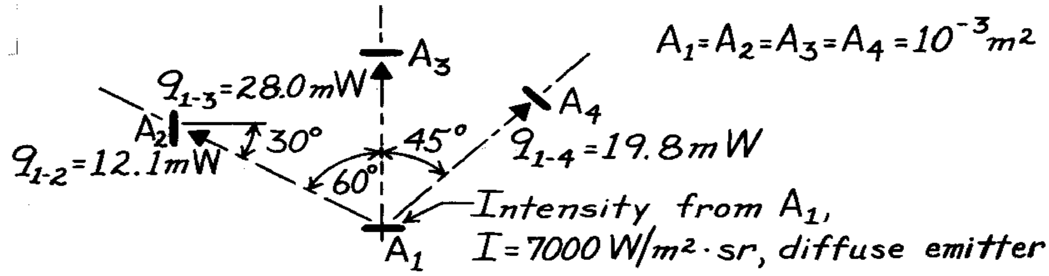
PROBLEM 12.1

KNOWN: Rate at which radiation is intercepted by each of three surfaces (see (Example 12.1).

FIND: Irradiation, G[W/m^2], at each of the three surfaces.

SCHEMATIC:



ANALYSIS: The irradiation at a surface is the rate at which radiation is incident on a surface per unit area of the surface. The irradiation at surface j due to emission from surface 1 is

$$G_j = \frac{q_{1-j}}{A_j}.$$

With $A_1 = A_2 = A_3 = A_4 = 10^{-3}$ m^2 and the incident radiation rates q_{1-j} from the results of Example 12.1, find

$$G_2 = \frac{12.1 \times 10^{-3}\ W}{10^{-3}\ m^2} = 12.1\ W/m^2$$ <

$$G_3 = \frac{28.0 \times 10^{-3}\ W}{10^{-3}\ m^2} = 28.0\ W/m^2$$ <

$$G_4 = \frac{19.8 \times 10^{-3}\ W}{10^{-3}\ m^2} = 19.8\ W/m^2.$$ <

COMMENTS: The irradiation could also be computed from Eq. 12.15, which, for the present situation, takes the form

$$G_j = I_1 \cos\theta_j\, \omega_{1-j}$$

where $I_1 = I = 7000$ W/m$^2\cdot$sr and ω_{1-j} is the solid angle subtended by surface 1 with respect to j. For example,

$$G_2 = I_1 \cos\theta_2\, \omega_{1-2}$$

$$G_2 = 7000\ W/m^2 \cdot sr \times$$

$$\cos 30° \frac{10^{-3}\ m^2 \times \cos 60°}{(0.5 m)^2}$$

$$G_2 = 12.1\ W/m^2.$$

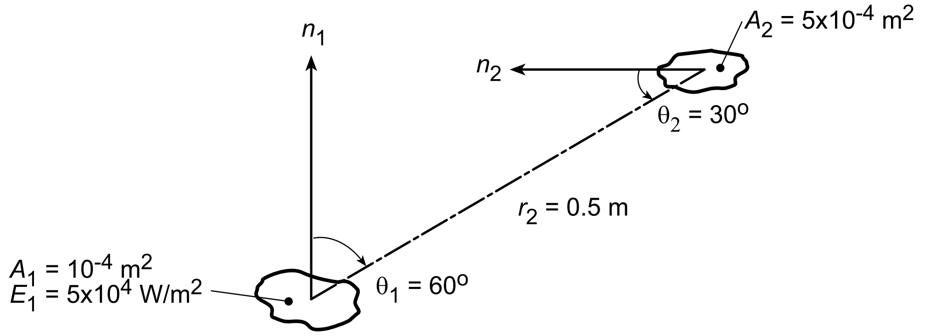
Note that, since A_1 is a diffuse radiator, the intensity I is independent of direction.

PROBLEM 12.2

KNOWN: A diffuse surface of area $A_1 = 10^{-4} m^2$ emits diffusely with total emissive power $E = 5 \times 10^4$ W/m^2 .

FIND: (a) Rate this emission is intercepted by small surface of area $A_2 = 5 \times 10^{-4}$ m^2 at a prescribed location and orientation, (b) Irradiation G_2 on A_2, and (c) Compute and plot G_2 as a function of the separation distance r_2 for the range $0.25 \le r_2 \le 1.0$ m for zenith angles $\theta_2 = 0$, 30 and 60°.

SCHEMATIC:



ASSUMPTIONS: (1) Surface A_1 emits diffusely, (2) A_1 may be approximated as a differential surface area and that $A_2 / r_2^2 << 1$.

ANALYSIS: (a) The rate at which emission from A_1 is intercepted by A_2 follows from Eq. 12.5 written on a total rather than spectral basis.

$$q_{1\to 2} = I_{e,1}(\theta,\phi) A_1 \cos\theta_1 d\omega_{2-1}. \tag{1}$$

Since the surface A_1 is diffuse, it follows from Eq. 12.13 that

$$I_{e,1}(\theta,\phi) = I_{e,1} = E_1/\pi \ . \tag{2}$$

The solid angle subtended by A_2 with respect to A_1 is

$$d\omega_{2-1} \approx A_2 \cdot \cos\theta_2 / r_2^2 \ . \tag{3}$$

Substituting Eqs. (2) and (3) into Eq. (1) with numerical values gives

$$q_{1\to 2} = \frac{E_1}{\pi} \cdot A_1 \cos\theta_1 \cdot \frac{A_2 \cos\theta_2}{r_2^2} = \frac{5\times 10^4 \text{ W/m}^2}{\pi \text{ sr}} \times \left(10^{-4} \text{m}^2 \times \cos 60^\circ\right) \times \left[\frac{5\times 10^{-4} \text{m}^2 \times \cos 30^\circ}{(0.5\text{m})^2}\right] \text{sr} \tag{4}$$

$$q_{1\to 2} = 15{,}915 \text{ W/m}^2\text{sr} \times \left(5\times 10^{-5} \text{m}^2\right) \times 1.732\times 10^{-3} \text{sr} = 1.378\times 10^{-3} \text{W}. \qquad <$$

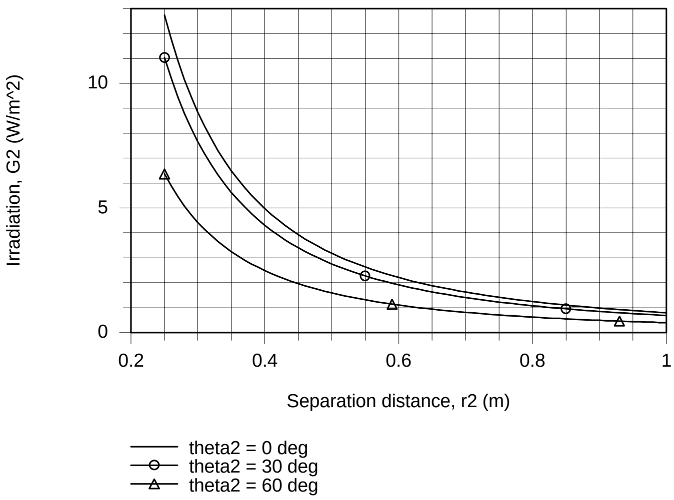
(b) From section 12, 2.3, the irradiation is the rate at which radiation is incident upon the surface per unit surface area,

$$G_2 = \frac{q_{1\to 2}}{A_2} = \frac{1.378\times 10^{-3} \text{W}}{5\times 10^{-4} \text{m}^2} = 2.76 \text{ W/m}^2 \tag{5} \qquad <$$

(c) Using the IHT workspace with the foregoing equations, the G_2 was computed as a function of the separation distance for selected zenith angles. The results are plotted below.

Continued...

PROBLEM 12.2 (Cont.)



For all zenith angles, G_2 decreases with increasing separation distance r_2 . From Eq. (3), note that $d\omega_{2-1}$ and, hence G_2, vary inversely as the square of the separation distance. For any fixed separation distance, G_2 is a maximum when $\theta_2 = 0°$ and decreases with increasing θ_2, proportional to cos θ_2.

COMMENTS: (1) For a diffuse surface, the intensity, I_e, is independent of direction and related to the emissive power as $I_e = E/\pi$. Note that π has the units of $[\mathrm{sr}]$ in this relation.

(2) Note that Eq. 12.5 is an important relation for determining the radiant power leaving a surface in a prescribed manner. It has been used here on a total rather than spectral basis.

(3) Returning to part (b) and referring to Figure 12.10, the irradiation on A2 may be expressed as

$$G_2 = I_{i,2} \cos\theta_2 \frac{A_1 \cos\theta_1}{r_2^2}$$

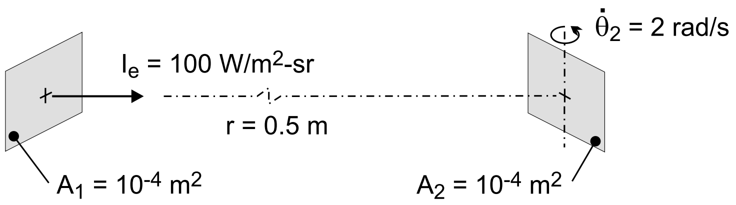
Show that the result is $G_2 = 2.76$ W/m^2. Explain how this expression follows from Eq. (12.15).

PROBLEM 12.003

KNOWN: Intensity and area of a diffuse emitter. Area and rotational frequency of a second surface, as well as its distance from and orientation relative to the diffuse emitter.

FIND: Energy intercepted by the second surface during a complete rotation.

SCHEMATIC:



ASSUMPTIONS: (1) A_1 and A_2 may be approximated as differentially small surfaces, (2) A_1 is a diffuse emitter.

ANALYSIS: From Eq. 12.5, the rate at which radiation emitted by A_1 is intercepted by A_2 is

$$q_{1-2} = I_e\, A_1 \cos\theta_1\, \omega_{2-1} = I_e\, A_1 \left(A_2 \cos\theta_2 / r^2\right)$$

where $\theta_1 = 0$ and θ_2 changes continuously with time. The amount of energy intercepted by both sides of A_2 during one rotation, ΔE, may be grouped into four equivalent parcels, each corresponding to rotation over an angular domain of $0 \le \theta_2 < \pi/2$. Hence, with $dt = d\theta_2/\dot{\theta}_2$, the radiant energy intercepted over the period T of one revolution is

$$\Delta E = \int_0^T q dt = \frac{4 I_e\, A_1}{\dot{\theta}_2}\left(\frac{A_2}{r^2}\right)\int_0^{\pi/2} \cos\theta_2 d\theta_2 = \frac{4 I_e\, A_1}{\dot{\theta}_2}\left(\frac{A_2}{r^2}\right)\sin\theta_2 \Big|_0^{\pi/2}$$

$$\Delta E = \frac{4 \times 100\ \text{W}/\text{m}^2 \cdot \text{sr} \times 10^{-4}\text{m}^2}{2\ \text{rad}/\text{s}}\left[\frac{10^{-4}\ \text{m}^2}{(0.50\text{m})^2}\right]\text{sr} = 8 \times 10^{-6}\ \text{J}$$

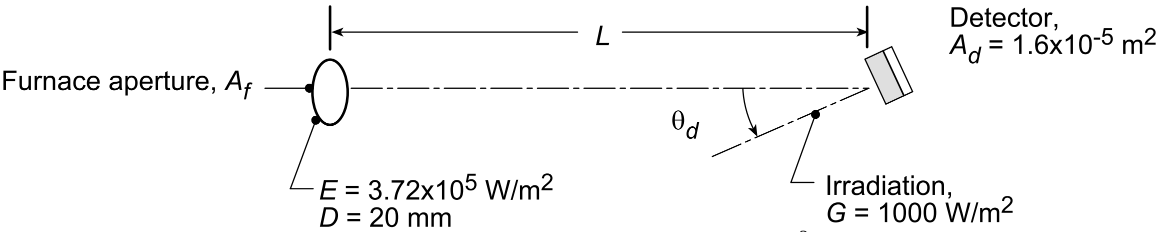
COMMENTS: The maximum rate at which A_2 intercepts radiation corresponds to $\theta_2 = 0$ and is $q_{max} = I_e\, A_1\, A_2/r^2 = 4 \times 10^{-6}$ W. The period of rotation is $T = 2\pi/\dot{\theta}_2 = 3.14$ s.

PROBLEM 12.004

KNOWN: Furnace with prescribed aperture and emissive power.

FIND: (a) Position of gauge such that irradiation is $G = 1000\ W/m^2$, (b) Irradiation when gauge is tilted $\theta_d = 20^o$, and (c) Compute and plot the gage irradiation, G, as a function of the separation distance, L, for the range $100 \le L \le 300$ mm and tilt angles of $\theta_d = 0$, 20, and 60^o.

SCHEMATIC:



ASSUMPTIONS: (1) Furnace aperture emits diffusely, (2) $A_d << L^2$.

ANALYSIS: (a) The irradiation on the detector area is defined as the power incident on the surface per unit area of the surface. That is

$$G = q_{f\to d}/A_d \qquad\qquad q_{f\to d} = I_e A_f \cos\theta_f\ \omega_{d-f} \qquad (1,2)$$

where $q_{f\to d}$ is the radiant power which leaves A_f and is intercepted by A_d. From Eqs. 12.2 and 12.5, ω_{d-f} is the solid angle subtended by surface A_d with respect to A_f,

$$\omega_{d-f} = A_d \cos\theta_d / L^2 . \qquad (3)$$

Noting that since the aperture emits diffusely, $I_e = E/\pi$ (see Eq. 12.14), and hence

$$G = (E/\pi) A_f \cos\theta_f \left(A_d \cos\theta_d / L^2\right) / A_d \qquad (4)$$

Solving for L^2 and substituting for the condition $\theta_f = 0^o$ and $\theta_d = 0^o$,

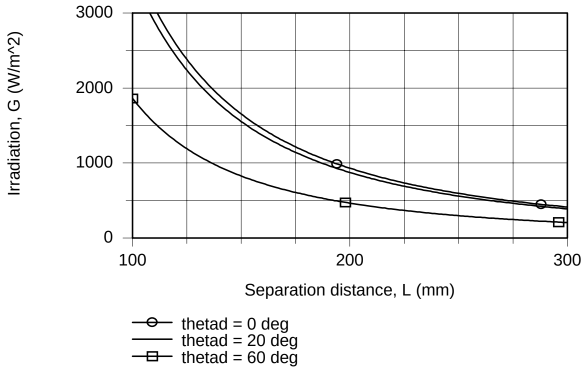
$$L^2 = E\cos\theta_f \cos\theta_d\ A_f / \pi G . \qquad (5)$$

$$L = \left[3.72\times10^5\ W/m^2 \times \frac{\pi}{4}(20\times10^{-3})^2\ m^2 / \pi\times1000\ W/m^2\right]^{1/2} = 193\ mm . \qquad <$$

(b) When $\theta_d = 20^o$, $q_{f\to d}$ will be reduced by a factor of $\cos\theta_d$ since ω_{d-f} is reduced by a factor $\cos\theta_d$. Hence,

$$G = 1000\ W/m^2 \times \cos\theta_d = 1000 W/m^2 \times \cos 20^o = 940\ W/m^2 . \qquad <$$

(c) Using the IHT workspace with Eq. (4), G is computed and plotted as a function of L for selected θ_d. Note that G decreases inversely as L^2. As expected, G decreases with increasing θ_d and in the limit, approaches zero as θ_d approaches 90^o.

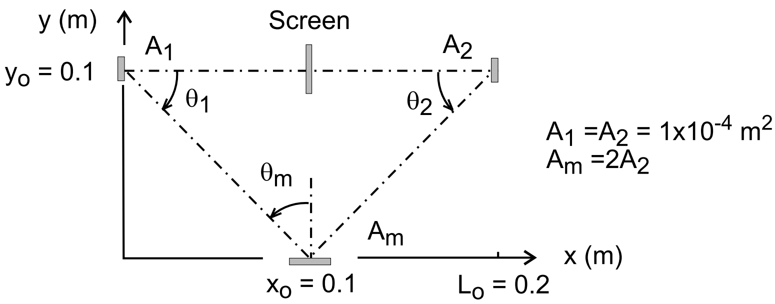


PROBLEM 12.005

KNOWN: Radiation from a diffuse radiant source A_1 with intensity $I_1 = 1.2 \times 10^5$ W/m^2·sr is incident on a mirror A_m, which reflects radiation onto the radiation detector A_2.

FIND: (a) Radiant power incident on A_m due to emission from the source, A_1, $q_{1\rightarrow m}$ (mW), (b) Intensity of radiant power leaving the perfectly reflecting, diffuse mirror A_m, I_m (W/m^2·sr), and (c) Radiant power incident on the detector A_2 due to the reflected radiation leaving A_m, $q_{m\rightarrow 2}$ (μW), (d) Plot the radiant power $q_{m\rightarrow 2}$ as a function of the lateral separation distance y_o for the range $0 \le y_o \le 0.2$ m; explain features of the resulting curve.

SCHEMATIC:



ASSUMPTIONS: (1) Surface A_1 emits diffusely, (2) Surface A_m does not emit, but reflects perfectly and diffusely, and (3) Surface areas are much smaller than the square of their separation distances.

ANALYSIS: (a) The radiant power leaving A_1 that is incident on A_m is

$$q_{1\rightarrow m} = I_1 \cdot A_1 \cdot \cos\theta_1 \cdot \Delta\omega_{m\text{-}1}$$

where $\omega_{m\text{-}1}$ is the solid angle A_m subtends with respect to A_1, Eq. 12.2,

$$\Delta\omega_{m\text{-}1} \equiv \frac{dA_n}{r^2} = \frac{A_m \cos\theta_m}{x_o^2 + y_o^2} = \frac{2\times10^{-4}\ \text{m}^2 \cdot \cos 45°}{\left[0.1^2 + 0.1^2\right]\text{m}^2} = 7.07\times10^{-3}\ \text{sr}$$

with $\theta_m = 90° - \theta_1$ and $\theta_1 = 45°$,

$$q_{1\rightarrow m} = 1.2\times10^5\ \text{W/m}^2\cdot\text{sr}\times1\times10^{-4}\ \text{m}^2\times\cos 45°\times7.07\times10^{-3}\ \text{sr} = 60\ \text{mW} \qquad <$$

(b) The intensity of radiation leaving A_m, after perfect and diffuse reflection, is

$$I_m = \left(q_{1\rightarrow m}/A_m\right)/\pi = \frac{60\times10^{-3}\ \text{W}}{\pi\times2\times10^{-4}\ \text{m}^2} = 95.5\ \text{W/m}^2\cdot\text{sr}$$

(c) The radiant power leaving A_m due to reflected radiation leaving A_m is

$$q_{m\rightarrow 2} = q_2 = I_m \cdot A_m \cdot \cos\theta_m \cdot \Delta\omega_{2-m}$$

where $\Delta\omega_{2\text{-}m}$ is the solid angle that A_2 subtends with respect to A_m, Eq. 12.2,

Continued …..

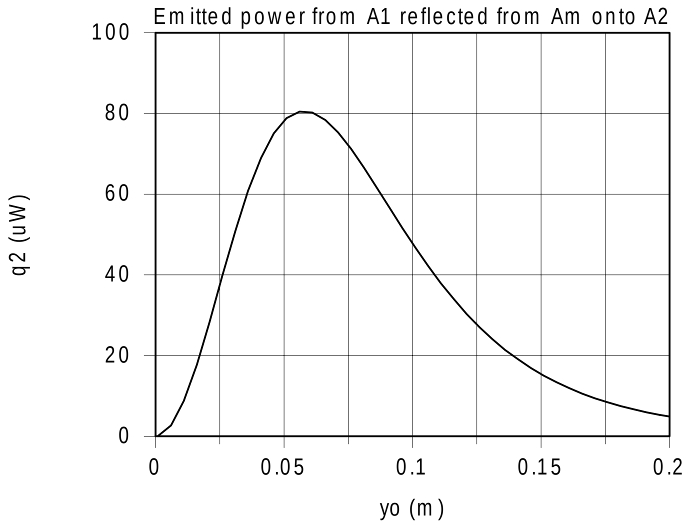
PROBLEM 12.005 (Cont.)

$$\Delta\omega_{2-m} \equiv \frac{dA_n}{r^2} = \frac{A_2 \cos\theta_2}{(L_o - x_o)^2 + y_o^2} = \frac{1\times10^{-4}\ m^2 \times \cos 45°}{\left[0.1^2 + 0.1^2\right] m^2} = 3.54\times10^{-3}\ sr$$

with $\theta_2 = 90° - \theta_m$

$$q_{m\to2} = q_2 = 95.5\ W/m^2 \cdot sr \times 2\times10^4\ m^2 \times \cos 45° \times 3.54\times10^{-3}\ sr = 47.8\ \mu W \quad <$$

(d) Using the foregoing equations in the *IHT* workspace, q_2 is calculated and plotted as a function of y_o for the range $0 \le y_o \le 0.2$ m.



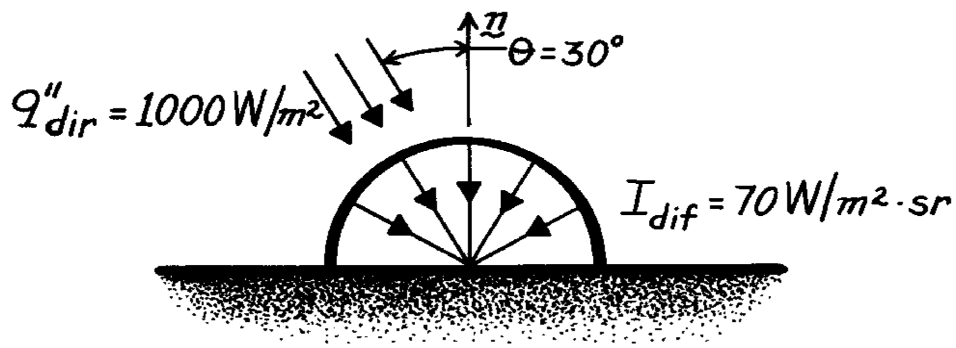
From the relations, note that q_2 is dependent upon the geometric arrangement of the surfaces in the following manner. For small values of y_o, that is, when $\theta_1 \approx 0°$, the cos θ_1 term is at a maximum, near unity. But, the solid angles $\Delta\omega_{m-1}$ and $\Delta\omega_{2-m}$ are very small. As y_o increases, the cos θ_1 term doesn't diminish as much as the solid angles increase, causing q_2 to increase. A maximum in the power is reached as the cos θ_1 term decreases and the solid angles increase. The maximum radiant power occurs when $y_o = 0.058$ m which corresponds to $\theta_1 = 30°$.

PROBLEM 12.6

KNOWN: Flux and intensity of direct and diffuse components, respectively, of solar irradiation.

FIND: Total irradiation.

SCHEMATIC:



ANALYSIS: Since the irradiation is based on the actual surface area, the contribution due to the direct solar radiation is

$$G_{dir} = q''_{dir} \cdot \cos\theta.$$

From Eq. 12.19 the contribution due to the diffuse radiation is

$$G_{dif} = \pi I_{dif}.$$

Hence

$$G = G_{dir} + G_{dif} = q''_{dir} \cdot \cos\theta + \pi I_{dif}$$

or

$$G = 1000\,\mathrm{W/m^2} \times 0.866 + \pi\,\mathrm{sr} \times 70\,\mathrm{W/m^2 \cdot sr}$$

$$G = (866 + 220)\,\mathrm{W/m^2}$$

or

$$G = 1086\,\mathrm{W/m^2}. \qquad <$$

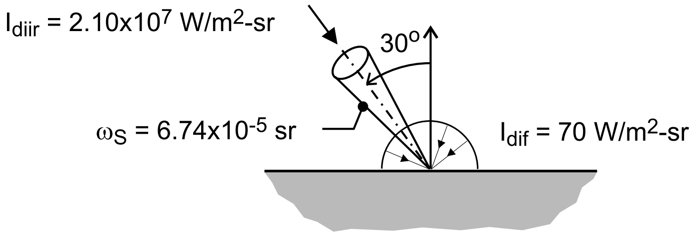
COMMENTS: Although a diffuse approximation is often made for the non-direct component of solar radiation, the actual directional distribution deviates from this condition, providing larger intensities at angles close to the direct beam.

PROBLEM 12.007

KNOWN: Daytime solar radiation conditions with direct solar intensity $I_{dir} = 2.10 \times 10^7$ W/m^2·sr within the solid angle subtended with respect to the earth, $\Delta\omega_S = 6.74 \times 10^{-5}$ sr, and diffuse intensity $I_{dif} = 70$ W/m^2·sr.

FIND: (a) Total solar irradiation at the earth's surface when the direct radiation is incident at 30°, and (b) Verify the prescribed value of $\Delta\omega_S$ recognizing that the diameter of the earth is $D_S = 1.39 \times 10^9$ m, and the distance between the sun and the earth is $r_{e\text{-}S} = 1.496 \times 10^{11}$ m (1 astronomical unity).

SCHEMATIC:



ANALYSIS: (a) The total solar irradiation is the sum of the diffuse and direct components,

$$G_S = G_{dif} + G_{dir} = (220 + 1226)\,W/m^2 = 1446\ W/m^2 \qquad <$$

From Eq. 12.19 the diffuse irradiation is

$$G_{dif} = \pi I_{dif} = \pi\,\text{sr} \times 70\ W/m^2 \cdot \text{sr} = 220\ W/m^2$$

The direct irradiation follows from Eq. 12.15, expressed in terms of the solid angle

$$G_{dir} = I_{dir} \cos\theta\, \Delta\omega_S$$

$$G_{dir} = 2.10 \times 10^7\ W/m^2 \cdot \text{sr} \times \cos 30° \times 6.74 \times 10^{-5}\ \text{sr} = 1226\ W/m^2$$

(b) The solid angle the sun subtends with respect to the earth is calculated from Eq. 12.2,

$$\Delta\omega_S = \frac{dA_n}{r^2} = \frac{\pi D_S^2/4}{r_{e\text{-}S}^2} = \frac{\pi\left(1.39 \times 10^9\ \text{m}\right)^2/4}{\left(1.496 \times 10^{11}\ \text{m}\right)^2} = 6.74 \times 10^{-5}\ \text{sr} \qquad <$$

where dA_n is the projected area of the sun and $r_{e\text{-}S}$, the distance between the earth and sun. We are assuming that $r_{e\text{-}S}^2 >> D_S^2$.

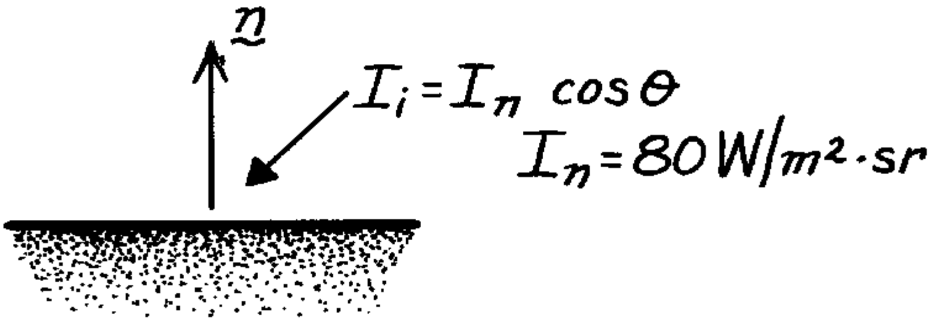
COMMENTS: Can you verify that the direct solar intensity, I_{dir}, is a reasonable value, assuming that the solar disk emits as a black body at 5800 K? $\left(I_{b,S} = \sigma T_S^4/\pi = \sigma(5800\ K)^4/\pi = 2.04 \times 10^7\ W/m^2 \cdot \text{sr}\right)$. Because of local cloud formations, it is possible to have an appreciable diffuse component. But it is not likely to have such a high direct component as given in the problem statement.

PROBLEM 12.8

KNOWN: Directional distribution of solar radiation intensity incident at earth's surface on an overcast day.

FIND: Solar irradiation at earth's surface.

SCHEMATIC:



ASSUMPTIONS: (1) Intensity is independent of azimuthal angle θ.

ANALYSIS: Applying Eq. 12.17 to the total intensity

$$G = \int_0^{2\pi}\int_0^{\pi/2} I_i(\theta)\cos\theta\sin\theta\,d\theta\,d\phi$$

$$G = 2\pi\, I_n \int_0^{\pi/2} \cos^2\theta \sin\theta\, d\theta$$

$$G = (2\pi\,\mathrm{sr})\times 80\,\mathrm{W/m^2\cdot sr}\left(-\frac{1}{3}\cos^3\theta\right)\Bigg|_0^{\pi/2}$$

$$G = -167.6\,\mathrm{W/m^2\cdot sr}\left(\cos^3\frac{\pi}{2} - \cos^3 0\right)$$

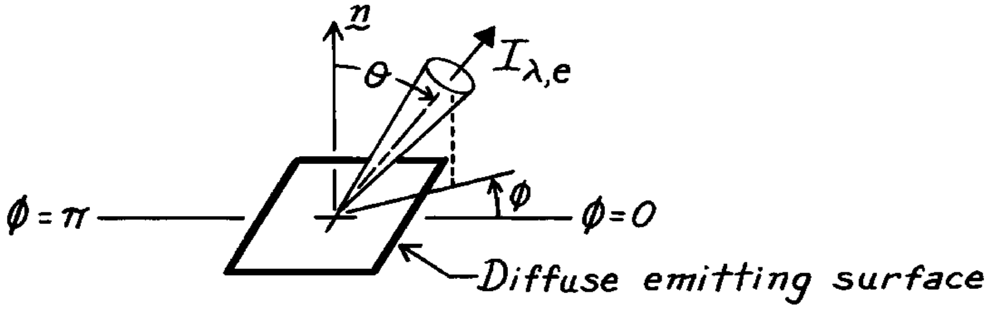
$$G = 167.6\,\mathrm{W/m^2}.$$ <

PROBLEM 12.9

KNOWN: Emissive power of a diffuse surface.

FIND: Fraction of emissive power that leaves surface in the directions $\pi/4 \le \theta \le \pi/2$ and $0 \le \phi \le \pi$.

SCHEMATIC:



ASSUMPTIONS: (1) Diffuse emitting surface.

ANALYSIS: According to Eq. 12.12, the total, hemispherical emissive power is

$$E = \int_0^\infty \int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,e}(\lambda, \theta, \phi) \cos\theta \sin\theta \, d\theta \, d\phi \, d\lambda.$$

For a diffuse surface $I_{\lambda,e}$ (λ, θ, ϕ) is independent of direction, and as given by Eq. 12.14,

$$E = \pi I_e.$$

The emissive power, which has directions prescribed by the limits on θ and ϕ, is

$$\Delta E = \int_0^\infty I_{\lambda,e}(\lambda) d\lambda \left[\int_0^\pi d\phi\right]\left[\int_{\pi/4}^{\pi/2} \cos\theta \sin\theta \, d\theta\right]$$

$$\Delta E = I_e [\phi]_0^\pi \times \left[\frac{\sin^2\theta}{2}\right]_{\pi/4}^{\pi/2} = I_e[\pi]\left[\frac{1}{2}\left(1 - 0.707^2\right)\right]$$

$$\Delta E = 0.25\pi I_e.$$

It follows that

$$\frac{\Delta E}{E} = \frac{0.25\pi I_e}{\pi I_e} = 0.25.$$ <

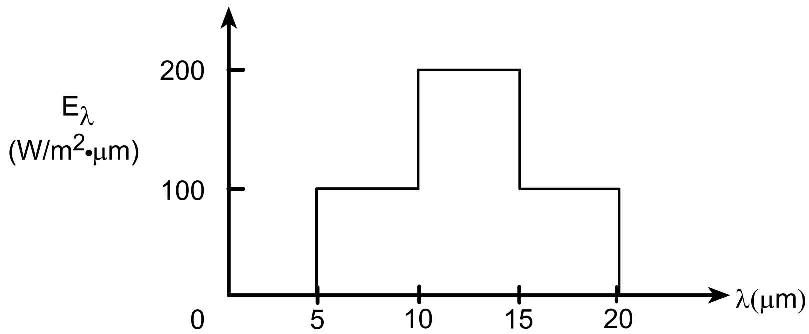
COMMENTS: The diffuse surface is an important concept in radiation heat transfer, and the directional independence of the intensity should be noted.

PROBLEM 12.10

KNOWN: Spectral distribution of E_λ for a diffuse surface.

FIND: (a) Total emissive power E, (b) Total intensity associated with directions $\theta = 0^o$ and $\theta = 30^o$, and (c) Fraction of emissive power leaving the surface in directions $\pi/4 \le \theta \le \pi/2$.

SCHEMATIC:



ASSUMPTIONS: (1) Diffuse emission.

ANALYSIS: (a) From Eq. 12.11 it follows that

$$E = \int_0^\infty E_\lambda(\lambda)\,d\lambda = \int_0^5 (0)\,d\lambda + \int_5^{10}(100)\,d\lambda + \int_{10}^{15}(200)\,d\lambda + \int_{15}^{20}(100)\,d\lambda + \int_{20}^{\infty}(0)\,d\lambda$$

$$E = 100\ W/m^2 \cdot \mu m\ (10 - 5)\ \mu m + 200 W/m^2 \cdot \mu m\ (15 - 10)\ \mu m + 100\ W/m^2 \cdot \mu m\ (20-15)\ \mu m$$

$$E = 2000\ W/m^2$$ <

(b) For a diffuse emitter, I_e is *independent* of θ and Eq. 12.14 gives

$$I_e = \frac{E}{\pi} = \frac{2000\ W/m^2}{\pi\ sr}$$

$$I_e = 637\ W/m^2 \cdot sr$$ <

(c) Since the surface is diffuse, use Eqs. 12.10 and 12.14,

$$\frac{E(\pi/4 \to \pi/2)}{E} = \frac{\int_0^{2\pi}\int_{\pi/4}^{\pi/2} I_e \cos\theta \sin\theta\, d\theta\, d\phi}{\pi I_e}$$

$$\frac{E(\pi/4 \to \pi/2)}{E} = \frac{\int_{\pi/4}^{\pi/2}\cos\theta\sin\theta\,d\theta \int_0^{2\pi} d\phi}{\pi} = \frac{1}{\pi}\left[\frac{\sin^2\theta}{2}\right]_{\pi/4}^{\pi/2} \phi\Big|_0^{2\pi}$$

$$\frac{E(\pi/4 \to \pi/2)}{E} = \frac{1}{\pi}\left[\frac{1}{2}(1^2 - 0.707^2)(2\pi - 0)\right] = 0.50$$ <

COMMENTS: (1) Note how a spectral integration may be performed in parts.

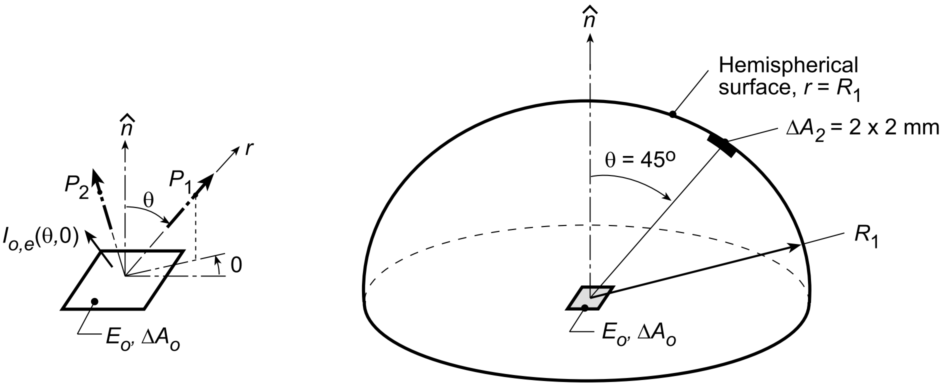
(2) In performing the integration of part (c), recognize the significance of the diffuse emission assumption for which the intensity is uniform in all directions.

PROBLEM 12.11

KNOWN: Diffuse surface ΔA_o, 5-mm square, with total emissive power $E_o = 4000$ W/m^2.

FIND: (a) Rate at which radiant energy is emitted by ΔA_o, q_{emit}; (b) Intensity $I_{o,e}$ of the radiation field emitted from the surface ΔA_o; (c) Expression for q_{emit} presuming knowledge of the intensity $I_{o,e}$ beginning with Eq. 12.10; (d) Rate at which radiant energy is incident on the hemispherical surface, r = R_1 = 0.5 m, due to emission from ΔA_o; (e) Rate at which radiant energy leaving ΔA_o is intercepted by the small area ΔA_2 located in the direction (40^o, ϕ) on the hemispherical surface using Eq. 12.5; also determine the irradiation on ΔA_2; (f) Repeat part (e), for the location (0^o, ϕ); are the irradiations at the two locations equal? and (g) Irradiation G_1 on the hemispherical surface at r = R_1 using Eq. 12.5.

SCHEMATIC:



ASSUMPTIONS: (1) Diffuse surface, ΔA_o, (2) Medium above ΔA_o is also non-participating, (3) $R_1^2 >> \Delta A_o, \Delta A_2$.

ANALYSIS: (a) The radiant power leaving ΔA_o by emission is

$$q_{emit} = E_o \cdot \Delta A_o = 4000 \text{ W/m}^2 (0.005 \text{ m} \times 0.005 \text{ m}) = 0.10 \text{ W} \qquad <$$

(b) The emitted intensity is $I_{o,e}$ and is independent of direction since ΔA_o is a diffuser emitter,

$$I_{o,e} = E_o / \pi = 1273 \text{ W/m}^2 \cdot \text{sr} \qquad <$$

The intensities at points P_1 and P_2 are also $I_{o,e}$ and the intensity in the directions shown in the schematic above will remain constant no matter how far the point is from the surface ΔA_o since the space is non-participating.

(c) From knowledge of $I_{o,e}$, the radiant power leaving ΔA_o from Eq. 12.10 is,

$$q_{emit} = \int_h I_{o,e} \Delta A_o \cos\theta \sin\theta d\theta d\phi = I_{o,e} \Delta A_o \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} \cos\theta \sin\theta d\theta d\phi = \pi I_{o,e} \Delta A_o = 0.10 \text{ W} \qquad <$$

(d) Defining control surfaces above ΔA_o and on A_1, the radiant power leaving ΔA_o must pass through A_1. That is,

$$q_{1,inc} = E_o \Delta A_o = 0.10 \text{ W} \qquad <$$

Recognize that the average irradiation on the hemisphere, A_1, where $A_1 = 2\pi R_1^2$, based upon the definition, Section 12.2.3,

$$\overline{G}_1 = q_{1,inc} / A_1 = E_o \Delta A_o / 2\pi R_1^2 = 63.7 \text{ mW/m}^2$$

where $q_{1,inc}$ is the radiant power incident on surface A_1.

Continued...

PROBLEM 12.11 (Cont.)

(e) The radiant power leaving ΔA_o intercepted by ΔA_2, where $\Delta A_2 = 4\times10^{-6}$ m^2, located at ($\theta = 45^o$, ϕ) as per the schematic, follows from Eq. 12.5,

$$q_{\Delta A_o \to \Delta A_2} = I_{o,e} \Delta A_o \cos\theta_o \Delta\omega_{2-o}$$

where $\theta_o = 45^o$ and the solid angle ΔA_2 subtends with respect to ΔA_o is

$$\Delta\omega_{2-o} = \Delta A_2 \cos\theta_2 \big/ R_1^2 = 4\times10^{-6}\,\mathrm{m}^2 \cdot 1\big/(0.5\mathrm{m})^2 = 1.60\times10^{-5}\,\mathrm{sr}$$

where $\theta_2 = 0^o$, the direction normal to ΔA_2,

$$q_{\Delta A_o \to \Delta A_2} = 1273\,\mathrm{W}\big/\mathrm{m}^2 \cdot \mathrm{sr} \times 25\times10^{-6}\,\mathrm{m}^2 \cos 45^o \times 1.60\times10^{-5}\,\mathrm{sr} = 3.60\times10^{-7}\ \mathrm{W} \qquad <$$

From the definition of irradiation, Section 12.2.3,

$$G_2 = q_{\Delta A_o \to \Delta A_2} \big/ \Delta A_2 = 90\,\mathrm{mW}\big/\mathrm{m}^2$$

(f) With ΔA_2, located at ($\theta = 0^o$, ϕ), where $\cos\theta_o = 1$, $\cos\theta_2 = 1$, find

$$\Delta\omega_{2-o} = 1.60\times10^{-5}\,\mathrm{sr} \qquad q_{\Delta A_o \to \Delta A_2} = 5.09\times10^{-7}\ \mathrm{W} \qquad G_2 = 127\,\mathrm{mW}\big/\mathrm{m}^2 \qquad <$$

Note that the irradiation on ΔA_2 when it is located at (0^o, ϕ) is larger than when ΔA_2 is located at (45^o, ϕ); that is, 127 mW/m^2 > 90 W/m^2. Is this intuitively satisfying?

(g) Using Eq. 12.15, based upon Figure 12.10, find

$$\overline{G}_1 = \int_h I_{1,i} dA_1 \cdot d\omega_{0-1} / A_1 = \pi I_{o,e}\, \Delta A_o / \Delta A_1 = 63.7\,\mathrm{mW}\big/\mathrm{m}^2 \qquad <$$

where the elemental area on the hemispherical surface A_1 and the solid angle ΔA_o subtends with respect to ΔA_1 are, respectively,

$$dA_1 = R_1^2 \sin\theta\, d\theta\, d\phi \qquad\qquad d\omega_{o-1} = \Delta A_o \cos\theta \big/ R_1^2$$

From this calculation you found that the *average* irradiation on the hemisphere surface, $r = R_1$, is $\overline{G}_1 = 63.7\,\mathrm{mW}\big/\mathrm{m}^2$. From parts (e) and (f), you found irradiations, G_2 on ΔA_2 at (0^o, ϕ) and (45^o, ϕ) as 127 mW/m^2 and 90 mW/m^2, respectively. Did you expect $\overline{G}_1$ to be less than either value for G_2? How do you explain this?

COMMENTS: (1) Note that from Parts (e) and (f) that the irradiation on A_1 is not uniform. Parts (d) and (g) give an average value.

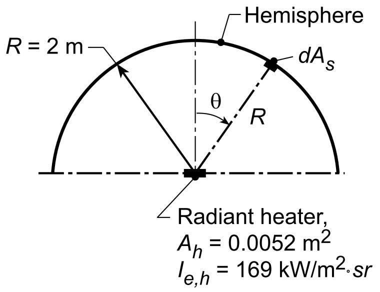
(2) What conclusions would you reach regarding G_1 if ΔA_o were a sphere?

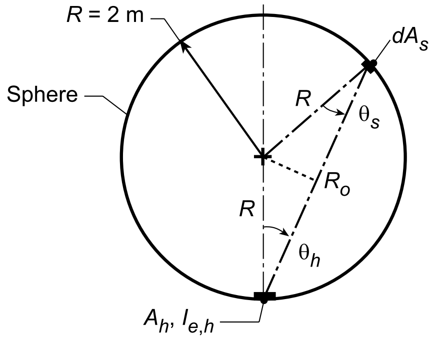
PROBLEM 12.12

KNOWN: Hemispherical and spherical arrangements for radiant heat treatment of a thin-film material. Heater emits diffusely with intensity $I_{e,h}$ = 169,000 W/ $m^2 \cdot sr$ and has an area 0.0052 m^2.

FIND: (a) Expressions for the irradiation on the film as a function of the zenith angle, θ, and (b) Identify arrangement which provides the more uniform irradiation, and hence better quality control for the treatment process.

SCHEMATIC:





ASSUMPTIONS: (1) Heater emits diffusely, (2) All radiation leaving the heater is absorbed by the thin film.

ANALYSIS: (a) The irradiation on any differential area, dA_s, due to emission from the heater, A_h , follows from its definition, Section 12.2.3,

$$G = \frac{q_{h \to s}}{dA_s} \tag{1}$$

Where $q_{h\to s}$ is the radiant heat rate leaving A_h and intercepted by dA_s. From Eq. 12.5,

$$q_{h \to s} = I_{e,h} \cdot dA_h \cos\theta_h \cdot \omega_{s-h} \tag{2}$$

where ω_{s-h} is the solid angle dA_s subtends with respect to any point on A_h. From the definition, Eq. 12.2,

$$\omega = \frac{dA_n}{r^2} \tag{3}$$

where dA_n is normal to the viewing direction and r is the separation distance.

For the hemisphere: Referring to the schematic above, the solid angle is

$$\omega_{s-h} = \frac{dA_s}{R^2}$$

and the irradiation distribution on the hemispheric surface as a function of θ_h is

$$G = I_{e,h} A_h \cos\theta_h / R^2 \tag{1}$$ <

For the sphere: From the schematic, the solid angle is

$$\omega_{s,h} = \frac{dA_s \cos\theta_s}{R_o^2} = \frac{dA_s}{4R^2 \cos\theta_h}$$

where R_o, from the geometry of sphere cord and radii with $\theta_s = \theta_h$, is

Continued...

PROBLEM 12.12 (Cont.)

$$R_o = 2R\cos\theta_h$$

and the irradiation distribution on the spherical surface as a function of θ_h is

$$G = I_{e,h} dA_h \big/ 4R^2 \qquad (2) <$$

(b) The spherical shape provides more uniform irradiation as can be seen by comparing Eqs. (1) and (2). In fact, for the spherical shape, the irradiation on the thin film is uniform and therefore provides for better quality control for the treatment process. Substituting numerical values, the irradiations are:

$$G_{hem} = 169{,}000\,\mathrm{W/m^2 \cdot sr} \times 0.0052\mathrm{m^2}\cos\theta_h \big/ (2\mathrm{m})^2 = 219.7\cos\theta_h\ \mathrm{W/m^2} \qquad (3)$$

$$G_{sph} = 169{,}000\,\mathrm{W/m^2 \cdot sr} \times 0.0052\,\mathrm{m^2} \big/ 4(2\mathrm{m})^2 = 54.9\,\mathrm{W/m^2} \qquad (4)$$

COMMENTS: (1) The radiant heat rate leaving the diffuse heater surface by emission is

$$q_{tot} = \pi I_{e,h} A_h = 276.1\mathrm{W}$$

The average irradiation on the *spherical surface*, $A_{sph} = 4\pi R^2$,

$$\bar{G}_{sph} = q_{tot} \big/ A_{sph} = 276.1\mathrm{W} \big/ 4\pi(2\mathrm{m})^2 = 54.9\,\mathrm{W/m^2}$$

while the average irradiation on the *hemispherical surface*, $A_{hem} = 2\pi R^2$ is

$$\bar{G}_{hem} = 276.1\mathrm{W} \big/ 2\pi(2\mathrm{m})^2 = 109.9\,\mathrm{W/m^2}$$

(2) Note from the foregoing analyses for the *sphere* that the result for $\bar{G}_{sph}$ is identical to that found as Eq. (4). That follows since the irradiation is uniform.

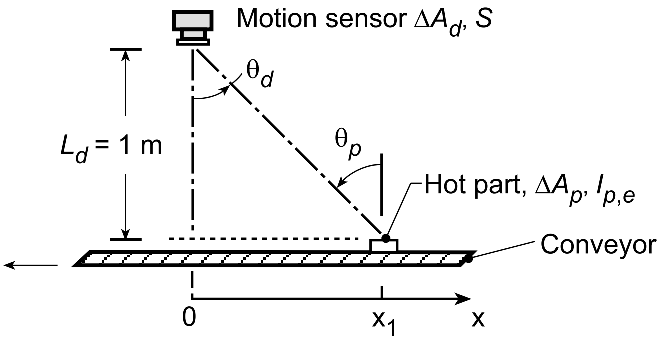
(3) Note that $\bar{G}_{hem} > \bar{G}_{sph}$ since the surface area of the hemisphere is half that of the sphere. Recognize that for the hemisphere thin film arrangement, the distribution of the irradiation is quite variable with a maximum at $\theta = 0°$ (top) and half the maximum value at $\theta = 30°$.

PROBLEM 12.13

KNOWN: Hot part, ΔA_p, located a distance x_1 from an origin directly beneath a motion sensor at a distance $L_d = 1$ m.

FIND: (a) Location x_1 at which sensor signal S_1 will be 75% that corresponding to x = 0, directly beneath the sensor, S_o, and (b) Compute and plot the signal ratio, S/S_o, as a function of the part position x_1 for the range $0.2 \le S/S_o \le 1$ for L_d = 0.8, 1.0 and 1.2 m; compare the x-location for each value of L_d at which $S/S_o = 0.75$.

SCHEMATIC:



ASSUMPTIONS: (1) Hot part is diffuse emitter, (2) $L_d^2 >> \Delta A_p, \Delta A_o$.

ANALYSIS: (a) The sensor signal, S, is proportional to the radiant power leaving ΔA_p and intercepted by ΔA_d,

$$S \sim q_{p\to d} = I_{p,e}\Delta A_p \cos\theta_p \Delta\omega_{d-p} \tag{1}$$

when

$$\cos\theta_p = \cos\theta_d = \frac{L_d}{R} = L_d \Big/ (L_d^2 + x_1^2)^{1/2} \tag{2}$$

$$\Delta\omega_{d-p} = \frac{\Delta A_d \cdot \cos\theta_d}{R^2} = \Delta A_d \cdot L_d \Big/ (L_d^2 + x_1^2)^{3/2} \tag{3}$$

Hence,

$$q_{p\to d} = I_{p,e}\Delta A_p \Delta A_d \frac{L_d^2}{(L_d^2 + x_1^2)^2} \tag{4}$$

It follows that, with S_o occurring when x= 0 and L_d = 1 m,

$$\frac{S}{S_o} = \frac{L_d^2 \Big/ (L_d^2 + x_1^2)^2}{L_d^2 \Big/ (L_d^2 + 0^2)^2} = \left[\frac{L_d^2}{L_d^2 + x_1^2}\right]^2 \tag{5}$$

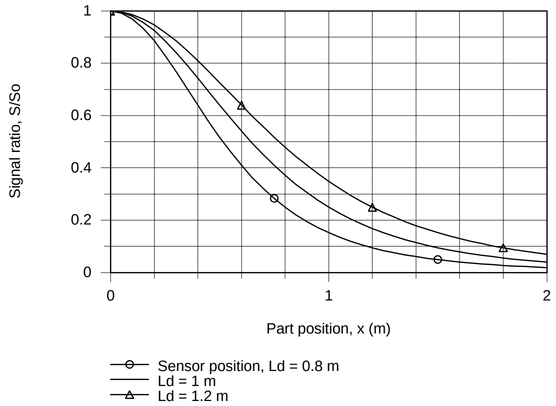
so that when $S/S_o = 0.75$, find,

$x_1 = 0.393$ m <

(b) Using Eq. (5) in the IHT workspace, the signal ratio, S/S_o, has been computed and plotted as a function of the part position x for selected L_d values.

Continued...

PROBLEM 12.13 (Cont.)



When the part is directly under the sensor, x = 0, $S/S_o = 1$ for all values of L_d. With increasing x, S/S_o decreases most rapidly with the smallest L_d. From the IHT model we found the part position x corresponding to $S/S_o = 0.75$ as follows.

S/S_o	L_d (m)	x_1 (m)
0.75	0.8	0.315
0.75	1.0	0.393
0.75	1.2	0.472

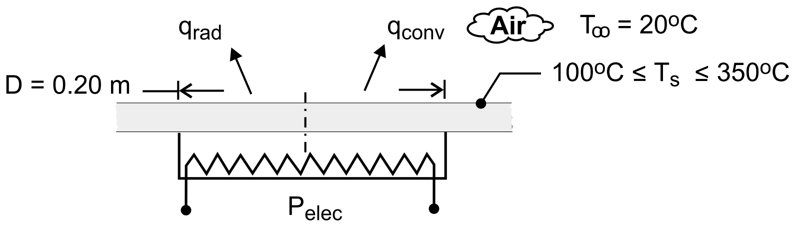
If the sensor system is set so that when S/S_o reaches 0.75 a process is initiated, the technician can use the above plot and table to determine at what position the part will begin to experience the treatment process.

PROBLEM 12.14

KNOWN: Diameter and temperature of burner. Temperature of ambient air. Burner efficiency.

FIND: (a) Radiation and convection heat rates, and wavelength corresponding to maximum spectral emission. Rate of electric energy consumption. (b) Effect of burner temperature on convection and radiation rates.

SCHEMATIC:



ASSUMPTIONS: (1) Burner emits as a blackbody, (2) Negligible irradiation of burner from surrounding, (3) Ambient air is quiescent, (4) Constant properties.

PROPERTIES: *Table A-4*, air (T_f = 408 K): k = 0.0344 W/m·K, $\nu = 27.4 \times 10^{-6}$ m^2/s, $\alpha = 39.7 \times 10^{-6}$ m^2/s, Pr = 0.70, $\beta = 0.00245$ K^{-1}.

ANALYSIS: (a) For emission from a black body

$$q_{rad} = A_s\ E_b = \left(\pi D^2/4\right)\sigma T^4 = \left[\pi\left(0.2m\right)^2/4\right]5.67\times10^{-8}\ W/m^2\cdot K^4\left(523K\right)^4 = 133\ W \qquad <$$

With L = A_s/P = D/4 = 0.05m and $Ra_L = g\beta(T_s - T_\infty)\ L^3/\alpha\nu$ = 9.8 m/s^2 × 0.00245 K^{-1} (230 K) (0.05m)^3/(27.4 × 39.7 × 10^{-12} m^4/s^2) = 6.35 × 10^5, Eq. (9.30) yields

$$\bar{h} = \frac{k}{L}\overline{Nu}_L = \left(\frac{k}{L}\right)0.54\ Ra_L^{1/4} = \left(\frac{0.0344\ W/m\cdot K}{0.05m}\right)0.54\left(6.35\times10^5\right)^{1/4} = 10.5\ W/m^2\cdot K$$

$$q_{conv} = \bar{h}A_s\left(T_s - T_\infty\right) = 19.4\ W/m^2\cdot K\left[\pi\left(0.2m\right)^2/4\right]230K = 75.7\ W \qquad <$$

The electric power requirement is then

$$P_{elec} = \frac{q_{rad} + q_{conv}}{\eta} = \frac{\left(133 + 75.7\right)W}{0.9} = 232\ W \qquad <$$

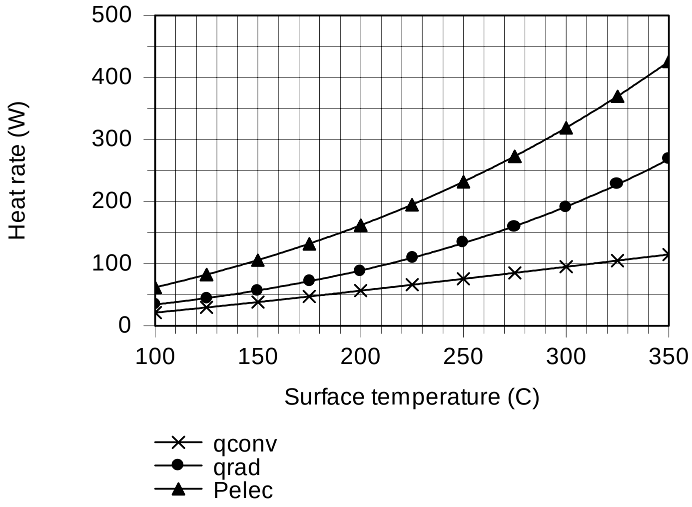
The wavelength corresponding to peak emission is obtained from Wien's law, Eq. (12.27)

$$\lambda_{max} = 2898\mu m\cdot K/523K = 5.54\mu m \qquad <$$

(b) As shown below, and as expected, the radiation rate increases more rapidly with temperature than the convection rate due to its stronger temperature dependence $\left(T_s^4\ \text{vs.}\ T_s^{5/4}\right)$.

Continued …..

PROBLEM 12.14(Cont.)



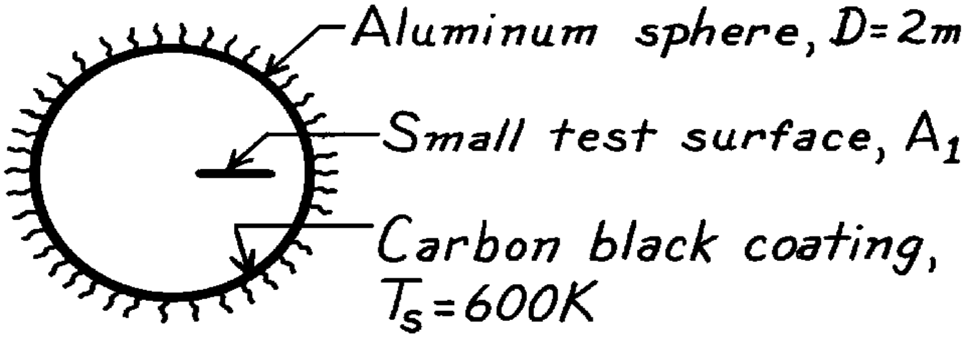
COMMENTS: If the surroundings are treated as a large enclosure with isothermal walls at $T_{sur} = T_\infty = 293$ K, irradiation of the burner would be $G = \sigma T_{sur}^4 = 418$ W/m^2 and the corresponding heat rate would be $A_s G = 13$ W. This input is much smaller than the energy outflows due to convection and radiation and is justifiably neglected.

PROBLEM 12.15

KNOWN: Evacuated, aluminum sphere (D = 2m) serving as a radiation test chamber.

FIND: Irradiation on a small test object when the inner surface is lined with carbon black and at 600K. What effect will surface coating have?

SCHEMATIC:



ASSUMPTIONS: (1) Sphere walls are isothermal, (2) Test surface area is small compared to the enclosure surface.

ANALYSIS: It follows from the discussion of Section 13.3 that this isothermal sphere is an enclosure behaving as a blackbody. For such a condition, see Fig. 12.12(c), the irradiation on a small surface within the enclosure is equal to the blackbody emissive power at the temperature of the enclosure. That is

$$G_1 = E_b\left(T_s\right) = \sigma T_s^4$$

$$G_1 = 5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\left(600\mathrm{K}\right)^4 = 7348\,\mathrm{W/m^2}. \qquad <$$

The irradiation is independent of the nature of the enclosure surface coating properties.

COMMENTS: (1) The irradiation depends only upon the enclosure surface temperature and is independent of the enclosure surface properties.

(2) Note that the test surface area must be small compared to the enclosure surface area. This allows for inter-reflections to occur such that the radiation field, within the enclosure will be uniform (diffuse) or isotropic.

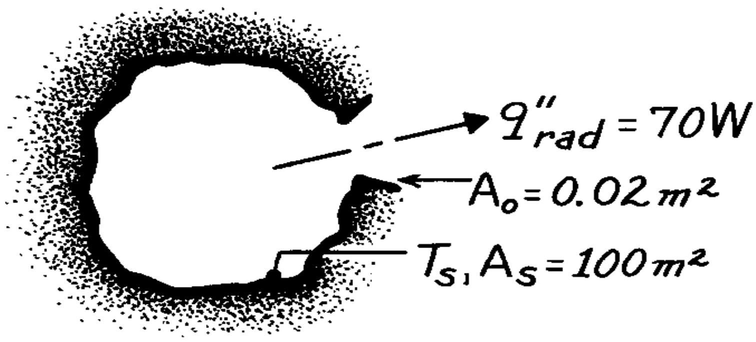
(3) The irradiation level would be the same if the enclosure were not evacuated since, in general, air would be a non-participating medium.

PROBLEM 12.16

KNOWN: Isothermal enclosure of surface area, A_s, and small opening, A_o, through which 70W emerges.

FIND: (a) Temperature of the interior enclosure wall if the surface is black, (b) Temperature of the wall surface having $\varepsilon = 0.15$.

SCHEMATIC:



ASSUMPTIONS: (1) Enclosure is isothermal, (2) $A_o \ll A_s$.

ANALYSIS: A characteristic of an isothermal enclosure, according to Section 12.3, is that the radiant power emerging through a small aperture will correspond to blackbody conditions. Hence

$$q_{rad} = A_o E_b(T_s) = A_o \sigma T_s^4$$

where q_{rad} is the radiant power leaving the enclosure opening. That is,

$$T_s = \left(\frac{q_{rad}}{A_o \sigma}\right)^{1/4} = \left(\frac{70\text{W}}{0.02\text{m}^2 \times 5.670 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4}\right)^{1/4} = 498\text{K}. \quad <$$

Recognize that the radiated power will be independent of the emissivity of the wall surface. As long as $A_o \ll A_s$ *and* the enclosure is isothermal, then the radiant power will depend only upon the temperature.

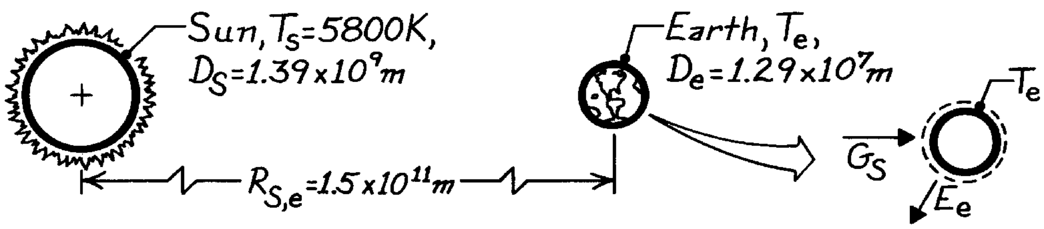
COMMENTS: It is important to recognize the unique characteristics of isothermal enclosures. See Fig. 12.12 to identify them.

PROBLEM 12.17

KNOWN: Sun has equivalent blackbody temperature of 5800 K. Diameters of sun and earth as well as separation distance are prescribed.

FIND: Temperature of the earth assuming the earth is black.

SCHEMATIC:



ASSUMPTIONS: (1) Sun and earth emit as blackbodies, (2) No attenuation of solar irradiation enroute to earth, and (3) Earth atmosphere has no effect on earth energy balance.

ANALYSIS: Performing an energy balance on the earth,

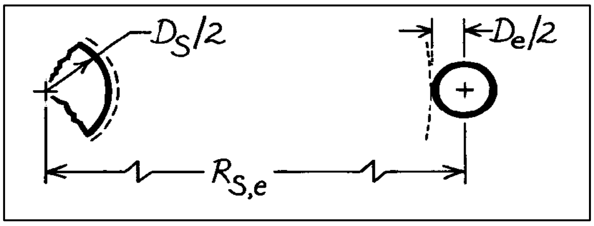
$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$A_{e,p} \cdot G_S = A_{e,s} \cdot E_b(T_e)$$

$$\left(\pi D_e^2/4\right) G_S = \pi D_e^2 \sigma T_e^4$$

$$T_e = \left(G_S/4\sigma\right)^{1/4}$$

where $A_{e,p}$ and $A_{e,s}$ are the projected area and total surface area of the earth, respectively. To determine the irradiation G_S at the earth's surface, equate the rate of emission from the sun to the rate at which this radiation passes through a spherical surface of radius $R_{S,e} - D_e/2$.



$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$\pi D_S^2 \cdot \sigma T_S^4 = 4\pi \left[R_{S,e} - D_e/2\right]^2 G_S$$

$$\pi\left(1.39\times10^9 \text{ m}\right)^2 \times 5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4 (5800 \text{ K})^4$$

$$= 4\pi\left[1.5\times10^{11} - 1.29\times10^7/2\right]^2 \text{ m}^2 \times G_S$$

$$G_S = 1377.5 \text{ W/m}^2.$$

Substituting numerical values, find

$$T_e = \left(1377.5 \text{ W/m}^2 / 4\times5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4\right)^{1/4} = 279 \text{ K}. \qquad <$$

COMMENTS: (1) The average earth's temperature is greater than 279 K since the effect of the atmosphere is to reduce the heat loss by radiation.

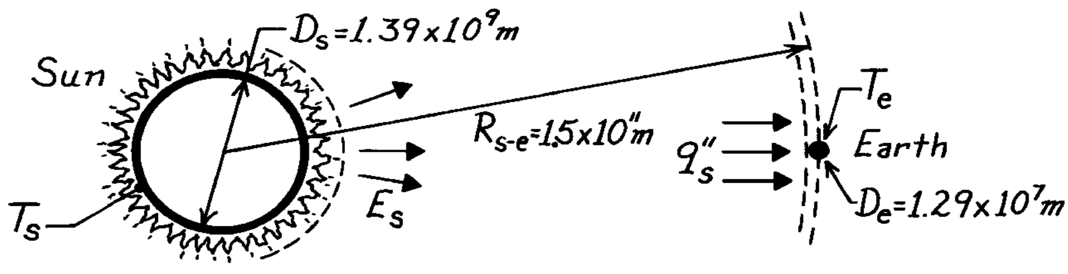
(2) Note carefully the different areas used in the earth energy balance. Emission occurs from the total spherical area, while solar irradiation is absorbed by the projected spherical area.

PROBLEM 12.18

KNOWN: Solar flux at outer edge of earth's atmosphere, 1353 W/m^2.

FIND: (a) Emissive power of sun, (b) Surface temperature of sun, (c) Wavelength of maximum solar emission, (d) Earth equilibrium temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Sun and earth emit as blackbodies, (2) No attenuation of solar radiation enroute to earth, (3) Earth atmosphere has no effect on earth energy balance.

ANALYSIS: (a) Applying conservation of energy to the solar energy crossing two concentric spheres, one having the radius of the sun and the other having the radial distance from the edge of the earth's atmosphere to the center of the sun

$$E_s\left(\pi D_s^2\right) = 4\pi\left(R_{s-e} - \frac{D_e}{2}\right)^2 q''_s.$$

Hence

$$E_s = \frac{4\left(1.5\times10^{11}\,\mathrm{m} - 0.65\times10^7\,\mathrm{m}\right)^2 \times 1353\,\mathrm{W/m^2}}{\left(1.39\times10^9\,\mathrm{m}\right)^2} = 6.302\times10^7\,\mathrm{W/m^2}. \qquad <$$

(b) From Eq. 12.28, the temperature of the sun is

$$T_s = \left(\frac{E_s}{\sigma}\right)^{1/4} = \left(\frac{6.302\times10^7\ \mathrm{W/m^2}}{5.67\times10^{-8}\ \mathrm{W/m^2\cdot K^4}}\right)^{1/4} = 5774\ \mathrm{K}. \qquad <$$

(c) From Wien's law, Eq. 12.27, the wavelength of maximum emission is

$$\lambda_{max} = \frac{C_3}{T} = \frac{2897.6\,\mu\mathrm{m\cdot K}}{5774\ \mathrm{K}} = 0.50\ \mu\mathrm{m}.$$

(d) From an energy balance on the earth's surface

$$E_e\left(\pi D_e^2\right) = q''_S\left(\pi D_e^2/4\right).$$

Hence, from Eq. 12.28,

$$T_e = \left(\frac{q''_S}{4\sigma}\right)^{1/4} = \left(\frac{1353\ \mathrm{W/m^2}}{4\times5.67\times10^{-8}\ \mathrm{W/m^2\cdot K^4}}\right)^{1/4} = 278\mathrm{K}. \qquad <$$

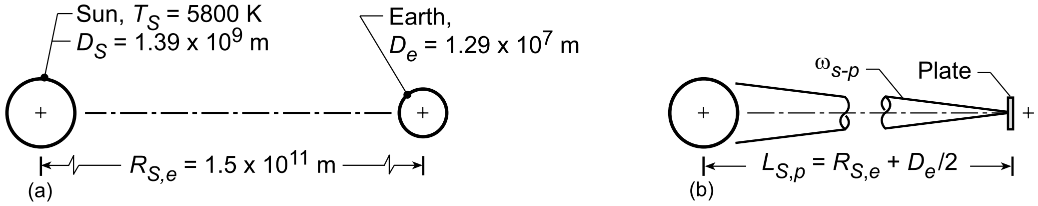
COMMENTS: The average earth temperature is higher than 278 K due to the shielding effect of the earth's atmosphere (transparent to solar radiation but not to longer wavelength earth emission).

PROBLEM 12.19

KNOWN: Small flat plate positioned just beyond the earth's atmosphere oriented such that its normal passes through the center of the sun. Pertinent earth-sun dimensions from Problem 12.18.

FIND: (a) Solid angle subtended by the sun about a point on the surface of the plate, (b) Incident intensity, I_i , on the plate using the known value of the solar irradiation about the earth's atmosphere, $G_S = 1353\ W/m^2$, and (c) Sketch of the incident intensity as a function of the zenith angle θ, where θ is measured from the normal to the plate.

SCHEMATIC:



ASSUMPTIONS: (1) Plate oriented normal to centerline between sun and earth, (2) Height of earth's atmosphere negligible compared to distance from the sun to the plate, (3) Dimensions of the plate are very small compared to sun-earth dimensions.

ANALYSIS: (a) The pertinent sun-earth dimensions are shown in the schematic (a) above while the position of the plate relative to the sun and the earth is shown in (b). The solid angle subtended by the sun with respect to any point on the plate follows from Eq. 12.2,

$$\omega_{S-p} = \frac{A_S \cos\theta_p}{L_{S,p}^2} = \frac{\left(\pi D_S^2/4\right)\cos\theta_p}{\left(R_{S,e} + D_{e/2}\right)^2} = \frac{\pi\left(1.39\times10^9\,m\right)^2/4\times1}{\left(1.5\times10^{11}m + 1.29\times10^7\,m/2\right)^2} = 6.74\times10^{-5}sr \quad (1) <$$

where A_S is the projected area of the sun (the solar disk), θ_p is the zenith angle measured between the plate normal and the centerline between the sun and earth, and $L_{S,p}$ is the separation distance between the plate at the sun's center.

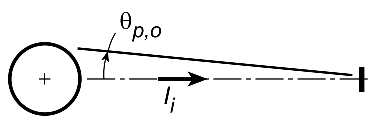
(b) The plate is irradiated by solar flux in the normal direction only (not diffusely). Using Eq. (12.7), the radiant power incident on the plate can be expressed as

$$G_S\Delta A_p = I_i \cdot \Delta A_p \cos\theta_p \cdot \omega_{S-p} \quad (2)$$

and the intensity I_i due to the solar irradiation G_S with $\cos\theta_p = 1$,

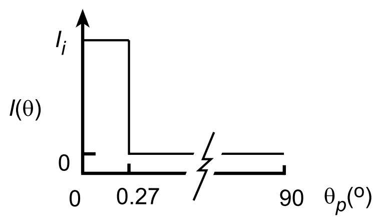
$$I_i = G_S/\omega_{S-p} = 1353\,W/m^2/6.74\times10^{-5}\,sr = 2.01\times10^7\,W/m^2\cdot sr \quad <$$

(c) As illustrated in the schematic to the right, the intensity I_i will be constant for the zenith angle range $0 \le \theta_p \le \theta_{p,o}$ where



$$\theta_{p,o} = \frac{D_S/2}{L_{S,p}} = \frac{1.39\times10^9\,m/2}{\left(1.5\times10^{11}m + 1.29\times10^7\,m/2\right)}$$

$$\theta_{p,o} = 4.633\times10^{-3}rad \approx 0.27^\circ$$



For the range $\theta_p > \theta_{p,o}$, the intensity will be zero. Hence the I_i as a function of θ_p will appear as shown to the right.

PROBLEM 12.20

KNOWN: Various surface temperatures.

FIND: (a) Wavelength corresponding to maximum emission for each surface, (b) Fraction of solar emission in UV, VIS and IR portions of the spectrum.

ASSUMPTIONS: (1) Spectral distribution of emission from each surface is approximately that of a blackbody, (2) The sun emits as a blackbody at 5800 K.

ANALYSIS: (a) From Wien's law, Eq. 12.27, the wavelength of maximum emission for blackbody radiation is

$$\lambda_{max} = \frac{C_3}{T} = \frac{2897.6\ \mu m \cdot K}{T}.$$

For the prescribed surfaces

Surface	Sun (5800K)	Tungsten (2500K)	Hot metal (1500K)	Skin (305K)	Cool metal (60K)	
$\lambda_{max}(\mu m)$	0.50	1.16	1.93	9.50	48.3	<

(b) From Fig. 12.3, the spectral regions associated with each portion of the spectrum are

Spectrum	**Wavelength limits, μm**
UV	**0.0 – 0.4**
VIS	**0.4 – 0.7**
IR	**0.7 - 100**

For T = 5800K and each of the wavelength limits, from Table 12.1 find:

$\lambda(\mu m)$	10^{-2}	0.4	0.7	10^{2}
$\lambda T(\mu m\cdot K)$	58	2320	4060	5.8×10^{5}
$F_{(0\to\lambda)}$	0	0.125	0.491	1

Hence, the fraction of the solar emission in each portion of the spectrum is:

$F_{UV} = 0.125 - 0 = 0.125$ <

$F_{VIS} = 0.491 - 0.125 = 0.366$ <

$F_{IR} = 1 - 0.491 = 0.509.$ <

COMMENTS: (1) Spectral concentration of surface radiation depends strongly on surface temperature.

(2) Much of the UV solar radiation is absorbed in the earth's atmosphere.

PROBLEM 12.21

KNOWN: Visible spectral region 0.47 μm (blue) to 0.65 μm (red). Daylight and incandescent lighting corresponding to blackbody spectral distributions from the solar disk at 5800 K and a lamp bulb at 2900 K, respectively.

FIND: (a) Band emission fractions for the visible region for these two lighting sources, and (b) wavelengths corresponding to the maximum spectral intensity for each of the light sources. Comment on the results of your calculations considering the rendering of true colors under these lighting conditions.

ASSUMPTIONS: Spectral distributions of radiation from the sources approximates those of blackbodies at their respective temperatures.

ANALYSIS: (a) From Eqs. 12.30 and 12.31, the band-emission fraction in the spectral range λ_1 to λ_2 at a blackbody temperature T is

$$F_{(\lambda 1-\lambda 2, T)} = F_{(0\to\lambda 2, T)} - F_{(0\to\lambda 1, T)}$$

where the $F_{(0\to\lambda T)}$ values can be read from Table 12.1 (or, more accurately calculated using the *IHT Radiation | Band Emission* tool)

Daylight source (T = 5800 K)

$$F_{(\lambda 1-\lambda 2, T)} = 0.4374 - 0.2113 = 0.2261 \qquad <$$

where at $\lambda_2 \cdot T = 0.65\ \mu m \times 5800\ K = 3770\ \mu m \cdot K$, find $F_{(0-\lambda T)} = 0.4374$, and at $\lambda_1 \cdot T = 0.47\ \mu m \times 5800\ K = 2726\ \mu m \cdot K$, find $F_{(0-\lambda T)} = 0.2113$.

Incandescent source (T = 2900 K)

$$F_{(\lambda 1-\lambda 2, T)} = 0.05098 - 0.00674 = 0.0442 \qquad <$$

(b) The wavelengths corresponding to the peak spectral intensity of these blackbody sources can be found using Wien's law, Eq. 12.27.

$$\lambda_{max} = C_3 / T = 2898\ \mu m \cdot K$$

For the daylight (d) and incandescent (i) sources, find

$$\lambda_{max,\ d} = 2898\ \mu m \cdot K / 5800\ K = 0.50\ \mu m \qquad <$$

$$\lambda_{max,\ i} = 2898\ \mu m \cdot K / 2800\ K = 1.0\ \mu m \qquad <$$

COMMENTS: (1) From the band-emission fraction calculation, part (a), note the substantial difference between the fractions for the daylight and incandescent sources. The fractions are a measure of the relative amount of total radiant power that is useful for lighting (visual illumination).

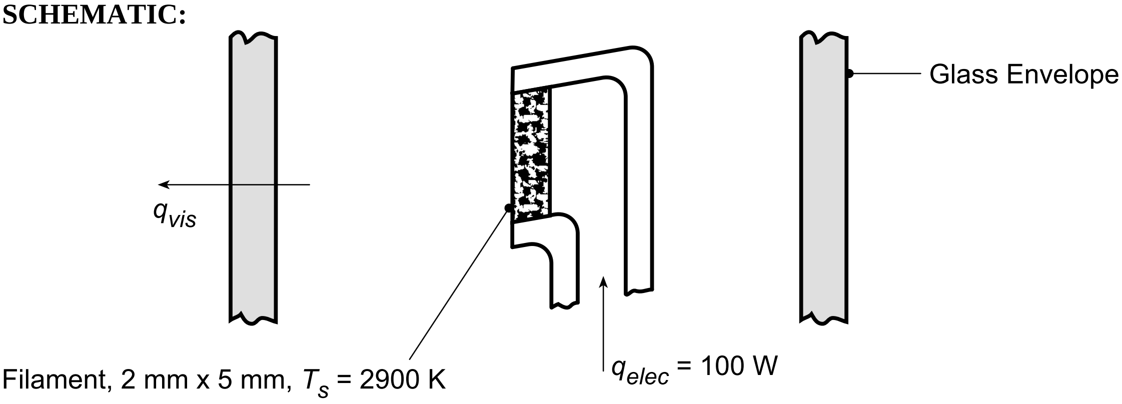
(2) For the daylight source, the peak of the spectral distribution is at 0.5 μm within the visible spectral region. In contrast, the peak for the incandescent source at 1 μm is considerably outside the visible region. For the daylight source, the spectral distribution is "flatter" (around the peak) than that for the incandescent source for which the spectral distribution is decreasing markedly with decreasing wavelength (on the short-wavelength side of the blackbody curve). The eye has a bell-shaped relative spectral response within the visible, and will therefore interpret colors differently under illumination by the two sources. In daylight lighting, the colors will be more "true," whereas with incandescent lighting, the shorter wavelength colors (blue) will appear less bright than the longer wavelength colors (red)

PROBLEM 12.22

KNOWN: Lamp with prescribed filament area and temperature radiates like a blackbody at 2900 K when consuming 100 W.

FIND: (a) Efficiency of the lamp for providing visible radiation, and (b) Efficiency as a function of filament temperature for the range 1300 to 3300 K.

SCHEMATIC:



ASSUMPTIONS: (1) Filament behaves as a blackbody, (2) Glass envelope transmits all visible radiation incident upon it.

ANALYSIS: (a) We define the efficiency of the lamp as the ratio of the radiant power within the visible spectrum (0.4 - 0.7 μm) to the electrical power required to operate the lamp at the prescribed temperature.

$$\eta = q_{vis}/q_{elec} \,.$$

The radiant power for a blackbody within the visible spectrum is given as

$$q_{vis} = F(0.4\mu m \rightarrow 0.7\mu m) A_s \sigma T_s^4 = \left[F_{(0\rightarrow 0.7\mu m)} - F_{(0\rightarrow 0.4\mu m)}\right] A_s \sigma T_s^4$$

using Eq. 12.31 to relate the band emission factors. From Table 12.1, find

$$\lambda_2 T_s = 0.7\ \mu m \times 2900\ K = 2030\ \mu m\cdot K, \qquad F_{(0\rightarrow 0.7\mu m)} = 0.0719$$

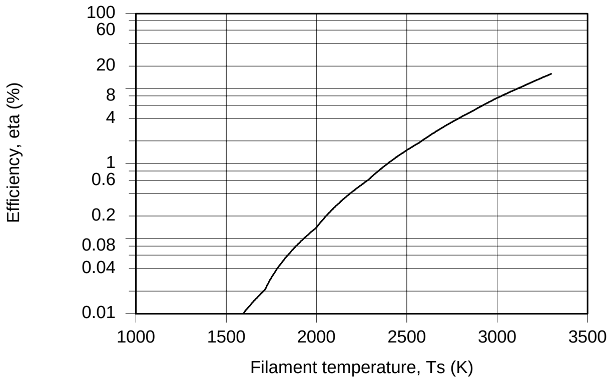
$$\lambda_1 T_s = 0.4\ \mu m \times 2900\ K = 1600\ \mu m\cdot K, \qquad F_{(0\rightarrow 0.4\mu m)} = 0.0018$$

The efficiency is then

$$\eta = [0.0719 - 0.0018] \times 2(2\times10^{-3} m \times 5\times10^{-3} m) 5.67\times10^{-8}\ W/m^2\cdot K^4 (2900\ K)^4 \,/\, 100\ W$$

$$\eta = 5.62\ W/100\ W = 5.6\%$$ <

(b) Using the *IHT Radiation Exchange Tool*, *Blackbody Emission Factor*, and Eqs. (1) and (2) above, a model was developed to compute and plot η as a function of T_s.



