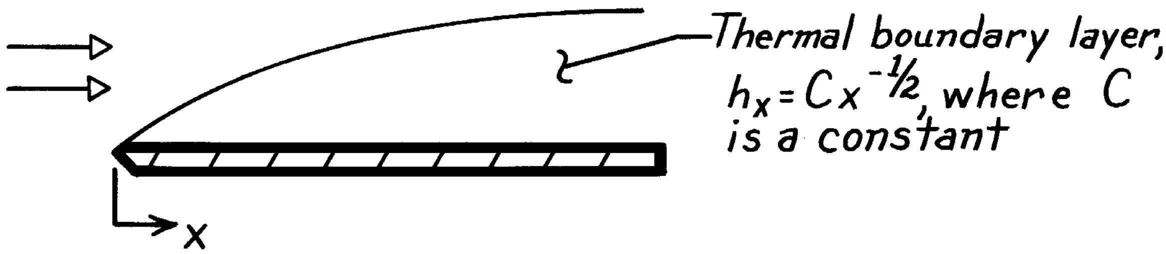
PROBLEM 6.1

KNOWN: Variation of h_x with x for laminar flow over a flat plate.

FIND: Ratio of average coefficient, $\bar{h}_x$, to local coefficient, h_x, at x.

SCHEMATIC:

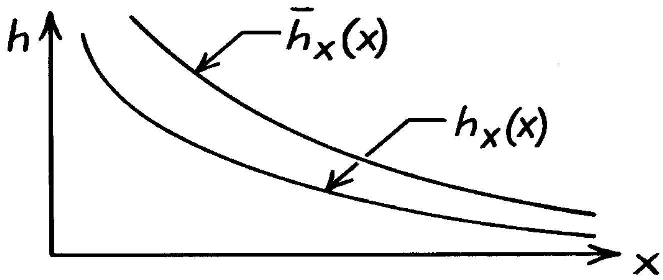


ANALYSIS: The average value of h_x between 0 and x is

$$\bar{h}_x = \frac{1}{x}\int_0^x h_x dx = \frac{C}{x}\int_0^x x^{-1/2} dx$$
$$\bar{h}_x = \frac{C}{x} 2x^{1/2} = 2Cx^{-1/2}$$
$$\bar{h}_x = 2h_x.$$

Hence, $$\frac{\bar{h}_x}{h_x} = 2.$$ <

COMMENTS: Both the local and average coefficients decrease with increasing distance x from the leading edge, as shown in the sketch below.

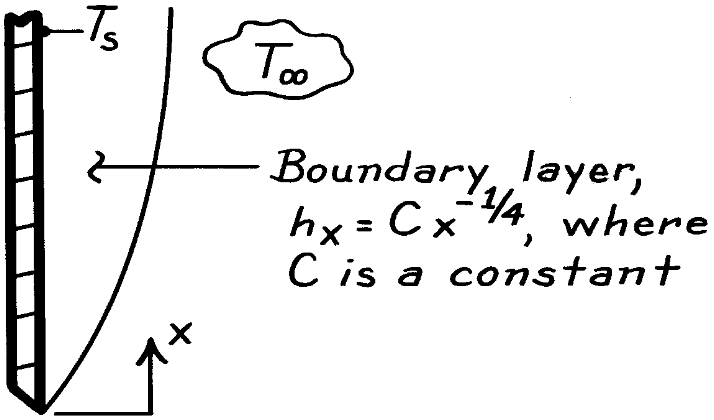


PROBLEM 6.2

KNOWN: Variation of local convection coefficient with x for free convection from a vertical heated plate.

FIND: Ratio of average to local convection coefficient.

SCHEMATIC:

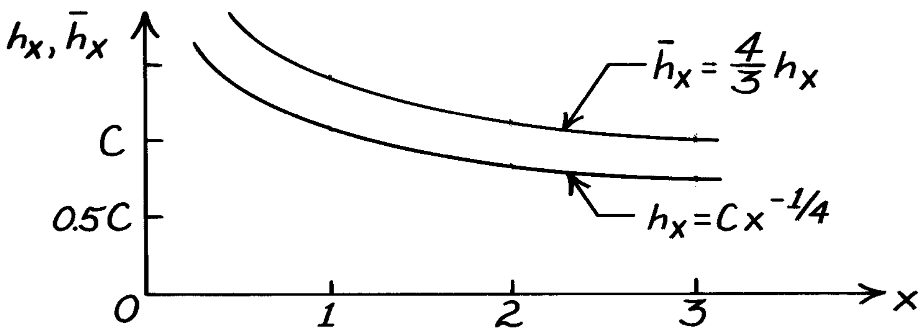


ANALYSIS: The average coefficient from 0 to x is

$$\overline{h}_x = \frac{1}{x}\int_0^x h_x dx = \frac{C}{x}\int_0^x x^{-1/4}dx$$
$$\overline{h}_x = \frac{4}{3}\frac{C}{x}x^{3/4} = \frac{4}{3}C\,x^{-1/4} = \frac{4}{3}h_x.$$

Hence,
$$\frac{\overline{h}_x}{h_x} = \frac{4}{3}.$$ <

The variations with distance of the local and average convection coefficients are shown in the sketch.



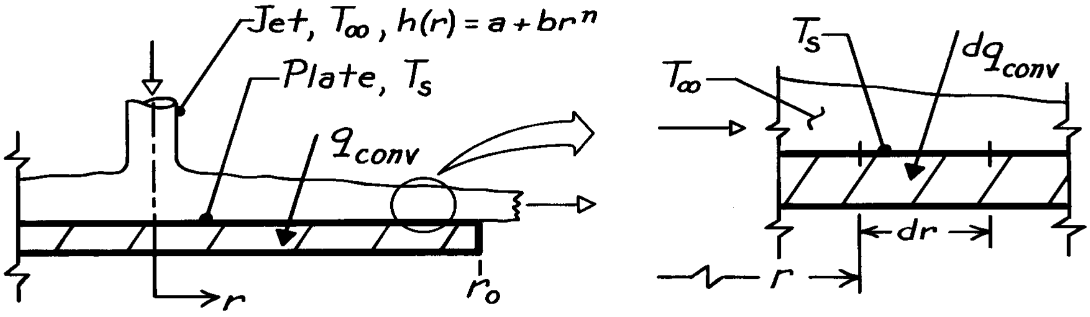
COMMENTS: Note that $\overline{h}_x / h_x = 4/3$ is independent of x. Hence the average coefficient for an entire plate of length L is $\overline{h}_L = \frac{4}{3}h_L$, where h_L is the local coefficient at x = L. Note also that the average *exceeds* the local. Why?

PROBLEM 6.3

KNOWN: Expression for the local heat transfer coefficient of a circular, hot gas jet at T_∞ directed normal to a circular plate at T_s of radius r_o.

FIND: Heat transfer rate to the plate by convection.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Flow is axisymmetric about the plate, (3) For h(r), a and b are constants and $n \neq -2$.

ANALYSIS: The convective heat transfer rate to the plate follows from Newton's law of cooling

$$q_{conv} = \int_A dq_{conv} = \int_A h(r)\cdot dA\cdot(T_\infty - T_s).$$

The local heat transfer coefficient is known to have the form,

$$h(r) = a + br^n$$

and the differential area on the plate surface is

$$dA = 2\pi r\, dr.$$

Hence, the heat rate is

$$q_{conv} = \int_0^{r_o} \left(a + br^n\right)\cdot 2\pi\ r\, dr\cdot(T_\infty - T_s)$$

$$q_{conv} = 2\pi(T_\infty - T_s)\left[\frac{a}{2}r^2 + \frac{b}{n+2}r^{n+2}\right]_0^{r_o}$$

$$q_{conv} = 2\pi\left[\frac{a}{2}r_o^2 + \frac{b}{n+2}r_o^{n+2}\right](T_\infty - T_s). \qquad <$$

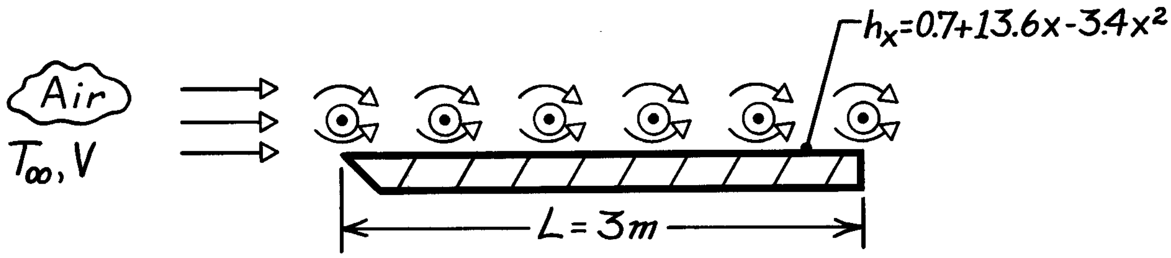
COMMENTS: Note the importance of the requirement, $n \neq -2$. Typically, the radius of the jet is much smaller than that of the plate.

PROBLEM 6.4

KNOWN: Distribution of local convection coefficient for obstructed parallel flow over a flat plate.

FIND: Average heat transfer coefficient and ratio of average to local at the trailing edge.

SCHEMATIC:



ANALYSIS: The average convection coefficient is

$$\bar{h}_L = \frac{1}{L}\int_0^L h_x dx = \frac{1}{L}\int_0^L \left(0.7 + 13.6x - 3.4x^2\right) dx$$

$$\bar{h}_L = \frac{1}{L}\left(0.7L + 6.8L^2 - 1.13L^3\right) = 0.7 + 6.8L - 1.13L^2$$

$$\bar{h}_L = 0.7 + 6.8(3) - 1.13(9) = 10.9\ \text{W/m}^2 \cdot \text{K}.$$ <

The local coefficient at x = 3m is

$$h_L = 0.7 + 13.6(3) - 3.4(9) = 10.9\ \text{W/m}^2 \cdot \text{K}.$$

Hence,

$$\bar{h}_L / h_L = 1.0.$$ <

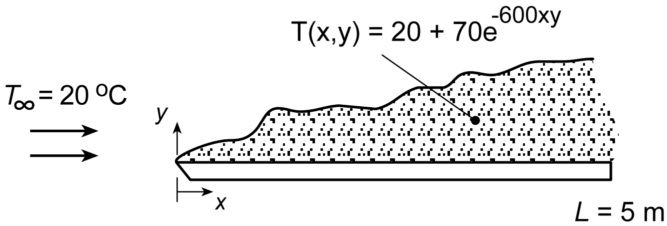
COMMENTS: The result $\bar{h}_L / h_L = 1.0$ is unique to x = 3m and is a consequence of the existence of a maximum for $h_x(x)$. The maximum occurs at x = 2m, where $\left(dh_x / dx\right) = 0$ and $\left(d^2h_x / dx^2 < 0.\right)$

PROBLEM 6.5

KNOWN: Temperature distribution in boundary layer for air flow over a flat plate.

FIND: Variation of local convection coefficient along the plate and value of average coefficient.

SCHEMATIC:



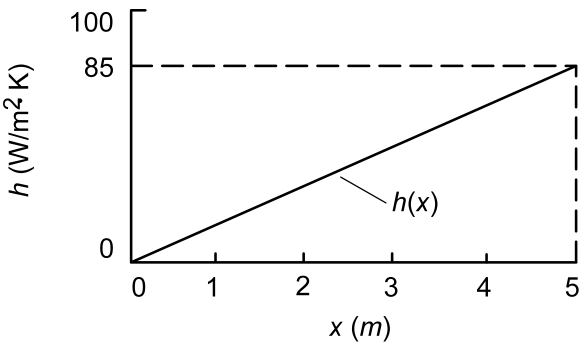
ANALYSIS: From Eq. 6.17,

$$h = -\frac{k\,\partial T/\partial y|_{y=0}}{(T_s - T_\infty)} = +\frac{k(70 \times 600x)}{(T_s - T_\infty)}$$

where $T_s = T(x,0) = 90°C$. Evaluating k at the arithmetic mean of the freestream and surface temperatures, $\overline{T} = (20 + 90)°C/2 = 55°C = 328\ K$, Table A.4 yields k = 0.0284 W/m·K. Hence, with $T_s - T_\infty = 70°C = 70\ K$,

$$h = \frac{0.0284\,W/m \cdot K(42,000x)K/m}{70\,K} = 17x\left(W/m^2 \cdot K\right)$$ <

and the convection coefficient increases linearly with x.



The average coefficient over the range 0 ≤ x ≤ 5 m is

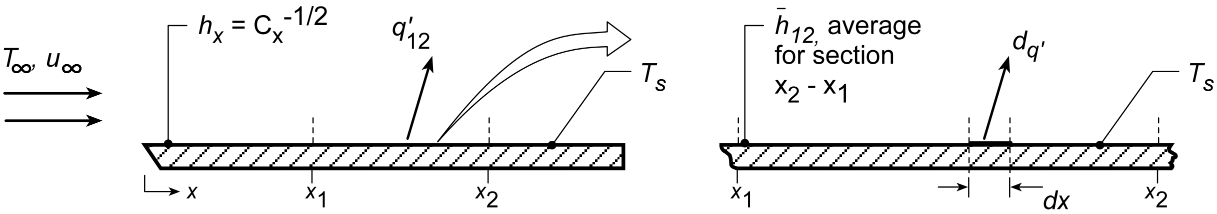
$$\overline{h} = \frac{1}{L}\int_0^L h\,dx = \frac{17}{5}\int_0^5 x\,dx = \frac{17}{5}\frac{x^2}{2}\bigg|_0^5 = 42.5\,W/m^2 \cdot K$$ <

PROBLEM 6.6

KNOWN: Variation of local convection coefficient with distance x from a heated plate with a uniform temperature T_s.

FIND: (a) An expression for the average coefficient $\bar{h}_{12}$ for the section of length $(x_2 - x_1)$ in terms of C, x_1 and x_2, and (b) An expression for $\bar{h}_{12}$ in terms of x_1 and x_2, and the average coefficients $\bar{h}_1$ and $\bar{h}_2$, corresponding to lengths x_1 and x_2, respectively.

SCHEMATIC:



ASSUMPTIONS: (1) Laminar flow over a plate with uniform surface temperature, T_s, and (2) Spatial variation of local coefficient is of the form $h_x = Cx^{-1/2}$, where C is a constant.

ANALYSIS: (a) The heat transfer rate per unit width from a longitudinal section, $x_2 - x_1$, can be expressed as

$$q'_{12} = \bar{h}_{12}\left(x_2 - x_1\right)\left(T_s - T_\infty\right) \tag{1}$$

where $\bar{h}_{12}$ is the average coefficient for the section of length $(x_2 - x_1)$. The heat rate can also be written in terms of the local coefficient, Eq. (6.3), as

$$q'_{12} = \int_{x_1}^{x_2} h_x dx\left(T_s - T_\infty\right) = \left(T_s - T_\infty\right)\int_{x_1}^{x_2} h_x dx \tag{2}$$

Combining Eq. (1) and (2),

$$\bar{h}_{12} = \frac{1}{\left(x_2 - x_1\right)}\int_{x_1}^{x_2} h_x dx \tag{3}$$

and substituting for the form of the local coefficient, $h_x = Cx^{-1/2}$, find that

$$\bar{h}_{12} = \frac{1}{\left(x_2 - x_1\right)}\int_{x_1}^{x_2} Cx^{-1/2}dx = \frac{C}{x_2 - x_1}\left[\frac{x^{1/2}}{1/2}\right]_{x_1}^{x_2} = 2C\frac{x_2^{1/2} - x_1^{1/2}}{x_2 - x_1} \tag{4}$$ <

(b) The heat rate, given as Eq. (1), can also be expressed as

$$q'_{12} = \bar{h}_2 x_2\left(T_s - T_\infty\right) - \bar{h}_1 x_1\left(T_s - T_\infty\right) \tag{5}$$

which is the difference between the heat rate for the plate over the section $(0 - x_2)$ and over the section $(0 - x_1)$. Combining Eqs. (1) and (5), find,

$$\bar{h}_{12} = \frac{\bar{h}_2 x_2 - \bar{h}_1 x_1}{x_2 - x_1} \tag{6}$$ <

COMMENTS: (1) Note that, from Eq. 6.6,

$$\bar{h}_x = \frac{1}{2}\int_0^x h_x dx = \frac{1}{x}\int_0^x Cx^{-1/2}dx = 2Cx^{-1/2} \tag{7}$$

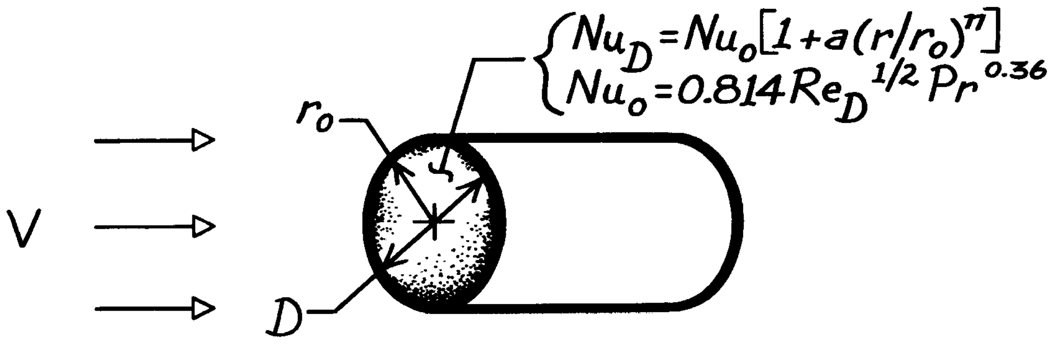
or $\bar{h}_x = 2h_x$. Substituting Eq. (7) into Eq. (6), see that the result is the same as Eq. (4).

PROBLEM 6.7

KNOWN: Radial distribution of local convection coefficient for flow normal to a circular disk.

FIND: Expression for average Nusselt number.

SCHEMATIC:



ASSUMPTIONS: Constant properties

ANALYSIS: The average convection coefficient is

$$\bar{h} = \frac{1}{A_s}\int_{A_s} h dA_s$$

$$\bar{h} = \frac{1}{\pi r_o^2}\int_0^{r_o} \frac{k}{D} Nu_o \left[1 + a\left(r/r_o\right)^n\right] 2\pi\ rdr$$

$$\bar{h} = \frac{kNu_o}{r_o^3}\left[\frac{r^2}{2} + \frac{ar^{n+2}}{(n+2)r_o^n}\right]_0^{r_o}$$

where Nu_o is the Nusselt number at the stagnation point (r = 0). Hence,

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = 2Nu_o\left[\frac{(r/r_o)^2}{2} + \frac{a}{(n+2)}\left(\frac{r}{r_o}\right)^{n+2}\right]_0^{r_o}$$

$$\overline{Nu}_D = Nu_o\left[1 + 2a/\left(n+2\right)\right]$$

$$\overline{Nu}_D = \left[1+2a/\left(n+2\right)\right]0.814 Re_D^{1/2} Pr^{0.36}.$$ <

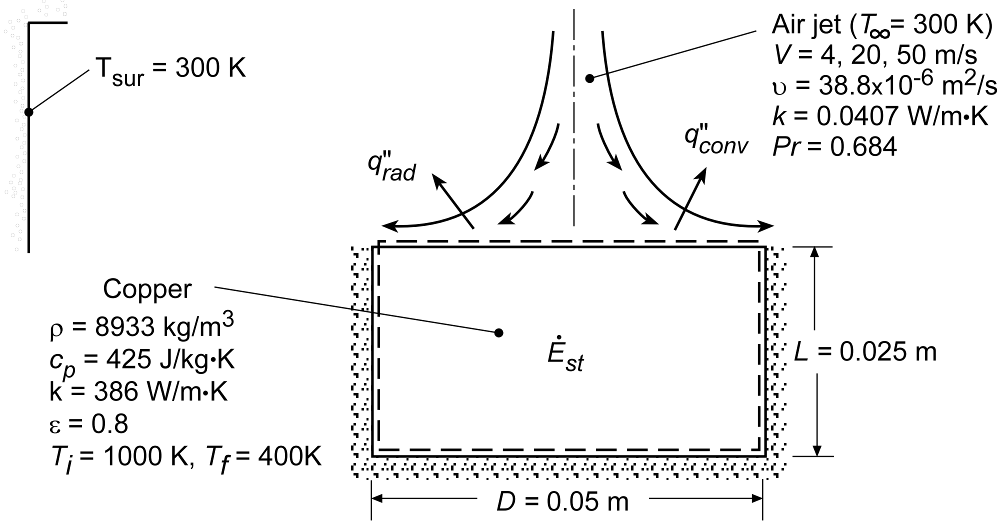
COMMENTS: The increase in h(r) with r may be explained in terms of the sharp turn which the boundary layer flow must make around the edge of the disk. The boundary layer accelerates and its thickness decreases as it makes the turn, causing the local convection coefficient to increase.

PROBLEM 6.8

KNOWN: Convection correlation and temperature of an impinging air jet. Dimensions and initial temperature of a heated copper disk. Properties of the air and copper.

FIND: Effect of jet velocity on temperature decay of disk following jet impingement.

SCHEMATIC:



ASSUMPTIONS: (1) Validity of lumped capacitance analysis, (2) Negligible heat transfer from sides and bottom of disk, (3) Constant properties.

ANALYSIS: Performing an energy balance on the disk, it follows that $\dot{E}_{st} = \rho V c\, dT/dt = -A_s\left(q''_{conv} + q''_{rad}\right)$. Hence, with V = A_sL,

$$\frac{dT}{dt} = -\frac{\overline{h}\left(T - T_\infty\right) + h_r\left(T - T_{sur}\right)}{\rho c L}$$

where, $h_r = \varepsilon\sigma\left(T + T_{sur}\right)\left(T^2 + T_{sur}^2\right)$ and, from the solution to Problem 6.7,

$$\overline{h} = \frac{k}{D}\overline{Nu}_D = \frac{k}{D}\left(1 + \frac{2a}{n+2}\right)0.814\,Re_D^{1/2}\,Pr^{0.36}$$

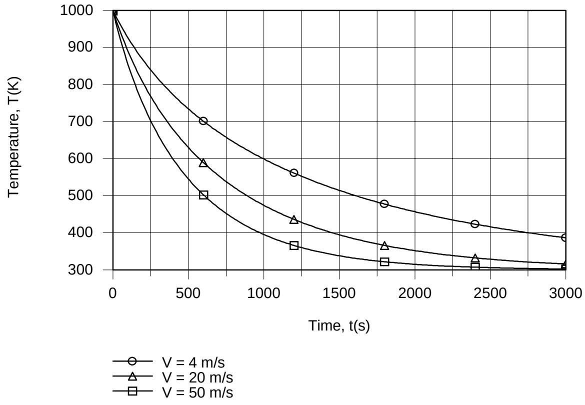
With a = 0.30 and n = 2, it follows that

$$\overline{h} = (k/D)\,0.936\,Re_D^{1/2}\,Pr^{0.36}$$

where Re_D = VD/ν. Using the *Lumped Capacitance Model* of IHT, the following temperature histories were determined.

Continued …..

PROBLEM 6.8 (Cont.)



The temperature decay becomes more pronounced with increasing V, and a final temperature of 400 K is reached at t = 2760, 1455 and 976s for V = 4, 20 and 50 m/s, respectively.

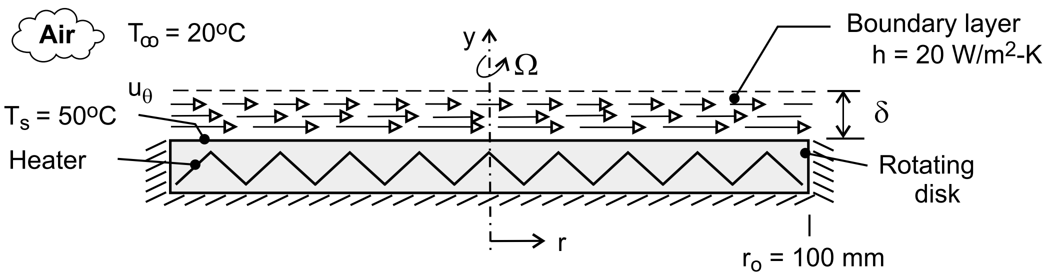
COMMENTS: The maximum Biot number, Bi = $\left(\bar{h}+h_r\right)L/k_{Cu}$, is associated with V = 50 m/s (maximum $\bar{h}$ of 169 $W/m^2 \cdot K$) and t = 0 (maximum h_r of 64 $W/m^2 \cdot K$), in which case the maximum Biot number is Bi = (233 $W/m^2 \cdot K$)(0.025 m)/(386 $W/m \cdot K$) = 0.015 < 0.1. Hence, the lumped capacitance approximation is valid.

PROBLEM 6.9

KNOWN: Local convection coefficient on rotating disk. Radius and surface temperature of disk. Temperature of stagnant air.

FIND: Local heat flux and total heat rate. Nature of boundary layer.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat transfer from back surface and edge of disk.

ANALYSIS: If the local convection coefficient is independent of radius, the local heat flux at every point on the disk is

$$q'' = h\left(T_s - T_\infty\right) = 20\,\text{W/m}^2\cdot\text{K}\left(50-20\right)^\circ\text{C} = 600\,\text{W/m}^2 \qquad <$$

Since h is independent of location, $\bar{h} = h = 20\,\text{W/m}^2\cdot\text{K}$ and the total power requirement is

$$P_{elec} = q = \bar{h}A_s\left(T_s - T_\infty\right) = \bar{h}\pi r_o^2\left(T_s - T_\infty\right)$$

$$P_{elec} = \left(20\,\text{W/m}^2\cdot\text{K}\right)\pi\left(0.1\text{m}\right)^2\left(50-20\right)^\circ\text{C} = 18.9\ \text{W} \qquad <$$

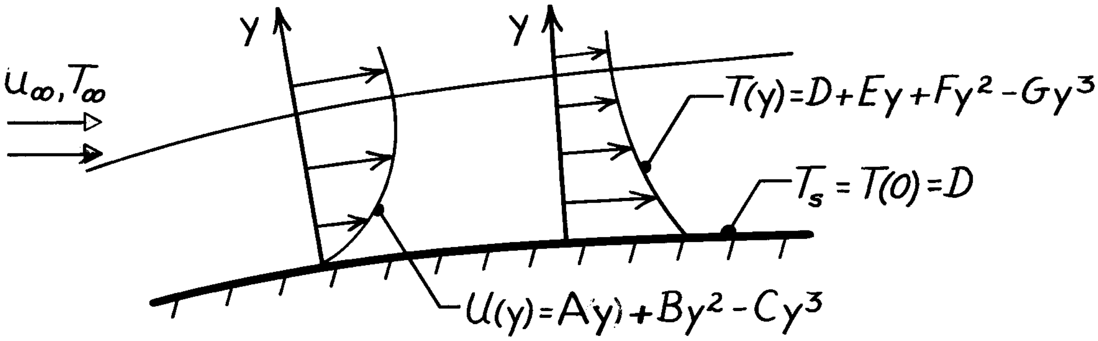
If the convection coefficient is independent of radius, the boundary layer must be of uniform thickness δ. Within the boundary layer, air flow is principally in the circumferential direction. The circumferential velocity component u_θ corresponds to the rotational velocity of the disk at the surface (y = 0) and increases with increasing r ($u_\theta = \Omega r$). The velocity decreases with increasing distance y from the surface, approaching zero at the outer edge of the boundary layer ($y \to \delta$).

PROBLEM 6.10

KNOWN: Form of the velocity and temperature profiles for flow over a surface.

FIND: Expressions for the friction and convection coefficients.

SCHEMATIC:



ANALYSIS: The shear stress at the wall is

$$\tau_s = \mu \left.\frac{\partial u}{\partial y}\right]_{y=0} = \mu \left[A + 2By - 3Cy^2\right]_{y=0} = A\mu.$$

Hence, the friction coefficient has the form,

$$C_f = \frac{\tau_s}{\rho u_\infty^2/2} = \frac{2A\mu}{\rho u_\infty^2}$$

$$C_f = \frac{2A\nu}{u_\infty^2}.$$ <

The convection coefficient is

$$h = \frac{-k_f \left(\partial T/\partial y\right)_{y=0}}{T_s - T_\infty} = \frac{-k_f \left[E + 2Fy - 3Gy^2\right]_{y=0}}{D - T_\infty}$$

$$h = \frac{-k_f E}{D - T_\infty}.$$ <

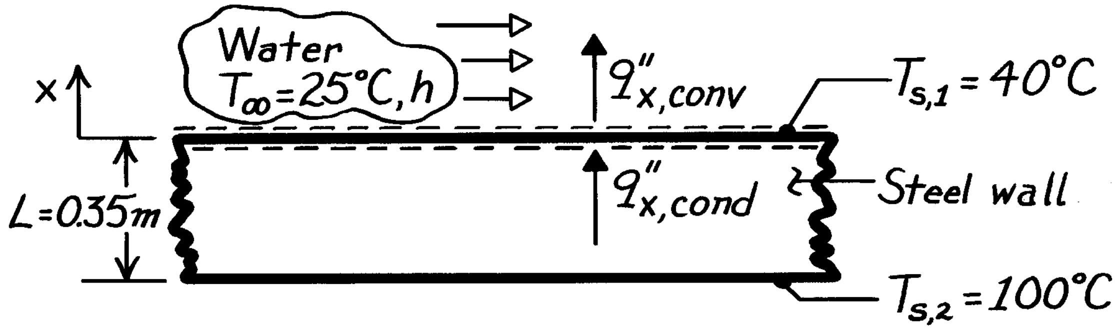
COMMENTS: It is a simple matter to obtain the important surface parameters from knowledge of the corresponding boundary layer profiles. However, it is rarely a simple matter to determine the form of the profile.

PROBLEM 6.11

KNOWN: Surface temperatures of a steel wall and temperature of water flowing over the wall.

FIND: (a) Convection coefficient, (b) Temperature gradient in wall and in water at wall surface.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer in x, (3) Constant properties.

PROPERTIES: *Table A-1*, Steel Type AISI 1010 (70°C = 343K), k_s = 61.7 W/m·K; *Table A-6*, Water (32.5°C = 305K), k_f = 0.62 W/m·K.

ANALYSIS: (a) Applying an energy balance to the control surface at x = 0, it follows that

$$q''_{x,cond} - q''_{x,conv} = 0$$

and using the appropriate rate equations,

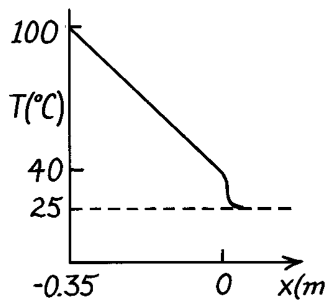
$$k_s \frac{T_{s,2} - T_{s,1}}{L} = h\left(T_{s,1} - T_\infty\right).$$

Hence,

$$h = \frac{k_s}{L}\frac{T_{s,2} - T_{s,1}}{T_{s,1} - T_\infty} = \frac{61.7\ \text{W/m}\cdot\text{K}}{0.35\text{m}}\frac{60^\circ\text{C}}{15^\circ\text{C}} = 705\ \text{W/m}^2\cdot\text{K}. \qquad <$$

(b) The gradient in the wall at the surface is

$$\left(dT/dx\right)_s = -\frac{T_{s,2} - T_{s,1}}{L} = -\frac{60^\circ\text{C}}{0.35\text{m}} = -171.4^\circ\text{C/m}.$$



In the water at x = 0, the definition of h gives

$$\left(dT/dx\right)_{f,x=0} = -\frac{h}{k_f}\left(T_{s,1} - T_\infty\right)$$

$$\left(dT/dx\right)_{f,x=0} = -\frac{705\ \text{W/m}^2\cdot\text{K}}{0.62\ \text{W/m}\cdot\text{K}}\left(15^\circ\text{C}\right) = -17{,}056^\circ\text{C/m}. \qquad <$$

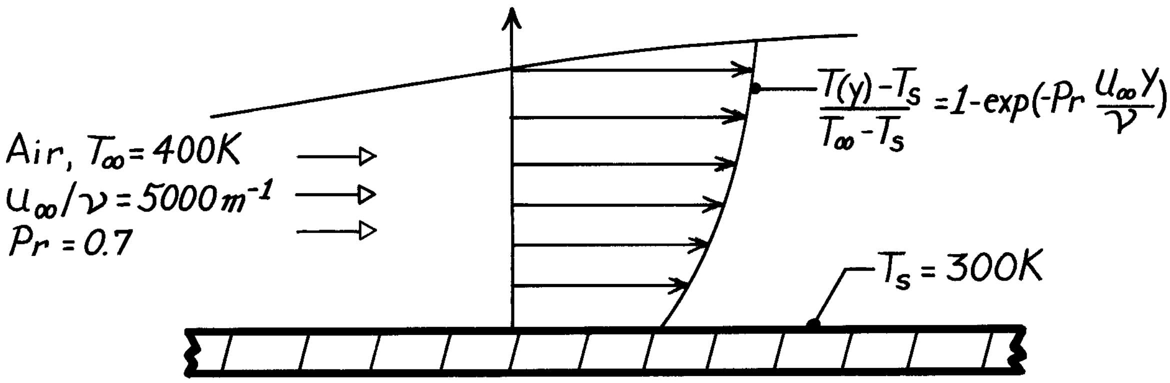
COMMENTS: Note the relative magnitudes of the gradients. Why is there such a large difference?

PROBLEM 6.12

KNOWN: Boundary layer temperature distribution.

FIND: Surface heat flux.

SCHEMATIC:



PROPERTIES: *Table A-4*, Air (T_s = 300K): k = 0.0263 W/m·K.

ANALYSIS: Applying Fourier's law at y = 0, the heat flux is

$$q_s'' = -k\left.\frac{\partial T}{\partial y}\right|_{y=0} = -k\left(T_\infty - T_s\right)\left[Pr\frac{u_\infty}{\nu}\right]\exp\left[-Pr\frac{u_\infty y}{\nu}\right]\Bigg|_{y=0}$$

$$q_s'' = -k\left(T_\infty - T_s\right)Pr\frac{u_\infty}{\nu}$$

$$q_s'' = -0.0263\ \text{W/m}\cdot\text{K}\left(100\text{K}\right)0.7\times 5000\ \text{1/m}.$$

$$q_s'' = -9205\ \text{W/m}^2.$$ <

COMMENTS: (1) Negative flux implies convection heat transfer to the surface.

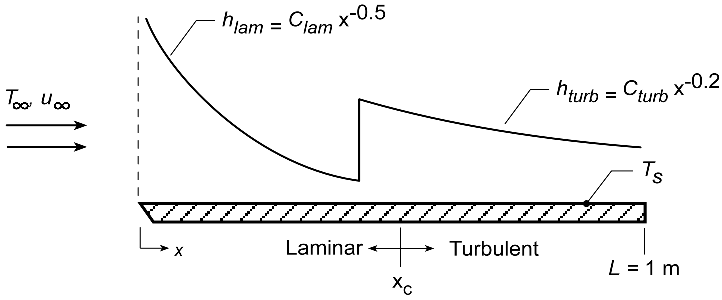
(2) Note use of k at T_s to evaluate q_s'' from Fourier's law.

