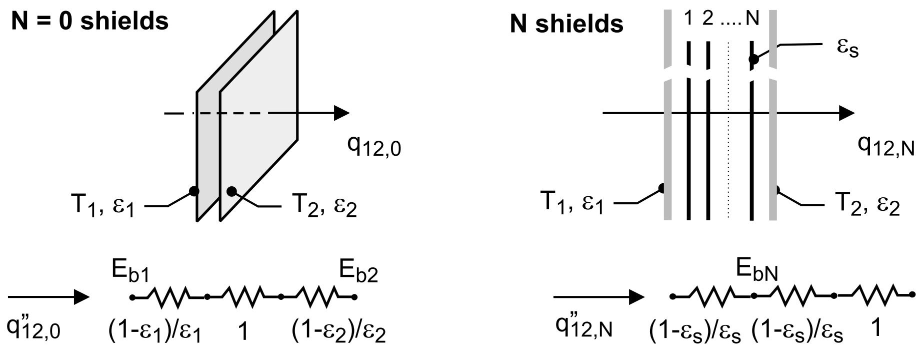
**KNOWN:** Two large, infinite parallel plates that are diffuse-gray with temperatures and emissivities of $T_1$ and $\varepsilon_1$ and $T_2$ and $\varepsilon_2$.

**FIND:** Show that the ratio of the radiation transfer rate with multiple shields, N, of emissivity $\varepsilon_s$ to that with no shields, N = 0, is

$$\frac{q_{12,N}}{q_{12,0}} = \frac{[1/\varepsilon_1 + 1/\varepsilon_2 - 1]}{[1/\varepsilon_1 + 1/\varepsilon_2 - 1] + N[2/\varepsilon_s - 1]}$$

where $q_{12,N}$ and $q_{12,0}$ represent the radiation heat rate with N and N = 0 shields, respectively.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Plane infinite planes with diffuse-gray surfaces and uniform radiosities, and (2) Shield has negligible thermal conduction resistance.

**ANALYSIS:** Representing the parallel plates by the resistance network shown above for the “no-shield” condition, N = 0, with $F_{12} = 1$, the heat rate per unit area follows from Eq. 13.24 (see also Fig. 13.11) as

$$q''_{12,0} = \frac{E_{b1} - E_{b2}}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} \tag{1}$$

With the addition of each shield as shown in the schematic above, three resistance elements are added to the network: two surface resistances, $(1 - \varepsilon_s)/\varepsilon_s$, and one space resistance, $1/F_{ij} = 1$. Hence, for the “N - shield” condition,

$$q''_{12,N} = \frac{E_{b1} - E_{b2}}{[1/\varepsilon_1 + 1/\varepsilon_2 - 1] + N[2(1-\varepsilon_s)/\varepsilon_s + 1]} \tag{2}$$

The ratio of the heat rates is obtained by dividing Eq. (2) by Eq. (1),

$$\frac{q''_{12,N}}{q_{12,0}} = \frac{[1/\varepsilon_1 + 1/\varepsilon_2 - 1]}{[1/\varepsilon_1 + 1/\varepsilon_2 - 1] + N[2/\varepsilon_s - 1]} \qquad <$$

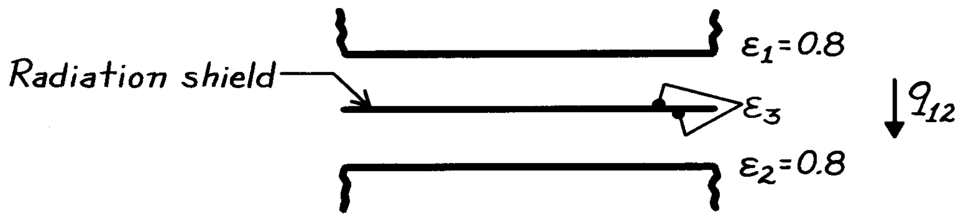
**COMMENTS:** Can you derive an expression to determine the temperature difference across pairs of the N-shields?

**PROBLEM 13.56**

**KNOWN:** Emissivities of two large, parallel surfaces.

**FIND:** Heat shield emissivity needed to reduce radiation transfer by a factor of 10.

**SCHEMATIC:**




**ASSUMPTIONS:** (a) Diffuse-gray surface behavior, (b) Negligible conduction resistance for shield, (c) Same emissivity on opposite sides of shield.

**ANALYSIS:** For this arrangement, $F_{13} = F_{32} = 1$.

*Without* (wo) the shield, it follows from Eq. 13.24,

$$(q_{12})_{wo} = \frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}.$$

*With* (w) the shield it follows from Eq. 13.28,

$$(q_{12})_{w} = \frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} + \frac{2}{\varepsilon_3} - 2}.$$

Hence, the heat rate ratio is

$$\frac{(q_{12})_w}{(q_{12})_{wo}} = 0.1 = \frac{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} + \frac{2}{\varepsilon_3} - 2} = \frac{\frac{1}{0.8} + \frac{1}{0.8} - 1}{\frac{1}{0.8} + \frac{1}{0.8} + \frac{2}{\varepsilon_3} - 2}.$$

Solving, find

$$\varepsilon_3 = 0.138. \qquad <$$

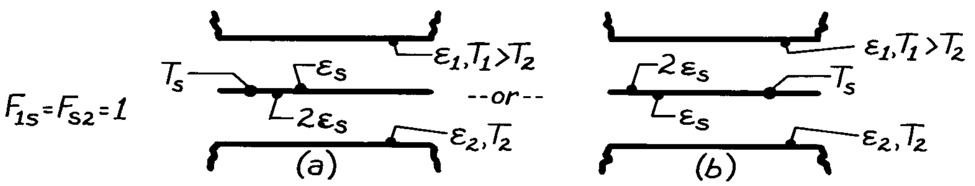
**COMMENTS:** The foregoing result is independent of $T_1$ and $T_2$. It is only necessary that the temperatures be maintained at fixed values, irrespective of whether or not the shield is in place.

## PROBLEM 13.57

**KNOWN:** Surface emissivities of a radiation shield inserted between parallel plates of prescribed temperatures and emissivities.

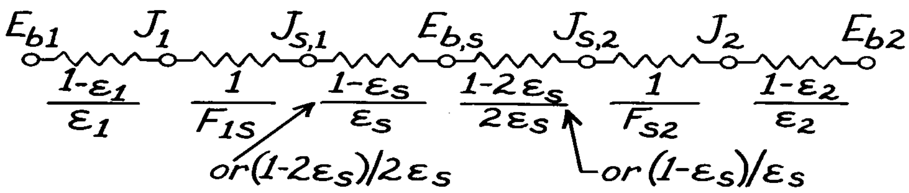
**FIND:** (a) Effect of shield orientation on radiation transfer, (b) Effect of shield orientation on shield temperature.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Diffuse-gray surface behavior, (2) Shield is isothermal.

**ANALYSIS:** (a) On a unit area basis, the network representation of the system is




Hence the total radiation resistance,

$$R = \frac{1-\varepsilon_1}{\varepsilon_1} + 1 + \frac{1-\varepsilon_s}{\varepsilon_s} + \frac{1-2\varepsilon_s}{2\varepsilon_s} + 1 + \frac{1-\varepsilon_2}{\varepsilon_2}$$

is independent of orientation. Since $q = (E_{b1} - E_{b2})/R$, the heat transfer rate is independent of orientation.

(b) Considering that portion of the circuit between $E_{b1}$ and $E_{bs}$, it follows that

$$q = \frac{E_{b1} - E_{bs}}{\frac{1-\varepsilon_1}{\varepsilon_1} + 1 + f(\varepsilon_s)}, \quad \text{where} \quad f(\varepsilon_s) = \frac{1-\varepsilon_s}{\varepsilon_s} \text{ or } \frac{1-2\varepsilon_s}{2\varepsilon_s}.$$

Hence,

$$E_{bs} = E_{b1} - \left[\frac{1-\varepsilon_1}{\varepsilon_1} + 1 + f(\varepsilon_s)\right] q.$$

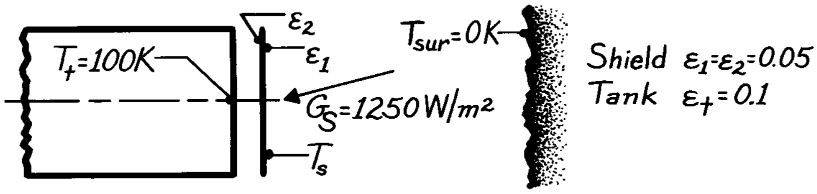
It follows that, since $E_{bs}$ increases with decreasing $f(\varepsilon_s)$ and $(1 - 2\varepsilon_s)/2\varepsilon_s < (1 - \varepsilon_s)/\varepsilon_s$, $E_{bs}$ is larger when the high emissivity ($2\varepsilon_s$) side faces plate 1. Hence $T_s$ is larger for case (b). <

# PROBLEM 13.58

**KNOWN:** End of propellant tank with radiation shield is subjected to solar irradiation in space environment.

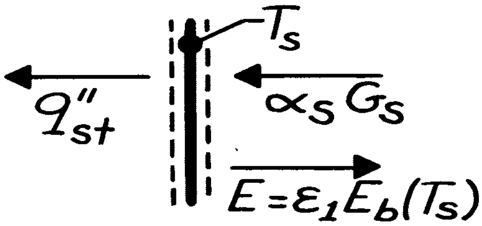
**FIND:** (a) Temperature of the shield, $T_s$, and (b) Heat flux to the tank, $q''_1\left(W/m^2\right)$.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) All surfaces are diffuse-gray, (2) View factor between shield and tank is unity, $F_{st}$ = 1, (3) Space surroundings are black at 0 K, (4) Resistance of shield for conduction is negligible.

**ANALYSIS:** (a) Perform a radiation balance on the shield. From the schematic,




$$\alpha_S G_S - \varepsilon_1 E_b\left(T_s\right) - q''_{st} = 0 \tag{1}$$

where $q''_{st}$ is the net heat exchange between the shield and the tank. Considering these two surfaces as large, parallel planes, from Eq. 13.24,

$$q''_{st} = \sigma\left(T_s^4 - T_t^4\right)/\left[1/\varepsilon_2 + 1/\varepsilon_1 - 1\right]. \tag{2}$$

Substituting $q''_{st}$ from Eq. (2) into Eq. (1), find

$$\alpha_s G_s - \varepsilon_1 \sigma T_s^4 - \sigma\left(T_s^4 - T_1^4\right)/\left[1/\varepsilon_2 + 1/\varepsilon_t - 1\right] = 0.$$

Solving for $T_s$, find

$$T_s = \left[\frac{\alpha_S G_S + \sigma T_t^4/\left[1/\varepsilon_2 + 1/\varepsilon_t - 1\right]}{\sigma\left(\varepsilon_1 + 1/\left[1/\varepsilon_2 + 1/\varepsilon_t - 1\right]\right)}\right]^{1/4}.$$

Since the shield is diffuse-gray, $\alpha_S = \varepsilon_1$ and then

$$T_s = \left[\frac{0.05 \times 1250\,W/m^2 + \sigma\left(100\right)^4 K^4/\left[1/0.05 + 1/0.1 - 1\right]}{\sigma\left(0.05 + 1/\left[1/0.05 + 1/0.1 - 1\right]\right)}\right]^{1/4} = 338\text{ K.} \quad <$$

(b) The heat flux to the tank can be determined from Eq. (2),

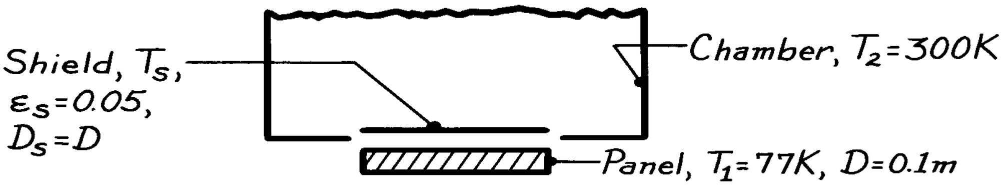
$$q''_{st} = 5.67 \times 10^{-8}\,W/m^2 \cdot K^4\left(338^4 - 100^4\right)K^4/\left[1/0.05 + 1/0.1 - 1\right] = 25.3\,W/m^2. \quad <$$

# PROBLEM 13.59

**KNOWN:** Black panel at 77 K in large vacuum chamber at 300 K with radiation shield having ε = 0.05.

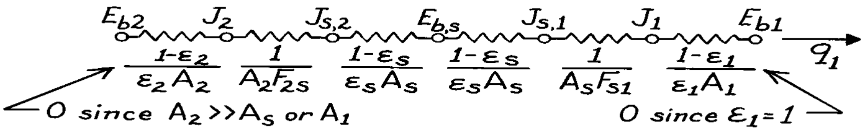
**FIND:** Net heat transfer by radiation to the panel.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Chamber is large compared to shield, (2) Shape factor between shield and plate is unity, (3) Shield is diffuse-gray, (4) Shield is thin, negligible thermal conduction resistance.

**ANALYSIS:** The arrangement lends itself to a network representation following Figs. 13.10 and 13.11.




Noting that $F_{2s} = F_{s1} = 1$, and that $A_2F_{2s} = A_sF_{s2}$, the heat rate is

$$q_1 = \left(E_{b2} - E_{b1}\right)/\Sigma R_i = \sigma\left(T_2^4 - T_1 4\right)/\left[\frac{1}{A_s} + 2\left(\frac{1-\varepsilon_s}{\varepsilon_s A_s}\right) + \frac{1}{A_s}\right].$$

Recognizing that $A_s = A_1$ and multiplying numerator and denominator by $A_1$ gives

$$q_1 = A_1\sigma\left(T_2 - T_1^4\right)\left[2 + 2\left(\frac{1-\varepsilon_s}{\varepsilon_s}\right)\right].$$

Substituting numerical values, find

$$q_1 = \frac{\pi 0.1^2\,m^2}{4} \times 5.67\times10^{-8}\,W/m^2\cdot K^4\left(300^4 - 77^4\right)K^4/\left[2 + 2\left(\frac{1-0.05}{0.05}\right)\right]$$

$$q_1 = 89.8\,mW.$$ <

**COMMENTS:** In using the network representation, be sure to designate direction of the net heat rate. In this situation, we have shown $q_1$ as the net rate *into* the surface $A_1$. The temperature of the shield, $T_s$ = 253 K, follows from the relation

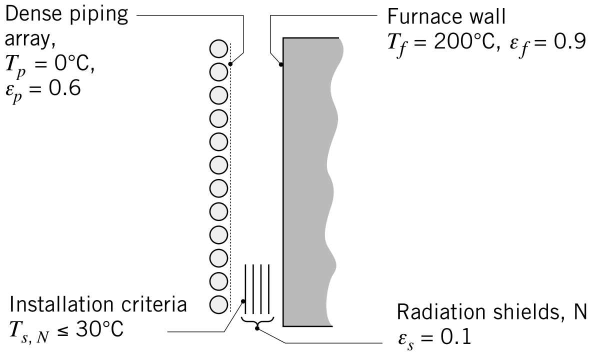
$$q_1 = \left(E_{bs} - E_{b1}\right)/\left[\frac{1-\varepsilon_s}{\varepsilon_s A_s} + \frac{1}{A_1 F_{s1}}\right].$$

## PROBLEM 13.60

**KNOWN:** Dense cryogenic piping array located close to furnace wall.

**FIND:** Number of radiation shields, N, to be installed such that the temperature of the shield closest to the array, $T_{s,N}$, is less than 30°C.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) The ice-covered dense piping array approximates a plane surface, (2) Piping array and furnace wall can be represented by infinite parallel plates, (3) Surfaces are diffuse-gray, and (4) Convection effects are negligible.

**ANALYSIS:** Treating the piping array and furnace wall as infinite parallel plates, the net heat rate by radiation exchange with N shields of identical emissivity, $\varepsilon_s$, on both sides follows from extending the network of Fig. 13.12 to account for the resistances of N shields. (See Problem 13.14 (S).) For each shield added, two surface resistances and one space resistance are added,

$$q_{fp} = \frac{\sigma\left(T_f^4 - T_p^4\right)A_f}{\left[1/\varepsilon_f + 1/\varepsilon_p - 1\right] + N\left[2/\varepsilon_s - 1\right]} \qquad (1)$$

where $\sigma = 5.67 \times 10^{-8}$ W/m$^2\cdot$K$^4$. The requirement that the N-th shield (next to the piping array) has a temperature $T_{s,N} \le 30°C$ will be satisfied when

$$q_{fp} \le \frac{\sigma\left(T_{s,N}4 - T_p4\right)A_f}{\left[1/\varepsilon_s + 1/\varepsilon_p - 1\right]} \qquad (2)$$

Using the foregoing equations in the *IHT* workspace, find that $T_{s,N}$ = 30°C when N = 8.60. So that $T_{s,N}$ is less than 30°C, the number of shields required is

$$N = 9$$ <

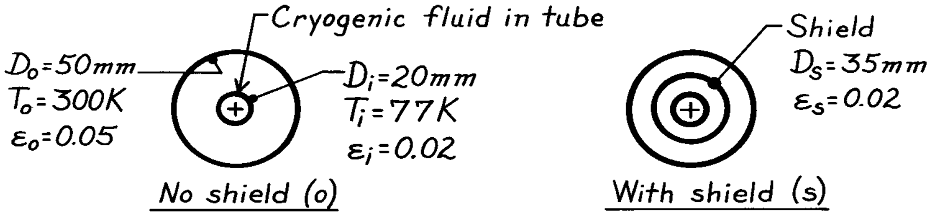
**COMMENTS:** Note that when N = 0, Eq. (1) reduces to the case of two parallel plates. Show for the case with one shield, N = 1, that Eq. (1) is identical to Eq. 13.28.

## PROBLEM 13.61

**KNOWN:** Concentric tube arrangement with diffuse-gray surfaces.

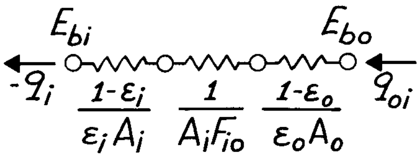
**FIND:** (a) Heat gain by the cryogenic fluid per unit length of the inner tube (W/m), (b) Change in heat gain if diffuse-gray shield with $\varepsilon_s$ = 0.02 is inserted midway between inner and outer surfaces.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Surfaces are diffuse-gray, (2) Space between tubes is evacuated.

**ANALYSIS:** (a) For the *no shield* case, the thermal circuit is shown at right. It follows that the net heat gain per unit tube length is



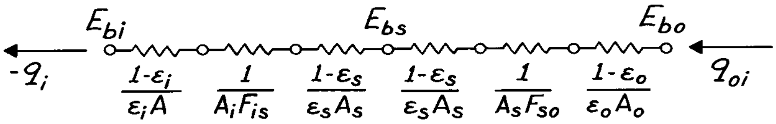

$$-q_1' = \frac{q_{oi}}{L} = \left(E_{bo} - E_{bi}\right) / \left[\frac{1-\varepsilon_o}{\varepsilon \pi D_o} + \frac{1}{\pi D_i F_{io}} + \frac{1-\varepsilon_i}{\varepsilon_i \pi D_i}\right]$$

where A = $\pi$DL. Note that $F_{io}$ = 1 and $E_b = \sigma T^4$ giving

$$-q_1' = 5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\left(300^4 - 77^4\right)\mathrm{K}^4 / \left[\frac{1-0.05}{0.05\pi\times50\times10^{-3}} + \frac{1}{\pi 20\times10^{-3}\times1} + \frac{1-0.02}{0.02\pi\times20\times10^{-3}}\right]\mathrm{m}^{-1}$$

$$-q_1' = 457\ \mathrm{W/m^2} / [121.0 + 15.9 + 779.8]\mathrm{m}^{-1} = 0.501\ \mathrm{W/m}.$$ <

(b) For the *with shield* case, the thermal circuit will include three additional resistances.




From the network, it follows that $-q_i = \left(E_{bo} - E_{bi}\right)/\Sigma R_t$. With $F_{is} = F_{so}$ = 1, find

$$-q_i' = 457\,\mathrm{W/m^2} / \left[121.0 + \frac{1}{\pi 35\times10^{-3}\times1} + \frac{2(1-0.02)}{0.02\pi 35\times10^{-3}} + 15.9 + 779.8\right]\mathrm{m}^{-1}$$

$$-q_i' = 457\,\mathrm{W/m^2} / [121.0 + 9.1 + 891.3 + 15.9 + 779.8]\mathrm{m}^{-1} = 0.251\,\mathrm{W/m}.$$

The change (percentage) in heat gain per unit length of the tube as a result of inserting the radiation shield is

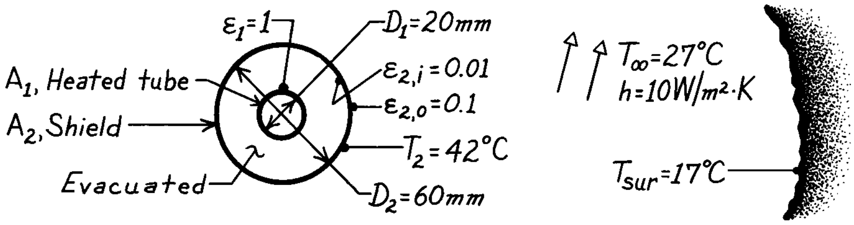
$$\frac{q_{i,s}' - q_{i,ns}'}{q_{i,ns}'}\times100 = \frac{(0.251 - 0.501)\,\mathrm{W/m}}{0.501\,\mathrm{W/m}}\times100 = -49\%.$$ <

# PROBLEM 13.62

**KNOWN:** Heated tube with radiation shield whose exterior surface is exposed to convection and radiation processes.

**FIND:** Operating temperature for the tube under the prescribed conditions.

**SCHEMATIC:**

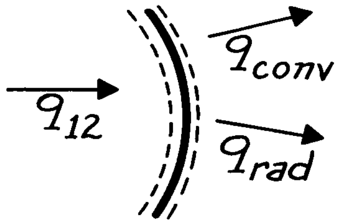



**ASSUMPTIONS:** (1) Steady-state conditions, (2) No convection in space between tube and shield, (3) Surroundings are large compared to the shield and are isothermal, (4) Tube and shield are infinitely long, (5) Surfaces are diffuse-gray, (6) Shield is isothermal.

**ANALYSIS:** Perform an energy balance on the shield.

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q_{12} - q_{conv} - q_{rad} = 0$$




where $q_{12}$ is the net radiation exchange between the tube and inner surface of the shield, which from Eq. 13.25 is,

$$-q_{12} = \frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_{2,i}}{\varepsilon_{2,i}}\frac{D_1}{D_2}}$$

Using appropriate rate equations for $q_{conv}$ and $q_{rad}$, the energy balance is

$$\frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{1 + \frac{1-\varepsilon_{2,i}}{\varepsilon_{2,i}}\frac{D_1}{D_2}} - hA_2\left(T_2 - T_\infty\right) - \varepsilon_{2,o}A_2\sigma\left(T_2^4 - T_{sur}^4\right) = 0$$

where $\varepsilon_1 = 1$. Substituting numerical values, with $A_1/A_2 = D_1/D_2$, and solving for $T_1$,

$$\frac{(20/60)\times 5.67\times10^{-8}\,W/m^2\cdot K^4\left(T_1^4 - 315^4\right)K^4}{1+(1-0.01/0.01)(20/60)} - 10\ W/m^2\cdot K(315-300)K$$

$$-0.1\times5.67\times10^{-8}\,W/m^2\cdot K^4\left(315^4 - 290^4\right)K^4 = 0$$

$$T_1 = 745\ K = 472°C. \quad <$$

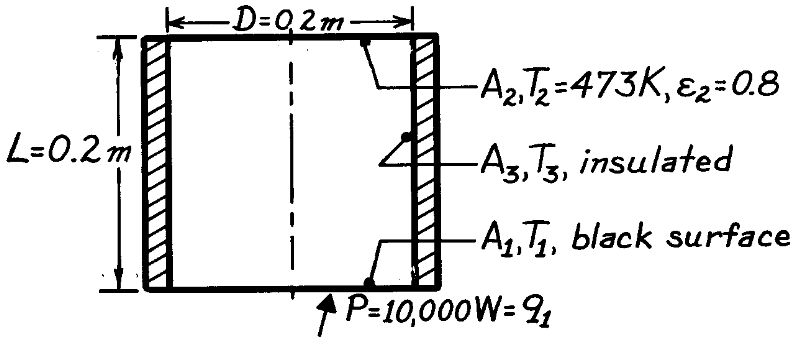
**COMMENTS:** Note that all temperatures are expressed in kelvins. This is a necessary practice when dealing with radiation and convection modes.

## PROBLEM 13.63

**KNOWN:** Cylindrical-shaped, three surface enclosure with lateral surface insulated.

**FIND:** Temperatures of the lower plate $T_1$ and insulated side surface $T_3$.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Surfaces have uniform radiosity or emissive power, (2) Upper and insulated surfaces are diffuse-gray, (3) Negligible convection.

**ANALYSIS:** Find the temperature of the lower plate $T_1$ from Eq. 13.30

$$q_1 = \frac{\sigma\left(T_1^4 - T_2^4\right)}{(1-\varepsilon_1)/\varepsilon_1 A_1 + \left[A_1 F_{12} + \left[(1/A_1 F_{13}) + (1/A_2 F_{23})\right]^{-1}\right]^{-1} + (1-\varepsilon_2)/\varepsilon_2 A_2}. \quad (1)$$

From Table 13.2 for parallel coaxial disks,

$$R_1 = r_1/L = 0.1/0.2 = 0.5 \qquad R_2 = r_2/L = 0.1/0.2 = 0.5$$

$$S = 1 + \left(1 + R_2^2\right)/R_1^2 = 1 + \left(1 + 0.5^2\right)/0.5^2 = 6.0$$

$$F_{12} = 1/2\left\{S - \left[S^2 - 4\left(r_2/r_1\right)^2\right]^{1/2}\right\} = 1/2\left\{6 - \left[6^2 - 4\left(0.5/0.5\right)^2\right]^{1/2}\right\} = 0.172.$$

Using the summation rule for the enclosure, $F_{13} = 1 - F_{12} = 1 - 0.172 = 0.828$, and from symmetry, $F_{23} = F_{13}$. With $A_1 = A_2 = \pi D^2/4 = \pi(0.2\text{ m})^2/4 = 0.03142\text{ m}^2$ and substituting numerical values into Eq. (1), obtain

$$10,000\text{ W} = \frac{0.03142\,\text{m}^2 \times 5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(T_1^4 - 473^4\right)\text{K}^4}{0 + \left[0.172 + \left[(1/0.828) + (1/0.172)\right]^{-1}\right]^{-1} + (1-0.8)/0.8}$$

$$10,000 = 4.540\times10^9\left(T_1^4 - 473^4\right) \qquad T_1 = 1225\text{ K}.$$ <

The temperature of the insulated side surface can be determined from the radiation balance, Eq. 13.31, with $A_1 = A_2$,

$$\frac{J_1 - J_3}{1/F_{13}} - \frac{J_3 - J_2}{1/F_{23}} = 0 \quad (2)$$

where $J_1 = \sigma T_1^4$ and $J_2$ can be evaluated from Eq. 13.19,

Continued …..

**PROBLEM 13.63 (Cont.)**

$$q_2 = \frac{E_{b2} - J_2}{(1-\varepsilon_2)/\varepsilon_2 A_2} \qquad -10,000\ \text{W} = \frac{5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\,(473\ \text{K})^4 - J_2}{(1-0.8)/\left(0.8\times0.03142\,\text{m}^2\right)}$$

find $J_2$ = 82,405 W/m$^2$. Substituting numerical values into Eq. (2),

$$\frac{5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\,(1225\ \text{K})^4 - J_3}{1/0.172} - \frac{J_3 - 82,405\ \text{W/m}^2}{1/0.172} = 0$$

find $J_3$ = 105,043 W/m$^2$. Hence, for this insulated, re-radiating (adiabatic) surface,

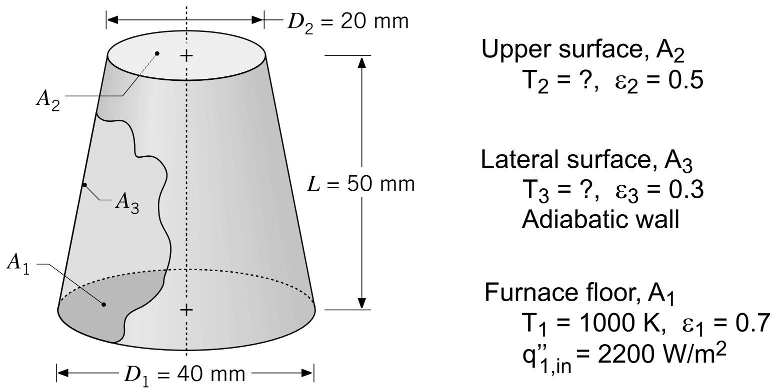
$$E_{b3} = \sigma T_3^4 = 105,043\ \text{W/m}^2 \qquad T_3 = 1167\ \text{K}.$$ <

## PROBLEM 13.64

**KNOWN:** Furnace in the form of a truncated conical section, floor (1) maintained at $T_1$ = 1000 K by providing a heat flux $q''_{1,in} = 2200\ W/m^2$; lateral wall (3) perfectly insulated; radiative properties of all surfaces specified.

**FIND:** (a) Temperature of the upper surface, $T_2$, and of the lateral wall $T_3$, and (b) $T_2$ and $T_3$ if all the furnace surfaces are black instead of diffuse-gray, with all other conditions remain unchanged. Explain effect of $\varepsilon_2$ on your results.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Furnace is a three-surface, diffuse-gray enclosure, (2) Surfaces have uniform radiosities, (3) Lateral surface is adiabatic, and (4) Negligible convection effects.

**ANALYSIS:** For the three-surface enclosure, write the radiation surface energy balances, Eq. 13.21, to find the radiosities of the three surfaces.

$$\frac{E_{b,1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} = \frac{J_1 - J_2}{1/A_1 F_{12}} + \frac{J_1 - J_3}{1/A_1 F_{13}} \tag{1}$$

$$\frac{E_{b,2} - J_2}{(1-\varepsilon_2)/\varepsilon_2 A_2} = \frac{J_2 - J_1}{1/A_2 F_{21}} + \frac{J_2 - J_3}{1/A_2 F_{23}} \tag{2}$$

$$\frac{E_{b,3} - J_3}{(1-\varepsilon_3)/\varepsilon_3 A_3} = \frac{J_3 - J_1}{1/A_3 F_{31}} + \frac{J_3 - J_2}{1/A_3 F_{32}} \tag{3}$$

where the blackbody emissive powers are of the form $E_b = \sigma T^4$ with $\sigma = 5.67 \times 10^{-8}$ W/m$^2$·K$^4$. From Eq. 13.19, the net radiation leaving $A_1$ is

$$q_1 = \frac{E_{b,1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} \tag{4}$$

$$q_1 = q''_{1,in} \cdot A_1 = 2200\ W/m^2 \times \pi (0.040\ m)^2 / 4 = 2.76\ W$$

Continued …..

## PROBLEM 13.64 (Cont.)

Since the lateral surface is adiabatic,

$$q_3 = \frac{E_{b,3} - J_3}{(1-\varepsilon_3)/\varepsilon_3 A_3} = 0 \tag{5}$$

from which we recognize $E_{b,3} = J_3$, but will find that as an outcome of the analysis. For the enclosure, N = 3, there are $N^2 = 9$ view factors, for which N (N - 1)/2 = 3 must be directly determined. Calculations for the $F_{ij}$ are summarized in Comments.

With the foregoing five relations, we can determine the five unknowns: $J_1$, $J_2$, $J_3$, $E_{b,2}$, and $E_{b,3}$. The temperatures $T_2$ and $T_3$ will be evaluated from the relation $E_b = \sigma T^4$. Using this analysis approach with the relations in the *IHT* workspace, the results for (a) the diffuse-gray surfaces and (b) black surfaces are tabulated below.

| | $J_1$ (kW/m$^2$) | $J_2$ (kW/m$^2$) | $J_3$ (kW/m$^2$) | $T_2$ (K) | $T_3$ (K) |
|---|---|---|---|---|---|
| (a) Diffuse-gray | 55.76 | 45.30 | 53.48 | 896 | 986 |
| (b) Black | 56.70 | 46.24 | 54.42 | 950 | 990 |

**COMMENTS:** (1) From the tabulated results, it follows that the temperatures of the lateral and top surfaces will be higher when the surfaces are black, rather than diffuse-gray as specified.

(2) From Eq. (5) for the net heat radiation leaving the lateral surface, $A_3$, the rate is zero since the wall is adiabatic. The consequences are that the blackbody emissive power and the radiosity are equal, and that the emissivity of the surface has no effect in the analysis. That is, this surface emits and absorbs at the same rate; the net is zero.

(3) For the enclosure, N = 3, there are $N^2 = 9$ view factors, for which

$$N\,(N-1)/2 = 3 \times 2/2 = 3$$

must be directly determined. We used the *IHT Tools | Radiation | View Factors Relations* model that sets up the summation rules and reciprocity relations for the N surfaces. The user is required to specify the 3 $F_{ij}$ that must be determined directly; by inspection, $F_{11} = F_{22} = 0$; and $F_{12}$ can be evaluated using the parallel coaxial disk relation, Table 13.2 (Fig. 13.5). This model is also provided in *IHT* to simplify the calculation task. The results of the view factor analysis are:

$$F_{12} = 0.03348 \qquad F_{13} = 0.9665$$

$$F_{21} = 0.1339 \qquad F_{23} = 0.8661$$

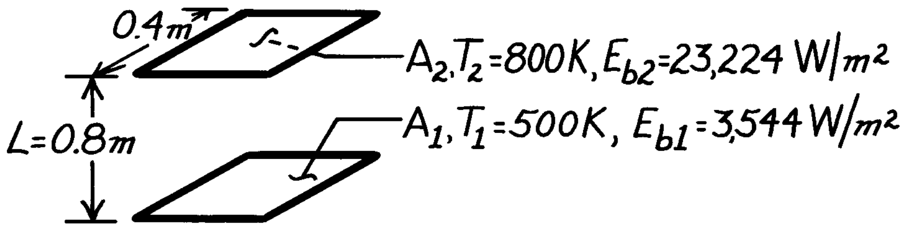
(4) An alternative method of solution for part (a) is to treat the enclosure of part (a) as described in Section 13.3.5. For part (b), the black enclosure analysis is described in Section 13.2. We chose to use the net radiation method, Section 13.3.1, to develop a general 3-surface enclosure code in *IHT* that can also handle black surfaces (caution: use $\varepsilon = 0.999$, not 1.000).

## PROBLEM 13.65

**KNOWN:** Two aligned, parallel square plates with prescribed temperatures.

**FIND:** Net radiative transfer from surface 1 for these plate conditions: (a) black, surroundings at 0 K, (b) black with connecting, re-radiating walls, (c) diffuse-gray with radiation-free surroundings at 0 K, (d) diffuse-gray with re-radiating walls.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Plates are black or diffuse-gray, (2) Surroundings are at 0 K.

**ANALYSIS:** (a) The view factor for the aligned, parallel plates follows from Fig. 13.4, X/L = 0.4 m/0.8 m = 0.5, Y/L = 0.4 m/0.8 m = 0.5, $F_{12} = F_{21} \approx 0.075$. When the plates are *black with surroundings at 0 K*, from Eq. 13.13,

$$q_1 = q_{12} + q_{1(sur)} = A_1F_{12}\left(E_{b1} - E_{b2}\right) + A_1F_{1(sur)}\left(E_{b1} - E_{b(sur)}\right)$$

$$q_1 = \left(0.4 \times 0.4\right)\text{m}^2\left[0.075\left(3544 - 23,224\right) + \left(1 - 0.075\right)\left(3544 - 0\right)\right]\text{W/m}^2 = 288\,\text{W}.$$ <

(b) When the plates are *black with connecting re-radiating walls*, from Eq. 13.30 with $F_{1R} = R_{2R} = 1 - F_{12} = 0.925$,

$$q_1 = \frac{A_1\left[E_{b1} - E_{b2}\right]}{\left[F_{12} + \left(1/F_{1R} + 1/F_{2R}\right)^{-1}\right]^{-1}} = \frac{\left(0.4\,\text{m}\right)^2\left[3544 - 23,224\right]\text{W/m}^2}{\left[0.075 + \left(1/0.925 + 1/0.925\right)^{-1}\right]^{-1}} = -1,692\,\text{W}.$$ <

(c) When the plates are *diffuse-gray ($\varepsilon_1$ = 0.6 and $\varepsilon_2$ = 0.8)* with the *surroundings at 0 K,* using Eq. 13.20 or Eq. 13.19, with $E_{b3} = J_3 = 0$,

$$q_1 = A_1F_{12}\left(J_1 - J_2\right) + A_1F_{13}\left(J_1 - J_3\right) = \left(E_{b1} - J_1\right)/\left[\left(1 - \varepsilon_1\right)/\varepsilon_1 A_1\right].$$

The radiosities must be determined from energy balances, Eq. 13.21, on each of the surfaces,

$$\frac{E_{b1} - J_1}{\left(1 - \varepsilon_1\right)/\varepsilon_1} = F_{12}\left(J_1 - J_2\right) + F_{13}\left(J_1 - J_3\right) \qquad \frac{E_{b2} - J_2}{\left(1 - \varepsilon_2\right)/\varepsilon_2} = F_{21}\left(J_2 - J_1\right) + F_{23}\left(J_2 - J_3\right)$$

$$\frac{3,544 - J_1}{\left(1 - 0.6\right)/0.6} = 0.075\left(J_1 - J_2\right) + 0.925 J_1 \qquad \frac{23,224 - J_2}{\left(1 - 0.8\right)/0.8} = 0.075\left(J_2 - J_1\right) + 0.925 J_2.$$

Find $J_1$ = 2682 W/m$^2$ and $J_2$ = 18,542 W/m$^2$. Combining these results,

$$q_1 = \left(0.4\,\text{m}\right)^2\left(0.075\right)\left(2682 - 18,542\right)\text{W/m}^2 + \left(0.4\,\text{m}\right)^2\left(0.925\right)\left(2682 - 0\right)\text{W/m}^2 = 207\ \text{W}.$$ <

(d) When the plates are *diffuse-gray with connecting re-radiating walls,* use Eq. 13.30,

$$q_1 = \frac{A_1\left[E_{b1} - E_{b2}\right]}{\left(1 - \varepsilon_1\right)/\varepsilon_1 + \left[F_{12} + \left(1/F_{1R} + 1/F_{2R}\right)^{-1}\right]^{-1} + \left(1 - \varepsilon_2\right)/\varepsilon_2}$$

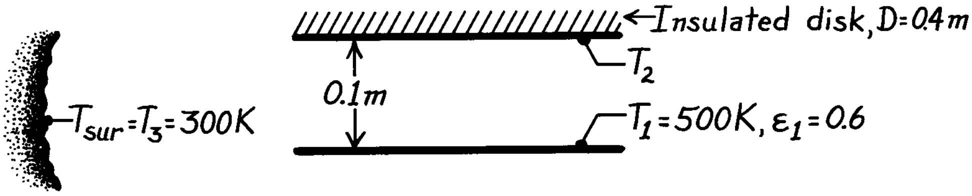
$$q_1 = \frac{\left(0.4\,\text{m}\right)^2\left[35444 - 23,244\right]\text{W/m}^2}{\left(1 - 0.6\right)/0.6 + \left[0.075 + \left(1/0.925 + 1/0.925\right)^{-1}\right]^{-1} + \left(1 - 0.8\right)/0.8} = -1133\,\text{W}.$$ <

## PROBLEM 13.66

**KNOWN:** Parallel, aligned discs located in a large room; one disk is insulated, the other is at a prescribed temperature.

**FIND:** Temperature of the insulated disc.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Surfaces are diffuse-gray, (2) Surroundings are large, with uniform temperature, behaving as a blackbody, (3) Negligible convection.

**ANALYSIS:** From an energy balance on surface $A_2$,

$$q_2 = 0 = \frac{J_2 - J_1}{1/A_2F_{21}} + \frac{J_2 - J_3}{1/A_2F_{23}}. \qquad (1)$$

Note that $q_2 = 0$ since the surface is adiabatic. Since $A_3$ is a blackbody, $J_3 = E_{b3} = \sigma T_3^4$; since $A_2$ is adiabatic, $J_2 = E_{b2} = \sigma T_2^4$. From Fig. 13.5 and the summation rule for surface $A_1$, find

$$F_{12} = 0.62 \text{ with } \frac{r_j}{L} = \frac{0.2}{0.1} = 2 \text{ and } \frac{L}{r_i} = \frac{0.1}{0.2} = 0.5, \qquad F_{13} = 1 - F_{12} = 1 - 0.62 = 0.38.$$

Hence, Eq. (1) with $J_3 = 5.67 \times 10^{-8} \times 300^4$ W/m$^2$ becomes

$$\frac{J_2 - J_1}{1/A_2 \times 0.62} + \frac{J_2 - 459.3\,\text{W/m}^2}{1/A_2 \times 0.38} = 0 \qquad -0.62J_1 + 1.00J_2 = 174.5 \qquad (2,3)$$

The radiation balance on surface $A_1$ with $E_{b3} = 5.67 \times 10^{-8} \times 500^4$ W/m$^2$ becomes

$$\frac{E_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} = \frac{J_1 - J_2}{1/A_1F_{12}} + \frac{J_1 - J_3}{1/A_1F_{13}} \qquad (4)$$

$$\frac{3543.8 - J_1}{(1-0.6)/0.6A_1} = \frac{J_1 - J_2}{1/A_1 \times 0.62} + \frac{J_1 - 459.3}{1/A_1 \times 0.38} \qquad 2.50J_1 - 0.62J_2 = 5490.2 \qquad (5,6)$$

Solve Eqs. (3) and (6) to find $J_2 = 1815$ W/m$^2$ and since $E_{b2} = J_2$,

$$T_2 = \left(\frac{E_{b2}}{\sigma}\right)^{1/4} = \left(\frac{1815\ \text{W/m}^2}{5.67 \times 10^{-8}\text{W/m}^2 \cdot \text{K}^4}\right)^{1/4} = 423\ \text{K}. \qquad <$$

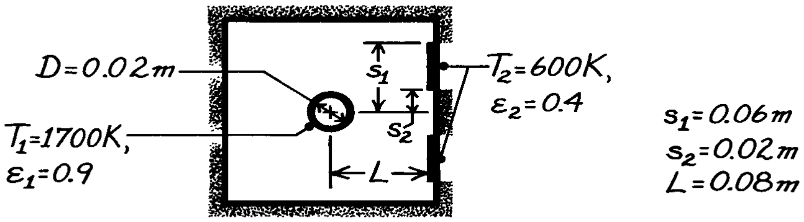
**COMMENTS:** A network representation would help to visualize the exchange relations. However, it is useful to approach the problem by recognizing there are two unknowns in the problem: $J_1$ and $J_2$; hence two radiation balances must be written. Note also the significance of $J_2 = E_{b2}$ and $J_3 = E_{b3}$.

## PROBLEM 13.67

**KNOWN:** Thermal conditions in oven used to cure strip coatings.

**FIND:** Electrical power requirement.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Diffuse-gray surfaces, (2) Furnace wall is reradiating, (3) Negligible end effects.

**ANALYSIS:** The net radiant power leaving the heater surface per unit length is

$$q_1' = \frac{E_{b1} - E_{b2}}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1'} + \dfrac{1}{A_1'F_{12} + \left[\left(1/A_1F_{1R}\right) + \left(1/A_2'F_{2R}\right)\right]^{-1}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2'}}$$

where $A_1' = \pi D = \pi(0.02\,\text{m}) = 0.0628\,\text{m}$ and $A_2' = 2(s_1 - s_2) = 0.08\,\text{m}$. The view factor between the heater and one of the strips is

$$F_{21} = \frac{D/2}{s_1 - s_2}\left[\tan^{-1}\frac{s_1}{L} - \tan^{-1}\frac{s_2}{L}\right] = \frac{0.01}{0.04}\left[\tan^{-1}\frac{0.06}{0.08} - \tan^{-1}\frac{0.02}{0.08}\right] = 0.10$$

and using the view factor relations find

$$A_1'F_{12} = A_2'F_{21} = 0.08\,\text{m} \times 0.10 = 0.008\,\text{m} \qquad F_{12} = (0.080/0.0628)0.10 = 0.127$$

$$F_{1R} = 1 - F_{12} = 1 - 0.127 = 0.873 \qquad F_{2R} = 1 - F_{21} = 1 - 0.10 = 0.90.$$

Hence, with $E_b = \sigma T^4$,

$$q_1' = \frac{5.67\times10^{-8}\left[(1700)^4 - (600)^4\right]}{\dfrac{1-0.9}{0.9\times0.0628} + \dfrac{1}{0.008 + \left[1/(0.0628\times0.873) + 1/(0.08\times0.90)\right]^{-1}} + \dfrac{1-0.4}{0.4\times0.08}}$$

$$q_1' = \frac{4.66\times10^5}{1.77 + 25.56 + 18.75} = 10{,}100\ \text{W/m}. \qquad <$$

**COMMENTS:** The radiosities for $A_1$ and $A_2$ follow from Eq. 13.19,

$$J_1 = E_{b1} - (1-\varepsilon_1)q_1'/\varepsilon_1 A_1' = 4.56\times10^5\ \text{W/m}^2$$

$$J_2 = E_{b2} + (1-\varepsilon_2)q_1'/\varepsilon_2 A_2' = 1.97\times10^5\ \text{W/m}^2.$$

From Eq. 13.31, find $J_R$ and hence $T_R$ as

$$0.0628\times0.873(J_1 - J_R) - 0.08\times0.90(J_R - J_2) = 0$$

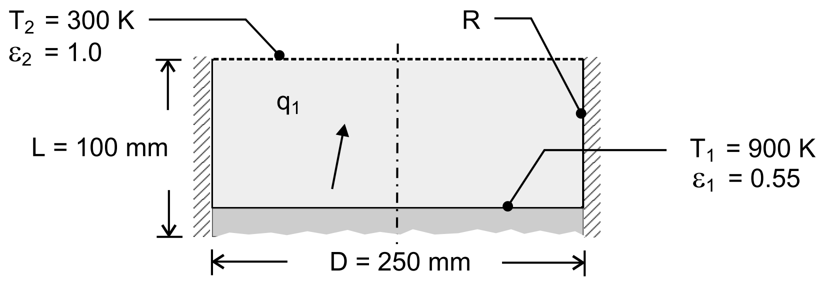
$$J_R = 3.08\times10^5\ \text{W/m}^2 = \sigma T_R^4 \qquad T_R = 1527\ \text{K}.$$

# PROBLEM 13.68

**KNOWN:** Surface temperature and emissivity of molten alloy and distance of surface from top of container. Container diameter.

**FIND:** Net rate of radiation heat transfer from surface of melt.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Opaque, diffuse, gray behavior for surface of melt, (2) Large surroundings may be represented by a hypothetical surface of temperature $T = T_{sur}$ and $\varepsilon = 1$, (3) Negligible convection at exposed side wall, (4) Adiatic side wall.

**ANALYSIS:** With negligible convection at an adiabatic side wall, the surface may be treated as reradiating. Hence, from Eq. (13.30), with $A_1 = A_2$,

$$q_1 = \frac{A_1\left(E_{b1} - E_{b2}\right)}{\dfrac{1-\varepsilon_1}{\varepsilon_1} + \dfrac{1}{F_{12} + \left[(1/F_{1R}) + (1/F_{2R})\right]^{-1}} + \dfrac{1-\varepsilon_2}{\varepsilon_2}}$$

With $\quad R_i = R_j = (D/2)/L = 1.25$ and $S = \left[1 + \left(1 + R_j^2\right)/R_i^2\right] = 2.640$, Table 13.2 yields

$$F_{12} = \frac{1}{2}\left\{S - \left[S^2 - 4\left(r_2/r_1\right)^2\right]^{1/2}\right\} = 0.458$$

Hence,

$$F_{1R} = F_{2R} = 1 - F_{12} = 0.542 \text{ and}$$

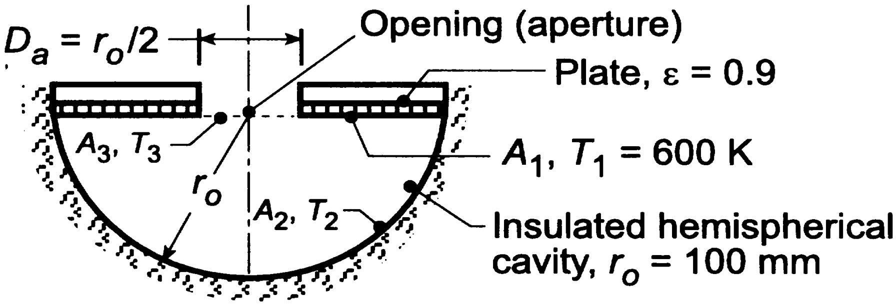
$$q_1 = \frac{\pi(0.25\text{m})^2 \times 5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\left(900^4 - 300^4\right)\text{K}^4}{\dfrac{1-0.55}{0.55} + \dfrac{1}{0.458(3.69)^{-1}} + 0} = 3295\,\text{W}$$ <

**PROBLEM 13.69**

**KNOWN:** Blackbody simulator design consisting of a heated circular plate with an opening over a well insulated hemispherical cavity.

**FIND:** (a) Radiant power leaving the opening (aperture), $D_a = r_o/2$, (b) Effective emissivity of the cavity, $\varepsilon_e$, defined as the ratio of the radiant power leaving the cavity to the rate at which the circular plate would emit radiation if it were black, (c) Temperature of hemispherical surface, $T_{hc}$, and (d) Compute and plot $\varepsilon_e$ and $T_{hc}$ as a function of the opening aperture in the circular plate, $D_a$, for the range $r_o/8 \le D_a \le r_o/2$, for plate emissivities of $\varepsilon_p = 0.5$, 0.7 and 0.9.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Plate and hemispherical surface are diffuse-gray, (2) Uniform radiosity over these same surfaces.

**ANALYSIS:** (a) The simulator can be treated as a three-surface enclosure with one re-radiating surface ($A_2$) and the opening ($A_3$) as totally absorbing with no emission into the cavity ($T_3$ = 300 K). The radiation leaving the cavity is the net radiation leaving $A_1$, $q_1$ which is equal to $-q_3$. Using Eq. 13.30,

$$q_{cav} = q_1 = -q_3 = \frac{\sigma\left(T_1^4 - T_3^4\right)}{(1-\varepsilon_1)/\varepsilon_1 A_1 + \left[A_1F_{13} + \left[(1/A_1F_{12}) + (1/A_3F_{32})\right]^{-1}\right]^{-1} + (1-\varepsilon_3)/\varepsilon_3 A_3} \quad (1)$$

Using the summation rule and reciprocity, evaluate the required view factors:

$$F_{11} + F_{12} + F_{13} = 1 \qquad F_{13} = 0 \qquad F_{12} = 1$$

$$F_{31} + F_{32} + F_{33} = 1 \qquad F_{32} = 1.$$

Substituting numerical values with $\varepsilon_3 = 1$, $T_3$ = 300 K, $A_1 = \pi\left(r_o^2 - (r_o/4)^2\right) = 15\pi r_o^2/16 = 2.945 \times 10^{-2}\ m^2$, $A_3 = \pi r_a^2 = \pi(r_o/4)^2 = 1.963 \times 10^{-3}\ m^2$ and $A_1/A_3 = 15$, and multiplying numerator and denominator by $A_1$,

$$q_{cav} = q_1 = \frac{A_1\sigma\left(T_1 - T_3^4\right)}{(1-\varepsilon_1)/\varepsilon_1 + \left\{F_{13} + \left[(1/F_{12}) + (A_1/A_3F_{32})\right]^{-1}\right\}^{-1} + 0} \quad (2)$$

Continued …..

## PROBLEM 13.69 (Cont.)

$$q_{cav} = q_1 = \frac{2.945\times10^{-2}\,m^2\times5.67\times10^{-8}\,W/m^2\cdot K^4\left(600^4-300^4\right)K^4}{(1-0.9)/0.9+\left\{0+\left[1+(15/1)\right]^{-1}\right\}^{-1}+0} = 12.6\,W \qquad <$$

(b) The effective emissivity is the ratio of the radiant power leaving the cavity to that from a blackbody having the area of the opening and temperature of the inner surface of the cavity. That is,

$$\varepsilon_e = \frac{q_{cav}}{A_3\sigma T_1^4} = \frac{12.6\,W}{1.963\times10^{-3}\,m^2\times5.67\times10^{-8}\,W/m^2\cdot K^4\times(600\,K)^4} = 0.873 \qquad (3) \qquad <$$

(c) From a radiation balance on $A_1$, find $J_1$,

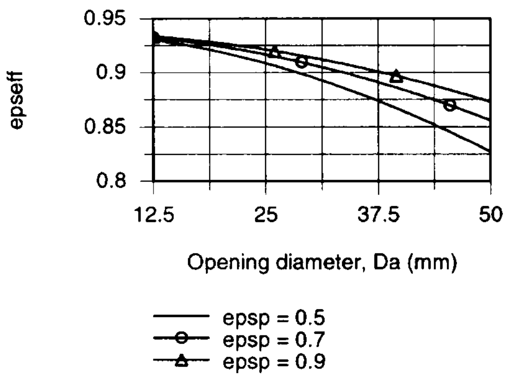
$$q_1 = 12.6W = \frac{E_{b1}-J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} = \frac{\sigma 600^4 - J_1}{(1-09)/0.9A_1} \qquad J_1 = 7301\,W/m^2 \qquad (4)$$

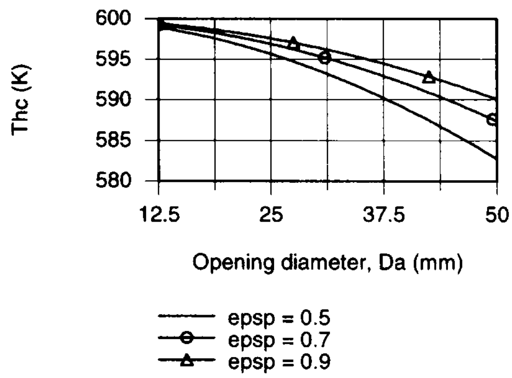
From a radiation balance on $A_2$ with $J_3 = E_{b3} = \sigma T_3^4 = 459.9\ W/m^2$ and $J_2 = \sigma T_2^4$, find

$$\frac{J_2-J_1}{(1/A_1F_{12})} + \frac{J_2-J_3}{(1/A_3F_{32})} = \frac{J_2-7301\,W/m^2}{\left(1/2.945\times10^{-2}\,m^2\right)} + \frac{J_2-459.9}{\left(1/1.963\times10^{-3}\,m^2\right)} = 0 \qquad (5)$$

$$J_2 = 6873\,W/m^2 \qquad T_2 = 590\,K. \qquad <$$

(d) Using the foregoing equations in the *IHT* workspace, $\varepsilon_e$ and $T_2$ were computed and plotted as a function of the opening, $D_a$, for selected plate emissivities, $\varepsilon_p$.




From the upper-left graph, $\varepsilon_e$ decreases with increasing opening, $D_a$, as expected. In the limit as $D_a \rightarrow 0$, $\varepsilon_3 \rightarrow 1$ since the cavity becomes a complete enclosure. From the upper-right graph, $T_{hc}$, the temperature of the re-radiating hemispherical surface decreases as $D_a$ increases. In the limit as $D_a \rightarrow 0$, $T_2$ will approach the plate temperature, $T_p = 600$ K. The effect of decreasing the plate emissivity is to decrease $\varepsilon_e$ and decrease $T_2$. Why is this so?

Continued …..

# PROBLEM 13.69 (Cont.)

**COMMENTS:** The *IHT Radiation, Tool, Radiation Tool, Radiation Exchange Analysis, Three-Surface Enclosure with Re-radiating Surface*, is especially convenient to perform the parametric analysis of part (c). A copy of the *IHT* workspace that can generate the above graphs is shown below.

