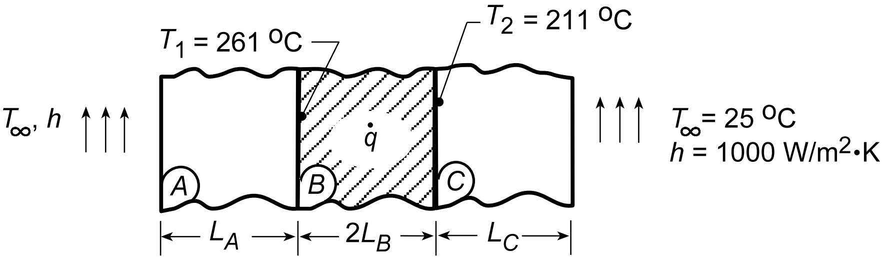
PROBLEM 3.73

KNOWN: Composite wall with outer surfaces exposed to convection process.

FIND: (a) Volumetric heat generation and thermal conductivity for material B required for special conditions, (b) Plot of temperature distribution, (c) T_1 and T_2, as well as temperature distributions corresponding to loss of coolant condition where h = 0 on surface A.

SCHEMATIC:



L_A = 30 mm
L_B = 30 mm
L_C = 20 mm
k_A = 25 W/m·K
k_C = 50 W/m·K

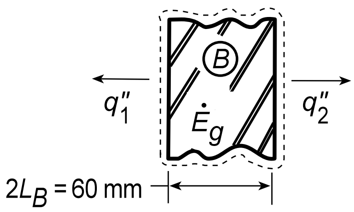
ASSUMPTIONS: (1) Steady-state, one-dimensional heat transfer, (2) Negligible contact resistance at interfaces, (3) Uniform generation in B; zero in A and C.

ANALYSIS: (a) From an energy balance on wall B,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st}$$

$$-q_1'' - q_2'' + 2\dot{q}L_B = 0$$

$$\dot{q}_B = \left(q_1'' + q_2''\right)/2L_B \ .$$



To determine the heat fluxes, q_1'' and q_2'', construct thermal circuits for A and C:

$$q_1'' = \left(T_1 - T_\infty\right)/\left(1/h + L_A/k_A\right) \qquad q_2'' = \left(T_2 - T_\infty\right)/\left(L_C/k_C + 1/h\right)$$

$$q_1'' = (261-25)^\circ\,C\Big/\left(\frac{1}{1000\,W/m^2\cdot K} + \frac{0.030\,m}{25\,W/m\cdot K}\right) \qquad q_2'' = (211-25)^\circ\,C\Big/\left(\frac{0.020\,m}{50\,W/m\cdot K} + \frac{1}{1000\,W/m^2\cdot K}\right)$$

$$q_1'' = 236^\circ C/(0.001+0.0012)\,m^2\cdot K/W \qquad q_2'' = 186^\circ C/(0.0004+0.001)\,m^2\cdot K/W$$

$$q_1'' = 107,273\,W/m^2 \qquad q_2'' = 132,857\,W/m^2$$

Using the values for q_1'' and q_2'' in Eq. (1), find

$$\dot{q}_B = \left(106,818 + 132,143\,W/m^2\right)\Big/2\times 0.030\,m = 4.00\times 10^6\,W/m^3 \ .$$ <

To determine k_B, use the general form of the temperature and heat flux distributions in wall B,

$$T(x) = -\frac{\dot{q}_B}{2k_B}x^2 + C_1 x + C_2 \qquad q_x''(x) = -k_B\left[-\frac{\dot{q}}{k_B}x + C_1\right] \qquad (1,2)$$

there are 3 unknowns, C_1, C_2 and k_B, which can be evaluated using three conditions,

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PROBLEM 3.73 (Cont.)

$$T(-L_B) = T_1 = -\frac{\dot{q}_B}{2k_B}(-L_B)^2 - C_1 L_B + C_2 \qquad \text{where } T_1 = 261°\text{C} \tag{3}$$

$$T(+L_B) = T_2 = -\frac{\dot{q}_B}{2k_B}(+L_B)^2 + C_1 L_B + C_2 \qquad \text{where } T_2 = 211°\text{C} \tag{4}$$

$$q''_x(-L_B) = -q''_1 = -k_B\left[-\frac{\dot{q}_B}{k_B}(-L_B) + C_1\right] \qquad \text{where } q''_1 = 107{,}273 \text{ W/m}^2 \tag{5}$$

Using IHT to solve Eqs. (3), (4) and (5) simultaneously with $\dot{q}_B$ = 4.00 × 10^6 W/m^3, find

$$k_B = 15.3\,\text{W/m}\cdot\text{K}$$ <

(b) Following the method of analysis in the *IHT Example 3.6, User-Defined Functions*, the temperature distribution is shown in the plot below. The important features are (1) Distribution is quadratic in B, but non-symmetrical; linear in A and C; (2) Because thermal conductivities of the materials are different, discontinuities exist at each interface; (3) By comparison of gradients at x = $-L_B$ and $+L_B$, find $q''_2 > q''_1$.

(c) Using the same method of analysis as for Part (c), the temperature distribution is shown in the plot below when h = 0 on the surface of A. Since the left boundary is adiabatic, material A will be isothermal at T_1. Find

T_1 = 835°C $\qquad$ T_2 = 360°C <