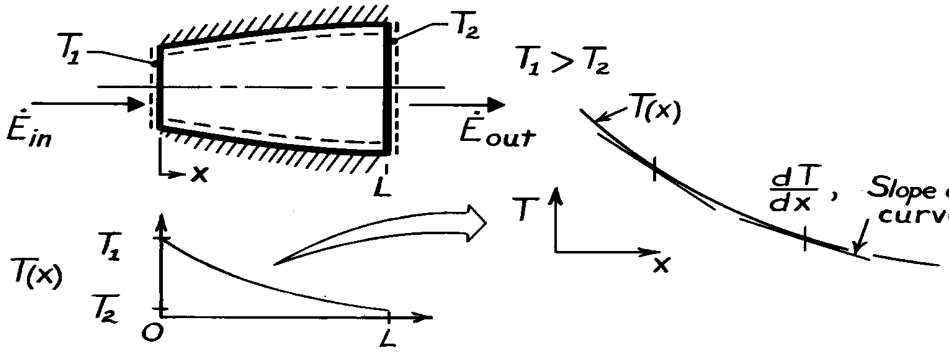
PROBLEM 2.1

KNOWN: Steady-state, one-dimensional heat conduction through an axisymmetric shape.

FIND: Sketch temperature distribution and explain shape of curve.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, one-dimensional conduction, (2) Constant properties, (3) No internal heat generation.

ANALYSIS: Performing an energy balance on the object according to Eq. 1.11a, $\dot{E}_{in} - \dot{E}_{out} = 0$, it follows that

$$\dot{E}_{in} - \dot{E}_{out} = q_x$$

and that $q_x \neq q_x(x)$. That is, the heat rate within the object is everywhere constant. From Fourier's law,

$$q_x = -kA_x \frac{dT}{dx},$$

and since q_x and k are both constants, it follows that

$$A_x \frac{dT}{dx} = \text{Constant.}$$

That is, the product of the cross-sectional area normal to the heat rate and temperature gradient remains a constant and independent of distance x. It follows that since A_x increases with x, then dT/dx must decrease with increasing x. Hence, the temperature distribution appears as shown above.

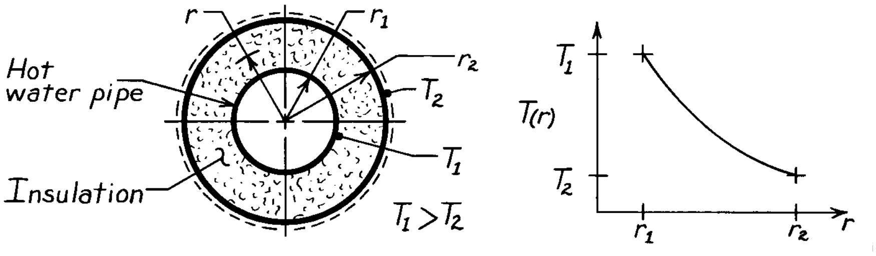
COMMENTS: (1) Be sure to recognize that dT/dx is the slope of the temperature distribution. (2) What would the distribution be when $T_2 > T_1$? (3) How does the heat flux, q''_x, vary with distance?

PROBLEM 2.2

KNOWN: Hot water pipe covered with thick layer of insulation.

FIND: Sketch temperature distribution and give brief explanation to justify shape.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional (radial) conduction, (3) No internal heat generation, (4) Insulation has uniform properties independent of temperature and position.

ANALYSIS: Fourier's law, Eq. 2.1, for this one-dimensional (cylindrical) radial system has the form

$$q_r = -kA_r \frac{dT}{dr} = -k(2\pi r\ell)\frac{dT}{dr}$$

where $A_r = 2\pi r\ell$ and ℓ is the axial length of the pipe-insulation system. Recognize that for steady-state conditions with no internal heat generation, an energy balance on the system requires $\dot{E}_{in} = \dot{E}_{out}$ since $\dot{E}_g = \dot{E}_{st} = 0$. Hence

$$q_r = \text{Constant.}$$

That is, q_r is independent of radius (r). Since the thermal conductivity is also constant, it follows that

$$r\left[\frac{dT}{dr}\right] = \text{Constant.}$$

This relation requires that the product of the radial temperature gradient, dT/dr, and the radius, r, remains constant throughout the insulation. For our situation, the temperature distribution must appear as shown in the sketch.

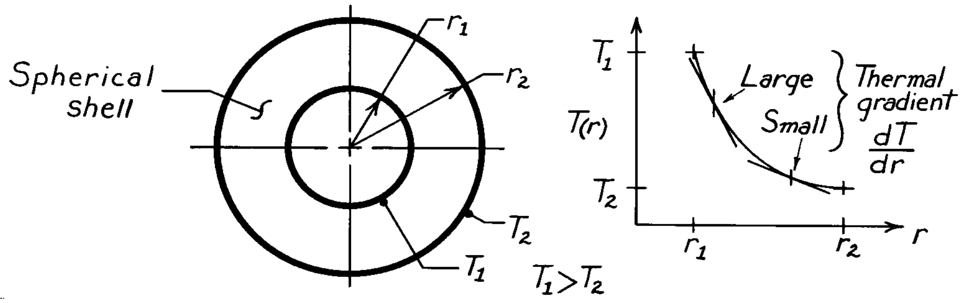
COMMENTS: (1) Note that, while q_r is a constant and independent of r, q''_r is not a constant. How does $q''_r(r)$ vary with r? (2) Recognize that the radial temperature gradient, dT/dr, decreases with increasing radius.

PROBLEM 2.3

KNOWN: A spherical shell with prescribed geometry and surface temperatures.

FIND: Sketch temperature distribution and explain shape of the curve.

SCHEMATIC:



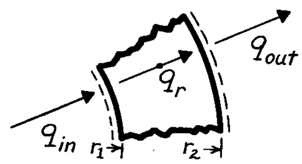
ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in radial (spherical coordinates) direction, (3) No internal generation, (4) Constant properties.

ANALYSIS: Fourier's law, Eq. 2.1, for this one-dimensional, radial (spherical coordinate) system has the form

$$q_r = -k\, A_r \frac{dT}{dr} = -k\left(4\pi r^2\right)\frac{dT}{dr}$$

where A_r is the surface area of a sphere. For steady-state conditions, an energy balance on the system yields $\dot{E}_{in} = \dot{E}_{out}$, since $\dot{E}_g = \dot{E}_{st} = 0$. Hence,

$$q_{in} = q_{out} = q_r \neq q_r(r).$$



That is, q_r is a constant, independent of the radial coordinate. Since the thermal conductivity is constant, it follows that

$$r^2\left[\frac{dT}{dr}\right] = \text{Constant.}$$

This relation requires that the product of the radial temperature gradient, dT/dr, and the radius squared, r^2, remains constant throughout the shell. Hence, the temperature distribution appears as shown in the sketch.

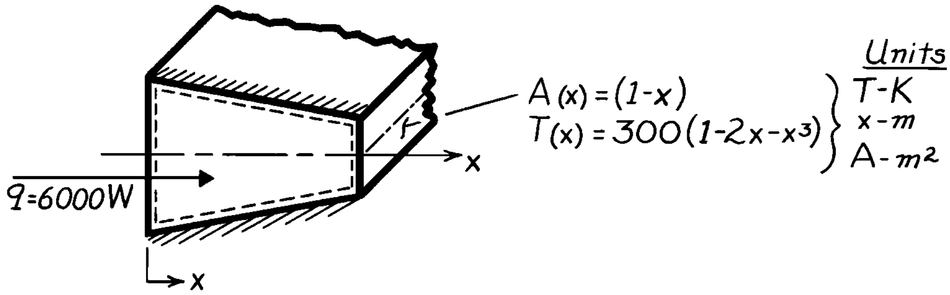
COMMENTS: Note that, for the above conditions, $q_r \neq q_r(r)$; that is, q_r is everywhere constant. How does q''_r vary as a function of radius?

PROBLEM 2.4

KNOWN: Symmetric shape with prescribed variation in cross-sectional area, temperature distribution and heat rate.

FIND: Expression for the thermal conductivity, k.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in x-direction, (3) No internal heat generation.

ANALYSIS: Applying the energy balance, Eq. 1.11a, to the system, it follows that, since $\dot{E}_{in} = \dot{E}_{out}$,

$$q_x = \text{Constant} \neq f(x).$$

Using Fourier's law, Eq. 2.1, with appropriate expressions for A_x and T, yields

$$q_x = -k\,A_x \frac{dT}{dx}$$
$$6000\text{W} = -k \cdot (1\text{-x})\text{m}^2 \cdot \frac{d}{dx}\left[300\left(1-2x-x^3\right)\right]\frac{\text{K}}{\text{m}}.$$

Solving for k and recognizing its units are W/m·K,

$$k = \frac{-6000}{(1\text{-x})\left[300\left(-2-3x^2\right)\right]} = \frac{20}{(1-x)\left(2+3x^2\right)}.$$ <

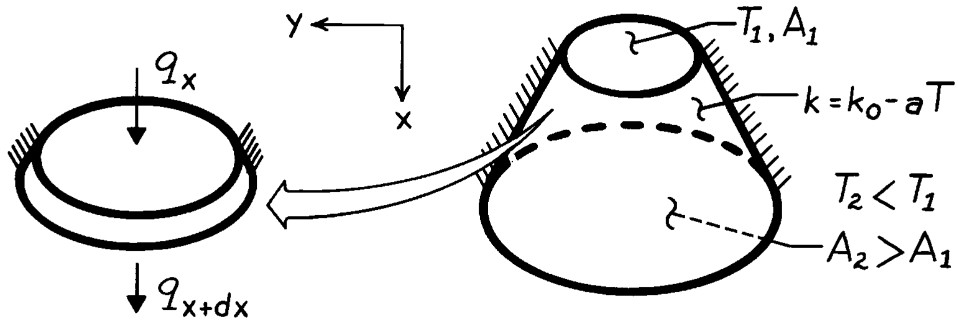
COMMENTS: (1) At x = 0, k = 10W/m·K and k → ∞ as x → 1. (2) Recognize that the 1-D assumption is an approximation which becomes more inappropriate as the area change with x, and hence two-dimensional effects, become more pronounced.

PROBLEM 2.5

KNOWN: End-face temperatures and temperature dependence of k for a truncated cone.

FIND: Variation with axial distance along the cone of q_x, q_x'', k, and dT / dx.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in x (negligible temperature gradients along y), (2) Steady-state conditions, (3) Adiabatic sides, (4) No internal heat generation.

ANALYSIS: For the prescribed conditions, it follows from conservation of energy, Eq. 1.11a, that for a differential control volume, $\dot{E}_{in} = \dot{E}_{out}$ or $q_x = q_{x+dx}$. Hence

q_x is independent of x.

Since A(x) *increases* with *increasing* x, it follows that $q_x'' = q_x / A(x)$ *decreases* with *increasing* x. Since T *decreases* with *increasing* x, k *increases* with *increasing* x. Hence, from Fourier's law, Eq. 2.2,

$$q_x'' = -k\,\frac{dT}{dx},$$

it follows that | dT/dx | *decreases* with increasing x.

PROBLEM 2.6

KNOWN: Temperature dependence of the thermal conductivity, k(T), for heat transfer through a plane wall.

FIND: Effect of k(T) on temperature distribution, T(x).

ASSUMPTIONS: (1) One-dimensional conduction, (2) Steady-state conditions, (3) No internal heat generation.

ANALYSIS: From Fourier's law and the form of k(T),

$$q''_x = -k\,\frac{dT}{dx} = -\left(k_o + aT\right)\frac{dT}{dx}. \tag{1}$$

The shape of the temperature distribution may be inferred from knowledge of $d^2T/dx^2 = d(dT/dx)/dx$. Since q''_x is independent of x for the prescribed conditions,

$$\frac{dq''_x}{dx} = -\frac{d}{dx}\left[\left(k_o + aT\right)\frac{dT}{dx}\right] = 0$$

$$-\left(k_o + aT\right)\frac{d^2T}{dx^2} - a\left[\frac{dT}{dx}\right]^2 = 0.$$

Hence,

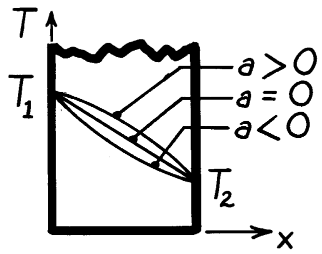
$$\frac{d^2T}{dx^2} = \frac{-a}{k_o + aT}\left[\frac{dT}{dx}\right]^2 \qquad \text{where} \begin{cases} k_o + aT = k > 0 \\ \left[\dfrac{dT}{dx}\right]^2 > 0 \end{cases}$$

from which it follows that for

$a > 0$: $d^2T/dx^2 < 0$

$a = 0$: $d^2T/dx^2 = 0$

$a < 0$: $d^2T/dx^2 > 0$.



COMMENTS: The shape of the distribution could also be inferred from Eq. (1). Since T decreases with increasing x,

$a > 0$: k decreases with increasing x = > | dT/dx | increases with increasing x

$a = 0$: $k = k_o$ = > dT/dx is constant

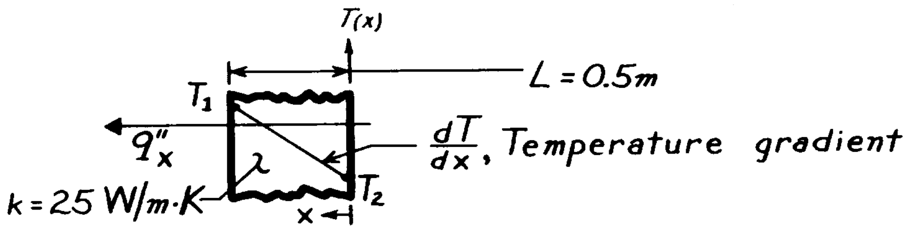
$a < 0$: k increases with increasing x = > | dT/dx | decreases with increasing x.

PROBLEM 2.7

KNOWN: Thermal conductivity and thickness of a one-dimensional system with no internal heat generation and steady-state conditions.

FIND: Unknown surface temperatures, temperature gradient or heat flux.

SCHEMATIC:



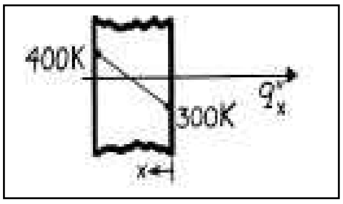
ASSUMPTIONS: (1) One-dimensional heat flow, (2) No internal heat generation, (3) Steady-state conditions, (4) Constant properties.

ANALYSIS: The rate equation and temperature gradient for this system are

$$q''_x = -k\frac{dT}{dx} \qquad \text{and} \qquad \frac{dT}{dx} = \frac{T_1 - T_2}{L}. \qquad (1,2)$$

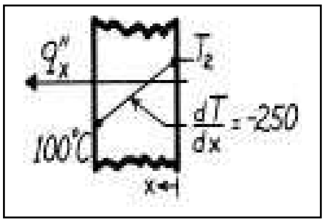
Using Eqs. (1) and (2), the unknown quantities can be determined.

