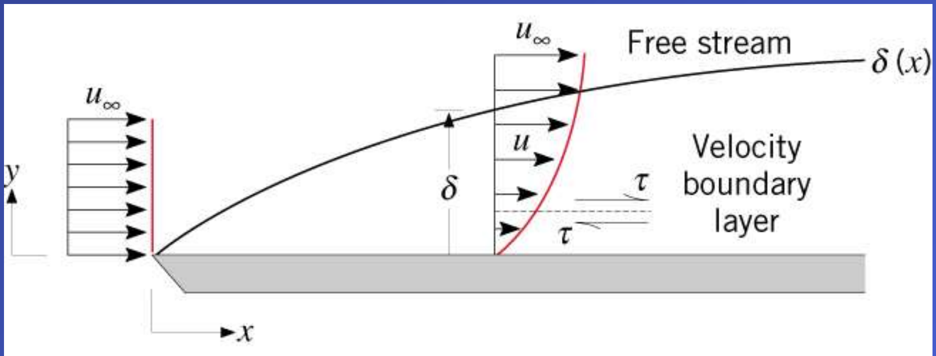
Introduction to Convection: Flow and Thermal Considerations

Chapter Six and Appendix E

Sections 6.1 to 6.9 and E.1 to E.3

Boundary Layers: Physical Features

- Velocity Boundary Layer



- A consequence of viscous effects associated with relative motion between a fluid and a surface.
- A region of the flow characterized by shear stresses and velocity gradients.
- A region between the surface and the free stream whose thickness δ increases in the flow direction.

$$\delta \rightarrow \frac{u(y)}{u_\infty} = 0.99$$

- Why does δ increase in the flow direction?

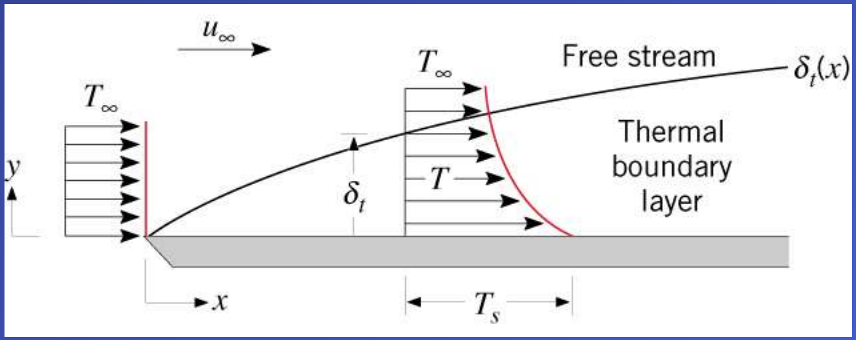
$$\tau_s = \mu \frac{\partial u}{\partial y}\bigg|_{y=0}$$

- Manifested by a surface shear stress τ_s that provides a drag force, F_D .

$$F_D = \int_{A_s} \tau_s dA_s$$

- How does τ_s vary in the flow direction? Why?

- Thermal Boundary Layer



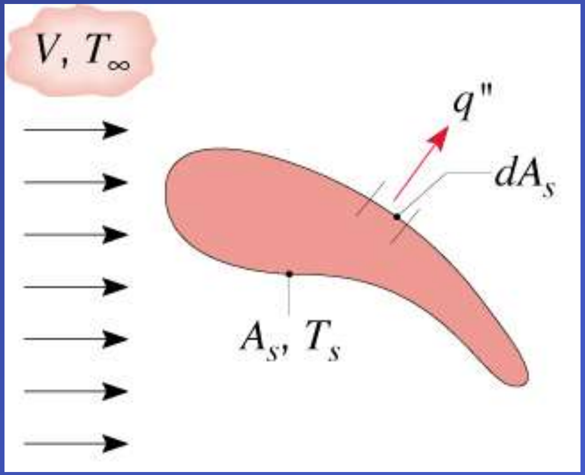
  - A consequence of heat transfer between the surface and fluid.
  - A region of the flow characterized by temperature gradients and heat fluxes.
  - A region between the surface and the free stream whose thickness δ_t increases in the flow direction.
  - Why does δ_t increase in the flow direction?
  - Manifested by a surface heat flux q_s'' and a convection heat transfer coefficient h .
  - If $\left(T_s - T_\infty\right)$ is constant, how do q_s'' and h vary in the flow direction?