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PROBLEM 12.22 (Cont.)

Note that the efficiency decreases markedly with reduced filament temperature. At 2900 K, η = 5.6% while at 2345 K, the efficiency decreases by more than a factor of five to η = 1%.

COMMENTS: (1) Based upon this analysis, less than 6% of the energy consumed by the lamp operating at 2900 K is converted to visible light. The transmission of the glass envelope will be less than unity, so the efficiency will be less than the calculated value.

(2) Most of the energy is absorbed by the glass envelope and then lost to the surroundings by convection and radiation. Also, a significant amount of power is conducted to the lamp base and into the lamp base socket.

(3) The IHT workspace used to generate the above plot is shown below.

```
// Radiation Exchange Tool - Blackbody Band Emission Factor:
/* The blackbody band emission factor, Figure 12.14 and Table 12.1, is */
FL1Ts = F_lambda_T(lambda1,Ts)            // Eq 12.30
// where units are lambda (micrometers, mum) and T (K)
/* The blackbody band emission factor, Figure 12.14 and Table 12.1, is */
FL2Ts = F_lambda_T(lambda2,Ts)            // Eq 12.30

// Efficiency and rate expressions:
eta = qvis / qelec                        // Eq. (1)
eta_pc = eta * 100                        // Efficiency, %
qelec = 100                               // Electrical power, W
qvis = (FL2Ts - FL1Ts) * As * sigma * Ts^4   // Eq (2)
sigma = 5.67e-8                           // Stefan-Boltzmann constant, W/m^2.K

//Assigned Variables:
Ts = 2900                     // Filament temperature, K
As = 0.005 * 0.005            // Filament area, m^2
lambda1 = 0.4                 // Wavelength, mum; lower limit of visible spectrum
lambda2 = 0.7                 // Wavelength, mum; upper limit of visible spectrum

/*Data Browser Results - Part (a):
FL1Ts       FL2Ts     eta       eta_pc   qvis    As      Ts     lambda1
lambda2     qelec     sigma
0.001771    0.07185   0.07026   7.026    7.026   2.5E-5  2900   0.4
0.7         100       5.67E-8  */
```

PROBLEM 12.23

KNOWN: Solar disc behaves as a blackbody at 5800 K.

FIND: (a) Fraction of total radiation emitted by the sun that is in the visible spectral region, (b) Plot the percentage of solar emission that is at wavelengths less than λ as a function of λ, and (c) Plot on the same coordinates the percentage of emission from a blackbody at 300 K that is at wavelengths less than λ as a function of λ; compare the plotted results with the upper abscissa scale of Figure 12.23.

ASSUMPTIONS: (1) Visible spectral region has limits $\lambda_1 = 0.40$ μm and $\lambda_2 = 0.70$ μm.

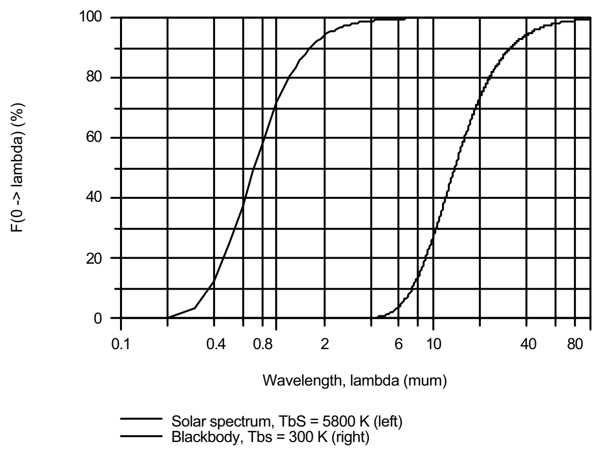
ANALYSIS: (a) Using the blackbody functions of Table 12.1, find $F_{(0\to\lambda)}$, the fraction of radiant flux leaving a black surface in the spectral interval $0\to\lambda$ as a function of the product λT. From the tabulated values for $F_{(0\to\lambda)}$ with T = 5800 K,

$\lambda_2 = 0.70\ \mu m$	$\lambda_2 T = 4060\ \mu m\cdot K$	$F_{(0\to\lambda_1)} = 0.4914$
$\lambda_1 = 0.40\ \mu m$	$\lambda_1 T = 2320\ \mu m\cdot K$	$F_{(0\to\lambda_2)} = 0.1245$

Hence, for the visible spectral region, the fraction of total emitted solar flux is

$$F_{(\lambda_1\to\lambda_2)} = F_{(0\to\lambda_2)} - F_{(0\to\lambda_1)} = 0.4914 - 0.1245 = 0.3669 \quad \text{or} \quad 37\%$$ <

(b,c) Using the *IHT Radiation Tool, Band Emission Factor*, $F_{(0-\lambda T)}$ are evaluated for the solar spectrum (T = 5800 K) and that for a blackbody temperature (T = 300 K) as a function of wavelength and are plotted below.



The left-hand curve in the plot represents the percentage of solar flux approximated as the 5800 K-blackbody spectrum in the spectral region less than λ. The right-hand curve represents the percentage of 300 K-blackbody flux in the spectral region less than λ. Referring to upper abscissa scale of Figure 12.23, for the solar flux, 75% of the solar flux is at wavelengths shorter than 1 μm. For the blackbody flux (300 K), 75% of the blackbody flux is at wavelengths shorter than 20 μm. These values are in agreement with points on the solar and 300K-blackbody curves, respectively, in the above plot.

PROBLEM 12.24

KNOWN: Thermal imagers operating in the spectral regions 3 to 5 μm and 8 to 14 μm.

FIND: (a) Band-emission factors for each of the spectral regions, 3 to 5 μm and 8 to 14 μm, for temperatures of 300 and 900 K, (b) Calculate and plot the band-emission factors for each of the spectral regions for the temperature range 300 to 1000 K; identify the maxima, and draw conclusions concerning the choice of an imager for an application; and (c) Considering imagers operating at the maximum-fraction temperatures found from the graph of part (b), determine the sensitivity (%) required of the radiation detector to provide a noise-equivalent temperature (NET) of 5°C.

ASSUMPTIONS: The sensitivity of the imager's radiation detector within the operating spectral region is uniform.

ANALYSIS: (a) From Eqs. 12.30 and 12.31, the band-emission fraction $F(\lambda 1 \rightarrow \lambda 2, T)$ for blackbody emission in the spectral range λ_1 to λ_2 for a temperature T is

$$F_{(\lambda 1 \rightarrow \lambda 2,\, T)} = F_{(0 \rightarrow \lambda 2,\, T)} - F_{(0 \rightarrow \lambda 1,\, T)}$$

Using the *IHT Radiation | Band Emission* tool (or Table 12.1), evaluate $F_{(0\text{-}\lambda T)}$ at appropriate $\lambda \cdot T$ products:

3 to 5 μm region

$$F_{(\lambda 1 - \lambda 2,\, 300\ \mathrm{K})} = 0.1375 - 0.00017 = 0.01359$$ <

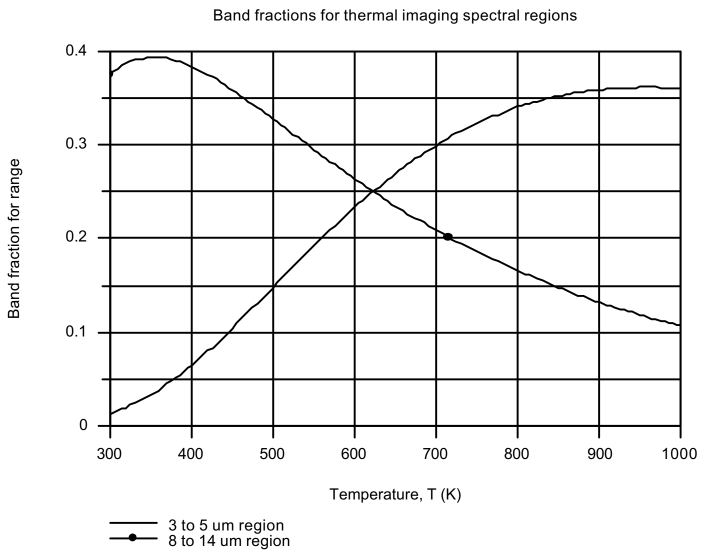
$$F_{(\lambda 1 - \lambda 2,\, 900\ \mathrm{K})} = 0.5640 - 0.2055 = 0.3585$$ <

8 to 14μm region

$$F_{(\lambda 1 - \lambda 2,\, 300\ \mathrm{K})} = 0.5160 - 0.1403 = 0.3758$$ <

$$F_{(\lambda 1 - \lambda 2,\, 900\ \mathrm{K})} = 0.9511 - 0.8192 = 0.1319$$ <

(b) Using the *IHT Radiation | Band Emission* tool, the band-emission fractions for each of the spectral regions is calculated and plotted below as a function of temperature.



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PROBLEM 12.24 (Cont.)

For the 3 to 5 μm imager, the band-emission factor increases with increasing temperature. For low temperature applications, not only is the radiant power $\left(\sigma T^4,\ T \approx 300\ \text{K}\right)$ low, but the band fraction is small. However, for high temperature applications, the imager operating conditions are more favorable with a large band-emission factor, as well as much higher radiant power $\left(\sigma T^4,\ T \rightarrow 900\ \text{K}\right)$.

For the 8 to 14 μm imager, the band-emission factor decreases with increasing temperature. This is a more favorable instrumentation feature, since the band-emission factor (proportionally more power) becomes larger as the radiant power decreases. This imager would be preferred over the 3 to 5 μm imager at lower temperatures since the band-emission factor is 8 to 10 times higher.

Recognizing that from Wien's law, the peaks of the blackbody curves for 300 and 900 K are approximately 10 and 3.3 μm, respectively, it follows that the imagers will receive the most radiant power when the peak target spectral distributions are close to the operating spectral region. It is good application practice to chose an imager having a spectral operating range close to the peak of the blackbody curve (or shorter than, if possible) corresponding to the target temperature.

The maxima band fractions for the 3 to 5 μm and 8 to 14 μm spectral regions correspond to temperatures of 960 and 355 K, respectively. Other application factors not considered (like smoke, water vapor, etc), the former imager is better suited with higher temperature scenes, and the latter with lower temperature scenes.

(c) Consider the 3 to 5 μm and 8 to 14 μm imagers operating at their band-emission peak temperatures, 355 and 960 K, respectively. The sensitivity S (% units) of the imager to resolve an NET of 5°C can be expressed as

$$S(\%) = \frac{F(\lambda 1 - \lambda 2,\ T1) - F(\lambda 1 - \lambda 2,\ T2)}{F(\lambda 1 - \lambda 2,\ T1)} \times 100$$

where T_1 = 355 or 960 K and T_2 = 360 or 965 K, respectively. Using this relation in the *IHT* workspace, find

$S_{3-5} = 0.035\%$ $\qquad\qquad$ $S_{8-14} = 0.023\%$ $\qquad\qquad$ $<$

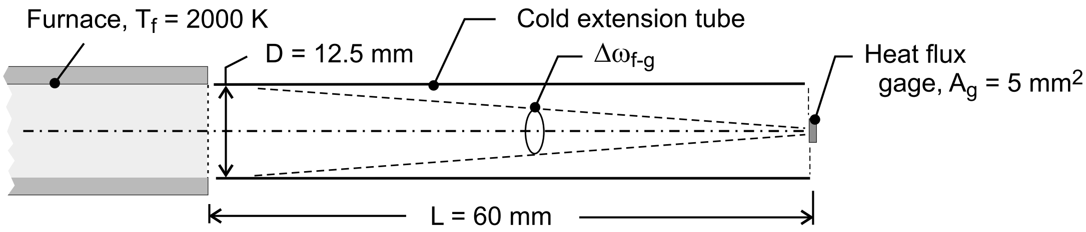
That is, we require the radiation detector (with its signal-processing system) to resolve the output signal with the foregoing precision in order to indicate a 5°C change in the scene temperature.

PROBLEM 12.25

KNOWN: Tube furnace maintained at T_f = 2000 K used to calibrate a heat flux gage of sensitive area 5 mm^2 mounted coaxial with the furnace centerline, and positioned 60 mm from the opening of the furnace.

FIND: (a) Heat flux (kW/m^2) on the gage, (b) Radiant flux in the spectral region 0.4 to 2.5 μm, the sensitive spectral region of a solid-state (photoconductive type) heat-flux gage, and (c) Calculate and plot the heat fluxes for each of the gages as a function of the furnace temperature for the range 2000 ≤ T_f ≤ 3000 K. Compare the values for the two types of gages; explain why the solid-state gage will always indicate systematically low values; does the solid-state gage performance improve, or become worse as the source temperature increases?

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Graphite tube furnace behaves as a blackbody, (3) Areas of gage and furnace opening are small relative to separation distance squared, and (4) Extension tube is cold relative to the furnace.

ANALYSIS: (a) The heat flux to the gage is equal to the irradiation, G_g, on the gage and can be expressed as (see Section 12.2.3)

$$G_g = I_f \cdot \cos\theta_g \cdot \Delta\omega_{f-g}$$

where $\Delta\omega_{f-g}$ is the solid angle that the furnace opening subtends relative to the gage. From Eq. 12.2, with $\theta_g = 0°$

$$\Delta\omega_{f-g} \equiv \frac{dA_n}{r^2} = \frac{A_f \cos\theta_g}{L^2} = \frac{\pi(0.0125\ \text{m})^2 / 4 \times 1}{(0.060\ \text{m})^2} = 3.409 \times 10^{-2}\ \text{sr}$$

The intensity of the radiation from the furnace is

$$I_f = E_{b,f}(T_f)/\pi = \sigma T_f^4/\pi = 5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\ (2000\ \text{K})^4/\pi = 2.888\times10^5\ \text{W/m}^2\cdot\text{sr}$$

Substituting numerical values,

$$G_g = 2.888\times10^5\ \text{W/m}^2\cdot\text{sr}\times1\times3.409\times10^{-2}\ \text{sr} = 9.84\ \text{kW/m}^2$$ <

(b) The solid-state detector gage, sensitive only in the spectral region λ_1 = 0.4 μm to λ_2 = 2.5 μm, will receive the band irradiation.

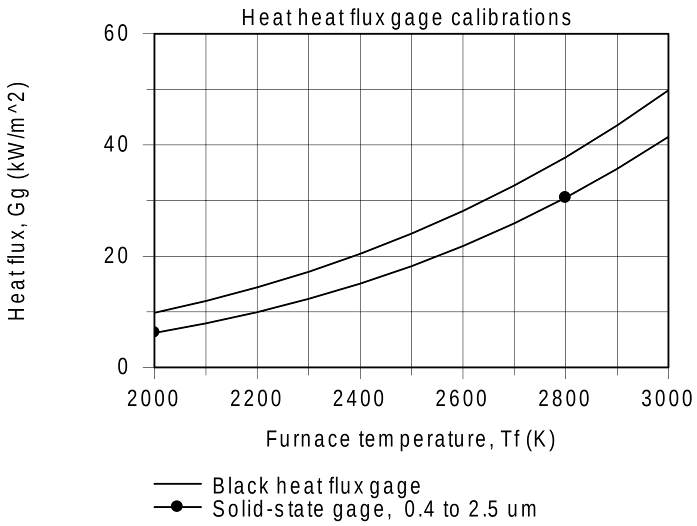
$$G_{g,\lambda1-\lambda2} = F_{(\lambda1\to\lambda2,\ Tf)}\cdot G_{g,b} = \left[F_{(0\to\lambda2,\ Tf)} - F_{(0\to\lambda1,\ Tf)}\right] G_{g,b}$$

Continued …..

where for $\lambda_1 T_f$ = 0.4 μm × 2000 K = 800 μm·K, $F_{(0 - \lambda 1)}$ = 0.0000 and for $\lambda_2 \cdot T_f$ = 2.5 μm × 2000 K = 5000 μm·K, $F_{(0 - \lambda 2)}$ = 0.6337. Hence,

$$G_{g,\lambda 1-\lambda 2} = [0.6337 - 0.0000] \times 9.84 \text{ kW/m}^2 = 6.24 \text{ kW/m}^2 \qquad <$$

(c) Using the foregoing equation in the *IHT* workspace, the heat fluxes for each of the gage types are calculated and plotted as a function of the furnace temperature.



For the black gage, the irradiation received by the gage, G_g, increases as the fourth power of the furnace temperature. For the solid-state gage, the irradiation increases slightly greater than the fourth power of the furnace temperature since the band-emission factor for the spectral region, $F_{(\lambda 1 - \lambda 2, Tf)}$, increases with increasing temperature. The solid-state gage will always indicate systematic low readings since its band-emission factor never approaches unity. However, the error will decrease with increasing temperature as a consequence of the corresponding increase in the band-emission factor.

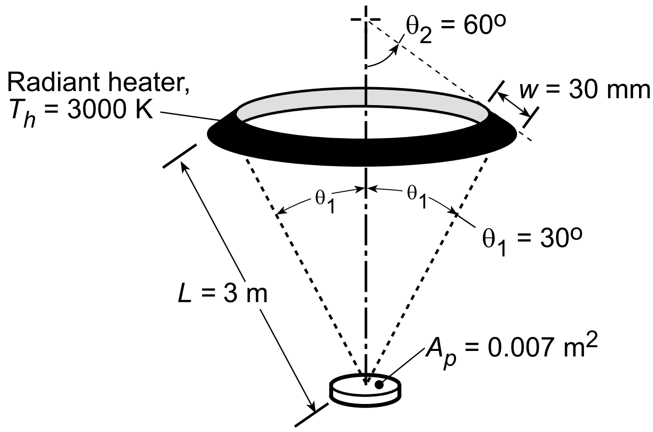
COMMENTS: For this furnace-gage geometrical arrangement, evaluating the solid angle, $\Delta\omega_{f - g}$, and the areas on a differential basis leads to results that are systematically high by 1%. Using the view factor concept introduced in Chapter 13 and Eq. 13.8, the results for the black and solid-state gages are 9.74 and 6.17 kW/m^2, respectively.

PROBLEM 12.26

KNOWN: Geometry and temperature of a ring-shaped radiator. Area of irradiated part and distance from radiator.

FIND: Rate at which radiant energy is incident on the part.

SCHEMATIC:



ASSUMPTIONS: (1) Heater emits as a blackbody.

ANALYSIS: Expressing Eq. 12.5 on the basis of the total radiation, $dq = I_e\, dA_h \cos\theta\, d\omega$, the rate at which radiation is incident on the part is

$$q_{h-p} = \int dq = I_e \int\int \cos\theta d\omega_{p-h} dA_h \approx I_e \cos\theta \cdot \omega_{p-h} \cdot A_h$$

Since radiation leaving the heater in the direction of the part is oriented normal to the heater surface, $\theta = 0$ and $\cos\theta = 1$. The solid angle subtended by the part with respect to the heater is $\omega_{p-h} = A_p \cos\theta_1/L^2$, while the area of the heater is $A_h \approx 2\pi r_h W = 2\pi(L \sin\theta_1)W$. Hence, with $I_e = E_b/\pi = \sigma T_h^4/\pi$,

$$q_{h-p} \approx \frac{5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\,(3000\,\mathrm{K})^4}{\pi} \times \frac{0.007\,\mathrm{m}^2\left(\cos 30^\circ\right)}{(3\,\mathrm{m})^2} \times 2\pi\,(1.5\,\mathrm{m})\,0.03\,\mathrm{m}$$

$$q_{h-p} \approx 278.4\,\mathrm{W}$$ <

COMMENTS: The foregoing representation for the double integral is an excellent approximation since $W << L$ and $A_p << L^2$.

PROBLEM 12.27

KNOWN: Spectral distribution of the emissive power given by Planck's law.

FIND: Approximations to the Planck distribution for the extreme cases when (a) $C_2/\lambda T >> 1$, Wien's law and (b) $C_2/\lambda T << 1$, Rayleigh-Jeans law.

ANALYSIS: Planck's law provides the spectral, hemispherical emissive power of a blackbody as a function of wavelength and temperature, Eq. 12.26,

$$E_{\lambda,b}(\lambda,T) = C_1/\lambda^5\left[\exp(C_2/\lambda T) - 1\right].$$

We now consider the extreme cases of $C_2/\lambda T >> 1$ and $C_2/\lambda T << 1$.

(a) When $C_2/\lambda T >> 1$ (or $\lambda T << C_2$), it follows $\exp(C_2/\lambda T) >> 1$. Hence, the –1 term in the denominator of the Planck law is insignificant, giving

$$E_{\lambda,b}(\lambda,T) \approx \left(C_1/\lambda^5\right)\exp(-C_2/\lambda T).$$ <

This approximate relation is known as *Wien's law.* The ratio of the emissive power by Wien's law to that by the Planck law is,

$$\frac{E_{\lambda,b,\text{Wien}}}{E_{\lambda,b,\text{Planck}}} = \frac{1/\exp(C_2/\lambda T)}{1/\left[\exp(C_2/\lambda T) - 1\right]}.$$

For the condition $\lambda T = \lambda_{max} T = 2898\ \mu m \cdot K$, $C_2/\lambda T = \dfrac{14388\ \mu m \cdot K}{2898\ \mu m \cdot K} = 4.966$ and

$$\frac{E_{\lambda,b}\big|_{\text{Wien}}}{E_{\lambda,b}\big|_{\text{Planck}}} = \frac{1/\exp(4.966)}{1/\left[\exp(4.966) - 1\right]} = 0.9930.$$ <

That is, for $\lambda T \le 2898\ \mu m \cdot K$, Wien's law is a good approximation to the Planck distribution.

(b) If $C_2/\lambda T << 1$ (or $\lambda T >> C_2$), the exponential term may be expressed as a series that can be approximated by the first two terms. That is,

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots \approx 1 + x \qquad \text{when} \qquad x << 1.$$

The *Rayleigh-Jeans* approximation is then

$$E_{\lambda,b}(\lambda,T) \approx C_1/\lambda^5\left[1 + (C_2/\lambda T) - 1\right] = C_1 T / C_2\lambda^4.$$

For the condition $\lambda T = 100{,}000\ \mu m \cdot K$, $C_2/\lambda T = 0.1439$

$$\frac{E_{\lambda,b,R-J}}{E_{\lambda,b,\text{Planck}}} = \frac{C_1 T / C_2\lambda^4}{C_1/\lambda^5}\left[\exp(C_2/\lambda T) - 1\right]^{-1} = (\lambda T/C_2)\left[\exp(C_2/\lambda T) - 1\right] = 1.0754.$$ <

That is, for $\lambda T \ge 100{,}000\ \mu m \cdot K$, the Rayleigh-Jeans law is a good approximation (better than 10%) to the Planck distribution.

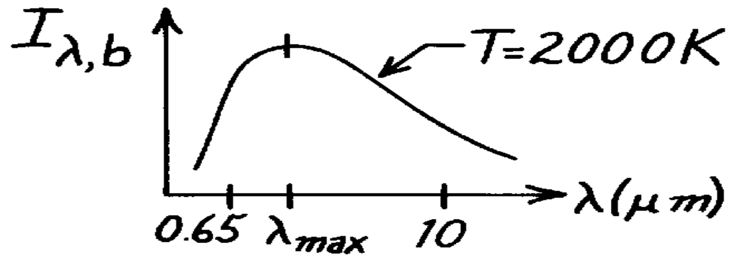
COMMENTS: The Wien law is used extensively in optical pyrometry for values of λ near 0.65 μm and temperatures above 700 K. The Rayleigh-Jeans law is of limited use in heat transfer but of utility for far infrared applications.

PROBLEM 12.28

KNOWN: Aperture of an isothermal furnace emits as a blackbody.

FIND: (a) An expression for the ratio of the fractional change in the spectral intensity to the fractional change in temperature of the furnace aperture, (b) Allowable variation in temperature of a furnace operating at 2000 K such that the spectral intensity at 0.65μm will not vary by more than 1/2%. Allowable variation for 10μm.

SCHEMATIC:



ASSUMPTIONS: (1) Furnace is isothermal and aperture radiates as a blackbody.

ANALYSIS: (a) The Planck spectral distribution, Eq. 12.26, is

$$I_\lambda(\lambda,T) = C_1/\pi\lambda^5\left[\exp(C_2/\lambda T)-1\right].$$

Taking natural logarithms of both sides, find $\ell n I_\lambda = \ell n\left[C_1/\pi\lambda^5\right] - \ell n\left[\exp(C_2/\lambda T)-1\right]$. Take the total derivative of both sides, but consider the λ variable as a constant.

$$\frac{dI_\lambda}{I_\lambda} = -\frac{d\left[\exp(C_2/\lambda T)-1\right]}{\left[\exp(C_2/\lambda T)-1\right]} = -\frac{\left\{\exp(C_2/\lambda T)\right\}(C_2/\lambda)\left(-1/T^2\right)dT}{\left[\exp(C_2/\lambda T)-1\right]}$$

$$\frac{dI_\lambda}{I_\lambda} = \frac{C_2}{\lambda T}\cdot\frac{\exp(C_2/\lambda T)}{\left[\exp(C_2/\lambda T)-1\right]}\cdot\frac{dT}{T} \quad \text{or} \quad \frac{d\,I_\lambda/I_\lambda}{dT/T} = \frac{C_2}{\lambda T}\cdot\frac{1}{1-\exp(-C_2/\lambda T)}. \qquad <$$

(b) If the furnace operates at 2000 K and the desirable fractional change of the spectral intensity is 0.5% at 0.65 μm, the allowable temperature variation is

$$\frac{dT}{T} = \frac{d\,I_\lambda}{I_\lambda}\Bigg/\left\{\frac{C_2}{\lambda T}\frac{1}{\left[1-\exp(-C_2/\lambda T)\right]}\right\}$$

$$\frac{dT}{T} = 0.005\Bigg/\left\{\frac{14{,}388\mu m\cdot K}{0.65\,\mu m\times 2000K}\Bigg/\left[1-\exp\left(\frac{-14{,}388\,\mu m\cdot K}{0.65\,\mu m\times 2000K}\right)\right]\right\} = 4.517\times10^{-4}.$$

That is, the allowable fractional variation in temperature is 0.045%; at 2000 K, the allowable temperature variation is

$$\Delta T \approx 4.517\times10^{-4}T = 4.517\times10^{-4}\times 2000K = 0.90K. \qquad <$$

Substituting with T = 2000 K and λ = 10 μm, find that

$$\frac{dT}{T} = 3.565\times10^{-3} \qquad \text{and} \qquad \Delta T \approx 3.565\times10^{-3}\,T = 7.1K. \qquad <$$

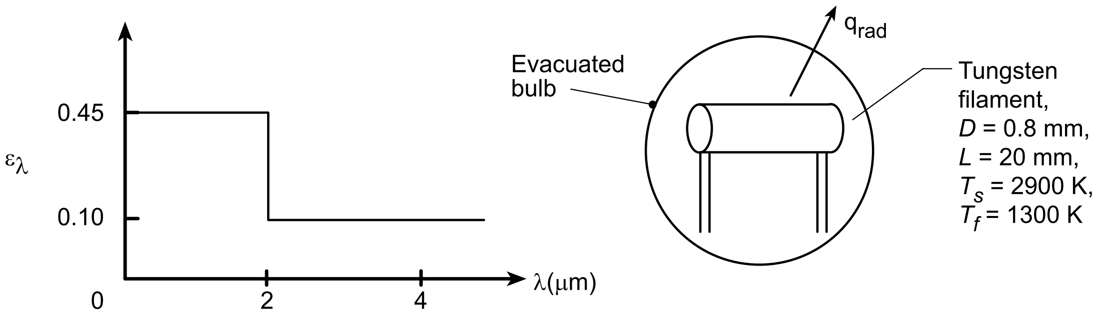
COMMENTS: Note that the power control requirements to satisfy the spectral intensity variation for 0.65 μm and 10 μm conditions are quite different. The peak of the blackbody curve for 2000 K is λ_{max} = 2898 μm·K/2000 K=1.45 μm.

PROBLEM 12.29

KNOWN: Spectral emissivity, dimensions and initial temperature of a tungsten filament.

FIND: (a) Total hemispherical emissivity, ε, when filament temperature is T_s = 2900 K; (b) Initial rate of cooling, dT_s/dt, assuming the surroundings are at T_{sur} = 300 K when the current is switched off; (c) Compute and plot ε as a function of T_s for the range $1300 \le T_s \le 2900$ K; and (d) Time required for the filament to cool from 2900 to 1300 K.

SCHEMATIC:



ASSUMPTIONS: (1) Filament temperature is uniform at any time (lumped capacitance), (2) Negligible heat loss by conduction through the support posts, (3) Surroundings large compared to the filament, (4) Spectral emissivity, density and specific heat constant over the temperature range, (5) Negligible convection.

PROPERTIES: *Table A-1*, Tungsten (2900 K); $\rho = 19,300\ \mathrm{kg/m^3}$, $c_p \approx 185\ \mathrm{J/kg \cdot K}$.

ANALYSIS: (a) The total emissivity at Ts = 2900 K follows from Eq. 12.38 using Table 12.1 for the band emission factors,

$$\varepsilon = \int_0^\infty \varepsilon_\lambda E_{\lambda,b}(T_s)d\lambda = \varepsilon_1 F_{(0\to 2\mu m)} + \varepsilon_2(1 - F_{0\to 2\mu m}) \qquad (1)$$

$$\varepsilon = 0.45 \times 0.72 + 0.1\,(1 - 0.72) = 0.352$$ <

where $F_{(0\to 2\mu m)} = 0.72$ at $\lambda T = 2\mu m \times 2900\ K = 5800\ \mu m \cdot K$.

(b) Perform an energy balance on the filament at the instant of time at which the current is switched off,

$$\dot{E}_{in} - \dot{E}_{out} = Mc_p \frac{dT_s}{dt}$$

$$A_s\left(\alpha G_{sur} - E\right) = A_s\left(\alpha \sigma T_s^4 - \varepsilon \sigma T_s^4\right) = Mc_p\, dT_s/dt$$

and find the change in temperature with time where $A_s = \pi DL$, $M = \rho\forall$, and $\forall = (\pi D^2/4)L$,

$$\frac{dT_s}{dt} = -\frac{\pi DL\sigma(\varepsilon T_s^4 - \alpha T_{sur}^4)}{\rho\left(\pi D^2/4\right)Lc_p} = -\frac{4\sigma}{\rho c_p D}\left(\varepsilon T_s^4 - \alpha T_{sur}^4\right)$$

$$\frac{dT_s}{dt} = -\frac{4\times 5.67\times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}(0.352\times 2900^4 - 0.1\times 300^4)K^4}{19,300\ \mathrm{kg/m^2} \times 185\ \mathrm{J/kg\cdot K} \times 0.0008\mathrm{m}} = -1977\ \mathrm{K/s}$$ <

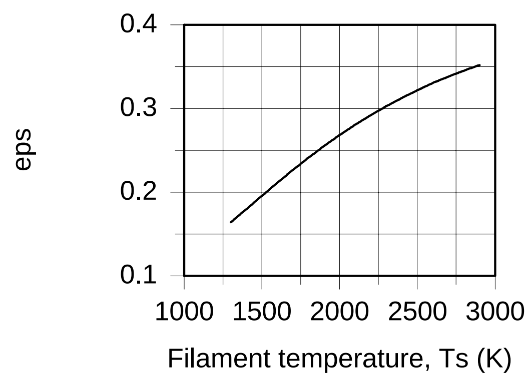
(c) Using the *IHT Tool*, *Radiation*, *Band Emission Factor*, and Eq. (1), a model was developed to calculate and plot ε as a function of T_s. See plot below.

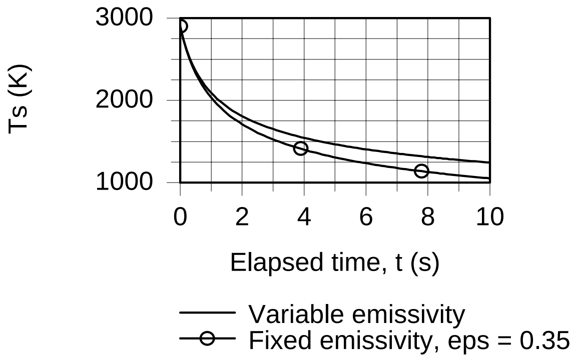
Continued...

PROBLEM 12.29 (Cont.)

(d) Using the IHT *Lumped Capacitance Model* along with the IHT workspace for part (c) to determine ε as a function of T_s, a model was developed to predict T_s as a function of cooling time. The results are shown below for the variable emissivity case (ε vs. T_s as per the plot below left) and the case where the emissivity is fixed at ε(2900 K) = 0.352. For the variable and fixed emissivity cases, the times to reach T_s = 1300 K are

t_{var} = 8.3 s $\qquad$ t_{fix} = 5.1 s <





COMMENTS: (1) From the ε vs. T_s plot, note that ε increases as T_s increases. Could you have surmised as much by looking at the spectral emissivity distribution, ε_λ vs. λ?

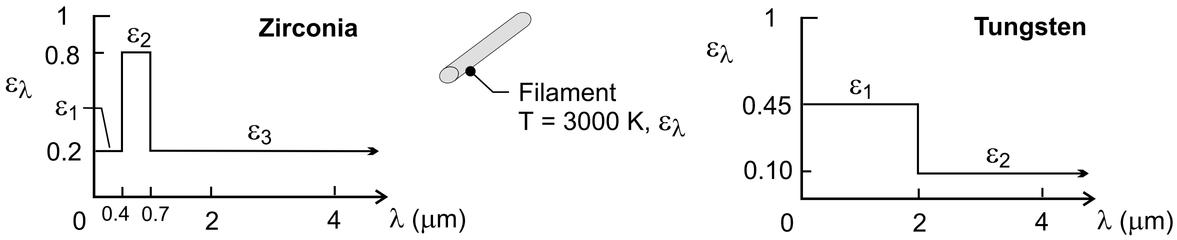
(2) How do you explain the result that $t_{var} > t_{fix}$?

PROBLEM 12.30

KNOWN: Spectral distribution of emissivity for zirconia and tungsten filaments. Filament temperature.

FIND: (a) Total emissivity of zirconia, (b) Total emissivity of tungsten and comparative power requirement, (c) Efficiency of the two filaments.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible reflection of radiation from bulb back to filament, (2) Equivalent surface areas for the two filaments, (3) Negligible radiation emission from bulb to filament.

ANALYSIS: (a) From Eq. (12.38), the emissivity of the zirconia is

$$\varepsilon = \int_o^\infty \varepsilon_\lambda \left(E_\lambda / E_b\right) d\lambda = \varepsilon_1 F_{(0\to0.4\mu m)} + \varepsilon_2 F_{(0.4\to0.7\mu m)} + \varepsilon_3 F_{(0.7\mu m\to\infty)}$$

$$\varepsilon = \varepsilon_1 F_{(0\to0.4\mu m)} + \varepsilon_2 \left(F_{(0\to0.7\mu m)} - F_{(0\to0.4\mu m)}\right) + \varepsilon_3 \left(1 - F_{(0\to0.7\mu m)}\right)$$

From Table 12.1, with T = 3000 K

$$\lambda T = 0.4\mu m \times 3000 \equiv 1200\mu m \cdot K: \quad F_{(0\to0.4\mu m)} = 0.0021$$

$$\lambda T = 0.7\mu m \times 3000\ K = 2100\mu m \cdot K: F_{(0\to0.7\mu m)} = 0.0838$$

$$\varepsilon = 0.2\times0.0021 + 0.8(0.0838 - 0.0021) + 0.2\times(1 - 0.0838) = 0.249$$ <

(b) For the tungsten filament,

$$\varepsilon = \varepsilon_1 F_{(0\to2\mu m)} + \varepsilon_2 \left(1 - F_{(0\to2\mu m)}\right)$$

With $\lambda T = 6000\mu m\cdot K$, $F(0 \to 2\mu m) = 0.738$

$$\varepsilon = 0.45\times0.738 + 0.1(1 - 0.738) = 0.358$$ <

Assuming, no reflection of radiation from the bulb back to the filament and with no losses due to natural convection, the power consumption per unit surface area of filament is $P''_{elec} = \varepsilon\sigma T^4$.

Continued …..

PROBLEM 12.30 (Cont.)

Zirconia: $P''_{elec} = 0.249 \times 5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}\,(3000\ \mathrm{K})^4 = 1.14 \times 10^6\ \mathrm{W/m^2}$

Tungsten: $P''_{elec} = 0.358 \times 5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}\,(3000\ \mathrm{K})^4 = 1.64 \times 10^6\ \mathrm{W/m^2}$

Hence, for an equivalent surface area and temperature, the tungsten filament has the largest power consumption. <

(c) Efficiency with respect to the production of visible radiation may be defined as

$$\eta_{vis} = \frac{\int_{0.4}^{0.7} \varepsilon_\lambda E_{\lambda,b}\, d\lambda}{E} = \frac{\int_{0.4}^{0.7} \varepsilon_\lambda \left(E_{\lambda,b} / E_b\right)}{\varepsilon} = \frac{\varepsilon_{vis}}{\varepsilon} F_{(0.4 \to 0.7 \mu m)}$$

With $F_{(0.4 \to 0.7\ \mu m)} = 0.0817$ for T = 3000 K,

Zirconia: $\eta_{vis} = (0.8/0.249)\,0.0817 = 0.263$

Tungsten: $\eta_{vis} = (0.45/0.358)\,0.0817 = 0.103$

Hence, the zirconia filament is the more efficient. <

COMMENTS: The production of visible radiation per unit filament surface area is $E_{vis} = \eta_{vis}\, P''_{elec}$. Hence,

Zirconia: $E_{vis} = 0.263 \times 1.14 \times 10^6\ \mathrm{W/m^2} = 3.00 \times 10^5\ \mathrm{W/m^2}$

Tungsten: $E_{vis} = 0.103 \times 1.64 \times 10^6\ \mathrm{W/m^2} = 1.69 \times 10^5\ \mathrm{W/m^2}$

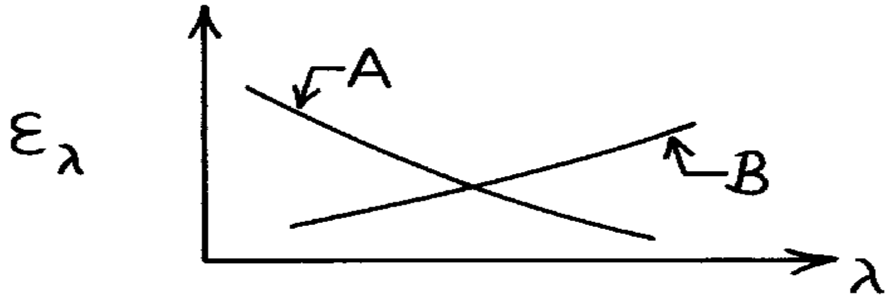
Hence, not only is the zirconia filament more efficient, but it also produces more visible radiation with less power consumption. This problem illustrates the benefits associated with carefully considering spectral surface characteristics in radiative applications.

PROBLEM 12.31

KNOWN: Variation of spectral, hemispherical emissivity with wavelength for two materials.

FIND: Nature of the variation with temperature of the total, hemispherical emissivity.

SCHEMATIC:



ASSUMPTIONS: (1) ε_λ is independent of temperature.

ANALYSIS: The total, hemispherical emissivity may be obtained from knowledge of the spectral, hemispherical emissivity by using Eq. 12.38

$$\varepsilon(T)=\frac{\int_0^\infty \varepsilon_\lambda(\lambda)E_{\lambda,b}(\lambda,T)d\lambda}{E_b(T)}=\int_0^\infty \varepsilon_\lambda(\lambda)\frac{E_{\lambda,b}(\lambda,T)}{E_b(T)}d\lambda.$$

We also know that the spectral emissive power of a blackbody becomes more concentrated at lower wavelengths with increasing temperature (Fig. 12.13). That is, the weighting factor, $E_{\lambda,b}(\lambda,T)/E_b(T)$ increases at lower wavelengths and decreases at longer wavelengths with increasing T. Accordingly,

Material A: $\varepsilon(T)$ increases with increasing T <

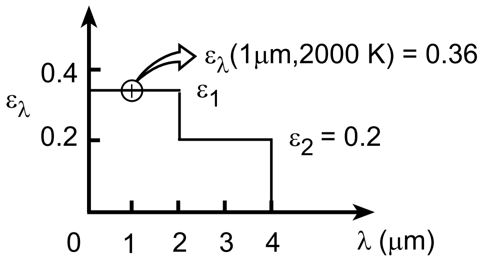
Material B: $\varepsilon(T)$ decreases with increasing T. <

PROBLEM 12.32

KNOWN: Metallic surface with prescribed spectral, directional emissivity at 2000 K and 1 μm (see Example 12.6) and additional measurements of the spectral, hemispherical emissivity.

FIND: (a) Total hemispherical emissivity, ε, and the emissive power, E, at 2000 K, (b) Effect of temperature on the emissivity.

SCHEMATIC:



ANALYSIS: (a) The total, hemispherical emissivity, ε, may be determined from knowledge of the spectral, hemispherical emissivity, ε_λ, using Eq. 12.38.

$$\varepsilon(\mathrm{T}) = \int_0^\infty \varepsilon_\lambda(\lambda)\mathrm{E}_{\lambda,\mathrm{b}}(\lambda,\mathrm{T})\,\mathrm{d}\lambda / \mathrm{E}_\mathrm{b}(\mathrm{T}) = \varepsilon_1 \int_0^{2\mu\mathrm{m}} \frac{\mathrm{E}_{\lambda,\mathrm{b}}(\lambda,\mathrm{T})\mathrm{d}\lambda}{\mathrm{E}_\mathrm{b}(\mathrm{T})} + \varepsilon_2 \int_{2\mu\mathrm{m}}^{4\mu\mathrm{m}} \frac{\mathrm{E}_{\lambda,\mathrm{b}}(\lambda,\mathrm{T})\mathrm{d}\lambda}{\mathrm{E}_\mathrm{b}(\mathrm{T})}$$

or from Eqs. 12.28 and 12.30,

$$\varepsilon(\mathrm{T}) = \varepsilon_1 \mathrm{F}_{(0\to\lambda_1)} + \varepsilon_2 \left[\mathrm{F}_{(0\to\lambda_2)} - \mathrm{F}_{(0\to\lambda_1)} \right]$$

From Table 12.1,

$$\lambda_1 = 2\,\mu\mathrm{m}, \quad \mathrm{T} = 2000\,\mathrm{K}: \quad \lambda_1\mathrm{T} = 4000\,\mu\mathrm{m}\cdot\mathrm{K}, \quad \mathrm{F}_{(0\to\lambda_1)} = 0.481$$

$$\lambda_2 = 4\,\mu\mathrm{m}, \quad \mathrm{T} = 2000\,\mathrm{K}: \quad \lambda_2\mathrm{T} = 8000\,\mu\mathrm{m}\cdot\mathrm{K}, \quad \mathrm{F}_{(0\to\lambda_2)} = 0.856$$

Hence,

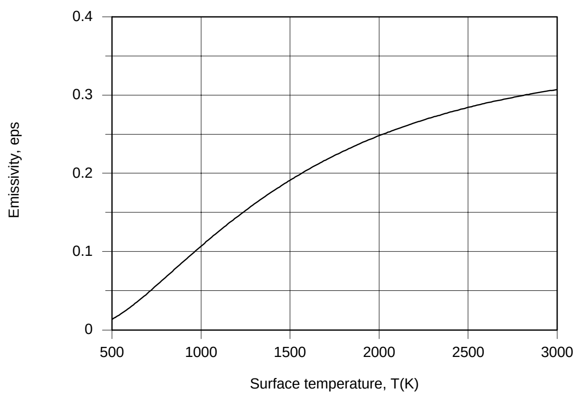
$$\varepsilon(\mathrm{T}) = 0.36\times0.481 + 0.20(0.856 - 0.481) = 0.25$$ <

From Eqs. 12.28 and 12.37, the total emissive power at 2000 K is

$$\mathrm{E}(2000\,\mathrm{K}) = \varepsilon\,(2000\,\mathrm{K}) \cdot \mathrm{E}_\mathrm{b}\,(2000\,\mathrm{K})$$

$$\mathrm{E}(2000\,\mathrm{K}) = 0.25\times5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\times(2000\,\mathrm{K})^4 = 2.27\times10^5\,\mathrm{W/m^2}\,.$$ <

(b) Using the *Radiation* Toolpad of IHT, the following result was generated.



Continued...

At $T \approx 500$ K, most of the radiation is emitted in the far infrared region ($\lambda > 4$ μm), in which case $\varepsilon \approx 0$. With increasing T, emission is shifted to lower wavelengths, causing ε to increase. As $T \to \infty$, $\varepsilon \to 0.36$.

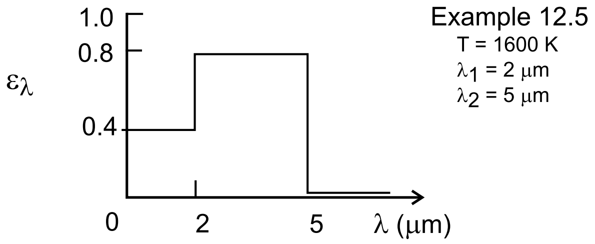
COMMENTS: Note that the value of ε_λ for $0 < \lambda \leq 2$ μm cannot be read directly from the ε_λ distribution provided in the problem statement. This value is calculated from knowledge of $\varepsilon_{\lambda,\theta}(\theta)$ in Example 12.6.

PROBLEM 12.33

KNOWN: Relationship for determining total, hemispherical emissivity, ε, by integration of the spectral emissivity distribution, ε_λ (Eq. 12.38).

FIND: Evaluate ε from ε_λ for the following cases: (a) Ex. 12.5, use the result to benchmark your code, (b) tungsten at 2800 K, and (c) aluminum oxide at 1400 K. Use the intrinsic function *INTEGRAL* of *IHT* as your solution tool.

SCHEMATIC:



ASSUMPTIONS: (1) Surfaces are diffuse emitters.

ANALYSIS: (a) Using *IHT* as the solution tool, Eq. 12.38 is entered into the workspace, and a look-up table created to represent the spectral emissivity distribution. See Comment 1 for the *IHT* annotated code. The result is ε = 0.558, in agreement with the analysis of Ex. 12.5 using the band-emission factors.

(b, c) Using the same code as for the benchmarking exercise in Part (a), but with new look-up table files (*.lut) representing the spectral distributions tabulated below, the total hemispherical emissivities for the tungsten at 2800 K and aluminum oxide at 1400 K are:

$$\varepsilon_W = 0.31 \qquad\qquad \varepsilon_{Al2O3} = 0.38$$ <

These results compare favorably with values of 0.29 and 0.41, respectively, from Fig. 12.19. See Comment 2.

Tungsten, 2800 K

λ (μm)	ε_λ	λ (μm)	ε_λ
0.3	0.47	2.0	0.26
0.4	0.48	4.0	0.17
0.5	0.47	6.0	0.05
0.6	0.44	8.0	0.03
1.0	0.38	10	0.03

Aluminum oxide, 1400 K

λ (μm)	ε_λ	λ (μm)	ε_λ
0.6	0.19	4.5	0.50
0.8	0.18	5	0.70
1.0	0.175	6	0.88
1.5	0.175	10	0.96
2	0.19	12.5	0.9
3	0.29	15	0.53
4	0.4	20	0.39

Continued …..

COMMENTS: (1) The *IHT* code to obtain ε from ε_λ for the case of Ex. 12.5 spectral distribution is shown below.

```
// Benchmarking use of INTEGRAL and LOOKUPVAL functions
// Calculating total emissivity from spectral distribution

/* Results:  integration from 0.05 to 15 by steps of 0.02, tabulated every 10
lLb       eps       eps_t     T         lambda    LLB
198.2     0.001     0.5579    1600      14.85     0.1982 */

// Emissivity integral, Eq. 12.38
eps_t = pi * INTEGRAL(lL,lambda) / (sigma * T^4)
sigma = 5.67 e-8

// Blackbody Spectral intensity, Tools | Radiation
/* From Planck's law, the blackbody spectral intensity is */
lL = eps *lLb
lLb = l_lambda_b(lambda, T, C1, C2)   // Eq. 12.25
// where units are lLb(W/m^2.sr.mum), lambda (mum) and T (K) with
C1 = 3.7420e8       // First radiation constant, W·mum^4/m^2
C2 = 1.4388e4       // Second radiation constant, mum·K
// and (mum) represents (micrometers).

// Emissivity function
eps = LOOKUPVAL(eps_L, 1, lambda, 2)
/* The table file name is eps_L.lut, with 2 columns and 6 rows.  See Help | Solver |
Lookup Tables | Lookupval
0.05      0.4
1.99      0.4
2         0.8
4.99      0.8
5         0.001
100       0.001     */

// Input variable
T = 1600
```

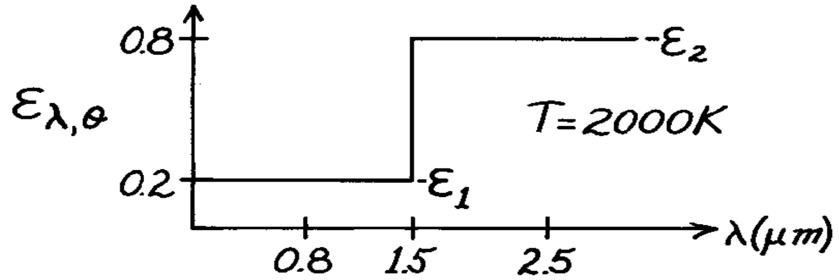
(2) For tungsten at 2800 K, the spectral limits for 98% of the blackbody spectrum are 0.51 and 8.3 μm. For aluminum at 1400 K, the spectral limits for 98% of the blackbody spectrum are 1.0 and 16.7 μm. For both cases, the foregoing tabulated spectral emissivity distributions are adequately represented for integration within the 98% limits.

PROBLEM 12.34

KNOWN: Spectral directional emissivity of a diffuse material at 2000K.

FIND: (a) Total, hemispherical emissivity, (b) Emissive power over the spectral range 0.8 to 2.5 μm and for directions $0 \le \theta \le \pi/6$.

SCHEMATIC:



ASSUMPTIONS: (1) Surface is diffuse emitter.

ANALYSIS: (a) Since the surface is diffuse, $\varepsilon_{\lambda,\theta}$ is independent of direction; from Eq. 12.36, $\varepsilon_{\lambda,\theta} = \varepsilon_\lambda$. Using Eq. 12.38,

$$\varepsilon(T) = \int_0^\infty \varepsilon_\lambda(\lambda) E_{\lambda,b}(\lambda,T)\,d\lambda / E_b(T)$$

$$E(T) = \int_0^{1.5} \varepsilon_1 E_{\lambda,b}(\lambda,2000)\,d\lambda / E_b + \int_{1.5}^\infty \varepsilon_2 E_{\lambda,b}(\lambda,2000)\,d\lambda / E_b.$$

Written now in terms of $F_{(0 \to \lambda)}$, with $F_{(0 \to 1.5)} = 0.2732$ at $\lambda T = 1.5 \times 2000 = 3000$ μm·K, (Table 12.1) find,

$$\varepsilon(2000\text{K}) = \varepsilon_1 \times F_{(0\to1.5)} + \varepsilon_2\left[1 - F_{(0\to1.5)}\right] = 0.2 \times 0.2732 + 0.8[1 - 0.2732] = 0.636. \qquad <$$

(b) For the prescribed spectral and geometric limits, from Eq. 12.12,

$$\Delta E = \int_{0.8}^{2.5} \int_0^{2\pi} \int_0^{\pi/6} \varepsilon_{\lambda,\theta}\, I_{\lambda,b}(\lambda,T) \cos\theta \sin\theta\, d\theta\, d\phi\, d\lambda$$

where $I_{\lambda,e}(\lambda, \theta, \phi) = \varepsilon_{\lambda,\theta} I_{\lambda,b}(\lambda,T)$. Since the surface is diffuse, $\varepsilon_{\lambda,\theta} = \varepsilon_\lambda$, and noting $I_{\lambda,b}$ is independent of direction and equal to $E_{\lambda,b}/\pi$, we can write

$$\Delta E = \left\{\int_0^{2\pi} \int_0^{\pi/6} \cos\theta \sin\theta\, d\theta\, d\phi\right\} \frac{E_b(T)}{\pi} \left\{\frac{\int_{0.8}^{1.5} \varepsilon_1 E_{\lambda,b}(\lambda,T)\,d\lambda}{E_b(T)} + \frac{\int_{1.5}^{2.5} \varepsilon_2 E_{\lambda,b}(\lambda,T)\,d\lambda}{E_b(T)}\right\}$$

or in terms $F_{(0 \to \lambda)}$ values,

$$\Delta E = \left\{\phi\Big|_0^{2\pi} \times \frac{\sin^2\theta}{2}\Big|_0^{\pi/6}\right\} \frac{\sigma T^4}{\pi} \left\{\varepsilon_1\left[F_{0\to1.5} - F_{0\to0.8}\right] + \varepsilon_2\left[F_{0\to2.5} - F_{0\to1.5}\right]\right\}.$$

From Table 12.1:	$\lambda T = 0.8 \times 2000 = 1600$ μm·K	$F_{(0 \to 0.8)} = 0.0197$
	$\lambda T = 2.5 \times 2000 = 5000$ μm·K	$F_{(0 \to 2.5)} = 0.6337$

$$\Delta E = \left\{2\pi \times \frac{\sin^2 \pi/6}{2}\right\} \frac{5.67 \times 10^{-8} \times 2000^4}{\pi} \frac{\text{W}}{\text{m}^2} \cdot \{0.2[0.2732 - 0.0197] + 0.8[0.6337 - 0.2732]\}$$

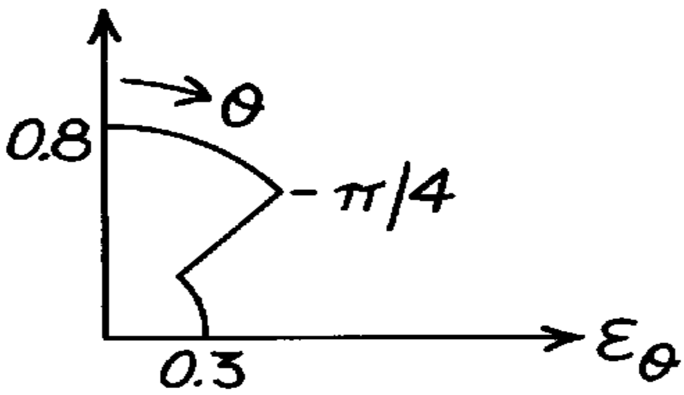
$$\Delta E = 0.25 \times \left(5.67 \times 10^{-8} \times 2000^4\right) \text{W/m}^2 \times 0.339 = 76.89\ \text{kW/m}^2. \qquad <$$

PROBLEM 12.35

KNOWN: Directional emissivity, ε_θ, of a selective surface.

FIND: Ratio of the normal emissivity, ε_n, to the hemispherical emissivity, ε.

SCHEMATIC:



ASSUMPTIONS: Surface is isotropic in ϕ direction.

ANALYSIS: From Eq. 12.36 written on a total, rather than spectral, basis, the hemispherical emissivity is

$$\varepsilon = 2\int_0^{\pi/2} \varepsilon_\theta(\theta)\cos\theta\sin\theta d\theta.$$

Recognizing that the integral can be expressed in two parts, find

$$\varepsilon = 2\left[\int_0^{\pi/4} \varepsilon(\theta)\cos\theta\sin\theta\, d\theta + \int_{\pi/4}^{\pi/2} \varepsilon(\theta)\cos\theta\sin\theta\, d\theta\right]$$

$$\varepsilon = 2\left[0.8\int_0^{\pi/4} \cos\theta\sin\theta\, d\theta + 0.3\int_{\pi/4}^{\pi/2} \cos\theta\sin\theta\, d\theta\right]$$

$$\varepsilon = 2\left[0.8\frac{\sin^2\theta}{2}\Bigg|_0^{\pi/4} + 0.3\frac{\sin^2\theta}{2}\Bigg|_{\pi/4}^{\pi/2}\right]$$

$$\varepsilon = 2\left[0.8\frac{1}{2}(0.50-0) + 0.3\times\frac{1}{2}(1-0.50)\right] = 0.550.$$

The ratio of the normal emissivity (ε_n) to the hemispherical emissivity is

$$\frac{\varepsilon_n}{\varepsilon} = \frac{0.8}{0.550} = 1.45.$$ <

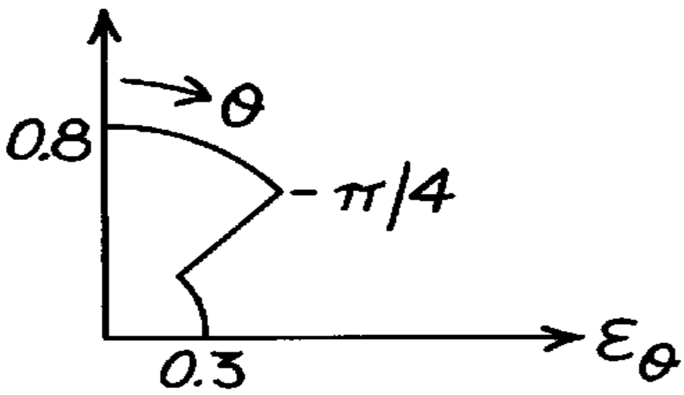
COMMENTS: Note that Eq. 12.36 assumes the directional emissivity is independent of the ϕ coordinate. If this is not the case, then Eq. 12.35 is appropriate.

PROBLEM 12.35

KNOWN: Directional emissivity, ε_θ, of a selective surface.

FIND: Ratio of the normal emissivity, ε_n, to the hemispherical emissivity, ε.

SCHEMATIC:



ASSUMPTIONS: Surface is isotropic in ϕ direction.

ANALYSIS: From Eq. 12.36 written on a total, rather than spectral, basis, the hemispherical emissivity is

$$\varepsilon = 2\int_0^{\pi/2} \varepsilon_\theta(\theta)\cos\theta\sin\theta \, d\theta.$$

Recognizing that the integral can be expressed in two parts, find

$$\varepsilon = 2\left[\int_0^{\pi/4} \varepsilon(\theta)\cos\theta\sin\theta\, d\theta + \int_{\pi/4}^{\pi/2} \varepsilon(\theta)\cos\theta\sin\theta\, d\theta\right]$$

$$\varepsilon = 2\left[0.8\int_0^{\pi/4} \cos\theta\sin\theta\, d\theta + 0.3\int_{\pi/4}^{\pi/2} \cos\theta\sin\theta\, d\theta\right]$$

$$\varepsilon = 2\left[0.8\frac{\sin^2\theta}{2}\Bigg|_0^{\pi/4} + 0.3\frac{\sin^2\theta}{2}\Bigg|_{\pi/4}^{\pi/2}\right]$$

$$\varepsilon = 2\left[0.8\frac{1}{2}(0.50-0) + 0.3\times\frac{1}{2}(1-0.50)\right] = 0.550.$$

The ratio of the normal emissivity (ε_n) to the hemispherical emissivity is

$$\frac{\varepsilon_n}{\varepsilon} = \frac{0.8}{0.550} = 1.45. \qquad <$$

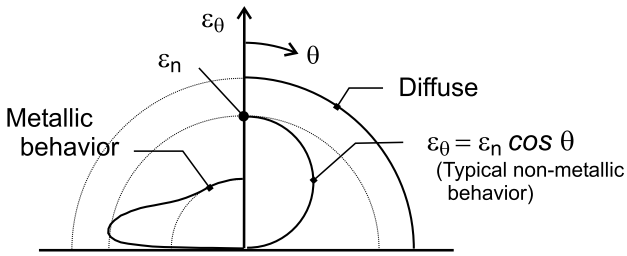
COMMENTS: Note that Eq. 12.36 assumes the directional emissivity is independent of the ϕ coordinate. If this is not the case, then Eq. 12.35 is appropriate.

PROBLEM 12.36

KNOWN: The total directional emissivity of non-metallic materials may be approximated as $\varepsilon_\theta = \varepsilon_n \cos\theta$ where ε_n is the total normal emissivity.

FIND: Show that for such materials, the total hemispherical emissivity, ε, is 2/3 the total normal emissivity.

SCHEMATIC:



ANALYSIS: From Eq. 12.36, written on a total rather than spectral basis, the hemispherical emissivity ε can be determined from the directional emissivity ε_θ as

$$\varepsilon = 2\int_0^{\pi/2} \varepsilon_\theta \cos\theta \sin\theta \, d\theta$$

With $\varepsilon_\theta = \varepsilon_n \cos\theta$, find

$$\varepsilon = 2\,\varepsilon_n \int_0^{\pi/2} \cos^2\theta \sin\theta \, d\theta$$

$$\varepsilon = -2\,\varepsilon_n \left(\cos^3\theta/3\right)\Big|_0^{\pi/2} = 2/3\,\varepsilon_n$$ <

COMMENTS: (1) Refer to Fig. 12.17 illustrating on cartesian coordinates representative directional distributions of the total, directional emissivity for nonmetallic and metallic materials. In the schematic above, we've shown ε_θ vs. θ on a polar plot for both types of materials, in comparison with a diffuse surface.

(2) See Section 12.4 for discussion on other characteristics of emissivity for materials.

PROBLEM 12.37

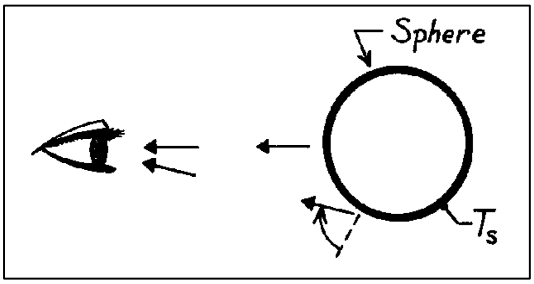
KNOWN: Incandescent sphere suspended in air within a darkened room exhibiting these characteristics:

initially: brighter around the rim
after some time: brighter in the center

FIND: Plausible explanation for these observations.

ASSUMPTIONS: (1) The sphere is at a uniform surface temperature, T_s.

ANALYSIS: Recognize that in observing the sphere by eye, emission from the central region is in a nearly normal direction. Emission from the rim region, however, has a large angle from the normal to the surface.



Note now the directional behavior, ε_θ, for conductors and non-conductors as represented in Fig. 12.17.

Assume that the sphere is fabricated from a *metallic* material. Then, the rim would appear brighter than the central region. This follows since ε_θ is higher at higher angles of emission.

If the metallic sphere oxidizes with time, then the ε_θ characteristics change. Then ε_θ at small angles of θ become larger than at higher angles. This would cause the sphere to appear brighter at the center portion of the sphere.

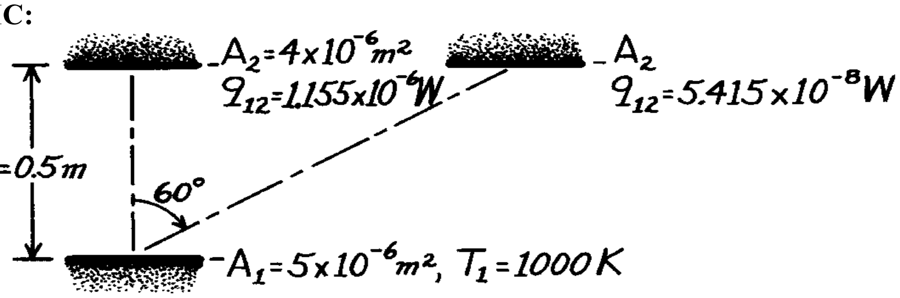
COMMENTS: Since the emissivity of non-conductors is generally larger than for metallic materials, you would also expect the oxidized sphere to appear brighter for the same surface temperature.

PROBLEM 12.38

KNOWN: Detector surface area. Area and temperature of heated surface. Radiant power measured by the detector for two orientations relative to the heated surface.

FIND: (a) Normal emissivity of heated surface, (b) Whether surface is a diffuse emitter.

SCHEMATIC:



ASSUMPTIONS: (1) Detector intercepts negligible radiation from surroundings, (2) A_1 and A_2 are differential surfaces.

ANALYSIS: The radiant power leaving the heated surface and intercepting the detector is

$$q_{12}(\theta) = I_1(\theta) A_1 \cos\theta\, \omega_{2-1}$$

$$I_1(\theta) = \varepsilon_1(\theta) I_{b,1} = \varepsilon_1(\theta)\sigma T_1^4/\pi \qquad \omega_{2-1} = \frac{A_2 \cos\theta}{(L/\cos\theta)^2}.$$

Hence,

$$q_{12}(\theta) = \varepsilon_1(\theta)\frac{\sigma T_1^4}{\pi} A_1 \cos\theta \frac{A_2 \cos\theta}{(L/\cos\theta)^2} \qquad \varepsilon_1(\theta) = \frac{q_{12}(\theta)\pi}{\sigma T_1^4}\frac{L^2}{A_1 A_2 (\cos\theta)^4}$$

(a) For the normal condition, $\theta = 0$, $\cos\theta = 1$, and $\varepsilon_1(\theta) \equiv \varepsilon_{1,n}$ is

$$\varepsilon_{1,n} = \frac{1.155\times10^{-6}\ \text{W}\,\pi}{5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4 (1000\text{K})^4} \frac{(0.5\text{m})^2}{5\times10^{-6}\ \text{m}^2 \times 4\times10^{-6}\ \text{m}^2}$$

$$\varepsilon_{1,n} = 0.80.$$ <

(b) For the orientation with $\theta = 60°$ and $\cos\theta = 0.5$, so that $\varepsilon_1(\theta = 60°)$ is

$$\varepsilon_1(60°) = \frac{5.415\times10^{-8}\ \text{W}\,\pi}{5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4 (1000\text{K})^4} \frac{(0.5\text{m})^2}{5\times10^{-6}\ \text{m}^2 \times 4\times10^{-6}\ \text{m}^2 (0.5)^4}$$

$$\varepsilon_1(60°) = 0.60.$$ <

Since $\varepsilon_{1,n} \neq \varepsilon_1(60°)$, the surface is not a diffuse emitter.

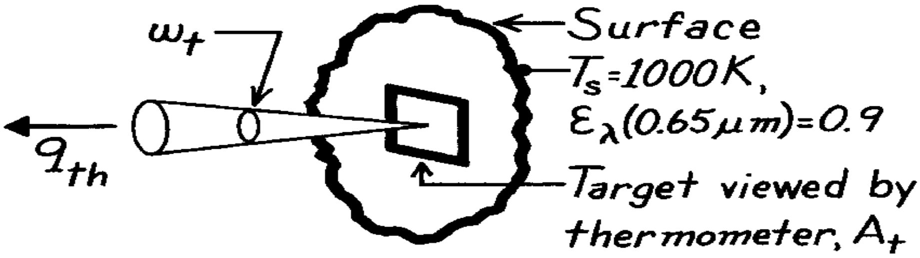
COMMENTS: Even if $\varepsilon_1(60°)$ were equal to $\varepsilon_{1,n}$, it could not be concluded there was diffuse emission until results were obtained for a wider range of θ.

PROBLEM 12.39

KNOWN: Radiation thermometer responding to radiant power within a prescribed spectral interval and calibrated to indicate the temperature of a blackbody.

FIND: (a) Whether radiation thermometer will indicate temperature greater than, less than, or equal to T_s when surface has $\varepsilon < 1$, (b) Expression for T_s in terms of spectral radiance temperature and spectral emissivity, (c) Indicated temperature for prescribed conditions of T_s and ε_λ.

SCHEMATIC:



ASSUMPTIONS: (1) Surface is a diffuse emitter, (2) Thermometer responds to radiant flux over interval $d\lambda$ about λ.

ANALYSIS: (a) The radiant power which reaches the radiation thermometer is

$$q_\lambda = \varepsilon_\lambda I_{\lambda,b}(\lambda, T_s) \cdot A_t \cdot \omega_t \tag{1}$$

where A_t is the area of the surface viewed by the thermometer (referred to as the target) and ω_t the solid angle through which A_t is viewed. The thermometer responds as if it were viewing a blackbody at T_λ, the spectral radiance temperature,

$$q_\lambda = I_{\lambda,b}(\lambda, T_\lambda) \cdot A_t \cdot \omega_t. \tag{2}$$

By equating the two relations, Eqs. (1) and (2), find

$$I_{\lambda,b}(\lambda, T_\lambda) = \varepsilon_\lambda I_{\lambda,b}(\lambda, T_s). \tag{3}$$

Since $\varepsilon_\lambda < 1$, it follows that $I_{\lambda,b}(\lambda, T_\lambda) < I_{\lambda,b}(\lambda, T_s)$ or that $T_\lambda < T_s$. That is, the thermometer will always indicate a temperature lower than the true or actual temperature for a surface with $\varepsilon < 1$.

(b) Using Wien's law in Eq. (3), find

$$I_\lambda(\lambda, T) = \frac{1}{\pi} C_1 \lambda^{-5} \exp(-C_2/\lambda T)$$

$$\frac{1}{\pi} C_1 \lambda^{-5} \exp(-C_2/\lambda T_\lambda) = \varepsilon_\lambda \cdot \frac{1}{\pi} C_1 \lambda^{-5} \exp(-C_2/\lambda T_s).$$

Canceling terms $(C_1\lambda^{-5}/\pi)$, taking natural logs of both sides of the equation and rearranging, the desired expression is

$$\frac{1}{T_s} = \frac{1}{T_\lambda} + \frac{\lambda}{C_2} \ell n \varepsilon_\lambda. \tag{4}$$ <

(c) For T_s = 1000K and ε = 0.9, from Eq. (4), the indicated temperature is

$$\frac{1}{T_\lambda} = \frac{1}{T_s} - \frac{\lambda}{C_2} \ln \varepsilon_\lambda = \frac{1}{1000\text{K}} - \frac{0.65\,\mu\text{m}}{14{,}388\,\mu\text{m}\cdot\text{K}} \ell n(0.9) \qquad T_\lambda = 995.3\text{K}.$$ <

That is, the thermometer indicates 5K less than the true temperature.

PROBLEM 12.40

KNOWN: Spectral distribution of emission from a blackbody. Uncertainty in measurement of intensity.

FIND: Corresponding uncertainities in using the intensity measurement to determine (a) the surface temperature or (b) the emissivity.

ASSUMPTIONS: Diffuse surface behavior.

ANALYSIS: From Eq. 12.25, the spectral intensity associated with emission may be expressed as

$$I_{\lambda,e} = \varepsilon_\lambda I_{\lambda,b} = \frac{\varepsilon_\lambda C_1/\pi}{\lambda^5\left[\exp(C_2/\lambda T)-1\right]}$$

(a) To determine the effect of temperature on intensity, we evaluate the derivative,

$$\frac{\partial I_{\lambda,e}}{\partial T} = -\frac{(\varepsilon_\lambda C_1/\pi)\lambda^5 \exp(C_2/\lambda T)\left(-C_2/\lambda T^2\right)}{\left\{\lambda^5\left[\exp(C_2/\lambda T)-1\right]\right\}^2}$$

$$\frac{\partial I_{\lambda,e}}{\partial T} = \frac{\left(C_2/\lambda T^2\right)\exp(C_2/\lambda T)}{\exp(C_2/\lambda T)-1} I_{\lambda,e}$$

Hence,

$$\frac{dT}{T} = \frac{1-\exp(-C_2/\lambda T)}{(C_2/\lambda T)} \frac{d I_{\lambda,e}}{I_{\lambda,e}}$$

With $\left(d I_{\lambda,e}/I_{\lambda,e}\right) = 0.1$, $C_2 = 1.439\times10^4\,\mu m\cdot K$ and $\lambda = 10\,\mu m$,

$$\frac{dT}{T} = \left[\frac{1-\exp(-1439K/T)}{1439K/T}\right]\times 0.1$$

T = 500 K: dT/T = 0.033 → 3.3% uncertainty <

T = 1000 K: dT/T = 0.053 → 5.5% uncertainty <

(b) To determine the effect of the emissivity on intensity, we evaluate

$$\frac{\partial I_{\lambda,e}}{\partial \varepsilon_\lambda} = I_{\lambda,b} = \frac{I_{\lambda,e}}{\varepsilon_\lambda}$$

Hence, $\dfrac{d\varepsilon_\lambda}{\varepsilon_\lambda} = \dfrac{dI_{\lambda,e}}{I_{\lambda,e}} = 0.10 \rightarrow 10\%$ uncertainty <

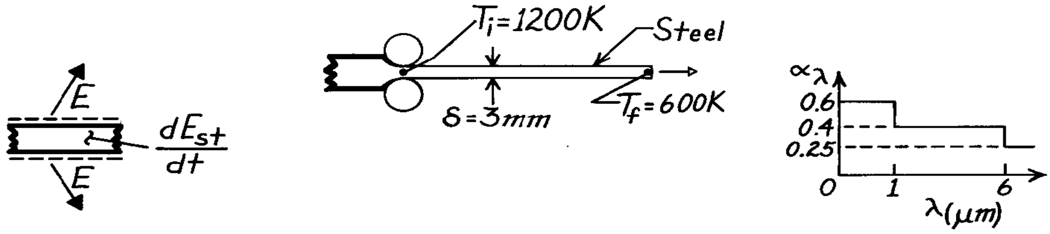
COMMENTS: The uncertainty in the temperature is less than that of the intensity, but increases with increasing temperature (and wavelength). In the limit as $C_2/\lambda T \rightarrow 0$, $\exp(-C_2/\lambda T) \rightarrow 1 - C_2/\lambda T$ and $dT/T \rightarrow d I_{\lambda,e}/I_{\lambda,e}$. The uncertainty in temperature then corresponds to that of the intensity measurement. The same is true for the uncertainty in the emissivity, irrespective of the value of T or λ.

PROBLEM 12.41

KNOWN: Temperature, thickness and spectral emissivity of steel strip emerging from a hot roller. Temperature dependence of total, hemispherical emissivity.

FIND: (a) Initial total, hemispherical emissivity, (b) Initial cooling rate, (c) Time to cool to prescribed final temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible conduction (in longitudinal direction), convection and radiation from surroundings, (2) Negligible transverse temperature gradients.

PROPERTIES: Steel (given): $\rho = 7900$ kg/m^3, c = 640 J/kg·K, $\varepsilon = 1200\varepsilon_i/T$ (K).

ANALYSIS: (a) The initial total hemispherical emissivity is

$$\varepsilon_i = \int_0^\infty \varepsilon_\lambda \left[E_{\lambda b}(1200)/E_b(1200)\right] d\lambda$$

and integrating by parts using values from Table 12.1, find

$$\lambda T = 1200\mu m\cdot K \rightarrow F_{(0-1\mu m)} = 0.002;\ \lambda T = 7200\mu m\cdot K \rightarrow F_{(0-6\mu m)} = 0.819$$

$$\varepsilon_i = 0.6\times 0.002 + 0.4(0.819 - 0.002) + 0.25(1 - 0.819) = 0.373. \qquad <$$

(b) From an energy balance on a unit surface area of strip (top and bottom),

$$-\dot{E}_{out} = dE_{st}/dt \qquad\qquad -2\varepsilon\sigma T^4 = d(\rho\delta c T)/dt$$

$$\left.\frac{dT}{dt}\right)_i = -\frac{2\varepsilon_i\sigma T_i^4}{\rho\delta c} = \frac{-2(0.373)5.67\times 10^{-8}\ W/m^2\cdot K^4 (1200\ K)^4}{7900\ kg/m^3(0.003\ m)(640\ J/kg\cdot K)} = -5.78\ K/s. \qquad <$$

(c) From the energy balance,

$$\frac{dT}{dt} = -\frac{2\varepsilon_i(1200/T)\sigma T^4}{\rho\delta c},\quad \int_{T_i}^{T_f}\frac{dT}{T^3} = -\frac{2400\varepsilon_i\sigma}{\rho\delta c}\int_0^t dt,\quad t = \frac{\rho\delta c}{4800\varepsilon_i\sigma}\left(\frac{1}{T_f^2} - \frac{1}{T_i^2}\right)$$

$$t = \frac{7900\ kg/m^3(0.003m)640\ J/kg\cdot K}{4800\ K\times 0.373\times 5.67\times 10^{-8}\ W/m^2\cdot K^4}\left(\frac{1}{600^2} - \frac{1}{1200^2}\right)K^{-2} = 311s. \qquad <$$

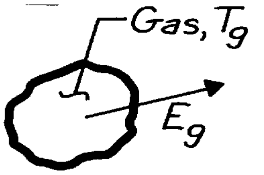
COMMENTS: Initially, from Eq. 1.9, $h_r = \varepsilon_i\sigma T_i^3 = 36.6$ W/m^2·K. Assuming a plate width of W = 1m, the Rayleigh number may be evaluated from $Ra_L = g\beta(T_i - T_\infty)(W/2)^3/\nu\alpha$. Assuming $T_\infty = 300$ K and evaluating properties at $T_f = 750$ K, $Ra_L = 2.4\times 10^8$. From Eq. 9.31, $Nu_L = 93$, giving $\bar{h} = 10$ W/m^2·K. Hence heat loss by radiation exceeds that associated with free convection. To check the validity of neglecting transverse temperature gradients, compute Bi = h(δ/2)/k. With h = 36.6 W/m^2·K and k = 28 W/m·K, Bi = 0.002 << 1. Hence the assumption is valid.

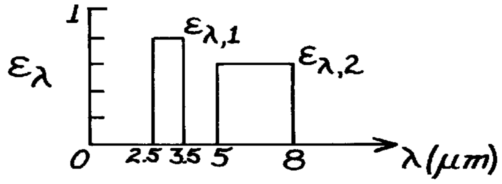
PROBLEM 12.42

KNOWN: Large body of nonluminous gas at 1200 K has emission bands between 2.5 – 3.5 μm and between 5 – 8 μm with effective emissivities of 0.8 and 0.6, respectively.

FIND: Emissive power of the gas.

SCHEMATIC:





ASSUMPTIONS: (1) Gas radiates only in specified bands, (2) Emitted radiation is diffuse.