(a) $$\frac{dT}{dx} = \frac{(400-300)\text{K}}{0.5\text{m}} = 200 \text{ K/m}$$



$$q''_x = -25\frac{\text{W}}{\text{m}\cdot\text{K}} \times 200\frac{\text{K}}{\text{m}} = -5000 \text{ W/m}^2.$$ <

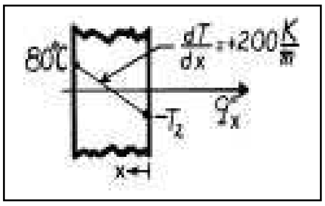
(b) $$q''_x = -25\frac{\text{W}}{\text{m}\cdot\text{K}} \times \left[-250\frac{\text{K}}{\text{m}}\right] = 6250 \text{ W/m}^2$$



$$T_2 = T_1 - L\left[\frac{dT}{dx}\right] = 1000^\circ\text{C-0.5m}\left[\text{-250}\frac{\text{K}}{\text{m}}\right]$$

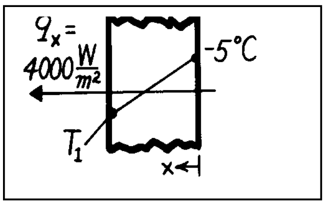
$$T_2 = 225^\circ\text{C}.$$ <

(c) $$q''_x = -25\frac{\text{W}}{\text{m}\cdot\text{K}} \times 200\frac{\text{K}}{\text{m}} = -5000 \text{ W/m}^2$$



$$T_2 = 80^\circ\text{C-0.5m}\left[200\frac{\text{K}}{\text{m}}\right] = -20^\circ\text{C}.$$ <

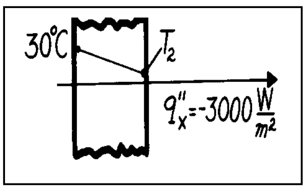
(d) $$\frac{dT}{dx} = -\frac{q''_x}{k} = -\frac{4000 \text{ W/m}^2}{25 \text{ W/m}\cdot\text{K}} = -160\frac{\text{K}}{\text{m}}$$



$$T_1 = L\left[\frac{dT}{dx}\right] + T_2 = 0.5\text{m}\left[\text{-160}\frac{\text{K}}{\text{m}}\right] + \left(-5^\circ\text{C}\right)\sqrt{a^2+b^2}$$

$$T_1 = -85^\circ\text{C}.$$ <

(e) $$\frac{dT}{dx} = -\frac{q''_x}{k} = -\frac{\left(-3000 \text{ W/m}^2\right)}{25 \text{ W/m}\cdot\text{K}} = 120\frac{\text{K}}{\text{m}}$$



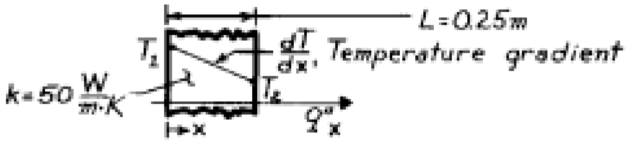
$$T_2 = 30^\circ\text{C-0.5m}\left[120\frac{\text{K}}{\text{m}}\right] = -30^\circ\text{C}.$$

PROBLEM 2.8

KNOWN: One-dimensional system with prescribed thermal conductivity and thickness.

FIND: Unknowns for various temperature conditions and sketch distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) No internal heat generation, (4) Constant properties.

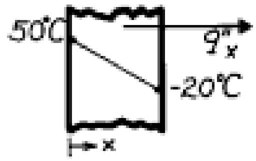
ANALYSIS: The rate equation and temperature gradient for this system are

$$q''_x = -k\frac{dT}{dx} \qquad \text{and} \qquad \frac{dT}{dx} = \frac{T_2 - T_1}{L}. \qquad (1,2)$$

Using Eqs. (1) and (2), the unknown quantities for each case can be determined.

(a)
$$\frac{dT}{dx} = \frac{(-20-50)\,K}{0.25m} = -280\ K/m$$

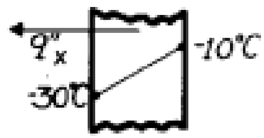
$$q''_x = -50\frac{W}{m\cdot K}\times\left[-280\frac{K}{m}\right] = 14.0\ kW/m^2.$$



<

(b)
$$\frac{dT}{dx} = \frac{(-10-(-30))\,K}{0.25m} = 80\ K/m$$

$$q''_x = -50\frac{W}{m\cdot K}\times\left[80\frac{K}{m}\right] = -4.0\ kW/m^2.$$

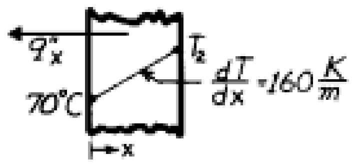


<

(c)
$$q''_x = -50\frac{W}{m\cdot K}\times\left[160\frac{K}{m}\right] = -8.0\ kW/m^2$$

$$T_2 = L\cdot\frac{dT}{dx} + T_1 = 0.25m\times\left[160\frac{K}{m}\right] + 70^\circ C.$$

$$T_2 = 110^\circ C.$$

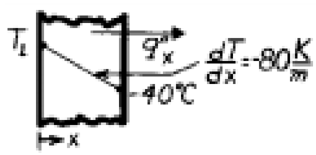


<

(d)
$$q''_x = -50\frac{W}{m\cdot K}\times\left[-80\frac{K}{m}\right] = 4.0\ kW/m^2$$

$$T_1 = T_2 - L\cdot\frac{dT}{dx} = 40^\circ C - 0.25m\left[-80\frac{K}{m}\right].\in$$

$$T_1 = 60^\circ C.$$



<

(e)
$$q''_x = -50\frac{W}{m\cdot K}\times\left[200\frac{K}{m}\right] = -10.0\ kW/m^2$$

$$T_1 = T_2 - L\cdot\frac{dT}{dx} = 30^\circ C - 0.25m\left[200\frac{K}{m}\right] = -20^\circ C.$$

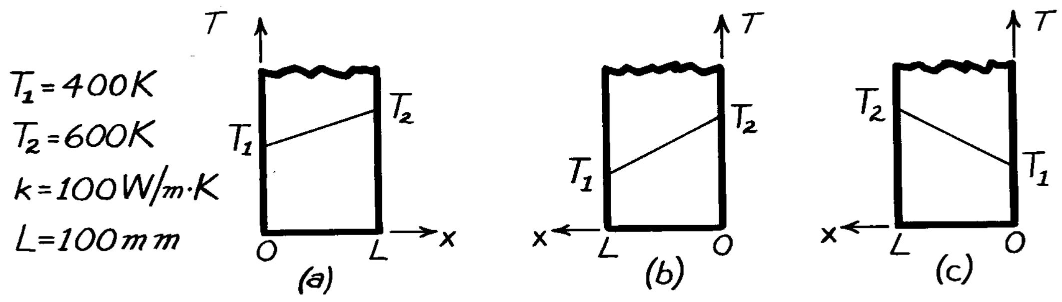
<

PROBLEM 2.9

KNOWN: Plane wall with prescribed thermal conductivity, thickness, and surface temperatures.

FIND: Heat flux, q''_x, and temperature gradient, dT/dx, for the three different coordinate systems shown.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional heat flow, (2) Steady-state conditions, (3) No internal generation, (4) Constant properties.

ANALYSIS: The rate equation for conduction heat transfer is

$$q''_x = -k\frac{dT}{dx}, \qquad (1)$$

where the temperature gradient is constant throughout the wall and of the form

$$\frac{dT}{dx} = \frac{T(L) - T(0)}{L}. \qquad (2)$$

Substituting numerical values, find the temperature gradients,

(a) $$\frac{dT}{dx} = \frac{T_2 - T_1}{L} = \frac{(600-400)K}{0.100m} = 2000\ K/m$$ <

(b) $$\frac{dT}{dx} = \frac{T_1 - T_2}{L} = \frac{(400-600)K}{0.100m} = -2000\ K/m$$ <

(c) $$\frac{dT}{dx} = \frac{T_2 - T_1}{L} = \frac{(600-400)K}{0.100m} = 2000\ K/m.$$ <

The heat rates, using Eq. (1) with k = 100 W/m·K, are

(a) $$q''_x = -100\frac{W}{m\cdot K}\times 2000\ K/m = -200\ kW/m^2$$ <

(b) $$q''_x = -100\frac{W}{m\cdot K}(-2000\ K/m) = +200\ kW/m^2$$ <

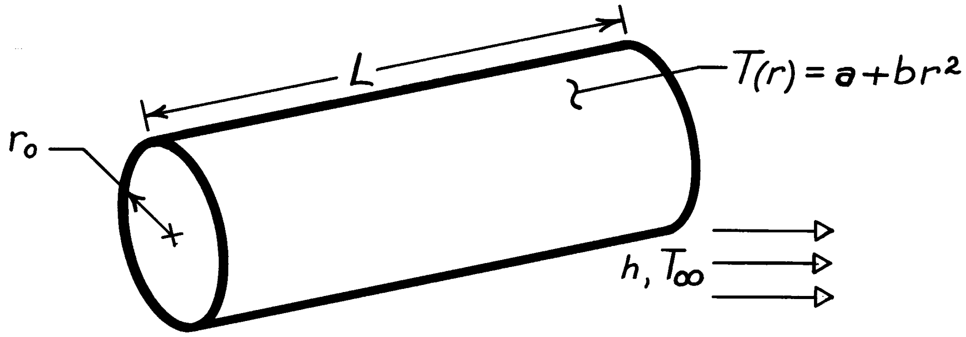
(c) $$q''_x = -100\frac{W}{m\cdot K}\times 2000\ K/m = -200\ kW/m^2$$ <

PROBLEM 2.10

KNOWN: Temperature distribution in solid cylinder and convection coefficient at cylinder surface.

FIND: Expressions for heat rate at cylinder surface and fluid temperature.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, radial conduction, (2) Steady-state conditions, (3) Constant properties.

ANALYSIS: The heat rate from Fourier's law for the radial (cylindrical) system has the form

$$q_r = -kA_r \frac{dT}{dr}.$$

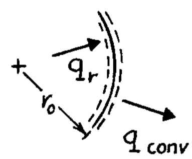
Substituting for the temperature distribution, $T(r) = a + br^2$,

$$q_r = -k(2\pi rL)\,2br \;=\; -4\pi kbLr^2.$$

At the outer surface ($r = r_o$), the conduction heat rate is

$$q_{r=r_o} = -4\pi kbLr_o^2. \qquad <$$

From a surface energy balance at $r = r_o$,



$$q_{r=r_o} = q_{conv} = h(2\pi r_o L)\left[T(r_o) - T_\infty\right],$$

Substituting for $q_{r=r_o}$ and solving for T_∞,

$$T_\infty \;=\; T(r_o) + \frac{2kbr_o}{h}$$

$$T_\infty \;=\; a + br_o^2 + \frac{2kbr_o}{h}$$

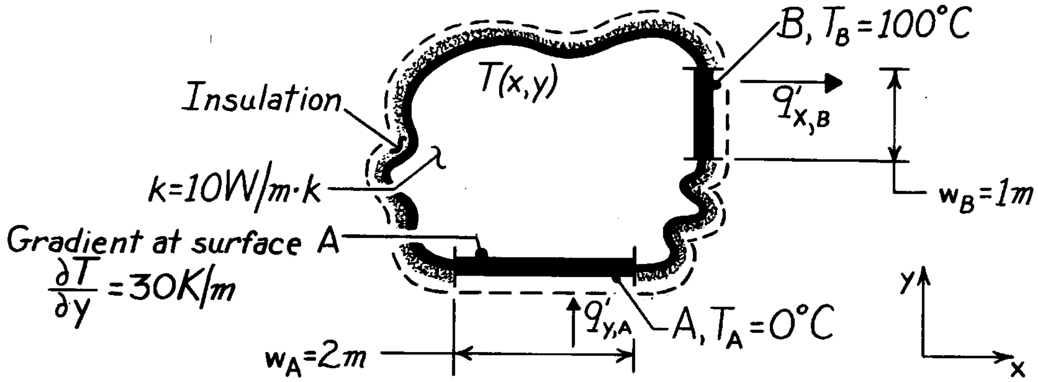
$$T_\infty \;=\; a + br_o\left[r_o + \frac{2k}{h}\right]. \qquad <$$

PROBLEM 2.11

KNOWN: Two-dimensional body with specified thermal conductivity and two isothermal surfaces of prescribed temperatures; one surface, A, has a prescribed temperature gradient.

FIND: Temperature gradients, $\partial T/\partial x$ and $\partial T/\partial y$, at the surface B.

SCHEMATIC:



ASSUMPTIONS: (1) Two-dimensional conduction, (2) Steady-state conditions, (3) No heat generation, (4) Constant properties.

ANALYSIS: At the surface A, the temperature gradient in the x-direction must be zero. That is, $(\partial T/\partial x)_A = 0$. This follows from the requirement that the heat flux vector must be normal to an isothermal surface. The heat rate at the surface A is given by Fourier's law written as

$$q'_{y,A} = -k \cdot w_A \left.\frac{\partial T}{\partial y}\right]_A = -10\frac{W}{m \cdot K} \times 2m \times 30\frac{K}{m} = -600 W/m.$$

On the surface B, it follows that

$$(\partial T/\partial y)_B = 0 \qquad <$$

in order to satisfy the requirement that the heat flux vector be normal to the isothermal surface B. Using the conservation of energy requirement, Eq. 1.11a, on the body, find

$$q'_{y,A} - q'_{x,B} = 0 \qquad \text{or} \qquad q'_{x,B} = q'_{y,A}.$$

Note that,

$$q'_{x,B} = -k \cdot w_B \left.\frac{\partial T}{\partial x}\right]_B$$

and hence

$$(\partial T/\partial x)_B = \frac{-q'_{y,A}}{k \cdot w_B} = \frac{-(-600 \text{ W/m})}{10 \text{ W/m} \cdot \text{K} \times 1\text{m}} = 60 \text{ K/m}. \qquad <$$

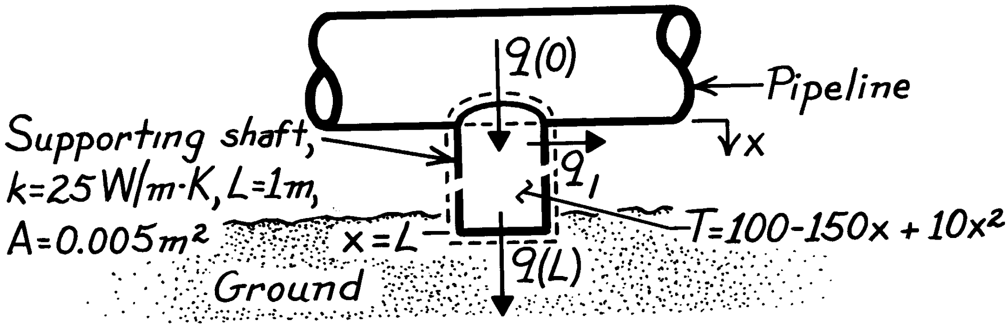
COMMENTS: Note that, in using the conservation requirement, $q'_{in} = +q'_{y,A}$ and $q'_{out} = +q'_{x,B}$.