$$\delta_t \rightarrow \frac{T_s - T\left(y\right)}{T_s - T_\infty} = 0.99$$

$$q_s'' = -k_f \frac{\partial T}{\partial y}\bigg|_{y=0}$$

$$h \equiv \frac{-k_f \partial T / \partial y\big|_{y=0}}{T_s - T_\infty}$$

Distinction between Local and Average Heat Transfer Coefficients



- Local Heat Flux and Coefficient:

$$q'' = h\left(T_s - T_\infty\right)$$

- Average Heat Flux and Coefficient for a Uniform Surface Temperature:

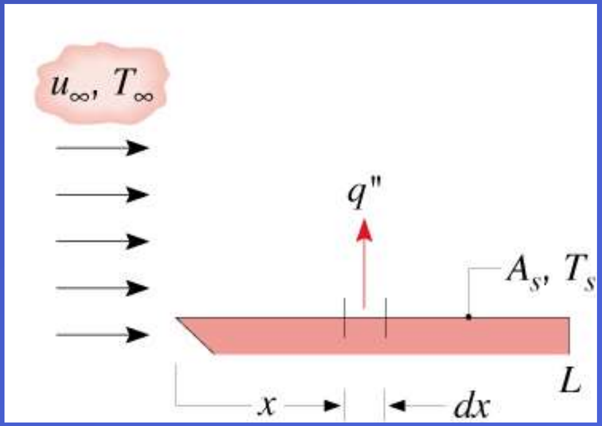
$$q = \bar{h} A_s \left(T_s - T_\infty\right)$$

$$q = \int_{A_s} q'' dA_s = \left(T_s - T_\infty\right) \int_{A_s} h dA_s$$

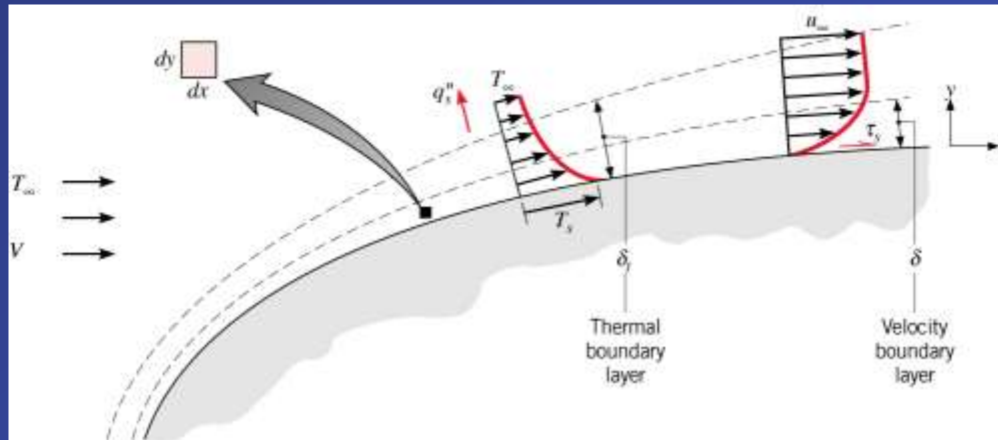
$$\bar{h} = \frac{1}{A_s} \int_{A_s} h dA_s$$

- For a flat plate in parallel flow:

$$\bar{h} = \frac{1}{L} \int_o^L h dx$$



The Boundary Layer Equations



- Consider concurrent velocity and thermal boundary layer development for steady, two-dimensional, incompressible flow with constant fluid properties $\left(\mu,\, c_p,\, k\right)$ and negligible body forces.

