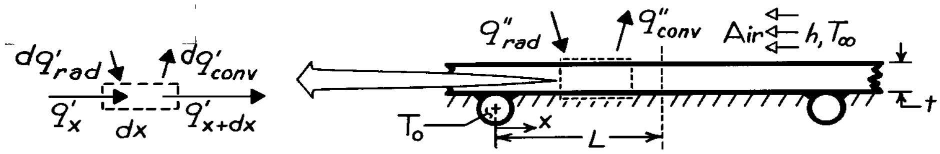
PROBLEM 3.100

KNOWN: Surface conditions and thickness of a solar collector absorber plate. Temperature of working fluid.

FIND: (a) Differential equation which governs plate temperature distribution, (b) Form of the temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Adiabatic bottom surface, (4) Uniform radiation flux and convection coefficient at top, (5) Temperature of absorber plate at x = 0 corresponds to that of working fluid.

ANALYSIS: (a) Performing an energy balance on the differential control volume,

$$q'_x + dq'_{rad} = q'_{x+dx} + dq'_{conv}$$

where
$$q'_{x+dx} = q'_x + \left(dq'_x / dx\right) dx$$
$$dq'_{rad} = q''_{rad} \cdot dx$$
$$dq'_{conv} = h\left(T - T_\infty\right) \cdot dx$$

Hence,
$$q''_{rad} dx = \left(dq'_x / dx\right) dx + h\left(T - T_\infty\right) dx.$$

From Fourier's law, the conduction heat rate per unit width is

$$q'_x = -k\,t\,dT/dx \qquad \frac{d^2T}{dx^2} - \frac{h}{kT}\left(T - T_\infty\right) + \frac{q''_{rad}}{kt} = 0. \qquad <$$

(b) Defining $\theta = T - T_\infty, d^2T/dx^2 = d^2\theta / dx^2$ and the differential equation becomes,

$$\frac{d^2\theta}{dx^2} - \frac{h}{kt}\theta + \frac{q''_{rad}}{kt} = 0.$$

It is a second-order, differential equation with constant coefficients and a source term, and its general solution is of the form

$$\theta = C_1 e^{+\lambda x} + C_2 e^{-\lambda x} + S/\lambda^2$$

where $\lambda = \left(h/kt\right)^{1/2}$, $\quad S = q''_{rad} / kt.$

Appropriate boundary conditions are:

$$\theta\left(0\right) = T_o - T_\infty \equiv \theta_0, \qquad d\theta/dx)_{x=L} = 0.$$

Hence,
$$\theta_o = C_1 + C_2 + S/\lambda^2$$

$$d\theta/dx)_{x=L} = C_1\,\lambda e^{+\lambda L} - C_2\,\lambda e^{-\lambda L} = 0 \qquad C_2 = C_1\,e^{2\lambda L}$$

Hence,
$$C_1 = \left(\theta_0 - S/\lambda^2\right)/\left(1 + e^{2\lambda L}\right) \qquad C_2 = \left(\theta_0 - S/\lambda^2\right)/\left(1 + e^{-2\lambda L}\right)$$

$$\theta = \left(\theta_0 - S/\lambda^2\right)\left[\frac{e^{\lambda x}}{1+e^{2\lambda L}} + \frac{e^{-\lambda x}}{1+e^{-2\lambda L}}\right] + S/\lambda^2. \qquad <$$