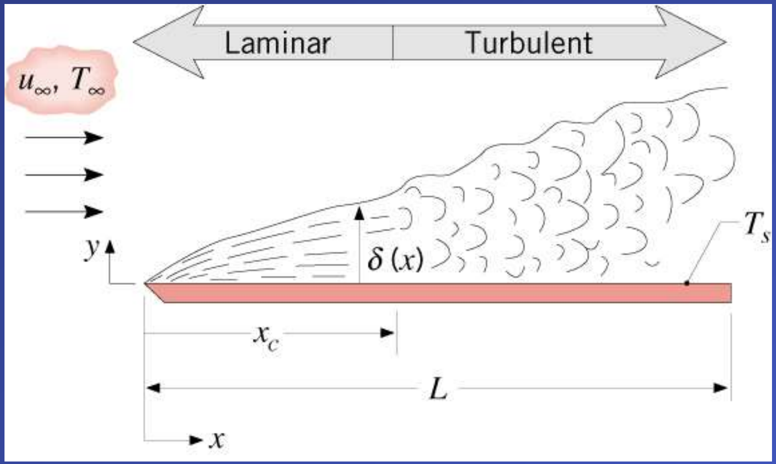
External Flow:
The Flat Plate in Parallel Flow

Chapter 7

Section 7.1 through 7.3

Appendix F

Physical Features



- As with all external flows, the boundary layers develop freely without constraint.
- Boundary layer conditions may be entirely laminar, laminar and turbulent, or entirely turbulent.
- To determine the conditions, compute

$$\mathrm{Re}_L = \frac{\rho u_\infty L}{\mu} = \frac{u_\infty L}{\nu}$$

and compare with the critical Reynolds number for transition to turbulence, $\mathrm{Re}_{x,c}$.

$$\mathrm{Re}_L < \mathrm{Re}_{x,c} \rightarrow \text{ laminar flow throughout}$$

$$\mathrm{Re}_L > \mathrm{Re}_{x,c} \rightarrow \text{ transition to turbulent flow at } x_c / L \equiv \mathrm{Re}_{x,c} / \mathrm{Re}_L$$

- Value of $\mathrm{Re}_{x,c}$ depends on free stream turbulence and surface roughness. Nominally,

$$\mathrm{Re}_{x,c} \approx 5 \times 10^5.$$

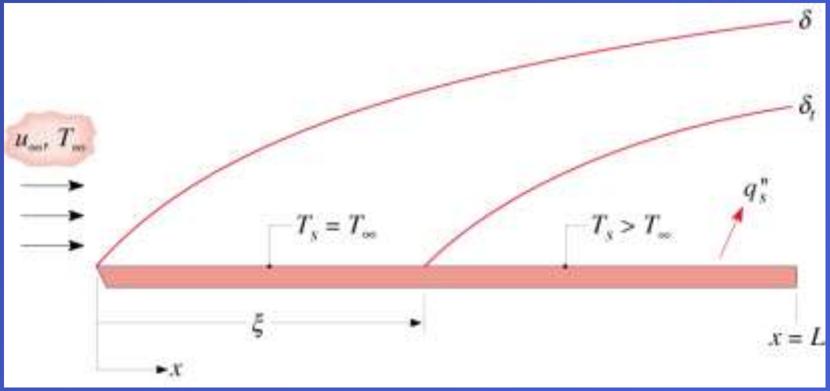
- If boundary layer is tripped at the leading edge

$$\mathrm{Re}_{x,c} = 0$$

and the flow is turbulent throughout.

- Surface thermal conditions are commonly idealized as being of uniform temperature T_s or uniform heat flux q_s''. Is it possible for a surface to be concurrently characterized by uniform temperature and uniform heat flux?

- Thermal boundary layer development may be delayed by an unheated starting length.



Equivalent surface and free stream temperatures for $x < \xi$ and uniform T_s (or q_s'') for $x > \xi$.

