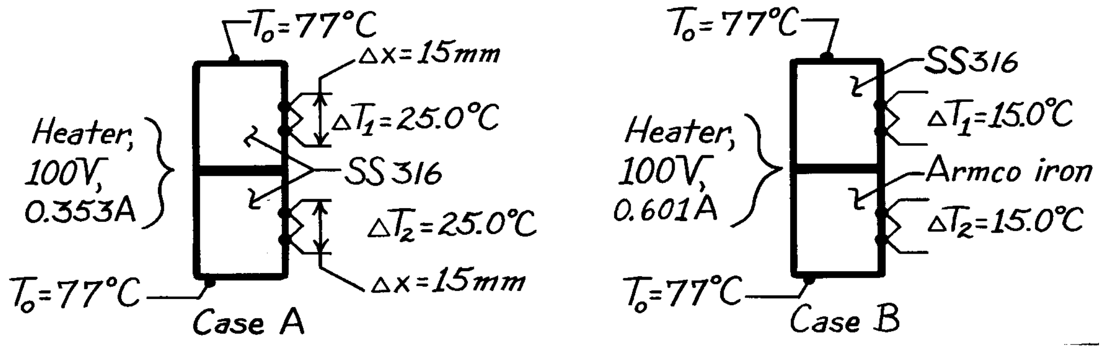
PROBLEM 2.17

KNOWN: Electrical heater sandwiched between two identical cylindrical (30 mm dia. × 60 mm length) samples whose opposite ends contact plates maintained at T_o.

FIND: (a) Thermal conductivity of SS316 samples for the prescribed conditions (A) and their average temperature, (b) Thermal conductivity of Armco iron sample for the prescribed conditions (B), (c) Comment on advantages of experimental arrangement, lateral heat losses, and conditions for which $\Delta T_1 \neq \Delta T_2$.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional heat transfer in samples, (2) Steady-state conditions, (3) Negligible contact resistance between materials.

PROPERTIES: *Table A.2,* Stainless steel 316 $(\bar{T} = 400\ K)$: $k_{ss} = 15.2\ W/m\cdot K$; Armco iron $(\bar{T} = 380\ K)$: $k_{iron} = 71.6\ W/m\cdot K$.

ANALYSIS: (a) For Case A recognize that half the heater power will pass through each of the samples which are presumed identical. Apply Fourier's law to a sample

$$q = kA_c \frac{\Delta T}{\Delta x}$$

$$k = \frac{q\Delta x}{A_c \Delta T} = \frac{0.5(100\,V \times 0.353\,A)\times 0.015\ m}{\pi(0.030\ m)^2/4 \times 25.0^\circ C} = 15.0\ W/m\cdot K. \qquad <$$

The total temperature drop across the length of the sample is $\Delta T_1(L/\Delta x)$ = 25°C (60 mm/15 mm) = 100°C. Hence, the heater temperature is T_h = 177°C. Thus the average temperature of the sample is

$$\bar{T} = (T_o + T_h)/2 = 127^\circ C = 400\ K \qquad <$$

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We compare the calculated value of k with the tabulated value (see above) at 400 K and note the good agreement.

(b) For Case B, we assume that the thermal conductivity of the SS316 sample is the same as that found in Part (a). The heat rate through the Armco iron sample is

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$$q_{iron} = q_{heater} - q_{ss} = 100\text{V} \times 0.601\text{A} - 15.0 \text{ W/m}\cdot\text{K} \times \frac{\pi(0.030 \text{ m})^2}{4} \times \frac{15.0^\circ\text{C}}{0.015 \text{ m}}$$

$$q_{iron} = (60.1 - 10.6)\text{W} = 49.5 \text{ W}$$

where

$$q_{ss} = k_{ss} A_c \Delta T_2 / \Delta x_2 .$$

Applying Fourier's law to the iron sample,

$$k_{iron} = \frac{q_{iron} \Delta x_2}{A_c \Delta T_2} = \frac{49.5 \text{ W} \times 0.015 \text{ m}}{\pi(0.030 \text{ m})^2 / 4 \times 15.0^\circ\text{C}} = 70.0 \text{ W/m}\cdot\text{K}. \qquad <$$

The total drop across the iron sample is 15°C(60/15) = 60°C; the heater temperature is (77 + 60)°C = 137°C. Hence the average temperature of the iron sample is

$$\overline{T} = (137 + 77)^\circ\text{C}/2 = 107^\circ\text{C} = 380 \text{ K}. \qquad <$$

We compare the computed value of k with the tabulated value (see above) at 380 K and note the good agreement.

(c) The principal advantage of having two identical samples is the assurance that all the electrical power dissipated in the heater will appear as equivalent heat flows through the samples. With only one sample, heat can flow from the backside of the heater even though insulated.

Heat leakage out the lateral surfaces of the cylindrically shaped samples will become significant when the sample thermal conductivity is comparable to that of the insulating material. Hence, the method is suitable for metallics, but must be used with caution on nonmetallic materials.

For any combination of materials in the upper and lower position, we expect $\Delta T_1 = \Delta T_2$. However, if the insulation were improperly applied along the lateral surfaces, it is possible that heat leakage will occur, causing $\Delta T_1 \neq \Delta T_2$.