- Apply conservation of mass, Newton's 2nd Law of Motion and conservation of energy to a differential control volume and invoke the boundary layer approximations.

Velocity Boundary Layer:

$$u \gg v$$

$$\frac{\partial u}{\partial y} \gg \frac{\partial u}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial v}{\partial x}$$

Thermal Boundary Layer:

$$\frac{\partial T}{\partial y} \gg \frac{\partial T}{\partial x}$$

- Conservation of Mass:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

In the context of flow through a differential control volume, what is the physical significance of the foregoing terms, if each is multiplied by the mass density of the fluid?

- Newton's Second Law of Motion:

x-direction :

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial u}\right) = -\frac{dp}{dx} + \mu\frac{\partial^2 u}{\partial y^2}$$

What is the physical significance of each term in the foregoing equation?

Why can we express the pressure gradient as dp/dx instead of $\partial p / \partial x$?

y-direction :

$$\frac{\partial p}{\partial y} = 0$$

- Conservation of Energy:

$$\rho c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2$$

What is the physical significance of each term in the foregoing equation?

What is the second term on the right-hand side called and under what conditions may it be neglected?

Boundary Layer Similarity

- As applied to the boundary layers, the principle of similitude is based on determining similarity parameters that facilitate application of results obtained for a surface experiencing one set of conditions to geometrically similar surfaces experiencing different conditions. (Recall how introduction of the similarity parameters *Bi* and *Fo* permitted generalization of results for transient, one-dimensional condition).

- Dependent boundary layer variables of interest are:

$$\tau_s \text{ and } q'' \text{ or } h$$

- For a prescribed geometry, the corresponding independent variables are:

Geometrical: Size (L), Location (x,y)

Hydrodynamic: Velocity (V)

Fluid Properties:

Hydrodynamic: ρ, μ

Thermal: c_p, k

Hence,

$$u = f\left(x, y, L, V, \rho, \mu\right)$$

$$\tau_s = f\left(x, L, V, \rho, \mu\right)$$

and

$$T = f\left(x, y, L, V, \rho, \mu, c_p, k\right)$$

$$h = f\left(x, L, V, \rho, \mu, c_p, k\right)$$

- Key similarity parameters may be inferred by non-dimensionalizing the momentum and energy equations.
- Recast the boundary layer equations by introducing dimensionless forms of the independent and dependent variables.

$$x^* \equiv \frac{x}{L} \qquad y^* \equiv \frac{y}{L}$$

$$u^* \equiv \frac{u}{V} \qquad v^* \equiv \frac{v}{V}$$

$$T^* \equiv \frac{T - T_s}{T_\infty - T_s}$$

- Neglecting viscous dissipation, the following normalized forms of the x-momentum and energy equations are obtained:

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{dp^*}{dx^*} + \frac{1}{\mathrm{Re}_L} \frac{\partial^2 u^*}{\partial y^{*2}}$$

$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{\mathrm{Re}_L \, \mathrm{Pr}} \frac{\partial^2 T^*}{\partial y^{*2}}$$

$$\mathrm{Re}_L \equiv \frac{\rho V L}{\mu} = \frac{VL}{\nu} \rightarrow$$ the Reynolds Number

$$\mathrm{Pr} \equiv \frac{c_p \mu}{k} = \frac{\nu}{\alpha} \rightarrow$$ the Prandtl Number

How may the Reynolds and Prandtl numbers be interpreted physically?

- For a prescribed geometry,

$$u^* = f\left(x^*, y^*, \mathrm{Re}_L\right)$$

$$\tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = \left(\frac{\mu V}{L}\right) \left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0}$$

The dimensionless shear stress, or local friction coefficient, is then

$$C_f \equiv \frac{\tau_s}{\rho V^2/2} = \frac{2}{\mathrm{Re}_L} \left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0}$$

$$\left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0} = f\left(x^*, \mathrm{Re}_L\right)$$

$$C_f = \frac{2}{\mathrm{Re}_L} f\left(x^*, \mathrm{Re}_L\right)$$

What is the functional dependence of the average friction coefficient, $\bar{C}_f$?