Similarity Solution for Laminar, Constant-Property Flow over an Isothermal Plate

- Based on premise that the dimensionless x-velocity component, u/u_∞, and temperature, $T^* \equiv \left[(T-T_s)/(T_\infty - T_s)\right]$, can be represented exclusively in terms of a dimensionless similarity parameter

$$\eta \equiv y\left(u_\infty/\nu x\right)^{1/2}$$

- Similarity permits transformation of the partial differential equations associated with the transfer of x-momentum and thermal energy to ordinary differential equations of the form

$$2\frac{d^3 f}{d\eta^3} + f\frac{d^2 f}{d\eta^2} = 0$$

where $\left(u/u_\infty\right) \equiv df/d\eta,$ and

$$\frac{d^2T^*}{d\eta^2} + \frac{\Pr}{2} f \frac{dT^*}{d\eta} = 0$$

- Subject to prescribed boundary conditions, numerical solutions to the momentum and energy equations yield the following results for important local boundary layer parameters:

  - with $u/u_\infty = 0.99$ at $\eta = 5.0$,

$$\delta = \frac{5.0}{\left(u_\infty / vx\right)^{1/2}} = \frac{5x}{\mathrm{Re}_x}$$

  - with $\tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = \mu u_\infty \sqrt{u_\infty / vx} \left.\frac{d^2 f}{d\eta^2}\right|_{\eta=0}$

    and $d^2 f / d\eta^2\big|_{\eta=0} = 0.332$,

$$C_{f,x} \equiv \frac{\tau_{s,x}}{\rho u_\infty^2 / 2} = 0.664\,\mathrm{Re}_x^{-1/2}$$

  - with $h_x = q_s'' / \left(T_s - T_\infty\right) = k \partial T^* / \partial y\big|_{y=0} = k\left(u_\infty / vx\right)^{1/2} dT^* / d\eta\big|_{\eta=0}$

    and $dT^* / d\eta\big|_{\eta=0} = 0.332\ \mathrm{Pr}^{1/3}$ for $\mathrm{Pr} > 0.6$,

$$Nu_x = \frac{h_x x}{k} = 0.332\ \mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}$$

    and

$$\frac{\delta}{\delta_t} = \mathrm{Pr}^{1/3}$$

- How would you characterize relative laminar velocity and thermal boundary layer growth for a gas? An oil? A liquid metal?

- How do the local shear stress and convection coefficient vary with distance from the leading edge?

- Average Boundary Layer Parameters:

$$\bar{\tau}_{s,x} \equiv \frac{1}{x}\int_0^x \tau_s dx$$

$$\overline{C_{f,x}} = 1.328\ Re_x^{-1/2}$$

$$\bar{h}_x = \frac{1}{x}\int_0^x h_x dx$$

$$\overline{Nu_x} = 0.664\ Re_x^{1/2}\ Pr^{1/3}$$

- The effect of variable properties may be considered by evaluating all properties at the film temperature.

$$T_f = \frac{T_s + T_\infty}{2}$$

Turbulent Flow

- Local Parameters:

Empirical Correlations
$$\begin{cases} C_{f,x} = 0.0592\ \mathrm{Re}_x^{-1/5} \\ Nu_x = 0.0296\ \mathrm{Re}_x^{4/5}\ \mathrm{Pr}^{1/3} \end{cases}$$

How do variations of the local shear stress and convection coefficient with distance from the leading edge for turbulent flow differ from those for laminar flow?

- Average Parameters:

$$\bar{h}_L = \frac{1}{L}\left(\int_0^{x_c} h_{lam}dx + \int_{x_c}^{L} h_{turb}dx\right)$$

Substituting expressions for the local coefficients and assuming $\mathrm{Re}_{x,c} = 5\times10^5$,

$$\bar{C}_{f,L} = \frac{0.074}{\mathrm{Re}_L^{1/5}} - \frac{1742}{\mathrm{Re}_L}$$

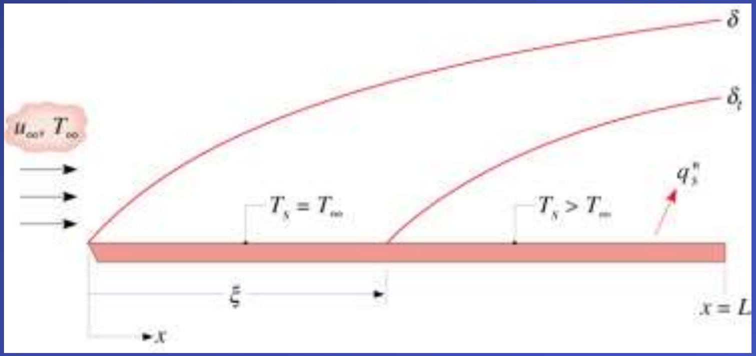
$$\overline{Nu}_L = \left(0.037\ \mathrm{Re}_L^{4/5} - 871\right)\mathrm{Pr}^{1/3}$$