ANALYSIS: The emissive power of the gas is

$$E_g = \varepsilon E_b(T_g) = \int_0^\infty \varepsilon_\lambda E_{\lambda,b}(T_g)\,d\lambda$$

$$E_g = \int_{2.5}^{3.5} \varepsilon_{\lambda,1} E_{\lambda,b}(T_g)\,d\lambda + \int_5^8 \varepsilon_{\lambda,2} E_{\lambda,b}(T_g)\,d\lambda$$

$$E_g = \left[\varepsilon_1 F_{(2.5-3.5\mu m)} + \varepsilon_2 F_{(5-8\mu m)}\right]\sigma T_g^4.$$

Using the blackbody function $F_{(0-\lambda T)}$ from Table 12.1 with T_g = 1200 K,

λT(μm·K)	2.5 × 1200 3000	3.5 × 1200 4200	5 × 1200 6000	8 × 1200 9600
$F_{(0-\lambda T)}$	0.273	0.516	0.738	0.905

so that

$$F_{(2.5-3.5\mu m)} = F_{(0-3.5\mu m)} - F_{(0-2.5\mu m)} = 0.516 - 0.273 = 0.243$$

$$F_{(5-8\mu m)} = F_{(0-8\mu m)} - F_{(0-5\mu m)} = 0.905 - 0.738 = 0.167.$$

Hence the emissive power is

$$E_g = [0.8 \times 0.243 + 0.6 \times 0.167]\,5.67 \times 10^{-8}\,W/m^2\cdot K^4\,(1200\ K)^4$$

$$E_g = 0.295 \times 117{,}573\ W/m^2 = 34{,}684\ W/m^2.$$ <

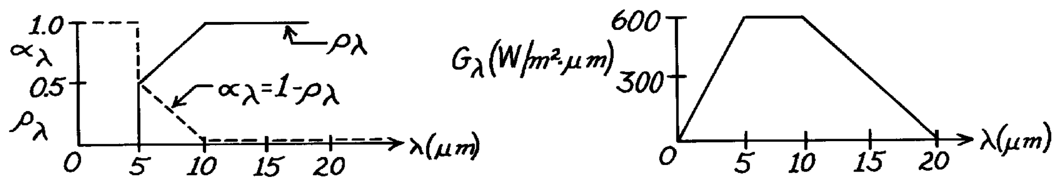
COMMENTS: Note that the effective emissivity for the gas is 0.295. This seems surprising since emission occurs only at the discrete bands. Since λ_{max} = 2.4 μm, all of the spectral emissive power is at wavelengths beyond the peak of blackbody radiation at 1200 K.

PROBLEM 12.43

KNOWN: An opaque surface with prescribed spectral, hemispherical reflectivity distribution is subjected to a prescribed spectral irradiation.

FIND: (a) The spectral, hemispherical absorptivity, (b) Total irradiation, (c) The absorbed radiant flux, and (d) Total, hemispherical absorptivity.

SCHEMATIC:



ASSUMPTIONS: (1) Surface is opaque.

ANALYSIS: (a) The spectral, hemispherical absorptivity, α_λ, for an opaque surface is given by Eq. 12.58,

$$\alpha_\lambda = 1 - \rho_\lambda \qquad <$$

which is shown as a dashed line on the ρ_λ distribution axes.

(b) The total irradiation, G, follows from Eq. 12.16 which can be integrated by parts,

$$G = \int_0^\infty G_\lambda d\lambda = \int_0^{5\,\mu m} G_\lambda d\lambda + \int_{5\,\mu m}^{10\,\mu m} G_\lambda d\lambda + \int_{10\,\mu m}^{20\,\mu m} G_\lambda d\lambda$$

$$G = \frac{1}{2}\times 600\frac{W}{m^2\cdot\mu m}(5-0)\mu m + 600\frac{W}{m^2\cdot\mu m}(10-5)\mu m + \frac{1}{2}\times 600\frac{W}{m^2\cdot\mu m}\times(20-10)\mu m$$

$$G = 7500\ W/m^2. \qquad <$$

(c) The absorbed irradiation follows from Eqs. 12.45 and 12.46 with the form

$$G_{abs} = \int_0^\infty \alpha_\lambda G_\lambda d\lambda = \alpha_1 \int_0^{5\,\mu m} G_\lambda d\lambda + G_{\lambda,2}\int_{5\mu m}^{10\,\mu m} \alpha_\lambda d\lambda + \alpha_3 \int_{10\mu m}^{20\,\mu m} G_\lambda d\lambda.$$

Noting that $\alpha_1 = 1.0$ for $\lambda = 0 \rightarrow 5$ µm, $G_{\lambda,2} = 600$ W/m^2·µm for $\lambda = 5 \rightarrow 10$ µm and $\alpha_3 = 0$ for $\lambda > 10$ µm, find that

$$G_{abs} = 1.0\left(0.5\times 600\ W/m^2\cdot\mu m\right)(5-0)\mu m + 600\ W/m^2\cdot\mu m\,(0.5\times 0.5)(10-5)\mu m + 0$$

$$G_{abs} = 2250\ W/m^2. \qquad <$$

(d) The total, hemispherical absorptivity is defined as the fraction of the total irradiation that is absorbed. From Eq. 12.45,

$$\alpha = \frac{G_{abs}}{G} = \frac{2250\ W/m^2}{7500\ W/m^2} = 0.30. \qquad <$$

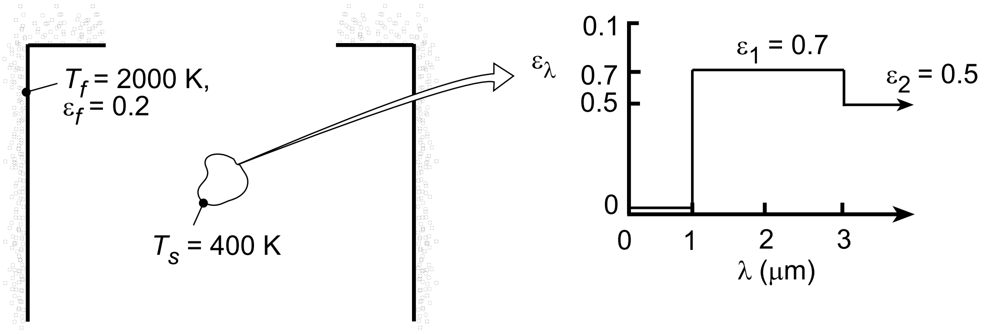
COMMENTS: Recognize that the total, hemispherical absorptivity, $\alpha = 0.3$, is for the given spectral irradiation. For a different G_λ, one would then expect a different value for α.

PROBLEM 12.44

KNOWN: Temperature and spectral emissivity of small object suspended in large furnace of prescribed temperature and total emissivity.

FIND: (a) Total surface emissivity and absorptivity, (b) Reflected radiative flux and net radiative flux to surface, (c) Spectral emissive power at λ = 2 μm, (d) Wavelength $\lambda_{1/2}$ for which one-half of total emissive power is in spectral region $\lambda \geq \lambda_{1/2}$.

SCHEMATIC:



ASSUMPTIONS: (1) Surface is opaque and diffuse, (2) Walls of furnace are much larger than object.

ANALYSIS: (a) The emissivity of the object may be obtained from Eq. 12.38, which is expressed as

$$\varepsilon(T_s) = \frac{\int_o^\infty \varepsilon_\lambda(\lambda) E_{\lambda,b}(\lambda, T_s)d\lambda}{E_b(T)} = \varepsilon_1\left[F_{(0\to3\mu m)} - F_{(0\to1\mu m)}\right] + \varepsilon_2\left[1 - F_{(0\to3\mu m)}\right]$$

where, with $\lambda_1 T_s$ = 400 μm·K and $\lambda_2 T_s$ = 1200 μm·K, $F_{(0\to1\mu m)} = 0$ and $F_{(0\to3\mu m)} = 0.002$. Hence,

$$\varepsilon(T_s) = 0.7(0.002) + 0.5(0.998) = 0.500$$ <

The absorptivity of the surface is determined by Eq. 12.46,

$$\alpha = \frac{\int_o^\infty \alpha_\lambda(\lambda) G_\lambda(\lambda)d\lambda}{\int_o^\infty G_\lambda(\lambda)d\lambda} = \frac{\int_o^\infty \alpha_\lambda(\lambda) E_{\lambda,b}(\lambda, T_f)d\lambda}{E_b(T_f)}$$

Hence, with $\lambda_1 T_f$ = 2000 μm·K and $\lambda_2 T_f$ = 6000 μm·K, $F_{(0\to1\mu m)} = 0.067$ and $F_{(0\to3\mu m)} = 0.738$. It follows that

$$\alpha = \alpha_1\left[F_{(0\to3\mu m)} - F_{(0\to1\mu m)}\right] + \alpha_2\left[1 - F_{(0\to3\mu m)}\right] = 0.7\times0.671 + 0.5\times0.262 = 0.601$$ <

(b) The reflected radiative flux is

$$G_{ref} = \rho G = (1-\alpha)E_b(T_f) = 0.399\times5.67\times10^{-8}\ \mathrm{W/m^2\cdot K^4}\,(2000\,\mathrm{K})^4 = 3.620\times10^5\ \mathrm{W/m^2}$$ <

The net radiative flux to the surface is

$$q''_{rad} = G - \rho G - \varepsilon E_b(T_s) = \alpha E_b(T_f) - \varepsilon E_b(T_s)$$

$$q''_{rad} = 5.67\times10^{-8}\ \mathrm{W/m^2\cdot K^4}\left[0.601(2000\,\mathrm{K})^4 - 0.500(400\,\mathrm{K})^4\right] = 5.438\times10^5\ \mathrm{W/m^2}$$ <

(c) At λ = 2 μm, λT_s = 800 K and, from Table 12.1, $I_{\lambda,b}(\lambda,T)/\sigma T^5 = 0.991\times10^{-7}\ (\mu m\cdot K\cdot sr)^{-1}$. Hence,

Continued...

PROBLEM 12.44 (Cont.)

$$I_{\lambda,b} = 0.991\times10^{-7}\times5.67\times10^{-8}\,\frac{\mathrm{W/m^2\cdot K^4}}{\mu\mathrm{m\cdot K\cdot sr}}\times(400\,\mathrm{K})^5 = 0.0575\,\frac{\mathrm{W}}{\mathrm{m^2\cdot\mu m\cdot sr}}$$

Hence, with $E_\lambda = \varepsilon_\lambda E_{\lambda,b} = \varepsilon_\lambda \pi I_{\lambda,b}$,

$$E_\lambda = 0.7(\pi\,\mathrm{sr})0.0575\,\mathrm{W/m^2\cdot\mu m\cdot sr} = 0.126\,\mathrm{W/m^2\cdot\mu m}$$ <

(d) From Table 12.1, $F_{(0\rightarrow\lambda)} = 0.5$ corresponds to $\lambda T_s \approx 4100\ \mu\mathrm{m\cdot K}$, in which case,

$$\lambda_{1/2} \approx 4100\,\mu\mathrm{m\cdot K}/400\,\mathrm{K} \approx 10.3\,\mu\mathrm{m}$$ <

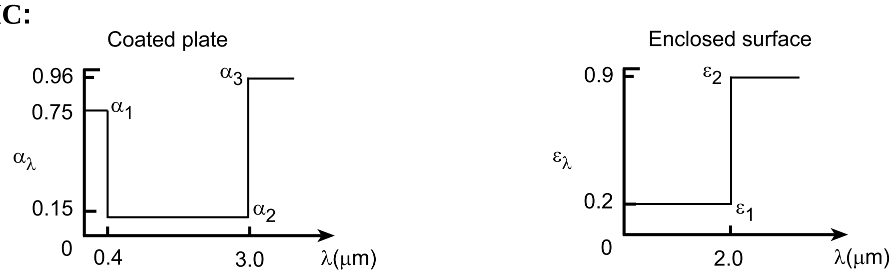
COMMENTS: Because of the significant difference between T_f and T_s, $\alpha \neq \varepsilon$. With increasing $T_s \rightarrow T_f$, ε would increase and approach a value of 0.601.

PROBLEM 12.45

KNOWN: Small flat plate maintained at 400 K coated with white paint having spectral absorptivity distribution (Figure 12.23) approximated as a stairstep function. Enclosure surface maintained at 3000 K with prescribed spectral emissivity distribution.

FIND: (a) Total emissivity of the enclosure surface, ε_{es}, and (b) Total emissivity, ε, and absorptivity, α, of the surface.

SCHEMATIC:



ASSUMPTIONS: (1) Coated plate with white paint is diffuse and opaque, so that $\alpha_\lambda = \varepsilon_\lambda$, (2) Plate is small compared to the enclosure surface, and (3) Enclosure surface is isothermal, diffuse and opaque.

ANALYSIS: (a) The total emissivity of the enclosure surface at T_{es} = 3000 K follows from Eq. 12.38 which can be expressed in terms of the bond emission factor, $F_{(0-\lambda T)}$, Eq. 12.30,

$$\varepsilon_{e,s} = \varepsilon_1 F_{(0-\lambda_1 T_{es})} + \varepsilon_2 \left[1 - F_{(0-\lambda_1 T_{es})}\right] = 0.2 \times 0.738 + 0.9[1 - 0.738] = 0.383$$ <

where, from Table 12.1, with $\lambda_1 T_{es}$ = 2 μm × 3000 K = 6000 μm·K, $F_{(0-\lambda T)}$ = 0.738.

(b) The total emissivity of the coated plate at T = 400 K can be expressed as

$$\varepsilon = \alpha_1 F_{(0-\lambda_1 T_s)} + \alpha_2 \left[F_{(0-\lambda_2 T_s)} - F_{(0-\lambda_1 T_s)}\right] + \alpha_3 \left[1 - F_{(0-\lambda_2 T_s)}\right]$$

$$\varepsilon = 0.75 \times 0 + 0.15[0.002134 - 0.000] + 0.96[1 - 0.002134] = 0.958$$ <

where, from Table 12.1, the band emission factors are: for $\lambda_1 T_s$ = 0.4 × 400 = 160 μm·K, find $F_{(0-\lambda_1 T_s)}$ = 0.000; for $\lambda_2 T_{es}$ = 3.0 × 400 = 1200 μm·K, find $F_{(0-\lambda_2 T_s)}$ = 0.002134. The total absorptivity for the irradiation due to the enclosure surface at T_{es} = 3000 K is

$$\alpha = \alpha_1 F_{(0-\lambda_1 T_{es})} + \alpha_2 \left[F_{(0-\lambda_2 T_{es})} - F_{(0-\lambda_2 T_{es})}\right] + \alpha_3 \left[1 - F_{(0-\lambda_2 T_{es})}\right]$$

$$\alpha = 0.75 \times 0.002134 + 0.15[0.8900 - 0.002134] + 0.96[1 - 0.8900] = 0.240$$ <

where, from Table 12.1, the band emission factors are: for $\lambda_1 T_{es}$ = 0.4 × 3000 = 1200 μm·K, find $F_{(0-\lambda_1 T_{es})}$ = 0.002134; for $\lambda_2 T_{es}$ = 3.0 × 3000 = 9000 μm·K, find $F_{(0-\lambda_2 T_{es})}$ = 0.8900.

COMMENTS: (1) In evaluating the total emissivity and absorptivity, remember that $\varepsilon = \varepsilon(\varepsilon_\lambda, T_s)$ and $\alpha = \alpha(\alpha_\lambda, G_\lambda)$ where T_s is the temperature of the surface and G_λ is the spectral irradiation, which if the surroundings are large and isothermal, $G_\lambda = E_{b,\lambda}(T_{sur})$. Hence, $\alpha = \alpha(\alpha_\lambda, T_{sur})$. For the opaque, diffuse surface, $\alpha_\lambda = \varepsilon_\lambda$.

(2) Note that the coated plate (white paint) has an absorptivity for the 3000 K-enclosure surface irradiation of α = 0.240. You would expect it to be a low value since the coating appears visually "white".

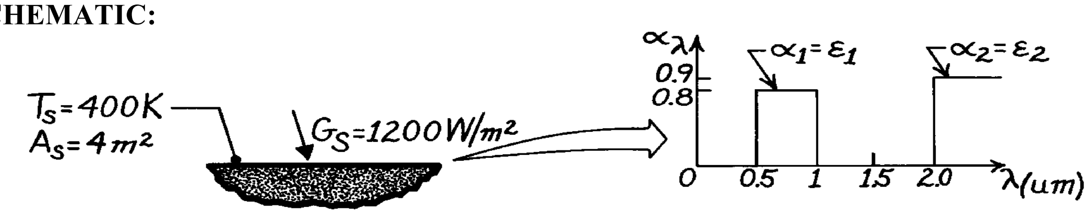
(3) The emissivity of the coated plate is quite high, ε = 0.958. Would you have expected this of a "white paint"? Most paints are oxide systems (high absorptivity at long wavelengths) with pigmentation (controls the "color" and hence absorptivity in the visible and near infrared regions).

PROBLEM 12.46

KNOWN: Area, temperature, irradiation and spectral absorptivity of a surface.

FIND: Absorbed irradiation, emissive power, radiosity and net radiation transfer from the surface.

SCHEMATIC:



ASSUMPTIONS: (1) Opaque, diffuse surface behavior, (2) Spectral distribution of solar radiation corresponds to emission from a blackbody at 5800 K.

ANALYSIS: The absorptivity to solar irradiation is

$$\alpha_s = \frac{\int_0^\infty \alpha_\lambda G_\lambda d\lambda}{G} = \frac{\int_0^\infty \alpha_\lambda E_{\lambda b}(5800\ K)\, d\lambda}{E_b} = \alpha_1 F_{(0.5\to 1\ \mu m)} + \alpha_2 F_{(2\to\infty)}.$$

From Table 12.1, $\lambda T = 2900\ \mu m \cdot K$: $F_{(0\to 0.5\ \mu m)} = 0.250$

$\lambda T = 5800\ \mu m \cdot K$: $F_{(0\to 1\ \mu m)} = 0.720$

$\lambda T = 11{,}600\ \mu m \cdot K$: $F_{(0\to 2\ \mu m)} = 0.941$

$$\alpha_s = 0.8(0.720 - 0.250) + 0.9(1 - 0.941) = 0.429.$$

Hence, $$G_{abs} = \alpha_S G_S = 0.429\left(1200\ W/m^2\right) = 515\ W/m^2.$$ <

The emissivity is

$$\varepsilon = \int_0^\infty \varepsilon_\lambda E_{\lambda b}(400\ K)\, d\lambda / E_b = \varepsilon_1 F_{(0.5\to 1\ \mu m)} + \varepsilon_2 F_{(2\to\infty)}.$$

From Table 12.1, $\lambda T = 200\ \mu m \cdot K$: $F_{(0\to 0.5\ \mu m)} = 0$

$\lambda T = 400\ \mu m \cdot K$: $F_{(0\to 1\ \mu m)} = 0$

$\lambda T = 800\ \mu m \cdot K$ $F_{(0\to 2\ \mu m)} = 0.$

Hence, $\varepsilon = \varepsilon_2 = 0.9$,

$$E = \varepsilon \sigma T_s^4 = 0.9 \times 5.67 \times 10^{-8}\ W/m^2 \cdot K^4 (400\ K)^4 = 1306\ W/m^2.$$ <

The radiosity is

$$J = E + \rho_S G_S = E + (1 - \alpha_s) G_S = [1306 + 0.571 \times 1200] W/m^2 = 1991\ W/m^2.$$ <

The net radiation transfer from the surface is

$$q_{net} = (E - \alpha_S G_S) A_s = (1306 - 515) W/m^2 \times 4\ m^2 = 3164\ W.$$ <

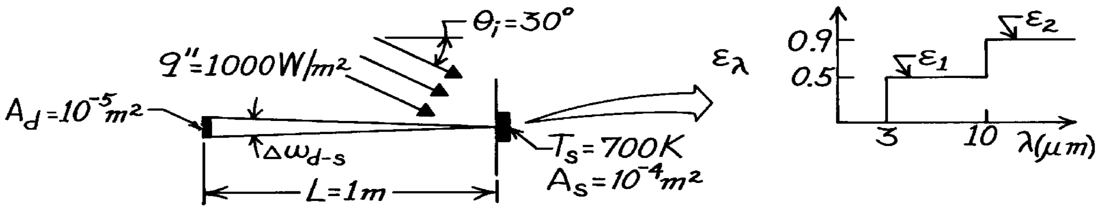
COMMENTS: Unless 3164 W are supplied to the surface by other means (for example, by convection), the surface temperature will decrease with time.

PROBLEM 12.47

KNOWN: Temperature and spectral emissivity of a receiving surface. Direction and spectral distribution of incident flux. Distance and aperture of surface radiation detector.

FIND: Radiant power received by the detector.

SCHEMATIC:



ASSUMPTIONS: (1) Target surface is diffuse, (2) $A_d/L^2 << 1$.

ANALYSIS: The radiant power received by the detector depends on emission and reflection from the target.

$$q_d = I_{e+r} A_s \cos\theta_{d-s} \Delta\omega_{d-s}$$

$$q_d = \frac{\varepsilon\sigma T_s^4 + \rho G}{\pi} A_s \frac{A_d}{L^2}$$

$$\varepsilon = \frac{\int_0^\infty \varepsilon_\lambda E_{\lambda b}(700\ K)\, d\lambda}{E_b(700\ K)} = \varepsilon_1 F_{(3\to 10\mu m)} + \varepsilon_2 F_{(10\to\infty)}.$$

From Table 12.1, $\lambda T = 2100\ \mu m\cdot K$: $F_{(0\to 3\ \mu m)} = 0.0838$

$\lambda T = 7000\ \mu m\cdot K$: $F_{(0\to 10\ \mu m)} = 0.8081$.

The emissivity can be expected as

$$\varepsilon = 0.5(0.8081 - 0.0838) + 0.9(1 - 0.8081) = 0.535.$$

Also,

$$\rho = \frac{\int_0^\infty \rho_\lambda G_\lambda d\lambda}{G} = \frac{\int_0^\infty (1-\varepsilon_\lambda) q''_\lambda d\lambda}{q''} = 1\times F_{(0\to 3\ \mu m)} + 0.5\times F_{(3\to 6\mu m)}$$

$$\rho = 1\times 0.4 + 0.5\times 0.6 = 0.70.$$

Hence, with $G = q''\cos\theta_i = 866\ W/m^2$,

$$q_d = \frac{0.535\times 5.67\times 10^{-8}\ W/m^2\cdot K^4 (700\ K)^4 + 0.7\times 866\ W/m^2}{\pi} 10^{-4} m^2 \frac{10^{-5} m^2}{(1m)^2}$$

$$q_d = 2.51\times 10^{-6}\ W.$$ <

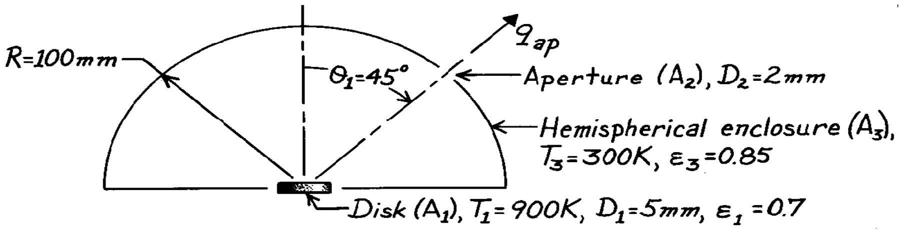
COMMENTS: A total radiation detector cannot discriminate between emitted and reflected radiation from a surface.

PROBLEM 12.48

KNOWN: Small disk positioned at center of an isothermal, hemispherical enclosure with a small aperture.

FIND: Radiant power [μW] leaving the aperture.

SCHEMATIC:



ASSUMPTIONS: (1) Disk is diffuse-gray, (2) Enclosure is isothermal and has area much larger than disk, (3) Aperture area is very small compared to enclosure area, (4) Areas of disk and aperture are small compared to radius squared of the enclosure.

ANALYSIS: The radiant power leaving the aperture is due to radiation leaving the disk and to irradiation on the aperture from the enclosure. That is,

$$q_{ap} = q_{1\to 2} + G_2 \cdot A_2. \tag{1}$$

The radiation leaving the disk can be written in terms of the radiosity of the disk. For the diffuse disk,

$$q_{1\to 2} = \frac{1}{\pi} J_1 \cdot A_1 \cos\theta_1 \cdot \omega_{2-1} \tag{2}$$

and with ε = α for the gray behavior, the radiosity is

$$J_1 = \varepsilon_1 E_b(T_1) + \rho G_1 = \varepsilon_1 \sigma T_1^4 + (1-\varepsilon_1)\sigma T_3^4 \tag{3}$$

where the irradiation G_1 is the emissive power of the black enclosure, $E_b(T_3)$; $G_1 = G_2 = E_b(T_3)$.
The solid angle ω_{2-1} follows from Eq. 12.2,

$$\omega_{2-1} = A_2 / R^2. \tag{4}$$

Combining Eqs. (2), (3) and (4) into Eq. (1) with $G_2 = \sigma T_3^4$, the radiant power is

$$q_{ap} = \frac{1}{\pi}\sigma\left[\varepsilon_1 T_1^4 + (1-\varepsilon_1)T_3^4\right] A_1 \cos\theta_1 \cdot \frac{A_2}{R^2} + A_2 \sigma T_3^4$$

$$q_{ap} = \frac{1}{\pi} 5.67\times10^{-8} \frac{W}{m^2\cdot K^4}\left[0.7(900K)^4 + (1-0.7)(300K)^4\right]\frac{\pi}{4}(0.005m)^2 \cos45°\times$$
$$\frac{\pi/4(0.002m)^2}{(0.100m)^2} + \frac{\pi}{4}(0.002m)^2\, 5.67\times10^{-8} W/m^2\cdot K^4 (300K)^4$$

$$q_{ap} = (36.2 + 0.19 + 1443)\mu W = 1479\,\mu W.$$ <

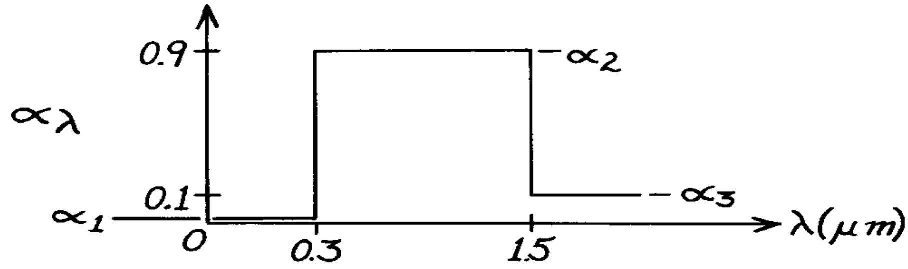
COMMENTS: Note the relative magnitudes of the three radiation components. Also, recognize that the emissivity of the enclosure ε_3 doesn't enter into the analysis. Why?

PROBLEM 12.49

KNOWN: Spectral, hemispherical absorptivity of an opaque surface.

FIND: (a) Solar absorptivity, (b) Total, hemispherical emissivity for $T_s = 340K$.

SCHEMATIC:



ASSUMPTIONS: (1) Surface is opaque, (2) $\varepsilon_\lambda = \alpha_\lambda$, (3) Solar spectrum has $G_\lambda = G_{\lambda,S}$ proportional to $E_{\lambda,b}$ (λ, 5800K).

ANALYSIS: (a) The solar absorptivity follows from Eq. 12.47.

$$\alpha_S = \int_0^\infty \alpha_\lambda(\lambda) E_{\lambda,b}(\lambda,5800K)\, d\lambda \Big/ \int_0^\infty E_{\lambda,b}(\lambda,5800K)\, d\lambda.$$

The integral can be written in three parts using $F_{(0\to\lambda)}$ terms.

$$\alpha_S = \alpha_1 F_{(0\to0.3)} + \alpha_2\left[F_{(0\to1.5)} - F_{(0\to0.3)}\right] + \alpha_3\left[1 - F_{(0\to1.5)}\right].$$

From Table 12.1,

$\lambda T = 0.3 \times 5800 = 1740\ \mu m \cdot K$	$F_{(0\to0.3\,\mu m)} = 0.0335$
$\lambda T = 1.5 \times 5800 = 8700\ \mu m \cdot K$	$F_{(0\to1.5\,\mu m)} = 0.8805.$

Hence,

$$\alpha_S = 0\times0.0355 + 0.9[0.8805 - 0.0335] + 0.1[1 - 0.8805] = 0.774.$$ <

(b) The total, hemispherical emissivity for the surface at 340K will be

$$\varepsilon = \int_0^\infty \varepsilon_\lambda(\lambda) E_{\lambda,b}(\lambda,340K)\, d\lambda \,/\, E_b(340K).$$

If $\varepsilon_\lambda = \alpha_\lambda$, then using the α_λ distribution above, the integral can be written in terms of $F_{(0\to\lambda)}$ values. It is readily recognized that since

$$F_{(0\to1.5\,\mu m,340K)} \approx 0.000 \qquad \text{at} \qquad \lambda T = 1.5\times340 = 510\,\mu m\cdot K$$

there is negligible spectral emissive power below 1.5 μm. It follows then that

$$\varepsilon = \varepsilon_\lambda = \alpha_\lambda = 0.1$$ <

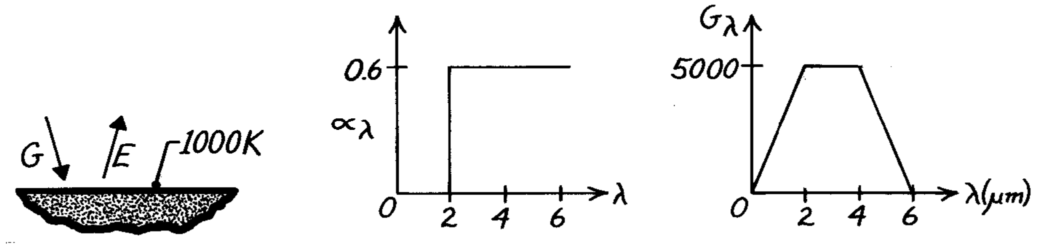
COMMENTS: The assumption $\varepsilon_\lambda = \alpha_\lambda$ can be satisfied if this surface were irradiated diffusely or if the surface itself were diffuse. Note that for this surface under the specified conditions of solar irradiation and surface temperature $\alpha_S \neq \varepsilon$. Such a surface is referred to as a spectrally selective surface.

PROBLEM 12.50

KNOWN: Spectral distribution of the absorptivity and irradiation of a surface at 1000 K.

FIND: (a) Total, hemispherical absorptivity, (b) Total, hemispherical emissivity, (c) Net radiant flux to the surface.

SCHEMATIC:



ASSUMPTIONS: (1) $\alpha_\lambda = \varepsilon_\lambda$.

ANALYSIS: (a) From Eq. 12.46,

$$\alpha = \frac{\int_0^\infty \alpha_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda} = \frac{\int_0^{2\mu m} \alpha_\lambda G_\lambda d\lambda + \int_2^{4\mu m} \alpha_\lambda G_\lambda d\lambda + \int_4^{6\mu m} \alpha_\lambda G_\lambda d\lambda}{\int_0^{2\mu m} G_\lambda d\lambda + \int_2^{4\mu m} G_\lambda d\lambda + \int_4^{6\mu m} G_\lambda d\lambda}$$

$$\alpha = \frac{0\times 1/2(2-0)5000 + 0.6(4-2)5000 + 0.6\times 1/2(6-4)5000}{1/2(2-0)5000 + (4-2)(5000) + 1/2(6-4)5000}$$

$$\alpha = \frac{9000}{20,000} = 0.45.$$ <

(b) From Eq. 12.38,

$$\varepsilon = \frac{\int_0^\infty \varepsilon_\lambda E_{\lambda,b} d\lambda}{E_b} = \frac{0\int_0^{2\mu m} E_{\lambda,b} d\lambda}{E_b} + \frac{0.6\int_2^\infty E_{\lambda,b} d\lambda}{E_b}$$

$$\varepsilon = 0.6F_{(2\mu m \to \infty)} = 0.6\left[1 - F_{(0\to 2\ \mu m)}\right].$$

From Table 12.1, with $\lambda T = 2\ \mu m \times 1000K = 2000\ \mu m\cdot K$, find $F_{(0 \to 2\ \mu m)} = 0.0667$. Hence,

$$\varepsilon = 0.6[1 - 0.0667] = 0.56.$$ <

(c) The net radiant heat flux *to* the surface is

$$q''_{rad,net} = \alpha G - E = \alpha G - \varepsilon\sigma T^4$$

$$q''_{rad,net} = 0.45\left(20,000 W/m^2\right) - 0.56\times 5.67\times 10^{-8} W/m^2\cdot K^4 \times (1000K)^4$$

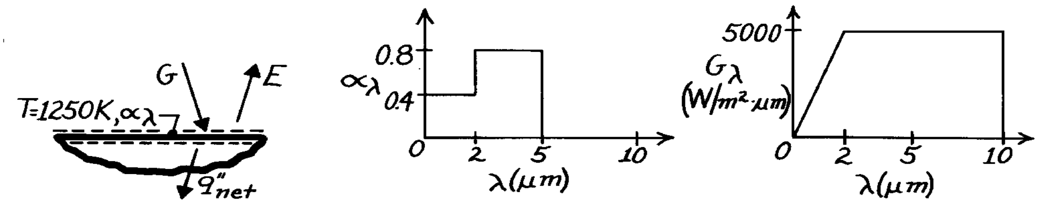
$$q''_{rad,net} = (9000 - 31,751)\ W/m^2 = -22,751 W/m^2.$$ <

PROBLEM 12.51

KNOWN: Spectral distribution of surface absorptivity and irradiation. Surface temperature.

FIND: (a) Total absorptivity, (b) Emissive power, (c) Nature of surface temperature change.

SCHEMATIC:



ASSUMPTIONS: (1) Opaque, diffuse surface behavior, (2) Convection effects are negligible.

ANALYSIS: (a) From Eqs. 12.45 and 12.46, the absorptivity is defined as

$$\alpha \equiv G_{abs}/G = \int_0^\infty \alpha_\lambda G_\lambda d\lambda / \int_0^\infty G_\lambda d\lambda.$$

The absorbed irradiation is,

$$G_{abs} = 0.4\left(5000\,W/m^2\cdot\mu m\times 2\,\mu m\right)/2 + 0.8\times 5000\,W/m^2\cdot\mu m\,(5-2)\,\mu m + 0 = 14,000\ W/m^2.$$

The irradiation is,

$$G = \left(2\,\mu m\times 5000\ W/m^2\cdot\mu m\right)/2 + (10-2)\,\mu m\times 5000\ W/m^2\cdot\mu m = 45,000\ W/m^2.$$

Hence, $\alpha = 14,000\ W/m^2 / 45,000\ W/m^2 = 0.311.$ <

(b) From Eq. 12.38, the emissivity is

$$\varepsilon = \int_0^\infty \varepsilon_\lambda E_{\lambda,b}\, d\lambda / E_b = 0.4\int_0^2 E_{\lambda,b}\, d\lambda / E_b + 0.8\int_2^5 E_{\lambda,b}\, d\lambda / E_b$$

From Table 12.1, $\lambda T = 2\ \mu m \times 1250K = 2500K$, $F_{(0-2)} = 0.162$

$\lambda T = 5\ \mu m \times 1250K = 6250K$, $F_{(0-5)} = 0.757.$

Hence, $\varepsilon = 0.4\times 0.162 + 0.8\,(0.757 - 0.162) = 0.54.$

$$E = \varepsilon E_b = \varepsilon\sigma T^4 = 0.54\times 5.67\times 10^{-8}\,W/m^2\cdot K^4\,(1250K)^4 = 74,751\,W/m^2.$$ <

(c) From an energy balance on the surface, the net heat flux *to* the surface is

$$q''_{net} = \alpha G - E = (14,000 - 74,751)\,W/m^2 = -60,751\,W/m^2.$$

Hence the temperature of the surface is *decreasing.* <

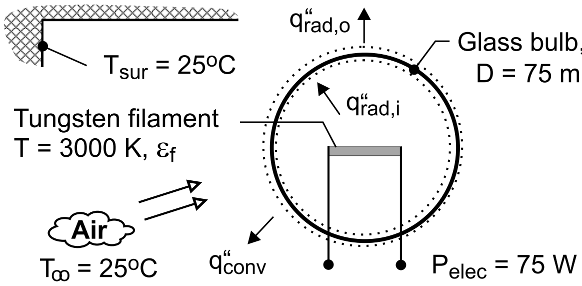
COMMENTS: Note that $\alpha \neq \varepsilon$. Hence the surface is not gray for the prescribed conditions.

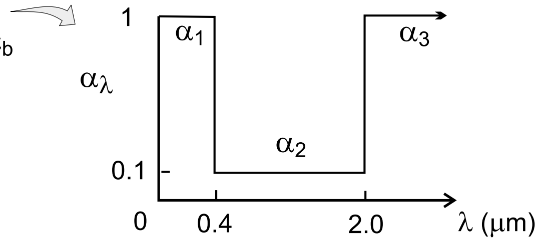
PROBLEM 12.52

KNOWN: Power dissipation temperature and distribution of spectral emissivity for a tungsten filament. Distribution of spectral absorptivity for glass bulb. Temperature of ambient air and surroundings. Bulb diameter.

FIND: Bulb temperature.

SCHEMATIC:





ASSUMPTIONS: (1) Steady-state, (2) Uniform glass temperature, T_s, and uniform irradiation of inner surface, (3) Surface of glass is diffuse, (4) Negligible absorption of radiation by filament due to emission from inner surface of bulb, (5) Net radiation transfer from outer surface of bulb is due to exchange with large surroundings, (6) Bulb temperature is sufficiently low to provide negligible emission at $\lambda < 2\mu$m, (7) Ambient air is quiescent.

PROPERTIES: *Table A-4*, air (assume T_f = 323 K): $\nu = 1.82 \times 10^{-5}$ m^2/s, $\alpha = 2.59 \times 10^{-5}$ m^2/s,k = 0.028 W/m·K, β = 0.0031 K^{-1}, Pr = 0.704.

ANALYSIS: From an energy balance on the glass bulb,

$$q''_{rad,i} = q''_{rad,o} + q''_{conv} = \varepsilon_b \sigma \left(T_s^4 - T_{sur}^4\right) + \bar{h}\left(T_s - T_\infty\right) \tag{1}$$

where $\varepsilon_b = \varepsilon_{\lambda > 2\mu m} = \alpha_{\lambda > 2\mu m} = 1$ and $\bar{h}$ is obtained from Eq. (9.35)

$$\overline{Nu}_D = 2 + \frac{0.589\ Ra_D^{1/4}}{\left[1 + \left(0.469/Pr\right)^{9/16}\right]^{4/9}} = \frac{\bar{h}D}{k} \tag{2}$$

with $Ra_D = g\beta\left(T_s - T_\infty\right)D^3/\nu\alpha$. Radiation absorption at the inner surface of the bulb may be expressed as

$$q''_{rad,i} = \alpha G = \alpha\left(P_{elec}/\pi D^2\right) \tag{3}$$

where, from Eq. (12.46),

$$\alpha = \alpha_1 \int_0^{0.4} \left(G_\lambda / G\right) d\lambda + \alpha_2 \int_{0.4}^{2.0} \left(G_\lambda / G\right) d\lambda + \alpha_3 \int_2^{\infty} \left(G_\lambda / G\right) d\lambda$$

Continued …..

The irradiation is due to emission from the filament, in which case $(G_\lambda/G) \sim (E_\lambda/E)_f = (\varepsilon_{f,\lambda}E_{\lambda,b}/\varepsilon_f E_b)$. Hence,

$$\alpha = (\alpha_1/\varepsilon_f)\int_0^{0.4}\varepsilon_{f,\lambda}(E_{\lambda,b}/E_b)d\lambda + (\alpha_2/\varepsilon_f)\int_{0.4}^{2.0}\varepsilon_{f,\lambda}(E_{\lambda,b}/E_b)d\lambda + (\alpha_3/\varepsilon_f)\int_2^{\infty}\varepsilon_{f,\lambda}(E_{\lambda,b}/E_b)d\lambda \quad (4)$$

where, from the spectral distribution of Problem 12.25, $\varepsilon_{f,\lambda} \equiv \varepsilon_1 = 0.45$ for $\lambda < 2\mu m$ and $\varepsilon_{f,\lambda} \equiv \varepsilon_2 = 0.10$ for $\lambda > 2\mu m$. From Eq. (12.38)

$$\varepsilon_f = \int_0^{\infty}\varepsilon_{f,\lambda}(E_{\lambda,b}/E_b)d\lambda = \varepsilon_1 F_{(0\to 2\mu m)} + \varepsilon_2\left(1 - F_{(0\to 2\mu m)}\right)$$

With $\lambda T_f = 2\mu m \times 3000\ K = 6000\ \mu m \cdot K$, $F_{(0\to 2\mu m)} = 0.738$ from Table 12.1. Hence,

$$\varepsilon_f = 0.45 \times 0.738 + 0.1(1 - 0.738) = 0.358$$

Equation (4) may now be expressed as

$$\alpha = (\alpha_1/\varepsilon_f)\varepsilon_1 F_{(0\to 0.4\mu m)} + (\alpha_2/\varepsilon_f)\varepsilon_1\left(F_{(0\to 2\mu m)} - F_{(0\to 0.4\mu m)}\right) + (\alpha_3/\varepsilon_f)\varepsilon_2\left(1 - F_{(0\to 2\mu m)}\right)$$

where, with $\lambda T = 0.4\mu m \times 3000\ K = 1200\ \mu m \cdot K$, $F_{(0\to 0.4\mu m)} = 0.0021$. Hence,

$$\alpha = (1/0.358)0.45 \times 0.0021 + (0.1/0.358)0.45 \times (0.738 - 0.0021) + (1/0.358)0.1(1 - 0.738) = 0.168$$

Substituting Eqs. (2) and (3) into Eq. (1), as well as values of $\varepsilon_b = 1$ and $\alpha = 0.168$, an iterative solution yields

$$T_s = 348.1\ K$$ <

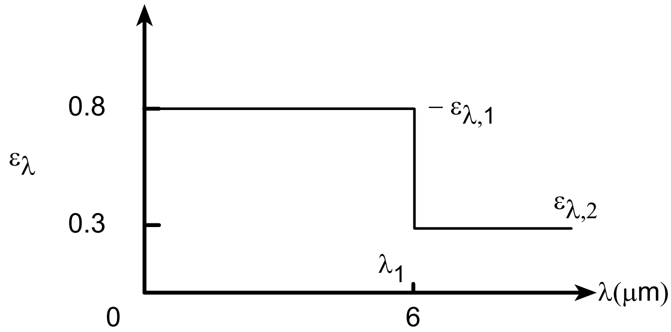
COMMENTS: For the prescribed conditions, $q''_{rad,i} = 713\ W/m^2$, $q''_{rad,o} = 385.5\ W/m^2$ and $q''_{conv} = 327.5\ W/m^2$.

PROBLEM 12.53

KNOWN: Spectral emissivity of an opaque, diffuse surface.

FIND: (a) Total, hemispherical emissivity of the surface when maintained at 1000 K, (b) Total, hemispherical absorptivity when irradiated by large surroundings of emissivity 0.8 and temperature 1500 K, (c) Radiosity when maintained at 1000 K and irradiated as prescribed in part (b), (d) Net radiation flux into surface for conditions of part (c), and (e) Compute and plot each of the parameters of parts (a)-(c) as a function of the surface temperature T_s for the range $750 < T_s \leq 2000$ K.

SCHEMATIC:



ASSUMPTIONS: (1) Surface is opaque, diffuse, and (2) Surroundings are large compared to the surface.

ANALYSIS: (a) When the surface is maintained at 1000 K, the total, hemispherical emissivity is evaluated from Eq. 12.38 written as

$$\varepsilon = \int_0^\infty \varepsilon_\lambda E_{\lambda,b}(T)\, d\lambda / E_b(T) = \varepsilon_{\lambda,1}\int_0^{\lambda_1} E_{\lambda,b}(T)\, d\lambda / E_b(T) + \varepsilon_{\lambda,2}\int_{\lambda 1}^\infty E_{\lambda,b}(T)\, d\lambda / E_b(T)$$

$$\varepsilon = \varepsilon_{\lambda,1} F_{(0-\lambda_1 T)} + \varepsilon_{\lambda,2}\left(1 - F_{(0-\lambda_1 T)}\right)$$

where for $\lambda T = 6\mu m \times 1000\ K = 6000 \mu m \cdot K$, from Table 12.1, find $F_{0-\lambda T} = 0.738$. Hence,

$$\varepsilon = 0.8 \times 0.738 + 0.3(1 - 0.738) = 0.669.$$ <

(b) When the surface is irradiated by large surroundings at $T_{sur} = 1500$ K, $G = E_b(T_{sur})$.
From Eq. 12.46,

$$\alpha = \int_0^\infty \alpha_\lambda G_\lambda\, d\lambda \Big/ \int_0^\infty G_\lambda\, d\lambda = \int_0^\infty \varepsilon_\lambda E_{\lambda,b}(T_{sur})\, d\lambda / E_b(T_{sur})$$

$$\alpha = \varepsilon_{\lambda,1} F_{(0-\lambda_1 T_{sur})} + \varepsilon_{\lambda,2}\left(1 - F_{(0-\lambda_1 T_{sur})}\right)$$

where for $\lambda_1 T_{sur} = 6\ \mu m \times 1500\ K = 9000\ \mu m \cdot K$, from Table 12.1, find $F_{(0-\lambda T)} = 0.890$. Hence,

$$\alpha = 0.8 \times 0.890 + 0.3\ (1 - 0.890) = 0.745.$$ <

Note that $\alpha_\lambda = \varepsilon_\lambda$ for all conditions and the emissivity of the surroundings is irrelevant.

(c) The radiosity for the surface maintained at 1000 K and irradiated as in part (b) is

$$J = \varepsilon E_b(T) + \rho G = \varepsilon E_b(T) + (1 - \alpha) E_b(T_{sur})$$

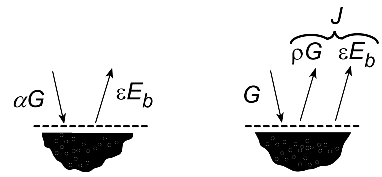
$$J = 0.669 \times 5.67 \times 10^{-8}\ W/m^2 \cdot K^4\ (1000\ K)^4 + (1 - 0.745)\ 5.67 \times 10^{-8}\ W/m^2 \cdot K^4\ (1500\ K)^4$$

$$J = (37{,}932 + 73{,}196)\ W/m^2 = 111{,}128\ W/m^2$$ <

Continued...

PROBLEM 12.53 (Cont.)

(d) The net radiation flux into the surface with $G = \sigma T_{sur}^4$ is

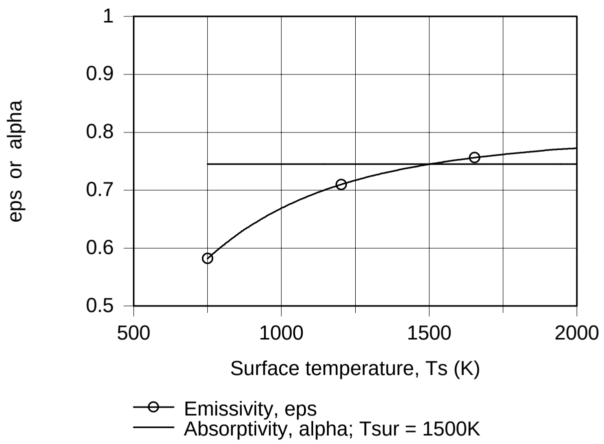


$$q''_{rad,in} = \alpha G - \varepsilon E_b(T) = G - J$$

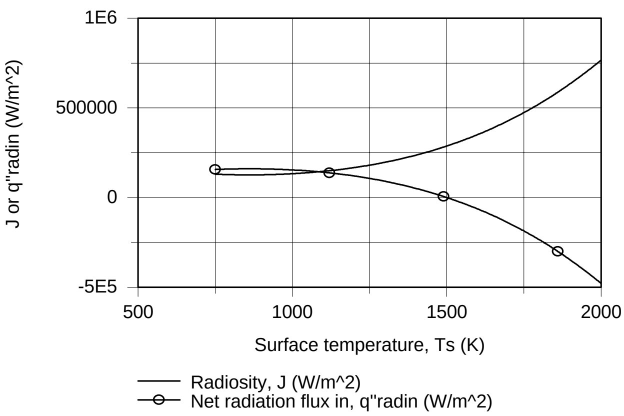
$$q''_{rad,in} = 5.67 \times 10^{-8}\ W/m^2 \cdot K\ (1500\ K)^4 - 111{,}128\ W/m^2$$

$$q''_{rad,in} = 175{,}915\ W/m^2. \quad <$$

(e) The foregoing equations were entered into the IHT workspace along with the *IHT Radiaton Tool, Band Emission Factor,* to evaluate $F_{(0-\lambda T)}$ values and the respective parameters for parts (a)-(d) were computed and are plotted below.



Note that the absorptivity, $\alpha = \alpha(\alpha_\lambda, T_{sur})$, remains constant as T_s changes since it is a function of α_λ (or ε_λ) and Tsur only. The emissivity $\varepsilon = \varepsilon(\varepsilon_\lambda, T_s)$ is a function of T_s and increases as T_s increases. Could you have surmised as much by looking at the spectral emissivity distribution? At what condition is $\varepsilon = \alpha$?



The radiosity, J_1 increases with increasing T_s since $E_b(T)$ increases markedly with temperature; the reflected irradiation, $(1 - \alpha)E_b(T_{sur})$ decreases only slightly as T_s increases compared to $E_b(T)$. Since G is independent of T_s, it follows that the variation of $q''_{rad,in}$ will be due to the radiosity change; note the sign difference.

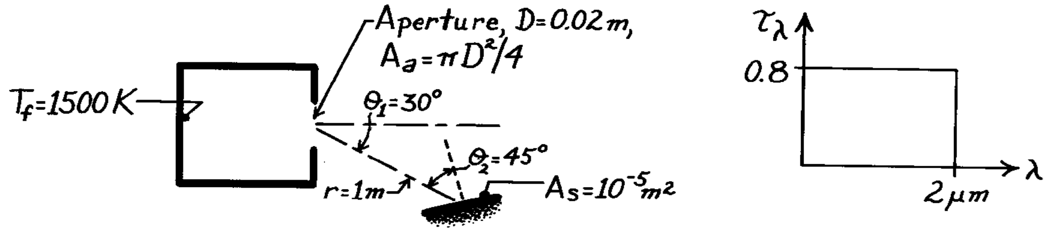
COMMENTS: We didn't use the emissivity of the surroundings ($\varepsilon = 0.8$) to determine the irradiation onto the surface. Why?

PROBLEM 12.54

KNOWN: Furnace wall temperature and aperture diameter. Distance of detector from aperture and orientation of detector relative to aperture.

FIND: (a) Rate at which radiation from the furnace is intercepted by the detector, (b) Effect of aperture window of prescribed spectral transmissivity on the radiation interception rate.

SCHEMATIC:



ASSUMPTIONS: (1) Radiation emerging from aperture has characteristics of emission from a blackbody, (2) Cover material is diffuse, (3) Aperture and detector surface may be approximated as infinitesimally small.

ANALYSIS: (a) From Eq. 12.5, the heat rate leaving the furnace aperture and intercepted by the detector is

$$q = I_e A_a \cos\theta_1\, \omega_{s-a}.$$

From Eqs. 12.14 and 12.28

$$I_e = \frac{E_b(T_f)}{\pi} = \frac{\sigma T_f^4}{\pi} = \frac{5.67\times10^{-8}(1500)^4}{\pi} = 9.14\times10^4\ \mathrm{W/m^2\cdot sr}.$$

From Eq. 12.2,

$$\omega_{s-a} = \frac{A_n}{r^2} = \frac{A_s\cdot\cos\theta_2}{r^2} = \frac{10^{-5}\ \mathrm{m^2}\times\cos45^\circ}{(1\mathrm{m})^2} = 0.707\times10^{-5}\ \mathrm{sr}.$$

Hence

$$q = 9.14\times10^4\,\mathrm{W/m^2\cdot sr}\left[\pi(0.02\mathrm{m})^2/4\right]\cos30^\circ\times0.707\times10^{-5}\mathrm{sr} = 1.76\times10^{-4}\mathrm{W}. \quad <$$

(b) With the window, the heat rate is

$$q = \tau\left(I_e A_a \cos\theta_1\, \omega_{s-a}\right)$$

where τ is the transmissivity of the window to radiation emitted by the furnace wall. From Eq. 12.55,

$$\tau = \frac{\int_0^\infty \tau_\lambda G_\lambda\, d\lambda}{\int_0^\infty G_\lambda\, d\lambda} = \frac{\int_0^\infty \tau_\lambda E_{\lambda,b}(T_f)\, d\lambda}{\int_0^\infty E_{\lambda,b}\, d\lambda} = 0.8\int_0^2 \left(E_{\lambda,b}/E_b\right) d\lambda = 0.8F_{(0\to2\mu\mathrm{m})}.$$

With $\lambda T = 2\ \mu\mathrm{m}\times1500\mathrm{K} = 3000\ \mu\mathrm{m\cdot K}$, Table 12.1 gives $F_{(0\to2\ \mu\mathrm{m})} = 0.273$. Hence, with $\tau = 0.273\times0.8 = 0.218$, find

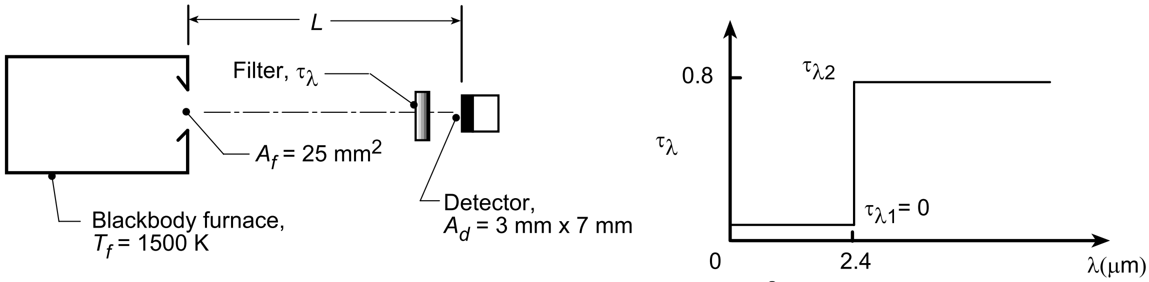
$$q = 0.218\times1.76\times10^{-4}\mathrm{W} = 0.384\times10^{-4}\mathrm{W}. \quad <$$

PROBLEM 12.55

KNOWN: Thermocouple is irradiated by a blackbody furnace at 1500 K with 25 mm^2 aperture. Optical fiber of prescribed spectral transmissivity in sight path.

FIND: (a) Distance L from the furnace detector should be positioned such that its irradiation is G = 50 W/m^2 and, (b) Compute and plot irradiation, G, vs separation distance L for the range $100 \le L \le 400$ mm for blackbody furnace temperatures of T_f = 1000, 1500 and 2000 K.

SCHEMATIC:



ASSUMPTIONS: (1) Furnace aperture emits diffusely, (2) $A_d << L^2$.

ANALYSIS: (a) The irradiation on the detector due to emission from the furnace which passes through the filter is defined as

$$G_d = q_{f\to d}/A_d = 50\,W/m^2 \tag{1}$$

where the power leaving the furnace and intercepted at the detector is

$$q_{f\to d} = [I_f \cdot A_f \cos\theta_f \cdot \omega_{d-f}]\tau_{filter} = \left[\frac{\sigma T^4}{\pi}\cdot A_f \cos\theta_f \cdot \frac{A_d}{L^2}\right]\tau_{filter}\,. \tag{2}$$

The transmittance of the filter is

$$\tau_{filter} = \tau_{\lambda 1}F_{0-\lambda T} + \tau_{\lambda 2}(1-F_{0-\lambda T}) = 0\times 0.4036 + 0.8(1-0.4036) = 0.477 \tag{3}$$

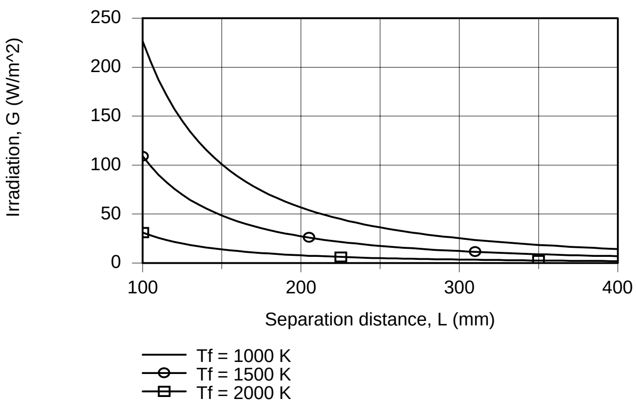
where $F_{0-\lambda T} = 0.4036$ with $\lambda T = 2.4\times 1500 = 3600\ \mu m\cdot K$ from Table 12.1. Combining Eqs. (1) and (2) and substituting numerical values,

$$G_d = (1/\pi)5.67\times 10^{-8}\,W/m^2\cdot K^4 \times (1500K)^4\ (25\times 10^{-6}m^2\times 1)(A_d/L^2)\times 0.477/A_d = 50\,W/m^2$$

find

$$L = 147\ mm.$$ <

(b) Using the foregoing equations in the IHT workspace along with the *IHT Radiation Tool*, *Band Emission Factor*, G was computed and plotted as a function of L for selected blackbody temperatures.



Continued...

PROBLEM 12.55 (Cont.)

The irradiation decreases with increasing separation distance x as the inverse square of the distance. At any fixed separation distance, the irradiation increases as T_f increases. In what manner will G depend upon T_f? Is $G \sim T_f^4$? Why not?

COMMENTS: The IHT workspace used to generate the above plot is shown below.

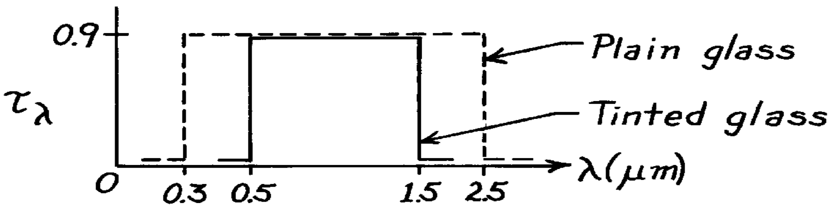
```
// Irradiation, Eq (2):
G = qfd / Ad
qfd = Ief * Af * omegadf * tauf
omegadf = Ad / L^2
Ief = Ebf / pi
Ebf = sigma * Tf^4
sigma = 5.67e-8
// Transmittance, Eq (3):
tauf = tau1 * FL1Tf  +  tau2 * ( 1 - FL1Tf )
/* The blackbody band emission factor, Figure 12.14 and Table 12.1, is */
FL1Tf = F_lambda_T(lambda1,Tf)    // Eq 12.30
// where units are lambda (micrometers, mum) and T (K)
// Assigned Variables:
G = 50                        // Irradiation on detector, W/m^2
Tf = 1000                     // Furnace temperature, K
//Tf = 1500
//Tf = 2000
Af = 25 * 1e-6                // Furnace aperture, m^2
Ad = 0.003 * 0.007            // Detector area, m^2
tau1 = 0                      // Spectral transmittance, <= lambda1
tau2 = 0.8                    // Spectral transmittance, >= lambda2
lambda1 = 2.4                 // Wavelength, mum
//L = 0.194                   // Separation distance, m
L_mm = L * 1000               // Separation distance, mm
/* Data Browser Results - Part (a)
Ebf     FL1Tf   Ief      L       omegadf   qfd     tauf     Ad      Af      Df      G
        Tf      lambda1  sigma   tau1      tau2
2.87E5  0.4036  9.137E4  0.1476  0.0009634         0.00105  0.4771  2.1E-5  2.5E-5  0.025
        50      1500     2.4     5.67E-8   0       0.8   */
```

PROBLEM 12.56

KNOWN: Spectral transmissivity of a plain and tinted glass.

FIND: (a) Solar energy transmitted by each glass, (b) Visible radiant energy transmitted by each with solar irradiation.

SCHEMATIC:



ASSUMPTIONS: (1) Spectral distribution of solar irradiation is proportional to spectral emissive power of a blackbody at 5800K.

ANALYSIS: To compare the energy transmitted by the glasses, it is sufficient to calculate the transmissivity of each glass for the prescribed spectral range when the irradiation distribution is that of the solar spectrum. From Eq. 12.55,

$$\tau_S = \int_0^\infty \tau_\lambda \cdot G_{\lambda,S} d\lambda \Big/ \int_0^\infty G_{\lambda,S} d\lambda = \int_0^\infty \tau_\lambda \cdot E_{\lambda,b}(\lambda, 5800\text{K})\, d\lambda / E_b(5800\text{K}).$$

Recognizing that τ_λ will be constant for the range $\lambda_1 \to \lambda_2$, using Eq. 12.31, find

$$\tau_S = \tau_\lambda \cdot F_{(\lambda_1 \to \lambda_2)} = \tau_\lambda \left[F_{(0\to\lambda_2)} - F_{(0\to\lambda_1)} \right].$$

(a) For the two glasses, the solar transmissivity, using Table 12.1 for F, is then

Plain glass: $\lambda_2 = 2.5$ μm $\quad \lambda_2 T = 2.5\ \mu\text{m} \times 5800\text{K} = 14{,}500\ \mu\text{m}\cdot\text{K}$ $\quad F_{(0\to\lambda 2)} = 0.966$

$\lambda_1 = 0.3$ μm $\quad \lambda_1 T = 0.3\ \mu\text{m} \times 5800\text{K} = 1{,}740\ \mu\text{m}\cdot\text{K}$ $\quad F_{(0\to\lambda_1)} = 0.033$

$\tau_S = 0.9\,[0.966 - 0.033] = 0.839.$ <

Tinted glass: $\lambda_2 = 1.5$ μm $\quad \lambda_2 T = 1.5\ \mu\text{m} \times 5800\text{K} = 8{,}700\ \mu\text{m}\cdot\text{K}$ $\quad F_{(0\to\lambda_2)} = 0.881$

$\lambda_1 = 0.5$ μm $\quad \lambda_1 T = 0.5\ \mu\text{m} \times 5800\text{K} = 2{,}900\ \mu\text{m}\cdot\text{K}$ $\quad F_{(0\to\lambda_1)} = 0.033$

$\tau_S = 0.9\,[0.886 - 0.250] = 0.568.$ <

(b) The limits of the visible spectrum are $\lambda_1 = 0.4$ and $\lambda_2 = 0.7$ μm. For the tinted glass, $\lambda_1 = 0.5$ μm rather than 0.4 μm. From Table 12.1,

$\lambda_2 = 0.7$ μm $\quad \lambda_2 T = 0.7\ \mu\text{m} \times 5800\text{K} = 4{,}060\ \mu\text{m}\cdot\text{K}$ $\quad F_{(0\to\lambda_2)} = 0.491$

$\lambda_1 = 0.5$ μm $\quad \lambda_1 T = 0.5\ \mu\text{m} \times 5800\text{K} = 2{,}900\ \mu\text{m}\cdot\text{K}$ $\quad F_{(0\to\lambda_1)} = 0.250$

$\lambda_1 = 0.4$ μm $\quad \lambda_1 T = 0.4\ \mu\text{m} \times 5800\text{K} = 2{,}320\ \mu\text{m}\cdot\text{K}$ $\quad F_{(0\to\lambda_1)} = 0.125$

Plain glass: $\tau_{vis} = 0.9\,[0.491 - 0.125] = 0.329$ <

Tinted glass: $\tau_{vis} = 0.9\,[0.491 - 0.250] = 0.217$ <

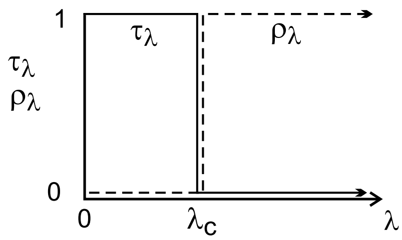
COMMENTS: For solar energy, the transmissivities are 0.839 for the plain glass vs. 0.568 for the plain and tinted glasses. Within the visible region, τ_{vis} is 0.329 vs. 0.217. Tinting reduces solar flux by 32% and visible solar flux by 34%.

PROBLEM 12.57

KNOWN: Spectral transmissivity and reflectivity of light bulb coating. Dimensions, temperature and spectral emissivity of a tungsten filament.

FIND: (a) Advantages of the coating, (b) Filament electric power requirement for different coating spectral reflectivities.

SCHEMATIC:



ASSUMPTIONS: (1) All of the radiation reflected from the inner surface of bulb is absorbed by the filament.

ANALYSIS: (a) For λ_c = 0.7 μm, the coating has two important advantages: (i) It transmits all of the visible radiation emitted by the filament, thereby maximizing the lighting efficiency. (ii) It returns all of the infrared radiation to the filament, thereby reducing the electric power requirement and conserving energy. (b) The power requirement is simply the amount of radiation transmitted by the bulb, or

$$P_{elec} = A_f\ E_{(0\to\lambda_c)} = \pi\left(DL + D^2/2\right)\int_0^{\lambda_c} \varepsilon_\lambda\ E_{\lambda,b}\ d\lambda$$

From the spectral distribution of Problem 12.25, ε_λ = 0.45 for both values of λ_c. Hence,

$$P_{elec} = \left\{\pi\left[0.0008\times 0.02 + (0.0008)^2/2\right]m^2\right\}0.45\,E_b\int_0^{\lambda_c}\left(E_{\lambda,b}/E_b\right)d\lambda$$

$$P_{elec} = 5.13\times 10^{-5}\,m^2\times 0.45\times 5.67\times 10^{-8}\ W/m^2\cdot K^4\,(3000\ K)^4\ F_{(0\to\lambda_c)}$$

$$P_{elec} = 106\ W\ F_{(0\to\lambda_c)}$$

For $\lambda_c = 0.7\mu m,\ \lambda_c T = 2100\,\mu m\cdot K$ and from Table 12.1, $F_{(0\to\lambda_c)} = 0.0838$. Hence,

$$\lambda_c = 0.7\,\mu m: P_{elec} = 106\ W\times 0.0838 = 8.88\ W \qquad <$$

For $\lambda_c = 2\,\mu m,\ \lambda_c T = 6000\,\mu m\cdot K$ and $F_{(0\to\lambda_c)} = 0.738$. Hence,

$$\lambda_c = 2.0\,\mu m: P_{elec} = 106\ W\times 0.738 = 78.2\ W \qquad <$$

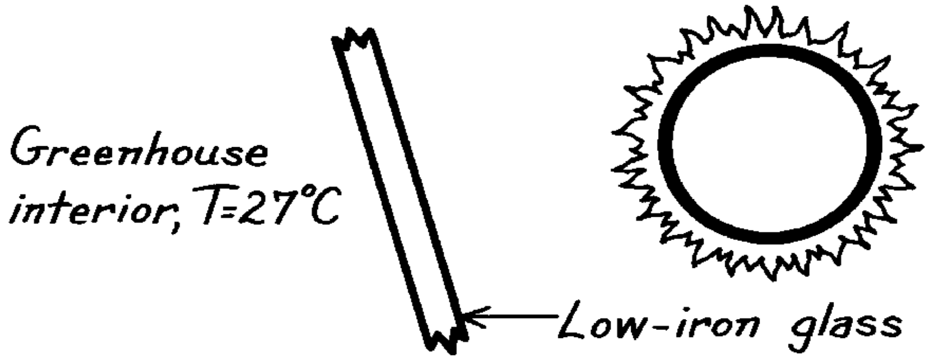
COMMENTS: Clearly, significant energy conservation could be realized with a reflective coating and λ_c = 0.7 μm. Although a coating with the prescribed spectral characteristics is highly idealized and does not exist, there are coatings that may be used to reflect a portion of the infrared radiation from the filament and to thereby provide some energy savings.

PROBLEM 12.58

KNOWN: Spectral transmissivity of low iron glass (see Fig. 12.24).

FIND: Interpretation of the greenhouse effect.

SCHEMATIC:



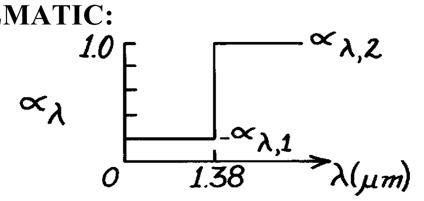
ANALYSIS: The glass affects the net radiation transfer to the contents of the greenhouse. Since most of the solar radiation is in the spectral region $\lambda < 3\ \mu m$, the glass will transmit a large fraction of this radiation. However, the contents of the greenhouse, being at a comparatively low temperature, emit most of their radiation in the medium to far infrared. This radiation is not transmitted by the glass. Hence the glass allows short wavelength solar radiation to enter the greenhouse, but does not permit long wavelength radiation to leave.

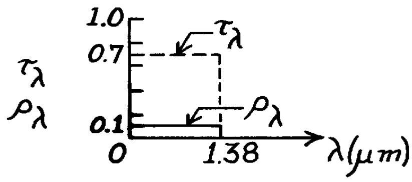
PROBLEM 12.59

KNOWN: Spectrally selective, diffuse surface exposed to solar irradiation.

FIND: (a) Spectral transmissivity, τ_λ, (b) Transmissivity, τ_S, reflectivity, ρ_S, and absorptivity, α_S, for solar irradiation, (c) Emissivity, ε, when surface is at T_s = 350K, (d) Net heat flux by radiation to the surface.

SCHEMATIC:





ASSUMPTIONS: (1) Surface is diffuse, (2) Spectral distribution of solar irradiation is proportional to $E_{\lambda,b}$ (λ, 5800K).

ANALYSIS: (a) Conservation of radiant energy requires, according to Eq. 12.56, that $\rho_\lambda + \alpha_\lambda + \tau_\lambda$ =1 or $\tau_\lambda = 1 - \rho_\lambda - \alpha_\lambda$. Hence, the spectral transmissivity appears as shown above (dashed line). Note that the surface is opaque for $\lambda > 1.38$ μm.

(b) The transmissivity to solar irradiation, G_S, follows from Eq. 12.55,

$$\tau_S = \int_0^\infty \tau_\lambda G_{\lambda,S} d\lambda / G_S = \int_0^\infty \tau_\lambda E_{\lambda,b}(\lambda,5800K)\, d\lambda / E_b(5800K)$$

$$\tau_S = \tau_{\lambda,b}\int_0^{1.38} E_{\lambda,b}(\lambda,5800K)\, d\lambda / E_b(5800K) = \tau_{\lambda,1} F_{(0\to\lambda_1)} = 0.7\times 8.56 = 0.599 \qquad <$$

where $\lambda_1 T_S = 1.38 \times 5800 = 8000$ μm·K and from Table 12.1, $F_{(0\to\lambda_1)} = 0.856$. From Eqs. 12.52 and 12.57,

$$\rho_S = \int_0^\infty \rho_\lambda G_{\lambda,S} d\lambda / G_S = \rho_{\lambda,1} F_{(0\to\lambda_1)} = 0.1\times 0.856 = 0.086 \qquad <$$

$$\alpha_S = 1 - \rho_S - \tau_S = 1 - 0.086 - 0.599 = 0.315. \qquad <$$

(c) For the surface at T_s = 350K, the emissivity can be determined from Eq. 12.38. Since the surface is diffuse, according to Eq. 12.65, $\alpha_\lambda = \varepsilon_\lambda$, the expression has the form

$$\varepsilon = \int_0^\infty \varepsilon_\lambda E_{\lambda,b}(T_s) d\lambda / E_b(T_s) = \int_0^\infty \alpha_\lambda E_{\lambda,b}(350K)\, d\lambda / E_b(350K)$$

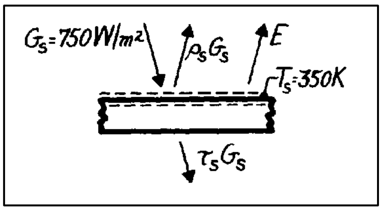
$$\varepsilon = \alpha_{\lambda,1} F_{(0-1.38\mu m)} + \alpha_{\lambda,2}\left[1 - F_{(0-1.38\mu m)}\right] = \alpha_{\lambda,2} = 1 \qquad <$$

where from Table 12.1 with $\lambda_1 T_S = 1.38 \times 350 = 483$ μm·K, $F_{(0-\lambda T)} \approx 0$.

(d) The net heat flux by radiation *to* the surface is determined by a radiation balance

$$q''_{rad} = G_S - \rho_S G_S - \tau_S G_S - E$$

$$q''_{rad} = \alpha_S G_S - E$$



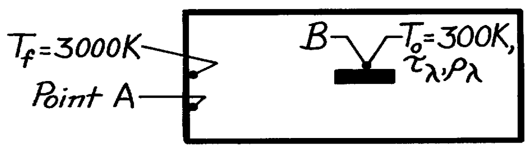
$$q''_{rad} = 0.315\times 750\ W/m^2 - 1.0\times 5.67\times 10^{-8}\ W/m^2\cdot K^4 (350K)^4 = -615\ W/m^2. \qquad <$$

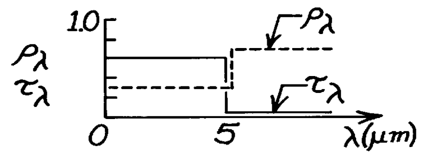
PROBLEM 12.60

KNOWN: Large furnace with diffuse, opaque walls (T_f, ε_f) and a small diffuse, spectrally selective object (T_o, τ_λ, ρ_λ).

FIND: For points on the furnace wall and the object, find ε, α, E, G and J.

SCHEMATIC:





ASSUMPTIONS: (1) Furnace walls are isothermal, diffuse, and gray, (2) Object is isothermal and diffuse.

ANALYSIS: Consider first the furnace wall (A). Since the wall material is diffuse and gray, it follows that

$$\varepsilon_A = \varepsilon_f = \alpha_A = 0.85.$$ <

The emissive power is

$$E_A = \varepsilon_A E_b(T_f) = \varepsilon_A \sigma T_f = 0.85 \times 5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4 (3000\ \text{K})^4 = 3.904 \times 10^6\ \text{W/m}^2.$$ <

Since the furnace is an isothermal enclosure, blackbody conditions exist such that

$$G_A = J_A = E_b(T_f) = \sigma T_f^4 = 5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4 (3000\text{K})^4 = 4.593 \times 10^6\ \text{W/m}^2.$$ <

Considering now the semitransparent, diffuse, spectrally selective object at T_o = 300 K. From the radiation balance requirement, find

$$\alpha_\lambda = 1 - \rho_\lambda - \tau_\lambda \quad \text{or} \quad \alpha_1 = 1 - 0.6 - 0.3 = 0.1 \quad \text{and} \quad \alpha_2 = 1 - 0.7 - 0.0 = 0.3$$

$$\alpha_B = \int_0^\infty \alpha_\lambda G_\lambda d\lambda / G = F_{0-\lambda T} \cdot \alpha_1 + (1 - F_{0-\lambda T}) \cdot \alpha_2 = 0.970 \times 0.1 + (1 - 0.970) \times 0.3 = 0.106$$ <

where $F_{0-\lambda T}$ = 0.970 at λT = 5 μm × 3000 K = 15,000 μm·K since G = $E_b(T_f)$. Since the object is diffuse, $\varepsilon_\lambda = \alpha_\lambda$, hence

$$\varepsilon_B = \int_0^\infty \varepsilon_\lambda E_{\lambda,b}(T_o) d\lambda / E_{b,o} = F_{0-\lambda T}\alpha_1 + (1 - F_{0-\lambda T}) \cdot \alpha_2 = 0.0138 \times 0.1 + (1 - 0.0138) \times 0.3 = 0.297$$ <

where $F_{0-\lambda T}$ = 0.0138 at λT = 5 μm × 300 K = 1500 μmK. The emissive power is

$$E_B = \varepsilon_B E_{b,B}(T_o) = 0.297 \times 5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4 (300\ \text{K})^4 = 136.5\ \text{W/m}^2.$$ <

The irradiation is that due to the large furnace for which blackbody conditions exist,

$$G_B = G_A = \sigma T_f^4 = 4.593 \times 10^6\ \text{W/m}^2.$$ <

The radiosity leaving point B is due to emission and reflected irradiation,

$$J_B = E_B + \rho_B G_B = 136.5\ \text{W/m}^2 + 0.3 \times 4.593 \times 10^6\ \text{W/m}^2 = 1.378 \times 10^6\ \text{W/m}^2.$$ <

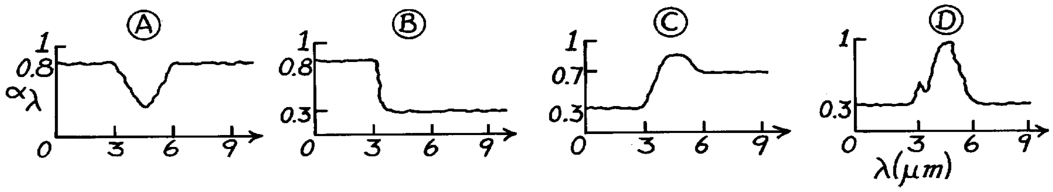
If we include transmitted irradiation, $J_B = E_B + (\rho_B + \tau_B)\, G_B = E_B + (1 - \alpha_B)\, G_B = 4.106 \times 10^6\ \text{W/m}^2$.
In the first calculation, note how we set $\rho_B \approx \rho_\lambda$ (λ < 5 μm).

PROBLEM 12.61

KNOWN: Spectral characteristics of four diffuse surfaces exposed to solar radiation.

FIND: Surfaces which may be assumed to be gray.

SCHEMATIC:



ASSUMPTIONS: (1) Diffuse surface behavior.

ANALYSIS: A gray surface is one for which α_λ and ε_λ are constant over the spectral regions of the irradiation and the surface emission.

For $\lambda = 3\ \mu m$ and $T = 5800K$, $\lambda T = 17,400\ \mu m \cdot K$ and from Table 12.1, find $F_{(0 \to \lambda)} = 0.984$. Hence, 98.4% of the solar radiation is in the spectral region below 3 μm.

For $\lambda = 6\ \mu m$ and $T = 300K$, $\lambda T = 1800\ \mu m \cdot K$ and from Table 12.1, find $F_{(0 \to \lambda)} = 0.039$. Hence, 96.1% of the surface emission is in the spectral region above 6 μm.

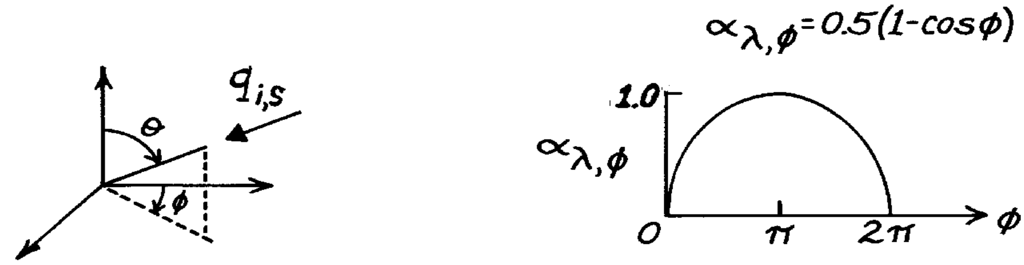
Hence:	Surface A is gray:	$\alpha_S \approx \varepsilon = 0.8$	<
	Surface B is not gray:	$\alpha_S \approx 0.8,\ \varepsilon \approx 0.3$	<
	Surface C is not gray:	$\alpha_S \approx 0.3,\ \varepsilon \approx 0.7$	<
	Surface D is gray:	$\alpha_S \approx \varepsilon = 0.3.$	<

PROBLEM 12.62

KNOWN: A gray, but directionally selective, material with $\alpha(\theta, \phi) = 0.5(1 - \cos\phi)$.