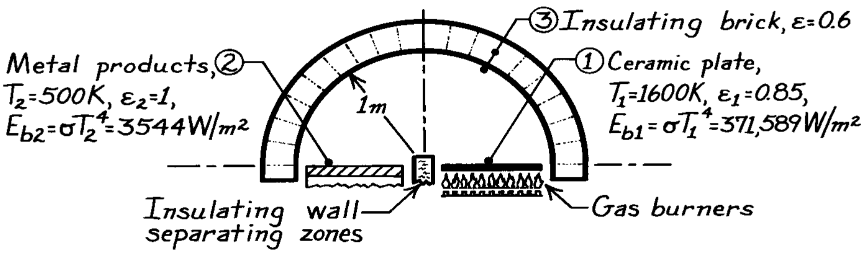
```
// Radiation Tool – Radiation Exchange Analyses, Reradiating Surface
/* For the three-surface enclosure A1, A3 and the reradiating surface A2, the net rate of radiation transfer
from the surface A1 to surface A3 is */
q1 = (Eb1 – Eb3) / ( (1 – eps1)/(eps1 * A1) + 1/(A1 * F13 + 1/(1/(A1 * F12) + 1/(A3 * F32))) + (1 –
eps3)/(eps3 * A3))  //  Eq 13.30
/* The net rate of radiation transfer from surface A3 to surface A1 is */
q3 = q1
/* From a radiation energy balance on A2, */
(J2 – J1) / (1/(A2 * F21)) + (J2 – J3)/(1/(A2 * F23) ) = 0          // Eq 13.31
/* where the radiosities J1 and J3 are determined from the radiation rate equations expressed in terms of
the surface resistances, Eq 13.22 */
q1 = (Eb1 – J1) / ((1 – eps1) / (eps1 * A1))
q3 = (Eb3 – J3) / ((1 – eps3) / (eps3 * A3))
// The blackbody emissive powers for A1 and A3 are
Eb1 = sigma * T1^4
Eb3 = sigma * T3^4
// For the reradiating surface,
J2 = Eb2
Eb2 = sigma * T2^4              // Stefan-Boltzmann constant, W/m^2·K^4
sigma = 5.67E-8

// Effective emissivity:
epseff = q1 / (A3 * Eb1)                    // Eq (3)

// Areas:
A1 = pi * ( ro^2 · ra^2)
A2 = 0.5 * pi * (2 * ro)^2                  // Hemisphere, As = 0.5 * pi * D^2
A3 = pi * ra^2

// Assigned Variables
T1 = 600                        // Plate temperature, K
eps1 = 0.9                      // Plate emissivity
T3 = 300                        // Opening temperature, K; Tsur
eps3 = 0.9999                   // Opening emissivity; not zero to avoid divide-by-zero error
ro = 0.1                        // Hemisphere radius, m
Da = 0.05                       // Opening diameter; range ro/8 to ro/2; 0.0125 to 0.050
Da_mm = Da * 1000                           // Scaling for plot
Ra = Da / 2                     // Opening radius
```

# PROBLEM 13.70

**KNOWN:** Long hemi-cylindrical shaped furnace comprised of three zones.

**FIND:** (a) Heat rate per unit length of the furnace which must be supplied by the gas burners and (b) Temperature of the insulating brick.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Surfaces are opaque, diffuse-gray or black, (2) Surfaces have uniform temperatures and radiosities, (3) Surface 3 is perfectly insulated, (4) Negligible convection, (5) Steady-state conditions.

**ANALYSIS:** (a) From an energy balance on the ceramic plate, the power required by the burner is $q'_{burners} = q'_1$, the net radiation leaving $A_1$; hence

$$q'_1 = A_1F_{12}\left(J_1 - J_2\right) + A_1F_{13}\left(J_1 - J_3\right) = 0 + A_1F_{13}\left(J_1 - J_3\right) \quad (1)$$

since $F_{12} = 0$. Note that $J_2 = E_{b2} = \sigma T_2^4$ and that $J_1$ and $J_3$ are unknown. Hence, we need to write two radiation balances.

$A_1$:
$$\frac{E_{b1} - J_1}{\left(1-\varepsilon_1\right)/\varepsilon_1 A_1} = q'_1 = 0 + A_1FF_{13}\left(J_1 - J_3\right) \quad (2)$$

$A_3$:
$$0 = A_3F_{31}\left(J_3 - J_1\right) + A_3F_{32}\left(J_3 - E_{b2}\right)$$

$$J_3 = \frac{J_1 + E_{b2}}{2} \quad (3)$$

since $F_{31} = F_{23}$. Substituting Eq. (3) into (2), find

$$\left(371,589 - J_1\right)/\left(1-0.85\right)/0.85 = 1\left[J_1 - \left(J_1 + 3,544\right)/2\right]$$

$$J_1 = 341,748\,\mathrm{W/m^2} \qquad J_3 = 172,646\,\mathrm{W/m^2}$$

using $E_{b1} = \sigma T_1^4 = 371,589\,\mathrm{W/m^2}$ and $E_{b2} = \sigma T_2^4 = 3544\,\mathrm{W/m^2}$. Substituting into Eq. (1), find

$$q'_1 = 1\ \mathrm{m} \times 1\left(341,748 - 172,646\right)\mathrm{W/m^2} = 169\ \mathrm{kW/m}.$$ <

(b) The temperature of the insulating brick, acting as a reradiating surface, is

$$J_3 = E_{b3} = \sigma T_3^4$$

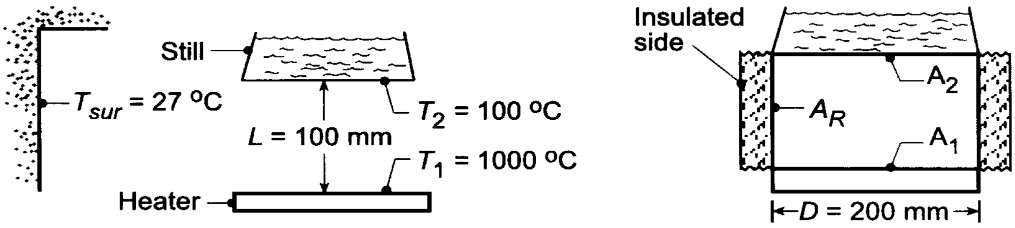
$$T_3 = \left(J_3/\sigma\right)^{1/4} = \left(172,646\,\mathrm{W/m^2}/5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\right)^{1/4} = 1320\ \mathrm{K}.$$ <

## PROBLEM 13.71

**KNOWN:** Steam producing still heated by radiation.

**FIND:** (a) Factor by which the vapor production could be increased if the cylindrical side of the heater were insulated rather than open to the surroundings, and (b) Compute and plot the net heat rate of radiation transfer to the still, as a function of the separation distance L for the range 15 ≤ L ≤ 100 mm for heater temperatures of 600, 800, 1000°C considering the cylindrical sides to be insulated.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Still and heater surfaces are black, (2) Surroundings are isothermal and large compared to still heater surfaces, (3) Insulation is diffuse-gray, (4) Negligible convection.

**ANALYSIS:** (a) The vapor production will be proportional to the net radiation exchange to the still. For the case when the sides are open (o) to the surroundings, the net radiation exchange leaving $A_2$ is from Eq. 13.13.

$$q_{2,o} = q_{21} + q_{2s} = A_2 F_{21} \sigma \left(T_2^4 - T_1^4\right) + A_2 F_{2s} \sigma \left(T_2^4 - T_{sur}^4\right)$$

where $F_{2s} = 1 - F_{21}$ and $F_{21}$ follows from Fig. 13.5 with $L/r_i$ = 100/100 = 1, $r_j/L$ = 100/100 = 1.

$$F_{21} = 0.38$$

With $A_2 = \pi D^2/4$, find

$$q_{2,o} = \frac{\pi(0.200\text{m})^2}{4} \times 5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4 \left\{0.38\left(373^4 - 1273^4\right)\text{K}^4 + (1-0.38)\left(373^4 - 300^4\right)\text{K}^4\right\}$$

$$q_{2,o} = -1752\,\text{W}. \qquad \leftarrow \text{w/o insulation}$$

With the cylindrical side insulated (i), a three-surface, re-radiating enclosure is formed. Eq. 13.30 can be used to evaluate $q_{2,i}$ and with $\varepsilon_2 = \varepsilon_1 = 1$, the relation is

$$q_{2,i} = \frac{\sigma\left(T_2^4 - T_1\right)^4}{\dfrac{1}{A_1F_{12} + \left[(1/A_1F_{1R}) + (1/A_2F_{2R})\right]^{-1}}} = A_1\left\{F_{12} + \left[1/F_{1R} + 1/F_{2R}\right]^{-1}\right\}\sigma\left(T_2^4 - T_1^4\right)$$

Recall $F_{12}$ = 0.38 and $F_{1R} = 1 - F_{12}$ = 1 – 0.38 = 0.62, giving

$$q_{2,i} = \frac{\pi(0.100\text{ m})^2}{4}\left\{0.38 + \left[1/0.62 + 1/0.62\right]^{-1}\right\}5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}\left(373^4 - 1273^4\right)\text{K}^4$$

$$q_{2,i} = -3204\text{ W}. \qquad \leftarrow \text{w insulation}$$

Continued …..

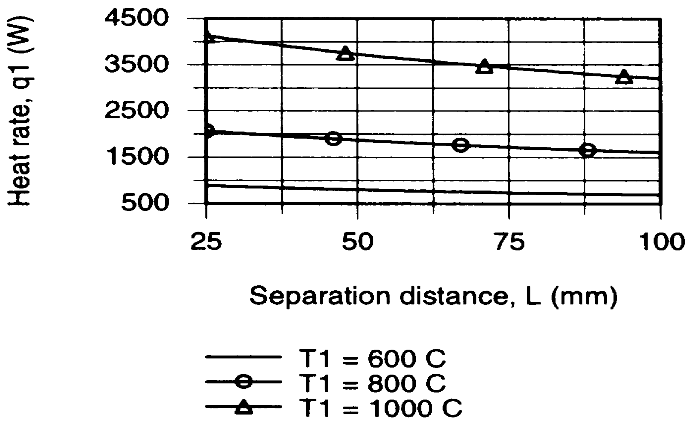
## PROBLEM 13.71 (Cont.)

Hence, the vapor production rate is increased by a factor

$$\frac{q_2)_{insul}}{q_2)_{open}} = \frac{3204\,\text{W}}{1752\,\text{W}} = 1.83$$

That is, the vapor production is increased by 83%. <

(b) The *IHT Radiation Tool – Radiation Exchange Analysis* for the *Three-Surface Enclosure* with a *reradiating surface* can be used directly to compute the net heat rate to the still, $q_1 = q_2$, as a function of the separation distance L for selected heater temperatures $T_1$. The results are plotted below.




Note that the heat rate for all values of $T_1$ decreases as expected with increasing separation distances, but not markedly. For any separation distance, increasing the heater temperature greatly influences the heat rate. For example, at L = 50 mm, increasing $T_1$ from 600 to 800 K, causes a nearly 6 fold increase in the heat rate. But increasing $T_1$ from 800 to 1000 K causes only a 2 fold increase in the heat rate.

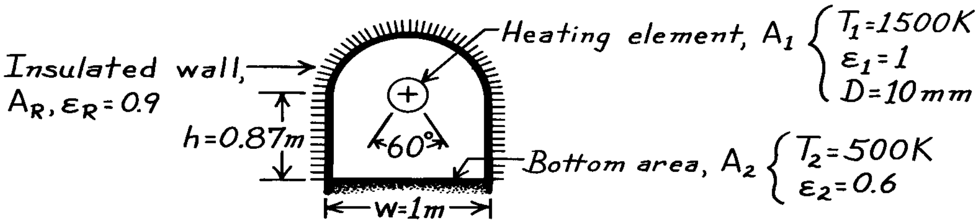
**COMMENTS:** When assigning the emissivity variables ($\varepsilon_1$, $\varepsilon_2$, $\varepsilon_3$) in the *IHT* model mentioned above, set $\varepsilon = 0.999$, rather than 1.0, to avoid a “division by zero” error message. You could also call up the *Radiation Tool, View Factor Coaxial Parallel Disk* to calculate $F_{12}$.

# PROBLEM 13.72

**KNOWN:** Furnace with cylindrical heater and re-radiating, insulated walls.

**FIND:** (a) Power required to maintain steady-state conditions, (b) Temperature of wall area.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Surfaces are diffuse-gray, (2) Furnace is of length $\ell$ where $\ell \gg$ w, (3) Convection is negligible.

**ANALYSIS:** (a) Consider the furnace as a three surface enclosure with the walls, $A_R$, represented as a re-radiating surface. The power that must be supplied to the heater is determined by Eq. 13.30.

$$q_1 = \frac{\sigma\left(T_1^4 - T_2^4\right)}{(1-\varepsilon_1)/\varepsilon_1 A_1 + \left[A_1 F_{12} + \left[(1/A_1 F_{1R}) + (1/A_2 F_{2R})\right]^{-1}\right]^{-1} + (1-\varepsilon_2)/\varepsilon_2 A_2}$$

Note that $A_1 = \pi d \ell$ and $A_2 = w\ell$. By inspection and the summation rule, find $F_{12}$ = 60°/360° = 0.167, $F_{1R} = 1 - F_{12} = 1 - 0.167 = 0.833$, and $F_{2R} \approx 1$. With $q_1' = q_1/\ell$,

$$q_1' = \frac{5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(1500^4 - 500^4\right)\text{K}^4}{0 + \left[\pi\left(10\times10^{-3}\right)\text{m}\times0.167 + \left[\left(1/\pi\left(10\times10^{-3}\,\text{m}\right)\times0.83\right) + (1/1\,\text{m}\times1)\right]^{-1}\right]^{-1} + (1-0.6)/0.6\times1\ \text{m}}$$

$$q_1' = 8518\ \text{W/m}.$$ <

(b) To determine the wall temperature, apply the radiation balance, Eq. 13.31,

$$\frac{J_1 - J_R}{(1/A_1 F_{1R})} = \frac{J_R - J_2}{(1/A_2 F_{2R})} \qquad \text{or} \qquad \frac{J_1 - J_R}{\left(1/\pi 10\times10^{-3}\,\text{m}\times0.833\right)} = \frac{J_R - J_2}{(1/1\text{m}\times1)}.$$

$$J_R = \sigma T_R^4 = (J_1 + 38.21 J_2)/39.21. \qquad (1)$$

Since $A_1$ is a blackbody, $J_1 = E_{b1} = \sigma T_1^4$. To determine $J_2$, use Eq. 13.19. Noting that $q_1' = -q_2'$, find

$$q_2 = (E_{b2} - J_2)/(1-\varepsilon_2)/\varepsilon_2 A_2 \qquad \text{or} \qquad J_2 = E_{b2} - q_2(1-\varepsilon_2)/\varepsilon_2 A_2$$

$$J_2 = 5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\,(500\ \text{K})^4 - \frac{(-8518\ \text{W/m})(1-0.6)}{0.6(1\text{m})} = 9222\ \text{W/m}^2.$$

Substituting this value for $J_2$ into Eq. (1), the wall temperature can be calculated.

$$J_R = \left(5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\,(1500\text{K})^4 + 38.21\times9222\,\text{W/m}^2\right)/39.21 = 16{,}308\,\text{W/m}^2$$

$$T_R = (J_R/\sigma)^{1/4} = \left(16{,}308\,\text{W/m}^2/5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\right)^{1/4} = 732\ \text{K}.$$ <

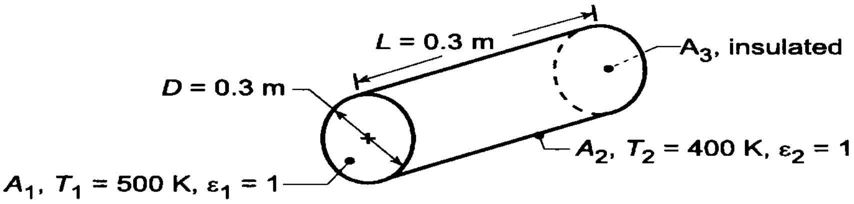
**COMMENTS:** Considering the entire wall as a single re-radiating surface may be a poor assumption since $J_R$ is not likely to be uniform over this large an area. It would be appropriate to consider several isothermal zones for improved accuracy.

## PROBLEM 13.73

**KNOWN:** Circular furnace with one end ($A_1$) and the lateral surface ($A_2$) black. Other end ($A_3$) is insulated.

**FIND:** (a) Net radiation heat transfer from each surface and (b) Temperature of $A_3$, and (c) Compute and plot $T_3$ as a function of tube length L for the range $0.1 \le L \le 0.5$ m with D = 0.3 m.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Surface $A_3$ is diffuse-gray, (2) Uniform radiosity over $A_3$.

**ANALYSIS:** (a) Since $A_3$ is insulated, the net radiation from $A_3$ is $q_3 = 0$. Using Eq. 13.30, find

$$q_1 = -q_2 = \frac{\sigma\left(T_1^4 - T_2^4\right)}{(1-\varepsilon_1)/\varepsilon_1 A_1 + \left[A_1F_{12} + \left[(1/A_1F_{13}) + (1/A_2F_{23})\right]^{-1}\right]^{-1} + (1-\varepsilon_2)/\varepsilon_2 A_2}$$

and since $\varepsilon_1 = \varepsilon_2 = 1$, $\qquad q_1 = -q_2 = \left\{A_1F_{12} + \left[(1/A_1F_{13}) + (1/A_2F_{23})\right]^{-1}\right\}\sigma\left(T_1^4 - T_2^4\right).$

Considering $A_1$ and $A_3$ as coaxial parallel disks, from Table 13.2 (Fig. 13.5) find $F_{13}$,

$$R_1 = r_1/L = 0.15/0.30 = 0.5 \qquad R_2 = r_2/L = 0.15/0.30 = 0.5$$

$$S = 1 + \left(1 + R_2^2\right)/R_1^2 = 1 + \left(1 + 0.5^2\right)/0.5^2 = 6$$

$$F_{13} = 0.5\left\{S - \left[S^2 - 4\left(r_2/r_1\right)^2\right]^{1/2}\right\} = 0.5\left\{6 - \left[6^2 - 4(0.5/0.5)\right]^{1/2}\right\} = 0.172.$$

From the summation rule and symmetry $\qquad F_{12} = 1 - F_{13} = 1 - 0.172 = 0.828.$

From reciprocity, with $F_{32} = F_{12}$

$$F_{23} = A_3F_{32}/A_2 = \left(\pi D^2/4\right)F_{12}/(\pi DL) = DF_{12}/4L = 0.3\,\text{m} \times 0.828/4 \times 0.3\,\text{m} = 0.207.$$

With $A_1 = \pi D^2/4 = \pi(0.3\text{ m})^2/4 = 0.07069\text{ m}^2$ and $A_2 = \pi DL = \pi(0.3\text{ m})(0.3\text{ m}) = 0.2827\text{ m}^2$,

$$q_1 = -q_2 = \left\{0.07069\,\text{m}^2 \times 0.828 + \left[\left(1/0.07069\,\text{m}^2 \times 0.172\right)\right.\right.$$

$$\left.\left. + \left(1/0.2827\,\text{m}^2 \times 0.207\right)\right]^{-1}\right\} 5.67 \times 10^{-8}\left(500^4 - 400^4\right)\text{K}^4 \qquad q_1 = -q_2 = 143\,\text{W} \qquad <$$

(b) From the radiation balance on $A_3$, Eq. 13.31, find $T_3$, with $J_1 = E_{b1}$ and $J_2 = E_{b2}$,

$$\frac{E_{b1} - J_3}{(1/A_1F_{13})} - \frac{J_3 - E_{b2}}{(1/A_2F_{23})} = 0 \qquad \frac{\sigma(500\text{ K})^4 - J_3}{(1/0.07069\,\text{m} \times 0.172)} - \frac{J_3 - \sigma(400\text{ K})^4}{\left(1/0.2827\,\text{m}^2 \times 0.207\right)} = 0$$

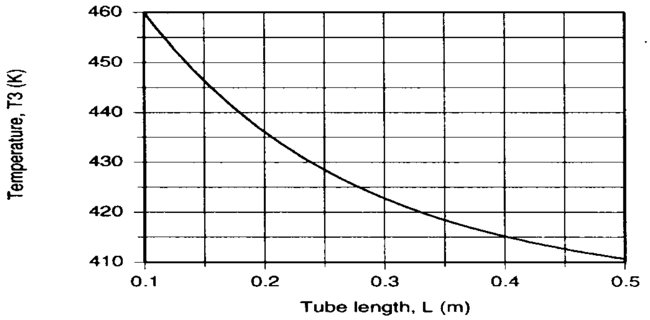
$$J_2 = E_{b3} = \sigma T_3^4 = 1811\text{ W/m}^2 \qquad T_3 = 423\text{ K}. \qquad <$$

(c) Using the *IHT Radiation Tools – Radiation Exchange Analysis, Three surface enclosure with a reradiating surface* and *View Factors, Three-dimensional geometries, Coaxial parallel disks – and*

Continued …..

## PROBLEM 13.73 (Cont.)

appropriate view factor relations developed in part (a), $T_3$ was computed as a function of L with the diameter, D = 0.3 m, and is plotted below.




Note that $T_3$ decreases with increasing tube length. In the limit as $L \to \infty$, $T_3$ will approach $T_2$. In the limit as $L \to 0$, $T_3$ will approach $T_1$. Is this intuitively satisfying?

**COMMENTS:** The *IHT* workspace used for the part (c) analysis is copied below.

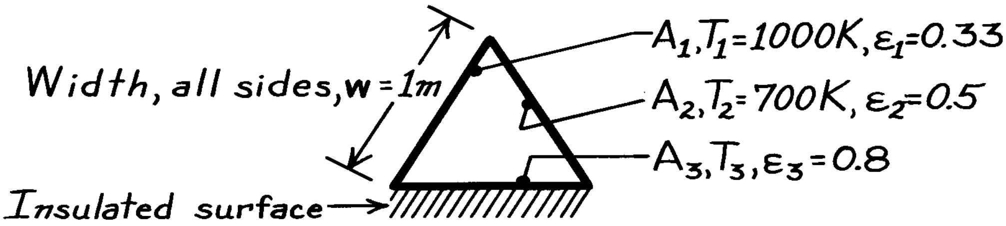
```
// Radiation Tool:
// Radiation Exchange Analysis, three-surface enclosure with a reradiating surface
/* For the three-surface enclosure A1, A2 and the reradiating surface A3, the net rate of radiation transfer
from the surface A1 to surface A2 is */
q1 = (Eb1 – Eb2) / ( (1 – eps1) / (eps1 * A1) + 1 / (A1 * F12 + 1 / (1 / (A1 * F13) + 1 / (A2 * F23))) + (1 –
eps2) / (eps2 * A2))              // Eq 13.30
/ * The net rate of radiation transfer from surface A2 to surface A1 is */
q2 = -q1
/ * From a radiation energy balance on A3, */
(J3 – J1) / (1 / (A3 * F31) ) + (J3 – J2) / (1 / (A3 * F32) ) = 0              // Eq 13.31
/* where the radiosities J1 and J2 are determined from the radiation rate equations expressed in terms of
the surface resistances, Eq 13.22 */
q1 = (Eb1 – J1) / ((1 – eps1) / (eps1 * A1))
q2 = (Eb2 – J2) / ((1 – eps2) / (eps2 * A2))
// The blackbody emissive powers for A1 and A2 are
Eb1 = sigma * T1^4
Eb2 = sigma * T2^4
// For the reradiating surface,
J3 = Eb3
Eb3 = sigma * T3^4
sigma = 5.67E-8               // Stefan-Boltzmann constant, W/m^2·K^4

// Radiation Tool – view factor, F13:
/* The view factor, F13, for coaxial parallel disks, is * /
F13 = 0.5 * (S – sqrt(S^2 – 4 * (r3 / r1)^2))
// where
R1 = r1 / L
R3 = r3 / L
r1 = D1 / 2
r3 = r1
S = 1 + (1 + R3^2) / R1^2
// See Table 13.2 for schematic of this three-dimensional geometry.
// View factor relations:
F12 = 1 – F13                 // Summation rule, A1
F32 = F12                     // Symmetry condition
F23 = A3 * F32 / A2           // Reciprocity relation
F31 = A1 * F13 / A3           // Reciprocity relation
// Areas:
A1 = pi * D1^2/4
A2 = pi * D2 * L
A3 = A1
// Assigned variables:
T1 = 500                      // Temperature, K
D1 = 0.3                      // Diameter, m
eps1 = 0.9999                 // Emissivity; avoiding  "divide-by-zero error"
D2 = D1                       // Diameter, m
eps2 = 0.9999                 // Emissivity; avoiding "divide-by-zero error"
T2 = 400                      // Temperature, K
L = 0.3                       // Length, m
```

## PROBLEM 13.74

**KNOWN:** Very long, triangular duct with walls that are diffuse-gray.

**FIND:** (a) Net radiation transfer from surface $A_1$ per unit length of duct, (b) The temperature of the insulated surface, (c) Influence of $\varepsilon_3$ on the results; comment on exactness of results.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Surfaces are diffuse-gray, (2) Duct is very long; end effects negligible.

**ANALYSIS:** (a) The duct approximates a three-surface enclosure for which the third surface ($A_3$) is re-radiating. Using Eq. 13.30 with $A_3 = A_R$, the net exchange is

$$q_1 = -q_2 = \frac{E_{b1} - E_{b2}}{\frac{(1-\varepsilon_1)}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12} + (1/A_1 F_{1T} + 1/A_2 F_{2R})^{-1}} + \frac{(1-\varepsilon_2)}{\varepsilon_2 A_2}} \quad (1)$$

From symmetry, $F_{12} = F_{1R} = F_{2R} = 0.5$. With $A_1 = A_2 = w \cdot \ell$, where $\ell$ is the length normal to the page and w = 1 m,

$$q_1' = q_1/\ell = (q_1/A_1)w$$

$$q_1' = \frac{(56,700 - 13,614)\,\mathrm{W/m^2} \times 1\,\mathrm{m}}{\frac{(1-0.33)}{0.33} + \frac{1}{0.5 + (1/0.5 + 1/0.5)^{-1}} + \frac{(1-0.5)}{0.5}} = 9874\ \mathrm{W/m}. \quad <$$

(b) From a radiation balance on $A_R$,

$$q_R = q_3 = 0 = \frac{E_{b3} - J_1}{(A_3 F_{31})^{-1}} + \frac{E_{b3} - J_2}{(A_3 F_{32})^{-1}} \qquad \text{or} \qquad E_{b3} = \frac{J_1 + J_2}{2}. \quad (2)$$

To evaluate $J_1$ and $J_2$, use Eq. 13.19,

$$J_i = E_{b,i} - \frac{q_i}{A_i}\frac{(1-\varepsilon_i)}{\varepsilon_i} \begin{cases} J_1 = 56,700 - (9874)\dfrac{1-0.33}{0.33} = 36,653\ \mathrm{W/m^2} \\[2ex] J_2 = 13,614 - (-9874)\dfrac{1-0.5}{0.5} = 23,488\ \mathrm{W/m^2} \end{cases}$$

From Eq. (2), now find

$$T_3 = (E_{b3}/\sigma)^{1/4} = ([J_1 + J_2]/2\sigma)^{1/4} = \left(\frac{(36,653 + 23,488)\,\mathrm{W/m^2}}{2\left(5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\right)}\right)^{1/4} = 853\ \mathrm{K}. \quad <$$

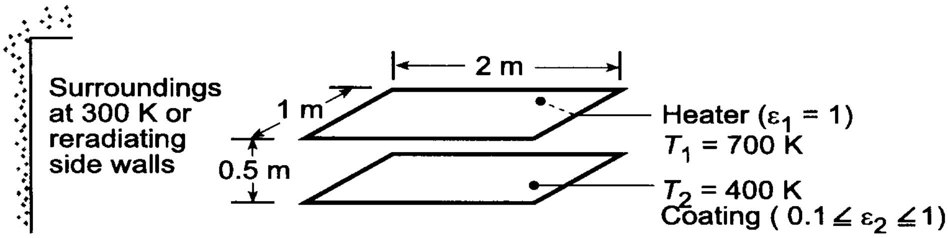
(c) Since $A_3$ is adiabatic or re-radiating, $J_3 = Eb_3$. Therefore, the value of $\varepsilon_3$ is of no influence on the radiation exchange or on $T_3$. In using Eq. (1), we require uniform radiosity over the surfaces. This requirement is not met near the corners. For best results we should subdivide the areas such that they represent regions of uniform radiosity. Of course, the analysis then becomes much more complicated.

## PROBLEM 13.75

**KNOWN:** Dimensions for aligned rectangular heater and coated plate. Temperatures of heater, plate and large surroundings.

**FIND:** (a) Electric power required to operate heater, (b) Heater power required if reradiating sidewalls are added, (c) Effect of coating emissivity and electric power.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state, (2) Blackbody behavior for surfaces and surroundings (Parts (a) and (b)).

**ANALYSIS:** (a) For $\varepsilon_1 = \varepsilon_2 = 1$, the net radiation leaving $A_1$ is

$$q_{elec} = q_1 = A_1F_{12}\sigma\left(T_1^4 - T_2^4\right) + A_1F_{1sur}\sigma\left(T_1^4 - T_{sur}^4\right).$$

From Fig. 13.4, with Y/L = 1/0.5 = 2 and X/L = 2/0.5 = 4, the view factors are $F_{12} \approx 0.5$ and $F_{sur} \approx 1 - 0.5 = 0.5$. Hence,

$$q_{elec} = \left(2\text{m}^2\right)0.5\times5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left[(700\ \text{K})^4 - (400\ \text{K})^4\right]$$
$$+\left(2\text{m}^2\right)0.5\times5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left[(700\ \text{K})^4 - (300\ \text{K})^4\right] = (12,162 + 13,154)\,\text{W} = 25,316\,\text{W}. \quad <$$

(b) With the reradiating walls, the net radiation leaving $A_1$ is $q_{elec} = q_1 = q_{12}$. From Eq. 13.30 with $\varepsilon_1 = \varepsilon_2 = 1$ and $A_1 = A_2$,

$$q_{elec} = A_1\sigma\left(T_1^4 - T_2^4\right)\left\{F_{12} + \left[(1/F_{1R}) + (1/F_{2R})\right]^{-1}\right\}$$

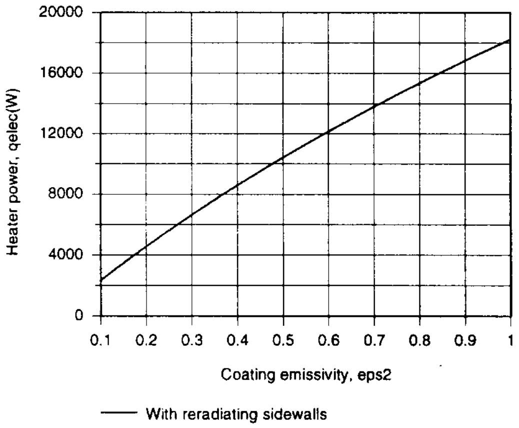
$$q_{elec} = \left(2\,\text{m}^2\right)5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left[(700\ \text{K})^4 - (400\ \text{K})^4\right]\times\left\{0.5 + \left[(1/0.5) + (1/0.5)\right]^{-1}\right\}$$

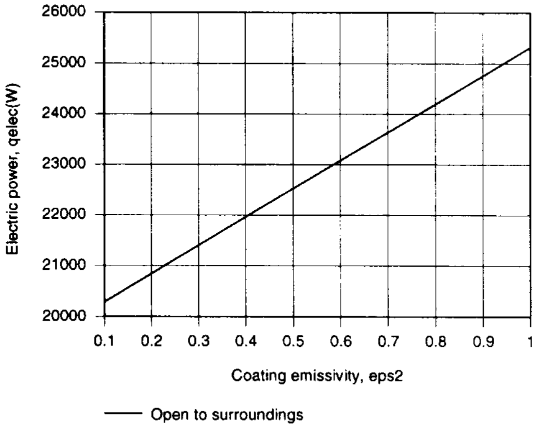
$$q_{elec} = 18,243\,\text{W}. \quad <$$

(c) Separately using the *IHT Radiation* Tool Pad for a three-surface enclosure, with one surface reradiating, and to perform a radiation exchange analysis for a three-surface enclosure, with one surface corresponding to large surroundings, the following results were obtained.

Continued …..

# PROBLEM 13.75 (Cont.)




In both cases, the required heater power decreases with decreasing $\varepsilon_2$, and the trend is attributed to a reduction in $\alpha_2 = \varepsilon_2$ and hence to a reduction in the rate at which radiant energy must be absorbed by the surface to maintain the prescribed temperature.

**COMMENTS:** With the reradiating walls in part (b), it follows from Eq. 13.31 that

$$J_R = E_{bR} = (J_1 + J_2)/2 = (E_{b1} + E_{b2})/2.$$

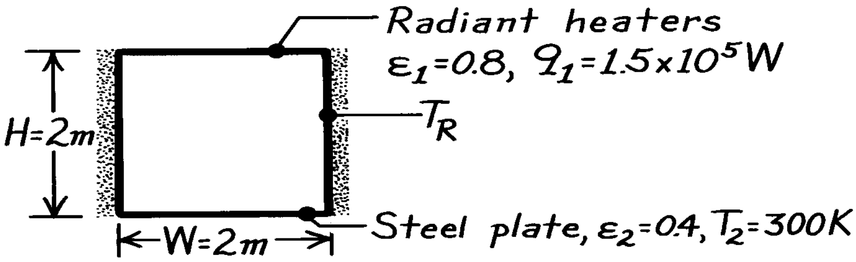
Hence, $T_R = 604$ K. The reduction in $q_{elec}$ resulting from use of the walls is due to the enhancement of radiation to the heater, which, in turn, is due to the presence of the high temperature walls.

## PROBLEM 13.76

**KNOWN:** Configuration and operating conditions of a furnace. Initial temperature and emissivity of steel plate to be treated.

**FIND:** (a) Heater temperature, (b) Sidewall temperature.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Opaque, diffuse-gray surface behavior, (3) Negligible convection, (4) Sidewalls are re-radiating.

**ANALYSIS:** (a) From Eq. 13.30

$$q_1 = \frac{E_{b1} - E_{b2}}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1 F_{12} + \left[(A_1 F_{1R})^{-1} + (A_2 F_{2R})^{-1}\right]^{-1}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}} = 1.5\times10^5\ \text{W}$$

Note that $A_1 = A_2 = 4\ \text{m}^2$ and $E_{b2} = \sigma T_2^4 = 5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4\ (300\ \text{K})^4 = 459\ \text{W/m}^2$. From Fig. 13.4, with X/L = Y/L = 1, $F_{12} = 0.2$; hence $F_{1R} = 1 - F_{12} = 0.8$, and $F_{2R} = F_{1R} = 0.8$. With $(1-\varepsilon_1)/\varepsilon_1 = 0.25$ and $(1 - \varepsilon_2)/\varepsilon_2 = 1.5$, find

$$\frac{1.5\times10^5\ \text{W}}{4\text{m}^2} = \frac{E_{b1} - 459\ \text{W/m}^2}{0.25 + \dfrac{1}{0.2 + [1.25 + 1.25]^{-1}} + 1.5} = \frac{E_{b1} - 459\ \text{W/m}^2}{3.417}$$

$$E_{b1} = 1.28\times10^5\ \text{W/m}^2 + 459\ \text{W/m}^2 = 1.29\times10^5\ \text{W/m}^2 = \sigma T_1^4$$

$$T_1 = \left(1.29\times10^5\ \text{W/m}^2 / 5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)^{1/4} = 1228\ \text{K}.$$

(b) From Eq. 13.31,it follows that, with $A_1F_{1R} = A_2F_{2R}$,

$$J_R = \sigma T_R^4 = (J_1 + J_2)/2$$

From Eq. 13.19, $$J_1 = E_{b1} - \frac{(1-\varepsilon_1)}{\varepsilon_1 A_1} q_1 = 1.29\times10^5\ \text{W/m}^2 - \frac{0.2\times1.5\times10^5\ \text{W}}{0.8\times4\text{m}^2}$$

$$J_1 = 1.196\times10^5\ \text{W/m}^2.$$

With $q_2 = q_1 = -\ 1.5 \times 10^5$ W,

$$J_2 = E_{b2} - \frac{(1-\varepsilon_2)}{\varepsilon_2 A_2} q_2 = 459\ \text{W/m}^2 + \frac{0.6}{0.4\times4\text{m}^2} 1.5\times10^5\ \text{W} = 5.67\times10^4\ \text{W/m}^2$$

$$T_R = \left(\frac{1.196\times10^5\ \text{W/m}^2 + 5.67\times10^4\ \text{W/m}^2}{2\times5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4}\right)^{1/4} = 1117\ \text{K}.$$ <

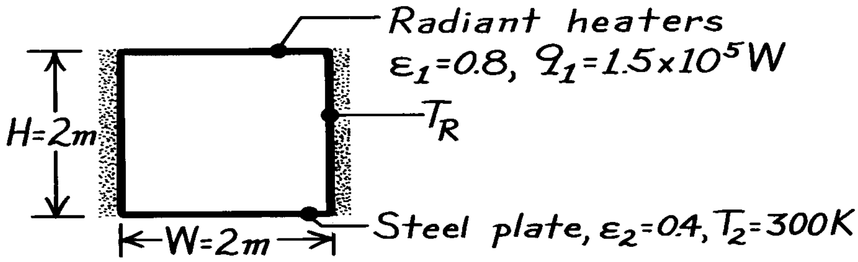
**COMMENTS:** (1) The above results are approximate, since the process is actually transient. (2) $T_1$ and $T_R$ will increase with time as $T_2$ increases.

## PROBLEM 13.76

**KNOWN:** Configuration and operating conditions of a furnace. Initial temperature and emissivity of steel plate to be treated.

**FIND:** (a) Heater temperature, (b) Sidewall temperature.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Opaque, diffuse-gray surface behavior, (3) Negligible convection, (4) Sidewalls are re-radiating.

**ANALYSIS:** (a) From Eq. 13.30

$$q_1 = \frac{E_{b1} - E_{b2}}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1F_{12} + \left[\left(A_1F_{1R}\right)^{-1} + \left(A_2F_{2R}\right)^{-1}\right]^{-1}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}} = 1.5\times10^5\ \text{W}$$

Note that $A_1 = A_2 = 4\ \text{m}^2$ and $E_{b2} = \sigma T_2^4 = 5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4\ (300\ \text{K})^4 = 459\ \text{W/m}^2$. From Fig. 13.4, with X/L = Y/L = 1, $F_{12} = 0.2$; hence $F_{1R} = 1 - F_{12} = 0.8$, and $F_{2R} = F_{1R} = 0.8$. With $(1-\varepsilon_1)/\varepsilon_1 = 0.25$ and $(1 - \varepsilon_2)/\varepsilon_2 = 1.5$, find

$$\frac{1.5\times10^5\ \text{W}}{4\text{m}^2} = \frac{E_{b1} - 459\ \text{W/m}^2}{0.25 + \dfrac{1}{0.2 + \left[1.25 + 1.25\right]^{-1}} + 1.5} = \frac{E_{b1} - 459\ \text{W/m}^2}{3.417}$$

$$E_{b1} = 1.28\times10^5\ \text{W/m}^2 + 459\ \text{W/m}^2 = 1.29\times10^5\ \text{W/m}^2 = \sigma T_1^4$$

$$T_1 = \left(1.29\times10^5\ \text{W/m}^2 / 5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)^{1/4} = 1228\ \text{K}.$$

(b) From Eq. 13.31,it follows that, with $A_1F_{1R} = A_2F_{2R}$,

$$J_R = \sigma T_R^4 = \left(J_1 + J_2\right)/2$$

From Eq. 13.19, $$J_1 = E_{b1} - \frac{(1-\varepsilon_1)}{\varepsilon_1 A_1}q_1 = 1.29\times10^5\ \text{W/m}^2 - \frac{0.2\times1.5\times10^5\ \text{W}}{0.8\times4\text{m}^2}$$

$$J_1 = 1.196\times10^5\ \text{W/m}^2.$$

With $q_2 = q_1 = -\ 1.5 \times 10^5$ W,

$$J_2 = E_{b2} - \frac{(1-\varepsilon_2)}{\varepsilon_2 A_2}q_2 = 459\ \text{W/m}^2 + \frac{0.6}{0.4\times4\text{m}^2}1.5\times10^5\ \text{W} = 5.67\times10^4\ \text{W/m}^2$$

$$T_R = \left(\frac{1.196\times10^5\ \text{W/m}^2 + 5.67\times10^4\ \text{W/m}^2}{2\times5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4}\right)^{1/4} = 1117\ \text{K}.$$ <

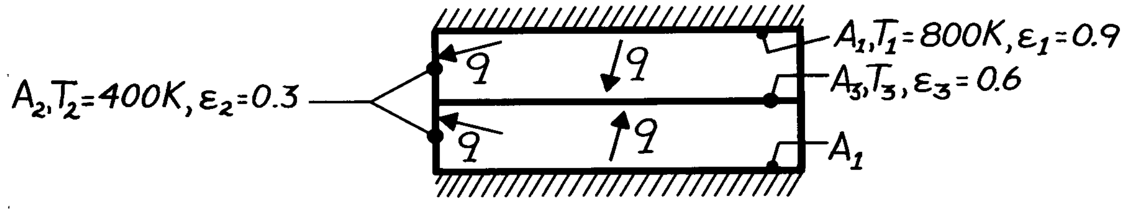
**COMMENTS:** (1) The above results are approximate, since the process is actually transient. (2) $T_1$ and $T_R$ will increase with time as $T_2$ increases.

## PROBLEM 13.77

**KNOWN:** Dimensions, surface radiative properties, and operating conditions of an electrical furnace.

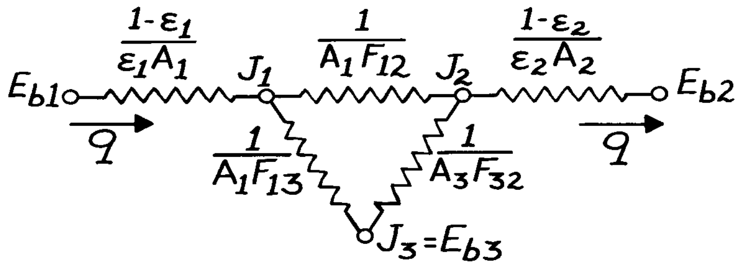
**FIND:** (a) Equivalent radiation circuit, (b) Furnace power requirement and temperature of a heated plate.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Opaque, diffuse-gray surfaces, (3) Negligible plate temperature gradients, (4) Back surfaces of heater are adiabatic, (5) Convection effects are negligible.

**ANALYSIS:** (a) Since there is symmetry about the plate, only one-half (top or bottom) of the system need be considered. Moreover, the plate must be adiabatic, thereby playing the role of a re-radiating surface.




<

(b) Note that $A_1 = A_3 = 4\ m^2$ and $A_2 = (0.5\ m \times 2\ m)4 = 4\ m^2$. From Fig. 13.4, with X/L = Y/L = 4, $F_{13} = 0.62$. Hence

$$F_{12} = 1 - F_{13} = 0.38, \quad \text{and} \quad F_{32} = F_{12} = 0.38.$$

It follows that

$$A_1F_{12} = 4(0.38) = 1.52\,m^2$$

$$A_1F_{13} = 4(0.62) = 2.48\,m^2, \qquad (1-\varepsilon_1)/\varepsilon_1A_1 = 0.1/3.6\,m^2 = 0.0278\,m^{-2}$$

$$A_3F_{32} = 4(0.38) = 1.52\,m^2, \qquad (1-\varepsilon_2)/\varepsilon_2A_2 = 0.7/1.2\,m^2 = 0.583\,m^{-2}.$$

Also,

$$E_{b1} = \sigma T_1^4 = 5.67\times10^{-8}\,W/m^2\cdot K^4\,(800\ K)^4 = 23{,}224\,W/m^2,$$

$$E_{b2} = \sigma T_2^4 = 5.67\times10^{-8}\,W/m^2\cdot K^4\,(400\ K)^4 = 1452\,W/m^2.$$

The system forms a three-surface enclosure, with one surface re-radiating. Hence the net radiation transfer from a single heater is, from Eq. 13.30,

$$q_1 = \frac{E_{b1} - E_{b2}}{\dfrac{1-\varepsilon_1}{\varepsilon_1A_1} + \dfrac{1}{A_1F_{12} + \left[1/A_1F_{13} + 1/A_3F_{32}\right]^{-1}} + \dfrac{1-\varepsilon_2}{\varepsilon_2A_2}}$$

Continued …..

$$q_1 = \frac{(23,224 - 1452)\,\mathrm{W/m^2}}{(0.0278 + 0.4061 + 0.583)\,\mathrm{m^{-2}}} = 21.4\,\mathrm{kW}.$$

The furnace power requirement is therefore $q_{elec} = 2q_1 = 43.8$ kW, with <

$$q_1 = \frac{E_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1}.$$

where

$$J_1 = E_{b1} - q_1 \frac{1-\varepsilon_1}{\varepsilon_1 A_1} = 23,224\,\mathrm{W/m^2} - 21,400\,\mathrm{W} \times 0.0278\,\mathrm{m^{-2}}$$

$$J_1 = 22,679\,\mathrm{W/m^2}.$$

Also,

$$J_2 = E_{b2} - q_2 \frac{1-\varepsilon_2}{\varepsilon_2 A_2} = 1,452\,\mathrm{W/m^2} - (-21,400\,\mathrm{W}) \times 0.583\,\mathrm{m^{-2}}$$

$$J_2 = 13,928\,\mathrm{W/m^2}.$$

From Eq. 13.31,

$$\frac{J_1 - J_3}{1/A_1F_{13}} = \frac{J_3 - J_2}{1/A_3F_{32}}$$

$$\frac{J_1 - J_3}{J_3 - J_2} = \frac{A_3F_{32}}{A_1F_{13}} = \frac{1.52}{2.48} = 0.613$$

$$1.613J_3 = J_1 + 0.613J_2 = 22,629 + 8537 = 31,166\,\mathrm{W/m^2}$$

$$J_3 = 19,321\,\mathrm{W/m^2}$$

Since $J_3 = E_{b3}$,

$$T_3 = (E_{b3}/\sigma)^{1/4} = \left(19,321/5.67 \times 10^{-8}\right)^{1/4} = 764\ \mathrm{K}.$$ <

**COMMENTS:** (1) To reduce $q_{elec}$, the sidewall temperature $T_2$, should be increased by insulating it from the surroundings. (2) The problem must be solved by simultaneously determining $J_1$, $J_2$ and $J_3$ from the radiation balances of the form

$$\frac{E_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} = A_1F_{12}(J_1 - J_2) + A_1F_{13}(J_1 - J_3)$$

$$\frac{E_{b2} - J_2}{(1-\varepsilon_2)/\varepsilon_2 A_2} = A_2F_{21}(J_2 - J_1) + A_2F_{23}(J_2 - J_3)$$

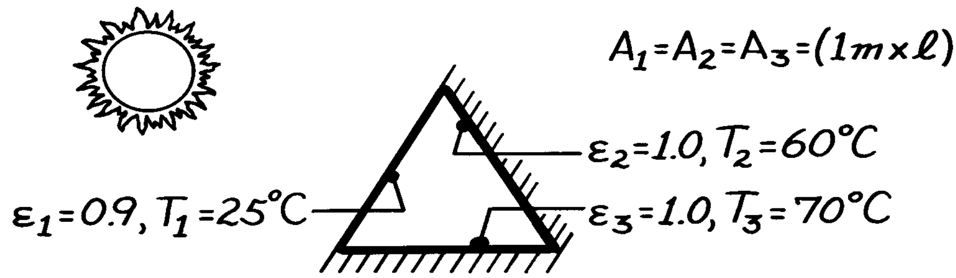
$$0 = A_1F_{13}(J_3 - J_1) + A_2F_{23}(J_3 - J_2).$$

**PROBLEM 13.78**

**KNOWN:** Geometry and surface temperatures and emissivities of a solar collector.

**FIND:** Net rate of radiation transfer to cover plate due to exchange with the absorber plates.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Isothermal surfaces with uniform radiosity, (2) Absorber plates behave as blackbodies, (3) Cover plate is diffuse-gray and opaque to thermal radiation exchange with absorber plates, (4) Duct end effects are negligible.

**ANALYSIS:** Applying Eq. 13.21 to the cover plate, it follows that

$$E_{b1} - J_1 = \frac{1-\varepsilon_1}{\varepsilon_1 A_1} \sum_{j=1}^{N} \frac{J_1 - J_j}{\left(A_i F_{ij}\right)^{-1}} = \frac{1-\varepsilon_1}{\varepsilon_1 A_1}\left[A_1 F_{12}\left(J_1 - J_2\right) + A_1 F_{13}\left(J_1 - J_3\right)\right].$$

From symmetry, $F_{12} = F_{13} = 0.5$. Also, $J_2 = E_{b2}$ and $J_3 = E_{b3}$. Hence

$$E_{b1} - J_1 = 0.0556\left(2J_1 - E_{b2} - E_{b3}\right)$$

or with $E_b = \sigma T^4$,

$$1.111 J_1 = E_{b1} + 0.0556\left(E_{b2} + E_{b3}\right)$$

$$1.111 J_1 = 5.67\times10^{-8}(298)^4\ \mathrm{W/m^2} + 0.0556\left(5.67\times10^{-8}\right)\left[(333)^4 + (343)^4\right]\mathrm{W/m^2}$$

$$J_1 = 476.64\ \mathrm{W/m^2}$$

From Eq. 13.19 the net rate of radiation transfer *from* the cover plate is then

$$q_1 = \frac{E_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} = \frac{5.67\times10^{-8}(298)^4 - 476.64}{(1-0.9)/0.9(\ell)} = (-265.5\ell)\,\mathrm{W}.$$

The net rate of radiation transfer *to* the cover plate per unit length is then

$$q_1' = \left(q_1/\ell\right) = 266\ \mathrm{W/m}.$$ <

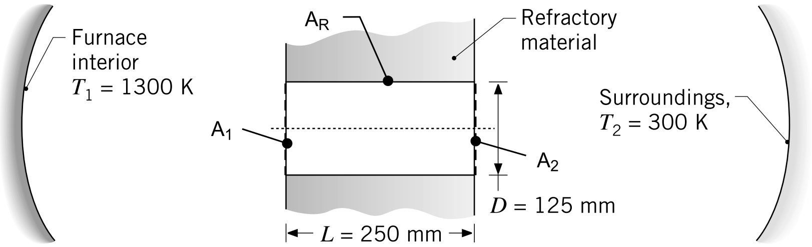
**COMMENTS:** Solar radiation effects are not relevant to the foregoing problem. All such radiation transmitted by the cover plate is completely absorbed by the absorber plate.

# PROBLEM 13.79

**KNOWN:** Cylindrical peep-hole of diameter D through a furnace wall of thickness L. Temperatures prescribed for the furnace interior and surroundings outside the furnace.

**FIND:** Heat loss by radiation through the peep-hole.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Furnace interior and exterior surroundings are large, isothermal surroundings for the peep-hole openings, (3) Furnace refractory wall is adiabatic and diffuse-gray with uniform radiosity.

**ANALYSIS:** The open-ends of the cylindrical peep-hole ($A_1$ and $A_2$) and the cylindrical lateral surface of the refractory material ($A_R$) form a diffuse-gray, three-surface enclosure. The hypothetical areas $A_1$ and $A_2$ behave as black surfaces at the respective temperatures of the large surroundings to which they are exposed. Since $A_r$ is adiabatic, it behaves as a re-radiating surface, and its emissivity has no effect on the analysis. From Eq. 13.30, the net radiation leaving $A_1$ passes through the enclosure into the outer surroundings.

$$q_1 = -q_2 = \frac{E_{b1} - E_{b2}}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12} + \left[(1/A_1 F_{1R}) + (1/A_2 F_{2R})\right]^{-1}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

Since $\varepsilon_1 = \varepsilon_2 = 1$, and with $E_b = \sigma T^4$ where $\sigma = 5.67 \times 10^{-8}$ W/m$^2$·K$^4$,

$$q_1 = \left\{A_1 F_{12} + \left[(1/A_1 F_{1R}) + (1/A_2 F_{2R})\right]^{-1}\right\}\sigma\left(T_1^4 - T_2^4\right)$$

where $A_1 = A_2 = \pi D^2/4$. The view factor $F_{12}$ can be determined from Table 13.2 (Fig. 13.5) for the coaxial parallel disks ($R_1 = R_2 = 125/(2 \times 250) = 0.25$ and $S = 17.063$) as

$$F_{12} = 0.05573$$

From the summation rule on $A_1$, with $F_{11} = 0$,

$$F_{11} + F_{12} + F_{1R} = 1$$

$$F_{1R} = 1 - F_{12} = 1 - 0.05573 = 0.9443$$

and from symmetry of the enclosure,

$$F_{2R} = F_{1R} = 0.9443$$

Substituting numerical values into the rate equation, find the heat loss by radiation through the peep-hole to the exterior surroundings as

$$q_{loss} = q_1 = 1046 \text{ W}$$ <

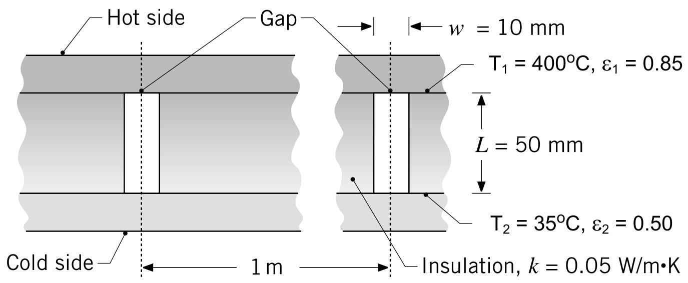
**COMMENTS:** If you held your hand 50 mm from the exterior opening of the peep-hole, how would that feel? It is standard, safe practice to use optical protection when viewing the interiors of high temperature furnaces as used in petrochemical, metals processing and power generation operations.

# PROBLEM 13.80

**KNOWN:** Composite wall comprised of two large plates separated by sheets of refractory insulation of thermal conductivity k = 0.05 W/m·K; gaps between the sheets of width w = 10 mm, located at 1 - m spacing, allow radiation transfer between the plates.

**FIND:** (a) Heat loss by radiation through the gap per unit length of the composite wall (normal to the page), and (b) fraction of the total heat loss through the wall that is due to radiation transfer through the gap.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Surfaces are diffuse-gray with uniform radiosities, (3) Refractory insulation surface in the gap is adiabatic, and (4) Heat flow through the wall is one-dimensional between the plates in the direction of the gap centerline.

**ANALYSIS:** (a) The gap of thickness w and infinite extent normal to the page can be represented by a diffuse-gray, three-surface enclosure formed by the plates $A_1$ and $A_2$ and the refractory walls, $A_R$. Since $A_R$ is adiabatic, it behaves as a re-radiating surface, and its emissivity has no effect on the analysis. From Eq. 13.30, the net radiation leaving the plate $A_1$ passes through the gap into plate $A_2$.

$$q_1 = -q_2 = \frac{E_{b1} - E_{b2}}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1 F_{12} + \left[(1/A_1 F_{1R}) + (1/A_2 F_{2R})\right]^{-1}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

where $E_b = \sigma T^4$ with $\sigma = 5.67 \times 10^{-8}$ W/m$^2$·K$^4$ and $A_1 = A_2 = w \cdot \ell$, but making $\ell$ = 1 m to obtain $q_1' \;(W/m)$.

The view factor $F_{12}$ can be determined from Table 13.2 (Fig. 13.4) for aligned parallel rectangles where $\overline{X} = X/L = \infty$ since $X \rightarrow \infty$ and $\overline{Y} = Y/L = W/L = 10/50 = 0.2$ giving

$$F_{12} = 0.09902$$

From the summation rule on $A_1$, with $F_{11} = 0$,

$$F_{11} + F_{12} + F_{1R} = 1 \qquad\qquad F_{1R} = 1 - F_{12} = 1 - 0.09902 = 0.901$$

and from symmetry of the enclosure,

$$F_{2R} = F_{1R} = 0.901.$$

Continued …..