PROBLEM 2.12

KNOWN: Length and thermal conductivity of a shaft. Temperature distribution along shaft.

FIND: Temperature and heat rates at ends of shaft.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in x, (3) Constant properties.

ANALYSIS: Temperatures at the top and bottom of the shaft are, respectively,

$$T(0) = 100°C \qquad T(L) = -40°C. \qquad <$$

Applying Fourier's law, Eq. 2.1,

$$q_x = -kA\frac{dT}{dx} = -25\ W/m \cdot K\left(0.005\ m^2\right)\left(-150+20x\right)^{\circ}C/m$$

$$q_x = 0.125\left(150\ -\ 20x\right)W.$$

Hence,

$$q_x(0) = 18.75\ W \qquad q_x(L) = 16.25\ W. \qquad <$$

The difference in heat rates, $q_x(0) > q_x(L)$, is due to heat losses q_ℓ from the side of the shaft.

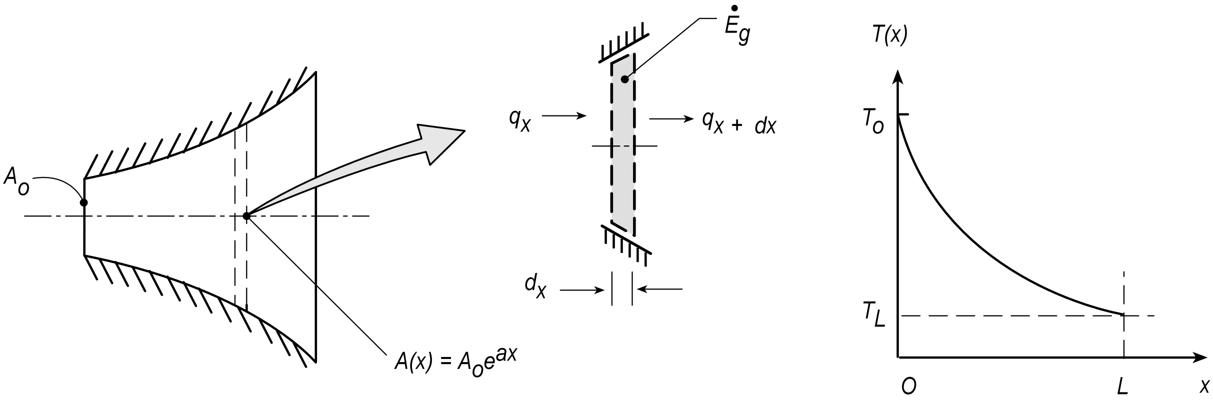
COMMENTS: Heat loss from the side requires the existence of temperature gradients over the shaft cross-section. Hence, specification of T as a function of only x is an approximation.

PROBLEM 2.13

KNOWN: A rod of constant thermal conductivity k and variable cross-sectional area $A_x(x) = A_o e^{ax}$ where A_o and a are constants.

FIND: (a) Expression for the conduction heat rate, $q_x(x)$; use this expression to determine the temperature distribution, T(x); and sketch of the temperature distribution, (b) Considering the presence of volumetric heat generation rate, $\dot{q} = \dot{q}_o \exp(-ax)$, obtain an expression for $q_x(x)$ when the left face, x = 0, is well insulated.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in the rod, (2) Constant properties, (3) Steady-state conditions.

ANALYSIS: Perform an energy balance on the control volume, A(x)·dx,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0$$

$$q_x - q_{x+dx} + \dot{q} \cdot A(x) \cdot dx = 0$$

The conduction heat rate terms can be expressed as a Taylor series and substituting expressions for $\dot{q}$ and A(x),

$$-\frac{d}{dx}(q_x) + \dot{q}_o \exp(-ax) \cdot A_o \exp(ax) = 0 \qquad (1)$$

$$q_x = -k \cdot A(x) \frac{dT}{dx} \qquad (2)$$

(a) With no internal generation, $\dot{q}_o = 0$, and from Eq. (1) find

$$-\frac{d}{dx}(q_x) = 0$$ <

indicating that the heat rate is constant with x. By combining Eqs. (1) and (2)

$$-\frac{d}{dx}\left(-k \cdot A(x) \frac{dT}{dx}\right) = 0 \qquad \text{or} \qquad A(x) \cdot \frac{dT}{dx} = C_1 \qquad (3)$$ <

Continued...

That is, the product of the cross-sectional area and the temperature gradient is a constant, independent of x. Hence, with T(0) > T(L), the temperature distribution is exponential, and as shown in the sketch above. Separating variables and integrating Eq. (3), the general form for the temperature distribution can be determined,

$$A_o \exp(ax) \cdot \frac{dT}{dx} = C_1$$

$$dT = C_1 A_o^{-1} \exp(-ax)\, dx$$

$$T(x) = -C_1 A_o a \exp(-ax) + C_2$$ <

We could use the two temperature boundary conditions, $T_o = T(0)$ and $T_L = T(L)$, to evaluate C_1 and C_2 and, hence, obtain the temperature distribution in terms of T_o and T_L.

(b) With the internal generation, from Eq. (1),

$$-\frac{d}{dx}(q_x) + \dot{q}_o A_o = 0 \qquad \text{or} \qquad q_x = \dot{q}_o A_o x$$ <

That is, the heat rate increases linearly with x.

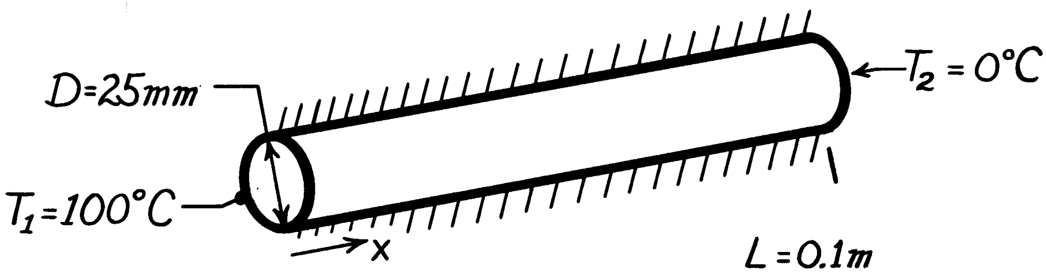
COMMENTS: In part (b), you could determine the temperature distribution using Fourier's law and knowledge of the heat rate dependence upon the x-coordinate. Give it a try!

PROBLEM 2.14

KNOWN: Dimensions and end temperatures of a cylindrical rod which is insulated on its side.

FIND: Rate of heat transfer associated with different rod materials.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction along cylinder axis, (2) Steady-state conditions, (3) Constant properties.

PROPERTIES: The properties may be evaluated from *Tables A-1* to *A-3* at a mean temperature of 50°C = 323K and are summarized below.

ANALYSIS: The heat transfer rate may be obtained from Fourier's law. Since the axial temperature gradient is linear, this expression reduces to

$$q = kA\frac{T_1 - T_2}{L} = k\frac{\pi(0.025\text{m})^2}{4}\frac{(100-0)^\circ\text{C}}{0.1\text{m}} = 0.491(\text{m}\cdot{}^\circ\text{C})\cdot k$$

	Cu (pure)	Al (2024)	St.St. (302)	SiN	Oak	Magnesia (85%)	Pyrex	
k(W/m·K)	401	177	16.3	14.9	0.19	0.052	1.4	
q(W)	197	87	8.0	7.3	0.093	0.026	0.69	<

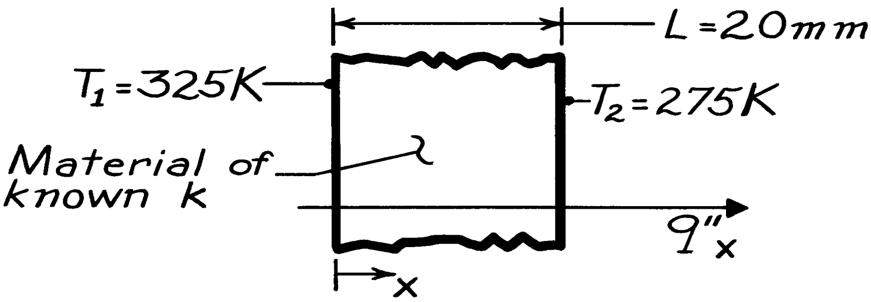
COMMENTS: The k values of Cu and Al were obtained by linear interpolation; the k value of St.St. was obtained by linear extrapolation, as was the value for SiN; the value for magnesia was obtained by linear interpolation; and the values for oak and pyrex are for 300 K.

PROBLEM 2.15

KNOWN: One-dimensional system with prescribed surface temperatures and thickness.

FIND: Heat flux through system constructed of these materials: (a) pure aluminum, (b) plain carbon steel, (c) AISI 316, stainless steel, (d) pyroceram, (e) teflon and (f) concrete.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) No heat generation, (4) Constant thermal properties.

PROPERTIES: The thermal conductivity is evaluated at the average temperature of the system, $T = (T_1+T_2)/2 = (325+275)K/2 = 300K$. Property values and table identification are shown below.

ANALYSIS: For this system, Fourier's law can be written as

$$q''_x = -k\frac{dT}{dx} = -k\frac{T_2 - T_1}{L}.$$

Substituting numerical values, the heat flux is

$$q''_x = -k\frac{(275\text{-}325)K}{20\times10^{-3}m} = +2500\frac{K}{m}\cdot k$$

where q''_x will have units W/m^2 if k has units W/m·K. The heat fluxes for each system follow.

Material	Thermal conductivity Table	k(W/m·K)	Heat flux $q''_x\left(kW/m^2\right)$	
(a) Pure Aluminum	A-1	237	593	<
(b) Plain carbon steel	A-1	60.5	151	
(c) AISI 316, S.S.	A-1	13.4	33.5	
(d) Pyroceram	A-2	3.98	9.95	
(e) Teflon	A-3	0.35	0.88	
(f) Concrete	A-3	1.4	3.5	

COMMENTS: Recognize that the thermal conductivity of these solid materials varies by more than two orders of magnitude.

PROBLEM 2.16

KNOWN: Different thicknesses of three materials: rock, 18 ft; wood, 15 in; and fiberglass insulation, 6 in.

FIND: The insulating quality of the materials as measured by the R-value.

PROPERTIES: *Table A-3* (300K):

Material	Thermal conductivity, W/m·K
Limestone	2.15
Softwood	0.12
Blanket (glass, fiber 10 kg/m^3)	0.048

ANALYSIS: The R-value, a quantity commonly used in the construction industry and building technology, is defined as

$$R \equiv \frac{L(\text{in})}{k\left(\text{Btu}\cdot\text{in}/\text{h}\cdot\text{ft}^2\cdot{}^\circ\text{F}\right)}.$$

The R-value can be interpreted as the thermal resistance of a 1 ft^2 cross section of the material. Using the conversion factor for thermal conductivity between the SI and English systems, the R-values are:

Rock, Limestone, 18 ft:

$$R=\frac{18\text{ ft}\times 12\dfrac{\text{in}}{\text{ft}}}{2.15\dfrac{\text{W}}{\text{m}\cdot\text{K}}\times 0.5778\dfrac{\text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F}}{\text{W/m}\cdot\text{K}}\times 12\dfrac{\text{in}}{\text{ft}}}=14.5\left(\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}\right)^{-1}$$

Wood, Softwood, 15 in:

$$R=\frac{15\text{ in}}{0.12\dfrac{\text{W}}{\text{m}\cdot\text{K}}\times 0.5778\dfrac{\text{Btu}/\text{h}\cdot\text{ft}\cdot{}^\circ\text{F}}{\text{W}/\text{m}\cdot\text{K}}\times 12\dfrac{\text{in}}{\text{ft}}}=18\left(\text{Btu}/\text{h}\cdot\text{ft}^2\cdot{}^\circ\text{F}\right)^{-1}$$

Insulation, Blanket, 6 in:

$$R=\frac{6\text{ in}}{0.048\dfrac{\text{W}}{\text{m}\cdot\text{K}}\times 0.5778\dfrac{\text{Btu}/\text{h}\cdot\text{ft}\cdot{}^\circ\text{F}}{\text{W}/\text{m}\cdot\text{K}}\times 12\dfrac{\text{in}}{\text{ft}}}=18\left(\text{Btu}/\text{h}\cdot\text{ft}^2\cdot{}^\circ\text{F}\right)^{-1}$$

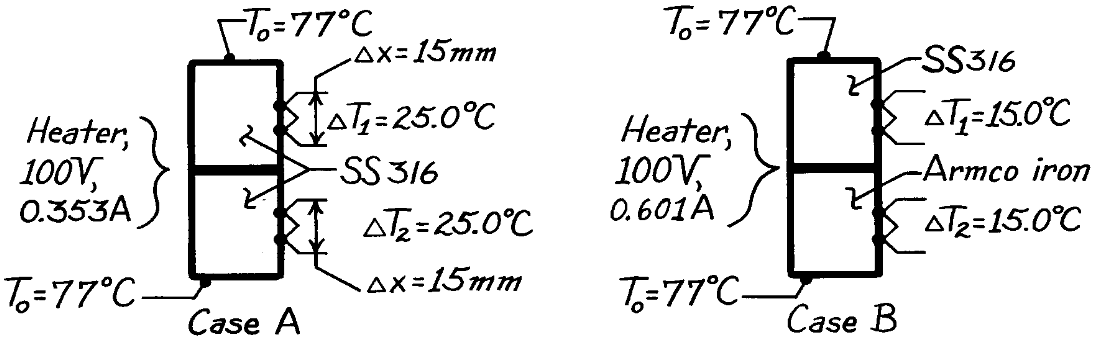
COMMENTS: The R-value of 19 given in the advertisement is reasonable.

PROBLEM 2.17

KNOWN: Electrical heater sandwiched between two identical cylindrical (30 mm dia. × 60 mm length) samples whose opposite ends contact plates maintained at T_o.

FIND: (a) Thermal conductivity of SS316 samples for the prescribed conditions (A) and their average temperature, (b) Thermal conductivity of Armco iron sample for the prescribed conditions (B), (c) Comment on advantages of experimental arrangement, lateral heat losses, and conditions for which $\Delta T_1 \neq \Delta T_2$.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional heat transfer in samples, (2) Steady-state conditions, (3) Negligible contact resistance between materials.

PROPERTIES: *Table A.2*, Stainless steel 316 $(\overline{T} = 400\text{ K})$: $k_{ss} = 15.2\text{ W/m}\cdot\text{K}$; Armco iron $(\overline{T} = 380\text{ K})$: $k_{iron} = 71.6\text{ W/m}\cdot\text{K}$.

ANALYSIS: (a) For Case A recognize that half the heater power will pass through each of the samples which are presumed identical. Apply Fourier's law to a sample

$$q = kA_c \frac{\Delta T}{\Delta x}$$

$$k = \frac{q\Delta x}{A_c \Delta T} = \frac{0.5(100\text{V} \times 0.353\text{A}) \times 0.015\text{ m}}{\pi(0.030\text{ m})^2 / 4 \times 25.0^\circ\text{C}} = 15.0\text{ W/m}\cdot\text{K}. \quad <$$

The total temperature drop across the length of the sample is $\Delta T_1(L/\Delta x)$ = 25°C (60 mm/15 mm) = 100°C. Hence, the heater temperature is T_h = 177°C. Thus the average temperature of the sample is

$$\overline{T} = (T_o + T_h)/2 = 127^\circ\text{C} = 400\text{ K} \quad <$$

.

