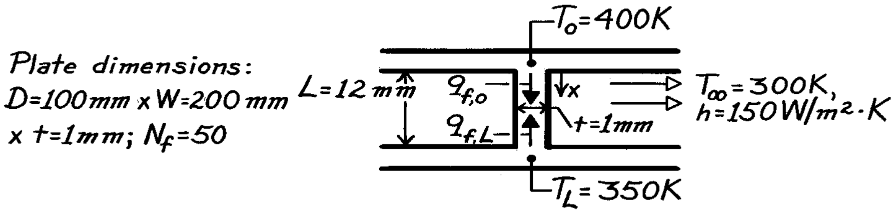
PROBLEM 3.130

KNOWN: Arrangement of fins between parallel plates. Temperature and convection coefficient of air flow in finned passages. Maximum allowable plate temperatures.

FIND: (a) Expressions relating fin heat transfer rates to end temperatures, (b) Maximum power dissipation for each plate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in fins, (3) Constant properties, (4) Negligible radiation, (5) All of the heat is dissipated to the air, (6) Uniform h, (7) Negligible variation in T_∞, (8) Negligible contact resistance.

PROPERTIES: *Table A.1*, Aluminum (pure), 375 K: k = 240 W/m·K.

ANALYSIS: (a) The general solution for the temperature distribution in fin is

$$\theta(x) \equiv T(x) - T_\infty = C_1 e^{mx} + C_2 e^{-mx}$$

Boundary conditions: $\theta(0) = \theta_o = T_o - T_\infty, \qquad \theta(L) = \theta_L = T_L - T_\infty.$

Hence $\quad \theta_o = C_1 + C_2 \qquad \theta_L = C_1 e^{mL} + C_2 e^{-mL}$

$$\theta_L = C_1 e^{mL} + (\theta_o - C_1) e^{-mL}$$

$$C_1 = \frac{\theta_L - \theta_o e^{-mL}}{e^{mL} - e^{-mL}} \qquad C_2 = \theta_o - \frac{\theta_L - \theta_o e^{-mL}}{e^{mL} - e^{-mL}} = \frac{\theta_o e^{mL} - \theta_L}{e^{mL} - e^{-mL}}.$$

Hence $\quad \theta(x) = \dfrac{\theta_L e^{mx} - \theta_o e^{m(x-L)} + \theta_o e^{m(L-x)} - \theta_L e^{-mx}}{e^{mL} - e^{-mL}}$

$$\theta(x) = \frac{\theta_o \left[e^{m(L-x)} - e^{-m(L-x)}\right] + \theta_L \left(e^{mx} - e^{-mx}\right)}{e^{mL} - e^{-mL}}$$

$$\theta(x) = \frac{\theta_o \sinh m(L-x) + \theta_L \sinh mx}{\sinh mL}.$$

The fin heat transfer rate is then

$$q_f = -kA_c \frac{dT}{dx} = -kDt\left[-\frac{\theta_o m}{\sinh mL}\cosh m(L-x) + \frac{\theta_L m}{\sinh mL}\cosh mx\right].$$

Hence $\quad q_{f,o} = kDt\left(\dfrac{\theta_o m}{\tanh mL} - \dfrac{\theta_L m}{\sinh mL}\right) \qquad$ <

$$q_{f,L} = kDt\left(\frac{\theta_o m}{\sinh mL} - \frac{\theta_L m}{\tanh mL}\right). \qquad <$$

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PROBLEM 3.130 (Cont.)

(b) $$m=\left(\frac{hP}{kA_c}\right)^{1/2}=\left(\frac{50\ \text{W/m}^2\cdot\text{K}\left(2\times0.1\ \text{m}+2\times0.001\ \text{m}\right)}{240\ \text{W/m}\cdot\text{K}\times0.1\ \text{m}\times0.001\ \text{m}}\right)^{1/2}=35.5\ \text{m}^{-1}$$

$$mL=35.5\ \text{m}^{-1}\times0.012\ \text{m}=0.43$$

$$\sinh\ mL=0.439 \qquad \tanh\ mL=0.401 \qquad \theta_o=100\ \text{K} \qquad \theta_L=50\ \text{K}$$

$$q_{f,o}=240\ \text{W/m}\cdot\text{K}\times0.1\ \text{m}\times0.001\ \text{m}\left(\frac{100\ \text{K}\times35.5\ \text{m}^{-1}}{0.401}-\frac{50\ \text{K}\times35.5\ \text{m}^{-1}}{0.439}\right)$$

$q_{f,o}=115.4\ \text{W}$ (*from* the top plate)

$$q_{f,L}=240\ \text{W/m}\cdot\text{K}\times0.1\ \text{m}\times0.001\ \text{m}\left(\frac{100\ \text{K}\times35.5\ \text{m}^{-1}}{0.439}-\frac{50\ \text{K}\times35.5\ \text{m}^{-1}}{0.401}\right)$$

$q_{f,L}=87.8\ \text{W}.$ (*into* the bottom plate)

Maximum power dissipations are therefore

$$q_{o,max}=N_f q_{f,o}+\left(W-N_f t\right)Dh\theta_o$$

$$q_{o,max}=50\times115.4\ \text{W}+\left(0.200-50\times0.001\right)\text{m}\times0.1\ \text{m}\times150\ \text{W/m}^2\cdot\text{K}\times100\ \text{K}$$

$$q_{o,max}=5770\ \text{W}+225\ \text{W}=5995\ \text{W}$$ <

$$q_{L,max}=-N_f q_{f,L}+\left(W-N_f t\right)Dh\theta_o$$

$$q_{L,max}=-50\times87.8\text{W}+\left(0.200-50\times0.001\right)\text{m}\times0.1\ \text{m}\times150\ \text{W/m}^2\cdot\text{K}\times50\ \text{K}$$

$$q_{L,max}=-4390\ \text{W}+112\text{W}=-4278\ \text{W}.$$ <

COMMENTS: (1) It is of interest to determine the air velocity needed to prevent excessive heating of the air as it passes between the plates. If the air temperature change is restricted to $\Delta T_\infty = 5$ K, its flowrate must be

$$\dot{m}_{air}=\frac{q_{tot}}{c_p\Delta T_\infty}=\frac{1717\ \text{W}}{1007\ \text{J/kg}\cdot\text{K}\times5\ \text{K}}=0.34\ \text{kg/s}.$$

Its mean velocity is then

$$V_{air}=\frac{\dot{m}_{air}}{\rho_{air}A_c}=\frac{0.34\ \text{kg/s}}{1.16\ \text{kg/m}^3\times0.012\ \text{m}\left(0.2-50\times0.001\right)\text{m}}=163\ \text{m/s}.$$

Such a velocity would be impossible to maintain. To reduce it to a reasonable value, e.g. 10 m/s, A_c would have to be increased substantially by increasing W (and hence the space between fins) and by increasing L. The present configuration is impractical from the standpoint that 1717 W could not be transferred to air in such a small volume.

(2) A negative value of $q_{L,max}$ implies that heat must be transferred from the bottom plate to the air to maintain the plate at 350 K.