## PROBLEM 13.80 (Cont.)

Substituting numerical values into the rate equation, find the heat loss through the gap due to radiation as

$$q'_{rad} = q'_1 = 37 \text{ W/m} \qquad <$$

(b) The conduction heat rate per unit length (normal to the page) for a 1 - m section is

$$q'_{cond} = k\,(1\text{ m})\frac{T_1 - T_2}{L} = 0.05 \text{ W/m}\cdot\text{K} \times 1\text{ m}\,\frac{(400-35)\text{K}}{0.050\text{ mm}}$$

$$q'_{cond} = 365 \text{ W/m}$$

The fraction of the total heat transfer through the 1 - m section due to radiation is

$$\frac{q'_{rad}}{q'_{tot}} = \frac{q'_{rad}}{q'_{cond} + q_{rad}} = \frac{37}{365+37} = 9.2\% \qquad <$$

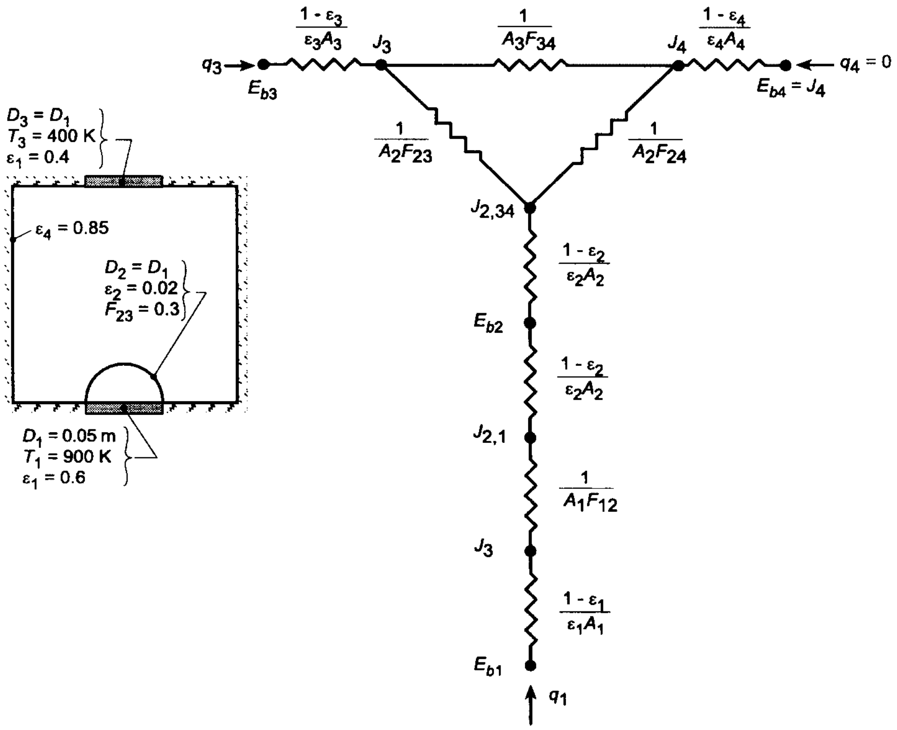
We conclude that if the installation process for the sheet insulation can be accomplished with a smaller gap, there is an opportunity to reduce the cost of operating the furnace.

# PROBLEM 13.81

**KNOWN:** Diameter, temperature and emissivity of a heated disk. Diameter and emissivity of a hemispherical radiation shield. View factor of shield with respect to a coaxial disk of prescribed diameter, emissivity and temperature.

**FIND:** (a) Equivalent circuit, (b) Net heat rate from the hot disk.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Surfaces may be approximated as diffuse/gray, (2) Surface 4 is reradiating, (3) Negligible convection.

**ANALYSIS:** (a) The equivalent circuit is shown in the schematic. Since surface 4 is treated as reradiating, the net transfer of radiation from surface 1 is equal to the net transfer of radiation to surface 3 ($q_1 = -q_3$).

(b) From the thermal circuit, the desired heat rate may be expressed as

$$q_1 = \frac{E_{b1} - E_{b3}}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1F_{12}} + \dfrac{2(1-\varepsilon_2)}{\varepsilon_2 A_2} + \left[A_2F_{23} + \dfrac{1}{\dfrac{1}{A_2F_{24}} + \dfrac{1}{A_3F_{34}}}\right]^{-1} + \dfrac{1-\varepsilon_1}{\varepsilon_3 A_3}}$$

where $A_1 = A_3 = \pi D_1^2/4 = \pi(0.05\text{ m})^2/4 = 1.963 \times 10^{3}\text{ m}^2$, $A_2 = \pi D_1^2/2 = 2A_1 = 3.925 \times 10^{-3}\text{ m}^2$, $F_{12} = 1$, and $F_{24} = 1 - F_{23} = 0.7$. With $F_{34} = 1 - F_{32} = 1 - F_{23}(A_2/A_3) = 1 - 0.3(2) = 0.4$, it follows that

Continued …..

## PROBLEM 13.81 (Cont.)

$$q_1 = \frac{A_1\sigma\left(T_1^4 - T_3^4\right)}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{F_{12}} + \dfrac{2(1-\varepsilon_2)}{\varepsilon_2}\dfrac{A_1}{A_2} + \left[\dfrac{A_2}{A_1}F_{23} + \dfrac{1}{\dfrac{A_1}{A_2F_{24}} + \dfrac{A_1}{A_3F_{34}}}\right]^{-1} + \dfrac{1-\varepsilon_3}{\varepsilon_3}}$$

$$q_1 = \frac{A_1\sigma\left(T_1^4 - T_3^4\right)}{0.667 + 1 + 49 + \left[0.6 + \dfrac{1}{\dfrac{1}{1.4} + \dfrac{1}{0.4}}\right]^{-1} + 1.5} = \frac{A_1\sigma\left(T_1^4 - T_3^4\right)}{0.667 + 1 + 49 + 1.098 + 1.5}$$

$$q_1 = 0.0188\left(1.963\times10^{-3}\,\mathrm{m}^2\right)5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\left(900^4 - 400^4\right)\mathrm{K}^4$$

$$q_1 = 1.32\,\mathrm{W}$$ <

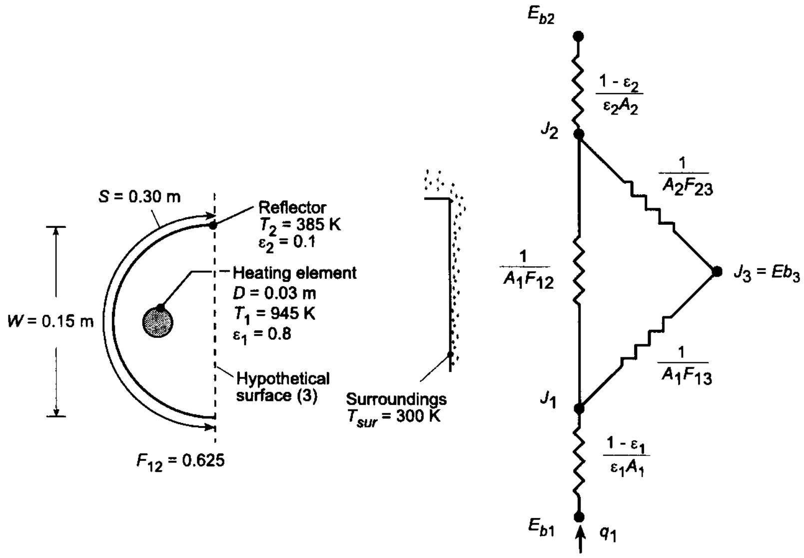
**COMMENTS:** Radiation transfer from 1 to 3 is impeded and enhanced, respectively, by the radiation shield and the reradiating walls. However, the dominant contribution to the total radiative resistance is made by the shield.

## PROBLEM 13.82

**KNOWN:** Diameter, temperature and emissivity of a cylindrical heater. Dimensions, temperature, and emissivity of a reflector. Temperature of large surroundings.

**FIND:** (a) Equivalent circuit and values of associated resistances and driving potentials, (b) Required electric power per unit length of heater.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Surfaces are diffuse-gray, (2) Surroundings form a large enclosure which may be represented by a hypothetical surface of temperature $T_3 = T_{sur}$ and emissivity $\varepsilon_3 = 1$.

**ANALYSIS:** (a) The circuit is shown in the schematic, where the hypothetical surface is part of a three-surface enclosure. On a unit length basis, the circuit resistances are

$$\frac{1-\varepsilon_1}{\varepsilon_1 A_1'} = \frac{0.2}{0.8(\pi)(0.03\,\text{m})} = 2.65\,\text{m}^{-1}$$ <

$$\frac{1}{A_1'F_{12}} = \frac{1}{\pi(0.03\,\text{m})0.625} = 16.98\,\text{m}^{-1}$$ <

$$\frac{1}{A_1'F_{13}} = \frac{1}{\pi(0.03\,\text{m})0.375} = 28.30\,\text{m}^{-1}$$ <

$$\frac{1}{A_2'F_{23}} = \frac{1}{A_3'F_{32}} = \frac{1}{(0.15\,\text{m})0.764} = 8.73\,\text{m}^{-1}$$ <

$$\frac{1-\varepsilon_2}{\varepsilon_2 A_2'} = \frac{0.9}{0.1(0.30\,\text{m})} = 30\,\text{m}^{-1}$$ <

The view factor $F_{13}$ is obtained from the summation rule, where $F_{13} = 1 - F_{12} = 1 - 0.625 = 0.375$. Similarly, $F_{32} = 1 - F_{31}$, where $F_{31} = F_{13}(A_1'/A_3') = 0.375(\pi D/W) = 0.375\pi(0.03\text{ m}/0.15\text{ m}) = 0.236$. Hence, $F_{32} = 0.764$. The potentials are

Continued …..

$$E_{b1} = \sigma T_1^4 = 5.67\times10^{-8}\,W/m^2\cdot K^4\,(945\ K)^4 = 45,220\,W/m^2$$ <

$$E_{b2} = \sigma T_2^4 = 5.67\times10^{-4}\,W/m^2\cdot K^4\,(385\ K)^4 = 1246\,W/m^2$$ <

$$E_{b3} = \sigma T_3^4 = 5.67\times10^{-8}\,W/m^2\cdot K^4\,(300\ K)^4 = 459\,W/m^2$$ <

(b) The required heater power may be obtained by applying Eq. (13.19) to node 1. Hence,

$$q_1' = \frac{E_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1'}$$

The radiosity may be obtained by applying radiation balances to nodes $J_1$ and $J_2$:

$$\frac{E_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1'} = \frac{J_1 - J_2}{1/A_1'F_{12}} + \frac{J_1 - E_{b3}}{1/A_1'F_{13}}$$

$$\frac{E_{b2} - J_2}{(1-\varepsilon_2)/\varepsilon_2 A_2'} = \frac{J_2 - J_1}{1/A_1'F_{12}} + \frac{J_2 - E_{b3}}{1/A_2'F_{23}}$$

Substituting the known potentials and solving, we obtain $J_1$ = 37,600 W/m$^2$ and $J_2$ = 11,160 W/m$^2$. Hence

$$q_1' = \frac{(45,220 - 37,600)\,W/m^2}{2.65\,m^{-1}} = 2875\,W/m$$ <

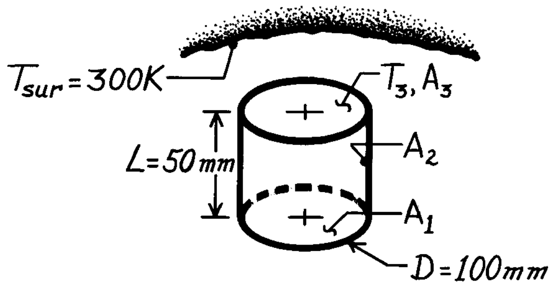
**COMMENTS:** Additional power would have to be supplied to compensate for heat transfer by free convection from the heater to the air.

**PROBLEM 13.83**

**KNOWN:** Cylindrical cavity with prescribed geometry, wall emissivity, and temperature.

**FIND:** Net radiation heat transfer from the cavity assuming the surroundings of the cavity are at 300 K.

**SCHEMATIC:**




$A_1 = A_3 = \pi D^2/4 = 7.85\times10^{-3}\,m^2$

$A_2 = \pi DL = 1.57\times10^{-2}\,m^2$

$F_{13} = F_{31} = 0.38 \qquad F_{32} = F_{12} = 0.62$

$F_{21} = F_{23} = 0.310$

$T_1 = T_2 = 1500\ K$

$\varepsilon_1 = \varepsilon_2 = 0.6$

**ASSUMPTIONS:** (1) Cavity interior surfaces are diffuse-gray, (2) Surroundings are much larger than the cavity opening $A_3$; $T_3 = T_{sur} = 300$ K and $\varepsilon_3 = 1$.

**ANALYSIS:** The net radiation heat transfer from the cavity is $-q_3$, which from Eq. 13.20 is,

$$q_3 = A_3F_{31}(J_3 - J_1) + A_3F_{32}(J_3 - J_2). \qquad (1)$$

While $J_3 = E_{b3}$ since $\varepsilon_3 = 1$, $J_1$ and $J_2$ are unknown and must be obtained from the radiation balances, Eq. 13.21.

$$\frac{E_{bi} - J_i}{(1-\varepsilon_i)/\varepsilon_i A_i} = \sum_{j=1}^{N} \frac{J_i - J_i}{(A_iF_{ij})^{-1}} \qquad (2)$$

From Fig. 13.5 with $L/r_1 = 0.050/0.050 = 1$ and $r_3/L = 1$, find $F_{13} = 0.38$. From summation rule and reciprocity: $F_{32} = F_{12} = 1 - F_{12} = 0.62$ and $F_{21} = F_{23} = (A_3\,F_{32})/A_2 = 0.310$. Note also, $E_{b1} = E_{b2} = \sigma T_1^4 = \sigma(1500K)^4 = 287{,}044\ W/m^2$ and $J_3 = E_{b3} = \sigma T_3^4 = 459.3\ W/m^2$.

$A_1$:
$$\frac{E_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} = \frac{J_1 - J_2}{(A_1F_{12})^{-1}} + \frac{J_1 - J_3}{(A_1F_{13})^{-1}}$$

$$\frac{287{,}044 - J_1}{(1-0.6)/0.6} = \frac{J_1 - J_2}{(0.62)^{-1}} + \frac{J_1 - 459.3}{(0.38)^{-1}} \qquad 2.5J_1 - 0.62J_2 = 430{,}741 \qquad (3)$$

$A_2$:
$$\frac{E_{b2} - J_2}{(1-\varepsilon_2)/\varepsilon_2 A_2} = \frac{J_2 - J_1}{(A_2F_{21})^{-1}} + \frac{J_2 - J_3}{(A_2F_{23})^{-1}}$$

$$\frac{287{,}044 - J_2}{(1-0.6)/0.6} = \frac{J_2 - J_1}{(0.31)^{-1}} + \frac{J_2 - 459.3}{(0.31)^{-1}} \qquad -0.31J_1 + 2.140J_2 = 430{,}708 \qquad (4)$$

Solving Eqs. (3) and (4) simultaneously, find $J_1 = 230{,}491\ W/m^2$ and $J_2 = 234{,}654\ W/m^2$, and from Eq. (1), find

$$q_3 = 7.854\times10^{-3}\,m^2\left[0.38(459.3 - 230{,}491) + 0.62(459.3 - 234{,}654)\right]W/m^2 = -1827\ W.$$

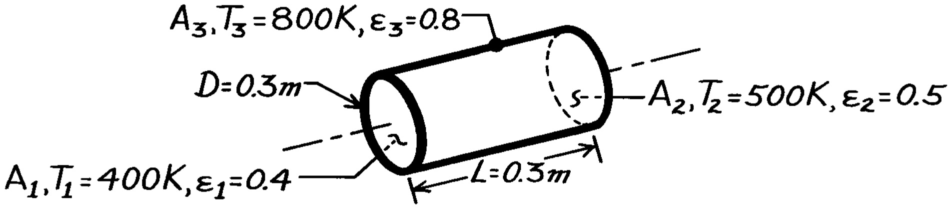
Hence, 1827 W are transferred from the cavity to the surroundings. <

**PROBLEM 13.84**

**KNOWN:** Circular furnace with prescribed temperatures and emissivities of the lateral and end surfaces.

**FIND:** Net radiative heat transfer from each surface.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Surfaces are isothermal and diffuse-gray.

**ANALYSIS:** To calculate the net radiation heat transfer from each surface, we need to determine its radiosity. First, evaluate terms which will be required.

$$E_{b1} = \sigma T_1^4 = 1452\ \mathrm{W/m^2} \qquad A_1 = A_2 = \pi D^2/4 = 0.07069\ \mathrm{m^2} \qquad F_{12} = F_{21} = 0.17$$

$$E_{b2} = \sigma T_2^4 = 3544\ \mathrm{W/m^2} \qquad A_3 = \pi DL = 0.2827\ \mathrm{m^2} \qquad F_{23} = F_{13} = 0.83$$

$$E_{b3} = \sigma T_3^4 = 23,224\ \mathrm{W/m^2}$$

The view factor $F_{12}$ results from Fig. 13.5 with $L/r_i = 2$ and $r_j/L = 0.5$. The radiation balances using Eq. 13.21, omitting units for convenience, are:

$$A_1: \quad \frac{1452 - J_1}{\dfrac{(1-0.4)}{0.4\times 0.07069}} = 0.07069\times 0.17(J_1 - J_2) + 0.07069\times 0.83(J_1 - J_3)$$

$$-2.500J_1 + 0.2550J_2 + 1.2450J_3 = -1452 \qquad (1)$$

$$A_2: \quad \frac{3544 - J_2}{\dfrac{(1-0.5)}{0.5\times 0.07069}} = 0.07069\times 0.17(J_2 - J_1) + 0.07069\times 0.83(J_2 - J_3)$$

$$-0.1700J_1 - 2.0000J_2 + 0.8300J_3 = -3544 \qquad (2)$$

$$A_3: \quad \frac{23,224 - J_3}{\dfrac{(1-0.8)}{0.8\times 0.2827}} = 0.07069\times 0.83(J_3 - J_1) + 0.07069\times 0.83(J_3 - J_2)$$

$$0.05189J_1 + 0.05189J_2 - 1.1037J_3 = -23,224 \qquad (3)$$

Solving Eqs. (1) – (3) simultaneously, find

$$J_1 = 12,877\ \mathrm{W/m^2} \qquad J_2 = 12,086\ \mathrm{W/m^2} \qquad J_3 = 22,216\ \mathrm{W/m^2}.$$

Using Eq. 13.22, the net radiation heat transfer for each surface follows:

$$q_i = \sum_{j=1}^{N} A_i F_{ij} (J_i - J_j)$$

$$A_1: \ q_1 = 0.07069\times 0.17(12,877 - 12,086)\,\mathrm{W} + 0.07069\times 0.83(12,877 - 22,216)\,\mathrm{W} = -538\ \mathrm{W} \quad <$$

$$A_2: \ q_2 = 0.07069\times 0.17(12,086 - 12,877)\,\mathrm{W} + 0.07069\times 0.83(12,086 - 22,216)\,\mathrm{W} = -603\ \mathrm{W} \quad <$$

$$A_3: \ q_3 = 0.07069\times 0.83(22,216 - 12,877)\,\mathrm{W} + 0.07069\times 0.83(22,216 - 12,086)\,\mathrm{W} = 1141\ \mathrm{W} \quad <$$

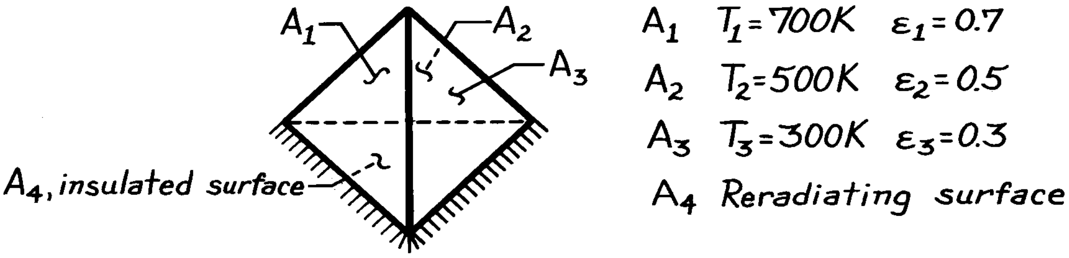
**COMMENTS:** Note that $\Sigma q_i = 0$. Also, note that $J_2 < J_1$ despite the fact that $T_2 > T_1$; note the role emissivity plays in explaining this.

## PROBLEM 13.85

**KNOWN:** Four surface enclosure with all sides of equal area; temperatures of three surfaces are specified while the fourth is re-radiating.

**FIND:** Temperature of the re-radiating surface $A_4$.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Surfaces are diffuse-gray, (2) Surfaces have uniform radiosities.

**ANALYSIS:** To determine the temperature of the re-radiating surface $A_4$, it is necessary to recognize that $J_4 = E_{b4} = \sigma T_4^4$ and that the $J_i$ (i = 1 to 4) values must be evaluated by simultaneously solving four radiation balances of the form, Eq. 13.21,

$$\frac{E_{bi} - J_i}{(1-\varepsilon_i)/\varepsilon_i A_i} = \sum_{j=1}^{N} \frac{J_i - j_j}{\left(A_i F_{ij}\right)^{-1}}$$

For simplicity, set $A_1 = A_2 = A_3 = A_4 = 1\ m^2$ and from symmetry, it follows that all view factors will be $F_{ij} = 1/3$. The necessary emissive powers are of the form $E_{bi} = \sigma T_1^4$.

$E_{b1} = \sigma(700\ K)^4 = 13{,}614\ W/m^2$, $E_{b2} = \sigma(500\ K)^4 = 3544\ W/m^2$, $E_{b3} = \sigma(300\ K)^4 = 459\ W/m^2$.

The radiation balances are:

$A_1$: $\frac{13,614 - J_1}{(1-0.7)/0.7} = \frac{1}{3}(J_1 - J_2) + \frac{1}{3}(J_1 - J_3) + \frac{1}{3}(J_1 - J_4);\ -1.42857J_1 + 0.14826J_2 + 0.14826J_3 + 0.14826J_4 = -13,614$

$A_2$: $\frac{3544 - J_2}{(1-0.5)/0.5} = \frac{1}{3}(J_2 - J_1) + \frac{1}{3}(J_2 - J_3) + \frac{1}{3}(J_2 - J_4)\quad 0.33333J_1 - 2.00000J_2 + 0.33333J_3 + 0.33333J_4 = -3544$

$A_3$: $\frac{459 - J_3}{(1-0.3)/0.3} = \frac{1}{3}(J_3 - J_1) + \frac{1}{3}(J_3 - J_2) + \frac{1}{3}(J_3 - J_4)\quad 0.77778J_1 + 0.77778J_2 - 3.33333J_3 + 0.77778J_4 = -459$

$A_4$: $0 = \frac{1}{3}(J_4 - J_1) + \frac{1}{3}(J_4 - J_2) + \frac{1}{3}(J_4 - J_3)\quad 0.33333J_1 + 0.33333J_2 + 0.33333J_3 - 1.00000J_4 = 0$

Solving this system of equations simultaneously, find

$J_1 = 11{,}572\ W/m^2$, $J_2 = 6031\ W/m^2$, $J_3 = 6088\ W/m^2$, $J_4 = 7897\ W/m^2$.

Since the radiosity and emissive power of the re-radiating surface are equal,

$$T_4^4 = J_4/\sigma$$

$$T_4 = \left(7897\ W/m^2 / 5.67\times10^{-8}\ W/m^2 \cdot K^4\right)^{1/4} = 611\ K. \qquad <$$

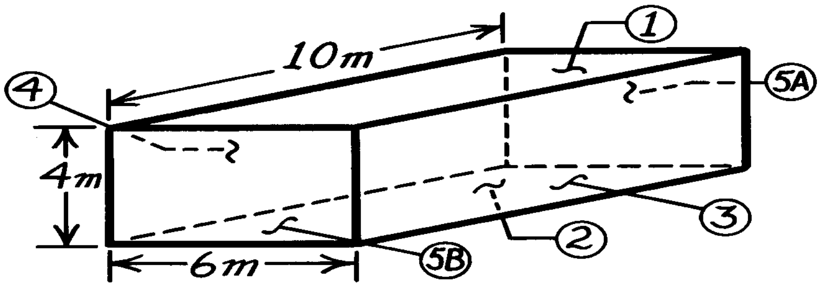
**COMMENTS:** Note the values of the radiosities; are their relative values what you would have expected? Is the value of $T_4$ reasonable?

**PROBLEM 13.86**

**KNOWN:** A room with electrical heaters embedded in ceiling and floor; one wall is exposed to the outdoor environment while the other three walls are to be considered as insulated.

**FIND:** Net radiation heat transfer from each surface.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Diffuse-gray surfaces, (2) Surfaces are isothermal and irradiated uniformly, (3) Negligible convection effects, (4) $A_5 = A_{5A} + A_{5B}$.

**ANALYSIS:** To determine the net radiation heat transfer from each surface, find the surface radiosities using Eq. 13.20.

$$q_i = \sum_{j=1}^{5} A_i F_{ij}\left(J_i - J_j\right) \qquad (1)$$

To determine the value of $J_i$, energy balances must be written for each of the five surfaces. For surfaces 1, 2, and 3, the form is given by Eq. 13.21.

$$\frac{E_{bi} - J_i}{(1-\varepsilon_i)/\varepsilon_i A_i} = \sum_{j=1}^{5} \frac{J_i - J_j}{\left(A_i F_{ij}\right)^{-1}} \qquad i = 1, 2, \text{ and } 3. \qquad (2)$$

For the insulated or adiabatic surfaces, Eq. 13.22 is appropriate with $q_i = 0$; that is

$$q_i = \sum_{j=1}^{N} \frac{J_i - J_j}{\left(A_i F_{ij}\right)^{-1}} = 0 \qquad i = 4 \text{ and } 5. \qquad (3)$$

In order to write the energy balances by Eq. (2) and (3), we will need to know view factors. Using Fig. 13.4 (parallel rectangles) or Fig. 13.5 (perpendicular rectangles) find:

| | | |
|---|---|---|
| $F_{12} = F_{21} = 0.39$ | $X/L = 10/4 = 2.5,$ | $Y/L = 6/4 = 1.5$ |
| $F_{13} = F_{14} = 0.19$ | $Z/X = 4/10 = 0.4,$ | $Y/X = 6/10 = 0.6$ |
| $F_{34} = F_{43} = 0.19$ | $X/L = 10/6 = 1.66,$ | $Y/L = 4/6 = 0.67$ |
| $F_{24} = F_{13} = 0.19$ | $Z/X = 4/10 = 0.4,$ | $Y/X = 6/10 = 0.6$ |

Note the use of symmetry in the above relations. Using reciprocity, find,

$$F_{32} = \frac{A_2}{A_3} F_{23} = \frac{A_2}{A_3} F_{13} = \frac{60}{40} \times 0.19 = 0.285; \qquad F_{31} = \frac{A_1}{A_3} F_{13} = \frac{60}{40} \times 0.19 = 0.285$$

$$F_{51} = \frac{A_1}{A_5} F_{15} = \frac{60}{48} \times 0.23 = 0.288; \qquad F_{53} = \frac{A_3}{A_5} F_{35} = \frac{40}{48} \times 0.25 = 0.208.$$

From the summation view factor relation,

$$F_{15} = 1 - F_{12} - F_{13} - F_{14} = 1 - 0.39 - 0.19 - 0.19 = 0.23$$

$$F_{35} = 1 - F_{31} - F_{32} - F_{34} = 1 - 0.285 - 0.285 - 0.19 = 0.24$$

Continued …..

Using Eq. (2), now write the energy balances for surfaces 1, 2, and 3. (Note $E_b = \sigma T^4$).

$$\frac{544.2 - J_1}{1 - 0.8/0.8 \times 60} = \frac{J_1 - J_2}{1/60 \times 0.39} + \frac{J_1 - J_3}{1/60 \times 0.19} + \frac{J_1 - J_4}{1/60 \times 0.19} + \frac{J_1 - J_5}{1/60 \times 0.23}$$

$$-1.2500J_1 + 0.0975J_2 + 0.0475J_3 + 0.570J_5 = -544.2 \qquad (4)$$

$$\frac{617.2 - J_2}{1 - 0.9/0.9 \times 60} = \frac{J_2 - J_1}{1/60 \times 0.39} + \frac{J_2 - J_3}{1/60 \times 0.19} + \frac{J_2 - J_4}{1/60 \times 0.19} + \frac{J_2 - J_5}{1/60 \times 0.23}$$

$$+0.0433J_1 - 1.111J_2 + 0.02111J_3 + 0.02111J_4 + 0.02556J_5 = -617.2 \qquad (5)$$

$$\frac{390.1 - J_3}{1 - 0.7/0.7 \times 40} = \frac{J_3 - J_1}{1/40 \times 0.285} + \frac{J_3 - J_2}{1/40 \times 0.285} + \frac{J_3 - J_4}{1/40 \times 0.19} + \frac{J_3 - J_5}{1/40 \times 0.24}$$

$$+0.1221J_1 + 0.1221J_2 - 1.4284J_3 + 0.08143J_4 + 0.1028J_5 = -390.1 \qquad (6)$$

Using Eq. (3), now write the energy balances for surfaces 4 and 5 noting $q_4 = q_5 = 0$.

$$0 = \frac{J_4 - J_1}{1/40 \times 0.285} + \frac{J_4 - J_2}{1/40 \times 0.285} + \frac{J_4 - J_3}{1/40 \times 0.19} + \frac{J_4 - J_5}{1/40 \times 0.24}$$

$$0.285J_1 + 0.285J_2 + 0.19J_3 - 1.0J_4 + 0.24J_5 = 0 \qquad (7)$$

$$0 = \frac{J_5 - J_1}{1/48 \times 0.288} + \frac{J_5 - J_2}{1/48 \times 0.288} + \frac{J_5 - J_3}{1/48 \times 0.208} + \frac{J_5 - J_4}{1/48 \times 0.208}$$

$$0.288J_1 + 0.288J_2 + 0.208J_3 + 0.208J_4 - 0.992J_5 = 0 \qquad (8)$$

Note that Eqs. (4) – (8) represent a set of simultaneous equations which can be written in matrix notation following treatment of Section 13.3.2. That is, [A] [J] = [C] with

$$A = \begin{bmatrix} -1.250 & 0.0975 & 0.0475 & 0.0475 & 0.0575 \\ 0.0433 & -1.111 & 0.02111 & 0.02111 & 0.02556 \\ 0.1221 & 0.1221 & -1.4284 & 0.08143 & 0.1028 \\ 0.285 & 0.285 & 0.190 & -1.000 & 0.240 \\ 0.288 & 0.288 & 0.208 & 0.208 & -0.992 \end{bmatrix} \qquad C = \begin{bmatrix} -544.2 \\ -617.2 \\ -390.1 \\ 0 \\ 0 \end{bmatrix} \qquad J = \begin{bmatrix} 545.1 \\ 607.9 \\ 441.5 \\ 542.3 \\ 5410 \end{bmatrix} \quad W/m^2$$

where the $J_i$ were found using a computer routine. The net radiation heat transfer from each of the surfaces can now be evaluated using Eq. (1).

$$q_1 = A_1F_{12}(J_1 - J_2) + A_1F_{13}(J_1 - J_3) + A_1F_{14}(J_1 - J_4) + A_1F_{15}(J_1 - J_5)$$

$$q_1 = 60\ m^2[0.39(545.1 - 607.9)$$

$$+0.19(545.1 - 441.5) + 0.19(545.1 - 542.3) + 0.23(545.1 - 541.0)]\ W/m^2 = -200\ W \quad <$$

$$q_2 = 60\ m^2[0.39(607.9 - 545.1)$$

$$+0.19(607.9 - 441.5) + 0.19(607.9 - 542.3) + 0.23(607.9 - 541.0)\ W/m^2 = 5037\ W \quad <$$

$$q_3 = 40\ m^2[0.285(441.5 - 545.1) + 0.285(441.5 - 607.9)$$

$$+0.19(441.5 - 542.3) + 0.24(441.5 - 541.0)]\ W/m^2 = -4{,}799\ W \quad <$$

Since $A_4$ and $A_5$ are insulated (adiabatic), $q_4 = q_5 = 0$. <

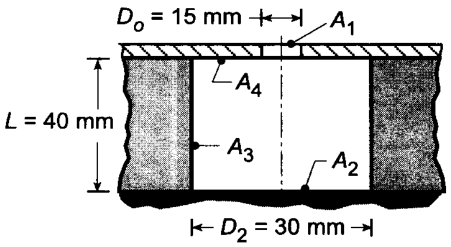
**COMMENTS:** (1) Note that the sum of $q_1 + q_2 + q_3 = +38$ W; this indicates a precision of less than 1% resulted from the solution of the equations. (2) The net radiation for the ceiling, $A_1$, is into the surface. Recognize that the embedded heaters function to offset heat losses to the room air by convection.

# PROBLEM 13.87

**KNOWN:** Cylindrical cavity closed at bottom with opening at top surface.

**FIND:** Rate at which radiation passes through cavity opening and effective emissivity for these conditions: (a) All interior surfaces are black and at 600 K, (b) Bottom surface of the cavity $\varepsilon = 0.6$, T = 600 K; other surfaces are re-radiating, (c) All surfaces are at 600 K with emissivity 0.6, (d) For the cavity configurations of parts (b) and (c), compute and plot $\varepsilon_e$ as a function of the interior surface emissivity over the range 0.6 to 1.0 with all other conditions remaining the same.

**SCHEMATIC:**




$$A_1 = \left(\pi D_1^2\right)/4 = 1.767 \times 10^{-4}\,\mathrm{m}^2$$

$$A_2 = \left(\pi D_2^2\right)/4 = 7.069 \times 10^{-4}\,\mathrm{m}^2$$

$$A_3 = \pi DL = 37.70 \times 10^{-4}\,\mathrm{m}^2$$

$$A_4 = A_2 - A_1 = 5.302 \times 10^{-4}\,\mathrm{m}^2$$

**ASSUMPTIONS:** (1) Surfaces are opaque, diffuse and gray, (2) Surfaces as subsequently defined have uniform radiosity, (3) Re-radiating surfaces are adiabatic, and (4) Surroundings are at 0 K so that $T_1 = 0$ K and $\varepsilon_1 = 1.0$.

**ANALYSIS:** Define the hypothetical surface $A_1$, the cavity opening, having $T_1 = 0$ K for which $E_{b.1} = J_1 = 0$. The radiant power passing through the cavity opening $A_1$ will be $-q_1$ due to exchange within the four-surface enclosure. The effective emissivity of the cavity is defined as the rate of the radiant power leaving the cavity to that from a blackbody having the same area of the cavity opening and at the temperature of the inner surface of the cavity. That is,

$$\varepsilon_e = -q_1 / A_1 \sigma T^4 \tag{1}$$

where T is the cavity surface temperature. Recognizing that the analysis will require knowledge of view factors, begin by evaluating them now.

For this four-surface enclosure (N=4), N(N-1)/2 = 6 view factors must be directly determined. The remaining $N^2 - 6 = 10$ can be determined by the summation rules and the reciprocity relations. By inspection,

(1-4): $F_{11} = 0$ $\quad F_{14} = 0$ $\quad F_{22} = 0$ $\quad F_{44} = 0$

Using the view factor equation for coaxial parallel disks, Table 13.2, or Fig. 13.5, evaluate $F_{21}$,

(5): $F_{21} = 0.030$ $\quad$ with $r_j/L = 7.5/40 = 0.188$, $L/r_i = 40/15 = 2.67$.

Considering the top and bottom surfaces, use the additive rule, Eq. 13.5,

(6): $F_{24} = F_{2(1.4)} - F_{21}$ (2)

where $F_{2(1.4)}$ can be evaluated using the coaxial parallel disk relations again

$F_{2(1.4)} = 0.111$ $\quad$ with $r_{(1.4)}/L = 15/40 = 0.375$, $L/r_2 = 40/15 = 2.67$

Substituting numerical values, find

$F_{24} = 0.111 - 0.030 = 0.081$

Continued …..

Using the summation rule for each surface, plus appropriate reciprocity relations, the remaining view factors can be determined. Written as a matrix, the $F_{ij}$ are

| | | | |
|---|---|---|---|
| 0* | 0.120 | 0.880 | 0 |
| 0.0300* | 0* | 0.889 | 0.0811* |
| 0.0413 | 0.167 | 0.667 | 0.125 |
| 0* | 0.108 | 0.892 | 0* |

The $F_{ij}$ shown with an asterisk were independently determined.

(a) When all the internal surfaces of the cavity are black at 600 K, the cavity opening emits as a black surface and the effective emissivity is unity. Using Eq. 13.14, the heat rate leaving $A_1$ is

$$q_1 = \sum_{i=1}^{3} A_1 F_{ij} \sigma \left(T_1^4 - T_1^4\right) = \sigma\left(T_1^4 - T^4\right) A_1 \left[F_{12} + F_{13} + F_{14}\right] \tag{3}$$

$$q_1 = 5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\left(0^4 - 600^4\right)\mathrm{K^4} \times 1.767\times10^{-4}\,\mathrm{m^2}\times\left[0.120+0.880+0\right] = -1.298\,\mathrm{W}$$

From Eq. (1), it follows that the effective emissivity must be unity.

$$\varepsilon_1 = 1$$

(b) When the bottom surface of the cavity is $T_2$ = 600 K with $\varepsilon_2$ = 0.6 and all other surfaces are re-radiating, an enclosure analysis to obtain $q_1$ involves use of Eqs. 13.21 and 13.22. The former will be used on $A_2$ and the latter on the remaining areas.

$A_2$:
$$\frac{E_{b2} - J_2}{\left(1-\varepsilon_2\right)/\varepsilon_2 A_2} = \frac{J_2 - J_1}{1/A_2F_{21}} + \frac{J_2 - J_3}{1/A_2F_{23}} + \frac{J_2 - J_4}{1/A_2F_{24}}$$

$$\frac{7384 - J_2}{\left(1-0.6\right)/0.6A_2} = \frac{J_2 - 0}{1/0.03A_2} + \frac{J_2 - J_3}{1/0.889A_2} + \frac{J_2 - J_4}{1/0.811A_2} \tag{4}$$

where $E_{b2} = \sigma T_2^4 = \sigma(600\mathrm{K})^4$ = 7348 W/m$^2$ and $J_1 = E_{b1} = \sigma T_1^4 = \sigma(0)^4$ = 0 W/m$^2$.

$A_3$:
$$0 = \frac{J_3 - J_1}{1/A_3F_{31}} + \frac{J_3 - J_2}{1/A_3F_{32}} + \frac{J_3 - J_4}{1/A_3F_{34}}$$

$$0 = \frac{J_3 - 0}{1/0.0413A_3} + \frac{J_3 - J_2}{1/0.167A_3} + \frac{J_3 - J_4}{1/0.125A_3} \tag{5}$$

$A_4$:
$$0 = \frac{J_4 - J_1}{1/A_4F_{41}} + \frac{J_4 - J_2}{1/A_4F_{42}} + \frac{J_4 - J_3}{1/A_4F_{43}}$$

$$0 = 0 + \frac{J_4 - J_2}{1/0.108A_4} + \frac{J_4 - J_3}{1/0.892A_4} \tag{6}$$

Solving Eqs. (4,5,6) simultaneously, find

| $J_1$ | $J_2$ | $J_3$ | $J_4$(W/m$^2$) |
|---|---|---|---|
| 0 | 6450 | 5284 | 5378 |

and the heat rate leaving surface $A_1$ is

Continued …..

$$q_1 = \frac{J_1 - J_2}{1/A_1F_{12}} + \frac{J_1 - J_3}{1/A_1F_{13}} + \frac{J_1 - J_4}{1/A_1F_{14}} = 0.9529\ \text{W} \qquad (7)$$

From Eq. (1), the cavity effective emissivity

$$\varepsilon_e = \frac{-q_1}{A_1\sigma T_2^4} = \frac{0.9529\,\text{W}}{1.767\times10^{-4}\,\text{m}^2\sigma\left(600\ \text{K}\right)^4} = 0.734 \qquad <$$

(c) When all the interior surfaces are at 600K ($T_2 = T_3 = T_4 = 600$ K) and $\varepsilon = 0.6$ ($\varepsilon_2 = \varepsilon_3 = \varepsilon_4 = 0.6$), apply the radiation surfaces energy balances using Eq. 13.21 to $A_2$, $A_3$, and $A_4$

$A_2$:

$$\frac{E_{b2} - J_2}{(1-\varepsilon_2)/\varepsilon_2 A_2} = \frac{J_2 - J_1}{1/A_2F_{21}} + \frac{J_2 - J_3}{1/A_2F_{23}} + \frac{J_2 - J_4}{1/A_2F_{24}}$$

$$\frac{7384 - J_2}{(1-0.6)/0.6A_2} = \frac{J_2 - 0}{1/0.03A_2} + \frac{J_2 - J_3}{1/0.889A_2} + \frac{J_2 - J_4}{1/0.811A_2} \qquad (8)$$

$A_3$:

$$\frac{E_{b3} - J_3}{(1-\varepsilon_3)/\varepsilon_3 A_3} = \frac{J_3 - J_1}{1/A_3F_{31}} + \frac{J_3 - J_2}{1/A_3F_{32}} + \frac{J_3 - J_4}{1/A_3F_{34}} = \frac{J_3 - 0}{1/0.0413A_3} + \frac{J_3 - J_2}{1/0.167A_3} + \frac{J_3 - J_4}{1/0.125A_3} \qquad (9)$$

A4:

$$\frac{E_{b4} - J_4}{(1-\varepsilon_4)/\varepsilon_4 A_4} = \frac{J_3 - J_1}{1/A_4F_{41}} + \frac{J_4 - J_2}{1/A_4F_{42}} + \frac{J_4 - J_3}{1/A_4F_{43}} = 0 + \frac{J_4 - J_2}{1/0.108A_4} + \frac{J_4 - J_3}{1/0.892A_4} \qquad (10)$$

Solving Eqs. (8, 9, 10) simultaneously, find

| $J_1$ | $J_2$ | $J_3$ | $J_4$ (W/m$^2$) |
|---|---|---|---|
| 0 | 7192 | 7164 | 7276 |

and the heat rate leaving surface $A_1$ is

$$q_1 = \frac{J_1 - J_2}{1/A_1F_{12}} + \frac{J_1 - J_3}{1/A_1F_{13}} + \frac{J_1 - J_4}{1/A_1F_{14}} = -1.267\ \text{W} \qquad <$$

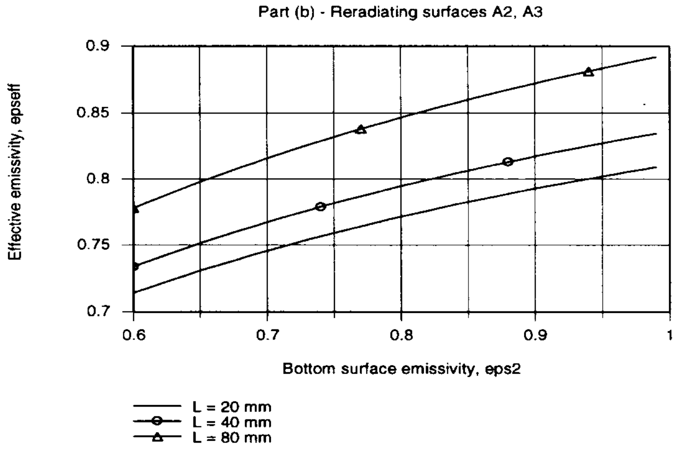
From Eq. (1), the cavity effectiveness is

$$\varepsilon_e = \frac{-q_1}{A_1\sigma T_2^4} = \frac{1.267\text{W}}{1.767\times10^{-4}\,\text{m}^2\sigma\left(600\ \text{K}\right)^4} = 0.976 \qquad <$$

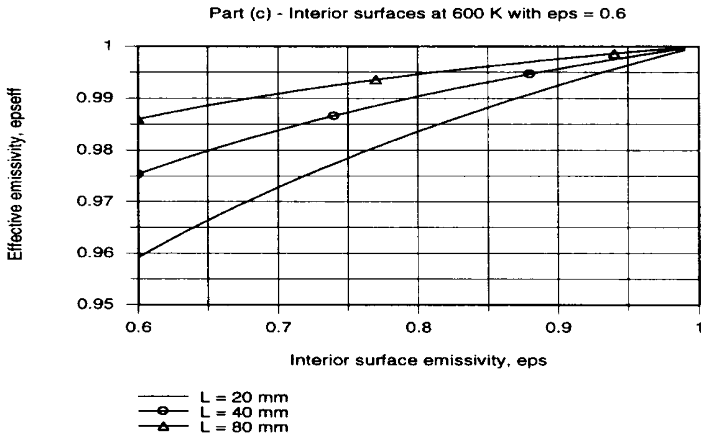
(d) For the cavity configurations of parts (b) and (c) and selected cavity depths, $\varepsilon_e$ was computed as a function of the interior surface emissivity $\varepsilon = \varepsilon_2 = \varepsilon_4$ using an *IHT* model. The model included the following tools: *Radiation-View Factors: Relations and Formulas (Coaxial parallel disks); Radiation-Radiation Surface Energy Balance Relations*. See comment 2 below.

Continued …..

## PROBLEM 13.87 (Cont.)




For the cavity configurations of part (b) – re –radiating surfaces $A_3$ and $A_4$ – the $\varepsilon_e$ vs. $\varepsilon_2$ plot shows that for all cavity depths, the effective emissivity increases as the emissivity of the bottom surface, $\varepsilon_2$, increases. Note that even when $\varepsilon_2 = 1$, the cavity effective emissivity is always less than unity. Why must that be so? The effect of increasing the cavity depth is to increase the effective emissivity.




For the cavity configuration of part (c) – all interior surfaces at 600 K and $\varepsilon = 0.6$ – the effective emissivity increases with increasing interior surface emissivity. In the limit when $\varepsilon \to 1$, $\varepsilon_e \to 1$ as expected, and the cavity performs as an isothermal enclosure. The effective emissivity increases with increasing cavity depth.

**COMMENTS:** (1) This arrangement of a cavity, referred to as a cylindrical cavity with a lid ($A_4$), is widely used for radiometric applications to calibrate radiometers, radiation thermometers, and heat flux gages. The effective emissivity can be improved by constructing the cavity with a conical bottom surface (rather than a flat bottom). Why do you think this is so?

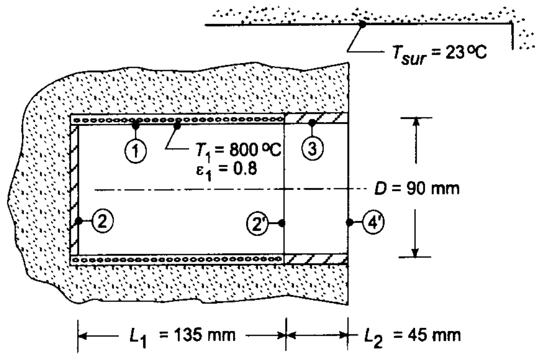
(2) The *IHT* model used to generate the part (b) graphical results is quite extensive. It is good practice to build the code in pieces, beginning with evaluation of the view factors. To avoid divide-by-zero errors, use small values for variables which are zero, such $F_{22} = 1e^{-20}$. Also, set unity emissivity values as 0.9999 rather than 1.0. The set of equations is very stiff, especially because of the re-radiating surface where $T_3$ and $T_4$ are unknowns. You should provide *Initial Guess* minimum values for $T_3$ and $T_4$ (> 0, positive) and unknown radiosities (> 0, positive).

# PROBLEM 13.88

**KNOWN:** Cylindrical furnace of diameter D = 90 mm and overall length L = 180 mm. Heating elements maintain the refractory liming ($\varepsilon = 0.8$) of section (1), $L_1$ = 135 mm, at $T_1$ = 800°C. The bottom (2) and upper (3) sections are refractory lined, but are insulated. Furnace operates in a spacecraft environment.

**FIND:** Power required to maintain the furnace operating conditions with the surroundings at 23°C.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) All surfaces are diffuse gray, (2) Uniform radiosity over the sections 1, 2, and 3, and (3) Negligible convection effects.

**ANALYSIS:** By defining the furnace opening as the hypothetical area $A_4$, the furnace can be represented as a four-surface enclosure as illustrated above. The power required to maintain $A_1$ at $T_1$ is $q_1$, the net radiation leaving $A_1$. To obtain $q_1$ following the methodology of Section 13.2.2, we must determine the radiosity at all surfaces by simultaneously solving the radiation energy balance equations for each surface which will be of the form, Eqs. 13.20 or 13.21.

$$q_1 = \frac{E_{bi} - J_i}{(1-\varepsilon_i)/\varepsilon_i A_i} = \sum_{j=1}^{N} \frac{J_j - J_j}{1/A_i F_{ij}} \qquad (1,2)$$

Since $\varepsilon_4 = 1$, $J_4 = E_{b4}$, so we only need to perform three energy balances, for $A_1$, $A_2$, and $A_3$, respectively

$A_1$: $$\frac{E_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} = \frac{J_1 - J_2}{1/A_1 F_{12}} + \frac{J_1 - J_3}{1/A_1 F_{13}} + \frac{J_1 - J_4}{1/A_1 F_{14}} \qquad (3)$$

$A_2$: $$0 = \frac{J_2 - J_1}{1/A_2 F_{21}} + \frac{J_2 - J_3}{1/A_2 F_{23}} + \frac{J_2 - J_4}{1/A_2 F_{24}} \qquad (4)$$

$A_3$: $$0 = \frac{J_3 - J_1}{1/A_3 F_{31}} + \frac{J_3 - J_2}{1/A_3 F_{32}} + \frac{J_3 - J_4}{1/A_3 F_{34}} \qquad (5)$$

Note that $q_2 = q_3 = 0$ since the surfaces are insulated (adiabatic). Recognize that in the above equation set, there are three equations and three unknowns: $J_1$, $J_2$, and $J_3$. From knowledge of $J_1$, $q_1$ can be determined using Eq. (1). Next we need to evaluate the view factors. There are $N^2 = 4^2 = 16$ view factors and N(N – 1)/2 = 6 must be independently evaluated, while the remaining can be determined by the summation rule and appropriate reciprocity relations. The six independently determined $F_{ij}$ are:

*By inspection:* (1) $F_{22} = 0$ (2) $F_{44} = 0$

*Coaxial parallel disks*: From Fig. 13.5 or Table 13.5,

Continued …..

## PROBLEM 13.88 (Cont.)

$$F_{24} = 0.5\left\{S - \left[S^2 - 4\left(r_4 / r_2\right)^2\right]^{1/2}\right\}$$

(3) $$F_{24} = 0.5\left\{18 - \left[18^2 - 4\left(1\right)^2\right]^{1/2}\right\} = 0.05573$$

$$S = 1 + \frac{1 + R_4^2}{R_2^2} = 1 + \frac{1 + 0.250^2}{0.250^2} = 18.00 \qquad R_2 = r_2 / L = 45/180 = 0.250 \qquad R_4 = r_4 / L = 0.250$$

*Enclosure 1-2-2′*: from the summation rule for $A_2$,

(4) $F_{21} = 1 - F_{22'} = 1 - 0.09167 = 0.9083$

where $F_{22'}$ can be evaluated from the coaxial parallel disk relation, Table 13.5. For these surfaces, $R_2 = r_2/L_1 = 45/135 = 0.333$, $R_{2'} = r_2/L_1 = 0.333$, and S = 11.00. From the summation rule for $A_1$,

(5) $F_{11} = 1 - F_{12} - F_{12'} = 1 - 0.1514 - 0.1514 = 0.6972$

and by symmetry $F_{12} = F_{12'}$ and using reciprocity

$$F_{12} = A_2F_{21} / A_1 = \left[\pi\left(0.090\text{m}\right)\left(2/4\right)\right] \times 0.9083 / \pi \times 0.090\text{m} \times 0.135\text{m} = 0.1514$$

*Enclosure 2′-3-4*: from the summation rule for $A_4$,

(6) $F_{43} = 1 - F_{42'} - F_{44} = 1 - 0.3820 - 0 = 0.6180$

where $F_{44} = 0$ and using the coaxial parallel disk relation from Table 13.5, with $R_4 = r_4/L_2 = 45/45 = 1$, $R_{2'} = r_2/L_2 = 1$, and S = 3.

*The View Factors*: Using summation rules and appropriate reciprocity relations, the remaining 10 view factors can be evaluated. Written in matrix form, the $F_{ij}$ are

| | | | |
|---|---|---|---|
| 0.6972* | 0.1514 | 0.09704 | 0.05438 |
| 0.9083* | 0* | 0.03597 | 0.05573* |
| 0.2911 | 0.01798 | 0.3819 | 0.3090 |
| 0.3262 | 0.05573 | 0.6180* | 0* |

The $F_{ij}$ shown with an asterisk were independently determined.

From knowledge of the relevant view factors, the energy balances, Eqs. (3, 4, 5), can be solved simultaneously to obtain the radiosities,

$$J_1 = 73,084\,\text{W/m}^2 \qquad J_2 = 67,723\,\text{W/m}^2 \qquad J_3 = 36,609\,\text{W/m}^2$$

The net heat rate leaving $A_1$ can be evaluated using Eq. (1) written as

$$q_1 = \frac{E_{b1} - J_1}{\left(1-\varepsilon_1\right)/\varepsilon_1 A_1} = \frac{\left(75,159 - 73,084\right)\text{W/m}^2}{\left(1-0.8\right)/0.8 \times 0.03817\,\text{m}^2} = 317\,\text{W} \qquad <$$

where $E_{b1} = \sigma T_1^4 = \sigma(800 + 273\text{K})^4 = 75,159\ \text{W/m}^2$ and $A_1 = \pi DL_1 = \pi \times 0.090\text{m} \times 0.135\text{m} = 0.03817\ \text{m}^2$.

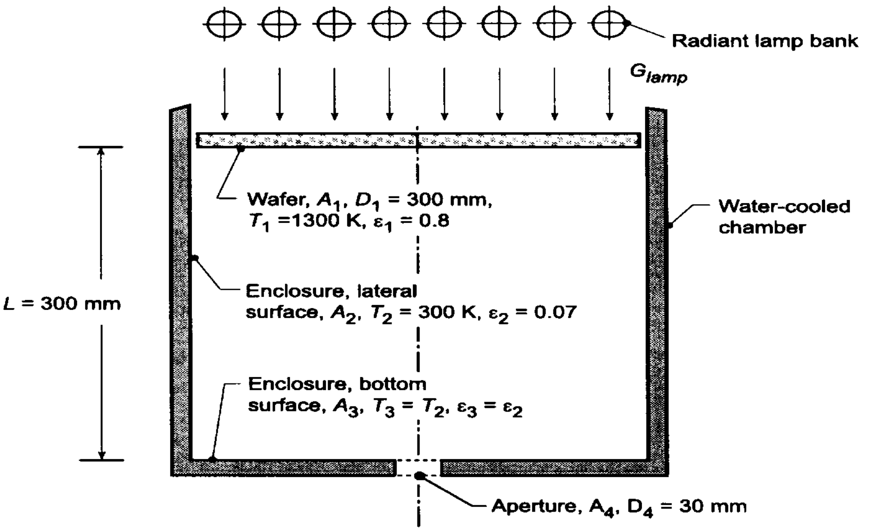
**COMMENTS:** (1) Recognize the importance of defining the furnace opening as the hypothetical area $A_4$ which completes the four-surface enclosure representing the furnace. The temperature of $A_4$ is that of the surroundings and its emissivity is unity since it absorbs all radiation incident on it. (2) To obtain the view factor matrix, we used the *IHT Tool*, *Radiation*, *View Factor Relations*, which permits you to specify the independently determined $F_{ij}$ and the tool will calculate the remaining ones.

# PROBLEM 13.89

**KNOWN:** Rapid thermal processing (RTP) tool consisting of a lamp bank to heat a silicon wafer with irradiation onto its front side. The backside of the wafer (1) is the top of a cylindrical enclosure whose lateral (2) and bottom (3) surfaces are water cooled. An aperture (4) on the bottom surface provides for optical access to the wafer.

**FIND:** (a) Lamp irradiation, $G_{lamp}$, required to maintain the wafer at 1300 K; heat removal rate by the cooling coil, and (b) Compute and plot the fractional difference $(E_{b1} - J_1)/E_{b1}$ as a function of the enclosure aspect ratio, L/D, for the range $0.5 \le L/D \le 2.5$ with D = 300 mm fixed for wafer emissivities of $\varepsilon_1 = 0.75$, 0.8, and 0.85; how sensitive is this parameter to the enclosure surface emissivity, $\varepsilon_2 = \varepsilon_3$.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Enclosure surfaces are diffuse, gray, (2) Uniform radiosity over the enclosure surfaces, (3) No heat losses from the top side of the wafer.

**ANALYSIS:** (a) The wafer-cylinder system can be represented as a four-surface enclosure. The aperture forms a hypothetical surface, $A_4$, at $T_4 = T_2 = T_3 = 300$ K with emissivity $\varepsilon_4 = 1$ since it absorbs all radiation incident on it. From an energy balance on the wafer, the absorbed lamp irradiation on the front side of the wafer, $\alpha_w G_{lamp}$, will be equal to the net radiation leaving the back-side (enclosure-side) of the wafer, $q_1$. To obtain $q_1$, following the methodology of Section 13.2.2, we must determine the radiosity of all the enclosure surfaces by simultaneously solving the radiation energy balance equations for each surface, which will be of the form, Eqs. 13.20 or 13.21.

$$q_i = \frac{E_{bi} - J_i}{(1-\varepsilon_i)/\varepsilon_i A_i} = \sum_{j=1}^{N} \frac{J_i - J_j}{1/A_i F_{ij}} \qquad (1,2)$$

Since $\varepsilon_4 = 1$, $J_4 = E_{b4}$, we only need to perform three energy balances, for $A_1$, $A_2$ and $A_3$, respectively,

$$A_1: \quad \frac{E_{b1} - J_1}{(1-\varepsilon_1)/A_1} = \frac{J_1 - J_2}{1/A_1F_{12}} + \frac{J_1 - J_3}{1/A_1F_{13}} + \frac{J_1 - J_4}{1/A_1F_{14}} \qquad (3)$$

$$A_2: \quad \frac{E_{b2} - J_1}{(1-\varepsilon_2)/A_2} = \frac{J_2 - J_1}{1/A_2F_{21}} + \frac{J_2 - J_3}{1/A_2F_{23}} + \frac{J_2 - J_4}{1/A_2F_{24}} \qquad (4)$$

Continued …..