We compare the calculated value of k with the tabulated value (see above) at 400 K and note the good agreement.

(b) For Case B, we assume that the thermal conductivity of the SS316 sample is the same as that found in Part (a). The heat rate through the Armco iron sample is

Continued …..

$$q_{iron} = q_{heater} - q_{ss} = 100\text{V} \times 0.601\text{A} - 15.0\ \text{W/m}\cdot\text{K} \times \frac{\pi(0.030\ \text{m})^2}{4} \times \frac{15.0°\text{C}}{0.015\ \text{m}}$$

$$q_{iron} = (60.1 - 10.6)\text{W} = 49.5\ \text{W}$$

where

$$q_{ss} = k_{ss} A_c \Delta T_2 / \Delta x_2 .$$

Applying Fourier's law to the iron sample,

$$k_{iron} = \frac{q_{iron}\Delta x_2}{A_c \Delta T_2} = \frac{49.5\ \text{W} \times 0.015\ \text{m}}{\pi(0.030\ \text{m})^2 / 4 \times 15.0°\text{C}} = 70.0\ \text{W/m}\cdot\text{K}. \qquad <$$

The total drop across the iron sample is 15°C(60/15) = 60°C; the heater temperature is (77 + 60)°C = 137°C. Hence the average temperature of the iron sample is

$$\overline{T} = (137\ +\ 77)°\text{C}/2 = 107°\text{C} = 380\ \text{K}. \qquad <$$

We compare the computed value of k with the tabulated value (see above) at 380 K and note the good agreement.

(c) The principal advantage of having two identical samples is the assurance that all the electrical power dissipated in the heater will appear as equivalent heat flows through the samples. With only one sample, heat can flow from the backside of the heater even though insulated.

Heat leakage out the lateral surfaces of the cylindrically shaped samples will become significant when the sample thermal conductivity is comparable to that of the insulating material. Hence, the method is suitable for metallics, but must be used with caution on nonmetallic materials.

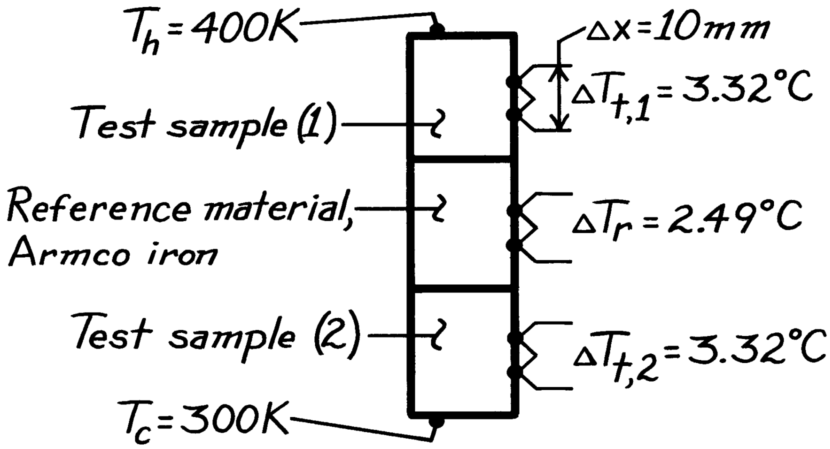
For any combination of materials in the upper and lower position, we expect $\Delta T_1 = \Delta T_2$. However, if the insulation were improperly applied along the lateral surfaces, it is possible that heat leakage will occur, causing $\Delta T_1 \neq \Delta T_2$.

PROBLEM 2.18

KNOWN: Comparative method for measuring thermal conductivity involving two identical samples stacked with a reference material.

FIND: (a) Thermal conductivity of test material and associated temperature, (b) Conditions for which $\Delta T_{t,1} \neq \Delta T_{t,2}$

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer through samples and reference material, (3) Negligible thermal contact resistance between materials.

PROPERTIES: *Table A.2,* Armco iron $\left(\overline{T} = 350\ K\right)$: $k_r = 69.2\ W/m\cdot K$.

ANALYSIS: (a) Recognizing that the heat rate through the samples and reference material, all of the same diameter, is the same, it follows from Fourier's law that

$$k_t \frac{\Delta T_{t,1}}{\Delta x} = k_r \frac{\Delta T_r}{\Delta x} = k_t \frac{\Delta T_{t,2}}{\Delta x}$$

$$k_t = k_r \frac{\Delta T_r}{\Delta T_t} = 69.2\ W/m\cdot K \frac{2.49^\circ C}{3.32^\circ C} = 51.9\ W/m\cdot K. \qquad <$$

We should assign this value a temperature of 350 K. $<$

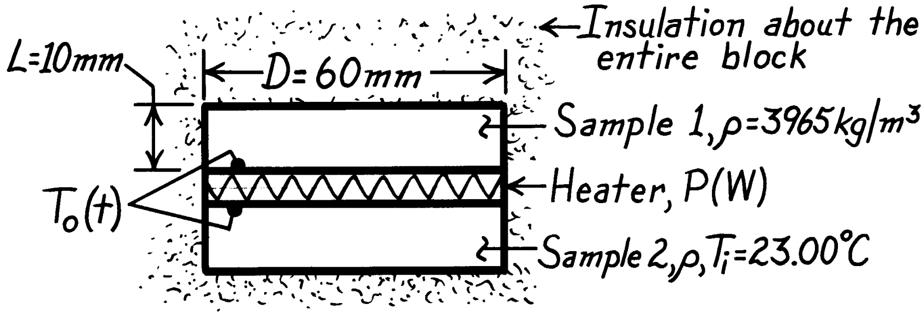
(b) If the test samples are identical in every respect, $\Delta T_{t,1} \neq \Delta T_{t,2}$ if the thermal conductivity is highly dependent upon temperature. Also, if there is heat leakage out the lateral surface, we can expect $\Delta T_{t,2} < \Delta T_{t,1}$. Leakage could be influential, if the thermal conductivity of the test material were less than an order of magnitude larger than that of the insulating material.

PROBLEM 2.19

KNOWN: Identical samples of prescribed diameter, length and density initially at a uniform temperature T_i, sandwich an electric heater which provides a uniform heat flux q''_o for a period of time Δt_o. Conditions shortly after energizing and a long time after de-energizing heater are prescribed.

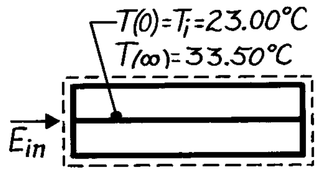
FIND: Specific heat and thermal conductivity of the test sample material. From these properties, identify type of material using Table A.1 or A.2.

SCHEMATIC:



ASSUMPTIONS: (1) One dimensional heat transfer in samples, (2) Constant properties, (3) Negligible heat loss through insulation, (4) Negligible heater mass.

ANALYSIS: Consider a control volume about the samples and heater, and apply conservation of energy over the time interval from t = 0 to ∞



$$E_{in} - E_{out} = \Delta E = E_f - E_i$$

$$P\Delta t_o - 0 = Mc_p\left[T(\infty) - T_i\right]$$

where energy inflow is prescribed by the Case A power condition and the final temperature T_f by Case B. Solving for c_p,

$$c_p = \frac{P\Delta t_o}{M\left[T(\infty) - T_i\right]} = \frac{15\text{ W} \times 120\text{ s}}{2 \times 3965\text{ kg/m}^3\left(\pi \times 0.060^2/4\right)\text{m}^2 \times 0.010\text{ m}\left[33.50 - 23.00\right]^\circ\text{C}}$$

$$c_p = 765\text{ J/kg} \cdot \text{K}$$ <

where $M = \rho V = 2\rho(\pi D^2/4)L$ is the mass of both samples. For Case A, the transient thermal response of the heater is given by

Continued …..

PROBLEM 2.19 (Cont.)

$$T_o(t) - T_i = 2q_o''\left[\frac{t}{\pi\rho c_p k}\right]^{1/2}$$

$$k = \frac{t}{\pi\rho c_p}\left[\frac{2q_o''}{T_o(t) - T_i}\right]^2$$

$$k = \frac{30\text{ s}}{\pi \times 3965\text{ kg/m}^3 \times 765\text{ J/kg}\cdot\text{K}}\left[\frac{2 \times 2653\text{ W/m}^2}{(24.57\ -\ 23.00)^\circ\text{C}}\right]^2 = 36.0\text{ W/m}\cdot\text{K} \qquad <$$

where

$$q_o'' = \frac{P}{2A_s} = \frac{P}{2(\pi D^2/4)} = \frac{15\text{ W}}{2(\pi \times 0.060^2/4)\text{m}^2} = 2653\text{ W/m}^2.$$

With the following properties now known,

$\rho = 3965$ kg/m^3 $\qquad$ $c_p = 765$ J/kg·K $\qquad$ $k = 36$ W/m·K

entries in Table A.1 are scanned to determine whether these values are typical of a metallic material. Consider the following,

- metallics with low ρ generally have higher thermal conductivities,
- specific heats of both types of materials are of similar magnitude,
- the low k value of the sample is typical of poor metallic conductors which generally have much higher specific heats,
- more than likely, the material is nonmetallic.

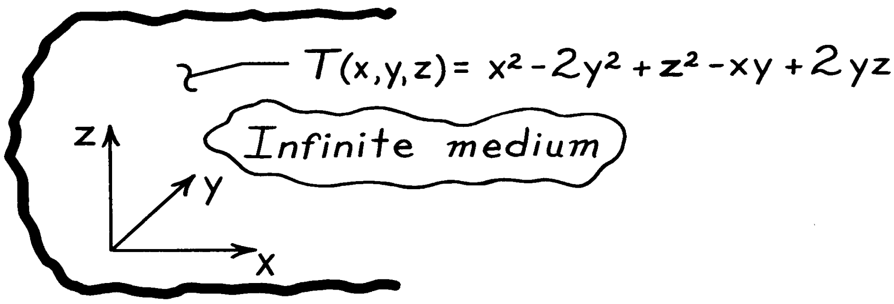
From Table A.2, the second entry, polycrystalline aluminum oxide, has properties at 300 K corresponding to those found for the samples. $<$

PROBLEM 2.20

KNOWN: Temperature distribution, T(x,y,z), within an infinite, homogeneous body at a given instant of time.

FIND: Regions where the temperature changes with time.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties of infinite medium and (2) No internal heat generation.

ANALYSIS: The temperature distribution throughout the medium, at any instant of time, must satisfy the heat equation. For the three-dimensional cartesian coordinate system, with constant properties and no internal heat generation, the heat equation, Eq. 2.15, has the form

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=\frac{1}{\alpha}\frac{\partial T}{\partial t}. \tag{1}$$

If T(x,y,z) satisfies this relation, conservation of energy is satisfied at every point in the medium. Substituting T(x,y,z) into the Eq. (1), first find the gradients, ∂T/∂x, ∂T/∂y, and ∂T/∂z.

$$\frac{\partial}{\partial x}(2x - y)+\frac{\partial}{\partial y}(-4y - x + 2z)+\frac{\partial}{\partial z}(2z+2y)=\frac{1}{\alpha}\frac{\partial T}{\partial t}.$$

Performing the differentiations,

$$2-4+2=\frac{1}{\alpha}\frac{\partial T}{\partial t}.$$

Hence,

$$\frac{\partial T}{\partial t}=0$$

which implies that, at the prescribed instant, the temperature is everywhere independent of time.

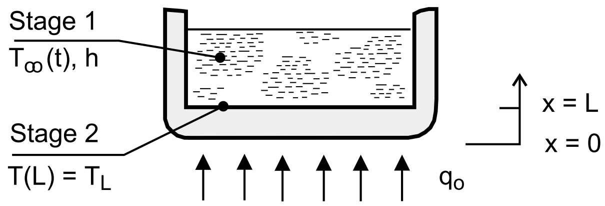
COMMENTS: Since we do not know the initial and boundary conditions, we cannot determine the temperature distribution, T(x,y,z), at any future time. We can only determine that, for this special instant of time, the temperature will not change.

PROBLEM 2.21

KNOWN: Diameter D, thickness L and initial temperature T_i of pan. Heat rate from stove to bottom of pan. Convection coefficient h and variation of water temperature $T_\infty(t)$ during Stage 1. Temperature T_L of pan surface in contact with water during Stage 2.

FIND: Form of heat equation and boundary conditions associated with the two stages.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in pan bottom, (2) Heat transfer from stove is uniformly distributed over surface of pan in contact with the stove, (3) Constant properties.

ANALYSIS:

Stage 1

Heat Equation: $$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Boundary Conditions: $$-k\left.\frac{\partial T}{\partial x}\right|_{x=0} = q_o'' = \frac{q_o}{\left(\pi D^2/4\right)}$$

$$-k\left.\frac{\partial T}{\partial x}\right|_{x=L} = h\left[T(L,t) - T_\infty(t)\right]$$

Initial Condition: $$T(x,0) = T_i$$

Stage 2

Heat Equation: $$\frac{d^2 T}{dx^2} = 0$$

Boundary Conditions: $$-k\left.\frac{dT}{dx}\right|_{x=0} = q_o''$$

$$T(L) = T_L$$

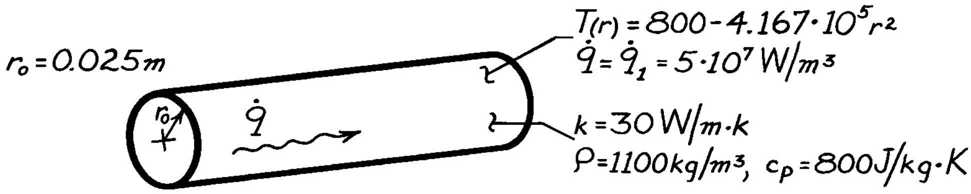
COMMENTS: Stage 1 is a transient process for which $T_\infty(t)$ must be determined separately. As a first approximation, it could be estimated by neglecting changes in thermal energy storage by the pan bottom and assuming that all of the heat transferred from the stove acted to increase thermal energy storage within the water. Hence, with $q \approx Mc_p\, dT_\infty/dt$, where M and c_p are the mass and specific heat of the water in the pan, $T_\infty(t) \approx (q/Mc_p)\, t$.

PROBLEM 2.22

KNOWN: Steady-state temperature distribution in a cylindrical rod having uniform heat generation of $\dot{q}_1 = 5\times10^7\ \text{W/m}^3$.

FIND: (a) Steady-state centerline and surface heat transfer rates per unit length, q_r'. (b) Initial time rate of change of the centerline and surface temperatures in response to a change in the generation rate from $\dot{q}_1$ to $\dot{q}_2 = 10^8\ \text{W/m}^3$.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in the r direction, (2) Uniform generation, and (3) Steady-state for $\dot{q}_1 = 5\times10^7\ \text{W/m}^3$.

