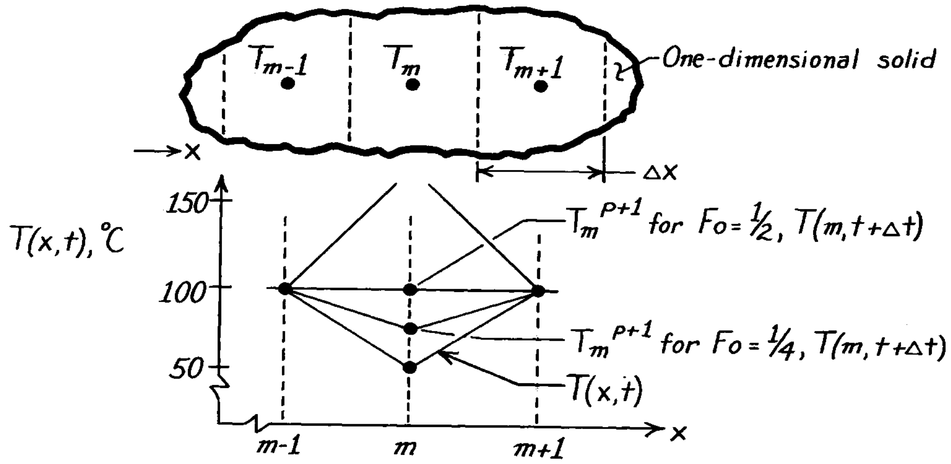
PROBLEM 5.93

KNOWN: Stability criterion for the explicit method requires that the coefficient of the T_m^p term of the one-dimensional, finite-difference equation be zero or positive.

FIND: For Fo > 1/2, the finite-difference equation will predict values of T_m^{p+1} which violate the Second law of thermodynamics. Consider the prescribed numerical values.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in x, (2) Constant properties, (3) No internal heat generation.

ANALYSIS: The explicit form of the finite-difference equation, Eq. 5.73, for an interior node is

$$T_m^{p+1} = \text{Fo}\left(T_{m+1}^p + T_{m-1}^p\right) + (1 - 2\ \text{Fo})T_m^p.$$

The stability criterion requires that the coefficient of T_m^p be zero or greater. That is,

$$(1 - 2\ \text{Fo}) \geq 0 \qquad \text{or} \qquad \text{Fo} \leq \frac{1}{2}.$$

For the prescribed temperatures, consider situations for which Fo = 1, ½ and ¼ and calculate T_m^{p+1}.

$$\text{Fo} = 1 \qquad T_m^{p+1} = 1(100 + 100)^\circ\text{C} + (1 - 2\times1)50^\circ\text{C} = 250^\circ\text{C}$$
$$\text{Fo} = 1/2 \qquad T_m^{p+1} = 1/2(100 + 100)^\circ\text{C} + (1 - 2\times1/2)50^\circ\text{C} = 100^\circ\text{C}$$
$$\text{Fo} = 1/4 \qquad T_m^{p+1} = 1/4(100 + 100)^\circ\text{C} + (1 - 2\times1/4)50^\circ\text{C} = 75^\circ\text{C}.$$

Plotting these distributions above, note that when Fo = 1, T_m^{p+1} is greater than 100°C, while for Fo = ½ and ¼ , $T_m^{p+1} \leq 100$°C. The distribution for Fo = 1 is thermodynamically impossible: heat is flowing into the node during the time period Δt, causing its temperature to rise; yet heat is flowing in the direction of increasing temperature. This is a violation of the Second law. When Fo = ½ or ¼, the node temperature increases during Δt, but the temperature gradients for heat flow are proper. This will be the case when Fo ≤ ½, verifying the stability criterion.