FIND: (a) Hemispherical absorptivity when irradiated with collimated solar flux in the direction (θ = 45° and ϕ = 0°) and (b) Hemispherical emissivity of the material.

SCHEMATIC:



ASSUMPTIONS: (1) Gray surface behavior.

ANALYSIS: (a) The surface has the directional absorptivity given as

$$\alpha(\theta,\phi) = \alpha_{\lambda,\phi} = 0.5[1-\cos\phi].$$

When irradiated in the direction θ = 45° and ϕ = 0°, the directional absorptivity for this condition is

$$\alpha(45°, 0°) = 0.5[1-\cos(0°)] = 0.$$ <

That is, the surface is completely reflecting (or transmitting) for irradiation in this direction.

(b) From Kirchhoff's law,

$$\alpha_{\theta,\phi} = \varepsilon_{\theta,\phi}$$

so that

$$\varepsilon_{\theta,\phi} = \alpha_{\theta,\phi} = 0.5(1-\cos\phi).$$

Using Eq. 12.35 find

$$\varepsilon = \frac{\int_0^{2\pi}\int_0^{\pi/2} \varepsilon_{\theta,\phi,\lambda} \cos\theta \sin\theta \, d\theta \, d\phi}{\int_0^{2\pi}\int_0^{\pi/2} \cos\theta \sin\theta \, d\theta \, d\phi}$$

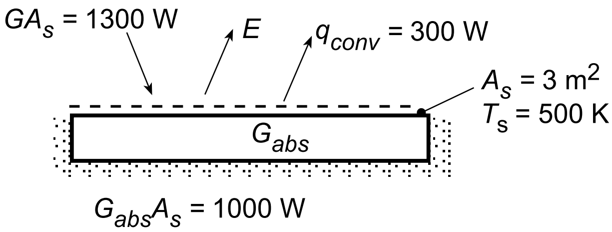
$$\varepsilon = \frac{\int_0^{2\pi} 0.5(1-\cos\phi)\, d\phi}{\int_0^{2\pi} d\phi} = \left.\frac{0.5(\phi - \sin\phi)}{2\pi}\right|_0^{2\pi} = 0.5.$$ <

PROBLEM 12.63

KNOWN: Area and temperature of an opaque surface. Rate of incident radiation, absorbed radiation and heat transfer by convection.

FIND: Surface irradiation, emissive power, radiosity, absorptivity, reflectivity and emissivity.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Adiabatic sides and bottom.

ANALYSIS: The irradiation, emissive power and radiosity are

$$G = 1300\,W/3m^2 = 433\,W/m^2$$ <

$$E = G_{abs} - q''_{conv} = (1000 - 300)\,W/3m^2 = 233\,W/m^2$$ <

$$J = E + G_{ref} = E + (G - G_{abs}) = [233 + (433 - 333)]\,W/m^2 = 333\,W/m^2$$ <

The absorptivity, reflectivity and emissivity are

$$\alpha = G_{abs}/G = \left(333\,W/m^2\right)/\left(433\,W/m^2\right) = 0.769$$ <

$$\rho = 1 - \alpha = 0.231$$ <

$$\varepsilon = E/E_b = E/\sigma T_s^4 = 233\,W/m^2 / 5.67\times10^{-8}\,W/m^2\cdot K^4\,(500\,K)^4 = 0.066$$ <

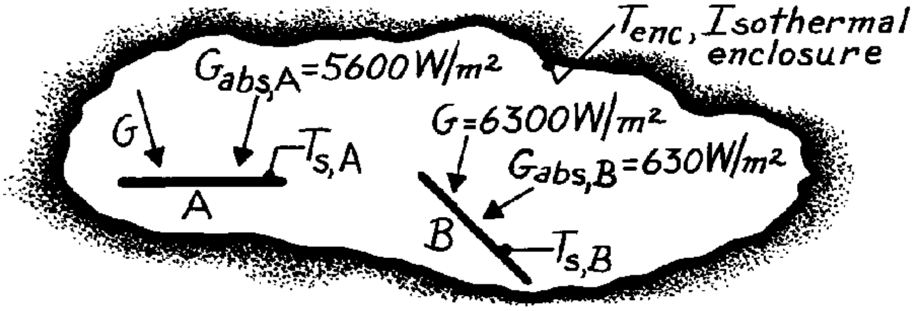
COMMENTS: The expression for E follows from a surface energy balance for which the absorbed irradiation is balanced by emission and convection.

PROBLEM 12.64

KNOWN: Isothermal enclosure at a uniform temperature provides a known irradiation on two small surfaces whose absorption rates have been measured.

FIND: (a) Net heat transfer rates and temperatures of the two surfaces, (b) Absorptivity of the surfaces, (c) Emissive power of the surfaces, (d) Emissivity of the surfaces.

SCHEMATIC:



ASSUMPTIONS: (1) Enclosure is at a uniform temperature and large compared to surfaces A and B, (2) Surfaces A and B have been in the enclosure a long time, (3) Irradiation to both surfaces is the same.

ANALYSIS: (a) Since the surfaces A and B have been within the enclosure a long time, thermal equilibrium conditions exist. That is,

$$q_{A,net} = q_{B,net} = 0.$$

Furthermore, the surface temperatures are the same as the enclosure, $T_{s,A} = T_{s,B} = T_{enc}$. Since the enclosure is at a uniform temperature, it follows that blackbody radiation exists within the enclosure (see Fig. 12.12) and

$$G = E_b\left(T_{enc}\right) = \sigma T_{enc}^4$$

$$T_{enc} = \left(G/\sigma\right)^{1/4} = \left(6300\,\mathrm{W/m^2}/5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\right)^{1/4} = 577.4\,\mathrm{K}. \qquad <$$

(b) From Eq. 12.45, the absorptivity is G_{abs}/G,

$$\alpha_A = \frac{5600\,\mathrm{W/m^2}}{6300\,\mathrm{W/m^2}} = 0.89 \qquad \alpha_B = \frac{630\,\mathrm{W/m^2}}{6300\,\mathrm{W/m^2}} = 0.10. \qquad <$$

(c) Since the surfaces experience zero net heat transfer, the energy balance is $G_{abs} = E$. That is, the absorbed irradiation is equal to the emissive power,

$$E_A = 5600\,\mathrm{W/m^2} \qquad E_B = 630\,\mathrm{W/m^2}. \qquad <$$

(d) The emissive power, E(T), is written as

$$E = \varepsilon E_b(T) = \varepsilon\sigma T^4 \qquad \text{or} \qquad \varepsilon = E/\sigma T^4.$$

Since the temperature of the surfaces and the emissive powers are known,

$$\varepsilon_A = 5600\,\mathrm{W/m^2}\Big/\left[5.67\times10^{-8}\,\frac{\mathrm{W}}{\mathrm{m^2\cdot K^4}}\left(577.4\,\mathrm{K}\right)^4\right] = 0.89 \qquad \varepsilon_B = 0.10. \qquad <$$

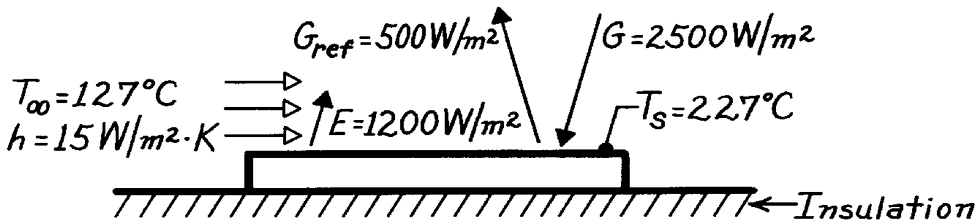
COMMENTS: Note for this equilibrium condition, $\varepsilon = \alpha$.

PROBLEM 12.65

KNOWN: Opaque, horizontal plate, well insulated on backside, is subjected to a prescribed irradiation. Also known are the reflected irradiation, emissive power, plate temperature and convection coefficient for known air temperature.

FIND: (a) Emissivity, absorptivity and radiosity and (b) Net heat transfer per unit area of the plate.

SCHEMATIC:



ASSUMPTIONS: (1) Plate is insulated on backside, (2) Plate is opaque.

ANALYSIS: (a) The total, hemispherical emissivity of the plate according to Eq. 12.37 is

$$\varepsilon = \frac{E}{E_b(T_s)} = \frac{E}{\sigma T_s^4} = \frac{1200\ \text{W/m}^2}{5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4 \times (227+273)^4\ \text{K}^4} = 0.34. \quad <$$

The total, hemispherical absorptivity is related to the reflectivity by Eq. 12.57 for an opaque surface. That is, $\alpha = 1 - \rho$. By definition, the reflectivity is the fraction of irradiation reflected, Eq. 12.51, such that

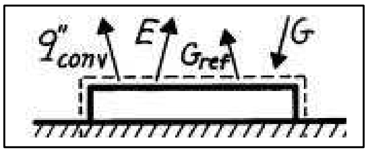
$$\alpha = 1 - G_{ref}/G = 1 - 500\ \text{W/m}^2 / \left(2500\ \text{W/m}^2\right) = 1 - 0.20 = 0.80. \quad <$$

The radiosity, J, is defined as the radiant flux leaving the surface by emission and reflection per unit area of the surface (see Section 12.24).

$$J = \rho G + \varepsilon E_b = G_{ref} + E = 500\ \text{W/m}^2 + 1200\ \text{W/m}^2 = 1700\ \text{W/m}^2. \quad <$$

(b) The net heat transfer is determined from an energy balance,

$$q''_{net} = q''_{in} - q''_{out} = G - G_{ref} - E - q''_{conv}$$



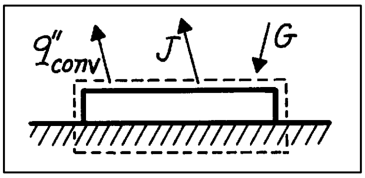
$$q''_{net} = (2500 - 500 - 1200)\ \text{W/m}^2 - 15\ \text{W/m}^2\cdot\text{K}(227 - 127)\text{K} = -700\ \text{W/m}^2. \quad <$$

An alternate approach to the energy balance using the radiosity,

$$q''_{net} = G - J - q''_{conv}$$

$$q''_{net} = (2500 - 1700 - 1500)\ \text{W/m}^2$$

$$q''_{net} = -700\ \text{W/m}^2.$$



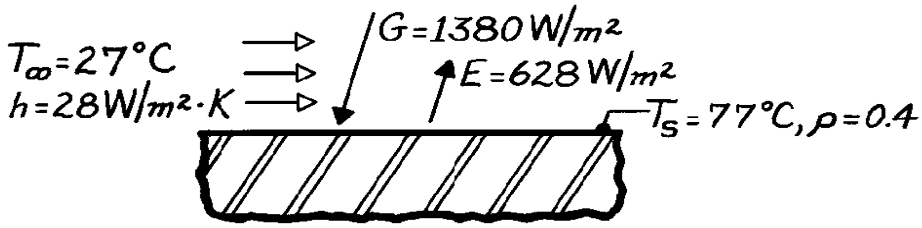
COMMENTS: (1) Since the net heat rate per unit area is negative, energy must be added to the plate in order to maintain it at T_s = 227°C. (2) Note that $\alpha \neq \varepsilon$. Hence, the plate is not a gray body. (3) Note the use of radiosity in performing energy balances. That is, considering only the radiation processes, $q''_{net} = G - J$.

PROBLEM 12.66

KNOWN: Horizontal, opaque surface at steady-state temperature of 77°C is exposed to a convection process; emissive power, irradiation and reflectivity are prescribed.

FIND: (a) Absorptivity of the surface, (b) Net radiation heat transfer rate for the surface; indicate direction, (c) Total heat transfer rate for the surface; indicate direction.

SCHEMATIC:

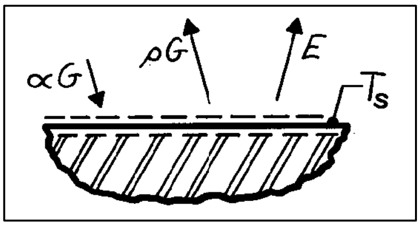


ASSUMPTIONS: (1) Surface is opaque, (2) Effect of surroundings included in the specified irradiation, (3) Steady-state conditions.

ANALYSIS: (a) From the definition of the thermal radiative properties and a radiation balance for an opaque surface on a total wavelength basis, according to Eq. 12.59,

$$\alpha = 1 - \rho = 1 - 0.4 = 0.6. \qquad <$$

(b) The net radiation heat transfer rate to the surface follows from a surface energy balance considering only radiation processes. From the schematic,

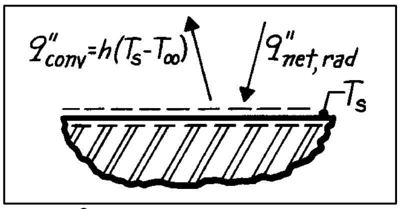


$$q''_{net,rad} = \left(\dot{E}''_{in} - \dot{E}''_{out}\right)_{rad}$$

$$q''_{net,rad} = G - \rho G - E = (1380 - 0.4 \times 1380 - 628) W/m^2 = 200 W/m^2. \qquad <$$

Since $q''_{net,rad}$ is positive, the net radiation heat transfer rate is *to* the surface.

(c) Performing a surface energy balance considering all heat transfer processes, the local heat transfer rate is



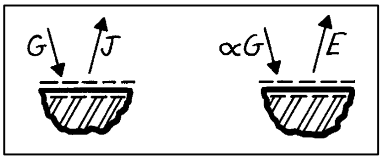
$$q''_{tot} = \left(\dot{E}''_{in} - \dot{E}''_{out}\right)$$

$$q''_{tot} = q''_{net,rad} - q''_{conv}$$

$$q''_{tot} = 200 W/m^2 - 28 W/m^2 \cdot K(77 - 27)K = -1200 W/m^2. \qquad <$$

The total heat flux is shown as a negative value indicating the heat flux is *from* the surface.

COMMENTS: (1) Note that the surface radiation balance could also be expresses as



$$q''_{net,rad} = G - J \qquad \text{or} \qquad \alpha G - E.$$

Note the use of radiosity to express the radiation flux leaving the surface.

(2) From knowledge of the surface emissive power and T_s, find the emissivity as

$$\varepsilon \equiv E/\sigma T_s^4 = 628 W/m^2 / \left(5.67 \times 10^{-8} W/m^2 \cdot K^4\right)(77 + 273)^4 K^4 = 0.74.$$

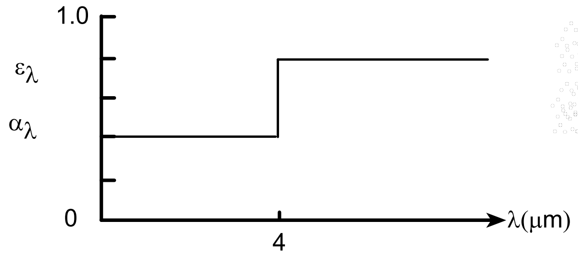
Since $\varepsilon \neq \alpha$, we know the surface is not gray.

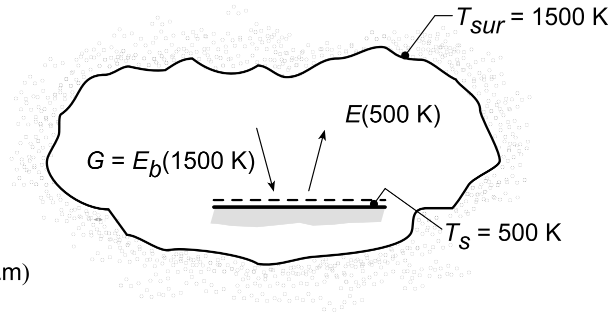
PROBLEM 12.67

KNOWN: Temperature and spectral characteristics of a diffuse surface at T_s = 500 K situated in a large enclosure with uniform temperature, T_{sur} = 1500 K.

FIND: (a) Sketch of spectral distribution of E_λ and $E_{\lambda,b}$ for the surface, (b) Net heat flux to the surface, $q''_{rad,in}$ (c) Compute and plot $q''_{rad,in}$ as a function of T_s for the range $500 \le T_s \le 1000$ K; also plot the heat flux for a diffuse, gray surface with total emissivities of 0.4 and 0.8; and (d) Compute and plot ε and α as a function of the surface temperature for the range $500 \le T_s \le 1000$ K.

SCHEMATIC:

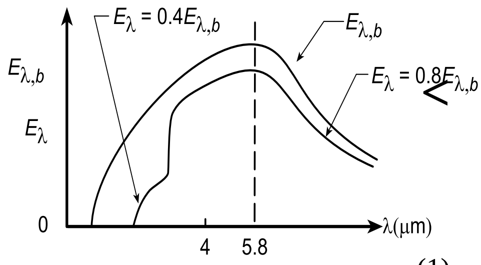




ASSUMPTIONS: (1) Surface is diffuse, (2) Convective effects are negligible, (3) Surface irradiation corresponds to blackbody emission at 1500 K.

ANALYSIS: (a) From Wien's law, Eq. 12.27, $\lambda_{max} T$ = 2897.6 μm·K. Hence, for blackbody emission from the surface at T_s = 500 K,

$$\lambda_{max} = \frac{2897.6\mu m \cdot K}{500\ K} = 5.80\mu m .$$



<

(b) From an energy balance on the surface, the net heat flux to the surface is

$q''_{rad,in}$ = αG – E = αE_b (1500 K) – εE_b (500 K). (1)

From Eq. 12.46,

$$\alpha = 0.4\int_0^4 \frac{E_{\lambda,b}(1500)}{E_b} d\lambda + 0.8\int_4^\infty \frac{E_{\lambda,b}(1500)}{E_b} d\lambda = 0.4F_{(0-4)} - 0.8[1 - F_{(0-4)}].$$

From Table 12.1 with λT = 4μm × 1500 K = 6000 μm·K, $F_{(0-4)}$ = 0.738, find

α = 0.4 × 0.738 + 0.8 (1 – 0.738) = 0.505.

From Eq. 12.38

$$\varepsilon = 0.4\int_0^4 \frac{E_{\lambda,b}(500)}{E_b} d\lambda + 0.8\int_4^\infty \frac{E_{\lambda,b}(500)}{E_b} d\lambda = 0.4F_{(0-4)} + 0.8[1 - F_{(0-4)}] .$$

From Table 12.1 with λT = 4μm × 500 K = 2000 μm·K, $F_{(0-4)}$ = 0.0667, find

ε = 0.4 × 0.0667 + 0.8 (1 – 0.0667) = 0.773.

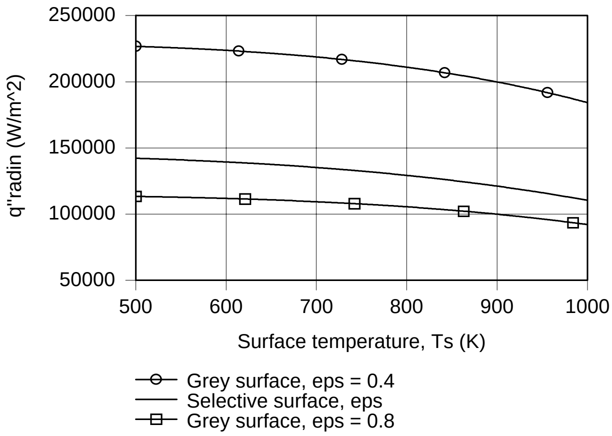
Hence, the net heat flux to the surface is

$$q''_{rad,in} = 5.67\times10^{-8}\ W/m^2 \cdot K^4 [0.505\times(1500\ K)^4 - 0.773\times(500\ K)^4] = 1.422\times10^5\ W/m^2 .$$ <

Continued...

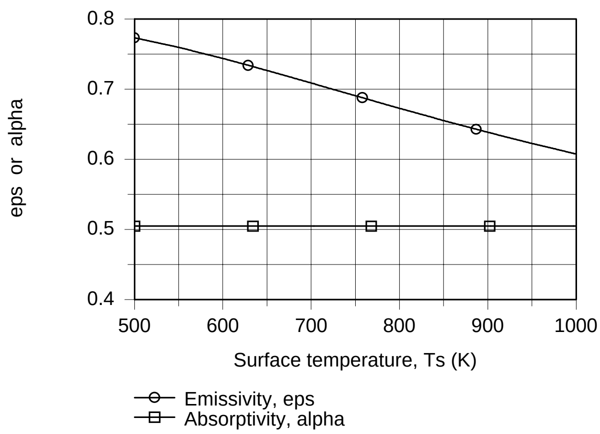
PROBLEM 12.67 (Cont.)

(c) Using the foregoing equations in the IHT workspace along with the *IHT Radiation Tool, Band Emission Factor*, $q''_{rad,in}$ was computed and plotted as a function of T_s.



The net radiation heat rate, $q''_{rad,in}$ decreases with increasing surface temperature since E increases with T_s and the absorbed irradiation remains constant according to Eq. (1). The heat flux is largest for the gray surface with $\varepsilon = 0.4$ and the smallest for the gray surface with $\varepsilon = 0.8$. As expected, the heat flux for the selective surface is between the limits of the two gray surfaces.

(d) Using the IHT model of part (c), the emissivity and absorptivity of the surface are computed and plotted below.



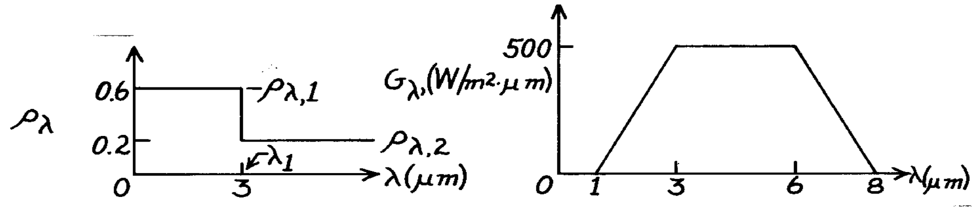
The absorptivity, $\alpha = \alpha(\alpha_\lambda, T_{sur})$, remains constant as T_s changes since it is a function of α_λ (or ε_λ) and T_{sur} only. The emissivity, $\varepsilon = \varepsilon(\varepsilon_\lambda, T_s)$ is a function of T_s and decreases as T_s increases. Could you have surmised as much by looking at the spectral emissivity distribution? Under what condition would you expect $\alpha = \varepsilon$?

PROBLEM 12.68

KNOWN: Opaque, diffuse surface with prescribed spectral reflectivity and at a temperature of 750K is subjected to a prescribed spectral irradiation, G_λ.

FIND: (a) Total absorptivity, α, (b) Total emissivity, ε, (c) Net radiative heat flux to the surface.

SCHEMATIC:



ASSUMPTIONS: (1) Opaque and diffuse surface, (2) Backside insulated.

ANALYSIS: (a) The total absorptivity is determined from Eq. 12.46 and 12.56,

$$\alpha_\lambda = 1-\rho_\lambda \qquad \text{and} \qquad \alpha = \int_0^\infty \alpha_\lambda G_\lambda d\lambda / G. \qquad (1,2)$$

Evaluating by separate integrals over various wavelength intervals.

$$\alpha = \frac{\left(1-\rho_{\lambda,1}\right)\int_1^3 G_\lambda\, d\lambda + \left(1-\rho_{\lambda,2}\right)\int_3^6 G_\lambda\, d\lambda + \left(1-\rho_{\lambda,2}\right)\int_6^8 G_\lambda\, d\lambda}{\int_1^3 G_\lambda\, d\lambda + \int_3^6 G_\lambda\, d\lambda + \int_6^8 G_\lambda\, d\lambda} = \frac{G_{abs}}{G}$$

$$G_{abs} = (1-0.6)\left[0.5\times 500\,\mathrm{W/m^2\cdot\mu m}(3-1)\mu m\right] + (1-0.2)\left[500\,\mathrm{W/m^2\cdot\mu m}(6-3)\mu m\right]$$
$$+(1-0.2)\left[0.5\times 500\,\mathrm{W/m^2\cdot\mu m}(8-6)\mu m\right]$$

$$G = 0.5\times 500\,\mathrm{W/m^2\cdot\mu m}\times(3-1)\mu m + 500\,\mathrm{W/m^2\cdot\mu m}(6-3)\mu m + 0.5\times 500\,\mathrm{W/m^2\cdot\mu m}(8-6)\mu m$$

$$\alpha = \frac{[200+1200+400]\,\mathrm{W/m^2}}{[500+1500+500]\,\mathrm{W/m^2}} = \frac{1800\,\mathrm{W/m^2}}{2500\,\mathrm{W/m^2}} = 0.720.$$ <

(b) The total emissivity of the surface is determined from Eq. 12.38 and 12.65,

$$\varepsilon_\lambda = \alpha_\lambda \qquad \text{and, hence} \qquad \varepsilon_\lambda = 1-\rho_\lambda. \qquad (3,4)$$

The total emissivity can then be expressed as

$$\varepsilon = \int_0^\infty \varepsilon_\lambda E_{\lambda,b}(\lambda,T_s)d\lambda / E_b(T_s) = \int_0^\infty (1-\rho_\lambda)E_{\lambda,b}(\lambda,T_s)d\lambda / E_b(T_s)$$

$$\varepsilon = \left(1-\rho_{\lambda,1}\right)\int_0^3 E_{\lambda,b}(\lambda,T_s)d\lambda / E_b(T_s) + \left(1-\rho_{\lambda,2}\right)\int_3^\infty E_{\lambda,b}(\lambda,T_s)d\lambda / E_b(T_s)$$

$$\varepsilon = \left(1-\rho_{\lambda,1}\right)F_{(0\to 3\,\mu m)} + \left(1-\rho_{\lambda,2}\right)\left[1-F_{(0\to 3\,\mu m)}\right]$$

$$\varepsilon = (1-0.6)\times 0.111 + (1-0.2)[1-0.111] = 0.756$$ <

where Table 12.1 is used to find $F_{(0-\lambda)} = 0.111$ for $\lambda_1 T_s = 3 \times 750 = 2250$ μm·K.

(c) The net radiative heat flux *to* the surface is

$$q''_{rad} = \alpha G - \varepsilon E_b(T_s) = \alpha G - \varepsilon\sigma T_s^4$$

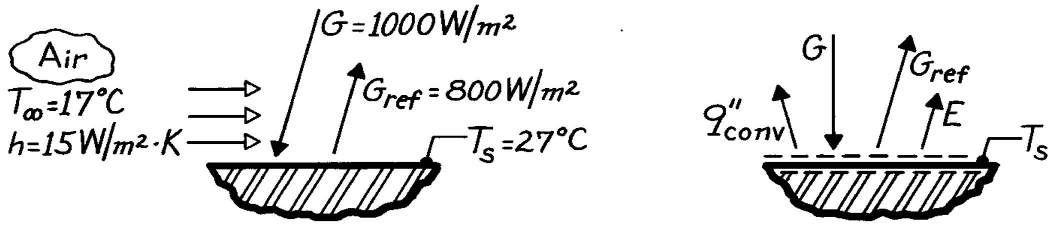
$$q''_{rad} = 0.720\times 2500\,\mathrm{W/m^2}$$
$$-0.756\times 5.67\times 10^{-8}\,\mathrm{W/m^2\cdot K^4}(750\mathrm{K})^4 = -11{,}763\,\mathrm{W/m^2}.$$ <

PROBLEM 12.69

KNOWN: Opaque, gray surface at 27°C with prescribed irradiation, reflected flux and convection process.

FIND: Net heat flux from the surface.

SCHEMATIC:



ASSUMPTIONS: (1) Surface is opaque and gray, (2) Surface is diffuse, (3) Effects of surroundings are included in specified irradiation.

ANALYSIS: From an energy balance on the surface, the net heat flux *from* the surface is

$$q''_{net} = \dot{E}''_{out} - \dot{E}''_{in}$$

$$q''_{net} = q''_{conv} + E + G_{ref} - G = h\left(T_s - T_\infty\right) + \varepsilon\sigma T_s^4 + G_{ref} - G. \qquad (1)$$

To determine ε, from Eq. 12.59 and Kirchoff's law for a diffuse-gray surface, Eq. 12.62,

$$\varepsilon = \alpha = 1 - \rho = 1 - \left(G_{ref}/G\right) = 1 - (800/1000) = 1 - 0.8 = 0.2 \qquad (2)$$

where from Eq. 12.51, $\rho = G_{ref}/G$. The net heat flux from the surface, Eq. (1), is

$$q''_{net} = 15\,W/m^2\cdot K(27-17)K + 0.2\times5.67\times10^{-8}\,W/m^2\cdot K^4(27+273)^4\,K^4$$
$$+800\,W/m^2 - 1000\,W/m^2$$

$$q''_{net} = (150 + 91.9 + 800 - 1000)\,W/m^2 = 42\,W/m^2. \qquad <$$

COMMENTS: (1) For this situation, the radiosity is

$$J = G_{ref} + E = (800 + 91.9)\,W/m^2 = 892\,W/m^2.$$

The energy balance can be written involving the radiosity (radiation leaving the surface) and the irradiation (radiation to the surface).

$$q''_{net} = J - G + q''_{conv} = (892 - 1000 + 150)\,W/m^2 = 42\,W/m^2.$$

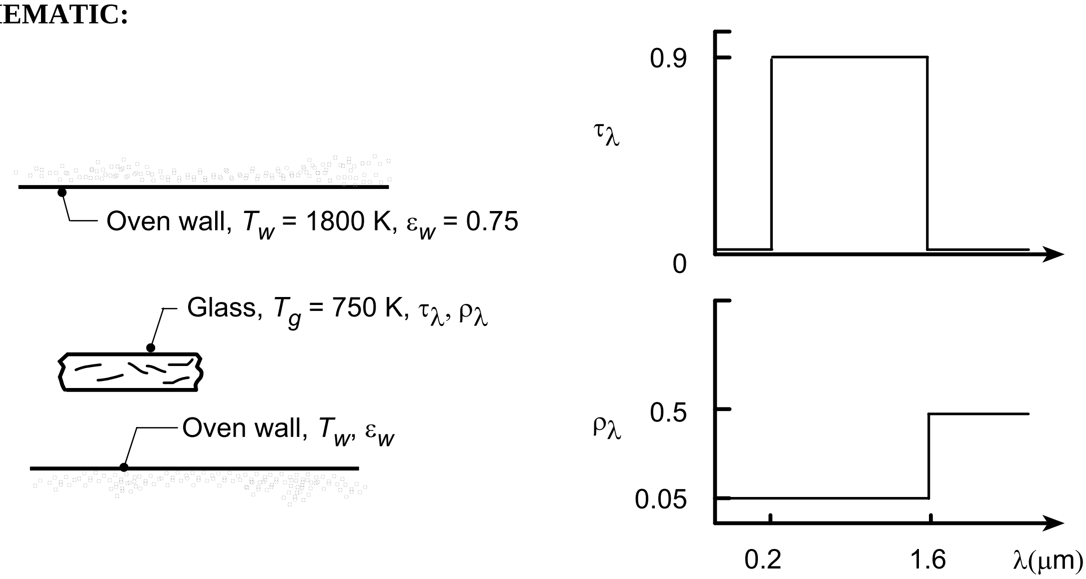
(2) Note the need to assume the surface is diffuse, gray and opaque in order that Eq. (2) is applicable.

PROBLEM 12.70

KNOWN: Diffuse glass at T_g = 750 K with prescribed spectral radiative properties being heated in a large oven having walls with emissivity of 0.75 and 1800 K.

FIND: (a) Total transmissivity r, total reflectivity ρ, and total emissivity ε of the glass; Net radiative heat flux to the glass, (b) $q''_{rad,in}$; and (c) Compute and plot $q''_{rad,in}$ as a function of glass temperatures for the range $500 \le T_g \le 800$ K for oven wall temperatures of T_w = 1500, 1800 and 2000 K.

SCHEMATIC:



ASSUMPTIONS: (1) Glass is of uniform temperature, (2) Glass is diffuse, (3) Furnace walls large compared to the glass; ε_w plays no role, (4) Negligible convection.

ANALYSIS: (a) From knowledge of the spectral transmittance,τ_w, and spectral reflectivity, ρ_λ, the following radiation properties are evaluated:

Total transmissivity, r: For the irradiation from the furnace walls, $G_\lambda = E_{\lambda,b}(\lambda, T_w)$. Hence

$$\tau = \int_0^\infty \tau_\lambda E_{\lambda,b}(\lambda, T_w)\, d\lambda \Big/ \sigma T_w^4 \approx \tau_{\lambda 1} F_{(0-\lambda T)} = 0.9 \times 0.25 = 0.225 .$$ <

where $\lambda T = 1.6\ \mu m \times 1800\ K = 2880\ \mu m \cdot K \approx 2898\ \mu m \cdot K$ giving $F_{(0-\lambda T)} \approx 0.25$.

Total reflectivity, ρ: With $G_\lambda = E_{\lambda,b}(\lambda, T_w)$, T_w = 1800 K, and $F_{0-\lambda T} = 0.25$,

$$\rho \approx \rho_{\lambda 1} F_{(0-\lambda T)} + \rho_{\lambda 2}\left(1 - F_{(0-\lambda T)}\right) = 0.05 \times 0.25 + 0.5(1 - 0.25) = 0.388$$ <

Total absorptivity, α : To perform the energy balance later, we'll need α. Employ the conservation expression,

$$\alpha = 1 - \rho - \tau = 1 - 0.388 - 0.225 = 0.387 .$$

Emissivity, ε: Based upon surface temperature T_g = 750 K, for

$$\lambda T = 1.6\ \mu m \times 750 K = 1200\ \mu m \cdot K, \qquad F_{0-\lambda T} \approx 0.002 .$$

Hence for $\lambda > 1.6\ \mu m$, $\varepsilon \approx \varepsilon_\lambda \approx 0.5$. <

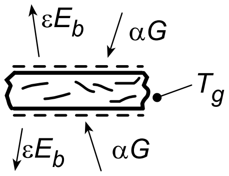
(b) Performing an energy balance on the glass, the net radiative heat flux by radiation into the glass is,

Continued...

PROBLEM 12.70 (Cont.)

$$q''_{net,in} = E''_{in} - E''_{out}$$

$$q''_{net,in} = 2\left(\alpha G - \varepsilon E_b\left(T_g\right)\right)$$

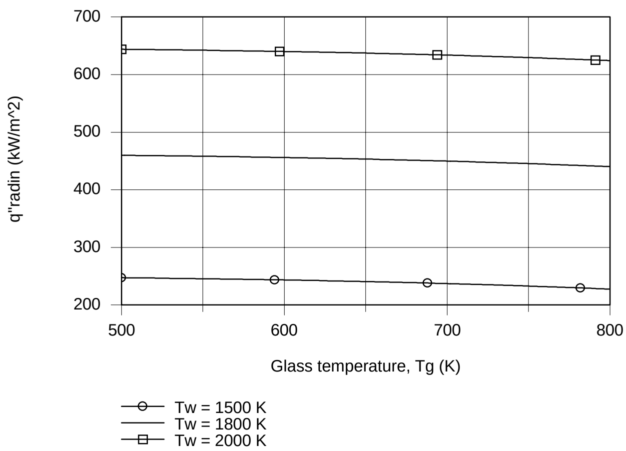


where $G = \sigma T_w^4$

$$q''_{net,in} = 2\left[0.387\sigma\left(1800K\right)^4 - 0.5\sigma\left(750K\right)^4\right]$$

$$q''_{net,in} = 442.8\,kW/m^2\ .$$

(b) Using the foregoing equations in the IHT Workspace along with the *IHT Radiation Tool, Band Emission Factor,* the net radiative heat flux, $q''_{rad,in}$, was computed and plotted as a function of T_g for selected wall temperatures T_w .



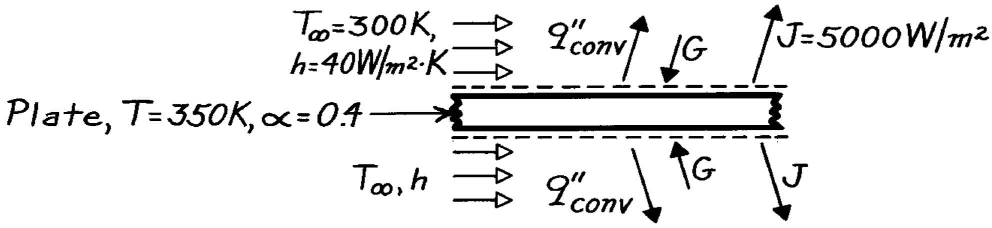
As the glass temperature increases, the rate of emission increases so we'd expect the net radiative heat rate into the glass to decrease. Note that the decrease is not very significant. The effect of increased wall temperature is to increase the irradiation and, hence the absorbed irradiation to the surface and the net radiative flux increase.

PROBLEM 12.71

KNOWN: Temperature, absorptivity, transmissivity, radiosity and convection conditions for a semitransparent plate.

FIND: Plate irradiation and total hemispherical emissivity.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform surface conditions.

ANALYSIS: From an energy balance on the plate

$$\dot{E}_{in} = \dot{E}_{out}$$

$$2G = 2q''_{conv} + 2J.$$

Solving for the irradiation and substituting numerical values,

$$G = 40\ \text{W/m}^2\cdot\text{K}(350-300)\text{K} + 5000\ \text{W/m}^2 = 7000\ \text{W/m}^2. \qquad <$$

From the definition of J,

$$J = E + \rho G + \tau G = E + (1-\alpha)G.$$

Solving for the emissivity and substituting numerical values,

$$\varepsilon = \frac{J-(1-\alpha)G}{\sigma T^4} = \frac{\left(5000\ \text{W/m}^2\right) - 0.6\left(7000\ \text{W/m}^2\right)}{5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4(350\text{K})^4} = 0.94. \qquad <$$

Hence,

$$\alpha \neq \varepsilon$$

and the surface is not gray for the prescribed conditions.

COMMENTS: The emissivity may also be determined by expressing the plate energy balance as

$$2\alpha G = 2q''_{conv} + 2E.$$

Hence

$$\varepsilon\sigma T^4 = \alpha G - h(T - T_\infty)$$

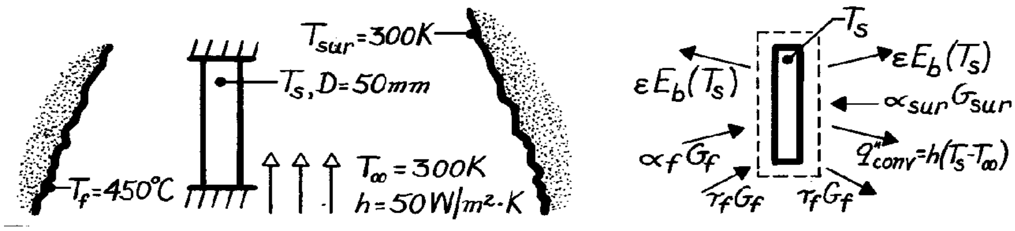
$$\varepsilon = \frac{0.4\left(7000\ \text{W/m}^2\right) - 40\ \text{W/m}^2\cdot\text{K}(50\ \text{K})}{5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4(350\ \text{K})^4} = 0.94.$$

PROBLEM 12.72

KNOWN: Material with prescribed radiative properties covering the peep hole of a furnace and exposed to surroundings on the outer surface.

FIND: Steady-state temperature of the cover, T_s; heat loss from furnace.

SCHEMATIC:



ASSUMPTIONS: (1) Cover is isothermal, no gradient, (2) Surroundings of the outer surface are large compared to cover, (3) Cover is insulated from its mount on furnace wall, (4) Negligible convection on interior surface.

PROPERTIES: Cover material (given): For irradiation from the furnace interior: $\tau_f = 0.8$, $\rho_f = 0$; For room temperature emission: $\tau = 0$, $\varepsilon = 0.8$.

ANALYSIS: Perform an energy balance identifying the modes of heat transfer,

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad \alpha_f G_f + \alpha_{sur} G_{sur} - 2\varepsilon E_b(T_s) - h(T_s - T_\infty) = 0. \tag{1}$$

Recognize that $\qquad G_f = \sigma T_f^4 \qquad G_{sur} = \sigma T_{sur}^4$. (2,3)

From Eq. 12.57, it follows that $\qquad \alpha_f = 1 - \tau_f - \rho_f = 1 - 0.8 - 0.0 = 0.2.$ (4)

Since the irradiation G_{sur} will have nearly the same spectral distribution as the emissive power of the cover, $E_b(T_s)$, and since G_{sur} is diffuse irradiation,

$$\alpha_{sur} = \varepsilon = 0.8. \tag{5}$$

This reasoning follows from Eqs. 12.65 and 12.66. Substituting Eqs. (2-5) into Eq. (1) and using numerical values,

$$0.2 \times 5.67 \times 10^{-8} (450 + 273)^4 \text{ W/m}^2 + 0.8 \times 5.67 \times 10^{-8} \times 300^4 \text{ W/m}^2 \tag{2-5}$$

$$-2 \times 0.8 \times 5.67 \times 10^{-8} T_s^4 \text{ W/m}^2 - 50 \text{ W/m}^2 \cdot \text{K}(T_s - 300)\text{K} = 0 \qquad <$$

$$9.072 \times 10^{-8} T_s^4 + 50 T_s = 18{,}466 \qquad \text{or} \qquad T_s = 344\text{K}.$$

The heat loss from the furnace (see energy balance schematic) is

$$q_{f,loss} = A_s\left[\alpha_f G_f + \tau_f G_f - \varepsilon E_b(T_s)\right] = \frac{\pi D^2}{4}\left[(\alpha_f + \tau_f) G_f - \varepsilon E_b(T_s)\right]$$

$$q_{f,loss} = \pi(0.050\text{m})^2/4\Big[(0.8 + 0.2)(723\text{K})^4$$

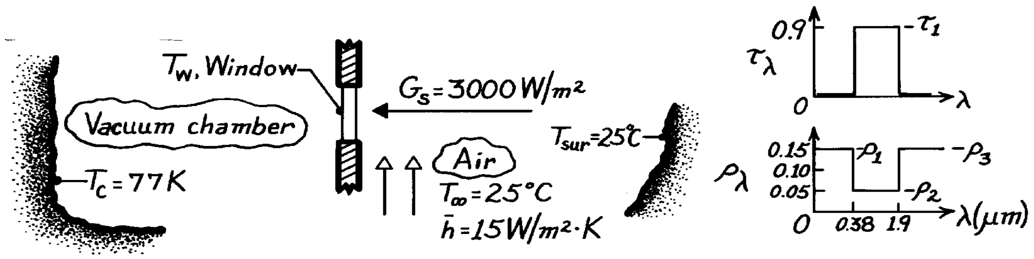
$$-0.8(344\text{K})^4\Big] 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 = 29.2 \text{ W}. \qquad <$$

PROBLEM 12.73

KNOWN: Window with prescribed τ_λ and ρ_λ mounted on cooled vacuum chamber passing radiation from a solar simulator.

FIND: (a) Solar transmissivity of the window material, (b) State-state temperature reached by window with simulator operating, (c) Net radiation heat transfer to chamber.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Diffuse behavior of window material, (3) Chamber and room surroundings large compared to window, (4) Solar simulator flux has spectral distribution of 5800K blackbody, (5) Window insulated from its mount, (6) Window is isothermal at T_w.

ANALYSIS: (a) Using Eq. 12.55 and recognizing that $G_{\lambda,S} \sim E_{b,\lambda}$ (λ, 5800K),

$$\tau_S = \tau_1 \int_{0.38}^{1.9} E_{\lambda,b}(\lambda, 5800\text{K})\, d\lambda / E_b(5800\text{K}) = \tau_1 \left[F_{(0\to 1.9\mu m)} - F_{(0\to 0.38\mu m)} \right].$$

From Table 12.1 at $\lambda T = 1.9 \times 5800 = 11{,}020$ μm·K, $F_{(0\to\lambda)} = 0.932$; at $\lambda T = 0.38 \times 5800$ μm·K = 2,204 μm·K, $F_{(0\to\lambda)} = 0.101$; hence

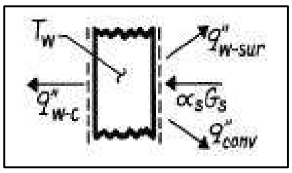
$$\tau_S = 0.90[0.932 - 0.101] = 0.748. \qquad <$$

Recognizing that later we'll need α_S, use Eq. 12.52 to find ρ_S

$$\rho_S = \rho_1 F_{(0\to 0.38\mu m)} + \rho_2 \left[F_{(0\to 1.9\mu m)} - F_{(0\to 0.38\mu m)} \right] + \rho_3 \left[1 - F_{(0\to 1.9\mu m)} \right]$$

$$\rho_S = 0.15 \times 0.101 + 0.05[0.932 - 0.101] + 0.15[1 - 0.932] = 0.067$$

$$\alpha_S = 1 - \rho_S - \tau_S = 1 - 0.067 - 0.748 = 0.185.$$



(b) Perform an energy balance on the window.

$$\alpha_S G_S - q''_{w-c} - q''_{w-sur} - q''_{conv} = 0$$

$$\alpha_S G_S - \varepsilon\sigma\left(T_w^4 - T_c^4\right) - \varepsilon\sigma\left(T_w^4 - T_{sur}^4\right) - \bar{h}\left(T_w - T_\infty\right) = 0.$$

Recognize that ρ_λ ($\lambda > 1.9$) = 0.15 and that $\varepsilon \approx 1 - 0.15 = 0.85$ since T_w will be near 300K. Substituting numerical values, find by trial and error,

$$0.185 \times 3000\ \text{W/m}^2 - 0.85 \times \sigma\left[2T_w^4 - 298^4 - 77^4\right]\text{k}^4 - 28\ \text{W/m}^2\cdot\text{K}\left(T_w - 298\right)\text{K} = 0$$

$$T_w = 302.6\text{K} = 29.6°\text{C}. \qquad <$$

(c) The net radiation transfer per unit area of the window to the vacuum chamber, excluding the transmitted simulated solar flux is

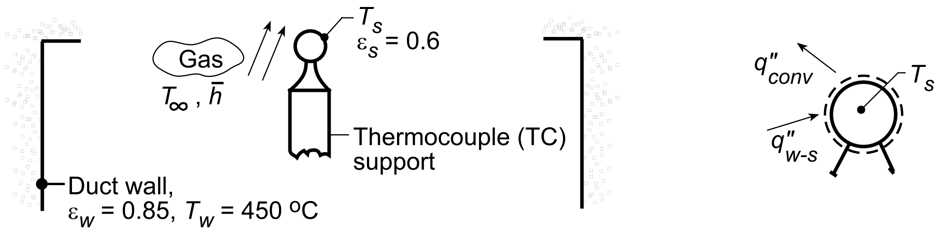
$$q''_{w-c} = \varepsilon\sigma\left(T_w^4 - T_c^4\right) = 0.85 \times 5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4\left[302.6^4 - 77^4\right]\text{K}^4 = 402\ \text{W/m}^2. \qquad <$$

PROBLEM 12.74

KNOWN: Reading and emissivity of a thermocouple (TC) located in a large duct to measure gas stream temperature. Duct wall temperature and emissivity; convection coefficient.

FIND: (a) Gas temperature, T_∞, (b) Effect of convection coefficient on measurement error.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat loss from TC sensing junction to support, (3) Duct wall much larger than TC, (4) TC surface is diffuse-gray.

ANALYSIS: (a) Performing an energy balance on the thermocouple, it follows that

$$q''_{w-s} - q''_{conv} = 0 .$$

where radiation exchange between the duct wall and the TC is given by Eq. 1.7. Hence,

$$\varepsilon_s \sigma (T_w^4 - T_s^4) - \bar{h}(T_s - T_\infty) = 0 .$$

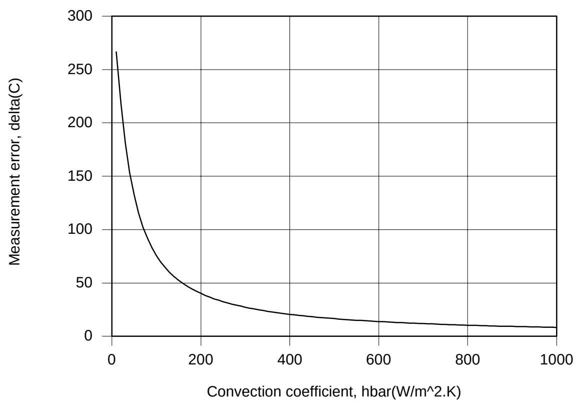
Solving for T_∞ with $T_s = 180^oC$,

$$T_\infty = T_s - \frac{\varepsilon_s \sigma}{\bar{h}}(T_w^4 - T_s^4)$$

$$T_\infty = (180+273)K - \frac{0.6(5.67\times10^{-8}\ W/m^2\cdot K^4)}{125\ W/m^2\cdot K}\left([450+273]^4 - [180+273]^4\right)K^4$$

$$T_\infty = 453\ K - 62.9\ K = 390\ K = 117^\circ C .$$ <

(b) Using the IHT *First Law* model for an *Isothermal Solid Sphere* to solve the foregoing energy balance for T_s, with $T_\infty = 125^oC$, the measurement error, defined as $\Delta T = T_s - T_\infty$, was determined and is plotted as a function of $\bar{h}$.



The measurement error is enormous ($\Delta T \approx 270^oC$) for $\bar{h} = 10\ W/m^2\cdot K$, but decreases with increasing $\bar{h}$. However, even for $\bar{h} = 1000\ W/m^2\cdot K$, the error ($\Delta T \approx 8°C$) is not negligible. Such errors must always be considered when measuring a gas temperature in surroundings whose temperature differs significantly from that of the gas.

Continued...

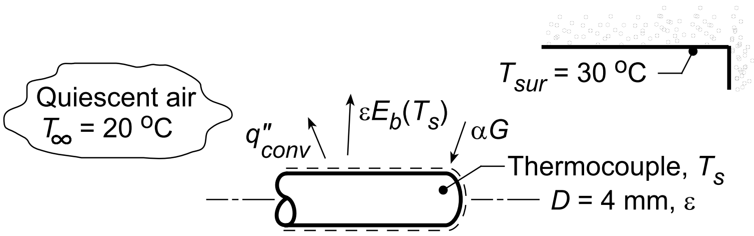
COMMENTS: (1) Because the duct wall surface area is much larger than that of the thermocouple, its emissivity is not a factor. (2) For such a situation, a shield about the thermocouple would reduce the influence of the hot duct wall on the indicated TC temperature. A low emissivity thermocouple coating would also help.

PROBLEM 12.75

KNOWN: Diameter and emissivity of a horizontal thermocouple (TC) sheath located in a large room. Air and wall temperatures.

FIND: (a) Temperature indicated by the TC, (b) Effect of emissivity on measurement error.

SCHEMATIC:



ASSUMPTIONS: (1) Room walls approximate isothermal, large surroundings, (2) Room air is quiescent, (3) TC approximates horizontal cylinder, (4) No conduction losses, (5) TC surface is opaque, diffuse and gray.

PROPERTIES: *Table A-4*, Air (assume T_s = 25 °C, $T_f = (T_s + T_\infty)/2 \approx 296$ K, 1 atm):
$\nu = 15.53\times10^{-6}\ \mathrm{m^2/s}$, $k = 0.026\ \mathrm{W/m\cdot K}$, $\alpha = 22.0\times10^{-6}\ \mathrm{m^2/s}$, $\mathrm{Pr} = 0.708$, $\beta = 1/T_f$.

ANALYSIS: (a) Perform an energy balance on the thermocouple considering convection and radiation processes. On a unit area basis, with $q''_{conv} = \bar{h}(T_s - T_\infty)$,

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$\alpha G - \varepsilon E_b(T_s) - \bar{h}(T_s - T_\infty) = 0 . \tag{1}$$

Since the surroundings are isothermal and large compared to the thermocouple, $G = E_b(T_{sur})$. For the gray-diffuse surface, $\alpha = \varepsilon$. Using the Stefan-Boltzman law, $E_b = \sigma T^4$, Eq. (1) becomes

$$\varepsilon\sigma(T_{sur}^4 - T_s^4) - \bar{h}(T_s - T_\infty) = 0 . \tag{2}$$

Using the Churchill-Chu correlation for a horizontal cylinder, estimate $\bar{h}$ due to free convection.

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = \left\{0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1+\left(0.559/\mathrm{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 , \quad Ra_D = \frac{g\beta\Delta T D^3}{\nu\alpha} . \tag{3,4}$$

To evaluate Ra_D and $\overline{Nu}_D$, assume $T_s = 25$°C, giving

$$Ra_D = \frac{9.8\ \mathrm{m/s^2}(1/296\ \mathrm{K})(25-20)\mathrm{K}(0.004\mathrm{m})^3}{15.53\times10^{-6}\ \mathrm{m^2/s}\times 22.0\times10^{-6}\ \mathrm{m^2/s}} = 31.0$$

$$\bar{h} = \frac{0.026\ \mathrm{W/m\cdot K}}{0.004\mathrm{m}}\left\{0.60 + \frac{0.387(31.0)^{1/6}}{\left[1+\left(0.559/0.708\right)^{9/16}\right]^{8/27}}\right\}^2 = 8.89\ \mathrm{W/m^2\cdot K} . \tag{5}$$

With ε = 0.4, the energy balance, Eq. (2), becomes

$$0.4\times5.67\times10^{-8}\ \mathrm{W/m^2\cdot K^4}[(30+273)^4 - T_s^4]\mathrm{K}^4 - 8.89\ \mathrm{W/m^2\cdot K}[T_s - (20+273)]\mathrm{K} = 0 \tag{6}$$

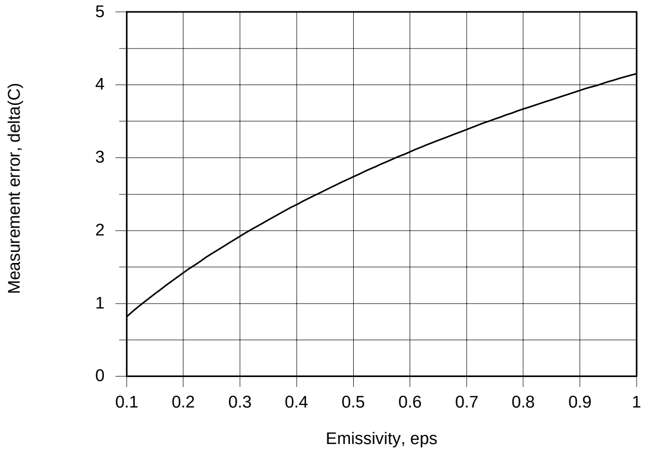
where all temperatures are in kelvin units. By trial-and-error, find

$T_s \approx 22.2$°C <

Continued...

PROBLEM 12.75 (Cont.)

(b) The thermocouple measurement error is defined as $\Delta T = T_s - T_\infty$ and is a consequence of radiation exchange with the surroundings. Using the IHT *First Law* Model for an *Isothermal Solid Cylinder* with the appropriate *Correlations* and *Properties* Toolpads to solve the foregoing energy balance for T_s, the measurement error was determined as a function of the emissivity.



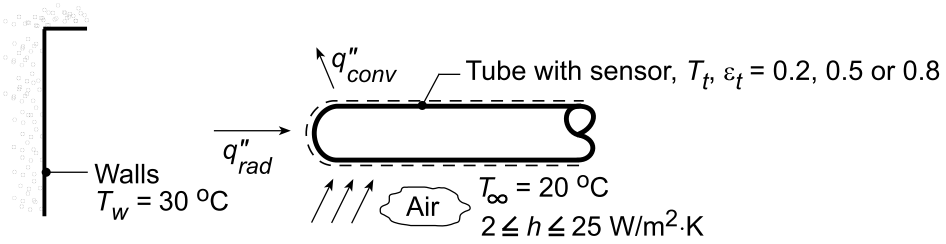
The measurement error decreases with decreasing ε, and hence a reduction in net radiation transfer from the surroundings. However, even for ε = 0.1, the error ($\Delta T \approx 1^oC$) is not negligible.

PROBLEM 12.76

KNOWN: Temperature sensor imbedded in a diffuse, gray tube of emissivity 0.8 positioned within a room with walls and ambient air at 30 and 20 °C, respectively. Convection coefficient is 5 $W/m^2 \cdot K$.

FIND: (a) Temperature of sensor for prescribed conditions, (b) Effect of surface emissivity and using a fan to induce air flow over the tube.

SCHEMATIC:



ASSUMPTIONS: (1) Room walls (surroundings) much larger than tube, (2) Tube is diffuse, gray surface, (3) No losses from tube by conduction, (4) Steady-state conditions, (5) Sensor measures temperature of tube surface.

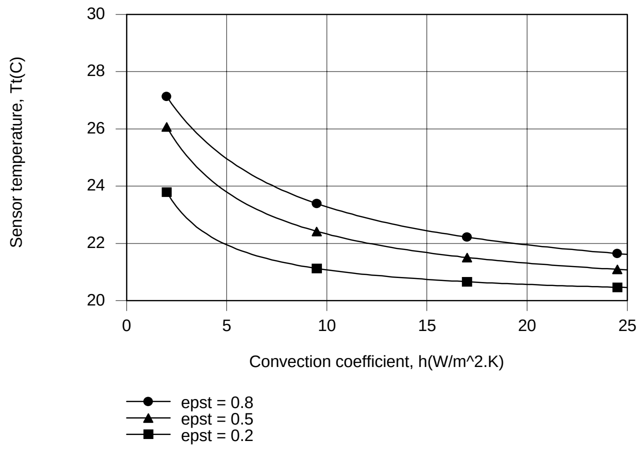
ANALYSIS: (a) Performing an energy balance on the tube, $\dot{E}_{in} - \dot{E}_{out} = 0$. Hence, $q''_{rad} - q''_{conv} = 0$, or $\varepsilon_t \sigma (T_w^4 - T_t^4) - h(T_t - T_w) = 0$. With h = 5 $W/m^2 \cdot K$ and ε_t = 0.8, the energy balance becomes

$$0.8 \times 5.67 \times 10^{-8}\, W/m^2 \cdot K^4 \left[(30+273)^4 - T_t^4 \right] K^4 = 5\, W/m^2 \cdot K \left[T_t - (20+273) \right] K$$

$$4.5360 \times 10^{-8} \left[303^4 - T_t^4 \right] = 5 \left[T_t - 293 \right]$$

which yields T_t = 298 K = 25°C. <

(b) Using the IHT *First Law* Model, the following results were determined.



The sensor temperature exceeds the air temperature due to radiation absorption, which must be balanced by convection heat transfer. Hence, the excess temperature $T_t - T_\infty$, may be reduced by increasing h or by decreasing α_t, which equals ε_t for a diffuse-gray surface, and hence the absorbed radiation.

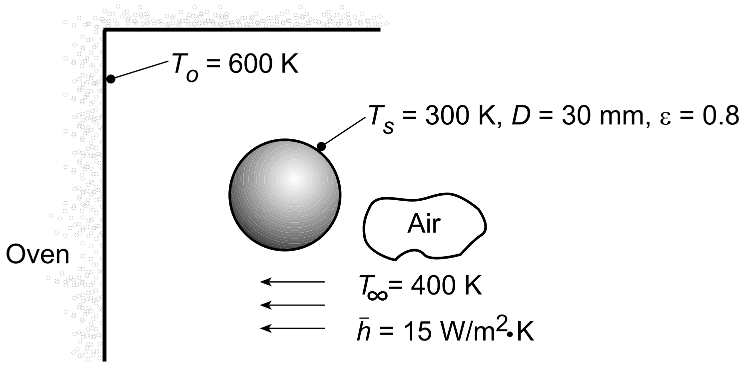
COMMENTS: A fan will increase the air velocity over the sensor and thereby increase the convection heat transfer coefficient. Hence, the sensor will indicate a temperature closer to T_∞

PROBLEM 12.77

KNOWN: Diffuse-gray sphere is placed in large oven with known wall temperature and experiences convection process.

FIND: (a) Net heat transfer rate to the sphere when its temperature is 300 K, (b) Steady-state temperature of the sphere, (c) Time required for the sphere, initially at 300 K, to come within 20 K of the steady-state temperature, and (d) Elapsed time of part (c) as a function of the convection coefficient for $10 \le h \le 25$ W/ $m^2 \cdot K$ for emissivities 0.2, 0.4 and 0.8.

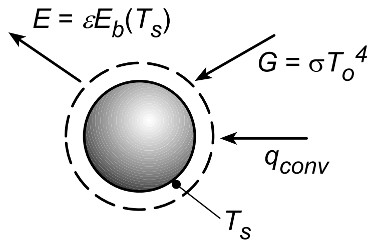
SCHEMATIC:



ASSUMPTIONS: (1) Sphere surface is diffuse-gray, (2) Sphere area is much smaller than the oven wall area, (3) Sphere surface is isothermal.

PROPERTIES: Sphere (Given) : $\alpha = 7.25 \times 10^{-5}$ m^2/s, k = 185 W/m·K.

ANALYSIS: (a) From an energy balance on the sphere find



$$q_{net} = q_{in} - q_{out}$$

$$q_{net} = \alpha G A_s + q_{conv} - E A_s$$

$$q_{net} = \alpha\sigma T_o^4 A_s + h A_s\left(T_\infty - T_s\right) - \varepsilon\sigma T_s^4 A_s. \qquad (1)$$

Note that the irradiation to the sphere is the emissive power of a blackbody at the temperature of the oven walls. This follows since the oven walls are isothermal and have a much larger area than the sphere area. Substituting numerical values, noting that $\alpha = \varepsilon$ since the surface is diffuse-gray and that $A_s = \pi D^2$, find

$$q_{net} = \Big[0.8\times5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\,(600\mathrm{K})^4 + 15\,\mathrm{W/m^2\cdot K}\times(400-300)\mathrm{K}$$

$$-0.8\times5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\,(300\mathrm{K})^4\Big]\pi\left(30\times10^{-3}\mathrm{m}\right)^2$$

$$q_{net} = [16.6 + 4.2 - 1.0]\mathrm{W} = 19.8\,\mathrm{W}. \qquad (1)$$ <

(b) For steady-state conditions, q_{net} in the energy balance of Eq. (1) will be zero,

$$0 = \alpha\sigma T_o^4 A_s + h A_s\left(T_\infty - T_{ss}\right) - \varepsilon\sigma T_{ss}^4 A_s \qquad (2)$$

Substitute numerical values and find the steady-state temperature as

$$T_{ss} = 538.2\mathrm{K}$$ <

Continued...

PROBLEM 12.77 (Cont.)

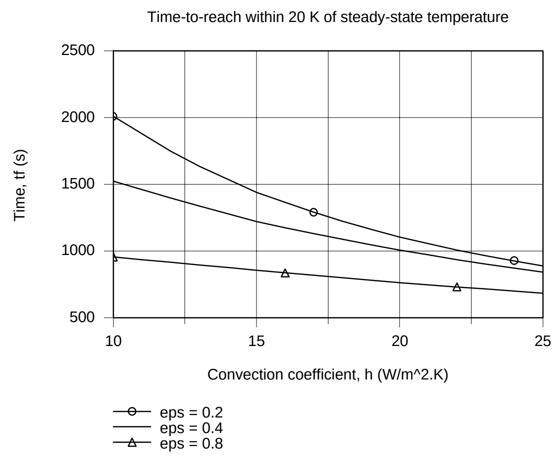
(c) Using the *IHT Lumped Capacitance Model* considering convection and radiation processes, the temperature- time history of the sphere, initially at $T_s(0) = T_i = 300$ K, can be determined. The elapsed time required to reach

$$T_s(t_o) = (538.2 - 20)\,K = 518.2K$$

was found as

$$t_o = 855s = 14.3\,min$$ <

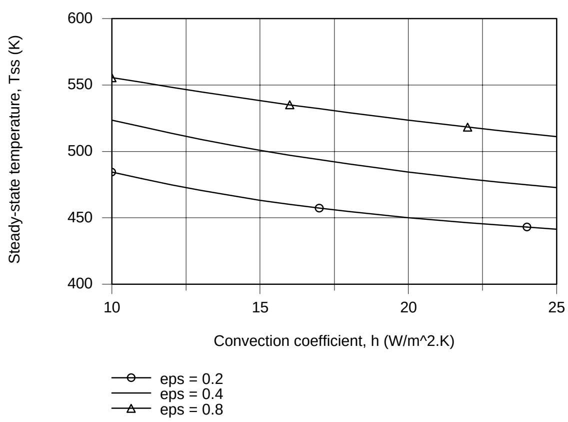
(d) Using the IHT model of part (c), the elapsed time for the sphere to reach within 20 K of its steady-state temperature, t_f, as a function of the convection coefficient for selected emissivities is plotted below.



For a fixed convection coefficient, t_f increases with decreasing ε since the radiant heat transfer into the sphere decreases with decreasing emissivity. For a given emissivity, the t_f decreases with increasing h since the convection heat rate increases with increasing h. However, the effect is much more significant with lower values of emissivity.

COMMENTS: (1) Why is t_f more strongly dependent on h for a lower sphere emissivity? Hint: Compare the relative heat rates by convection and radiation processes.

(2) The steady-state temperature, T_{ss}, as a function of the convection coefficient for selected emmissivities calculated using (2) is plotted below. Are these results consistent with the above plot of t_f vs h ?

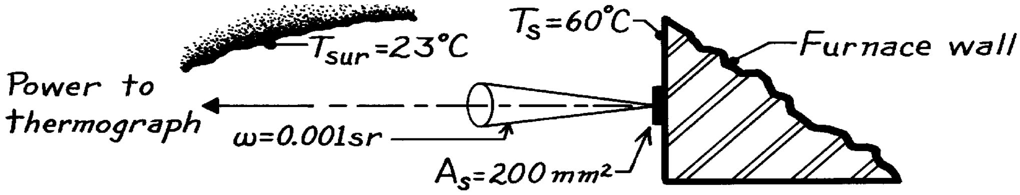


PROBLEM 12.78

KNOWN: Thermograph with spectral response in 9 to 12 μm region views a target of area 200mm^2 with solid angle 0.001 sr in a normal direction.

FIND: (a) For a black surface at 60°C, the emissive power in 9 – 12 μm spectral band, (b) Radiant power (W), received by thermograph when viewing black target at 60°C, (c) Radiant power (W) received by thermograph when viewing a gray, diffuse target having ε = 0.7 and considering the surroundings at T_{sur} = 23°C.

SCHEMATIC:



ASSUMPTIONS: (1) Wall is diffuse, (2) Surroundings are black with T_{sur} = 23°C.

ANALYSIS: (a) Emissive power in spectral range 9 to 12 μm for a 60°C black surface is

$$E_t \equiv E_b(9-12\,\mu m) = E_b\left[F(0\to 12\,\mu m) - F(0-9\,\mu m)\right]$$

where $E_b(T_s) = \sigma T_s^4$. From Table 12.1:

$$\lambda_2 T_s = 12\times(60+273) \approx 4000\,\mu m\ K, \qquad F(0\text{–}12\,\mu m) = 0.491$$

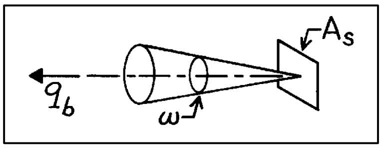
$$\lambda_1 T_s = 9\times(60+273) \approx 3000\,\mu m\ K, \qquad F(0-9\,\mu m) = 0.273.$$

Hence

$$E_t = 5.667\times 10^{-8}\ W/m^2\cdot K^4 \times (60+273)^4\ K^4\,[0.491-0.273] = 144.9\ W/m^2. \qquad <$$

(b) The radiant power, q_b (w), received by the thermograph from a black target is determined as

$$q_b = \frac{E_t}{\pi}\cdot A_s\cos\theta_1\cdot\omega$$



where E_t = emissive power in 9 – 12 μm spectral region, part (a) result
A_s = target area viewed by thermograph, 200mm^2 (2×10^{-4} m^2)
ω = solid angle thermograph aperture subtends when viewed from the target, 0.001 sr
θ = angle between target area normal and view direction, 0°.

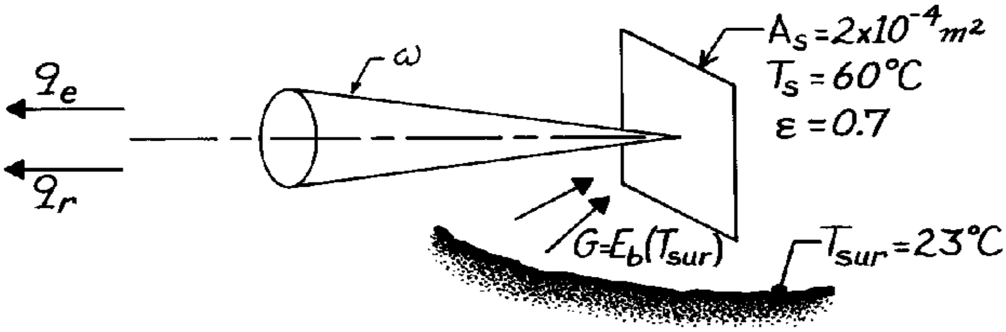
Hence,

$$q_b = \frac{144.9\ W/m^2}{\pi\ sr}\times\left(2\times 10^{-4}\ m^2\right)\times\cos 0°\times 0.001\,sr = 9.23\,\mu W. \qquad <$$

Continued …..

PROBLEM 12.78 (Cont.)

(c) When the target is a gray, diffuse emitter, ε = 0.7, the thermograph will receive emitted power from the target and reflected irradiation resulting from the surroundings at T_{sur} = 23°C. Schematically:



The power is expressed as

$$q = q_e + q_r = \varepsilon\, q_b + I_r \cdot A_s \cos\theta_1 \cdot \omega \left[F_{(0\to 12\,\mu m)} - F_{(0\to 9\,\mu m)} \right]$$

where

q_b = radiant power from black surface, part (b) result

$F_{(0\text{ - }\lambda)}$ = band emission fraction for T_{sur} = 23°C; using Table 12.1

$\lambda_2\, T_{sur}$ = 12 × (23 + 273) = 3552 μm·K, $F_{(0-\lambda_2)} = 0.394$

$\lambda_1\, T_{sur}$ = 9 × (23 + 273) = 2664 μm·K, $F_{(0-\lambda_1)} = 0.197$

I_r = reflected intensity, which because of diffuse nature of surface

$$I_r = \rho \frac{G}{\pi} = (1-\varepsilon)\frac{E_b(T_{sur})}{\pi}.$$

Hence

$$q = 0.7\times 9.23\,\mu W + (1-0.7)\frac{5.667\times10^{-8}\ W/m^2\cdot K^4 \times (273+23)^4\ K}{\pi\ sr}$$

$$\times\left(2\times10^{-4}\ m^2\right)\times \cos 0°\times 0.001 sr\left[0.394-0.197\right]$$

$$q = 6.46\ \mu W + 1.64\ \mu W = 8.10\ \mu W.$$ <

COMMENTS: (1) Comparing the results of parts (a) and (b), note that the power to the thermograph is slightly less for the gray surface with ε = 0.7. From part (b) see that the effect of the irradiation is substantial; that is, 1.64/8.10 ≈ 20% of the power received by the thermograph is due to reflected irradiation. Ignoring such effects leads to misinterpretation of temperature measurements using thermography.

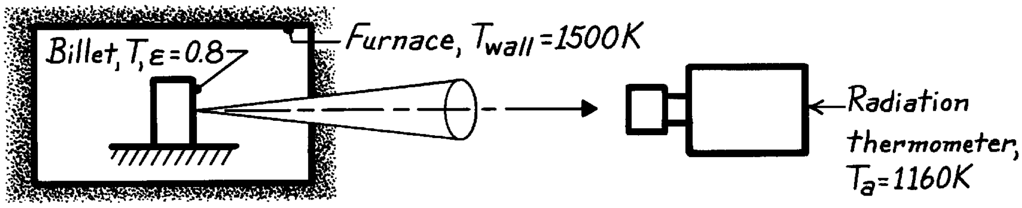
(2) Many thermography devices have a spectral response in the 3 to 5 μm wavelength region as well as 9 – 12 μm.

PROBLEM 12.79

KNOWN: Radiation thermometer (RT) viewing a steel billet being heated in a furnace.

FIND: Temperature of the billet when the RT indicates 1160K.

SCHEMATIC:



ASSUMPTIONS: (1) Billet is diffuse-gray, (2) Billet is small object in large enclosure, (3) Furnace behaves as isothermal, large enclosure, (4) RT is a radiometer sensitive to total (rather than a prescribed spectral band) radiation and is calibrated to correctly indicate the temperature of a black body, (5) RT receives radiant power originating from the target area on the billet.

ANALYSIS: The radiant power reaching the radiation thermometer (RT) is proportional to the radiosity of the billet. For the diffuse-gray billet within the large enclosure (furnace), the radiosity is

$$J = \varepsilon E_b(T) + \rho G = \varepsilon E_b(T) + (1-\varepsilon) E_b(T_w)$$

$$J = \varepsilon \sigma T^4 + (1-\varepsilon)\sigma T_w^4 \tag{1}$$

where $\alpha = \varepsilon$, $G = E_b(T_w)$ and $E_b = \sigma T^4$. When viewing the billet, the RT indicates T_a = 1100K, referred to as the apparent temperature of the billet. That is, the RT *indicates* the billet is a blackbody at T_a for which the radiosity will be

$$E_b(T_a) = J_a = \sigma T_a^4. \tag{2}$$

Recognizing that $J_a = J$, set Eqs. (1) and (2) equal to one another and solve for T, the billet true temperature.

$$T = \left[\frac{1}{\varepsilon} T_a^4 - \frac{1-\varepsilon}{\varepsilon} T_w^4\right]^{1/4}.$$

Substituting numerical values, find

$$T = \left[\frac{1}{0.8}(1160\text{K})^4 - \frac{1-0.8}{0.8}(1500\text{K})^4\right]^{1/4} = 999\text{K}.$$ <

COMMENTS: (1) The effect of the reflected wall irradiation from the billet is to cause the RT to indicate a temperature higher than the true temperature.

(2) What temperature would the RT indicate when viewing the furnace wall assuming the wall emissivity were 0.85?

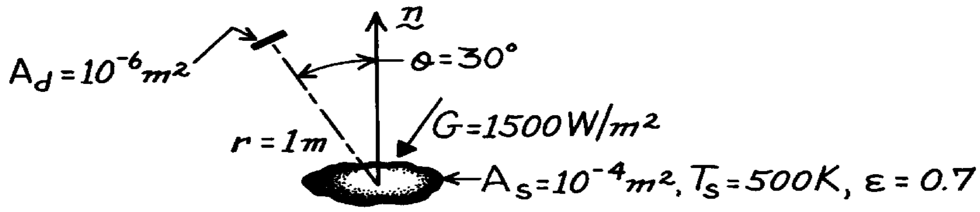
(3) What temperature would the RT indicate if the RT were sensitive to spectral radiation at 0.65 μm instead of total radiation? Hint: in Eqs. (1) and (2) replace the emissive power terms with spectral intensity. Answer: 1365K.

PROBLEM 12.80

KNOWN: Irradiation and temperature of a small surface.

FIND: Rate at which radiation is received by a detector due to emission and reflection from the surface.

SCHEMATIC:



ASSUMPTIONS: (1) Opaque, diffuse-gray surface behavior, (2) A_s and A_d may be approximated as differential areas.

ANALYSIS: Radiation intercepted by the detector is due to emission and reflection from the surface, and from the definition of the intensity, it may be expressed as

$$q_{s-d} = I_{e+r} A_s \cos\theta \, \Delta\omega.$$

The solid angle intercepted by A_d with respect to a point on A_s is

$$\Delta\omega = \frac{A_d}{r^2} = 10^{-6} \text{ sr}.$$

Since the surface is diffuse it follows from Eq. 12.24 that

$$I_{e+r} = \frac{J}{\pi}$$

where, since the surface is opaque and gray ($\varepsilon = \alpha = 1 - \rho$),

$$J = E + \rho G = \varepsilon E_b + (1 - \varepsilon) G.$$

Substituting for E_b from Eq. 12.28

$$J = \varepsilon \sigma T_s^4 + (1 - \varepsilon) G = 0.7 \times 5.67 \times 10^{-8} \frac{W}{m^2 \cdot K^4} (500K)^4 + 0.3 \times 1500 \frac{W}{m^2}$$

or

$$J = (2481 + 450) W/m^2 = 2931 W/m^2.$$

Hence

$$I_{e+r} = \frac{2931 W/m^2}{\pi \text{ sr}} = 933 W/m^2 \cdot sr$$

and

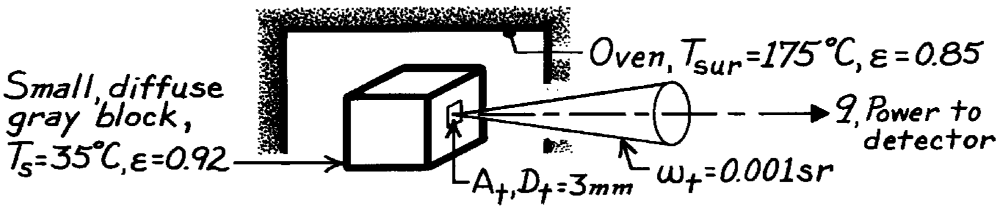
$$q_{s-d} = 933 \text{ W}/m^2 \cdot sr \left(10^{-4} \text{ m}^2 \times 0.866\right) 10^{-6} sr = 8.08 \times 10^{-8} \text{ W}. \qquad <$$

PROBLEM 12.81

KNOWN: Small, diffuse, gray block with ε = 0.92 at 35°C is located within a large oven whose walls are at 175°C with ε = 0.85.

FIND: Radiant power reaching detector when viewing (a) a deep hole in the block and (b) an area on the block's surface.

SCHEMATIC:



ASSUMPTIONS: (1) Block is isothermal, diffuse, gray and small compared to the enclosure, (2) Oven is isothermal enclosure.

ANALYSIS: (a) The small, deep hole in the isothermal block approximates a blackbody at T_s. The radiant power to the detector can be determined from Eq. 12.54 written in the form:

$$q = I_e \cdot A_t \cdot \omega_t = \frac{\sigma T_s^4}{\pi} \cdot A_t \cdot \omega_t$$

$$q = \frac{1}{\pi \text{ sr}}\left[5.67\times10^{-8}\times(35+273)^4\right]\frac{\text{W}}{\text{m}^2}\times\frac{\pi\left(3\times10^{-3}\right)^2 \text{m}^2}{4}\times0.001\text{sr} = 1.15\,\mu\text{W} \quad <$$

where $A_t = \pi D_t^2/4$. Note that the hole diameter must be greater than 3mm diameter.