For $\mathrm{Re}_{x,c} = 0$ or $L \gg x_c \left(\mathrm{Re}_L \gg \mathrm{Re}_{x,c}\right)$,

$$\bar{C}_{f,L} = 0.074\ \mathrm{Re}_L^{-1/5}$$

$$\overline{Nu}_L = 0.037\ \mathrm{Re}_L^{4/5}\ \mathrm{Pr}^{1/3}$$

Special Cases: Unheated Starting Length (USL) and/or Uniform Heat Flux



For both uniform surface temperature (UST) and uniform surface heat flux (USF), the effect of the USL on the local Nusselt number may be represented as follows:

$$Nu_x = \frac{Nu_x\big|_{\xi=0}}{\left[1-\left(\xi/x\right)^a\right]^b}$$

$$Nu_x\big|_{\xi=0} = C\,\mathrm{Re}_x^m\,\mathrm{Pr}^{1/3}$$

	Laminar		Turbulent	
	UST	USF	UST	USF
a	3/4	3/4	9/10	9/10
b	1/3	1/3	1/9	1/9
C	0.332	0.453	0.0296	0.0308
m	1/2	1/2	4/5	4/5

Sketch the variation of h_x versus $\left(x-\xi\right)$ for two conditions: $\xi > 0$ and $\xi = 0$. What effect does an USL have on the local convection coefficient?

- UST:

$$q_s'' = h_x\left(T_s - T_\infty\right) \quad q = \bar{h}_L A_s\left(T_s - T_\infty\right)$$

$$\overline{Nu}_L = \overline{Nu}_L\Big|_{\xi=0} \frac{L}{\left(L-\xi\right)}\left[1-\left(\xi/L\right)^{(2p+1)/(2p+2)}\right]^{2p/(2p+1)}$$

$p = 1$ for laminar flow throughout

$p = 4$ for turbulent flow throughout

$\bar{h}_L \rightarrow$ numerical integration for laminar/turbulent flow

$$\bar{h}_L = \frac{1}{L}\left[\int_{\xi}^{x_c} h_{1am}dx + \int_{x_c}^{L} h_{turb}dx\right]$$

- USF:

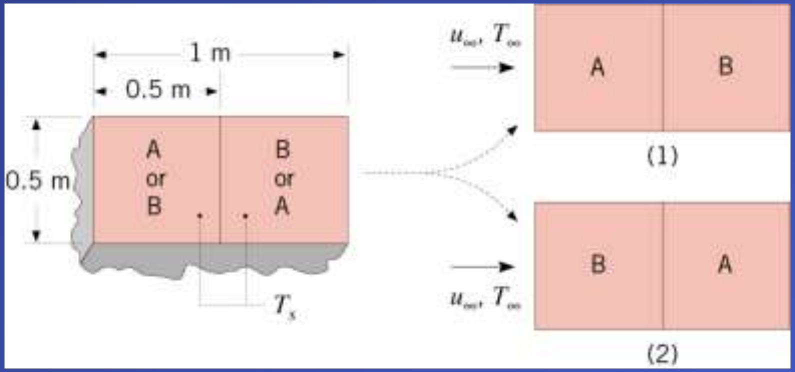
$$T_s = T_\infty + \frac{q_s''}{h_x} \qquad q = q_s'' A_s$$

- Treatment of Non-Constant Property Effects:

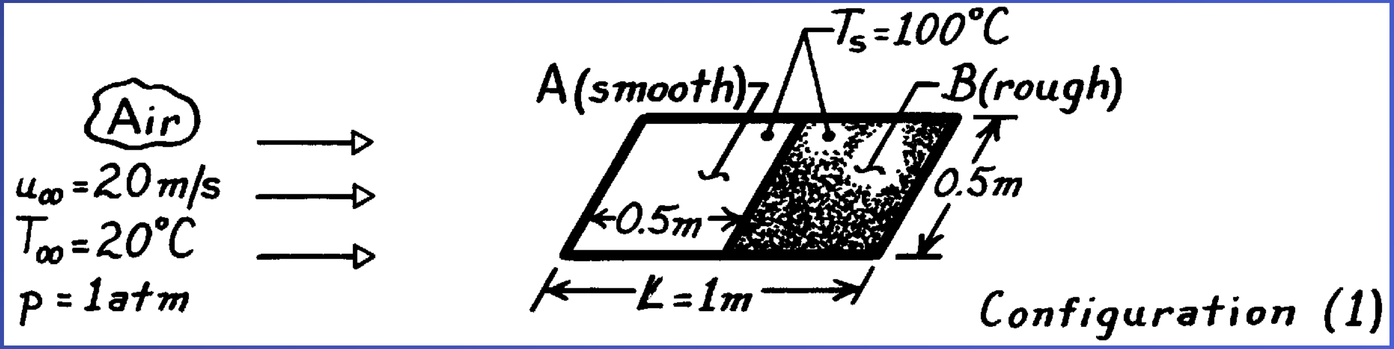
Evaluate properties at the film temperature.