- For a prescribed geometry,

$$T^* = f\left(x^*, y^*, \mathrm{Re}_L, \mathrm{Pr}\right)$$

$$h = \frac{-k_f \partial T / \partial y\big|_{y=0}}{T_s - T_\infty} = -\frac{k_f}{L}\frac{\left(T_\infty - T_s\right)}{\left(T_s - T_\infty\right)}\frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0} = +\frac{k_f}{L}\frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0}$$

The dimensionless local convection coefficient is then

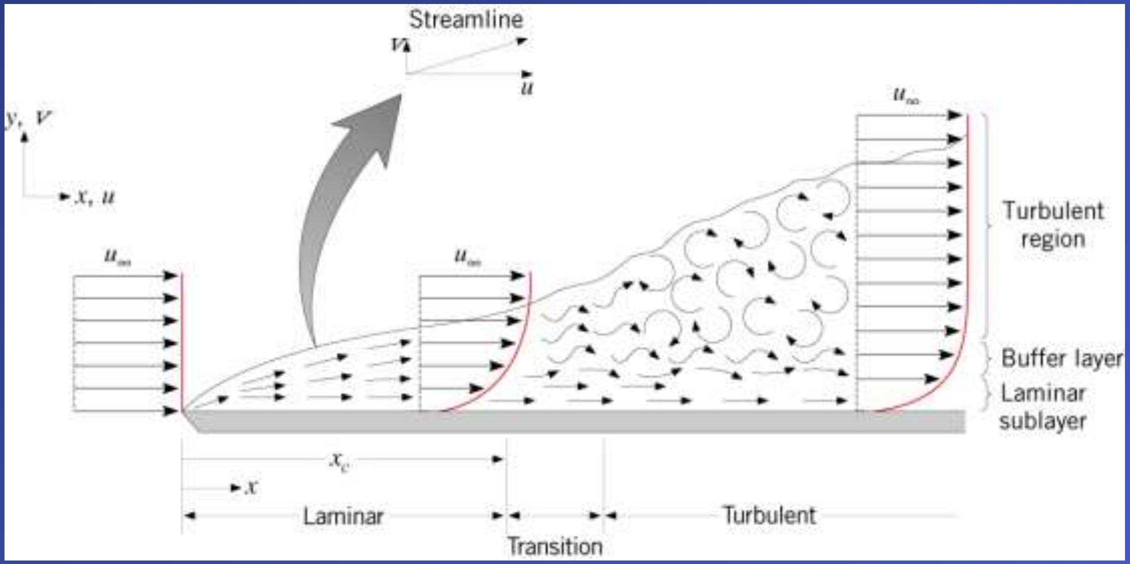
$$Nu \equiv \frac{hL}{k_f} = \frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0} = f\left(x^*, \mathrm{Re}_L, \mathrm{Pr}\right)$$

$Nu \rightarrow$ local Nusselt number

What is the functional dependence of the average Nusselt number?

How does the Nusselt number differ from the Biot number?

Boundary Layer Transition



- How would you characterize conditions in the laminar region of boundary layer development? In the turbulent region?

- What conditions are associated with transition from laminar to turbulent flow?

- Why is the Reynolds number an appropriate parameter for quantifying transition from laminar to turbulent flow?