PROBLEM 6.13

KNOWN: Air flow over a flat plate of length L = 1 m under conditions for which transition from laminar to turbulent flow occurs at x_c = 0.5m based upon the critical Reynolds number, $Re_{x,c} = 5\times 10^5$. Forms for the local convection coefficients in the laminar and turbulent regions.

FIND: (a) Velocity of the air flow using thermophysical properties evaluated at 350 K, (b) An expression for the average coefficient $\overline{h}_{lan}(x)$, as a function of distance from the leading edge, x, for the laminar region, $0 \le x \le x_c$, (c) An expression for the average coefficient $\overline{h}_{turb}(x)$, as a function of distance from the leading edge, x, for the turbulent region, $x_c \le x \le L$, and (d) Compute and plot the local and average convection coefficients, h_x and $\overline{h}_x$, respectively, as a function of x for $0 \le x \le L$.

SCHEMATIC:



ASSUMPTIONS: (1) Forms for the local coefficients in the laminar and turbulent regions, $h_{lam} = C_{lam}x^{-0.5}$ and $h_{tirb} = C_{turb}x^{-0.2}$ where C_{lam} = 8.845 W/m$^{3/2}$·K, C_{turb} = 49.75 W/m^2·K$^{0.8}$, and x has units (m).

PROPERTIES: *Table A.4*, Air (T = 350 K): k = 0.030 W/m·K, ν = 20.92 × 10^{-6} m^2/s, Pr = 0.700.

ANALYSIS: (a) Using air properties evaluated at 350 K with x_c = 0.5 m,

$$Re_{x,c} = \frac{u_\infty x_c}{\nu} = 5\times10^5 \qquad u_\infty = 5\times10^5 \nu/x_c = 5\times10^5\times20.92\times10^{-6}\,\mathrm{m^2/s}/0.5\,\mathrm{m} = 20.9\,\mathrm{m/s} \quad <$$

(b) From Eq. 6.5, the average coefficient in the laminar region, $0 \le x \le x_c$, is

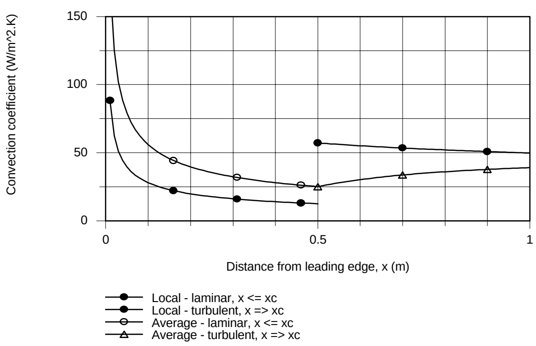
$$\overline{h}_{lam}(x) = \frac{1}{x}\int_0^x h_{lam}(x)dx = \frac{1}{x}C_{lam}\int_o^x x^{-0.5}dx = \frac{1}{x}C_{lam}x^{0.5} = 2C_{lam}x^{-0.5} = 2h_{lam}(x) \qquad (1) \quad <$$

(c) The average coefficient in the turbulent region, $x_c \le x \le L$, is

$$\overline{h}_{turb}(x) = \frac{1}{x}\left[\int_0^{x_c} h_{lam}(x)dx + \int_{x_c}^{x} h_{turb}(x)dx\right] = \left[C_{lam}\left.\frac{x^{0.5}}{0.5}\right|_0^{x_c} + C_{turb}\left.\frac{x^{0.8}}{0.8}\right|_{x_c}^{x}\right]$$

$$\overline{h}_{turb}(x) = \frac{1}{x}\left[2C_{lam}x_c^{0.5} + 1.25C_{turb}\left(x^{0.8} - x_c^{0.8}\right)\right] \qquad (2) \quad <$$

(d) The local and average coefficients, Eqs. (1) and (2) are plotted below as a function of x for the range $0 \le x \le L$.

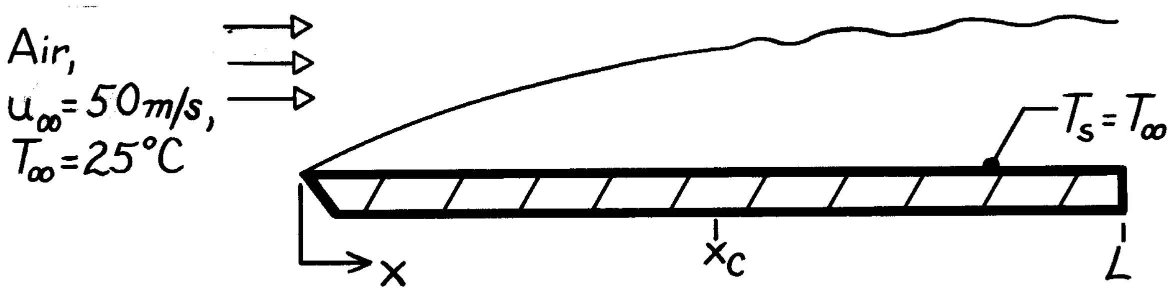


PROBLEM 6.14

KNOWN: Air speed and temperature in a wind tunnel.

FIND: (a) Minimum plate length to achieve a Reynolds number of 10^8, (b) Distance from leading edge at which transition would occur.

SCHEMATIC:



ASSUMPTIONS: (1) Isothermal conditions, $T_s = T_\infty$.

PROPERTIES: *Table A-4*, Air (25°C = 298K): $\nu = 15.71 \times 10^{-6} m^2/s$.

ANALYSIS: (a) The Reynolds number is

$$Re_x = \frac{\rho\, u_\infty x}{\mu} = \frac{u_\infty x}{\nu}.$$

To achieve a Reynolds number of 1×10^8, the minimum plate length is then

$$L_{min} = \frac{Re_x\, \nu}{u_\infty} = \frac{1\times10^8\left(15.71\times10^{-6} m^2/s\right)}{50\ m/s}$$

$$L_{min} = 31.4\ m. \qquad <$$

(b) For a transition Reynolds number of 5×10^5

$$x_c = \frac{Re_{x,c}\, \nu}{u_\infty} = \frac{5\times10^5\ \left(15.71\times10^{-6} m^2/s\right)}{50\ m/s}$$

$$x_c = 0.157\ m. \qquad <$$

COMMENTS: Note that

$$\frac{x_c}{L} = \frac{Re_{x,c}}{Re_L}$$

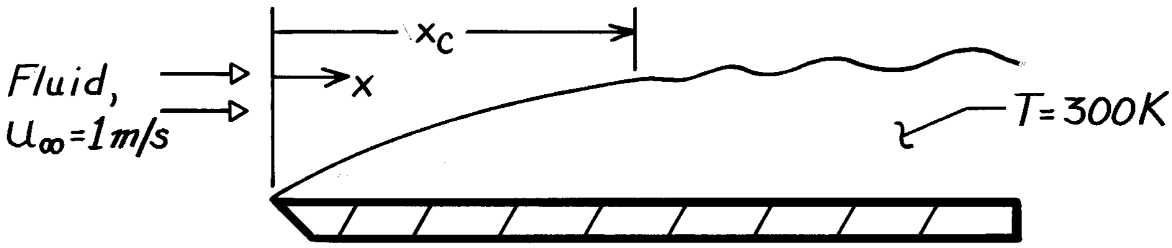
This expression may be used to quickly establish the location of transition from knowledge of $Re_{x,c}$ and Re_L.

PROBLEM 6.15

KNOWN: Transition Reynolds number. Velocity and temperature of atmospheric air, water, engine oil and mercury flow over a flat plate.

FIND: Distance from leading edge at which transition occurs for each fluid.

SCHEMATIC:



ASSUMPTIONS: Transition Reynolds number is $Re_{x,c} = 5\times10^5$.

PROPERTIES: For the fluids at T = 300K;

Fluid	*Table*	*ν(m²/s)*
Air (1 atm)	A-4	15.89×10^{-6}
Water	A-6	0.858×10^{-6}
Engine Oil	A-5	550×10^{-6}
Mercury	A-5	0.113×10^{-6}

ANALYSIS: The point of transition is

$$x_c = Re_{x,c}\frac{\nu}{u_\infty} = \frac{5\times10^5}{1\text{ m/s}}\nu.$$

Substituting appropriate viscosities, find

Fluid	*x_c(m)*
Air	7.95
Water	0.43
Oil	275
Mercury	0.06

<

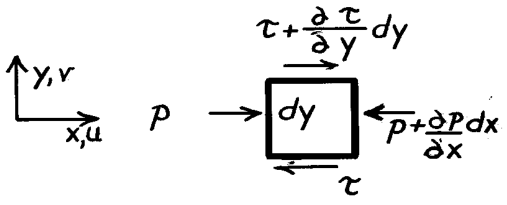
COMMENTS: Due to the effect which viscous forces have on attenuating the instabilities which bring about transition, the distance required to achieve transition increases with increasing ν.

PROBLEM 6.16

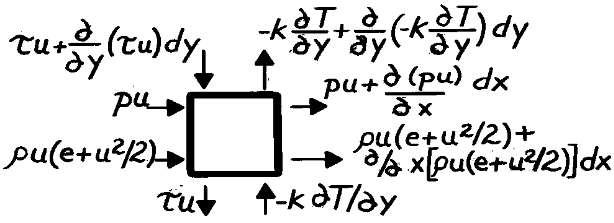
KNOWN: Two-dimensional flow conditions for which v = 0 and T = T(y).

FIND: (a) Verify that u = u(y), (b) Derive the x-momentum equation, (c) Derive the energy equation.

SCHEMATIC:



Pressure & shear forces



Energy fluxes

ASSUMPTIONS: (1) Steady-state conditions, (2) Incompressible fluid with constant properties, (3) Negligible body forces, (4) v = 0, (5) T = T(y) or $\partial T/\partial x = 0$, (6) Thermal energy generation occurs only by viscous dissipation.

ANALYSIS: (a) From the mass continuity equation, it follows from the prescribed conditions that $\partial u/\partial x = 0$. Hence u = u(y).

(b) From Newton's second law of motion, $\Sigma F_x =$ (Rate of increase of fluid momentum)$_x$,

$$\left[p-\left[p+\frac{\partial p}{\partial x}dx\right]\right]dy\cdot 1+\left[-\tau+\left[\tau+\frac{\partial \tau}{\partial y}dy\right]\right]dx\cdot 1=\left\{(\rho u)u+\frac{\partial}{\partial x}[(\rho u)u]dx\right\}dy\cdot 1-(\rho u)u\,dy\cdot 1$$

Hence, with $\tau=\mu(\partial u/\partial y)$, it follows that

$$-\frac{\partial p}{\partial x}+\frac{\partial \tau}{\partial y}=\frac{\partial}{\partial x}[(\rho u)u]=0 \qquad \frac{\partial p}{\partial x}=\mu\frac{\partial^2 u}{\partial y^2}.$$ <

(c) From the conservation of energy requirement and the prescribed conditions, it follows that $\dot{E}_{in}-\dot{E}_{out}=0$, or

$$\left[pu+\rho u\left(e+u^2/2\right)\right]dy\cdot 1+\left[-k\frac{\partial T}{\partial y}+\tau u+\frac{\partial(\tau u)}{\partial y}dy\right]dx\cdot 1$$

$$-\left\{pu+\frac{\partial}{\partial x}(pu)dx+\rho u\left(e+u^2/2\right)+\frac{\partial}{\partial x}\left[\rho u\left(e+u^2/2\right)\right]dx\right\}dy\cdot 1-\left[\tau u-k\frac{\partial T}{\partial y}+\frac{\partial}{\partial y}\left[-k\frac{\partial T}{\partial y}\right]dy\right]dx\cdot 1=0$$

or, $$\frac{\partial(\tau u)}{\partial y}-\frac{\partial}{\partial x}(pu)-\frac{\partial}{\partial x}\left[\rho u\left(e+u^2/2\right)\right]+\frac{\partial}{\partial y}\left[k\frac{\partial T}{\partial y}\right]=0$$

$$\tau\frac{\partial u}{\partial y}+u\frac{\partial \tau}{\partial y}-u\frac{\partial p}{\partial x}+k\frac{\partial^2 T}{\partial y^2}=0.$$

Noting that the second and third terms cancel from the momentum equation,

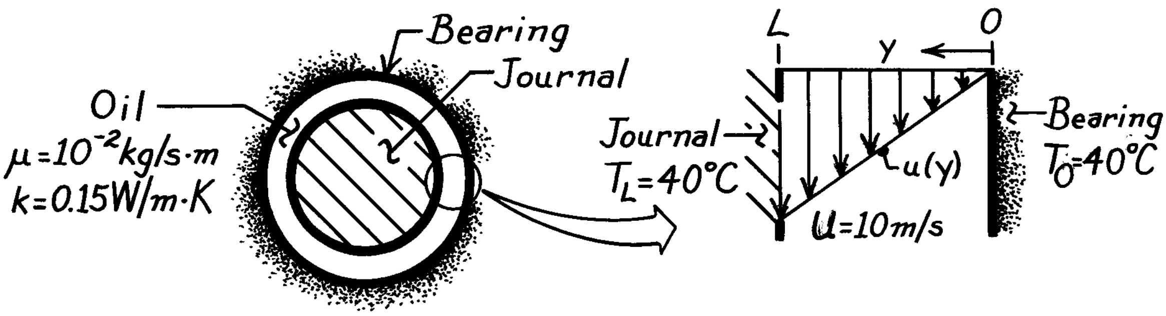
$$\mu\left[\frac{\partial u}{\partial y}\right]^2+k\left[\frac{\partial^2 T}{\partial y^2}\right]=0.$$ <

PROBLEM 6.17

KNOWN: Oil properties, journal and bearing temperatures, and journal speed for a lightly loaded journal bearing.

FIND: Maximum oil temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Incompressible fluid with constant properties, (3) Clearance is much less than journal radius and flow is Couette.

ANALYSIS: The temperature distribution corresponds to the result obtained in the text Example on Couette flow,

$$T(y) = T_0 + \frac{\mu}{2k} U^2 \left[\frac{y}{L} - \left[\frac{y}{L} \right]^2 \right].$$

The position of maximum temperature is obtained from

$$\frac{dT}{dy} = 0 = \frac{\mu}{2k} U^2 \left[\frac{1}{L} - \frac{2y}{L^2} \right]$$

or, $\quad y = L/2.$

The temperature is a maximum at this point since $d^2T/dy^2 < 0$. Hence,

$$T_{max} = T(L/2) = T_0 + \frac{\mu}{2k} U^2 \left[\frac{1}{2} - \frac{1}{4} \right] = T_0 + \frac{\mu U^2}{8k}$$

$$T_{max} = 40^\circ C + \frac{10^{-2} kg/s \cdot m (10 m/s)^2}{8 \times 0.15 \ W/m \cdot K}$$

$$T_{max} = 40.83^\circ C.$$ <

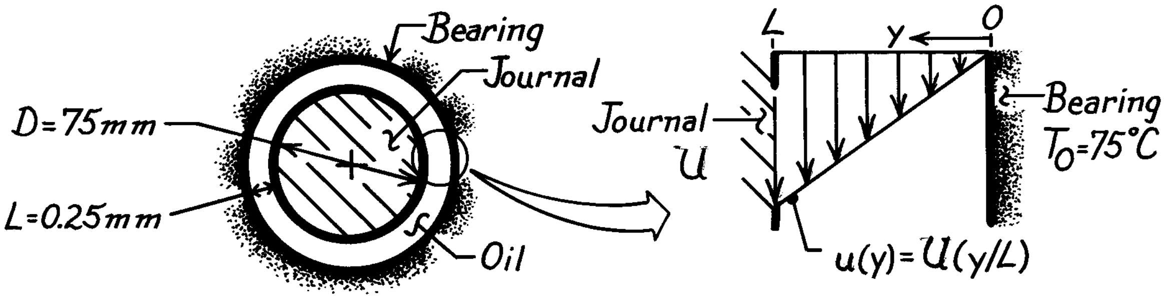
COMMENTS: Note that T_{max} *increases* with *increasing* μ and U, *decreases* with *increasing* k, and is *independent* of L.

PROBLEM 6.18

KNOWN: Diameter, clearance, rotational speed and fluid properties of a lightly loaded journal bearing. Temperature of bearing.

FIND: (a) Temperature distribution in the fluid, (b) Rate of heat transfer from bearing and operating power.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Incompressible fluid with constant properties, (3) Couette flow.

PROPERTIES: Oil (Given): $\rho = 800\ \text{kg/m}^3$, $\nu = 10^{-5}\ \text{m}^2/\text{s}$, $k = 0.13\ \text{W/m·K}$; $\mu = \rho\nu = 8 \times 10^{-3}\ \text{kg/s·m}$.

ANALYSIS: (a) For Couette flow, the velocity distribution is linear, $u(y) = U(y/L)$, and the energy equation and general form of the temperature distribution are

$$k\frac{d^2T}{dy^2} = -\mu\left[\frac{du}{dy}\right]^2 = -\mu\left[\frac{U}{L}\right]^2 \qquad T = -\frac{\mu}{2k}\left[\frac{U}{L}\right]^2 y^2 + \frac{C_1}{k}y + C_2.$$

Considering the boundary conditions $dT/dy)_{y=L} = 0$ and $T(0) = T_0$, find $C_2 = T_0$ and $C_1 = \mu U^2/L$. Hence,

$$T = T_0 + \left(\mu U^2\right)/k\left[(y/L) - 1/2(y/L)^2\right]. \qquad <$$

(b) Applying Fourier's law at y = 0, the rate of heat transfer per unit length to the bearing is

$$q' = -k(\pi D)\left.\frac{dT}{dy}\right]_{y=0} = -(\pi D)\frac{\mu U^2}{L} = -\left(\pi \times 75\times10^{-3}\text{m}\right)\frac{8\times10^{-3}\text{kg/s}\cdot\text{m}\ (14.14\ \text{m/s})^2}{0.25\times10^{-3}\text{m}} = -1507.5\ \text{W/m}$$

where the velocity is determined as

$$U = (D/2)\omega = 0.0375\text{m}\times3600\ \text{rev/min}\ (2\pi\ \text{rad/rev})/(60\ \text{s/min}) = 14.14\ \text{m/s}.$$

The journal power requirement is

$$P' = F'_{(y=L)}U = \tau_{s(y=L)}\cdot\pi D\cdot U$$

$$P' = 452.5\text{kg/s}^2\cdot\text{m}\ \left(\pi\times75\times10^{-3}\text{m}\right)14.14\text{m/s} = 1507.5\text{kg}\cdot\text{m/s}^3 = 1507.5\text{W/m} \qquad <$$

where the shear stress at y = L is

$$\tau_{s(y=L)} = \mu(\partial u/\partial y)_{y=L} = \mu\frac{U}{L} = 8\times10^{-3}\text{kg/s}\cdot\text{m}\left[\frac{14.14\ \text{m/s}}{0.25\times10^{-3}\text{m}}\right] = 452.5\ \text{kg/s}^2\cdot\text{m}.$$

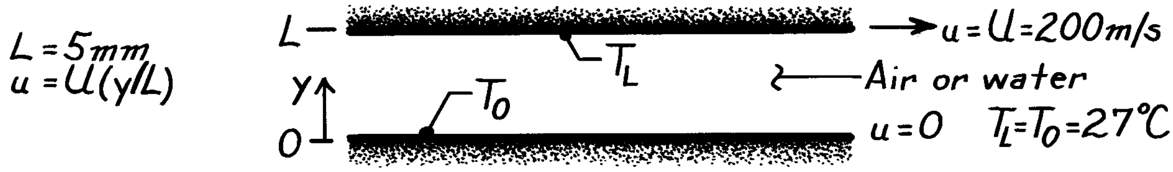
COMMENTS: Note that $q' = P'$, which is consistent with the energy conservation requirement.

PROBLEM 6.19

KNOWN: Conditions associated with the Couette flow of air or water.

FIND: (a) Force and power requirements per unit surface area, (b) Viscous dissipation, (c) Maximum fluid temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Fully-developed Couette flow, (2) Incompressible fluid with constant properties.

PROPERTIES: *Table A-4*, Air (300K): $\mu = 184.6 \times 10^{-7}$ N·s/m^2, k = 26.3×10^{-3} W/m·K; *Table A-6*, Water (300K): $\mu = 855 \times 10^{-6}$ N·s/m^2, k = 0.613 W/m·K.

ANALYSIS: (a) The force per unit area is associated with the shear stress. Hence, with the linear velocity profile for Couette flow, $\tau = \mu\left(\mathrm{du/dy}\right) = \mu\left(\mathrm{U/L}\right)$.

Air: $$\tau_{\mathrm{air}} = 184.6\times10^{-7}\ \mathrm{N\cdot s/m^2} \times \frac{200\ \mathrm{m/s}}{0.005\ \mathrm{m}} = 0.738\ \mathrm{N/m^2}$$ <

Water: $$\tau_{\mathrm{water}} = 855\times10^{-6}\ \mathrm{N\cdot s/m^2} \times \frac{200\ \mathrm{m/s}}{0.005\ \mathrm{m}} = 34.2\ \mathrm{N/m^2}.$$

With the required power given by P/A = $\tau \cdot \mathrm{U}$,

Air: $$\left(\mathrm{P/A}\right)_{\mathrm{air}} = \left(0.738\ \mathrm{N/m^2}\right)200\ \mathrm{m/s} = 147.6\ \mathrm{W/m^2}$$ <

Water: $$\left(\mathrm{P/A}\right)_{\mathrm{water}} = \left(34.2\ \mathrm{N/m^2}\right)200\ \mathrm{m/s} = 6840\ \mathrm{W/m^2}.$$

(b) The viscous dissipation is $\mu\Phi = \mu\left(\mathrm{du/dy}\right)^2 = \mu\left(\mathrm{U/L}\right)^2$. Hence,

Air: $$\left(\mu\Phi\right)_{\mathrm{air}} = 184.6\times10^{-7}\ \frac{\mathrm{N\cdot s}}{\mathrm{m^2}}\left[\frac{200\ \mathrm{m/s}}{0.005\ \mathrm{m}}\right]^2 = 2.95\times10^4\ \mathrm{W/m^3}$$ <

Water: $$\left(\mu\Phi\right)_{\mathrm{water}} = 855\times10^{-6}\ \frac{\mathrm{N\cdot s}}{\mathrm{m^2}}\left[\frac{200\ \mathrm{m/s}}{0.005\ \mathrm{m}}\right]^2 = 1.37\times10^6\ \mathrm{W/m^3}.$$

(c) From the solution to Part 4 of the text Example, the location of the maximum temperature corresponds to y_{max} = L/2. Hence, $T_{max} = T_0 + \mu U^2/8k$ and

Air: $$\left(T_{max}\right)_{\mathrm{air}} = 27^\circ\mathrm{C} + \frac{184.6\times10^{-7}\ \mathrm{N\cdot s/m^2}\left(200\ \mathrm{m/s}\right)^2}{8\times0.0263\ \mathrm{W/m\cdot K}} = 30.5^\circ\mathrm{C}$$ <

Water: $$\left(T_{max}\right)_{\mathrm{water}} = 27^\circ\mathrm{C} + \frac{855\times10^{-6}\ \mathrm{N\cdot s/m^2}\left(200\ \mathrm{m/s}\right)^2}{8\times0.613\ \mathrm{W/m\cdot K}} = 34.0^\circ\mathrm{C}.$$

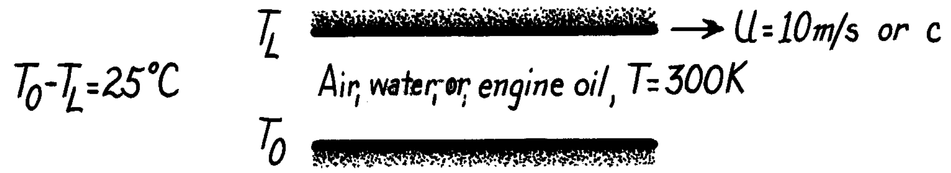
COMMENTS: (1) The viscous dissipation associated with the entire fluid layer, $\mu\Phi\left(\mathrm{LA}\right)$, must equal the power, P. (2) Although $\left(\mu\Phi\right)_{\mathrm{water}} >> \left(\mu\Phi\right)_{\mathrm{air}}$, $k_{\mathrm{water}} >> k_{\mathrm{air}}$. Hence, $T_{\mathrm{max,water}} \approx T_{\mathrm{max,air}}$.

PROBLEM 6.20

KNOWN: Velocity and temperature difference of plates maintaining Couette flow. Mean temperature of air, water or oil between the plates.

FIND: (a) Pr·Ec product for each fluid, (b) Pr·Ec product for air with plate at sonic velocity.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Couette flow, (3) Air is at 1 atm.

PROPERTIES: *Table A-4*, Air (300K, 1atm), c_p = 1007 J/kg·K, Pr = 0.707, γ = 1.4, R= 287.02 J/kg·K; *Table A-6*, Water (300K): c_p = 4179 J/kg·K, Pr = 5.83; *Table A-5*, Engine oil (300K), c_p = 1909 J/kg·K, Pr = 6400.

ANALYSIS: The product of the Prandtl and Eckert numbers is dimensionless,

$$\mathrm{Pr}\cdot\mathrm{Ec} = \mathrm{Pr}\frac{U^2}{c_p\Delta T} \sim \frac{m^2/s^2}{(J/kg\cdot K)K} \sim \frac{m^2/s^2}{\left(kg\cdot m^2/s^2\right)/kg}.$$

Substituting numerical values, find

	Air	*Water*	*Oil*
Pr·Ec	0.0028	0.0056	13.41

<

(b) For an ideal gas, the speed of sound is

$$c = (\gamma\ R\ T)^{1/2}$$

where R, the gas constant for air, is R_u/M = 8.315 kJ/kmol·K/(28.97 kg/kmol) = 287.02 J/kg·K. Hence, at 300K for air,

$$U = c = (1.4\times 287.02\ J/kg\cdot K\times 300K)^{1/2} = 347.2\ m/s.$$

For sonic velocities, it follows that

$$\mathrm{Pr}\cdot\mathrm{Ec} = 0.707\frac{(347.2\ m/s)^2}{1007 J/kg\cdot K\times 25K} = 3.38.$$ <

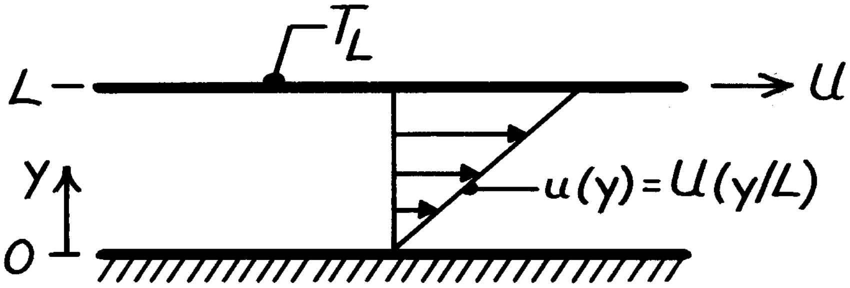
COMMENTS: From the above results it follows that viscous dissipation effects must be considered in the high speed flow of gases and in oil flows at moderate speeds. For Pr·Ec to be less than 0.1 in air with ΔT = 25°C, U should be $\lesssim$ 60 m/s.

PROBLEM 6.21

KNOWN: Couette flow with moving plate isothermal and stationary plate insulated.

FIND: Temperature of stationary plate and heat flux at the moving plate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Incompressible fluid with constant properties, (3) Couette flow.

ANALYSIS: The energy equation is given by

$$0 = k\left[\frac{\partial^2 T}{\partial y^2}\right] + \mu\left[\frac{\partial u}{\partial y}\right]^2$$

Integrating twice find the general form of the temperature distribution,

$$\frac{\partial^2 T}{\partial y^2} = -\frac{\mu}{k}\left[\frac{U}{L}\right]^2 \qquad \frac{\partial T}{\partial y} = -\frac{\mu}{k}\left[\frac{U}{L}\right]^2 y + C_1$$

$$T(y) = -\frac{\mu}{2k}\left[\frac{U}{L}\right]^2 y^2 + C_1 y + C_2.$$

Consider the boundary conditions to evaluate the constants,

$$\partial T/\partial y\Big|_{y=0} = 0 \;\rightarrow\; C_1 = 0 \text{ and } T(L) = T_L \;\rightarrow\; C_2 = T_L + \frac{\mu}{2k}U^2.$$

Hence, the temperature distribution is

$$T(y) = T_L + \left[\frac{\mu U^2}{2k}\right]\left[1 - \left[\frac{y}{L}\right]^2\right].$$

The temperature of the lower plate (y = 0) is

$$T(0) = T_L + \left[\frac{\mu U^2}{2k}\right]. \qquad <$$

The heat flux to the upper plate (y = L) is

$$q''(L) = -k\frac{\partial T}{\partial y}\Big|_{y=L} = \frac{\mu U^2}{L}. \qquad <$$

COMMENTS: The heat flux at the top surface may also be obtained by integrating the viscous dissipation over the fluid layer height. For a control volume about a unit area of the fluid layer,

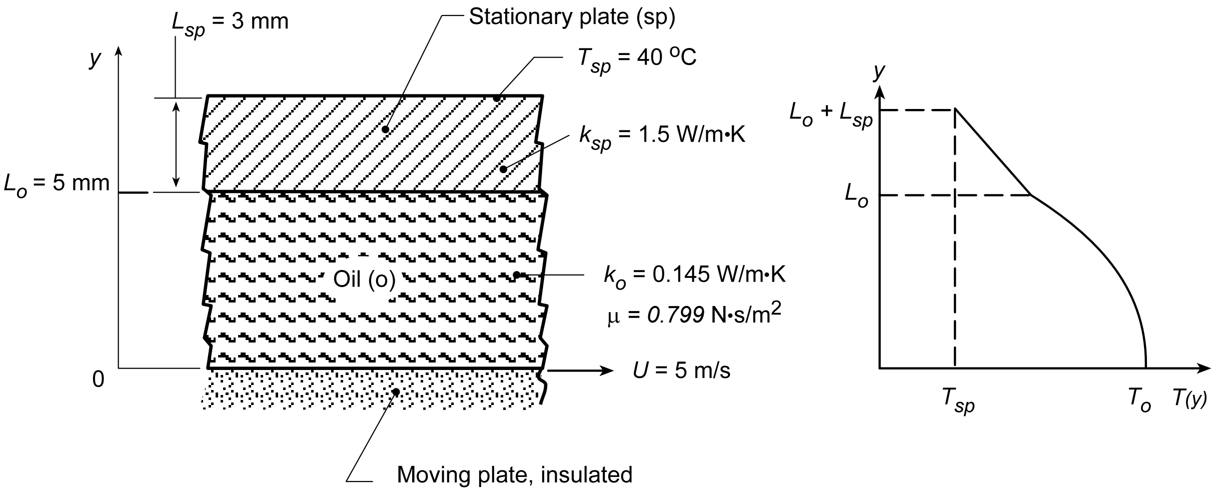
$$\dot{E}''_g = \dot{E}''_{out} \qquad \int_0^L \mu\left[\frac{\partial u}{\partial y}\right]^2 dy = q''(L) \qquad q''(L) = \frac{\mu U^2}{L}.$$

PROBLEM 6.22

KNOWN: Couette flow with heat transfer. Lower (insulated) plate moves with speed U and upper plate is stationary with prescribed thermal conductivity and thickness. Outer surface of upper plate maintained at constant temperature, $T_{sp} = 40°C$.

FIND: (a) On T-y coordinates, sketch the temperature distribution in the oil and the stationary plate, and (b) An expression for the temperature at the lower surface of the oil film, $T(0) = T_o$, in terms of the plate speed U, the stationary plate parameters (T_{sp}, k_{sp}, L_{sp}) and the oil parameters (μ, k_o, L_o). Determine this temperature for the prescribed conditions.

SCHEMATIC:



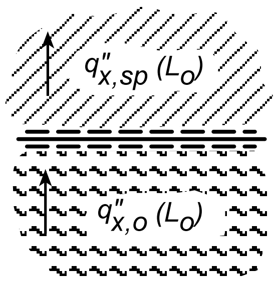
ASSUMPTIONS: (1) Steady-state conditions, (2) Fully developed Couette flow and (3) Incompressible fluid with constant properties.

ANALYSIS: (a) The temperature distribution is shown above with these key features: linear in plate, parabolic in oil film, discontinuity at plate-oil interface, and zero gradient at lower plate surface.

(b) From Example 6.4, the general solution to the conservation equations for the temperature distribution in the oil film is

$$T_o(y) = -Ay^2 + C_3 y + C_4 \qquad \text{where} \qquad A = \frac{\mu}{2k_o}\left(\frac{U}{L_o}\right)^2$$

and the boundary conditions are,



At y = 0, insulated boundary $\qquad \left.\frac{dT_o}{dy}\right)_{y=0} = 0; \qquad C_3 = 0$

At y = L_o, heat fluxes in oil and plate are equal, $\qquad q''_o(L_o) = q''_{sp}(L_o)$

Continued...

PROBLEM 6.22 (Cont.)

$$-k\left.\frac{dT_o}{dy}\right)_{y=L_o} = \frac{T_o(L_o)-T_{sp}}{R_{sp}} \qquad \left\{\begin{array}{l} \left.\dfrac{dT_o}{dy}\right)_{y=L} = -2AL_o \\ R_{sp} = L_{sp}/k_{sp} \end{array}\right. \qquad T_o(L) = -AL_o^2 + C_4$$

$$C_4 = T_{sp} + AL_o^2\left[1+2\frac{k_o}{L_o}\frac{L_{sp}}{k_{sp}}\right]$$

Hence, the temperature distribution at the lower surface is

$$T_o(0) = -A\cdot 0 + C_4$$

$$T_o(0) = T_{sp} + \frac{\mu}{2k_o}U^2\left[1+2\frac{k_o}{L_o}\frac{L_{sp}}{k_{sp}}\right] \qquad <$$

Substituting numerical values, find

$$T_o(0) = 40^\circ C + \frac{0.799\,N\cdot s/m^2}{2\times 0.145\,W/m\cdot K}(5\,m/s)^2\left[1+2\frac{0.145}{5}\times\frac{3}{1.5}\right] = 116.9^\circ C \qquad <$$

COMMENTS: (1) Give a physical explanation about why the maximum temperature occurs at the lower surface.