$$T_f = \frac{T_s + T_\infty}{2}$$

Problem 7.21: Preferred orientation (corresponding to lower heat loss) and the corresponding heat rate for a surface with adjoining smooth and roughened sections.



SCHEMATIC:



ASSUMPTIONS: (1) Surface B is sufficiently rough to trip the boundary layer when in the upstream position (Configuration 2); (2) $Re_{x,c} \approx 5\times10^5$ for flow over A in Configuration 1.

PROPERTIES: *Table A-4*, Air (T_f = 333K, 1 atm): $\nu = 19.2 \times 10^{-6}$ m^2/s, k = 28.7×10^{-3} W/m·K, Pr = 0.7.

ANALYSIS: Since Configuration (2) results in a turbulent boundary layer over the entire surface, the lowest heat transfer is associated with Configuration (1).

Find

$$Re_L = \frac{u_\infty L}{\nu} = \frac{20\ \text{m/s} \times 1\text{m}}{19.2 \times 10^{-6}\text{m}^2/\text{s}} = 1.04 \times 10^6.$$

Hence in Configuration (1), transition will occur just before the rough surface (x_c = 0.48m).

$$\overline{Nu}_{L,1} = \left[0.037\left(1.04 \times 10^6\right)^{4/5} - 871\right] 0.7^{1/3} = 1366$$

For Configuration (1):

$$\frac{\bar{h}_{L,1} L}{k} = \overline{Nu}_{L,1} = 1366.$$

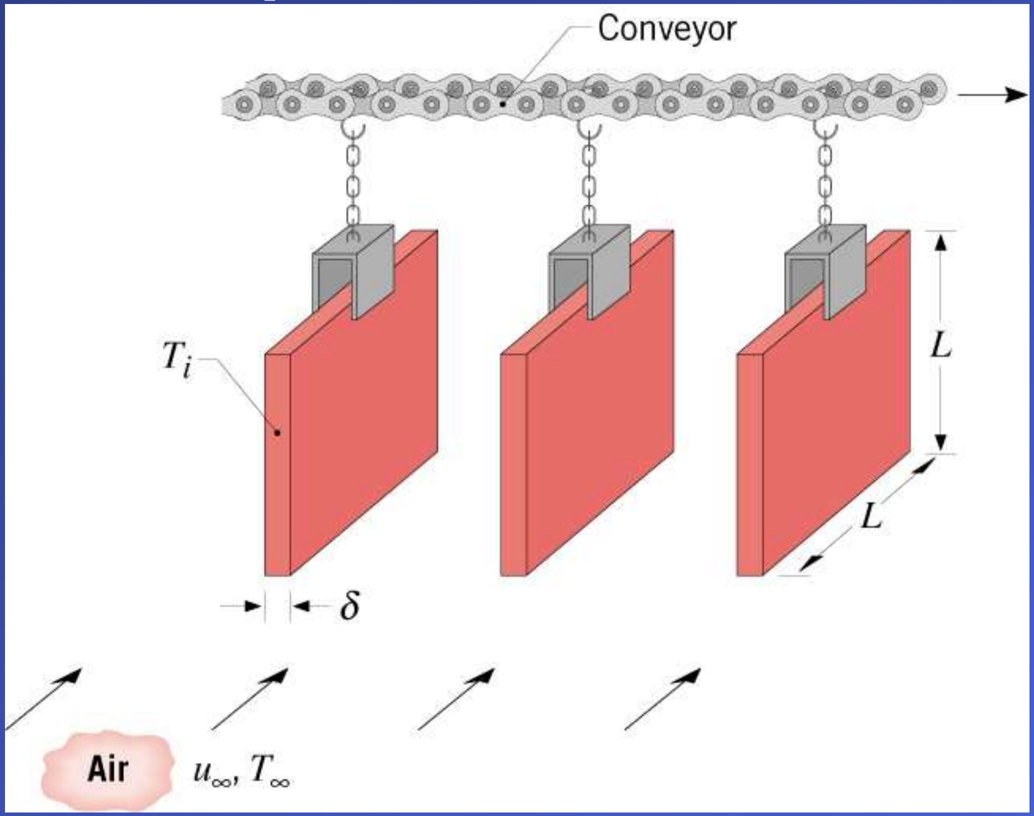
Hence

$$\bar{h}_{L,1} = 1366\left(28.7 \times 10^{-3}\,\text{W/m} \cdot \text{K}\right)/1\text{m} = 39.2\ \text{W/m}^2 \cdot \text{K}$$