ANALYSIS: (a) From the rate equations for cylindrical coordinates,

$$q_r'' = -k\frac{\partial T}{\partial r} \qquad q = -kA_r\frac{\partial T}{\partial r}.$$

Hence,

$$q_r = -k(2\pi rL)\frac{\partial T}{\partial r}$$

or

$$q_r' = -2\pi kr\frac{\partial T}{\partial r}$$

where ∂T/∂r may be evaluated from the prescribed temperature distribution, T(r).

At r = 0, the gradient is (∂T/∂r) = 0. Hence, from Eq. (1) the heat rate is

$$q_r'(0) = 0. \qquad <$$

At $r = r_o$, the temperature gradient is

$$\left.\frac{\partial T}{\partial r}\right]_{r=r_o} = -2\left[4.167\times10^5\ \frac{K}{m^2}\right](r_o) = -2\left(4.167\times10^5\right)(0.025m)$$

$$\left.\frac{\partial T}{\partial r}\right]_{r=r_o} = -0.208\times10^5\ K/m.$$

Continued …..

Hence, the heat rate at the outer surface ($r = r_o$) per unit length is

$$q'_r(r_o) = -2\pi[30\ \mathrm{W/m\cdot K}](0.025\mathrm{m})\left[-0.208\times10^5\ \mathrm{K/m}\right]$$

$$q'_r(r_o) = 0.980\times10^5\ \mathrm{W/m}. \qquad <$$

(b) Transient (time-dependent) conditions will exist when the generation is changed, and for the prescribed assumptions, the temperature is determined by the following form of the heat equation, Eq. 2.20

$$\frac{1}{r}\frac{\partial}{\partial r}\left[kr\frac{\partial T}{\partial r}\right] + \dot{q}_2 = \rho c_p \frac{\partial T}{\partial t}$$

Hence

$$\frac{\partial T}{\partial t} = \frac{1}{\rho c_p}\left[\frac{1}{r}\frac{\partial}{\partial r}\left[kr\frac{\partial T}{\partial r}\right] + \dot{q}_2\right].$$

However, initially (at t = 0), the temperature distribution is given by the prescribed form, $T(r) = 800 - 4.167\times10^5 r^2$, and

$$\frac{1}{r}\frac{\partial}{\partial r}\left[kr\frac{\partial T}{\partial r}\right] = \frac{k}{r}\frac{\partial}{\partial r}\left[r\left(-8.334\times10^5\cdot r\right)\right]$$

$$= \frac{k}{r}\left(-16.668\times10^5\cdot r\right)$$

$$= 30\ \mathrm{W/m\cdot K}\left[-16.668\times10^5\ \mathrm{K/m^2}\right]$$

$$= -5\times10^7\ \mathrm{W/m^3}\ \left(\text{the original } \dot{q} = \dot{q}_1\right).$$

Hence, everywhere in the wall,

$$\frac{\partial T}{\partial t} = \frac{1}{1100\ \mathrm{kg/m^3}\times 800\ \mathrm{J/kg\cdot K}}\left[-5\times10^7 + 10^8\right]\mathrm{W/m^3}$$

or

$$\frac{\partial T}{\partial t} = 56.82\ \mathrm{K/s}. \qquad <$$

COMMENTS: (1) The value of $(\partial T/\partial t)$ will decrease with increasing time, until a new steady-state condition is reached and once again $(\partial T/\partial t) = 0$.

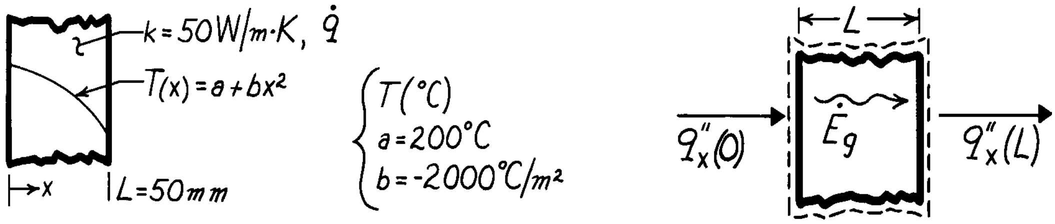
(2) By applying the energy conservation requirement, Eq. 1.11a, to a unit length of the rod for the steady-state condition, $\dot{E}'_{in} - E'_{out} + \dot{E}'_{gen} = 0.$ Hence $q'_r(0) - q'_r(r_o) = -\dot{q}_1\left(\pi r_o^2\right).$

PROBLEM 2.23

KNOWN: Temperature distribution in a one-dimensional wall with prescribed thickness and thermal conductivity.

FIND: (a) The heat generation rate, $\dot{q}$, in the wall, (b) Heat fluxes at the wall faces and relation to $\dot{q}$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat flow, (3) Constant properties.

ANALYSIS: (a) The appropriate form of the heat equation for steady-state, one-dimensional conditions with constant properties is Eq. 2.15 re-written as

$$\dot{q} = -k\frac{d}{dx}\left[\frac{dT}{dx}\right]$$

Substituting the prescribed temperature distribution,

$$\dot{q} = -k\frac{d}{dx}\left[\frac{d}{dx}\left(a+bx^2\right)\right] = -k\frac{d}{dx}\left[2bx\right] = -2bk$$

$$\dot{q} = -2\left(-2000^\circ\mathrm{C/m^2}\right)\times 50\ \mathrm{W/m\cdot K} = 2.0\times 10^5\ \mathrm{W/m^3}.$$ <

(b) The heat fluxes at the wall faces can be evaluated from Fourier's law,

$$q''_x(x) = -k\left.\frac{dT}{dx}\right]_x .$$

Using the temperature distribution T(x) to evaluate the gradient, find

$$q''_x(x) = -k\frac{d}{dx}\left[a+bx^2\right] = -2kbx.$$

The fluxes at x = 0 and x = L are then

$$q''_x(0) = 0$$ <

$$q''_x(L) = -2kbL = -2\times 50\mathrm{W/m\cdot K}\left(-2000^\circ\mathrm{C/m^2}\right)\times 0.050\mathrm{m}$$

$$q''_x(L) = 10{,}000\ \mathrm{W/m^2}.$$ <

COMMENTS: From an overall energy balance on the wall, it follows that, for a unit area,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0 \qquad q''_x(0) - q''_x(L) + \dot{q}L = 0$$

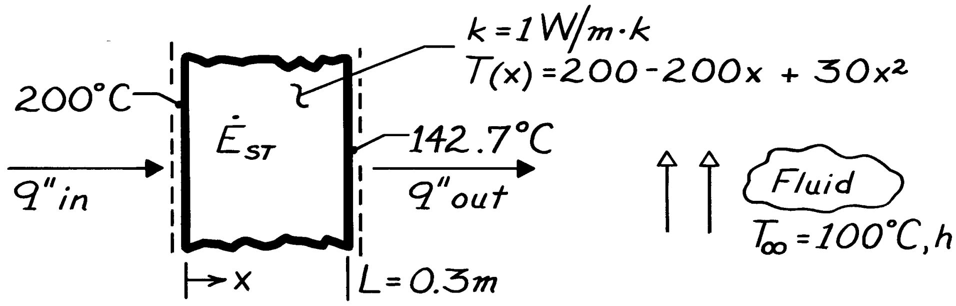
$$\dot{q} = \frac{q''_x(L) - q''_x(0)}{L} = \frac{10{,}000\ \mathrm{W/m^2} - 0}{0.050\mathrm{m}} = 2.0\times 10^5\mathrm{W/m^3}.$$

PROBLEM 2.24

KNOWN: Wall thickness, thermal conductivity, temperature distribution, and fluid temperature.

FIND: (a) Surface heat rates and rate of change of wall energy storage per unit area, and (b) Convection coefficient.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in x, (2) Constant k.

ANALYSIS: (a) From Fourier's law,

$$q''_x = -k\frac{\partial T}{\partial x} = (200 - 60x)\cdot k$$

$$q''_{in} = q''_{x=0} = 200\frac{°C}{m}\times 1\frac{W}{m\cdot K} = 200\ W/m^2$$ <

$$q''_{out} = q''_{x=L} = (200 - 60\times 0.3)°C/m\ \times\ 1\ W/m\cdot K = 182\ W/m^2.$$ <

Applying an energy balance to a control volume about the wall, Eq. 1.11a,

$$\dot{E}''_{in} - \dot{E}''_{out} = \dot{E}''_{st}$$

$$\dot{E}''_{st} = q''_{in} - q''_{out} = 18\ W/m^2.$$ <

(b) Applying a surface energy balance at x = L,

$$q''_{out} = h[T(L) - T_\infty]$$

$$h = \frac{q''_{out}}{T(L) - T_\infty} = \frac{182\ W/m^2}{(142.7 - 100)°C}$$

$$h = 4.3\ W/m^2\cdot K.$$ <

COMMENTS: (1) From the heat equation,

$$(\partial T/\partial t) = (k/\rho c_p)\ \partial^2 T/\partial x^2 = 60(k/\rho c_p),$$

it follows that the temperature is increasing with time at every point in the wall.

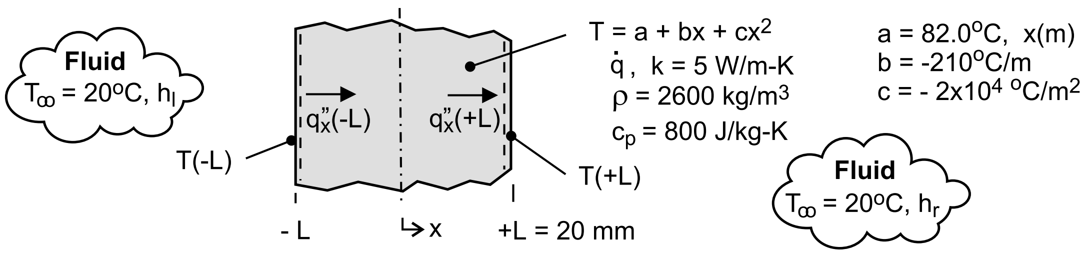
(2) The value of h is small and is typical of free convection in a gas.

PROBLEM 2.25

KNOWN: Analytical expression for the steady-state temperature distribution of a plane wall experiencing uniform volumetric heat generation $\dot{q}$ while convection occurs at both of its surfaces.

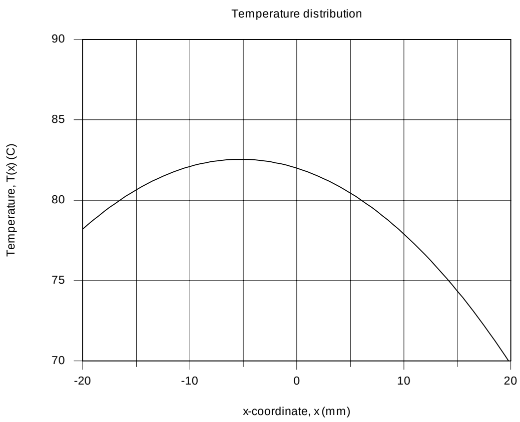
FIND: (a) Sketch the temperature distribution, T(x), and identify significant physical features, (b) Determine $\dot{q}$, (c) Determine the surface heat fluxes, $q''_x(-L)$ and $q''_x(+L)$; how are these fluxes related to the generation rate; (d) Calculate the convection coefficients at the surfaces x = L and x = +L, (e) Obtain an expression for the heat flux distribution, $q''_x(x)$; explain significant features of the distribution; (f) If the source of heat generation is suddenly deactivated ($\dot{q}$ = 0), what is the rate of change of energy stored at this instant; (g) Determine the temperature that the wall will reach eventually with $\dot{q} = 0$; determine the energy that must be removed by the fluid per unit area of the wall to reach this state.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform volumetric heat generation, (3) Constant properties.

ANALYSIS: (a) Using the analytical expression in the Workspace of IHT, the temperature distribution appears as shown below. The significant features include (1) parabolic shape, (2) maximum does not occur at the mid-plane, T(-5.25 mm) = 83.3°C, (3) the gradient at the x = +L surface is greater than at x = -L. Find also that T(-L) = 78.2°C and T(+L) = 69.8°C for use in part (d).



(b) Substituting the temperature distribution expression into the appropriate form of the heat diffusion equation, Eq. 2.15, the rate of volumetric heat generation can be determined.

$$\frac{d}{dx}\left(\frac{dT}{dx}\right)+\frac{\dot{q}}{k}=0 \qquad \text{where} \qquad T(x)=a+bx+cx^2$$

$$\frac{d}{dx}(0+b+2cx)+\frac{\dot{q}}{k}=(0+2c)+\frac{\dot{q}}{k}=0$$

Continued …..

PROBLEM 2.25 (Cont.)

$$\dot{q} = -2ck = -2\left(-2\times10^{4}{}^\circ\mathrm{C/m^2}\right)5\,\mathrm{W/m\cdot K} = 2\times10^{5}\,\mathrm{W/m^3} \qquad <$$

(c) The heat fluxes at the two boundaries can be determined using Fourier's law and the temperature distribution expression.

$$q''_x(x) = -k\frac{dT}{dx} \qquad \text{where} \qquad T(x) = a + bx + cx^2$$

$$q''_x(-L) = -k\left[0 + b + 2cx\right]_{x=-L} = -\left[b - 2cL\right]k$$

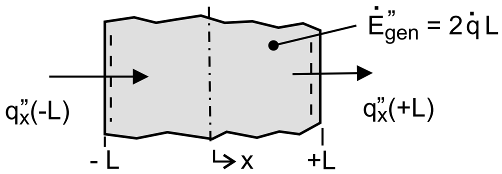
$$q''_x(-L) = -\left[-210^\circ\mathrm{C/m} - 2\left(-2\times10^{4}{}^\circ\mathrm{C/m^2}\right)0.020\mathrm{m}\right]\times5\,\mathrm{W/m\cdot K} = -2950\,\mathrm{W/m^2} \qquad <$$

$$q''_x(+L) = -\left(b + 2cL\right)k = +5050\,\mathrm{W/m^2} \qquad <$$

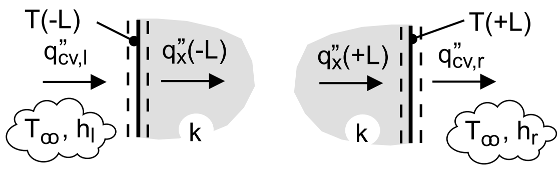
From an overall energy balance on the wall as shown in the sketch below, $\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = 0$,

$$+q''_x(-L) - q''_x(+L) + 2\dot{q}L \overset{?}{=} 0 \qquad \text{or} \qquad -2950\,\mathrm{W/m^2} - 5050\,\mathrm{W/m^2} + 8000\,\mathrm{W/m^2} = 0$$

where $2\dot{q}L = 2\times2\times10^{5}\,\mathrm{W/m^3}\times0.020\ \mathrm{m} = 8000\,\mathrm{W/m^2}$, so the equality is satisfied



Part (c) Overall energy balance



Part (d) Surface energy balances

(d) The convection coefficients, h_l and h_r, for the left- and right-hand boundaries (x = -L and x= +L, respectively), can be determined from the convection heat fluxes that are equal to the conduction fluxes at the boundaries. See the surface energy balances in the sketch above. See also part (a) result for T(-L) and T(+L).

$$q''_{cv,\ell} = q''_x(-L)$$

$$h_l\left[T_\infty - T(-L)\right] = h_l\left[20 - 78.2\right]\mathrm{K} = -2950\,\mathrm{W/m^2} \qquad h_l = 51\,\mathrm{W/m^2\cdot K} \qquad <$$

$$q''_{cv,r} = q''_x(+L)$$

$$h_r\left[T(+L) - T_\infty\right] = h_r\left[69.8 - 20\right]\mathrm{K} = +5050\,\mathrm{W/m^2} \qquad h_r = 101\,\mathrm{W/m^2\cdot K} \qquad <$$

(e) The expression for the heat flux distribution can be obtained from Fourier's law with the temperature distribution

$$q''_x(x) = -k\frac{dT}{dx} = -k\left[0 + b + 2cx\right]$$

$$q''_x(x) = -5\,\mathrm{W/m\cdot K}\left[-210^\circ\mathrm{C/m} + 2\left(-2\times10^{4}{}^\circ\mathrm{C/m^2}\right)\right]x = 1050 + 2\times10^{5}x \qquad <$$

Continued …..