(b) When the detector views an area on the surface of the block, the radiant power reaching the detector will be due to emission and reflected irradiation originating from the enclosure walls. In terms of the radiosity, Section 12.24, we can write using Eq. 12.24,

$$q = I_{e+r} \cdot A_t \cdot \omega_t = \frac{J}{\pi} \cdot A_t \cdot \omega_t.$$

Since the surface is diffuse and gray, the radiosity can be expressed as

$$J = \varepsilon E_b(T_s) + \rho G = \varepsilon E_b(T_s) + (1-\varepsilon)E_b(T_{sur})$$

recognizing that $\rho = 1 - \varepsilon$ and $G = E_b(T_{sur})$. The radiant power is

$$q = \frac{1}{\pi}\left[\varepsilon E_b(T_s) + (1-\varepsilon)E_b(T_{sur})\right] \cdot A_t \cdot \omega_t$$

$$q = \frac{1}{\pi \text{ sr}}\left[0.92\times5.67\times10^{-8}(35+273)^4 + (1-0.92)\times5.67\times10^{-8}(175+273)^4\right]\text{W/m}^2\times$$

$$\frac{\pi\left(3\times10^{-3}\right)^2 \text{m}^2}{4}\times0.001\text{sr} = 1.47\,\mu\text{W}. \quad <$$

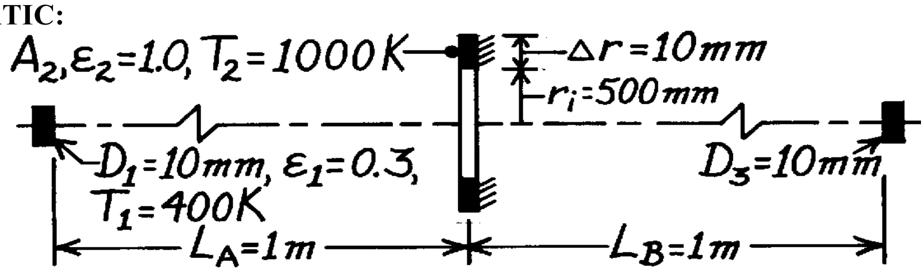
COMMENTS: The effect of reflected irradiation when ε < 1 is important for objects in enclosures. The practical application is one of measuring temperature by radiation from objects within furnaces.

PROBLEM 12.82

KNOWN: Diffuse, gray opaque disk (1) coaxial with a ring-shaped disk (2), both with prescribed temperatures and emissivities. Cooled detector disk (3), also coaxially positioned at a prescribed location.

FIND: Rate at which radiation is incident on the detector due to emission and reflection from A_1.

SCHEMATIC:



ASSUMPTIONS: (1) A_1 is diffuse-gray, (2) A_2 is black, (3) A_1 and $A_3 << R^2$, the distance of separation, (4) $\Delta r << r_i$, such that $A_2 \approx 2\pi r_i \Delta r$, and (5) Backside of A_2 is insulated.

ANALYSIS: The radiant power leaving A_1 intercepted by A_3 is of the form

$$q_{1\to3} = (J_1/\pi) A_1 \cos\theta_1 \cdot \omega_{3-1}$$

where for this configuration of A_1 and A_3,

$$\theta_1 = 0° \qquad \omega_{3-1} = A_3 \cos\theta_3 / (L_A + L_B)^2 \qquad \theta_3 = 0°.$$

Hence,

$$q_{1\to3} = (J_1/\pi) A_1 \cdot A_3 / (L_A + L_B)^2 \qquad J_1 = \rho G_1 + \varepsilon E_b(T_1) = \rho G_1 + \varepsilon\sigma T_1^4.$$

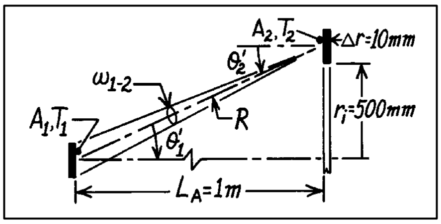
The irradiation on A_1 due to emission from A_2, G_1, is

$$G_1 = q_{2\to1}/A_1 = (I_2 \cdot A_2 \cos\theta_2' \cdot \omega_{1-2})/A_1$$

where

$$\omega_{1-2} = A_1 \cos\theta_1' / R^2$$

is constant over the surface A_2. From geometry,



$$\theta_1' = \theta_2' = \tan^{-1}\left[(r_i + \Delta r/2)/L_A\right] = \tan^{-1}\left[(0.500 + 0.005)/1.000\right] = 26.8°$$

$$R = L_A/\cos\theta_1' = 1\text{m}/\cos 26.8° = 1.12\text{m}.$$

Hence,

$$G_1 = \left(\sigma T_2^4/\pi\right) A_2 \cos 26.8° \cdot \left[A_1 \cos 26.8°/(1.12\text{m})^2\right]/A_1 = 360.2\ \text{W/m}^2$$

using $A_2 = 2\pi r_i \Delta r = 3.142 \times 10^{-2}\ \text{m}^2$ and

$$J_1 = (1-0.3)\times 360.2\ \text{W/m}^2 + 0.3\times 5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4 (400\ \text{K})^4 = 687.7\ \text{W/m}^2.$$

Hence the radiant power is

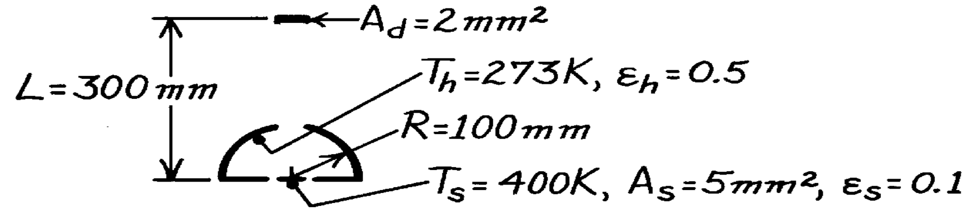
$$q_{1\to3} = \left(687.7\ \text{W/m}^2/\pi\right)\left[\pi(0.010\ \text{m})^2/4\right]^2/(1\ \text{m} + 1\ \text{m})^2 = 337.6\times10^{-9}\ \text{W}. \quad <$$

PROBLEM 12.83

KNOWN: Area and emissivity of opaque sample in hemispherical enclosure. Area and position of detector which views sample through an aperture. Sample and enclosure temperatures.

FIND: (a) Detector irradiation, (b) Spectral distribution and maximum intensities.

SCHEMATIC:



ASSUMPTIONS: (1) Diffuse-gray surfaces, (2) Hemispherical enclosure forms a blackbody cavity about the sample, $A_h >> A_s$, (3) Detector field of view is limited to sample surface.

ANALYSIS: (a) The irradiation can be evaluated as $G_d = q_{s\text{-}d}/A_d$ and $q_{s\text{-}d} = I_{s(e+r)}\ A_s\ \omega_{d\text{-}s}$. Evaluating parameters: $\omega_{d\text{-}s} \approx A_d/L^2 = 2\ mm^2/(300\ mm)^2 = 2.22 \times 10^{-5}$ sr, find

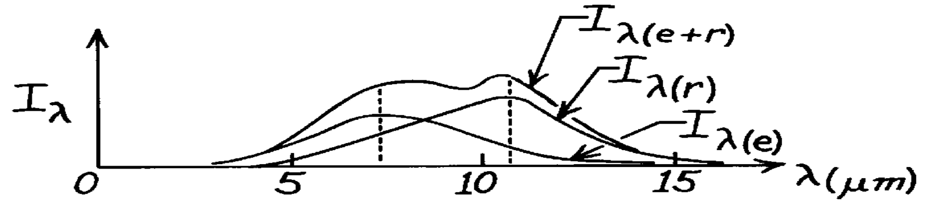
$$I_{s(e)} = \frac{E_s}{\pi} = \frac{\varepsilon_s \sigma T_s^4}{\pi} = \frac{0.1\left(5.67\times10^{-8}\ W/m^2\cdot K^4\right)(400\ K)^4}{\pi\ sr} = 46.2\ W/m^2\cdot sr$$

$$I_{s(r)} = \frac{\rho_s G_s}{\pi} = \frac{(1-\varepsilon_s)\sigma T_h^4}{\pi} = \frac{0.9\left(5.67\times10^{-8} W/m^2\cdot K^4\right)(273\ K)^4}{\pi\ sr} = 90.2\ W/m^2\cdot sr$$

$$q_{s-d} = (46.2+90.2)\,W/m^2\cdot sr\left(5\times10^{-6} m^2 \times 2.22\times10^{-5} sr\right) = 1.51\times10^{-8}\ W$$

$$G_d = 1.51\times10^{-8}\ W/2\times10^{-6}\ m^2 = 7.57\times10^{-3}\ W/m^2. \qquad <$$

(b) Since $\lambda_{max} T = 2898$ μm·K, it follows that $\lambda_{max(e)} = 2898$ μm·K/400 K = 7.25 μm and $\lambda_{max(r)} = 2898$ μm·K/273 K = 10.62 μm.



<

$\lambda = 7.25$ μm: Table 12.1 → $I_{\lambda,b}(400\ K) = 0.722 \times 10^{-4}\ (5.67 \times 10^{-8})(400)^5 = 41.9\ W/m^2\cdot\mu m\cdot sr$

$I_{\lambda,b}(273\ K) = 0.48 \times 10^{-4}\ (5.67 \times 10^{-8})(273)^5 = 4.1\ W/m^2\cdot\mu m\cdot sr$

$I_\lambda = I_{\lambda,e} + I_{\lambda,r} = \varepsilon_s I_{\lambda,b}(400\ K) + \rho I_{\lambda,b}(273K) = 0.1 \times 41.9 + 0.9 \times 4.1 = 7.90\ W/m^2\cdot\mu m\cdot sr$ <

$\lambda = 10.62$ μm: Table 12.1 → $I_{\lambda,b}(400\ K) = 0.53 \times 10^{-4}\ (5.67 \times 10^{-8})(400)^5 = 30.9\ W/m^2\cdot\mu m\cdot sr$

$I_{\lambda,b}(273\ K) = 0.722 \times 10^{-4}\ (5.67 \times 10^{-8})(273)^5 = 6.2\ W/m^2\cdot\mu m\cdot sr$

$I_\lambda = 0.1 \times 30.9 + 0.9 \times 6.2 = 8.68\ W/m^2\cdot\mu m\cdot sr.$ <

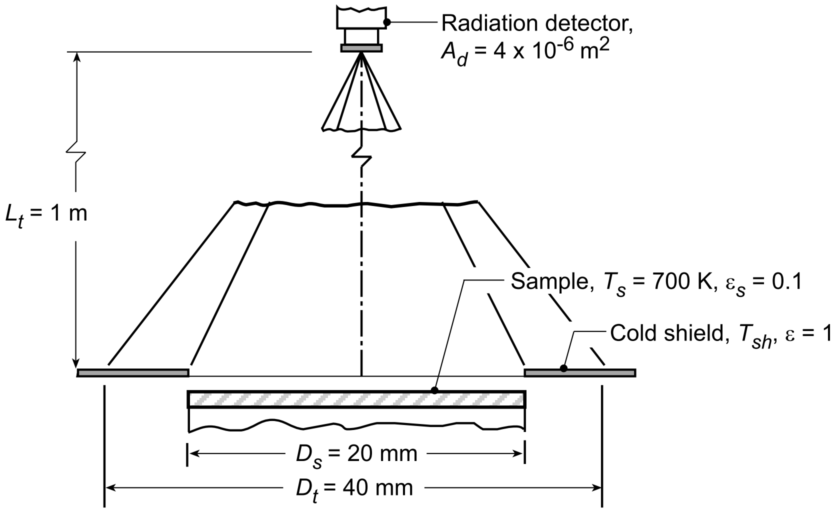
COMMENTS: Although Th is substantially smaller than T_s, the high sample reflectivity renders the reflected component of J_s comparable to the emitted component.

PROBLEM 12.84

KNOWN: Sample at T_s = 700 K with ring-shaped cold shield viewed normally by a radiation detector.

FIND: (a) Shield temperature, T_{sh}, required so that its emitted radiation is 1% of the total radiant power received by the detector, and (b) Compute and plot T_{sh} as a function of the sample emissivity for the range $0.05 \le \varepsilon \le 0.35$ subject to the parametric constraint that the radiation emitted from the cold shield is 0.05, 1 or 1.5% of the total radiation received by the detector.

SCHEMATIC:



ASSUMPTIONS: (1) Sample is diffuse and gray, (2) Cold shield is black, and (3) $A_d, D_s^2, D_t^2 << L_t^2$.

ANALYSIS: (a) The radiant power intercepted by the detector from within the target area is

$$q_d = q_{s\to d} + q_{sh\to d}$$

The contribution from the sample is

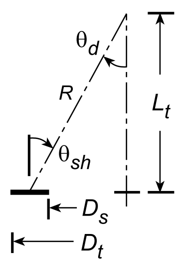
$$q_s \to d = I_{s,e} A_s \cos\theta_s \,\Delta\omega_{d-s} \qquad \theta_s = 0^\circ$$

$$I_{s,e} = \varepsilon_s E_b / \pi = \varepsilon_s \sigma T_s^4 / \pi$$

$$\Delta\omega_{d-s} = \frac{A_d \cos\theta_d}{L_t^2} = \frac{A_d}{L_t^2} \qquad \theta_d = 0^\circ$$

$$q_{s\to d} = \varepsilon_s \sigma T_s^4 A_s A_d / \pi L_t^2 \tag{1}$$

The contribution from the ring-shaped cold shield is



$$q_{sh\to d} = I_{sh,e} A_{sh} \cos\theta_{sh} \Delta\omega_{d-sh}$$

$$I_{sh,e} = E_b / \pi = \sigma T_{sh}^4 / \pi$$

and, from the geometry of the shield -detector,

$$A_{sh} = \frac{\pi}{4}\left(D_t^2 - D_s^2\right)$$

$$\cos\theta_{sh} = L_t \Big/ \left[\left(\bar{D}/2\right)^2 = L_t^2\right]^{1/2}$$

Continued...

PROBLEM 12.84 (Cont.)

where $\bar{D} = (D_s + D_t)/2$

$$\Delta\omega_{d-sh} = \frac{A_d \cos\theta_d}{R^2} \qquad \cos\theta_d = \cos\theta_{sh}$$

where $R = \left[L_t^2 + \bar{D}^2 \right]^{1/2}$

$$q_{sh \to d} = \frac{\sigma T_{sh}^4}{\pi} A_{sh} \left[\frac{L_t}{\left[((D_s + D_t)/4)^2 + L_t^2 \right]^{1/2}} \right]^2 \frac{A_d}{\left[((D_s + D_t)/4)^2 + L_t^2 \right]} \qquad (2)$$

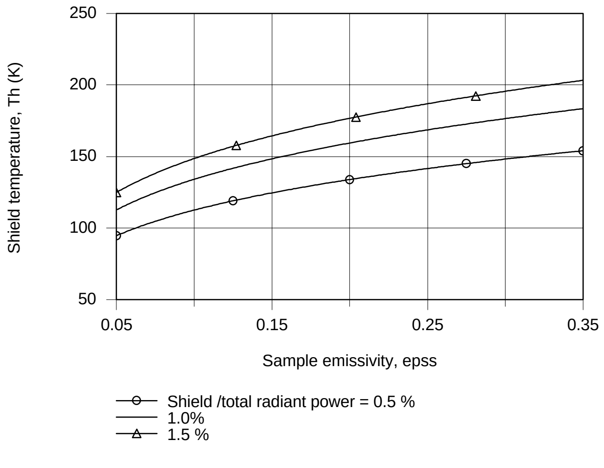
The requirement that the emitted radiation from the cold shield is 1% of the total radiation intercepted by the detector is expressed as

$$\frac{q_{sh-d}}{q_{tot}} = \frac{q_{sh-d}}{q_{sh-d} + q_{s-d}} = 0.01 \qquad (3)$$

By evaluating Eq. (3) using Eqs. (1) and (3), find

$$T_{sh} = 134\,K$$ <

(b) Using the foregoing equations in the IHT workspace, the required shield temperature for $q_{sh - d}/q_{tot}$ = 0.5, 1 or 1.5% was computed and plotted as a function of the sample emissivity.



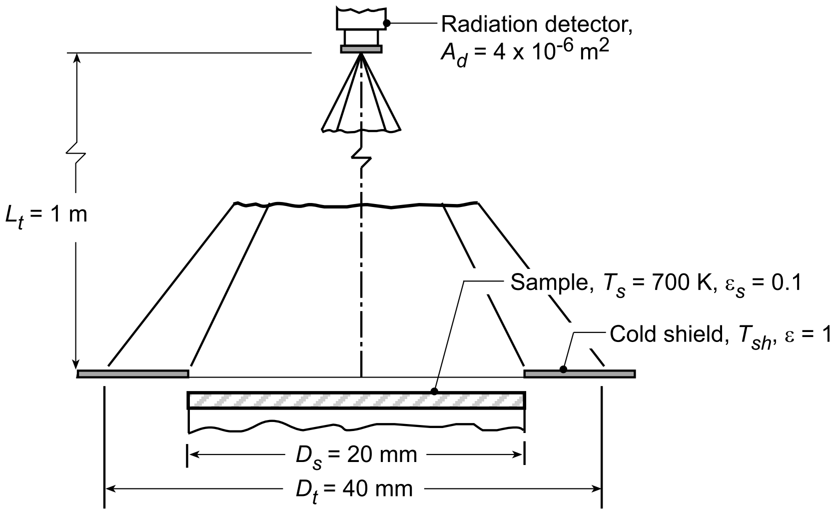
As the shield emission-to-total radiant power ratio decreases (from 1.5 to 0.5%) , the required shield temperature decreases. The required shield temperature increases with increasing sample emissivity for a fixed ratio.

PROBLEM 12.84

KNOWN: Sample at T_s = 700 K with ring-shaped cold shield viewed normally by a radiation detector.

FIND: (a) Shield temperature, T_{sh}, required so that its emitted radiation is 1% of the total radiant power received by the detector, and (b) Compute and plot T_{sh} as a function of the sample emissivity for the range $0.05 \le \varepsilon \le 0.35$ subject to the parametric constraint that the radiation emitted from the cold shield is 0.05, 1 or 1.5% of the total radiation received by the detector.

SCHEMATIC:



ASSUMPTIONS: (1) Sample is diffuse and gray, (2) Cold shield is black, and (3) $A_d, D_s^2, D_t^2 << L_t^2$.

ANALYSIS: (a) The radiant power intercepted by the detector from within the target area is

$$q_d = q_{s\to d} + q_{sh\to d}$$

The contribution from the sample is

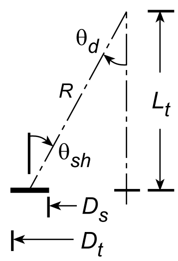
$$q_s \to d = I_{s,e} A_s \cos\theta_s \,\Delta\omega_{d-s} \qquad \theta_s = 0^\circ$$

$$I_{s,e} = \varepsilon_s E_b / \pi = \varepsilon_s \sigma T_s^4 / \pi$$

$$\Delta\omega_{d-s} = \frac{A_d \cos\theta_d}{L_t^2} = \frac{A_d}{L_t^2} \qquad \theta_d = 0^\circ$$

$$q_{s\to d} = \varepsilon_s \sigma T_s^4 A_s A_d / \pi L_t^2 \tag{1}$$

The contribution from the ring-shaped cold shield is



$$q_{sh\to d} = I_{sh,e} A_{sh} \cos\theta_{sh} \Delta\omega_{d-sh}$$

$$I_{sh,e} = E_b / \pi = \sigma T_{sh}^4 / \pi$$

and, from the geometry of the shield -detector,

$$A_{sh} = \frac{\pi}{4}\left(D_t^2 - D_s^2\right)$$

$$\cos\theta_{sh} = L_t \Big/ \left[\left(\bar{D}/2\right)^2 = L_t^2\right]^{1/2}$$

Continued...

PROBLEM 12.84 (Cont.)

where $\bar{D} = (D_s + D_t)/2$

$$\Delta\omega_{d-sh} = \frac{A_d \cos\theta_d}{R^2} \qquad \cos\theta_d = \cos\theta_{sh}$$

where $R = \left[L_t^2 + \bar{D}^2\right]^{1/2}$

$$q_{sh\to d} = \frac{\sigma T_{sh}^4}{\pi} A_{sh} \left[\frac{L_t}{\left[((D_s + D_t)/4)^2 + L_t^2\right]^{1/2}}\right]^2 \frac{A_d}{\left[((D_s + D_t)/4)^2 + L_t^2\right]} \tag{2}$$

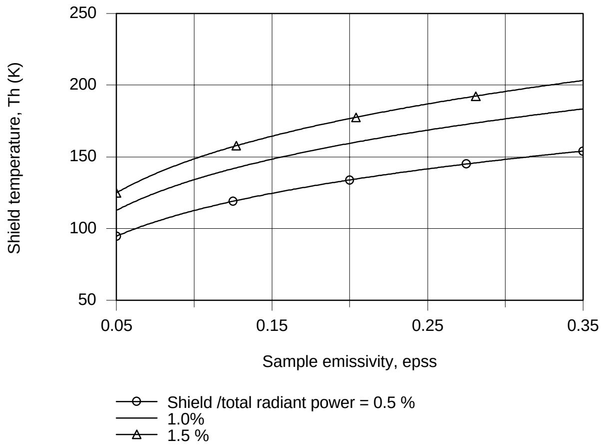
The requirement that the emitted radiation from the cold shield is 1% of the total radiation intercepted by the detector is expressed as

$$\frac{q_{sh-d}}{q_{tot}} = \frac{q_{sh-d}}{q_{sh-d} + q_{s-d}} = 0.01 \tag{3}$$

By evaluating Eq. (3) using Eqs. (1) and (3), find

$$T_{sh} = 134\,\text{K}$$ <

(b) Using the foregoing equations in the IHT workspace, the required shield temperature for q_{sh-d}/q_{tot} = 0.5, 1 or 1.5% was computed and plotted as a function of the sample emissivity.



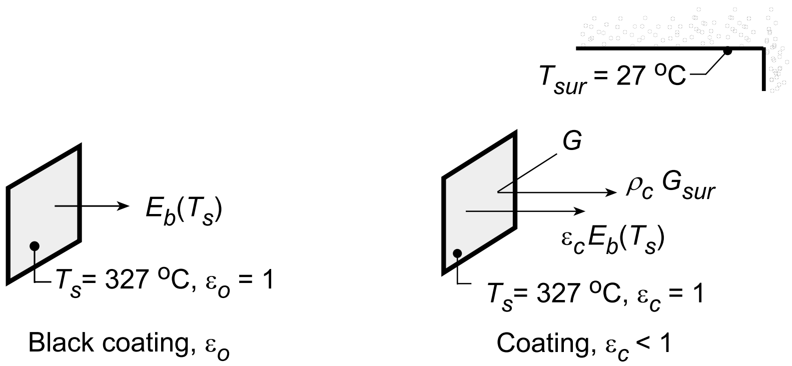
As the shield emission-to-total radiant power ratio decreases (from 1.5 to 0.5%) , the required shield temperature decreases. The required shield temperature increases with increasing sample emissivity for a fixed ratio.

PROBLEM 12.85

KNOWN: Infrared scanner (radiometer) with a 3- to 5-micrometer spectral bandpass views a metal plate maintained at T_s = 327°C having four diffuse, gray coatings of different emissivities. Surroundings at T_{sur} = 87°C.

FIND: (a) Expression for the scanner output signal, S_o, in terms of the responsivity, R (μV·m^2/W), the black coating (ε_o = 1) emissive power and appropriate band emission fractions; assuming R = 1 μV·m^2/W, evaluate S_o(V); (b) Expression for the output signal, S_c, in terms of the responsivity R, the blackbody emissive power of the coating, the blackbody emissive power of the surroundings, the coating emissivity, ε_c, and appropriate band emission fractions; (c) Scanner signals, S_c (μV), when viewing with emissivities of 0.8, 0.5 and 0.2 assuming R = 1 μV·m^2/W; and (d) Apparent temperatures which the scanner will indicate based upon the signals found in part (c) for each of the three coatings.

SCHEMATIC:



ASSUMPTIONS: (1) Plate has uniform temperature, (2) Surroundings are isothermal and large compared to the plate, and (3) Coatings are diffuse and gray so that ε = α and ρ = 1 - ε.

ANALYSIS: (a) When viewing the black coating (ε_o = 1), the scanner output signal can be expressed as

$$S_o = RF_{(\lambda_1-\lambda_2,T_s)}E_b(T_s) \tag{1}$$

where R is the responsivity (μV·m^2/W), $E_b(T_s)$ is the blackbody emissive power at T_s and $F_{(\lambda_1-\lambda_2,T_s)}$ is the fraction of the spectral band between λ_1 and λ_2 in the spectrum for a blackbody at T_s,

$$F_{(\lambda_1-\lambda_2,T_s)} = F_{(0-\lambda_2,T_s)} - F_{(0-\lambda_1,T_s)} \tag{2}$$

where the band fractions Eq. 12.38 are evaluated using Table 12.1 with $\lambda_1 T_s$ = 3 μm (327 + 273)K = 1800 μm·K and $\lambda_2 T_s$ = 5 μm (327 + 273) = 3000 μm·K. Substituting numerical values with R = 1 μV·m^2/W, find

$$S_o = 1\mu V\cdot m^2/W\,[0.2732 - 0.0393]\,5.67\times10^{-8}\ W/m^2\cdot K^4\,(600K)^4$$

$$S_o = 1718\,\mu V$$ <

(b) When viewing one of the coatings ($\varepsilon_c < \varepsilon_o$ = 1), the scanner output signal as illustrated in the schematic above will be affected by the emission and reflected irradiation from the surroundings,

$$S_c = R\left\{F_{(\lambda_1-\lambda_2,T_s)}\varepsilon_c E_b(T_s) + F_{(\lambda_1-\lambda_2,T_{sur})}\rho_c G_c\right\} \tag{3}$$

where the reflected irradiation parameters are

Continued...

$$\rho_c = 1 - \varepsilon_c \qquad G_c = \sigma T_{sur}^4 \tag{4,5}$$

and the related band fractions are

$$F_{(\lambda_1-\lambda_2, T_{sur})} = F_{(0-\lambda_2, T_{sur})} - F_{(0-\lambda_1, T_{sur})} \tag{6}$$

Combining Eqs. (2-6) above, the scanner output signal when viewing a coating is

$$S_c = R\left\{\left[F_{(0-\lambda_2 T_s)} - F_{(0-\lambda_1 T_s)}\right]\varepsilon_c \sigma T_s^4 + \left[F_{(0-\lambda_2 T_{sur})} - F_{(0-\lambda_2 T_{sur})}\right](1-\varepsilon_c)\sigma T_{sur}^4\right\} \tag{7}$$

(c) Substituting numerical values into Eq. (7), find

$$S_c = 1\,\mu V \cdot m^2/W\left\{[0.2732 - 0.0393]\varepsilon_c \sigma\,(600K)^4 + [0.0393 - 0.0010](1-\varepsilon_c)\sigma\,(360K)^4\right\}$$

where for $\lambda_2 T_{sur}$ = 5 μm × 360 K = 1800 μm·K, $F_{(0-\lambda_2 T_{sur})}$ = 0.0393 and $\lambda_1 T_{sur}$ = 3 μm × 360 K = 1080 μm·K, $F_{(0-\lambda_1 T_{sur})}$ = 0.0010. For ε_c = 0.80, find

$$S_c\,(\varepsilon_c = 0.8) = 1\,\mu V \cdot m^2/W\,\{1375 + 7.295\}\,W/m^2 = 1382\,\mu V$$ <

$$S_c\,(\varepsilon_c = 0.5) = 1\,\mu V \cdot m^2/W\,\{859.4 + 18.238\}\,W/m^2 = 878\,\mu V$$ <

$$S_c\,(\varepsilon_c = 0.2) = 1\,\mu V \cdot m^2/W\,\{343.8 + 29.180\}\,W/m^2 = 373\,\mu V$$ <

(d) The scanner calibrated against a black surface (ε_1 = 1) interprets the radiation reaching the detector by emission and reflected radiation from a coating target ($\varepsilon_c < \varepsilon_o$) as that from a blackbody at an apparent temperature T_a. That is,

$$S_c = RF_{(\lambda_1-\lambda_2, T_a)} E_b\,(T_a) \;= R\left[F_{(0-\lambda_2 T_a)} - F_{(0-\lambda_1 T_a)}\right]\sigma T_a^4 \tag{8}$$

For each of the coatings in part (c), solving Eq. (8) using the IHT workspace with the *Radiation Tool*, *Band Emission Factor*, the following results were obtained,

ε_c	S_c (μV)	T_a (K)	$T_a - T_s$ (K)
0.8	1382	579.3	-20.7
0.5	878	539.2	-60.8
0.2	373	476.7	-123.3

COMMENTS: (1) From part (c) results for S_c, note that the contribution of the reflected irradiation becomes relatively more significant with lower values of ε_c.

(2) From part (d) results for the apparent temperature, note that the error, $(T - T_a)$, becomes larger with decreasing ε_c. By rewriting Eq. (8) to include the emissivity of the coating,

$$S_c' = R\left[F_{(0-\lambda_2 T_a)} - F_{(0-\lambda_1 T_a)}\right]\varepsilon_c \sigma T_a^4$$

The apparent temperature T_a' will be influenced only by the reflected irradiation. The results correcting only for the emissivity, ε_c, are

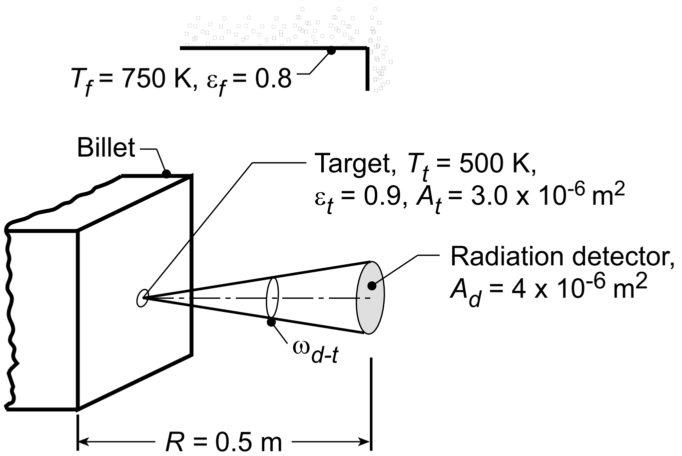
ε_c	0.8	0.5	0.2
$T_a'(K)$	600.5	602.2	608.5
$T_a' - T_s$ (K)	+0.5	+2.2	+8.5

PROBLEM 12.86

KNOWN: Billet at T_t = 500 K which is diffuse, gray with emissivity ε_t = 0.9 heated within a large furnace having isothermal walls at T_f = 750 K with diffuse, gray surface of emissivity ε_f = 0.8. Radiation detector with sensitive area $A_d = 5.0 \times 10^{-4}\ m^2$ positioned normal to and at a distance R = 0.5 m from the billet. Detector receives radiation from a billet target area $A_t = 3.0 \times 10^{-6}\ m^2$.

FIND: (a) Symbolic expressions and numerical values for the following radiation parameters associated with the target surface (t): irradiation on the target, G_t; intensity of the reflected irradiation leaving the target, $I_{t,ref}$; emissive power of the target, E_t; intensity of the emitted radiation leaving the target, $I_{t,emit}$; and radiosity of the target J_t; and (b) Expression and numerical value for the radiation which leaves the target in the spectral region $\lambda \geq 4$ μm and is intercepted by the radiation detector, $q_{t\to d}$; write the expression in terms of the target reflected and emitted intensities $I_{t,ref}$ and $I_{t,emit}$, respectively, as well as other geometric and radiation parameters.

SCHEMATIC:



ASSUMPTIONS: (1) Furnace wall is isothermal and large compared to the billet, (2) Billet surface is diffuse gray, and (3) $A_t, A_d << R^2$.

ANALYSIS: (a) Expressions and numerical values for radiation parameters associated with the target are:

Irradiation, G_t: due to blackbody emission from the furnace walls which are isothermal and large relative to the billet target,

$$G_t = E_b(T_f) = \sigma T_f^4 = 5.67\times10^{-8}\ W/m^2\cdot K^4\,(750\,K)^4 = 17,940\ W/m^2 \qquad <$$

Intensity of reflected irradiation, $I_{t,ref}$: since the billet is diffuse, $I = G_i/\pi$ from Eq. 12.19, and diffuse-gray, $\rho_t = 1 - \varepsilon_t$,

$$I_{t,ref} = \rho_t G_t/\pi = (1-\varepsilon_t)G_t/\pi$$

$$I_{t,ref} = (1-0.9)\times19,740\ W/m^2/\pi = 571\ W/m^2\cdot sr \qquad <$$

Emissive power, E_t: from the Stefan-Boltzmann law, Eq. 12.28, and the definition of the total emissivity, Eq. 12.37,

$$E_t = \varepsilon_t E_b(T_t) = \varepsilon_t \sigma T_t^4$$

$$E_t = 0.9\times5.67\times10^{-8}\ W/m^2\cdot K^4\,(500\,K)^4 = 3189\ W/m^2 \qquad <$$

Continued...

PROBLEM 12.86 (Cont.)

Intensity of emitted radiation, $I_{t,emit}$: since the billet is diffuse, $I_e = E/\pi$ from Eq. 12.14,

$$I_{t,emit} = E_t/\pi = 3189\,W/m^2/\pi = 1015\,W/m^2 \cdot sr \qquad <$$

Radiosity, J_t: the radiosity accounts for the emitted radiation and reflected portion of the irradiation; for the diffuse surface, from Eq. 12.24,

$$J_t = \pi\left(I_{t,ref} + I_{t,emit}\right)$$

$$J_t = \pi\,sr\,(571 + 1015)\,W/m^2 \cdot sr = 4983\,W/m^2 \qquad <$$

(b) The radiant power in the spectral region $\lambda \geq 4\ \mu m$ leaving the target which is intercepted by the detector follows from Eq. 12.5,

$$q_{t\to d} = \left[F_{ref}\,I_{t,ref} + F_{emit}I_{t,emit}\right]A_t\cos\theta_t\,\omega_{d-t}$$

The F factors account for the fraction of the total spectral region for $\lambda \geq 4\ \mu m$,

$$F_{ref} = 1 - F\left(0-\lambda T_f\right) = 1 - 0.2732 = 0.727$$

$$F_{emit} = 1 - F\left(0-\lambda T_t\right) = 1 - 0.06673 = 0.933$$

where from Eq. 12.30 and Table 12.1, for $\lambda T_f = 4\ \mu m \times 750\ K = 3000\ \mu m\cdot K$, $F(0 - \lambda T_f) = 0.2732$ and for $\lambda T_t = 4\ \mu m \times 500\ K = 2000\ \mu m\cdot K$, $F(0 - \lambda T_t) = 0.06673$. Since the radiation detector is normal to the billet, $\cos\theta_t = 1$. The solid angle subtended by the detector area with respect to the target area is

$$\omega_{d-t} = \frac{A_d\cos\theta_d}{R^2} = \frac{5\times10^{-4}\,m^2\times1}{(0.5\,m)^2} = 2.00\times10^{-3}\,sr$$

Hence, the radiant power is

$$q_{t\to d} = (0.727\times571 + 0.933\times1015)\,W/m^2\cdot sr\times3.0\times10^{-6}\,m^2\times1\times2.00\times10^{-3}\,sr$$

$$q_{t\to d} = (2.491 + 5.682)\times10^{-6}\,W = 8.17\,\mu W \qquad <$$

COMMENTS: (1) Why doesn't the emissivity of the furnace walls, ε_f, affect the target irradiation?

(2) Note the importance of the diffuse, gray assumption for the billet target surface. In what ways was the assumption used in the analysis?

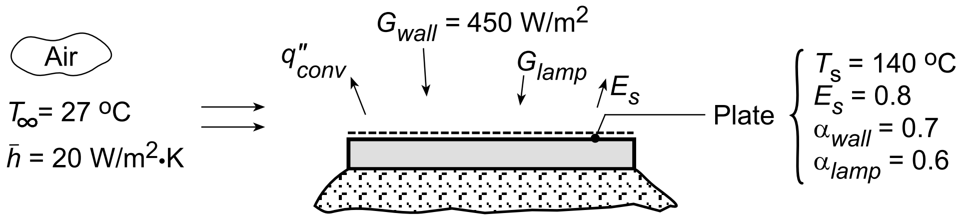
(3) From the calculation of the radiant power to the detector, $q_{t\to d}$, note that the contribution of the reflected irradiation is nearly a third of the total.

PROBLEM 12.87

KNOWN: Painted plate located inside a large enclosure being heated by an infrared lamp bank.

FIND: (a) Lamp irradiation required to maintain plot at $T_s = 140^oC$ for the prescribed convection and enclosure irradiation conditions, (b) Compute and plot the lamp irradiation, G_{lamp}, required as a function of the plate temperature, T_s, for the range $100 \le T_s \le 300\ ^oC$ and for convection coefficients of h = 15, 20 and 30 $W/m^2\cdot K$, and (c) Compute and plot the air stream temperature, T_∞, required to maintain the plate at 140^oC as a function of the convection coefficient h for the range $10 \le h \le 30\ W/m^2\cdot K$ with a lamp irradiation $G_{lamp} = 3000\ W/m^2$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) No losses on backside of plate.

ANALYSIS: (a) Perform an energy balance on the plate, per unit area,

$$\dot{E}_{in} - \dot{E}_{out} = 0 \tag{1}$$

$$\alpha_{wall} \cdot G_{wall} + \alpha_{lamp} G_{lamp} - q''_{conv} - E_s = 0 \tag{2}$$

where the emissive power of the surface and convective fluxes are

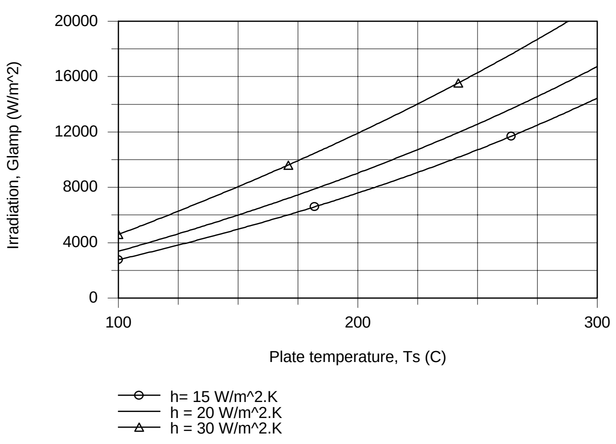
$$E_s = \varepsilon_s E_b(T_s) = \varepsilon_s \cdot \sigma T_s^4 \qquad q''_{conv} = h(T_s - T_\infty) \tag{3,4}$$

Substituting values, find the lamp irradiation

$$0.7 \times 450\ W/m^2 + 0.6 \times G_{lamp} - 20\ W/m^2 \cdot K(413 - 300\ K$$
$$-0.8 \times 5.67 \times 10^{-8}\ W/m^2 \cdot K^4\ (413\ K)^4 = 0 \tag{5}$$

$G_{lamp} = 5441\ W/m^2$ <

(b) Using the foregoing equations in the IHT workspace, the irradiation, G_{lamp}, required to maintain the plate temperature in the range $100 \le T_s \le 300\ ^oC$ for selected convection coefficients was computed. The results are plotted below.

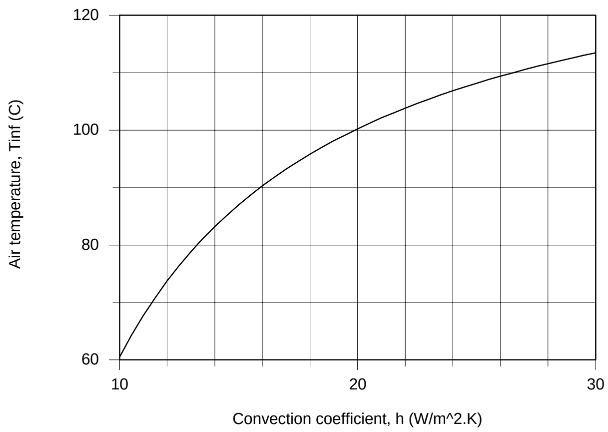


Continued...

PROBLEM 12.87 (Cont.)

As expected, to maintain the plate at higher temperatures, the lamp irradiation must be increased. At any plate operating temperature condition, the lamp irradiation must be increased if the convection coefficient increases. With forced convection (say, $h \geq 20\ W/m^2 \cdot K$) of the airstream at 27^oC, excessive irradiation levels are required to maintain the plate above the cure temperature of 140^oC.

(c) Using the IHT model developed for part (b), the airstream temperature, T_∞, required to maintain the plate at $T_s = 140^oC$ as a function of the convection coefficient with $G_{lamp} = 3000\ W/m^2 \cdot K$ was computed and the results are plotted below.



As the convection coefficient increases, for example by increasing the airstream velocity over the plate, the required air temperature must increase. Give a physical explanation for why this is so.

COMMENTS: (1) For a spectrally selective surface, we should expect the absorptivity to depend upon the spectral distribution of the source and $\alpha \neq \varepsilon$.

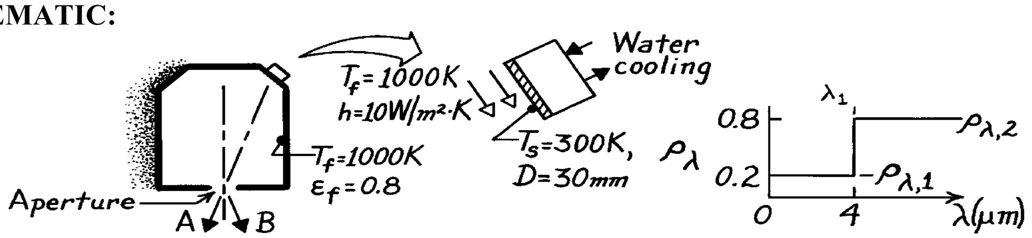
(2) Note the new terms used in this problem; use your Glossary, Section 12.9 to reinforce their meaning.

PROBLEM 12.88

KNOWN: Small sample of reflectivity, ρ_λ, and diameter, D, is irradiated with an isothermal enclosure at T_f.

FIND: (a) Absorptivity, α, of the sample with prescribed ρ_λ, (b) Emissivity, ε, of the sample, (c) Heat removed by coolant to the sample, (d) Explanation of why system provides a measure of ρ_λ.

SCHEMATIC:



ASSUMPTIONS: (1) Sample is diffuse and opaque, (2) Furnace is an isothermal enclosure with area much larger than the sample, (3) Aperture of furnace is small.

ANALYSIS: (a) The absorptivity, α, follows from Eq. 12.42, where the irradiation on the sample is $G = E_b(T_f)$ and $\alpha_\lambda = 1 - \rho_\lambda$.

$$\alpha = \int_0^\infty \alpha_\lambda G_\lambda \, d\lambda / G = \int_0^\infty (1-\rho_\lambda) E_{\lambda,b}(\lambda, 1000K) \, d\lambda / E_b(1000K)$$

$$\alpha = (1-\rho_{\lambda,1}) F_{(0\to\lambda_1)} + (1-\rho_{\lambda,2})\left[1 - F_{(0\to\lambda_1)}\right].$$

Using Table 12.1 for $\lambda_1 T_f = 4 \times 1000 = 4000\ \mu m \cdot K$, $F_{(0-\lambda)} = 0.491$ giving

$$\alpha = (1-0.2)\times 0.491 + (1-0.8)\times(1-0.491) = 0.49.$$ <

(b) The emissivity, ε, follows from Eq. 12.37 with $\varepsilon_\lambda = \alpha_\lambda = 1 - \rho_\lambda$ since the sample is diffuse.

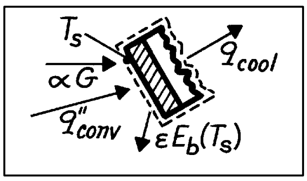
$$\varepsilon = E(T_s)/E_b(T_s) = \int_0^\infty \varepsilon_\lambda E_{\lambda,b}(\lambda, 300K) \, d\lambda / E_b(300K)$$

$$\varepsilon = (1-\rho_{\lambda,1}) F_{(0-\lambda_1)} + (1-\rho_{\lambda,2})\left[1 - F_{(0\to\lambda_1)}\right].$$

Using Table 12.1 for $\lambda_1 T_s = 4 \times 300 = 1200\ \mu m \cdot K$, $F_{(0-\lambda)} = 0.002$ giving

$$\varepsilon = (1-0.2)\times 0.002 + (1-0.8)\times(1-0.002) = 0.20.$$

(c) Performing an energy balance on the sample, the heat removal rate by the cooling water is



$$q_{cool} = A_s\left[\alpha G + q''_{conv} - \varepsilon E_b(T_s)\right]$$

where $\quad G = E_b(T_f) = E_b(1000K)$

$$q''_{conv} = h(T_f - T_s) \qquad A_s = \pi D^2/4$$

$$q_{cool} = (\pi/4)(0.03m)^2\Big[0.49\times 5.67\times 10^{-8} W/m^2\cdot K^4 \times (1000K)^4$$

$$+10\, W/m^2\cdot K(1000-300)K - 0.20\times 5.67\times 10^{-8} W/m^2\cdot K^4 \times (300K)^4\Big] = 24.4\ W.$$ <

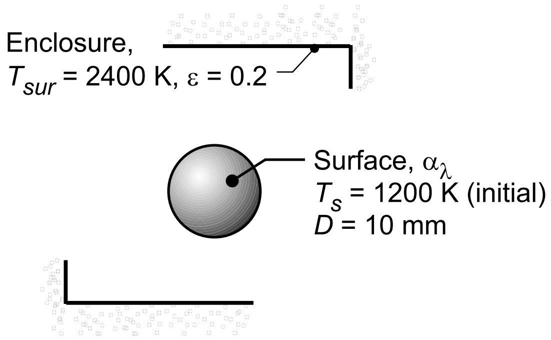
(d) Assume that reflection makes the dominant contribution to the radiosity of the sample. When viewing in the direction A, the spectral radiant power is proportional to $\rho_\lambda G_\lambda$. In direction B, the spectral radiant power is proportional to $E_{\lambda,b}(T_f)$. Noting that $G_\lambda = E_{\lambda,b}(T_f)$, the ratio gives ρ_λ.

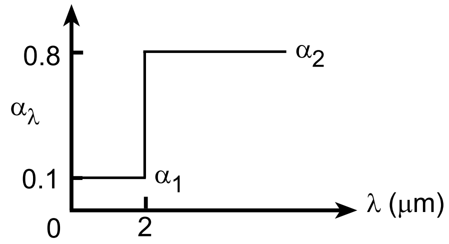
PROBLEM 12.89

KNOWN: Small, opaque surface initially at 1200 K with prescribed α_λ distribution placed in a large enclosure at 2400 K.

FIND: (a) Total, hemispherical absorptivity of the sample surface, (b) Total, hemispherical emissivity, (c) α and ε after long time has elapsed, (d) Variation of sample temperature with time.

SCHEMATIC:





ASSUMPTIONS: (1) Surface is diffusely radiated, (2) Enclosure is much larger than surface and at a uniform temperature.

PROPERTIES: *Table A.1*, Tungsten (T ≈ 1800 K): ρ = 19,300 kg/m^3, c_p = 163 J/kg·K, k ≈ 102 W/m·K.

ANALYSIS: (a) The total, hemispherical absorptivity follows from Eq. 12.46, where $G_\lambda = E_{\lambda,b}(T_{sur})$. That is, the irradiation corresponds to the spectral emissive power of a blackbody at the enclosure temperature and is independent of the enclosure emissivity.

$$\alpha = \int_0^\infty \alpha_\lambda G_\lambda \, d\lambda \Big/ \int_0^\infty G_\lambda \, d\lambda = \int_0^\infty \alpha_\lambda E_{\lambda,b}(\lambda, T_{sur}) d\lambda / E_b(T_{sur})$$

$$\alpha = \alpha_1 \int_0^{2\mu m} E_{\lambda,b}(\lambda, T_{sur}) d\lambda / \sigma T_{sur}^4 + \alpha_2 \int_{2\mu m}^\infty E_{\lambda,b}(\lambda, T_{sur}) d\lambda \Big/ \sigma T_{sur}^4$$

$$\alpha = \alpha_1 F_{(0\to 2\mu m)} + \alpha_2 \left[1 - F_{(0\to 2\mu m)}\right] = 0.1 \times 0.6076 + 0.8[1 - 0.6076] = 0.375$$ <

where at $\lambda T = 2 \times 2400 = 4800\,\mu m \cdot K$, $F_{(0\to 2\mu m)} = 0.6076$ from Table 12.1.

(b) The total, hemispherical emissivity follows from Eq. 12.38,

$$\varepsilon = \int_0^\infty \varepsilon_\lambda E_{\lambda,b}(\lambda, T_s) d\lambda \Big/ \int_0^\infty E_{\lambda,b}(\lambda, T_s) d\lambda .$$

Since the surface is diffuse, $\varepsilon_\lambda = \alpha_\lambda$ and the integral can be expressed as

$$\varepsilon = \alpha_1 \int_0^{2\mu m} E_{\lambda,b}(\lambda, T_s) d\lambda \Big/ \sigma T_s^4 + \alpha_2 \int_{2\mu m}^\infty E_{\lambda,b}(\lambda, T_s) d\lambda \Big/ \sigma T_s^4$$

$$\varepsilon = \alpha_1 F_{(0\to 2\mu m)} + \alpha_2 \left[1 - F_{(0\to 2\mu m)}\right] = 0.1 \times 0.1403 + 0.8[1 - 0.1403] = 0.702$$ <

where at $\lambda T = 2 \times 1200 = 2400\,\mu m \cdot K$, find $F_{(0\to 2\mu m)} = 0.1403$ from Table 12.1.

(c) After a long period of time, the surface will be at the temperature of the enclosure. This condition of thermal equilibrium is described by Kirchoff's law, for which

$\varepsilon = \alpha = 0.375.$ <

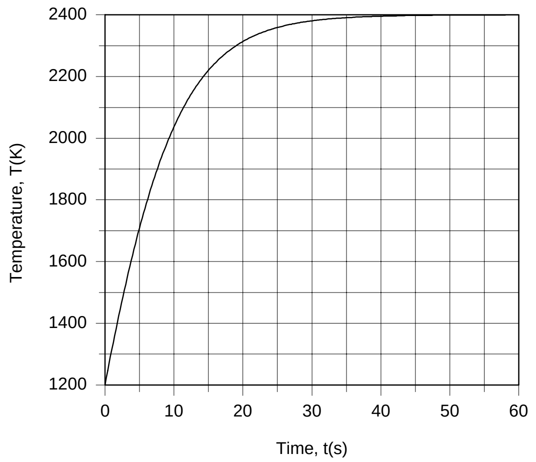
Continued...

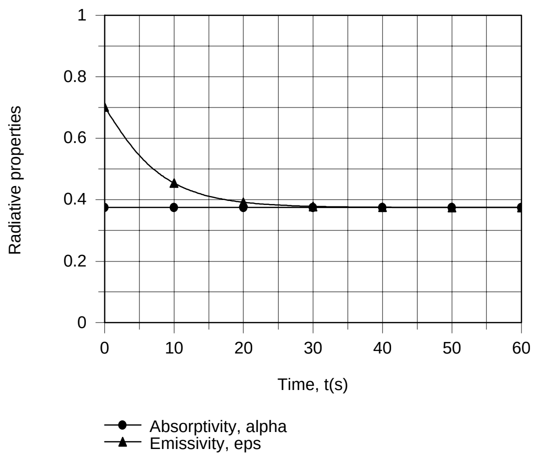
PROBLEM 12.89 (Cont.)

(d) Using the IHT *Lumped Capacitance* Model, the energy balance relation is of the form

$$\rho c_p \forall \frac{dT}{dt} = A_s[\alpha G - \varepsilon(T) E_b(T)]$$

where T = T_s, $\forall = \pi D^3/6$, $A_s = \pi D^2$ and $G = \sigma T_{sur}^4$. Integrating over time in increments of Δt = 0.5s and using the *Radiation* Toolpad to determine ε(t),
the following results are obtained.





The temperature of the specimen increases rapidly with time and achieves a value of 2399 K within t ≈ 47s. The emissivity decreases with increasing time, approaching the absorptivity as T approaches T_{sur}.

COMMENTS: (1) Recognize that α always depends upon the spectral irradiation distribution, which, in this case, corresponds to emission from a blackbody at the temperature of the enclosure.

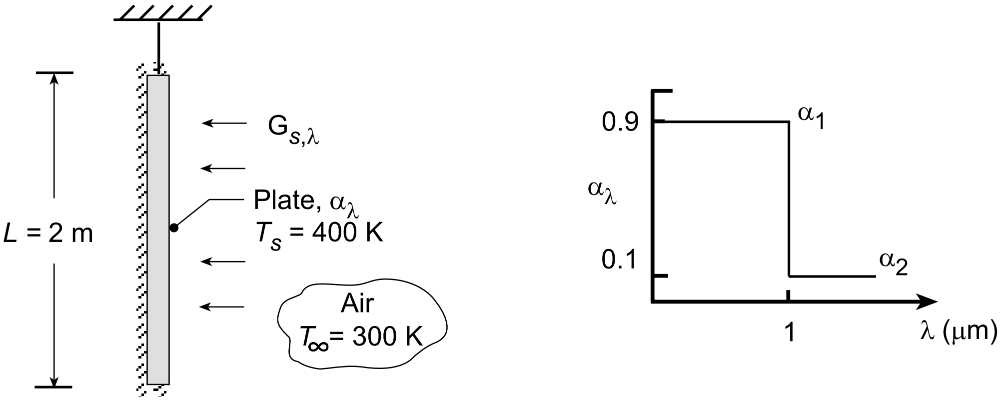
(2) With $h_r = \varepsilon\sigma(T + T_{sur})(T^2 + T_{sur}^2) = 0.375\sigma 4T_{sur}^3 = 1176\, W/m^2 \cdot K$, $Bi = h_r\,(r_o/3)/k$ $= (1176\, W/m^2 \cdot K)\ \ .667\times 10^{-3}\, m/102\, W/m \cdot K = 0.0192 << 1$, use of the lumped capacitance model is justified.

PROBLEM 12.90

KNOWN: Vertical plate of height L = 2 m suspended in quiescent air. Exposed surface with diffuse coating of prescribed spectral absorptivity distribution subjected to simulated solar irradiation, $G_{S,\lambda}$. Plate steady-state temperature T_s = 400 K.

FIND: (a) Plate emissivity, ε, plate absorptivity, α, plate irradiation, G, and using an appropriate correlation, the free convection coefficient, $\bar{h}$, and (b) Plate steady-state temperature if the irradiation found in part (a) were doubled.

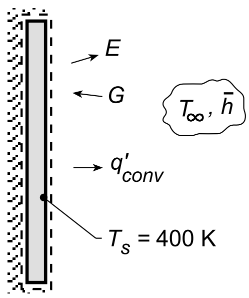
SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Ambient air is extensive, quiescent, (3) Spectral distribution of the simulated solar irradiation, $G_{S,\lambda}$, proportional to that of a blackbody at 5800 K, (4) Coating is opaque, diffuse, and (5) Plate is perfectly insulated on the edges and the back side, and (6) Plate is isothermal.

PROPERTIES: *Table A.4,* Air (T_f = 350 K, 1 atm): $\nu = 20.92 \times 10^{-6}$ m^2/s, k = 0.030 W/m·K, $\alpha = 29.90 \times 10^{-6}$ m^2/s, Pr = 0.700.

ANALYSIS: (a) Perform an energy balance on the plate as shown in the schematic on a per unit plate width basis,



$$\dot{E}_{in} - E_{out} = 0$$

$$\left[\alpha G - \varepsilon\sigma T_s^4 - \bar{h}\left(T_s - T_\infty\right)\right] L = 0 \qquad (1)$$

where α and ε are determined from knowledge of α_λ and $\bar{h}$ is estimated from an appropriate correlation.

Plate total emmissivity: From Eq. 12.38 written in terms of the band emission factor, $F_{(0-\lambda T)}$, Eq. 12.30,

$$\varepsilon = \alpha_1 F_{\left(0-\lambda_1 T_s\right)} + \alpha_2\left[1 - F_{\left(0-\lambda_1 T_s\right)}\right]$$

$$\varepsilon = 0.9 \times 0 + 0.1[1 - 0] = 0.1$$ <

where, from Table 12.1, with λ, T_s = 1μm × 400 K = 400 μm·K, F(0-λT) = 0.000.

Plate absorptivity: With the spectral distribution of simulated solar irradiation proportional to E_b (T_s = 5800 K),

Continued...

$$\alpha = \alpha_1 F_{(0-\lambda_1 T_s)} + \alpha_2 \left[1 - F_{(0-\lambda_1 T_s)}\right]$$

$$\alpha = 0.9 \times 0.7202 + 0.1[1 - 0.7202] = 0.676$$ <

where, from Table 12.1, with $\lambda_1 T_s = 5800$ μm·K, $F_{(0-\lambda T)} = 0.7202$.

Estimating the free convection coefficient, $\bar{h}$ *:* Using the Churchill-Chu correlation Eq. (9.26) with properties evaluated at $T_f = (T_s + T_\infty)/2 = 350$ K,

$$Ra_L = \frac{g\beta (T_s - T_\infty) L^3}{\nu\alpha}$$

$$Ra_L = \frac{9.8\,\text{m}/\text{s}^2\,(1/350\,\text{K}) \times 100\,\text{K}\,(2\,\text{m})^3}{20.92 \times 10^{-6}\,\text{m}^2/\text{s} \times 29.90 \times 10^{-6}\,\text{m}^2/\text{s}} = 3.581 \times 10^{10}$$

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1 + (0.492/Pr)^{9/16}\right]^{8/27}}\right\}^2$$

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1 + (0.492/0.700)^{9/16}\right]^{8/27}}\right\}^2 = 377.6$$

$$\bar{h}_L = \overline{Nu}_L\, k/L = 377.6 \times 0.030\,\text{W}/\text{m}\cdot\text{K}/2\,\text{m} = 5.66\,\text{W}/\text{m}^2\cdot\text{K}$$ <

Irradiation on the Plate: Substituting numerical values into Eq. (1), find G.

$$0.676G - 0.1 \times 5.67 \times 10^{-8}\,\text{W}/\text{m}^2\cdot\text{K}^4\,(400\,\text{K})^4 - 5.66\,\text{W}/\text{m}^2\cdot\text{K}\,(400 - 300)\,\text{K} = 0$$

$$G = 1052\,\text{W}/\text{m}^2$$ <

(b) If the irradiation were doubled, G = 2104 W/m^2, the plate temperature T_s will increase, of course, requiring re-evaluation of ε and $\bar{h}$. Since α depends upon the irradiation distribution, and not the plate temperature, α will remain the same. As a first approximation, assume ε = 0.1 and $\bar{h}$ = 5.66 W/m^2·K and with G = 2104 W/m^2, use Eq. (1) to find

$$T_s \approx 492\,\text{K}$$ <

With $T_f = (T_s + T_\infty)/2 = (492 + 300)\text{K}/2 \approx 400$ K, use the correlation to reevaluate $\bar{h}$. For T_s = 492 K, ε = 0.1 is yet a good assumption. We used IHT with the foregoing equations in part (a) and found these results.

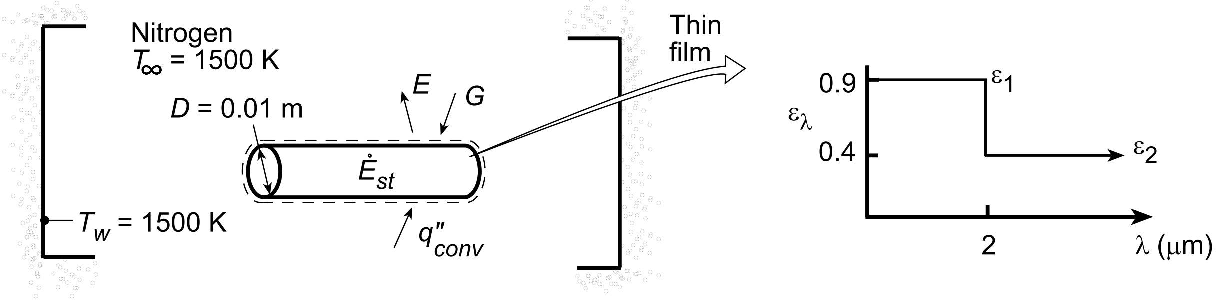
$$T_s = 477\text{K} \qquad T_f = 388.5 \qquad \bar{h} = 6.38\,\text{W}/\text{m}^2\cdot\text{K} \qquad \varepsilon = 0.1$$ <

PROBLEM 12.91

KNOWN: Diameter and initial temperature of copper rod. Wall and gas temperature.

FIND: (a) Expression for initial rate of change of rod temperature, (b) Initial rate for prescribed conditions, (c) Transient response of rod temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Applicability of lumped capacitance approximation, (2) Furnace approximates a blackbody cavity, (3) Thin film is diffuse and has negligible thermal resistance, (4) Properties of nitrogen approximate those of air (Part c).

PROPERTIES: *Table A.1*, copper (T = 300 K): c_p = 385 J/kg·K, ρ = 8933 kg/m^3, k = 401 W/m·K. *Table A.4*, nitrogen (p = 1 atm, T_f = 900 K): ν = 100.3 × 10^{-6} m^2/s, α = 139 × 10^{-6} m^2/s, k = 0.0597 W/m·K, Pr = 0.721.

ANALYSIS: (a) Applying conservation of energy at an instant of time to a control surface about the cylinder, $\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$, where energy inflow is due to natural convection and radiation from the furnace wall and energy outflow is due to emission. Hence, for a unit cylinder length,

$$q_{conv} + q_{rad,net} = \frac{\rho\pi D^2}{4} c_p \frac{dT}{dt}$$

where

$$q_{conv} = \bar{h}(\pi D)(T_\infty - T)$$

$$q_{rad,net} = \pi D(\alpha G - \varepsilon E_b) = \pi D[\alpha E_b(T_w) - \varepsilon E_b(T)]$$

Hence, at t = 0 (T = T_i),

$$dT/dt)_i = (4/\rho c_p D)[\bar{h}(T_\infty - T_i) + \alpha E_b(T_w) - \varepsilon E_b(T_i)]$$

(b) With $Ra_D = \frac{g\beta(T_\infty - T_i)D^3}{\alpha\nu} = \frac{9.8\,m/s^2\,(1/900\,K)(1200\,K)(0.01\,m)^3}{100.3\times139\times10^{-12}\,m^4/s^2} = 937$, Eq. (9.34) yields

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1+(0.559/Pr)^{9/16}\right]^{8/27}}\right\}^2 = 2.58$$

$$\bar{h} = k\frac{\overline{Nu}_D}{D} = \frac{(0.0597\,W/m\cdot K)2.58}{0.01\,m} = 15.4\,W/m^2\cdot K$$

With T = T_i = 300 K, λT = 600 μm·K yields $F_{(0\to\lambda)}$ = 0, in which case $\varepsilon = \varepsilon_1 F_{(0\to\lambda)} + \varepsilon_2\left[1 - F_{(0\to\lambda)}\right] = 0.4$.

With T = T_w = 1500 K, λT = 3000 K yields $F_{(0\to\lambda)}$ = 0.273. Hence, with $\alpha_\lambda = \varepsilon_\lambda$, $\alpha = \varepsilon_1 F_{(0\to\lambda)} + \varepsilon_2[1 - F_{(0\to\lambda)}]$ = 0.9(0.273) + 0.4(1 - 0.273) = 0.537. It follows that

Continued...

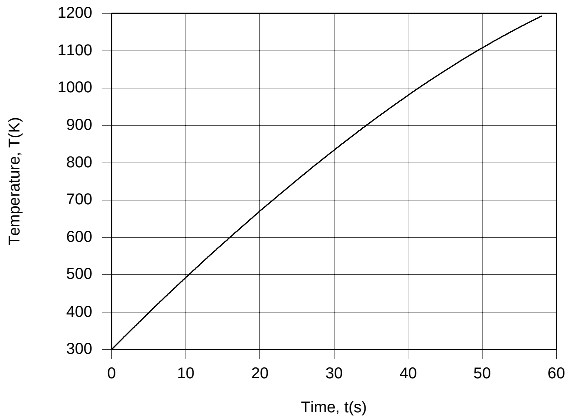
$$\left.\frac{dT}{dt}\right)_i = \frac{4}{8933\dfrac{kg}{m^3}\left(385\dfrac{J}{kg\cdot K}\right)0.01\,m}\left[15\frac{W}{m^2\cdot K}(1500-300)\,K\right.$$

$$\left. +0.537\times5.67\times10^{-4}\frac{W}{m^2\cdot K^4}(1500\,K)^4 - 0.4\times5.67\times10^{-8}\frac{W}{m^2\cdot K^4}(300\,K)^4\right]$$

$$dT/dt)_i = 1.163\times10^{-4}\,m^2\cdot K/J\,[18,480+154,140-180]\,W/m^2 = 20\,K/s \qquad <$$

Defining a pseudo radiation coefficient as $h_r = (\alpha G - \varepsilon E_b)/(T_w - T_i) = (153,960\ W/m^2)/1200\ K = 128.3\ W/m^2\cdot K$, $Bi = (h + h_r)(D/4)/k = 143.7\ W/m^2\cdot K\ (0.0025\ m)/401\ W/m\cdot K = 0.0009$. Hence, the lumped capacitance approximation is appropriate.

(c) Using the IHT *Lumped Capacitance* Model with the *Correlations*, *Radiation* and *Properties* (copper and air) Toolpads, the transient response of the rod was computed for $300 \le T < 1200$ K, where the upper limit was determined by the temperature range of the copper property table.



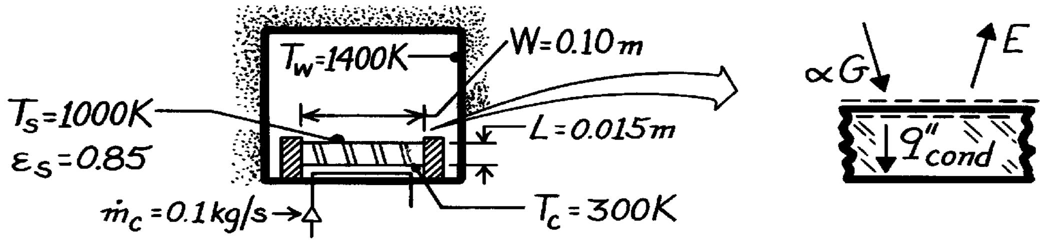
The rate of change of the rod temperature, dT/dt, decreases with increasing temperature, in accordance with a reduction in the convective and *net* radiative heating rates. Note, however, that even at $T \approx 1200$ K, αG, which is fixed, is large relative to q''_{conv} and εE_b and dT/dt is still significant.

PROBLEM 12.92

KNOWN: Temperatures of furnace wall and top and bottom surfaces of a planar sample. Dimensions and emissivity of sample.

FIND: (a) Sample thermal conductivity, (b) Validity of assuming uniform bottom surface temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in sample, (3) Constant k, (4) Diffuse-gray surface, (5) Irradiation equal to blackbody emission at 1400K.

PROPERTIES: *Table A-6*, Water coolant (300K): $c_{p,c} = 4179$ J/kg·K

ANALYSIS: (a) From energy balance at top surface,

$$\alpha G - E = q''_{cond} = k_s\left(T_s - T_c\right)/L$$

where $E = \varepsilon_s \sigma T_s^4, G = \sigma T_w^4, \alpha = \varepsilon_s$ giving

$$\varepsilon_s \sigma T_w^4 - \varepsilon_s \sigma T_s^4 = k_s\left(T_s - T_c\right)/L.$$

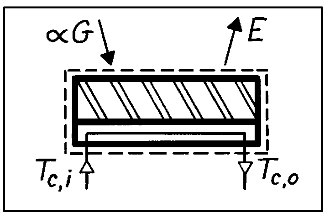
Solving for the thermal conductivity and substituting numerical values, find

$$k_s = \frac{\varepsilon_s L \sigma}{T_s - T_c}\left(T_w^4 - T_s^4\right)$$

$$k_s = \frac{0.85 \times 0.015\text{m} \times 5.67 \times 10^{-8}\text{W/m}^2 \cdot \text{K}^4}{(1000 - 300)\text{K}}\left[(1400\text{K})^4 - (1000\text{K})^4\right]$$

$$k_s = 2.93\ \text{W/m} \cdot \text{K}. \qquad <$$

(b) Non-uniformity of bottom surface temperature depends on coolant temperature rise. From the energy balance



$$q = \dot{m}_c c_{p,c} \Delta T_c = (\alpha G - E) W^2$$

$$\Delta T_c = 0.85 \times 5.67 \times 10^{-8}\text{W/m}^2 \cdot \text{K}^4\left[1400^4 - 1000^4\right]\text{K}^4 (0.10\text{m})^2 / 0.1\text{kg/s} \times 4179\ \text{J/kg} \cdot \text{K}$$

$$\Delta T_c = 3.3\text{K}. \qquad <$$

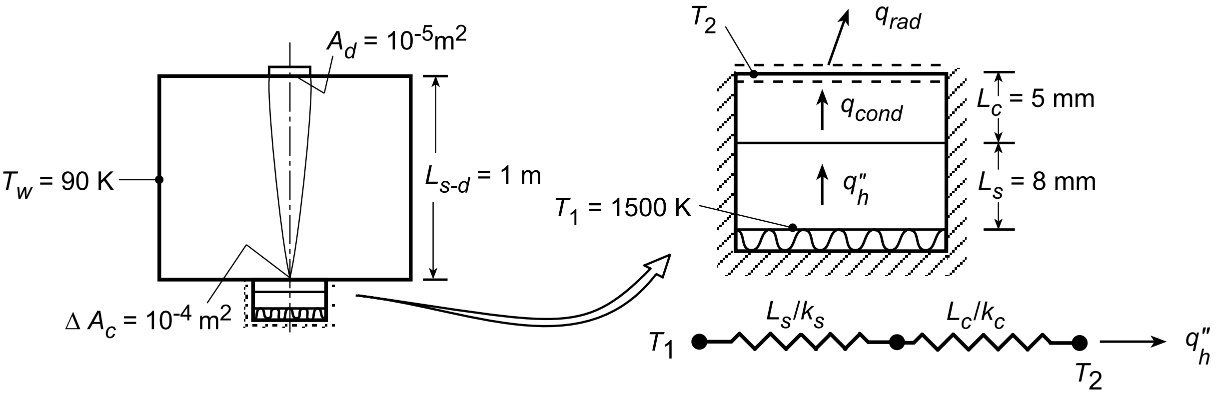
The variation in T_c (~ 3K) is small compared to ($T_s - T_c$) ≈ 700K. Hence it is not large enough to introduce significant error in the k determination.

PROBLEM 12.93

KNOWN: Thicknesses and thermal conductivities of a ceramic/metal composite. Emissivity of ceramic surface. Temperatures of vacuum chamber wall and substrate lower surface. Receiving area of radiation detector, distance of detector from sample, and sample surface area viewed by detector.

FIND: (a) Ceramic top surface temperature and heat flux, (b) Rate at which radiation emitted by the ceramic is intercepted by detector, (c) Effect of an interfacial (ceramic/substrate) contact resistance on sample top and bottom surface temperatures.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction in sample, (2) Constant properties, (3) Chamber forms a blackbody enclosure at T_w, (4) Ceramic surface is diffuse/gray, (5) Negligible interface contact resistance for part (a).

PROPERTIES: Ceramic: k_c = 60 W/m·K, ε_c = 0.8. Substrate: k_s = 25 W/m·K.

ANALYSIS: (a) From an energy balance at the exposed ceramic surface, $q''_{cond} = q''_{rad}$, or

$$\frac{T_1 - T_2}{(L_s/k_s) + (L_c/k_c)} = \varepsilon_c \sigma \left(T_2^4 - T_w^4\right)$$

$$\frac{1500\,\text{K} - T_2}{\dfrac{0.008\,\text{m}}{25\,\text{W/m·K}} + \dfrac{0.005\,\text{m}}{60\,\text{W/m·K}}} = 0.8 \times 5.67 \times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4 \left(T_2^4 - 90^4\right)\text{K}^4$$

$$3.72 \times 10^6 - 2479 T_2 = 4.54 \times 10^{-8} T_2^4 - 2.98$$

$$4.54 \times 10^{-8} T_2^4 + 2479 T_2 = 3.72 \times 10^6$$

Solving, we obtain

$$T_2 = 1425\,\text{K} \qquad <$$

$$q''_h = \frac{T_1 - T_2}{(L_s/k_s) + (L_c/k_c)} = \frac{(1500 - 1425)\,\text{K}}{4.033 \times 10^{-4}\,\text{m}^2 \cdot \text{K/W}} = 1.87 \times 10^5\,\text{W/m}^2 \qquad <$$

(b) Since the ceramic surface is diffuse, the total intensity of radiation emitted in all directions is $I_e = \varepsilon_c E_b(T_s)/\pi$. Hence, the rate at which *emitted* radiation is intercepted by the detector is

$$q_{c(em)-d} = I_e \Delta A_c \left(A_d / L_{s-d}^2\right)$$

$$q_{c(em)-d} = \frac{0.8 \times 5.67 \times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\,(1425\,\text{K})^4}{\pi\,\text{sr}} \times 10^{-4}\,\text{m}^2 \times 10^{-5}\,\text{sr} = 5.95 \times 10^{-5}\,\text{W}$$

Continued...

PROBLEM 12.93 (Cont.)

(c) With the development of an interfacial thermal contact resistance and fixed values of q''_h and T_w, (i) T_2 remains the same (its value is determined by the requirement that $q''_h = \varepsilon_c \sigma \left(T_2^4 - T_w^4\right)$, while (ii) T_1 increases (its value is determined by the requirement that $q''_h = \left(T_1 - T_2\right)/R''_{tot}$, where R''_{tot} = [(L_s/k_s) + $R''_{t,c}$ + (L_c/k_c)]; if q''_h and T_2 are fixed, T_1 must increase with increasing R''_{tot}).

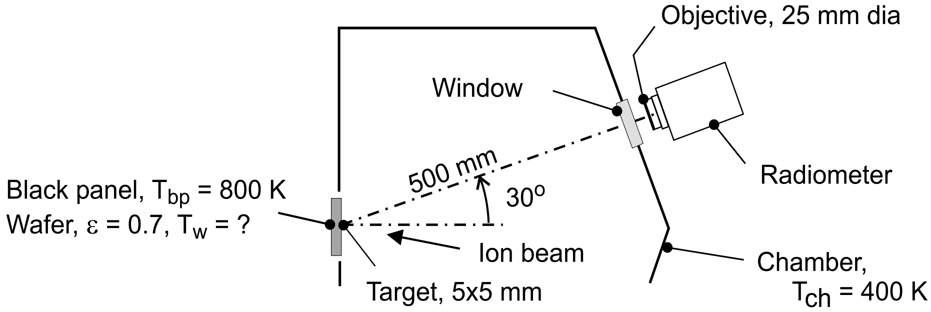
COMMENTS: The detector will also see radiation which is reflected from the ceramic. The corresponding radiation rate is $q_{c(reflection)-d} = \rho_c G_c \Delta A_c \, A_d / L^2_{s-d} = 0.2\,\sigma(90\text{ K})^4 \times 10^{-4}\text{ m}^2 \times (10^{-5}\text{ sr}) = 7.44 \times 10^{-10}$ W. Hence, reflection is negligible.

PROBLEM 12.94

KNOWN: Wafer heated by ion beam source within large process-gas chamber with walls at uniform temperature; radiometer views a 5 × 5 mm target on the wafer. Black panel mounted in place of wafer in a pre-production test of the equipment.

FIND: (a) Radiant power (μW) received by the radiometer when the black panel temperature is T_{bp} = 800 K and (b) Temperature of the wafer, T_w, when the ion beam source is adjusted so that the radiant power received by the radiometer is the same as that of part (a)

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Chamber represents large, isothermal surroundings, (3) Wafer is opaque, diffuse-gray, and (4) Target area << square of distance between target and radiometer objective.

ANALYSIS: (a) The radiant power leaving the black-panel target and reaching the radiometer as illustrated in the schematic below is

$$q_{bp-rad} = \left[E_{b,bp}\left(T_{bp}\right)/\pi\right] A_t \cos\theta_t \cdot \Delta\omega_{rad-t} \tag{1}$$

where $\theta_t = 0°$ and the solid angle the radiometer subtends with respect to the target follows from Eq. 12.2,

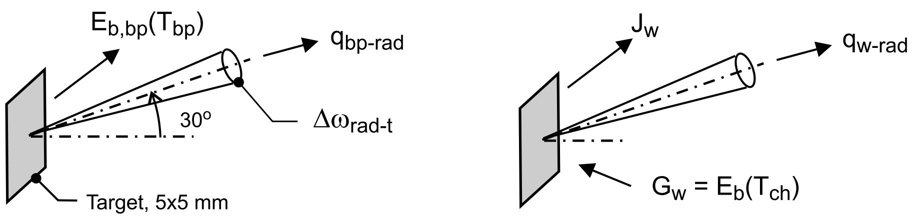
$$\Delta\omega_{rad-t} = \frac{dA_n}{r^2} = \frac{\left(\pi D_o^2/4\right)}{r^2} = \frac{\pi(0.025\text{ m})^2/4}{(0.500\text{ m})^2} = 1.964\times10^{-3}\text{ sr}$$

With $E_{b,bp} = \sigma T_{bp}^4$, find

$$q_{bp-rad} = \left[5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4(800\text{ K})^4/\pi\text{ sr}\right]$$

$$\times(0.005\text{ m})^2\times\cos 30°\times1.964\times10^{-3}\text{ sr}$$

$$q_{bp-rad} = 314\ \mu\text{W}$$ <



(a) Black panel, T_{bp} = 800 K **(b) Wafer, ε_w = 0.7, T_w = ?**

Continued …..

(b) With the wafer mounted, the ion beam source is adjusted until the radiometer receives the same radiant power as with part (a) for the black panel. The power reaching the radiometer is expressed in terms of the wafer radiosity,

$$q_{w-rad} = \left[J_w / \pi\right] A_t \cos\theta_t \cdot \Delta\omega_{rad-t} \qquad (2)$$

Since $q_{w-rad} = q_{bp-rad}$ (see Eq. (1)), recognize that

$$J_w = E_{b,bp}\left(T_{bp}\right) \qquad (3)$$

where the radiosity is

$$J_w = \varepsilon_w E_{b,w}(T_w) + \rho_w G_w = \varepsilon_w E_{b,w}(T_w) + (1-\varepsilon_w) E_b(T_{ch}) \qquad (4)$$

and G_w is equal to the blackbody emissive power at T_{ch}. Using Eqs. (3) and (4) and substituting numerical values, find

$$\sigma T_{bp}^4 = \varepsilon_w \sigma T_w^4 + (1-\varepsilon_w)\sigma T_{ch}^4$$

$$(800\ \text{K})^4 = 0.7\ T_w^4 + 0.3(400\ \text{K})^4$$

$$T_w = 871\ \text{K}$$ <

COMMENTS: (1) Explain why T_w is higher than 800 K, the temperature of the black panel, when the radiometer receives the same radiant power for both situations.

(2) If the chamber walls were cold relative to the wafer, say near liquid nitrogen temperature, T_{ch} = 80 K, and the test repeated with the same indicated radiometer power, is the wafer temperature higher or lower than 871 K?

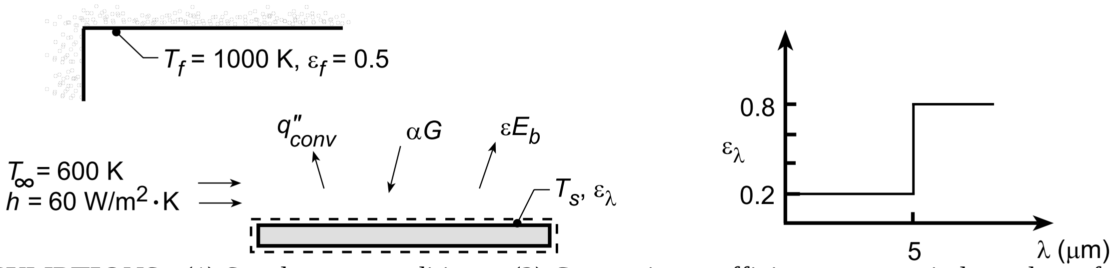
(3) If the chamber walls were maintained at 800 K, and the test repeated with the same indicated radiometer power, what is the wafer temperature?

