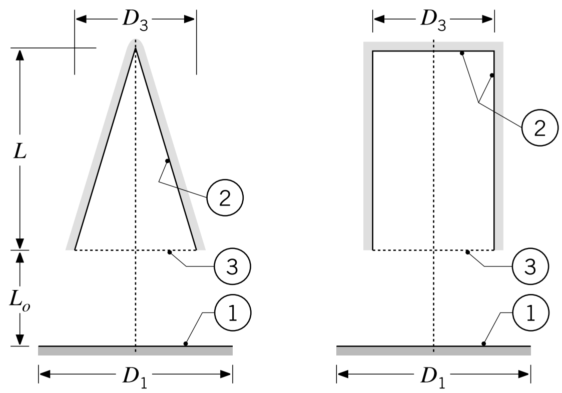
PROBLEM 13.4

KNOWN: Right circular cone and right-circular cylinder of same diameter D and length L positioned coaxially a distance L_o from the circular disk A_1; hypothetical area corresponding to the openings identified as A_3.

FIND: (a) Show that $F_{21} = (A_1/A_2)\, F_{13}$ and $F_{22} = 1 - (A_3/A_2)$, where F_{13} is the view factor between two, coaxial parallel disks (Table 13.2), for both arrangements, (b) Calculate F_{21} and F_{22} for $L = L_o = 50$ mm and $D_1 = D_3 = 50$ mm; compare magnitudes and explain similarities and differences, and (c) Magnitudes of F_{21} and F_{22} as L increases and all other parameters remain the same; sketch and explain key features of their variation with L.

SCHEMATIC:



(*a*) Right-circular cone (*b*) Right-circular cylinder

ASSUMPTIONS: (1) Diffuse surfaces with uniform radiosities, and (2) Inner base and lateral surfaces of the cylinder treated as a single surface, A_2.

ANALYSIS: (a) For both configurations,

$$F_{13} = F_{12} \tag{1}$$

since the radiant power leaving A_1 that is intercepted by A_3 is likewise intercepted by A_2. Applying reciprocity between A_1 and A_2,

$$A_1\, F_{12} = A_2\, F_{21} \tag{2}$$

Substituting from Eq. (1), into Eq. (2), solving for F_{21}, find

$$F_{21} = (A_1 / A_2) F_{12} = (A_1 / A_2) F_{13}$$ <

Treating the cone and cylinder as two-surface enclosures, the summation rule for A_2 is

$$F_{22} + F_{23} = 1 \tag{3}$$

Apply reciprocity between A_2 and A_3, solve Eq. (3) to find

$$F_{22} = 1 - F_{23} = 1 - (A_3 / A_2) F_{32}$$

and since $F_{32} = 1$, find

$$F_{22} = 1 - A_3 / A_2$$ <

Continued …..

PROBLEM 13.4 (Cont.)

(b) For the specified values of L, L_o, D_1 and D_2, the view factors are calculated and tabulated below. Relations for the areas are:

Disk-cone: $A_1 = \pi D_1^2 / 4$ $\quad A_2 = \pi D_3 / 2\left(L^2 + (D_3/2)^2\right)^{1/2}$ $\quad A_3 = \pi D_3^2 / 4$

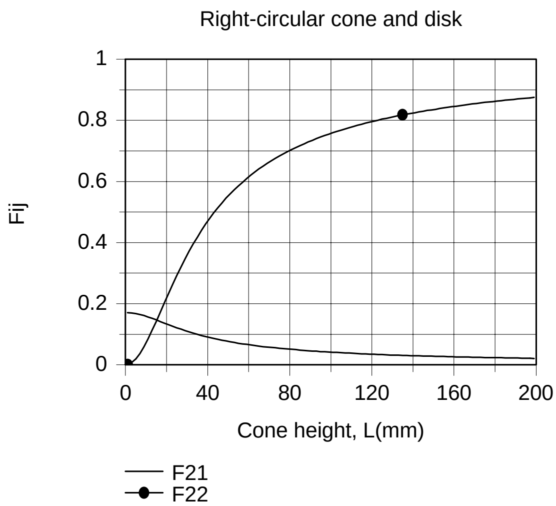
Disk-cylinder: $A_1 = \pi D_1^2 / 4$ $\quad A_2 = \pi D_3^2 / 4 + \pi D_3 L$ $\quad A_3 = \pi D_3^2 / 4$

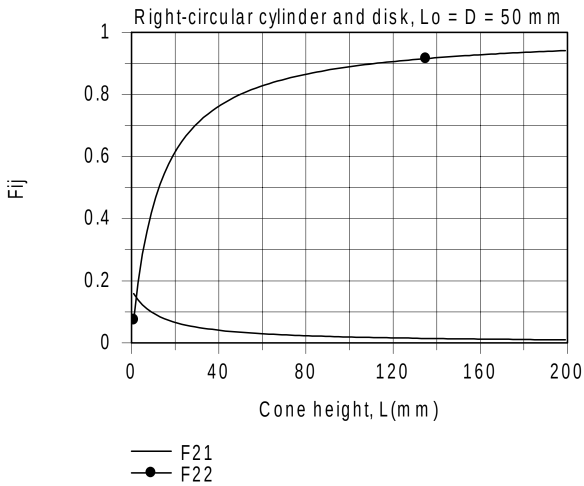
The view factor F_{13} is evaluated from Table 13.2, coaxial parallel disks (Fig. 13.5); find $F_{13} = 0.1716$.

	F_{21}	F_{22}
Disk-cone	0.0767	0.553
Disk-cylinder	0.0343	0.800

It follows that F_{21} is greater for the disk-cone (a) than for the cylinder-cone (b). That is, for (a), surface A_2 sees more of A_1 and less of itself than for (b). Notice that F_{22} is greater for (b) than (a); this is a consequence of $A_{2,b} > A_{2,a}$.

(c) Using the foregoing equations in the IHT workspace, the variation of the view factors F_{21} and F_{22} with L were calculated and are graphed below.





Note that for both configurations, when L = 0, find that $F_{21} = F_{13} = 0.1716$, the value obtained for coaxial parallel disks. As L increases, find that $F_{22} \rightarrow 1$; that is, the interior of both the cone and cylinder see mostly each other. Notice that the changes in both F_{21} and F_{22} with increasing L are greater for the disk-cylinder; F_{21} decreases while F_{22} increases.

COMMENTS: From the results of part (b), why isn't the sum of F_{21} and F_{22} equal to unity?