## PROBLEM 13.89 (Cont.)

$A_3$: $$\frac{E_{b3}-J_3}{(1-\varepsilon_3)/A_3}=\frac{J_3-J_1}{1/A_3F_{31}}+\frac{J_3-J_2}{1/A_3F_{32}}+\frac{J_3-J_4}{1/A_3F_{34}} \qquad (5)$$

Recognize that in the above equation set, there are three equations and three unknowns: $J_1$, $J_2$, and $J_3$. From knowledge of the radiosities, the desired heat rate can be determined using Eq. (1). The required *lamp irradiation*,

$$\alpha_w\ G_{lamp}A_1 = q_1 = \frac{E_{b1}-J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} \qquad (6)$$

and the heat removal rate by the cooling coil, $q_{coil}$, on surfaces $A_2$ and $A_3$, is

$$q_{coil} = -(q_2+q_3) \qquad (7)$$

where the net radiation leaving $A_2$ and $A_3$ are, from Eq. (1),

$$q_2 = \frac{E_{b2}-J_2}{(1-\varepsilon_2)/\varepsilon_2 A_2} \qquad q_1 = \frac{E_{b3}-J_3}{(1-\varepsilon_3)/\varepsilon_3 A_3} \qquad (8,9)$$

The surface areas are expressed as

$$A_1 = \pi D_1^2/4 = 0.07069\,m^2 \qquad A_2 = \pi D_1 L = 0.2827 \qquad (10,11)$$

$$A_3 = \pi\left(D_1^2 - D_4^2\right) = 0.06998\,m^2 \qquad A_2 = \pi D_4^2/4 = 0.0007069\,m^2 \qquad (12,13)$$

Next evaluate the view factors. There are $N^2 = 4^2 = 16$ and $N(N-1)/2 = 6$ must be independently evaluated, and the remaining can be determined by summation rules and reciprocity relations. The six independently determined $F_{ij}$ are:

*By inspection*: (1) $F_{11} = 0$ (2) $F_{33} = 0$ (3) $F_{44} = 0$ (4) $F_{34} = 0$

*Coaxial parallel disks*: from Fig. 13.5 or Table 13.5,

$$F_{14} = 0.5\left\{S-\left[S^2-4(r_4/r_1)^2\right]^{1/2}\right\}$$

(5) $$F_{14} = 0.5\left\{5.01-\left[5.01^2-4(15/150)^2\right]^{1/2}\right\} = 0.001997$$

$$S = 1+\frac{1+R_4^2}{R_1^2} = 1+\frac{1+0.05^2}{0.5^2} = 5.010$$

$$R_1 = r_1/L = 150/300 = 0.5 \qquad R_4 = 15/300 = 0.05$$

*Coaxial parallel disks*: from the composite surface rule, Eq. 13.5,

(6) $F_{13} = F_{1(3,4)} - F_{14} = 0.17157 - 0.01997 = 0.1696$

where $F_{1(3,4)}$ can be evaluated from the coaxial parallel disk relation, Table 13.5. For these surfaces, $R_1 = r_1/L = 150/300 = 0.5$, $R_{(3,4)} = r_3/L = 150/300 = 0.5$, and $S = 6.000$.

*The view factors*: Using summation rules and reciprocity relations, the remaining 10 view factors can be evaluated. Written in matrix form, the $F_{ij}$ are

Continued …..

| | | | |
|---|---|---|---|
| 0* | 0.8284 | 0.1696 | 0.001997* |
| 0.2071 | 0.5858 | 0.2051 | 0.002001 |
| 0.1713 | 0.8287 | 0* | 0* |
| 0.1997 | 0.8003 | 0* | 0* |

The $F_{ij}$ shown with an asterisk were independently determined.

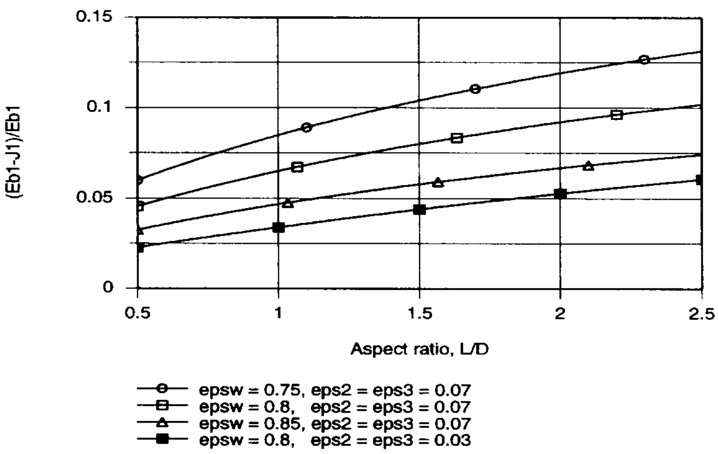
From knowledge of the relevant view factors, the energy balances, Eqs. (3, 4, 5) can be solved simultaneously to obtain the radiosities,

| $J_1$ | $J_2$ | $J_3$ | $J_4$ ($W/m^2$) |
|---|---|---|---|
| $1.514\times10^5$ | $1.097\times10^5$ | $1.087\times10^5$ | 576.8 |

From Eqs. (6) and (7), the required lamp irradiation and cooling-coil heat removal rate are

$$G_{lamp} = 52,650\,W/m^2 \qquad q_{coil} = 2.89\ kW$$ <

(b) If the enclosure were perfectly reflecting, the radiosity of the wafer, $J_1$, would be equal to its blackbody emissive power. For the conditions of part (a), $J_1 = 1.514 \times 10^5$ W/m$^2$ and $E_{b1} = 1.619 \times 10^5$ W/m$^2$. As such, the radiosity would be independent of $\varepsilon_w$ thereby minimizing effects due to variation of that property from wafer-to-wafer. Using the foregoing analysis in the *IHT* workspace (see Comment 1 below), the fractional difference, $(E_{b1} - J_1)/E_{b1}$, was computed and plotted as a function of L/D, the aspect ratio of the enclosure.




Note that as the aspect ratio increases, the fractional difference between the wafer blackbody emissive power and the radiosity increases. As the enclosure gets larger, (L/D increases), more power supplied to the wafer is transferred to the water-cooled walls. For any L/D condition, the effect of increasing the wafer emissivity is to reduce the fractional difference. That is, as $\varepsilon_w$ increases, the radiosity increases. The lowest curve on the above plot corresponds to the condition $\varepsilon_2 = \varepsilon_3 = 0.03$, rather than 0.07 as used in the $\varepsilon_w$ parameter study. The effect of reducing $\varepsilon_2$ is substantial, nearly halving the fractional difference. We conclude that the "best" cavity is one with a low aspect ratio and low emissivity (high reflectivity) enclosure walls.

**COMMENTS:** The *IHT* model developed to perform the foregoing analysis is shown below. Since the model utilizes several *IHT Tools*, good practice suggests the code be built in stages. In the first stage, the view factors were evaluated; the bottom portion of the code. Note that you must set the $F_{ij}$ which

Continued …..

are zero to a value such as 1e-20 rather than 0. In the second stage, the enclosure exchange analysis was added to the code to obtain the radiosities and required heat rate. Finally, the equations necessary to obtain the fractional difference and perform the parameter analysis were added.

```
// Enclosure Performance Parameter:
Eb1J1 = (Eb1 - J1 ) / Eb1
LoverD = L / D1
// Energy Balances - Wafer and water-cooled surfaces, Eqs (6) and (7):
alphaw * Glamp * A1 = q1   // Energy balance on wafer
alphaw = eps1              // Wafer absorptivity to lamp irradiation
qcoil = - (q2 + q3)                     // Heat rate to the cooling coil, W
// Radiation Exchange Analysis Tool - Surface Energy Balances:
/* The net heat rate leaving A1 in terms of the surface resistance is */
q1 = (Eb1 - J1) / ((1 - eps1) / (eps1 * A1))      // Eq 13.19
/* The net heat rate leaving A1 in terms of the net exchanges between enclosure surfaces is */
q1 = q12 + q13 + q14
/* where the net exchange rates expressed in terms of the space resistances are, Eq 13.20 and 13.22, */
q12 = (J1 - J2) / (1 / (A1 * F12))
q13 = (J1 - J3) / (1 / (A1 * F13))
q14 = (J1 - J4) / (1 / (A1 * F14))
/* The net heat rate leaving A2 in terms of the surface resistance is */
q2 = (Eb2 - J2) / ((1 - eps2) / (eps2 * A2))      // Eq 13.19
/* The net heat rate leaving A2 in terms of the net exchanges between enclosure surfaces is */
q2 = q21 + q23 + q24
/* where the net exchange rates expressed in terms of the space resistances are, Eq 13.20 and 13.22, */
q21 = (J2 - J1) / (1 / (A2 * F21))
q23 = (J2 - J3) / (1 / (A2 * F23))
q24 = (J2 - J4) / (1 / (A2 * F24))
/* The net heat rate leaving A3 in terms of the surface resistance is */
q3 = (Eb3 - J3) / ((1 - eps3) / (eps3 * A3))      // Eq 13.19
/* The net heat rate leaving A3 in terms of the net exchanges between enclosure surfaces is */
q3 = q31 + q32 + q34
/* where the net exchange rates expressed in terms of the space resistances are, Eq 13.20 and 13.22, */
q31 = (J3 - J1) / (1 / (A3 * F31))
q32 = (J3 - J2) / (1 / (A3 * F32))
q34 = (J3 - J4) / (1 / (A3 * F34))
/* The net heat rate leaving A4 in terms of the surface resistance is */
q4 = (Eb4 - J4) / ((1 - eps4) / (eps4 * A4))      // Eq 13.19
/* The net heat rate leaving A4 in terms of the net exchanges between enclosure surfaces is */
q4 = q41 + q42 + q43
/* where the net exchange rates expressed in terms of the space resistances are, Eq 13.20 and 13.22, */
q41 = (J4 - J1) / (1 / (A4 * F41))
q42 = (J4 - J2) / (1 / (A4 * F42))
q43 = (J4 - J3) / (1 / (A4 * F43))
// Emissive Powers:
Eb1 = sigma * T1^4          // Blackbody emissive power, W/m^2
Eb2 = sigma * T2^4
Eb3 = sigma * T3^4
Eb4 = sigma * T4^4
sigma = 5.67e-8  // Stefan-Boltzmann constant, W/m^2.K^4
// Assigned Variables - Thermal Parameters Only:
T1 = 1300                   // Wafer temperature, K
eps1 = 0.8                  // Wafer emissivity
//eps1 = 0.75
//eps1 = 0.85
T2 = 300                    // Lateral surface temperature, K
eps2 = 0.07                 // Enclosure emissivity
//eps2 = 0.03
T3 = 300                    // Bottom surface temperature, K
eps3 = 0.07                 // Enclosure emissivity
//eps3 = 0.03
T4 = 300                    // Aperture surface temperature, K
eps4 = 0.999                // Aperture emissivity; not zero to avoid divide-by-zero error
```

Continued …..

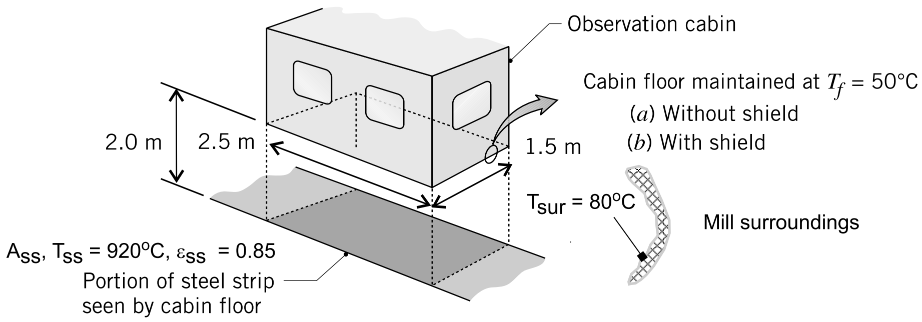
```
// Radiation Exchange Analysis Tool - View Factor Relations:
/* The summation rule for an N-surface enclosure, Eq 13.4, is */
F11 + F12 + F13 + F14 = 1
F21 + F22 + F23 + F24 = 1
F31 + F32 + F33 + F34 = 1
F41 + F42 + F43 + F44 = 1
/* Then N * (N - 1) / 2 reciprocity relations associated with an N-surface enclosure, Eq 13.3, are */
A1 * F12 = A2 * F21
A1 * F13 = A3 * F31
A1 * F14 = A4 * F41
A2 * F23 = A3 * F32
A2 * F24 = A4 * F42
A3 * F34 = A4 * F43
// Areas:
A1 =pi * D1^2 / 4               // Wafer, m^2
A2 = pi * D1 * L                // Lateral surface, m^2
A3 = pi * (D1^2 - D4^2) / 4     // Bottom surface, m^2
A4 = pi * D4^2 / 4              // Aperture, m^2
// Assigned Variables - Geometry Only:
D1 = 0.300                      // Wafer diameter, m
D4 = 0.030                      // Aperture diameter, m
L = 0.300                       // Enclosure height, m
// Independently determined Fij - by inspection:
F11 = 1e-20                     // Not zero to avoid divide-by-zero error
F33 = 1e-20
F44 = 1e-20
F34 = 1e-20
// Other independently determined Fij:
/* The view factor, F14, for coaxial parallel disks, is */
F14 = 0.5 * (Sa - sqrt(Sa^2 - 4*(r4 / r1)^2))
// where
R1 = r1 / L
R4 = r4 / L
r1 = D1 / 2
r4 = D4 / 2
Sa = 1 + (1 + R4^2) / R1^2
// Composite surface relation to find F13:
F134 = F13 + F14
/* The view factor, F1(34), for coaxial parallel disks, is */
F134 = 0.5 * (Sb - sqrt(Sb^2 - 4*(r34 / r1)^2))
// where
//R1 = r1 / L
R34 = r34 / L
r34 = r1
Sb = 1 + (1 + R34^2) / R1^2
```

## PROBLEM 13.90

**KNOWN:** Observation cabin located in a hot-strip mill directly over the line; cabin floor (f) exposed to steel strip (ss) at $T_{ss}$ = 920°C and to mill surroundings at $T_{sur}$ = 80°C.

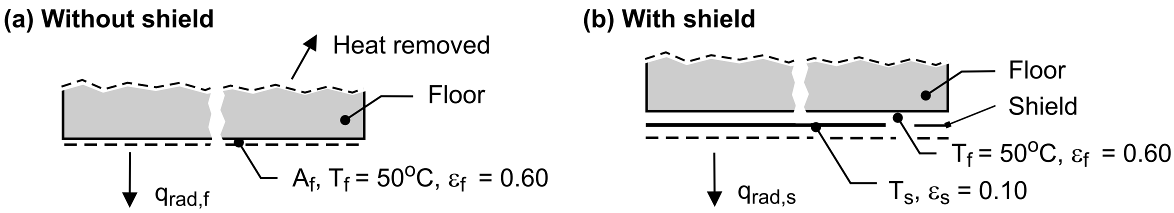
**FIND:** Coolant system heat removal rate required to maintain the cabin floor at $T_f$ = 50°C for the following conditions: (a) when the floor is directly exposed to the steel strip and (b) when a radiation shield (s) $\varepsilon_s$ = 0.10 is installed between the floor and the strip.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Cabin floor (f) or shield (s), steel strip (ss), and mill surroundings (sur) form a three-surface, diffuse-gray enclosure, (2) Surfaces with uniform radiosities, (3) Mill surroundings are isothermal, black, (4) Floor-shield configuration treated as infinite parallel planes, and (5) Negligible convection heat transfer to the cabin floor.

**ANALYSIS:** A gray-diffuse, three-surface enclosure is formed by the cabin floor (f) (or radiation shield, s), steel strip (ss), and the mill surroundings (sur). The heat removal rate required to maintain the cabin floor at $T_f$ = 50°C is equal to - $q_f$ (or, -$q_s$), where $q_f$ or $q_s$ is the net radiation leaving the floor or shield. The schematic below represents the details of the surface energy balance on the floor and shield for the conditions *without the shield* (floor exposed) and *with the shield* (floor shielded from strip).




(a) *Without the shield*. Radiation surface energy balances, Eq. 13.21, are written for the floor (f) and steel strip (ss) surfaces to determine their radiosities.

$$\frac{E_{b,f} - J_f}{(1-\varepsilon_f)/\varepsilon_f A_f} = \frac{J_f - J_{ss}}{1/A_f F_{f-ss}} + \frac{J_f - E_{b,sur}}{1/A_f F_{f-sur}} \tag{1}$$

$$\frac{E_{b,ss} - J_{ss}}{(1-\varepsilon_{ss})/\varepsilon_{ss} A_{ss}} = \frac{J_{ss} - J_f}{1/A_{ss} F_{ss-f}} + \frac{J_{ss} - E_{b,sur}}{1/A_{ss} F_{ss\text{-}sur}} \tag{2}$$

Since the surroundings (sur) are black, $J_{sur} = E_{b,sur}$. The blackbody emissive powers are expressed as $E_b = \sigma T^4$ where $\sigma = 5.67 \times 10^{-8}$ W/m$^2\cdot$K$^4$. The net radiation leaving the floor, Eq. 13.20, is

$$q_f = A_f F_{f-ss}(J_f - J_{ss}) + A_f F_{f-sur}(J_f - E_{b,sur}) \tag{3}$$

Continued …..

## PROBLEM 13.90 (Cont.)

The required view factors for the analysis are contained in the summation rule for the areas $A_f$ and $A_{ss}$,

$$F_{f-ss} + F_{f-sur} = 1 \qquad F_{ss-f} + F_{ss-sur} = 1 \tag{4,5}$$

$F_{f-ss}$ can be evaluated from Fig. 13.4 (Table 13.2) for the aligned parallel rectangles geometry. By symmetry, $F_{ss-f} = F_{f-ss}$, and with the summation rule, all the view factors are determined. Using the foregoing relations in the *IHT* workspace, the following results were obtained:

$$F_{f-ss} = 0.1864 \qquad J_f = 7959\ \mathrm{W/m^2}$$

$$F_{f-sur} = 0.8136 \qquad J_{ss} = 97.96\ \mathrm{kW/m^2}$$

and the heat removal rate required of the coolant system (cs) is

$$q_{cs} = -q_f = 41.3\ \ \mathrm{kW} \qquad <$$

(b) *With the shield*. Radiation surface energy balances are written for the shield (s) and steel strip (ss) to determine their radiosities.

$$\frac{E_{b,s} - J_s}{(1-\varepsilon_s)/\varepsilon_s A_s} = \frac{J_s - J_{ss}}{1/A_s F_{s-ss}} + \frac{J_s - E_{b,sur}}{1/A_s F_{s-sur}} \tag{6}$$

$$\frac{E_{b,ss} - J_{ss}}{(1-\varepsilon_{ss})/\varepsilon_{ss} A_{ss}} = \frac{J_{ss} - J_s}{1/A_{ss} F_{ss-s}} + \frac{J_{ss} - E_{b,sur}}{1/A_{ss} F_{ss-sur}} \tag{7}$$

The net radiation leaving the shield is

$$q_s = A_{ss} F_{ss-s} \left(J_{ss} - J_s\right) + A_{ss} F_{ss-sur} \left(J_{ss} - E_{b,sur}\right) \tag{8}$$

Since the temperature of the shield is unknown, an additional relation is required. The heat transfer from the shield (s) to the floor (f) - the coolant heat removal rate - is

$$-q_s = \frac{\sigma\left(T_s^4 - T_f^4\right)A_s}{1 - 1/\varepsilon_s - 1/\varepsilon_f} \tag{9}$$

where the floor-shield configuration is that of infinite parallel planes, Eq. 13.24. Using the foregoing relations in the *IHT* workspace, with appropriate view factors from part (a), the following results were obtained

$$J_s = 18.13\ \ \mathrm{kW/m^2} \qquad J_{ss} = 98.20\ \ \mathrm{kW/m^2} \qquad T_s = 377^\circ\mathrm{C}$$

and the heat removal rate required of the coolant system is

$$q_{cs} = -q_s = 6.55\ \ \mathrm{kW} \qquad <$$

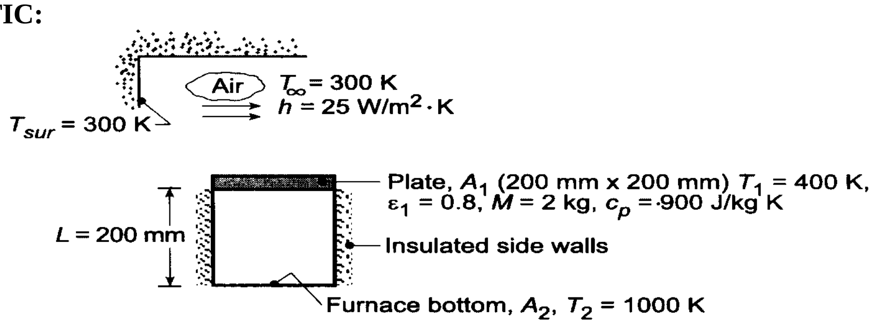
**COMMENTS:** The effect of the shield is to reduce the coolant system heat rate by a factor of nearly seven. Maintaining the integrity of the reflecting shield ($\varepsilon_s = 0.10$) operating at nearly 400°C in the mill environment to prevent corrosion or oxidation may be necessary.

# PROBLEM 13.91

**KNOWN:** Opaque, diffuse-gray plate with $\varepsilon_1 = 0.8$ is at $T_1 = 400$ K at a particular instant. The bottom surface of the plate is subjected to radiative exchange with a furnace. The top surface is subjected to ambient air and large surroundings.

**FIND:** (a) Net radiative heat transfer to the bottom surface of the plate for $T_1 = 400$ K, (b) Change in temperature of the plate with time, $dT_1/dt$, and (c) Compute and plot $dT_1/dt$ as a function of $T_1$ for the range $350 \le T_1 \le 900$ K; determine the steady-state temperature of the plate.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Plate is opaque, diffuse-gray and isothermal, (2) Furnace bottom behaves as a blackbody while sides are perfectly insulated, (3) Surroundings are large compared to the plate and behave as a blackbody.

**ANALYSIS:** (a) Recognize that the plate ($A_1$), furnace bottom ($A_2$) and furnace side walls ($A_R$) form a three-surface enclosure with one surface being re-radiating. The net radiative heat transfer *leaving* $A_1$ follows from Eq. 13.30 written as

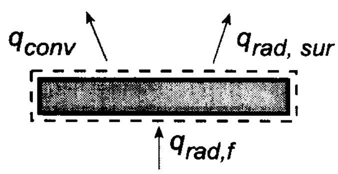
$$q_1 = \frac{E_{b1} - E_{b2}}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1 F_{12} + \left(1/A_1 F_{1R} + 1/A_2 F_{2R}\right)^{-1}}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2} \qquad (1)$$

From Fig. 13.4 with X/L = 0.2/0.2 = 1 and Y/L = 0.2/0.2 = 1, it follows that $F_{12} = 0.2$ and $F_{1R} = 1 - F_{12} = 1 - 0.2 = 0.8$. Hence, with $F_{1R} = F_{2R}$ (by symmetry) and $\varepsilon_2 = 1$.

$$q_1 = \frac{5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\left(400^4 - 1000^4\right)\mathrm{K}^4}{\dfrac{1-0.8}{0.8\times0.4\mathrm{m}^2} + \dfrac{1}{0.4\mathrm{m}^2\times0.20 + \left(2/0.04\mathrm{m}^2\times0.8\right)^{-1}}} = -1153\,\mathrm{W} \qquad <$$

It follows the net radiative exchange to the plate is, $q_{rad\cdot f} = 1153$ W.

(b) Perform now an energy balance on the plate written as




$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$$

$$q_{rad.f} - q_{conv} - q_{rad,sur} = Mc_p \frac{dT_1}{dt}$$

$$q_{rad.f} - hA_s\left(T_1 - T_\infty\right) - \varepsilon_1 A_1 \sigma\left(T_1^4 - T_{sur}^4\right) = Mc_p \frac{dT_1}{dt}. \quad (2)$$

Substituting numerical values and rearranging to obtain dT/dt, find

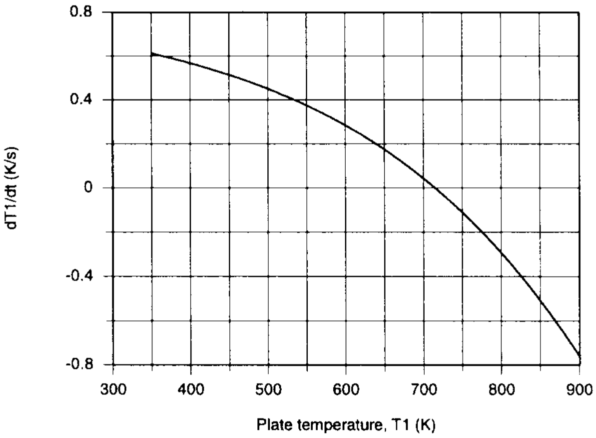
Continued …..

$$\frac{dT_1}{dt} = \frac{1}{2\,\text{kg} \times 900\,\text{J/kg}\cdot\text{K}}\Big[+1153\text{W} - 25\,\text{W/m}^2\cdot\text{K} \times 0.04\,\text{m}^2\,(400-300)\text{K}$$

$$-0.8 \times 0.04\text{m}^2 \times 5.67 \times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(400^4 - 300^4\right)\text{K}^4\Big] \quad <$$

$$\frac{dT_1}{dt} = 0.57\,\text{K/s}.$$

(c) With Eqs. (1) and (2) in the *IHT* workspace, $dT_1/dt$ was computed and plotted as a function of $T_1$.




When $T_1$ = 400 K, the condition of part (b), we found $dT_1/dt$ = 0.57 K/s which indicates the plate temperature is increasing with time. For $T_1$ = 900 K, $dT_1/dt$ is a negative value indicating the plate temperature will decrease with time. The steady-state condition corresponds to $dT_1/dt = 0$ for which

$$T_{1,ss} = 715\text{ K} \quad <$$

**COMMENTS:** Using the *IHT Radiation Tools – Radiation Exchange Analysis, Three Surface Enclosure with Re-radiating Surface and View Factors, Aligned Parallel Rectangle* – the above analysis can be performed. A copy of the workspace follows:

```
// Energy Balance on the Plate, Equation 2:
M * cp * dTdt = - q1 – h * A1 * (T1 – Tinf) – eps1 * A1 * sigma * (T1^4 – Tsur^4)

/* Radiation Tool – Radiation Exchange Analysis,
Three-Surface Enclosure with Reradiating Surface: */
/* For the three-surface enclosure A1, A2 and the reradiating surface AR, the net rate of radiation transfer
from the surface A1 to surface A2 is */
q1 = (Eb1 – Eb2) / ( (1 – eps1) / (eps1 * A1) + 1 / (A1 * F12 + 1/(1/(A1 * F1R) + 1/(A2 * F2R))) + (1 –
eps2) / (eps2 * A2))  //  Eq 13.30
/* The net rate of radiation transfer from surface A2 to surface A1 is */
q2 = -q1
/* From a radiation energy balance on AR, */
(JR – J1) / (1/(AR * FR1)) + (JR – J2) / (1/(AR *FR2)) = 0      // Eq 13.31
/* where the radiosities J1 and J2 are determined from the radiation rate equations expressed in terms of
the surface resistances, Eq 13.22 */
q1 = (Eb1 – J1) / ((1 – eps1) / (eps1 * A1))
q2 = (Eb2 – J2) / ((1-eps2) / (eps2 * A2))
// The blackbody emissive powers for A1 and A2 are
Eb1 = sigma * T1^4
Eb2 = sigma * T2^4
// For the reradiating surface,
JR = EbR
```

Continued …..

## PROBLEM 13.91 (Cont.)

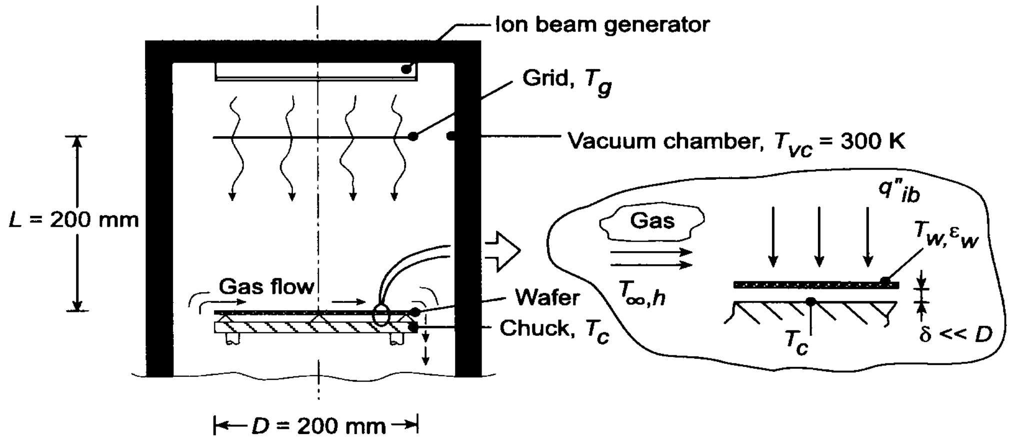
```
EbR = sigma *TR^4
sigma = 5.67E-8                  // Stefan-Boltzmann constant, W/m^2·K^4

// Radiation Tool – View Factor:
/* The view factor, F12, for aligned parallel rectangles, is */
F12 = Fij_APR(Xbar, Ybar)
// where
Xbar = X/L
Ybar = Y/L
// See Table 13.2 for schematic of this three-dimensional geometry.

// View Factors Relations:
F1R = 1 – F12
FR1 = F1R * A1 / AR
FR2 = FR1
A1 = X * Y
A2 = X * Y
AR = 2 * (X * Z + Y * Z)
Z = L
F2R = F1R

// Assigned Variables:
T1 = 400                         // Plate temperature, K
eps1 = 0.8                       // Plate emissivity
T2 = 1000                                  // Bottom temperature, K
eps2 = 0.9999                    // Bottom surface emissivity
X = 0.2                          // Plate dimension, m
Y = 0.2                          // Plate dimension, m
L = 0.2                          // Plate separation distance, m
M = 2                            // Mass, kg
cp = 900                         // Specific heat, J/kg.K,
h = 25                           // Convection coefficient, W/m^2.K
Tinf = 300                                 // Ambient air temperature, K
Tsur = 300                       // Surroundings temperature, K
```

# PROBLEM 13.92

**KNOWN:** Tool for processing silicon wafer within a vacuum chamber with cooled walls. Thin wafer is radiatively coupled on its back side to a chuck which is electrically heated. The top side is irradiated by an ion beam flux and experiences convection with the process gas and radioactive exchange with the ion-beam *grid* control surface and the chamber walls.

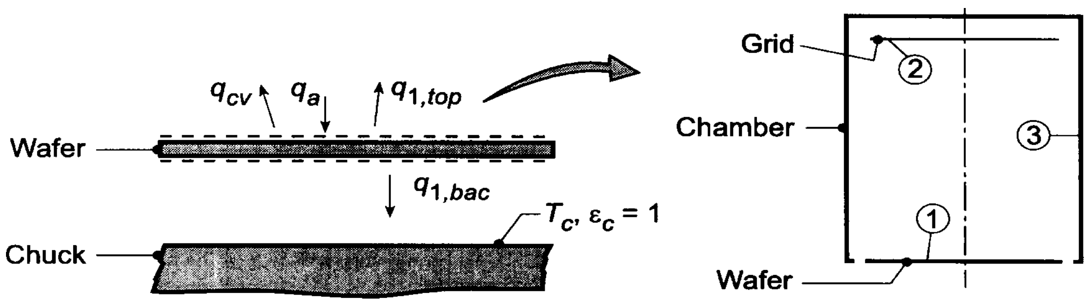
**FIND:** (a) Show control surfaces and all relevant processes on a schematic of the wafer, and (b) Perform an energy balance on the wafer and determine the chuck temperature $T_c$ required to maintain the prescribed conditions.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Wafer is diffuse, gray, (3) Separation distance between the wafer and chuck is much smaller than the wafer and chuck diameters, (4) Negligible convection in the gap between the wafer and chuck; convection occurs on the wafer top surface with the process gas, (5) Surfaces forming the three-surface enclosure – wafer ($\varepsilon_w$ = 0.8), grid ($\varepsilon_g$ = 1), and chamber walls ($\varepsilon_c$ = 1) have uniform radiosity and are diffuse, gray, and (6) the chuck surface is black.

**ANALYSIS:** (a) The wafer is shown schematically above in relation to the key components of the tool: the ion beam generator, the grid which is used to control the ion beam flux, $q''_{ib}$, the chuck which aids in controlling the wafer temperature and the process gas flowing over the wafer top surface. The schematic below shows the control surfaces on the top and back surfaces of the wafer along with the relevant thermal processes: $q_{cv}$, convection between the wafer and process gas; $q_a$, applied heat source due to absorption of the ion beam flux, $q''_{ib}$; $q_{1,top}$, net radiation leaving the top surface of the wafer (1) which is part of the three-surface enclosure – grid (2) and chamber walls (3), and; $q_{1,bac}$, net radiation leaving the backside of the wafer (w) which is part of a two-surface enclosure formed with the chuck (c).

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Continued …..

## PROBLEM 13.92 (Cont.)

(b) Referring to the schematic and the identified thermal processes, the energy balance on the wafer has the form,

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$-q_{cv} + q_a - q_{1,bac} - q_{1,top} = 0 \qquad (1)$$

where each of the processes are evaluated as follows:

*Convection with the process gas*: with $A_w = \pi D_4^2 = \pi (0.200\text{m})^2 / 4 = 0.03142\,\text{m}^2$,

$$q_{cv} = hA_w (T_w - T_\infty) = 10\,\text{W/m}^2 \times 0.03142\text{m}^2 \times (700 - 500)\text{K} = 62.84\,\text{W} \qquad (2)$$

*Applied heat source – ion beam*:

$$q_a = q''_{ib} A_w = 600\,\text{W/m}^2 \times 0.3142\text{m}^2 = 18.85\,\text{W} \qquad (3)$$

*Net radiation heat rate, back side*; *enclosure (w,c):* for the two-surface enclosure comprised of the back side of the wafer (w) and the chuck, (c), Eq. 13.28, yields

$$q_{1,bac} = \frac{\sigma\left(T_w^4 - T_c^4\right)}{(1-\varepsilon_w)/\varepsilon_w A_w + 1/A_w F_{wc} + (1-\varepsilon_c)/\varepsilon_c A_c}$$

and since the wafer-chuck approximate large parallel plates, $F_{wc} = 1$, and since the chuck is black, $\varepsilon_c = 1$,

$$q_{1,bac} = \frac{\sigma\left(T_w^4 - T_c^4\right)A_w}{(1-\varepsilon_w)/\varepsilon_w + 1} \qquad (4)$$

$$q_{1,bac} = \frac{0.03142\text{m}^2 \times \sigma\left(700^4 - T_c^4\right)\text{K}^4}{(1-0.6)/0.6+1} = 1.069\times10^{-9}\left(700^4 - T_c^4\right)$$

*Net radiation heat rate, top surface; enclosure (1, 2, 3)*: from the surface energy balance on $A_1$, Eq. 13.20.

$$q_{1,top} = \frac{E_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} \qquad (5)$$

where $\varepsilon_1 = \varepsilon_w$, $A_1 = A_w$, $E_{b1} = \sigma T_1^4$ and the radiosity can be evaluated by an enclosure analysis following the methodology of Section 13.2.2. From the energy balance, Eq. 13.21,

$$\frac{E_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} = \frac{J_1 - J_2}{1/A_1F_{12}} + \frac{J_1 - J_3}{1/A_1F_{13}} \qquad (6)$$

where $J_2 = E_{b2} = \sigma T_g^4$ and $J_3 = E_{b3} = \sigma T_{vc}^4$ since both surfaces are black ($\varepsilon_g = \varepsilon_{vc} = 1$). The view factor $F_{12}$ can be computed from the relation for coaxial parallel disks, Table 13.5.

$$F_{12} = 0.5\left\{S - \left[S^2 - 4(r_2/r_1)^2\right]^{1/2}\right\} = 0.5\left\{6.0 - \left[6.0^2 - 4(1)^2\right]^{1/2}\right\} = 0.1716$$

$$S = 1 + \frac{1+R_2^2}{R_1^2} = 1 + \frac{1+0.5^2}{0.5^2} = 6.00$$

Continued …..

## PROBLEM 13.92 (Cont.)

$$R_1 = r_1 / L = 100 / 200 = 0.5 \qquad\qquad R_4 = r_4 / L = 0.5$$

The view factor $F_{13}$ follows from the summation rule applied to $A_1$,

$$F_{13} = 1 - F_{12} = 1 - 0.1716 = 0.8284$$

Substituting numerical values into Eq. (6), with $T_1 = T_w = 700$ K, $T_2 = T_g = 500$ K, and $T_3 = T_{vc} =$ 300 K, find $J_1$,

$$\frac{\sigma T_1^4 - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} = \frac{J_1 - \sigma T_g^4}{1/F_{12}} + \frac{J_1 - \sigma T_{vc}^4}{1/F_{13}} \tag{7}$$

$$J_1 = 8564 \text{ W/m}^2$$

Using Eq. (5), find $q_{1,top}$ with $E_{b2} = \sigma T_w^4 = 13{,}614 \text{ W/m}^2$ and $A_1 = A_w$,

$$q_{1,top} = \frac{(13{,}614 - 8564)\text{ W/m}^2}{(1-0.6)/\left(0.6 \times 0.03142 \text{m}^2\right)} = 238 \text{ W}$$

Evaluating $T_c$ from the energy balance on the wafer, Eq. (1), and substituting appropriate expressions for each of the processes, find

$$-62.84 \text{ W/m}^2 + 18.85\text{W} - 1.069 \times 10^{-9}\left(700^4 - T_c^4\right) - 238 \text{ W} = 0$$

$$T_c = 842.5 \text{ K} \qquad <$$

From Eq. (4), with $T_c$ = 815 K, the electrical power required to maintain the chuck is

$$P_c = -q_{1,bac} = 1.069 \times 10^{-9}\left(700^4 - 842.5\right) = 282 \text{ W}$$

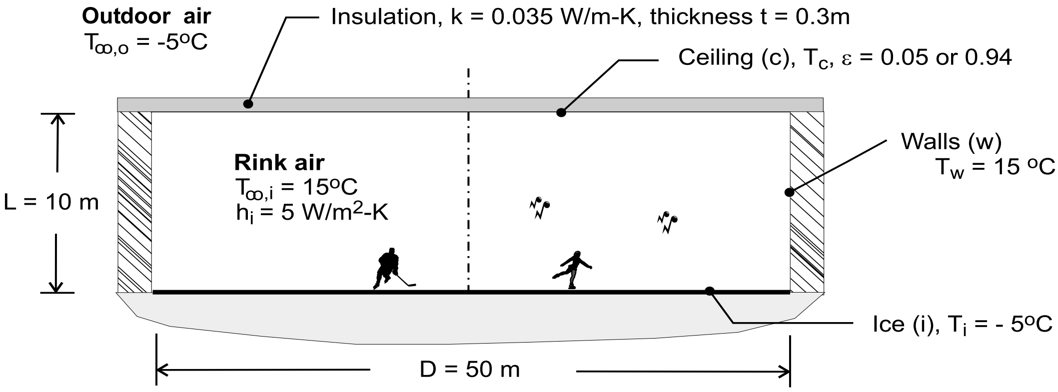
**COMMENTS:** Recognize that the method of analysis is centered about an energy balance on the wafer. Identifying the processes and representing them on the energy balance schematic is a vital step in developing the strategy for a solution. This methodology introduced in Section 1.3.3 becomes important, if not essential, in analyzing complicated physical systems.

# PROBLEM 13.93

**KNOWN:** Ice rink with prescribed ice, rink air, wall, ceiling and outdoor air conditions.

**FIND:** (a) Temperature of the ceiling, $T_c$, having an emissivity of 0.05 (highly reflective panels) or 0.94 (painted panels); determine whether condensation will occur for either or both ceiling panel types if the relative humidity of the rink air is 70%, and (b) Calculate and plot the ceiling temperature as a function of ceiling insulation thickness for $0.1 \le t \le 1$ m, identify conditions for which condensation will occur on the ceiling.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Rink comprised of the ice, walls and ceiling approximates a three-surface, diffuse-gray enclosure, (2) Surfaces have uniform radiosities, (3) Ice surface and walls are black, (4) Panels are diffuse-gray, and (5) Thermal resistance for convection on the outdoor side of the ceiling is negligible compared to the conduction thermal resistance of the ceiling insulation.

**PROPERTIES:** *Psychometric chart* (Atmospheric pressure; dry bulb temperature, $T_{db} = T_{\infty,i}$ = 15°C; relative humidity, RH = 70%): Dew point temperature, $T_{dp}$ = 9.4°C.

**ANALYSIS:** The energy balance on the ceiling illustrated in the schematic below has the form

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$-q_o - q_{conv,c} - q_{rad,c} = 0 \tag{1}$$

where the rate equations for each process are

$$q_o = \left(T_c - T_{\infty,o}\right)/R_{cond} \qquad\qquad R_{cond} = t/kA_c \tag{2,3}$$

$$q_{conv,c} = h\,A_c\left(T_c - T_{\infty,i}\right) \tag{4}$$

$$q_{rad,c} = \varepsilon\,E_b\left(T_c\right)A_c - \alpha\,A_w\,F_{wc}\,E_b\left(T_w\right) - \alpha\,A_i\,F_{ic}\,E_b\left(T_i\right) \tag{5}$$

The blackbody emissive powers are $E_b = \sigma T^4$ where $\sigma = 5.67 \times 10^{-8}$ W/m²·K⁴. Since the ceiling panels are diffuse-gray, $\alpha = \varepsilon$. The view factors required of Eq. (5): determine $F_{ic}$ (ice to ceiling) from Table 13.2 (Fig. 13.5) for parallel, coaxial disks

$$F_{ic} = 0.672$$

and $F_{wc}$ (wall to ceiling) from the summation rule on the ice (i) and the reciprocity rule,

$$F_{ic} + F_{iw} = 1 \qquad\qquad F_{iw} = F_{cw} \text{ (symmetry)}$$

$$F_{cw} = 1 - F_{ic}$$

$$F_{wc} = \left(A_c/A_w\right)F_{cw} = \left(A_c/A_w\right)\left(1 - F_{ic}\right) = 0.410$$

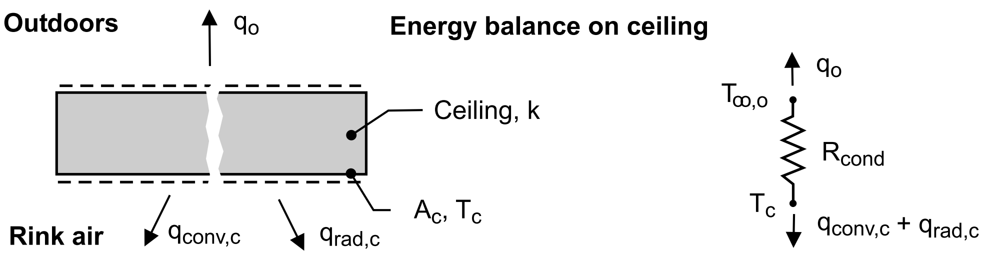
Continued …..

where $A_c = \pi D^2/4$ and $A_w = \pi DL$.

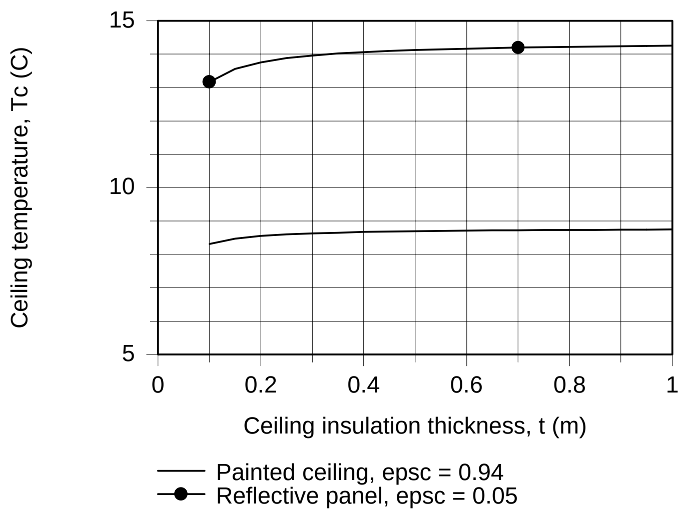
Using the foregoing energy balance, Eq. (1), and the rate equations, Eqs. (2-5), the ceiling temperature is calculated using radiative properties for the two panel types,

| *Ceiling panel* | $\varepsilon$ | $T_c$ (°C) | | |
|---|---|---|---|---|
| Reflective | 0.05 | 14.0 | | |
| Paint | 0.94 | 8.6 | $T_c < T_{dp}$ | < |

The dew point is 9.4°C corresponding to a relative humidity of 70% with (dry bulb) air temperature of 15°C. Condensation will occur on the painted panel since $T_c < T_{dp}$.




(b) The equations required of the analysis above were solved using *IHT*. The analysis is extended to calculate the ceiling temperatures for a range of insulation thickness and the results plotted below.




For the reflective panel ($\varepsilon$ = 0.05), the ceiling surface temperature is considerably above the dew point. Therefore, condensation will not occur for the range of insulation thickness shown. For the painted panel ($\varepsilon$ = 0.94), the ceiling surface temperature is always below the dew point. We expect condensation to occur for the range of insulation thickness shown.

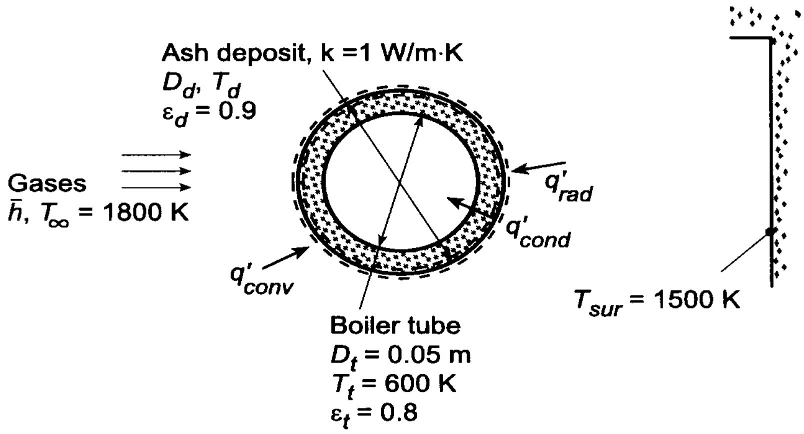
**COMMENTS:** From the analysis, recognize that the radiative exchange between the ice and the ceiling is the dominant process for influencing the ceiling temperature. With the reflective panel, the rate is reduced nearly 20 times that with the painted panel. With the painted panel ceiling, for most of the conditions likely to exist in the rink, condensation will occur.

## PROBLEM 13.94

**KNOWN:** Diameter, temperature and emissivity of boiler tube. Thermal conductivity and emissivity of ash deposit. Convection coefficient and temperature of gas flow over the tube. Temperature of surroundings.

**FIND:** (a) Rate of heat transfer to tube without ash deposit, (b) Rate of heat transfer with an ash deposit of diameter $D_d$ = 0.06 m, (c) Effect of deposit diameter and convection coefficient on heat rate and contributions due to convection and radiation.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Diffuse/gray surface behavior, (2) Surroundings form a large enclosure about the tube and may be approximated as a blackbody, (3) One-dimensional conduction in ash, (4) Steady-state.

**ANALYSIS:** (a) Without an ash deposit, the heat rate per unit tube length may be calculated directly.

$$q' = \bar{h}\pi D_t\left(T_\infty - T_t\right) + \varepsilon_t \sigma \pi D_t\left(T_{sur}^4 - T_t^4\right)$$

$$q' = 100\ \text{W/m}^2\cdot\text{K}\,(\pi)\,0.05\,\text{m}\,(1800-600)\,\text{K} + 0.8\times 5.67\times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\,(\pi)(0.05\,\text{m})\left(1500^4 - 600^4\right)\text{K}$$

$$q' = (18{,}850 + 35{,}150)\,\text{W/m} = 54{,}000\ \text{W/m} \qquad <$$

(b) Performing an energy balance for a control surface about the outer surface of the ash deposit, $q'_{conv} + q'_{rad} = q'_{cond}$, or

$$\bar{h}\pi D_d\left(T_\infty - T_d\right) + \varepsilon_d \sigma \pi D_d\left(T_{sur}^4 - T_d^4\right) = \frac{2\pi k\left(T_d - T_t\right)}{\ln\left(D_d/D_t\right)}$$

Hence, canceling π and considering an ash deposit for which $D_d$ = 0.06 m,

$$100\,\text{W/m}^2\cdot\text{K}\,(0.06\,\text{m})(1800 - T_d)\,\text{K} + 0.9\times 5.67\times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\,(0.06\,\text{m})\left(1500^4 - T_d^4\right)\text{K}^4$$

$$= \frac{2(1\ \text{W/m}\cdot\text{K})(T_d - 600)\,\text{K}}{\ln(0.06/0.05)}$$

A trial-and-error solution yields $T_d \approx 1346$ K, from which it follows that

$$q' = \bar{h}\pi D_d\left(T_\infty - T_d\right) + \varepsilon_d \sigma \pi D_d\left(T_{sur}^4 - T_d^4\right)$$

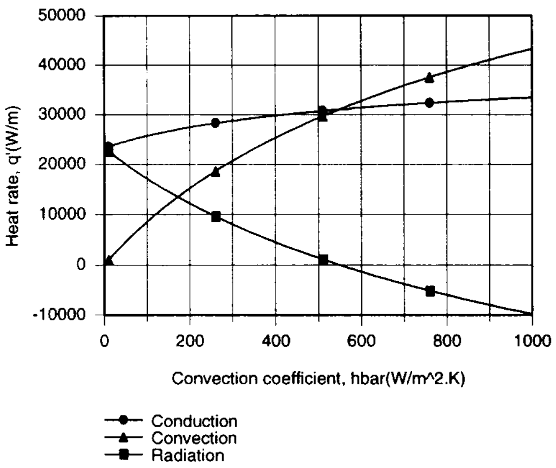
$$q' = 100\,\text{W/m}^2\cdot\text{K}\,(\pi)\,0.06\,\text{m}\,(1800-1346)\,\text{K} + 0.9\times 5.67\times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\,(\pi)\,0.06\,\text{m}\left(1500^4 - 1346^4\right)\text{K}^4$$

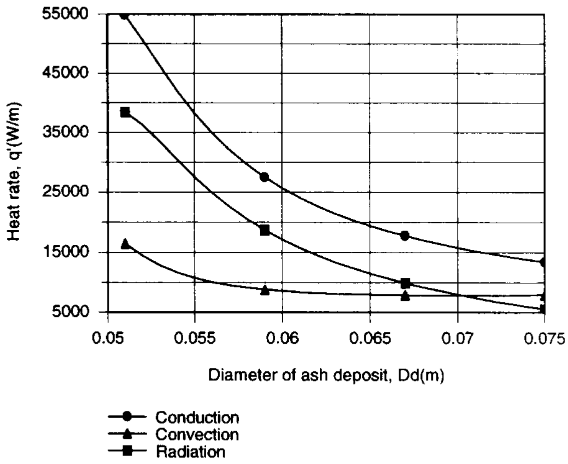
Continued …..

## PROBLEM 13.94 (Cont.)

$$q' = (8560 + 17{,}140)\,\mathrm{W/m} = 25{,}700\,\mathrm{W/m} \qquad <$$

(c) The foregoing energy balance was entered into the *IHT* workspace and parametric calculations were performed to explore the effects of $\bar{h}$ and $D_d$ on the heat rates.




For $D_d = 0.06$ m and $10 \le \bar{h} \le 1000\ \mathrm{W/m^2 \cdot K}$, the heat rate to the tube, $q'_{cond}$, as well as the contribution due to convection, $q'_{conv}$, increase with increasing $\bar{h}$. However, because the outer surface temperature $T_d$ also increases with $\bar{h}$, the contribution due to radiation decreases and becomes negative (heat transfer from the surface) when $T_d$ exceeds 1500 K at $\bar{h} = 540\ \mathrm{W/m^2 \cdot K}$. Both the convection and radiation heat rates, and hence the conduction heat rate, increase with decreasing $D_d$, as $T_d$ decreases and approaches $T_t = 600$ K. However, even for $D_d = 0.051$ m (a deposit thickness of 0.5 mm), $T_d = 773$ K and the ash provides a significant resistance to heat transfer.

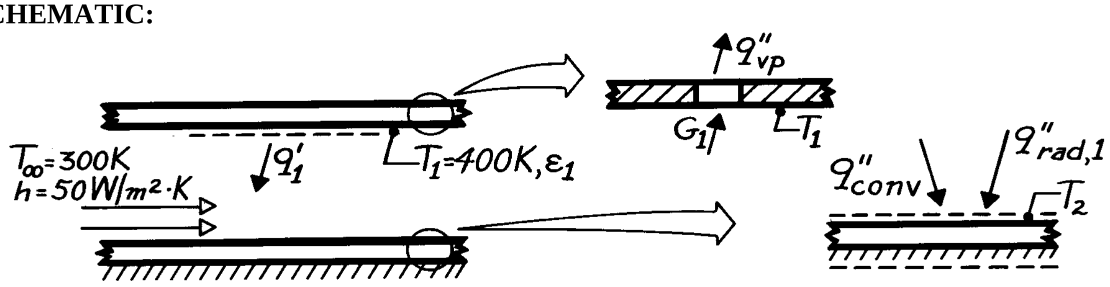
**COMMENTS:** Boiler operation in an energy efficient manner dictates that ash deposits be minimized.

## PROBLEM 13.95

**KNOWN:** Two parallel, large, diffuse-gray surfaces; top one maintained at $T_1$ while lower one is insulated and experiences convection.

**FIND:** (a) Temperature of lower surface, $T_2$, when $\varepsilon_1 = \varepsilon_2 = 0.5$ and (b) Radiant flux leaving the viewing port.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Surfaces are large, diffuse-gray, (2) Lower surface experiences convection and radiation exchange, backside is perfectly insulated.

**ANALYSIS:** (a) Perform an energy balance on the lower surface, giving

$$q''_{conv} + q''_{rad,1} = 0 \tag{1}$$

where the latter term is equal to $q''_1$ or $q''_{12}$, the net radiant power per unit area exchanged between surfaces 1 and 2. For this two surface enclosure,

$$q''_1 = \frac{E_b(T_1) - E_b(T_2)}{(1-\varepsilon_1)/\varepsilon_1 + 1/F_{12} + (1-\varepsilon_2)/\varepsilon_2} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{(1-\varepsilon_1)/\varepsilon_1 + 1 + (1-\varepsilon_2)/\varepsilon_2} \tag{2}$$

with $F_{12} = 1$. Combining Eqs. (1) and (2),

$$h(T_\infty - T_2) + \sigma\left(T_1^4 - T_2^4\right)/\left[(1-\varepsilon_1)/\varepsilon_1 + 1 + (1-\varepsilon_2)/\varepsilon_2\right] = 0 \tag{3}$$

Substituting numerical values with $\varepsilon_1 = \varepsilon_2 = 0.5$,

$$50 \text{ W/m}^2 \cdot \text{K}(300 - T_2)\text{K} + 5.67\times10^{-8}\text{W/m}^2 \cdot \text{K}^4\left(400^4 - T_2^4\right)\text{K}^4/[1+1+1] = 0$$

$$T_2 \approx 306 \text{ K.}$$ <

(b) The radiant flux leaving the viewing port is $q''_{vp} = G_1$. From an energy balance on the upper plate

$$q''_1 = E_1 - \alpha_1 G_1$$

where $q''_1 = q''_{1-2}$, net exchange by radiation. But

$$q''_1 = (1/3)\sigma\left(T_1^4 - T_2^4\right)$$

$$E_1 = \varepsilon E_{b1} = 0.5\sigma T_1^4.$$

Hence, the flux is

$$G_1 = (E_1 - q_1)/\alpha = (1/0.5)\left[0.5\sigma T_1^4 - (1/3)\sigma\left(T_1^4 - T_2^4\right)\right]$$

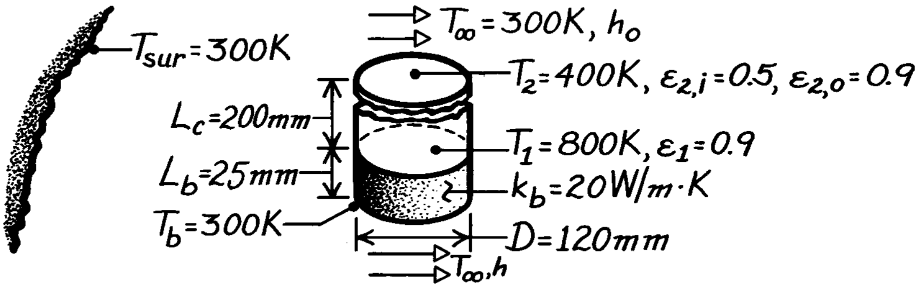
$$G_1 = 2\sigma\left[(0.5 - 0.333)T_1^4 + 0.333 T_2^4\right] = 816 \text{ W/m}^2.$$ <

# PROBLEM 13.96

**KNOWN:** Dimensions, emissivities and temperatures of heated and cured surfaces at opposite ends of a cylindrical cavity. External conditions.

