PROBLEM 2.6

KNOWN: Temperature dependence of the thermal conductivity, k(T), for heat transfer through a plane wall.

FIND: Effect of k(T) on temperature distribution, T(x).

ASSUMPTIONS: (1) One-dimensional conduction, (2) Steady-state conditions, (3) No internal heat generation.

ANALYSIS: From Fourier's law and the form of k(T),

$$q''_x = -k\,\frac{dT}{dx} = -\left(k_o + aT\right)\frac{dT}{dx}. \tag{1}$$

The shape of the temperature distribution may be inferred from knowledge of $d^2T/dx^2 = d(dT/dx)/dx$. Since q''_x is independent of x for the prescribed conditions,

$$\frac{dq''_x}{dx} = -\frac{d}{dx}\left[\left(k_o + aT\right)\frac{dT}{dx}\right] = 0$$

$$-\left(k_o + aT\right)\frac{d^2T}{dx^2} - a\left[\frac{dT}{dx}\right]^2 = 0.$$

Hence,

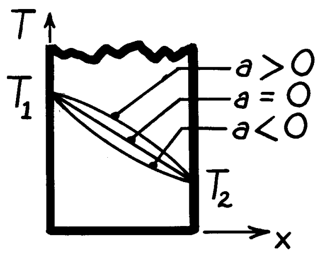
$$\frac{d^2T}{dx^2} = \frac{-a}{k_o + aT}\left[\frac{dT}{dx}\right]^2 \qquad \text{where } \begin{cases} k_o + aT = k > 0 \\ \left[\dfrac{dT}{dx}\right]^2 > 0 \end{cases}$$

from which it follows that for

$$a > 0: \; d^2T/dx^2 < 0$$

$$a = 0: \; d^2T/dx^2 = 0$$

$$a < 0: \; d^2T/dx^2 > 0.$$



COMMENTS: The shape of the distribution could also be inferred from Eq. (1). Since T decreases with increasing x,

a > 0: k decreases with increasing x = > | dT/dx | increases with increasing x

a = 0: $k = k_o$ = > dT/dx is constant

a < 0: k increases with increasing x = > | dT/dx | decreases with increasing x.