- Transition criterion for a flat plate in parallel flow:

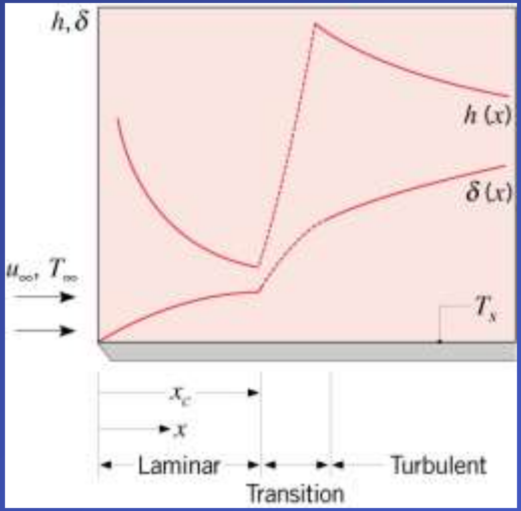
$$Re_{x,c} \equiv \frac{\rho u_\infty x_c}{\mu} \rightarrow \text{ critical Reynolds number}$$

$x_c \rightarrow$ location at which transition to turbulence begins

$$10^5 \lesssim Re_{x,c} \lesssim 3 \text{ x } 10^6$$

What may be said about transition if $Re_L < Re_{x,c}$? If $Re_L > Re_{x,c}$?

- Effect of transition on boundary layer thickness and local convection coefficient:



Why does transition provide a significant increase in the boundary layer thickness?

Why does the convection coefficient decay in the laminar region? Why does it increase significantly with transition to turbulence, despite the increase in the boundary layer thickness? Why does the convection coefficient decay in the turbulent region?

The Reynolds Analogy

- Equivalence of dimensionless momentum and energy equations for negligible pressure gradient (dp*/dx*~0) and Pr~1:

$$\underbrace{u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*}}_{\text{Advection terms}} = \underbrace{\frac{1}{\text{Re}} \frac{\partial^2 u^*}{\partial y^{*2}}}_{\text{Diffusion}}$$

$$\overbrace{u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*}} = \overbrace{\frac{1}{\text{Re}} \frac{\partial^2 T^*}{\partial y^{*2}}}$$

- Hence, for equivalent boundary conditions, the solutions are of the same form:

$$u^* = T^*$$

$$\left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0} = \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0}$$

$$C_f \frac{\text{Re}}{2} = Nu$$

or, with the Stanton number defined as,

$$St \equiv \frac{h}{\rho V c_p} = \frac{Nu}{Re\,Pr}$$

With Pr = 1, the Reynolds analogy, which relates important parameters of the velocity and thermal boundary layers, is

$$\frac{C_f}{2} = St$$

- Modified Reynolds (Chilton-Colburn) Analogy:
  - An empirical result that extends applicability of the Reynolds analogy:

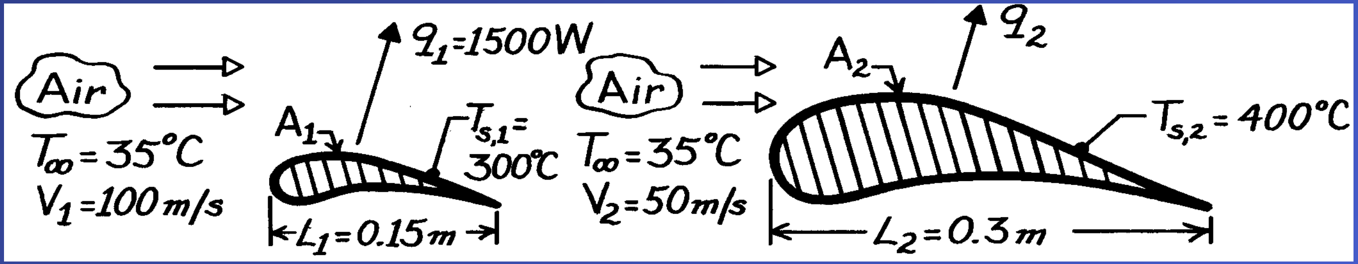
$$\frac{C_f}{2} = St\,\mathrm{Pr}^{2/3} \equiv j_H \qquad 0.6 < \mathrm{Pr} < 60$$

Colburn j factor for heat transfer

  - Applicable to laminar flow if dp*/dx* ~ 0.
  - Generally applicable to turbulent flow without restriction on dp*/dx*.