The distribution is linear with the x-coordinate. The maximum temperature will occur at the location where $q''_x(x_{max}) = 0$,

$$x_{max} = -\frac{1050\,W/m^2}{2\times10^5\ W/m^3} = -5.25\times10^{-3}m = -5.25\,mm \qquad <$$

(f) If the source of the heat generation is suddenly deactivated so that $\dot{q}$ = 0, the appropriate form of the heat diffusion equation for the ensuing transient conduction is

$$k\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) = \rho\, c_p \frac{\partial T}{\partial t}$$

At the instant this occurs, the temperature distribution is still $T(x) = a + bx + cx^2$. The right-hand term represents the rate of energy storage per unit volume,

$$\dot{E}''_{st} = k\frac{\partial}{\partial x}[0+b+2cx] = k[0+2c] = 5\,W/m\cdot K\times2\left(-2\times10^4\,°C/m^2\right) = -2\times10^5\ W/m^3 \qquad <$$

(g) With no heat generation, the wall will eventually ($t \to \infty$) come to equilibrium with the fluid, $T(x,\infty) = T_\infty = 20°C$. To determine the energy that must be removed from the wall to reach this state, apply the conservation of energy requirement over an interval basis, Eq. 1.11b. The "initial" state is that corresponding to the steady-state temperature distribution, T_i, and the "final" state has $T_f = 20°C$. We've used T_∞ as the reference condition for the energy terms.

$$E''_{in} - E''_{out} = \Delta E''_{st} = E''_f - E''_i \qquad \text{with} \qquad E''_{in} = 0.$$

$$-E''_{out} = \rho\, c_p 2L(T_f - T_\infty) - \rho\, c_p \int_{-L}^{+L}(T_i - T_\infty)dx$$

$$E''_{out} = \rho\, c_p \int_{-L}^{+L}\left[a + bx + cx^2 - T_\infty\right]dx = \rho\, c_p\left[ax + bx^2/2 + cx^3/3 - T_\infty x\right]_{-L}^{+L}$$

$$E''_{out} = \rho\, c_p\left[2aL + 0 + 2cx^3/3 - 2T_\infty L\right]$$

$$E''_{out} = 2600\,kg/m^3\times800\,J/kg\cdot K\Big[2\times82°C\times0.020m + 2\left(-2\times10^4\,°C/m^2\right)$$

$$(0.020m)^3/3 - 2(20°C)0.020m\Big]$$

$$E''_{out} = 4.94\times10^6\ J/m^2 \qquad <$$

COMMENTS: (1) In part (a), note that the temperature gradient is larger at x = + L than at x = - L. This is consistent with the results of part (c) in which the conduction heat fluxes are evaluated.

Continued …..

PROBLEM 2.25 (Cont.)

(2) In evaluating the conduction heat fluxes, $q''_x(x)$, it is important to recognize that this flux is in the positive x-direction. See how this convention is used in formulating the energy balance in part (c).

(3) It is good practice to represent energy balances with a schematic, clearly defining the system or surface, showing the CV or CS with dashed lines, and labeling the processes. Review again the features in the schematics for the energy balances of parts (c & d).

(4) Re-writing the heat diffusion equation introduced in part (b) as

$$-\frac{d}{dx}\left(-k\frac{dT}{dx}\right)+\dot{q}=0$$

recognize that the term in parenthesis is the heat flux. From the differential equation, note that if the differential of this term is a constant $(\dot{q}/k)$, then the term must be a linear function of the x-coordinate. This agrees with the analysis of part (e).

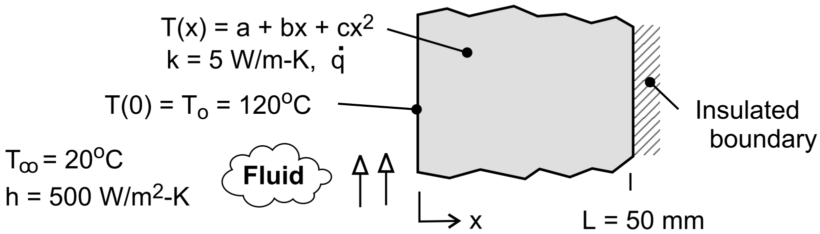
(5) In part (f), we evaluated $\dot{E}_{st}$, the rate of energy change stored in the wall at the instant the volumetric heat generation was deactivated. Did you notice that $\dot{E}_{st} = -2\times10^5\ W/m^3$ is the same value of the deactivated $\dot{q}$? How do you explain this?

PROBLEM 2.26

KNOWN: Steady-state conduction with uniform internal energy generation in a plane wall; temperature distribution has quadratic form. Surface at x=0 is prescribed and boundary at x = L is insulated.

FIND: (a) Calculate the internal energy generation rate, $\dot{q}$, by applying an overall energy balance to the wall, (b) Determine the coefficients a, b, and c, by applying the boundary conditions to the prescribed form of the temperature distribution; plot the temperature distribution and label as Case 1, (c) Determine new values for a, b, and c for conditions when the convection coefficient is halved, and the generation rate remains unchanged; plot the temperature distribution and label as Case 2; (d) Determine new values for a, b, and c for conditions when the generation rate is doubled, and the convection coefficient remains unchanged ($h = 500\ W/m^2 \cdot K$); plot the temperature distribution and label as Case 3.

SCHEMATIC:



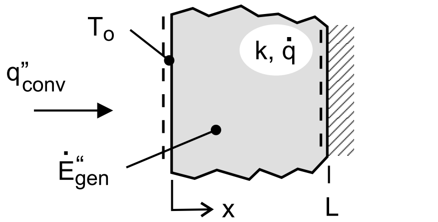
ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction with constant properties and uniform internal generation, and (3) Boundary at x = L is adiabatic.

ANALYSIS: (a) The internal energy generation rate can be calculated from an overall energy balance on the wall as shown in the schematic below.

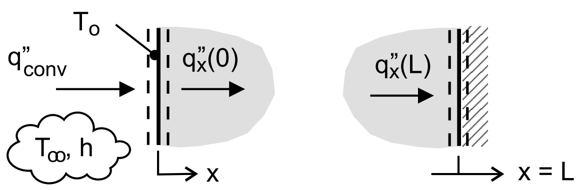
$$\dot{E}''_{in} - \dot{E}''_{out} + \dot{E}''_{gen} = 0 \qquad \text{where} \qquad \dot{E}''_{in} = q''_{conv}$$

$$h\left(T_\infty - T_o\right) + \dot{q}\,L = 0 \tag{1}$$

$$\dot{q} = -h\left(T_\infty - T_o\right)/L = -500\,W/m^2 \cdot K\left(20-120\right)°C/0.050\ m = 1.0\times10^6\ W/m^3 \quad <$$



(a) Overall energy balance



(b) Surface energy balances

(b) The coefficients of the temperature distribution, $T(x) = a + bx + cx^2$, can be evaluated by applying the boundary conditions at x = 0 and x = L. See Table 2.1 for representation of the boundary conditions, and the schematic above for the relevant surface energy balances.

Boundary condition at x = 0, convection surface condition

$$\dot{E}''_{in} - \dot{E}''_{out} = q''_{conv} - q''_x(0) = 0 \qquad \text{where} \qquad q''_x(0) = -k\left.\frac{dT}{dx}\right)_{x=0}$$

$$h\left(T_\infty - T_o\right) - \left[-k\left(0 + b + 2cx\right)_{x=0}\right] = 0$$

Continued …..

$$b = -h\left(T_\infty - T_o\right)/k = -500\,W/m^2\cdot K\left(20-120\right)°C/5\,W/m\cdot K = 1.0\times10^4\,K/m \quad <$$

Boundary condition at x = L, adiabatic or insulated surface

$$\dot{E}_{in} - \dot{E}_{out} = -q''_x\left(L\right) = 0 \qquad \text{where} \qquad q''_x\left(L\right) = -k\left.\frac{dT}{dx}\right)_{x=L}$$

$$k\left[0 + b + 2cx\right]_{x=L} = 0 \tag{3}$$

$$c = -b/2L = -1.0\times10^4\,K/m/\left(2\times0.050m\right) = -1.0\times10^5\,K/m^2 \quad <$$

Since the surface temperature at x = 0 is known, $T(0) = T_o = 120°C$, find

$$T\left(0\right) = 120°C = a + b\cdot0 + c\cdot0 \qquad \text{or} \qquad a = 120°C \tag{4} \quad <$$

Using the foregoing coefficients with the expression for T(x) in the Workspace of IHT, the temperature distribution can be determined and is plotted as Case 1 in the graph below.

(c) Consider Case 2 when the convection coefficient is halved, $h_2 = h/2 = 250\ W/m^2\cdot K$, $\dot{q} = 1\times10^6\ W/m^3$ and other parameters remain unchanged except that $T_o \neq 120°C$. We can determine a, b, and c for the temperature distribution expression by repeating the analyses of parts (a) and (b).

Overall energy balance on the wall, see Eqs. (1,4)

$$a = T_o = \dot{q}L/h + T_\infty = 1\times10^6\,W/m^3\times0.050m/250\,W/m^2\cdot K + 20°C = 220°C \quad <$$

Surface energy balance at x = 0, see Eq. (2)

$$b = -h\left(T_\infty - T_o\right)/k = -250\,W/m^2\cdot K\left(20-220\right)°C/5\,W/m\cdot K = 1.0\times10^4\,K/m \quad <$$

Surface energy balance at x = L, see Eq. (3)

$$c = -b/2L = -1.0\times10^4\,K/m/\left(2\times0.050m\right) = -1.0\times10^5\,K/m^2 \quad <$$

The new temperature distribution, T_2 (x), is plotted as Case 2 below.

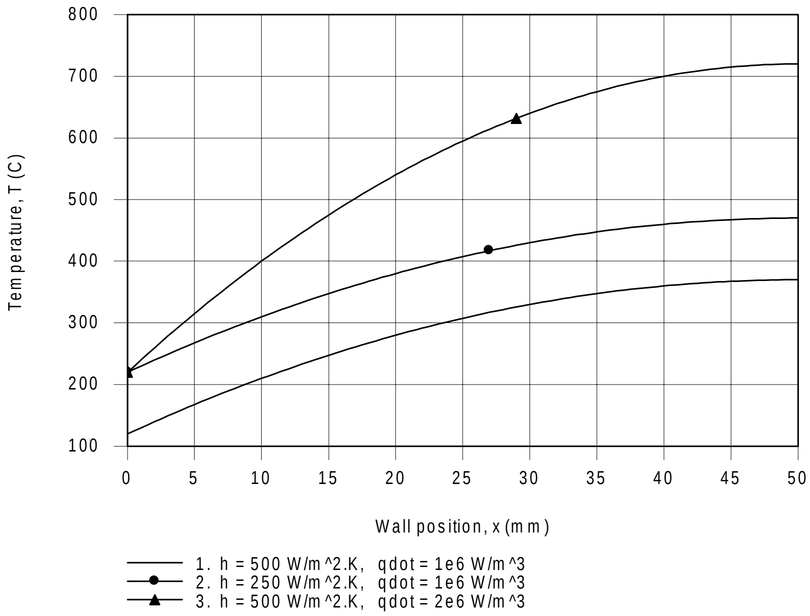
(d) Consider Case 3 when the internal energy volumetric generation rate is doubled, $\dot{q}_3 = 2\dot{q} = 2\times10^6\,W/m^3$, $h = 500\ W/m^2\cdot K$, and other parameters remain unchanged except that $T_o \neq 120°C$. Following the same analysis as part (c), the coefficients for the new temperature distribution, T (x), are

$$a = 220°C \qquad b = 2\times10^4\,K/m \qquad c = -2\times10^5\,K/m^2 \quad <$$

and the distribution is plotted as Case 3 below.

Continued …..

PROBLEM 2.26 (Cont.)



COMMENTS: Note the following features in the family of temperature distributions plotted above. The temperature gradients at x = L are zero since the boundary is insulated (adiabatic) for all cases. The shapes of the distributions are all quadratic, with the maximum temperatures at the insulated boundary.

By halving the convection coefficient for Case 2, we expect the surface temperature T_o to increase relative to the Case 1 value, since the same heat flux is removed from the wall $(\dot{q}L)$ but the convection resistance has increased.

By doubling the generation rate for Case 3, we expect the surface temperature T_o to increase relative to the Case 1 value, since double the amount of heat flux is removed from the wall $(2\dot{q}L)$.

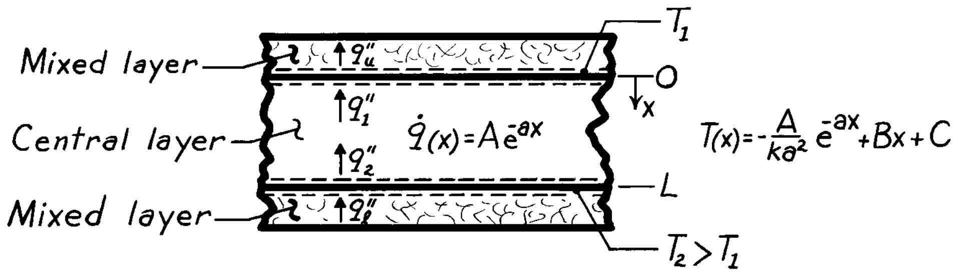
Can you explain why T_o is the same for Cases 2 and 3, yet the insulated boundary temperatures are quite different? Can you explain the relative magnitudes of T(L) for the three cases?

PROBLEM 2.27

KNOWN: Temperature distribution and distribution of heat generation in central layer of a solar pond.

FIND: (a) Heat fluxes at lower and upper surfaces of the central layer, (b) Whether conditions are steady or transient, (c) Rate of thermal energy generation for the entire central layer.

