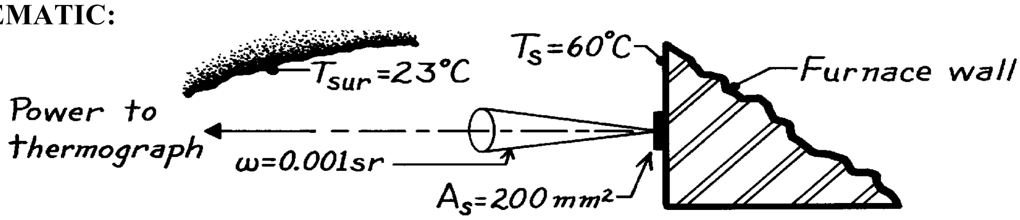
PROBLEM 12.78

KNOWN: Thermograph with spectral response in 9 to 12 μm region views a target of area 200mm^2 with solid angle 0.001 sr in a normal direction.

FIND: (a) For a black surface at 60°C, the emissive power in 9 – 12 μm spectral band, (b) Radiant power (W), received by thermograph when viewing black target at 60°C, (c) Radiant power (W) received by thermograph when viewing a gray, diffuse target having ε = 0.7 and considering the surroundings at T_{sur} = 23°C.

SCHEMATIC:



ASSUMPTIONS: (1) Wall is diffuse, (2) Surroundings are black with T_{sur} = 23°C.

ANALYSIS: (a) Emissive power in spectral range 9 to 12 μm for a 60°C black surface is

$$E_t \equiv E_b(9-12\,\mu m) = E_b\left[F(0\to 12\,\mu m) - F(0-9\,\mu m)\right]$$

where $E_b(T_s) = \sigma T_s^4$. From Table 12.1:

$$\lambda_2 T_s = 12\times(60+273) \approx 4000\,\mu m\ K, \qquad F(0\text{–}12\,\mu m) = 0.491$$

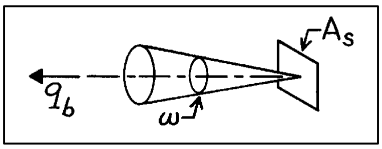
$$\lambda_1 T_s = 9\times(60+273) \approx 3000\,\mu m\ K, \qquad F(0-9\,\mu m) = 0.273.$$

Hence

$$E_t = 5.667\times10^{-8}\ W/m^2\cdot K^4 \times (60+273)^4\ K^4\,[0.491-0.273] = 144.9\ W/m^2. \qquad <$$

(b) The radiant power, q_b (w), received by the thermograph from a black target is determined as

$$q_b = \frac{E_t}{\pi}\cdot A_s\cos\theta_1\cdot\omega$$



where E_t = emissive power in 9 – 12 μm spectral region, part (a) result
A_s = target area viewed by thermograph, 200mm^2 (2×10^{-4} m^2)
ω = solid angle thermograph aperture subtends when viewed from the target, 0.001 sr
θ = angle between target area normal and view direction, 0°.

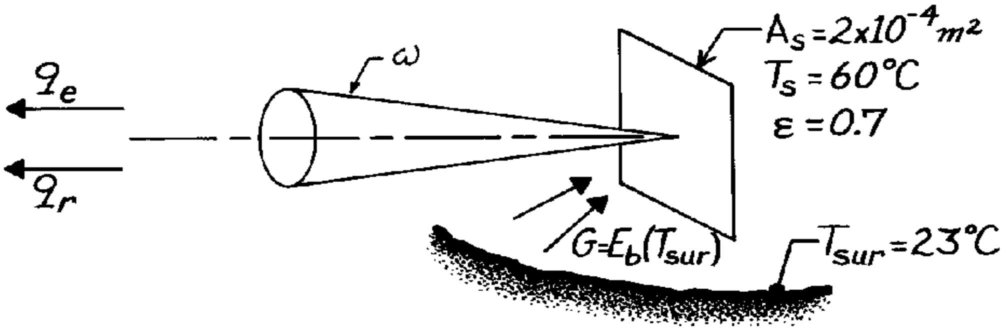
Hence,

$$q_b = \frac{144.9\ W/m^2}{\pi\ sr}\times\left(2\times10^{-4}\ m^2\right)\times\cos0°\times0.001\,sr = 9.23\,\mu W. \qquad <$$

Continued …..

PROBLEM 12.78 (Cont.)

(c) When the target is a gray, diffuse emitter, $\varepsilon = 0.7$, the thermograph will receive emitted power from the target and reflected irradiation resulting from the surroundings at $T_{sur} = 23°C$. Schematically:



The power is expressed as

$$q = q_e + q_r = \varepsilon\, q_b + I_r \cdot A_s \cos\theta_1 \cdot \omega \left[F_{(0\to12\,\mu m)} - F_{(0\to9\,\mu m)} \right]$$

where

q_b = radiant power from black surface, part (b) result

$F_{(0-\lambda)}$ = band emission fraction for $T_{sur} = 23°C$; using Table 12.1

$\lambda_2\, T_{sur} = 12 \times (23 + 273) = 3552\ \mu m\cdot K$, $F_{(0-\lambda_2)} = 0.394$

$\lambda_1\, T_{sur} = 9 \times (23 + 273) = 2664\ \mu m\cdot K$, $F_{(0-\lambda_1)} = 0.197$

I_r = reflected intensity, which because of diffuse nature of surface

$$I_r = \rho \frac{G}{\pi} = (1-\varepsilon)\frac{E_b(T_{sur})}{\pi}.$$

Hence

$$q = 0.7 \times 9.23\,\mu W + (1-0.7)\frac{5.667\times10^{-8}\ W/m^2\cdot K^4 \times (273+23)^4\ K}{\pi\ sr}$$

$$\times\left(2\times10^{-4}\ m^2\right)\times\cos0°\times0.001\,sr\,[0.394-0.197]$$

$$q = 6.46\,\mu W + 1.64\,\mu W = 8.10\,\mu W.$$ <

COMMENTS: (1) Comparing the results of parts (a) and (b), note that the power to the thermograph is slightly less for the gray surface with $\varepsilon = 0.7$. From part (b) see that the effect of the irradiation is substantial; that is, $1.64/8.10 \approx 20\%$ of the power received by the thermograph is due to reflected irradiation. Ignoring such effects leads to misinterpretation of temperature measurements using thermography.

(2) Many thermography devices have a spectral response in the 3 to 5 μm wavelength region as well as 9 – 12 μm.