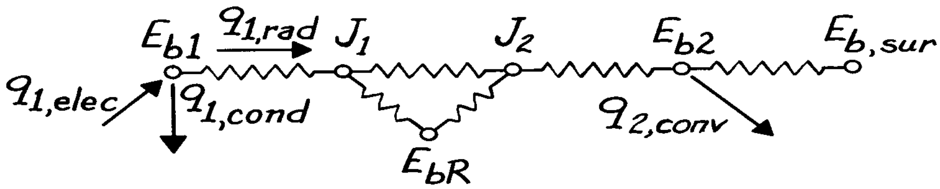
**FIND:** Required heater power and outside convection coefficient.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Opaque, diffuse-gray surfaces, (3) Negligible convection within cavity, (4) Isothermal disk and heater surfaces, (5) One-dimensional conduction in base, (6) Negligible contact resistance between heater and base, (7) Sidewall is reradiating.

**ANALYSIS:** The equivalent circuit is




From an energy balance on the heater surface, $q_{1,elec} = q_{1,cond} + q_{1,rad}$,

$$q_{1,elec} = k_b\left(\pi D^2/4\right)\frac{T_1 - T_b}{L_b} + \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1F_{12} + \left[(1/A_1F_{1R}) + (1/A_2F_{2R})\right]^{-1}} + \dfrac{1-\varepsilon_{2,i}}{\varepsilon_{2,i}A_2}}$$

where $A_1 = A_2 = \pi D^2/4 = \pi(0.12\text{ m})^2/4 = 0.0113\text{ m}^2$ and from Fig. 13.5, with $L_c/r_1 = 3.33$ and $r_2/L_c = 0.3$ find $F_{12} = F_{21} = 0.077$; hence, $F_{1R} = F_{2R} = 0.923$. The required heater power is

$$q_{1,elec} = 20\text{ W/m}\cdot\text{K}\times 0.0113\text{ m}^2\,\frac{(800-300)\text{K}}{0.025\text{ m}}$$

$$+\frac{0.0113\text{ m}^2\times 5.67\times10^{-8}\text{W/m}^2\cdot\text{K}^4\left(800^4 - 400^4\right)\text{K}^4}{\dfrac{1-0.9}{0.9} + \dfrac{1}{0.077 + \left[(1/0.923) + (1/0.923)\right]^{-1}} + \dfrac{1-0.5}{0.5}}$$

$$q_{1,elec} = 4521\text{ W} + 82.9\text{ W} = 4604\text{ W.}$$ <

An energy balance for the disk yields, $q_{rad,2} = q_{rad,1} = h_oA_2\left(T_2 - T_\infty\right) + \varepsilon_{2,o}A_2\sigma\left(T_2^4 - T_{sur}^4\right)$,

$$h_o = \frac{82.9\text{ W} - 0.9\times0.0113\text{ m}^2\times5.67\times10^{-8}\text{W/m}^2\cdot\text{K}^4\left(400^4 - 300^4\right)\text{K}^4}{0.0113\text{ m}^2\times100\text{ K}} = 64\text{ W/m}^2\cdot\text{K.}$$ <

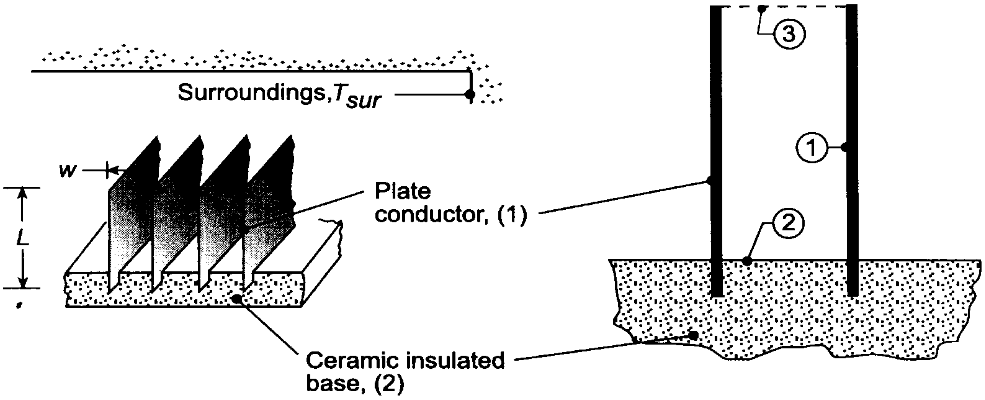
**COMMENTS:** Conduction through the ceramic base represents an enormous system loss. The base should be insulated to greatly reduce this loss and hence the electric power input.

# PROBLEM 13.97

**KNOWN:** Electrical conductors in the form of parallel plates having one edge mounted to a ceramic insulated base. Plates exposed to large, isothermal surroundings, $T_{sur}$. Operating temperature is $T_1$ = 500 K.

**FIND:** (a) Electrical power dissipated in a conductor plate per unit length, $q_1'$, considering only radiative exchange with the surroundings; temperature of the ceramic insulated base $T_2$; and, (b) $q_1'$ and $T_2$ when the surfaces experience convection with an airstream at $T_\infty$ = 300 K and a convection coefficient of h = 24 W/m$^2$·K.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Conductor surfaces are diffuse, gray, (2) Conductor and ceramic insulated base surfaces have uniform temperatures and radiosities, (3) Surroundings are large, isothermal.

**ANALYSIS:** (a) Define the opening between the conductivities as the hypothetical area $A_3$ at the temperature of the surroundings, $T_{sur}$, with an emissivity $\varepsilon_3 = 1$ since all the radiation incident on the area will be absorbed. The conductor (1)-base (2)-opening (3) form a three surface enclosure with one surface re-radiating (2). From Section 13.3.5 and Eq. 13.30, the net radiation leaving the conductor surface $A_1$ is

$$q_1 = \frac{E_{b1} - E_{b2}}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1 F_{13} + \left[(1/A_1 F_{12}) + (1/A_3 F_{32})\right]^{-1}} + \dfrac{1-\varepsilon_3}{\varepsilon_3 A_3}} \qquad (1)$$

where $E_{b1} = \sigma T_1^4$ and $E_{b1} = \sigma T_3^4$. The view factors are evaluated as follows:

$F_{32}$: use the relation for two aligned parallel rectangles, Table 13.2 or Fig. 13.4,

$$\bar{X} = X/L = w/L = 10/40 = 0.25 \qquad \bar{Y} = Y/L = \infty$$

$$F_{32} = 0.1231$$

$F_{13}$: applying reciprocity between $A_1$ and $A_3$, where $A_1 = 2L\ell = 2 \times 0.040\text{ m}\ell = 0.080\ \ell$ and $A_3 = w\ell = 0.010\ \ell$ and $\ell$ is the length of the conductors normal to the page, $\ell$ >> L or w,

$$F_{13} = \frac{A_3 F_{31}}{A_1} = 0.010\ell \times 0.8769 / 0.080\ell = 0.1096$$

where $F_{31}$ can be obtained by using the summation rule on $A_3$,

$$F_{31} = 1 - F_{32} = 1 - 0.1231 = 0.8769$$

$F_{12}$: by symmetry $\qquad F_{12} = F_{13} = 0.1096$

Continued …..

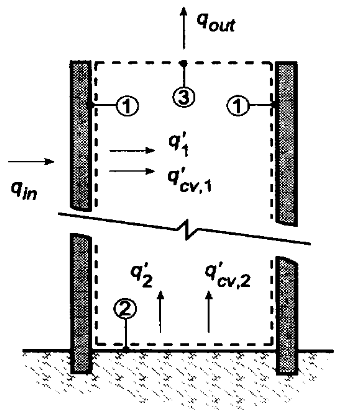
# PROBLEM 13.97 (Cont.)

Substituting numerical values into Eq. (1), the net radiation leaving the conductor is

$$q_1 = \frac{5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\left(500^4 - 300^4\right)\mathrm{K^4}}{\dfrac{1-0.8}{0.8\times0.080\ell} + \dfrac{1}{0.080\ell\times0.1096 + \left[(1/0.080\ell\times0.1096)+(1/0.010\ell\times0.123)\right]^{-1}} + 0}$$

$$q_1' = q_1/\ell = \frac{(3544-459.3)\,\mathrm{W}}{3.1250+101.557+0} = 29.5\ \mathrm{W/m} \qquad <$$

(b) Consider now convection processes occurring at the conductor (1) and base (2) surfaces, and perform energy balances as illustrated in the schematic below.




*Surface 1*: The heat rate from the conductor includes convection and the net radiation heat rates,

$$q_{in} = q_{cv,1} + q_1 = h\,A_1\left(T_1 - T_\infty\right) + \frac{E_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} \qquad (2)$$

and the radiosity $J_1$ can be determined from the radiation energy balance, Eq. 13.21,

$$\frac{E_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} = \frac{J_1 - J_2}{1/A_1F_{12}} + \frac{J_1 - J_3}{1/A_1F_{13}} \qquad (3)$$

where $J_3 = E_{b3} = \sigma T_3^4$ since $A_3$ is black.

*Surface 2*: Since the surface is insulated (adiabatic), the energy balance has the form

$$0 = q_{cv,2} + q_2 = hA_2\left(T_2 - T_\infty\right) + \frac{E_{b2} - J_2}{1-\varepsilon_2/\varepsilon_2 A_2} \qquad (4)$$

and the radiosity $J_2$ can be determined from the radiation energy balance, Eq. 13.21,

$$\frac{E_{b2} - J_2}{(1-\varepsilon_2)/\varepsilon_2 A_2} = \frac{J_2 - J_1}{1/A_2F_{21}} + \frac{J_2 - J_3}{1/A_2F_{23}} \qquad (5)$$

There are 4 equations, Eqs. (2-5), with 4 unknowns: $J_2$, $J_2$, $T_2$ and $q_1$. Substituting numerical values, the simultaneous solution to the set yields

$$J_1 = 3417\ \mathrm{W/m^2} \qquad J_2 = 1745\ \mathrm{W/m^2} \qquad T_2 = 352\ \mathrm{K} \qquad q_{in}' = 441\ \mathrm{W/m} \qquad <$$

**COMMENTS:** (1) The effect of convection is substantial, increasing the heat removal rate from 29.5 W to 441 W for the combined modes.

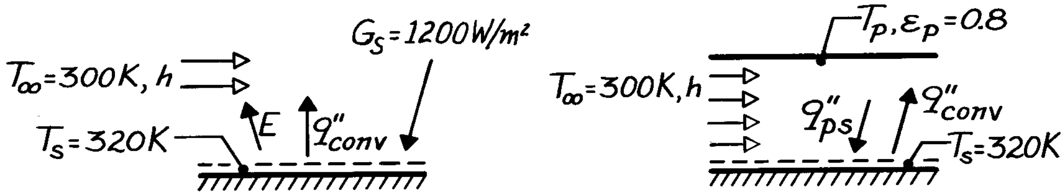
(2) With the convection process, the current carrying capacity of the conductors can be increased. Another advantage is that, with the presence of convection, the ceramic base operates at a cooler temperature: 352 K vs. 483 K.

## PROBLEM 13.98

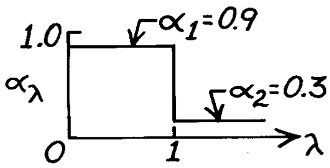
**KNOWN:** Surface temperature and spectral radiative properties. Temperature of ambient air. Solar irradiation or temperature of shield.

**FIND:** (a) Convection heat transfer coefficient when surface is exposed to solar radiation, (b) Temperature of shield needed to maintain prescribed surface temperature.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Surface is diffuse ($\alpha_\lambda = \varepsilon_\lambda$), (2) Bottom of surface is adiabatic, (3) Atmospheric irradiation is negligible,
(4) With shield, convection coefficient is unchanged and radiation losses at ends are negligible (two-surface enclosure).




**ANALYSIS:** (a) From a surface energy balance,

$$\alpha_S G_S = \varepsilon_s \sigma T_s^4 + h\left(T_s - T_\infty\right).$$

Emission occurs mostly at long wavelengths, hence $\varepsilon_s = \alpha_2 = 0.3$. However,

$$\alpha_S = \frac{\int_0^\infty \alpha_\lambda E_{\lambda,b}\left(\lambda,\ 5800\ \mathrm{K}\right)d\lambda}{E_b} = \alpha_1 F_{(0-1\mu m)} + \alpha_2 F_{(1-\infty)}$$

and from Table 12.1 at $\lambda T$ = 5800 μm·K, $F_{(0\text{-}1\mu m)}$ = 0.720 and hence, $F_{(1\text{ - }\infty)}$ = 0.280 giving

$$\alpha = 0.9\times 0.72 + 0.3\times 0.280 = 0.732.$$

Hence

$$h = \frac{\alpha_S G_S - \varepsilon\sigma T_s^4}{T_s - T_\infty} = \frac{0.732\left(1200\ \mathrm{W/m^2}\right) - 0.3\times 5.67\times 10^{-8}\,\mathrm{W/m^2\cdot K^4}\left(320\ \mathrm{K}\right)^4}{20\ \mathrm{K}}$$

$$h = 35\ \mathrm{W/m^2\cdot K}. \qquad <$$

(b) Since the plate emits mostly at long wavelengths, $\alpha_s = \varepsilon_s = 0.3$. Hence radiation exchange is between two diffuse-gray surfaces.

$$q''_{ps} = \frac{\sigma\left(T_p^4 - T_s^4\right)}{1/\varepsilon_p + 2/\varepsilon_s - 1} = q''_{conv} = h\left(T_s - T_\infty\right)$$

$$T_p^4 = \left(h/\sigma\right)\left(T_s - T_\infty\right)\left(1/\varepsilon_p + 1/\varepsilon_s - 1\right) + T_s^4$$

$$T_p^4 = \frac{35\ \mathrm{W/m^2\cdot K}\left(20\ \mathrm{K}\right)}{5.67\times 10^{-8}\,\mathrm{W/m^2\cdot K^4}}\left(\frac{1}{0.8} + \frac{1}{0.3} - 1\right) + \left(320\ \mathrm{K}\right)^4 \qquad T_p = 484\ \mathrm{K}. \qquad <$$

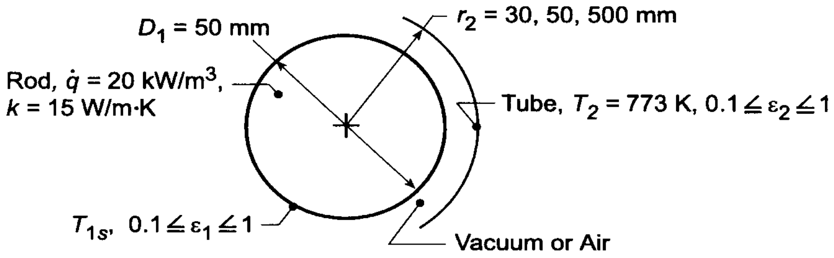
**COMMENTS:** For $T_p$ = 484 K and $\lambda$ = 1 μm, $\lambda T$ = 484 μm·K and $F_{(0\text{-}\lambda)}$ = 0.000. Hence assumption of $\alpha_s$ = 0.3 is excellent.

# PROBLEM 13.99

**KNOWN:** Long uniform rod with volumetric energy generation positioned coaxially within a larger circular tube maintained at 500°C.

**FIND:** (a) Center $T_1(0)$ and surface $T_{1s}$ temperatures of the rod for evacuated space, (b) $T_1(0)$ and $T_{1s}$ for airspace, (c) Effect of tube diameter and emissivity on $T_1(0)$ and $T_{1s}$.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) All surfaces are diffuse-gray.

**PROPERTIES:** *Table A-4*, Air ($\overline{T}$ = 780 K): $\nu = 81.5 \times 10^{-6}$ m$^2$/s, k = 0.0563 W/m·K, $\alpha = 115.6 \times 10^{-6}$ m$^2$/s, $\beta$ = 0.00128K$^{-1}$, Pr = 0.706.

**ANALYSIS:** (a) The net heat exchange by radiation between the rod and the tube is

$$q'_{12} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\left(1-\varepsilon_1\right)/\varepsilon_1\pi D_1 + 1/\pi D_1 F_{12} + \left(1-\varepsilon_2\right)/\varepsilon_2\pi D_2} \tag{1}$$

and, from an energy balance on the rod, $-\dot{E}'_{out} + \dot{E}'_{gen} = 0,$ or

$$q'_{12} = \dot{q}\left(\pi D_1^2/4\right). \tag{2}$$

Combining Eqs. (1) and (2) and substituting numerical values, with $F_{12}$ = 1, we obtain

$$\dot{q} = \frac{4}{D_1}\left[\frac{\sigma\left(T_1^4 - T_2^4\right)}{\left(1-\varepsilon_1\right)/\varepsilon_1 + 1 + \left[\left(1-\varepsilon_2\right)/\varepsilon_2\right]\left(D_1/D_2\right)}\right]$$

$$20\times10^3\,\frac{W}{m^3} = \frac{4}{0.050\text{m}}\left[\frac{5.67\times10^{-8}\,W/m^2\cdot K^4\left(T_{1s}^4 - 773^4\right)K^4}{\left(1-0.2\right)/0.2 + 1 + \left[\left(1-0.2\right)/0.2\right]\left(0.050/0.060\right)}\right]$$

$$= 54.4\times10^{-8}\left(T_{1s}^4 - 773^4\right)\,W/m^3$$

$$T_{1s} = 792\ K. \qquad <$$

From Eq. 3.53, the rod center temperature is

$$T_1(0) = \frac{\dot{q}\left(D_1/2\right)^2}{4k} + T_{1s}$$

$$T_1(0) \approx \frac{20\times10^3\,W/m^3\left(0.050\,m/2\right)^2}{4\times15\,W/m\cdot K} + 792\ K = 0.21\ K + 792\ K = 792.2\ K. \qquad <$$

(b) The convection heat rate is given by Eqs. 9.58 to 9.60. However, assuming a maximum possible value of $(T_{s1} - T_2)$ = 19 K, $Ra_L = g\beta\,(T_{s,1} - T_2)L^3/\alpha\nu$ = 9.8 m/s$^2$ (0.00128 K$^{-1}$)19 K (0.005 m)$^3$/115.6 × 81.5 × 10$^{-12}$ m$^4$/s$^2$ = 3.16 and $Ra_c^*$ = {[ln($D_2/D_1$)]$^4$/$L^3$[$(D_1)^{-3/5}$ + $(D_2)^{-3/5}$]$^5$} $Ra_L$ = {[ln(1.2)]$^4$/(0.005 m)$^3$ [(0.05 m)$^{-3/5}$

Continued …..

$+(0.06\text{ m})^{-3/5}]^{5}\}$ 3.16 = 0.14. It follows that buoyancy driven flow is negligible and heat transfer across the airspace is by conduction. Hence, from Eq. 3.27, $q'_{cond} = 2\pi k(T_{1s} - T_2)/\ln(r_2/r_1)$.

$$q'_{cond} = \frac{2\pi k(T_{1s} - T_2)}{\ln(r_2/r_1)} = \frac{2\pi(0.0563\text{ W/m}\cdot\text{K})(T_{1s} - 773)\text{K}}{\ln(30/25)} = 1.94(T_{1s} - 773)$$

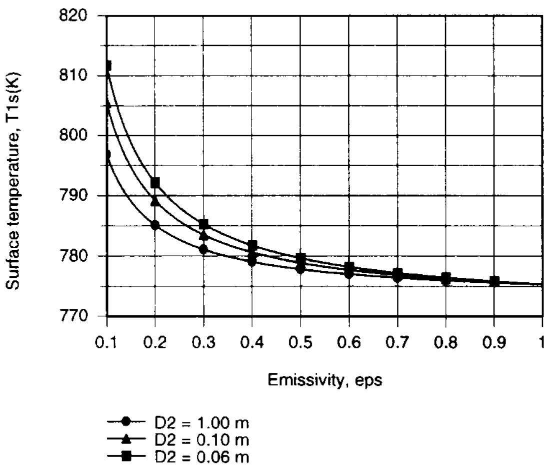
The energy balance then becomes $\dot{q}\left(\pi D_1^2/4\right) = q'_{12} + q'_{cond}$, or

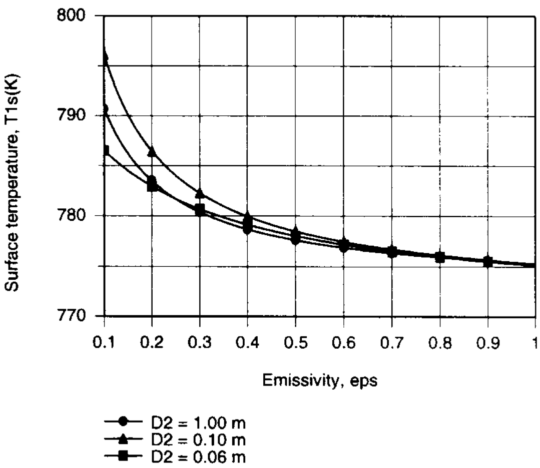
$$\dot{q} = \left(4/\pi D_1^2\right)\left(q'_{12} + q'_{cond}\right)$$

$$2\times10^4 = \left[54.4\times10^{-8}\left(T_{1s}^4 - 773^4\right) + 988\left(T_{1s} - 773\right)\right]$$

$$T_{1s} = 783\text{ K} \qquad T_1(0) = 783.2\text{ K}$$ <

(c) Entering the foregoing model and the prescribed properties of air into the *IHT* workspace, the parametric calculations were performed for $D_2$ = 0.06 m and $D_2$ = 0.10 m. For $D_2$ = 1.0 m, $Ra_c^* > 100$ and heat transfer across the airspace is by free convection, instead of conduction. In this case, convection was evaluated by entering Eqs. 9.58 – 9.60 into the workspace. The results are plotted as follows.




The first graph corresponds to the evacuated space, and the surface temperature decreases with increasing $\varepsilon_1 = \varepsilon_2$, as well as with $D_2$. The increased emissivities enhance the effectiveness of emission at surface 1 and absorption at surface 2, both which have the effect of reducing $T_{1s}$. Similarly, with increasing $D_2$, more of the radiation emitted from surface 1 is ultimately absorbed at 2 (less of the radiation reflected by surface 2 is intercepted by 1). The second graph reveals the expected effect of a reduction in $T_{1s}$ with inclusion of heat transfer across the air. For small emissivities ($\varepsilon_1 = \varepsilon_2 < 0.2$), conduction across the air is significant relative to radiation, and the small conduction resistance corresponding to $D_2$ = 0.06 m yields the smallest value of $T_{1s}$. However, with increasing $\varepsilon$, conduction/convection effects diminish relative to radiation and the trend reverts to one of decreasing $T_{1s}$ with increasing $D_2$.

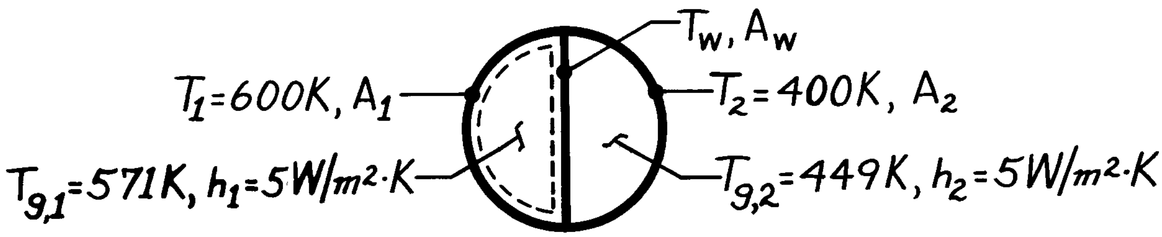
**COMMENTS:** For this situation, the temperature variation *within* the rod is small and independent of surface conditions.

## PROBLEM 13.100

**KNOWN:** Side wall and gas temperatures for adjoining semi-cylindrical ducts. Gas flow convection coefficients.

**FIND:** (a) Temperature of intervening wall, (b) Verification of gas temperature on one side.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) All duct surfaces may be approximated as blackbodies, (2) Fully developed conditions, (3) Negligible temperature difference across intervening wall, (4) Gases are nonparticipating media.

**ANALYSIS:** (a) Applying an energy balance to a control surface about the wall yields

$$\dot{E}_{in} = \dot{E}_{out}.$$

Assuming $T_{g,1} > T_w > T_{g,2}$, it follows that

$$q_{rad(1 \to w)} + q_{conv(g1 \to w)} = q_{rad(w \to 2)} + q_{conv(w \to g2)}$$

$$A_1 F_{1w} \sigma \left(T_1^4 - T_w^4\right) + hA_w \left(T_{g,1} - T_w\right) = A_w F_{w2} \sigma \left(T_w^4 - T_2^4\right) + hA_w \left(T_w - T_{g,2}\right)$$

and with

$$A_1 F_{1w} = A_w F_{w1} = A_w F_{w2} = A_w$$

and substituting numerical values,

$$2\sigma T_w^4 + 2hT_w = \sigma \left(T_1^4 + T_2^4\right) + h\left(T_{g,1} + T_{g,2}\right)$$

$$11.34 \times 10^{-8} T_w^4 + 10 T_w = 13,900.$$

Trial-and-error solution yields

$$T_w \approx 526 \text{ K}. \qquad <$$

(b) Applying an energy balance to a control surface about the hot gas (g,1) yields

$$\dot{E}_{in} = \dot{E}_{out}$$

$$hA_1 \left(T_1 - T_{g,1}\right) = hA_w \left(T_{g,1} - T_w\right)$$

or

$$T_1 - T_{g,1} = \left[D / (\pi D / 2)\right]\left(T_{g,1} - T_w\right)$$

$$29°C = 29°C. \qquad <$$

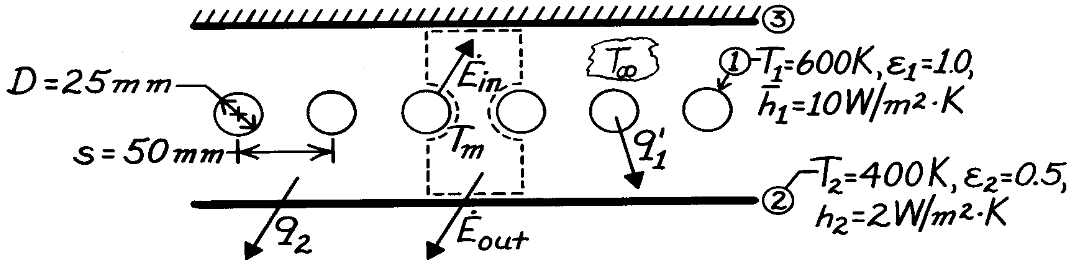
**COMMENTS:** Since there is no change in any of the temperatures in the axial direction, this scheme simply provides for energy transfer from side wall 1 to side wall 2.

# PROBLEM 13.101

**KNOWN:** Temperature, dimensions and arrangement of heating elements between two large parallel plates, one insulated and the other of prescribed temperature. Convection coefficients associated with elements and bottom surface.

**FIND:** (a) Temperature of gas enclosed by plates, (b) Element electric power requirement, (c) Rate of heat transfer to 1 m × 1m section of panel.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Diffuse-gray surfaces, (2) Negligible end effects since the surfaces form an enclosure, (3) Gas is nonparticipating, (4) Surface 3 is reradiating with negligible conduction and convection.

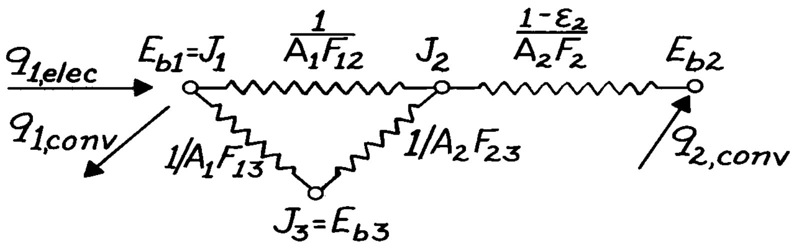
**ANALYSIS:** (a) Performing an energy balance for a unit control surface about the gas space, $\dot{E}_{in} - \dot{E}_{out} = 0.$

$$\overline{h}_1 \pi D (T_1 - T_m) - \overline{h}_2 s (T_m - T_2) = 0$$

$$T_m = \frac{\overline{h}\pi D T_1 + \overline{h}_2 s T_2}{\overline{h}_1 \pi D + \overline{h}_2 s} = \frac{10\ \text{W/m}^2\cdot\text{K}\pi(0.025\ \text{m})600\ \text{K} + 2\ \text{W/m}^2\cdot\text{K}(0.05\ \text{m})400\ \text{K}}{10\ \text{W/m}^2\cdot\text{K}\pi(0.025\ \text{m}) + 2\ \text{W/m}^2\cdot\text{K}(0.05\ \text{m})}$$

$$T_m = 577\ \text{K}.$$ <

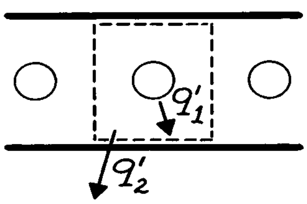
(b) The equivalent thermal circuit is




The energy balance on surface 1 is

$$q'_{1,elec} = q'_{1,conv} + q'_{1,rad}$$

where $q'_{1,rad}$ can be evaluated by considering a unit cell of the form




$$A'_1 = \pi D = \pi(0.025\ \text{m}) = 0.0785\ \text{m}$$
$$A'_2 = A'_3 = s = 0.05\ \text{m}$$

Continued …..

## PROBLEM 13.101 (Cont.)

The view factors are:

$$F_{21} = 1 - \left[1 - (D/s)^2\right]^{1/2} + (D/s)\tan^{-1}\left[\left(s^2 - D^2\right)/D^2\right]^{1/2}$$

$$F_{21} = 1 - [1 - 0.25]^{1/2} + 0.5\tan^{-1}(4-1)^{1/2} = 0.658 = F_{31}$$

$$F_{23} = 1 - F_{21} = 0.342 = F_{32}.$$

For the unit cell,

$$A_2'F_{21} = sF_{21} = 0.05\ \text{m} \times 0.658 = 0.0329\ \text{m} = A_1'F_{12} = A_3'F_{31} = A_1'F_{13}$$

$$A_2'F_{23} = sF_{23} = 0.05\ \text{m} \times 0.342 = 0.0171\ \text{m} = A_3'F_{32}.$$

Hence,

$$q'_{1,\text{rad}} = \frac{E_{b1} - E_{b2}}{R'_{\text{equiv}} + (1-\varepsilon_2)/\varepsilon_2 A_2'}$$

$$R'^{-1}_{\text{equiv}} = A_1'F_{12} + \frac{1}{1/A_1'F_{13} + 1/A_2'F_{23}} = \left(0.0329 + \frac{1}{(0.0329)^{-1} + (0.0171)^{-1}}\right)\text{m}$$

$$R'_{\text{equiv}} = 22.6\ \text{m}^{-1}.$$

Hence

$$q'_{1,\text{rad}} = \frac{5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(600^4 - 400^4\right)\text{K}^4}{\left[22.6 + (1-0.5)/0.5\times0.05\right]\text{m}^{-1}} = 138.3\ \text{W/m}$$

$$q'_{1,\text{conv}} = \bar{h}_1\pi D(T_1 - T_m) = 10\ \text{W/m}^2\cdot\text{K}\,\pi(0.025\ \text{m})(600-577)\text{K} = 17.8\ \text{W/m}$$

$$q'_{1,\text{elec}} = (138.3 + 17.8)\text{W/m} = 156\ \text{W/m}.$$ <

(c) Since all energy added via the heating elements must be transferred to surface 2,

$$q_2' = q_1'.$$

Hence, since there are 20 elements in a 1 m wide strip,

$$q_{2(1\text{m}\times1\text{m})} = 20 \times q'_{1,\text{elec}} = 3120\ \text{W}.$$ <

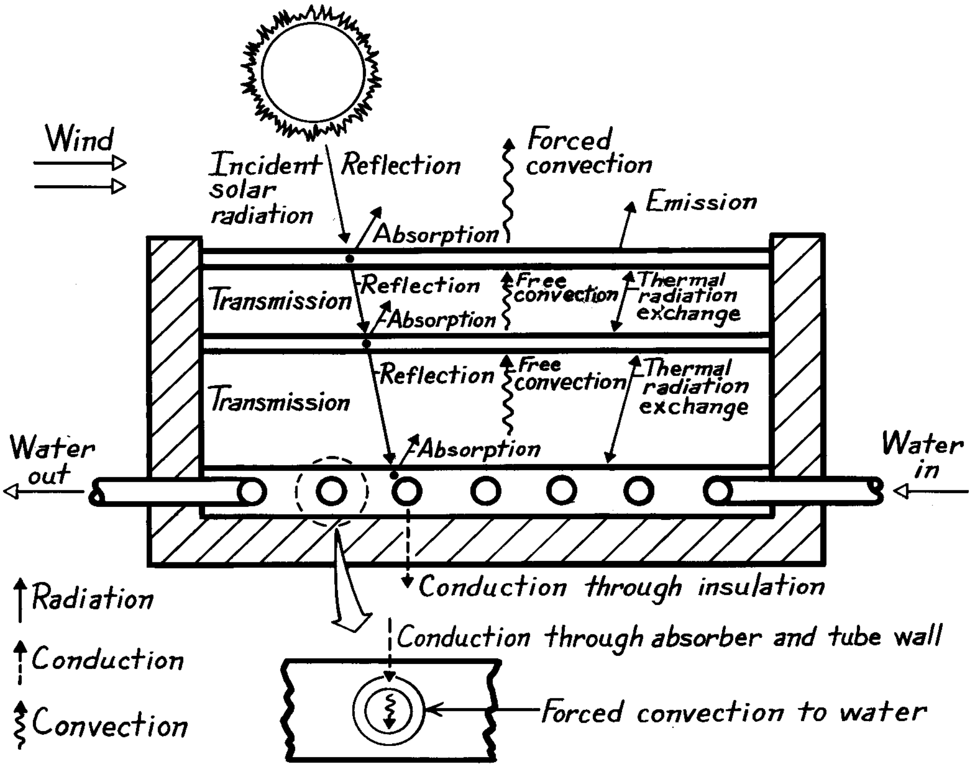
**COMMENTS:** The bottom panel would have to be cooled (from below) by a heat sink which could dissipate 3120 W/m$^2$.

## PROBLEM 13.102

**KNOWN:** Flat plate solar collector configuration.

**FIND:** Relevant heat transfer processes.

**SCHEMATIC:**




The incident solar radiation will experience transmission, reflection and absorption at each of the cover plates. However, it is desirable to have plates for which absorption and reflection are minimized and transmission is maximized. Glass of low iron content is a suitable material. Solar radiation incident on the absorber plate may be absorbed and reflected, but it is desirable to have a coating which maximizes absorption at short wavelengths.

Energy losses from the absorber plate are associated with radiation, convection and conduction. Thermal radiation exchange occurs between the absorber and the adjoining cover plate, between the two cover plates, and between the top cover plate and the surroundings. To minimize this loss, it is desirable that the emissivity of the absorber plate be small at long wavelengths. Energy is also transferred by free convection from the absorber plate to the first cover plate and between cover plates. It is transferred by free or forced convection to the atmosphere. Energy is also transferred by conduction from the absorber through the insulation.

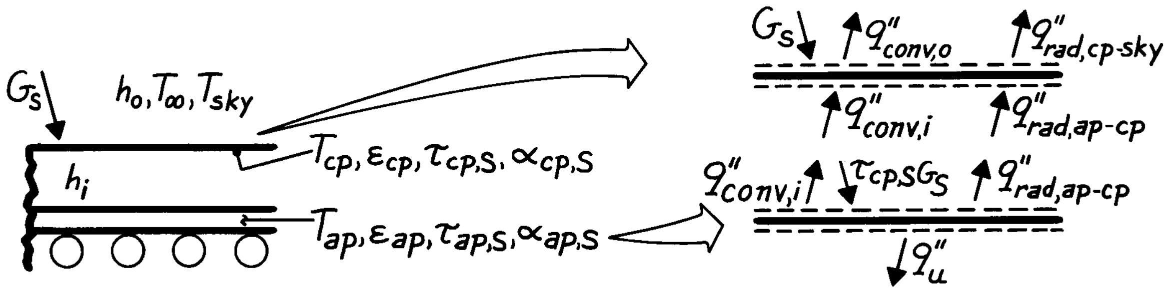
The foregoing processes provide for heat loss from the absorber, and it is desirable to minimize these losses. The difference between the solar radiation absorbed by the absorber and the energy loss by radiation, convection and conduction is the energy which is transferred to the working fluid. This transfer occurs by conduction through the absorber and the tube wall and by forced convection from the tube wall to the fluid.

## PROBLEM 13.103

**KNOWN:** Operating conditions of a flat plate solar collector.

**FIND:** Expressions for determining the rate at which useful energy is collected per unit area.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Uniform surface heat fluxes and temperatures, (3) Opaque, diffuse-gray surface behavior for long-wave thermal radiation, (4) Complete absorption of solar radiation by absorber plate ($\alpha_{ap,S} = 1$).

**ANALYSIS:** From an energy balance on the absorber plate, $\dot{E}''_{in} = \dot{E}''_{out}$,

$$\alpha_{ap,S}\left(\tau_{cp,S}\right)G_S = q''_u + q''_{conv,i} + q''_{rad,ap-cp}.$$

Hence with complete absorption of solar radiation by the absorber plate,

$$q''_u = \tau_{cp,S}G_S - h_i\left(T_{ap} - T_{cp}\right) - \frac{\sigma\left(T_{ap}^4 - T_{cp}^4\right)}{1/\varepsilon_{ap} + 1/\varepsilon_{cp} - 1} \qquad (1) \quad <$$

where $F_{ap-cp} \approx 1$ and Eq. 13.24 is used to obtain $q''_{rad,ap-cp}$. To determine $q''_u$ from Eq. (1), however, $T_{cp}$ must be known. From an energy balance on the cover plate,

$$\alpha_{cp,S}G_S + q''_{conv,i} + q''_{rad,ap-cp} = q''_{conv,o} + q''_{rad,cp-sky}$$

or

$$\alpha_{cp,S}G_S + h_i\left(T_{ap} - T_{cp}\right) + \frac{\sigma\left(T_{ap}^4 - T_{cp}^4\right)}{1/\varepsilon_{ap} + 1/\varepsilon_c - 1}$$

$$= h_o\left(T_{cp} - T_\infty\right) + \varepsilon_{cp}\sigma\left(T_{cp}^4 - T_{sky}^4\right). \qquad (2) \quad <$$

Eq. (2) may be used to obtain $T_{cp}$.

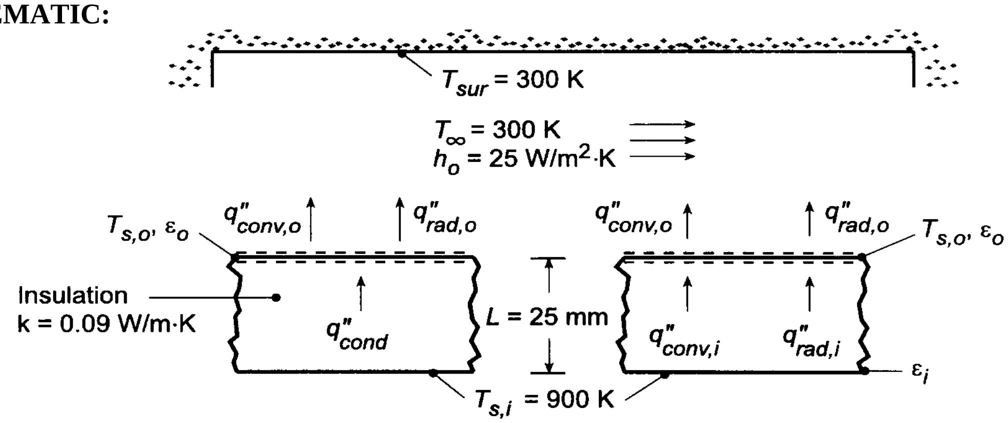
**COMMENTS:** With $T_{ap}$ presumed to be known, $T_{cp}$ may be evaluated from Eq. (2) and $q''_u$ from Eq. (1).

# PROBLEM 13.104

**KNOWN:** Ceiling temperature of furnace. Thickness, thermal conductivity, and/or emissivities of alternative thermal insulation systems. Convection coefficient at outer surface and temperature of surroundings.

**FIND:** (a) Mathematical model for each system, (b) Temperature of outer surface $T_{s,o}$ and heat loss $q''$ for each system and prescribed conditions, (c) Effect of emissivity on $T_{s,o}$ and $q''$.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state, (2) Diffuse/gray surfaces, (3) Surroundings form a large enclosure about the furnace, (4) Radiation in air space corresponds to a two-surface enclosure of large parallel plates.

**PROPERTIES:** *Table A-4*, air ($T_f$ = 730 K): k = 0.055 W/m·K, $\alpha = 1.09 \times 10^{-4}$ m$^2$/s, $\nu = 7.62 \times 10^{-5}$ m$^2$/s, $\beta = 0.001335$ K$^{-1}$, Pr = 0.702.

**ANALYSIS:** (a) To obtain $T_{s,o}$ and $q''$, an energy balance must be performed at the outer surface of the shield.

*Insulation*: $\quad q''_{cond} = q''_{conv,o} + q''_{rad,o} = q''$

$$k\frac{\left(T_{s,i} - T_{s,o}\right)}{L} = h_o\left(T_{s,o} - T_\infty\right) + \varepsilon_o\sigma\left(T_{s,o}^4 - T_{sur}^4\right)$$

*Air Space*: $\quad q''_{conv,i} + q''_{rad,i} = q''_{conv,o} + q''_{rad,o} = q''$

$$h_i\left(T_{s,i} - T_{s,o}\right) + \frac{\sigma\left(T_{s,i}^4 - T_{s,o}^4\right)}{\frac{1}{\varepsilon_i} + \frac{1}{\varepsilon_o} - 1} = h_o\left(T_{s,o} - T_\infty\right) + \varepsilon_o\sigma\left(T_{s,o}^4 - T_{sur}^4\right)$$

where Eq. 13.24 has been used to evaluate $q''_{rad,i}$ and $h_i$ is given by Eq. 9.49

$$\overline{Nu}_L = \frac{h_i L}{k} = 0.069 Ra_L^{1/3} Pr^{0.074}$$

(b) For the prescribed conditions ($\varepsilon_i = \varepsilon_o = 0.5$), the following results were obtained.

*Insulation*: The energy equation becomes

$$\frac{0.09\ \text{W/m}\cdot\text{K}\left(900 - T_{s,o}\right)\text{K}}{0.025\,\text{m}} = 25\ \text{W/m}^2\cdot\text{K}\left(T_{s,o} - 300\right)\text{K} + 0.5 \times 5.67 \times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(T_{s,o}^4 - 300^4\right)\text{K}^4$$

Continued …..

and a trial-and-error solution yields

$$T_{s,o} = 366\ \text{K} \qquad q'' = 1920\ \text{W/m}^2$$ <

*Air-Space*: The energy equation becomes

$$h_i\left(900 - T_{s,o}\right)\text{K} + \frac{5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(900^4 - T_{s,o}^4\right)\text{K}^4}{3}$$
$$= 25\,\text{W/m}^2\cdot\text{K}\left(T_{s,o} - 300\right)\text{K} + 0.5\times5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(T_{s,o}^4 - 300^4\right)\text{K}^4$$

where

$$h_i = \frac{0.055\ \text{W/m}\cdot\text{K}}{0.025\ \text{m}}0.069\,Ra_L^{1/3}\,Pr^{0.074} \qquad (1)$$

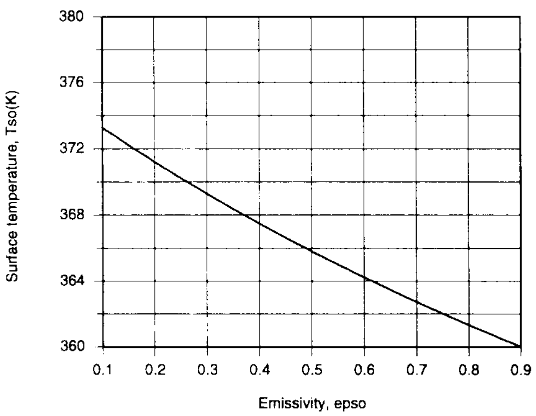
and $Ra_L = g\beta(T_{s,i} - T_{s,o})L^3/\alpha\nu$. A trial-and-error solution, which includes reevaluation of the air properties, yields

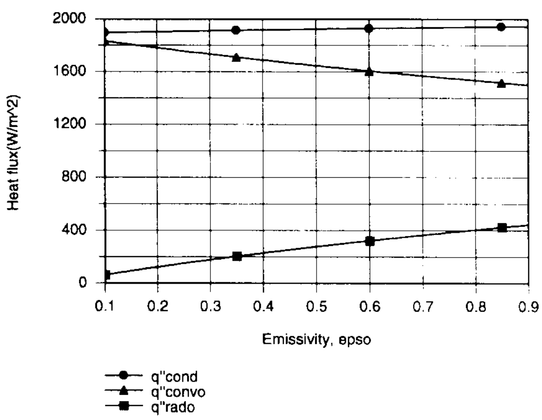
$$T_{s,o} = 598\ \text{K} \qquad q'' = 10{,}849\ \text{W/m}^2$$ <

The inner and outer heat fluxes are $q''_{conv,i} = 867\ \text{W/m}^2$, $q''_{rad,i} = 9982\ \text{W/m}^2$, $q''_{conv,o} = 7452$ W/m$^2$, and $q''_{rad,o} = 3397\ \text{W/m}^2$.

(c) Entering the foregoing models into the *IHT* workspace, the following results were generated.

*Insulation*:

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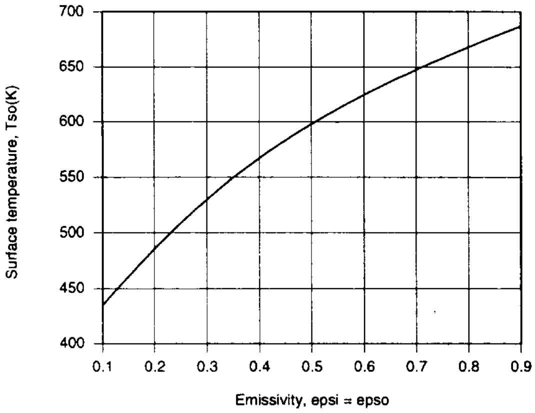
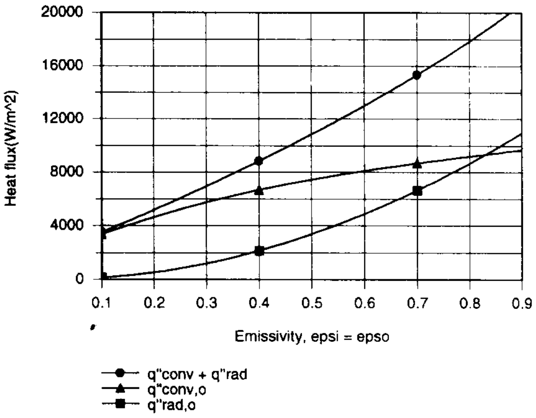
## PROBLEM 13.104 (Cont.)

As expected, the outer surface temperature decreases with increasing $\varepsilon_o$. However, the reduction in $T_{s,o}$ is not large since heat transfer from the outer surface is dominated by convection.




In this case $T_{s,o}$ increases with increasing $\varepsilon_o = \varepsilon_i$ and the effect is significant. The effect is due to an increase in radiative transfer from the inner surface, with $q''_{rad,i} = q''_{conv,i} = 1750\ \text{W/m}^2$ for $\varepsilon_o = \varepsilon_i = 0.1$ and $q''_{rad,i} = 20{,}100\ \text{W/m}^2 >> q''_{conv,i} = 523\ \text{W/m}^2$ for $\varepsilon_o = \varepsilon_i = 0.9$. With the increase in $T_{s,o}$, the total heat flux increases, along with the relative contribution of radiation $\left(q''_{rad,o}\right)$ to heat transfer from the outer surface.

**COMMENTS:** (1) With no insulation or radiation shield and $\varepsilon_i = 0.5$, radiative and convective heat fluxes from the ceiling are 18,370 and 15,000 W/m$^2$, respectively. Hence, a significant reduction in the heat loss results from use of the insulation or the shield, although the insulation is clearly more effective.

(2) Rayleigh numbers associated with free convection in the air space are well below the lower limit of applicability of Eq. (1). Hence, the correlation was used outside its designated range, and the error associated with evaluating $h_i$ may be large.

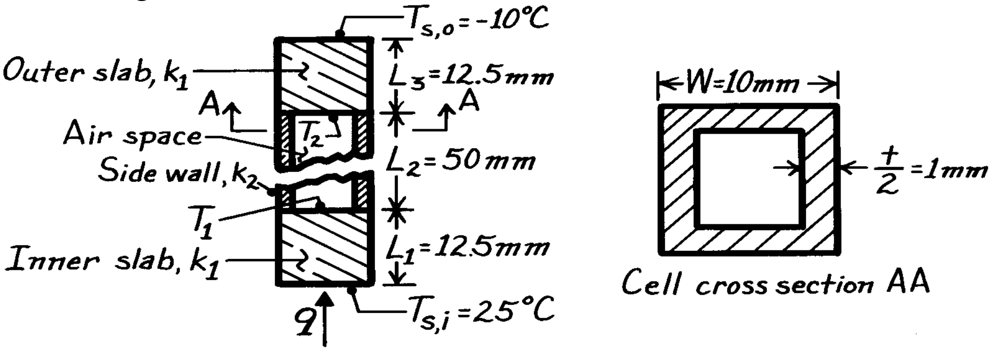
(3) The *IHT* solver had difficulty achieving convergence in the first calculation performed for the radiation shield, since the energy balance involves two nonlinear terms due to radiation and one due to convection. To obtain a solution, a fixed value of $Ra_L$ was prescribed for Eq. (1), while a second value of $Ra_{L,2} \equiv g\beta(T_{s,i} - T_{s,o})L^3/\alpha\nu$ was computed from the solution. The prescribed value of $Ra_L$ was replaced by the value of $Ra_{L,2}$ and the calculations were repeated until $Ra_{L,2} = Ra_L$.

# PROBLEM 13.105

**KNOWN:** Dimensions of a composite insulation consisting of honeycomb core sandwiched between solid slabs.

**FIND:** Total thermal resistance.

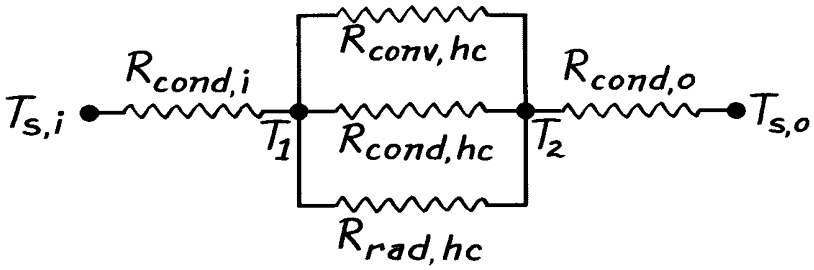
**SCHEMATIC:** Because of the repetitive nature of the honeycomb core, the cell sidewalls will be adiabatic. That is, there is no lateral heat transfer from cell to cell, and it suffices to consider the heat transfer across a single cell.




**ASSUMPTIONS:** (1) One-dimensional, steady-state conditions, (2) Equivalent conditions for each cell, (3) Constant properties, (4) Diffuse, gray surface behavior.

**PROPERTIES:** *Table A-3*, Particle board (low density): $k_1 = 0.078$ W/m·K; Particle board (high density): $k_2 = 0.170$ W/m·K; For both board materials, $\varepsilon = 0.85$; *Table A-4*, Air ($\overline{T} \approx 7.5°C$, 1 atm): $\nu = 14.15 \times 10^{-6}$ m$^2$/s, $k = 0.0247$ W/m·K, $\alpha = 19.9 \times 10^{-6}$ m$^2$/s, Pr = 0.71, $\beta = 3.57 \times 10^{-3}$ K$^{-1}$.

**ANALYSIS:** The total resistance of the composite is determined by conduction, convection and radiation processes occurring within the honeycomb and by conduction across the inner and outer slabs. The corresponding thermal circuit is shown.




The total resistance of the composite and equivalent resistance for the honeycomb are

$$R = R_{cond,i} + R_{eq} + R_{cond,o} \qquad\qquad R_{eq}^{-1} = \left(R_{cond}^{-1} + R_{conv}^{-1} + R_{rad}^{-1}\right)_{hc}.$$

The component resistances may be evaluated as follows. The inner and outer slabs are plane walls, for which the thermal resistance is given by Eq. 3.6. Hence, since $L_1 = L_3$ and the slabs are constructed from low-density particle board.

$$R_{cond,i} = R_{cond,o} = \frac{L_1}{k_1 W^2} = \frac{0.0125\ \text{m}}{0.078\ \text{W/m}\cdot\text{K}\left(0.01\ \text{m}\right)^2} = 1603\ \text{K/W}.$$

Continued …..

**PROBLEM 13.105 (Cont.)**

Similarly, applying Eq. 3.6 to the side walls of the cell

$$R_{cond,hc} = \frac{L_2}{k_2\left[W^2 - (W-t)^2\right]} = \frac{L_2}{k_2\left(2Wt - t^2\right)}$$

$$= \frac{0.050\ \text{m}}{0.170\ \text{W/m}\cdot\text{K}\left[2\times 0.01\ \text{m}\times 0.002\ \text{m} - (0.002\ \text{m})^2\right]} = 8170\ \text{K/W}.$$

From Eq. 3.9 the convection resistance associated with the cellular airspace may be expressed as

$$R_{conv,hc} = 1/h(W-t)^2.$$

The cell forms an enclosure that may be classified as a horizontal cavity heated from below, and the appropriate form of the Rayleigh number is $Ra_L = g\beta(T_1 - T_2)L_2^3/\alpha\nu$. To evaluate this parameter, however, it is necessary to *assume* a value of the cell temperature difference. As a first approximation, $T_1 - T_2 = 15°C - (-5°C) = 20°C$,

$$Ra_L = \frac{9.8\ \text{m/s}^2\left(3.57\times 10^{-3}\text{K}^{-1}\right)(20\ \text{K})(0.05\ \text{m})^3}{19.9\times 10^{-6}\text{m}^2/\text{s}\times 14.15\times 10^{-6}\text{m}^2/\text{s}} = 3.11\times 10^5.$$

Applying Eq. 9.49 as a first approximation, it follows that

$$h = (k/L_2)\left[0.069 Ra_L^{1/3}\ Pr^{0.074}\right] = \frac{0.0247\ \text{W/m}\cdot\text{K}}{0.05\ \text{m}}\left[0.069\left(3.11\times 10^5\right)^{1/3}(0.71)^{0.074}\right] = 2.25\ \text{W/m}^2\cdot\text{K}.$$

The convection resistance is then

$$R_{conv,hc} = \frac{1}{2.25\ \text{W/m}^2\cdot\text{K}(0.01\ \text{m} - 0.002\ \text{m})^2} = 6944\ \text{K/W}.$$

The resistance to heat transfer by radiation may be obtained by first noting that the cell forms a three-surface enclosure for which the sidewalls are reradiating. The net radiation heat transfer between the end surfaces of the cell is then given by Eq. 13.30. With $\varepsilon_1 = \varepsilon_2 = \varepsilon$ and $A_1 = A_2 = (W - t)^2$, the equation reduces to

$$q_{rad} = \frac{(W-t)^2\sigma\left(T_1^4 - T_2^4\right)}{2(1/\varepsilon - 1) + \left[F_{12} + \left[(F_{1R} + F_{2R})/F_{1R}F_{2R}\right]^{-1}\right]}.$$

However, with $F_{1R} = F_{2R} = (1 - F_{12})$, it follows that

$$q_{rad} = \frac{(W-t)^2\sigma\left(T_1^4 - T_2^4\right)}{2\left(\frac{1}{\varepsilon} - 1\right) + \left[F_{12} + \frac{(1-F_{12})^2}{2(1-F_{12})}\right]^{-1}} = \frac{(W-t)^2\sigma\left(T_1^4 - T_2^4\right)}{2\left(\frac{1}{\varepsilon} - 1\right) + \frac{2}{1+F_{12}}}.$$

The view factor $F_{12}$ may be obtained from Fig. 13.4, where

$$\frac{X}{L} = \frac{Y}{L} = \frac{W-t}{L_2} = \frac{10\ \text{mm} - 2\ \text{mm}}{50\ \text{mm}} = 0.16.$$

Hence, $F_{12} \approx 0.01$. Defining the radiation resistance as

$$R_{rad,hc} = \frac{T_1 - T_2}{q_{rad}}$$

it follows that

Continued …..

