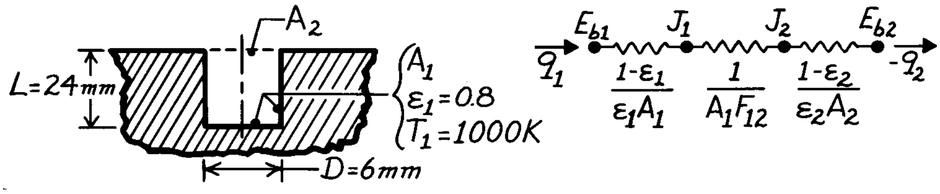
PROBLEM 13.42

KNOWN: Dimensions and temperature of a flat-bottomed hole.

FIND: (a) Radiant power leaving the opening, (b) Effective emissivity of the cavity, ε_e, (c) Limit of ε_e as depth of hole increases.

SCHEMATIC:



ASSUMPTIONS: (1) Hypothetical surface A_2 is a blackbody at 0 K, (2) Cavity surface is isothermal, opaque and diffuse-gray.

ANALYSIS: Approximating A_2 as a blackbody at 0 K implies that all of the radiation incident on A_2 from the cavity results (directly or indirectly) from emission by the walls and escapes to the surroundings. It follows that for A_2, $\varepsilon_2 = 1$ and $J_2 = E_{b2} = 0$.

(a) From the thermal circuit, the rate of radiation loss through the hole (A_2) is

$$q_1 = \left(E_{b1} - E_{b2}\right) / \left[\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}\right]. \qquad (1)$$

Noting that $F_{21} = 1$ and $A_1 F_{12} = A_2 F_{21}$, also that

$$A_1 = \pi D^2/4 + \pi D L = \pi D\left(D/4 + L\right) = \pi\left(0.006\,\text{m}\right)\left(0.006\,\text{m}/4 + 0.024\,\text{m}\right) = 4.807\times10^{-4}\,\text{m}^2$$

$$A_2 = \pi D^2/4 = \pi\left(0.006\,\text{m}\right)^2/4 = 2.827\times10^{-5}\,\text{m}^2.$$

Substituting numerical values with $E_b = \sigma T^4$, find

$$q_1 = 5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(1000^4 - 0\right)\text{K}^4 / \left[\frac{1-0.8}{0.8\times4.807\times10^{-4}\,\text{m}^2} + \frac{1}{2.827\times10^{-5}\,\text{m}^2} + 0\right]$$

$$q_1 = 1.580\ \text{W}. \qquad <$$

(b) The effective emissivity, ε_e, of the cavity is defined as the ratio of the radiant power leaving the cavity to that from a blackbody having the same area of the cavity opening and at the temperature of the inner surfaces of the cavity. For the cavity above,

$$\varepsilon_e = \frac{q_1}{A_2 \sigma T_1^4}$$

$$\varepsilon_e = 1.580\,\text{W} / 2.827\times10^{-5}\,\text{m}^2\left(5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\right)\left(1000\ \text{K}\right)^4 = 0.986. \qquad <$$

(c) As the depth of the hole increases, the term $(1 - \varepsilon_1)/\varepsilon_1 A_1$ goes to zero such that the remaining term in the denominator of Eq. (1) is $1/A_1 F_{12} = 1/A_2 F_{21}$. That is, as L increases, $q_1 \to A_2 F_{21} E_{b1}$. This implies that $\varepsilon_e \to 1$ as L increases. For L/D = 10, one would expect $\varepsilon_e = 0.999$ or better.