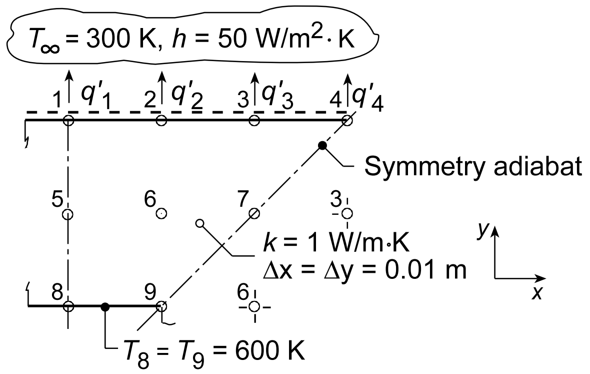
PROBLEM 4.46

KNOWN: Nodal temperatures from a steady-state, finite-difference analysis for a one-eighth symmetrical section of a square channel.

FIND: (a) Beginning with properly defined control volumes, derive the finite-difference equations for nodes 2, 4 and 7, and determine T_2, T_4 and T_7, and (b) Heat transfer loss per unit length from the channel, q'.

SCHEMATIC:



Node	T(°C)
1	430
3	394
6	492
8,9	600

ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction, (3) No internal volumetric generation, (4) Constant properties.

ANALYSIS: (a) Define control volumes about the nodes 2, 4, and 7, taking advantage of symmetry where appropriate and performing energy balances, $\dot{E}_{in} - \dot{E}_{out} = 0$, with $\Delta x = \Delta y$,

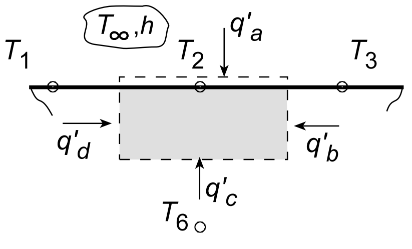
Node 2: $q'_a + q'_b + q'_c + q'_d = 0$

$$h\Delta x\left(T_\infty - T_2\right) + k\left(\Delta y/2\right)\frac{T_3 - T_2}{\Delta x} + k\Delta x\frac{T_6 - T_2}{\Delta y} + k\left(\Delta y/2\right)\frac{T_1 - T_2}{\Delta x} = 0$$

$$T_2 = \left[0.5T_1 + 0.5T_3 + T_6 + \left(h\Delta x/k\right)T_\infty\right] / \left[2 + \left(h\Delta x/k\right)\right]$$

$$T_2 = \left[0.5\times 430 + 0.5\times 394 + 492 + \left(50\,\mathrm{W/m^2\cdot K}\times 0.01\,\mathrm{m}/1\,\mathrm{W/m\cdot K}\right)300\right]\mathrm{K} \Big/ \left[2 + 0.50\right]$$

$$T_2 = 422\,\mathrm{K} \qquad <$$



Node 4: $q'_a + q'_b + q'_c = 0$

$$h\left(\Delta x/2\right)\left(T_\infty - T_4\right) + 0 + k\left(\Delta y/2\right)\frac{T_3 - T_4}{\Delta x} = 0$$

$$T_4 = \left[T_3 + \left(h\Delta x/k\right)T_\infty\right] / \left[1 + \left(h\Delta x/k\right)\right]$$

$$T_4 = \left[394 + 0.5\times 300\right]\mathrm{K} / \left[1 + 0.5\right] = 363\,\mathrm{K} \qquad <$$

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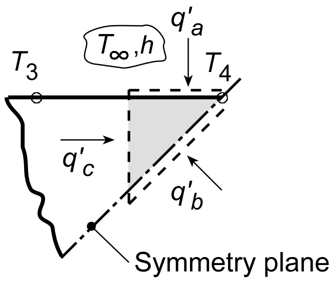

Node 7: From the first schematic, recognizing that the diagonal is a symmetry adiabat, we can treat node 7 as an interior node, hence

$$T_7 = 0.25(T_3 + T_3 + T_6 + T_6) = 0.25(394 + 394 + 492 + 492)\,\text{K} = 443\,\text{K}$$ <

(b) The heat transfer loss from the upper surface can be expressed as the sum of the convection rates from each node as illustrated in the first schematic,

$$q'_{cv} = q'_1 + q'_2 + q'_3 + q'_4$$

$$q'_{cv} = h(\Delta x/2)(T_1 - T_\infty) + h\Delta x(T_2 - T_\infty) + h\Delta x(T_3 - T_\infty) + h(\Delta x/2)(T_4 - T_\infty)$$

$$q'_{cv} = 50\,\text{W/m}^2\cdot\text{K} \times 0.1\,\text{m}\left[(430-300)/2 + (422-300) + (394-300) + (363-300)/2\right]\text{K}$$

$$q'_{cv} = 156\,\text{W/m}$$ <

COMMENTS: (1) Always look for symmetry conditions which can greatly simplify the writing of the nodal equation as was the case for Node 7.

(2) Consider using the *IHT Tool*, *Finite-Difference Equations*, for *Steady-State*, *Two-Dimensional* heat transfer to determine the nodal temperatures T_1 - T_7 when only the boundary conditions T_8, T_9 and (T_∞,h) are specified.