PROBLEM 12.95

KNOWN: Spectrally selective workpiece placed in an oven with walls at T_f = 1000 K experiencing convection with air at T_∞ = 600 K.

FIND: (a) Steady-state temperature, T_s, by performing an energy balance on the workpiece; show control surface identifying all relevant processes; (b) Compute and plot T_s as a function of the convection coefficient h, for the range $10 \le h \le 120\ \mathrm{W/m^2 \cdot K}$; on the same plot, show the behavior for diffuse surfaces of emissivity 0.2 and 0.8.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Convection coefficient constant, independent of temperature, (3) Workpiece is diffusely irradiated by oven wall which is large isothermal surroundings, (4) Spectral emissivity independent of workpiece temperature.

ANALYSIS: (a) Performing an energy balance on the workpiece,

$$\dot{E}_{in} - \dot{E}_{out} = \alpha G - \varepsilon E_b - q''_{conv} = 0 \qquad \text{and} \qquad G = E_b(T_f), \tag{1}$$

$$\alpha E_b(T_f) - \varepsilon E_b(T_s) - h(T_s - T_\infty) = 0 \tag{2}$$

where the total absorptivity is, with $\alpha_\lambda = \varepsilon_\lambda$, <

$$\alpha = F_{(0\to 5\mu m, 1000\ K)} \cdot \varepsilon_1 + \left(1 - F_{(0\to 5\mu m, 1000\ K)}\right)\varepsilon_2 = 0.6337 \times 0.2 + (1 - 0.6337) \times 0.8 = 0.419$$

using Table 12.1 with $\lambda T = 5 \times 1000 = 5000\ \mu m \cdot K$ for which $F_{(0-\lambda T)} = 0.6337$. The total emissivity is,

$$\varepsilon = F_{(0\to 5\mu m \cdot T_s)}\varepsilon_1 + \left(1 - F_{(0\to 5\times T_s)}\right)\cdot \varepsilon_2 \tag{3}$$

which requires knowing T_s. Hence, an iterative solution is required, beginning by assuming a value of T_s to find ε using Eq. (3), and then using that value in Eq. (2) to find T_s. The result is,

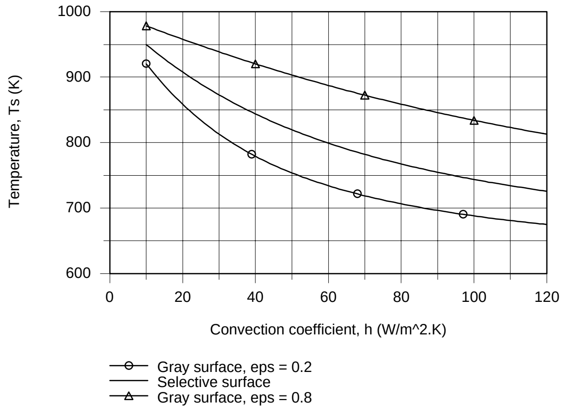
T_s = 800 K <

for which ε = 0.512.

(b) Using the IHT workspace with the foregoing equations and the *Radiation Exchange Tool*, *Band Emission Factor*, a model was developed to calculate T_s as a function of the convection coefficient. Additionally, T_s was plotted for the cases when the workpiece is diffuse, gray with ε = 0.2 and 0.8

Continued...

PROBLEM 12.95 (Cont.)



For the workpiece with the selective surface (ε_λ as shown in the schematic), the temperature decreases with increasing convection coefficient. For the gray surface, ε = 0.2 or 0.8, the temperature is lower and higher, respectively, than that of the workpiece. Recall from part (c), at $h = 60\ \mathrm{W/m^2{\cdot}K}$, ε = 0.512 and α = 0.419, so that it is understandable why the curve for the workpiece is between that for the two gray surfaces.

COMMENTS: (1) For the conditions in part (b), make a sketch of the workpiece emissivity and the absorptivity as a function of its temperature.

(2) The IHT workspace model used to generate the plot is shown below. Note how we used this model to also calculate T_s vs h for the gray surfaces by adjusting λ_1.

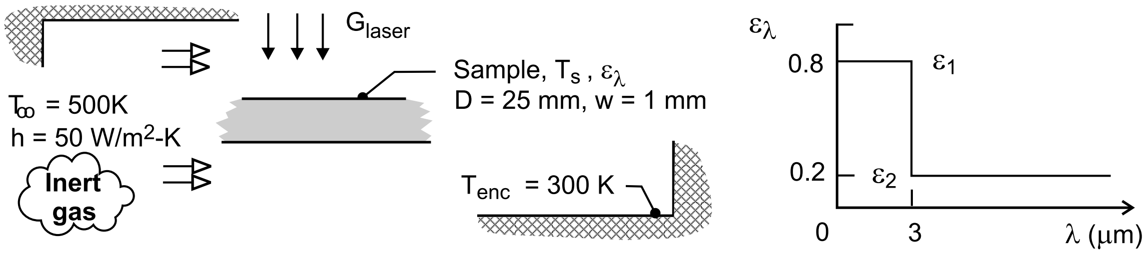
```
// Energy Balance:
alpha * Gf  - eps * Ebs  -  h * (Ts - Tinf) = 0
Gf = Ebf                          // Irradiation from furnace, W/m^2
Ebf = sigma * Tf^4                // Blackbody emissive power, W/m^2; furnace wall
Ebs = sigma * Ts^4                // Blackbody emissive power, W/m^2; workpiece
sigma = 5.67e-8                   // Stefan-Boltzmann constant, W/m^2.K^4

// Radiation Tool - Band Emission Factor, Total emissivity and absorptivity
eps = FL1Ts * eps1 + (1 - FL1Ts) * eps2
/* The blackbody band emission factor, Figure 12.14 and Table 12.1, is */
FL1Ts = F_lambda_T(lambda1,Ts)          // Eq 12.30
// where units are lambda (micrometers, mum) and T (K)
alpha = FL1Tf * eps1 + (1 - FL1Tf) * eps2
FL1Tf = F_lambda_T(lambda1,Tf)          // Eq 12.30

// Assigned Variables:
Ts > 0              // Workpiece temperature, K; assures positive value for Ts
Tf = 1000           // Furnace wall temperature, K
Tinf = 600          // Air temperature, K
h = 60              // Convection coefficient, W/m^2.K
eps1 = 0.2          // Spectral emissivity, 0 <= epsL <= 5 micrometers
eps2 = 0.8          // Spectral emissivity, 5 <= epsL <= infinity
//lambda1 = 5       // Wavelength, micrometers; spectrally selective workpiece
//lambda1 = 1e6     // Wavelength; gray surface for which eps = 0.2
lambda1 = 0.5       // Wavelength; gray surface for which eps = 0.8
```

PROBLEM 12.96

KNOWN: Laser-materials-processing apparatus. Spectrally selective sample heated to the operating temperature $T_s = 2000$ K by laser irradiation (0.5 μm), G_{laser}, experiences convection with an inert gas and radiation exchange with the enclosure.

FIND: (a) Total emissivity of the sample, ε ; (b) Total absorptivity of the sample, α, for irradiation from the enclosure; (c) Laser irradiation required to maintain the sample at $T_s = 2000$ K by performing an energy balance on the sample; (d) Sketch of the sample emissivity during the cool-down process when the laser and inert gas flow are deactivated; identify key features including the emissivity for the final condition ($t \rightarrow \infty$); and (e) Time-to-cool the sample from the operating condition at T_s (0) = 2000 K to a safe-to-touch temperature of T_s (t) = 40°C; use the lumped capacitance method and include the effects of convection with inert gas ($T_\infty = 300$ K , h = 50 W/ $m^2 \cdot K$) as well as radiation exchange $T_{enc} = T_\infty$.

SCHEMATIC:



ASSUMPTIONS: (1) Enclosure is isothermal and large compared to the sample, (2) Sample is opaque and diffuse, but spectrally selective, so that $\varepsilon_\lambda = \alpha_\lambda$, (3) Sample is isothermal.

PROPERTIES: Sample (Given) $\rho = 3900$ kg/m^3, $c_p = 760$ J/kg , k = 45 W/m·K.

ANALYSIS: (a) The total emissivity of the sample, ε, at $T_s = 2000$ K follows from Eq. 12.38 which can be expressed in terms of the band emission factor, $F_{(0\text{-}\lambda,T)}$ Eq. 12.30,

$$\varepsilon = \varepsilon_1 F_{(0-\lambda_1 T_s)} + \varepsilon_2 \left[1 - F_{(0-\lambda_1 T_s)}\right] \qquad (1)$$

$$\varepsilon = 0.8 \times 0.7378 + 0.2[1 - 0.7378] = 0.643$$ <

where from Table 12.1, with $\lambda_1 T_s = 3\mu m \times 2000$ K = 6000 μm·K, $F_{(0\text{-}\lambda T)} = 0.7378$.

(b) The total absorptivity of the sample, α, for irradiation from the enclosure at $T_{enc} = 300$ K, is

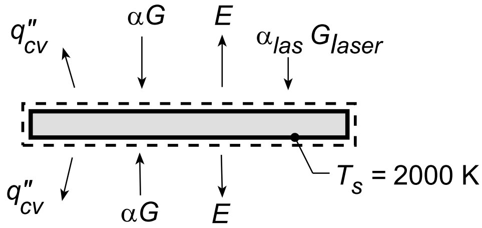
$$\alpha = \varepsilon_1 F_{(0-\lambda_1 T_{enc})} + \varepsilon_2 \left[1 - F_{(0-\lambda_1 T_{enc})}\right] \qquad (2)$$

$$\alpha = 0.8 \times 0 + 0.2[1 - 0] = 0.200$$ <

where, from Table 12.1, with $\lambda_1 T_{enc} = 3$ μm × 300 K = 900 μm·K, $F_{(0\text{-}\lambda T)} = 0$.

Continued...

(c) The energy balance on the sample, on a per unit area basis, as shown in the schematic at the right is



$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$+\alpha_{las} G_{laser} + 2\alpha G - 2\varepsilon E_b\left(T_s\right) - q''_{cv} = 0$$

$$\alpha_{las} G_{laser} + 2\alpha\sigma T_{enc}^4 - 2\varepsilon\tau T_s^4 - 2h\left(T_s - T_\infty\right) = 0 \qquad (3)$$

Recognizing that $\alpha_{las}(0.5\ \mu m) = 0.8$, and substituting numerical values find,

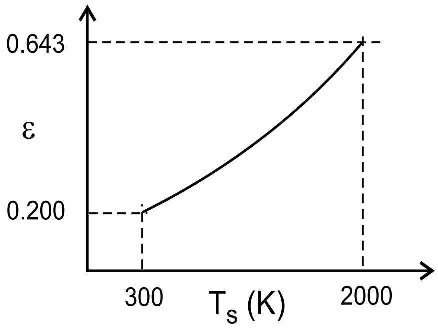
$$0.8 \times G_{laser} + 2 \times 0.200 \times 5.67 \times 10^{-8}\ W/m^2 \cdot K^4 \left(300\ K\right)^4$$

$$-2 \times 0.643 \times 5.67 \times 10^{-8}\ W/m^2 \cdot K^4 \left(2000\ K\right)^4 - 2 \times 50\ W/m^2 \cdot K\left(2000 - 500\right)K = 0$$

$$0.8 \times G_{laser} = \left[-184.6 + 1.167 \times 10^6 + 1.500 \times 10^5\right] W/m^2$$

$$G_{laser} = 1646\ kW/m^2 \qquad <$$

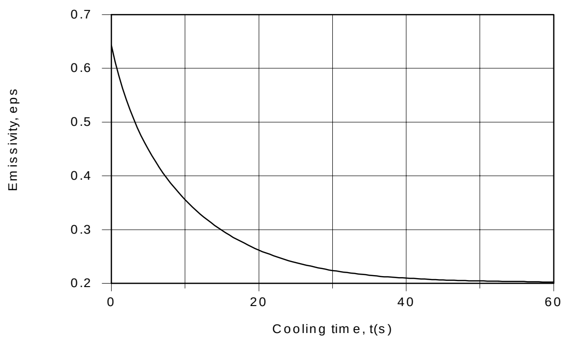
(d) During the cool-down process, the total emissivity ε will decrease as the temperature decreases, T_s (t). In the limit, $t \to \infty$, the sample will reach that of the enclosure, $T_s(\infty) = T_{enc}$ for which $\varepsilon = \alpha = 0.200$.

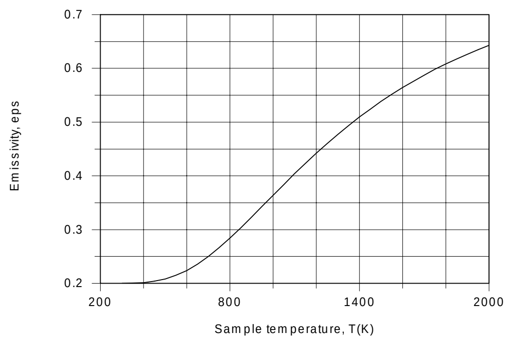


(e) Using the *IHT Lumped Capacitance Model* considering radiation exchange (T_{enc} = 300 K) and convection (T_∞ = 300 K, h = 50 W/m²·K) and evaluating the emissivity using Eq. (1) with the *Radiation Tool*, *Band Emission Factors*, the temperature-time history was determined and the time-to-cool to T(t) = 40°C was found as

$$t = 119\ s \qquad <$$

COMMENTS: (1) From the IHT model used for part (e), the emissivity as a function of cooling time and sample temperature were computed and are plotted below. Compare these results to your sketch of part (c).





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PROBLEM 12.96 (Cont.)

(2) The IHT workspace model to perform the lumped capacitance analysis with variable emissivity is shown below.

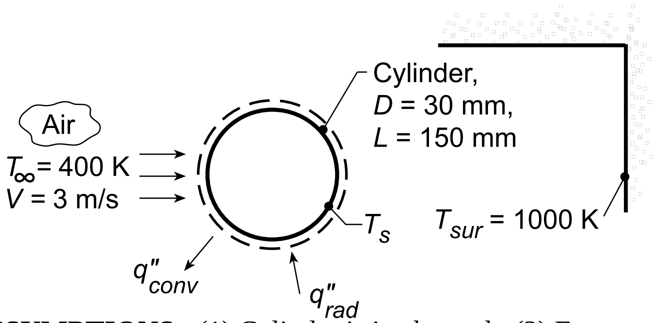
```
// Lumped Capacitance Model - convection and emission/irradiation radiation processes:
/* Conservation of energy requirement on the control volume, CV.  */
Edotin - Edotout = Edotst
Edotin = As * ( + Gabs)
Edotout = As * ( + q''cv + E )
Edotst = rho * vol * cp * Der(T,t)
T_C = T - 273
// Absorbed irradiation from large surroundings on CS
Gabs = alpha * G
G =  sigma * Tsur^4
sigma = 5.67e-8                    // Stefan-Boltzmann constant, W/m^2·K^4
// Emissive power of CS
E = eps * Eb
Eb = sigma * T^4
//sigma = 5.67e-8                  // Stefan-Boltzmann constant, W/m^2·K^4
//Convection heat flux for control surface CS
q''cv = h * ( T - Tinf )
/* The independent variables for this system and their assigned numerical values are */
As =     2 * 1          // surface area, m^2; unit area, top and bottom surfaces
vol =  1 * w             // vol, m^3
w = 0.001                // sample thickness, m
rho =   3900            // density, kg/m^3
cp =   760               // specific heat, J/kg·K
// Convection heat flux, CS
h =    50                // convection coefficient, W/m^2·K
Tinf =  300              // fluid temperature, K
// Emission, CS
//eps =   0.5            // emissivity; value used to test the model initially
// Irradiation from large surroundings, CS
alpha = 0.200            // absorptivity; from Part (b); remains constant during cool-down
Tsur =   300             // surroundings temperature, K

// Radiation Tool - Band emission factor:
eps = eps1 * FL1T  + eps2 * ( 1 - FL1T )
/* The blackbody band emission factor, Figure 12.14 and Table 12.1, is */
FL1T = F_lambda_T(lambda1,T)    // Eq 12.30
// where units are lambda (micrometers, mum) and T (K)
lambda1 = 3              // wavelength, mum
eps1 = 0.8               // spectral emissivity; for lambda < lambda1
eps2 = 0.2               // spectral emissivity; for lambda > lambda1
```

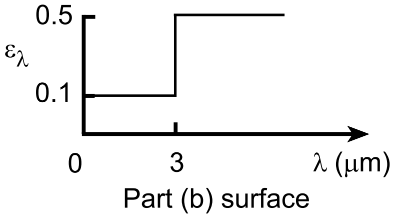
PROBLEM 12.97

KNOWN: Cross flow of air over a cylinder placed within a large furnace.

FIND: (a) Steady-state temperature of the cylinder when it is diffuse and gray with $\varepsilon = 0.5$, (b) Steady-state temperature when surface has spectral properties shown below, (c) Steady-state temperature of the diffuse, gray cylinder if air flow is parallel to the cylindrical axis, (d) Effect of air velocity on cylinder temperature for conditions of part (a).

SCHEMATIC:





ASSUMPTIONS: (1) Cylinder is isothermal, (2) Furnace walls are isothermal and very large in area compared to the cylinder, (3) Steady-state conditions.

PROPERTIES: *Table A.4*, Air ($T_f \approx 600$ K): $\nu = 52.69\times10^{-6}\ \mathrm{m^2/s}$, $k = 46.9 \times 10^{-3}$ W/m·K, Pr = 0.685.

ANALYSIS: (a) When the cylinder surface is gray and diffuse with $\varepsilon = 0.5$, the energy balance is of the form, $q''_{rad} - q''_{conv} = 0$. Hence,

$$\varepsilon\sigma(T_{sur}^4 - T_s^4) - \bar{h}(T_s - T_\infty) = 0 .$$

The heat transfer coefficient, $\bar{h}$, can be estimated from the Churchill-Bernstein correlation,

$$\overline{Nu}_D = (\bar{h}D/k) = 0.3 + \frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re_D}{282,000}\right)^{5/8}\right]^{4/5}$$

where $Re_D = VD/\nu = 3\,\mathrm{m/s}\times30\times10^{-3}\,\mathrm{m}/52.69\times10^{-6}\,\mathrm{m^2/s} = 1710$. Hence,

$$\overline{Nu}_D = 20.8$$

$$\bar{h} = 20.8\times46.9\times10^{-3}\,\mathrm{W/m\cdot K}/30\times10^{-3}\,\mathrm{m} = 32.5\,\mathrm{W/m^2\cdot K} .$$

Using this value of $\bar{h}$ in the energy balance expression, we obtain

$$0.5\times5.67\times10^{-8}(1000^4 - T_s^4)\,\mathrm{W/m^2} - 32.5\,\mathrm{W/m^2\cdot K}(T_s - 400)\,\mathrm{K} = 0$$

which yields $T_s \approx 839$ K. <

(b) When the cylinder has the spectrally selective behavior, the energy balance is written as

$$\alpha G - \varepsilon E_b(T_s) - q''_{conv} = 0$$

where $G = E_b(T_{sur})$. With $\alpha = \int_0^\infty \alpha_\lambda G_\lambda\, d\lambda / G$,

$$\alpha = 0.1\times F_{(0\to3)} + 0.5\times\left(1 - F_{(0\to3)}\right) = 0.1\times0.273 + 0.5(1-0.273) = 0.391$$

where, using Table 12.1 with $\lambda T = 3\times1000 = 3000$ μm·K, $F_{(0\to3)} = 0.273$. Assuming T_s is such that emission in the spectral region $\lambda < 3$ μm is negligible, the energy balance becomes

Continued...

$$0.391\times5.67\times10^{-8}\times1000^4\,\mathrm{W/m^2} - 0.5\times5.67\times10^{-8}\times T_s^4\,\mathrm{W/m^2} - 32.5\,\mathrm{W/m^2\cdot K}(T_s - 400)\,\mathrm{K} = 0$$

which yields $T_s \approx 770$ K. <

Note that, for $\lambda T = 3 \times 770 = 2310\ \mu\mathrm{m\cdot K}$, $F_{(0\rightarrow\lambda)} \approx 0.11$; hence the assumption of $\varepsilon = 0.5$ is acceptable. Note also that the value of $\bar{h}$ based upon $T_f = 600$ K is also acceptable.

(c) When the cylinder is diffuse-gray with air flow in the longitudinal direction, the characteristic length for convection is different. Assume conditions can be modeled as flow over a flat plate of L = 150 mm. With

$$Re_L = VL/\nu = 3\,\mathrm{m/s}\times150\times10^{-3}\,\mathrm{m}/52.69\times10^{-6}\,\mathrm{m^2/s} = 8540$$

$$\overline{Nu}_L = (\bar{h}L/k) = 0.664\,Re_L^{1/2}\,Pr^{1/3} = 0.664(8540)^{1/2}0.685^{1/3} = 54.1$$

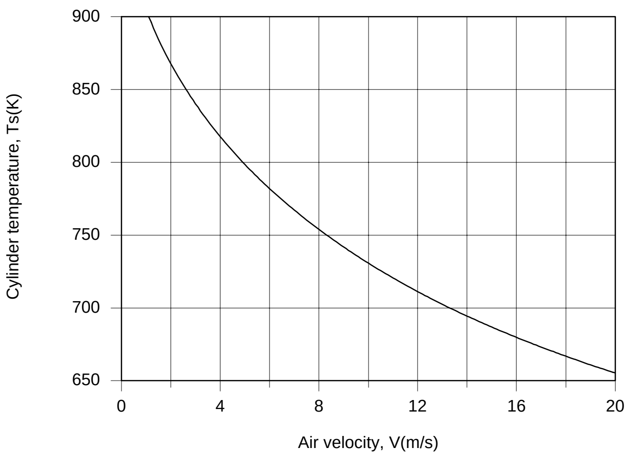
$$\bar{h} = 54.1\times0.0469\,\mathrm{W/m\cdot K}/0.150\,\mathrm{m} = 16.9\,\mathrm{W/m^2\cdot K}\,.$$

The energy balance now becomes

$$0.5\times5.667\times10^{-8}\,\mathrm{W/m^2\cdot K^4}(1000^4 - T_s^4)\mathrm{K^4} - 16.9\,\mathrm{W/m^2\cdot K}(T_s - 400)\mathrm{K} = 0$$

which yields $T_s \approx 850$ K. <

(b) Using the IHT *First Law* Model with the *Correlations* and *Properties* Toolpads, the effect of velocity may be determined and the results are as follows:



Since the convection coefficient increases with increasing V (from 18.5 to 90.6 $\mathrm{W/m^2\cdot K}$ for $1 \le V \le 20$ m/s), the cylinder temperature decreases, since a smaller value of $(T_s - T_\infty)$ is needed to dissipate the absorbed irradiation by convection.

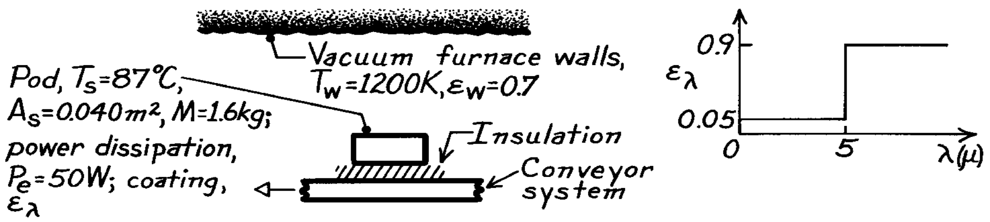
COMMENTS: The cylinder temperature exceeds the air temperature due to absorption of the incident radiation. The cylinder temperature would approach T_∞ as $\bar{h} \rightarrow \infty$ and/or $\alpha \rightarrow 0$. If $\alpha \rightarrow 0$ and $\bar{h}$ has a small to moderate value, would T_s be larger than, equal to, or less than T_∞? Why?

PROBLEM 12.98

KNOWN: Instrumentation pod, initially at 87°C, on a conveyor system passes through a large vacuum brazing furnace. Inner surface of pod surrounded by a mass of phase-change material (PCM). Outer surface with special diffuse, opaque coating of ε_λ. Electronics in pod dissipate 50 W.

FIND: How long before all the PCM changes to the liquid state?

SCHEMATIC:



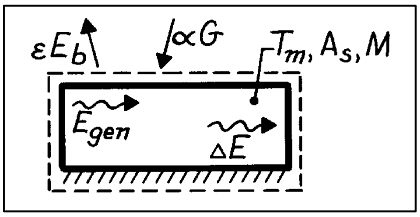
ASSUMPTIONS: (1) Surface area of furnace walls much larger than that of pod, (2) No convection, (3) No heat transfer to pod from conveyor, (4) Pod coating is diffuse, opaque, (5) Initially pod internal temperature is uniform at T_{pcm} = 87°C and remains so during time interval Δt_m, (6) Surface area provided is that exposed to walls.

PROPERTIES: Phase-change material, PCM (given): Fusion temperature, T_f = 87°C, h_{fg} = 25 kJ/kg.

ANALYSIS: Perform an energy balance on the pod for an interval of time Δt_m which corresponds to the time for which the PCM changes from solid to liquid state,

$$E_{in} - E_{out} + E_{gen} = \Delta E$$

$$\left[\left(\alpha G - \varepsilon E_b\right)A_s + P_e\right]\Delta t_m = M h_{fg}$$



where P_e is the electrical power dissipation rate, M is the mass of PCM, and h_{fg} is the heat of fusion of PCM.

Irradiation: $G = \sigma T_w = 5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4 (1200\ \text{K})^4 = 117{,}573\ \text{W/m}^2$

Emissive power: $E_b = \sigma T_m^4 = \sigma\left(87 + 273\right)^4 = 952\ \text{W/m}^2$

Emissivity: $\varepsilon = \varepsilon_1 F_{(0-\lambda T)} + \varepsilon_2(1 - F_{(0-\lambda T)})$ $\lambda T = 5 \times 360 = 1800\ \mu\text{m}\cdot\text{K}$

$\varepsilon = 0.05 \times 0.0393 + 0.9\ (1 - 0.0393)$ $F_{0-\lambda T} = 0.0393$ (Table 12.1)

$\varepsilon = 0.867$

Absorptivity: $\alpha = \alpha_1 F_{(0-\lambda T)} + \alpha_2(1 - F_{(0-\lambda T)})$ $\lambda T = 5 \times 1200 = 6000\ \mu\text{m}\cdot\text{K}$

$\alpha = 0.005 \times 0.7378 + 0.9\ (1 - 0.7378)$ $F_{0-\lambda T} = 0.7378$ (Table 12.1)

$\alpha = 0.273$

Substituting numerical values into the energy balance, find,

$$\left[\left(0.273 \times 117{,}573 - 0.867 \times 952\right)\text{W/m}^2 \times 0.040\ \text{m}^2 + 50\ \text{W}\right]\Delta t_m = 1.6\text{kg} \times 25 \times 10^3\ \text{J/kg}$$

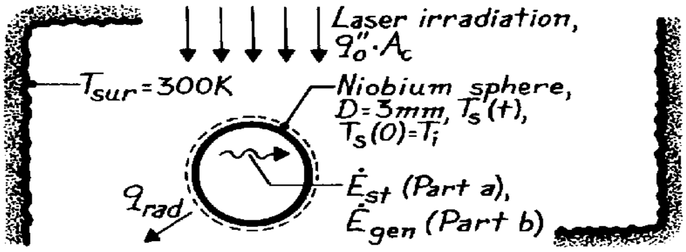
$$\Delta t_m = 30.7\ \text{s} = 0.51\,\text{min}. \qquad <$$

PROBLEM 12.99

KNOWN: Niobium sphere, levitated in surroundings at 300 K and initially at 300 K, is suddenly irradiated with a laser (10 W/m^2) and heated to its melting temperature.

FIND: (a) Time required to reach the melting temperature, (b) Power required from the RF heater causing uniform volumetric generation to maintain the sphere at the melting temperature, and (c) Whether the spacewise isothermal sphere assumption is realistic for these conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Niobium sphere is spacewise isothermal and diffuse-gray, (2) Initially sphere is at uniform temperature T_i, (3) Constant properties, (4) Sphere is small compared to the uniform temperature surroundings.

PROPERTIES: *Table A-1*, Niobium ($\overline{T}$ = (300 + 2741)K/2 = 1520 K): T_{mp} = 2741 K, ρ = 8570 kg/m^3, c_p = 324 J/kg·K, k = 72.1 W/m·K.

ANALYSIS: (a) Following the methodology of Section 5.3 for general lumped capacitance analysis, the time required to reach the melting point T_{mp} may be determined from an energy balance on the sphere,

$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st} \qquad\qquad q''_o \cdot A_c - \varepsilon\sigma A_s\left(T^4 - T_{sur}^4\right) = Mc_p\left(dT/dt\right)$$

where $A_c = \pi D^2/4$, $A_s = \pi D^2$, and $M = \rho V = \rho(\pi D^3/6)$. Hence,

$$q''_o\left(\pi D^2/4\right) - \varepsilon\sigma\left(\pi D^2\right)\left(T^4 - T_{sur}^4\right) = \rho\left(\pi D^3/6\right)c_p\left(dT/dt\right).$$

Regrouping, setting the limits of integration, and integrating, find

$$\left[\frac{q''_o}{4\varepsilon\sigma} + T_{sur}^4\right] - T^4 = \frac{\rho D c}{6\varepsilon\sigma}\frac{dT}{dt} \qquad\qquad b\int_0^t dt = \int_{T_i}^{T_{mp}} \frac{dT}{\left(a^4 - T^4\right)}$$

$$\text{where} \quad a^4 = \frac{q''_o}{4\varepsilon\sigma} + T_{sur}^4 = \frac{10\ \text{W/mm}^2\left(10^3\ \text{mm/m}\right)^2}{4\times 0.6\times 5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}} + \left(300\ \text{K}\right)^4 \qquad a = 2928\text{K}$$

$$b = \frac{6\varepsilon\sigma}{\rho D c_p} = \frac{6\times 0.6\times 5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4}{8570\ \text{kg/m}^3\times 0.003\ \text{m}\times 324\ \text{J/kg}\cdot\text{K}} = 2.4504\times 10^{-11}\ \text{K}^{-1}\cdot\text{s}^{-1}$$

which from Eq. 5.18, has the solution

$$t = \frac{1}{4ba^3}\left\{\ln\left|\frac{a + T_{mp}}{a - T_{mp}}\right| - \ln\left|\frac{a + T_i}{a - T_i}\right| + 2\left[\tan^{-1}\left(\frac{T_{mp}}{a}\right) - \tan^{-1}\left(\frac{T_i}{a}\right)\right]\right\}$$

Continued …..

$$t = \frac{1}{4\left(2.4504\times10^{-11}\ \mathrm{K}^{-1}\cdot\mathrm{s}^{-1}\right)(2928\ \mathrm{K})^3}\left\{\ln\left|\frac{2928+2741}{2928-2741}\right| - \ln\left|\frac{2928+300}{2928-300}\right|\right.$$

$$\left. +2\left[\tan^{-1}\left(\frac{2741}{2928}\right) - \tan^{-1}\left(\frac{300}{2928}\right)\right]\right\}$$

$$t = 0.40604\left(3.4117 - 0.2056 + 2[0.7524 - 0.1021]\right) = 1.83\mathrm{s}.$$ <

(b) The power required of the RF heater to induce a uniform volumetric generation to sustain steady-state operation at the melting point follows from an energy balance on the sphere,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = 0 \qquad -\varepsilon\sigma A_s\left(T_{mp}^4 - T_{sur}^4\right) = -\dot{E}_{gen}$$

$$\dot{E}_{gen} = \dot{q}V = 0.6\times5.67\times10^{-8}\mathrm{W/m^2\cdot K^4}\left(\pi 0.003^2\right)\mathrm{m}^2\left(2741^4 - 300^4\right)\mathrm{K}^4 = 54.3\mathrm{W}.$$ <

(c) The lumped capacitance method is appropriate when

$$Bi = \frac{h_r L_c}{k} = \frac{h_r(D/6)}{k} < 0.1$$

where h_r is the linearized radiation coefficient, which has the largest value when $T = T_{mp} = 2741$ K,

$$h_r = \varepsilon\sigma\left(T + T_{sur}\right)\left(T^2 + T_{sur}^2\right)$$

$$h_r = 0.6\times5.67\times10^{-8}\mathrm{W/m^2\cdot K^4}(2741+300)\left(2741^2+300^2\right)\mathrm{K}^3 = 787\ \mathrm{W/m^2\cdot K}.$$

Hence, since

$$Bi = 787\ \mathrm{W/m^2\cdot K}\left(0.003\mathrm{m}/3\right)/72.1\mathrm{W/m\cdot K} = 1.09\times10^{-2}$$

we conclude that the transient analysis using the lumped capacitance method is satisfactory. <

COMMENTS: (1) Note that at steady-state conditions with internal generation, the difference in temperature between the center and surface, is

$$T_o = T_s = \frac{\dot{q}(D/2)^2}{6k}$$

and with $V = \pi D^3/6$, from the part (b) results,

$$\dot{q} = \dot{E}_{gen}/V = 54.3\ \mathrm{W}/\left(\pi\times0.003^3/6\right)\mathrm{m}^3 = 3.841\times10^9\ \mathrm{W/m^3}.$$

Find using an approximate value for the thermal conductivity in the liquid state,

$$\Delta T = T_o - T_s = \frac{3.841\times10^9\ \mathrm{W/m^3}\left(0.03\mathrm{m}/2\right)^2}{6\times80\ \mathrm{W/m\cdot K}} = 18\mathrm{K}.$$

We conclude that the sphere is very nearly isothermal even under these conditions.

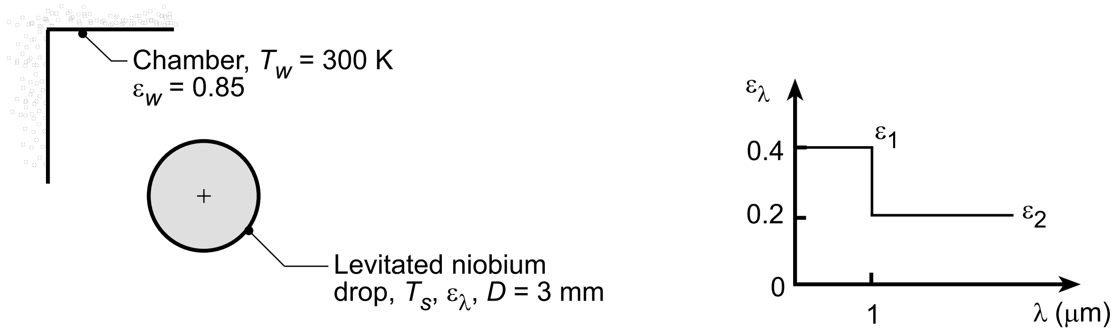
(2) The relation for ΔT in the previous comment follows from solving the heat diffusion equation written for the one-dimensional (spherical) radial coordinate system. See the deviation in Section 3.4.2 for the cylindrical case (Eq. 3.53).

PROBLEM 12.100

KNOWN: Spherical niobium droplet levitated in a vacuum chamber with cool walls. Niobium surface is diffuse with prescribed spectral emissivity distribution. Melting temperature, T_{mp} = 2741 K.

FIND: Requirements for maintaining the drop at its melting temperature by two methods of heating: (a) Uniform internal generation rate, $\dot{q}$ (W/m^3), using a radio frequency (RF) field, and (b) Irradiation, G_{laser}, (W/mm^2), using a laser beam operating at 0.632 μm; and (c) Time for the drop to cool to 400 K if the heating method were terminated.

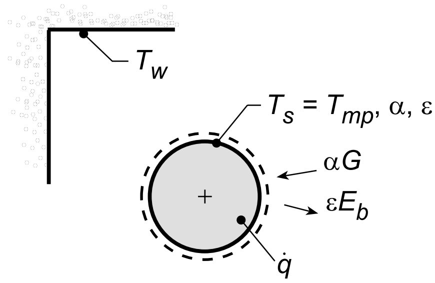
SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions during the heating processes, (2) Chamber is isothermal and large relative to the drop, (3) Niobium surface is diffuse but spectral selective, (4) $\dot{q}$ is uniform, (5) Laser bean diameter is larger than the droplet, (6) Drop is spacewise isothermal during the cool down.

PROPERTIES: *Table A.1*, Niobium ($\overline{T}$ = (2741 + 400)K/2 ≈ 1600 K): ρ = 8570 kg/m^3, c_p = 336 J/kg·K, k = 75.6 W/m·K.

ANALYSIS: (a) For the RF field-method of heating, perform an energy balance on the drop considering volumetric generation, irradiation and emission,



$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0$$

$$\left[\alpha G - \varepsilon E_b\left(T_s\right)\right]A_s + \dot{q}\forall = 0 \qquad (1)$$

where $A_s = \pi D^2$ and $V = \pi D^3/6$. The irradiation and blackbody emissive power are,

$$G = \sigma T_w^4 \qquad E_b = \sigma T_s^4 \qquad (2,3)$$

The absorptivity and emissivity are evaluated using Eqs. 12.46 and 12.48, respectively, with the band emission fractions, Eq. 12.30, and

$$\alpha = \alpha\left(\alpha_\lambda, T_w\right) = \varepsilon_1 F\left(0-\lambda_1 T_w\right) + \varepsilon_2\left[1 - F\left(0-\lambda_1 T_w\right)\right] \qquad (4)$$

$$\alpha = 0.4 \times 0.000 + 0.2\left(1-0.000\right) = 0.2$$

where, from Table 12.1, with $\lambda_1 T_w = 1\ \mu m \times 300\,K = 300\ \mu m\cdot K$, F(0 - λT) = 0.000.

$$\varepsilon = \varepsilon\left(\varepsilon_\lambda, T_s\right) = \varepsilon_1 F\left(0-\lambda_1 T_s\right) + \varepsilon_2\left[1 - F\left(0-\lambda_1 T_s\right)\right] \qquad (5)$$

$$\varepsilon = 0.4 \times 0.2147 + 0.2\left(1-0.2147\right) = 0.243$$

with $\lambda_1 T_s = 1\ \mu m \times 2741 K = 2741\ \mu m\cdot K$, F(0 - λT) = 0.2147. Substituting numerical values with T_s = T_{mp} = 2741 K and T_w = 300 K, find

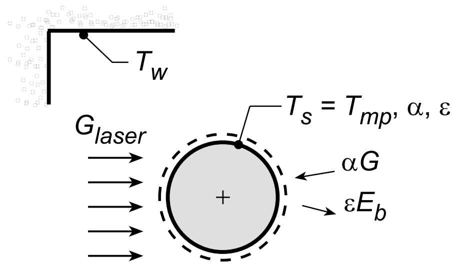
Continued...

$$\left[0.2\times5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\,(300\,\mathrm{K})^4 - 0.243\times5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\,(2741\,\mathrm{K})^4\right]$$

$$\pi\,(0.003\,\mathrm{m})^2 + \dot{q}\pi\,(0.003\,\mathrm{m})^3/6 = 0$$

$$\dot{q} = [-91.85 + 777{,}724]\,\mathrm{W/m^2}\,(6/0.003\,\mathrm{m}) = 1.556\times10^9\,\mathrm{W/m^3}$$ <

(b) For the laser-beam heating method, performing an energy balance on the drop considering absorbed laser irradiation, irradiation from the enclosure walls and emission,



$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$[\alpha G - \varepsilon E_b(T_s)]A_s + \alpha_{las}G_{laser}A_p = 0 \qquad (6)$$

where A_p represents the projected area of the droplet,

$$A_p = \pi D^2/4 \qquad (7)$$

Laser irradiation at 10.6 μm. Recognize that for the laser irradiation, G_{laser} (10.6 μm), the spectral absorptivity is

$$\alpha_{las}(10.6\,\mu\mathrm{m}) = 0.2$$

Substituting numerical values onto the energy balance, Eq. (6), find

$$\left[0.2\times\sigma\times(300\text{ K})^4 - 0.243\times\sigma\times(2741\text{ K})^4\right]\pi\,(0.003\text{ m})^2$$

$$+0.2\times G_{laser}\times\pi\,(0.003\text{ m})^2/4 = 0$$

$$G_{laser}(10.6\,\mu\mathrm{m}) = 1.56\times10^7\text{ W/m}^2 = 15.6\text{ W/mm}^2$$ <

Laser irradiation at 0.632 μm. For laser irradiation at 0.632 μm, the spectral absorptivity is

$$\alpha_{laser}(0.632\,\mu\mathrm{m}) = 0.4$$

Substituting numerical values into the energy balance, find

$$G_{laser}(0.632\,\mu\mathrm{m}) = 7.76\times10^6\,\mathrm{W/m^2} = 7.8\text{ W/mm}^2$$ <

(c) With the method of heating terminated, the drop experiences only radiation exchange and begins cooling. Using the *IHT Lumped Capacitance Model* with irradiation and emission processes and the *Radiation Tool*, *Band Emission Factor* for estimating the emissivity as a function of drop temperature, Eq. (5), the time-to-cool to 400 K from an initial temperature, $T_s(0) = T_{mp}$ = 2741 K was found as

$$T_s(t) = 400\,\mathrm{K} \qquad t = 772\text{ s} = 12.9\text{ min}$$ <

COMMENTS: (1) Why doesn't the emissivity of the chamber wall, ε_w, affect the irradiation onto the drop?

(2) The validity of the lumped capacitance method can be determined by evaluating the Biot number,

Continued …..

PROBLEM 12.100 (Cont.)

$$\mathrm{Bi} = \frac{\overline{h}D/6}{k} = \frac{185\,\mathrm{W/m^2\cdot K} \times 0.003\,\mathrm{m}/6}{75.6\,\mathrm{W/m\cdot K}} = 0.007$$

where we estimated an average radiation coefficient as

$$\overline{h}_{rad} \approx 4\varepsilon\sigma\overline{T}^3 = 4 \times 0.2 \times 5.67 \times 10^{-8}\,\mathrm{W/m^2\cdot K^4}\,(1600\,\mathrm{K})^3 = 185\,\mathrm{W/m^2\cdot K}$$

following the estimation method described in Problem 1.20. Since Bi << 0.1, the lumped capacitance method was appropriate.

(3) In the IHT model of part (c), the emissivity was calculated as a function of $T_s(t)$ varying from 0.243 at$T_s = T_{mp}$ to 0.200 at T_s = 300 K. If we had done an analysis assuming the drop were diffuse, gray with $\alpha = \varepsilon = 0.2$, the time-to-cool would be t = 773 s. How do you explain that this simpler approach predicts a time-to-cool that is in good agreement with the result of part (c)?

(4) A copy of the IHT workspace with the model of part (c) is shown below.

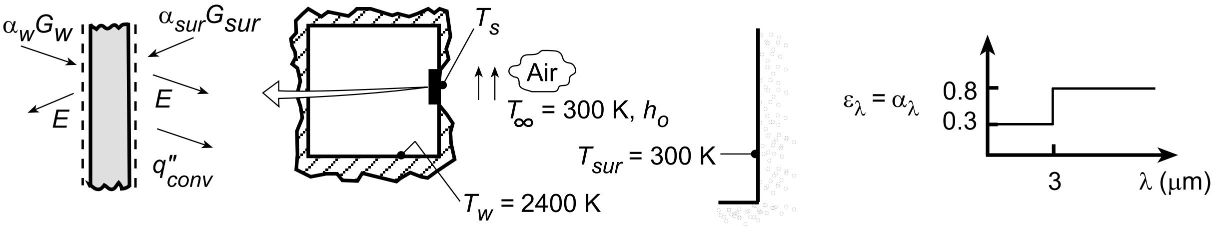
```
// Lumped Capacitance Model: Irradiation and Emission
/* Conservation of energy requirement on the control volume, CV.  */
Edotin - Edotout = Edotst
Edotin = As * ( + Gabs)
Edotout = As * ( + E )
Edotst = rho * vol * cp * Der(T,t)
// Absorbed irradiation from large surroundings on CS
Gabs = alpha * G
G =  sigma * Tsur^4
sigma = 5.67e-8      // Stefan-Boltzmann constant, W/m^2·K^4
// Emissive power of CS
E = eps * Eb
Eb = sigma * T^4
//sigma = 5.67e-8    // Stefan-Boltzmann constant, W/m^2·K^4
/* The independent variables for this system and their assigned numerical values are */
As =    pi * D^2          // surface area, m^2
vol =  pi * D^3 / 6       // vol, m^3
D =   0.003               // sphere diameter, m
rho =  8570               // density, kg/m^3
cp =  336                 // specific heat, J/kg·K
// Emission, CS
//eps =   0.4             // emissivity
// Irradiation from large surroundings, CS
alpha =  0.2              // absorptivity
Tsur =   300              // surroundings temperature, K

// Radiation Tool - Band Emission Fractions
eps = eps1 * FL1T + eps2 * ( 1 - FL1T )
/* The blackbody band emission factor, Figure 12.14 and Table 12.1, is */
FL1T = F_lambda_T(lambda1,T)            // Eq 12.30
// where units are lambda (micrometers, mum) and T (K)
lambda1 = 1                        // wavelength, mum
eps1 = 0.4                         // spectral emissivity, lambda < lambda1
eps2 = 0.2                         // spectral emissivity, lambda > lambda1
```

PROBLEM 12.101

KNOWN: Temperatures of furnace and surroundings separated by ceramic plate. Maximum allowable temperature and spectral absorptivity of plate.

FIND: (a) Minimum value of air-side convection coefficient, h_o, (b) Effect of h_o on plate temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Diffuse surface, (2) Negligible temperature gradients in plate, (3) Negligible inside convection, (4) Furnace and surroundings act as blackbodies.

ANALYSIS: (a) From a surface energy balance on the plate, $\alpha_w G_w + \alpha_{sur} G_{sur} = 2E + q''_{conv}$. Hence,

$\alpha_w \sigma T_w^4 + \alpha_{sur} \sigma T_{sur}^4 = 2\varepsilon\sigma T_s^4 + h_o (T_s - T_\infty)$.

$$h_o = \frac{\alpha_w \sigma T_w^4 + \alpha_{sur} \sigma T_{sur}^4 - 2\varepsilon\sigma T_s^4}{(T_s - T_\infty)}$$

Evaluating the absorptivities and emissivity,

$$\alpha_w = \int_0^\infty \alpha_\lambda G_\lambda \, d\lambda / G = \int_0^\infty \alpha_\lambda E_{\lambda b}(T_w)/E_b(T_w) d\lambda = 0.3F_{(0-3\mu m)} + 0.8\left[1 - F_{(0-3\mu m)}\right]$$

With $\lambda T_w = 3\,\mu m \times 2400\text{ K} = 7200\,\mu m \cdot K$, Table 12.1 $\rightarrow F_{(0-3\mu m)} = 0.819$. Hence,

$$\alpha_w = 0.3 \times 0.819 + 0.8(1 - 0.819) = 0.391$$

Since T_{sur} = 300 K, irradiation from the surroundings is at wavelengths well above 3 μm. Hence,

$$\alpha_{sur} = \int_0^\infty \alpha_\lambda E_{\lambda b}(T_{sur})/E_b(T_{sur}) d\lambda \approx 0.800.$$

The emissivity is $\varepsilon = \int_0^\infty \varepsilon_\lambda E_{\lambda b}(T_s)/E_b(T_s) d\lambda = 0.3F_{(0-3\mu m)} + 0.8\left[1 - F_{(0-3\mu m)}\right]$. With $\lambda T_s = 5400\,\mu m \cdot K$, Table 12.1 $\rightarrow F_{(0-3\mu m)} = 0.680$. Hence, $\varepsilon = 0.3 \times 0.68 + 0.8(1 - 0.68) = 0.460$.

For the maximum allowable value of T_s = 1800 K, it follows that

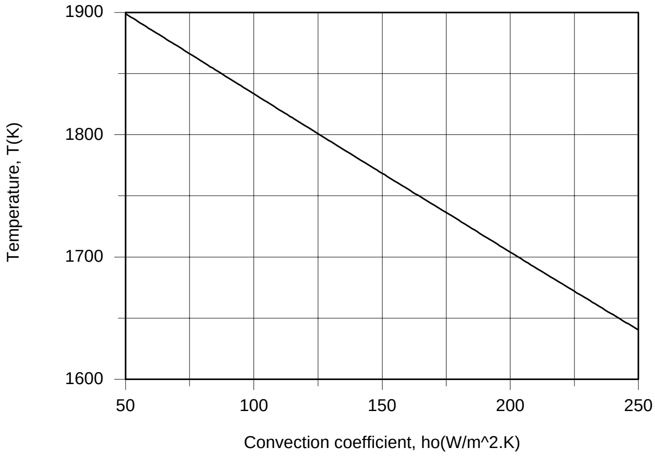
$$h_o = \frac{0.391 \times 5.67 \times 10^{-8}(2400)^4 + 0.8 \times 5.67 \times 10^{-8}(300)^4 - 2 \times 0.46 \times 5.67 \times 10^{-8}(1800)^4}{(1800 - 300)}$$

$$h_o = \frac{7.335 \times 10^5 + 3.674 \times 10^2 - 5.476 \times 10^5}{1500} = 126\text{ W/m}^2 \cdot K. \quad <$$

(b) Using the IHT *First Law* Model with the *Radiation* Toolpad, parametric calculations were performed to determine the effect of h_o.

Continued...

PROBLEM 12.101 (Cont.)



With increasing h_o, and hence enhanced convection heat transfer at the outer surface, the plate temperature is reduced.

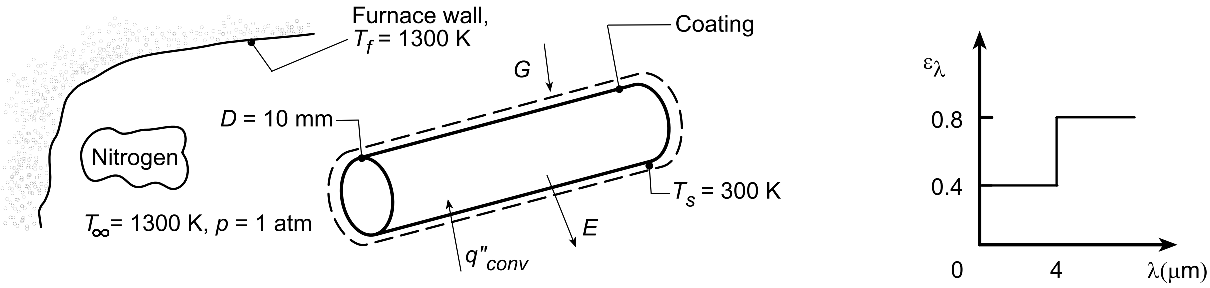
COMMENTS: (1) The surface is not gray. (2) The required value of $h_o \geq 126\,\mathrm{W/m^2 \cdot K}$ is well within the range of air cooling.

PROBLEM 12.102

KNOWN: Spectral radiative properties of thin coating applied to long circular copper rods of prescribed diameter and initial temperature. Wall and atmosphere conditions of furnace in which rods are inserted.

FIND: (a) Emissivity and absorptivity of the coated rods when their temperature is T_s = 300 K, (b) Initial rate of change of their temperature, dT_s/dt, (c) Emissivity and absorptivity when they reach steady-state temperature, and (d) Time required for the rods, initially at T_s = 300 K, to reach 1000 K.

SCHEMATIC:



ASSUMPTIONS: (1) Rod temperature is uniform, (2) Nitrogen is quiescent, (3) Constant properties, (4) Diffuse, opaque surface coating, (5) Furnace walls form a blackbody cavity about the cylinders, $G = E_b(T_f)$, (6) Negligible end effects.

PROPERTIES: *Table A.1*, Copper (300 K): ρ = 8933 kg/m^3, c_p = 385 J/kg·K, k = 401 W/m·K; *Table A.4*, Nitrogen (T_f = 800 K, 1 atm): $\nu = 82.9 \times 10^{-6}$ m^2/s, k = 0.0548 W/m·K, $\alpha = 116 \times 10^{-6}$ m^2/s, Pr = 0.715, $\beta = (T_f)^{-1} = 1.25 \times 10^{-3}$ K^{-1}.

ANALYSIS: (a) The total emissivity of the copper rod, ε, at T_s = 300 K follows from Eq. 12.38 which can be expressed in terms of the band emission factor, $F(0 - \lambda T)$, Eq. 12.30,

$$\varepsilon = \varepsilon_1 F(0-\lambda_1 T_s) + \varepsilon_2 [1 - F(0 - \lambda_1 T_s)] \qquad (1)$$

$$\varepsilon = 0.4 \times 0.0021 + 0.8[1 - 0.0021] = 0.799 \qquad <$$

where, from Table 12.1, with $\lambda_1 T_s$ = 4 μm × 300 K = 1200 μm·K, $F(0 - \lambda T) = 0.0021$. The total absorptivity, α, for irradiation for the furnace walls at T_f = 1300 K, is

$$\alpha = \varepsilon_1 F(0-\lambda_1 T_f) + \varepsilon_2 [1 - F(0 - \lambda_1 T_f)] \qquad (2)$$

$$\alpha = 0.4 \times 0.6590 + 0.8[1 - 0.6590] = 0.536 \qquad <$$

where, from Table 12.1, with $\lambda_1 T_f$ = 4 μm × 1300 K = 5200 K, $F(0 - \lambda T) = 0.6590$.

(b) From an energy balance on a control volume about the rod,

$$\dot{E}_{st} = \rho c_p \left(\pi D^2/4\right) L\,(dT/dt) = \dot{E}_{in} - \dot{E}_{out} = \pi D L\left[\alpha G + \bar{h}(T_\infty - T_s) - E\right]$$

$$dT_s/dt = 4\left[\alpha G + \bar{h}(T_\infty - T_s) - \varepsilon\sigma T_s^4\right]\Big/\rho c_p D\,. \qquad (3)$$

With

$$Ra_D = \frac{g\beta(T_\infty - T_s)D^3}{\nu\alpha} = \frac{9.8\,m^2/s\left(1.25\times10^{-3}\,K^{-1}\right)1000\,K\,(0.01\,m)^3}{82.9\times10^{-6}\,m^2/s \times 116\times10^{-6}\,m^2/s} = 1274 \qquad (4)$$

Eq. 9.34 gives

Continued...

$$\bar{h} = \frac{0.0548}{0.01\,\text{m}}\left\{0.60 + \frac{0.387(1274)^{1/6}}{\left[1+(0.559/0.715)^{9/16}\right]^{8/27}}\right\}^2 = 15.1\,\text{W/m}^2\cdot\text{K} \qquad (5)$$

With values of ε and α from part (a), the rate of temperature change with time is

$$dT_s/dt = \frac{4\left[0.53\times5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\times(1300\,\text{K})^4 + 15.1\,\text{W/m}^2\cdot\text{K}\times1000\,\text{K} - 0.8\times5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}\times(300\,\text{K})^4\right]}{8933\,\text{kg/m}^3\times385\,\text{J/kg}\cdot\text{K}\times0.01\,\text{m}}$$

$$dT_s/dt = 1.16\times10^{-4}\,[85{,}829 + 15{,}100 - 3767]\,\text{K/s} = 11.7\,\text{K/s}. \qquad <$$

(c) Under steady-state conditions, $T_s = T_\infty = T_f = 1300$ K. For this situation, ε = α, hence

$$\varepsilon = \alpha = 0.536 \qquad <$$

(d) The time required for the rods, initially at $T_s(0) = 300$ K, to reach 1000 K can be determined using the lumped capacitance method. Using the *IHT Lumped Capacitance Model*, considering convection, irradiation and emission processes; the *Correlations Tool, Free Convection, Horizontal Cylinder*; *Radiation Tool, Band Emission Fractions*; and a user-generated *Lookup Table Function* for the nitrogen thermophysical properties, find

$$T_s(t_o) = 1000\ \text{K} \qquad t_o = 81.8\ \text{s} \qquad <$$

COMMENTS: (1) To determine the validity of the lumped capacitance method to this heating process, evaluate the approximate Biot number, Bi = $\bar{h}D/k$ = 15 W/m^2·K × 0.010 m/401 W/m·K = 0.0004. Since Bi << 0.1, the method is appropriate.

(2) The IHT workspace with the model used for part (c) is shown below.

```
// Lumped Capacitance Model - irradiation, emission, convection
/* Conservation of energy requirement on the control volume, CV.  */
Edotin - Edotout = Edotst
Edotin = As * ( + Gabs)
Edotout = As * ( + q''cv + E )
Edotst = rho * vol * cp * Der(Ts,t)
//Convection heat flux for control surface CS
q''cv = h * ( Ts - Tinf )
// Emissive power of CS
E = eps * Eb
Eb = sigma * Ts^4
sigma = 5.67e-8                 // Stefan-Boltzmann constant, W/m^2·K^4
// Absorbed irradiation from large surroundings on CS
Gabs = alpha * G
G = sigma * Tf^4
/* The independent variables for this system and their assigned numerical values are */
As =   pi * D * 1               // surface area, m^2
vol =   pi * D^2 / 4 * 1        // vol, m^3
rho =   8933                    // density, kg/m^3
cp =    433                     // specific heat, J/kg·K; evaluated at 800 K
// Convection heat flux, CS
//h =                   // convection coefficient, W/m^2·K
Tinf =   1300           // fluid temperature, K
// Emission, CS
//eps =                 // emissivity
// Irradiation from large surroundings, CS
//alpha =               // absorptivity
Tf =   1300             // surroundings temperature, K
```

Continued...

PROBLEM 12.102 (Cont.)

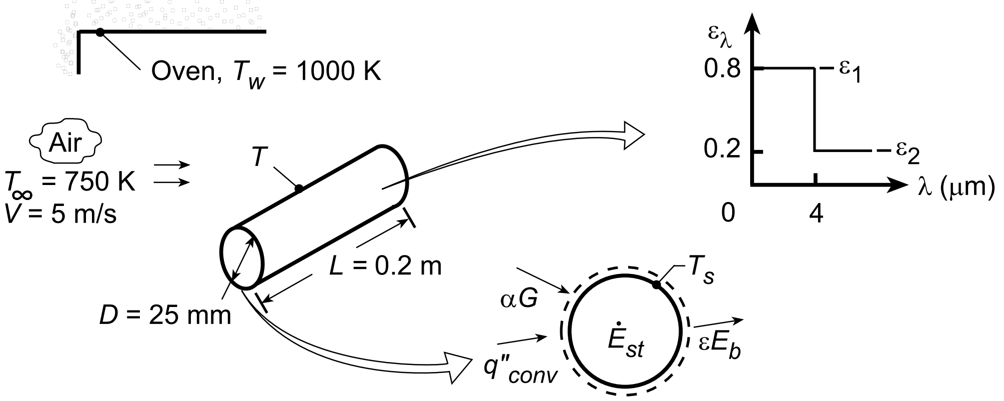
```
// Radiative Properties Tool - Band Emission Fraction
eps = eps1 * FL1Ts + eps2 * (1 - FL1Ts)
/* The blackbody band emission factor, Figure 12.14 and Table 12.1, is */
FL1Ts = F_lambda_T(lambda1,Ts)                    // Eq 12.30
// where units are lambda (micrometers, mum) and T (K)
alpha = eps1 * FL1Tf  + eps2 * (1- FL1Tf)
/* The blackbody band emission factor, Figure 12.14 and Table 12.1, is */
FL1Tf = F_lambda_T(lambda1,Tf)                    // Eq 12.30

// Assigned Variables:
D = 0.010          // Cylinder diameter, m
eps1 = 0.4         // Spectral emissivity for lambda < lambda1
eps2 = 0.8         // Spectral emissivity for lambda > lambda1
lambda1 = 4        // Wavelength, mum

// Correlations Tool - Free Convection, Cylinder, Horizontal:
NuDbar = NuD_bar_FC_HC(RaD,Pr)                    // Eq 9.34
NuDbar = h * D / k
RaD = g * beta * deltaT * D^3 / (nu * alphan)     //Eq 9.25
deltaT = abs(Ts - Tinf)
g = 9.8     // gravitational constant, m/s^2
// Evaluate properties at the film temperature, Tf.
Tff =Tfluid_avg(Tinf,Ts)

// Properties Tool - Nitrogen: Lookup Table Function "nitrog"
nu = lookupval (nitrog, 1, Tff, 2)
k = lookupval (nitrog, 1, Tff, 3)
alphan =  lookupval (nitrog, 1, Tff, 4)
Pr = lookupval (nitrog, 1, Tff, 5)
beta = 1 / Tff
/* Lookup table function, nitrog; from Table A.4 1 atm):
Columns: T(K), nu(m^2/s), k(W/m.K), alpha(m^2/s), Pr
300     1.586E-5        0.0259    2.21E-5   0.716
350     2.078E-5        0.0293    2.92E-5   0.711
400     2.616E-5        0.0327    3.71E-5   0.704
450     3.201E-5        0.0358    4.56E-5   0.703
500     3.824E-5        0.0389    5.47E-5   0.7
550     4.17E-5         0.0417    6.39E-5   0.702
600     5.179E-5        0.0446    7.39E-5   0.701
700     6.671E-5        0.0499    9.44E-5   0.706
800     8.29E-5         0.0548    0.000116 0.715
900     0.0001003       0.0597    0.000139 0.721
1000    0.0001187       0.0647    0.000165 0.721    */
```

PROBLEM 12.103

KNOWN: Large combination convection-radiation oven heating a cylindrical product of a prescribed spectral emissivity.

FIND: (a) Initial heat transfer rate to the product when first placed in oven at 300 K, (b) Steady-state temperature of the product, (c) Time to achieve a temperature within 50°C of the steady-state temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Cylinder is opaque-diffuse, (2) Oven walls are very large compared to the product, (3) Cylinder end effects are negligible, (4) ε_λ is dependent of temperature.

PROPERTIES: *Table A-4*, Air (T_f = 525 K, 1 atm): $\nu = 42.2\times10^{-6}\ \text{m}^2/\text{s}, k = 0.0423\ \text{W/m}\cdot\text{K}$, Pr = 0.684; ($T_f$ = 850 K (assumed), 1 atm): $\nu = 93.8\times10^{-6}\ \text{m}^2/\text{s}$, k = 0.0596 W/m·K, Pr = 0.716.

ANALYSIS: (a) The net heat rate to the product is $q_{net} = A_s(q''_{conv} + \alpha G - \varepsilon E_b)$, or

$$q_{net} = \pi DL[\bar{h}(T_\infty - T) + \alpha G - \varepsilon\sigma T^4] \qquad (1)$$

Evaluating properties at T_f = 525 K, $Re_D = VD/\nu = 5\ \text{m/s}\times0.025\ \text{m}/42.2\times10^{-6}\ \text{m}^2/\text{s} = 2960$, and the Churchill-Bernstein correlation yields

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = 0.3 + \frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{[1+(0.4/Pr)^{2/3}]^{1/4}}\left[1+\left(\frac{Re_D}{282{,}000}\right)^{5/8}\right]^{4/5} = 27.5$$

Hence,

$$\bar{h} = \frac{0.0423\ \text{W/m}\cdot\text{K}}{0.025\text{m}}\times27.5 = 46.5\ \text{W/m}^2\cdot\text{K}.$$

The total, hemispherical emissivity of the diffuse, spectrally selective surface follows from Eq. 12.38,

$\varepsilon = \int_0^\infty \varepsilon_\lambda(\lambda, T_s)E_{\lambda,b}\big/\sigma T_s^4 = \varepsilon_1 F_{(0\to4\mu m)} + \varepsilon_2(1 - F_{(0\to4\mu m)})$, where $\lambda T = 4\ \mu\text{m}\times300\ \text{K} = 1200$ μm·K and $F_{(0-\lambda T)} = 0.002$ (Table 12.1). Hence, $\varepsilon = 0.8\times0.002 + 0.2(1-0.002) = 0.201$.

The absorptivity is for irradiation from the oven walls which, because they are large and isothermal, behave as a black surface at 1000 K. From Eq. 12.46, with $G_\lambda = E_{\lambda,b}$ (λ, 1000 K) and $\alpha_\lambda = \varepsilon_\lambda$,

$$\alpha = \varepsilon_1 F_{(0\to4\mu m)} + \varepsilon_2(1 - F_{(0\to4\mu m)}) = 0.8\times0.481 + 0.2(1-0.481) = 0.489$$

where, for $\lambda T = 4\times1000 = 4000$ μm·K from Table 12.1, $F_{(0-\lambda T)} = 0.481$. From Eq. (1) the net initial heat rate is $q_{net} = \pi\times0.025\text{m}\times0.2\ \text{m}[46.5\ \text{W/m}^2\cdot\text{K}(750-300)\ \text{K} + 0.489\sigma(1000)^4\text{K}^4 - 0.201\sigma(300\ \text{K})^4]$

Continued...

PROBLEM 12.103 (Cont.)

$q = 763$ W. <

(b) For the steady-state condition, the net heat rate will be zero, and the energy balance yields,

$$0 = \overline{h}\,(T_\infty - T) + \alpha G - \varepsilon\sigma T^4 \qquad (2)$$

Evaluating properties at an assumed film temperature of $T_f = 850$ K, $Re_D = VD/\nu = 5\text{ m/s} \times 0.025\text{ m}/93.8 \times 10^{-6}\text{ m}^2\text{/s} = 1333$, and the Churchill-Bernstein correlation yields $\overline{Nu}_D = 18.6$. Hence, $\overline{h} = 18.6\ (0.0596\text{ W/m}\cdot\text{K})/0.025\text{ m} = 44.3\text{ W/m}^2\cdot\text{K}$. Since irradiation from the oven walls is fixed, the absorptivity is unchanged, in which case $\alpha = 0.489$. However, the emissivity depends on the product temperature. *Assuming* T = 950 K, we obtain

$$\varepsilon = \varepsilon_1 F_{(0\to 4\mu m)} + \varepsilon_2 (1 - F_{(0\to 4\mu m)}) = 0.8 \times 0.443 + 0.2(1 - 0.443) = 0.466$$

where for $\lambda T = 4 \times 950 = 3800\ \mu\text{m}\cdot\text{K}$, $F_{0-\lambda T} = 0.443$, from Table 12.1. Substituting values into Eq. (2) with $\sigma = 5.67 \times 10^{-8}\text{ W/m}^2\cdot\text{K}^4$,

$$0 = 44.3\,(750 - T) + 0.489\,\sigma\,(1000\text{ K})^4 - 0.466\,\sigma\,T^4.$$

A trial-and-error solution yields $T \approx 930$ K. <

(c) Using the IHT *Lumped Capacitance* Model with the *Correlations*, *Properties* (for copper and air) and *Radiation* Toolpads, the transient response of the cylinder was computed and the time to reach T = 880 K is

$t \approx 537$ s. <

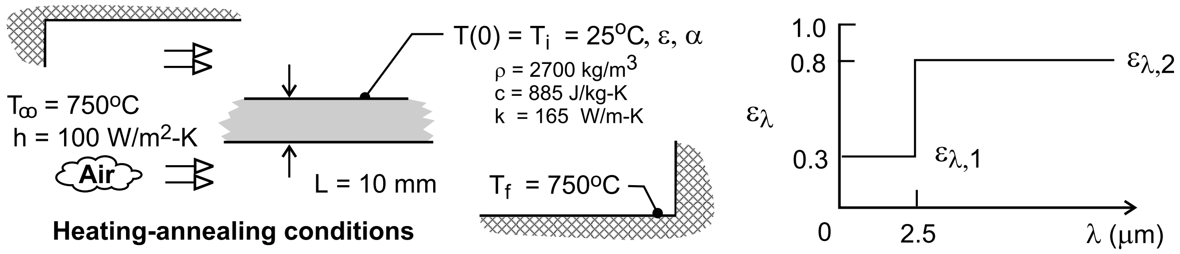
COMMENTS: Note that $\overline{h}$ is relatively insensitive to T, while ε is not. At T = 930 K, $\varepsilon = 0.456$.

PROBLEM 12.104

KNOWN: Workpiece, initially at 25°C, to be annealed at a temperature above 725°C for a period of 5 minutes and then cooled; furnace wall temperature and convection conditions; cooling surroundings and convection conditions.

FIND: (a) Emissivity and absorptivity of the workpiece at 25°C when it is placed in the furnace, (b) Net heat rate per unit area into the workpiece for this initial condition; change in temperature with time, dT/dt, for the workpiece; (c) Calculate and plot the emissivity of the workpiece as a function of temperature for the range 25 to 750°C using the *Radiation | Band Emission* tool in *IHT*, (d) The time required for the workpiece to reach 725°C assuming the applicability of the lumped-capacitance method using the *DER(T,t)* function in *IHT* to represent the temperature-time derivative in your energy balance; (e) Calculate the time for the workpiece to cool from 750°C to a safe-to-touch temperature of 40°C if the cool surroundings and cooling air temperature are 25°C and the convection coefficient is 100 $W/m^2 \cdot K$; and (f) Assuming that the workpiece temperature increases from 725 to 750°C during the five-minute annealing period, sketch (don't plot) the temperature history of the workpiece from the start of heating to the end of cooling; identify key features of the process; determine the total time requirement; and justify the lumped-capacitance method of analysis.

SCHEMATIC:



ASSUMPTIONS: (1) Workpiece is opaque and diffuse, (2) Spectral emissivity is independent of temperature, and (3) Furnace and cooling environment are large isothermal surroundings.

ANALYSIS: (a) Using Eqs. 12.38 and 12.46, ε and α can be determined using band-emission factors, Eq. 12.30 and 12.31.

Emissivity, workpiece at 25°C

$$\varepsilon = \varepsilon_{\lambda 1} \cdot F_{(0-\lambda T)} + \varepsilon_{\lambda 2}\left(1 - F_{(0-\lambda T)}\right)$$

$$\varepsilon = 0.3 \times 1.6 \times 10^{-5} + 0.8 \times \left(1 - 1.6 \times 10^{-5}\right) = 0.8 \qquad <$$

where $F_{(0-\lambda T)}$ is determined from Table 12.1 with $\lambda T = 2.5\ \mu m \times 298\ K = 745\ \mu m \cdot K$.

Absorptivity, furnace temperature T_f = 750°C

$$\alpha = \varepsilon_{\lambda 1} \cdot F_{(0-\lambda,\ T)} + \varepsilon_2 \cdot \left(1 - F_{(0-\lambda,\ T)}\right)$$

$$\alpha = 0.3 \times 0.174 + 0.8 \times (1 - 0.174) = 0.713 \qquad <$$

where $F_{(0\ -\ \lambda T)}$ is determined with $\lambda T = 2.5\ \mu m \times 1023\ K = 2557.5\ \mu m \cdot K$.

(b) For the initial condition, $T(0) = T_i$, the energy balance shown schematically below is written in terms of the net heat rate *in*,

Continued …..

$$\dot{E}''_{in} - \dot{E}''_{out} = \dot{E}''_{st} \qquad \text{and} \qquad q''_{net,in} = \dot{E}''_{in} - \dot{E}''_{out}$$

$$q''_{net,in} = 2\left[q''_{cv} - \varepsilon\, E_b(T_i) + \alpha E_b(T_f)\right]$$

where G = E_b (T_f). Substituting numerical values,

$$q''_{net,in} = 2\left[h(T_\infty - T_i) - \varepsilon\sigma T_i^4 + \alpha\sigma T_f^4\right]$$

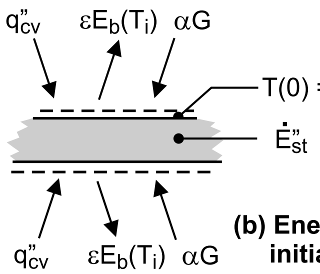
$$q''_{net,in} = 2\left[100\ W/m^2\cdot K\ (750-25)K - 0.8\times 5.67\times 10^{-8}\ W/m^2\cdot K^4 (298\ K)^4 \right.$$
$$\left. + \ 0.713\times 5.67\times 10^{-8}\ W/m^2\cdot K^4\ (1023\ K)^4\right]$$

$$q''_{net,in} = 2\times 116.4\ kW/m^2 = 233\ kW/m^2 \qquad <$$

Considering the energy storage term,

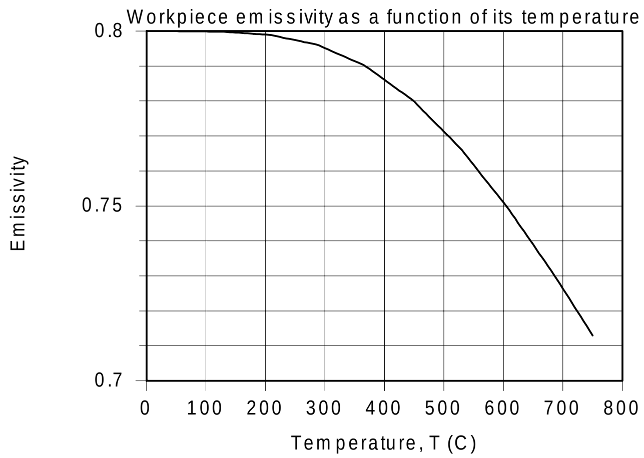
$$\dot{E}''_{st} = \rho c L\left(\frac{dT}{dt}\right)_i = q''_{net,in}$$

$$\left.\frac{dT}{dt}\right)_i = \frac{q''_{net,in}}{\rho c L} = \frac{233\ kW/m^2}{2700\ kg/m^3 \times 885\ J/kg\cdot K \times 0.010\ m} = 9.75\ K/s \qquad <$$



(b) Energy balance, initial condition

(c) With the relation for ε of Part (a) in the *IHT* workspace, and using the *Radiation* | *Band Emission* tool, ε as a function of workpiece temperature is calculated and plotted below.



Continued …..

As expected, ε decreases with increasing T, and when $T = T_f = 750°C$, $\varepsilon = \alpha = 0.713$. Why is that so?

(d) The energy balance of Part (b), using the lumped capacitance method with the *IHT DER (T,t)* function, has the form,

$$2\left[h\left(T_\infty - T\right) - \varepsilon\sigma T^4 + \alpha\sigma T_f^4\right] = \rho c L \text{ DER }(T, t)$$

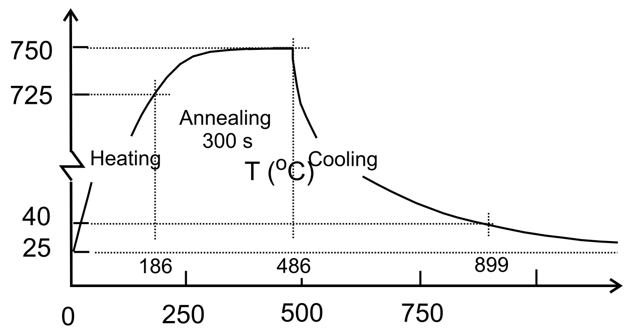
where ε = ε (T) from Part (c). From a plot of T vs. t (not shown) in the *IHT* workspace, find

$$T(t_a) = 725^\circ C \quad \text{when} \quad t_a = 186 \text{ s} \qquad <$$

(e) The time to cool the workpiece from 750°C to the safe-to-touch temperature of 40°C can be determined using the *IHT* code from Part (d). The cooling conditions are $T_\infty = 25°C$ and h = 100 $W/m^2 \cdot K$ with $T_{sur} = 25°C$. The emissivity is still evaluated using the relation of Part (c), but the absorptivity, which depends upon the surrounding temperature, is α = 0.80. From the results in the *IHT* workspace, find

$$T(t_c) = 40^\circ C \qquad \text{when} \qquad t_c = 413 \text{ s} \qquad <$$

(f) Assuming the workpiece temperature increases from 725°C to 750°C during a five-minute annealing period, the temperature history is as shown below.



The workpiece heats from 25°C to 725°C in t_a = 186 s, anneals for a 5-minute period during which the temperature reaches 750°C, followed by the cool-down process which takes 413 s. The total required time is

$$t = t_a + 5 \times 60 \text{ s} + t_c = (186 + 300 + 413)\text{s} = 899 \text{ s} = 15 \text{ min} \qquad <$$

Continued …..

PROBLEM 12.104 (Cont.)

The Biot number based upon convection only is

$$\mathrm{Bi} = \frac{h_{cv}(L/2)}{k} = \frac{100\ \mathrm{W/m^2 \cdot K} \times 0.005\ \mathrm{m}}{165\ \mathrm{W/m \cdot K}} = 0.003 << 0.1$$

so the lumped-capacitance method of analysis is appropriate.

COMMENTS: The *IHT* code to obtain the heating time, including emissivity as a function of the workpiece temperature, Part (b), is shown below, complete except for the input variables.

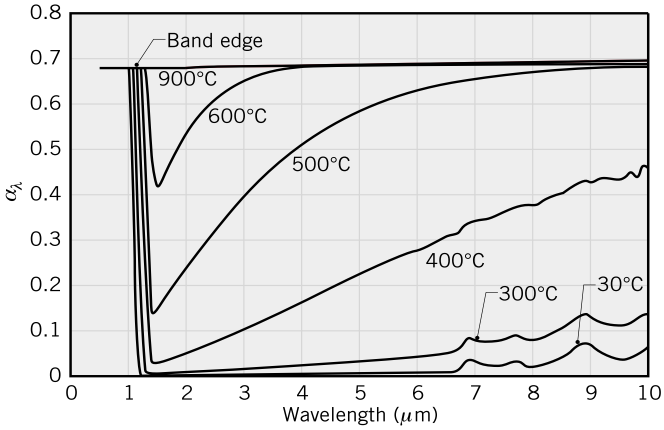
```
/* Analysis. The radiative properties and net heat flux in are calculated when the workpiece is
just inserted into the furnace. The workpiece experiences emission, absorbed irradiation and
convection processes. See Help | Solver | Intrinsic Functions for information on DER(T, t). */

/* Results - conditions at t = 186 s, Ts C - 725 C
FL1T      T_C       Tf        L         Tf_C      Tinf_C    eps1      eps2      h         k
          lambda1   rho       t         T
0.1607    725.1     1023      0.01      750       750       0.3       0.8       100       165
          2.5       2700      186       998.1     */

// Energy Balance
2 * ( h * (Tinf - T) + alpha * G - eps * sigma * T^4) = rho * cp * L * DER(T,t)
sigma = 5.67e-8
G = sigma * Tf^4

// Emissivity and absorptivity
eps = FL1T * eps1 + (1 - FL1T) * eps2
FL1T = F_lambda_T(lambda1, T)          // Eq 12.30
alpha = 0.713

// Temperature conversions
T_C = T - 273                    // For customary units, graphical output
Tf_C = Tf - 273
Tinf_C = Tinf - 273
```

PROBLEM 12.105

KNOWN: For the semiconductor silicon, the spectral distribution of absorptivity, α_λ, at selected temperatures. High-intensity, tungsten halogen lamps having spectral distribution approximating that of a blackbody at 2800 K.

FIND: (a) 1%-limits of the spectral band that includes 98% of the blackbody radiation corresponding to the spectral distribution of the lamps; spectral region for which you need to know the spectral absorptivity; (b) Sketch the variation of the total absorptivity as a function of silicon temperature; explain key features; (c) Calculate the total absorptivity at 400, 600 and 900°C for the lamp irradiation; explain results and the temperature dependence; (d) Calculate the total emissivity of the wafer at 600 and 900°C; explain results and the temperature dependence; and (e) Irradiation on the upper surface required to maintain the wafer at 600°C in a vacuum chamber with walls at 20°C. Use the *Look-up Table* and *Integral Functions* of *IHT* to perform the necessary integrations.

