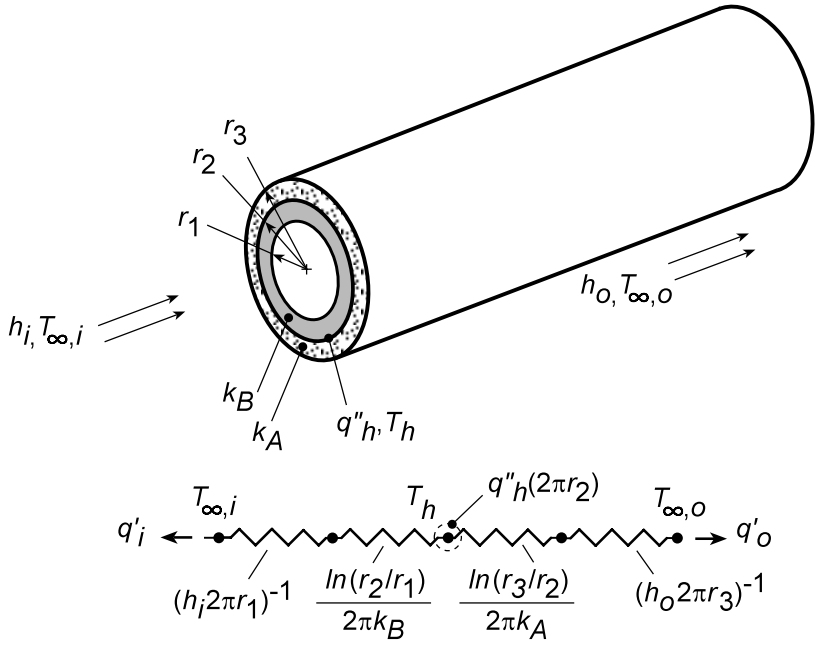
PROBLEM 3.46

KNOWN: Conditions associated with a composite wall and a thin electric heater.

FIND: (a) Equivalent thermal circuit, (b) Expression for heater temperature, (c) Ratio of outer and inner heat flows and conditions for which ratio is minimized.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction, (2) Constant properties, (3) Isothermal heater, (4) Negligible contact resistance(s).

ANALYSIS: (a) On the basis of a unit axial length, the circuit, thermal resistances, and heat rates are as shown in the schematic.

(b) Performing an energy balance for the heater, $\dot{E}_{in} = \dot{E}_{out}$, it follows that

$$q''_h(2\pi r_2) = q'_i + q'_o = \frac{T_h - T_{\infty,i}}{(h_i 2\pi r_1)^{-1} + \frac{\ln(r_2/r_1)}{2\pi k_B}} + \frac{T_h - T_{\infty,o}}{(h_o 2\pi r_3)^{-1} + \frac{\ln(r_3/r_2)}{2\pi k_A}} \quad <$$

(c) From the circuit,

$$\frac{q'_o}{q'_i} = \frac{(T_h - T_{\infty,o})}{(T_h - T_{\infty,i})} \times \frac{(h_i 2\pi r_1)^{-1} + \frac{\ln(r_2/r_1)}{2\pi k_B}}{(h_o 2\pi r_3)^{-1} + \frac{\ln(r_3/r_2)}{2\pi k_A}} \quad <$$

To reduce q'_o/q'_i, one could increase k_B, h_i, and r_3/r_2, while reducing k_A, h_o and r_2/r_1.

COMMENTS: Contact resistances between the heater and materials A and B could be important.