$$q_1 = \bar{h}_{L,1} A\left(T_s - T_\infty\right) = 39.2\ \text{W/m}^2 \cdot \text{K}\left(0.5\text{m} \times 1\text{m}\right)\left(100 - 20\right)\text{K} \quad = 1568\ \text{W} \qquad <$$

Comment: Note that $\overline{Nu}_{L,2} = 0.037\left(1.04 \times 10^6\right)^{4/5} (0.7)^{1/3} = 2139 > \overline{Nu}_{L,1}$.

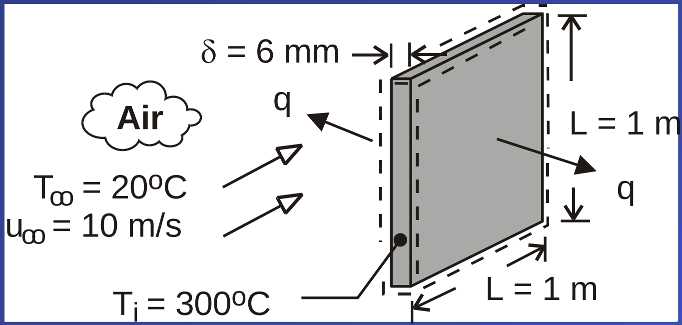
Problem 7.25: Convection cooling of steel plates on a conveyor by air in parallel flow.



KNOWN: Plate dimensions and initial temperature. Velocity and temperature of air in parallel flow over plates.

FIND: Initial rate of heat transfer from plate. Rate of change of plate temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible radiation, (2) Negligible effect of conveyor velocity on boundary layer development, (3) Plates are isothermal, (4) Negligible heat transfer from edges of plate, (5) $Re_{x,c} = 5 \times 10^5$.

PROPERTIES: *Table A-1*, AISI 1010 steel (573K): k_p = 49.2 W/m·K, c = 549 J/kg·K, ρ = 7832 kg/m^3. *Table A-4*, Air (p = 1 atm, T_f = 433K): ν = 30.4 × 10^{-6} m^2/s, k = 0.0361 W/m·K, Pr = 0.688.

ANALYSIS: The initial rate of heat transfer from a plate is

$$q = 2\bar{h} A_s \left(T_i - T_\infty\right) = 2\bar{h} L^2 \left(T_i - T_\infty\right)$$

With $Re_L = u_\infty L/\nu = 10\,m/s \times 1m/30.4 \times 10^{-6} m^2/s = 3.29 \times 10^5$, flow is laminar over the entire surface.

Hence,

$$\overline{Nu}_L = 0.664 Re_L^{1/2} Pr^{1/3} = 0.664\left(3.29 \times 10^5\right)^{1/2} (0.688)^{1/3} = 336$$

$$\bar{h} = (k/L)\overline{Nu}_L = (0.0361\,W/m \cdot K/1m)336 = 12.1\,W/m^2 \cdot K$$

$$q = 2 \times 12.1\,W/m^2 \cdot K (1m)^2 (300 - 20)°C = 6780\,W$$

Performing an energy balance at an instant of time for a control surface about the plate, $-\dot{E}_{out} = \dot{E}_{st}$,

$$\rho \delta L^2 c \left.\frac{dT}{dt}\right|_i = -\bar{h}\, 2L^2 \left(T_i - T_\infty\right)$$

$$\left.\frac{dT}{dt}\right|_i = -\frac{2\left(12.1\,W/m^2 \cdot K\right)(300-20)°C}{7832\ kg/m^3 \times 0.006m \times 549\,J/kg \cdot K} = -0.26°C/s$$

COMMENTS: (1) With $Bi = \bar{h}(\delta/2)/k_p = 7.4 \times 10^{-4}$, use of the lumped capacitance method is appropriate.

(2) Despite the large plate temperature and the small convection coefficient, if adjoining plates are in close proximity, radiation exchange with the surroundings will be small and the assumption of negligible radiation is justifiable.