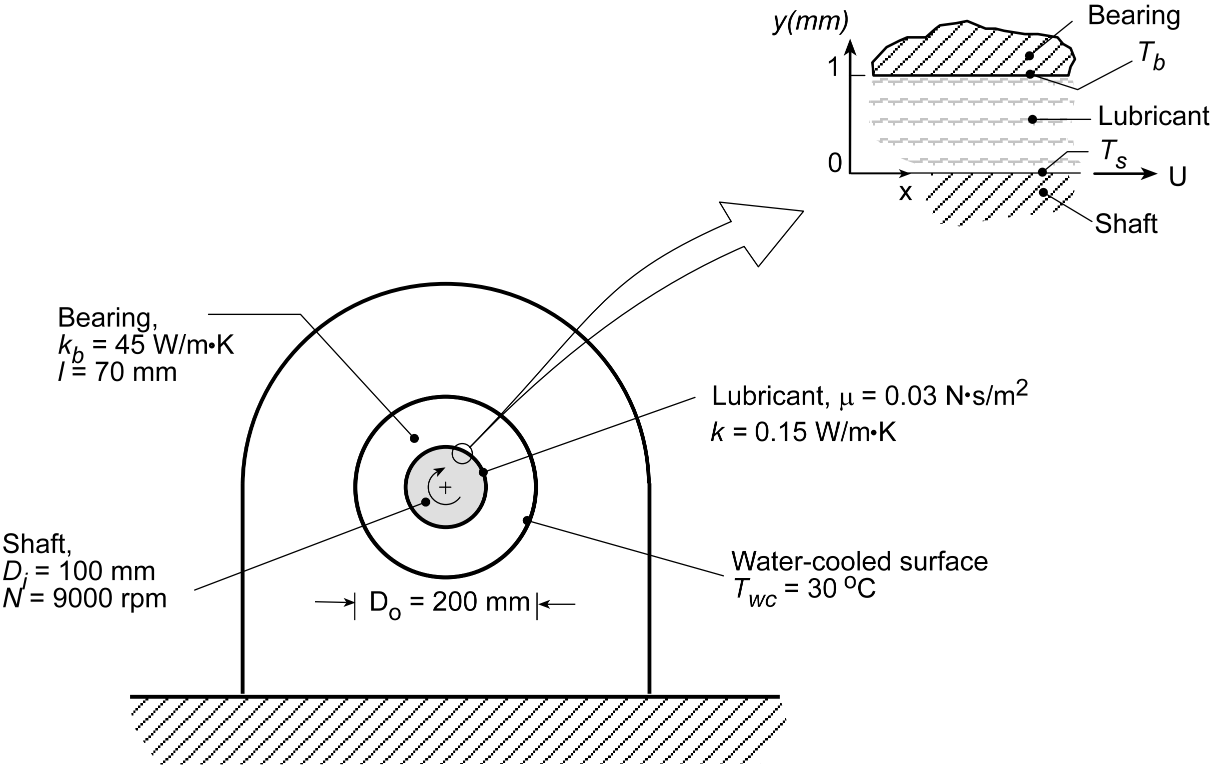
(2) Sketch the temperature distribution if the upper plate moved with a speed U while the lower plate is stationary and all other conditions remain the same.

PROBLEM 6.23

KNOWN: Shaft of diameter 100 mm rotating at 9000 rpm in a journal bearing of 70 mm length. Uniform gap of 1 mm separates the shaft and bearing filled with lubricant. Outer surface of bearing is water-cooled and maintained at T_{wc} = 30°C.

FIND: (a) Viscous dissipation in the lubricant, $\mu\Phi(W/m^3)$, (b) Heat transfer rate from the lubricant, assuming no heat lost through the shaft, and (c) Temperatures of the bearing and shaft, T_b and T_s.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Fully developed Couette flow, (3) Incompressible fluid with constant properties, and (4) Negligible heat lost through the shaft.

ANALYSIS: (a) The viscous dissipation, μΦ, Eq. 6.40, for Couette flow from Example 6.4, is

$$\mu\Phi = \mu\left(\frac{du}{dy}\right)^2 = \mu\left(\frac{U}{L}\right)^2 = 0.03\,N\cdot s/m^2\left(\frac{47.1\,m/s}{0.001\,m}\right)^2 = 6.656\times10^7\ W/m^3 \qquad <$$

where the velocity distribution is linear and the tangential velocity of the shaft is

$$U = \pi DN = \pi\,(0.100\,m)\times 9000\,rpm\times(min/60s) = 47.1\,m/s\,.$$

(b) The heat transfer rate from the lubricant volume ∀ through the bearing is

$$q = \mu\Phi\cdot\forall = \mu\Phi\,(\pi D\cdot L\cdot\ell) = 6.65\times10^7\ W/m^3\,(\pi\times0.100\,m\times0.001\,m\times0.070\,m) = 1462\,W \qquad <$$

where ℓ = 70 mm is the length of the bearing normal to the page.

Continued...

PROBLEM 6.23 (Cont.)

(c) From Fourier's law, the heat rate through the bearing material of inner and outer diameters, D_i and D_o, and thermal conductivity k_b is, from Eq. (3.27),

$$q_r = \frac{2\pi \ell k_b \left(T_b - T_{wc}\right)}{\ln\left(D_o / D_i\right)}$$

$$T_b = T_{wc} + \frac{q_r \ln\left(D_o / D_i\right)}{2\pi \ell k_b}$$

$$T_b = 30^\circ C + \frac{1462\,W \ln\left(200/100\right)}{2\pi \times 0.070\,m \times 45\,W/m\cdot K} = 81.2^\circ C$$ <

To determine the temperature of the shaft, $T(0) = T_s$, first the temperature distribution must be found beginning with the general solution, Example 6.4,

$$T\left(y\right) = -\frac{\mu}{2k}\left(\frac{U}{L}\right)^2 y^2 + C_3 y + C_4$$

The boundary conditions are, at y = 0, the surface is adiabatic

$$\left.\frac{dT}{dy}\right)_{y=0} = 0 \qquad\qquad C_3 = 0$$

and at y = L, the temperature is that of the bearing, T_b

$$T\left(L\right) = T_b = -\frac{\mu}{2k}\left(\frac{U}{L}\right)^2 L^2 + 0 + C_4 \qquad\qquad C_4 = T_b + \frac{\mu}{2k}U^2$$

Hence, the temperature distribution is

$$T\left(y\right) = T_b + \frac{\mu}{2k}U^2\left(1 - \frac{y^2}{L^2}\right)$$

and the temperature at the shaft, y = 0, is

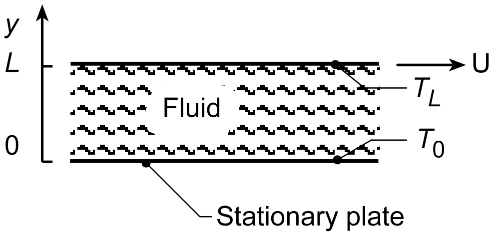
$$T_s = T\left(0\right) = T_b + \frac{\mu}{2k}U^2 = 81.3^\circ C + \frac{0.03\,N\cdot s/m^2}{2 \times 0.15\,W/m\cdot K}\left(47.1\,m/s\right)^2 = 303^\circ C$$ <

PROBLEM 6.24

KNOWN: Couette flow with heat transfer.

FIND: (a) Dimensionless form of temperature distribution, (b) Conditions for which top plate is adiabatic, (c) Expression for heat transfer to lower plate when top plate is adiabatic.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) incompressible fluid with constant properties, (3) Negligible body forces, (4) Couette flow.

ANALYSIS: (a) From Example 6.4, the temperature distribution is

$$T = T_0 + \frac{\mu}{2k} U^2 \left[\frac{y}{L} - \left(\frac{y}{L} \right)^2 \right] + \left(T_L - T_0 \right) \frac{y}{L}$$

$$\frac{T - T_0}{T_L - T_0} = \frac{\mu U^2}{2k\left(T_L - T_0\right)} \left[\frac{y}{L} - \left(\frac{y}{L} \right)^2 \right] + \frac{y}{L}$$

or, with

$$\theta \equiv \left(T - T_0\right) / T_L - T_0 , \qquad \eta \equiv y/L ,$$

$$\Pr \equiv c_p \mu / k , \qquad Ec \equiv U^2 / c_p \left(T_L - T_0\right)$$

$$\theta = \frac{\Pr \cdot Ec}{2}\left(\eta - \eta^2\right) + \eta = \eta\left[1 + \frac{1}{2}\Pr \cdot Ec\left(1 - \eta\right)\right] \qquad (1)$$ <

(b) For there to be zero heat transfer at the top plate, $dT/dy)_{y=L} = 0$. Hence,

$$\left.\frac{d\theta}{d\eta}\right)_{\eta=1} \cdot \frac{T_L - T_0}{L} = \frac{\Pr \cdot Ec}{2}\left.\left(1 - 2\eta\right)\right|_{\eta=1} + 1 = -\frac{\Pr \cdot Ec}{2} + 1 = 0$$

There is no heat transfer at the top plate if,

$$Ec \cdot \Pr = 2. \qquad (2)$$ <

(c) The heat transfer rate to the lower plate (per unit area) is

$$q_0'' = -k \left.\frac{dT}{dy}\right|_{y=0} = -k \frac{\left(T_L - T_0\right)}{L} \left.\frac{d\theta}{d\eta}\right|_{\eta=0}$$

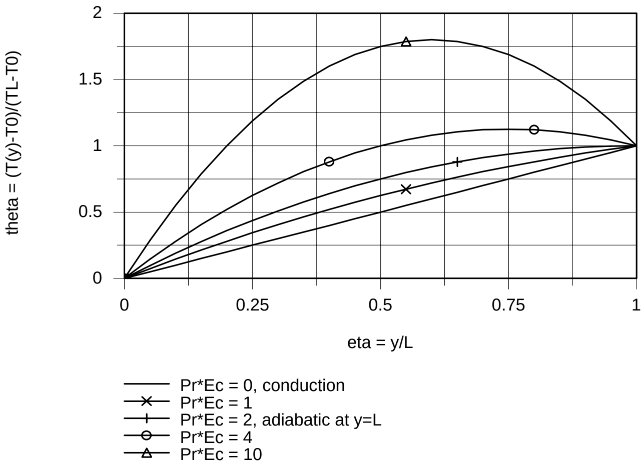
$$q_o'' = -k \frac{T_L - T_0}{L} \left[\frac{\Pr \cdot Ec}{2} \left.\left(1 - 2\eta\right)\right|_{\eta=0} + 1 \right]$$

$$q_0'' = -k \frac{T_L - T_0}{L} \left(\frac{\Pr \cdot Ec}{2} + 1 \right) = -2k\left(T_L - T_0\right)/L$$ <

Continued...

PROBLEM 6.24 (Cont.)

(d) Using Eq. (1), the dimensionless temperature distribution is plotted as a function of dimensionless distance, η = y/L. When Pr·Ec = 0, there is no dissipation and the temperature distribution is linear, so that heat transfer is by conduction only. As Pr ·Ec increases, viscous dissipation becomes more important. When Pr·Ec = 2, heat transfer to the upper plate is zero. When Pr·Ec > 2, the heat rate is out of the oil film at both surfaces.

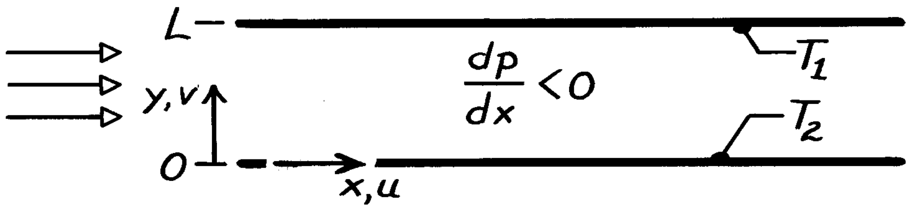


PROBLEM 6.25

KNOWN: Steady, incompressible, laminar flow between infinite parallel plates at different temperatures.

FIND: (a) Form of continuity equation, (b) Form of momentum equations and velocity profile. Relationship of pressure gradient to maximum velocity, (c) Form of energy equation and temperature distribution. Heat flux at top surface.

SCHEMATIC:



ASSUMPTIONS: (1) Two-dimensional flow (no variations in z) between infinite, parallel plates, (2) Negligible body forces, (3) No internal energy generation, (4) Incompressible fluid with constant properties.

ANALYSIS: (a) For two-dimensional, steady conditions, the continuity equation is

$$\frac{\partial(\rho\,u)}{\partial\,x}+\frac{\partial(\rho\,v)}{\partial\,y}=0.$$

Hence, for an incompressible fluid (constant ρ) in parallel flow (v = 0),

$$\frac{\partial\,u}{\partial\,x}=0.$$ <

The flow is fully developed in the sense that, irrespective of y, u is independent of x.

(b) With the above result and the prescribed conditions, the momentum equations reduce to

$$0=-\frac{\partial\,p}{\partial\,x}+\mu\frac{\partial^2 u}{\partial\,y^2}\qquad\qquad 0=-\frac{\partial\,p}{\partial\,y}$$ <

Since p is independent of y, ∂p/∂x = dp/dx is independent of y and

$$\mu\frac{\partial^2 u}{\partial\,y^2}=\mu\frac{d^2u}{dy^2}=\frac{dp}{dx}.$$

Since the left-hand side can, at most, depend only on y and the right-hand side is independent of y, both sides must equal the same constant C. That is,

$$\mu\frac{d^2u}{dy^2}=C.$$

Hence, the velocity distribution has the form

$$u(y)=\frac{C}{2\mu}y^2+C_1y+C_2.$$

Using the boundary conditions to evaluate the constants,

$$u(0)=0\quad\rightarrow\quad C_2=0\quad\text{and}\quad u(L)=0\quad\rightarrow\quad C_1=-CL/2\mu.$$

Continued …..

The velocity profile is $$u(y) = \frac{C}{2\mu}\left(y^2 - Ly\right).$$

The profile is symmetric about the midplane, in which case the maximum velocity exists at y = L/2. Hence,

$$u(L/2) = u_{max} = \frac{C}{2\mu}\left[-\frac{L^2}{4}\right] \qquad \text{or} \qquad u_{max} = -\frac{L^2}{8\mu}\frac{dp}{dx}. \qquad <$$

(c) For fully developed thermal conditions, $(\partial T/\partial x) = 0$ and temperature depends only on y. Hence with v = 0, $\partial u/\partial x = 0$, and the prescribed assumptions, the energy equation becomes

$$\rho\, u\frac{\partial i}{\partial x} = k\frac{d^2T}{dy^2} + u\frac{dp}{dx} + \mu\left[\frac{du}{dy}\right]^2.$$

With $i = e + p/\rho$, $$\frac{\partial i}{\partial x} = \frac{\partial e}{\partial x} + \frac{1}{\rho}\frac{dp}{dx} \quad \text{where} \quad \frac{\partial e}{\partial x} = \frac{\partial e}{\partial T}\frac{\partial T}{\partial x} + \frac{\partial e}{\partial \rho}\frac{\partial \rho}{\partial x} = 0.$$

Hence, the energy equation becomes $$0 = k\frac{d^2T}{dy^2} + \mu\left[\frac{du}{dy}\right]^2. \qquad <$$

With $du/dy = (C/2\mu)(2y - L)$, it follows that

$$\frac{d^2T}{dy^2} = -\frac{C^2}{4k\mu}\left(4y^2 - 4Ly + L^2\right).$$

Integrating twice,

$$T(y) = -\frac{C^2}{4k\mu}\left[\frac{y^4}{3} - \frac{2Ly^3}{3} + \frac{L^2y^2}{2}\right] + C_3y + C_4$$

Using the boundary conditions to evaluate the constants,

$$T(0) = T_2 \quad \rightarrow \quad C_4 = T_2 \quad \text{and} \quad T(L) = T_1 \quad \rightarrow \quad C_3 = \frac{C^2L^3}{24k\mu} + \frac{(T_1 - T_2)}{L}.$$

Hence, $$T(y) = T_2 + \left[\frac{y}{L}\right](T_1 - T_2) - \frac{C^2}{4k\mu}\left[\frac{y^4}{3} - \frac{2Ly^3}{3} + \frac{L^2y^2}{2} - \frac{L^3y}{6}\right]. \qquad <$$

From Fourier's law,

$$q''(L) = -k\left.\frac{\partial T}{\partial y}\right|_{y=L} = \frac{k}{L}(T_2 - T_1) + \frac{C^2}{4\mu}\left[\frac{4}{3}L^3 - 2L^3 + L^3 - \frac{L^3}{6}\right]$$

$$q''(L) = \frac{k}{L}(T_2 - T_1) + \frac{C^2L^3}{24\mu}. \qquad <$$

COMMENTS: The third and second terms on the right-hand sides of the temperature distribution and heat flux, respectively, represents the effects of viscous dissipation. If C is large (due to large μ or u_{max}), viscous dissipation is significant. If C is small, conduction effects dominate.

PROBLEM 6.26

KNOWN: Pressure independence of μ, k and c_p.

FIND: Pressure dependence of ν and α for air at 350K and p = 1, 10 atm.

ASSUMPTIONS: Perfect gas behavior for air.

PROPERTIES: *Table A-4*, Air (350K, 1 atm): $\nu = 20.92 \times 10^{-6} m^2/s$, $\alpha = 29.9 \times 10^{-6} m^2/s$.

ANALYSIS: The kinematic viscosity and thermal diffusivity are, respectively,

$$\nu = \mu / \rho \qquad \alpha = k/\rho\, c_p.$$

Hence, ν and α are inversely proportional to ρ.

For an *incompressible liquid*, ρ is constant.

Hence ν and α are independent of pressure. <

For a *perfect gas*, $\rho = p/RT$.

Hence, ρ is directly proportional to p, in which case ν and α vary inversely with pressure. It follows that ν and α are inversely proportional to pressure. <

To calculate ν or α for a *perfect* gas at $p \neq 1$ atm,

$$\nu(p) = \nu(1 \text{ atm}) \cdot \frac{1}{p}$$
$$\alpha(p) = \alpha(1 \text{ atm}) \cdot \frac{1}{p}$$

Hence, for air at 350K,

p(atm)	$\nu(m^2/s)$	$\alpha(m^2/s)$
1	20.92×10^{-6}	29.9×10^{-6}
10	2.09×10^{-6}	2.99×10^{-6}

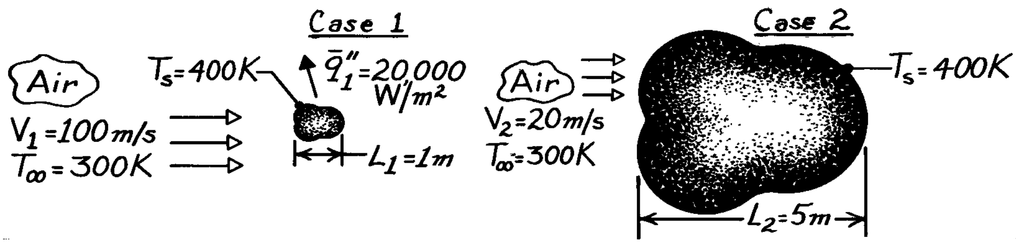
COMMENTS: For the incompressible liquid and the perfect gas, $Pr = \nu/\alpha$ is independent of pressure.

PROBLEM 6.27

KNOWN: Characteristic length, surface temperature and average heat flux for an object placed in an airstream of prescribed temperature and velocity.

FIND: Average convection coefficient if characteristic length of object is increased by a factor of five and air velocity is decreased by a factor of five.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties.

ANALYSIS: For a particular geometry,

$$\overline{Nu}_L = f(Re_L, Pr).$$

The Reynolds numbers for each case are

Case 1: $$Re_{L,1} = \frac{V_1 L_1}{\nu_1} = \frac{(100\text{m/s})1\text{m}}{\nu_1} = \frac{100\ \text{m}^2/\text{s}}{\nu_1}$$

Case 2: $$Re_{L,2} = \frac{V_2 L_2}{\nu_2} = \frac{(20\text{m/s})5\text{m}}{\nu_2} = \frac{100\ \text{m}^2/\text{s}}{\nu_2}.$$

Hence, with $\nu_1 = \nu_2$, $Re_{L,1} = Re_{L,2}$. Since $Pr_1 = Pr_2$, it follows that

$$\overline{Nu}_{L,2} = \overline{Nu}_{L,1}.$$

Hence,

$$\bar{h}_2 L_2 / k_2 = \bar{h}_1 L_1 / k_1$$

$$\bar{h}_2 = \bar{h}_1 \frac{L_1}{L_2} = 0.2\ \bar{h}_1.$$

For *Case 1*, using the rate equation, the convection coefficient is

$$q_1 = \bar{h}_1 A_1 (T_s - T_\infty)_1$$

$$\bar{h}_1 = \frac{(q_1/A_1)}{(T_s - T_\infty)_1} = \frac{q''_1}{(T_s - T_\infty)_1} = \frac{20{,}000\ \text{W/m}^2}{(400-300)\text{K}} = 200\ \text{W/m}^2\cdot\text{K}.$$

Hence, it follows that for *Case 2*

$$\bar{h}_2 = 0.2\times 200\ \text{W/m}^2\cdot\text{K} = 40\ \text{W/m}^2\cdot\text{K}.$$ <

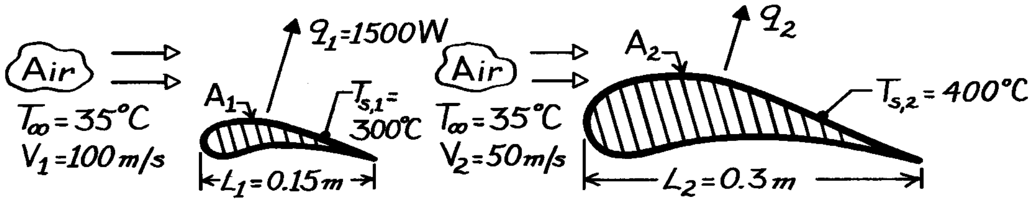
COMMENTS: If $Re_{L,2}$ were *not* equal to $Re_{L,1}$, it would be necessary to know the specific form of f(Re_L, Pr) before $\bar{h}_2$ could be determined.

PROBLEM 6.28

KNOWN: Heat transfer rate from a turbine blade for prescribed operating conditions.

FIND: Heat transfer rate from a larger blade operating under different conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Surface area A is directly proportional to characteristic length L, (4) Negligible radiation, (5) Blade shapes are geometrically similar.

ANALYSIS: For a prescribed geometry,

$$\overline{Nu} = \frac{\bar{h}L}{k} = f\left(Re_L, Pr\right).$$

The Reynolds numbers for the blades are

$$Re_{L,1} = \left(V_1 L_1 / \nu\right) = 15/\nu \qquad Re_{L,2} = \left(V_2 L_2 / \nu\right) = 15/\nu.$$

Hence, with constant properties, $Re_{L,1} = Re_{L,2}$. Also, $Pr_1 = Pr_2$. Therefore,

$$\overline{Nu}_2 = \overline{Nu}_1$$
$$\left(\bar{h}_2 L_2 / k\right) = \left(\bar{h}_1 L_1 / k\right)$$
$$\bar{h}_2 = \frac{L_1}{L_2}\bar{h}_1 = \frac{L_1}{L_2}\frac{q_1}{A_1\left(T_{s,1} - T_\infty\right)}.$$

Hence, the heat rate for the *second blade* is

$$q_2 = \bar{h}_2 A_2\left(T_{s,2} - T_\infty\right) = \frac{L_1}{L_2}\ \frac{A_2}{A_1}\ \frac{\left(T_{s,2} - T_\infty\right)}{\left(T_{s,1} - T_\infty\right)} q_1$$
$$q_2 = \frac{T_{s,2} - T_\infty}{T_{s,1} - T_\infty} q_1 = \frac{(400 - 35)}{(300 - 35)}(1500\text{ W})$$

$$q_2 = 2066\text{ W}.$$ <

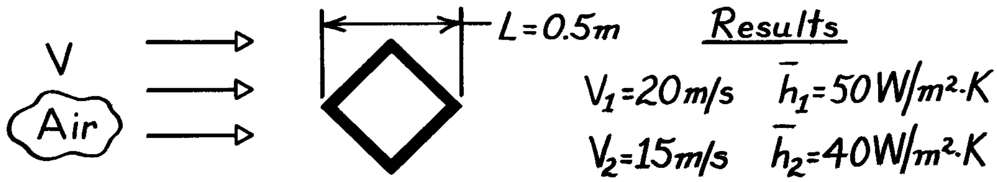
COMMENTS: The slight variation of ν from Case 1 to Case 2 would cause $Re_{L,2}$ to differ from $Re_{L,1}$. However, for the prescribed conditions, this non-constant property effect is small.

PROBLEM 6.29

KNOWN: Experimental measurements of the heat transfer coefficient for a square bar in cross flow.

FIND: (a) $\bar{h}$ for the condition when L = 1m and V = 15m/s, (b) $\bar{h}$ for the condition when L = 1m and V = 30m/s, (c) Effect of defining a side as the characteristic length.

SCHEMATIC:



ASSUMPTIONS: (1) Functional form $\overline{Nu} = CRe^m Pr^n$ applies with C, m, n being constants, (2) Constant properties.

ANALYSIS: (a) For the experiments and the condition L = 1m and V = 15m/s, it follows that Pr as well as C, m, and n are constants. Hence

$$\bar{h}L \ \alpha \ (VL)^m.$$

Using the experimental results, find m. Substituting values

$$\frac{\bar{h}_1 L_1}{\bar{h}_2 L_2} = \left[\frac{V_1 L_1}{V_2 L_2}\right]^m \qquad \frac{50\times 0.5}{40\times 0.5} = \left[\frac{20\times 0.5}{15\times 0.5}\right]^m$$

giving m = 0.782. It follows then for L = 1m and V = 15m/s,

$$\bar{h} = \bar{h}_1 \frac{L_1}{L}\left[\frac{V\cdot L}{V_1 \cdot L_1}\right]^m = 50\frac{W}{m^2\cdot K}\times\frac{0.5}{1.0}\left[\frac{15\times 1.0}{20\times 0.5}\right]^{0.782} = 34.3 W/m^2\cdot K. \quad <$$

(b) For the condition L = 1m and V = 30m/s, find

$$\bar{h} = \bar{h}_1 \frac{L_1}{L}\left[\frac{V\cdot L}{V_1 \cdot L_1}\right]^m = 50\frac{W}{m^2\cdot K}\times\frac{0.5}{1.0}\left[\frac{30\times 1.0}{20\times 0.5}\right]^{0.782} = 59.0 W/m^2\cdot K. \quad <$$

(c) If the characteristic length were chosen as a side rather than the diagonal, the value of C would change. However, the coefficients m and n would not change.

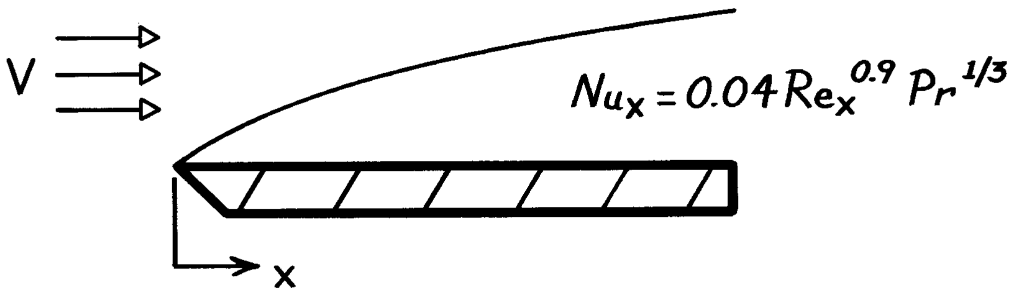
COMMENTS: The foregoing Nusselt number relation is used frequently in heat transfer analysis, providing appropriate scaling for the effects of length, velocity, and fluid properties on the heat transfer coefficient.

PROBLEM 6.30

KNOWN: Local Nusselt number correlation for flow over a roughened surface.

FIND: Ratio of average heat transfer coefficient to local coefficient.

SCHEMATIC:



ANALYSIS: The local convection coefficient is obtained from the prescribed correlation,

$$h_x = Nu_x \frac{k}{x} = 0.04 \frac{k}{x} Re_x^{0.9} Pr^{1/3}$$

$$h_x = 0.04\, k \left[\frac{V}{\nu}\right]^{0.9} Pr^{1/3} \frac{x^{0.9}}{x} \equiv C_1 x^{-0.1}.$$

To determine the average heat transfer coefficient for the length zero to x,

$$\bar{h}_x \equiv \frac{1}{x}\int_0^x h_x\, dx = \frac{1}{x} C_1 \int_0^x x^{-0.1} dx$$

$$\bar{h}_x = \frac{C_1}{x}\frac{x^{0.9}}{0.9} = 1.11\, C_1\, x^{-0.1}.$$

Hence, the ratio of the average to local coefficient is

$$\frac{\bar{h}_x}{h_x} = \frac{1.11\, C_1\, x^{-0.1}}{C_1\, x^{-0.1}} = 1.11.$$ <

COMMENTS: Note that $\overline{Nu}_x / Nu_x$ is also equal to 1.11. Note, however, that

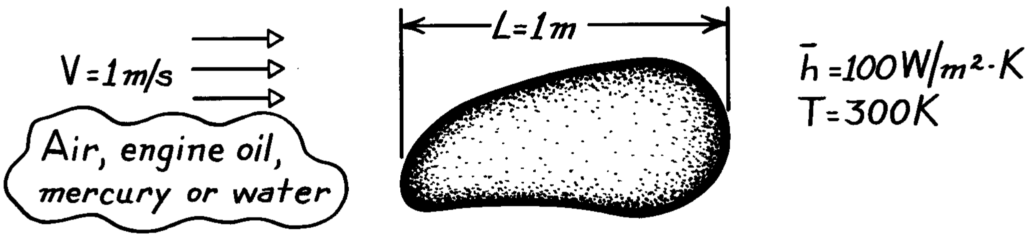
$$\overline{Nu}_x \neq \frac{1}{x}\int_0^x Nu_x\, dx.$$

PROBLEM 6.31

KNOWN: Freestream velocity and average convection heat transfer associated with fluid flow over a surface of prescribed characteristic length.

FIND: Values of $\overline{Nu}_L$, Re_L, Pr, $\bar{j}_H$ for (a) air, (b) engine oil, (c) mercury, (d) water.

SCHEMATIC:



PROPERTIES: For the fluids at 300K:

Fluid	Table	$\nu(m^2/s)$	$k(W/m\cdot K)$	$\alpha(m^2/s)$	Pr
Air	A.4	15.89×10^{-6}	0.0263	22.5×10^{-7}	0.71
Engine Oil	A.5	550×10^{-6}	0.145	0.859×10^{-7}	6400
Mercury	A.5	0.113×10^{-6}	8.54	45.30×10^{-7}	0.025
Water	A.6	0.858×10^{-6}	0.613	1.47×10^{-7}	5.83

ANALYSIS: The appropriate relations required are

$$\overline{Nu}_L = \frac{\bar{h}L}{k} \qquad Re_L = \frac{VL}{\nu} \qquad Pr = \frac{\nu}{\alpha} \qquad \bar{j}_H = \bar{St}Pr^{2/3} \qquad \bar{St} = \frac{\overline{Nu}_L}{Re_L\,Pr}$$

Fluid	$\overline{Nu}_L$	Re_L	Pr	$\bar{j}_H$
Air	3802	6.29×10^4	0.71	0.068
Engine Oil	690	1.82×10^3	6403	0.0204
Mercury	11.7	8.85×10^6	0.025	4.52×10^{-6}
Water	163	1.17×10^6	5.84	7.74×10^{-5}

<

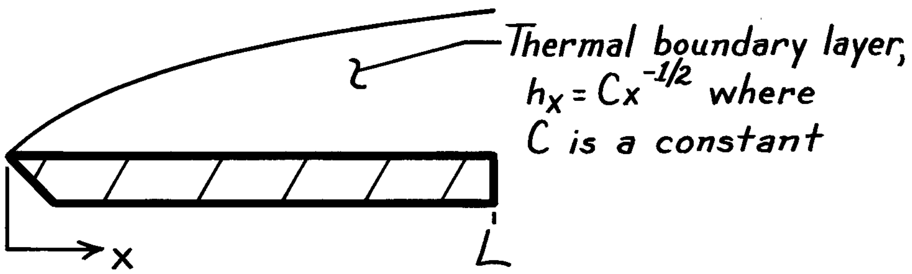
COMMENTS: Note the wide range of Pr associated with the fluids.

PROBLEM 6.32

KNOWN: Variation of h_x with x for flow over a flat plate.

FIND: Ratio of average Nusselt number for the entire plate to the local Nusselt number at x = L.

SCHEMATIC:



ANALYSIS: The expressions for the local and average Nusselt numbers are

$$Nu_L = \frac{h_L L}{k} = \frac{\left(CL^{-1/2}\right)L}{k} = \frac{CL^{1/2}}{k}$$

$$\overline{Nu}_L = \frac{\overline{h}_L L}{k}$$

where

$$\overline{h}_L = \frac{1}{L}\int_0^L h_x\, dx = \frac{C}{L}\int_0^L x^{-1/2}dx = \frac{2C}{L}L^{1/2} = 2\, CL^{-1/2}.$$

Hence,

$$\overline{Nu}_L = \frac{2\, CL^{-1/2}(L)}{k} = \frac{2\, CL^{1/2}}{k}$$

and

$$\frac{\overline{Nu}_L}{Nu_L} = 2.$$ <

COMMENTS: Note the manner in which $\overline{Nu}_L$ is defined in terms of $\overline{h}_L$. Also note that

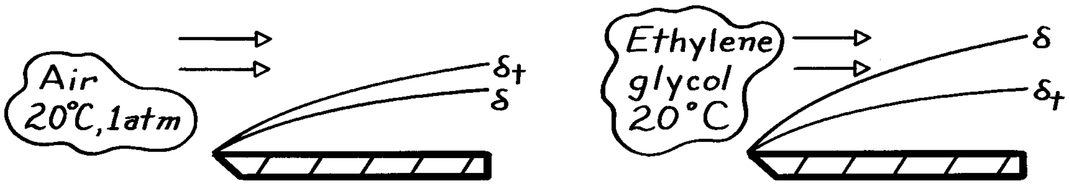
$$\overline{Nu}_L \neq \frac{1}{L}\int_0^L Nu_x\, dx.$$

PROBLEM 6.33

KNOWN: Laminar boundary layer flow of air at 20°C and 1 atm having $\delta_t = 1.13\ \delta$.

FIND: Ratio δ / δ_t when fluid is ethylene glycol for same conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Laminar flow.

PROPERTIES: *Table A-4*, Air (293K, 1 atm): Pr = 0.709; *Table A-5*, Ethylene glycol (293K): Pr = 211.

ANALYSIS: The Prandtl number strongly influences relative growth of the velocity, δ, and thermal, δ_t, boundary layers. For laminar flow, the approximate relationship is given by

$$\mathrm{Pr}^n \approx \frac{\delta}{\delta_t}$$

where n is a positive coefficient. Substituting the values for air

$$(0.709)^n = \frac{1}{1.13}$$

find that n = 0.355. Hence, for ethylene glycol it follows that

$$\frac{\delta}{\delta_t} = \mathrm{Pr}^{0.355} = 211^{0.355} = 6.69. \qquad <$$

COMMENTS: (1) For laminar flow, generally we find n = 0.33. In which case, $\delta / \delta_t = 5.85$.

