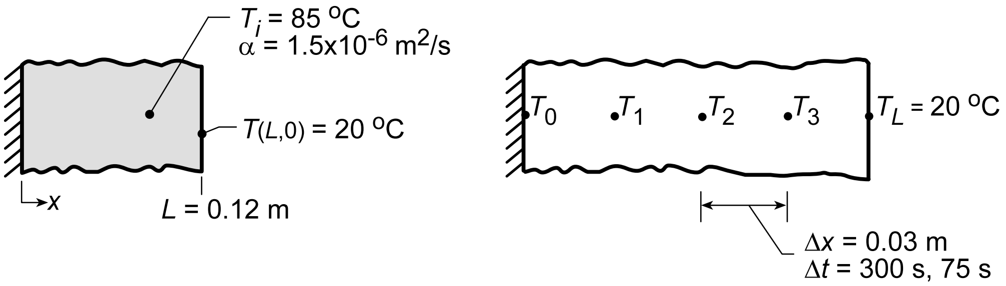
PROBLEM 5.98

KNOWN: A 0.12 m thick wall, with thermal diffusivity 1.5×10^{-6} m^2/s, initially at a uniform temperature of 85°C, has one face suddenly lowered to 20°C while the other face is perfectly insulated.

FIND: (a) Using the explicit finite-difference method with space and time increments of Δx = 30 mm and Δt = 300s, determine the temperature distribution within the wall 45 min after the change in surface temperature; (b) Effect of Δt on temperature histories of the surfaces and midplane.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional transient conduction, (2) Constant properties.

ANALYSIS: (a) The finite-difference equations for the interior points, nodes 0, 1, 2, and 3, can be determined from Eq. 5.73,

$$T_m^{p+1} = Fo\left(T_{m-1}^p + T_{m+1}^p\right) + (1 - 2\,Fo)T_m^p \qquad (1)$$

with

$$Fo = \alpha \Delta t / \Delta x^2 = 1.5 \times 10^{-6}\, m^2/s \times 300 s / (0.03\,m)^2 = 1/2. \qquad (2)$$

Note that the stability criterion, Eq. 5.74, Fo ≤ 1/2, is satisfied. Hence, combining Eqs. (1) and (2), $T_m^{p+1} = 1/2\left(T_{m-1}^p + T_{m+1}^p\right)$ for m = 0, 1, 2, 3. Since the adiabatic plane at x = 0 can be treated as a symmetry plane, $T_{m-1} = T_{m+1}$ for node 0 (m = 0). The finite-difference solution is generated in the table below using t = p·Δt = 300 p (s) = 5 p (min).

p	t(min)	T_0	T_1	T_2	T_3	T_L(°C)
0	0	85	85	85	85	20
1		85	85	85	52.5	20
2	10	85	85	68.8	52.5	20
3		85	76.9	68.8	44.4	20
4	20	76.9	76.9	60.7	44.4	20
5		76.9	68.8	60.7	40.4	20
6	30	68.8	68.8	54.6	40.4	20
7		68.8	61.7	54.6	37.3	20
8	40	61.7	61.7	49.5	37.3	20
9	45	61.7	55.6	49.5	34.8	20

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The temperature distribution can also be determined from the Heisler charts. For the wall,

$$Fo = \frac{\alpha t}{L^2} = \frac{1.5 \times 10^{-6}\, m^2/s \times (45 \times 60) s}{(0.12\,m)^2} = 0.28 \qquad \text{and} \qquad Bi^{-1} = \frac{k}{hL} = 0.$$

Continued...

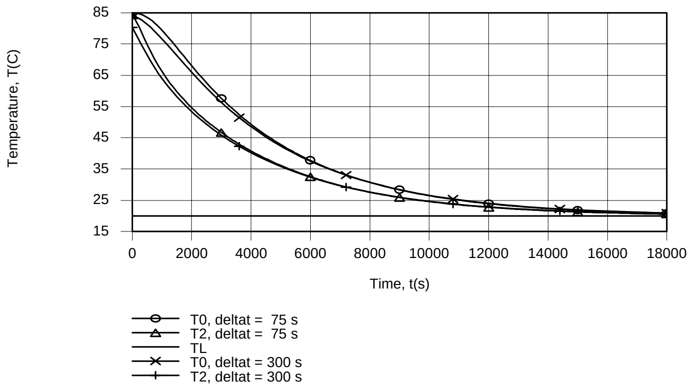
PROBLEM 5.98 (Cont.)

From Figure D.1, for $Bi^{-1} = 0$ and $Fo = 0.28$, find $\theta_o/\theta_i \approx 0.55$. Hence, for x = 0

$$\frac{T_o - T_\infty}{T_i - T_\infty} = \frac{\theta_o}{\theta_i} \quad \text{or} \quad T_o = T(0,t) = T_\infty + \frac{\theta_o}{\theta_i}(T_i - T_\infty) = 20^\circ C + 0.55(85 - 20)^\circ C = 55.8^\circ C .$$

This value is to be compared with 61.7°C for the finite-difference method.

(b) Using the IHT *Finite-Difference Equation* Tool Pad for *One-Dimensional Transient Conduction*, temperature histories were computed and results are shown for the insulated surface (T0) and the midplane, as well as for the chilled surface (TL).



Apart from small differences during early stages of the transient, there is excellent agreement between results obtained for the two time steps. The temperature decay at the insulated surface must, of course, lag that of the midplane.