## PROBLEM 13.105 (Cont.)

$$R_{rad,hc} = \frac{2(1/\varepsilon - 1) + 2/(1+F_{12})}{(W-t)^2 \sigma (T_1^2 + T_2^2)(T_1 + T_2)}$$

where $(T_1^4 - T_2^4) = (T_1^2 + T_2^2)(T_1 + T_2)(T_1 - T_2)$. Accordingly,

$$R_{rad,hc} = \frac{\left[2\left(\frac{1}{0.85} - 1\right) + \frac{2}{1+0.01}\right]}{(0.01\ \text{m} - 0.002\ \text{m})^2 \times 5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left[(288\ \text{K})^2 + (268\ \text{K})^2\right](288+268)\text{K}}$$

where, again, it is *assumed* that $T_1$ = 15°C and $T_2$ = -5°C. From the above expression, it follows that

$$R_{rad,hc} = \frac{0.353 + 1.980}{3.123\times10^{-4}} = 7471\ \text{K/W}.$$

In summary the component resistances are

$$R_{cond,i} = R_{cond,o} = 1603\ \text{K/W}$$

$$R_{cond,hc} = 8170\ \text{K/W} \qquad R_{conv,hc} = 6944\ \text{K/W} \qquad R_{rad,hc} = 7471\ \text{K/W}.$$

The equivalent resistance is then

$$R_{eq} = \left(\frac{1}{8170} + \frac{1}{6944} + \frac{1}{7471}\right)^{-1} = 2498\ \text{K/W}$$

and the total resistance is

$$R = 1603 + 2498 + 1603 = 5704\ \text{K/W}.$$ <

**COMMENTS:** (1) The problem is iterative, since values of $T_1$ and $T_2$ were assumed to calculate $R_{conv,hc}$ and $R_{rad,hc}$. To check the validity of the assumed values, we first obtain the heat transfer rate q from the expression

$$q = \frac{T_{s,1} - T_{s,2}}{R} = \frac{25°\text{C} - (-10°\text{C})}{5704\ \text{K/W}} = 6.14\times10^{-3}\text{W}.$$

Hence

$$T_1 = T_{s,i} - qR_{cond,i} = 25°\text{C} - 6.14\times10^{-3}\text{W}\times1603\ \text{K/W} = 15.2°\text{C}$$

$$T_2 = T_{s,o} + qR_{cond,o} = -10°\text{C} + 6.14\times10^{-3}\text{W}\times1603\ \text{K/W} = -0.2°\text{C}.$$

Using these values of $T_1$ and $T_2$, $R_{conv,hc}$ and $R_{rad,hc}$ should be recomputed and the process repeated until satisfactory agreement is obtained between the initial and computed values of $T_1$ and $T_2$.

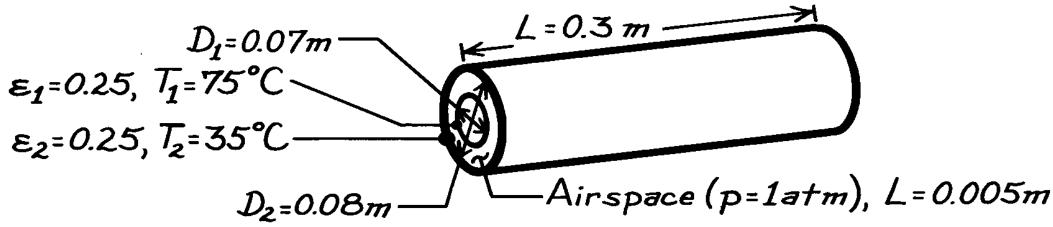
(2) The resistance of a section of low density particle board 75 mm thick ($L_1 + L_2 + L_3$) of area $W^2$ is 9615 K/W, which exceeds the total resistance of the composite by approximately 70%. Accordingly, use of the honeycomb structure offers no advantages as an insulating material. Its effectiveness as an insulator could be improved ($R_{eq}$ increased) by reducing the wall thickness t to increase $R_{cond}$, evacuating the cell to increase $R_{conv}$, and/or decreasing ε to increase $R_{rad}$. A significant increase in $R_{rad,hc}$ could be achieved by aluminizing the top and bottom surfaces of the cell.

## PROBLEM 13.106

**KNOWN:** Dimensions and surface conditions of a cylindrical thermos bottle filled with hot coffee and lying horizontally.

**FIND:** Heat loss.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Negligible heat loss from ends (long infinite cylinders), (3) Diffuse-gray surface behavior.

**PROPERTIES:** *Table A-4*, Air ($T_f = (T_1 + T_2)/2 = 328$ K, 1 atm): $k = 0.0284$ W/m·K, $\nu = 23.74 \times 10^{-6}$ m$^2$/s, $\alpha = 26.6 \times 10^{-6}$ m$^2$/s, Pr = 0.0703, $\beta = 3.05 \times 10^{-3}$ K$^{-1}$.

**ANALYSIS:** The heat transfer across the air space is

$$q = q_{rad} + q_{conv}.$$

From Eq. 13.25 for concentric cylinders

$$q_{rad} = \frac{\sigma(\pi D_1 L)\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{r_1}{r_2}\right)} = \frac{5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\pi(0.07\times0.3)\text{m}^2\left(348^4 - 308^4\right)\text{K}^4}{4 + 3(0.035/0.04)}$$

$$q_{rad} = 3.20\ \text{W}.$$

From Eq. 9.25,

$$Ra_L = \frac{g\beta(T_1 - T_2)L^3}{\alpha\nu} = \frac{9.8\ \text{m/s}^2\left(3.05\times10^{-3}\text{K}^{-1}\right)(40\ \text{K})(0.005\ \text{m})^3}{26.6\times10^{-6}\text{m}^2/\text{s}\times23.74\times10^{-6}\text{m}^2/\text{s}} = 236.7.$$

Hence from Eq. 9.60

$$Ra_c^* = \frac{\left[\ln(D_2/D_1)\right]^4 Ra_L}{L^3\left(D_1^{-0.6} + D_2^{-0.6}\right)^5} = \frac{\left[\ln(0.08/0.07)\right]^4 236.7}{(0.005\ \text{m})^3\left(0.07^{-0.6} + 0.08^{-0.6}\right)^5\ \text{m}^{-3}} = 7.85.$$

However, the implication of such a small value of $Ra_c^*$ is that free convection effects are negligible. Heat transfer across the airspace is therefore by conduction ($k_{eff} = k$). From Eq. 3.27

$$q_{cond} = \frac{2\pi Lk(T_1 - T_2)}{\ln(r_2/r_1)} = \frac{2\pi\times0.3\ \text{m}\times0.0284\ \text{W/m}\cdot\text{K}(75-35)\text{K}}{\ln(0.04/0.035)} = 16.04\ \text{W}.$$

Hence the total heat loss is

$$q = q_{rad} + q_{cond} = 19.24\ \text{W}.$$ <

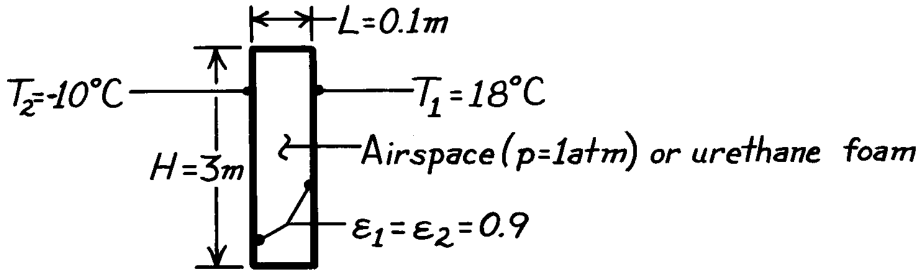
**COMMENTS:** (1) End effects could be considered in a more detailed analysis, (2) Conduction losses could be eliminated by evacuating the annulus.

# PROBLEM 13.107

**KNOWN:** Thickness and height of a vertical air space. Emissivity and temperature of adjoining surfaces.

**FIND:** (a) Heat loss per unit area across the space, (b) Heat loss per unit area if space is filled with urethane foam.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Diffuse-gray surface behavior, (3) Air space is a vertical cavity, (4) Constant properties, (5) One-dimensional conduction across foam.

**PROPERTIES:** *Table A-4*, Air ($T_f$ = 4°C, 1 atm): $\nu = 13.84 \times 10^{-6}$ m$^2$/s, k = 0.0245 W/m·K, $\alpha$ = $19.5 \times 10^{-6}$ m$^2$/s, Pr = 0.71, $\beta = 3.61 \times 10^{-3}$ K$^{-1}$; *Table A-3*, Urethane foam: k = 0.026 W/m·K.

**ANALYSIS:** (a) With the air space, heat loss is by radiation and free convection or conduction. From Eq. 13.24,

$$q''_{rad} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} = \frac{5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\left(291^4 - 263^4\right)\mathrm{K^4}}{1.222} = 110.7\ \mathrm{W/m^2}.$$

With

$$Ra_L = \frac{g\beta\left(T_1 - T_2\right)L^3}{\nu\alpha} = \frac{9.8\ \mathrm{m^2/s}\left(3.61\times10^{-3}\mathrm{K^{-1}}\right)(18+10)\mathrm{K}\,(0.1\ \mathrm{m})^3}{13.84\times10^{-6}\,\mathrm{m^2/s}\times19.5\times10^{-6}\mathrm{m^2/s}} = 3.67\times10^6$$

and H/L = 30, Eq. 9.53 may be used as a first approximation to obtain

$$\overline{Nu}_L = 0.046 Ra_L^{1/3} = 0.046\left(3.67\times10^6\right)^{1/3} = 7.10$$

$$\bar{h} = \frac{k}{L}\overline{Nu}_L = \frac{0.0245\ \mathrm{W/m\cdot K}}{0.1\ \mathrm{m}} 7.10 = 1.74\ \mathrm{W/m^2\cdot K}.$$

The convection heat flux is

$$q''_{conv} = \bar{h}\left(T_1 - T_2\right) = 1.74\ \mathrm{W/m^2\cdot K}(18+10)\mathrm{K} = 48.7\ \mathrm{W/m^2}.$$

The heat loss is then

$$q'' = q''_{rad} + q''_{conv} = 110.7 + 48.7 = 159\ \mathrm{W/m^2}.$$

(b) With the foam, heat loss is by conduction and

$$q'' = q''_{cond} = \frac{k}{L}\left(T_1 - T_2\right) = \frac{0.026\ \mathrm{W/m\cdot K}}{0.1\ \mathrm{m}}(18+10)\mathrm{K} = 7.3\ \mathrm{W/m^2}. \quad <$$

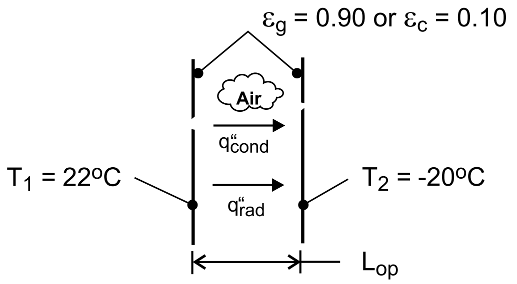
**COMMENTS:** Use of the foam insulation reduces the heat loss considerably. Note the significant effect of radiation.

# PROBLEM 13.108

**KNOWN:** Temperatures and emissivity of window panes and critical Rayleigh number for onset of convection in air space.

**FIND:** (a) The conduction heat flux across the air gap for the optimal spacing, (b) The total heat flux for uncoated panes, (c) The total heat flux if one or both of the panes has a low-emissivity coating.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Critical Rayleigh number is $Ra_{L,c}$ = 2000, (2) Constant properties, (3) Radiation exchange between large (infinite), parallel, diffuse-gray surfaces.

**PROPERTIES:** *Table A-4*, air [T = ($T_1$ + $T_2$)/2 = 1°C = 274 K]: $\nu = 13.6 \times 10^{-6}$ m²/s, k = 0.0242 W/m·K, $\alpha = 19.1 \times 10^{-6}$ m²/s, $\beta = 0.00365$ K$^{-1}$.

**ANALYSIS:** (a) With $Ra_{L,c} = g\beta\left(T_1 - T_2\right)L_{op}^3/\alpha\nu$

$$L_{op} = \left[\frac{\alpha\nu\, Ra_{L,c}}{g\beta\left(T_1 - T_2\right)}\right]^{1/3} = \left[\frac{19.1\times13.6\times10^{-12}\,m^4/s^2\times2000}{9.8\,m/s^2\left(0.00365\ K^{-1}\right)42°C}\right]^{1/3} = 0.0070m$$

The conduction heat flux is then

$$q''_{cond} = k\left(T_1 - T_2\right)/L_{op} = 0.0242\,W/m\cdot K\left(42°C\right)/0.0070m = 145.2\ W/m^2 \qquad <$$

(b) For conventional glass ($\varepsilon_g$ = 0.90), Eq. (13.24) yields,

$$q''_{rad} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{2}{\varepsilon_g} - 1} = \frac{5.67\times10^{-8}\,W/m^2\cdot K^4\left(295^4 - 253^4\right)K^4}{1.222} = 161.3\ W/m^2$$

and the total heat flux is

$$q''_{tot} = q''_{cond} + q''_{rad} = 306.5\ W/m^2 \qquad <$$

(c) With only one surface coated,

$$q''_{rad} = \frac{5.67\times10^{-8}W/m^2\cdot K^4\left(295^4 - 253^4\right)}{\frac{1}{0.90} + \frac{1}{0.10} - 1} = 19.5\ W/m^2$$

Continued …..

$$q''_{tot} = 164.7\ \mathrm{W/m^2} \qquad <$$

With both surfaces coated,

$$q''_{rad} = \frac{5.67\times10^{-8}\ \mathrm{W/m^2\cdot K^4}\left(295^4 - 253^4\right)}{\dfrac{1}{0.10}+\dfrac{1}{0.10}-1} = 10.4\ \mathrm{W/m^2}$$

$$q''_{tot} = 155.6\ \mathrm{W/m^2} \qquad <$$

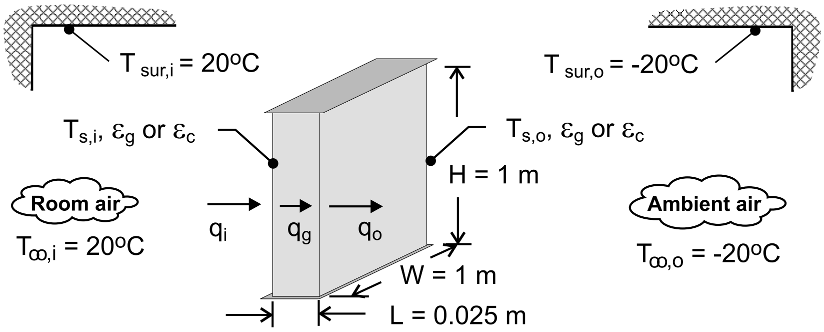
**COMMENTS:** Without any coating, radiation makes a large contribution (53%) to the total heat loss. With one coated pane, there is a significant reduction (46%) in the total heat loss. However, the benefit of coating both panes is marginal, with only an additional 3% reduction in the total heat loss.

## PROBLEM 13.109

**KNOWN:** Dimensions and emissivity of double pane window. Thickness of air gap. Temperatures of room and ambient air and the related surroundings.

**FIND:** (a) Temperatures of glass panes and rate of heat transfer through window, (b) Heat rate if gap is evacuated. Heat rate if special coating is applied to window.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state, (2) Negligible glass pane thermal resistance, (3) Constant properties, (4) Diffuse-gray surface behavior, (5) Radiation exchange between interior window surfaces may be approximated as exchange between infinite parallel plates, (6) Interior and exterior surroundings are very large.

**PROPERTIES:** *Table A-4*, Air (p = 1 atm). Obtained from using *IHT* to solve for conditions of Part (a): $T_{f,i}$ = 287.4 K: $\nu_i = 14.8 \times 10^{-6}$ m$^2$/s, $k_i$ = 0.0253 W/m·K, $\alpha_i = 20.8 \times 10^{-6}$ m$^2$/s, $Pr_i$ = 0.71, $\beta_i$ = 0.00348 K$^{-1}$. $\overline{T} = (T_{s,i} + T_{s,o})/2$ = 273.7 K: $\nu = 13.6 \times 10^{-6}$ m$^2$/s, k = 0.0242 W/m·K, $\alpha = 19.0 \times 10^{-6}$ m$^2$/s, Pr = 0.71, $\beta$ = 0.00365 K$^{-1}$. $T_{f,o}$ = 259.3 K: $\nu_o = 12.3 \times 10^{-6}$ m$^2$/s, $k_o$ = 0.023 W/m·K, $\alpha_o = 17.1 \times 10^{-6}$ m$^2$/s, $Pr_o$ = 0.72, $\beta_o$ = 0.00386 K$^{-1}$.

**ANALYSIS:** (a) The heat flux through the window may be expressed as

$$q'' = q''_{rad,i} + q''_{conv,i} = \varepsilon_g \sigma \left(T_{sur,i}^4 - T_{s,i}^4\right) + \overline{h}_i \left(T_{\infty,i} - T_{s,i}\right) \tag{1}$$

$$q'' = q''_{rad,gap} + q''_{conv,gap} = \frac{\sigma\left(T_{s,i}^4 - T_{s,o}^4\right)}{\dfrac{1}{\varepsilon_g} + \dfrac{1}{\varepsilon_g} - 1} + \overline{h}_{gap}\left(T_{s,i} - T_{s,o}\right) \tag{2}$$

$$q'' = q''_{rad,o} + q''_{conv,o} = \varepsilon_g \sigma \left(T_{s,o}^4 - T_{sur,o}^4\right) + \overline{h}_o \left(T_{s,o} - T_{\infty,o}\right) \tag{3}$$

where radiation exchange between the window panes is determined from Eq. (13.24) and radiation exchange with the surroundings is determined from Eq. (13.27). The inner and outer convection coefficients, $\overline{h}_i$ and $\overline{h}_o$, are determined from Eq. (9.26), and $\overline{h}_{gap}$ is obtained from Eq. (9.52).

The foregoing equations may be solved for the three unknowns $\left(q'', T_{s,i}, T_{s,o}\right)$. Using the *IHT* software to effect the solution, we obtain

$$T_{s,i} = 281.8 \text{ K} = 8.8°\text{C}$$ <

Continued …..

## PROBLEM 13.109 (Cont.)

$$T_{s,o} = 265.6\ \text{K} = -7.4°\text{C} \qquad <$$

$$q = 91.3\ \text{W} \qquad <$$

(b) If the air space is evacuated $\left(\overline{h}_g = 0\right)$, we obtain

$$T_{s,i} = 283.6\ \text{K} = 10.6°\text{C} \qquad <$$

$$T_{s,o} = 263.8\ \text{K} = 9.2°\text{C} \qquad <$$

$$q = 75.5\ \text{W} \qquad <$$

If the space is not evacuated but the coating is applied to inner surfaces of the window panes,

$$T_{s,i} = 285.9\ \text{K} = 12.9°\text{C} \qquad <$$

$$T_{s,o} = 261.3\ \text{K} = -11.7°\text{C} \qquad <$$

$$q = 55.9\ \text{W} \qquad <$$

If the space is evacuated and the coating is applied,

$$T_{s,i} = 291.7\ \text{K} = 18.7°\text{C} \qquad <$$

$$T_{s,o} = 254.7\ \text{K} = -18.3°\text{C} \qquad <$$

$$q = 9.0\ \text{W} \qquad <$$

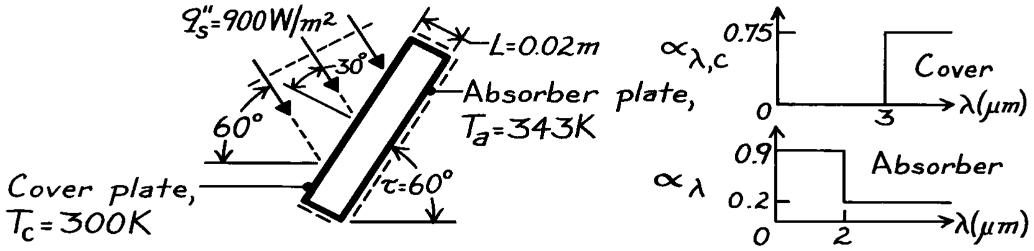
**COMMENTS:** (1) For the conditions of part (a), the convection and radiation heat fluxes are comparable at the inner and outer surfaces of the window, but because of the comparatively small convection coefficient, the radiation flux is approximately twice the convection flux across the air gap. (2) As the resistance across the air gap is progressively increased (evacuated, coated, evacuated and coated), the temperatures of the inner and outer panes increase and decrease, respectively, and the heat loss decreases. (3) Clearly, there are significant energy savings associated with evacuation of the gap and application of the coating. (4) In all cases, solutions were obtained using the temperature-dependent properties of air provided by the software. The property values listed in the **PROPERTIES** section of this solution pertain to the conditions of part (a).

## PROBLEM 13.110

**KNOWN:** Absorber and cover plate temperatures and spectral absorptivities for a flat plate solar collector. Collector orientation and solar flux.

**FIND:** (a) Rate of solar radiation absorption per unit area, (b) Heat loss per unit area.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Adiabatic sides and bottom, (3) Cover is transparent to solar radiation, (4) Sun emits as a blackbody at 5800 K, (5) Cover and absorber plates are diffuse-gray to long wave radiation, (6) Negligible end effects, (7) L << width and length.

**PROPERTIES:** *Table A-4*, Air ($T = T_a + T_c)/2 = 321.5$ K, 1 atm): $\nu = 18.05 \times 10^{-6}$ m²/s, k = 0.0279 W/m·K, $\alpha = 25.7 \times 10^{-6}$ m²/s.

**ANALYSIS:** (a) The absorbed solar irradiation is

$$G_{S,abs} = \alpha_{S,a} G_S$$

where

$$G_S = q_S'' \cos 30° = 900 \times 0.866 = 779.4 \text{ W/m}^2$$

$$\alpha_{S,a} = \frac{\int_0^\infty \alpha_{\lambda,a} G_{\lambda,S} d\lambda}{G_S} = \frac{\int_0^\infty \alpha_{\lambda,a} E_{\lambda,b}(5800 \text{ K}) d\lambda}{E_b(5800 \text{ K})}$$

$$\alpha_{S,a} = \alpha_{\lambda,a,1} F_{(0\to 2\ \mu m)} + \alpha_{\lambda,a2} F_{(2\to\infty)}.$$

For $\lambda T = 2\ \mu m \times 5800$ K = 11,600 μm·K from Table 12.1, $F_{(0\to 2\lambda T)} = 0.941$, find

$$\alpha_{S,a} = 0.9 \times 0.941 + 0.2 \times (1 - 0.941) = 0.859.$$

Hence

$$G_{S,abs} = 0.859 \times 779.4 = 669 \text{ W/m}^2. \qquad <$$

(b) The heat loss per unit area from the collector is

$$q''_{loss} = q''_{conv} + q''_{rad}.$$

The convection heat flux is

$$q''_{conv} = \bar{h}(T_a - T_c)$$

Continued …..

## PROBLEM 13.110 (Cont.)

and with

$$Ra_L = \frac{g\beta\left(T_a - T_c\right)L^3}{\alpha\nu}$$

$$Ra_L = \frac{9.8\ \text{m/s}^2 \times \left(321.5\ \text{K}\right)^{-1}\left(343-300\right)\text{K}\left(0.02\ \text{m}\right)^3}{18.05\times10^{-6}\text{m}^2/\text{s}\times25.7\times10^{-6}\text{m}^2/\text{s}} = 22,604$$

find from Eq. 9.54 with

$$H/L > 12,\ \tau < \tau^*,\ \cos\ \tau = 0.5,\ Ra_L\ \cos\ \tau = 11,302$$

$$\overline{Nu}_L = 1 + 1.44\left[1 - \frac{1708}{11,302}\right]\left[1 - \frac{1708\left(\sin\ 108°\right)^{1.6}}{11,302}\right] + \left[\left(\frac{11,302}{5830}\right)^{1/3} - 1\right]$$

$$\bar{h} = \overline{Nu}_L\ \frac{k}{L} = 2.30\times\frac{0.0279\ \text{W/m}\cdot\text{K}}{0.02\ \text{m}} = 3.21\ \text{W/m}^2\cdot\text{K}.$$

Hence, the convective heat flux is

$$q''_{conv} = 3.21\ \text{W/m}^2\cdot\text{K}\left(343-300\right)\text{K} = 138.0\ \text{W/m}^2.$$

The radiative exchange can be determined from Eq. 13.24 treating the cover and absorber plates as a two-surface enclosure,

$$q''_{rad} = \frac{\sigma\left(T_a^4 - T_c^4\right)}{1/\varepsilon_a + 1/\varepsilon_c - 1} = \frac{5.67\times10^{-8}\text{W/m}^2\cdot\text{K}^4\left[\left(343\ \text{K}\right)^4 - \left(300\ \text{K}\right)^4\right]}{1/0.2 + 1/0.75 - 1}$$

$$q''_{rad} = 61.1\ \text{W/m}^2.$$

Hence, the total heat loss per unit area from the collector

$$q''_{loss} = \left(138.0 + 61.1\right) = 199\ \text{W/m}^2.$$ <

**COMMENTS:** (1) Non-solar components of radiation transfer are concentrated at long wavelength for which $\alpha_a = \varepsilon_a = 0.2$ and $\alpha_c = \varepsilon_c = 0.75$.

(2) The collector efficiency is

$$\eta = \frac{669.3 - 199.1}{669.3}\times100 = 70\%.$$

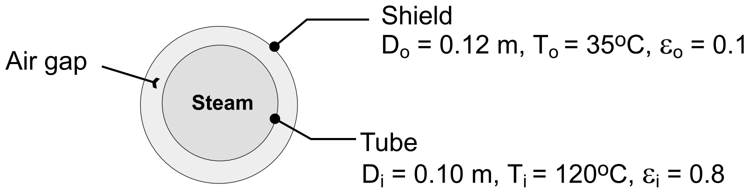
This value is uncharacteristically high due to specification of nearly optimum $\alpha_a(\lambda)$ for absorber.

## PROBLEM 13.111

**KNOWN:** Diameters and temperatures of a heated tube and a radiation shield.

**FIND:** (a) Total heat loss per unit length of tube, (b) Effect of shield diameter on heat rate.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Opaque, diffuse-gray surfaces, (2) Negligible end effects.

**PROPERTIES:** *Table A-4*, Air ($T_f$ = 77.5°C ≈ 350 K): k = 0.030 W/m·K, Pr = 0.70, $\nu = 20.92 \times 10^{-6}$ $m^2/s$, $\alpha = 29.9 \times 10^{-6}$ $m^2/s$, $\beta = 0.00286$ $K^{-1}$.

**ANALYSIS:** (a) Heat loss from the tube is by radiation and free convection

$$q' = q'_{rad} + q'_{conv}$$

From Eq. (13.25)

$$q'_{rad} = \frac{\sigma(\pi D_i)\left(T_i^4 - T_o^4\right)}{\dfrac{1}{\varepsilon_i} + \dfrac{1-\varepsilon_o}{\varepsilon_o}\left(\dfrac{r_i}{r_o}\right)}$$

or

$$q'_{rad} = \frac{5.67\times10^{-8}\dfrac{W}{m\cdot K^4}(\pi\times0.1m)\left(393^4 - 308^4\right)K^4}{\dfrac{1}{0.8} + \dfrac{0.9}{0.1}\left(\dfrac{0.05}{0.06}\right)} = 30.2\ \frac{W}{m}$$

$$Ra_L = \frac{g\beta(T_i - T_o)L^3}{\nu\alpha} = \frac{9.8\,m/s^2\times0.00286\ K^{-1}(85\ K)(0.01m)^3}{\left(20.92\times10^{-6}m^2/s\right)\left(29.9\times10^{-6}m^2/s\right)} = 3809$$

Hence from Eq. (9.60)

$$Ra_c^* = \frac{\left[\ell n(D_o/D_i)\right]^4 Ra_L}{L^3\left(D_i^{-3/5} + D_o^{-3/5}\right)^5} = \frac{\left[\ell n(0.12/0.10)\right]^4 3809}{(0.01m)^3\left[(0.1m)^{-0.6} + (0.12m)^{-0.6}\right]^5} = 171.6$$

and from Eq. (9.59)

$$k_{eff} = 0.386\ k\left(\frac{Pr}{0.861 + Pr}\right)^{1/4}\left(Ra_c^*\right)^{1/4}$$

Continued …..

## PROBLEM 13.111 (Cont.)

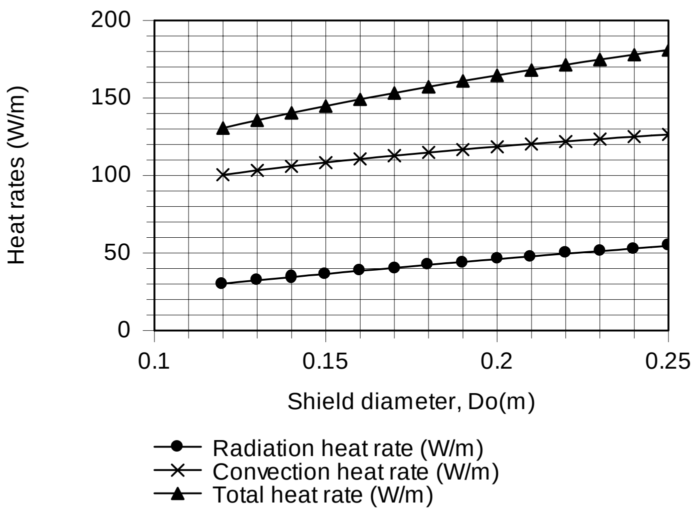
$$k_{eff} = 0.386\left(0.03\frac{W}{m\cdot K}\right)\left(\frac{0.7}{0.861+0.7}\right)^{1/4}(171.6)^{1/4} = 0.0343\frac{W}{m\cdot K}$$

Hence from Eq. (9.58)

$$q'_{conv} = \frac{2\pi\, k_{eff}}{\ell n\left(D_o / D_i\right)}\left(T_i - T_o\right) = \frac{2\pi\left(0.0343\frac{W}{m\cdot K}\right)}{\ell n\left(0.12/0.10\right)}(120-35)K = 100.5\frac{W}{m}$$

$$q' = (30.2+100.5)\frac{W}{m} = 130.7\frac{W}{m}$$

(b) As shown below, both convection and radiation, and hence the total heat rate, increase with increasing shield diameter. In the limit as $D_o \to \infty$, the radiation rate approaches that corresponding to net transfer between a small surface and large surroundings at $T_o$. The rate is independent of $\varepsilon$.



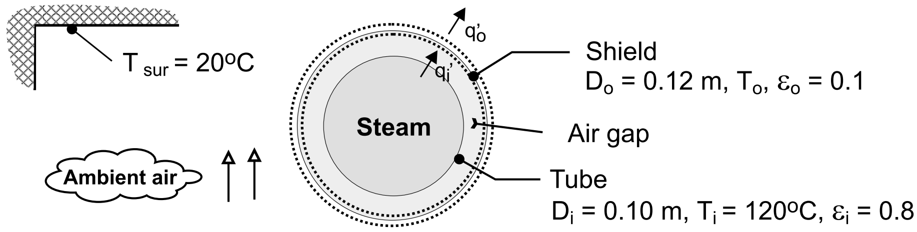

**COMMENTS:** Designation of a shield temperature is arbitrary. The temperature depends on the nature of the environment external to the shield.

# PROBLEM 13.112

**KNOWN:** Diameters of heated tube and radiation shield. Tube surface temperature and temperature of ambient air and surroundings.

**FIND:** Temperature of radiation shield and heat loss per unit length of tube.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Opaque, diffuse-gray surfaces, (2) Negligible end effects, (3) Large surroundings, (4) Quiescent air, (5) Steady-state.

**PROPERTIES:** Determined from use of *IHT* software for iterative solution. Air, $(T_i + T_o)/2 =$ 362.7 K: $\nu_i = 2.23 \times 10^{-5}$ m$^2$/s, $k_i = 0.031$ W/m·K, $\alpha_i = 3.20 \times 10^{-5}$ m$^2$/s, $\beta_i = 0.00276$ K$^{-1}$, $Pr_i =$ 0.698. Air, $T_f = 312.7$ K: $\nu_o = 1.72 \times 10^{-5}$ m$^2$/s, $k_o = 0.027$ W/m·K, $\alpha_o = 2.44 \times 10^{-5}$ m$^2$/s, $\beta_o =$ 0.0032 K$^{-1}$, $Pr_o = 0.705$.

**ANALYSIS:** From an energy balance on the radiation shield, $q_i' = q_o'$ or $q_{rad,i}' + q_{conv,i}'$ $= q_{rad,o}' + q_{conv,o}'$. Evaluating the inner and outer radiation rates from Eqs. (13.25) and (13.27), respectively, and the convection heat rate in the air gap from Eq. (9.58),

$$\frac{\sigma \pi D_i \left(T_i^4 - T_o^4\right)}{\dfrac{1}{\varepsilon_i} + \dfrac{1-\varepsilon_o}{\varepsilon_o}\left(\dfrac{D_i}{D_o}\right)} + \frac{2\pi\, k_{eff}\, (T_i - T_o)}{\ell n (Do/Di)} = \sigma \pi D_o \varepsilon_o \left(T_o^4 - T_{sur}^4\right) + \pi D_o \bar{h}_o (T_o - T_\infty)$$

From Eqs. (9.59) and (9.60)

$$k_{eff} = 0.386\; k_i \left(\frac{Pr_i}{0.861 + Pr_i}\right)^{1/4} \left(Ra_c^*\right)^{1/4}$$

$$Ra_c^* = \frac{\left[\ell n (D_o / D_i)\right]^4 Ra_L}{L^3 \left(D_i^{-3/5} + D_o^{-3/5}\right)^5}$$

where $Ra_L = g\, \beta_i\, (T_i - T_o) L^3 / \nu_i\, \alpha_i$ and $L = (D_o - D_i)/2$. From Eq. (9.34), the convection coefficient on the outer surface of the shield is

$$\bar{h}_o = \frac{k_o}{D_o}\left\{0.60 + \frac{0.387\; Ra_D^{1/6}}{\left[1 + (0.559/Pr)^{9/16}\right]^{8/27}}\right\}^2$$

Continued …..

The solution to the energy balance is obtained using the *IHT* software, and the result is

$$T_o = 332.5\ \text{K} = 59.5^\circ\text{C} \qquad <$$

The corresponding value of the heat loss is

$$q_i' = 88.7\ \text{W/m} \qquad <$$

**COMMENTS:** (1) The radiation and convection heat rates are $q'_{rad,i} = 23.7\ \text{W/m}$, $q'_{rad,o} = 10.4\ \text{W/m}$, $q'_{conv,i} = 65.0\ \text{W/m}$, and $q'_{conv,o} = 78.3\ \text{W/m}$. Convection is clearly the dominant mode of heat transfer.

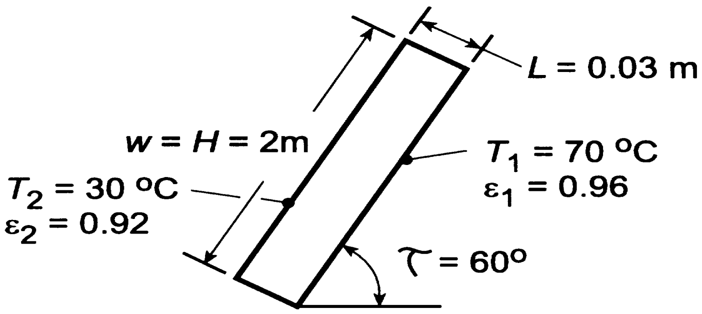
(2)With a value of $T_o = 59.5^\circ\text{C} > 35^\circ\text{C}$, the heat loss is reduced (88.7 W/m compared to 130.7 W/m if the shield is at 35°C).

## PROBLEM 13.113

**KNOWN:** Dimensions and inclination angle of a flat-plate solar collector. Absorber and cover plate temperatures and emissivities.

**FIND:** (a) Rate of heat transfer by free convection and radiation, (b) Effect of the absorber plate temperature on the heat rates.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Diffuse-gray, opaque surface behavior.

**PROPERTIES:** *Table A-4*, air $\left(\overline{T} = \left(T_1 + T_2\right)/2 = 323\ K\right)$: $\nu = 18.2 \times 10^{-6}\ m^2/s$, k = 0.028 W/m·K, $\alpha = 25.9 \times 10^{-6}\ m^2/s$, Pr = 704, $\beta = 0.0031\ K^{-1}$.

**ANALYSIS:** (a) The convection heat rate is

$$q_{conv} = \overline{h}A\left(T_1 - T_2\right)$$

where A = wH=4 $m^2$ and, with H/L > 12 and $\tau < \tau^* = 70$ deg, $\overline{h}$ is given by Eq. 9.54. With a Rayleigh number of

$$Ra_L = \frac{g\beta\left(T_1 - T_2\right)L^3}{\alpha\nu} = \frac{9.8\,m/s^2\left(0.0031\ K^{-1}\right)\left(40°C\right)\left(0.03\,m\right)^3}{25.9\times10^{-6}m^2/s\times18.2\times10^{-6}m^2/s} = 69,600$$

$$\overline{Nu}_L = 1 + 1.44\left[1 - \frac{1708}{0.5\left(69,600\right)}\right]\left[1 - \frac{1708\left(0.923\right)}{0.5\left(69,600\right)}\right] + \left[\left(\frac{0.5\times69,600}{5830}\right)^{1/3} - 1\right]$$

$$\overline{Nu}_L = 1 + 1.44\left[0.951\right]\left[0.955\right] + 0.814 = 3.12$$

$$\overline{h} = \left(k/L\right)\overline{Nu}_L = \left(0.028\ W/m\cdot K/0.03\,m\right)3.12 = 2.91\ W/m^2\cdot K$$

$$q_{conv} = 2.91\ W/m^2\cdot K\left(4\ m^2\right)\left(70 - 30\right)°C = 466\ W \qquad <$$

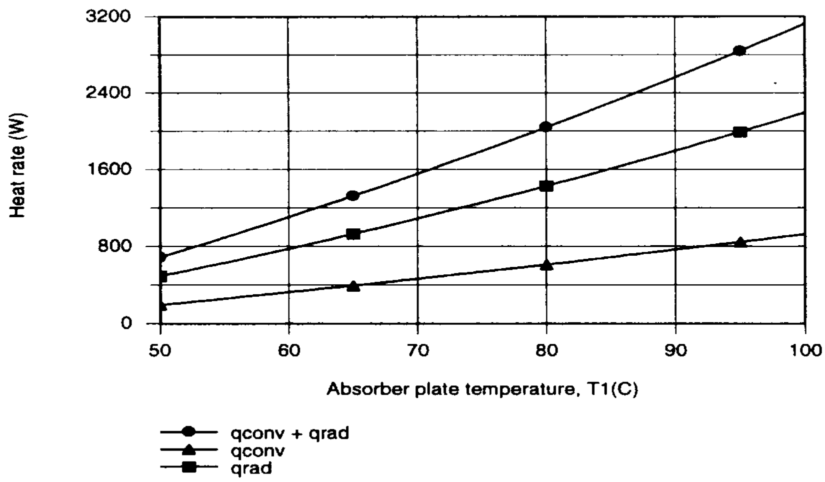
The net rate of radiation exchange is given by Eq. 13.24.

$$q = \frac{A\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} = \frac{\left(4\ m^2\right)5.67\times10^{-8}W/m^2\cdot K^4\left(343^4 - 303^4\right)K^4}{\frac{1}{0.96} + \frac{1}{0.92} - 1} = 1088\ W \qquad <$$

(b) The effect of the absorber plate temperature was determined by entering Eq. 9.54 into the *IHT* workspace and using the *Properties* and *Radiation* Toolpads.

Continued …..

# PROBLEM 13.113 (Cont.)




As expected, the convection and radiation losses increase with increasing $T_i$, with the $T^4$ dependence providing a more pronounced increase for the radiation.

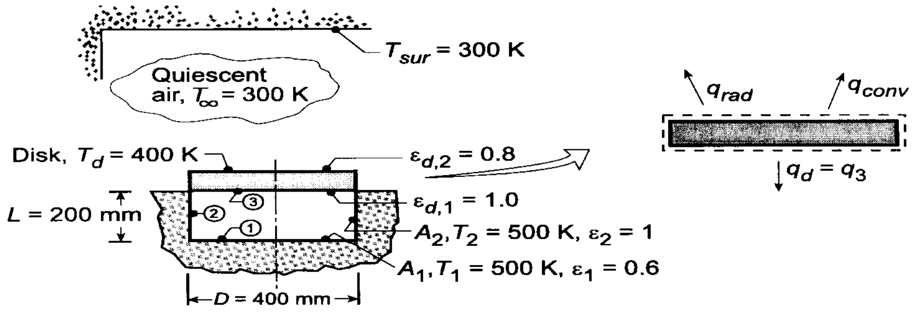
**COMMENTS:** To minimize heat losses, it is obviously desirable to operate the absorber plate at the lowest possible temperature. However, requirements for the outlet temperature of the working fluid may dictate operation at a low flow rate and hence an elevated plate temperature.

## PROBLEM 13.114

**KNOWN:** Disk heated by an electric furnace on its lower surface and exposed to an environment on its upper surface.

**FIND:** (a) Net heat transfer to (or from) the disk $q_{net,d}$ when $T_d$ = 400 K and (b) Compute and plot $q_{net,d}$ as a function of disk temperature for the range $300 \le T_d \le 500$ K; determine steady-state temperature of the disk.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Disk is isothermal; negligible thermal resistance, (3) Surroundings are isothermal and large compared to the disk, (4) Non-black surfaces are gray-diffuse, (5) Furnace-disk forms a 3-surface enclosure, (6) Negligible convection in furnace, (7) Ambient air is quiescent.

**PROPERTIES:** *Table A-4*, Air ($T_f = (T_d + T_\infty)/2$ = 350 K,1 atm): $\nu = 20.92 \times 10^{-6}$ m$^2$/s, k = 0.30 W/m·K, $\alpha = 29.9 \times 10^{-6}$ m$^2$/s.

**ANALYSIS:** (a) Perform an energy balance on the disk identifying: $q_{rad}$ as the net radiation exchange between the disk and surroundings; $q_{conv}$ as the convection heat transfer; and $q_3$ as the net radiation leaving the disk within the 3-surface enclosure.

$$q_{net,d} = \dot{E}_{in} - \dot{E}_{out} = -q_{rad} - q_{conv} - q_3 \qquad (1)$$

*Radiation exchange with surroundings*: The rate equation is of the form

$$q_{rad} = \varepsilon_{d,2} A_d \sigma \left(T_d^4 - T_{sur}^4\right) \qquad (2)$$

$$q_{rad} = 0.8(\pi/4)(0.400\,\text{m})^2\, 5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(400^4 - 300^4\right)\text{K}^4 = 99.8\ \text{W}.$$

*Free convection*: The rate equation is of the form

$$q_{conv} = \bar{h} A_d \left(T_d - T_\infty\right) \qquad (3)$$

where $\bar{h}$ can be estimated by an appropriate convection correlation. Find first,

$$Ra_L = g\beta\Delta T L^3/\nu\alpha \qquad (4)$$

$$Ra_L = 9.8\,\text{m/s}^2\,(1/350\ \text{K})(400-300)\text{K}\,(0.400\,\text{m}/4)^3 / 20.92\times10^{-6}\,\text{m/s}^2 \times 29.9\times10^{-6}\,\text{m}^2/\text{s}$$

$$Ra_L = 4.476\times10^6$$

where L = $A_c$/P = D/4. For the upper surface of a heated plate for which $10^4 \le Ra_L \le 10^7$, Eq. 9.30 is the appropriate correlation,

Continued …..

$$\overline{Nu}_L = \bar{h}L/k = 0.54\,Ra_L^{1/4} \quad (5)$$

$$\bar{h} = 0.030\,W/m\cdot K/(0.400\,m/4)\times 0.54\left(4.476\times 10^6\right)^{1/4} = 7.45\,W/m^2\cdot K$$

Hence, from Eq. (3),

$$q_{conv} = 7.45\,W/m^2\cdot K\,(\pi/4)(0.400\,m)^2\,(400-300)\,K = 93.6\ W.$$

*Furnace-disk enclosure*: From Eq. 13.20, the net radiation leaving the disk is

$$q_3 = \frac{J_3 - J_1}{(A_3F_{31})^{-1}} + \frac{J_3 - J_2}{(A_3F_{32})^{-1}} = A_3\left[F_{31}(J_3 - J_1) + F_{32}(J_3 - J_2)\right]. \quad (6)$$

The view factor $F_{32}$ can be evaluated from the *coaxial parallel disks* relation of Table 13.1 or from Fig. 13.5.

$$R_i = r_i/L = 200\ mm/200\ mm = 1,$$

$$R_j = r_j/L = 1,$$

$$S = 1 + \left(1 + R_j^2\right)/R_j^2 = 1 + \left(1 + 1^2\right)1^2 = 3$$

$$F_{31} = 1/2\left\{S - \left[S^2 - 4\left(r_j/r_i\right)^2\right]^{1/2}\right\} = 1/2\left\{3 - \left[3^2 - 4(1)^2\right]^{1/2}\right\} = 0.382. \quad (7)$$

From summation rule, $F_{32} = 1 - F_{33} - F_{31} = 0.618$ with $F_{33} = 0$. Since surfaces $A_2$ and $A_3$ are black,

$$J_2 = E_{b2} = \sigma T_2^4 = \sigma\,(500\ K)^4 = 3544\ W/m^2$$

$$J_3 = E_{b3} = \sigma T_3^4 = \sigma\,(400\ K)^4 = 1452\ W/m^2.$$

To determine $J_1$, use Eq. 13.21, the radiation balance equation for $A_1$, noting that $F_{12} = F_{32}$ and $F_{13} = F_{31}$,

$$\frac{E_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} = \frac{J_1 - J_2}{(A_1F_{12})^{-1}} + \frac{J_1 - J_3}{(A_1F_{13})^{-1}}$$

$$\frac{3544 - J_1}{(1-0.6)/0.6} = \frac{J_1 - 3544}{(0.618)^{-1}} + \frac{J_1 - 1452}{(0.382)^{-1}} \qquad J_1 = 3226\ W/m^2. \quad (8)$$

Substituting numerical values in Eq. (6), find

$$q_3 = (\pi/4)(0.400\,m)^2\left[0.382(1452-3226)\,W/m^2 + 0.618(1452-3544)\,W/m^2\right] = -248\ W.$$

Returning to the overall energy balance, Eq. (1), the net heat transfer to the disk is

$$q_{net,d} = -99.8\ W - 93.6\,W - (-248\,W) = +54.6\ W$$ <

That is, there is a net heat transfer rate *into* the disk.

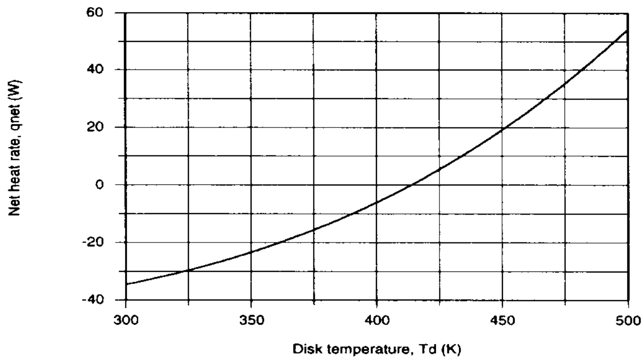
(b) Using the energy balance, Eq. (1), and the rate equation, Eqs. (2) and (3) with the *IHT Radiation Tool, Radiation, Exchange Analysis, Radiation surface energy balances* and the *Correlation Tool, Free Convection, Horizontal Plate (Hot surface up)*, the analysis was performed to obtain $q_{net,d}$ as a function of $T_d$. The results are plotted below.

The steady-state condition occurs when $q_{net,d} = 0$ for which

$$T_d = 413\ K$$ <

Continued …..

## PROBLEM 13.114 (Cont.)




**COMMENTS:** The *IHT* workspace for the foregoing analysis is shown below.

```
// Radiation Tool - Three Surface Enclosure, Furnace Disk Enclosure:
/* The net heat rate leaving A1 in terms of the surface resistance is */
q1 = (Eb1 - J1) / ((1 - eps1) / (eps1 * A1))     // Eq 13.19
/* The net heat rate leaving A1 in terms of the net exchanges between enclosure surfaces is */
q1 = q12 + q13
/* where the net exchange rates expressed in terms of the space resistances are, Eq 13.20 and 13.22, */
q12 = (J1 - J2) / (1 / (A1 * F12))
q13 = (J1 - J3) / (1 / (A1 * F13))
/* The net heat rate leaving A2 in terms of the surface resistance is */
q2 = (Eb2 - J2) / ((1 - eps2) / (eps2 * A2))     // Eq 13.19
/* The net heat rate leaving A2 in terms of the net exchanges between enclosure surfaces is */
q2 = q21 + q23
/* where the net exchange rates expressed in terms of the space resistances are, Eq 13.20 and 13.22, */
q21 = (J2 - J1) / (1 / (A2 * F21))
q23 = (J2 - J3) / (1 / (A2 * F23))
/* The net heat rate leaving A3 in terms of the surface resistance is */
q3 = (Eb3 - J3) / ((1 - eps3) / (eps3 * A3))     // Eq 13.19
/* The net heat rate leaving A3 in terms of the net exchanges between enclosure surfaces is */
q3 = q31 + q32
/* where the net exchange rates expressed in terms of the space resistances are, Eq 13.20 and 13.22, */
q31 = (J3 - J1) / (1 / (A3 * F31))
q32 = (J3 - J2) / (1 / (A3 * F32))

// Emissive Powers:
Eb1 = sigma * T1^4
Eb2 = sigma * T2^4
Eb3 = sigma * T3^4
sigma = 5.67e-8               // Stefan-Boltzmann constant, W/m^2.K^4

// Radiation Tool - View Factor:
/* The view factor, F12, for coaxial parallel disks, is */
F13 = 0.5 * (S - sqrt(S^2 - 4*(r3 / r1)^2))
// where
R1 = r1 / L
R3 = r3 / L
S = 1 + (1 + R3^2) / R1^2
// See Table 13.2 for schematic of this three-dimensional geometry.

// Other View Factors and Areas Required:
F12 = 1 - F13                 // Summation rule, A1
F21 = A1 * F12 / A2           // Reciprocity rule
F23 = F21                     // Symmetry condition
F31 = F13                     // Symmetry condition
F32 = F12                     // Symmetry condition
A1 = pi * r1^2                // Surface area, m^2
A2 = pi * r1 * L              // Surface area, m^2
A3 = pi * r3^2                // Surface area, m^2

// Overall plate energy balance, Eqs (1,2,3):
qnet = - qrad - qcv - q3
qrad = eps32 * A3 * sigma * (T3^4 - Tsur^4)
qcv = hLbar * A3 * (T3 - Tinf)
```

Continued …..

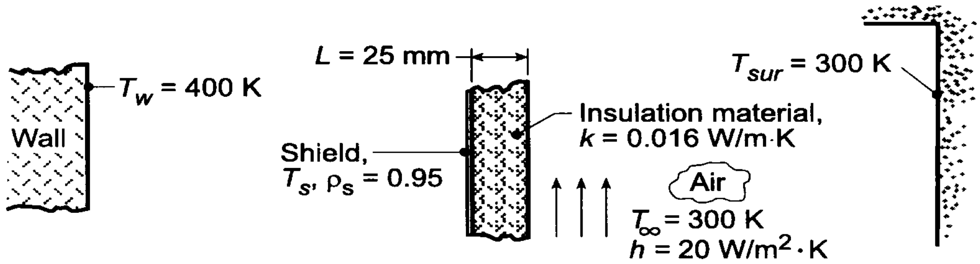
```
// Convection Tool - Free Convection, Flat Plate:
// Hot Surface Up (HSU) or Cold Surface Down (CSD)
NuLbar3 = NuL_bar_FC_HP_HSU(RaL3)     // Eq 9.30 or 31
NuLbar3 = hLbar * L3 / k3
RaL3 = g * beta3 * deltaT3 * L3^3 / (nu3 * alpha3)     //Eq 9.25
deltaT3 = abs(T3 - Tinf)
g = 9.8                               // gravitational constant, m/s^2
// Evaluate properties at the film temperature, Tf1.
Tf = Tfluid_avg(Tinf,T3)
// The characteristic length, surface area and perimeter are
L3 = As3 / P3                         // Eq 9.29
As3 =pi * r3^2
P3 = pi * r3
// Properties Tool - Air
// Air property functions : From Table A.4
// Units: T(K); 1 atm pressure
nu3 = nu_T("Air",Tf)                  // Kinematic viscosity, m^2/s
k3 = k_T("Air",Tf)                    // Thermal conductivity, W/m·K
alpha3 = alpha_T("Air",Tf)            // Thermal diffusivity, m^2/s
Pr3 = Pr_T("Air",Tf)                  // Prandtl number
beta3 = 1/Tf                          // Volumetric coefficient of expansion, K^(-1); ideal gas

// Assigned Variables
r1 = 0.2                              // Radius, m
r3 = 0.2                              // Radius, m
L = 0.2                               // Separation distance, m
T1 = 500                              // Temperature, K
eps1 = 0.6                            // Emissivity
T2 = 500                              // Temperature, K
eps2 = 0.999                          // Emissivity; avoiding 'division by zero error'
T3 = 400                              // Temperature, K
eps32 = 0.8                           // Emissivity; upper surface
eps3 = 0.999                          // Emissivity; lower surface, enclosure side
Tinf = 300                            // Ambient air temperature, K
Tsur = 300                            // Surroundings temperature, K
```

## PROBLEM 13.115

**KNOWN:** Radiation shield facing hot wall at $T_w$ = 400 K is backed by an insulating material of known thermal conductivity and thickness which is exposed to ambient air and surroundings at 300 K.

**FIND:** (a) Heat loss per unit area from the hot wall, (b) Radiosity of the shield, and (c) Perform a parameter sensitivity analysis on the insulation system considering effects of shield reflectivity $\rho_s$, insulation thermal conductivity k, overall coefficient h, on the heat loss from the hot wall.

**SCHEMATIC:**

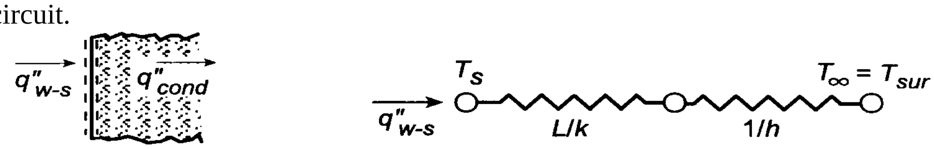



**ASSUMPTIONS:** (1) Wall is black surface of uniform temperature, (2) Shield and wall behave as parallel infinite plates, (3) Negligible convection in region between shield and wall, (4) Shield is diffuse-gray and very thin, (5) Prescribed coefficient h = 10 W/m$^2$·K is for convection and radiation.

**ANALYSIS:** (a) Perform an energy balance on the shield to obtain

$$q''_{w-s} = q''_{cond}$$

But the insulating material and the convection process at the exposed surface can be represented by a thermal circuit.