(2) Recognize the physical importance of $\nu > \alpha$, which gives large values of the Prandtl number, and causes $\delta > \delta_t$.

PROBLEM 6.34

KNOWN: Air, water, engine oil or mercury at 300K in laminar, parallel flow over a flat plate.

FIND: Sketch of velocity and thermal boundary layer thickness.

ASSUMPTIONS: (1) Laminar flow.

PROPERTIES: For the fluids at 300K:

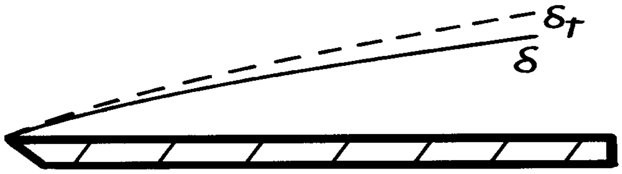
Fluid	Table	Pr
Air	A.4	0.71
Water	A.6	5.83
Engine Oil	A.5	6400
Mercury	A.5	0.025

ANALYSIS: For laminar, boundary layer flow over a flat plate.

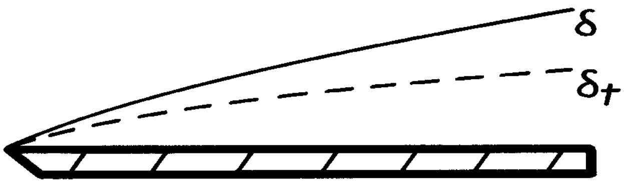
$$\frac{\delta}{\delta_t} \sim \mathrm{Pr}^n$$

where n > 0. Hence, the boundary layers appear as shown below.

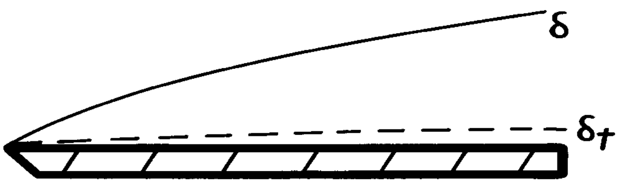
Air:



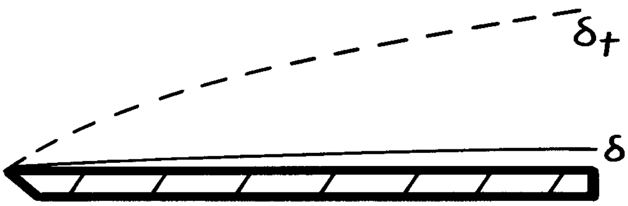
Water:



Engine Oil:



Mercury:



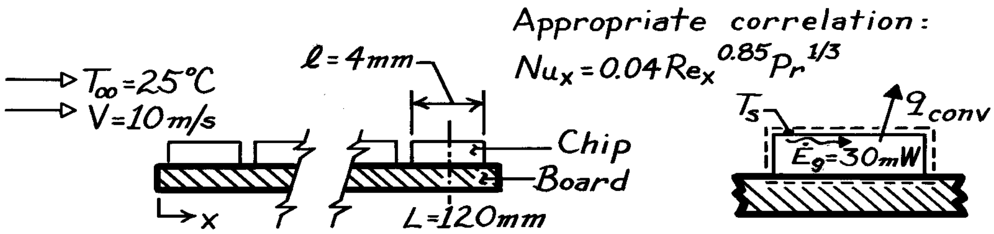
COMMENTS: Although Pr strongly influences relative boundary layer development in laminar flow, its influence is weak for turbulent flow.

PROBLEM 6.35

KNOWN: Expression for the local heat transfer coefficient of air at prescribed velocity and temperature flowing over electronic elements on a circuit board and heat dissipation rate for a 4 × 4 mm chip located 120mm from the leading edge.

FIND: Surface temperature of the chip surface, T_s.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Power dissipated within chip is lost by convection across the upper surface only, (3) Chip surface is isothermal, (4) The average heat transfer coefficient for the chip surface is equivalent to the local value at x = L.

PROPERTIES: *Table A-4*, Air (assume T_s = 45°C, T_f = (45 + 25)/2 = 35°C = 308K, 1atm): ν = 16.69 × 10^{-6} m^2/s, k = 26.9 × 10^{-3} W/m·K, Pr = 0.703.

ANALYSIS: From an energy balance on the chip (see above),

$$q_{conv} = \dot{E}_g = 30W. \qquad (1)$$

Newton's law of cooling for the upper chip surface can be written as

$$T_s = T_\infty + q_{conv} / \bar{h}\, A_{chip} \qquad (2)$$

where $A_{chip} = \ell^2$. Assume that the *average* heat transfer coefficient $(\bar{h})$ over the chip surface is equivalent to the *local* coefficient evaluated at x = L. That is, $\bar{h}_{chip} \approx h_x(L)$ where the local coefficient can be evaluated from the special correlation for this situation,

$$Nu_x = \frac{h_x x}{k} = 0.04\left[\frac{Vx}{\nu}\right]^{0.85} Pr^{1/3}$$

and substituting numerical values with x = L, find

$$h_x = 0.04\frac{k}{L}\left[\frac{VL}{\nu}\right]^{0.85} Pr^{1/3}$$

$$h_x = 0.04\left[\frac{0.0269\ W/m\cdot K}{0.120\ m}\right]\left[\frac{10\ m/s \times 0.120\ m}{16.69\times 10^{-6}\ m^2/s}\right]^{0.85}(0.703)^{1/3} = 107\ W/m^2\cdot K.$$

The surface temperature of the chip is from Eq. (2),

$$T_s = 25°C + 30\times 10^{-3}\ W/107\ W/m^2\cdot K \times (0.004m)^2 = 42.5°C. \qquad <$$

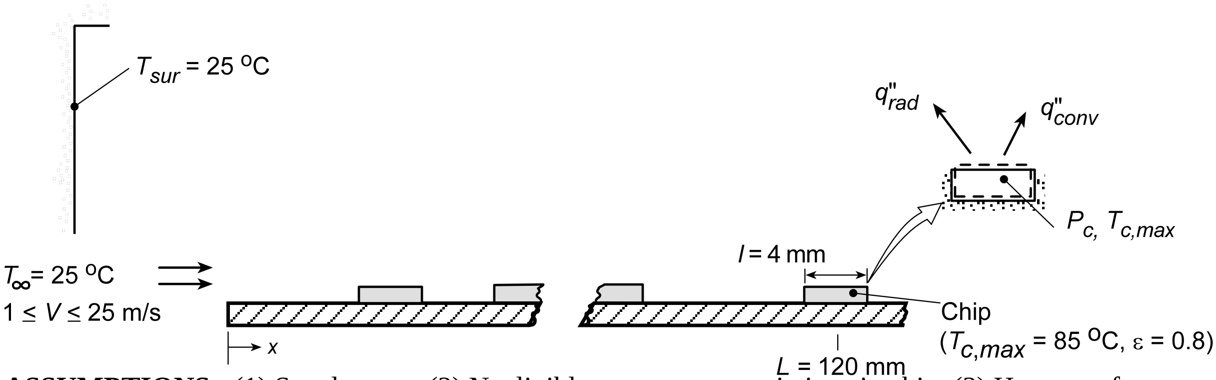
COMMENTS: (1) Note that the estimated value for T_f used to evaluate the air properties was reasonable. (2) Alternatively, we could have evaluated $\bar{h}_{chip}$ by performing the integration of the local value, h(x).

PROBLEM 6.36

KNOWN: Location and dimensions of computer chip on a circuit board. Form of the convection correlation. Maximum allowable chip temperature and surface emissivity. Temperature of cooling air and surroundings.

FIND: Effect of air velocity on maximum power dissipation, first without and then with consideration of radiation effects.

SCHEMATIC:



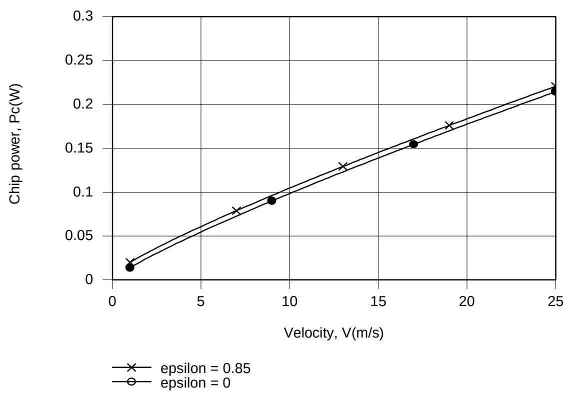
ASSUMPTIONS: (1) Steady-state, (2) Negligible temperature variations in chip, (3) Heat transfer exclusively from the top surface of the chip, (4) The local heat transfer coefficient at x = L provides a good approximation to the average heat transfer coefficient for the chip surface.

PROPERTIES: *Table A.4*, air ($\overline{T} = (T_\infty + T_c)/2$ = 328 K): $\nu = 18.71 \times 10^{-6}$ m^2/s, k = 0.0284 W/m·K, Pr = 0.703.

ANALYSIS: Performing an energy balance for a control surface about the chip, we obtain $P_c = q_{conv} + q_{rad}$, where $q_{conv} = \overline{h}A_s(T_c - T_\infty)$, $q_{rad} = h_r A_s(T_c - T_{sur})$, and $h_r = \varepsilon\sigma(T_c + T_{sur})(T_c^2 + T_{sur}^2)$. With $\overline{h} \approx h_L$, the convection coefficient may be determined from the correlation provided in Problem 6.35 ($Nu_L = 0.04\,Re_L^{0.85}\,Pr^{1/3}$). Hence,

$$P_c = \ell^2\left[0.04(k/L)Re_L^{0.85}\,Pr^{1/3}(T_c - T_\infty) + \varepsilon\sigma(T_c + T_{sur})(T_c^2 + T_{sur}^2)(T_c - T_{sur})\right]$$

where $Re_L = VL/\nu$. Computing the right side of this expression for ε = 0 and ε = 0.85, we obtain the following results.



Since h_L increases as $V^{0.85}$, the chip power must increase with V in the same manner. Radiation exchange increases P_c by a fixed, but small (6 mW) amount. While h_L varies from 14.5 to 223 W/m^2·K over the prescribed velocity range, h_r = 6.5 W/m^2·K is a constant, independent of V.

COMMENTS: Alternatively, $\overline{h}$ could have been evaluated by integrating h_x over the range 118 ≤ x ≤ 122 mm to obtain the appropriate average. However, the value would be extremely close to $h_{x=L}$.

PROBLEM 6.37

KNOWN: Form of Nusselt number for flow of air or a dielectric liquid over components of a circuit card.

FIND: Ratios of time constants associated with intermittent heating and cooling. Fluid that provides faster thermal response.

PROPERTIES: Prescribed. Air: $k = 0.026$ W/m·K, $\nu = 2 \times 10^{-5}$ m^2/s, Pr = 0.71. Dielectric liquid: $k = 0.064$ W/m·K, $\nu = 10^{-6}$ m^2/s, Pr = 25.

ANALYSIS: From Eq. 5.7, the thermal time constant is

$$\tau_t = \frac{\rho \forall c}{\overline{h} A_s}$$

Since the only variable that changes with the fluid is the convection coefficient, where

$$\overline{h} = \frac{k}{L}\overline{Nu}_L = \frac{k}{L} C \, Re_L^m \, Pr^n = \frac{k}{L} C \left(\frac{VL}{\nu}\right)^m Pr^n$$

the desired ratio reduces to

$$\frac{\tau_{t,air(a)}}{\tau_{t,dielectric(d)}} = \frac{\overline{h}_d}{\overline{h}_a} = \frac{k_d}{k_a}\left(\frac{\nu_a}{\nu_d}\right)^m \left(\frac{Pr_d}{Pr_a}\right)^n$$

$$\frac{\tau_{t,a}}{\tau_{t,d}} = \frac{0.064}{0.026}\left(\frac{2 \times 10^{-5}}{10^{-6}}\right)^{0.8}\left(\frac{25}{0.71}\right)^{0.33} = 88.6$$

Since its time constant is nearly two orders of magnitude smaller than that of the air, the dielectric liquid is clearly the fluid of choice.

COMMENTS: The accelerated testing procedure suggested by this problem is commonly used to test the durability of electronic packages.

PROBLEM 6.38

KNOWN: Form of the Nusselt number correlation for forced convection and fluid properties.

FIND: Expression for figure of merit F_F and values for air, water and a dielectric liquid.

PROPERTIES: Prescribed. Air: k = 0.026 W/m·K, $\nu = 1.5 \times 10^{-5}$ m^2/s, Pr = 0.70. Water: k = 0.600 W/m·K, $\nu = 10^{-6}$ m^2/s, Pr = 5.0. Dielectric liquid: k = 0.064 W/m·K, $\nu = 10^{-6}$ m^2/s, Pr = 25

ANALYSIS: With $Nu_L \sim Re_L^m \, Pr^n$, the convection coefficient may be expressed as

$$h \sim \frac{k}{L}\left(\frac{VL}{\nu}\right)^m Pr^n \sim \frac{V^m}{L^{1-m}}\left(\frac{kPr^n}{\nu^m}\right)$$

The figure of merit is therefore

$$F_F = \frac{kPr^n}{\nu^m} \qquad <$$

and for the three fluids, with m = 0.80 and n = 0.33,

$F_F\left(W \cdot s^{0.8} / m^{2.6} \cdot K\right)$	Air	Water	Dielectric	
	167	64,400	11,700	<

Water is clearly the superior heat transfer fluid, while air is the least effective.

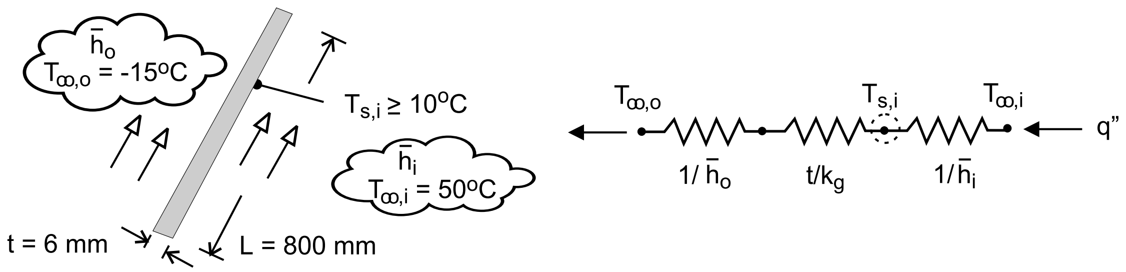
COMMENTS: The figure of merit indicates that heat transfer is enhanced by fluids of large k, large Pr and small ν.

PROBLEM 6.39

KNOWN: Ambient, interior and dewpoint temperatures. Vehicle speed and dimensions of windshield. Heat transfer correlation for external flow.

FIND: Minimum value of convection coefficient needed to prevent condensation on interior surface of windshield.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional heat transfer, (3) Constant properties.

PROPERTIES: *Table A-3*, glass: k_g = 1.4 W/m·K. Prescribed, air: k = 0.023 W/m·K, $\nu = 12.5 \times 10^{-6}$ m^2/s, Pr = 0.70.

ANALYSIS: From the prescribed thermal circuit, conservation of energy yields

$$\frac{T_{\infty,i} - T_{s,i}}{1/\bar{h}_i} = \frac{T_{s,i} - T_{\infty,o}}{t/k_g + 1/\bar{h}_o}$$

where $\bar{h}_o$ may be obtained from the correlation

$$\overline{Nu}_L = \frac{\bar{h}_o L}{k} = 0.030\, Re_L^{0.8}\, Pr^{1/3}$$

With V = (70 mph × 1585 m/mile)/3600 s/h = 30.8 m/s, Re_D = (30.8 m/s × 0.800 m)/12.5 × 10^{-6} m^2/s = 1.97 × 10^6 and

$$\bar{h}_o = \frac{0.023\,\text{W/m}\cdot\text{K}}{0.800\text{ m}} 0.030\left(1.97\times10^6\right)^{0.8}(0.70)^{1/3} = 83.1\,\text{W/m}^2\cdot\text{K}$$

From the energy balance, with $T_{s,i} = T_{dp}$ = 10°C

$$\bar{h}_i = \frac{\left(T_{s,i} - T_{\infty,o}\right)}{\left(T_{\infty,i} - T_{s,i}\right)}\left(\frac{t}{k_g} + \frac{1}{\bar{h}_o}\right)^{-1}$$

$$\bar{h}_i = \frac{(10+15)°\text{C}}{(50-10)°\text{C}}\left(\frac{0.006\text{ m}}{1.4\,\text{W/m}\cdot\text{K}} + \frac{1}{83.1\,\text{W/m}^2\cdot\text{K}}\right)^{-1}$$

$$\bar{h}_i = 38.3\,\text{W/m}^2\cdot\text{K}$$ <

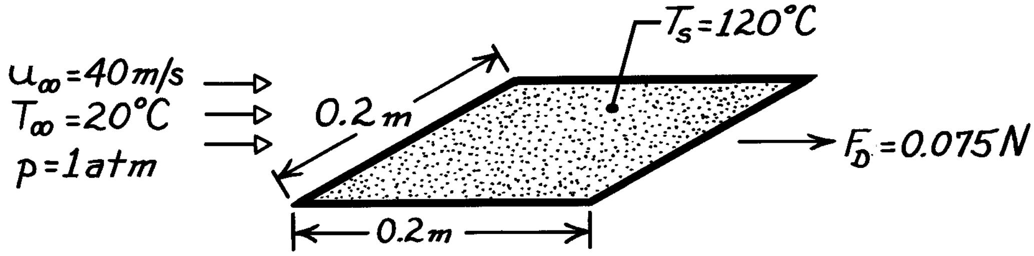
COMMENTS: The output of the fan in the automobile's heater/defroster system must maintain a velocity for flow over the inner surface that is large enough to provide the foregoing value of $\bar{h}_i$. In addition, the output of the heater must be sufficient to maintain the prescribed value of $T_{\infty,i}$ at this velocity.

PROBLEM 6.40

KNOWN: Drag force and air flow conditions associated with a flat plate.

FIND: Rate of heat transfer from the plate.

SCHEMATIC:



ASSUMPTIONS: (1) Chilton-Colburn analogy is applicable.

PROPERTIES: *Table A-4*, Air (70°C,1 atm): ρ = 1.018 kg/m^3, c_p = 1009 J/kg·K, Pr = 0.70, $\nu = 20.22 \times 10^{-6}$m^2/s.

ANALYSIS: The rate of heat transfer from the plate is

$$q = 2\bar{h}\left(L^2\right)\left(T_s - T_\infty\right)$$

where $\bar{h}$ may be obtained from the Chilton-Colburn analogy,

$$\bar{j}_H = \frac{\bar{C}_f}{2} = \overline{St}\,Pr^{2/3} = \frac{\bar{h}}{\rho\, u_\infty\, c_p} Pr^{2/3}$$

$$\frac{\bar{C}_f}{2} = \frac{1}{2}\frac{\bar{\tau}_s}{\rho\, u_\infty^2/2} = \frac{1}{2}\frac{(0.075\text{ N}/2)/(0.2\text{m})^2}{1.018\text{ kg/m}^3(40\text{ m/s})^2/2} = 5.76\times10^{-4}.$$

Hence,

$$\bar{h} = \frac{C_f}{2}\rho\, u_\infty\, c_p\, Pr^{-2/3}$$

$$\bar{h} = 5.76\times10^{-4}\left(1.018\text{kg/m}^3\right)40\text{m/s}\ (1009\text{J/kg}\cdot\text{K})\ (0.70)^{-2/3}$$

$$\bar{h} = 30\text{ W/m}^2\cdot\text{K}.$$

The heat rate is

$$q = 2\left(30\text{ W/m}^2\cdot\text{K}\right)(0.2\text{m})^2\ (120-20)^\circ\text{C}$$

$$q = 240\text{ W.}$$ <

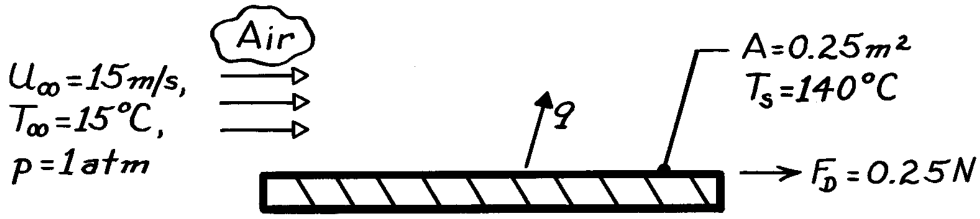
COMMENTS: Although the flow is laminar over the entire surface ($Re_L = u_\infty L/\nu$ = 40 m/s × 0.2m/20.22 × 10^{-6}m^2/s = 4.0 × 10^5), the pressure gradient is zero and the Chilton-Colburn analogy is applicable to *average*, as well as *local*, surface conditions. Note that the only contribution to the drag force is made by the surface shear stress.

PROBLEM 6.41

KNOWN: Air flow conditions and drag force associated with a heater of prescribed surface temperature and area.

FIND: Required heater power.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Reynolds analogy is applicable, (3) Bottom surface is adiabatic.

PROPERTIES: *Table A-4*, Air (T_f = 350K, 1atm): ρ = 0.995 kg/m^3, c_p = 1009 J/kg·K, Pr = 0.700.

ANALYSIS: The average shear stress and friction coefficient are

$$\bar{\tau}_s = \frac{F_D}{A} = \frac{0.25\text{ N}}{0.25\text{ m}^2} = 1\text{ N/m}^2$$

$$\bar{C}_f = \frac{\bar{\tau}_s}{\rho\, u_\infty^2/2} = \frac{1\text{ N/m}^2}{0.995\text{ kg/m}^3(15\text{m/s})^2/2} = 8.93\times10^{-3}.$$

From the Reynolds analogy,

$$\overline{\text{St}} = \frac{\bar{h}}{\rho\, u_\infty c_p} = \frac{\bar{C}_f}{2}\text{Pr}^{-2/3}.$$

Solving for $\bar{h}$ and substituting numerical values, find

$$\bar{h} = 0.995\text{ kg/m}^3(15\text{m/s})\ 1009\text{ J/kg}\cdot\text{K}\left(8.93\times10^{-3}/2\right)(0.7)^{-2/3}$$

$$\bar{h} = 85\text{ W/m}^2\cdot\text{K}.$$

Hence, the heat rate is

$$q = \bar{h}\,A\,(T_s - T_\infty) = 85\text{W/m}^2\cdot\text{K}\left(0.25\text{m}^2\right)(140-15)^\circ\text{C}$$

$$q = 2.66\text{ kW}.$$ <

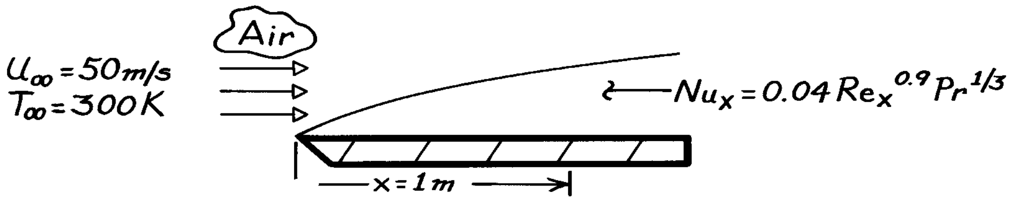
COMMENTS: Due to bottom heat losses, which have been assumed negligible, the actual power requirement would exceed 2.66 kW.

PROBLEM 6.42

KNOWN: Heat transfer correlation associated with parallel flow over a rough flat plate. Velocity and temperature of air flow over the plate.

FIND: Surface shear stress l m from the leading edge.

SCHEMATIC:



ASSUMPTIONS: (1) Modified Reynolds analogy is applicable, (2) Constant properties.

PROPERTIES: *Table A-4*, Air (300K, 1atm): $\nu = 15.89 \times 10^{-6} m^2/s$, Pr = 0.71, $\rho = 1.16 kg/m^3$.

ANALYSIS: Applying the Chilton-Colburn analogy

$$\frac{C_f}{2} = St_x Pr^{2/3} = \frac{Nu_x}{Re_x Pr} Pr^{2/3} = \frac{0.04 Re_x^{0.9} Pr^{1/3}}{Re_x Pr} Pr^{2/3}$$

$$\frac{C_f}{2} = 0.04 Re_x^{-0.1}$$

where

$$Re_x = \frac{u_\infty x}{\nu} = \frac{50 \text{ m/s} \times 1\text{m}}{15.89 \times 10^{-6} \text{m}^2/\text{s}} = 3.15 \times 10^6.$$

Hence, the friction coefficient is

$$C_f = 0.08 \left(3.15 \times 10^6\right)^{-0.1} = 0.0179 = \tau_s / \left(\rho u_\infty^2 / 2\right)$$

and the surface shear stress is

$$\tau_s = C_f \left(\rho u_\infty^2 / 2\right) = 0.0179 \times 1.16 \text{kg/m}^3 (50 \text{ m/s})^2 / 2$$

$$\tau_s = 25.96 \text{ kg/m} \cdot \text{s}^2 = 25.96 \text{ N/m}^2. \qquad <$$

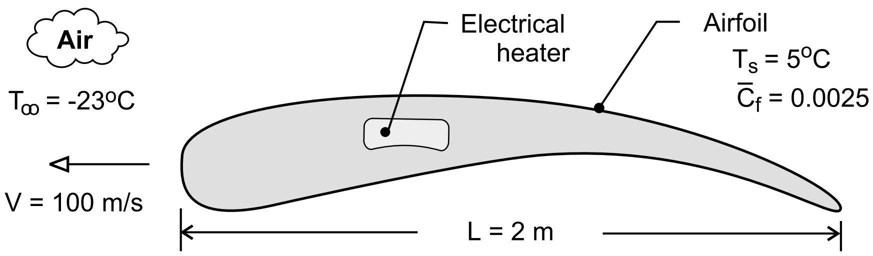
COMMENTS: Note that turbulent flow will exist at the designated location.

PROBLEM 6.43

KNOWN: Nominal operating conditions of aircraft and characteristic length and average friction coefficient of wing.

FIND: Average heat flux needed to maintain prescribed surface temperature of wing.

SCHEMATIC:



ASSUMPTIONS: (1) Applicability of modified Reynolds analogy, (2) Constant properties.

PROPERTIES: Prescribed, Air: $\nu = 16.3 \times 10^{-6}$ m^2/s, k = 0.022 W/m·K, Pr = 0.72.

ANALYSIS: The average heat flux that must be maintained over the surface of the air foil is $\overline{q}'' = \overline{h}\left(T_s - T_\infty\right)$, where the average convection coefficient may be obtained from the modified Reynolds analogy.

$$\frac{\overline{C}_f}{2} = \text{St}\,\text{Pr}^{2/3} = \frac{\overline{\text{Nu}}_L}{\text{Re}_L\,\text{Pr}}\text{Pr}^{2/3} = \frac{\overline{\text{Nu}}_L}{\text{Re}_L\,\text{Pr}^{1/3}}$$

Hence, with $\text{Re}_L = VL/\nu = 100\,\text{m/s}\left(2\text{m}\right)/16.3\times10^{-6}\text{m}^2/\text{s} = 1.23\times10^7$,

$$\overline{\text{Nu}}_L = \frac{0.0025}{2}\left(1.23\times10^7\right)\left(0.72\right)^{1/3} = 13,780$$

$$\overline{h} = \frac{k}{L}\overline{\text{Nu}}_L = \frac{0.022\,\text{W/m}\cdot\text{K}}{2\text{m}}\left(13,780\right) = 152\,\text{W/m}^2\cdot\text{K}$$

$$\overline{q}'' = 152\,\text{W/m}^2\cdot\text{K}\left[5-\left(-23\right)\right]°\text{C} = 4260\,\text{W/m}^2$$ <

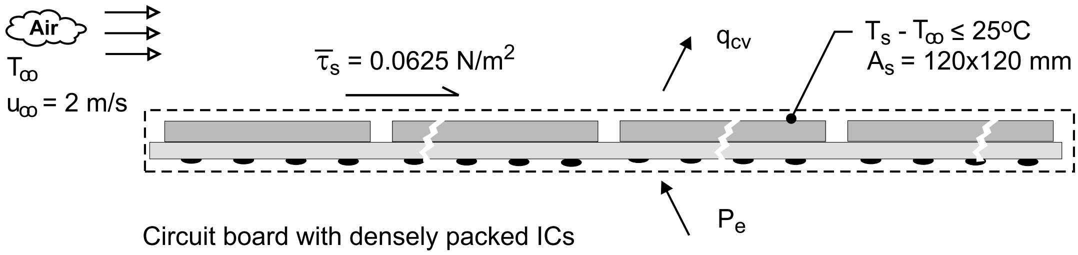
COMMENTS: If the flow is turbulent over the entire airfoil, the modified Reynolds analogy provides a good measure of the relationship between surface friction and heat transfer. The relation becomes more approximate with increasing laminar boundary layer development on the surface and increasing values of the magnitude of the pressure gradient.

PROBLEM 6.44

KNOWN: Average frictional shear stress of $\overline{\tau}_s = 0.0625$ N/m^2 on upper surface of circuit board with densely packed integrated circuits (ICs)

FIND: Allowable power dissipation from the upper surface of the board if the average surface temperature of the ICs must not exceed a rise of 25°C above ambient air temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) The modified Reynolds analogy is applicable, (3) Negligible heat transfer from bottom side of the circuit board, and (4) Thermophysical properties required for the analysis evaluated at 300 K,

PROPERTIES: *Table A-4*, Air (T_f = 300 K, 1 atm): ρ = 1.161 kg/m^3, c_p = 1007 J/kg·K, Pr = 0.707.

ANALYSIS: The power dissipation from the circuit board can be calculated from the convection rate equation assuming an excess temperature (T_s - T_∞) = 25°C.

$$q = \overline{h}\, A_s \left(T_s - T_\infty\right) \tag{1}$$

The average convection coefficient can be estimated from the Reynolds analogy and the measured average frictional shear stress $\overline{\tau}_s$.

$$\frac{\overline{C}_f}{2} = \overline{St}\ Pr^{2/3} \qquad \overline{C}_f = \frac{\overline{\tau}_s}{\rho\ V^2/2} \qquad \overline{St} = \frac{\overline{h}}{\rho\ V\ c_p} \tag{2,3,4}$$

With V = u_∞ and substituting numerical values, find $\overline{h}$.

$$\frac{\overline{\tau}_s}{\rho\ V^2} = \frac{\overline{h}}{\rho\ V c_p} Pr^{2/3}$$

$$\overline{h} = \frac{\overline{\tau}_s\ c_p}{V} Pr^{-2/3}$$

$$\overline{h} = \frac{0.0625\ N/m^2 \times 1007\ J/kg \cdot K}{2\ m/s}\left(0.707\right)^{-2/3} = 39.7\ W/m^2 \cdot K$$

Substituting this result into Eq. (1), the allowable power dissipation is

$$q = 39.7\ W/m^2 \cdot K \times \left(0.120 \times 0.120\right) m^2 \times 25\ K = 14.3\ W$$ <

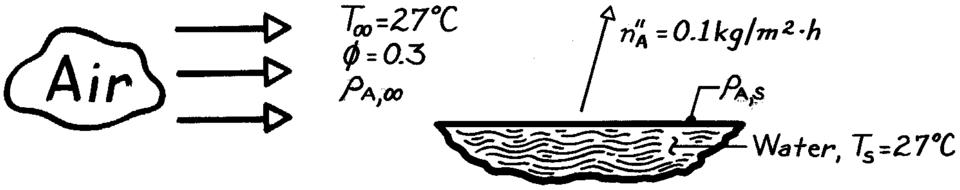
COMMENTS: For this analyses using the modified or Chilton-Colburn analogy, we found C_f = 0.0269 and St = 0.0170. Using the Reynolds analogy, the results are slightly different with $\overline{h} = 31.5\ W/m^2 \cdot K$ and q = 11.3 W.

PROBLEM 6.45

KNOWN: Evaporation rate of water from a lake.

FIND: The convection mass transfer coefficient, $\overline{h}_m$.

SCHEMATIC:



ASSUMPTIONS: (1) Equilibrium at water vapor-liquid surface, (2) Isothermal conditions, (3) Perfect gas behavior of water vapor, (4) Air at standard atmospheric pressure.

PROPERTIES: *Table A-6*, Saturated water vapor (300K): $p_{A,sat} = 0.03531$ bar, $\rho_{A,sat} = 1/v_g = 0.02556\ kg/m^3$.

ANALYSIS: The convection mass transfer (evaporation) rate equation can be written in the form

$$\overline{h}_m = \frac{n''_A}{\left(\rho_{A,s} - \rho_{A,\infty}\right)}$$

where

$$\rho_{A,s} = \rho_{A,sat},$$

the saturation density at the temperature of the water and

$$\rho_{A,\infty} = \phi \rho_{A,sat}$$

which follows from the definition of the relative humidity, $\phi = p_A/p_{A,sat}$ and perfect gas behavior. Hence,

$$\overline{h}_m = \frac{n''_A}{\rho_{A,sat}(1-\phi)}$$

and substituting numerical values, find

$$\overline{h}_m = \frac{0.1\ kg/m^2 \cdot h \times 1/3600\ s/h}{0.02556\ kg/m^3 (1-0.3)} = 1.55\times10^{-3}\ m/s. \qquad <$$

COMMENTS: (1) From knowledge of $p_{A,sat}$, the perfect gas law could be used to obtain the saturation density.

$$\rho_{A,sat} = \frac{p_{A,sat} M_A}{\Re T} = \frac{0.03531\ bar \times 18\ kg/kmol}{8.314\times10^{-2} m^3 \cdot bar/kmol \cdot K (300K)} = 0.02548\ kg/m^3.$$

This value is within 0.3% of that obtained from Table A-6.

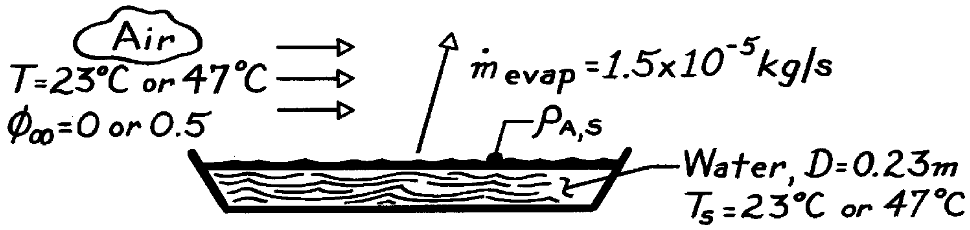
(2) Note that psychrometric charts could also be used to obtain $\rho_{A,sat}$ and $\rho_{A,\infty}$.

PROBLEM 6.46

KNOWN: Evaporation rate from pan of water of prescribed diameter. Water temperature. Air temperature and relative humidity.

FIND: (a) Convection mass transfer coefficient, (b) Evaporation rate for increased relative humidity, (c) Evaporation rate for increased temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Water vapor is saturated at liquid interface and may be approximated as a perfect gas.

