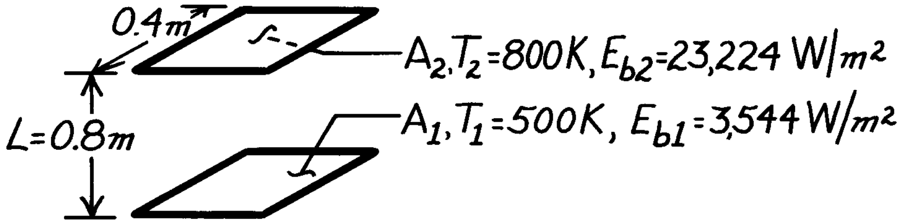
PROBLEM 13.65

KNOWN: Two aligned, parallel square plates with prescribed temperatures.

FIND: Net radiative transfer from surface 1 for these plate conditions: (a) black, surroundings at 0 K, (b) black with connecting, re-radiating walls, (c) diffuse-gray with radiation-free surroundings at 0 K, (d) diffuse-gray with re-radiating walls.

SCHEMATIC:



ASSUMPTIONS: (1) Plates are black or diffuse-gray, (2) Surroundings are at 0 K.

ANALYSIS: (a) The view factor for the aligned, parallel plates follows from Fig. 13.4, X/L = 0.4 m/0.8 m = 0.5, Y/L = 0.4 m/0.8 m = 0.5, $F_{12} = F_{21} \approx 0.075$. When the plates are *black with surroundings at 0 K*, from Eq. 13.13,

$$q_1 = q_{12} + q_{1(sur)} = A_1F_{12}\left(E_{b1} - E_{b2}\right) + A_1F_{1(sur)}\left(E_{b1} - E_{b(sur)}\right)$$

$$q_1 = (0.4\times0.4)\,\text{m}^2\left[0.075(3544 - 23,224) + (1-0.075)(3544-0)\right]\text{W/m}^2 = 288\,\text{W}. \quad <$$

(b) When the plates are *black with connecting re-radiating walls*, from Eq. 13.30 with $F_{1R} = R_{2R} = 1 - F_{12} = 0.925$,

$$q_1 = \frac{A_1\left[E_{b1} - E_{b2}\right]}{\left[F_{12} + \left(1/F_{1R} + 1/F_{2R}\right)^{-1}\right]^{-1}} = \frac{(0.4\,\text{m})^2\left[3544 - 23,224\right]\text{W/m}^2}{\left[0.075 + (1/0.925 + 1/0.925)^{-1}\right]^{-1}} = -1,692\,\text{W}. \quad <$$

(c) When the plates are *diffuse-gray (ε_1 = 0.6 and ε_2 = 0.8)* with the *surroundings at 0 K*, using Eq. 13.20 or Eq. 13.19, with $E_{b3} = J_3 = 0$,

$$q_1 = A_1F_{12}\left(J_1 - J_2\right) + A_1F_{13}\left(J_1 - J_3\right) = \left(E_{b1} - J_1\right)/\left[(1-\varepsilon_1)/\varepsilon_1A_1\right].$$

The radiosities must be determined from energy balances, Eq. 13.21, on each of the surfaces,

$$\frac{E_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1} = F_{12}\left(J_1 - J_2\right) + F_{13}\left(J_1 - J_3\right) \qquad \frac{E_{b2} - J_2}{(1-\varepsilon_2)/\varepsilon_2} = F_{21}\left(J_2 - J_1\right) + F_{23}\left(J_2 - J_3\right)$$

$$\frac{3,544 - J_1}{(1-0.6)/0.6} = 0.075\left(J_1 - J_2\right) + 0.925J_1 \qquad \frac{23,224 - J_2}{(1-0.8)/0.8} = 0.075\left(J_2 - J_1\right) + 0.925J_2.$$

Find J_1 = 2682 W/m^2 and J_2 = 18,542 W/m^2. Combining these results,

$$q_1 = (0.4\,\text{m})^2(0.075)(2682 - 18,542)\,\text{W/m}^2 + (0.4\,\text{m})^2(0.925)(2682 - 0)\,\text{W/m}^2 = 207\ \text{W}. \quad <$$

(d) When the plates are *diffuse-gray with connecting re-radiating walls*, use Eq. 13.30,

$$q_1 = \frac{A_1\left[E_{b1} - E_{b2}\right]}{(1-\varepsilon_1)/\varepsilon_1 + \left[F_{12} + \left(1/F_{1R} + 1/F_{2R}\right)^{-1}\right]^{-1} + (1-\varepsilon_2)/\varepsilon_2}$$

$$q_1 = \frac{(0.4\,\text{m})^2\left[35444 - 23,244\right]\text{W/m}^2}{(1-0.6)/0.6 + \left[0.075 + (1/0.925 + 1/0.925)^{-1}\right]^{-1} + (1-0.8)/0.8} = -1133\,\text{W}. \quad <$$