SCHEMATIC:



ASSUMPTIONS: (1) Central layer is stagnant, (2) One-dimensional conduction, (3) Constant properties

ANALYSIS: (a) The desired fluxes correspond to conduction fluxes in the central layer at the lower and upper surfaces. A general form for the conduction flux is

$$q''_{cond} = -k\frac{\partial T}{\partial x} = -k\left[\frac{A}{ka}e^{-ax} + B\right].$$

Hence,

$$q''_l = q''_{cond(x=L)} = -k\left[\frac{A}{ka}e^{-aL} + B\right] \quad q''_u = q''_{cond(x=0)} = -k\left[\frac{A}{ka} + B\right].$$ <

(b) Conditions are steady if $\partial T/\partial t = 0$. Applying the heat equation,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad -\frac{A}{k}e^{-ax} + \frac{A}{k}e^{-ax} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Hence conditions are *steady* since

$$\partial T/\partial t = 0 \qquad \text{(for all } 0 \le x \le L).$$ <

(c) For the central layer, the energy generation is

$$\dot{E}''_g = \int_0^L \dot{q}\, dx = A\int_0^L e^{-ax}\, dx$$

$$\dot{E}_g = -\frac{A}{a}e^{-ax}\Big|_0^L = -\frac{A}{a}\left(e^{-aL} - 1\right) = \frac{A}{a}\left(1 - e^{-aL}\right).$$ <

Alternatively, from an overall energy balance,

$$q''_2 - q''_1 + \dot{E}''_g = 0 \qquad \dot{E}''_g = q''_1 - q''_2 = \left(-q''_{cond(x=0)}\right) - \left(-q''_{cond(x=L)}\right)$$

$$\dot{E}_g = k\left[\frac{A}{ka} + B\right] - k\left[\frac{A}{ka}e^{-aL} + B\right] = \frac{A}{a}\left(1 - e^{-aL}\right).$$

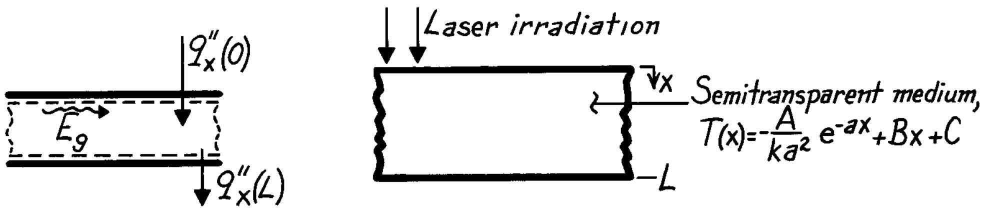
COMMENTS: Conduction is in the negative x-direction, necessitating use of minus signs in the above energy balance.

PROBLEM 2.28

KNOWN: Temperature distribution in a semi-transparent medium subjected to radiative flux.

FIND: (a) Expressions for the heat flux at the front and rear surfaces, (b) Heat generation rate $\dot{q}(x)$, (c) Expression for absorbed radiation per unit surface area in terms of A, a, B, C, L, and k.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in medium, (3) Constant properties, (4) All laser irradiation is absorbed and can be characterized by an internal volumetric heat generation term $\dot{q}(x)$.

ANALYSIS: (a) Knowing the temperature distribution, the surface heat fluxes are found using Fourier's law,

$$q''_x = -k\left[\frac{dT}{dx}\right] = -k\left[-\frac{A}{ka^2}(-a)e^{-ax} + B\right]$$

Front Surface, x=0: $$q''_x(0) = -k\left[+\frac{A}{ka}\cdot 1 + B\right] = -\left[\frac{A}{a} + kB\right] \qquad <$$

Rear Surface, x=L: $$q''_x(L) = -k\left[+\frac{A}{ka}e^{-aL} + B\right] = -\left[\frac{A}{a}e^{-aL} + kB\right]. \qquad <$$

(b) The heat diffusion equation for the medium is

$$\frac{d}{dx}\left(\frac{dT}{dx}\right) + \frac{\dot{q}}{k} = 0 \qquad \text{or} \qquad \dot{q} = -k\frac{d}{dx}\left(\frac{dT}{dx}\right)$$

$$\dot{q}(x) = -k\frac{d}{dx}\left[+\frac{A}{ka}e^{-ax} + B\right] = Ae^{-ax}. \qquad <$$

(c) Performing an energy balance on the medium,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0$$

recognize that $\dot{E}_g$ represents the absorbed irradiation. On a unit area basis

$$\dot{E}''_g = -\dot{E}''_{in} + \dot{E}''_{out} = -q''_x(0) + q''_x(L) = +\frac{A}{a}\left(1 - e^{-aL}\right). \qquad <$$

Alternatively, evaluate $\dot{E}''_g$ by integration over the volume of the medium,

$$\dot{E}''_g = \int_0^L \dot{q}(x)dx = \int_0^L Ae^{-ax}dx = -\frac{A}{a}\left[e^{-ax}\right]_0^L = \frac{A}{a}\left(1 - e^{-aL}\right).$$

PROBLEM 2.29

KNOWN: Steady-state temperature distribution in a one-dimensional wall of thermal conductivity, $T(x) = Ax^3 + Bx^2 + Cx + D$.

FIND: Expressions for the heat generation rate in the wall and the heat fluxes at the two wall faces (x = 0,L).

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat flow, (3) Homogeneous medium.

ANALYSIS: The appropriate form of the heat diffusion equation for these conditions is

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0 \quad \text{or} \quad \dot{q} = -k\frac{d^2T}{dx^2}.$$

Hence, the generation rate is

$$\dot{q} = -k\frac{d}{dx}\left[\frac{dT}{dx}\right] = -k\frac{d}{dx}\left[3Ax^2 + 2Bx + C + 0\right]$$

$$\dot{q} = -k\left[6Ax + 2B\right] \qquad <$$

which is linear with the coordinate x. The heat fluxes at the wall faces can be evaluated from Fourier's law,

$$q''_x = -k\frac{dT}{dx} = -k\left[3Ax^2 + 2Bx + C\right]$$

using the expression for the temperature gradient derived above. Hence, the heat fluxes are:

Surface x=0:

$$q''_x(0) = -kC \qquad <$$

Surface x=L:

$$q''_x(L) = -k\left[3AL^2 + 2BL + C\right]. \qquad <$$

COMMENTS: (1) From an overall energy balance on the wall, find

$$\dot{E}''_{in} - \dot{E}''_{out} + \dot{E}''_g = 0$$

$$q''_x(0) - q''_x(L) + \dot{E}''_g = (-kC) - (-k)\left[3AL^2 + 2BL + C\right] + \dot{E}''_g = 0$$

$$\dot{E}''_g = -3AkL^2 - 2BkL.$$

From integration of the volumetric heat rate, we can also find $\dot{E}''_g$ as

$$\dot{E}''_g = \int_0^L \dot{q}(x)dx = \int_0^L -k\left[6Ax + 2B\right]dx = -k\left[3Ax^2 + 2Bx\right]_0^L$$

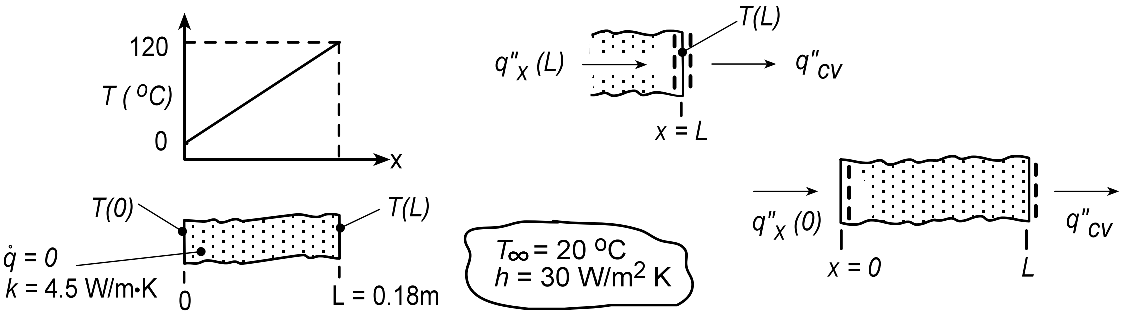
$$\dot{E}''_g = -3AkL^2 - 2BkL.$$

PROBLEM 2.30

KNOWN: Plane wall with no internal energy generation.

FIND: Determine whether the prescribed temperature distribution is possible; explain your reasoning. With the temperatures T(0) = 0°C and T_∞ = 20°C fixed, compute and plot the temperature T(L) as a function of the convection coefficient for the range $10 \le h \le 100\ W/m^2 \cdot K$.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) No internal energy generation, (3) Constant properties, (4) No radiation exchange at the surface x = L, and (5) Steady-state conditions.

ANALYSIS: (a) Is the prescribed temperature distribution possible? If so, the energy balance at the surface x = L as shown above in the Schematic, must be satisfied.

$$\dot{E}_{in} - \dot{E}_{out} \ ?=?\ 0 \qquad\qquad q''_x(L) - q''_{cv} \ ?=?\ 0 \qquad\qquad (1,2)$$

where the conduction and convection heat fluxes are, respectively,

$$q''_x(L) = -k\left.\frac{dT}{dx}\right)_{x=L} = -k\frac{T(L)-T(0)}{L} = -4.5\,W/m\cdot K \times (120-0)^\circ C/0.18\,m = -3000\,W/m^2$$

$$q''_{cv} = h\left[T(L) - T_\infty\right] = 30\,W/m^2\cdot K \times (120-20)^\circ C = 3000\,W/m^2$$

Substituting the heat flux values into Eq. (2), find (-3000) - (3000) ≠ 0 and therefore, the temperature distribution is not possible.

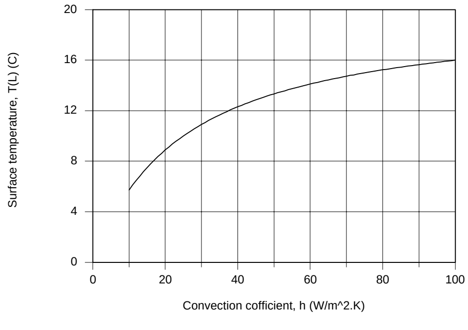
(b) With T(0) = 0°C and T_∞ = 20°C, the temperature at the surface x = L, T(L), can be determined from an overall energy balance on the wall as shown above in the Schematic,

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad q''_x(0) - q''_{cv} = 0 \qquad -k\frac{T(L)-T(0)}{L} - h\left[T(L) - T_\infty\right] = 0$$

$$-4.5\,W/m\cdot K\left[T(L) - 0^\circ C\right]/0.18\,m - 30\,W/m^2\cdot K\left[T(L) - 20^\circ C\right] = 0$$

$$T(L) = 10.9^\circ C \qquad\qquad <$$

Using this same analysis, T(L) as a function of the convection coefficient can be determined and plotted. We don't expect T(L) to be linearly dependent upon h. Note that as h increases to larger values, T(L) approaches T_∞. To what value will T(L) approach as h decreases?

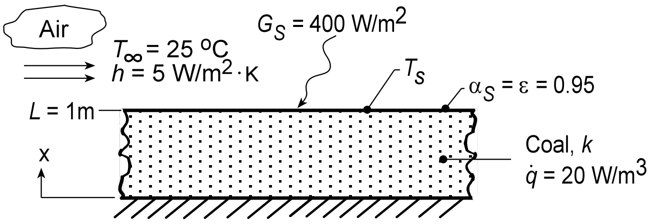


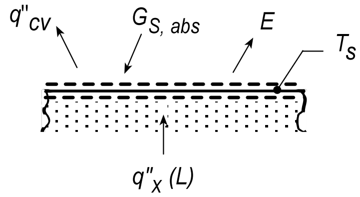
PROBLEM 2.31

KNOWN: Coal pile of prescribed depth experiencing uniform volumetric generation with convection, absorbed irradiation and emission on its upper surface.

FIND: (a) The appropriate form of the heat diffusion equation (HDE) and whether the prescribed temperature distribution satisfies this HDE; conditions at the bottom of the pile, x = 0; sketch of the temperature distribution with labeling of key features; (b) Expression for the conduction heat rate at the location x = L; expression for the surface temperature T_s based upon a surface energy balance at x = L; evaluate T_S and T(0) for the prescribed conditions; (c) Based upon typical daily averages for G_S and h, compute and plot T_S and T(0) for (1) h = 5 W/m^2·K with $50 \le G_S \le 500$ W/m^2, (2) G_S = 400 W/m^2 with $5 \le h \le 50$ W/m^2·K.

SCHEMATIC:





ASSUMPTIONS: (1) One-dimensional conduction, (2) Uniform volumetric heat generation, (3) Constant properties, (4) Negligible irradiation from the surroundings, and (5) Steady-state conditions.

PROPERTIES: *Table A.3*, Coal (300K): k = 0.26 W/m.K

ANALYSIS: (a) For one-dimensional, steady-state conduction with uniform volumetric heat generation and constant properties the heat diffusion equation (HDE) follows from Eq. 2.16,

$$\frac{d}{dx}\left(\frac{dT}{dx}\right)+\frac{\dot{q}}{k}=0 \qquad (1) <$$

Substituting the temperature distribution into the HDE, Eq. (1),

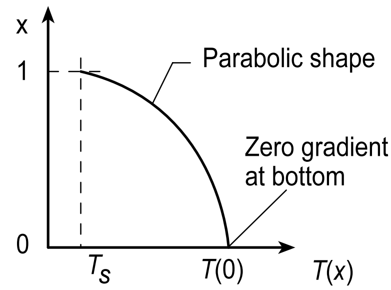
$$T(x)=T_S+\frac{\dot{q}L^2}{2k}\left(1-\frac{x^2}{L^2}\right) \qquad \frac{d}{dx}\left[0+\frac{\dot{q}L^2}{2k}\left(0-\frac{2x}{L^2}\right)\right]+\frac{\dot{q}}{k}\,?=?\,0 \qquad (2,3)$$

we find that it does indeed satisfy the HDE for all values of x. <

From Eq. (2), note that the temperature distribution must be quadratic, with maximum value at x = 0. At x = 0, the heat flux is

$$q''_x(0)=-k\left.\frac{dT}{dx}\right)_{x=0}=-k\left[0+\frac{\dot{q}L^2}{2k}\left(0-\frac{2x}{L^2}\right)\right]_{x=0}=0$$

so that the gradient at x = 0 is zero. Hence, the bottom is insulated.



(b) From an overall energy balance on the pile, the conduction heat flux at the surface must be

$$q''_x(L)=\dot{E}''_g=\dot{q}L \qquad <$$

Continued...

PROBLEM 2.31 (Cont.)