PROPERTIES: *Table A-6*, Saturated water vapor (T_s = 296K): $\rho_{A,sat} = v_g^{-1} = (49.4\ m^3/kg)^{-1}$ = 0.0202 kg/m^3; (T_s = 320 K): $\rho_{A,sat} = v_g^{-1} = \left(13.98\ m^3/kg\right)^{-1} = 0.0715\ kg/m^3$.

ANALYSIS: (a) Since evaporation is a convection mass transfer process, the rate equation has the form $\dot{m}_{evap} = \bar{h}_m A\left(\rho_{A,s} - \rho_{A,\infty}\right)$ and the mass transfer coefficient is

$$\bar{h}_m = \frac{\dot{m}_{evap}}{\left(\pi\ D^2/4\right)\left(\rho_{A,s} - \rho_{A,\infty}\right)} = \frac{1.5\times10^{-5} kg/s}{(\pi/4)(0.23\ m)^2\ 0.0202\ kg/m^3} = 0.0179\ m/s \qquad <$$

with $T_s = T_\infty$ = 23°C and ϕ_∞ = 0.

(b) If the relative humidity of the ambient air is increased to 50%, the ratio of the evaporation rates is

$$\frac{\dot{m}_{evap}\left(\phi_\infty = 0.5\right)}{\dot{m}_{evap}\left(\phi_\infty = 0\right)} = \frac{\bar{h}_m A\left[\rho_{A,s}\left(T_s\right) - \phi_\infty \rho_{A,s}\left(T_\infty\right)\right]}{\bar{h}_m A\ \rho_{A,s}\left(T_s\right)} = 1 - \phi_\infty \frac{\rho_{A,s}\left(T_\infty\right)}{\rho_{A,s}\left(T_s\right)}.$$

Hence, $\dot{m}_{evap}\left(\phi_\infty = 0.5\right) = 1.5\times10^{-5} kg/s \left[1 - 0.5\frac{0.0202\ kg/m^3}{0.0202\ kg/m^3}\right] = 0.75\times10^{-5} kg/s.$

(c) If the temperature of the ambient air is increased from 23°C to 47°C, with ϕ_∞ = 0 for both cases, the ratio of the evaporation rates is

$$\frac{\dot{m}_{evap}\left(T_s = T_\infty = 47^\circ C\right)}{\dot{m}_{evap}\left(T_s = T_\infty = 23^\circ C\right)} = \frac{\bar{h}_m A \rho_{A,s}\left(47^\circ C\right)}{\bar{h}_m A \rho_{A,s}\left(23^\circ C\right)} = \frac{\rho_{A,s}\left(47^\circ C\right)}{\rho_{A,s}\left(23^\circ C\right)}.$$

Hence, $\dot{m}_{evap}\left(T_s = T_\infty = 47^\circ C\right) = 1.5\times10^{-5} kg/s \dfrac{0.0715\ kg/m^3}{0.0202\ kg/m^3} = 5.31\times10^{-5} kg/s.$ $\qquad <$

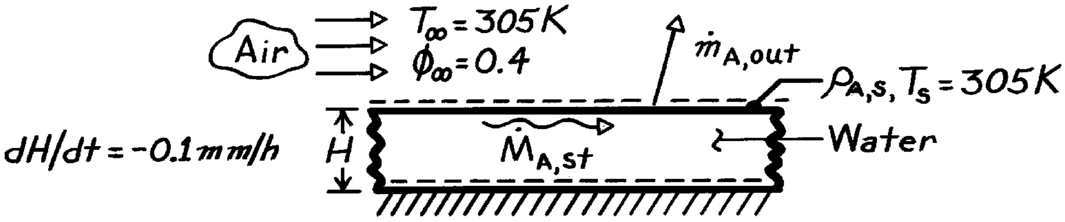
COMMENTS: Note the highly nonlinear dependence of the evaporation rate on the water temperature. For a 24°C rise in $T_s, \dot{m}_{evap}$ increases by 350%.

PROBLEM 6.47

KNOWN: Water temperature and air temperature and relative humidity. Surface recession rate.

FIND: Mass evaporation rate per unit area. Convection mass transfer coefficient.

SCHEMATIC:



ASSUMPTIONS: (1) Water vapor may be approximated as a perfect gas, (2) No water inflow; outflow is only due to evaporation.

PROPERTIES: *Table A-6*, Saturated water: Vapor (305K), $\rho_g = v_g^{-1} = 0.0336 \text{ kg/m}^3$; Liquid (305K), $\rho_f = v_f^{-1} = 995 \text{ kg/m}^3$.

ANALYSIS: Applying conservation of species to a control volume about the water,

$$-\dot{M}_{A,out} = \dot{M}_{A,st}$$
$$-\dot{m}''_{evap}A = \frac{d}{dt}(\rho_f V) = \frac{d}{dt}(\rho_f AH) = \rho_f A\frac{dH}{dt}.$$

Substituting numerical values, find

$$\dot{m}''_{evap} = -\rho_f \frac{dH}{dt} = -995\text{kg/m}^3\left(-10^{-4}\text{m/h}\right)\ (1/3600 \text{ s/h})$$

$$\dot{m}''_{evap} = 2.76\times10^{-5}\text{kg/s}\cdot\text{m}^2.$$ <

Because evaporation is a convection mass transfer process, it also follows that

$$\dot{m}''_{evap} = n''_A$$

or in terms of the rate equation,

$$\dot{m}''_{evap} = h_m\left(\rho_{A,s} - \rho_{A,\infty}\right) = h_m\left[\rho_{A,sat}(T_s) - \phi_\infty\rho_{A,sat}(T_\infty)\right]$$
$$\dot{m}''_{evap} = h_m\rho_{A,sat}(305K)\ (1-\phi_\infty),$$

and solving for the convection mass transfer coefficient,

$$h_m = \frac{\dot{m}''_{evap}}{\rho_{A,sat}(305K)\ (1-\phi_\infty)} = \frac{2.76\times10^{-5}\text{kg/s}\cdot\text{m}^2}{0.0336 \text{ kg/m}^3(1-0.4)}$$

$$h_m = 1.37\times10^{-3}\text{m/s}.$$ <

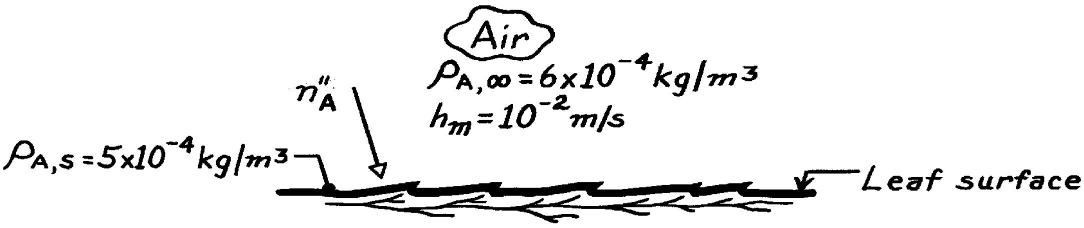
COMMENTS: Conservation of species has been applied in exactly the same way as a conservation of energy. Note the sign convention.

PROBLEM 6.48

KNOWN: CO_2 concentration in air and at the surface of a green leaf. Convection mass transfer coefficient.

FIND: Rate of photosynthesis per unit area of leaf.

SCHEMATIC:



ANALYSIS: Assuming that the CO_2 (species A) is consumed as a reactant in photosynthesis at the same rate that it is transferred across the atmospheric boundary layer, the rate of photosynthesis per unit leaf surface area is given by the rate equation,

$$n''_A = h_m\left(\rho_{A,\infty} - \rho_{A,s}\right).$$

Substituting numerical values, find

$$n''_A = 10^{-2}\,\text{m/s}\left(6\times10^{-4} - 5\times10^{-4}\right)\text{kg/m}^3$$

$$n''_A = 10^{-6}\,\text{kg/s}\cdot\text{m}^2. \qquad <$$

COMMENTS: (1) It is recognized that CO_2 transport is from the air to the leaf, and ($\rho_{A,s}$ - $\rho_{A,\infty}$) in the rate equation has been replaced by ($\rho_{A,\infty}$ - $\rho_{A,s}$).

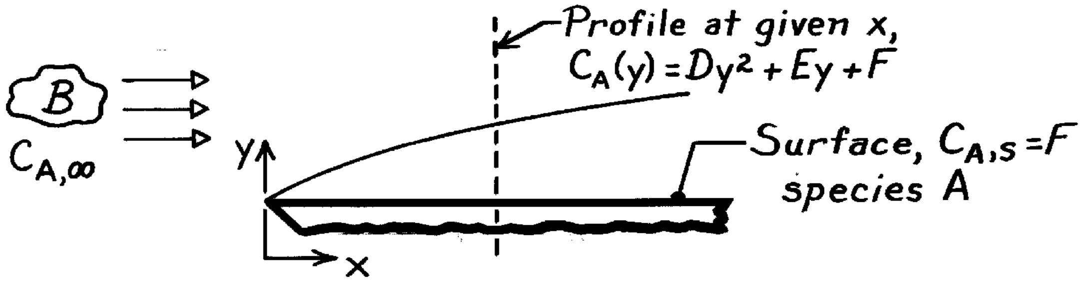
(2) The atmospheric concentration of CO_2 is known to be increasing by approximately 0.3% per year. This increase in $\rho_{A,\infty}$ will have the effect of increasing the photosynthesis rate and hence plant biomass production.

PROBLEM 6.49

KNOWN: Species concentration profile, $C_A(y)$, in a boundary layer at a particular location for flow over a surface.

FIND: Expression for the mass transfer coefficient, h_m, in terms of the profile constants, $C_{A,\infty}$ and D_{AB}. Expression for the molar convection flux, N''_A.

SCHEMATIC:



ASSUMPTIONS: (1) Parameters D, E, and F are constants at any location x, (2) D_{AB}, the mass diffusion coefficient of A through B, is known.

ANALYSIS: The convection mass transfer coefficient is defined in terms of the concentration gradient at the wall,

$$h_m(x) = -D_{AB} \frac{\partial C_A / \partial y)_{y=0}}{\left(C_{A,s} - C_{A,\infty}\right)}.$$

The gradient at the surface follows from the profile, $C_A(y)$,

$$\left.\frac{\partial C_A}{\partial y}\right]_{y=0} = \frac{\partial}{\partial y}\left(Dy^2 + Ey + F\right)_{y=0} = +E.$$

Hence,

$$h_m(x) = -\frac{D_{AB}E}{\left(C_{A,s} - C_{A,\infty}\right)} = \frac{-D_{AB}E}{\left(F - C_{A,\infty}\right)}.$$ <

The molar flux follows from the rate equation,

$$N''_A = h_m\left(C_{A,s} - C_{A,\infty}\right) = \frac{-D_{AB}E}{\left(C_{A,s} - C_{A,\infty}\right)} \cdot \left(C_{A,s} - C_{A,\infty}\right).$$

$$N''_A = -D_{AB}E.$$ <

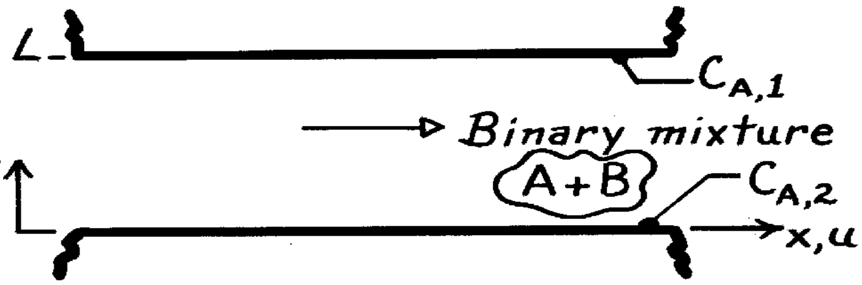
COMMENTS: It is important to recognize that the influence of species B is present in the property D_{AB}. Otherwise, all the parameters relate to species A.

PROBLEM 6.50

KNOWN: Steady, incompressible flow of binary mixture between infinite parallel plates with different species concentrations.

FIND: Form of species continuity equation and concentration distribution. Species flux at upper surface.

SCHEMATIC:



ASSUMPTIONS: (1) Two-dimensional flow, (2) No chemical reactions, (3) Constant properties.

ANALYSIS: For fully developed conditions, $\partial C_A/\partial x = 0$. Hence with v = 0, the species conservation equation reduces to

$$\frac{d^2 C_A}{dy^2} = 0. \qquad <$$

Integrating twice, the general form of the species concentration distribution is

$$C_A(y) = C_1 y + C_2.$$

Using appropriate boundary conditions and evaluating the constants,

$$C_A(0) = C_{A,2} \quad \rightarrow \quad C_2 = C_{A,2}$$
$$C_A(L) = C_{A,1} \quad \rightarrow \quad C_1 = \left(C_{A,1} - C_{A,2}\right)/L,$$

the concentration distribution is

$$C_A(y) = C_{A,2} + (y/L)\left(C_{A,1} - C_{A,2}\right). \qquad <$$

From Fick's law, the species flux is

$$N''_A(L) = -D_{AB}\left.\frac{dC_A}{dy}\right|_{y=L}$$

$$N''_A(L) = \frac{D_{AB}}{L}\left(C_{A,2} - C_{A,1}\right). \qquad <$$

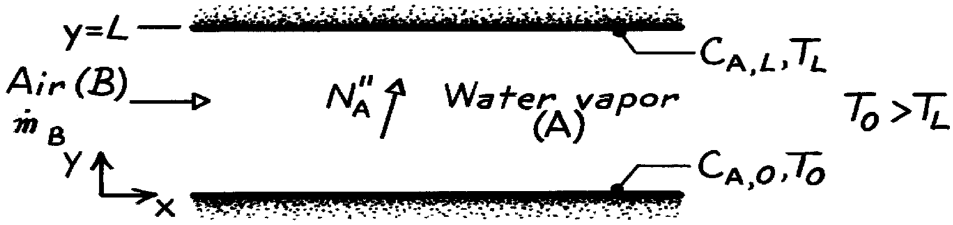
COMMENTS: An analogy between heat and mass transfer exists if viscous dissipation is negligible. The energy equation is then $d^2T/dy^2 = 0$. Hence, both heat and species transfer are influenced only by diffusion. Expressions for T(y) and $q''(L)$ are analogous to those for $C_A(y)$ and $N''_A(L)$.

PROBLEM 6.51

KNOWN: Flow conditions between two parallel plates, across which vapor transfer occurs.

FIND: (a) Variation of vapor molar concentration between the plates and mass rate of water production per unit area, (b) Heat required to sustain the process.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Fully developed, incompressible flow with constant properties, (3) Negligible body forces, (4) No chemical reactions, (5) All work interactions, including viscous dissipation, are negligible.

ANALYSIS: (a) The flow will be fully developed in terms of the vapor concentration field, as well as the velocity and temperature fields. Hence

$$\frac{\partial C_A}{\partial x} = 0 \qquad \text{or} \qquad C_A(x,y) = C_A(y).$$

Also, with $\partial C_A/\partial t = 0$, $\dot{N}_A = 0$, $v = 0$ and constant D_{AB}, the species conservation equation reduces to

$$\frac{d^2 C_A}{dy^2} = 0.$$

Separating and integrating twice,

$$C_A(y) = C_1(y) + C_2.$$

Applying the boundary conditions,

$$C_A(0) = C_{A,0} \qquad \rightarrow \qquad C_2 = C_{A,0}$$

$$C_A(L) = C_{A,L} \qquad \rightarrow \qquad C_{A,L} = C_1 L + C_2 \qquad C_1 = -\frac{C_{A,0} - C_{A,L}}{L}$$

find the species concentration distribution,

$$C_A(y) = C_{A,0} - \left(C_{A,0} - C_{A,L}\right)(y/L).$$ <

From Fick's law, Eq. 6.19, the species transfer rate is

$$N''_A = N''_{A,s} = -D_{AB} \left.\frac{\partial C_A}{\partial y}\right]_{y=0} = D_{AB}\frac{C_{A,0} - C_{A,L}}{L}.$$

Continued …..

Multiplying by the molecular weight of water vapor, M_A, the mass rate of water production per unit area is

$$n''_A = M_A N''_A = M_A D_{AB} \frac{C_{A,0} - C_{A,L}}{L}. \qquad <$$

(b) Heat must be supplied to the bottom surface in an amount equal to the latent and sensible heat transfer from the surface,

$$q'' = q''_{lat} + q''_{sen}$$
$$q'' = n''_{A,s}\ h_{fg} + \left[-k \frac{dT}{dy} \right]_{y=0}.$$

The temperature distribution may be obtained by solving the energy equation, which, for the prescribed conditions, reduces to

$$\frac{d^2T}{dy^2} = 0.$$

Separating and integrating twice,

$$T(y) = C_1 y + C_2.$$

Applying the boundary conditions,

$$T(0) = T_0 \qquad \rightarrow \qquad C_2 = T_0$$
$$T(L) = T_L \qquad \rightarrow \qquad C_1 = (T_1 - T_0)/L$$

find the temperature distribution,

$$T(y) = T_0 - (T_0 - T_L)\, y/L.$$

Hence,

$$-k \frac{dT}{dy}\bigg]_{y=0} = k \frac{(T_0 - T_L)}{L}.$$

Accordingly,

$$q'' = M_A D_{AB} \frac{C_{A,0} - C_{A,L}}{L} h_{fg} + k \frac{(T_0 - T_L)}{L}. \qquad <$$

COMMENTS: Despite the existence of the flow, species and energy transfer across the air are uninfluenced by advection and transfer is only by diffusion. If the flow were not fully developed, advection would have a significant influence on the species concentration and temperature fields and hence on the rate of species and energy transfer. The foregoing results would, of course, apply in the case of no air flow. The physical condition is an example of Poiseuille flow with heat and mass transfer.

PROBLEM 6.52

KNOWN: The conservation equations, Eqs. E.24 and E.31.

FIND: (a) Describe physical significance of terms in these equations, (b) Identify approximations and special conditions used to reduce these equations to the boundary layer equations, Eqs. 6.33 and 6.34, (c) Identify the conditions under which these two boundary layer equations have the same form and, hence, an analogy will exist.

ANALYSIS: (a) The energy conservation equation, Eq. E.24, has the form

$$\rho u \frac{\partial i}{\partial x} + \rho v \frac{\partial i}{\partial y} = \frac{\partial}{\partial x}\left[k \frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[k \frac{\partial T}{\partial y}\right] + \left[u \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y}\right] + \mu \Phi + \dot{q}.$$

1a 1b 2a 2b 3 4 5

The terms, as identified, have the following phnysical significance:

1. Change of enthalpy (thermal + flow work) advected in x and y directions, <
2. Change of conduction flux in x and y directions,
3. Work done by static pressure forces,
4. Word done by viscous stresses,
5. Rate of energy generation.

The species mass conservation equation for a constant total concentration has the form

$$u \frac{\partial C_A}{\partial x} + v \frac{\partial C_A}{\partial y} = \frac{\partial}{\partial x}\left[D_{AB} \frac{\partial C_A}{\partial x}\right] + \frac{\partial}{\partial y}\left[D_{AB} \frac{\partial C_A}{\partial y}\right] + \dot{N}_A$$

1a 1b 2a 2b 3

1. Change in species transport due to advection in x and y directions, <
2. Change in species transport by diffusion in x and y directions, and
3. Rate of species generation.

(b) The special conditions used to reduce the above equations to the boundary layer equations are: *constant properties, incompressible flow, non-reacting species* $\left(\dot{N}_A = 0\right)$, *without internal heat generation* $\left(\dot{q} = 0\right)$, *species diffusion has negligible effect on the thermal boundary layer,* $u(\partial p/\partial x)$ *is negligible.* The approximations are,

Velocity boundary layer $\left\{ u >> v \right.$ $\quad \frac{\partial u}{\partial y} >> \frac{\partial u}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial v}{\partial x}$

Thermal b.1.: $\left\{ \frac{\partial T}{\partial y} >> \frac{\partial T}{\partial x} \right.$ Concentration b.1.: $\left\{ \frac{\partial C_A}{\partial y} >> \frac{\partial C_A}{\partial x} \right.$.

The resulting simplified boundary layer equations are

$$\underbrace{u \frac{\partial T}{\partial x}}_{1a} + \underbrace{v \frac{\partial T}{\partial y}}_{1b} = \underbrace{\alpha \frac{\partial^2 T}{\partial y^2}}_{2a} + \underbrace{\frac{\nu}{c}\left[\frac{\partial u}{\partial y}\right]^2}_{3} \qquad \underbrace{u \frac{\partial C_A}{\partial x}}_{1c} + \underbrace{v \frac{\partial C_A}{\partial y}}_{1d} = \underbrace{D_{AB} \frac{\partial^2 C_A}{\partial y^2}}_{2b}$$ <

where the terms are: 1. Advective transport, 2. Diffusion, and 3. Viscous dissipation.

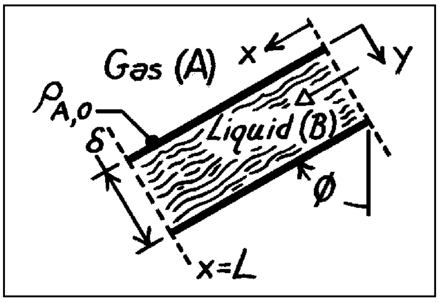
(c) When viscous dissipation effects are negligible, the two boundary layer equations have identical form. If the boundary conditions for each equation are of the same form, an analogy between heat and mass (species) transfer exists.

PROBLEM 6.53

KNOWN: Thickness and inclination of a liquid film. Mass density of gas in solution at free surface of liquid.

FIND: (a) Liquid momentum equation and velocity distribution for the x-direction. Maximum velocity, (b) Continuity equation and density distribution of the gas in the liquid, (c) Expression for the local Sherwood number, (d) Total gas absorption rate for the film, (e) Mass rate of NH_3 removal by a water film for prescribed conditions.

SCHEMATIC:



NH_3 (A) – Water (B)
L = 2m
δ = 1 mm
D = 0.05m
W = πD = 0.157m
$\rho_{A,o}$ = 25 kg/m^3
$D_{AB} = 2 \times 10^{-9}$ m^2/s
$\phi = 0°$

ASSUMPTIONS: (1) Steady-state conditions, (2) The film is in fully developed, laminar flow, (3) Negligible shear stress at the liquid-gas interface, (4) Constant properties, (5) Negligible gas concentration at x = 0 and y = δ, (6) No chemical reactions in the liquid, (7) Total mass density is constant, (8) Liquid may be approximated as semi-infinite to gas transport.

PROPERTIES: *Table A-6*, Water, liquid (300K): $\rho_f = 1/v_f = 997$ kg/m^3, $\mu = 855 \times 10^{-6}$ N·s/m^2, $\nu = \mu/\rho_f = 0.855 \times 10^{-6}$ m^2/s.

ANALYSIS: (a) For fully developed flow (v = w = 0, $\partial u/\partial x = 0$), the x-momentum equation is

$$0 = \partial\tau_{yx}/\partial y + X \quad \text{where} \quad \tau_{yx} = \mu(\partial u/\partial y) \quad \text{and} \quad X = (\rho g)\cos\phi.$$

That is, the momentum equation reduces to a balance between gravitational and shear forces. Hence,

$$\mu\left(\partial^2 u/\partial y^2\right) = -(\rho g)\cos\phi.$$

Integrating, $\quad \partial u/\partial y = -(g\cos\phi/\nu)y + C_1 \qquad u = -(g\cos\phi/2\nu)y^2 + C_1 y + C_2.$

Applying the boundary conditions,

$$\partial u/\partial y)_{y=0} = 0 \qquad \rightarrow \qquad C_1 = 0$$

$$u(\delta) = 0 \qquad \rightarrow \qquad C_2 = g\cos\phi\,\frac{\delta^2}{2\nu}.$$

Hence, $$u = \frac{g\cos\phi}{2\nu}\left(\delta^2 - y^2\right) = \frac{g\cos\phi\,\delta^2}{2\nu}\left[1-(y/\delta)^2\right]$$ <

and the maximum velocity exists at y = 0,

$$u_{max} = u(0) = \left(g\cos\phi\,\delta^2\right)/2\nu.$$ <

(b) Species transport within the liquid is influenced by diffusion in the y-direction and convection in the x-direction. Hence, the species continuity equation with u assumed equal to u_{max} throughout the region of gas penetration is

Continued …..

PROBLEM 6.53 (Cont.)

$$u\frac{\partial \rho_A}{\partial x} = D_{AB}\frac{\partial^2 \rho_A}{\partial y^2} \qquad \frac{\partial^2 \rho_A}{\partial y^2} = \frac{u_{max}}{D_{AB}}\frac{\partial \rho_A}{\partial x}.$$

Appropriate boundary conditions are: $\rho_A(x,0) = \rho_{A,o}$ and $\rho_A(x,\infty) = 0$ and the entrance condition is: $\rho_A(0,y) = 0$. The problem is therefore analogous to transient conduction in a semi-infinite medium due to a sudden change in surface temperature. From Section 5.7, the solution is then

$$\frac{\rho_A - \rho_{A,o}}{0-\rho_{A,o}} = \text{erf}\frac{y}{2\left(D_{AB}x/u_{max}\right)^{1/2}} \qquad \rho_A = \rho_{A,o}\text{erfc}\frac{y}{2\left(D_{AB}x/u_{max}\right)^{1/2}} \quad <$$

(c) The Sherwood number is defined as

$$Sh_x = \frac{h_{m,x}x}{D_{AB}} \quad \text{where} \quad h_{m,x} \equiv \frac{n''_{A,x}}{\rho_{A,o}} = \frac{-D_{AB}\partial\rho_A/\partial y)_{y=0}}{\rho_{A,o}}$$

$$\left.\frac{\partial \rho_A}{\partial y}\right|_{y=0} = -\rho_{A,o}\frac{2}{(\pi)^{1/2}}\exp\left[-\frac{y^2 u_{max}}{4D_{AB}x}\right]\left.\frac{1}{2\left(D_{AB}x/u_{max}\right)^{1/2}}\right|_{y=0} = -\rho_{A,o}\left[\frac{u_{max}}{\pi D_{AB}x}\right]^{1/2}.$$

Hence,

$$h_{m,x} = \left[\frac{u_{max}D_{AB}}{\pi x}\right]^{1/2} \qquad Sh_x = \frac{1}{(\pi)^{1/2}}\left[\frac{u_{max}x}{D_{AB}}\right]^{1/2} = \frac{1}{(\pi)^{1/2}}\left[\frac{u_{max}x}{\nu}\right]^{1/2}\left[\frac{\nu}{D_{AB}}\right]^{1/2}$$

and with $Re_x \equiv u_{max}\,x/\nu$,

$$Sh_x = \left[1/(\pi)^{1/2}\right] Re_x^{1/2}\ Sc^{1/2} = 0.564\ Re_x^{1/2}\ Sc^{1/2}. \quad <$$

(d) The total gas absorption rate may be expressed as

$$n_A = \bar{h}_{m,x}\left(W\cdot L\right)\rho_{A,o}$$

where the average mass transfer convection coefficient is

$$\bar{h}_{m,x} = \frac{1}{L}\int_0^L h_{m,x}dx = \frac{1}{L}\left[\frac{u_{max}D_{AB}}{\pi}\right]^{1/2}\int_0^L\frac{dx}{x^{1/2}} = \left[\frac{4u_{max}D_{AB}}{\pi L}\right]^{1/2}.$$

Hence, the absorption rate per unit width is

$$n_A/W = \left(4u_{max}\ D_{AB}L/\pi\right)^{1/2}\rho_{A,o}. \quad <$$

(e) From the foregoing results, it follows that the ammonia absorption rate is

$$n_A = \left[\frac{4u_{max}D_{AB}L}{\pi}\right]^{1/2} W\rho_{A,o} = \left[\frac{4\,g\cos\phi\,\delta^2 D_{AB}L}{2\pi\nu}\right]^{1/2} W\rho_{A,o}.$$

Substituting numerical values,

$$n_A = \left[\frac{4\times 9.8\text{ m/s}^2\times 1\times\left(10^{-3}\text{m}\right)^2\ \left(2\times 10^{-9}\text{m}^2/\text{s}\right)2\text{m}}{2\pi\times 0.855\times 10^{-6}\text{m}^2/\text{s}}\right]^{1/2}(0.157\text{m})25\text{ kg/m}^3 = 6.71\times 10^{-4}\text{kg/s}. \quad <$$

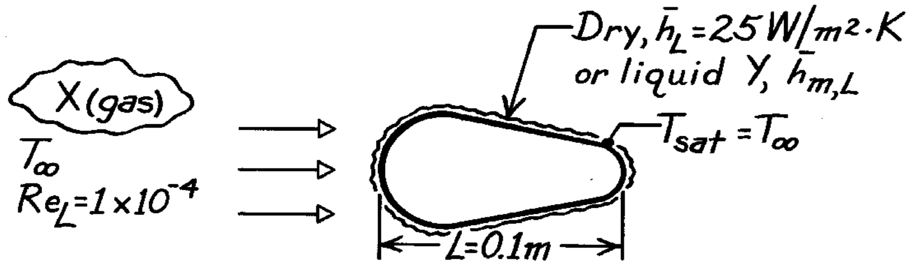
COMMENTS: Note that $\rho_{A,o} \neq \rho_{A,\infty}$, where $\rho_{A,\infty}$ is the mass density of the gas phase. The value of $\rho_{A,o}$ depends upon the pressure of the gas and the solubility of the gas in the liquid.

PROBLEM 6.54

KNOWN: Cross flow of gas X over object with prescribed characteristic length L, Reynolds number, and average heat transfer coefficient. Thermophysical properties of gas X, liquid Y, and vapor Y.

FIND: Average mass transfer coefficient for same object when impregnated with liquid Y and subjected to same flow conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Heat and mass transfer analogy is applicable, (2) Vapor Y behaves as perfect gas

PROPERTIES:

(Given)	$\nu(m^2/s)$	k(W/m·K)	$\alpha(m^2/s)$
Gas X	21×10^{-6}	0.030	29×10^{-6}
Liquid Y	3.75×10^{-7}	0.665	1.65×10^{-7}
Vapor Y	4.25×10^{-5}	0.023	4.55×10^{-5}

Mixture of gas X - vapor Y: Sc = 0.72

ANALYSIS: The heat-mass transfer analogy may be written as

$$\overline{Nu}_L = \frac{\bar{h}_L L}{k} = f\left(Re_L,\ Pr\right) \qquad \overline{Sh}_L = \frac{\bar{h}_{m,L} L}{D_{AB}} = f\left(Re_L, Sc\right)$$

The flow conditions are the same for both situations. Check values of Pr and Sc. For Pr, the properties are those for gas X (B).

$$Pr = \frac{\nu_B}{\alpha_B} = \frac{21\times10^{-6} m^2/s}{29\times10^{-6} m^2/s} = 0.72$$

while Sc = 0.72 for the gas X (B) - vapor Y (A) mixture. It follows for this situation

$$\overline{Nu}_L = \frac{\bar{h}_L L}{k} = \overline{Sh}_L = \frac{\bar{h}_{m,L} L}{D_{AB}} \qquad \text{or} \qquad \bar{h}_{m,L} = \bar{h}_L \frac{D_{AB}}{k}.$$

Recognizing that

$$D_{AB} = \nu_B / Sc = 21.6\times10^{-6} m^2/s \big/ (0.72) = 30.0\times10^{-6} m^2/s$$

and substituting numerical values, find

$$\bar{h}_{m,L} = 25\ W/m^2\cdot K \times \frac{30.0\times10^{-6} m^2/s}{0.030\ W/m\cdot K} = 0.0250\ m/s. \qquad <$$

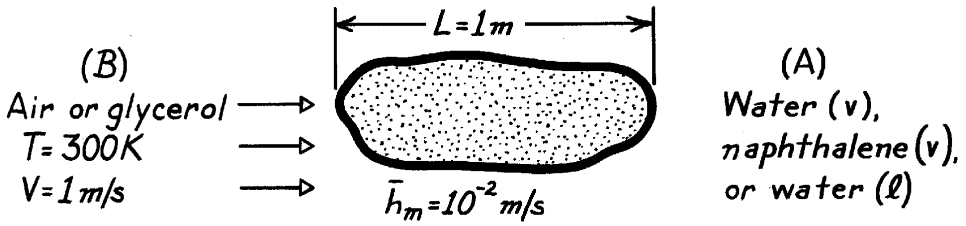
COMMENTS: Note that none of the thermophysical properties of liquid or vapor Y are required for the solution. Only the gas X properties and the Schmidt number (gas X - vapor Y) are required.

PROBLEM 6.55

KNOWN: Free stream velocity and average convection mass transfer coefficient for fluid flow over a surface of prescribed characteristic length.

FIND: Values of $\overline{Sh}_L$, Re_L, Sc and $\bar{j}_m$ for (a) air flow over water, (b) air flow over naphthalene, and (c) warm glycerol over ice.

SCHEMATIC:



PROPERTIES: For the fluids at 300K:

Table	*Fluid(s)*	$\nu(m^2/s)\times10^{-6}$	$D_{AB}(m^2/s)$
A-4	Air	15.89	-
A-5	Glycerin	634	-
A-8	Water vapor - Air	-	0.26×10^{-4}
A-8	Naphthalene - Air	-	0.62×10^{-5}
A-8	Water - Glycerol	-	0.94×10^{-9}

ANALYSIS: (a) *Water (vapor) - Air:*

$$\overline{Sh}_L=\frac{\bar{h}_m L}{D_{AB}}=\frac{(0.01\text{m/s})1\text{m}}{0.26\times10^{-4}\text{m}^2/\text{s}}=385$$

$$Re_L=\frac{VL}{\nu}=\frac{(1\text{ m/s})1\text{m}}{15.89\times10^{-6}\text{m}^2/\text{s}}=6.29\times10^4$$

$$Sc=\frac{\nu}{D_{AB}}=\frac{0.16\times10^{-6}\text{m}^2/\text{s}}{0.26\times10^{-6}\text{m}^2/\text{s}}=0.62$$

$$\bar{j}_m=St_m Sc^{2/3}=\frac{h_m}{V}Sc^{2/3}=\frac{0.01\text{ m/s}}{1\text{ m/s}}(0.62)^{2/3}=0.0073. \qquad <$$

(b) *Naphthalene (vapor) - Air:*

$$\overline{Sh}_L=1613 \qquad Re_L=6.29\times10^4 \qquad Sc=2.56 \qquad \bar{j}_m=0.0187. \qquad <$$

(c) *Water (1iquid) - Glycerol:*

$$\overline{Sh}_L=1.06\times10^7 \qquad Re_L=1577 \qquad Sc=6.74\times10^5 \qquad \bar{j}_m=76.9. \qquad <$$

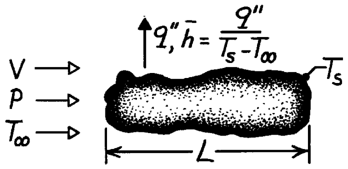
COMMENTS: Note the association of ν with the freestream fluid B.