In equation form, using Eq.13.24 for the wall and shield,

$$q''_{w-s} = \frac{\sigma\left(T_w^4 - T_s^4\right)}{1/\varepsilon_w + 1/\varepsilon_s - 1} = \frac{T_s - T_\infty}{L/k + 1/h} \qquad (1,2)$$

$$\frac{\sigma\left(400^4 - T_s^4\right)}{1 + 1/0.05 - 1} = \frac{\left(T_s - 300\right)K}{\left(0.025/0.016 + 1/10\right)m^2 \cdot K/W}$$

$$T_s = 350\ K.$$

where $\varepsilon_s = 1 - \rho_s$. Hence,

$$q''_{w-s} = \frac{\left(350 - 300\right)K}{\left(0.025/0.016 + 1/10\right)m^2 \cdot K/W} = 30\ W/m^2.$$ <

(b) The radiosity of the shield follows form the definition,

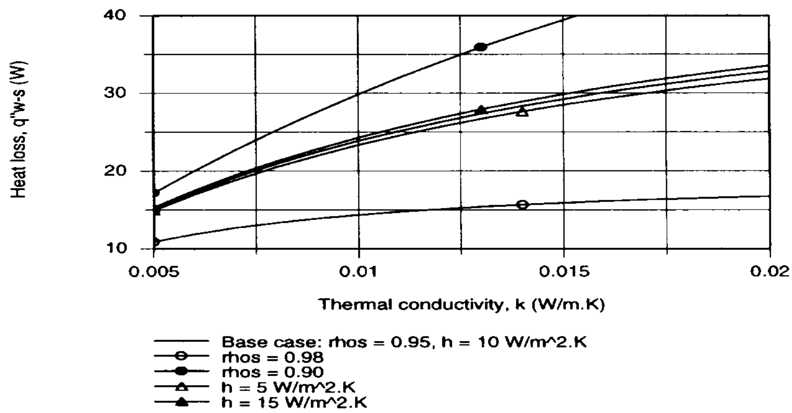
$$J_s = \rho_s G_s + \varepsilon_s E_b\left(T_s\right) = \rho_s\left(\sigma T_w^4\right) + \left(1 - \rho_s\right)\left(\sigma T_s^4\right). \qquad (3)$$

$$J_s = 0.95\sigma\left(400\ K\right)^4 + \left(1 - 0.95\right)\sigma\left(350\ K\right)^4 = 1421\ W/m^2.$$ <

with $\sigma = 5.67 \times 10^{-8}$ W/m$^2$·K$^4$.

Continued …..

(c) Using the Eqs. (1) and (2) in the *IHT* workspace, $q''_{w-s}$ can be computed and plotted for selected ranges of the insulation system variables, $\rho_s$, k, and h. Intuitively we know that $q''_{w-s}$ will decrease with increasing $\rho_s$, decreasing k and decreasing h. We chose to generate the following family of curves plotting $q''_{w-s}$ vs. k for selected values of $\rho_s$ and h.



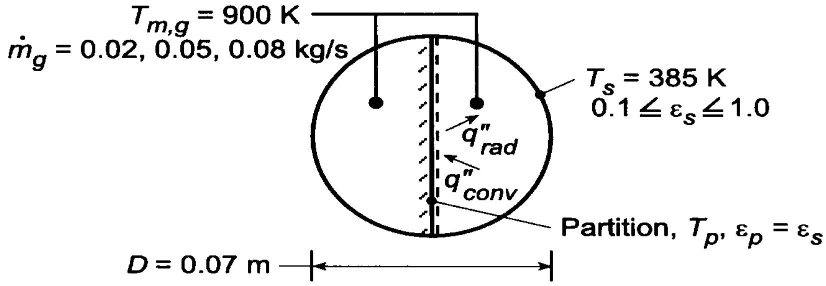

Considering the base condition with variable k, reducing k by a factor of 3, the heat loss is reduced by a factor of 2. The effect of changing h (4 to 24 $W/m^2 \cdot K$) has little influence on the heat loss. However, the effect of shield reflectivity change is very significant. With $\rho_s = 0.98$, probably the upper limit of a practical reflector-type shield, the heat loss is reduced by a factor of two. To improve the performance of the insulation system, it is most advantageous to increase $\rho_s$ and decrease k.

## PROBLEM 13.116

**KNOWN:** Diameter and surface temperature of a fire tube. Gas low rate and temperature. Emissivity of tube and partition.

**FIND:** (a) Heat transfer per unit tube length, $q'$, without the partition, (b) Partition temperature, $T_p$, and heat rate with the partition, (c) Effect of flow rate and emissivity on $q'$ and $T_p$. Effect of emissivity on radiative and convective contributions to $q'$.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Fully-developed flow in duct, (2) Diffuse/gray surface behavior, (3) Negligible gas radiation.

**PROPERTIES:** *Table A-4*, air ($T_{m,g}$ = 900 K): $\mu = 398 \times 10^{-7}$ N·s/m$^2$, k = 0.062 W/m·K, Pr = 0.72; air ($T_s$ = 385 K): $\mu = 224 \times 10^{-7}$ N·s/m$^2$.

**ANALYSIS:** (a) Without the partition, heat transfer to the tube wall is only by convection. With $\dot{m}$ = 0.05 kg/s and $Re_D = 4\,\dot{m}_g/\pi D\mu = 4(0.05\ \text{kg/s})/\pi(0.07\ \text{m})398\times10^{-7}\ \text{N}\cdot\text{s/m}^2 = 22{,}850$, the flow is turbulent. From Eq. (8.61),

$$Nu_D = 0.027\,Re_D^{4/5}\,Pr^{1/3}\left(\mu/\mu_s\right)^{0.14} = 0.027(22{,}850)^{4/5}(0.72)^{1/3}(398/224)^{0.14} = 80.5$$

$$h = \frac{k}{D}Nu_D = \frac{0.062\ \text{W/m}\cdot\text{K}}{0.07\ \text{m}}80.5 = 71.3\ \text{W/m}^2\cdot\text{K}$$

$$q' = h\pi D\left(T_{m,g} - T_s\right) = 71.3\,\text{W/m}^2\cdot\text{K}(\pi)0.07\ \text{m}(900-385) = 8075\ \text{W/m} \quad <$$

(b) The temperature of the partition is determined from an energy balance which equates net radiation exchange with the tube wall to convection from the gas. Hence, $q''_{rad} = q''_{conv}$, where from Eq. 13.23,

$$q''_{rad} = \frac{\sigma\left(T_p^4 - T_s^4\right)}{\dfrac{1-\varepsilon_p}{\varepsilon_p} + \dfrac{1}{F_{ps}} + \dfrac{1-\varepsilon_s}{\varepsilon_s}\dfrac{A_p}{A_s}}$$

where $F_{12}$ = 1 and $A_p/A_s = D/(\pi D/2) = 2/\pi = 0.637$. The flow is now in a noncircular duct for which $D_h = 4A_c/P = 4(\pi D^2/8)/(\pi D/2+D) = \pi D/(\pi + 2) = 0.611$ D = 0.0428 m and $\dot{m}_{1/2} = \dot{m}_g/2$ = 0.025 kg/s. Hence, $Re_D = \dot{m}_{1/2}D_h/A_c\mu = \dot{m}_{1/2}D_h/(\pi D^2/8)\mu$ = 8(0.025 kg/s) (0.0428 m)/$\pi$(0.07 m)$^2$ 398 $\times$ $10^{-7}$ N·s/m$^2$ = 13,970 and

$$Nu_D = 0.027(13{,}970)^{4/5}(0.72)^{1/3}(398/224)^{0.14} = 54.3$$

$$h = \frac{k}{D_h}Nu_D = \frac{0.062\ \text{W/m}\cdot\text{K}}{0.0428\ \text{m}}54.3 = 78.7\ \text{W/m}^2\cdot\text{K}$$

Continued …..

**PROBLEM 13.116 (Cont.)**

Hence, with $\varepsilon_s = \varepsilon_p = 0.5$ and $q''_{conv} = h\left(T_{m,g} - T_p\right)$,

$$\frac{5.67\times10^{-8}\,W/m^2\cdot K^4\left(T_p^4 - 385^4\right)K^4}{1+1+0.637} = 78.7\ W/m^2\cdot K\left(900 - T_p\right)K$$

$$21.5\times10^{-8}T_p^4 + 78.7T_p - 71{,}302 = 0$$

A trial-and-error solution yields

$$T_p = 796\ K$$ <

The heat rate to one-half of the tube is then

$$q'_{1/2} = q'_{ps} + q'_{conv} = \frac{D\sigma\left(T_p^4 - T_s^4\right)}{\frac{1-\varepsilon_p}{\varepsilon_p} + \frac{1}{F_{ps}} + \frac{1-\varepsilon_s}{\varepsilon_s}\frac{A_p}{A_s}} + h\left(\pi D/2\right)\left(T_{m,g} - T_s\right)$$

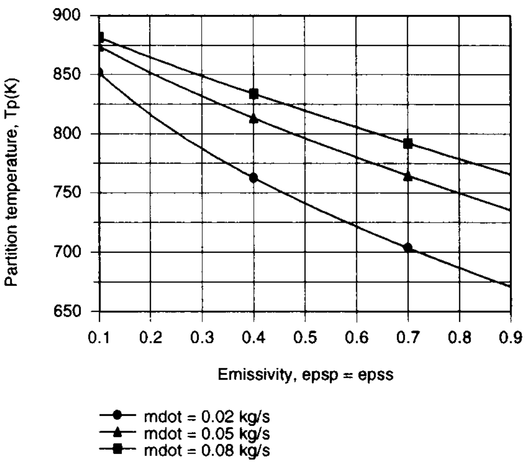
$$q'_{1/2} = \frac{0.07\ m\left(5.67\times10^{-8}W/m^2\cdot K^4\right)\left(796.4^4 - 385^4\right)K^4}{2.637} + 78.7\ W/m^2\cdot K\left(0.110\ m\right)\left(900-385\right)K$$

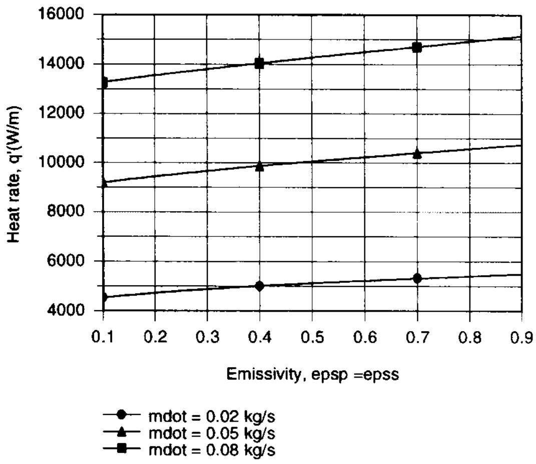
$$q'_{1/2} = 572\ W/m + 4458\ W/m = 5030\ W/m$$

The heat rate for the entire tube is

$$q' = 2q'_{1/2} = 10{,}060\ W/m$$ <

(c) The foregoing model was entered into the *IHT* workspace, and parametric calculations were performed to obtain the following results.

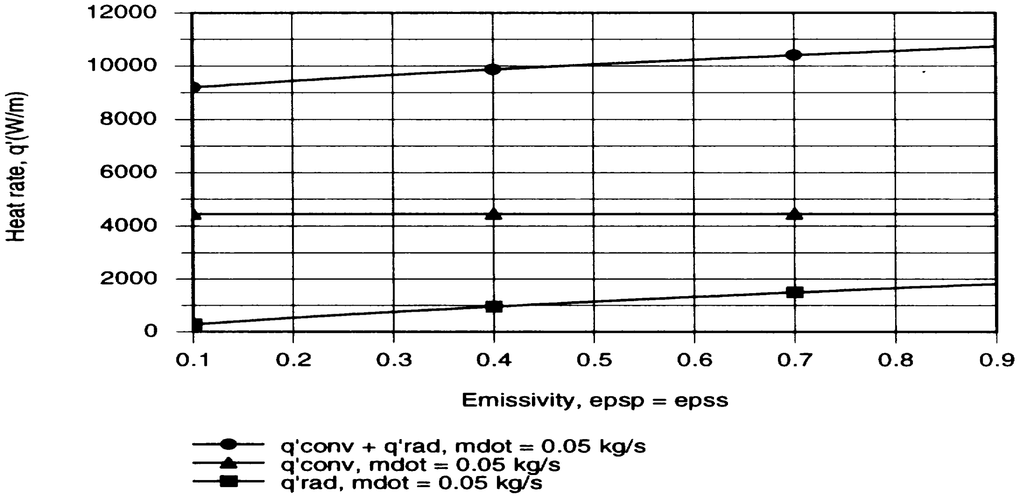



Radiation transfer from the partition increases with increasing $\varepsilon_p = \varepsilon_s$, thereby reducing $T_p$ while increasing $q'$. Since h increases with increasing $\dot{m}$, $T_p$ and $q'$ also increase with $\dot{m}$.

Continued …..

## PROBLEM 13.116 (Cont.)




Although the radiative contribution to the heat rate increases with increasing $\varepsilon_p = \varepsilon_s$, it still remains small relative to convection.

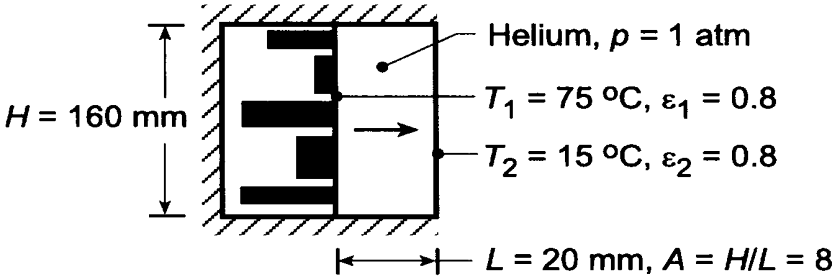
**COMMENTS:** Contrasting the heat rate predicted for part (b) with that for part (a), it is clear that use of the partition enhances heat transfer to the tube. However, the effect is due primarily to an increase in h and secondarily to the addition of radiation.

## PROBLEM 13.117

**KNOWN:** Height and width of a two-dimensional cavity filled with helium. Temperatures and emissivities of opposing vertical plates.

**FIND:** (a) Heat rate per unit length, (b) Effect of L on heat rate.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Isothermal plates, (3) Diffuse-gray surfaces, (4) Reradiating cavity sidewalls.

**PROPERTIES:** *Table A-4*, Helium (T = 318 K, 1 atm): $\nu = 136 \times 10^{-6}\ m^2/s$, k = 0.158 W/m·K, $\alpha = 201 \times 10^{-6}\ m^2/s$, Pr = 0.679, $\beta = 0.00314\ K^{-1}$.

**ANALYSIS:** (a) The power generated by the electronics leaving the surface $A_1$ is $q' = q'_{conv} + q'_{rad}$, or

$$q' = \bar{h}H\left(T_1 - T_2\right) + \frac{H\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-\varepsilon_1}{\varepsilon_1} \frac{1}{F_{12} + \left[\left(1/F_{1R}\right) + \left(1/F_{2R}\right)\right]^{-1}} + \frac{1-\varepsilon_2}{\varepsilon_2}}$$

The free convection coefficient can be estimated using Eq. 9.50 with

$$Ra_L = \frac{g\beta\left(T_1 - T_2\right)L^3}{\alpha\nu} = \frac{9.8\,m/s^2\left(0.00314\ K^{-1}\right)60\ K\left(0.02\,m\right)^3}{201 \times 10^{-6} \times 136 \times 10^{-6}\,m^4/s^2} = 540$$

However, since $Ra_L < 1000$, free convection effects can be neglected, in which case heat transfer is by conduction and $\overline{Nu}_L = 1$. Hence,

$$\bar{h} = \overline{Nu}_L\left(k/L\right) = 1.0\left(0.158\ W/m \cdot K/0.02m\right) = 7.9\ W/m^2 \cdot K.$$

The view factor can be found from Fig. 13.4 with X/L = 8 and Y/L = ∞. Hence, $F_{12} = 0.9$ and $F_{1R} = F_{2R} = 0.1$. It follows that

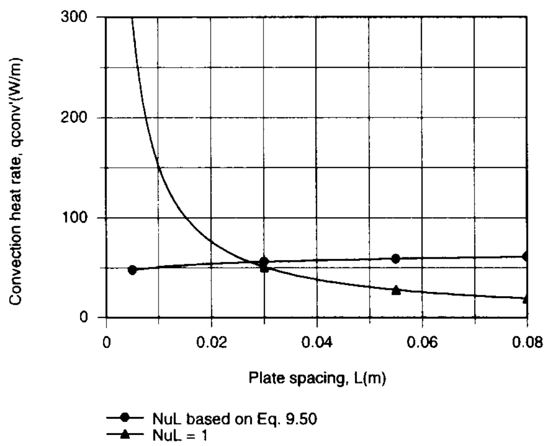
$$q' = 7.9\ W/m^2 \cdot K\left(0.16\,m\right)\left(60°C\right) + \frac{0.16\,m \times 5.67 \times 10^{-8}\,W/m^2 \cdot K\left(348^4 - 288^4\right)K^4}{0.25 + \frac{1}{0.9 + \left[10 + 10\right]^{-1}} + 0.25}$$

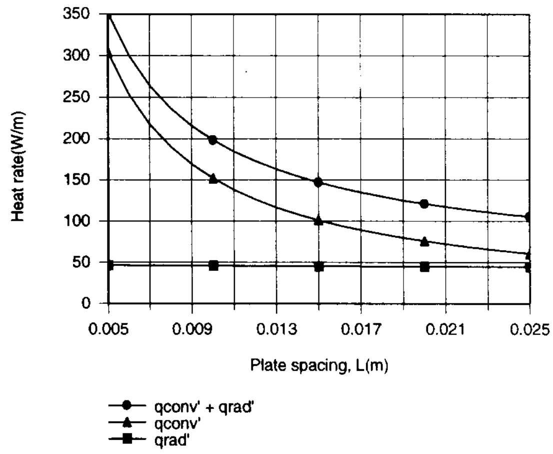
$$q' = 75.8\ W/m + 45.5\ W/m = 121\ W/m.$$ <

Continued …..

## PROBLEM 13.117 (Cont.)

(b) To assess the effect of plate spacing on convection heat transfer, $q'_{conv} = \overline{h}H(T_1 - T_2)$ was computed by using both Eq. 9.50 and the conduction limit $\left(\overline{Nu}_L = 1\right)$ to determine $\overline{h}$. These expressions were entered into the *IHT* workspace, and the *Radiation* Toolpad was used to obtain the appropriate radiation rate equation and view factor.




The cross-over at L = 28 mm marks the plate spacing below and above which, respectively, the conduction limit and Eq. 9.50 are applicable. Although there is a slight increase in $q'_{conv}$ with increasing L for L > 28 mm, the increase pales by comparison with that corresponding to a reduction in L for L < 28 mm. As the two plates are brought closer to each other in the conduction limit, the reduction in the corresponding thermal resistance significantly increases the heat rate. The total heat rate and the conduction and radiative components are also plotted for $5 \le L \le 25$ mm. There is an increase in $q'_{rad}$ with decreasing L, due to an increase in $F_{12}$ ($F_{12} = 0.97$ for L = 5 mm). However, the increase is small, and conduction is the dominant heat transfer mode.

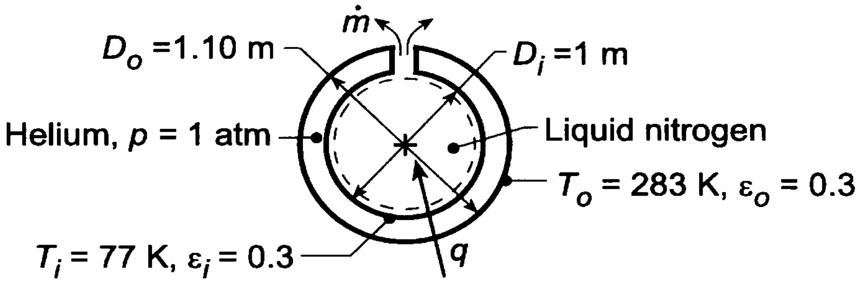
**COMMENTS:** Even for small values of L, the total heat rate is small and the scheme is poorly suited for electronic cooling. Note that helium is preferred over air on the basis of its larger thermal conductivity.

## PROBLEM 13.118

**KNOWN:** Diameters, temperatures, and emissivities of concentric spheres.

**FIND:** Rate at which nitrogen is vented from the inner sphere. Effect of radiative properties on evaporation rate.

**SCHEMATIC:**




**ASSUMPTIONS:** Diffuse-gray surfaces.

**PROPERTIES:** Liquid nitrogen (given): $h_{fg} = 2 \times 10^5$ J/kg; *Table A-4*, Helium ($\overline{T} = (T_i + T_o)/2 =$ 180 K, 1 atm): $\nu = 51.3 \times 10^{-6}$ m$^2$/s, k = 0.107 W/m·K, $\alpha = 76.2 \times 10^{-6}$ m$^2$/s, Pr = 0.673, $\beta =$ 0.00556 K$^{-1}$.

**ANALYSIS:** (a) Performing an energy balance for a control surface about the liquid nitrogen, it follows that $q = q_{conv} + q_{rad} = \dot{m}h_{fg}$. From the Raithby-Hollands expressions for free convection between concentric spheres, $q_{conv} = k_{eff}\pi(D_i D_o/L)(T_o - T_i)$, where

$$k_{eff} = 0.74k\left(\frac{Pr}{0.861+Pr}\right)^{1/4}\left[\frac{L}{\left(D_oD_i\right)^4\left(D_i^{-7/5}+D_o^{-7/5}\right)^5}\right]^{1/4}$$

$$Ra_L = \frac{g\beta\left(T_o - T_i\right)L^3}{\nu\alpha} = \frac{9.8\,m/s^2\left(0.00556\ K^{-1}\right)\left(206\ K\right)\left(0.05m\right)^3}{\left(51.3\times10^{-6}m^2/s\right)\left(76.2\times10^{-6}m^2/s\right)} = 3.589\times10^5.$$

Hence,

$$k_{eff} = 0.74\left(0.107\,W/m\cdot K\right)\left(\frac{0.673}{0.861+0.673}\right)^{1/4}\left[\frac{0.05}{(1.1)^4}\frac{3.589\times10^5}{(1+0.875)^5}\right]^{1/4} = 0.309\ W/m\cdot K.$$

The heat rate by convection is

$$q_{conv} = \left(0.309\ W/m\cdot K\right)\pi\left(1.10\,m^2/0.05\,m\right)206\ K = 4399\ W.$$

From Table 13.3,

$$q_{rad} = q_{oi} = \frac{\sigma\pi D_1^2\left(T_o^4 - T_i4\right)}{1/\varepsilon_i + \left(\left(1-\varepsilon_o\right)/\varepsilon_o\right)\left(D_i/D_o\right)^2}$$

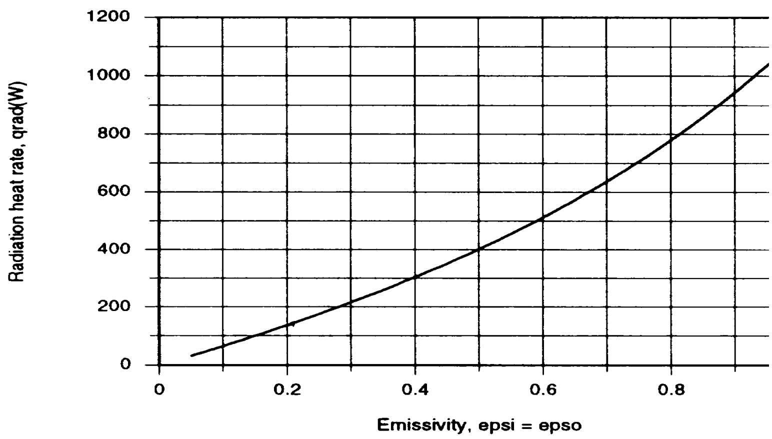
$$= \frac{\left(5.67\times10^{-8}W/m^2\cdot K^4\right)\pi\left(1\ m\right)^2\left(283^4 - 77^4\right)K^4}{1/0.3+\left(0.7/0.3\right)\left(1/1.1\right)^2} = 216\ W.$$

Continued …..

## PROBLEM 13.118 (Cont.)

Hence, $\dot{m} = q / h_{fg} = (4399 + 216)\,W / 2\times10^5\,J/kg = 0.023\,kg/s.$ <

With the cavity evacuated, *IHT* was used to compute the radiation heat rate as a function of $\varepsilon_i = \varepsilon_o$.




Clearly, significant advantage is associated with reducing the emissivities and $q_{rad}$ = 31.8 W for $\varepsilon_i$ = $\varepsilon_o$ = 0.05.

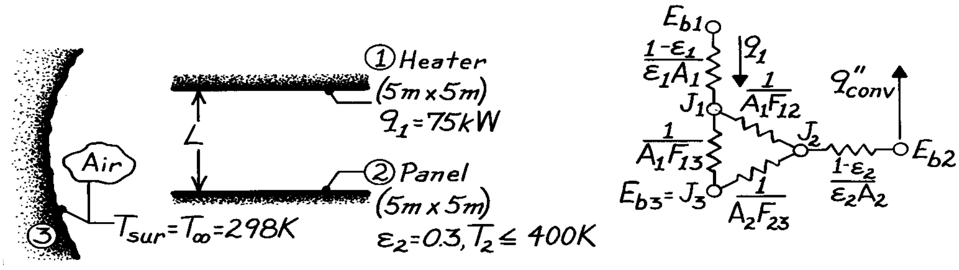
**COMMENTS:** The convection heat rate is too large. It could be reduced by replacing He with a gas of smaller k, a cryogenic insulator (Table A.3), or a vacuum. Radiation effects are second order for small values of the emissivity.

## PROBLEM 13.119

**KNOWN:** Dimensions, emissivity and upper temperature limit of coated panel. Arrangement and power dissipation of a radiant heater. Temperature of surroundings.

**FIND:** (a) Minimum panel-heater separation, neglecting convection, (b) Minimum panel-heater separation, including convection.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Top and bottom surfaces of heater and panel, respectively, are adiabatic, (2) Bottom and top surfaces of heater and panel, respectively are diffuse-gray, (3) Surroundings form a large enclosure about the heater-panel arrangement, (4) Steady-state conditions, (5) Heater power is dissipated entirely as radiation (negligible convection), (6) Air is quiescent and convection from panel may be approximated as free convection from a horizontal surface, (7) Air is at atmospheric pressure.

**PROPERTIES:** *Table A-4*, Air ($T_f$ = (400 + 298)/2 ≈ 350 K, 1 atm): $\nu = 20.9 \times 10^{-6}$ m²/s, k = 0.03 W/m·K, Pr = 0.700, $\alpha = 29.9 \times 10^{-6}$ m²/s, $\beta = 2.86 \times 10^{-3}$ K$^{-1}$.

**ANALYSIS:** (a) Neglecting convection effects, the panel constitutes a floating potential for which the net radiative transfer must be zero. That is, the panel behaves as a re-radiating surface for which $E_{b2} = J_2$. Hence

$$q_1 = \frac{J_1 - E_{b2}}{1/A_1F_{12}} + \frac{J_1 - E_{b3}}{1/A_1F_{13}} \tag{1}$$

and evaluating terms

$$E_{b2} = \sigma T_2^4 = 5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\,(400\ \text{K})^4 = 1452\ \text{W/m}^2$$

$$E_{b3} = \sigma T_3^4 = 5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\,(298\ \text{K})^4 = 447\ \text{W/m}^2$$

$$F_{13} = 1 - F_{12} \qquad\qquad A_1 = 25\ \text{m}^2$$

find that

$$\frac{75{,}000\ \text{W}}{25\ \text{m}^2} = \frac{J_1 - 1452}{1/F_{12}} + \frac{J_1 - 447}{1/(1 - F_{12})}$$

$$3000\ \text{W/m}^2 = F_{12}(J_1 - 1452) + (J_1 - 447) - F_{12}(J_1 - 447)$$

$$J_1 = 3447 + 1005F_{12}. \tag{2}$$

Performing a radiation balance on the panel yields

$$\frac{J_1 - E_{b2}}{1/A_1F_{12}} = \frac{E_{b2} - E_{b3}}{1/A_2F_{23}}.$$

Continued …..

## PROBLEM 13.119 (Cont.)

With $A_1 = A_2$ and $F_{23} = 1 - F_{12}$

$$F_1\left(J_1 - 1452\right) = \left(1 - F_{12}\right)\left(1452 - 447\right)$$

or

$$447F_{12} = F_{12}J_1 - 1005. \qquad (3)$$

Substituting for $J_1$ from Eq. (2), find

$$447F_{12} = F_{12}\left(3447 + 1005F_{12}\right) - 1005$$

$$1005F_{12}^2 + 3000F_{12} - 1005 = 0$$

$$F_{12} = 0.30.$$

Hence from Fig. 13.4, with X/L = Y/L and $F_{ij} = 0.3$,

$$X/L \approx 1.45$$

$$L \approx 5\ \text{m}/1.45 = 3.45\ \text{m}. \qquad <$$

(b) Accounting for convection from the panel, the net radiation transfer is no longer zero at this surface and $E_{b2} \neq J_2$. It then follows that

$$q_1 = \frac{J_1 - J_2}{1/A_1F_{12}} + \frac{J_1 - E_{b3}}{1/A_1F_{13}} \qquad (4)$$

where, from an energy balance on the panel,

$$\frac{J_2 - E_{b2}}{\left(1-\varepsilon_2\right)/\varepsilon_2 A_2} = q_{conv,2} = \bar{h}A_2\left(T_2 - T_\infty\right). \qquad (5)$$

With $L \equiv A_s/P = 25\ \text{m}^2/20\ \text{m} = 1.25\ \text{m}$,

$$Ra_L = \frac{g\beta\left(T_s - T_\infty\right)L^3}{\nu\alpha} = \frac{9.8\ \text{m/s}^2\left(2.86\times10^{-3}\ \text{K}^{-1}\right)\left(102\ \text{K}\right)\left(1.25\ \text{m}\right)^3}{\left(20.9\times29.9\right)10^{-12}\ \text{m}^4/\text{s}^2} = 8.94\times10^9.$$

Hence

$$\overline{Nu}_L = 0.15Ra_L^{1/3} = 0.15\left(8.94\times10^9\right)^{1/3} = 311$$

$$\bar{h} = 311\ k/L = 311\frac{0.03\ \text{W/m}\cdot\text{K}}{1.25\ \text{m}} = 7.46\ \text{W/m}^2\cdot\text{K}$$

$$q''_{conv,2} = 7.46\ \text{W/m}^2\cdot\text{K}\left(102\ \text{K}\right) = 761\ \text{W/m}^2.$$

From Eq. (5)

$$J_2 = E_{b2} + \frac{1-\varepsilon_2}{\varepsilon_2}q''_{conv,2} = 1452 + \frac{0.7}{0.3}761 = 3228\ \text{W/m}^2.$$

Continued …..

## PROBLEM 13.119 (Cont.)

From Eq. (4),

$$\frac{75{,}000}{25} = \frac{J_1 - 3228}{1/F_{12}} + \frac{J_1 - 447}{1/\left(1 - F_{12}\right)}$$

$$3000 = F_{12}\left(J_1 - 3228\right) + J_1 - 447 - F_{12}\left(J_1 - 447\right)$$

$$J_1 = 3447 + 2781F_{12}. \qquad (6)$$

From an energy balance on the panel,

$$\frac{J_1 - J_2}{1/A_1F_{12}} + \frac{E_{b3} - J_2}{1/A_2F_{23}} = \frac{J_2 - E_{b2}}{\left(1 - \varepsilon_2\right)/\varepsilon_2 A_2} = q_{conv,2}$$

$$F_{12}\left(J_1 - 3228\right) + \left(1 - F_{12}\right)\left(447 - 3228\right) = 761$$

$$F_{12}J_1 - 447F_{12} = 3542.$$

Substituting from Eq. (6),

$$F_{12}\left(3447 + 2781F_{12}\right) - 447F_{12} = 3542$$

$$2781F_{12}^2 + 3000F_{12} - 3542 = 0$$

$$F_{12} = 0.71.$$

Hence from Fig. 13.4, with X/L = Y/L and $F_{ij}$ = 0.71,

$$X/L = 5.7$$

$$L \approx 5\ \text{m}/5.7 = 0.88\ \text{m}. \qquad <$$

**COMMENTS:** (1) The results are independent of the heater surface radiative properties.

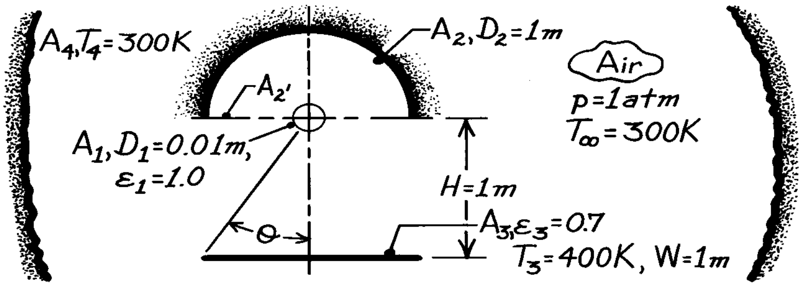
(2) Convection at the heater surface would reduce the heat rate $q_1$ available for radiation exchange and hence reduce the value of L.

# PROBLEM 13.120

**KNOWN:** Diameter and emissivity of rod heater. Diameter and position of reflector. Width, emissivity, temperature and position of coated panel. Temperature of air and large surroundings.

**FIND:** (a) Equivalent thermal circuit, (b) System of equations for determining heater and reflector temperatures. Values of temperatures for prescribed conditions, (c) Electrical power needed to operate heater.

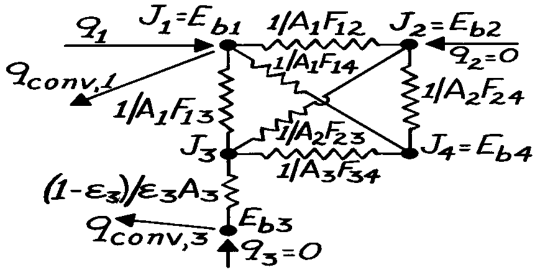
**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Diffuse-gray surfaces, (3) Large surroundings act as blackbody, (4) Surfaces are infinitely long (negligible end effects), (5) Air is quiescent, (6) Negligible convection at reflector, (7) Reflector and panel are perfectly insulated.

**PROPERTIES:** *Table A-4*, Air ($T_f$ = 350 K, 1 atm): k = 0.03 W/m·K, $\nu = 20.9 \times 10^{-6}$ m$^2$/s, $\alpha = 29.9 \times 10^{-6}$ m$^2$/s, Pr = 0.70; ($T_f$ = (1295 + 300)/2 = 800 K): k = 0.0573 W/m·K, $\nu = 84.9 \times 10^{-6}$ m$^2$/s, $\alpha = 120 \times 10^{-6}$ m$^2$/s.

**ANALYSIS:** (a) We have assumed blackbody behavior for $A_1$ and $A_4$; hence, $J = E_b$. Also, $A_2$ is insulated and has negligible convection; hence q = 0 and $J_2 = E_{b2}$. The equivalent thermal circuit is:




(b) Performing surface energy balances at 1, 2 and 3:

$$q_1 - q_{conv,1} = \frac{E_{b1} - E_{b2}}{1/A_1F_{12}} + \frac{E_{b1} - J_3}{1/A_1F_{13}} + \frac{E_{b1} - E_{b4}}{1/A_1F_{14}} \quad (1)$$

$$0 = \frac{E_{b1} - E_{b2}}{1/A_2F_{21}} + \frac{J_3 - E_{b2}}{1/A_2F_{23}} + \frac{E_{b4} - E_{b2}}{1/A_2F_{24}} \quad (2)$$

$$\frac{J_3 - E_{b3}}{(1-\varepsilon_3)/\varepsilon_3A_3} = \frac{E_{b1} - J_3}{1/A_3F_{31}} + \frac{E_{b2} - J_3}{1/A_3F_{32}} + \frac{E_{b4} - J_3}{1/A_3F_{34}} \quad (3a)$$

where

$$\frac{J_3 - E_{b3}}{(1-\varepsilon_3)/\varepsilon_3A_3} = q_{conv,3}. \quad (3b)$$

Continued …..

Solution procedure with $E_{b3}$ and $E_{b4}$ known: Evaluate $q_{conv,3}$ and use Eq. (3b) to obtain $J_3$; Solve Eqs. (2) and (3a) simultaneously for $E_{b1}$ and $E_{b2}$ and hence $T_1$ and $T_2$; Evaluate $q_{conv,1}$ and use Eq. (1) to obtain $q_1$.

For *free convection* from a heated, horizontal plate:

$$L_c = \frac{A_s}{P} = \frac{(W \times L)}{(2L + 2W)} \approx \frac{W}{2} = 0.5\ \text{m}$$

$$Ra_L = \frac{g\beta(T_3 - T_\infty)L_c^3}{\alpha\nu} = \frac{9.8\ \text{m/s}^2\,(350\ \text{K})^{-1}(100\ \text{K})(0.5\ \text{m})^3}{20.9 \times 29.9 \times 10^{-12}\,\text{m}^4/\text{s}^2} = 5.6 \times 10^8$$

$$\overline{Nu}_L = 0.15 Ra_L^{1/3} = 0.15\left(5.6 \times 10^8\right)^{1/3} = 123.6$$

$$\overline{h}_3 = \frac{k}{L_c}\overline{Nu}_L = \frac{0.03\ \text{W/m}\cdot\text{K} \times 123.6}{0.5\ \text{m}} = 7.42\ \text{W/m}^2\cdot\text{K}.$$

$$q''_{conv,3} = \overline{h}_3(T_3 - T_\infty) = 742\ \text{W/m}^2.$$

Hence, with

$$E_{b3} = \sigma T_3^4 = 5.67 \times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\,(400\ \text{K})^4 = 1451\ \text{W/m}^2$$

using Eq. (3b) find

$$J_3 = E_{b3} + \frac{1-\varepsilon_3}{\varepsilon_3 A_3} q_{conv,3} = (1451 + [0.3/0.7]742) = 1769\ \text{W/m}^2.$$

*View Factors*: From symmetry, it follows that $F_{12} = 0.5$. With $\theta = \tan^{-1}(W/2)/H = \tan^{-1}(0.5) = 26.57°$, it follows that

$$F_{13} = 2\theta/360 = 0.148.$$

From summation and reciprocity relations,

$$F_{14} = 1 - F_{12} - F_{13} = 0.352$$

$$F_{21} = (A_1/A_2)F_{12} = (2D_1/D_2)F_{12} = 0.02 \times 0.5 = 0.01$$

$$F_{23} = (A_3/A_2)F_{32} = (2/\pi)(F_{32'} - F_{31}).$$

For X/L = 1, Y/L ≈ ∞, find from Fig. 13.4 that $F_{32'} \approx 0.42$. Also find,

$$F_{31} = (A_1/A_3)F_{13} = (\pi \times 0.01/1)0.148 = 0.00465 \approx 0.005$$

$$F_{23} = (2/\pi)(0.42 - 0.005) = 0.264$$

$$F_{22} \approx 1 - F_{22'} = 1 - (A'_2/A_2)F_{2'2} = 1 - (2/\pi) = 0.363$$

$$F_{24} = 1 - F_{21} - F_{22} - F_{23} = 0.363$$

Continued …..

## PROBLEM 13.120 (Cont.)

$F_{31} = 0.005, \qquad F_{32} = 0.415$

$F_{34} = 1 - F_{32'} = 1 - 0.42 = 0.58.$

With $E_{b4} = \sigma T_4^4 = 5.67\times10^{-8}\ W/m^2\cdot K^4\left(300\ K\right)^4 = 459\ W/m^2$,

Eq. (3a) $\rightarrow$ 0.005($E_{b1}$ – 1769) + 0.415($E_{b2}$ – 1769) + 0.58(459 – 1769) = 742

$0.005E_{b1} + 0.415Eb2 = 2245$ (4)

Eq. (2) $\rightarrow$ 0.01($E_{b1}$ – $E_{b2}$) + 0.264(1769 – $E_{b2}$) + 0.363(459 – $E_{b2}$) = 0

$0.01E_{b1} - 0.637E_{b2} + 633.6 = 0.$ (5)

Hence, manipulating Eqs. (4) and (5), find

$E_{b2} = 0.0157E_{b1} + 994.7$

$0.005E_{b1} + \left(0.415\right)\left(0.0157E_{b1} + 994.7\right) = 2245.$

$E_{b1} = 159{,}322\ W/m^2 \qquad T_1 = \left(E_{b1}/\sigma\right)^{1/4} = 1295\ K$ <

$E_{b2} = 0.0157\left(159{,}322\right) + 994.7 = 3496\ W/m^2 \qquad T_2 = \left(E_{b2}/\sigma\right)^{1/4} = 498\ K.$ <

(c) With $T_1$ = 1295 K, then $T_f$ = (1295 + 300)/2 ≈ 800 K, and using

$$Ra_D = \frac{g\beta\left(T_1 - T_\infty\right)D_1^3}{\alpha\nu} = \frac{9.8\ m/s^2\left(1/800\ K\right)\left(1295-300\right)K\left(0.01\ m\right)^3}{120\times84.9\times10^{-12}m^4/s^2} = 1196$$

$$\overline{Nu}_D = 0.85Ra_D^{0.188} = 0.85\left(1196\right)^{0.188} = 3.22$$

$$\overline{h}_1 = \left(k/D_1\right)\overline{Nu}_D = \left(0.0573/0.01\right)\times3.22 = 18.5\ W/m^2\cdot K.$$

The convection heat flux is

$$q''_{conv,1} = \overline{h}_1\left(T_1 - T_\infty\right) = 18.5\left(1295-300\right) = 18{,}407\ W/m^2,$$

Using Eq. (1), find

$$q''_1 = q''_{conv,1} + F_{12}\left(E_{b1} - E_{b2}\right) + F_{13}\left(E_{b1} - J_3\right) + F_{14}\left(E_{b1} - E_{b4}\right)$$

$$q''_1 = 18{,}407 + 0.5\left(159{,}322 - 3496\right) + 0.148\left(159{,}322 - 1769\right) + 0.352\left(159{,}322 - 459\right)$$

$$q''_1 = 18{,}407 + \left(77{,}913 + 23{,}314 + 55{,}920\right)$$

$$q''_1 = 18{,}407 + 236{,}381 = 254{,}788\ W/m^2$$

$q'_1 = \pi D_1 q''_1 = \pi\left(0.01\right)254{,}788 = 8000\ W/m.$ <

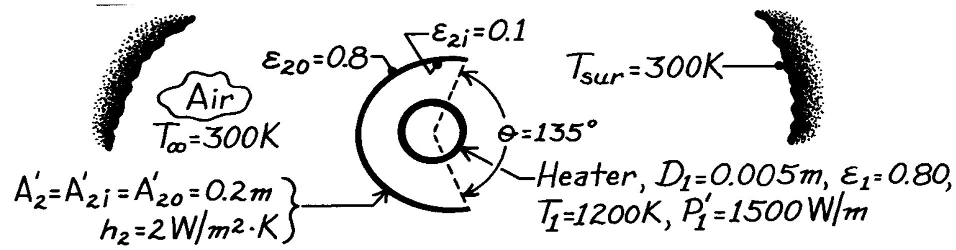
**COMMENTS:** Although convection represents less than 8% of the net radiant transfer from the heater, it is equal to the net radiant transfer to the panel. Since the reflector is a re-radiating surface, results are independent of its emissivity.

## PROBLEM 13.121

**KNOWN:** Temperature, power dissipation and emissivity of a cylindrical heat source. Surface emissivities of a parabolic reflector. Temperature of air and surroundings.

**FIND:** (a) Radiation circuit, (b) Net radiation transfer from the heater, (c) Net radiation transfer from the heater to the surroundings, (d) Temperature of reflector.

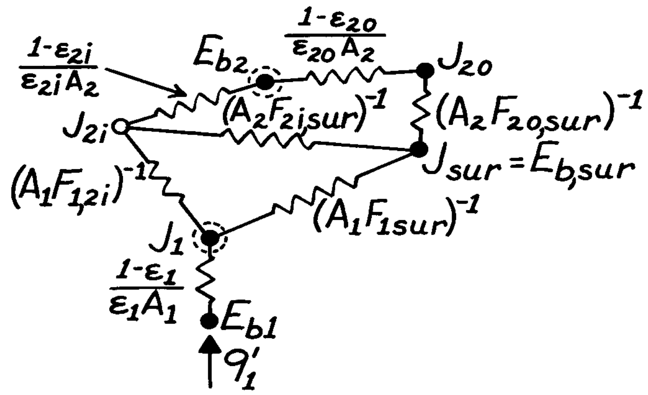
**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Heater and reflector are in quiescent and infinite air, (3) Surroundings are infinitely large, (4) Reflector is thin (isothermal), (5) Diffuse-gray surfaces.

**PROPERTIES:** *Table A-4*, Air ($T_f$ = 750 K, 1 atm): $\nu = 76.37 \times 10^{-6}$ m²/s, k = 0.0549 W/m·K, $\alpha$ = $109 \times 10^{-6}$ m²/s, Pr = 0.702.

**ANALYSIS:** (a) The thermal circuit is




(b) Energy transfer from the heater is by radiation and free convection. Hence,

$$P_1' = q_1' + q_{1,conv}'$$

where

$$q_{1,conv}' = \bar{h}\pi D_1 \left(T_1 - T_\infty\right)$$

and

$$Ra_D = \frac{g\beta\left(T_1 - T_\infty\right)D^3}{\nu\alpha} = \frac{9.8\ \text{m/s}^2\left(750\ \text{K}\right)^{-1}\left(900\ \text{K}\right)\left(0.005\ \text{m}\right)^3}{76.37\times109\times10^{-12}\,\text{m}^4/\text{s}^2} = 176.6.$$

Using the Churchill and Chu correlation, find

$$\overline{Nu}_D = \left\{0.6 + \frac{0.387 Ra_D^{1/6}}{\left[1+\left(0.559/\text{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387\left(176.6\right)^{1/6}}{\left[1+\left(0.559/0.702\right)^{9/16}\right]^{8/27}}\right\}^2 = 1.85$$

$$\bar{h} = \overline{Nu}_D\left(k/D\right) = 1.85\left(0.0549\ \text{W/m}\cdot\text{K}/0.005\ \text{m}\right) = 20.3\ \text{W/m}^2\cdot\text{K}.$$

Continued …..

## PROBLEM 13.121 (Cont.)

Hence,

$$q'_{1,conv} = 20.3\ W/m^2 \cdot K\pi(0.005\ m)(1200-300)K = 287\ W/m$$

$$q'_1 = 1500\ W/m - 287\ W/m = 1213\ W/m. \qquad <$$

(c) The net radiative heat transfer from the heater to the surroundings is

$$q'_{1(sur)} = A'_1 F_{1sur}(J_1 - J_{sur}).$$

The view factor is

$$F_{1sur} = (135/360) = 0.375$$

and the radiosities are

$$J_{sur} = \sigma T_{sur}^4 = 5.67\times10^{-8} W/m^2\cdot K^4 (300\ K)^4 = 459\ W/m^2$$

$$J_1 = E_{b1} - q'_1(1-\varepsilon_1)\varepsilon_1 A'_1 = 5.67\times10^{-8} W/m^2\cdot K^4(1200\ K)^4$$
$$-1213\ W/m[0.2/0.8\pi(0.005\ m)]$$

$$J_1 = 98,268\ W/m^2.$$

Hence

$$q'_{1(sur)} = \pi(0.005\ m)0.375(98,268-459)W/m^2 = 576\ W/m. \qquad <$$

(d) Perform an energy balance on the reflector,

$$q'_{2i} = q'_{2o} + q'_{2,conv}$$

$$\frac{J_{2i}-E_{b2}}{(1-\varepsilon_{2i})/\varepsilon_{2i}A'_2} = \frac{E_{b2}-J_{sur}}{(1-\varepsilon_{2o})/\varepsilon_{2o}A'_2 + 1/A'_2F_{2o(sur)}} + 2\bar{h}_2A'_2(T_2 - T_\infty).$$

The radiosity of the reflector is

$$J_{2i} = J_1 - \frac{q'_{1(2i)}}{A'_1F_{1(2i)}} = 98,268\ W/m^2 - \frac{(1213-576)W/m}{\pi(0.005\ m)(225/360)}$$

$$J_{2i} = 33,384\ W/m^2.$$

Hence

$$\frac{33,384 - 5.67\times10^{-8}\left(T_2^4\right)}{(0.9/0.1\times0.2\ m)} = \frac{5.67\times10^{-8}\left(T_2^4\right)-459}{(0.2/0.8\times0.2\ m)+(1/0.2m\times1)} + 2\times0.4(T_2-300)$$

$$741.9 - 0.126\times10^{-8}T_2^4 = 0.907\times10^{-8}T_2^4 - 73.4 + 0.8T_2 - 240$$

$$1.033\times10^{-8}T_2^4 + 0.8T_2 = 1005$$

and from a trial and error solution, find

$$T_2 = 502\ K. \qquad <$$

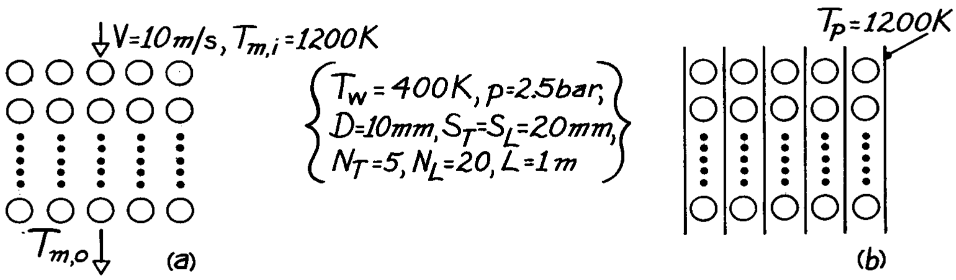
**COMMENTS:** Choice of small $\varepsilon_{2i}$ and large $\varepsilon_{2o}$ insures that most of the radiation from heater is reflected to surroundings and reflector temperature remains low.

## PROBLEM 13.122

**KNOWN:** Geometrical conditions associated with tube array. Tube wall temperature and pressure of water flowing through tubes. Gas inlet velocity and temperature when heat is transferred from products of combustion in cross-flow, or temperature of electrically heated plates when heat is transferred by radiation from the plates.

**FIND:** (a) Steam production rate for gas flow without heated plates, (b) Steam production rate with heated plates and no gas flow, (c) Effects of inserting unheated plates with gas flow.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Negligible gas radiation, (3) Tube and plate surfaces may be approximated as blackbodies, (4) Gas outlet temperature is 600 K.

**PROPERTIES:** *Table A-4*, Air ($\overline{T}$ = 900 K, 1 atm): $\rho = 0.387$ kg/m$^3$, $c_p = 1121$ J/kg·K, $\nu = 102.9 \times 10^{-6}$ m$^2$/s, k = 0.062 W/m·K, Pr = 0.720; (T = 400 K): Pr = 0.686; (T = 1200 K): $\rho = 0.29$ kg/m$^3$; *Table A-6*, Sat. water (2.5 bars): $h_{fg} = 2.18 \times 10^6$ J/kg.

**ANALYSIS:** (a) With

$$V_{max} = \left[S_T / \left(S_T - D\right)\right] V = 20 \text{ m/s}$$

$$Re_D = \frac{V_{max} D}{\nu} = \frac{20 \text{ m/s}\left(0.01 \text{ m}\right)}{102.9 \times 10^{-6} \text{m}^2/\text{s}} = 1944$$

and from the Zhukauskas correlation with C = 0.27 and m = 0.63,

$$\overline{Nu}_D = 0.27\left(1944\right)^{0.63}\left(0.720\right)^{0.36}\left(0.720/0.686\right)^{1/4} = 28.7$$

$$\overline{h} = 0.062 \text{ W/m}\cdot\text{K} \times 28.7 / 0.01 \text{ m} = 178 \text{ W/m}^2\cdot\text{K}.$$

The outlet temperature may be evaluated from

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{\overline{h}A}{\dot{m}c_p}\right) = \exp\left(-\frac{\overline{h}N\pi DL}{\rho V N_T S_T L c_p}\right)$$

$$\frac{400 - T_{m,o}}{400 - 1200} = \exp\left(-\frac{178 \text{ W/m}^2\cdot\text{K} \times 100 \times \pi \times 0.01 \text{ m}}{0.29 \text{ kg/m}^3 \times 10 \text{ m/s} \times 5 \times 0.02 \text{ m} \times 1121 \text{ J/kg}\cdot\text{K}}\right)$$

$$T_{m,o} = 543 \text{ K}.$$

Continued …..

## PROBLEM 13.122 (Cont.)

With

$$\Delta T_{\ell m} = \frac{\left(T_s - T_{m,i}\right) - \left(T_s - T_{m,o}\right)}{\ln\left(T_s - T_{m,i}\right)/\left(T_s - T_{m,o}\right)\big]} = \frac{-800 - (-143)}{\ln(800/143)} = -382\ \text{K}$$

find

$$q = \bar{h} A \Delta T_{\ell m} = 178\ \text{W/m}^2 \cdot \text{K}\,(100)\pi\,(0.01\ \text{m})1\ \text{m}\,(-382\ \text{K})$$

$$q = -214\ \text{kW}.$$

If the water enters and leaves as saturated liquid and vapor, respectively, it follows that $-q = \dot{m}\, h_{fg}$, hence

$$\dot{m} = \frac{214{,}000\ \text{W}}{2.18\times10^6\ \text{J/kg}} = 0.098\ \text{kg/s}. \qquad <$$

(b) The radiation exchange between the plates and tube walls is

$$q = \left[A_p F_{ps}\sigma\left(T_p^4 - T_s^4\right)\right]\cdot 2\cdot N_T$$

where the factor of 2 is due to radiation transfer from two plates. The view factor and area are

$$F_{ps} = 1 - \left[1 - (D/S)^2\right]^{1/2} + (D/S)\tan^{-1}\left[\left(S^2 - D^2\right)/D^2\right]^{1/2}$$

$$F_{ps} = 1 - 0.866 + 0.5\tan^{-1}1.732 = 1 - 0.866 + 0.524$$

$$F_{ps} = 0.658$$

$$A_p = N_L \cdot S_L \cdot 1\ \text{m} = 20\times0.02\ \text{m}\times1\ \text{m} = 0.40\ \text{m}^2.$$

Hence,

$$q = 5\times\left[0.80\ \text{m}^2\times0.658\times5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(1200^4 - 400^4\right)\text{K}^4\right]$$

$$q = 305{,}440\ \text{W}$$

and the steam production rate is

$$\dot{m} = \frac{305{,}440\ \text{W}}{2.18\times10^6\ \text{J/kg}} = 0.140\ \text{kg/s}. \qquad <$$

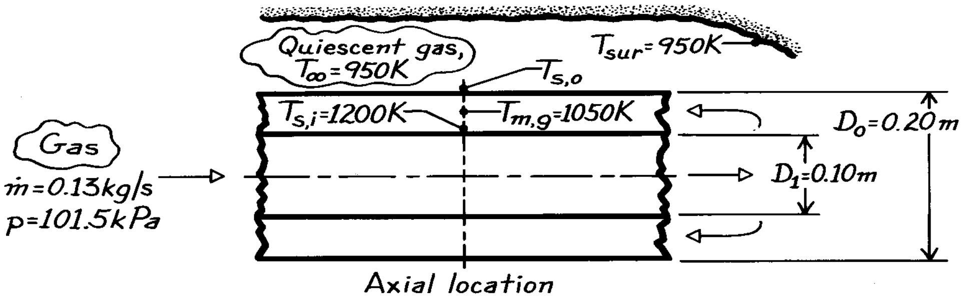
(c) The plate temperature is determined by an energy balance for which convection to the plate from the gas is equal to net radiation transfer from the plate to the tube. Conditions are complicated by the fact that the gas transfers energy to both the plate and the tubes, and its decay is not governed by a simple exponential. Insertion of the plates enhances heat transfer to the tubes and thereby increases the steam generation rate. However, for the prescribed conditions, the effect would be small, since in case (a), the heat transfer is already ≈ 80% of the maximum possible transfer.

# PROBLEM 13.123

**KNOWN:** Gas-fired radiant tube located within a furnace having quiescent gas at 950 K. At a particular axial location, inner wall and gas temperature measured by thermocouples.

**FIND:** Temperature of the outer tube wall at the axial location where the thermocouple measurements are being made.

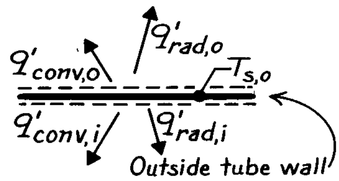
**SCHEMATIC:**




**ASSUMPTIONS:** (1) Silicon carbide tube walls have negligible thermal resistance and are diffuse-gray, (2) Tubes are positioned horizontally, (3) Gas is radiatively non-participating and quiescent, (4) Furnace gas behaves as ideal gas, $\beta = 1/T$.

**PROPERTIES:** Gas (given): $\rho = 0.32$ kg/m$^3$, $\nu = 130 \times 10^{-6}$ m$^2$/s, k = 0.070 W/m·K, Pr = 0.72, $\alpha = \nu/\text{Pr} = 1.806 \times 10^{-4}$ m$^2$/s.