Problem 6.28: Determination of heat transfer rate for prescribed turbine blade operating conditions from wind tunnel data obtained for a geometrically similar but smaller blade. The blade surface area may be assumed to be directly proportional to its characteristic length $\left(A_s \propto L\right)$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Surface area A is directly proportional to characteristic length L, (4) Negligible radiation, (5) Blade shapes are geometrically similar.

ANALYSIS: For a prescribed geometry,

$$\overline{Nu} = \frac{\overline{h}L}{k} = f\left(Re_L, Pr\right).$$

The Reynolds numbers for the blades are

$$Re_{L,1} = \left(V_1 L_1 / \nu_1\right) = 15\,m^2/s \big/ \nu_1 \qquad Re_{L,2} = \left(V_2 L_2 / \nu_2\right) = 15\,m^2/s \big/ \nu_2.$$

Hence, with constant properties $\left(\nu_1 = \nu_2\right)$, $Re_{L,1} = Re_{L,2}$. Also, $Pr_1 = Pr_2$

Therefore,

$$\overline{Nu}_2 = \overline{Nu}_1$$

$$\left(\bar{h}_2 L_2 / k_2\right) = \left(\bar{h}_1 L_1 / k_1\right)$$

$$\bar{h}_2 = \frac{L_1}{L_2}\bar{h}_1 = \frac{L_1}{L_2}\frac{q_1}{A_1\left(T_{s,1} - T_\infty\right)}$$

The heat rate for the *second blade* is then

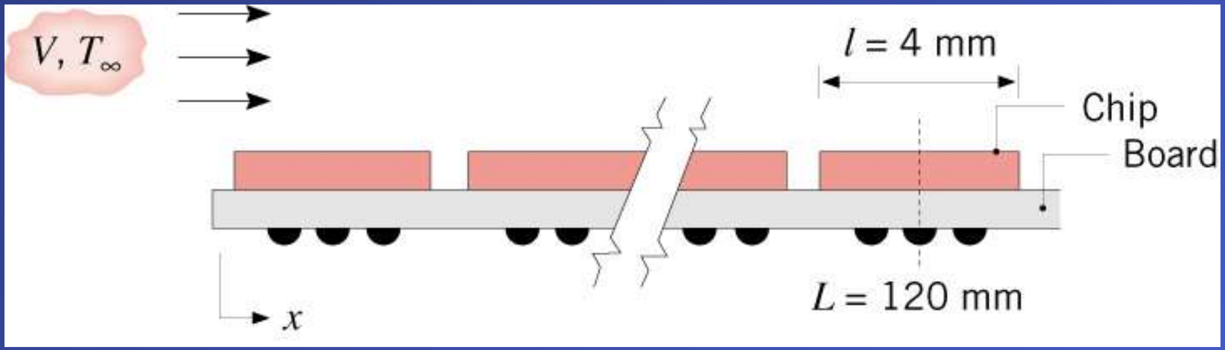
$$q_2 = \bar{h}_2 A_2 \left(T_{s,2} - T_\infty\right) = \frac{L_1}{L_2}\frac{A_2}{A_1}\frac{\left(T_{s,2} - T_\infty\right)}{\left(T_{s,1} - T_\infty\right)} q_1$$

$$q_2 = \frac{T_{s,2} - T_\infty}{T_{s,1} - T_\infty} q_1 = \frac{(400 - 35)}{(300 - 35)}(1500\ W)$$

$$q_2 = 2066\ W.$$

COMMENTS: (i) The variation in ν from Case 1 to Case 2 would cause $Re_{L,2}$ to differ from $Re_{L,1}$. However, for air and the prescribed temperatures, this non-constant property effect is small. (ii) If the Reynolds numbers were not equal $\left(Re_{L,1} \neq Re_{L2}\right)$, knowledge of the specific form of $f\left(Re_{L,}\ Pr\right)$ would be needed to determine h_2.