PROBLEM 6.56

KNOWN: Characteristic length, surface temperature, average heat flux and airstream conditions associated with an object of irregular shape.

FIND: Whether similar behavior exists for alternative conditions, and average convection coefficient for similar cases.

SCHEMATIC:



Case:	1	2	3	4	5
L,m	1	2	2	2	2
V, m/s	100	50	50	50	250
p, atm	1	1	0.2	1	0.2
T_∞, K	275	275	275	300	300
Ts, K	325	325	325	300	300
q'', W/m^2	12,000	-	-	-	-
$\bar{h}$, W/m$^2\cdot$K	240	-	-	-	-
$D_{AB}\times 10^{+4}$, m^2/s	-	-	-	1.12	1.12

ASSUMPTIONS: (1) Heat and mass transfer analogy is applicable; that is, $f(Re_L,Pr) = f(Re_L,Sc)$, see Eqs. 6.57 and 6.61.

PROPERTIES: *Table A-4*, Air (300K, 1 atm): $\nu_1 = 15.89\times10^{-6}\,m^2/s$, $Pr_1 = 0.71$, $k_1 = 0.0263\ W/m\cdot K$.

ANALYSIS: $Re_{L,1} = V_1L_1/\nu_1 = (100\ m/s\times 1m)/15.89\times10^{-6}\,m^2/s = 6.29\times10^6$ and $Pr_1 = 0.71$.

Case 2: $$Re_{L,2} = \frac{V_2L_2}{\nu_2} = \frac{50\ m/s\times 2m}{15.89\times10^{-6}m^2/s} = 6.29\times10^6, \qquad Pr_2 = 0.71.$$

From Eq. 6.57 it follows that Case 2 is analogous to Case 1. Hence $\overline{Nu}_2 = \overline{Nu}_1$ and

$$\bar{h}_2 = \frac{\bar{h}_1L_1}{k_1}\frac{k_2}{L_2} = \bar{h}_1\frac{L_1}{L_2} = 240\frac{W}{m^2\cdot K}\frac{1m}{2m} = 120\ W/m^2\cdot K.$$ <

Case 3: With p = 0.2 atm, $\nu_3 = 79.45\times10^{-6}\,m^2/s$ and $Re_{L,3} = \frac{V_3L_3}{\nu_3} = \frac{50\ m/s\times 2m}{79.45\times10^{-6}m^2/s} = 1.26\times10^6$, $Pr_3 = 0.71$.

Since $Re_{L,3} \neq Re_{L,1}$, Case 3 is not analogous to Case 1. <

Case 4: $$Re_{L,4} = Re_{L,1},\ Sc_4 = \frac{\nu_4}{D_{AB,4}} = \frac{15.89\times10^{-6}m^2/s}{1.12\times10^{-4}m^2/s} = 0.142 \neq Pr_1.$$

Hence, Case 4 is not analogous to Case 1. <

Case 5: $$Re_{L,5} = \frac{V_5L_5}{\nu_5} = \frac{250\ m/s\ \times\ 2m}{79.45\times10^{-6}m^2/s} = 6.29\times10^6 = Re_{L,1}$$

$$Sc_5 = \frac{\nu_5}{D_{AB,5}} = \frac{79.45\times10^{-6}m^2/s}{1.12\times10^{-4}m^2/s} = 0.71 = Pr_1.$$

Hence, conditions are analogous to Case 1, and with $\overline{Sh}_5 = \overline{Nu}_1$,

$$h_{m,5} = h_1\frac{L_1}{L_5}\frac{D_{AB,5}}{k_1} = 240\frac{W}{m^2\cdot K}\times\frac{1m}{2m}\times\frac{1.12\times10^{-4}m^2/s}{0.0263\ W/m\cdot K} = 0.51\ m/s.$$ <

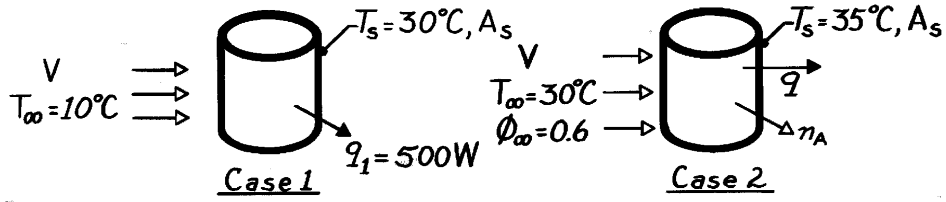
COMMENTS: Note that Pr, k and Sc are independent of pressure, while ν and D_{AB} vary inversely with pressure.

PROBLEM 6.57

KNOWN: Surface temperature and heat loss from a runner's body on a cool, spring day. Surface temperature and ambient air-conditions for a warm summer day.

FIND: (a) Water loss on summer day, (b) Total heat loss on summer day.

SCHEMATIC:



ASSUMPTIONS: (1) Heat and mass transfer analogy is applicable. Hence, from Eqs. 6.57 and 6.61, $f(Re_L,Pr)$ is of same form as $f(Re_L,Sc)$, (2) Negligible surface evaporation for Case 1, (3) Constant properties, (4) Water vapor is saturated for Case 2 surface and may be approximated as a perfect gas.

PROPERTIES: Air (given): $\nu = 1.6\times10^{-5}$ m^2/s, k = 0.026 W/m·K, Pr = 0.70; Water vapor - air (given): $D_{AB} = 2.3\times10^{-5}$m^2/s; *Table A-6,* Saturated water vapor (T_∞ = 303K): $\rho_{A,sat} = v_g^{-1} = 0.030$ kg/m^3; $(T_s = 308K)$: $\rho_{A,sat} = v_g^{-1} = 0.039$ kg/m^3, $h_{fg} = 2419$ kJ/kg.

ANALYSIS: (a) With $Re_{L,2} = Re_{L,1}$ and $Sc=\nu/D_{AB} = 1.6\times10^{-5}m^2/s/2.3\times10^{-5}m^2/s=0.70=Pr$, it follows that $\overline{Sh}_L = \overline{Nu}_L$. Hence

$$\bar{h}_m L/D_{AB} = \bar{h}L/k$$

$$\bar{h}_m = \bar{h}\frac{D_{AB}}{k} = \frac{q_1}{A_s(T_s - T_\infty)_1}\;\frac{D_{AB}}{k} = \frac{500\text{ W}}{A_s(20K)}\;\frac{2.3\times10^{-5}m^2/s}{0.026\text{ W/m}\cdot K} = \left[\frac{0.0221}{A_s}\right]\text{m/s.}$$

Hence, from the rate equation, with A_s as the wetted surface

$$n_A = \bar{h}_m A_s(\rho_{A,s} - \rho_{A,\infty}) = \left[\frac{0.0221}{A_s}\right]\frac{m}{s}A_s\left[\rho_{A,sat}(T_{s,2}) - \phi_\infty\rho_{A,sat}(T_{\infty,2})\right]$$

$$n_A = 0.0221\, m^3/s(0.039 - 0.6\times0.030)kg/m^3 = 4.64\times10^{-4}kg/s. \qquad <$$

(b) The total heat loss for Case 2 is comprised of sensible and latent contributions, where

$$q_2 = q_{sen} + q_{lat} = \bar{h}A_s(T_{s,2} - T_{\infty,2}) + n_A\, h_{fg}.$$

Hence, with $\bar{h}A_s = q_1/(T_{s,1} - T_{\infty,1}) = 25$ W/K,

$$q_2 = 25\text{ W/K }(35-30)^\circ C + 4.64\times10^{-4}kg/s\times2.419\times10^6\text{ J/kg}$$

$$q_2 = 125\text{ W} + 1122\text{ W} = 1247\text{ W.} \qquad <$$

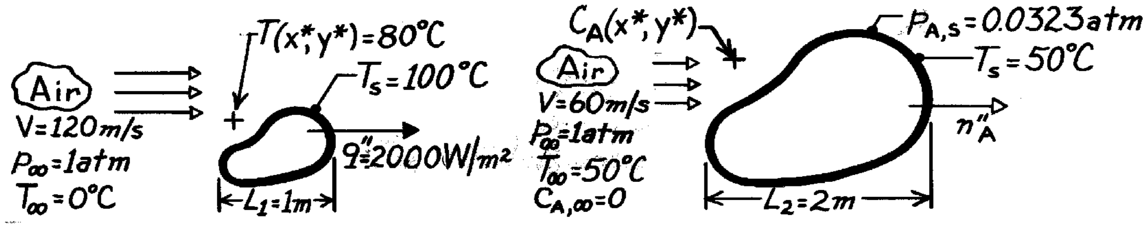
COMMENTS: Note the significance of the evaporative cooling effect.

PROBLEM 6.58

KNOWN: Heat transfer results for an irregularly shaped object.

FIND: (a) The concentration, C_A, and partial pressure, p_A, of vapor in an airstream for a drying process of an object of similar shape, (b) Average mass transfer flux, $n''_A\left(\text{kg/s}\cdot\text{m}^2\right)$.

SCHEMATIC:



Case 1: Heat Transfer Case 2: Mass Transfer

ASSUMPTIONS: (1) Heat-mass transfer analogy applies, (b) Perfect gas behavior.

PROPERTIES: *Table A-4*, Air (323K, 1 atm): $\nu = 18.20\times10^{-6}\,\text{m}^2\text{/s}$, Pr = 0.703, k = 28.0×10^{-3} W/m·K; Plastic vapor (given): $\mathcal{M}_A = 82$ kg/kmol, $p_{sat}(50°\text{C}) = 0.0323$ atm, $D_{AB} = 2.6\times^{-5}\,\text{m}^2\text{/s}$.

ANALYSIS: (a) Calculate Reynolds numbers

$$Re_1 = \frac{V_1 L_1}{\nu} = \frac{120\text{ m/s}\times 1\text{m}}{18.2\times10^{-6}\text{m}^2\text{/s}} = 6.59\times10^6$$

$$Re_2 = \frac{60\text{ m/s}\times 2\text{m}}{18.2\times10^{-6}\text{m}^2\text{/s}} = 6.59\times10^6.$$

Note that

$$Pr_1 = 0.703 \qquad Sc_2 = \frac{\nu}{D_{AB}} = \frac{18.2\times10^{-6}\text{m}^2\text{/s}}{2.6\times10^{-5}\text{m}^2\text{/s}} = 0.700.$$

Since $Re_1 = Re_2$ and $Pr_1 = Sc_2$, the dimensionless solutions to the energy and species equations are identical. That is, from Eqs. 6.54 and 6.58,

$$T^*\left(x^*,y^*\right) = C_A^*\left(x^*,y^*\right)$$

$$\frac{T-T_s}{T_\infty - T_s} = \frac{C_A - C_{A,s}}{C_{A,\infty}-C_{A,s}} \tag{1}$$

where T^* and C_A^* are defined by Eqs. 6.37 and 6.38, respectively. Now, determine

$$C_{A,s} = \frac{p_{A,sat}}{\Re T} = \left(0.0323\text{ atm}/8.205\times10^{-2}\text{m}^3\cdot\text{atm/kmol}\cdot\text{K}\times\left(273+50\right)\text{K}\right)$$

$$C_{A,s} = 1.219\times10^{-3}\text{ kmol/kg.}$$

Continued …..

Substituting numerical values in Eq. (1),

$$C_A = C_{A,s} + \left(C_{A,\infty} - C_{A,s}\right)\frac{T - T_s}{T_\infty - T_s}$$

$$C_A = 1.219\times10^{-3}\ \text{kmol/m}^3 + \left(0 - 1.219\times10^{-3}\right)\ \text{kmol/m}^3\,\frac{(80-100)^\circ\text{C}}{(0-100)^\circ\text{C}}$$

$$C_A = 0.975\times10^{-3}\ \text{kmol/m}^3. \qquad <$$

The vapor pressure is then

$$p_A = C_A\,\Re T\ =\ 0.0258\ \text{atm}. \qquad <$$

(b) For case 1, $q'' = 2000\ \text{W/m}^2$. The rate equations are

$$q'' = \bar{h}\left(T_s - T_\infty\right) \tag{2}$$

$$n''_A = \bar{h}_m\left(C_{A,s} - C_{A,\infty}\right)\mathcal{M}_A. \tag{3}$$

From the analogy

$$\overline{Nu}_L = \overline{Sh}_L \qquad \rightarrow \qquad \frac{\bar{h}\,L_1}{k} = \frac{\bar{h}_m\,L_2}{D_{AB}} \quad \text{or} \quad \frac{\bar{h}}{\bar{h}_m} = \frac{L_2}{L_1}\,\frac{k}{D_{AB}}. \tag{4}$$

Combining Eqs. (2) - (4),

$$n''_A = q''\frac{\bar{h}_m}{\bar{h}}\,\frac{\left(C_{A,s} - C_{A,\infty}\right)\mathcal{M}_A}{\left(T_s - T_\infty\right)} = q''\frac{L_1 D_{AB}}{L_2 k}\,\frac{\left(C_{A,s} - C_{A,\infty}\right)\mathcal{M}_A}{\left(T_s - T_\infty\right)}$$

which numerically gives

$$n''_A = 2000\ \text{W/m}^2\ \frac{1\text{m}\left(2.6\times10^{-5}\text{m}^2/\text{s}\right)}{2\text{m}\left(28\times10^{-3}\text{W/m}\cdot\text{K}\right)}\ \frac{\left(1.219\times10^{-3} - 0\right)\text{kmol/m}^3\ \left(82\ \text{kg/kmol}\right)}{(100\text{-}0)\ \text{K}}$$

$$n''_A = 9.28\times10^{-4}\,\text{kg/s}\cdot\text{m}^2. \qquad <$$

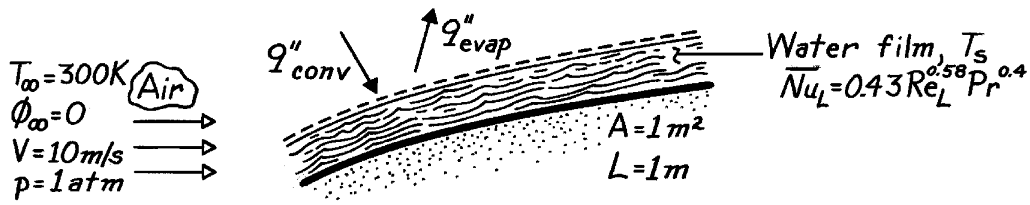
COMMENTS: Recognize that the analogy between heat and mass transfer applies when the conservation equations and boundary conditions are of the same form.

PROBLEM 6.59

KNOWN: Convection heat transfer correlation for flow over a contoured surface.

FIND: (a) Evaporation rate from a water film on the surface, (b) Steady-state film temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (b) Constant properties, (c) Negligible radiation, (d) Heat and mass transfer analogy is applicable.

PROPERTIES: *Table A-4*, Air (300K, 1 atm): $k = 0.0263$ W/m·K, $\nu = 15.89\times10^{-6}$ m²/s, Pr = 0.707; *Table A-6*, Water ($T_s \approx 280$K): $v_g = 130.4$ m³/kg, $h_{fg} = 2485$ kJ/kg; *Table A-8*, Water-air ($T \approx 298$K): $D_{AB} = 0.26\times10^{-4}$ m²/s.

ANALYSIS: (a) The mass evaporation rate is

$$\dot{m}_{evap} = n_A = \overline{h}_m\ A\left[\rho_{A,sat}(T_s) - \phi_\infty\ \rho_{A,sat}(T_\infty)\right] = \overline{h}_m\ A\ \rho_{A,sat}(T_s).$$

From the heat and mass transfer analogy: $\qquad \overline{Sh}_L = 0.43\ Re_L^{0.58}\ Sc^{0.4}$

$$Re_L = \frac{VL}{\nu} = \frac{(10\ m/s)\ 1m}{15.89\times10^{-6} m^2/s} = 6.29\times10^5 \qquad Sc = \frac{\nu}{D_{AB}} = \frac{15.89\times10^{-6} m^2/s}{26\times10^{-6} m^2/s} = 0.61$$

$$\overline{Sh}_L = 0.43\left(6.29\times10^5\right)^{0.58}(0.61)^{0.4} = 814$$

$$\overline{h}_m = \frac{D_{AB}}{L}\overline{Sh}_L = \frac{0.26\times10^{-4} m^2/s}{1m}(814) = 0.0212\ m/s$$

$$\rho_{A,sat}(T_s) = v_g(T_s)^{-1} = 0.0077\ kg/m^3.$$

Hence, $\qquad \dot{m}_{evap} = 0.0212 m/s \times 1m^2 \times 0.0077 kg/m^3 = 1.63\times10^{-4} kg/s.$ $\qquad$ <

(b) From a surface energy balance, $q''_{conv} = q''_{evap}$, or

$$\overline{h}_L(T_\infty - T_s) = \dot{m}''_{evap} h_{fg} \qquad T_s = T_\infty - \frac{\left(\dot{m}''_{evap} h_{fg}\right)}{\overline{h}_L}.$$

With $\qquad \overline{Nu}_L = 0.43\left(6.29\times10^5\right)^{0.58}(0.707)^{0.4} = 864$

$$\overline{h}_L = \frac{k}{L}\overline{Nu}_L = \frac{0.0263\ W/m\cdot K}{1m}864 = 22.7\ W/m^2\cdot K.$$

Hence, $\qquad T_s = 300K - \dfrac{1.63\times10^{-4} kg/s\cdot m^2\left(2.485\times10^6\ J/kg\right)}{22.7\ W/m^2\cdot K} = 282.2K.$ $\qquad$ <

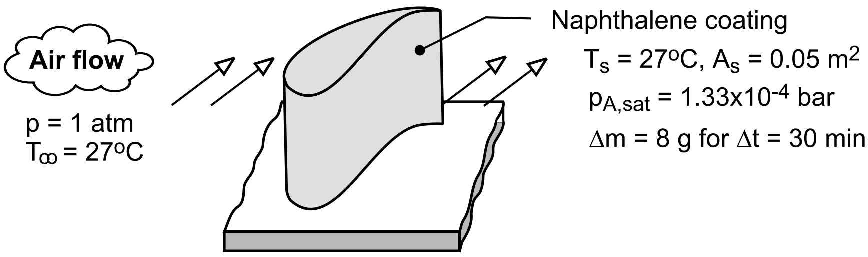
COMMENTS: The saturated vapor density, $\rho_{A,sat}$, is strongly temperature dependent, and if the initial guess of T_s needed for its evaluation differed from the above result by more than a few degrees, the density would have to be evaluated at the new temperature and the calculations repeated.

PROBLEM 6.60

KNOWN: Surface area and temperature of a coated turbine blade. Temperature and pressure of air flow over the blade. Molecular weight and saturation vapor pressure of the naphthalene coating. Duration of air flow and corresponding mass loss of naphthalene due to sublimation.

FIND: Average convection heat transfer coefficient.

SCHEMATIC:



ASSUMPTIONS: (1) Applicability of heat and mass transfer analogy, (2) Negligible change in A_s due to mass loss, (3) Naphthalene vapor behaves as an ideal gas, (4) Solid/vapor equilibrium at surface of coating, (5) Negligible vapor density in freestream of air flow.

PROPERTIES: *Table A-4*, Air (T = 300K): $\rho = 1.161$ kg/m^3, $c_p = 1007$ J/kg·K, $\alpha = 22.5 \times 10^{-6}$ m^2/s. *Table A-8*, Naphthalene vapor/air (T = 300K): $D_{AB} = 0.62 \times 10^{-5}$ m^2/s.

ANALYSIS: From the rate equation for convection mass transfer, the average convection mass transfer coefficient may be expressed as

$$\overline{h}_m = \frac{n_a}{A_s\left(\rho_{A,s} - \rho_{A,\infty}\right)} = \frac{\Delta m / \Delta t}{A_s \ \rho_{A,s}}$$

where

$$\rho_{A,s} = \rho_{A,sat}\left(T_s\right) = \frac{\mathcal{M}_a \, p_{A,sat}}{\Re T_s} = \frac{(128.16\,\text{kg/kmol})1.33\times10^{-4}\,\text{bar}}{0.08314\,\text{m}^3\cdot\text{bar/kmol}\cdot\text{K}\ (300\text{K})} = 6.83\times10^{-4}\,\text{kg/m}^3$$

Hence,

$$\overline{h}_m = \frac{0.008\,\text{kg}/\left(30\,\text{min}\times60\text{s/min}\right)}{0.05\text{m}^2\left(6.83\times10^{-4}\,\text{kg/m}^3\right)} = 0.13\,\text{m/s}$$

Using the heat and mass transfer analogy with n = 1/3, we then obtain

$$\overline{h} = \overline{h}_m \rho c_p Le^{2/3} = \overline{h}_m \rho c_p \left(\frac{\alpha}{D_{AB}}\right)^{2/3} = 0.130\,\text{m/s}\left(1.161\,\text{kg/m}^3\right)\times$$

$$1007\,\text{J/kg}\cdot\text{K}\left(22.5\times10^{-6}/0.62\times10^{-5}\right)^{2/3} = 359\,\text{W/m}^2\cdot\text{K}$$ <

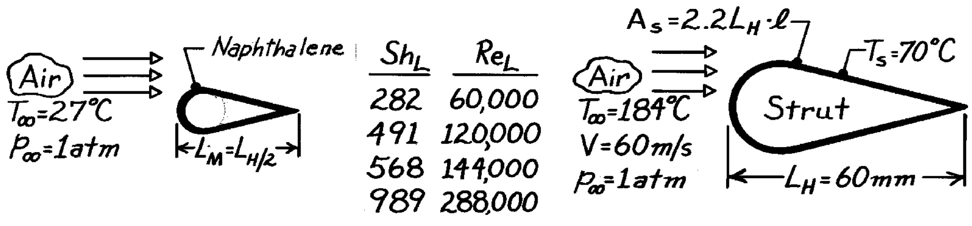
COMMENTS: The naphthalene sublimation technique has been used extensively to determine convection coefficients associated with complex flows and geometries.

PROBLEM 6.61

KNOWN: Mass transfer experimental results on a half-sized model representing an engine strut.

FIND: (a) The coefficients C and m of the correlation $\overline{\mathrm{Sh}}_L = C\mathrm{Re}_L^m \mathrm{Sc}^{1/3} \dfrac{n!}{r!(n-r)!}$ for the mass transfer results, (b) Average heat transfer coefficient, $\bar{h}$, for the full-sized strut with prescribed operating conditions, (c) Change in total heat rate if characteristic length L_H is doubled.

SCHEMATIC:



Mass transfer   Heat transfer

ASSUMPTIONS: Analogy exists between heat and mass transfer.

PROPERTIES: *Table A-4*, Air $\left(\bar{T} = (T_\infty + T_s)/2 = 400\mathrm{K},\ 1\ \mathrm{atm}\right)$: $\nu = 26.41\times10^{-6}\mathrm{m}^2/\mathrm{s}$, k = 0.0338 W/m·K, Pr = 0.690; $\left(\bar{T} = 300\mathrm{K}\right)$: $\nu_B = 15.89\times10^{-6}\ \mathrm{m}^2/\mathrm{s}$; *Table A-8,* Naphthalene-air (300K, 1 atm): $D_{AB} = 0.62\times10^{-5}\mathrm{m}^2/\mathrm{s}$, $\mathrm{Sc} = \nu_B/D_{AB} = 15.89\times10^{-6}\mathrm{m}^2/\mathrm{s}/0.62\times10^{-5}\mathrm{m}^2/\mathrm{s} = 2.56.$

ANALYSIS: (a) The correlation for the mass transfer experimental results is of the form $\overline{\mathrm{Sh}}_L = C\mathrm{Re}_L^m\ \mathrm{Sc}^{1/3}$. The constants C,m may be evaluated from two data sets of $\overline{\mathrm{Sh}}_L$ and Re_L; choosing the middle sets (2,3):

$$\frac{\left(\overline{\mathrm{Sh}}_L\right)_2}{\left(\overline{\mathrm{Sh}}_L\right)_3} = \frac{\left(\mathrm{Re}_L\right)_2^m}{\left(\mathrm{Re}_L\right)_3^m} \quad \text{or } m = \frac{\log\left[\mathrm{Sh}_L)_2/\mathrm{Sh}_L)_3\right]}{\log\left[\mathrm{Re}_L)_2/\mathrm{Re}_L)_3\right]} = \frac{\log\left[491/568\right]}{\log\left[120{,}000/144{,}000\right]} = 0.80. \qquad <$$

Then, using set 2, find $C = \dfrac{\overline{\mathrm{Sh}}_L)_2}{\mathrm{Re}_L^m)_2\ \mathrm{Sc}^{1/3}} = \dfrac{491}{(120{,}000)^{0.8}\ 2.56^{1/3}} = 0.031.$ $<$

(b) For the heat transfer analysis of the strut, the correlation will be of the form $\overline{\mathrm{Nu}}_L = \bar{h}_L \cdot L_H/k = 0.031\ \mathrm{Re}_L^{0.8}\ \mathrm{Pr}^{1/3}$ where $\mathrm{Re}_L = V L_H/\nu$ and the constants C,m were determined in Part (a). Substituting numerical values,

$$\bar{h}_L = \overline{\mathrm{Nu}}_L \cdot \frac{k}{L_H} = 0.031\left[\frac{60\ \mathrm{m/s}\times0.06\ \mathrm{m}}{26.41\times10^{-6}\mathrm{m}^2/\mathrm{s}}\right]^{0.8} 0.690^{1/3}\ \frac{0.0338\ \mathrm{W/m\cdot K}}{0.06\ \mathrm{m}} = 198\ \mathrm{W/m}^2\cdot\mathrm{K}. \qquad <$$

(c) The total heat rate for the strut of characteristic length L_H is $q = \bar{h}\ A_s\left(T_s - T_\infty\right)$, where A_s = 2.2 L_H·l and

$$\bar{h} \sim \overline{\mathrm{Nu}}_L \cdot L_H^{-1} \sim \mathrm{RE}_L^{0.8} \cdot L_H^{-1} \sim L_H^{0.8} \cdot L_H^{-1} \sim L_H^{-0.2} \qquad A_s \sim L_H$$

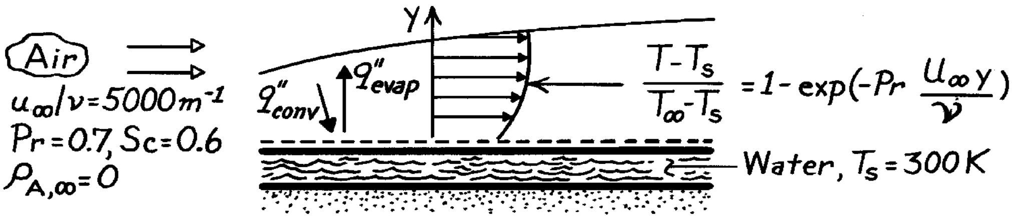
Hence, $q \sim \bar{h}\cdot A_s \sim \left(L_H^{-0.2}\right)\ \left(L_H\right) \sim L_H^{0.8}$. If the characteristic length were doubled, the heat rate would be increased by a factor of $(2)^{0.8}$ = 1.74. $<$

PROBLEM 6.62

KNOWN: Boundary layer temperature distribution for flow of dry air over water film.

FIND: Evaporative mass flux and whether net energy transfer is to or from the water.

SCHEMATIC:



ASSUMPTIONS: (1) Heat and mass transfer analogy is applicable, (2) Water is well insulated from below.

PROPERTIES: *Table A-4*, Air (T_s = 300K, 1 atm): k = 0.0263 W/m·K; *Table A-6*, Water vapor (T_s = 300K): $\rho_{A,s} = v_g^{-1} = 0.0256\ \text{kg/m}^3,\ h_{fg} = 2.438\times10^6\ \text{J/kg}$; *Table A-8*, Air-water vapor $(T_s = 300\text{K})$: $D_{AB} = 0.26\times10^{-4}\text{m}^2/\text{s}$.

ANALYSIS: From the heat and mass transfer analogy,

$$\frac{\rho_A - \rho_{A,s}}{\rho_{A,\infty} - \rho_{A,s}} = 1 - \exp\left[-Sc\frac{u_\infty y}{\nu}\right].$$

Using Fick's law at the surface (y = 0), the species flux is

$$n''_A = -D_{AB}\frac{\partial \rho_A}{\partial y}\bigg|_{y=0} = +\rho_{A,s}\ D_{AB}\ Sc\ \frac{u_\infty}{\nu}$$

$$n''_A = 0.0256\ \text{kg/m}^3 \times 0.26\times10^{-4}\text{m}^2/\text{s} \times (0.6)5000\ \text{m}^{-1} = 2.00\times10^{-3}\ \text{kg/s}\cdot\text{m}^2.$$

The net heat flux to the water has the form

$$q''_{net} = q''_{conv} - q''_{evap} = +k\ \frac{\partial T}{\partial y}\bigg|_{y=0} - n''_A\ h_{fg} = k(T_\infty - T_s)\ Pr\ \frac{u_\infty}{\nu} - n''_A\ h_{fg}$$

and substituting numerical values, find

$$q''_{net} = 0.0263\frac{\text{W}}{\text{m}\cdot\text{K}}(100\text{K})\ 0.7 \times 5000\ \text{m}^{-1} - 2\times10^{-3}\frac{\text{kg}}{\text{s}\cdot\text{m}^2}\times2.438\times10^6\ \text{J/kg}$$

$$q''_{net} = 9205\ \text{W/m}^2 - 4876\ \text{W/m}^2 = 4329\ \text{W/m}^2.$$

Since $q''_{net} > 0$, the net heat transfer is to the water. <

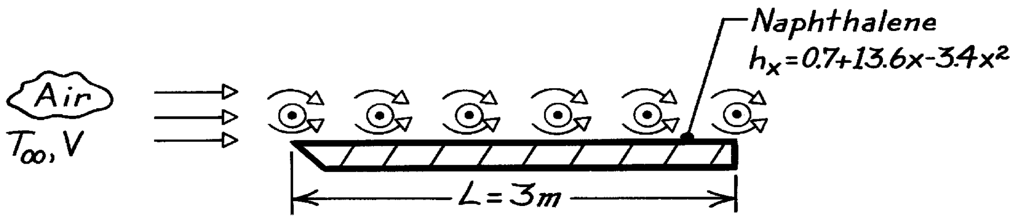
COMMENTS: Note use of properties (D_{AB} and k) evaluated at T_s to determine surface fluxes.

PROBLEM 6.63

KNOWN: Distribution of local convection heat transfer coefficient for obstructed flow over a flat plate with surface and air temperatures of 310K and 290K, respectively.

FIND: Average convection mass transfer coefficient.

SCHEMATIC:



ASSUMPTIONS: Heat and mass transfer analogy is applicable.

PROPERTIES: *Table A-4*, Air $\left(T_f = \left(T_s + T_\infty\right)/2 = \left(310 + 290\right)K/2 = 300\text{ K, 1 atm}\right)$: $k = 0.0263\text{ W/m}\cdot\text{K}$, $\nu = 15.89\times10^{-6}\text{m}^2/\text{s}$, $\text{Pr} = 0.707$. *Table A-8*, Air-napthalene (300K, 1 atm): $D_{AB} = 0.62\times10^{-5}\text{m}^2/\text{s}$, $\text{Sc} = \nu/D_{AB} = 2.56$.

ANALYSIS: The average heat transfer coefficient is

$$\bar{h}_L = \frac{1}{L}\int_0^L h_x dx = \frac{1}{L}\int_0^L \left(0.7 + 13.6x - 3.4x^2\right)dx$$

$$\bar{h}_L = 0.7 + 6.8L - 1.13L^2 = 10.9\text{ W/m}^2\cdot\text{K}.$$

Applying the heat and mass transfer analogy with n = 1/3, Equation 6.66 yields

$$\frac{\overline{Nu}_L}{Pr^{1/3}} = \frac{\overline{Sh}_L}{Sc^{1/3}}$$

Hence,

$$\frac{\bar{h}_{m,L}L}{D_{AB}} = \frac{\bar{h}_L L}{k}\frac{Sc^{1/3}}{Pr^{1/3}}$$

$$\bar{h}_{m,L} = \bar{h}_L\frac{D_{AB}}{k}\frac{Sc^{1/3}}{Pr^{1/3}} = 10.9\text{ W/m}^2\cdot\text{K}\frac{0.62\times10^{-5}\text{m}^2/\text{s}}{0.0263\text{ W/m}\cdot\text{K}}\left(\frac{2.56}{0.707}\right)^{1/3}$$

$$\bar{h}_{m,L} = 0.00395\text{ m/s}.$$ <

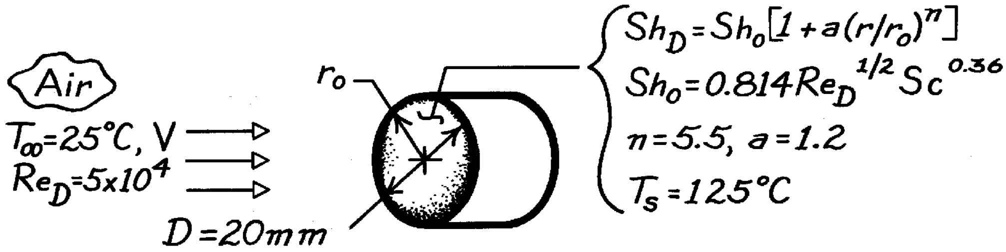
COMMENTS: The napthalene sublimation method provides a useful tool for determining local convection coefficients.

PROBLEM 6.64

KNOWN: Radial distribution of local Sherwood number for uniform flow normal to a circular disk.

FIND: (a) Expression for average Nusselt number. (b) Heat rate for prescribed conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Applicability of heat and mass transfer analogy.

PROPERTIES: *Table A-4*, Air $\left(\bar{T} = 75^\circ C = 348\ K\right)$: k = 0.0299 W/m·K, Pr = 0.70.