**ANALYSIS:** Consider a segment of the outer tube at the
Prescribed axial location and perform an energy balance,




$$\dot{E}'_{in} - \dot{E}'_{out} = 0$$

$$q'_{rad,i} + q'_{conv,i} - q'_{rad,o} - q'_{conv,o} = 0 \qquad (1)$$

The heat rates by *radiative* transfer are:

*Inside*: For long concentric cylinders, Eq. 13.25,

$$q'_{rad,i} = \frac{\sigma\pi D_i\left(T_{s,i}^4 - T_{s,o}^4\right)}{1/\varepsilon_1 + (1-\varepsilon_2)/\varepsilon_2\,(D_i/D_o)}$$

$$q'_{rad,i} = \frac{5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\,\pi\,(0.10\text{ m})\left(1200^4 - T_{s,o}^4\right)\text{K}^4}{1/0.6 + (1-0.6)/0.6\,(0.10/0.20)}$$

$$q'_{rad,i} = 8.906\times10^{-9}\left(1200^4 - T_{s,o}^4\right). \qquad (2)$$

*Outside*: For the outer tube surface to large surroundings,

$$q'_{rad,o} = \varepsilon\pi D_o\sigma\left(T_{s,o}^4 - T_{sur}^4\right) = 0.6\pi\,(0.20\text{ m})\,5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(T_{s,o}^4 - 950^4\right)\text{K}^4$$

$$q'_{rad,o} = 2.138\times10^{-8}\left(T_{s,o}^4 - 950^4\right). \qquad (3)$$

The heat rates by *convection processes* are:

Continued …..

## PROBLEM 13.123 (Cont.)

*Inside*: The rate equation is

$$q'_{conv,i} = h_i \pi D_o \left(T_{m,g} - T_{s,o}\right). \qquad (4)$$

Find the Reynolds number with $A_c = \pi\left(D_o^2 - D_i^2\right)/4$ and $D_h = 4A_c/P$,

$$Re_D = u_m D_h/\nu \quad u_m = \dot{m}/\rho A_c = 0.13\ \text{kg/s}\Big/\left[0.32\ \text{kg/m}^3 \times \pi/4\left(0.2^2 - 0.1^2\right)\text{m}^2\right] = 17.2\ \text{m/s}$$

$$D_h = \frac{4(\pi/4)\left(D_o^4 - D_i^2\right)}{\pi\left(D_o + D_i\right)} = \frac{\pi\left(0.2^2 - 0.1^2\right)\text{m}^2}{(0.2+0.1)\text{m}} = 0.100\ \text{m} \qquad Re_D = \frac{17.2\ \text{m/s} \times 0.100\,\text{m}}{130\times10^{-6}\text{m}^2/\text{s}} = 13,231.$$

The flow is turbulent and assumed to be fully developed; from the Dittus-Boelter correlation,

$$Nu_D = hD_h/k = 0.023\,Re_D^{0.8}\,Pr^{0.3}$$

$$h_i = \frac{0.070\ \text{W/m}\cdot\text{K}}{0.100\ \text{m}} \times 0.023(13,231)^{0.8}(0.720)^{0.3} = 28.9\ \text{W/m}^2\cdot\text{K}$$

Substituting into Eq. (4),

$$q'_{conv,i} = 28.9\ \text{W/m}^2\cdot\text{K} \times \pi(0.20\ \text{m})\left(1050 - T_{s,o}\right)\text{K} = 18.16\left(1050 - T_{s,o}\right). \qquad (5)$$

*Outside*: The rate equation is

$$q'_{conv,o} = h_o \pi D_o \left(T_{s,o} - T_\infty\right).$$

Evaluate the Rayleigh number *assuming* $T_{s,o}$ = 1025 K so that $T_f$ = 987 K,

$$Ra_D = \frac{g\beta\Delta T D_o^3}{\nu\alpha} = \frac{9.8\ \text{m}^2/\text{s}^2\,(1/987\ \text{K})(1025-950)\text{K}\,(0.20\ \text{m})^3}{130\times10^{-6}\text{m}^2/\text{s}\times1.806\times10^{-4}\text{m}^2/\text{s}} = 2.537\times10^5.$$

For a horizontal tube, using Eq. 9.33 and Table 9.1,

$$Nu_D = h_o D_o/k = C Ra_D^n = 0.48\left(2.537\times10^5\right)^{1/4} = 10.77$$

$$h_o = (0.070\ \text{W/m}\cdot\text{K})/0.20\ \text{m} \times 10.77 = 3.77\ \text{W/m}^2\cdot\text{K}.$$

Substituting into Eq. (6)

$$q'_{conv,o} = 3.77\ \text{W/m}^2\cdot\text{K} \times \pi(0.20\ \text{m})\left(T_{s,o} - 950\right)\text{K} = 2.369\left(T_{s,o} - 950\right). \qquad (7)$$

Returning to the energy balance relation on the outer tube, Eq. (1), substitute for the individual rates from Eqs. (2, 5, 3, 7),

$$8.906\times10^{-9}\left(1200^4 - T_{s,o}^4\right) + 18.16\left(1050 - T_{s,o}\right) - 2.138\times10^{-8}\left(T_{s,o}^4 - 950^4\right) - 2.369\left(T_{s,o} - 950\right) = 0 \qquad (8)$$

By trial-and-error, find $T_{s,o}$ = 1040 K. <

**COMMENTS:** (1) Recall that in estimating $h_o$ we assumed $T_{s,o}$ = 1025 K, such that ΔT = 75 K (vs. 92 K using $T_{s,o}$ = 1042 K) for use in evaluating the Rayleigh number. For an improved estimate of $T_{s,o}$, it would be advisable to recalculate $h_o$.

(2) Note from Eq. (8) that the radiation processes dominate the heat transfer rate:

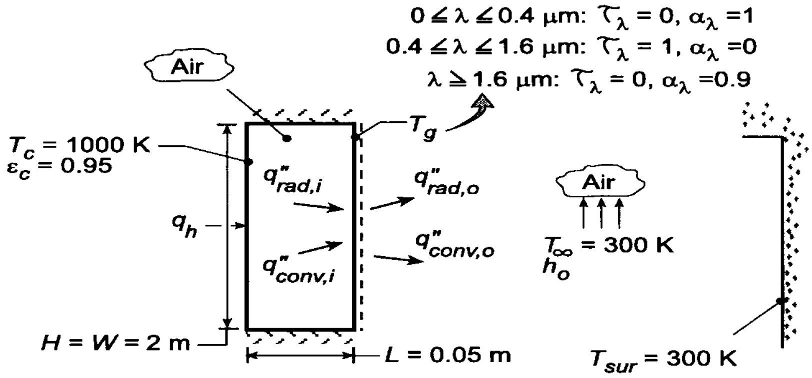
| | $q'_{rad}$ (W/m) | $q'_{conv}$ (W/m) |
|---|---|---|
| *Inside* | 7948 | 136 |
| *Outside* | 7839 | 219 |

## PROBLEM 13.124

**KNOWN:** Temperature and emissivity of ceramic plate which is separated from a glass plate of equivalent height and width by an air space. Temperature of air and surroundings on opposite side of glass. Spectral radiative properties of glass.

**FIND:** (a) Transmissivity of glass, (b) Glass temperature $T_g$ and total heat rate $q_h$, (c) Effect of external forced convection on $T_g$ and $q_h$.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Spectral distribution of emission from ceramic approximates that of a blackbody, (2) Glass surface is diffuse, (3) Atmospheric air in cavity and ambient, (4) Cavity may be approximated as a two-surface enclosure with infinite parallel plates, (5) Glass is isothermal.

**PROPERTIES:** *Table A-4*, air (p = 1 atm): Evaluated at $\overline{T} = (T_c + T_g)/2$ and $T_f = (T_g + T_\infty)/2$ using *IHT Properties* Toolpad.

**ANALYSIS:** (a) The total transmissivity of the glass is

$$\tau = \frac{\int_0^\infty \tau_\lambda E_{\lambda b} d\lambda}{E_b} = \int_{\lambda_1=0.4\mu m}^{\lambda_2=1.6\mu m} \left(E_{\lambda,b}/E_b\right) d\lambda = F_{(0\to\lambda_2)} - F_{(0\to\lambda_1)}$$

With $\lambda_2 T = 1600$ μm·K and $\lambda_1 T = 400$ μm·K, respectively, Table 12.1 yields $F_{(0\to\lambda_2)} = 0.0197$ and $F_{(0\to\lambda_1)} = 0.0$. Hence,

$$\tau = 0.0197 \qquad <$$

With so little transmission of radiation from the ceramic, the glass plate may be assumed to be opaque to a good approximation. Since more than 98% of the incident radiation is at wavelengths exceeding 1.6 μm, for which $\alpha_\lambda = 0.9$, $\alpha_g \approx 0.9$. Also, since $T_g < T_c$, nearly 100% of emission from the glass is at $\lambda > 1.6$ μm, for which $\varepsilon_\lambda = \alpha_\lambda = 0.9$, $\varepsilon_g = 0.9$ and the glass may be approximated as a gray surface.

(b) The glass temperature may be obtained from an energy balance of the form $q''_{conv,i} + q''_{rad,i} = q''_{conv,o} + q''_{rad,o}$. Using Eqs. 13.24 and 13.27 to evaluate $q''_{rad,i}$ and $q''_{rad,o}$, respectively, it follows that

$$\overline{h}_i\left(T_c - T_g\right) + \frac{\sigma\left(T_c^4 - T_g^4\right)}{\frac{1}{\varepsilon_c} + \frac{1}{\varepsilon_g} - 1} = \overline{h}_o\left(T_g - T_\infty\right) + \varepsilon_g\sigma\left(T_g^4 - T_{sur}^4\right)$$

Continued …..

## PROBLEM 13.124 (Cont.)

where, assuming $10^4 \le Ra_L \le 10^7$, $\bar{h}_i$ and $\bar{h}_o$ are given by Eqs. 9.52 and 9.26, respectively,

$$\bar{h}_i = \frac{k_i}{L} Ra_L^{1/4} Pr_i^{0.012} (H/L)^{-0.3}$$

$$\bar{h}_o = \frac{k_o}{H} \left\{ 0.825 + \frac{0.387 Ra_H^{1/6}}{\left[1 + (0.492/Pr_o)^{9/16}\right]^{8/27}} \right\}^2$$

with $Ra_L = g\beta_i (T_c - T_g)L^3/\nu_i\alpha_i$ and $Ra_H = g\beta_o (T_g - T_\infty) H^3/\nu_o\alpha_o$. Entering the energy balance into the *IHT* workspace and using the *Correlations, Properties* and *Radiation* Toolpads to evaluate the convection and radiation terms, the following result is obtained.

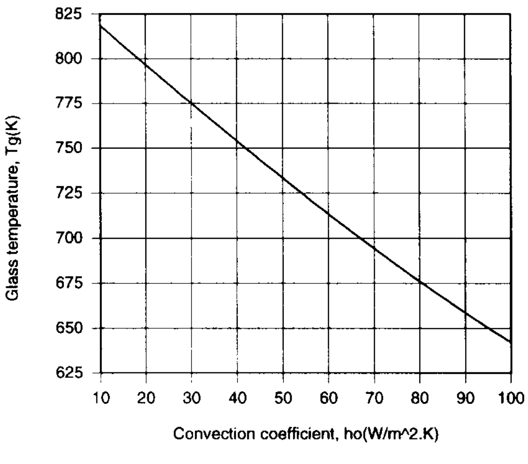
$$T_g = 825 \text{ K}$$ <

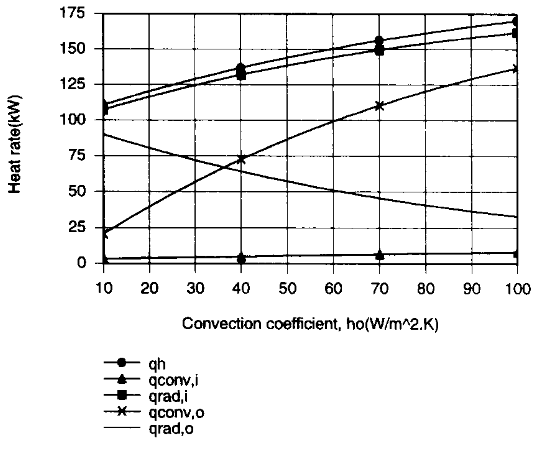
The corresponding value of $q_h$ is

$$q_h = 108 \text{ kW}$$ <

where $q_{conv,i}$ = 3216 W, $q_{rad,i}$ = 104.7 kW, $q_{convo,o}$ = 15,190 W and $q_{rad,o}$ = 92.8 kW. The convection coefficients are $\bar{h}_i = 4.6 \text{ W/m}^2 \cdot \text{K}$ and $\bar{h}_o = 7.2 \text{ W/m}^2 \cdot \text{K}$.

(c) For the prescribed range of $\bar{h}_o$, *IHT* was used to generate the following results.



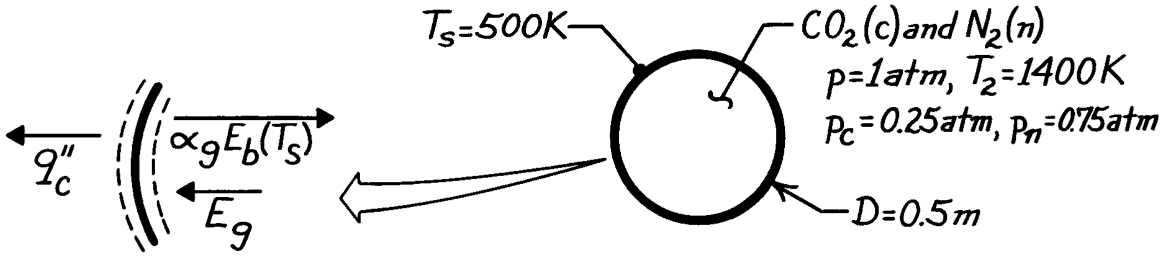

With increasing $\bar{h}_o$, the glass is cooled more effectively and $T_g$ must decrease. With decreasing $T_g$, $q_{conv,i}$, $q_{rad,i}$ and hence $q_h$ must increase. Note that radiation makes the dominant contribution to heat transfer across the airspace. Although $q_{rad,o}$ decreases with decreasing $T_g$, the increase in $q_{conv,o}$ exceeds the reduction in $q_{rad,o}$.

## PROBLEM 13.125

**KNOWN:** Conditions associated with a spherical furnace cavity.

**FIND:** Cooling rate needed to maintain furnace wall at a prescribed temperature.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state, (2) Blackbody behavior for furnace wall, (3) $N_2$ is non-radiating.

**ANALYSIS:** From an energy balance on a unit surface area of the furnace wall, the cooling rate per unit area must equal the absorbed irradiation from the gas ($E_g$) minus the portion of the wall's emissive power absorbed by the gas

$$q_c'' = E_g - \alpha_g E_b(T_s)$$

$$q_c'' = \varepsilon_g \sigma T_g^4 - \alpha_g \sigma T_s^4.$$

Hence, for the entire furnace wall,

$$q_c = A_s \sigma \left(\varepsilon_g T_g^4 - \alpha_g T_s^4\right).$$

The gas emissivity, $\varepsilon_g$, follows from Table 13.4 with

$$L_e = 0.65D = 0.65 \times 0.5\text{ m} = 0.325\text{ m} = 1.066\text{ ft}.$$

$$p_c L_e = 0.25\text{ atm} \times 1.066\text{ ft} = 0.267\text{ ft}-\text{atm}$$

and from Fig. 13.18, find $\varepsilon_g = \varepsilon_c = 0.09$. From Eq. 13.42,

$$\alpha_g = \alpha_c = C_c \left(\frac{T_g}{T_s}\right)^{0.45} \times \varepsilon_c \left(T_s, p_c L_e \left[T_s / T_g\right]\right).$$

With $C_c = 1$ from Fig. 13.19,

$$\alpha_g = 1(1400/50)^{0.45} \times \varepsilon_c (500\text{K, } 0.095\text{ ft}-\text{atm})$$

where, from Fig. 13.18,

$$\varepsilon_c (500\text{K, } 0.095\text{ ft}-\text{atm}) = 0.067.$$

Hence

$$\alpha_g = 1(1400/500)^{0.45} \times 0.067 = 0.106$$

and the heat rate is

$$q_c = \pi (0.5\text{ m})^2\, 5.67 \times 10^{-8}\text{W/m}^2 \cdot \text{K}^4 \left[0.09(1400\text{ K})^4 - 0.106(500\text{ K})^4\right]$$

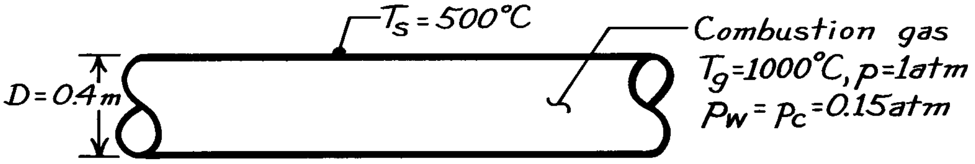
$$q_c = 15.1\text{ kW}.$$ <

# PROBLEM 13.126

**KNOWN:** Diameter and gas pressure, temperature and composition associated with a gas turbine combustion chamber.

**FIND:** Net radiative heat flux between the gas and the chamber surface.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Blackbody behavior for chamber surface, (3) Remaining species are non-radiating, (4) Chamber may be approximated as an infinitely long tube.

**ANALYSIS:** From Eq. 13.40 the net rate of radiation transfer to the surface is

$$q_{net} = A_s\sigma\left(\varepsilon_g T_g^4 - \alpha_g T_s^4\right) \qquad \text{or} \qquad q'_{net} = \pi D\sigma\left(\varepsilon_g T_g^4 - \alpha_g T_s^4\right)$$

with $A_s = \pi DL$. From Table 13.4, $L_e = 0.95D = 0.95 \times 0.4$ m = 0.380 m = 1.25 ft. Hence, $p_w L_e = p_c L_e = 0.152$ atm $\times$ 1.25 ft = 0.187 atm-ft.

$$\text{Fig.13.16 } \left(T_g = 1273 \text{ K}\right), \rightarrow \varepsilon_w \approx 0.069.$$

$$\text{Fig.13.18 } \left(T_g = 1273 \text{ K}\right), \rightarrow \varepsilon_c \approx 0.085.$$

$$\text{Fig.13.20 } \left(p_w / \left(p_c + p_w\right) = 0.5,\ L_c\left(p_w + p_c\right) = 0.375 \text{ ft} - \text{atm},\ T_g \geq 930°\text{C}\right), \rightarrow \Delta\varepsilon \geq 0.01.$$

From Eq. 13.38,

$$\varepsilon_g = \varepsilon_w + \varepsilon_c - \Delta\varepsilon = 0.069 + 0.085 - 0.01 \approx 0.144.$$

From Eq. 13.41 for the water vapor,

$$\alpha_w = C_w\left(T_g / T_s\right)^{0.45} \times \varepsilon_w\left(T_s, p_w L_c\left[T_s / T_g\right]\right)$$

where from Fig. 13.16 (773 K, 0.114 ft-atm), $\rightarrow \varepsilon_w \approx 0.083$,

$$\alpha_w = 1\left(1273/773\right)^{0.45} \times 0.083 = 0.104.$$

From Eq. 13.42, using Fig. 13.18 (773 K, 0.114 ft-atm), $\rightarrow \varepsilon_c \approx 0.08$,

$$\alpha_c = 1\left(1273/773\right)^{0.45} \times 0.08 = 0.100.$$

From Fig. 13.20, the correction factor for water vapor at carbon dioxide mixture,

$$\left(p_w / \left(p_c + p_w\right) = 0.1, L_e\left(p_w + p_c\right) = 0.375,\ T_g \approx 540°\text{C}\right), \rightarrow \Delta\alpha \approx 0.004$$

and using Eq. 13.43

$$\alpha_g = \alpha_w + \alpha_c - \Delta\alpha = 0.104 + 0.100 - 0.004 \approx 0.200.$$

Hence, the heat rate is

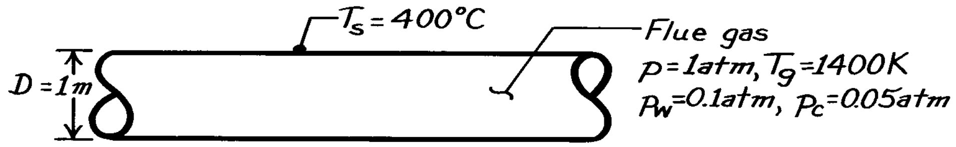
$$q'_{net} = \pi\left(0.4 \text{ m}\right)5.67 \times 10^{-8} \text{W/m}^2 \cdot \text{K}^4\left[0.144\left(1273\right)^4 - 0.200\left(773\right)^4\right] = 21.9 \text{ kW/m.} \quad <$$

**PROBLEM 13.127**

**KNOWN:** Pressure, temperature and composition of flue gas in a long duct of prescribed diameter.

**FIND:** Net radiative flux to the duct surface.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Duct surface behaves as a blackbody, (3) Other gases are non-radiating, (4) Flue may be approximated as an infinitely long tube.

**ANALYSIS:** With $A_s = \pi DL$, it follows from Eq. 13.40 that

$$q'_{net} = \pi D\sigma\left(\varepsilon_g T_g^4 - \alpha_g T_s^4\right)$$

From Table 13.4, $L_e = 0.95D = 0.95 \times 1\text{ m} = 0.95\text{ m} = 3.12\text{ ft}$. Hence

$$p_w L_e = 0.12\text{ atm} \times 3.12\text{ m} = 0.312\text{ atm} - \text{ft}$$

$$p_c L_e = 0.05\text{ atm} \times 3.12\text{ m} = 0.156\text{ atm} - \text{ft}.$$

With $T_g = 1400$ K, Fig. 13.16 $\rightarrow \varepsilon_w = 0.083$; Fig. 13.18 $\rightarrow \varepsilon_c = 0.072$. With $p_w/(p_c + p_w) = 0.67$, $L_e(p_w + p_c) = 0.468$ atm-ft, $T_g \geq 930°C$, Fig. 13.20 $\rightarrow \Delta\varepsilon = 0.01$. Hence from Eq. 13.38,

$$\varepsilon_g = \varepsilon_w + \varepsilon_c - \Delta\varepsilon = 0.083 + 0.072 - 0.01 = 0.145.$$

From Eq. 13.41,

$$\alpha_w = C_w\left(T_g / T_s\right)^{0.45} \times \varepsilon_w\left(T_s, p_w L_e\left[T_s / T_g\right]\right)$$

$$\alpha_w = 1(1400/400)^{0.45} \times \varepsilon_w \text{ Fig. 13.16} \rightarrow \varepsilon_w\left(400\text{ K, } 0.0891\text{ atm} - \text{ft}\right) = 0.1$$

$$\alpha_w = 0.176.$$

From Eq. 13.42,

$$\alpha_c = C_c\left(T_g / T_s\right)^{0.45} \times \varepsilon_c\left(T_s,\ p_c L_e T_s / T_g\right)$$

$$\alpha_c = 1(1400/400)^{0.45} \times \varepsilon_c \text{ Fig. 13.18} \rightarrow \varepsilon_c\left(400\text{ K, } 0.0891\text{ atm} - \text{ft}\right) = 0.053$$

$$\alpha_c = 0.093.$$

With $p_w/(p_c + p_w) = 0.67$, $L_e(p_w + p_c) = 0.468$ atm-ft, $T_g \approx 125°C$, Fig. 13.20 gives

$$\Delta\alpha \approx 0.003.$$

Hence from Eq. 13.43,

$$\alpha_g = \alpha_w + \alpha_c - \Delta\alpha = 0.176 + 0.093 - 0.003 = 0.266.$$

The heat rate per unit length is

$$q'_{net} = \pi\,(1\text{ m})\,5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4\left[0.145(1400\text{ K})^4 - 0.266(400\text{ K})^4\right]$$

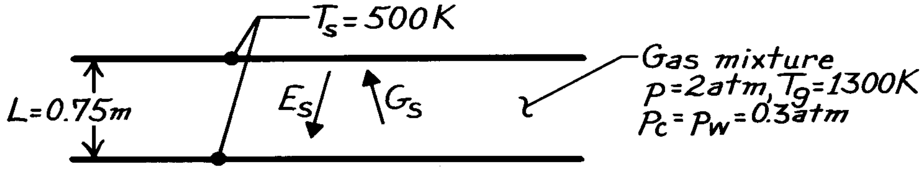
$$q'_{net} = 98\text{ kW/m}.$$ <

## PROBLEM 13.128

**KNOWN:** Gas mixture of prescribed temperature, pressure and composition between large parallel plates of prescribed separation.

**FIND:** Net radiation flux to the plates.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Furnace wall behaves as a blackbody, (3) $O_2$ and $N_2$ are non-radiating, (4) Negligible end effects.

**ANALYSIS:** The net radiative flux to a plate is

$$q''_{s,1} = G_s - E_s = \varepsilon_g \sigma T_g^4 - \left(1 - \tau_g\right) \sigma T_s^4$$

where $G_s = \varepsilon_g \sigma T_g^4 + \tau_g E_s$, $E_s = \sigma T_s^4$ and $\tau_g = 1 - \alpha_g\left(T_s\right)$. From Table 13.4, $L_e = 1.8L = 1.8 \times 0.75$ m = 1.35 m = 4.43 ft. Hence $p_w L_e = p_c L_e = 1.33$ atm-ft. From Figs. 3.16 and 3.18 find $\varepsilon_w \approx 0.22$ and $\varepsilon_c \approx 0.16$ for p = 1 atm. With $(p_w + p)/2 = 1.15$ atm, Fig. 13.17 yields $C_w \approx 1.40$ and from Fig. 13.19, $C_c \approx 1.08$. Hence, the gas emissivities are

$$\varepsilon_w = C_w \varepsilon_w \left(1 \text{ atm}\right) \approx 1.40 \times 0.22 = 0.31 \qquad \varepsilon_c = C_c \varepsilon_c \left(1 \text{ atm}\right) \approx 1.08 \times 0.16 = 0.17.$$

From Fig. 13.20 with $p_w/(p_c + p_w) = 0.5$, $L_e(p_c + p_w) = 2.66$ atm-ft and $T_g > 930°C$, $\Delta\varepsilon \approx 0.047$. Hence, from Eq. 13.38,

$$\varepsilon_g = \varepsilon_w + \varepsilon_c - \Delta\varepsilon \approx 0.31 + 0.17 - 0.047 \approx 0.43.$$

To evaluate $\alpha_g$ at $T_s$, use Eq. 13.43 with

$$\alpha_w = C_w \left(T_g / T_s\right)^{0.45} \varepsilon_w \left(T_s, p_w L_2 T_s / T_g\right) = C_w \left(1300/500\right)^{0.45} \varepsilon_w \left(500,\ 0.51\right)$$

$$\alpha_w \approx 1.40 \left(1300/500\right)^{0.45} 0.22 = 0.47$$

$$\alpha_c = C_c \left(1300/500\right)^{0.45} \varepsilon_c \left(500,\ 0.51\right) \approx 1.08 \left(1300/500\right)^{0.45} 0.11 = 0.18.$$

From Fig. 13.20, with $T_g \approx 125°C$ and $L_e(p_w + p_c) = 2.66$ atm-ft, $\Delta\alpha = \Delta\varepsilon \approx 0.024$. Hence

$$\alpha_g = \alpha_w + \alpha_c - \Delta\alpha \approx 0.47 + 0.18 - 0.024 \approx 0.63 \quad \text{and} \quad \tau_g = 1 - \alpha_g \approx 0.37.$$

Hence, the heat flux from Eq. (1) is

$$q''_{s,1} = 0.43 \times 5.67 \times 10^{-8} \text{W/m}^2 \cdot \text{K}^4 \left(1300 \text{ K}\right)^4 - 0.63 \times 5.67 \times 10^{-8} \text{W/m}^2 \cdot \text{K}^4 \left(500 \text{ K}\right)^4$$

$$q''_{s,1} \approx 67.4 \text{ kW/m}^2.$$

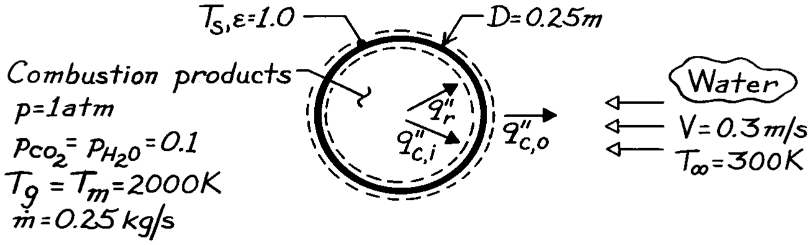
The net radiative flux to both plates is then $q''_{s,2} \approx 134.8 \text{ kW/m}^2$. $<$

# PROBLEM 13.129

**KNOWN:** Flow rate, temperature, pressure and composition of exhaust gas in pipe of prescribed diameter. Velocity and temperature of external coolant.

**FIND:** Pipe wall temperature and heat flux.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) L/D >> 1 (infinitely long pipe), (2) Negligible axial gradient for gas temperature, (3) Gas is in fully developed flow, (4) Gas thermophysical properties are those of air, (5) Negligible pipe wall thermal resistance, (6) Negligible pipe wall emission.

**PROPERTIES:** *Table A-4*: Air ($T_m$ = 2000 K, 1 atm): $\rho$ = 0.174 kg/m$^3$, $\mu = 689 \times 10^{-7}$ kg/m·s, k = 0.137 W/m·K, Pr = 0.672; *Table A-6*: Water ($T_\infty$ = 300 K): $\rho$ = 997 kg/m$^3$, $\mu = 855 \times 10^{-6}$ kg/s·m, k = 0.613 W/m·K, Pr = 5.83.

**ANALYSIS:** Performing an energy balance for a control surface about the pipe wall,

$$q''_r + q''_{c,i} = q''_{c,o}$$

$$\varepsilon_g \sigma T_g^4 + h_i\left(T_m - T_s\right) = \overline{h}_o\left(T_s - T_\infty\right)$$

The gas emissivity is

$$\varepsilon_g = \varepsilon_w + \varepsilon_c = \Delta\varepsilon$$

where

$$L_e = 0.95D = 0.238\ \text{m} = 0.799\ \text{ft}$$

$$p_c L_e = p_w L_e = 0.1\ \text{atm} \times 0.238\ \text{m} = 0.0238\ \text{atm} - \text{m} = 0.0779\ \text{atm} - \text{ft}$$

and from Fig. 13.16 $\rightarrow \varepsilon_w \approx 0.017$; Fig. 13.18 $\rightarrow \varepsilon_c \approx 0.031$; Fig. 13.20 $\rightarrow \Delta\varepsilon \approx 0.001$. Hence $\varepsilon_g$ = 0.047. Estimating the *internal flow convection coefficient*, find

$$Re_D = \frac{4\ \dot{m}}{\pi D \mu} = \frac{4 \times 0.25\ \text{kg/s}}{\pi\left(0.25\ \text{m}\right) 689 \times 10^{-7}\ \text{kg/m}\cdot\text{s}} = 18,480$$

and for turbulent flow,

$$Nu_D = 0.023 Re_D^{4/5} Pr^{0.3} = 0.023\left(18,480\right)^{4/5}\left(0.672\right)^{0.3} = 52.9$$

$$h_i = Nu_D \frac{k}{D} = 52.9 \frac{0.137\ \text{W/m}\cdot\text{K}}{0.25\ \text{m}} = 29.0\ \text{W/m}^2 \cdot \text{K}.$$

Continued …..

Estimating the *external convection coefficient*, find

$$\mathrm{Re}_D = \frac{\rho VD}{\mu} = \frac{997\ \mathrm{kg/m^3} \times 0.3\ \mathrm{m/s} \times 0.25\ \mathrm{m}}{855 \times 10^{-6}\ \mathrm{kg/s \cdot m}} = 87{,}456.$$

Hence

$$\overline{\mathrm{Nu}}_D = 0.26\ \mathrm{Re}_D^{0.6}\ \mathrm{Pr}^{0.37}\left(\mathrm{Pr}/\mathrm{Pr}_s\right)^{1/4}.$$

Assuming $\mathrm{Pr}/\mathrm{Pr}_s \approx 1$,

$$\overline{\mathrm{Nu}}_D = 0.26(87{,}456)^{0.6}(5.83)^{0.37} = 461$$

$$\bar{h}_o = \overline{\mathrm{Nu}}_D\,(k/D) = 461(0.613\ \mathrm{W/m \cdot K}/0.25\ \mathrm{m}) = 1129\,\mathrm{W/m^2 \cdot K}.$$

Substituting numerical values in the energy balance, find

$$0.047 \times 5.67 \times 10^{-8}\,\mathrm{W/m^2 \cdot K^4}\,(2000\ \mathrm{K})^4 + 29\ \mathrm{W/m^2 \cdot K}\,(2000 - T_s)\,\mathrm{K}$$

$$= 1129\ \mathrm{W/m^2 \cdot K}\,(T_s - 300)\,\mathrm{K}$$

$$T_s = 380\ \mathrm{K}. \qquad <$$

The heat flux due to convection is

$$q''_{c,i} = h_i\,(T_m - T_s) = 29\ \mathrm{W/m^2 \cdot K}\,(2000 - 379.4)\,\mathrm{K} = 46{,}997\ \mathrm{W/m^2}$$

and the total heat flux is

$$q''_s = q''_r + q''_{c,i} = 42{,}638 + 46{,}997 = 89{,}640\ \mathrm{W/m^2}. \qquad <$$

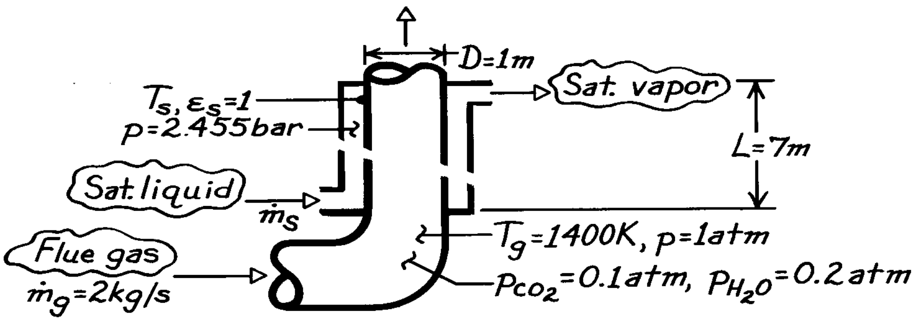
**COMMENTS:** Contributions of gas radiation and convection to the wall heat flux are approximately the same. Small value of $T_s$ justifies neglecting emission from the pipe wall to the gas. $\mathrm{Pr}_s = 1.62$ for $T_s = 380 \rightarrow (\mathrm{Pr}/\mathrm{Pr}_s)1/4 = 1.38$. Hence the value of $\bar{h}_o$ should be corrected. The value would $\uparrow$, and $T_s$ would $\downarrow$.

## PROBLEM 13.130

**KNOWN:** Flowrate, composition and temperature of flue gas passing through inner tube of an annular waste heat boiler. Boiler dimensions. Steam pressure.

**FIND:** Rate at which saturated liquid can be converted to saturated vapor, $\dot{m}_s$.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Inner wall is thin and steam side convection coefficient is very large; hence $T_s = T_{sat}(2.455\text{ bar})$, (2) For calculation of gas radiation, inner tube is assumed infinitely long and gas is approximated as isothermal at $T_g$.

**PROPERTIES:** Flue gas (given): $\mu = 530 \times 10^{-7}$ kg/s·m, k = 0.091 W/m·K, Pr = 0.70; *Table A-6*, Saturated water (2.455 bar): $T_s$ = 400 K, $h_{fg}$ = 2183 kJ/kg.

**ANALYSIS:** The steam generation rate is

$$\dot{m}_s = q / h_{fg} = \left(q_{conv} + q_{rad}\right) / h_{fg}$$

where

$$q_{rad} = A_s \sigma \left(\varepsilon_g T_g^4 - \alpha_g T_s^4\right)$$

with

$$\varepsilon_g = \varepsilon_w + \varepsilon_c - \Delta\varepsilon \qquad\qquad \alpha_g = \alpha_w + \alpha_c - \Delta\alpha.$$

From Table 13.4, find $L_e$ = 0.95D = 0.95 m = 3.117 ft. Hence

$$p_w L_e = 0.2\text{ atm} \times 3.117\text{ ft} = 0.623\text{ ft}-\text{atm}$$

$$p_c L_e = 0.1\text{ atm} \times 3.117\text{ ft} = 0.312\text{ ft}-\text{atm}.$$

From Fig. 13.16, find $\varepsilon_w \approx 0.13$ and Fig. 13.18 find $\varepsilon_c \approx 0.095$. With $p_w/(p_c + p_w) = 0.67$ and $L_e(p_w + p_c)$ = 0.935 ft-atm, from Fig. 13.20 find $\Delta\varepsilon \approx 0.036 \approx \Delta\alpha$. Hence $\varepsilon_g \approx 0.13 + 0.095 - 0.036 = 0.189$. Also, with $p_w L_e(T_s/T_g) = 0.2\text{ atm} \times 0.95\text{ m}(400/1400) = 0.178$ ft-atm and $T_s$ = 400 K, Fig. 13.16 yields $\varepsilon_w \approx 0.14$. With $p_c L_e(T_s/T_g) = 0.1\text{ atm} \times 0.95\text{ m}(400/1400) = 0.089$ ft-atm and $T_s$ = 400 K, Fig. 13.18 yields $\varepsilon_c \approx 0.067$. Hence

$$\alpha_w = \left(T_g / T_s\right)^{0.45} \varepsilon_w \left(T_s,\ p_w L_e T_s / T_g\right)$$

$$\alpha_w = \left(1400 / 400\right)^{0.45} 0.14 = 0.246$$

and

$$\alpha_c = \left(T_g / T_s\right)^{0.65} \varepsilon_c \left(T_s, p_c L_e T_s / T_g\right)$$

Continued …..

$$\alpha_c = (1400/400)^{0.65}\, 0.067 = 0.151$$

$$\alpha_g = 0.246 + 0.151 - 0.036 = 0.361.$$

Hence

$$q_{rad} = \pi(1\text{ m})7\text{ m} \times 5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left[0.189(1400\text{ K})^4 - 0.361(400\text{ K})^4\right]$$

$$q_{rad} = (905.3 - 11.5)\,\text{kW} = 893.8\text{ kW}.$$

For convection,

$$q_{conv} = \bar{h}\pi DL\left(T_g - T_s\right)$$

with

$$Re_D = \frac{4\dot{m}}{\pi D\mu} = \frac{4\times2\text{ kg/s}}{\pi\times1\text{ m}\times530\times10^{-7}\,\text{kg/s}\cdot\text{m}} = 48,047$$

and assuming fully developed turbulent flow throughout the tube, the Dittus-Boelter correlation gives

$$\overline{Nu}_D = 0.023\,Re_D^{4/5}\,Pr^{0.3} = 0.023(48,047)^{4/5}(0.70)^{0.3} = 115$$

$$\bar{h} = (k/D)\overline{Nu}_D = (0.091\text{ W/m}\cdot\text{K}/1\text{ m})115 = 10.5\text{ W/m}^2\cdot\text{K}.$$

Hence

$$q_{conv} = 10.5\text{ W/m}^2\cdot\text{K}\,\pi(1\text{ m})7\text{ m}(1400-400)\,\text{K} = 230.1\text{ kW}$$

and the vapor production rate is

$$\dot{m}_s = \frac{q}{h_{fg}} = \frac{(893.8+230.1)\,\text{kW}}{2183\text{ kJ/kg}} = \frac{1123.9\text{ kW}}{2183\text{ kJ/kg}}$$

$$\dot{m}_s = 0.515\text{ kg/s}.$$ <

**COMMENTS:** (1) Heat transfer is dominated by radiation, which is typical of heat recovery devices having a large gas volume.

(2) A more detailed analysis would account for radiation exchange involving the ends (upstream and downstream) of the inner tube.

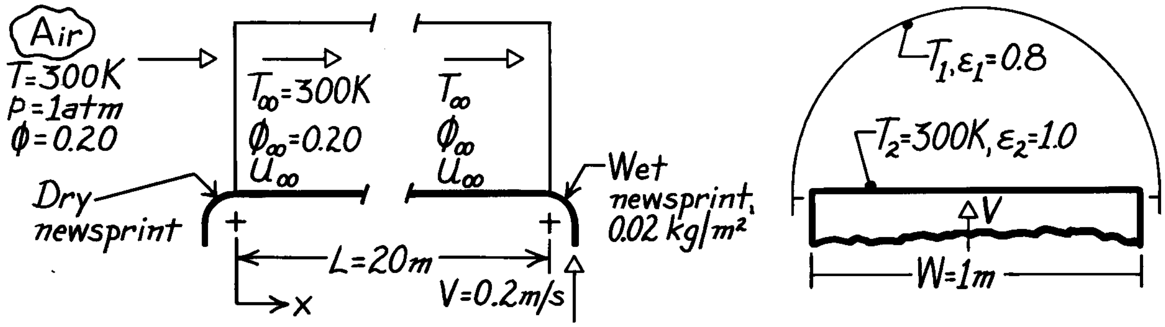
(3) Using a representative specific heat of $c_p$ = 1.2 kJ/kg·K, the temperature drop of the gas passing through the tube would be $\Delta T_g$ = 1123.9 kW/(2 kg/s × 1.2 kJ/kg·K) = 468 K.

## PROBLEM 13.131

**KNOWN:** Wet newsprint moving through a drying oven.

**FIND:** Required evaporation rate, air velocity and oven temperature.

**SCHEMATIC:**

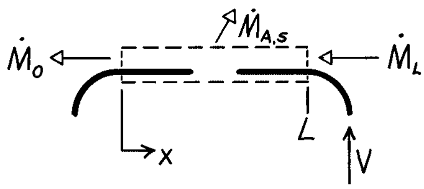



**ASSUMPTIONS:** (1) Steady-state conditions, (2) Negligible freestream turbulence, (3) Heat and mass transfer analogy applicable, (4) Oven and newsprint surfaces are diffuse gray, (5) Oven end effects negligible.

**PROPERTIES:** *Table A-6*, Water vapor (300 K, 1 atm): $\rho_{sat} = 1/v_g = 0.0256$ kg/m$^3$, $h_{fg} = 2438$ kJ/kg; *Table A-4*, Air (300 K, 1 atm): $\eta = 15.89 \times 10^{-6}$ m$^2$/s; *Table A-8*, Water vapor-air (300 K, 1 atm): $D_{AB} = 0.26 \times 10^{-4}$ m$^2$/s, $S_c = \eta/D_{AB} = 0.611$.

**ANALYSIS:** The evaporation rate required to completely dry the newsprint having a water content of $m''_A = 0.02\ \text{kg/m}^2$ as it enters the oven (x = L) follows from a species balance on the newsprint.

$$\dot{M}_{A,in} - \dot{M}_{A,out} = \dot{M}_{st}$$




$$\dot{M}_L - \dot{M}_0 - \dot{M}_{A,s} = 0.$$

The rate at which moisture enters in the newsprint is

$$\dot{M}_L = m''_A VW$$

hence,

$$\dot{M}_{A,s} = m''_A VW = 0.02\,\text{kg/m}^2 \times 0.2\,\text{m/s} \times 1\ \text{m} = 4\times10^{-3}\text{kg/s}.$$ <

The required velocity of the airstream through the oven, $u_\infty$, can be determined from a convection analysis. From the rate equation,

$$\dot{M}_{A,s} = \bar{h}_m WL\left(\rho_{A,s} - \rho_{A,\infty}\right) = \bar{h}_m WL\rho_{A,sat}\left(1-\phi_\infty\right)$$

$$\bar{h}_m = \dot{M}_{A,s} / WL\rho_{A,sat}\left(1-\phi_\infty\right)$$

$$\bar{h}_m = 4\times10^{-3}\text{kg/s}/1\ \text{m}\times20\ \text{m}\times0.0256\ \text{kg/m}^3\left(1-0.2\right) = 9.77\times10^{-3}\text{m/s}.$$

Now determine what flow velocity is required to produce such a coefficient. Assume flow over a flat plate with

$$\overline{Sh}_L = \bar{h}_m L/D_{AB} = 9.77\times10^{-3}\text{m/s}\times20\ \text{m}/0.26\times10^{-4}\text{m}^2/\text{s} = 7515$$

Continued …..

and

$$\mathrm{Re}_L = \left[\overline{\mathrm{Sh}}_L / 0.664\mathrm{Sc}^{1/3}\right]^2 = \left[7515/0.664(0.611)^{1/3}\right]^2 = 1.78\times10^8.$$

Since $\mathrm{Re}_L > \mathrm{Re}_{Lc} = 5 \times 10^5$, the flow must be turbulent. Using the correlation for mixed laminar and turbulent flow conditions, find

$$\mathrm{Re}_L^{4/5} = \left[\overline{\mathrm{Sh}}_L / \mathrm{Sc}^{1/3} + 871\right]/0.037$$

$$\mathrm{Re}_L^{4/5} = \left[7515/(0.611)^{1/3} + 871\right]/0.037$$

$$\mathrm{Re}_L = 5.95\times10^6$$

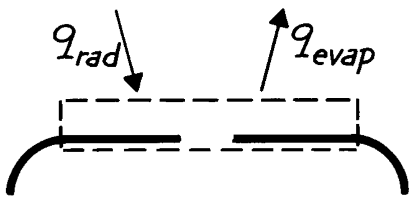
noting $\mathrm{Re}_L > \mathrm{Re}_{Lc}$. Recognize that $u_\infty^*$ is the velocity relative to the newsprint,

$$u_\infty^* = \mathrm{Re}_L \nu / L = 5.95\times10^6 \times 15.89\times10^{-6}\,\mathrm{m}^2/\mathrm{s}/20\,\mathrm{m} = 4.73\ \mathrm{m/s}.$$

The air velocity relative to the oven will be,

$$u_\infty = u_\infty^* - V = (4.73 - 0.2)\,\mathrm{m/s} = 4.53\ \mathrm{m/s}. \qquad <$$

The temperature required of the oven surface follows from an energy balance on the newsprint. Find




$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q_{rad} - q_{evap} = 0$$

where

$$q_{evap} = \dot{M}_{A,s} h_{fg} = 4.0\times10^{-3}\,\mathrm{kg/s} \times 2438\times10^3\,\mathrm{J/kg} = 9752\ \mathrm{W}$$

and the radiation exchange is that for a two surface enclosure, Eq. 13.23,

$$q_{rad} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{(1-\varepsilon_1)/\varepsilon_1 A_1 + 1/A_1F_{12} + (1-\varepsilon_2)/\varepsilon_2 A_2}.$$

Evaluate,

$$A_1 = \pi/2\ \mathrm{WL}, \qquad A_2 = \mathrm{WL}, \qquad F_{21} = 1, \quad \text{and} \quad A_1F_{12} = A_2F_{21} = \mathrm{WL}$$

hence, with $\varepsilon_1 = 0.8$,

$$q_{rad} = \sigma \mathrm{WL}\left(T_1^4 - T_2^4\right)/\left[(1/2\pi) + 1\right]$$

$$T_1^4 = T_2^4 + q_{rad}\left[(1/2\pi) + 1\right]/\sigma \mathrm{WL}$$

$$T_1^4 = (300\ \mathrm{K})^4 + 9752\ \mathrm{W}\left[(1/2\pi + 1)\right]/5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4} \times 1\ \mathrm{m} \times 20\ \mathrm{m}$$

$$T_1 = 367\ \mathrm{K}. \qquad <$$

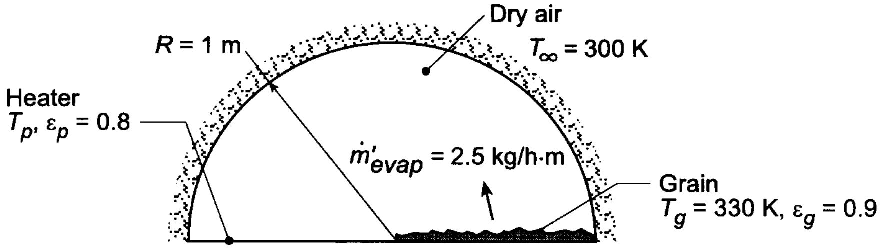
**COMMENTS:** Note that there is no convection heat transfer since $T_\infty = T_s = 300$ K.

## PROBLEM 13.132

**KNOWN:** Configuration of grain dryer. Emissivities of grain bed and heater surface. Temperature of grain.

**FIND:** (a)Temperature of heater required for specified drying rate, (b) Convection mass transfer coefficient required to sustain evaporation, (c) Validity of assuming negligible convection heat transfer.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Diffuse/gray surfaces, (2) Oven wall is a reradiating surface, (3) Negligible convection heat transfer, (4) Applicability of heat/mass transfer analogy, (5) Air is dry.

**PROPERTIES:** *Table A-6*, saturated water (T = 330 K): $v_g = 8.82\ m^3/kg$, $h_{fg} = 2.366 \times 10^6$ J/kg. *Table A-4*, air (assume T ≈ 300 K): ρ = 1.614 kg/m$^3$, $c_p$ = 1007 J/kg·K, α = 22.5 × 10$^{-6}$ m$^2$/s. *Table A-8*, $H_2O$(v) – air (T = 298 K): $D_{AB} = 0.26 \times 10^{-4}\ m^2/s$.

**ANALYSIS:** (a) Neglecting convection, the energy required for evaporation must be supplied by net radiation transfer from the heater plate to the grain bed. Hence,

$$q'_{rad} = \dot{m}'_{evap} h_{fg} = (2.5\ kg/h \cdot m)\left(2.366 \times 10^6 J/kg\right)/3600 s/h = 1643\ W/m$$

where $q'_{rad}$ is given by Eq. 13.30. With $A'_p = A'_g \equiv A'$,

$$q'_{rad} = \frac{A'\left(E_{bp} - E_{bg}\right)}{\dfrac{1-\varepsilon_p}{\varepsilon_p} + \dfrac{1}{F_{pg} + \left[\left(1/F_{pR}\right) + \left(1/F_{gR}\right)\right]^{-1}}} + \frac{1-\varepsilon_g}{\varepsilon_g}$$

where $A'$ = R = 1 m, $F_{pg}$ = 0 and $F_{pR} = F_{gR}$ = 1. Hence,

$$q'_{rad} = \frac{\sigma\left(T_p^4 - 320^4\right)}{0.25 + 2 + 0.111} = 2.40 \times 10^{-8}\left(T_p^4 - 320^4\right) = 1643\ W/m$$

$$2.40 \times 10^{-8} T_p^4 - 2518 = 1643$$

$$T_p = 530\ K$$ <

(b) The evaporation rate is given by Eq. 6.12, and with $A'_s = 1$ m, $n'_A = \dot{m}'_{evap}$, and $\rho_{A,\infty}$ = 0,

Continued …..

$$h_m = \frac{n'_A}{A'_s \rho_{A,s}} = \frac{n'_A v_g}{A'_s} = \frac{2.5\,\text{kg/h}\cdot\text{m}}{1\,\text{m}} \times \frac{1}{3600\text{ s}} \times 8.82\,\frac{\text{m}^3}{\text{kg}} = 6.13\times10^{-3}\,\text{m/s}$$ <

(c) From the heat and mass transfer analogy, Eq. 6.92,

$$h = h_m \rho c_p Le^{2/3}$$

where Le = $\alpha/D_{AB}$ = 22.5/26.0 = 0.865. Hence

$$h = 6.13\times10^{-3}\,\text{m/s}\left(1.161\,\text{kg/m}^3\right)1007\ \text{J/kg}\cdot\text{K}\,(0.865)^{2/3} = 6.5\ \text{W/m}^2\cdot\text{K}.$$

The corresponding convection heat transfer rate is

$$q'_{conv} = hA'\left(T_g - T_\infty\right) = 6.5\,\text{W/m}^2\cdot\text{K}\,(1\text{ m})(330-300)\,\text{K} = 195\,\text{W/m}$$

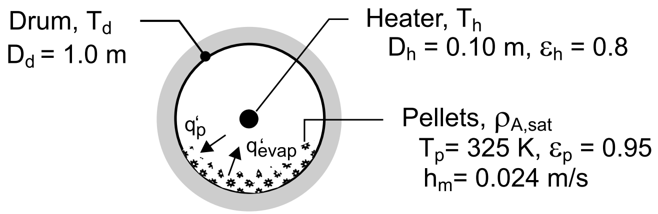
Since $q'_{conv} << q'_{rad}$, the assumption of negligible convection heat transfer is reasonable.

## PROBLEM 13.133

**KNOWN:** Diameters of coaxial cylindrical drum and heater. Heater emissivity. Temperature and emissivity of pellets covering bottom half of drum. Convection mass transfer coefficient associated with flow of dry air over the pellets.

**FIND:** (a) Evaporation rate per unit length of drum, (b) Surface temperatures of heater and top half of drum.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state, (2) Negligible heat transfer from ends of drum, (3) Diffuse-gray surface behavior, (4) Negligible heat loss from the drum to the surroundings, (5) Negligible convection heat transfer from interior surfaces of the drum, (6) Pellet surface area corresponds to that of bottom half of drum.

**PROPERTIES** *Table A-6*, sat. water (T = 325 K): $\rho_{A,sat} = v_g^{-1}$ = 0.0904 kg/m$^3$, $h_{fg}$ = 2378 kJ/kg.

**ANALYSIS:** (a) The evaporation rate is

$$n'_A = h_m(\pi D_d/2)\left[\rho_{A,sat}(T_p) - \rho_{A,\infty}\right]$$

$$n'_A = 0.024\ \text{m/s}(\pi \times 1\text{m}/2) \times 0.0904\ \text{kg/m}^3 = 0.00341\ \text{kg/s}\cdot\text{m}$$ <

(b) From an energy balance on the surface of the pellets,

$$q'_p = q'_{evap} = n'_A h_{fg} = 0.00341\ \text{kg/s}\cdot\text{m} \times 2.378 \times 10^6\ \text{J/kg} = 8109\ \text{W/m}$$ <

where $q'_p$ may be determined from analysis of radiation transfer in a three surface enclosure. Since the top half of the enclosure may be treated as reradiating, net radiation transfer to the pellets may be obtained from Eq. 13.30, which takes the form

$$q'_p = \frac{E_{bh} - E_{bp}}{\dfrac{1-\varepsilon_h}{\varepsilon_h A'_h} + \dfrac{1}{A'_h F_{hp} + \left[(1/A'_h F_{hd}) + (1/A'_p F_{pd})\right]^{-1}} + \dfrac{1-\varepsilon_p}{\varepsilon_p A'_p}}$$

where $F_{hp} = F_{hd} = 0.5$, $A'_h = \pi D_h$ and $A'_p = \pi D_d/2$.

The view factor $F_{pd}$ may be obtained from the summation rule,

$$F_{pd} = 1 - F_{ph} - F_{pp}$$

Continued …..

**PROBLEM 13.133 (Cont.)**

where $F_{ph} = A'_h F_{hp} / A'_p = (\pi D_h \times 0.5)/(\pi D_d / 2) = 0.10$ and

$$F_{pp} = 1 - (2/\pi)\left\{\left[1-(0.1)^2\right]^{1/2} + 0.1 \sin^{-1}(0.1)\right\} = 0.360$$

Hence, $F_{pd}$ = 1 – 0.10 – 0.360 = 0.540, and the expression for the heat rate yields

$$8109\ \mathrm{W/m} = \frac{E_{bh} - \sigma(325\ \mathrm{K})^4}{\dfrac{0.25}{\pi \times 0.1\mathrm{m}} + \dfrac{1}{\pi\left\{0.1\mathrm{m} \times 0.5 + \left[(0.1\mathrm{m} \times 0.5)^{-1} + (0.5\mathrm{m} \times 0.54)^{-1}\right]^{-1}\right\}} + \dfrac{0.053}{\pi \times 0.5\mathrm{m}}}$$

$$E_{bh} = \sigma T_h^4 = 35,359\ \mathrm{W/m^2}$$

$$T_h = 889\ \mathrm{K} \qquad <$$

Applying Eq. (13.19) to surfaces h and p,

$$J_h = E_{bh} - q'_h (1-\varepsilon_h)/\varepsilon_h A'_h = 35,359\ \mathrm{W/m^2} - 6,453\ \mathrm{W/m^2} = 28,906\ \mathrm{W/m^2}$$

$$J_p = E_{bp} + q'_p (1-\varepsilon_p)/\varepsilon_p A'_p = 633\ \mathrm{W/m^2} + 272\ \mathrm{W/m^2} = 905\ \mathrm{W/m^2}$$

Hence, from

$$\frac{J_h - J_d}{(A'_h F_{hd})^{-1}} - \frac{J_d - J_p}{(A'_p F_{pd})^{-1}} = 0$$

$$\frac{28,906\ \mathrm{W/m^2} - J_d}{(\pi \times 0.1\mathrm{m} \times 0.5)^{-1}} - \frac{J_d - 905\ \mathrm{W/m^2}}{(\pi \times 0.5\mathrm{m} \times 0.54)^{-1}} = 0$$

$$J_d = \sigma T_d^4 = 24,530\ \mathrm{W/m^2}$$

$$T_d = 811\ \mathrm{K} \qquad <$$

**COMMENTS:** The required value of $T_h$ could be reduced by increasing $D_h$, although care must be taken to prevent contact of the plastic with the heater.