SCHEMATIC:



ASSUMPTIONS: (1) Silicon is a diffuse emitter, (2) Chamber is large, isothermal surroundings for the wafer, (3) Wafer is isothermal.

ANALYSIS: (a) From Eqs. 12.30 and 12.31, using Table 12.1 for the band emission factors, $F_{(0-\lambda T)}$, equal to 0.01 and 0.99 are:

$$F_{(0\to\lambda 1\cdot T)} = 0.01 \text{ at } \lambda_1 \cdot T = 1437\ \mu m \cdot K$$

$$F_{(0\to\lambda 2\cdot T)} = 0.99 \text{ at } \lambda_2 \cdot T = 23{,}324\ \mu m \cdot K$$

So that we have λ_1 and λ_2 limits for several temperatures, the following values are tabulated.

T(°C)	T(K)	λ_1(μm)	λ_2(μm)
-	2800	0.51	8.33
400	673	2.14	34.7
600	873	1.65	26.7
900	1173	1.23	19.9

<

For the 2800 K blackbody lamp irradiation, we need to know the spectral absorptivity over the spectral range 0.51 to 8.33 μm in order to include 98% of the radiation.

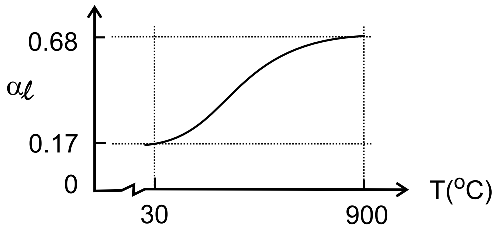
(b) The spectral absorptivity is calculated from Eq. 12.46 in which the spectral distribution of the lamp irradiation G_λ is proportional to the blackbody spectral emissive power $E_{\lambda,b}(\lambda, T)$ at the temperature of lamps, T_ℓ 2800 K.

$$\alpha_\ell = \frac{\int_0^\infty \alpha_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda} = \frac{\int_0^\infty \alpha_\lambda\ E_{\lambda,b}(\lambda,\ 2800\ K)}{\sigma T_\ell^4}$$

Continued …..

PROBLEM 12.105 (Cont.)

For 2800 K, the peak of the blackbody curve is at 1 μm; the limits of integration for 98% coverage are 0.5 to 8.3 μm according to part (a) results. Note that α_λ increases at all wavelengths with temperature, until around 900°C where the behavior is gray. Hence, we'd expect the total absorptivity of the wafer for lamp irradiation to appear as shown in the graph below.



At 900°C, since the wafer is gray, we expect $\alpha_\ell = \alpha_\lambda \approx 0.68$. Near room temperature, since $\alpha_\lambda \approx 0$ beyond the band edge, α_ℓ is dependent upon α_λ in the spectral region below and slightly beyond the peak. From the blackbody tables, the band emission fraction to the short-wavelength side of the peak is 0.25. Hence, estimate $\alpha_\ell \approx 0.68 \times 0.25 = 0.17$ at these low temperatures. The increase of α_ℓ with temperature is at first moderate, since the longer wavelength region is less significant than is the shorter region. As temperature increases, the α_λ closer to the peak begin to change more noticeably, explaining the greater dependence of α_ℓ on temperature.

(c) The integration of part (b) can be performed numerically using the *IHT INTEGRAL* function and specifying the spectral absorptivity in a *Lookup Table* file (*.lut). The code is shown in the Comments (1) and the results are:

T_w(°C)	400	600	900	
α_ℓ	0.30	0.59	0.68	<

(d) The total emissivity can be calculated from Eq. 12.38, recognizing that $\varepsilon_\lambda = \alpha_\lambda$ and that for silicon temperatures of 600 and 900°C, the 1% limits for the spectral integration are 1.65 - 26.7 μm and 1.23 - 19.9 μm, respectively. The integration is performed in the same manner as described in part (c); see Comments (2).

T(°C)	600	900	
ε	0.66	0.68	<

(e) From an energy balance on the silicon wafer with irradiation on the upper surface as shown in the schematic below, calculate the irradiation required to maintain the wafer at 600°C.

$$\dot{E}''_{in} - \dot{E}''_{out} = 0 \qquad \alpha_\ell G_\ell - 2\left[\varepsilon E_b\left(T_w\right) - \alpha_{sur} E_b\left(T_{sur}\right)\right] = 0$$

Recognize that α_{sur} corresponds to the spectral distribution of $E_{\lambda,b}(T_{sur})$; that is, upon α_λ for long wavelengths ($\lambda_{max} \approx 10$ μm). We assume $\alpha_{sur} \approx 0.1$, and with $T_{sur} = 20$°C, find

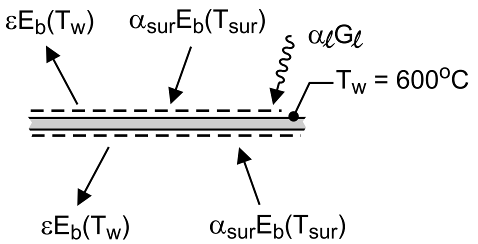
$$0.59\ G_\ell - 2\sigma\left[0.66(600+273)^4 K^4 - 0.1(20+273)^4 K^4\right] = 0$$

Continued …..

PROBLEM 12.105 (Cont.)

$$G_\ell = 73.5 \text{ kW/m}^2 \qquad <$$

where $E_b(T) = \sigma T^4$ and $\sigma = 5.67 \times 10^{-8}$ W/m$^2 \cdot$K^4.



COMMENTS: (1) The *IHT* code to obtain the total absorptivity for the lamp irradiation, α_ℓ for a wafer temperature of 400°C is shown below. Similar look-up tables were written for the spectral absorptivity for 600 and 800°C.

```
/* Results; integration for total absorptivity of lamp irradiation
T = 400 C; find abs_t = 0.30
ILb      absL     abs_t    C1        C2        T        sigma     lambda
1773     0.45     0.3012   3.742E8   1.439E4   2800     5.67E-8   10        */

// Input variables
T = 2800            // Lamp blackbody distribution

// Total absorptivity integral, Eq. 12.46
abs_t = pi * integral (ILsi, lambda) / (sigma * T^4)          // See Help | Solver
sigma = 5.67e-8

// Blackbody spectral intensity, Tools | Radiation
/* From Planck's law, the blackbody spectral intensity is */
ILsi = absL * ILb
ILb = I_lambda_b(lambda, T, C1, C2)     // Eq. 12.25
// where units are ILb(W/m^2.sr.mum), lambda (mum) and T (K) with
C1 = 3.7420e8       // First radiation constant, W·mum^4/m^2
C2 = 1.4388e4       // Second radiation constant, mum·K
// and (mum) represents (micrometers).

// Spectral absorptivity function
absL = LOOKUPVAL(abs_400, 1, lambda, 2)       // Silicon spectral data at 400 C
//absL = LOOKUPVAL(abs_600, 1, lambda, 2)     // Silicon spectral data at 600 C
//absL = LOOKUPVAL(abs_900, 1, lambda, 2)     // Silicon spectral data at 900 C

// Lookup table values for Si spectral data at 600 C
/* The table file name is abs_400.lut, with 2 columns and 10 rows
0.5      0.68
1.2      0.68
1.3      0.025
2        0.05
3        0.1
4        0.17
5        0.22
6        0.28
8        0.37
10       0.45 */
```

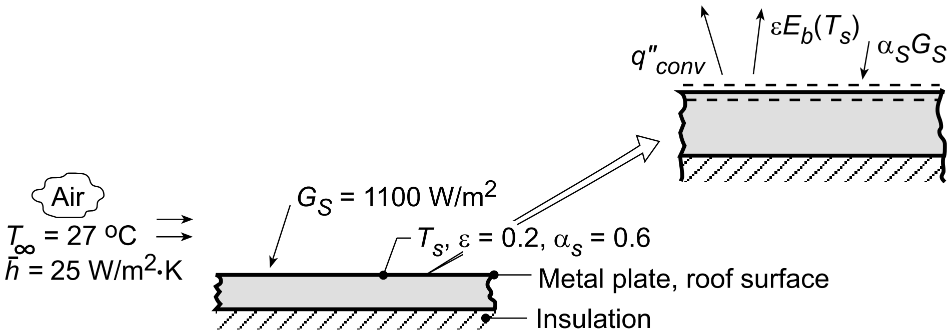
(2) The *IHT* code to obtain the total emissivity for a wafer temperature of 600°C has the same organization as for obtaining the total absorptivity. We perform the integration, however, with the blackbody spectral emissivity evaluated at the wafer temperature (rather than the lamp temperature). The same look-up file for the spectral absorptivity created in the part (c) code can be used.

PROBLEM 12.106

KNOWN: Solar irradiation of 1100 W/m^2 incident on a flat roof surface of prescribed solar absorptivity and emissivity; air temperature and convection heat transfer coefficient.

FIND: (a) Roof surface temperature, (b) Effect of absorptivity, emissivity and convection coefficient on temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Back-side of plate is perfectly insulated, (3) Negligible irradiation to plate by atmospheric (sky) emission.

ANALYSIS: (a) Performing a surface energy balance on the exposed side of the plate,

$$\alpha_S G_S - q''_{conv} - \varepsilon E_b(T_s) = 0 \qquad \alpha_S G_S - \bar{h}(T_s - T_\infty) - \varepsilon\sigma T_s^4 = 0$$

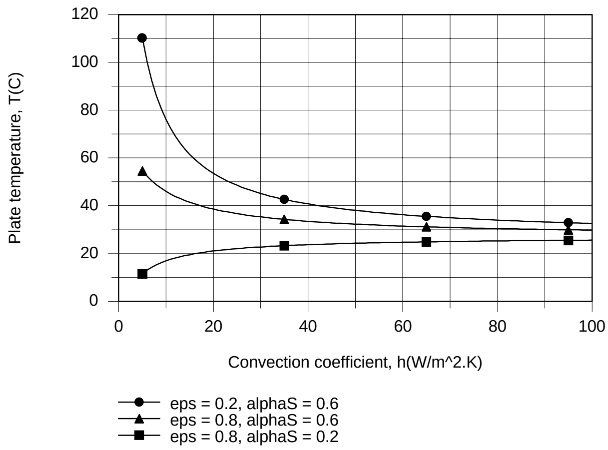
Substituting numerical values and using absolute temperatures,

$$0.6\times1100\frac{W}{m^2} - 25\frac{W}{m^2\cdot K}(T_s - 300)K - 0.2(5.67\times10^{-8}\ W/m^2\cdot K^4)T_s^4 = 0$$

Regrouping , $8160 = 25T_s + 1.1340\times10^{-8}T_s^4$, and performing a trial-and-error solution,

$T_s = 321.5\ K = 48.5^{\circ}C$. <

(b) Using the IHT *First Law* Model for a plane wall, the following results were obtained.



Irrespective of the value of $\bar{h}$, T decreases with increasing ε (due to increased emission) and decreasing α_S (due to reduced absorption of solar energy). For moderate to large α_S and/or small ε (net radiation transfer to the surface) T decreases with increasing $\bar{h}$ due to enhanced cooling by convection. However, for small α_S and large ε, emission exceeds absorption, dictating convection heat transfer to the surface and hence $T < T_\infty$. With increasing $\bar{h}$, $T \to T_\infty$, irrespective of the values of α_S and ε.

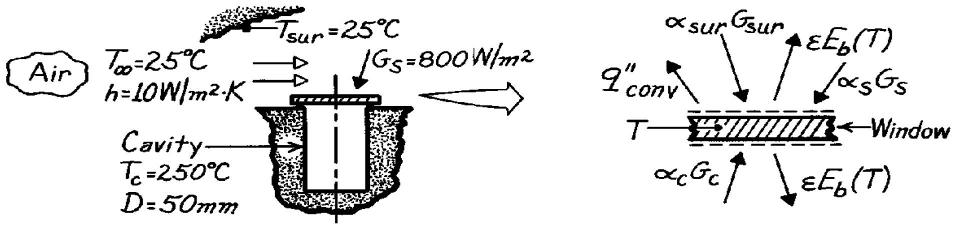
COMMENTS: To minimize the roof temperature, the value of ε/α_S should be maximized.

PROBLEM 12.107

KNOWN: Cavity with window whose outer surface experiences convection and radiation.

FIND: Temperature of the window and power required to maintain cavity at prescribed temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Cavity behaves as a blackbody, (3) Solar spectral distribution is that of a blackbody at 5800K, (4) Window is isothermal, (5) Negligible convection on lower surface of window.

PROPERTIES: Window material: $0.2 \le \lambda \le 4$ μm, $\tau_\lambda = 0.9$, $\rho_\lambda = 0$, hence $\alpha_\lambda = 1 - \tau_\lambda = 0.1$; 4 μm < λ, $\tau_\lambda = 0$, $\alpha = \varepsilon = 0.95$, diffuse-gray, opaque

ANALYSIS: To determine the window temperature, perform an energy balance on the window,

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$\left[\alpha_{sur}G_{sur} + \alpha_S G_S - \varepsilon E_b - q''_{conv}\right]_{upper} + \left[\alpha_c G_c - \varepsilon E_b(T)\right]_{lower} = 0. \quad (1)$$

Calculate the absorptivities for various irradiation conditions using Eq. 12.46,

$$\alpha = \int_0^\infty \alpha_\lambda G_\lambda d\lambda \Big/ \int_0^\infty G_\lambda d\lambda \quad (2)$$

where $G(\lambda)$ is the spectral distribution of the irradiation.

Surroundings, α_{sur}: $G_{sur} = E_b(T_{sur}) = \sigma T_{sur}^4$

$$\alpha_{sur} = 0.1\left[F_{(0\to 4\mu m)} - F_{(0\to 0.2\mu m)}\right] + 0.95\left[1 - F_{(0\to 4\mu m)}\right]$$

where from Table 12.1, with $T = T_{sur} = (25 + 273)K = 298K$,

$$\lambda T = 0.2\mu m \times 298K = 59.6\mu m\cdot K, \quad F_{(0-\lambda T)} = 0.000$$

$$\lambda T = 4\mu m \times 298K = 1192\mu m\cdot K, \quad F_{(0-\lambda T)} = 0.002$$

$$\alpha_{sur} = 0.1[0.002 - 0.000] + 0.95[1 - 0.002] = 0.948. \quad (3)$$

Solar, α_S: $G_S \sim E_b$ (5800K)

$$\alpha_S = 0.1\left[F_{(0\to 4\mu m)} - F_{(0\to 0.2\mu m)}\right] + 0.95\left[1 - F_{(0\to 4\mu m)}\right]$$

where from Table 12.1, with T = 5800K,

$$\lambda T = 0.2\mu m \times 5800K = 1160\mu m\cdot K, \quad F_{(0-\lambda T)} = 0.002$$

$$\lambda T = 4\mu m \times 5800K = 23{,}200\mu m\cdot K, \quad F_{(0-\lambda T)} = 0.990$$

$$\alpha_S = 0.1[0.990 - 0.002] + 0.95[1 - 0.990] = 0.108. \quad (4)$$

Continued …..

Cavity, α_c: $G_c = E_b(T_c) = \sigma T_c^4$

$$\alpha_c = 0.1\left[F_{(0\to 4\mu m)} - F_{(0\to 0.2\mu m)}\right] + 0.95\left[1 - F_{(0\to 4\mu m)}\right]$$

where from Table 12.1 with $T_c = 250°C = 523K$,

$$\lambda T = 0.2\mu m \times 523K = 104.6\mu m\cdot K, \quad F_{0\to\lambda T} = 0.000$$

$$\lambda T = 4\mu m \times 523K = 2092\mu m\cdot K \quad F_{0\to\lambda T} = 0.082$$

$$\alpha_c = 0.1[0.082 - 0.000] + 0.95[1 - 0.082] = 0.880. \tag{5}$$

To determine the *emissivity* of the window, we need to know its temperature. However, we know that T will be less than T_c and the long wavelength behavior will dominate. That is,

$$\varepsilon \approx \varepsilon_\lambda(\lambda > 4\mu m) = 0.95. \tag{6}$$

With these radiative properties now known, the energy equation, Eq. (1) can now be evaluated using $q''_{conv} = h(T - T_\infty)$ with all temperatures in kelvin units.

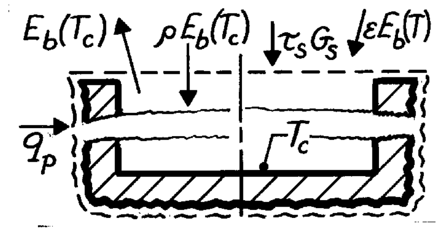
$$0.948\times\sigma(298K)^4 + 0.108\times 800\,W/m^2 - 0.95\times\sigma T^4 - 10\,W/m^2\cdot K(T - 298K)$$
$$+0.880\sigma(523K)^4 - 0.95\times\sigma T^4 = 0$$

$$1.077\times 10^{-7}T^4 + 10T - 7223 = 0.$$

Using a trial-and-error approach, find the window temperature as

$$T = 413K = 139°C.$$ <

To determine the power required to maintain the cavity at $T_c = 250°C$, perform an energy balance on the cavity.



$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q_p + A_c[\rho E_b(T_c) + \tau_S G_S + \varepsilon E_b(T) - E_b(T_c)] = 0.$$

For simplicity, we have assumed the window opaque to irradiation from the surroundings. It follows that

$$\tau_S = 1 - \rho_S - \alpha_S = 1 - 0 - 0.108 = 0.892$$

$$\rho = 1 - \alpha = 1 - \varepsilon = 1 - 0.95 = 0.05.$$

Hence, the power required to maintain the cavity, when $A_c = (\pi/4)D^2$, is

$$q_p = A_c\left[\sigma T_c^4 - \rho\sigma T_c^4 - \tau_s G_s - \varepsilon\sigma T^4\right]$$

$$q_p = \frac{\pi}{4}(0.050m)^2\left[\sigma(523K)^4 - 0.05\sigma(523K)^4 - 0.892\times 800\,W/m^2 - 0.95\sigma(412K)^4\right]$$

$$q_p = 3.47W.$$ <

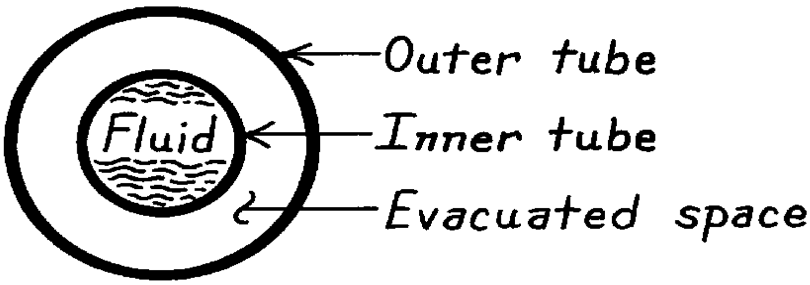
COMMENTS: Note that the assumed value of $\varepsilon = 0.95$ is not fully satisfied. With T = 412K, we would expect $\varepsilon = 0.929$. Hence, an iteration may be appropriate.

PROBLEM 12.108

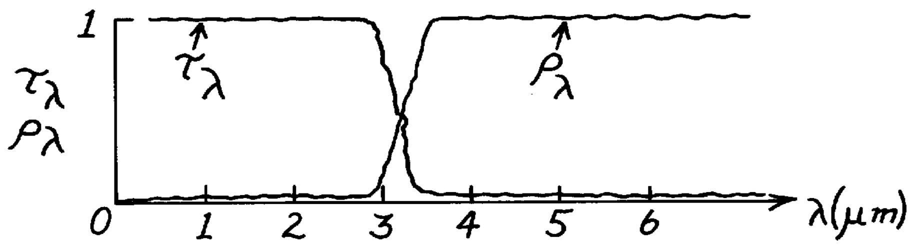
KNOWN: Features of an evacuated tube solar collector.

FIND: Ideal surface spectral characteristics.

SCHEMATIC:

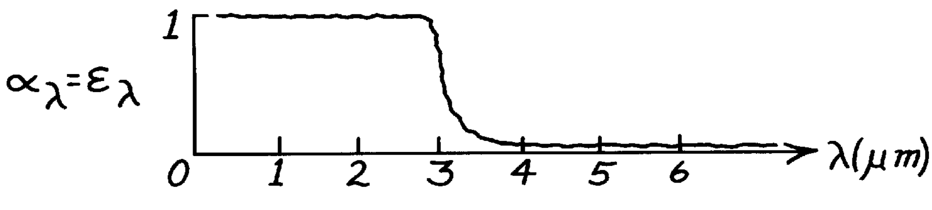


ANALYSIS: The outer tube should be transparent to the incident solar radiation, which is concentrated in the spectral region $\lambda \leq 3\mu m$, but it should be opaque and highly reflective to radiation emitted by the outer surface of the inner tube, which is concentrated in the spectral region above $3\mu m$. Accordingly, ideal spectral characteristics for the outer tube are



Note that large ρ_λ is desirable for the outer, as well as the inner, surface of the outer tube. If the surface is diffuse, a large value of ρ_λ yields a small value of $\varepsilon_\lambda = \alpha_\lambda = 1 - \rho_\lambda$. Hence losses due to emission from the outer surface to the surroundings would be negligible.

The opaque outer surface of the inner tube should absorb all of the incident solar radiation ($\lambda \leq 3\mu m$) and emit little or no radiation, which would be in the spectral region $\lambda > 3\mu m$. Accordingly, assuming diffuse surface behavior, ideal spectral characteristics are:

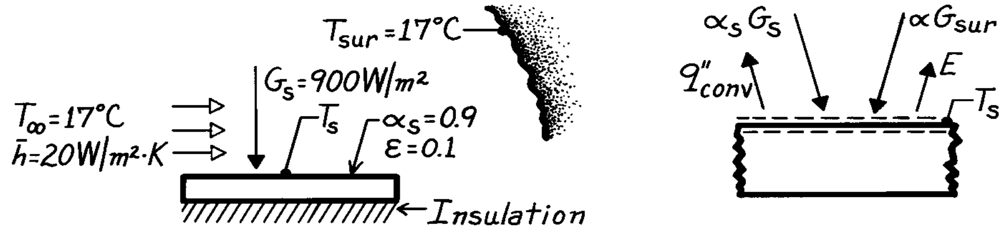


PROBLEM 12.109

KNOWN: Plate exposed to solar flux with prescribed solar absorptivity and emissivity; convection and surrounding conditions also prescribed.

FIND: Steady-state temperature of the plate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Plate is small compared to surroundings, (3) Backside of plate is perfectly insulated, (4) Diffuse behavior.

ANALYSIS: Perform a surface energy balance on the top surface of the plate.

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$\alpha_S G_S + \alpha G_{sur} - q''_{conv} - \varepsilon E_b(T_s) = 0$$

Note that the effect of the surroundings is to provide an irradiation, G_{sur}, on the plate; since the spectral distribution of G_{sur} and $E_{\lambda,b}$ (T_s) are nearly the same, accordiing to Kirchoff's law, $\alpha = \varepsilon$. Recognizing that $G_{sur} = \sigma T_{sur}^4$ and using Newton's law of cooling, the energy balance is

$$\alpha_S G_S + \varepsilon\sigma T_{sur}^4 - \bar{h}(T_s - T_\infty) - \varepsilon \cdot \sigma T_s^4 = 0.$$

Substituting numerical values,

$$0.9\times 900\ \mathrm{W/m^2} + 0.1\times 5.67\times 10^{-8}\mathrm{W/m^2\cdot K}\times(17+273)^4\ \mathrm{K}^4$$
$$-20\,\mathrm{W/m^2\cdot K}\,(T_s - 290)\mathrm{K} - 0.1\left(5.67\times 10^{-8}\mathrm{W/m^2\cdot K^4}\right)T_s^4 = 0$$

$$6650\ \mathrm{W/m^2} = 20\,T_s + 5.67\times 10^{-9}\,T_s^4.$$

From a trial-and-error solution, find

$$T_s = 329.2\ \mathrm{K}.$$ <

COMMENTS: (1) When performing an analysis with both convection and radiation processes present, all temperatures must be expressed in absolute units (K).

(2) Note also that the terms $\alpha\ G_{sur}$ - $\varepsilon\ E_b$ (T_s) could be expressed as a radiation exchange term, written as

$$q''_{rad} = q/A = \varepsilon\sigma\left(T_{sur}^4 - T_s^4\right).$$

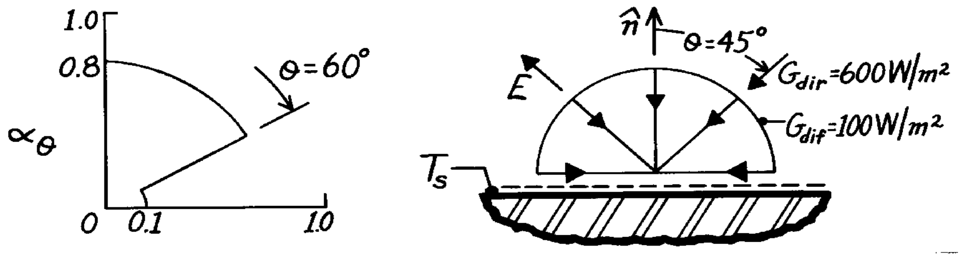
The conditions for application of this relation were met and are namely: surroundings much larger than surface, diffuse surface, and spectral distributions of irradiation and emission are similar (or the surface is gray).

PROBLEM 12.110

KNOWN: Directional distribution of α_θ for a horizontal, opaque, gray surface exposed to direct and diffuse irradiation.

FIND: (a) Absorptivity to direct radiation at 45° and to diffuse radiation, and (b) Equilibrium temperature for specified direct and diffuse irradiation components.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Opaque, gray surface behavior, (3) Negligible convection at top surface and perfectly insulated back surface.

ANALYSIS: (a) From knowledge of α_θ (θ) – see graph above – it is evident that the absorptivity of the surface to the direct radiation (45°) is

$$\alpha_{dir} = \alpha_\theta\left(45°\right) = 0.8. \qquad <$$

The absorptivity to the diffuse radiation is the hemispherical absorptivity given by Eq. 12.44. Dropping the λ subscript,

$$\alpha_{dir} = 2\int_0^{\pi/2} \alpha_\theta\left(\theta\right)\cos\theta\sin\theta\, d\theta \qquad (1)$$

$$\alpha_{dir} = 2\left[0.8\left.\frac{\sin^2\theta}{2}\right|_0^{\pi/3} + 0.1\left.\frac{\sin^2\theta}{2}\right|_{\pi/3}^{\pi/2}\right]$$

$$\alpha_{dir} = 0.625. \qquad <$$

(b) Performing a surface energy balance,

$$\dot{E}''_{in} - \dot{E}''_{out} = 0$$

$$\alpha_{dir}\, G_{dir} + \alpha_{dif}\, G_{dif} - \varepsilon\sigma T_s^4 = 0. \qquad (2)$$

The total, hemispherical emissivity may be obtained from Eq. 12.36 where again the subscript may be deleted. Since this equation is of precisely the same form as Eq. 12.44 – see Eq. (1) above – and since $\alpha_\theta = \varepsilon_\theta$, it follows that

$$\varepsilon = \alpha_{dif} = 0.625$$

and from Eq. (2), find

$$T_s^4 = \frac{\left(0.8\times600 + 0.625\times100\right)\mathrm{W/m^2}}{0.625\times5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}} = 1.53\times10^{10}\,\mathrm{K}^4, \qquad T_s = 352\ \mathrm{K}. \qquad <$$

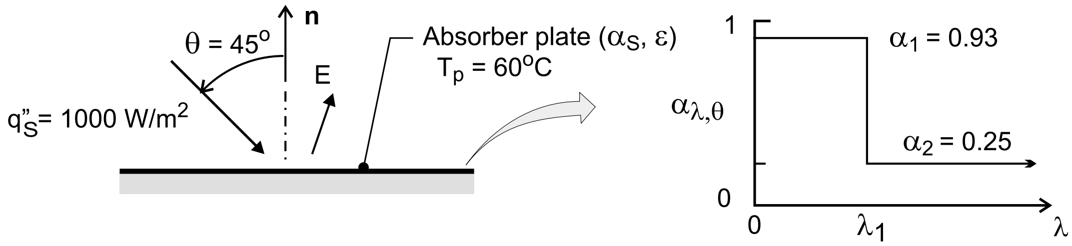
COMMENTS: In assuming *gray* surface behavior, spectral effects are not present, and total and spectral properties are identical. However, the surface is *not diffuse* and hence hemispherical and directional properties differ.

PROBLEM 12.111

KNOWN: Plate temperature and spectral and directional dependence of its absorptivity. Direction and magnitude of solar flux.

FIND: (a) Expression for total absorptivity, (b) Expression for total emissivity, (c) Net radiant flux, (d) Effect of cut-off wavelength associated with directional dependence of the absorptivity.

SCHEMATIC:



ASSUMPTIONS: (1) Diffuse component of solar flux is negligible, (2) Spectral distribution of solar radiation may be approximated as that from a blackbody at 5800 K, (3) Properties are independent of azimuthal angle ϕ.

ANALYSIS: (a) For $\lambda < \lambda_c$ and $\theta = 45°$, $\alpha_\lambda = \alpha_1 \cos\theta = 0.707\ \alpha_1$. From Eq. (12.47) the total absorptivity is then

$$\alpha_S = 0.707\ \alpha_1 \left\{ \frac{\int_0^{\lambda_c} E_{\lambda,b}\ (\lambda, 5800\ K)d\lambda}{E_b} \right\} + \alpha_2 \left\{ \frac{\int_{\lambda_c}^{\infty} E_{\lambda,b}\ (\lambda, 5800\ K)d\lambda}{E_b} \right\}$$

$$\alpha_S = 0.707\ \alpha_1\ F_{(0\to\lambda_c)} + \alpha_2 \left[1 - F_{(0\to\lambda_c)}\right]$$ <

For the prescribed value of λ_c, $\lambda_c T$ = 11,600 μm·K and, from Table 12.1, $F_{(0\to\lambda c)}$ = 0.941. Hence,

$$\alpha_S = 0.707 \times 0.93 \times 0.941 + 0.25(1 - 0.941) = 0.619 + 0.015 = 0.634$$ <

(b) With $\varepsilon_{\lambda,\theta} = \alpha_{\lambda,o}$, Eq (12.36) may be used to obtain ε_λ for $\lambda < \lambda_c$.

$$\varepsilon_\lambda(\lambda, T) = 2\alpha_1 \int_0^{\pi/2} \cos^2\theta \sin\theta\, d\theta = -2\alpha_1 \frac{\cos^3\theta}{3}\bigg|_0^{\pi/2} = \frac{2}{3}\alpha_1$$

From Eq. (12.38),

$$\varepsilon = 0.667\alpha_1 \frac{\int_0^{\lambda_c} E_{\lambda,b}(\lambda, T_p)d\lambda}{E_b} + \alpha_2 \frac{\int_{\lambda_c}^{\infty} E_{\lambda,b}(\lambda, T_p)d\lambda}{E_b}$$

$$\varepsilon = 0.667\alpha_1 F_{(0\to\lambda_c)} + \alpha_2\left[1 - F_{(0-\lambda_c)}\right]$$ <

For λ_c = 2 μm and T_p = 333 K, $\lambda_c T$ = 666 μm·K and, from Table 12.1, $F_{(0\text{-}\lambda c)}$ = 0. Hence,

$$\varepsilon = \alpha_2 = 0.25$$ <

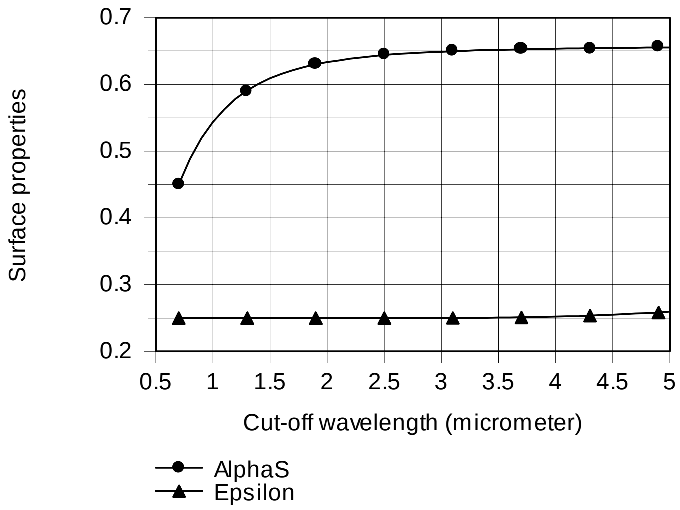
Continued …..

PROBLEM 12.111 (Cont.)

(c) $$q''_{net} = \alpha_S q''_S - \varepsilon\sigma T_p^4 = 634\ \mathrm{W/m^2} - 0.25 \times 5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4} (333\ \mathrm{K})^4$$

$$q''_{net} = 460\ \mathrm{W/m^2}$$ <

(d) Using the foregoing model with the Radiation/Band Emission Factor option of *IHT*, the following results were obtained for α_S and ε. The absorptivity increases with increasing λ_c, as more of the incident solar radiation falls within the region of $\alpha_1 > \alpha_2$. Note, however, the limit at $\lambda \approx 3\ \mu$m, beyond which there is little change in α_S. The emissivity also increases with increasing λ_c, as more of the emitted radiation is at wavelengths for which $\varepsilon_1 = \alpha_1 > \varepsilon_2 = \alpha_2$. However, the surface temperature is low, and even for $\lambda_c = 5\ \mu$m, there is little emission at $\lambda < \lambda_c$. Hence, ε only increases from 0.25 to 0.26 as λ_c increases from 0.7 to 5.0 μm.



The net heat flux increases from 276 W/m^2 at $\lambda_c = 2\ \mu$m to a maximum of 477 W/m^2 at $\lambda_c = 4.2\ \mu$m and then decreases to 474 W/m^2 at $\lambda_c = 5\ \mu$m. The existence of a maximum is due to the upper limit on the value of α_S and the increase in ε with λ_c.

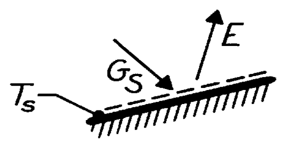
COMMENTS: Spectrally and directionally selective coatings may be used to enhance the performance of solar collectors.

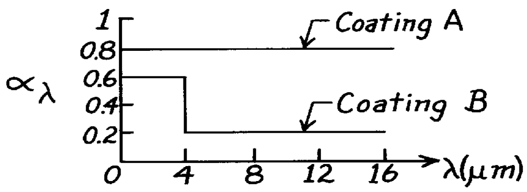
PROBLEM 12.112

KNOWN: Spectral distribution of α_λ for two roof coatings.

FIND: Preferred coating for summer and winter use. Ideal spectral distribution of α_λ.

SCHEMATIC:





ASSUMPTIONS: (1) Opaque, diffuse surface behavior, (2) Negligible convection effects and heat transfer from bottom of roof, negligible atmospheric irradiation, (3) Steady-state conditions.

ANALYSIS: From an energy balance on the roof surface

$$\varepsilon \sigma T_s^4 = \alpha_S G_S.$$

Hence

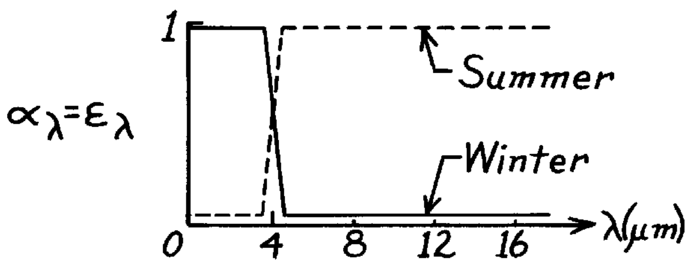
$$T_s = \left(\frac{\alpha_S}{\varepsilon} \frac{G_S}{\sigma} \right)^{1/4}.$$

Solar irradiation is concentrated in the spectral region $\lambda < 4\mu m$, while surface emission is concentrated in the region $\lambda > 4\mu m$. Hence, with $\alpha_\lambda = \varepsilon_\lambda$

Coating A: $\alpha_S \approx 0.8$, $\varepsilon \approx 0.8$

Coating B: $\alpha_S \approx 0.6$, $\varepsilon \approx 0.2$.

Since $(\alpha_S/\varepsilon)_A = 1 < (\alpha_S/\varepsilon)_B = 3$, Coating A would result in the lower roof temperature and is preferred for summer use. In contrast, Coating B is preferred for winter use. The ideal coating is one which minimizes (α_S/ε) in the summer and maximizes it in the winter.

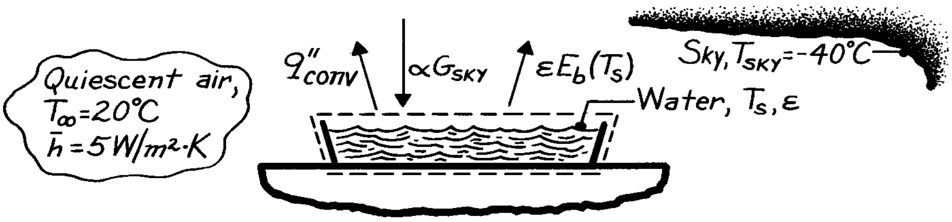


PROBLEM 12.113

KNOWN: Shallow pan of water exposed to night desert air and sky conditions.

FIND: Whether water will freeze.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Bottom of pan is well insulated, (3) Water surface is diffuse-gray, (4) Sky provides blackbody irradiation, $G_{sky} = \sigma T_{sky}^4$.

PROPERTIES: *Table A-11*, Water (300 K): ε = 0.96.

ANALYSIS: To estimate the water surface temperature for these conditions, begin by performing an energy balance on the pan of water considering convection and radiation processes.

$$\dot{E}''_{in} - \dot{E}''_{out} = 0$$

$$\alpha G_{sky} - \varepsilon E_b - \bar{h}(T_s - T_\infty) = 0$$

$$\varepsilon \sigma \left(T_{sky}^4 - T_s^4\right) - \bar{h}(T_s - T_\infty) = 0.$$

Note that, from Eq. 12.64, $G_{sky} = \sigma T_{sky}^4$ and from Assumption 3, $\alpha = \varepsilon$. Substituting numerical values, with all temperatures in kelvin units, the energy balance is

$$0.96 \times 5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \cdot \text{K}^4}\left[(-40+273)^4 - T_s^4\right]\text{K}^4 - 5\frac{\text{W}}{\text{m}^2 \cdot \text{K}}\left[T_s - (20+273)\right]\text{K} = 0$$

$$5.443 \times 10^{-8}\left[233^4 - T_s^4\right] - 5\left[T_s - 293\right] = 0.$$

Using a trial-and-error approach, find the water surface temperature,

$$T_s = 268.5\text{K}. \qquad <$$

Since T_s < 273 K, it follows that the water surface will freeze under the prescribed air and sky conditions.

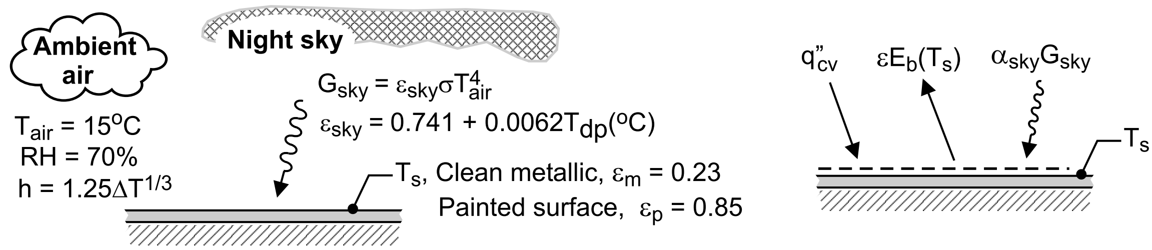
COMMENTS: If the heat transfer coefficient were to increase as a consequence of wind, freezing might not occur. Verify that for the given T_∞ and T_{sky}, that if $\bar{h}$ increases by more than 40%, freezing cannot occur.

PROBLEM 12.114

KNOWN: Flat plate exposed to night sky and in ambient air at T_{air} = 15°C with a relative humidity of 70%. Radiation from the atmosphere or sky estimated as a fraction of the blackbody radiation corresponding to the near-ground air temperature, $G_{sky} = \varepsilon_{sky}\, \sigma\, T_{air}$, and for a clear night, ε_{sky} = 0.741 + 0.0062 T_{dp} where T_{dp} is the dew point temperature (°C). Convection coefficient estimated by correlation, $\bar{h}\left(W/m^2\cdot K\right) = 1.25\Delta T^{1/3}$ where ΔT is the plate-to-air temperature difference (K).

FIND: Whether dew will form on the plate if the surface is (a) clean metal with ε_m = 0.23 and (b) painted with ε_p = 0.85.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Surfaces are diffuse, gray, and (3) Backside of plate is well insulated.

PROPERTIES: *Psychrometric charts* (Air), T_{dp} = 9.4°C for dry bulb temperature 15°C and relative humidity 70%.

ANALYSIS: From the schematic above, the energy balance on the plate is

$$\dot{E}''_{in} - \dot{E}''_{out} = 0$$

$$\alpha_{sky}\, G_{sky} + q''_{cv} - \varepsilon\, E_b\, (T_s) = 0$$

$$\varepsilon\left[\left(0.741 + 0.0062\ T_{dp}\left({}^\circ C\right)\right)\sigma T_{air}^4\right] + 1.25\left(T_{air} - T_s\right)^{4/3}\ W/m^2 - \varepsilon\sigma T_s^4\ W/m^2 = 0$$

where $G_{sky} = \varepsilon_{sky}\, \sigma\, T_{air}$, ε_{sky} = 0.741 + 0.062 T_{dp} (°C); T_{dp} has units (°C); and, other temperatures in kelvins. Since the surface is diffuse-gray, $\alpha_{sky} = \varepsilon$.

(a) *Clean metallic surface*, ε_m = 0.23

$$0.23\left[\left(0.741 + 0.0062\ T_{dp}\left({}^\circ C\right)\right)\sigma\left(15 + 273\right)^4 K^4\right]$$

$$+1.25\left(289\ - T_{s,m}\right)^{4/3}\ W/m^2 - 0.23\ \sigma\, T_{s,m}^4\ W/m^2 = 0$$

$$T_{s,m} = 282.7\ K = 9.7^\circ C$$ <

(b) *Painted surface, ε_p = 0.85* $T_{s,p}$ = 278.5 K = 5.5°C <

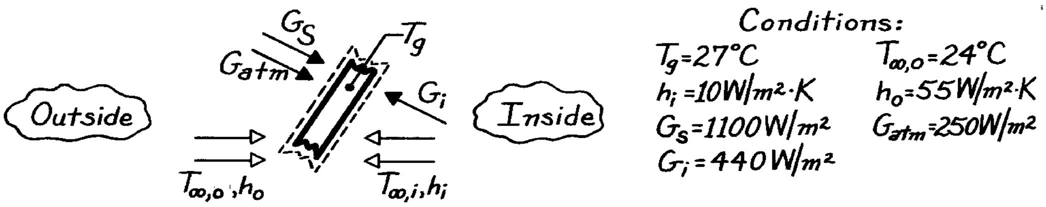
COMMENTS: For the painted surface, ε_p = 0.85, find that $T_s < T_{dp}$, so we expect dew formation. For the clean, metallic surface, $T_s > T_{dp}$, so we do not expect dew formation.

PROBLEM 12.115

KNOWN: Glass sheet, used on greenhouse roof, is subjected to solar flux, G_S, atmospheric emission, G_{atm}, and interior surface emission, G_i, as well as to convection processes.

FIND: (a) Appropriate energy balance for a unit area of the glass, (b) Temperature of the greenhouse ambient air, $T_{\infty,i}$, for prescribed conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Glass is at a uniform temperature, T_g, (2) Steady-state conditions.

PROPERTIES: Glass: $\tau_\lambda = 1$ for $\lambda \le 1\mu m$; $\tau_\lambda = 0$ and $\alpha_\lambda = 1$ for $\lambda > 1\ \mu m$.

ANALYSIS: (a) Performing an energy balance on the glass sheet with $\dot{E}_{in} - \dot{E}_{out} = 0$ and considering two convection processes, emission and three absorbed irradiation terms, find

$$\alpha_S G_S + \alpha_{atm} G_{atm} + h_o\left(T_{\infty,o} - T_g\right) + \alpha_i G_i + h_i\left(T_{\infty,i} - T_g\right) - 2\varepsilon\sigma T_g^4 = 0 \qquad (1)$$

where α_S = solar absorptivity for absorption of $G_{\lambda,S} \sim E_{\lambda,b}$ (λ, 5800K)

$\alpha_{atm} = \alpha_i$ = absorptivity of long wavelength irradiation ($\lambda >> 1\ \mu m$) ≈ 1

$\varepsilon = \alpha_\lambda$ for $\lambda >> 1\ \mu m$, emissivity for long wavelength emission ≈ 1

(b) For the prescribed conditions, $T_{\infty,i}$ can be evaluated from Eq. (1). As noted above, $\alpha_{atm} = \alpha_i = 1$ and $\varepsilon = 1$. The solar absorptivity of the glass follows from Eq. 12.47 where $G_{\lambda,S} \sim E_{\lambda,b}$ (λ, 5800K),

$$\alpha_S = \int_0^\infty \alpha_\lambda G_{\lambda,S}\, d\lambda / G_s = \int_0^\infty \alpha_\lambda E_{\lambda,b}(\lambda, 5800K)\, d\lambda / E_b(5800K)$$

$$\alpha_S = \alpha_1 F_{(0\to 1\mu m)} + \alpha_2\left[1 - F_{(0\to 1\mu m)}\right] = 0 \times 0.720 + 1.0[1 - 0.720] = 0.28.$$

Note that from Table 12.1 for $\lambda T = 1\ \mu m \times 5800K = 5800\ \mu m \cdot K$, $F_{(0-\lambda)} = 0.720$. Substituting numerical values into Eq. (1),

$$0.28 \times 1100 W/m^2 + 1 \times 250 W/m^2 + 55 W/m^2 \cdot K(24 - 27)K + 1 \times 440 W/m^2 +$$
$$10 W/m^2 \cdot K\left(T_{\infty,i} - 27\right)K - 2 \times 1 \times 5.67 \times 10^{-8} W/m^2 \cdot K(27 + 273)^4 K^4 = 0$$

find that

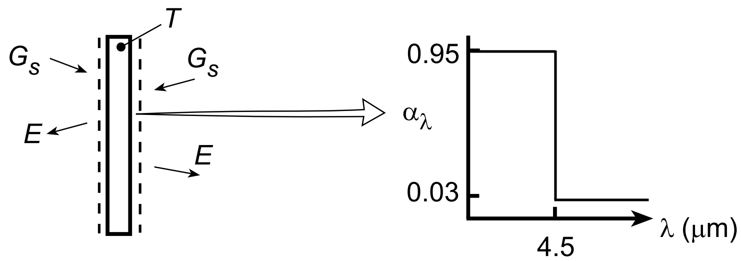
$$T_{\infty,i} = 35.5°C. \qquad <$$

PROBLEM 12.116

KNOWN: Plate temperature and spectral absorptivity of coating.

FIND: (a) Solar irradiation, (b) Effect of solar irradiation on plate temperature, total absorptivity, and total emissivity.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Opaque, diffuse surface, (3) Isothermal plate, (4) Negligible radiation from surroundings.

ANALYSIS: (a) Performing an energy balance on the plate, $2\alpha_S G_S - 2E = 0$ and

$$\alpha_S G_S - \varepsilon\sigma T^4 = 0$$

For $\lambda T = 4.5\ \mu m \times 2000\ K = 9000\ \mu m\cdot K$, Table 12.1 yields $F_{(o\to\lambda)} = 0.890$. Hence,

$$\varepsilon = \varepsilon_1 F_{(0\to\lambda)} + \varepsilon_2\left(1 - F_{(o\to\lambda)}\right) = 0.95\times0.890 + 0.03(1-0.890) = 0.849$$

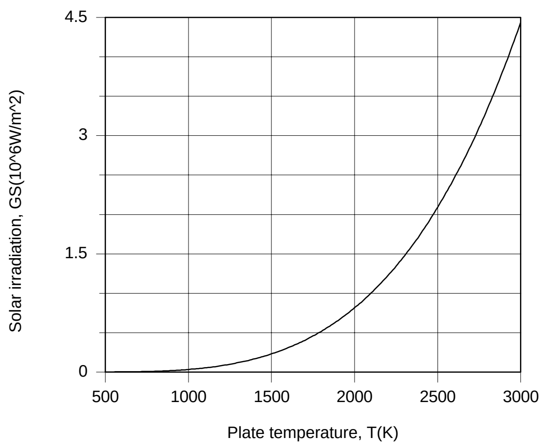
For $\lambda T = 4.5\ \mu m \times 5800\ K = 26{,}100$, $F_{(o\to\lambda)} = 0.993$. Hence,

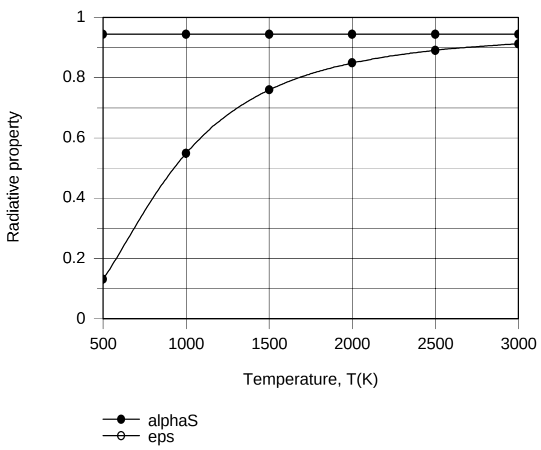
$$\alpha_S = \alpha_1 F_{(0\to\lambda)} + \alpha_2\left(1 - F_{(0\to\lambda)}\right) = 0.95\times0.993 + 0.03\times0.007 = 0.944$$

Hence,

$$G_S = (\varepsilon/\alpha_S)\sigma T^4 = (0.849/0.944)5.67\times10^{-8}\ W/m^2\cdot K^4\,(2000\,K)^4 = 8.16\times10^5\ W/m^2 \quad <$$

(b) Using the IHT *First Law* Model and the *Radiation* Toolpad, the following results were obtained.





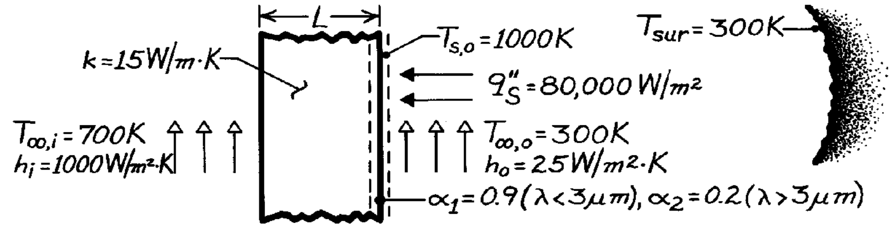
The required solar irradiation increases with T to the fourth power. Since α_S is determined by the spectral distribution of solar radiation, its value is fixed. However, with increasing T, the spectral distribution of emission is shifted to lower wavelengths, thereby increasing the value of ε.

PROBLEM 12.117

KNOWN: Thermal conductivity, spectral absorptivity and inner and outer surface conditions for wall of central solar receiver.

FIND: Minimum wall thickness needed to prevent thermal failure. Collector efficiency.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Outer surface is opaque and diffuse, (3) Spectral distribution of solar radiation corresponds to blackbody emission at 5800 K.

ANALYSIS: From an energy balance at the outer surface, $\dot{E}_{in} = \dot{E}_{out}$,

$$\alpha_S q''_S + \alpha_{sur} G_{sur} = \varepsilon\sigma T_{s,o}^4 + h_o\left(T_{s,o} - T_{\infty,o}\right) + \frac{T_{s,o} - T_{\infty,i}}{(L/k) + (1/h_i)}$$

Since radiation from the surroundings is in the far infrared, $\alpha_{sur} = 0.2$. From Table 12.1, $\lambda T = (3\ \mu m \times 5800\ K) = 17{,}400\ \mu m\cdot K$, find $F_{(0\to 3\mu m)} = 0.979$. Hence,

$$\alpha_S = \frac{\int_0^\infty \alpha_\lambda E_{\lambda,b}(5800\ K)\,d\lambda}{E_b} = \alpha_1 F_{(0\to 3\mu m)} + \alpha_2 F_{(3\to\infty)} = 0.9(0.979) + 0.2(0.021) = 0.885.$$

From Table 12.1, $\lambda T = (3\ \mu m \times 1000\ K) = 3000\ \mu m\cdot K$, find $F_{(0\to 3\mu m)} = 0.273$. Hence,

$$\varepsilon_S = \frac{\int_0^\infty \varepsilon_\lambda E_{\lambda,b}(1000\ K)\,d\lambda}{E_b} = \varepsilon_1 F_{(0\to 3)} + \varepsilon_2 F_{(3\to\infty)} = 0.9(0.273) + 0.2(0.727) = 0.391.$$

Substituting numerical values in the energy balance, find

$$0.885\left(80{,}000\ W/m^2\right) + 0.2 \times 5.67\times 10^{-8}\ W/m^2\cdot K^4\,(300\ K)^4 = 0.391\times 5.67\times 10^{-8}\ W/m^2\cdot K^4\,(1000\ K)^4$$

$$+25\ W/m^2\cdot K\,(700\ K) + (300\ K)/\left[(L/15\ W/m\cdot K) + \left(1/1000\ W/m^2\cdot K\right)\right]$$

$$L = 0.129\ m.$$ <

The corresponding collector efficiency is

$$\eta = \frac{q''_{use}}{q''_S} = \left[\frac{T_{s,o} - T_{\infty,i}}{(L/k) + (1/h_i)}\right] / q''_S$$

$$\eta = \left[\frac{300\ K}{(0.129\ m/15\ W/m\cdot K) + \left(0.001 m^2\cdot K/W\right)}\right] / 80{,}000\ W/m^2 = 0.391 \text{ or } 39.1\%.$$ <

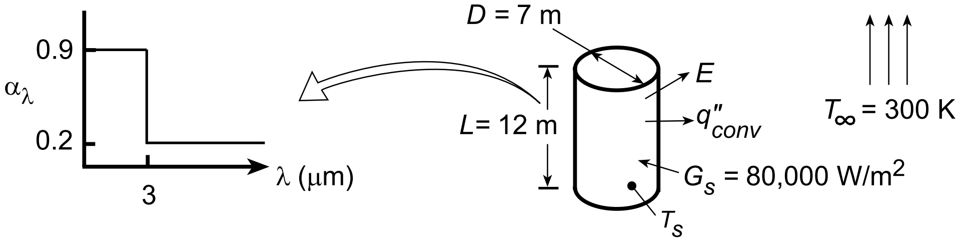
COMMENTS: The collector efficiency could be increased and the outer surface temperature reduced by decreasing the value of L.

PROBLEM 12.118

KNOWN: Dimensions, spectral absorptivity, and temperature of solar receiver. Solar irradiation and ambient temperature.

FIND: (a) Rate of energy collection q and collector efficiency η, (b) Effect of receiver temperature on q and η.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Uniform irradiaton, (3) Opaque, diffuse surface.

PROPERTIES: *Table A.4*, air (T_f = 550 K): $\nu = 45.6 \times 10^{-6}$ m²/s, k = 0.0439 W/m·K, $\alpha = 66.7 \times 10^{-6}$ m²/s, Pr = 0.683.

ANALYSIS: (a) The rate of heat transfer to the receiver is q = $A_S\left(\alpha_S G_S - E - q''_{conv}\right)$, or

$$q = \pi DL\left[\alpha_S G_S - \varepsilon\sigma T_s^4 - \overline{h}\left(T_s - T_\infty\right)\right]$$

For $\lambda T = 3\ \mu m \times 5800$ K = 17,400, $F_{(0\to\lambda)} = 0.979$. Hence,

$$\alpha_S = \alpha_1 F_{(0\to\lambda)} + \alpha_2\left(1 - F_{(0\to\lambda)}\right) = 0.9\times0.979 + 0.2(0.021) = 0.885$$

For $\lambda T = 3\ \mu m \times 800$ K = 2400 μm·K, $F_{(0\to\lambda)} = 0.140$. Hence,

$$\varepsilon = \varepsilon_1 F_{(0\to\lambda)} + \varepsilon_2\left(1 - F_{(0\to\lambda)}\right) = 0.9\times0.140 + 0.2(0.860) = 0.298.$$

With $Ra_L = g\beta(T_s - T_\infty)L^3/\alpha\nu = 9.8\ m/s^2(1/550\ K)(500\ K)(12\ m)^3/66.7 \times 10^{-6}\ m^2/s \times 45.6 \times 10^{-6}\ m^2/s = 5.06 \times 10^{12}$, Eq. 9.26 yields

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1 + (0.492/Pr)^{9/16}\right]^{8/27}}\right\}^2 = 1867$$

$$\overline{h} = \overline{Nu}_L \frac{k}{L} = 1867 \frac{0.0439\ W/m\cdot K}{12\ m} = 6.83\ W/m^2\cdot K$$

Hence,

$$q = \pi(7\ m \times 12\ m)\left[0.885\times80,000\ W/m^2 - 0.298\times5.67\times10^{-8}\ W/m^2\cdot K^4\,(800\ K)^4 - 6.83\ W/m^2\cdot K\,(500\ K)\right]$$

$$q = 263.9\ m^2\,(70,800 - 6,920 - 3415)\ W/m^2 = 1.60\times10^7\ W$$ <

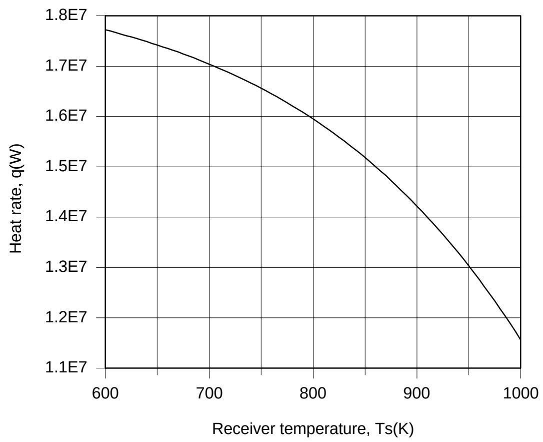
The collector efficiency is $\eta = q/A_s G_S$. Hence

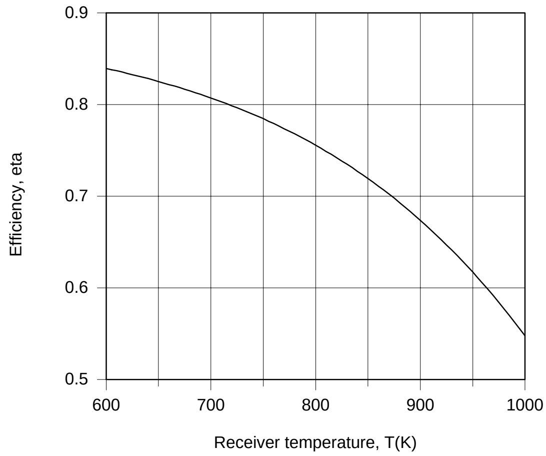
$$\eta = \frac{1.60\times10^7\ W}{263.9\ m^2\left(80,000\ W/m^2\right)} = 0.758$$ <

Continued …..

PROBLEM 12.118 (Cont.)

(b) The IHT *Correlations*, *Properties* and *Radiation* Toolpads were used to obtain the following results.





Losses due to emission and convection increase with increasing T_s, thereby reducing q and η.

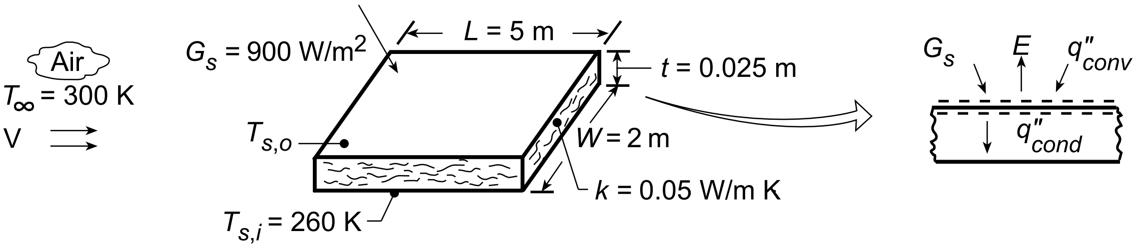
COMMENTS: The increase in radiation emission is due to the increase in T_s, as well as to the effect of T_s on ε, which increases from 0.228 to 0.391 as T_s increases from 600 to 1000 K.

PROBLEM 12.119

KNOWN: Dimensions and construction of truck roof. Roof interior surface temperature. Truck speed, ambient air temperature, and solar irradiation.

FIND: (a) Preferred roof coating, (b) Roof surface temperature, (c) Heat load through roof, (d) Effect of velocity on surface temperature and heat load.

SCHEMATIC:



ASSUMPTIONS: (1) Turbulent boundary layer development over entire roof, (2) Constant properties, (3) Negligible atmospheric (sky) irradiation, (4) Negligible contact resistance.

PROPERTIES: *Table A.4*, Air ($T_{s,o} \approx 300$ K, 1 atm): $\nu = 15\times10^{-6}\ \mathrm{m^2/s}$, $k = 0.026\ \mathrm{W/m\cdot K}$, Pr = 0.71.

ANALYSIS: (a) To minimize heat transfer through the roof, minimize solar absorption relative to surface emission. Hence, use zinc oxide white for which $\alpha_S = 0.16$ and $\varepsilon = 0.93$. <

(b) Performing an energy balance on the outer surface of the roof, $\alpha_S G_S + q''_{conv} - E - q''_{cond} = 0$, it follows that

$$\alpha_S G_S + \bar{h}(T_\infty - T_{s,o}) = \varepsilon\sigma T_{s,o}^4 + (k/t)(T_{s,o} - T_{s,i})$$

where it is assumed that convection is from the air to the roof. With

$$Re_L = \frac{VL}{\nu} = \frac{30\ \mathrm{m/s}(5\ \mathrm{m})}{15\times10^{-6}\ \mathrm{m^2/s}} = 10^7$$

$$\overline{Nu}_L = 0.037\, Re_L^{4/5}\, Pr^{1/3} = 0.037(10^7)^{4/5}(0.71)^{1/3} = 13{,}141$$

$$\bar{h} = \overline{Nu}_L(k/L) = 13{,}141(0.026\ \mathrm{W/m\cdot K}/5\ \mathrm{m} = 68.3\ \mathrm{W/m^2\cdot K}\ .$$

Substituting numerical values in the energy balance and solving by trial-and-error, we obtain

$T_{s,o}$ = 295.2 K. <

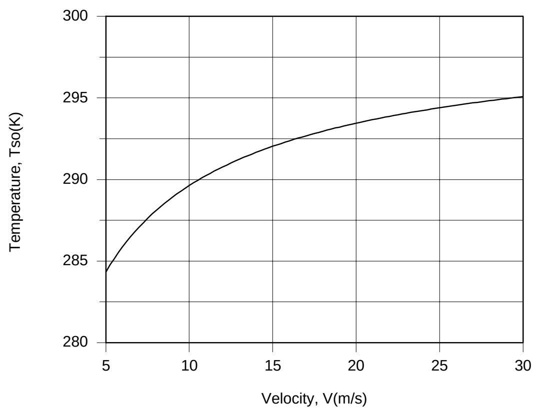
(c) The heat load through the roof is

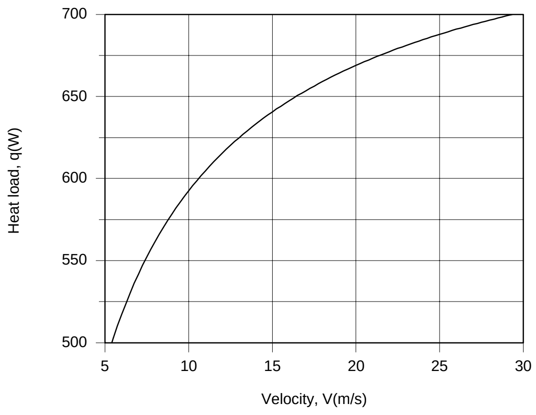
$$q = (kA_s/t)(T_{s,o} - T_{s,i}) = (0.05\ \mathrm{W/m\cdot K}\times 10\ \mathrm{m^2}/0.025\ \mathrm{m})35.2\ \mathrm{K} = 704\ \mathrm{W}\ .$$ <

(d) Using the IHT *First Law* Model with the *Correlations* and *Properties* Toolpads, the following results are obtained.

Continued...

PROBLEM 12.119 (Cont.)





The surface temperature and heat load decrease with decreasing V due to a reduction in the convection heat transfer coefficient and hence convection heat transfer from the air.

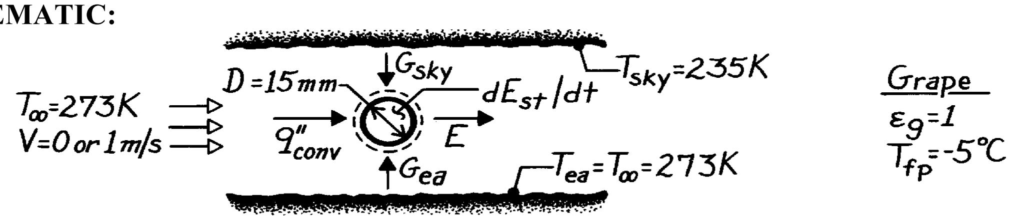
COMMENTS: The heat load would increase with increasing α_S/ε.

PROBLEM 12.120

KNOWN: Sky, ground, and ambient air temperatures. Grape of prescribed diameter and properties.

FIND: (a) General expression for rate of change of grape temperature, (b) Whether grapes will freeze in quiescent air, (c) Whether grapes will freeze for a prescribed air speed.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible temperature gradients in grape, (2) Uniform blackbody irradiation over top and bottom hemispheres, (3) Properties of grape are those of water at 273 K, (4) Properties of air are constant at values for T_∞, (5) Negligible buoyancy for V = 1 m/s.

PROPERTIES: *Table A-6*, Water (273 K): c_p = 4217 J/kg·K, ρ = 1000 kg/m^3; *Table A-4*, Air (273 K, 1 atm): $\nu = 13.49 \times 10^{-6}$ m^2/s, k = 0.0241 W/m·K, $\alpha = 18.9 \times 10^{-6}$ m^2/s, Pr = 0.714, $\beta = 3.66 \times 10^{-3}$ K^{-1}.

ANALYSIS: (a) Performing an energy balance for a control surface about the grape,

$$\frac{dE_{st}}{dt} = \rho_g \frac{\pi D^3}{6} c_{p\cdot g} \frac{dT_g}{dt} = \bar{h}\pi D^2 \left(T_\infty - T_g\right) + \frac{\pi D^2}{2}\left(G_{ea} + G_{sky}\right) - E\pi D^2.$$

Hence, the rate of temperature change with time is

$$\frac{dT_g}{dt} = \frac{6}{\rho_g c_{p\cdot g} D}\left[\bar{h}\left(T_\infty - T_g\right) + \sigma\left(\left(T_{ea}^4 + T_{sky}^4\right)/2 - \varepsilon_g T_g^4\right)\right].$$ <

(b) The grape freezes if $dT_g/dt < 0$ when $T_g = T_{fp} = 268$ K. With

$$Ra_D = \frac{g\beta\left(T_\infty - T_g\right)D^3}{\alpha\nu} = \frac{9.8\ \text{m/s}^2\left(3.66\times10^{-3}\ \text{K}^{-1}\right)5\text{K}\left(0.015\ \text{m}\right)^3}{18.9\times10^{-6}\times13.49\times10^{-6}\ \text{m}^4/\text{s}^2} = 2374$$

using Eq. 9.35 find

$$\overline{Nu}_D = 2 + \frac{0.589(2374)^{1/4}}{\left[1+(0.469/\text{Pr})^{9/16}\right]^{4/9}} = 5.17$$

$$\bar{h} = (k/D)\,\overline{Nu}_D = \left[(0.0241\,\text{W/m}\cdot\text{K})/(0.015\text{m})\right]5.17 = 8.31\,\text{W/m}^2\cdot\text{K}.$$

Hence, the rate of temperature change is

$$\frac{dT_g}{dt} = \frac{6}{\left(1000\ \text{kg/m}^3\right)4217\ \text{J/kg}\cdot\text{K}\left(0.015\ \text{m}\right)}\left[8.31\,\text{W/m}^2\cdot\text{K}(5\ \text{K})\right.$$

$$\left. +5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\left[\left(273^4+235^4\right)/2-268^4\right]\text{K}^4\right.$$

Continued …..

$$\frac{dT_g}{dt} = 9.49\times10^{-5}\ \mathrm{K\cdot m^2/J}\left[41.55 - 48.56\right]\mathrm{W/m^2} = -6.66\times10^{-4}\ \mathrm{K/s}$$ <

and since $dT_g/dt < 0$, the grape *will freeze*.

(c) For V = 1 m/s,

$$Re_D = \frac{VD}{\nu} = \frac{1\ \mathrm{m/s}\left(0.015\ \mathrm{m}\right)}{13.49\times10^{-6}\ \mathrm{m^2/s}} = 1112.$$

Hence with $(\mu/\mu_s)^{1/4} = 1$,

$$\overline{Nu}_D = 2 + \left(0.4Re_D^{1/2} + 0.06Re_D^{2/3}\right)Pr^{0.4} = 21.8$$

$$\bar{h} = \overline{Nu}_D\,\frac{k}{D} = 21.8\frac{0.0241}{0.015} = 35\ \mathrm{W/m^2\cdot K}.$$

Hence the rate of temperature change with time is

$$\frac{dT_g}{dt} = 9.49\times10^{-5}\ \mathrm{K\cdot m^2/J}\left[35\ \mathrm{W/m^2\cdot K}\left(5\ \mathrm{K}\right) - 48.56\ \mathrm{W/m^2}\right] = 0.012\ \mathrm{K/s}$$

and since $dT_g/dt > 0$, the grape *will not freeze*. <

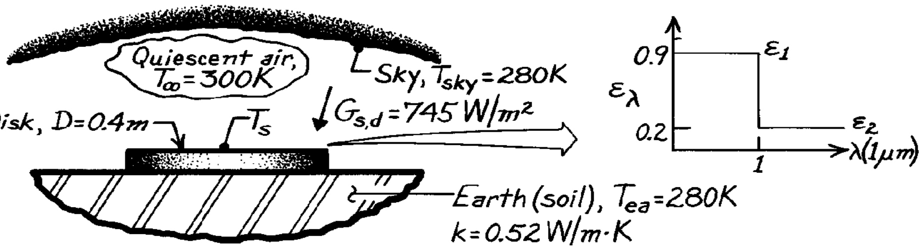
COMMENTS: With $Gr_D = Ra_D/Pr = 3325$ and $Gr_D/Re_D^2 = 0.0027$, the assumption of negligible buoyancy for V = 1 m/s is reasonable.

PROBLEM 12.121

KNOWN: Metal disk exposed to environmental conditions and placed in good contact with the earth.

FIND: (a) Fraction of direct solar irradiation absorbed, (b) Emissivity of the disk, (c) Average free convection coefficient of the disk upper surface, (d) Steady-state temperature of the disk (confirm the value 340 K).

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Disk is diffuse, (3) Disk is isothermal, (4) Negligible contact resistance between disk and earth, (5) Solar irradiance has spectral distribution of $E_{\lambda,b}$ (λ, 5800 K).

PROPERTIES: *Table A-4*, Air (1 atm, $T_f = (T_s + T_\infty)/2 = (340 + 300)$ K/2 = 320 K): $\nu = 17.90 \times 10^{-6}$ m^2/s, k = 0.0278 W/m·K, $\alpha = 25.5 \times 10^{-6}$ m^2/s, Pr = 0.704.

ANALYSIS: (a) The solar absorptivity follows from Eq. 12.49 with $G_{\lambda,S}$ α $E_{\lambda,b}$ (λ, 5800 K), and $\alpha_\lambda = \varepsilon_\lambda$ since the disk surface is diffuse.

$$\alpha_S = \int_0^\infty \alpha_\lambda E_{\lambda,b}(\lambda,\ 5800\text{K}) / E_b(5800\ \text{K})$$

$$\alpha_S = \varepsilon_1 F_{(0\to 1\mu\text{m})} + \varepsilon_2\left(1 - f_{(0\to 1\mu\text{m})}\right).$$

From Table 12.1 with

$$\lambda T = 1\ \mu\text{m} \times 5800\ \text{K} = 5800\ \mu\text{m}\cdot\text{K} \quad \text{find} \quad F_{(0\to\lambda T)} = 0.720$$

giving

$$\alpha_S = 0.9 \times 0.720 + 0.2(1 - 0.720) = 0.704.$$ <

Note this value is appropriate for diffuse or direct solar irradiation since the surface is diffuse.

(b) The emissivity of the disk depends upon the surface temperature T_s which we believe to be 340 K. (See part (d)). From Eq. 12.38,

$$\varepsilon = \int_0^\infty \varepsilon_\lambda E_{\lambda,b}(\lambda,\ T_s) / E_b(T_s)$$

$$\varepsilon = \varepsilon_1 F_{(0\to 1\mu\text{m})} + \varepsilon_2\left(1 - F_{(0\to 1\ \mu\text{m})}\right)$$

Continued …..

PROBLEM 12.121 (Cont.)

From Table 12.1 with

$$\lambda T = 1\ \mu m \times 340\ K = 340\ \mu m \cdot K \qquad \text{find} \qquad F_{(0\to\lambda T)} = 0.000$$

giving

$$\varepsilon = 0.9 \times 0.000 + 0.2(1 - 0.000) = 0.20.$$ <

(c) The disk is a hot surface facing upwards for which the free convection correlation of Eq. 9.30 is appropriate. Evaluating properties at $T_f = (T_s + T_\infty)/2 = 320$ K,

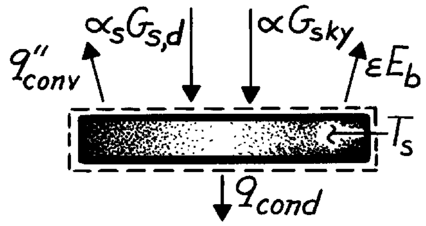
$$Ra_L = g\beta\Delta T L^3/\nu\alpha \qquad \text{where} \quad L = A_s/P = D/4$$

$$Ra_L = 9.8\ m/s^2(1/320\ K)(340-300)K(0.4\ m/4)^3/17.90\times10^{-6}\ m^2/s\times25.5\times10^{-6}\ m^2/s = 3.042\times10^6$$

$$\overline{Nu}_L = \bar{h}L/k = 0.54 Ra_L^{1/4} \qquad 10^4 \le Ra_L \le 10^7$$

$$\bar{h} = 0.0278\ W/m\cdot K/(0.4\ m/4)\times0.54\left(3.042\times10^6\right)^{1/4} = 6.37\ W/m^2\cdot K.$$ <

(d) To determine the steady-state temperature, perform an energy balance on the disk.



$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$$

$$\left(\alpha_S G_{s,d} + \alpha G_{sky} - \varepsilon E_b - q''_{conv}\right)A_s - q_{cond} = 0.$$

Since G_{sky} is predominately long wavelength radiation, it follows that $\alpha = \varepsilon$. The conduction heat rate between the disk and the earth is

$$q_{cond} = kS(T_s - T_{ea}) = k(2D)(T_s - T_{ea})$$

where S, the conduction shape factor, is that of an isothermal disk on a semi-infinite medium, Table 4.1. Substituting numerical values, with $A_s = \pi D^2/4$,

$$\Big[0.704\times745\ W/m^2 + 0.20\sigma(280\ K)^4 - 0.20\sigma T_s^4$$

$$-6.3\ W/m^2\cdot K(T_s - 300\ K)\Big]\pi/4(0.4\ m)^2 - 0.52\ W/m\cdot K(2\times0.4\ m)(T_s - 280\ K) = 0$$

$$65.908\ W + 8.759\ W - 1.425\times10^{-9}T_s^4 - 0.792(T_s - 300) - 0.416(T_s - 280) = 0.$$

By trial-and-error, find

$$T_s \approx 339\ K.$$ <

so indeed the assumed value of 340 K was proper.

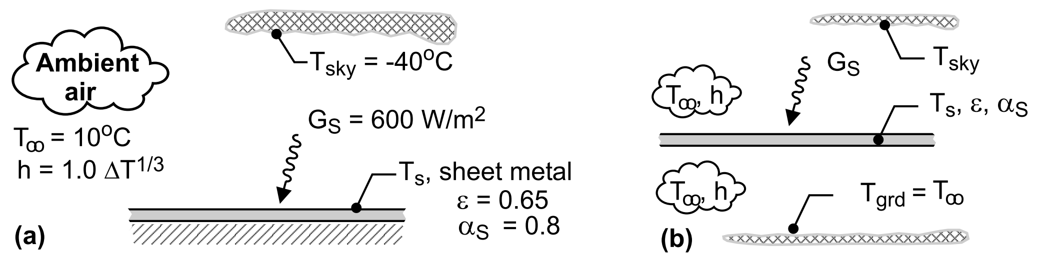
COMMENTS: Note why it is not necessary for this situation to distinguish between direct and diffuse irradiation. Why does $\alpha_{sky} = \varepsilon$?

PROBLEM 12.122

KNOWN: Shed roof of weathered galvanized sheet metal exposed to solar insolation on a cool, clear spring day with ambient air at - 10°C and convection coefficient estimated by the empirical correlation $\overline{h} = 1.0\ \Delta T^{1/3}$ (W/m^2·K with temperature units of kelvins).

FIND: Temperature of the roof, T_s, (a) assuming the backside is well insulated, and (b) assuming the backside is exposed to ambient air with the same convection coefficient relation and experiences radiation exchange with the ground, also at the ambient air temperature. Comment on whether the roof will be a comfortable place for the neighborhood cat to snooze for these conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) The roof surface is diffuse, spectrally selective, (3) Sheet metal is thin with negligible thermal resistance, and (3) Roof is a small object compared to the large isothermal surroundings represented by the sky and the ground.

ANALYSIS: (a) For the backside-insulated condition, the energy balance, represented schematically below, is

$$\dot{E}''_{in} - \dot{E}''_{out} = 0$$

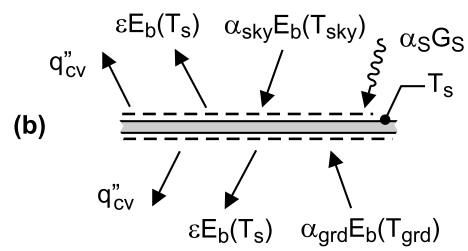
$$\alpha_{sky}\ E_b\left(T_{sky}\right) + \alpha_S G_S - q''_{cv} - \varepsilon\, E_b\left(T_s\right) = 0$$

$$\alpha_{sky}\sigma T_{sky}^4 + \alpha_S G_S - 1.0\left(T_s - T_\infty\right)^{4/3} - \varepsilon\sigma T_s^4 = 0$$

With $\alpha_{sky} = \varepsilon$ (see Comment 2) and $\sigma = 5.67\times10^{-8}\ \mathrm{W/m^2\cdot K^4}$, find T_s.

$$0.65\ \sigma(233\ \mathrm{K})^4\ \mathrm{W/m^2} + 0.8\times600\ \mathrm{W/m^2} - 1.0\left(T_s - 283\ \mathrm{K}\right)^{4/3}\ \mathrm{W/m^2} - 0.65\ \sigma T_s^4 = 0$$

$$T_s = 312.5\ \mathrm{K} = 39.5^\circ\mathrm{C}$$ <



Energy balances: backside condition-
(a) insulated, (b) exposed to air/ground

Continued …..