ANALYSIS: (a) From the heat and mass transfer analogy, Equation 6.66,

$$\frac{\overline{Nu}_D}{Pr^{0.36}} = \frac{\overline{Sh}_D}{Sc^{0.36}}$$

where

$$\overline{Sh}_D = \frac{1}{A_s}\int_{A_s} Sh_D(r)dA_s = \frac{Sh_o}{\pi\ r_o^2}2\pi\int_0^{r_o}\left[1+a\left(r/r_o\right)^n\right]rdr$$

$$\overline{Sh}_D = \frac{2Sh_o}{r_o^2}\left[\frac{r^2}{2}+\frac{ar^{n+2}}{(n+2)r_o^n}\right]_0^{r_o} = Sh_o\left[1+2a/(n+2)\right]$$

Hence,

$$\overline{Nu}_D = 0.814\left[1+2a/(n+2)\right]Re_D^{1/2}Pr^{0.36}.$$ <

(b) The heat rate for these conditions is

$$q = \bar{h}A\left(T_s - T_\infty\right) = 0.814\left[1+2a/(n+2)\right]\frac{k}{D}Re_D^{1/2}Pr^{0.36}\frac{\left(\pi\ D^2\right)}{4}\left(T_s - T_\infty\right)$$

$$q = 0.814(1+2.4/7.5)0.0299\ W/m\cdot K\left(\pi 0.02\ m/4\right)\left(5\times10^4\right)^{1/2}(0.07)^{0.36}\left(100^\circ C\right)$$

$$q = 9.92\ W.$$ <

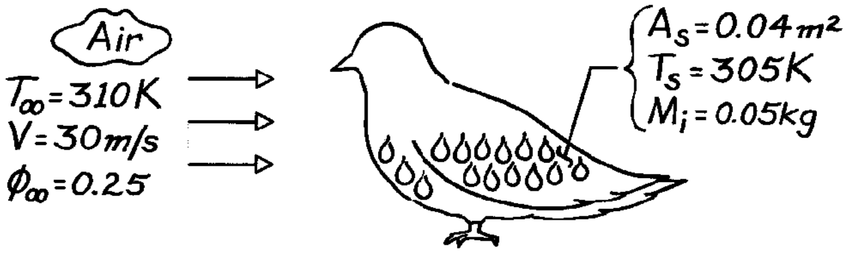
COMMENTS: The increase in h(r) with r may be explained in terms of the sharp turn which the boundary layer flow must make around the edge of the disk. The boundary layer accelerates and its thickness decreases as it makes the turn, causing the local convection coefficient to increase.

PROBLEM 6.65

KNOWN: Convection heat transfer correlation for wetted surface of a sand grouse. Initial water content of surface. Velocity of bird and ambient air conditions.

FIND: Flight distance for depletion of 50% of initial water content.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Vapor behaves as a perfect gas, (3) Constant properties, (4) Applicability of heat and mass transfer analogy.

PROPERTIES: Air (given): $\nu = 16.7\times10^{-6} m^2/s$; Air-water vapor (given): $D_{AB} = 0.26\times10^{-4} m^2/s$; *Table A-6*, Water vapor (T_s = 305 K): v_g = 29.74 m^3/kg; (T_s = 310 K), v_g = 22.93 m^3/kg.

ANALYSIS: The maximum flight distance is

$$X_{max} = Vt_{max}$$

where the time to deplete 50% of the initial water content ΔM is

$$t_{max} = \frac{\Delta M}{\dot{m}_{evap}} = \frac{\Delta M}{\overline{h}_m A_s \left(\rho_{A,s} - \rho_{A,\infty}\right)}.$$

The mass transfer coefficient is

$$\overline{h}_m = \overline{Sh}_L \frac{D_{AB}}{L} = 0.034 Re_L^{4/5} Sc^{1/3} \frac{D_{AB}}{L}$$

$$Sc = \nu/D_{AB} = 0.642, \qquad L = \left(A_s\right)^{1/2} = 0.2 \text{ m}$$

$$Re_L = \frac{VL}{\nu} = \frac{30 \text{ m/s}\times 0.2 \text{ m}}{16.7\times10^{-6} m^2/s} = 3.59\times10^5$$

$$\overline{h}_m = 0.034\left(3.59\times10^5\right)^{4/5}(0.642)^{1/3}\left(0.26\times10^{-4} m^2/s/0.2 \text{ m}\right) = 0.106 \text{ m/s}.$$

Hence,

$$t_{max} = \frac{0.025 \text{ kg}}{0.106 \text{ m/s}\left(0.04 \text{ m}^2\right)\left[(29.74)^{-1} - 0.25(22.93)^{-1}\right]\text{kg/m}^3} = 259 \text{ s}$$

$$X_{max} = 30 \text{ m/s}(259 \text{ s}) = 7785 \text{ m} = 7.78 \text{ km}.$$ <

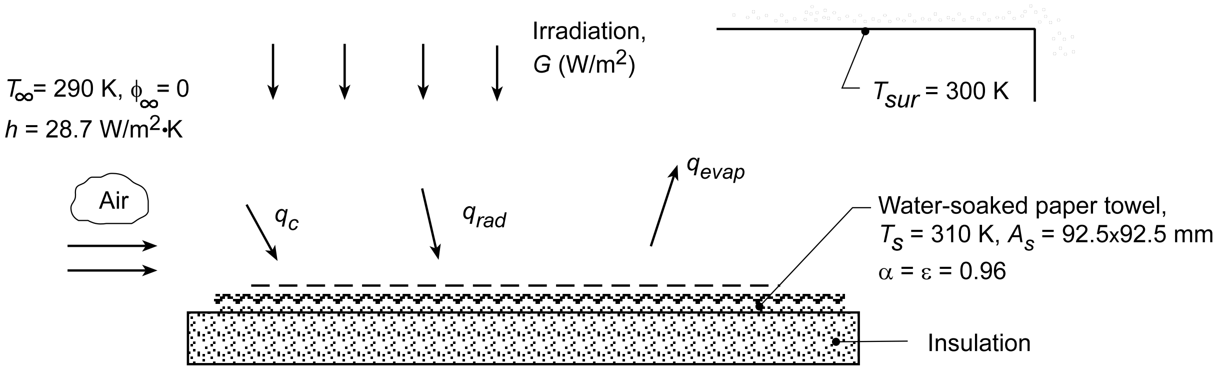
COMMENTS: Evaporative heat loss is balanced by convection heat transfer from air. Hence, $T_s < T_\infty$.

PROBLEM 6.66

KNOWN: Water-soaked paper towel experiences simultaneous heat and mass transfer while subjected to parallel flow of air, irradiation from a radiant lamp bank, and radiation exchange with surroundings. Average convection coefficient estimated as $\overline{h}$ = 28.7 W/m^2·K.

FIND: (a) Rate at which water evaporates from the towel, n_A (kg/s), and (b) The net rate of radiation transfer, q_{rad} (W), to the towel. Determine the irradiation G (W/m^2).

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Vapor behaves as an ideal gas, (3) Constant properties, (4) Towel experiences radiation exchange with the large surroundings as well as irradiation from the lamps, (5) Negligible heat transfer from the bottom side of the towel, and (6) Applicability of the heat-mass transfer analogy.

PROPERTIES: *Table A.4*, Air (T_f = 300 K): ρ = 1.1614 kg/m^3, c_p = 1007 J/kg·K, α = 22.5 × 10^{-6} m^2/s; *Table A.6*, Water (310 K): $\rho_{A,s} = \rho_g = 1/v_g$ = 1/22.93 = 0.0436 kg/m^3, h_{fg} = 2414 kJ/kg.

ANALYSIS: (a) The evaporation rate from the towel is

$$n_A = \overline{h}_m A_s \left(\rho_{A,s} - \rho_{A,\infty}\right)$$

where $\overline{h}_m$ can be determined from the heat-mass transfer analogy, Eq. 6.92, with n = 1/3,

$$\frac{h}{h_m} = \rho c_p Le^{2/3} = \rho c_p \left(\frac{\alpha}{D_{AB}}\right)^{2/3} = 1.614\,\text{kg}/\text{m}^3 \times 1007\,\text{J/kg}\cdot\text{K}\left(\frac{22.5\times10^{-6}}{0.26\times10^{-4}}\right)^{2/3} = 1062\,\text{J}/\text{m}^3\cdot\text{K}$$

$$h_m = 28.7\,\text{W}/\text{m}^2\cdot\text{K}/1062\,\text{J}/\text{m}^3\cdot\text{K} = 0.0270\,\text{m/s}$$

The evaporation rate is

$$n_A = 0.0270\,\text{m/s}\times(0.0925\times0.0925)\text{m}^2\,(0.0436-0)\,\text{kg}/\text{m}^3 = 1.00\times10^{-5}\,\text{kg/s} \qquad <$$

(b) Performing an energy balance on the towel considering processes of evaporation, convection and radiation, find

$$\dot{E}_{in} - \dot{E}_{out} = q_{conv} - q_{evap} + q_{rad} = 0$$

$$\overline{h}A_s\left(T_\infty - T_s\right) - n_A h_{fg} + q_{rad} = 0$$

$$q_{rad} = 1.00\times10^{-5}\,\text{kg/s}\times2414\times10^3\,\text{J/kg} - 27.8\,\text{W}/\text{m}^2\,(0.0925\,\text{m})^2\,(290-310)\text{K}$$

$$q_{rad} = 2414\,\text{W} + 4.76\,\text{W} = 28.9\,\text{W} \qquad <$$

Continued...

The net radiation heat transfer to the towel is comprised of the absorbed irradiation and the net exchange between the surroundings and the towel,

$$q_{rad} = \alpha G A_s + \varepsilon A_s \sigma \left(T_{sur}^4 - T_s^4\right)$$

$$28.9\,\text{W} = 0.96G\,(0.0925\,\text{m})^2 + 0.96 \times (0.0925\,\text{m})^2\, 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4 \left(300^4 - 310^4\right)\text{K}^4$$

Solving, find the irradiation from the lamps,

$$G = 3583\ \text{W/m}^2. \qquad <$$

COMMENTS: (1) From the energy balance in Part (b), note that the heat rate by convection is considerably smaller than that by evaporation.

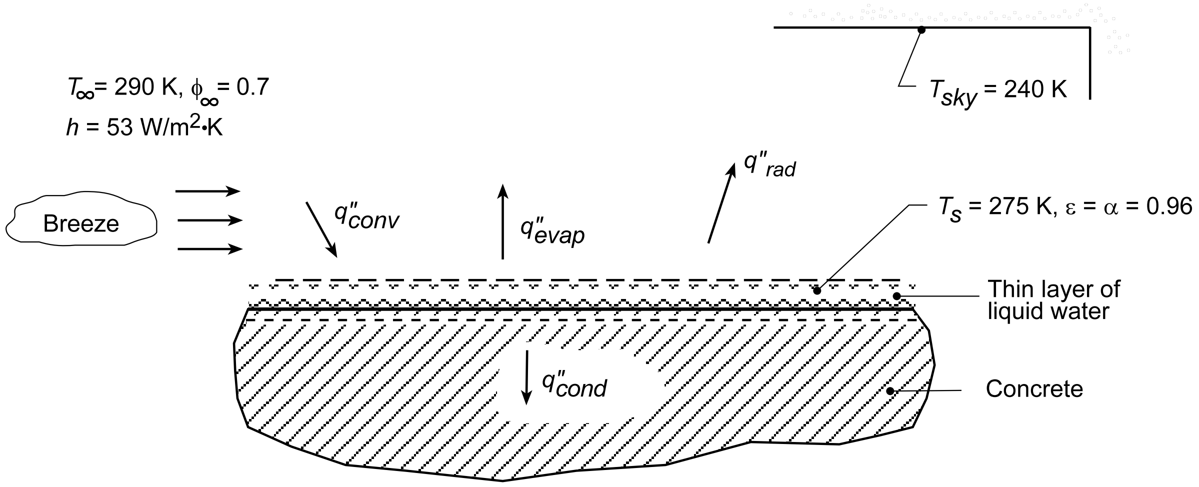
(2) As we'll learn in Chapter 12, the lamp irradiation found in Part (c) is nearly 3 times that of solar irradiation to the earth's surface.

PROBLEM 6.67

KNOWN: Thin layer of water on concrete surface experiences evaporation, convection with ambient air, and radiation exchange with the sky. Average convection coefficient estimated as $\bar{h}$ = 53 W/m^2·K.

FIND: (a) Heat fluxes associated with convection, q''_{conv}, evaporation, q''_{evap}, and radiation exchange with the sky, q''_{rad}, (b) Use results to explain why the concrete is wet instead of dry, and (c) Direction of heat flow and the heat flux by conduction into or out of the concrete.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Vapor behaves as an ideal gas, (3) Constant properties, (4) Water surface is small compare to large, isothermal surroundings (sky), and (4) Applicability of the heat-mass transfer analogy.

PROPERTIES: *Table A.4*, Air ($T_f = (T_\infty + T_s)/2$ = 282.5 K): ρ = 1.243 kg/m^3, c_p = 1007 J/kg·K, α = 2.019 ×10^5 m^2/s; *Table A.8*, Water-air (T_f = 282.5 K): $D_{AB} = 0.26 \times 10^{-4}$ m^2/s $(282.5/298)^{3/2} = 0.24 \times 10^{-4}$ m^2/s; *Table A.6*, Water (T_s = 275 K): $\rho_{A,s} = \rho_g = 1/v_g$ = 1/181.7 = 0.0055 kg/m^3, h_{fg} = 2497 kJ/kg; *Table A.6*, Water (T_∞ = 290 K): $\rho_{A,s}$ = 1/69.7 = 0.0143 kg/m^3.

ANALYSIS: (a) The heat fluxes associated with the processes shown on the schematic are

Convection:

$$q''_{conv} = \bar{h}\left(T_\infty - T_s\right) = 53\,\mathrm{W/m^2\cdot K}\left(290-275\right)\mathrm{K} = +795\,\mathrm{W/m^2} \qquad <$$

Radiation Exchange:

$$q''_{rad} = \varepsilon\sigma\left(T_s^4 - T_{sky}\right) = 0.96\times5.76\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\left(275^4-240^4\right)\mathrm{K^4} = +131\,\mathrm{W/m^2} \qquad <$$

Evaporation:

$$q''_{evap} = n''_A h_{fg} = -2.255\times10^{-4}\,\mathrm{kg/s\cdot m^2}\times2497\times10^3\,\mathrm{J/kg} = -563.1\,\mathrm{W/m^2} \qquad <$$

where the evaporation rate *from* the surface is

$$n''_A = \bar{h}_m\left(\rho_{A,s} - \rho_{A,\infty}\right) = 0.050\,\mathrm{m/s}\left(0.0055 - 0.7\times0.0143\right)\mathrm{kg/m^3} = -2.255\times10^{-4}\,\mathrm{kg/s\cdot m^2}$$

Continued...

and where the mass transfer coefficient is evaluated from the heat-mass transfer analogy, Eq. 6.92, with n = 1/3,

$$\frac{\overline{h}}{\overline{h}_m} = \rho c_p Le^{2/3} = \rho c_p \left(\frac{\alpha}{D_{AB}}\right)^{2/3} = 1.243\,kg/m^3 \times 1007\,J/kg\cdot K\left(\frac{2.019\times10^{-5}}{0.26\times10^{-4}}\right)^{2/3}$$

$$\frac{\overline{h}}{\overline{h}_m} = 1058\,J/m^3\cdot K$$

$$\overline{h}_m = 53\,W/m^2\cdot K/1058\,J/m^3\cdot K = 0.050\,m/s$$

(b) From the foregoing evaporation calculations, note that water vapor from the air is condensing on the liquid water layer. That is, vapor is being transported to the surface, explaining why the concrete surface is wet, even without rain.

(c) From an overall energy balance on the water film considering conduction in the concrete as shown in the schematic,

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q''_{conv} - q''_{evap} - q''_{rad} - q''_{cond} = 0$$

$$q''_{cond} = q''_{conv} - q''_{evap} - q''_{rad}$$

$$q''_{cond} = 1795\,W/m^3 - \left(-563.1\,W/m^2\right) - \left(+131\,W/m^2\right) = 1227\,W/m^2 \qquad <$$

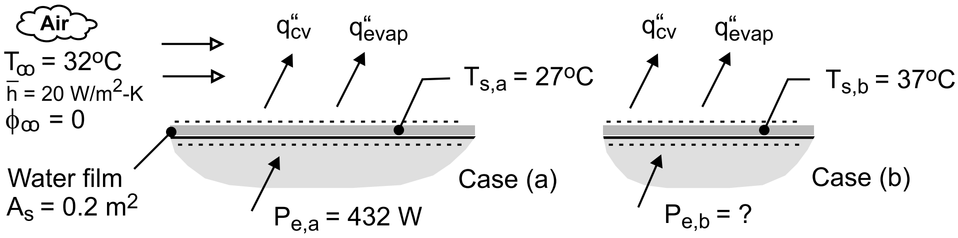
The heat flux by conduction is *into* the concrete.

PROBLEM 6.68

KNOWN: Heater power required to maintain wetted (water) plate at 27°C, and average convection coefficient for specified dry air temperature, case (a).

FIND: Heater power required to maintain the plate at 37°C for the same dry air temperature if the convection coefficients remain unchanged, case (b).

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Convection coefficients unchanged for different plate temperatures, (3) Air stream is dry at atmospheric pressure, and (4) Negligible heat transfer from the bottom side of the plate.

PROPERTIES: *Table A-6*, Water ($T_{s,a}$ = 27°C = 300 K): $\rho_{A,s} = 1/v_g = 0.02556\ \text{kg/m}^3$, $h_{fg} = 2.438 \times 10^6$ J/kg; Water ($T_{s,b}$ = 37°C = 310 K): $\rho_{A,s} = 1/v_g = 0.04361\ \text{kg/m}^3$, $h_{fg} = 2.414 \times 10^6$ J/kg.

ANALYSIS: For *case (a)* with T_s = 27°C and P_e = 432 W, perform an energy balance on the plate to determine the mass transfer coefficient h_m.

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$P_{e,a} - \left(q''_{evap} + q''_{cv}\right) A_s = 0$$

Substituting the rate equations and appropriate properties,

$$P_{e,a} - \left[\bar{h}_m\left(\rho_{A,s} - \rho_{A,\infty}\right)h_{fg} + \bar{h}\left(T_{s,a} - T_\infty\right)\right]A_s = 0$$

$$432\ \text{W} + \Big[\bar{h}_m\left(0.0256\ \text{kg/m}^3 - 0\right)\times 2.438\times 10^6\ \text{J/kg} +$$

$$20\ \text{W/m}^2\cdot\text{K}\left(27-32\right)\text{K}\Big]\times 0.2\ \text{m}^2 = 0$$

where $\rho_{A,s}$ and h_{fg} are evaluated at T_s = 27°C = 300 K. Find,

$$\bar{h}_m = 0.0363\ \text{m/s}$$

For case (b), with T_s = 37°C and the same values for $\bar{h}$ and $\bar{h}_m$, perform an energy balance to determine the heater power required to maintain this condition.

$$P_{e,b} - \left[\bar{h}_m\left(\rho_{A,s} - 0\right)h_{fg} + \bar{h}\left(T_{s,b} - T_\infty\right)\right]A_s = 0$$

$$P_{e,b} - \Big[0.0363\ \text{m/s}\left(0.04361 - 0\right)\text{kg/m}^3 \times 2.414\times 10^6\ \text{J/kg} +$$

$$20\ \text{W/m}^2\cdot\text{K}\left(37-32\right)\Big]\times 0.2\ \text{m}^2 = 0$$

$$P_{e,b} = 784\ \text{W}$$ <

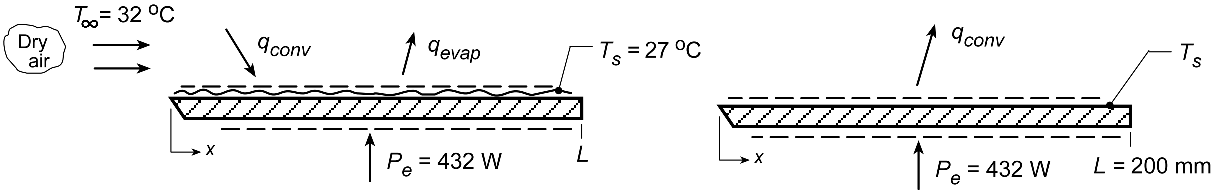
where $\rho_{A,a}$ and h_{fg} are evaluated at T_s = 37°C = 310 K.

PROBLEM 6.69

KNOWN: Dry air at 32°C flows over a wetted plate of width 1 m maintained at a surface temperature of 27°C by an embedded heater supplying 432 W.

FIND: (a) The evaporation rate of water from the plate, n_A (kg/h) and (b) The plate temperature T_s when all the water is evaporated, but the heater power remains the same.

SCHEMATIC:



(a) Wetted plate (a) Dry plate

ASSUMPTIONS: (1) Steady-state conditions, (2) Vapor behaves as an ideal gas, (3) Constant properties, and (4) Applicability of the heat-mass transfer analogy.

PROPERTIES: *Table A.4*, Air (T_f = (32 + 27)°C/2 = 302.5 K): ρ = 1.153 kg/m^3, c_p = 1007 J/kg·K, α = 2.287 $\times 10^5$ m^2/s; *Table A.8*, Water-air ($T_f \approx$ 300 K): D_{AB} = 0.26 $\times$ 10^{-4} m^2/s; *Table A.6*, Water (T_s = 27°C = 300 K): $\rho_{A,s} = 1/v_g$ = 1/39.13 = 0.0256 kg/m^3, h_{fg} = 2438 kJ/kg.

ANALYSIS: (a) Perform an energy balance on the wetted plate to obtain the evaporation rate, n_A.

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad\qquad P_e + q_{conv} - q_{evap} = 0$$

$$P_e + \overline{h}A_s\left(T_\infty - T_s\right) - n_A h_{fg} = 0 \tag{1}$$

In order to find $\overline{h}$, invoke the heat-mass transfer analogy, Eq. (6.92) with n = 1/3,

$$\frac{\overline{h}}{\overline{h}_m} = \rho c_p \mathrm{Le}^{2/3} = \rho c_p\left(\frac{\alpha}{D_{AB}}\right)^{2/3} = 1.153\,\mathrm{kg/m^3} \times 1007\,\mathrm{J/kg\cdot K}\left(\frac{2.287\times10^{-5}}{0.26\times10^{-4}}\right)^{2/3} = 1066\,\mathrm{J/m^3\cdot K} \tag{2}$$

The evaporation rate equation

$$n_A = \overline{h}_m A_s\left(\rho_{A,s} - \rho_{A,\infty}\right)$$

Substituting Eqs. (2) and (3) into Eq. (1), find $\overline{h}_m$

$$P_e + \left(1066\,\mathrm{J/m^3\cdot K}\,h_m\right)A_s\left(T_\infty - T_s\right) - h_m A_s\left(\rho_{A,s} - \rho_{A,\infty}\right)h_{fg} = 0 \tag{4}$$

$$432\,\mathrm{W} + \left[1066\,\mathrm{J/m^3\cdot K}\left(32-27\right)\mathrm{K} - \left(0.0256-0\right)\mathrm{kg/m^3} \times 2438\times10^3\,\mathrm{J/kg}\right]\left(0.200\times1\right)\mathrm{m^2}\cdot h_m = 0$$

$432 + [5330 - 62{,}413] \times 0.20\, h_m = 0$

$h_m = 0.0378$ m/s

Using Eq. (3), find

$$n_A = 0.0378\,\mathrm{m/s}\left(0.200\times1\right)\mathrm{m^2}\left(0.0256-0\right)\mathrm{kg/m^3} = 1.94\times10^{-4}\,\mathrm{kg/s} = 0.70\,\mathrm{kg/h} \qquad <$$

(b) When the plate is dry, all the power must be removed by convection,

$P_e = q_{conv} = \overline{h}\, A_s(T_s - T_\infty)$

Assuming $\overline{h}$ is the same as for conditions with the wetted plate,

$$T_s = T_\infty + P_e/\overline{h}\,A_s = T_\infty + P_e/\left(1066 h_m\right)A_s$$

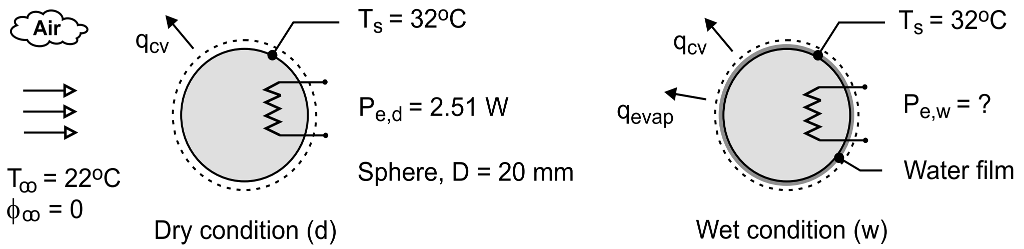
$$T_s = 32^\circ\mathrm{C} + 432\,\mathrm{W}\Big/\left(1066\times0.0378\,\mathrm{W/m^2\cdot K}\times0.200\,\mathrm{m^2}\right) = 85.6^\circ\mathrm{C} \qquad <$$

PROBLEM 6.70

KNOWN: Surface temperature of a 20-mm diameter sphere is 32°C when dissipating 2.51 W in a dry air stream at 22°C.

FIND: Power required by the imbedded heater to maintain the sphere at 32°C if its outer surface has a thin porous covering saturated with water for the same dry air temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Heat and mass transfer analogy is applicable, (3) Heat transfer convection coefficient is the same for the dry and wet condition, and (3) Properties of air and the diffusion coefficient of the air-water vapor mixture evaluated at 300 K.

PROPERTIES: *Table A-4*, Air (300 K, 1 atm): ρ = 1.1614 kg/m^3, c_p = 1007 J/kg·K, $\alpha = 22.5 \times 10^{-6}$ m^2/s; *Table A-8*, Water-air mixture (300 K, 1 atm): $D_{A\text{-}B} = 0.26 \times 10^{-4}$ m^2/s; *Table A-4*, Water (305 K, 1 atm): $\rho_{A,s} = 1/v_g$ = 0.03362 kg/m^3, $h_{fg} = 2.426 \times 10^6$ J/kg.

ANALYSIS: For the *dry case (d)*, perform an energy balance on the sphere and calculate the heat transfer convection coefficient.

$$\dot{E}_{in} - \dot{E}_{out} = P_{e,d} - q_{cv} = 0 \qquad P_{e,d} - \bar{h} A_s (T_s - T_\infty) = 0$$

$$2.51\ \text{W} - \bar{h}\pi (0.020\ \text{m})^2 \times (32 - 22)\text{K} = 0 \qquad \bar{h} = 200\ \text{W/m}^2\cdot\text{K}$$

Use the heat-mass analogy, Eq. (6.67) with n = 1/3, to determine $\bar{h}_m$.

$$\frac{\bar{h}}{\bar{h}_m} = \rho c_p \left(\frac{\alpha}{D_{AB}} \right)^{2/3}$$

$$\frac{200\ \text{W/m}^2\cdot\text{K}}{\bar{h}_m} = 1.1614\ \text{kg/m}^3 \times 1007\ \text{J/kg}\cdot\text{K} \left(\frac{22.5\times10^6\text{m}^2/\text{s}}{0.26\times10^6\text{m}^2/\text{s}} \right)^{2/3}$$

$$\bar{h}_m = 0.188\ \text{m/s}$$

For the *wet case* (*w*), perform an energy balance on the wetted sphere using values for $\bar{h}$ and $\bar{h}_m$ to determine the power required to maintain the same surface temperature.

$$\dot{E}_{in} - \dot{E}_{out} = P_{e,w} - q_{cv} - q_{evap} = 0$$

$$P_{e,w} - \left[\bar{h}(T_s - T_\infty) + \bar{h}_m (\rho_{A,s} - \rho_{A,\infty}) h_{fg} \right] A_s = 0$$

$$P_{e,w} - \Big[200\ \text{W/m}^2\cdot\text{K}(32 - 22)\text{K} +$$

$$0.188\ \text{m/s}(0.03362 - 0)\text{kg/m}^3 \times 2.426\times10^6\text{J/kg} \Big] \pi (0.020\,\text{m})^2 = 0$$

$$P_{e,w} = 21.8\ \text{W} \qquad <$$

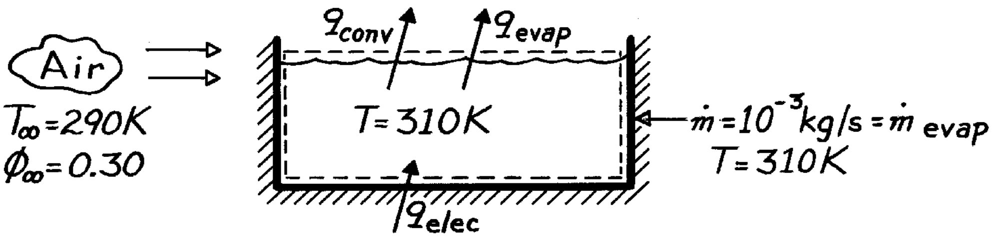
COMMENTS: Note that $\rho_{A,s}$ and h_{fg} for the mass transfer rate equation are evaluated at T_s = 32°C = 305 K, not 300 K. The effect of evaporation is to require nearly 8.5 times more power to maintain the same surface temperature.

PROBLEM 6.71

KNOWN: Operating temperature, ambient air conditions and make-up water requirements for a hot tub.

FIND: Heater power required to maintain prescribed conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Side wall and bottom are adiabatic, (2) Heat and mass transfer analogy is applicable.

PROPERTIES: *Table A-4,* Air $(\bar{T} = 300\text{K, 1 atm})$: ρ = 1.161 kg/m^3, c_p = 1007 J/kg·K, α = 22.5× 10^{-6} m^2/s; *Table A-6*, Sat. water vapor (T = 310K): h_{fg} = 2414 kJ/kg, $\rho_{A,sat}(T) = 1/v_g$ = (22.93m^3/kg)$^{-1}$ = 0.0436 kg/m^3; (T_∞ = 290K): $\rho_{A,sat}(T_\infty) = 1/v_g$ = (69.7 m^3/kg)$^{-1}$ = 0.0143 kg/m^3; *Table A-8*, Air-water vapor (298K): D_{AB} = 26 × 10^{-6} m^2/s.

ANALYSIS: Applying an energy balance to the control volume,

$$q_{elec} = q_{conv} + q_{evap} = \bar{h}A(T - T_\infty) + \dot{m}_{evap}\, h_{fg}(T).$$

Obtain $\bar{h}A$ from Eq. 6.67 with n = 1/3,

$$\frac{\bar{h}}{\bar{h}_m} = \frac{\bar{h}A}{\bar{h}_m A} = \rho\, c_p \mathrm{Le}^{2/3}$$

$$\bar{h}A = \bar{h}_m A\, \rho\, c_p \mathrm{Le}^{2/3} = \frac{\dot{m}_{evap}}{\rho_{A,sat}(T) - \phi_\infty\, \rho_{A,sat}(T_\infty)}\rho\, c_p \mathrm{Le}^{2/3}.$$

Substituting numerical values,

$$\mathrm{Le} = \alpha/D_{AB} = \left(22.5\times10^{-6}\mathrm{m}^2/\mathrm{s}\right)/26\times10^{-6}\mathrm{m}^2/\mathrm{s} = 0.865$$

$$\bar{h}A = \frac{10^{-3}\ \mathrm{kg/s}}{[0.0436 - 0.3\times0.0143]\ \mathrm{kg/m^3}}1.161\frac{\mathrm{kg}}{\mathrm{m}^3}\times1007\frac{\mathrm{J}}{\mathrm{kg\cdot K}}(0.865)^{2/3}$$

$$\bar{h}A = 27.0\ \mathrm{W/K}.$$

Hence, the required heater power is

$$q_{elec} = 27.0\mathrm{W/K}(310 - 290)\mathrm{K} + 10^{-3}\mathrm{kg/s}\times2414\mathrm{kJ/kg}\times1000\mathrm{J/kJ}$$

$$q_{elec} = (540 + 2414)\ \mathrm{W} = 2954\ \mathrm{W}.$$ <

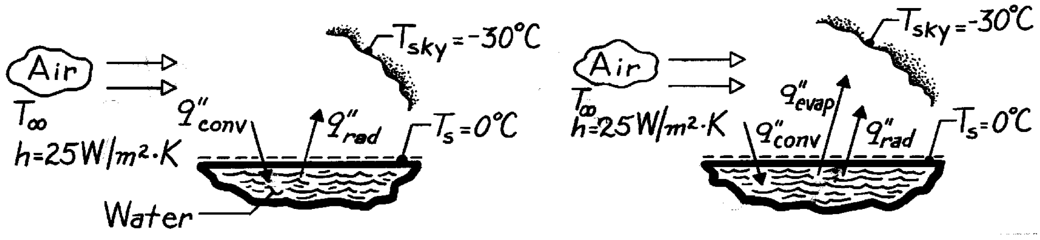
COMMENTS: The evaporative heat loss is dominant.

PROBLEM 6.72

KNOWN: Water freezing under conditions for which the air temperature exceeds 0°C.

FIND: (a) Lowest air temperature, T_∞, before freezing occurs, neglecting evaporation, (b) The mass transfer coefficient, h_m, for the evaporation process, (c) Lowest air temperature, T_∞, before freezing occurs, including evaporation.

SCHEMATIC:



No evaporation With evaporation

ASSUMPTIONS: (1) Steady-state conditions, (2) Water insulated from ground, (3) Water surface has $\varepsilon = 1$, (4) Heat-mass transfer analogy applies, (5) Ambient air is dry.

PROPERTIES: *Table A-4*, Air ($T_f \approx 2.5°C \approx 276K$, 1 atm): $\rho = 1.2734\ kg/m^3$, $c_p = 1006\ J/kg\cdot K$, $\alpha = 19.3 \times 10^{-6} m^2/s$; *Table A-6*, Water vapor (273.15K): $h_{fg} = 2502\ kJ/kg$, $\rho_g = 1/v_g = 4.847 \times 10^{-3} kg/m^3$; *Table A-8*, Water vapor - air (298K): $D_{AB} = 0.26 \times 10^{-4} m^2/s$.

ANALYSIS: (a) Neglecting evaporation and performing an energy balance,

$$q''_{conv} - q''_{rad} = 0$$

$$h\left(T_\infty - T_s\right) - \varepsilon\sigma\left(T_s^4 - T_{sky}^4\right) = 0 \quad \text{or} \quad T_\infty = T_s + \left(\varepsilon\sigma/h\right)\left(T_s^4 - T_{sky}^4\right)$$

$$T_\infty = 0°C + \frac{1\times 5.667\times 10^{-8}\ W/m^2\cdot K^4}{25\ W/m^2\cdot K}\left[\left(0+273\right)^4 - \left(-30+273\right)^4\right] = 4.69°C. \qquad <$$

(b) Invoking the heat-mass transfer analogy in the form of Eq. 6.67 with n = 1/3,

$$\frac{h}{h_m} = \rho\, c_p Le^{2/3} \quad \text{or} \quad h_m = h/\rho\, c_p Le^{2/3} \quad \text{where} \quad Le = \alpha/D_{AB}$$

$$h_m = \left(25\ W/m^2\cdot K\right)/1.273\ kg/m^3\left(1006\ J/kg\cdot K\right)\left[\frac{19.3\times 10^{-6} m^2/s}{0.26\times 10^{-4} m^2/s}\right]^{2/3} = 0.0238\ m/s. \qquad <$$

(c) Including evaporation effects and performing an energy balance gives $q''_{conv} - q''_{rad} - q''_{evap} = 0$

where $q''_{evap} = \dot{m}'' h_{fg} = h_m\left(\rho_{A,s} - \rho_{A,\infty}\right)h_{fg}$, $\rho_{A,s} = \rho_g$ and $\rho_{A,\infty} = 0$. Hence,

$$T_\infty = T_s + \left(\varepsilon\sigma/h\right)\left(T_s^4 - T_{sky}^4\right) + \left(h_m/h\right)\left(\rho_g - 0\right)h_{fg}$$

$$T_\infty = 4.69°C + \frac{0.0238\ m/s}{25\ W/m^2\cdot K}\times 4.847\times 10^{-3} kg/m^3 \times 2.502\times 10^6 J/kg$$

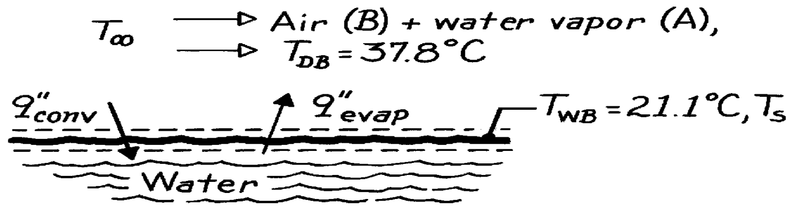
$$T_\infty = 4.69°C + 11.5°C = 16.2°C. \qquad <$$

PROBLEM 6.73

KNOWN: Wet-bulb and dry-bulb temperature for water vapor-air mixture.