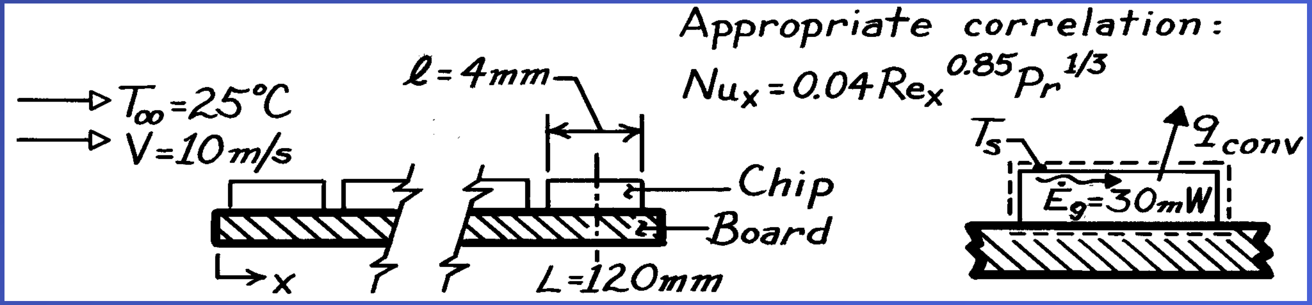
Problem 6.35: Use of a *local* Nusselt number correlation to estimate the surface temperature of a chip on a circuit board.



KNOWN: Expression for the local heat transfer coefficient of air at prescribed velocity and temperature flowing over electronic elements on a circuit board and heat dissipation rate for a 4 × 4 mm chip located 120mm from the leading edge.

FIND: Surface temperature of the chip surface, T_S.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Power dissipated within chip is lost by convection across the upper surface only, (3) Chip surface is isothermal, (4) The average heat transfer coefficient for the chip surface is equivalent to the local value at x = L.

PROPERTIES: *Table A-4*, Air (Evaluate properties at the *average temperature* of air in the boundary layer. Assuming $T_s = 45°C$, $T_{ave} = (45 + 25)/2 = 35°C = 308K$. Also, p = 1atm): $\nu = 16.69 \times 10^{-6} m^2/s$, $k = 26.9 \times 10^{-3}$ W/m·K, Pr = 0.703.

ANALYSIS: From an energy balance on the chip,

$$q_{conv} = \dot{E}_g = 30mW.$$

Newton's law of cooling for the upper chip surface can be written as

$$T_s = T_\infty + q_{conv} / \bar{h} A_{chip} \qquad (2)$$

where $A_{chip} = \ell^2$.

Assuming that the *average* heat transfer coefficient $\left(\bar{h}\right)$ over the chip surface is equivalent to the *local* coefficient evaluated at x = L, that is, $\bar{h}_{chip} \approx h_x\left(L\right)$, the local coefficient can be evaluated by applying the prescribed correlation at x = L.

$$Nu_x = \frac{h_x x}{k} = 0.04\left[\frac{Vx}{\nu}\right]^{0.85} Pr^{1/3}$$

$$h_L = 0.04\frac{k}{L}\left[\frac{VL}{\nu}\right]^{0.85} Pr^{1/3}$$

$$h_L = 0.04\left[\frac{0.0269\ \text{W/m}\cdot\text{K}}{0.120\ \text{m}}\right]\left[\frac{10\ \text{m/s}\times 0.120\ \text{m}}{16.69\times 10^{-6}\ \text{m}^2/\text{s}}\right]^{0.85}(0.703)^{1/3} = 107\ \text{W/m}^2\cdot\text{K}.$$

From Eq. (2), the surface temperature of the chip is

$$T_s = 25^\circ\text{C} + 30\times 10^{-3}\ \text{W}/107\ \text{W/m}^2\cdot\text{K}\times(0.004\text{m})^2 = 42.5^\circ\text{C}.$$

COMMENTS: (1) The estimated value of T_{ave} used to evaluate the air properties is reasonable.

(2) How else could $\overline{h}_{chip}$ have been evaluated? Is the assumption of $\overline{h} = h_L$ reasonable?