(b) With the backside exposed to convection with the ambient air and radiation exchange with the ground, the energy balance, represented schematically above, is

$$\alpha_{sky} E_b\left(T_{sky}\right) + \alpha_{grd} E_b\left(T_{grd}\right) + \alpha_S G_S - 2q''_{cv} - 2\varepsilon E_b\left(T_s\right) = 0$$

Substituting numerical values, recognizing that $T_{grd} = T_\infty$, and $\alpha_{grd} = \varepsilon$ (see Comment 2), find T_s.

$$0.65\ \sigma(233\ \mathrm{K})^4\ \mathrm{W/m^2} + 0.65\ \sigma(283\ \mathrm{K})^4 \mathrm{W/m^2} + 0.8 \times 600\ \mathrm{W/m^2}$$
$$-2 \times 1.0\ (T_s - 283\ \mathrm{K})^{4/3}\ \mathrm{W/m^2} - 2 \times 0.65\ \sigma T_s^4 = 0$$

$$T_s = 299.5\ \mathrm{K} = 26.5^\circ\mathrm{C}$$ <

COMMENTS: (1) For the insulated-backside condition, the cat would find the roof quite warm remembering that 43°C represents a safe-to-touch temperature. For the exposed-backside condition, the cat would find the roof comfortable, certainly compared to an area not exposed to the solar insolation (that is, exposed only to the ambient air through convection).

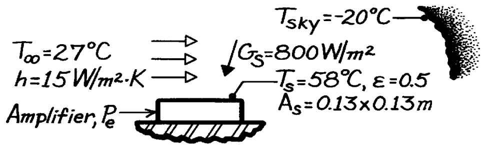
(2) For this spectrally selective surface, the absorptivity for the sky irradiation is equal to the emissivity, $\alpha_{sky} = \varepsilon$, since the sky irradiation and surface emission have the same approximate spectral regions. The same reasoning applies for the absorptivity of the ground irradiation, $\alpha_{grd} = \varepsilon$.

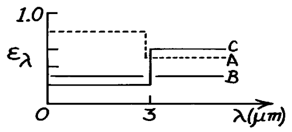
PROBLEM 12.123

KNOWN: Amplifier operating and environmental conditions.

FIND: (a) Power generation when $T_s = 58°C$ with diffuse coating $\varepsilon = 0.5$, (b) Diffuse coating from among three (A, B, C) which will give greatest reduction in T_s, and (c) Surface temperature for the conditions with coating chosen in part (b).

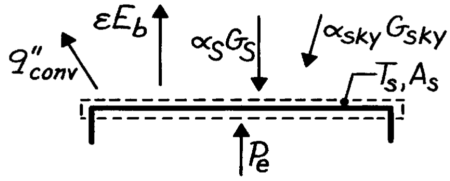
SCHEMATIC:





ASSUMPTIONS: (1) Environmental conditions remain the same with all surface coatings, (2) Coatings A, B, C are opaque, diffuse.

ANALYSIS: (a) Performing an energy balance on the amplifier's exposed surface, $\dot{E}_{in} - \dot{E}_{out} = 0$, find



$$P_e + A_s\left[\alpha_S G_S + \alpha_{sky} G_{sky} - \varepsilon E_b - q''_{conv}\right] = 0$$

$$P_e = A_s\left[\varepsilon\sigma T_s^4 + h\left(T_s - T_\infty\right) - \alpha_S G_S - \alpha_{sky}\sigma T_{sky}^4\right]$$

$$P_e = 0.13\times 0.13\ m^2\left[0.5\times\sigma(331)^4 + 15(331-300) - 0.5\times 800 - 0.5\times\sigma(253)^4\right] W/m^2$$

$$P_e = 0.0169 m^2\ [0.5\times 680.6 + 465 - 0.5\times 800 - 0.5\times 232.3] W/m^2 = 4887\ \ W. \quad <$$

(b) From above, recognize that we seek a coating with low α_S and high ε to decrease T_s. Further, recognize that α_S is determined by values of $\alpha_\lambda = \varepsilon_\lambda$ for $\lambda < 3$ μm and ε by values of ε_λ for $\lambda > 3$ μm. Find approximate values as

Coating	A	B	C
ε	0.5	0.3	0.6
α_S	0.8	0.3	0.2
α_S/ε	1.6	1	0.333

Note also that $\alpha_{sky} \approx \varepsilon$. We conclude that coating C is likely to give the lowest T_s since its α_S/ε is substantially lower than for B and C. While α_{sky} for C is twice that of B, because G_{sky} is nearly 25% that of G_S, we expect coating C to give the lowest T_s.

(c) With the values of α_S, α_{sky} and ε for coating C from part (b), rewrite the energy balance as

$$P_e / A_s + \alpha_S G_S + \alpha_{sky}\sigma T_{sky}^4 - \varepsilon\sigma T_s^4 - h\left(T_s - T_\infty\right) = 0$$

$$4.887\ W/(0.13\ m)^2 + 0.2\times 800\ W/m^2 + 0.6\times 232.3\ W/m^2 - 0.6\times\sigma T_s^4 - 15\left(T_s - 300\right) = 0$$

Using trial-and-error, find $T_s = 316.5\ K = 43.5°C$. $<$

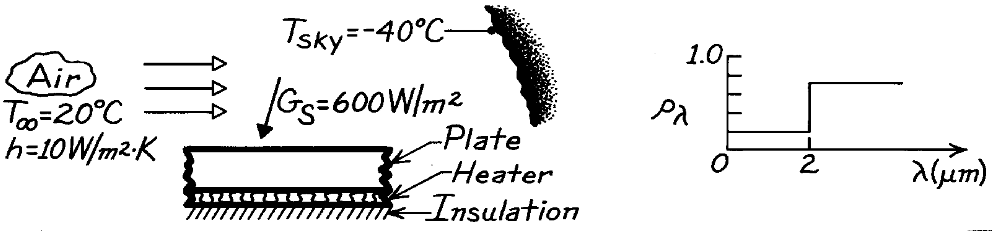
COMMENTS: (1) Using coatings A and B, find T_s = 71 and 54°C, respectively. (2) For more precise values of α_S, α_{sky} and ε, use $T_s = 43.5°C$. For example, at $\lambda T_s = 3 \times (43.5 + 273) = 950$ μm·K, $F_{0-\lambda T} = 0.000$ while at $\lambda T_{solar} = 3 \times 5800 = 17{,}400$ μm·K, $F_{0-\lambda T} \approx 0.98$; we conclude little effect will be seen.

PROBLEM 12.124

KNOWN: Opaque, spectrally-selective horizontal plate with electrical heater on backside is exposed to convection, solar irradiation and sky irradiation.

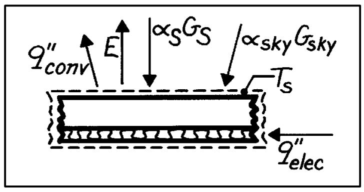
FIND: Electrical power required to maintain plate at 60°C.

SCHEMATIC:



ASSUMPTIONS: (1) Plate is opaque, diffuse and uniform, (2) No heat lost out the backside of heater.

ANALYSIS: From an energy balance on the plate-heater system, per unit area basis,



$$\dot{E}''_{in} - \dot{E}''_{out} = 0$$

$$q''_{elec} + \alpha_S G_S + \alpha G_{sky} - \varepsilon E_b(T_s) - q''_{conv} = 0$$

where $G_{sky} = \sigma T_{sky}^4$, $E_b = \sigma T_s^4$, and $q''_{conv} = h(T_s - T_\infty)$. The *solar absorptivity* is

$$\alpha_S = \int_0^\infty \alpha_\lambda G_{\lambda,S} d\lambda \Big/ \int_0^\infty G_{\lambda,S} d\lambda = \int_0^\infty \alpha_\lambda E_{\lambda,b}(\lambda, 5800\text{ K}) d\lambda \Big/ \int_0^\infty E_{\lambda,b}(\lambda, 5800\text{K}) d\lambda$$

where $G_{\lambda,S} \sim E_{\lambda,b}$ (λ, 5800 K). Noting that $\alpha_\lambda = 1 - \rho_\lambda$,

$$\alpha_S = (1-0.2) F_{(0-2\mu m)} + (1-0.7)\left(1 - F_{(0-2\mu m)}\right)$$

where at $\lambda T = 2\ \mu m \times 5800\ K = 11{,}600\ \mu m{\cdot}K$, find from Table 12.1, $F_{(0-\lambda T)} = 0.941$,

$$\alpha_S = 0.80 \times 0.941 + 0.3(1 - 0.941) = 0.771.$$

The *total, hemispherical emissivity* is

$$\varepsilon = (1-0.2) F_{(0-2\mu m)} + (1-0.7)\left(1 - F_{(0-2\mu m)}\right).$$

At $\lambda T = 2\ \mu m \times 333\ K = 666\ K$, find $F_{(0-\lambda T)} \approx 0.000$; hence $\varepsilon = 0.30$. The *total, hemispherical absorptivity* for sky irradiation is $\alpha = \varepsilon = 0.30$ since the surface is gray for this emission and irradiation process. Substituting numerical values,

$$q''_{elec} = \varepsilon\sigma T_s^4 + h(T_s - T_\infty) - \alpha_S G_S - \alpha\sigma T_{sky}^4$$

$$q''_{elec} = 0.30 \times \sigma (333\text{ K})^4 + 10\text{ W/m}^2\cdot\text{K}(60-20)°\text{C} - 0.771 \times 600\text{ W/m}^2 - 0.30 \times \sigma(233\text{ K})^4$$

$$q''_{elec} = 209.2\text{ W/m}^2 + 400.0\text{ W/m}^2 - 462.6\text{ W/m}^2 - 50.1\text{ W/m}^2 = 96.5\text{ W/m}^2.$$ <

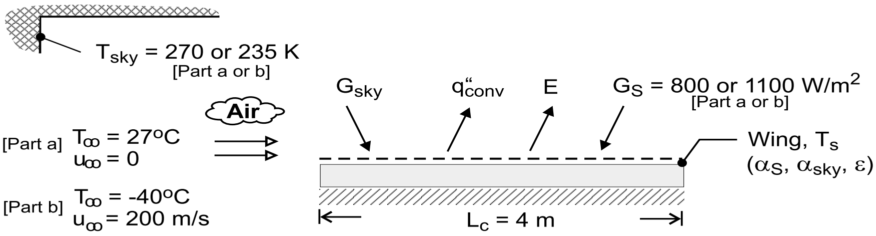
COMMENTS: (1) Note carefully why $\alpha_{sky} = \varepsilon$ for the sky irradiation.

PROBLEM 12.125

KNOWN: Chord length and spectral emissivity of wing. Ambient air temperature, sky temperature and solar irradiation for ground and in-flight conditions. Flight speed.

FIND: Temperature of top surface of wing for (a) ground and (b) in-flight conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Negligible heat transfer from back of wing surface, (3) Diffuse surface behavior, (4) Negligible solar radiation for $\lambda > 3\ \mu$m ($\alpha_S = \alpha_{\lambda \le 3\ \mu m} = \varepsilon_{\lambda \le 3\ \mu m} = 0.6$), (5) Negligible sky radiation and surface emission for $\lambda \le 3\ \mu$m ($\alpha_{sky} = \alpha_{\lambda > 3\ \mu m} = \varepsilon_{\lambda > 3\ \mu m} = 0.3 = \varepsilon$), (6) Quiescent air for ground condition, (7) Air foil may be approximated as a flat plate, (8) Negligible viscous heating in boundary layer for in-flight condition, (9) The wing span W is much larger than the chord length L_c, (10) In-flight transition Reynolds number is 5×10^5.

PROPERTIES: Part (a). *Table A-4*, air ($T_f \approx 325$ K): $\nu = 1.84 \times 10^{-5}$ m^2/s, $\alpha = 2.62 \times 10^{-5}$ m^2/s, k = 0.0282 W/m·K, $\beta = 0.00307$. Part (b). Given: $\rho = 0.470$ kg/m^3, $\mu = 1.50 \times 10^{-5}$ N·s/m^2, k = 0.021 W/m·K, Pr = 0.72.

ANALYSIS: For both ground and in-flight conditions, a surface energy balance yields

$$\alpha_{sky} G_{sky} + \alpha_S G_S = \varepsilon\sigma T_s^4 + \bar{h}\left(T_s - T_\infty\right) \quad (1)$$

where $\alpha_{sky} = \varepsilon = 0.3$ and $\alpha_S = 0.6$.

(a) For the ground condition, $\bar{h}$ may be evaluated from Eq. 9.30 or 9.31, where $L = A_s/P = L_c \times W/2\,(L_c + W) \approx L_c/2 = 2$m and $Ra_L = g\beta\,(T_s - T_\infty)\,L^3/\nu\alpha$. Using the *IHT* software to solve Eq. (1) and accounting for the effect of temperature-dependent properties, the surface temperature is

$$T_s = 350.6\ \text{K} = 77.6°\text{C} \quad <$$

where $Ra_L = 2.52 \times 10^{10}$ and $\bar{h} = 6.2$ W/m^2·K. Heat transfer from the surface by emission and convection is 257.0 and 313.6 W/m^2, respectively.

(b) For the in-flight condition, $Re_L = \rho u_\infty L_c/\mu = 0.470$ kg/m$^3 \times 200$ m/s $\times$ 4m/1.50×10^{-5} N·s/m^2 = 2.51×10^7. For mixed, laminar/turbulent boundary layer conditions (Section 7.2.3 of text) and a transition Reynolds number of $Re_{x,c} = 5 \times 10^5$.

$$\overline{Nu}_L = \left(0.037\, Re_L^{4/5} - 871\right) Pr^{1/3} = 26,800$$

$$\bar{h} = \frac{k}{L}\overline{Nu}_L = \frac{0.021\,\text{W/m}\cdot\text{K} \times 26,800}{4\text{m}} = 141\ \text{W/m}^2\cdot\text{K}$$

Substituting into Eq. (1), a trial-and-error solution yields

$$T_s = 237.7\ \text{K} = -35.3°\text{C} \quad <$$

Heat transfer from the surface by emission and convection is now 54.3 and 657.6 W/m^2, respectively.

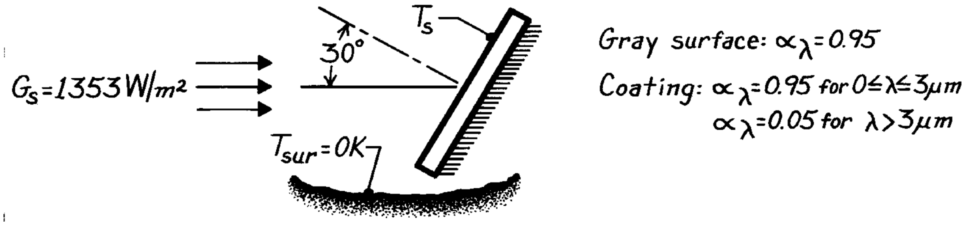
COMMENTS: The temperature of the wing is strongly influenced by the convection heat transfer coefficient, and the large coefficient associated with flight yields a surface temperature that is within 5°C of the air temperature.

PROBLEM 12.126

KNOWN: Spectrally selective and gray surfaces in earth orbit are exposed to solar irradiation, G_S, in a direction 30° from the normal to the surfaces.

FIND: Equilibrium temperature of each plate.

SCHEMATIC:



ASSUMPTIONS: (1) Plates are at uniform temperature, (2) Surroundings are at 0K, (3) Steady-state conditions, (4) Solar irradiation has spectral distribution of $E_{\lambda,b}(\lambda, 5800K)$, (5) Back side of plate is insulated.

ANALYSIS: Noting that the solar irradiation is directional (at 30° from the normal), the radiation balance has the form

$$\alpha_S G_S \cos\theta - \varepsilon E_b(T_s) = 0. \tag{1}$$

Using $E_b(T_s) = \sigma T_s^4$ and solving for T_s, find

$$T_s = \left[(\alpha_S/\varepsilon)\,(G_S\,\cos\theta/\sigma)\right]^{1/4}. \tag{2}$$

For the *gray surface*, $\alpha_S = \varepsilon = \alpha_\lambda$ and the temperature is independent of the magnitude of the absorptivity.

$$T_s = \left(\frac{0.95}{0.95}\times\frac{1353\ \mathrm{W/m^2}\times\cos 30°}{5.67\times10^{-8}\ \mathrm{W/m^2\cdot K^4}}\right)^{1/4} = 379\ \mathrm{K}.$$ <

For the *selective surface*, $\alpha_S = 0.95$ since nearly all the solar spectral power is in the region $\lambda < 3\mu m$. The value of ε depends upon the surface temperature T_s and would be determined by the relation.

$$\varepsilon = 0.95\ F_{(0\to\lambda T_s)} + 0.05\left[1 - F_{(0\to\lambda T_s)}\right] \tag{3}$$

where $\lambda = 3\mu m$ and T_s is as yet unknown. To find T_s, a trial-and-error procedure as follows will be used: (1) assume a value of T_s, (2) using Eq. (3), calculate ε with the aid of Table 12.1 evaluating $F_{(0\to\lambda T)}$ at $\lambda T_s = 3\mu m\cdot T_s$, (3) with this value of ε, calculate T_s from Eq. (2) and compare with assumed value of T_s. The results of the iterations are:

T_s(K), assumed value	633	700	666	650	655
ε, from Eq. (3)	0.098	0.125	0.110	0.104	0.106
T_s(K), from Eq. (2)	656	629	650	659	656

Hence, for the coating, $T_s \approx 656$K. <

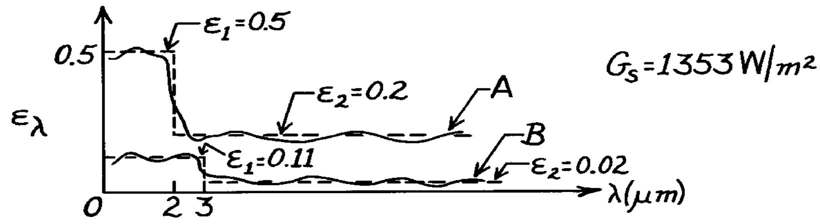
COMMENTS: Note the role of the ratio α_s/ε in determining the equilibrium temperature of an isolated plate exposed to solar irradiation in space. This is an important property of the surface in spacecraft thermal design and analysis.

PROBLEM 12.127

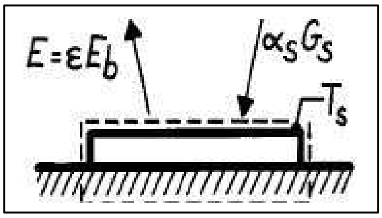
KNOWN: Spectral, hemispherical emissivity distributions for two panels subjected to solar flux in the deep space environment.

FIND: Steady-state temperatures of the panels.

SCHEMATIC:



ASSUMPTIONS: (1) Surfaces are opaque and diffuse, (2) Panels are oriented normal to solar flux with backside insulated, (3) Steady-state conditions, (4) No convection.



ANALYSIS: An energy balance on the panel is

$$q''_{in} - q''_{out} = 0$$

$$\alpha_S G_S - \varepsilon E_b(T_s) = \alpha_S G_S - \varepsilon\sigma T_s^4 = 0$$

$$T_s = \left[(\alpha_S/\varepsilon)(G_S/\sigma)\right]^{1/4}.$$

For each panel determine α_S and ε. Recognizing that $G_{\lambda,S} \sim E_{\lambda,b}$ (λ, 5800K), the solar absorptivity from Eq. 12.47 is

$$\alpha_S = \frac{\int_0^\infty \alpha_\lambda G_{\lambda,S}\, d\lambda}{\int_0^\infty G_{\lambda,S}\, d\lambda} = \frac{\int_0^\infty \varepsilon_\lambda E_{\lambda,b}(\lambda,5800\text{K})}{E_b(5800\text{K})}.$$

Note that $\varepsilon_\lambda = \alpha_\lambda$ since the surface is diffuse. Using Eq. 12.65 and Table 12.1 find

Surface A: $\alpha_S = \varepsilon_1 F_{(0\to 2\mu m)} + \varepsilon_2\left[1 - F_{(0\to 2\mu m)}\right]\}$ $\qquad \lambda T = 2\times 5800 = 11{,}600\mu m\cdot K,$

$\alpha_S = 0.5\times 0.940 + 0.2[1-0.940] = 0.482\}$ $\qquad F_{(0\to 2\mu m)} = 0.940$

Surface B: $\alpha_S = 0.11\times 0.979 + 0.02[1-0.979] = 0.108\}$ $\qquad \lambda T = 3\times 5800 = 17{,}400\mu m\cdot K,$ $F_{(0\to 3\mu m)} = 0.979.$

To determine the total emissivity, we need to know T_s. If $T_s \le 400$K, then for $\lambda T = 3\ \mu m \times 400K = 1200K$, $F_{(0\to\lambda)} = 0.002$. That is, there is negligible power for $\lambda < 3\ \mu m$ if $T_s \le 400$K, and hence

Surface A: $\varepsilon \approx \varepsilon_2 = 0.2$ $\qquad$ *Surface B*: $\varepsilon \approx \varepsilon_2 = 0.02.$

Substituting the solar absorptivity and emissivity values, find

Surface A: $$T_s = \left(\frac{0.482}{0.20}\times\frac{1353\ \text{W/m}^2}{5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}}\right)^{1/4} = 499\text{K}$$ <

Surface B: $$T_s = \left(\frac{0.108}{0.02}\times\frac{1353\ \text{W/m}^2}{5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}}\right)^{1/4} = 599\text{K}.$$ <

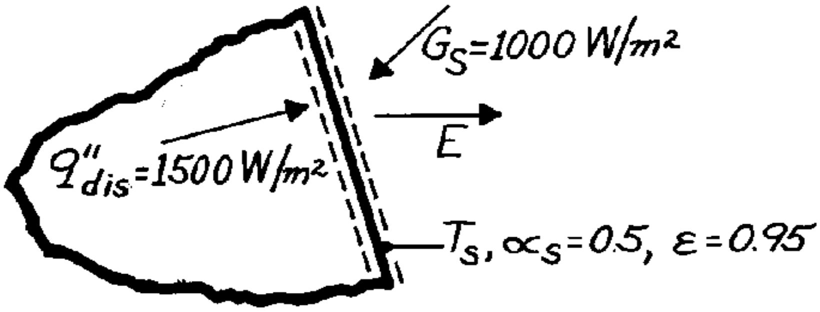
COMMENTS: (1) Note the assumption that $T_s \le 400$K used for finding ε is not satisfied; for better precision, it is necessary to perform an iterative solution. (2) Note the importance of the α_S/ε ratio which determines the surface temperature.

PROBLEM 12.128

KNOWN: Radiative properties and operating conditions of a space radiator.

FIND: Equilibrium temperature of the radiator.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible irradiation due to earth emission.

ANALYSIS: From a surface energy balance, $\dot{E}''_{in} - \dot{E}''_{out} = 0.$

$$q''_{dis} + \alpha_S G_S - E = 0.$$

Hence

$$T_s = \left(\frac{q''_{dis} + \alpha_S G_S}{\varepsilon \sigma} \right)^{1/4}$$

$$T_s = \left(\frac{1500\,\mathrm{W/m^2} + 0.5 \times 1000\,\mathrm{W/m^2}}{0.95 \times 5.67 \times 10^{-8}\,\mathrm{W/m^2 \cdot K^4}} \right)^{1/4}$$

or

$$T_s = 439\,\mathrm{K}.$$ <

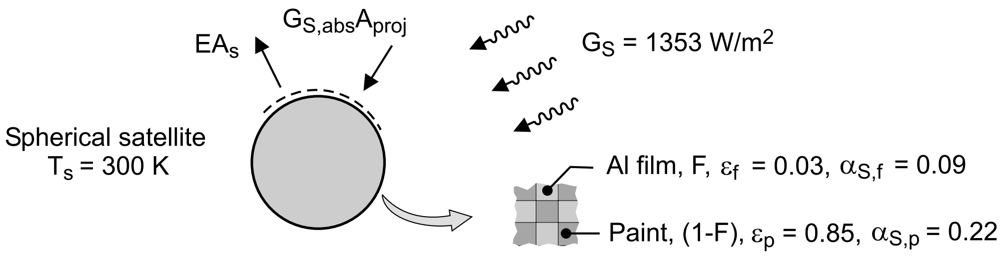
COMMENTS: *Passive* thermal control of spacecraft is practiced by using surface coatings with desirable values of α_S and ε.

PROBLEM 12.129

KNOWN: Spherical satellite exposed to solar irradiation of 1353 m^2; surface is to be coated with a checker pattern of evaporated aluminum film, (fraction, F) and white zinc-oxide paint (1 - F).

FIND: The fraction F for the checker pattern required to maintain the satellite at 300 K.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Satellite is isothermal, and (3) No internal power dissipation.

ANALYSIS: Perform an energy balance on the satellite, as illustrated in the schematic, identifying absorbed solar irradiation on the projected area, A_p, and emission from the spherical area A_s.

$$\dot{E}_{in} - \dot{E}_{out} = o$$

$$\left(F \cdot \alpha_{S,f} + (1-F) \cdot \alpha_{S,p}\right) G_S \, A_p - \left(F \cdot \varepsilon_f + (1-F) \cdot \varepsilon_p\right) E_b \, (T_s) \, A_s = 0$$

where $A_p = \pi D^2 / 4$, $A_s = \pi D^2$, $E_b = \sigma T^4$ and $\sigma = 5.67 \times 10^{-8}$ $W/m^2 \cdot K^4$. Substituting numerical values, find F.

$$\left(F \times 0.09 + (1-F) \times 0.22\right) \times 1353 \; W/m^2 \times (1/4)$$
$$- \left(F \times 0.03 + (1-F) \times 0.85\right) \sigma (300 \; K)^4 \times 1 = 0$$

$$F = 0.95$$ <

COMMENTS: (1) If the thermal control engineer desired to maintain the spacecraft at 325 K, would the fraction F (aluminum film) be increased or decreased? Verify your opinion with a calculation.

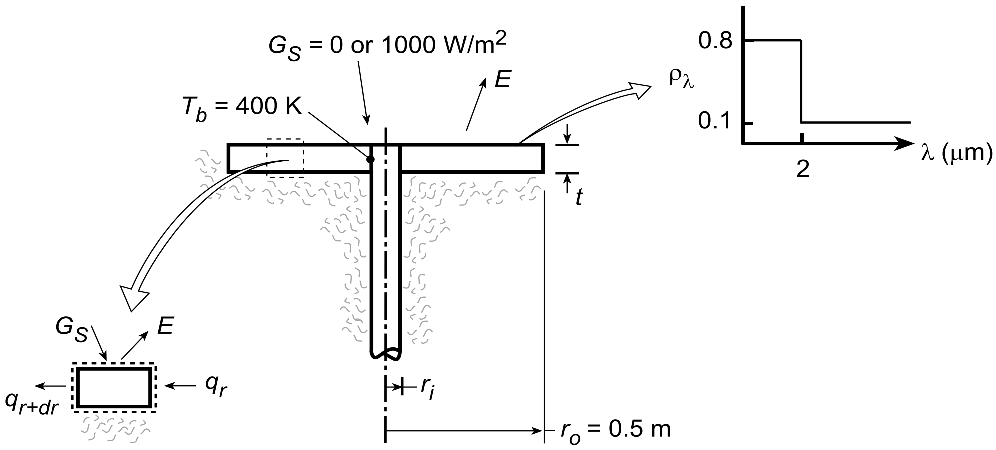
(2) If the internal power dissipation per unit surface area is 150 W/m^2, what fraction F will maintain the satellite at 300 K?

PROBLEM 12.130

KNOWN: Inner and outer radii, spectral reflectivity, and thickness of an annular fin. Base temperature and solar irradiation.

FIND: (a) Rate of heat dissipation if $\eta_f = 1$, (b) Differential equation governing radial temperature distribution in fin if $\eta_f < 1$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional radial conduction, (3) Adiabatic tip and bottom surface, (4) Opaque, diffuse surface ($\alpha_\lambda = 1 - \rho_\lambda,\ \varepsilon_\lambda = \alpha_\lambda$).

ANALYSIS: (a) If $\eta_f = 1$, $T(r) = T_b = 400$ K across the entire fin and

$$q_f = \left[\varepsilon E_b(T_b) - \alpha_S G_S\right]\pi r_o^2$$

With $\lambda T = 2\ \mu m \times 5800\ K = 11{,}600\ \mu m\cdot K$, $F_{(0\to 2\mu m)} = 0.941$. Hence $\alpha_S = \alpha_1 F_{(0\to 2\mu m)} + \alpha_2\left[1 - F_{(0\to 2\mu m)}\right] = 0.2 \times 0.941 + 0.9 \times 0.059 = 0.241$. With $\lambda T = 2\ \mu m \times 400\ K = 800\ \mu m\cdot K$, $F_{(0\to 2\mu m)} = 0$ and $\varepsilon = 0.9$. Hence, for $G_S = 0$,

$$q_f = 0.9 \times 5.67 \times 10^{-8}\ W/m^2\cdot K^4\,(400\,K)^4\,\pi\,(0.5\,m)^2 = 1026\,W \qquad <$$

and for $G_S = 1000\ W/m^2$,

$$q_f = 1026\,W - 0.241\left(1000\,W/m^2\right)\pi\,(0.5\,m)^2 = (1026 - 189)\,W = 837\,W \qquad <$$

(b) Performing an energy balance on a differential element extending from r to r+dr, we obtain

$$q_r + \alpha_S G_S\,(2\pi r dr) - q_{r+dr} - E\,(2\pi r dr) = 0$$

where

$$q_r = -k\,(dT/dr)\,2\pi r t \qquad \text{and} \qquad q_{r+dr} = q_r + (dq_r/dr)\,dr\,.$$

Hence,

$$\alpha_S G_S\,(2\pi r dr) - d\left[-k\,(dT/dr)\,2\pi r t\right]dr - E\,(2\pi r dr) = 0$$

$$2\pi r t k \frac{d^2T}{dr^2} + 2\pi t k \frac{dT}{dr} + \alpha_S G_S 2\pi r - E 2\pi r = 0$$

$$kt\left(\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr}\right) + \alpha_S G_S - \varepsilon\sigma T^4 = 0 \qquad <$$

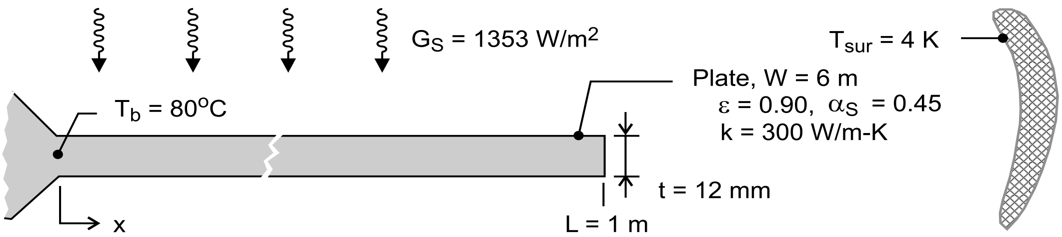
COMMENTS: The radiator should be constructed of a light weight, high thermal conductivity material (aluminum).

PROBLEM 12.131

KNOWN: Rectangular plate, with prescribed geometry and thermal properties, for use as a radiator in a spacecraft application. Radiator exposed to solar radiation on upper surface, and to deep space on both surfaces.

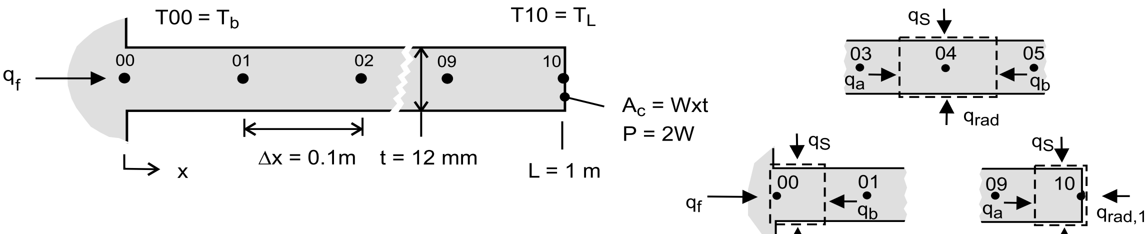
FIND: Using a computer-based, finite-difference method with a space increment of 0.1 m, find the tip temperature, T_L, and rate of heat rejection, q_f, when the base temperature is maintained at 80°C for the cases: (a) when exposed to the sun, (b) on the dark side of the earth, not exposed to the sun; and (c) when the thermal conductivity is extremely large. Compare the case (c) results with those obtained from a hand calculation assuming the radiator is at a uniform temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (b) Plate-radiator behaves as an extended surface with one-dimensional conduction, and (c) Radiating tip condition.

ANALYSIS: The finite-difference network with 10 nodes and a space increment Δx = 0.1 m is shown in the schematic below. The finite-difference equations (FDEs) are derived for an interior node (nodes 01 - 09) and the tip node (10). The energy balances are represented also in the schematic below where q_a and q_b represent conduction heat rates, q_S represents the absorbed solar radiation, and q_{rad} represents the radiation exchange with outer space.



Finite-difference network, energy balances

Interior node 04

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q_a + q_b + q_S + q_{rad} = 0$$

$$kA_c\left(T_{03} - T_{04}\right)/\Delta x + kA_c\left(T_{05} - T_{04}\right)/\Delta x$$

$$+\alpha_S\, G_S\,(P/2)\Delta x + \varepsilon P\Delta x\sigma\left(T_{sur}^4 - T_{04}^4\right) = 0$$

where $P = 2W$ and $A_c = W \cdot t$.

Tip node 10

$$q_a + q_S + q_{rad,1} + q_{rad,2} = 0$$

$$kA_c\left(T_{09} - T_{10}\right)/\Delta x + \alpha_S G_S\,(P/2)\,(\Delta x/2)$$

$$+\varepsilon\, A_c\sigma\left(T_{sur}^4 - T_{10}^4\right) + \varepsilon P(\Delta x/2)\sigma\left(T_{sur}^4 - T_{04}^4\right) = 0$$

Continued …..

Heat rejection, q_f. From an energy balance on the base node 00,

$$q_f + q_{01} + q_S + q_{rad} = 0$$

$$q_f + kA_c\left(T_{01} - T_{00}\right)/\Delta x + \alpha_S G_S \left(P/2\right)\left(\Delta x/2\right)$$
$$+\ \varepsilon P\left(\Delta x/2\right)\sigma\left(T_{sur}^4 - T_{00}^4\right) = 0$$

The foregoing nodal equations and the heat rate expression were entered into the *IHT* workspace to obtain solutions for the three cases. See Comment 2 for the *IHT* code, and Comment 1 for code validation remarks.

Case	k(W/m·K)	G_S(W/m^2)	T_L(°C)	q_f(W)	
a	300	1353	30.5	2766	<
b	300	0	-7.6	4660	<
c	1×10^{10}	0	80.0	9557	

COMMENTS: (1) Case (c) using the *IHT* code with $k = 1 \times 10^{10}$ W/m·K corresponds to the condition of the plate at the uniform temperature of the base; that is $T(x) = T_b$. For this condition, the heat rejection from the upper and lower surfaces and the tip area can be calculated as

$$q_{f,u} = \varepsilon\sigma\left(T_b^4 - T_{sur}^4\right)\left[P \cdot L + A_c\right]$$

$$q_{f,u} = 0.65\ \sigma\left[\left(80 + 273\right)^4 - 4^4\right] \mathrm{W/m^2}\left[12 + 6 \times 0.012\right]\mathrm{m^2}$$

$$q_{f,u} = 9565\ \mathrm{W/m^2}$$

Note that the heat rejection rate for the uniform plate is in excellent agreement with the result of the FDE analysis when the thermal conductivity is made extremely large. We have confidence that the code is properly handling the conduction and radiation processes; but, we have not exercised the portion of the code dealing with the absorbed irradiation. What analytical solution/model could you use to validate this portion of the code?

(2) Selection portions are shown below of the *IHT* code with the 10-nodal FDEs for the temperature distribution and the heat rejection rate.

```
// Finite-difference equations
// Interior nodes, 01 to 09
k * Ac * (T00 - T01) / deltax + k * Ac * (T02 - T01) / deltax + absS * GS * P/2 * deltax + eps * P *
deltax * sigma * (Tsur^4 - T01^4) = 0
…..
…..
k * Ac * (T03 - T04) / deltax + k * Ac * (T05 - T04) / deltax + absS * GS * P/2 * deltax + eps * P *
deltax * sigma * (Tsur^4 - T04^4) = 0
…..
…..
k * Ac * (T08 - T09) / deltax + k * Ac * (T10 - T09) / deltax + absS * GS * P/2 * deltax + eps * P *
deltax * sigma * (Tsur^4 - T09^4) = 0

// Tip node 10
k* Ac * (T09 - T10) / deltax + absS * GS * P/2 * (deltax / 2) + eps * P * (deltax / 2) * sigma *
(Tsur^4 - T10^4) - eps * Ac * sigma * (Tsur^4 - T00^4) = 0

// Rejection heat rate, energy balance on base node
qf + k * Ac * (T01 - T00) / deltax + absS * GS * (P/4) * (deltax /2) + eps * (P * deltax /2) *
sigma * (Tsur^4 - T00^4) = 0
```

Continued …..

(3) To determine the validity of the one-dimensional, extended surface analysis, calculate the Biot number estimating the linearized radiation coefficient based upon the uniform plate condition, T_b = 80°C.

$$\mathrm{Bi} = h_{rad}(t/2)/k$$

$$h_{rad} = \varepsilon\sigma(T_b + T_{sur})\left(T_b^2 + T_{sur}^2\right) \approx \varepsilon\sigma T_b^3 = 2.25\ \mathrm{W/m^2 \cdot K}$$

$$\mathrm{Bi} = 2.25\ \mathrm{W/m^2 \cdot K}(0.012\ \mathrm{m}/2)/300\ \mathrm{W/m \cdot K} = 4.5\times 10^{-5}$$

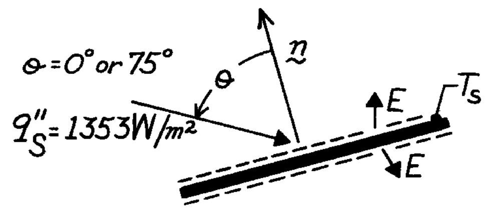
Since Bi << 0.1, the assumption of one-dimensional conduction is appropriate.

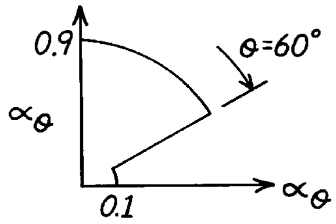
PROBLEM 12.132

KNOWN: Directional absorptivity of a plate exposed to solar radiation on one side.

FIND: (a) Ratio of normal absorptivity to hemispherical emissivity, (b) Equilibrium temperature of plate at 0° and 75° orientation relative to sun's rays.

SCHEMATIC:





ASSUMPTIONS: (1) Surface is gray, (2) Properties are independent of ϕ.

ANALYSIS: (a) From the prescribed α_θ (θ), $\alpha_n = 0.9$. Since the surface is gray, $\varepsilon_\theta = \alpha_\theta$. Hence from Eq. 12.36, which applies for total as well as spectral properties.

$$\varepsilon = 2\int_0^{\pi/2} \varepsilon_\theta \cos\theta \sin\theta \, d\theta = 2\left[0.9\frac{\sin^2\theta}{2}\Bigg|_0^{\pi/3} + 0.1\frac{\sin^2\theta}{2}\Bigg|_{\pi/3}^{\pi/2}\right]$$

$$\varepsilon = 2\left[0.9(0.375) + 0.1(0.5 - 0.375)\right] = 0.70.$$

Hence

$$\frac{\alpha_n}{\varepsilon} = \frac{0.9}{0.7} = 1.286.$$

(b) Performing an energy balance on the plate,

$$\alpha_\theta \, q_S'' \cos\theta - 2\varepsilon\sigma T_S^4 = 0$$

or

$$T_S = \left[\frac{\alpha_\theta}{2\varepsilon\sigma} q_S'' \cos\theta\right]^{1/4}.$$

Hence for $\theta = 0°$, $\alpha_\theta = 0.9$ and $\cos\theta = 1$,

$$T_S = \left[\frac{0.9}{2 \times 0.7 \times 5.67 \times 10^{-8}} \times 1353\right]^{1/4} = 352\text{K}. \qquad <$$

For $\theta = 75°$, $\alpha_\theta = 0.1$ and $\cos\theta = 0.259$

$$T_S = \left[\frac{0.1}{2 \times 0.7 \times 5.67 \times 10^{-8}} \times 1353 \times 0.259\right]^{1/4} = 145\text{K}. \qquad <$$

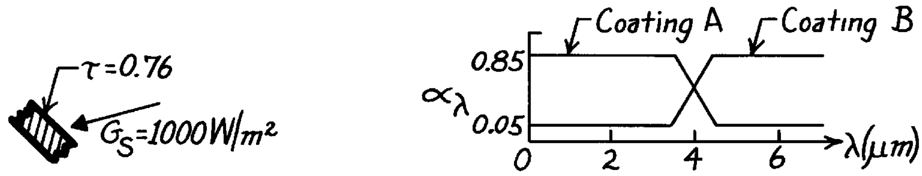
COMMENTS: Since the surface is not diffuse, its absorptivity depends on the directional distribution of the incident radiation.

PROBLEM 12.133

KNOWN: Transmissivity of cover plate and spectral absorptivity of absorber plate for a solar collector.

FIND: Absorption rate for prescribed solar flux and preferred absorber plate coating.

SCHEMATIC:



ASSUMPTIONS: (1) Solar irradiation of absorber plate retains spectral distribution of blackbody at 5800K, (2) Coatings are diffuse.

ANALYSIS: At the absorber plate we wish to maximize solar radiation absorption and minimize losses due to emission. The solar radiation is concentrated in the spectral region $\lambda < 4\mu m$, and for a representative plate temperature of $T \le 350K$, emission from the plate is concentrated in the spectral region $\lambda > 4\mu m$. Hence,

Coating A is vastly superior. <

With $G_{\lambda,S} \sim E_{\lambda,b}$ (5800K), it follows from Eq. 12.47

$$\alpha_A \approx 0.85\ F_{(0-4\mu m)} + 0.05\ F_{(4\mu m-\infty)}.$$

From Table 12.1, $\lambda T = 4\mu m \times 5800K = 23{,}200\mu m{\cdot}K$,

$$F_{(0-4\mu m)} \approx 0.99.$$

Hence

$$\alpha_A = 0.85(0.99) + 0.05(1-0.99) \approx 0.85.$$

With $G_S = 1000\ W/m^2$ and $\tau = 0.76$ (Ex. 12.9), the absorbed solar flux is

$$G_{S,abs} = \alpha_A(\tau G_S) = 0.85\left(0.76 \times 1000\ W/m^2\right)$$

$$G_{S,abs} = 646\ W/m^2.$$ <

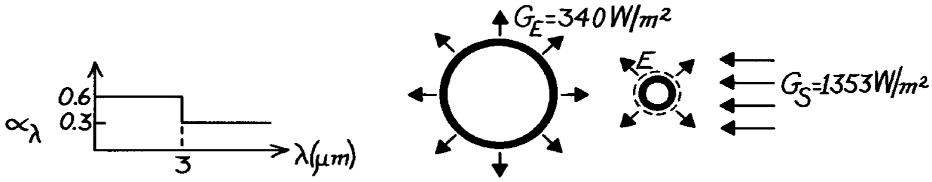
COMMENTS: Since the absorber plate emits in the infrared ($\lambda > 4\mu m$), its emissivity is $\varepsilon_A \approx 0.05$. Hence $(\alpha/\varepsilon)_A = 17$. A large value of α/ε is desirable for solar absorbers.

PROBLEM 12.134

KNOWN: Spectral distribution of coating on satellite surface. Irradiation from earth and sun.

FIND: (a) Steady-state temperature of satellite on dark side of earth, (b) Steady-state temperature on bright side.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Opaque, diffuse-gray surface behavior, (3) Spectral distributions of earth and solar emission may be approximated as those of blackbodies at 280K and 5800K, respectively, (4) Satellite temperature is less than 500K.

ANALYSIS: Performing an energy balance on the satellite,

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$\alpha_E G_E \left(\pi D^2/4\right) + \alpha_S G_S \left(\pi D^2/4\right) - \varepsilon \sigma T_s^4 \left(\pi D^2\right) = 0$$

$$T_s = \left(\frac{\alpha_E G_E + \alpha_S G_S}{4\varepsilon\sigma}\right)^{1/4}.$$

From Table 12.1, with 98% of radiation below 3μm for λT = 17,400μm·K,

$$\alpha_S \cong 0.6.$$

With 98% of radiation above 3μm for λT = 3μm × 500K = 1500μm·K,

$$\varepsilon \approx 0.3 \qquad \alpha_E \approx 0.3.$$

(a) On *dark* side,

$$T_s = \left(\frac{\alpha_E G_E}{4\varepsilon\sigma}\right)^{1/4} = \left(\frac{0.3 \times 340\,\text{W/m}^2}{4 \times 0.3 \times 5.67 \times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4}\right)^{1/4}$$

$$T_s = 197\ \text{K}.$$ <

(b) On *bright* side,

$$T_s = \left(\frac{\alpha_E G_E + \alpha_S G_S}{4\varepsilon\sigma}\right)^{1/4} = \left(\frac{0.3 \times 340\ \text{W/m}^2 + 0.6 \times 1353\,\text{W/m}^2}{4 \times 0.3 \times 5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4}\right)^{1/4}$$

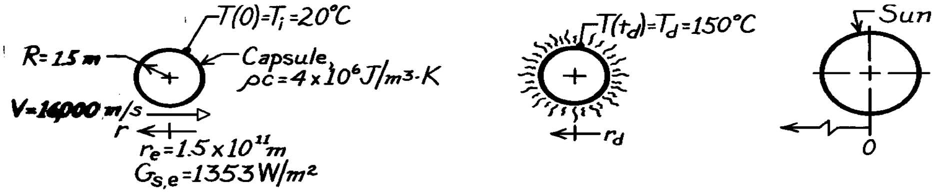
$$T_s = 340\text{K}.$$ <

PROBLEM 12.135

KNOWN: Space capsule fired from earth orbit platform in direction of sun.

FIND: (a) Differential equation predicting capsule temperature as a function of time, (b) Position of capsule relative to sun when it reaches its destruction temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Capsule behaves as lumped capacitance system, (2) Capsule surface is black, (3) Temperature of surroundings approximates absolute zero, (4) Capsule velocity is constant.

ANALYSIS: (a) To find the temperature as a function of time, perform an energy balance on the capsule considering absorbed solar irradiation and emission,

$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st} \qquad G_S \cdot \pi R^2 - \sigma T^4 \cdot 4\pi R^2 = \rho c (4/3)\pi R^3 (dT/dt). \tag{1}$$

Note the use of the projected capsule area (πR^2) and the surface area ($4\pi R^2$). The solar irradiation will increase with decreasing radius (distance toward the sun) as

$$G_S(r) = G_{S,e}(r_e/r)^2 = G_{S,e}(r_e/(r_e - Vt))^2 = G_{S,e}(1/(1 - Vt/r_e))^2 \tag{2}$$

where r_e is the distance of earth orbit from the sun and $r = r_e - Vt$. Hence, Eq. (1) becomes

$$\frac{dT}{dt} = \frac{3}{\rho c R}\left[\frac{G_{S,e}}{4(1 - Vt/r_e)^2} - \sigma T^4\right].$$

The rate of temperature change is

$$\frac{dT}{dt} = \frac{3}{\left(4\times10^6 \text{ J/m}^3\cdot\text{K}\times1.5\text{m}\right)}\left[\frac{1353 \text{ W/m}^2}{4\left(1 - 16\times10^3 \text{ m/s}\times t/1.5\times10^{11}\text{m}\right)^2} - \sigma T^4\right]$$

$$\frac{dT}{dt} = 1.691\times10^{-4}\left(1 - 1.067\times10^{-7}\,t\right)^{-2} - 2.835\times10^{-14}\,T^4$$

where T[K] and t(s). For the initial condition, t = 0, with T = 20°C = 293K,

$$\frac{dT}{dt}(0) = -3.984\times10^{-5} \text{ K/s}. \qquad <$$

That is, the capsule will cool for a period of time and then begin to heat.

(b) The differential equation cannot be explicitly solved for temperature as a function of time. Using a numerical method with a time increment of $\Delta t = 5 \times 10^5$ s, find

$$T(t) = 150°C = 423 \text{ K} \qquad \text{at} \qquad t \approx 5.5\times10^6 \text{s}. \qquad <$$

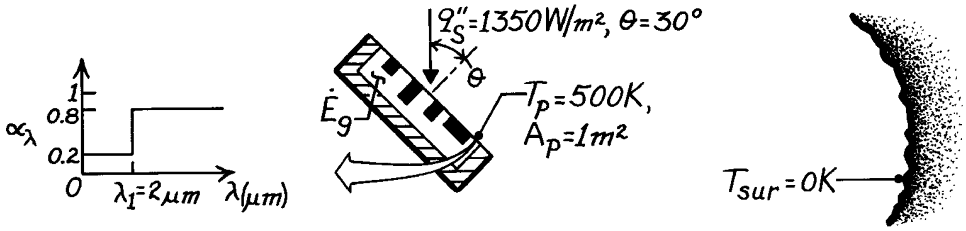
Note that in this period of time the capsule traveled $(r_e - r) = Vt = 16 \times 10^3$ m/s $\times 5.5 \times 10^6 = 1.472 \times 10^{10}$ m. That is, $r = 1.353 \times 10^{11}$ m.

PROBLEM 12.136

KNOWN: Dimensions and spectral absorptivity of radiator used to dissipate heat to outer space. Radiator temperature. Magnitude and direction of incident solar flux.

FIND: Power dissipation within radiator.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat loss through sides and bottom of compartment, (3) Opaque, diffuse surface.

ANALYSIS: Applying conservation of energy to a control surface about the compartment yields

$$\dot{E}_{in} + \dot{E}_g = \dot{E}_{out}$$

$$\dot{E}_g = \left(\varepsilon \sigma T_p^4 - \alpha G_s\right) A.$$

The emissivity can be expressed as

$$\varepsilon = \int_0^\infty \varepsilon_\lambda \left(E_{\lambda,b} / E_b\right) d\lambda = \varepsilon_1 F_{(0\to\lambda_1)} + \varepsilon_2 F_{(\lambda_1\to\infty)}.$$

From Table 12.1: $\lambda_1 T = 1000\ \mu m\cdot K \to F_{(0\to\lambda_1)} = 0.000321$

$$\varepsilon = 0.2(0.000321) + 0.8(1 - 0.00321) = 0.8.$$

The absorptivity can be expressed as

$$\alpha = \int_0^\infty \alpha_\lambda (G_\lambda / G) d\lambda = \int_0^\infty \alpha_\lambda \left[E_{\lambda,b}(5800\ K) / E_b(5800\ K)\right] d\lambda.$$

From Table 12.1: $\lambda_1 T = 11{,}600\ \mu m \cdot K \to F_{(0\to\lambda_1)} = 0.941$,

$$\alpha = 0.2(0.941) + 0.8(0.059) = 0.235.$$

Hence,

$$\dot{E}_g = \left[0.8 \times 5.67 \times 10^{-8}\ W/m^2 \cdot K^4 \times (500\ K)^4 - 0.235\ \cos 30^\circ \left(1350\ W/m^2\right)\right] 1\ m^2$$

$$\dot{E}_g = 2560\ W.$$ <

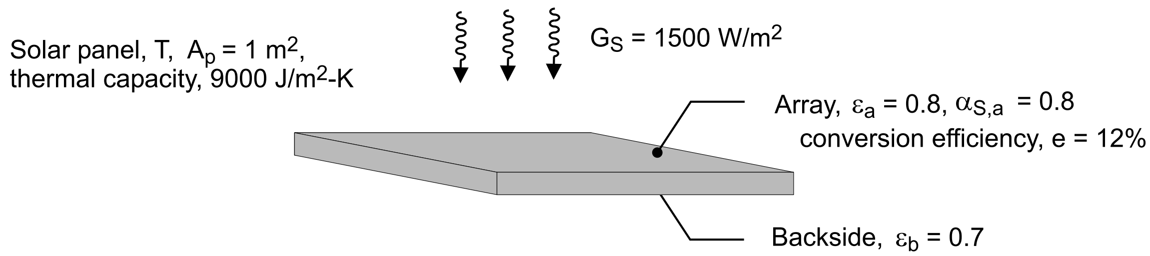
COMMENTS: Solar irradiation and plate emission are concentrated at short and long wavelength portions of the spectrum. Hence, $\alpha \neq \varepsilon$ and the surface is not gray for the prescribed conditions.

PROBLEM 12.137

KNOWN: Solar panel mounted on a spacecraft of area 1 m^2 having a solar-to-electrical power conversion efficiency of 12% with specified radiative properties.

FIND: (a) Steady-state temperature of the solar panel and electrical power produced with solar irradiation of 1500 W/m^2, (b) Steady-state temperature if the panel were a thin plate (no solar cells) with the same radiative properties and for the same prescribed conditions, and (c) Temperature of the solar panel 1500 s after the spacecraft is eclipsed by the earth; thermal capacity of the panel per unit area is 9000 $J/m^2 \cdot K$.

SCHEMATIC:



ASSUMPTIONS: (1) Solar panel and thin plate are isothermal, (2) Solar irradiation is normal to the panel upper surface, and (3) Panel has unobstructed view of deep space at 0 K.

ANALYSIS: (a) The energy balance on the solar panel is represented in the schematic below and has the form

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$\alpha_S G_S \cdot A_p - (\varepsilon_a + \varepsilon_b)\, E_b\left(T_{sp}\right) \cdot A_p - P_{elec} = 0 \qquad (1)$$

where $E_b(T) = \sigma T^4$, $\sigma = 5.67 \times 10^{-8}$ $W/m^2 \cdot K^4$, and the electrical power produced is

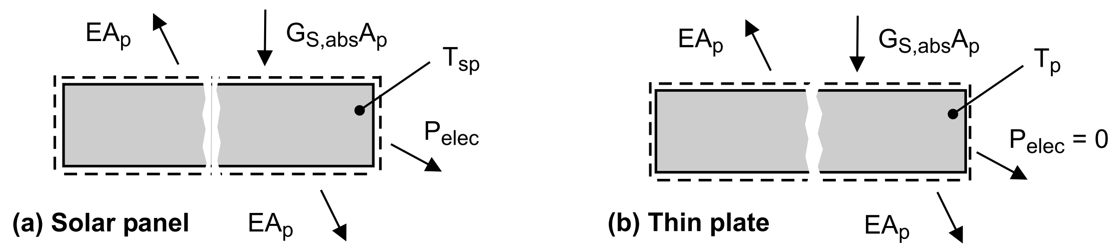
$$P_{elec} = e \cdot G_S \cdot A_p \qquad (2)$$

$$P_{elec} = 0.12 \times 1500\ W/m^2 \times 1\ m^2 = 180\ W$$ <

Substituting numerical values into Eq. (1), find

$$0.8 \times 1500\ W/m^2 \times 1\ m^2 - (0.8 + 0.7)\sigma T_{sp}^4 \times 1\ m^2 - 180\ W = 0$$

$$T_{sp} = 330.9\ K = 57.9^\circ C$$ <



(b) The energy balance for the thin plate shown in the schematic above follows from Eq. (1) with P_{elec} = 0 yielding

$$0.8 \times 1500\ W/m^2 \times /m^2 - (0.8 + 0.7)\sigma T_p^4 \times 1\ m^2 = 0 \qquad (3)$$

$$T_p = 344.7\ K = 71.7^\circ C$$ <

Continued …..

(c) Using the lumped capacitance method, the energy balance on the solar panel as illustrated in the schematic below has the form

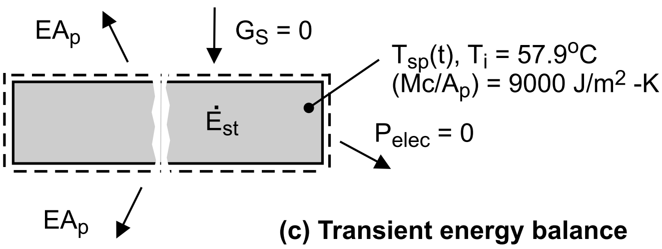
$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$$

$$-(\varepsilon_a + \varepsilon_b)\sigma T_{sp}^4 \cdot A_p = TC'' \cdot A_p \frac{dT_{sp}}{dt} \qquad (4)$$

where the thermal capacity per unit area is $TC'' = (Mc / A_p) = 9000\ J/m^2 \cdot K$.

Eq. 5.18 provides the solution to this differential equation in terms of t = t (T_i, T_{sp}). Alternatively, use Eq. (4) in the *IHT* workspace (see Comment 4 below) to find

$$T_{sp}(1500\ s) = 242.6\ K = -30.4^\circ C$$ <



(c) Transient energy balance

COMMENTS: (1) For part (a), the energy balance could be written as

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0$$

where the energy generation term represents the *conversion process from thermal energy to electrical energy*. That is,

$$\dot{E}_g = -e \cdot G_S \cdot A_p$$

(2) The steady-state temperature for the thin plate, part (b), is higher than for the solar panel, part (a). This is to be expected since, for the solar panel, some of the absorbed solar irradiation (thermal energy) is converted to electrical power.

(3) To justify use of the lumped capacitance method for the transient analysis, we need to know the effective thermal conductivity or internal thermal resistance of the solar panel.

(4) Selected portions of the *IHT* code using the *Models Lumped | Capacitance* tool to perform the transient analysis based upon Eq. (4) are shown below.

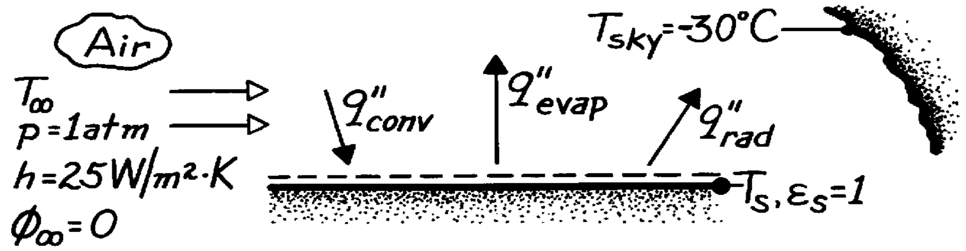
```
// Energy balance, Model | Lumped Capacitance
/ * Conservation of energy requirement on the control volume, CV. * /
Edotin - Edotout = Edotst
Edotin = 0
Edotout = Ap * (+q"rad)
Edostat = rhovolcp * Ap * Der(T,t)
// rhovolcp = rho * vol * cp        // thermal capacitance per unit area, J/m^2·K

// Radiation exchange between Cs and large surroundings
q"rad = (eps_a + eps_b) * sigma * (T^4 - Tsur^4)
sigma = 5.67e-8    // Stefan-Boltzmann constant, W/m^2·K^4

// Initial condition
// Ti = 57.93 + 273 = 330.9                // From part (a), steady-state condition
T_C = T - 273
```

PROBLEM 12.138

KNOWN: Effective sky temperature and convection heat transfer coefficient associated with a thin layer of water.

FIND: Lowest air temperature for which the water will not freeze (without and with evaporation).

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Bottom of water is adiabatic, (3) Heat and mass transfer analogy is applicable, (4) Air is dry.

PROPERTIES: *Table A-4*, Air (273 K, 1 atm): $\rho = 1.287$ kg/m^3, $c_p = 1.01$ kJ/kg·K, $\nu = 13.49 \times 10^{-6}$ m^2/s, Pr = 0.72; *Table A-6*, Saturated vapor ($T_s = 273$ K): $\rho_A = 4.8 \times 10^{-3}$ kg/m^3, $h_{fg} = 2502$ kJ/kg; *Table A-8*, Vapor-air (298 K): $D_{AB} \approx 0.36 \times 10^{-4}$ m^2/s, $Sc = \nu/D_{AB} = 0.52$.

ANALYSIS: Without evaporation, the surface heat loss by radiation must be balanced by heat gain due to convection. An energy balance gives

$$q''_{conv} = q''_{rad} \qquad \text{or} \qquad h\left(T_\infty - T_s\right) = \varepsilon_s \sigma \left(T_s^4 - T_{sky}^4\right).$$

At freezing, $T_s = 273$ K. Hence

$$T_\infty = T_s + \frac{\varepsilon_s \sigma}{h}\left(T_s^4 - T_{sky}^4\right) = 273\text{ K} + \frac{5.67 \times 10^{-8}\text{ W/m}^2 \cdot \text{K}^4}{25\text{ W/m}^2 \cdot \text{K}}\left[274^4 - 243^4\right]\text{K}^4 = 4.69°\text{C}. \quad <$$

With evaporation, the surface energy balance is now

$$q''_{conv} = q''_{evap} + q''_{rad} \text{ or } h\left(T_\infty - T_s\right) = h_m\left[\rho_{A,sat}\left(T_s\right) - \rho_{A,\infty}\right]h_{fg} + \varepsilon_s \sigma\left(T_s^4 - T_{sky}^4\right).$$

$$T_\infty = T_s + \frac{h_m}{h}\rho_{A,sat}\left(T_s\right)h_{fg} + \frac{\varepsilon_s \sigma}{h}\left(T_s^4 - T_{sky}^4\right).$$

Substituting from Eq. 6.92, with $n \approx 0.33$,

$$h_m / h = \left(\rho c_p Le^{0.67}\right)^{-1} = \left[\rho c_p \left(Sc/Pr\right)^{0.67}\right]^{-1} = \left[1.287\text{ kg/m}^3 \times 1010\text{J/kg} \cdot \text{K}\left(0.52/0.72\right)^{0.67}\right]^{-1} = 9.57 \times 10^{-4}\text{ m}^3 \cdot \text{K/J},$$

$$T_\infty = 273\text{K} + 9.57 \times 10^{-4}\text{m}^3 \cdot \text{K/J} \times 4.8 \times 10^{-3}\text{kg/m}^3 \times 2.5 \times 10^6\text{J/kg} + 4.69\text{ K} = 16.2°\text{C}. \quad <$$

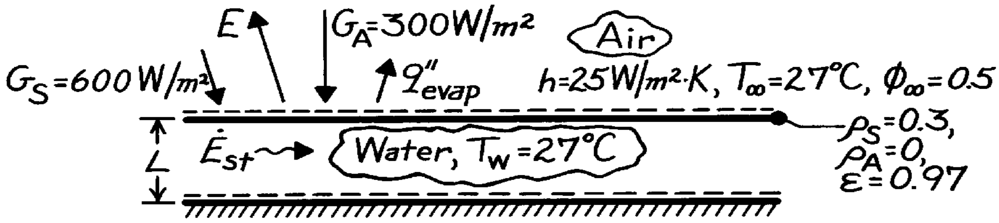
COMMENTS: The existence of clear, cold skies and dry air will allow water to freeze for ambient air temperatures well above 0°C (due to radiative and evaporative cooling effects, respectively). The lowest air temperature for which the water will not freeze increases with decreasing ϕ_∞, decreasing T_{sky} and decreasing h.

PROBLEM 12.139

KNOWN: Temperature and environmental conditions associated with a shallow layer of water.

FIND: Whether water temperature will increase or decrease with time.

SCHEMATIC:



ASSUMPTIONS: (1) Water layer is well mixed (uniform temperature), (2) All non-reflected radiation is absorbed by water, (3) Bottom is adiabatic, (4) Heat and mass transfer analogy is applicable, (5) Perfect gas behavior for water vapor.

PROPERTIES: *Table A-4*, Air (T = 300 K, 1 atm): ρ_a = 1.161 kg/m^3, $c_{p,a}$ = 1007 J/kg·K, Pr = 0.707; *Table A-6*, Water (T = 300 K, 1 atm): ρ_w = 997 kg/m^3, $c_{p,w}$ = 4179 J/kg·K; Vapor (T = 300 K, 1 atm): $\rho_{A,sat}$ = 0.0256 kg/m^3, h_{fg} = 2.438 × 10^6 J/kg; *Table A-8*, Water vapor-air (T = 300 K, 1 atm): $D_{AB} \approx 0.26 \times 10^{-4}$ m^2/s; with $\nu_a = 15.89 \times 10^{-6}$ m^2/s from *Table A-4*, Sc = ν_a/D_{AB} = 0.61.

ANALYSIS: Performing an energy balance on a control volume about the water,

$$\dot{E}_{st} = \left(G_{S,abs} + G_{A,abs} - E - q''_{evap}\right)A$$

$$\frac{d\left(\rho_w c_{p,w} L A T_w\right)}{dt} = \left[(1-\rho_s)G_S + (1-\rho_A)G_A - \varepsilon\sigma T_w^4 - h_m h_{fg}\left(\rho_{A,sat} - \rho_{A,\infty}\right)\right]A$$

or, with $T_\infty = T_w$, $\rho_{A,\infty} = \phi_\infty \rho_{A,sat}$ and

$$\rho_w c_{p,w} L \frac{dT_w}{dt} = (1-\rho_s)G_S + (1-\rho_A)G_A - \varepsilon\sigma T_w^4 - h_m h_{fg}(1-\phi_\infty)\rho_{A,sat}.$$

From Eq. 6.92, with a value of n = 1/3,

$$h_m = \frac{h}{\rho_a c_{p,a} Le^{1-n}} = \frac{h}{\rho_a c_{p,a}(Sc/Pr)^{1-n}} = \frac{25\,W/m^2\cdot K(0.707)^{2/3}}{1.161\,kg/m^3 \times 1007\,J/kg\cdot K(0.61)^{2/3}} = 0.0236\,m/s.$$

Hence

$$\rho_w c_{p,w} L \frac{dT_w}{dt} = (1-0.3)600 + (1-0)300 - 0.97\times 5.67\times 10^{-8}(300)^4$$
$$-0.0236 \times 2.438 \times 10^6 (1-0.5)0.0256$$

$$\rho_w c_{p,w} L \frac{dT_w}{dt} = (420 + 300 - 445 - 736)\,W/m^2 = -461\,W/m^2.$$

Hence the water will *cool*. <

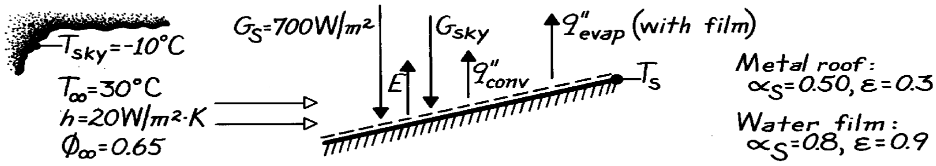
COMMENTS: (1) Since $T_w = T_\infty$ for the prescribed conditions, there is no convection of sensible energy. However, as the water cools, there will be convection heat transfer from the air. (2) If L = 1m, $(dT_w/dt) = -461/(997 \times 4179 \times 1) = -1.11 \times 10^{-4}$ K/s.

PROBLEM 12.140

KNOWN: Environmental conditions for a metal roof with and without a water film.

FIND: Roof surface temperature (a) without the film, (b) with the film.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Diffuse-gray surface behavior in the infrared (for the metal, $\alpha_{sky} = \varepsilon = 0.3$; for the water, $\alpha_{sky} = \varepsilon = 0.9$), (3) Adiabatic roof bottom, (4) Perfect gas behavior for vapor.

PROPERTIES: *Table A-4*, Air ($T \approx 300$ K): $\rho = 1.16$ kg/m^3, $c_p = 1007$ J/kg·K, $\alpha = 22.5 \times 10^{-6}$ m^2/s; *Table A-6*, Water vapor ($T \approx 303$ K): $v_g = 32.4$ m^3/kg or $\rho_{A,sat} = 0.031$ kg/m^3; *Table A-8*, Water vapor-air ($T = 298$ K): $D_{AB} = 0.26 \times 10^{-4}$ m^2/s.

ANALYSIS: (a) From an energy balance on the metal roof

$$\alpha_S G_S + \alpha_{sky} G_{sky} = E + q''_{conv}$$

$$0.5\left(700\ \text{W/m}^2\right) + 0.3\times5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\,(263\ \text{K})^4$$

$$= 0.3\times5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\left(T_s^4\right) + 20\ \text{W/m}^2\cdot\text{K}\,(T_s - 303\ \text{K})$$

$$431\ \text{W/m}^2 = 1.70\times10^{-8}\,T_s^4 + 20\,(T_s - 303). \qquad <$$

From a trial-and-error solution, $T_s = 316.1\ \text{K} = 43.1°\text{C}$.

(b) From an energy balance on the water film,

$$\alpha_S G_S + \alpha_{sky} G_{sky} = E + q''_{conv} + q''_{evap}$$

$$0.8\left(700\ \text{W/m}^2\right) + 0.9\times5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\,(263\ \text{K})^4 = 0.9\times5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\left(T_s^4\right)$$

$$+20\ \text{W/m}^2\cdot\text{K}\,(T_s - 303) + h_m\left(\rho_{A,sat}(T_s) - 0.65\times0.031\,\text{kg/m}^3\right)h_{fg}.$$

From Eq. 6.92, assuming $n = 0.33$,

$$h_m = \frac{h}{\rho c_p \text{Le}^{0.67}} =$$

$$\frac{h}{\rho c_p\left(\alpha / D_{AB}\right)^{0.67}} = \frac{20\ \text{W/m}^2\cdot\text{K}}{1.16\ \text{kg/m}^3\times1007\ \text{J/kg}\cdot\text{K}\left(0.225\times10^{-4}/0.260\times10^{-4}\right)^{0.67}} = 0.019\text{m/s}.$$

$$804\ \text{W/m}^2 = 5.10\times10^{-8}\,T_s^4 + 20(T_s - 303) + 0.019\left[\rho_{A,sat}(T_s) - 0.020\right]h_{fg}.$$

From a trial-and-error solution, obtaining $\rho_{A,sat}(T_s)$ and h_{fg} from Table A-6 for each assumed value of T_s, it follows that

$$T_s = 302.2\ \text{K} = 29.2°\text{C}. \qquad <$$

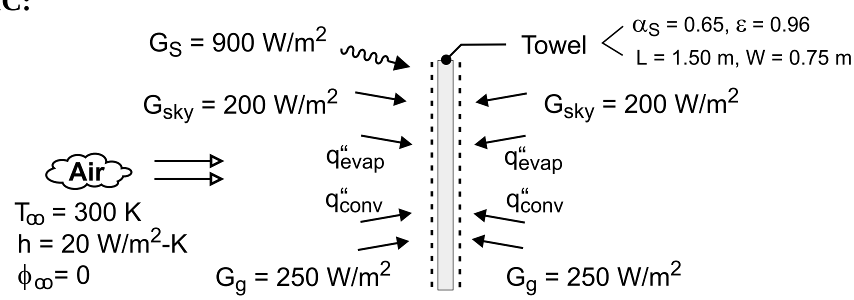
COMMENTS: (1) The film is an effective coolant, reducing T_s by 13.9°C. (2) With the film $E \approx 425$ W/m^2, $q''_{conv} \approx -16$ W/m^2 and $q''_{evap} \approx 428$ W/m^2.

PROBLEM 12.141

KNOWN: Solar, sky and ground irradiation of a wet towel. Towel dimensions, emissivity and solar absorptivity. Temperature, relative humidity and convection heat transfer coefficient associated with air flow over the towel.

FIND: Temperature of towel and evaporation rate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Diffuse-gray surface behavior of towel in the infrared ($\alpha_{sky} = \alpha_g = \varepsilon = 0.96$), (3) Perfect gas behavior for vapor.

PROPERTIES: *Table A-4*, Air (T ≈ 300 K): $\rho = 1.16$ kg/m^3, $c_p = 1007$ J/kg·K, $\alpha = 0.225 \times 10^{-4}$ m^2/s; *Table A-6*, Water vapor (T_∞ = 300 K): $\rho_{A,sat} = 0.0256$ kg/m^3; *Table A-8*, Water vapor/air (T = 298 K): $D_{AB} = 0.26 \times 10^{-4}$ m^2/s.

ANALYSIS: From an energy balance on the towel, it follows that

$$\alpha_S G_S + 2\alpha_{sky} G_{sky} + 2\alpha_g G_g = 2E + 2q''_{evap} + 2q''_{conv}$$

$$0.65 \times 900 \text{W/m}^2 + 2 \times 0.96 \times 200 \text{ W/m}^2 \times 2 \times 0.96 \times 250 \text{ W/m}^2$$

$$= 2 \times 0.96\, \sigma T_s^4 + 2n''_A h_{fg} + 2h(T_s - T_\infty) \quad (1)$$

where $\quad n''_A = h_m\left[\rho_{A,sat}(T_s) - \phi_\infty \rho_{A,sat}(T_\infty)\right]$

From the heat and mass transfer analogy, Eq. 6.67, with an assumed exponent of n = 1/3,

$$h_m = \frac{h}{\rho c_p (\alpha / D_{AB})^{2/3}} = \frac{20 \text{ W/m}^2 \cdot \text{K}}{1.16 \text{kg/m}^3 (1007 \text{ J/kg} \cdot \text{K})\left(\frac{0.225}{0.260}\right)^{2/3}} = 0.0189 \text{ m/s}$$

From a trial-and-error solution, we find that for T_s = 298 K, $\rho_{A,sat} = 0.0226$ kg/m^3, $h_{fg} = 2.442 \times 10^6$ J/kg and $n''_A = 1.380 \times 10^{-4}$ kg/s·m^2. Substituting into Eq. (1),

$$(585 + 384 + 480) \text{W/m}^2 = 2 \times 0.96 \times 5.67 \times 10^{-8} \text{W/m}^2 \cdot \text{K}^4 (298 \text{ K})^4$$

$$+2 \times 1.380 \times 10^{-4} \text{kg/s} \cdot \text{m}^2 \times 2.442 \times 10^6 \text{ J/kg}$$

$$+2 \times 20 \text{ W/m}^2 \cdot \text{K}(-2 \text{ K})$$

$$1449 \text{W/m}^2 = (859 + 674 - 80) \text{W/m}^2 = 1453 \text{W/m}^2$$

The equality is satisfied to a good approximation, in which case

$$T_s \approx 298 \text{ K} = 25°\text{C} \quad <$$

and $\quad n_A = 2A_s n''_A = 2(1.50 \times 0.75)\text{m}^2 \left(1.38 \times 10^{-4} \text{ kg/s} \cdot \text{m}^2\right) = 3.11 \times 10^{-4} \text{ kg/s} \quad <$

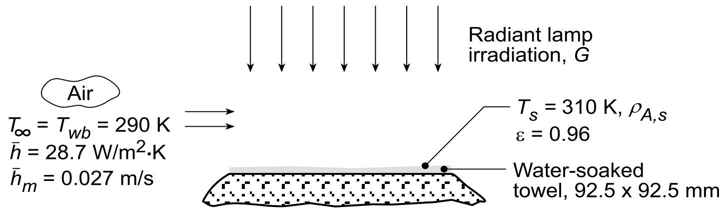
COMMENTS: Note that the temperature of the air exceeds that of the towel, in which case convection heat transfer is to the towel. Reduction of the towel's temperature below that of the air is due to the evaporative cooling effect.

PROBLEM 12.142

KNOWN: Wet paper towel experiencing forced convection heat and mass transfer and irradiation from radiant lamps. Prescribed convection parameters including wet and dry bulb temperature of the air stream, T_{wb} and T_∞, average heat and mass transfer coefficients, $\overline{h}$ and $\overline{h}_m$. Towel temperature T_s.

FIND: (a) Vapor densities, $\rho_{A,s}$ and $\rho_{A,\infty}$; the evaporation rate n_A (kg/s); and the net rate of radiation transfer to the towel q_{rad} (W); and (b) Emissive power E, the irradiation G, and the radiosity J, using the results from part (a).

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat loss from the bottom side of the towel, (3) Uniform irradiation on the towel, and (4) Water surface is diffuse, gray.

PROPERTIES: *Table A.6*, Water (T_s = 310 K): h_{fg} = 2414 kJ/kg.

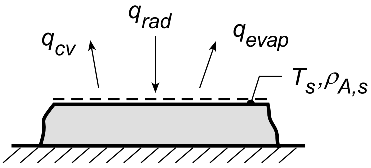
ANALYSIS: (a) Since $T_{wb} = T_\infty$, the free stream contains water vapor at its saturation condition. The water vapor at the surface is saturated since it is in equilibrium with the liquid in the towel. From Table A.6,

T (K)	v_g (m³/kg)	ρ_g (kg/m³)
$T_\infty = 290$	69.7	$\rho_{A,\infty} = 1.435 \times 10^{-2}$
$T_s = 310$	22.93	$\rho_{A,s} = 4.361 \times 10^{-2}$

Using the mass transfer convection rate equation, the water evaporation rate from the towel is

$$n_A = \overline{h}_m A_s\left(\rho_{A,s} - \rho_{A,\infty}\right) = 0.027\,\text{m/s}\,(0.0925\,\text{m})^2\,(4.361 - 1.435)\times 10^{-2}\,\text{kg/m}^3 = 6.76\times 10^{-6}\,\text{kg/s} \quad <$$

To determine the net radiation heat rate q''_{rad}, perform an energy balance on the water film,



$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad q_{rad} - q_{cv} - q_{evap} = 0$$

$$q_{rad} = q_{cv} + q_{evap} = \overline{h}_s A_s\left(T_s - T_\infty\right) + n_A h_{fg}$$

and substituting numerical values find

$$q_{rad} = 28.7\,\text{W/m}^2\cdot\text{K}\,(0.0925\,\text{m})^2\,(310 - 290)\,\text{K} + 6.76\times 10^{-6}\,\text{kg/s}\times 2414\times 10^3\,\text{J/kg}$$

$$q_{rad} = (4.91 + 16.32)\,\text{W} = 21.2\,\text{W} \quad <$$

(b) The radiation parameters for the towel surface are now evaluated. The emissive power is

$$E = \varepsilon E_b(T_s) = \varepsilon\sigma T_s^4 = 0.96\times 5.67\times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\,(310\,\text{K})^4 = 502.7\,\text{W/m}^2 \quad <$$

To determine the irradiation G, recognize that the net radiation heat rate can be expressed as,

$$q_{rad} = (\alpha G - E)A_s \qquad 21.2\,\text{W} = (0.96G - 502.7)\,\text{W/m}^2\times(0.0925\,\text{m})^2 \qquad G = 3105\ \text{W/m}^2 \quad <$$

where α = ε since the water surface is diffuse, gray. From the definition of the radiosity,

$$J = E + \rho G = [502.7 + (1 - 0.96)\times 3105]\,\text{W/m}^2 = 626.9\,\text{W/m}^2 \quad <$$

where ρ = 1 - α = 1 - ε.

COMMENTS: An alternate method to evaluate J is to recognize that q''_{rad} = G - J.