FIND: (a) Partial pressure, p_A, and relative humidity, ϕ, using Carrier's equation, (b) p_A and ϕ using psychrometric chart, (c) Difference between air stream, T_∞, and wet bulb temperatures based upon evaporative cooling considerations.

SCHEMATIC:



ASSUMPTIONS: (1) Evaporative cooling occurs at interface, (2) Heat-mass transfer analogy applies, (3) Species A and B are perfect gases.

PROPERTIES: *Table A-6*, Water vapor: $p_{A,sat}$ (21.1°C) = 0.02512 bar, $p_{A,sat}$ (37.8°C) = 0.06603 bar, h_{fg} (21.1°C) = 2451 kJ/kg; *Table A-4*, Air ($T_{am} = [T_{WB} + T_{DB}]/2 \cong 300\text{K}$, 1 atm): $\alpha = 22.5 \times 10^{-6}\ \text{m}^2/\text{s}$, c_p = 1007 J/kg·K; *Table A-8*, Air-water vapor (298K): $D_{AB} = 0.26 \times 10^{-4}\ \text{m}^2/\text{s}$.

ANALYSIS: (a) Carrier's equation has the form

$$p_V = p_{gw} - \frac{\left(p - p_{gw}\right)\ \left(T_{DB} - T_{WB}\right)}{1810 - T_{WB}}$$

where p_V = partial pressure of vapor in air stream, bar

p_{gw} = sat. pressure at TWB = 21.1°C, 0.02512 bar

p = total pressure of mixture, 1.033 bar

T_{DB} = dry bulb temperature, 37.8°C

T_{WB} = wet bulb temperature, 21.1°C.

Hence,

$$p_V = 0.02512\ \text{bar} - \frac{\left(1.013 - 0.02512\right)\text{bar} \times \left(37.8 - 21.1\right)^\circ \text{C}}{1810 - \left(21.1 + 273.1\right)\text{K}} = 0.0142\ \text{bar}.$$

The relative humidity, ϕ, is then

$$\phi \equiv \frac{p_A}{p_{A,sat}} = \frac{p_V}{p_A\left(37.8^\circ\text{C}\right)} = \frac{0.0142\ \text{bar}}{0.06603\ \text{bar}} = 0.214.$$ <

(b) Using a psychrometric chart

$$\left.\begin{aligned} T_{WB} &= 21.1^\circ\text{C} = 70^\circ\text{F} \\ T_{DB} &= 37.8^\circ\text{C} = 100^\circ\text{F} \end{aligned}\right\} \qquad \phi \approx 0.225$$ <

$$p_V = \phi p_{sat} = 0.225 \times 0.06603\ \text{bar} = 0.0149\ \text{bar}.$$ <

Continued …..

(c) An application of the heat-mass transfer analogy is the process of evaporative cooling which occurs when air flows over water. The change in temperature is estimated by Eq. 6.73.

$$\left(T_\infty - T_s\right) = \frac{\left(M_A / M_B\right) h_{fg}}{c_p Le^{2/3}} \left[\frac{p_{A,sat}\left(T_s\right)}{p} - \frac{p_{A,\infty}}{p}\right]$$

where c_p and Le are evaluated at $T_{am} = (T_\infty + T_s)/2$ and $p_{A,\infty} = p_v$, as determined in Part (a). Substituting numerical values, using $Le = \alpha / D_{AB}$,

$$\left(T_\infty - T_s\right) = \frac{\left(18\ \text{kg/kmol}/29\ \text{kg/kmol}\right) \times 2451 \times 10^3\ \dfrac{\text{J}}{\text{kg}}}{1007\ \text{J/kg}\cdot\text{K}\left[\dfrac{22.5\times10^{-6}\text{m}^2/\text{s}}{0.26\times10^{-4}\text{m}^2/\text{s}}\right]^{2/3}} \left[\frac{0.02491\ \text{bar}}{1.013\ \text{bar}} - \frac{0.0149\ \text{bar}}{1.013\ \text{bar}}\right]$$

$$\left(T_\infty - T_s\right) = 17.6^\circ\text{C}.$$ <

Note that c_p and α are associated with the air.

COMMENTS: The following table compares results from the two calculation methods.

	Carrier's Eq.	*Psychrometric Chart*
p_v (bar)	0.0142	0.0149
ϕ	0.214	0.225

Evaporative Cooling $\quad T_\infty - T_s = 17.6°\text{C}$

Observed Difference $\quad T_{DB} - T_{WB} = 16.7°\text{C}$

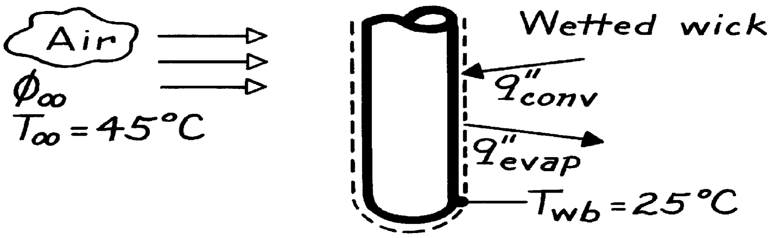
% Difference: $\dfrac{17.6 - 16.7}{16.7} \times 100 = 5.4\%.$

PROBLEM 6.74

KNOWN: Wet and dry bulb temperatures.

FIND: Relative humidity of air.

SCHEMATIC:



ASSUMPTIONS: (1) Perfect gas behavior for vapor, (2) Steady-state conditions, (3) Negligible radiation, (4) Negligible conduction along thermometer.

PROPERTIES: *Table A-4*, Air (308K, 1 atm): $\rho = 1.135$ kg/m^3, $c_p = 1007$ J/kg·K, $\alpha = 23.7 \times 10^{-6}$ m^2/s; *Table A-6*, Saturated water vapor (298K): $v_g = 44.25$ m^3/kg, $h_{fg} = 2443$ kJ/kg; (318K): $v_g = 15.52$ m^3/kg; *Table A-8*, Air-vapor (1 atm, 298K): $D_{AB} = 0.26 \times 10^{-4}$ m^2/s, D_{AB} (308K) = 0.26×10^{-4} m^2/s × $(308/298)^{3/2}$ = 0.27×10^{-4} m^2/s, Le = α/D_{AB} = 0.88.

ANALYSIS: From an energy balance on the wick, Eq. 6.71 follows from Eq. 6.68. Dividing Eq. 6.71 by $\rho_{A,sat}(T_\infty)$,

$$\frac{T_\infty - T_s}{\rho_{A,sat}(T_\infty)} = h_{fg}\left[\frac{h_m}{h}\right]\left[\frac{\rho_{A,sat}(T_s)}{\rho_{A,sat}(T_\infty)} - \frac{\rho_{A,\infty}}{\rho_{A,sat}(T_\infty)}\right].$$

With $\left[\rho_{A,\infty}/\rho_{A,sat}(T_\infty)\right] \approx \phi_\infty$ for a perfect gas and h/h_m given by Eq. 6.67,

$$\phi_\infty = \frac{\rho_{A,sat}(T_s)}{\rho_{A,sat}(T_\infty)} - \frac{\rho\, c_p}{Le^{2/3}\rho_{A,sat}(T_\infty)h_{fg}}(T_\infty - T_s).$$

Using the property values, evaluate

$$\frac{\rho_{A,sat}(T_s)}{\rho_{A,sat}(T_\infty)} = \frac{v_g T_\infty}{v_g(T_s)} = \frac{15.52}{44.25} = 0.351$$

$$\rho_{A,sat}(T_\infty) = \left(15.52\ \text{m}^3/\text{kg}\right)^{-1} = 0.064\ \text{kg/m}^3.$$

Hence,

$$\phi_\infty = 0.351\ -\ \frac{1.135\ \text{kg/m}^3\ (1007\ \text{J/kg}\cdot\text{K})}{(0.88)^{2/3}\ 0.064\ \text{kg/m}^3\ \left(2.443\times10^6\ \text{J/kg}\right)}(45-25)\text{K}$$

$$\phi_\infty = 0.351 - 0.159 = 0.192.$$ <

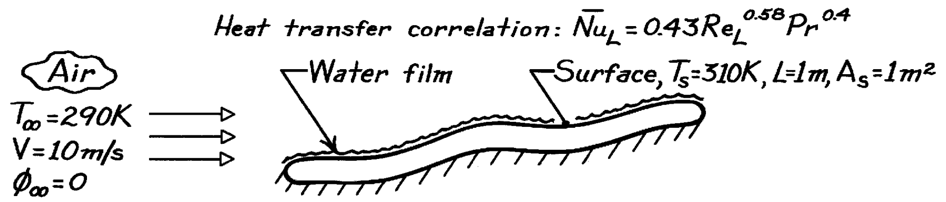
COMMENTS: Note that latent heat must be evaluated at the surface temperature (evaporation occurs at the surface).

PROBLEM 6.75

KNOWN: Heat transfer correlation for a contoured surface heated from below while experiencing air flow across it. Flow conditions and steady-state temperature when surface experiences evaporation from a thin water film.

FIND: (a) Heat transfer coefficient and convection heat rate, (b) Mass transfer coefficient and evaporation rate (kg/h) of the water, (c) Rate at which heat must be supplied to surface for these conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Heat-mass transfer analogy applies, (3) Correlation requires properties evaluated at $T_f = (T_s + T_\infty)/2$.

PROPERTIES: *Table A-4*, Air ($T_f = (T_s + T_\infty)/2 = (290 + 310)K/2 = 300$ K, 1 atm): $\nu = 15.89 \times 10^{-6} m^2/s$, k = 0.0263 W/m·K, Pr = 0.707; *Table A-8*, Air-water mixture (300 K, 1 atm): $D_{AB} = 0.26 \times 10^{-4} m^2/s$; *Table A-6*, Sat. water (T_s = 310 K): $\rho_{A,sat} = 1/v_g = 1/22.93 m^3/kg = 0.04361 kg/m^3$, h_{fg} = 2414 kJ/kg.

ANALYSIS: (a) To characterize the flow, evaluate Re_L at T_f

$$Re_L = \frac{VL}{\nu} = \frac{10 \text{ m/s} \times 1 \text{ m}}{15.89 \times 10^{-6} \text{m}^2/\text{s}} = 6.293 \times 10^5$$

and substituting into the prescribed correlation for this surface, find

$$\overline{Nu}_L = 0.43\left(6.293 \times 10^5\right)^{0.58} (0.707)^{0.4} = 864.1$$

$$\bar{h}_L = \frac{\overline{Nu}_L \cdot k}{L} = \frac{864.1 \times 0.0263 \text{ W/m} \cdot \text{K}}{1 \text{ m}} = 22.7 \text{ W/m}^2 \cdot \text{K}. \qquad <$$

Hence, the convection heat rate is

$$q_{conv} = \bar{h}_L A_s (T_s - T_\infty)$$

$$q_{conv} = 22.7 \text{ W/m}^2 \cdot \text{K} \times 1 \text{ m}^2 (310 - 290) \text{K} = 454 \text{ W} \qquad <$$

(b) Invoking the heat-mass transfer analogy

$$\overline{Sh}_L = \frac{\bar{h}_m L}{D_{AB}} = 0.43 Re_L^{0.58} Sc^{0.4}$$

where

$$Sc = \frac{\nu}{D_{AB}} = \frac{15.89 \times 10^{-6} \text{m}^2/\text{s}}{0.26 \times 10^{-4} \text{m}^2/\text{s}} = 0.611$$

and ν is evaluated at T_f. Substituting numerical values, find

Continued …..

PROBLEM 6.75 (Cont.)

$$\overline{Sh}_L = 0.43\left(6.293\times10^5\right)^{0.58}\left(0.611\right)^{0.4} = 815.2$$

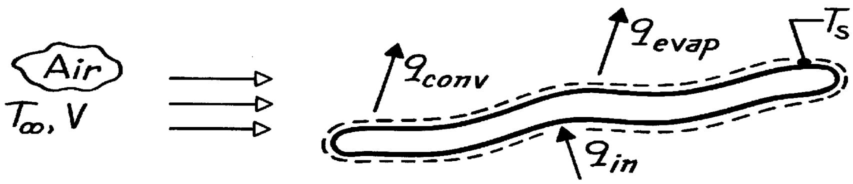
$$\bar{h}_m = \frac{\overline{Sh}_L \cdot D_{AB}}{L} = \frac{815.2\times0.26\times10^{-4}\,m^2/s}{1\ m} = 2.12\times10^{-2}\ m/s. \qquad <$$

The evaporation rate, with $\rho_{A,s} = \rho_{A,sat}(T_s)$, is

$$\dot{m} = \bar{h}_m A_s\left(\rho_{A,s} - \rho_{A,\infty}\right)$$
$$\dot{m} = 2.12\times10^{-2}\ m/s\times1\ m^2\left(0.04361\ -\ 0\right)kg/m^3$$

$$\dot{m} = 9.243\times10^{-4} kg/s = 3.32\ kg/h. \qquad <$$

(c) The rate at which heat must be supplied to the plate to maintain these conditions follows from an energy balance.



$$\dot{E}_{in} - \dot{E}_{out} = 0$$
$$q_{in} - q_{conv} - q_{evap} = 0$$

where q_{in} is the heat supplied to sustain the losses by convection and evaporation.

$$q_{in} = q_{conv} + q_{evap}$$
$$q_{in} = \bar{h}_L A_s\left(T_s - T_\infty\right) + \dot{m}h_{fg}$$
$$q_{in} = 454\ W\ +\ 9.243\times10^{-4}\ kg/s\times2414\times10^3\ J/kg$$

$$q_{in} = \left(254 + 2231\right)W = 2685\ W. \qquad <$$

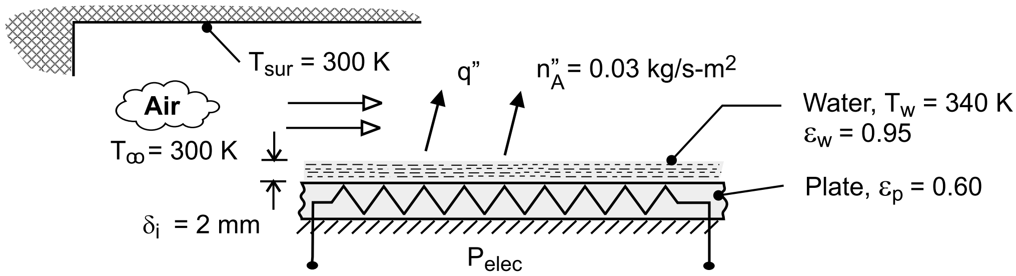
COMMENTS: Note that the loss from the surface by evaporation is nearly 5 times that due to convection.

PROBLEM 6.76

KNOWN: Thickness, temperature and evaporative flux of a water layer. Temperature of air flow and surroundings.

FIND: (a) Convection mass transfer coefficient and time to completely evaporate the water, (b) Convection heat transfer coefficient, (c) Heater power requirement per surface area, (d) Temperature of dry surface if heater power is maintained.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Applicability of heat and mass transfer analogy with n = 1/3, (3) Radiation exchange at surface of water may be approximated as exchange between a small surface and large surroundings, (4) Air is dry ($\rho_{A,\infty} = 0$), (5) Negligible heat transfer from unwetted surface of the plate.

PROPERTIES: *Table A-6*, Water (T_w = 340K): $\rho_f = 979\ \text{kg/m}^3$, $\rho_{A,sat} = v_g^{-1} = 0.174\ \text{kg/m}^3$, $h_{fg} = 2342\ \text{kJ/kg}$. Prescribed, Air: $\rho = 1.08\ \text{kg/m}^3$, $c_p = 1008$ J/kg·K, k = 0.028 W/m·K. Vapor/Air: $D_{AB} = 0.29 \times 10^{-4}\ \text{m}^2/\text{s}$. Water: $\varepsilon_w = 0.95$. Plate: $\varepsilon_p = 0.60$.

ANALYSIS: (a) The convection mass transfer coefficient may be determined from the rate equation $n''_A = h_m\left(\rho_{A,s} - \rho_{A,\infty}\right)$, where $\rho_{A,s} = \rho_{A,sat}(T_w)$ and $\rho_{A,\infty} = 0$. Hence,

$$h_m = \frac{n''_A}{\rho_{A,sat}} = \frac{0.03\,\text{kg/s}\cdot\text{m}^2}{0.174\,\text{kg/m}^3} = 0.172\,\text{m/s} \qquad <$$

The time required to completely evaporate the water is obtained from a mass balance of the form $-n''_A = \rho_f\, d\delta/dt$, in which case

$$\rho_f \int_{\delta_i}^{0} d\delta = -n''_A \int_0^t dt$$

$$t = \frac{\rho_f \delta_i}{n''_A} = \frac{979\,\text{kg/m}^3(0.002\text{m})}{0.03\,\text{kg/s}\cdot\text{m}^2} = 65.3\text{s} \qquad <$$

(b) With n = 1/3 and $Le = \alpha/D_{AB} = k/\rho c_p D_{AB}$ = 0.028 W/m·K/(1.08 kg/m^3 × 1008 J/kg·K × 0.29 × 10^{-4} m^2/s) = 0.887, the heat and mass transfer analogy yields

$$h = \frac{k\,h_m}{D_{AB}Le^{1/3}} = \frac{0.028\,\text{W/m}\cdot\text{K}(0.172\,\text{m/s})}{0.29\times10^{-4}\text{m}^2/\text{s}(0.887)^{1/3}} = 173\,\text{W/m}^2\cdot\text{K} \qquad <$$

The electrical power requirement per unit area corresponds to the rate of heat loss from the water. Hence,

Continued …..

PROBLEM 6.76 (Cont.)

$$P''_{elec} = q''_{evap} + q''_{conv} + q''_{rad} = n''_A h_{fg} + h\left(T_w - T_\infty\right) + \varepsilon_w \sigma \left(T_w^4 - T_{sur}^4\right)$$

$$P''_{elec} = 0.03\,kg/s\cdot m^2 \left(2.342\times10^6\ J/kg\right) + 173\,W/m^2\cdot K\,(40K) + 0.95\times5.67\times10^{-8}\,W/m^2\cdot K^4\left(340^4 - 300^4\right)$$

$$P''_{elec} = 70,260\,W/m^2 + 6920\,W/m^2 + 284\,W/m^2 = 77,464\,W/m^2 \qquad <$$

(c) After complete evaporation, the steady-state temperature of the plate is determined from the requirement that

$$P''_{elec} = h\left(T_p - T_\infty\right) + \varepsilon_p \sigma \left(T_p^4 - T_{sur}^4\right)$$

$$77,464\,W/m^2 = 173\,W/m^2\cdot K\left(T_p - 300\right) + 0.60\times5.67\times10^{-8}\,W/m^2\cdot K^4\left(T_p^4 - 300^4\right)$$

$$T_p = 702K = 429°C \qquad <$$

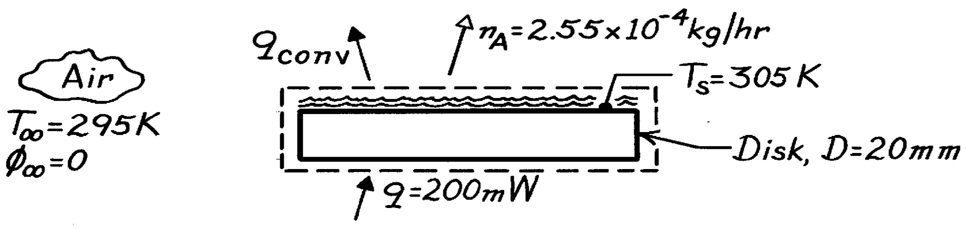
COMMENTS: The evaporative heat flux is the dominant contributor to heat transfer from the water layer, with convection of sensible energy being an order of magnitude smaller and radiation exchange being negligible. Without evaporation (a dry surface), convection dominates and is approximately an order of magnitude larger than radiation.

PROBLEM 6.77

KNOWN: Heater power required to maintain water film at prescribed temperature in dry ambient air and evaporation rate.

FIND: (a) Average mass transfer convection coefficient $\overline{h}_m$, (b) Average heat transfer convection coefficient $\overline{h}$, (c) Whether values of $\overline{h}_m$ and $\overline{h}$ satisfy the heat-mass analogy, and (d) Effect on evaporation rate and disc temperature if relative humidity of the ambient air were increased from 0 to 0.5 but with heater power maintained at the same value.

SCHEMATIC:



ASSUMPTIONS: (1) Water film and disc are at same temperature; (2) Mass and heat transfer coefficient are independent of ambient air relative humidity, (3) Constant properties.

PROPERTIES: *Table A-6*, Saturated water (305 K): $v_g = 29.74\ m^3/kg$, $h_{fg} = 2426 \times 10^3\ J/kg$; *Table A-4*, Air $(\overline{T} = 300\ K,\ 1\ atm)$: $k = 0.0263\ W/m{\cdot}K$, $\alpha = 22.5 \times 10^{-6}\ m^2/s$, *Table A-8*, Air-water vapor (300 K, 1 atm): $D_{AB} = 0.26 \times 10^{-4}\ m^2/s$.

ANALYSIS: (a) Using the mass transfer convection rate equation,

$$n_A = \overline{h}_m A_s\left(\rho_{A,s} - \rho_{A,\infty}\right) = \overline{h}_m A_s\, \rho_{A,sat}\left(1-\phi_\infty\right)$$

and evaluating $\rho_{A,s} = \rho_{A,sat}$ (305 K) = $1/v_g$ (305 K) with $\phi_\infty \sim \rho_{A,\infty} = 0$, find

$$\overline{h}_m = \frac{n_A}{A_s\left(\rho_{A,s} - \rho_{A,\infty}\right)}$$

$$\overline{h}_m = \frac{2.55\times10^{-4}\text{kg/hr}/\left(3600\text{s/hr}\right)}{\left(\pi\left(0.020\text{ m}\right)^2/4\right)\left(1/29.74-0\right)\text{kg/m}^3} = 6.71\times10^{-3}\ \text{m/s}. \qquad <$$

(b) Perform an overall energy balance on the disc,

$$q = q_{conv} + q_{evap} = \overline{h}A_s\left(T_s - T_\infty\right) + n_A h_{fg}$$

and substituting numerical values with h_{fg} evaluated at T_s, find $\overline{h}$:

$$200\times10^{-3}\ \text{W} = \overline{h}\pi\left(0.020\text{ m}\right)^2/\,4\left(305-295\right)\text{K} + 7.083\times10^{-8}\ \text{kg/s}\times2426\times10^3\ \text{J/kg}$$

$$\overline{h} = 8.97\ \text{W/m}^2\cdot\text{K}. \qquad <$$

Continued …..

(c) The heat-mass transfer analogy, Eq. 6.67, requires that

$$\frac{\overline{h}}{h_m} \stackrel{?}{=} \frac{k}{D_{AB}}\left(\frac{D_{AB}}{\alpha}\right)^{1/3}.$$

Evaluating k and D_{AB} at $\overline{T} = (T_s + T_\infty)/2 = 300\text{ K}$ and substituting numerical values,

$$\frac{8.97\text{ W/m}^2\cdot\text{K}}{6.71\times10^{-3}\text{ m/s}} = 1337 \neq \frac{0.0263\text{ W/m}\cdot\text{K}}{0.26\times10^{-4}\text{ m}^2/\text{s}}\left(\frac{0.26\times10^{-4}\text{ m}^2/\text{s}}{22.5\times10^{-6}\text{m}^2/\text{s}}\right)^{1/3} = 1061$$

Since the equality is not satisfied, we conclude that, for this situation, the analogy is only approximately met (≈ 30%).

(d) If $\phi_\infty = 0.5$ instead of 0.0 and q is unchanged, n_A will decrease by nearly a factor of two, as will $n_A h_{fg} = q_{evap}$. Hence, since q_{conv} must increase and $\overline{h}$ remains nearly constant, $T_s - T_\infty$ must increase. Hence, T_s will increase.

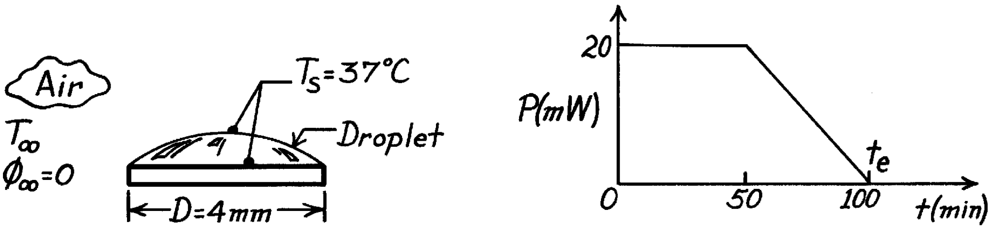
COMMENTS: Note that in part (d), with an increase in T_s, h_{fg} decreases, but only slightly, and $\rho_{A,sat}$ increases. From a trial-and-error solution assuming constant values for $\overline{h}_m$ and h, the disc temperature is 315 K for $\phi_\infty = 0.5$.

PROBLEM 6.78

KNOWN: Power-time history required to completely evaporate a droplet of fixed diameter maintained at 37°C.

FIND: (a) Average mass transfer convection coefficient when droplet, heater and dry ambient air are at 37°C and (b) Energy required to evaporate droplet if the dry ambient air temperature is 27°C.

SCHEMATIC:



ASSUMPTIONS: (1) Wetted surface area of droplet is of fixed diameter D, (2) Heat-mass transfer analogy is applicable, (3) Heater controlled to operate at constant temperature, $T_s = 37°C$, (4) Mass of droplet same for part (a) and (b), (5) Mass transfer coefficients for parts (a) and (b) are the same.

PROPERTIES: *Table A-6*, Saturated water (37°C = 310 K): $h_{fg} = 2414$ kJ/kg, $\rho_{A,sat} = 1/v_g = 1/22.93 = 0.04361$ kg/m^3; *Table A-8*, Air-water vapor (T_s = 37°C = 310 K, 1 atm): $D_{AB} = 0.26 \times 10^{-6}$ m^2/s$(310/289)^{3/2} = 0.276 \times 10^{-6}$ m^2/s; *Table A-4*, Air ($\overline{T}$ = (27 + 37)°C/2 = 305 K, 1 atm): $\rho = 1.1448$ kg/m^3, $c_p = 1008$ J/kg·K, $\nu = 16.39 \times 10^{-6}$ m^2/s, Pr = 0.706.

ANALYSIS: (a) For the isothermal conditions (37°C), the electrical energy Q required to evaporate the droplet during the interval of time $\Delta t = t_e$ follows from the area under the P-t curve above,

$$Q = \int_0^{t_e} P dt = \left[20\times10^{-3}\ \text{W}\times(50\times60)\text{s} + 0.5\times20\times10^{-3}\ \text{W}(100-50)\times60\text{s}\right]$$
$$Q = 90\ \text{J}.$$

From an overall energy balance during the interval of time $\Delta t = t_e$, the mass loss due to evaporation is

$$Q = Mh_{fg} \quad \text{or} \quad M = Q/h_{fg}$$
$$M = 90\ \text{J}/2414\times10^3\ \text{J/kg} = 3.728\times10^{-5}\ \text{kg}.$$

To obtain the average mass transfer coefficient, write the rate equation for an interval of time $\Delta t = t_e$,

$$M = \dot{m}\cdot t_e = \overline{h}_m A_s\left(\rho_{A,s} - \rho_{A,\infty}\right)\cdot t_e = \overline{h}_m A_s \rho_{A,s}\left(1-\phi_\infty\right)\cdot t_e$$

Substituting numerical values with $\phi_\infty = 0$, find

$$3.278\times10^{-5}\ \text{kg} = \overline{h}_m\left(\pi(0.004\ \text{m})^2/4\right)0.04361\ \text{kg/m}^3\times(100\times60)\text{s}$$

Continued …..

$$\overline{h}_m = 0.0113 \text{ m/s.}$$ <

(b) The energy required to evaporate the droplet of mass M = 3.728 × 10^{-5} kg follows from an overall energy balance,

$$Q = Mh_{fg} + \overline{h}A_s\left(T_s - T_\infty\right)$$

where $\overline{h}$ is obtained from the heat-mass transfer analogy, Eq. 6.67, using n = 1/3,

$$\frac{\overline{h}}{h_m} = \frac{k}{D_{AB}Le^n} = \rho\, c_p Le^{2/3}$$

where

$$Sc = \frac{\nu}{D_{AB}} = \frac{16.39\times10^{-6}\ \text{m}^2/\text{s}}{0.276\times10^{-4}\ \text{m}^2/\text{s}} = 0.594$$

$$Le = \frac{Sc}{Pr} = \frac{0.594}{0.706} = 0.841.$$

Hence,

$$\overline{h} = 0.0113\ \text{m/s}\times1.1448\ \text{kg/m}^3\times1008\ \text{J/kg}\cdot\text{K}\left(0.841\right)^{2/3} = 11.62\ \text{W/m}^2\cdot\text{K.}$$

and the energy requirement is

$$Q = 3.728\times10^{-5}\ \text{kg}\times2414\ \text{kJ/kg} + 11.62\ \text{W/m}^2\cdot\text{K}\left(\pi\left(0.004\ \text{m}\right)^2/4\right)\left(37-27\right)^\circ\text{C}$$

$$Q = \left(90.00+0.00145\right)\text{J} = 90\ \text{J.}$$ <

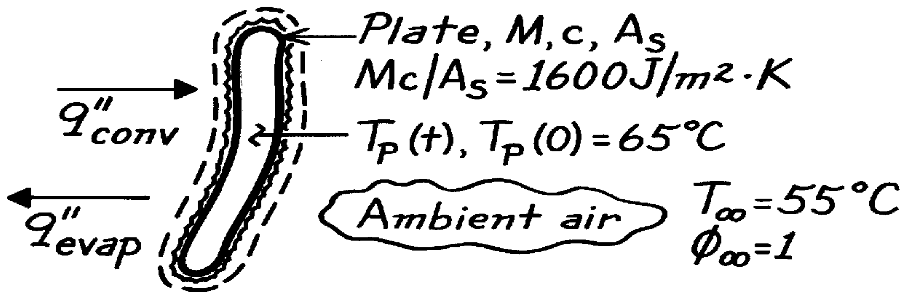
The energy required to meet the convection heat loss is very small compared to that required to sustain the evaporative loss.

PROBLEM 6.79

KNOWN: Initial plate temperature T_p (0) and saturated air temperature (T_∞) in a dishwasher at the start of the dry cycle. Thermal mass per unit area of the plate $Mc/A_s = 1600\ J/m^2 \cdot K$.

FIND: (a) Differential equation to predict plate temperature as a function of time during the dry cycle and (b) Rate of change in plate temperature at the start of the dry cycle assuming the average convection heat transfer coefficient is $3.5\ W/m^2 \cdot K$.

SCHEMATIC:



ASSUMPTIONS: (1) Plate is spacewise isothermal, (2) Negligible thermal resistance of water film on plate, (3) Heat-mass transfer analogy applies.

PROPERTIES: *Table A-4*, Air ($\bar{T} = (55 + 65)°C/2 = 333\ K$, 1 atm): $\rho = 1.0516\ kg/m^3$, $c_p = 1008\ J/kg \cdot K$, Pr = 0.703, $\nu = 19.24 \times 10^{-6}\ m^2/s$; *Table A-6*, Saturated water vapor, ($T_s = 65°C = 338\ K$): $\rho_A = 1/v_g = 0.1592\ kg/m^3$, $h_{fg} = 2347\ kJ/kg$; ($T_s = 55°C = 328\ K$): $\rho_A = 1/v_g = 0.1029\ kg/m^3$; *Table A-8*, Air-water vapor ($T_s = 65°C = 338\ K$, 1 atm): $D_{AB} = 0.26 \times 10^{-4}\ m^2/s\ (338/298)^{3/2} = 0.314 \times 10^{-4}\ m^2/s$.

ANALYSIS: (a) Perform an energy balance on a rate basis on the plate,

$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st} \qquad q''_{conv} - q''_{evap} = (Mc/A_s)(dT_p/dt).$$

Using the rate equations for the heat and mass transfer fluxes, find

$$\bar{h}\left[T_\infty - T_p(t)\right] - \bar{h}_m\left[\rho_{A,s}(T_s) - \rho_{A,\infty}(T_\infty)\right]h_{fg} = (Mc/A_s)(dT/dt). \qquad <$$

(b) To evaluate the change in plate temperature at t = 0, the start of the drying process when T_p (0) = 65°C and T_∞ = 55°C, evaluate $\bar{h}_m$ from knowledge of $\bar{h} = 3.5\ W/m^2 \cdot K$ using the heat-mass transfer analogy, Eq. 6.67, with n = 1/3,

$$\frac{\bar{h}}{\bar{h}_m} = \rho\, c_p Le^{2/3} = \rho\, c_p\left(\frac{Sc}{Pr}\right)^{2/3} = \rho\, c_p\left(\frac{\nu / D_{AB}}{Pr}\right)^{2/3}$$

and evaluating thermophysical properties at their appropriate temperatures, find

$$\frac{3.5\ W/m^2 \cdot K}{\bar{h}_m} = 1.0516\ kg/m^3 \times 1008\ J/kg \cdot K\left(\frac{19.24 \times 10^{-6} m^2/s / 0.314 \times 10^{-4} m^2/s}{0.703}\right)^{2/3} \qquad \bar{h}_m = 3.619 \times 10^{-3} m/s.$$

Substituting numerical values into the conservation expression of part (a), find

$$3.5\ W/m^2 \cdot K(55-65)°C - 3.619 \times 10^{-3} m/s(0.1592 - 0.1029)kg/m^3 \times 2347 \times 10^3\ J/kg = 1600\ J/m^2 \cdot K(dT_p/dt)$$

$$dT_p/dt = -[35.0 + 478.2]W/m^2 \cdot K/1600\ J/m^2 \cdot K = -0.32\ K/s. \qquad <$$

COMMENTS: This rate of temperature change will not be sustained for long, since, as the plate cools, the rate of evaporation (which dominates the cooling process) will diminish.