From a surface energy balance per unit area shown in the Schematic above,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0 \qquad q''_x(L) - q''_{cv} + G_{S,abs} - E = 0$$

$$\dot{q}L - h\left(T_s - T_\infty\right) + 0.95G_S - \varepsilon\sigma T_s^4 = 0 \tag{4}$$

$$20\,\text{W/m}^3 \times 1\,\text{m} - 5\,\text{W/m}^2\cdot\text{K}\left(T_s - 298\,\text{K}\right) + 0.95 \times 400\,\text{W/m}^2 - 0.95 \times 5.67 \times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4 T_s^4 = 0$$

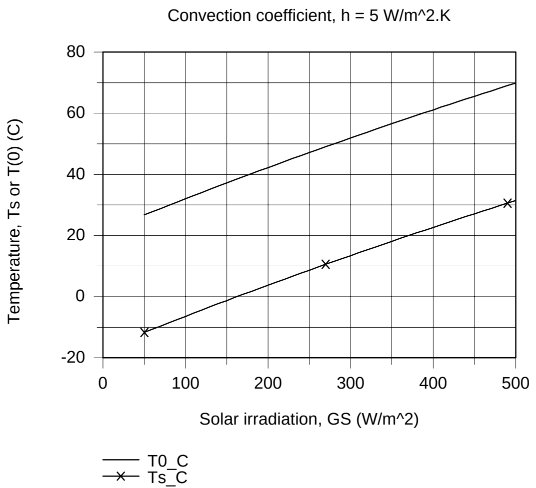
$$T_s = 295.7\text{ K} = 22.7°\text{C}$$ <

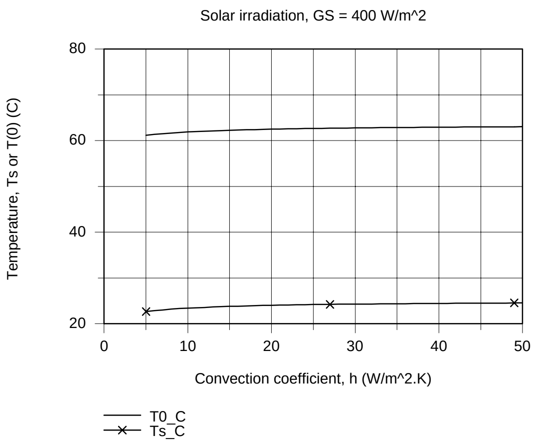
From Eq. (2) with x = 0, find

$$T(0) = T_s + \frac{\dot{q}L^2}{2k} = 22.7°\text{C} + \frac{30\,\text{W/m}^2 \times (1\,\text{m})^2}{2 \times 0.26\,\text{W/m}\cdot\text{K}} = 61.1°\text{C} \tag{5}$$ <

where the thermal conductivity for coal was obtained from Table A.3.

(c) Two plots are generated using Eq. (4) and (5) for T_s and T(0), respectively; (1) with h = 5 W/m^2·K for $50 \le G_S \le 500$ W/m^2 and (2) with G_S = 400 W/m^2 for $5 \le h \le 50$ W/m^2·K.





From the T vs. h plot with G_S = 400 W/m^2, note that the convection coefficient does not have a major influence on the surface or bottom coal pile temperatures. From the T vs. G_S plot with h = 5 W/m^2·K, note that the solar irradiation has a very significant effect on the temperatures. The fact that T_s is less than the ambient air temperature, T_∞, and, in the case of very low values of G_S, below freezing, is a consequence of the large magnitude of the emissive power E.

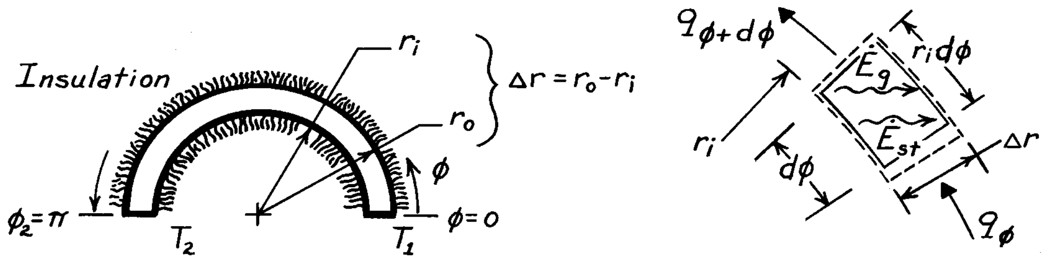
COMMENTS: In our analysis we ignored irradiation from the sky, an environmental radiation effect you'll consider in Chapter 12. Treated as large isothermal surroundings, $G_{sky} = \sigma T_{sky}^4$ where T_{sky} = -30°C for very clear conditions and nearly air temperature for cloudy conditions. For low G_S conditions we should consider G_{sky}, the effect of which will be to predict higher values for T_s and T(0).

PROBLEM 2.32

KNOWN: Cylindrical system with negligible temperature variation in the r,z directions.

FIND: (a) Heat equation beginning with a properly defined control volume, (b) Temperature distribution $T(\phi)$ for steady-state conditions with no internal heat generation and constant properties, (c) Heat rate for Part (b) conditions.

SCHEMATIC:



ASSUMPTIONS: (1) T is independent of r,z, (2) $\Delta r = (r_o - r_i) << r_i$.

ANALYSIS: (a) Define the control volume as $V = r_i d\phi \cdot \Delta r \cdot L$ where L is length normal to page. Apply the conservation of energy requirement, Eq. 1.11a,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st} \qquad q_\phi - q_{\phi+d\phi} + \dot{q}V = \rho Vc\frac{\partial T}{\partial t} \qquad (1,2)$$

where
$$q_\phi = -k(\Delta r \cdot L)\frac{\partial T}{r_i \partial \phi} \qquad q_{\phi+d\phi} = q_\phi + \frac{\partial}{\partial \phi}(q_\phi)d\phi. \qquad (3,4)$$

Eqs. (3) and (4) follow from Fourier's law, Eq. 2.1, and from Eq. 2.7, respectively. Combining Eqs. (3) and (4) with Eq. (2) and canceling like terms, find

$$\frac{1}{r_i^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \dot{q} = \rho c\frac{\partial T}{\partial t}. \qquad (5) <$$

Since temperature is independent of r and z, this form agrees with Eq. 2.20.

(b) For steady-state conditions with $\dot{q} = 0$, the heat equation, (5), becomes

$$\frac{d}{d\phi}\left[k\frac{dT}{d\phi}\right] = 0. \qquad (6)$$

With constant properties, it follows that $dT/d\phi$ is constant which implies $T(\phi)$ is linear in ϕ. That is,

$$\frac{dT}{d\phi} = \frac{T_2 - T_1}{\phi_2 - \phi_1} = +\frac{1}{\pi}(T_2 - T_1) \quad \text{or} \quad T(\phi) = T_1 + \frac{1}{\pi}(T_2 - T_1)\phi. \qquad (7,8) <$$

(c) The heat rate for the conditions of Part (b) follows from Fourier's law, Eq. (3), using the temperature gradient of Eq. (7). That is,

$$q_\phi = -k(\Delta r \cdot L)\frac{1}{r_i}\left[+\frac{1}{\pi}(T_2 - T_1)\right] = -k\left[\frac{r_o - r_i}{\pi r_i}\right]L(T_2 - T_1). \qquad (9) <$$

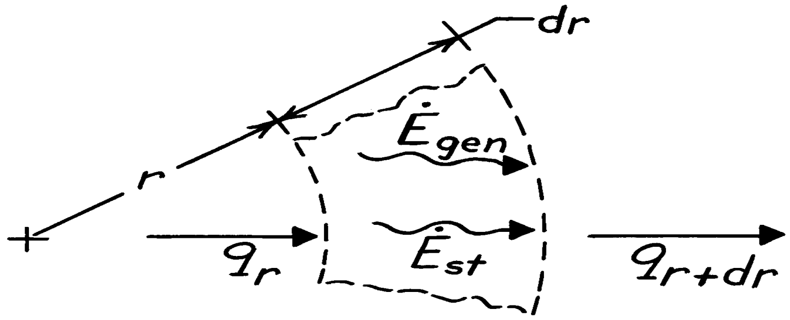
COMMENTS: Note the expression for the temperature gradient in Fourier's law, Eq. (3), is $\partial T/r_i\partial\phi$ not $\partial T/\partial\phi$. For the conditions of Parts (b) and (c), note that q_ϕ is independent of ϕ; this is first indicated by Eq. (6) and confirmed by Eq. (9).

PROBLEM 2.33

KNOWN: Heat diffusion with internal heat generation for one-dimensional cylindrical, radial coordinate system.

FIND: Heat diffusion equation.

SCHEMATIC:



ASSUMPTIONS: (1) Homogeneous medium.

ANALYSIS: Control volume has volume, $V = A_r \cdot dr = 2\pi r \cdot dr \cdot 1$, with unit thickness normal to page. Using the conservation of energy requirement, Eq. 1.11a,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st}$$

$$q_r - q_{r+dr} + \dot{q}V = \rho V c_p \frac{\partial T}{\partial t}.$$

Fourier's law, Eq. 2.1, for this one-dimensional coordinate system is

$$q_r = -kA_r \frac{\partial T}{\partial r} = -k \times 2\pi r \cdot 1 \times \frac{\partial T}{\partial r}.$$

At the outer surface, r+dr, the conduction rate is

$$q_{r+dr} = q_r + \frac{\partial}{\partial r}(q_r)dr = q_r + \frac{\partial}{\partial r}\left[-k \cdot 2\pi r \cdot \frac{\partial T}{\partial r}\right]dr.$$

Hence, the energy balance becomes

$$q_r - \left[q_r + \frac{\partial}{\partial r}\left[-k2\pi r\frac{\partial T}{\partial r}\right]dr\right] + \dot{q} \cdot 2\pi r dr = \rho \cdot 2\pi r dr \cdot c_p \frac{\partial T}{\partial t}$$

Dividing by the factor $2\pi r\ dr$, we obtain

$$\frac{1}{r}\frac{\partial}{\partial r}\left[kr\frac{\partial T}{\partial r}\right] + \dot{q} = \rho c_p \frac{\partial T}{\partial t}.$$ <

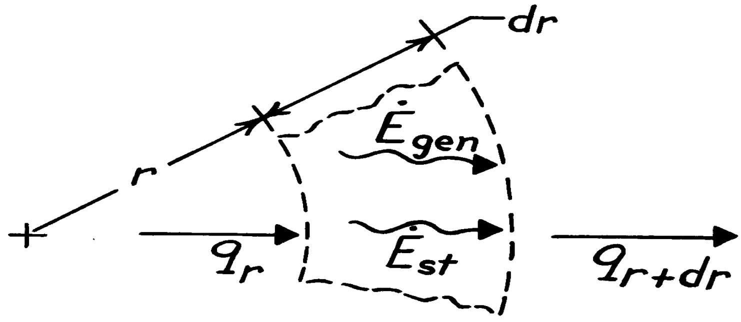
COMMENTS: (1) Note how the result compares with Eq. 2.20 when the terms for the ϕ,z coordinates are eliminated. (2) Recognize that we did not require $\dot{q}$ and k to be independent of r.

PROBLEM 2.34

KNOWN: Heat diffusion with internal heat generation for one-dimensional spherical, radial coordinate system.

FIND: Heat diffusion equation.

SCHEMATIC:



ASSUMPTIONS: (1) Homogeneous medium.

ANALYSIS: Control volume has the volume, $V = A_r \cdot dr = 4\pi r^2 dr$. Using the conservation of energy requirement, Eq. 1.11a,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st}$$

$$q_r - q_{r+dr} + \dot{q}V = \rho V c_p \frac{\partial T}{\partial t}.$$

Fourier's law, Eq. 2.1, for this coordinate system has the form

$$q_r = -kA_r \frac{\partial T}{\partial r} = -k \cdot 4\pi r^2 \cdot \frac{\partial T}{\partial r}.$$

At the outer surface, r+dr, the conduction rate is

$$q_{r+dr} = q_r + \frac{\partial}{\partial r}(q_r)dr = q_r + \frac{\partial}{\partial r}\left[-k \cdot 4\pi r^2 \cdot \frac{\partial T}{\partial r}\right]dr.$$

Hence, the energy balance becomes

$$q_r - \left[q_r + \frac{\partial}{\partial r}\left[-k \cdot 4\pi r^2 \cdot \frac{\partial T}{\partial r}\right]dr\right] + \dot{q} \cdot 4\pi r^2 dr = \rho \cdot 4\pi r^2 dr \cdot c_p \frac{\partial T}{\partial t}.$$

Dividing by the factor $4\pi r^2 dr$, we obtain

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[kr^2 \frac{\partial T}{\partial r}\right] + \dot{q} = \rho c_p \frac{\partial T}{\partial t}.$$ <

COMMENTS: (1) Note how the result compares with Eq. 2.23 when the terms for the θ,ϕ directions are eliminated.

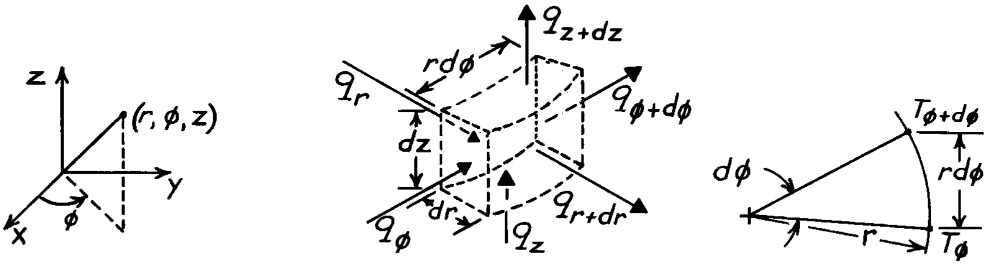
(2) Recognize that we did not require $\dot{q}$ and k to be independent of the coordinate r.

PROBLEM 2.35

KNOWN: Three-dimensional system – described by cylindrical coordinates (r,ϕ,z) – experiences transient conduction and internal heat generation.

FIND: Heat diffusion equation.

SCHEMATIC: See also Fig. 2.9.



ASSUMPTIONS: (1) Homogeneous medium.

ANALYSIS: Consider the differential control volume identified above having a volume given as V = dr·rdϕ·dz. From the conservation of energy requirement,

$$q_r - q_{r+dr} + q_\phi - q_{\phi+d\phi} + q_z - q_{z+dz} + \dot{E}_g = \dot{E}_{st}. \qquad (1)$$

The generation and storage terms, both representing volumetric phenomena, are

$$\dot{E}_g = \dot{q}V = \dot{q}(dr \cdot rd\phi \cdot dz) \qquad \dot{E}_g = \rho Vc\partial T/\partial t = \rho(dr \cdot rd\phi \cdot dz)c\,\partial T/\partial t. \qquad (2,3)$$

Using a Taylor series expansion, we can write

$$q_{r+dr} = q_r + \frac{\partial}{\partial r}(q_r)dr, \quad q_{\phi+d\phi} = q_\phi + \frac{\partial}{\partial \phi}(q_\phi)d\phi, \quad q_{z+dz} = q_z + \frac{\partial}{\partial z}(q_z)dz. \qquad (4,5,6)$$

Using Fourier's law, the expressions for the conduction heat rates are

$$q_r = -kA_r\partial T/\partial r = -k(rd\phi \cdot dz)\partial T/\partial r \qquad (7)$$

$$q_\phi = -kA_\phi\partial T/r\partial\phi = -k(dr \cdot dz)\partial T/r\partial\phi \qquad (8)$$

$$q_z = -kA_z\partial T/\partial z = -k(dr \cdot rd\phi)\partial T/\partial z. \qquad (9)$$

Note from the above, right schematic that the gradient in the ϕ-direction is ∂T/r∂ϕ and not ∂T/∂ϕ. Substituting Eqs. (2), (3) and (4), (5), (6) into Eq. (1),

$$-\frac{\partial}{\partial r}(q_r)dr - \frac{\partial}{\partial\phi}(q_\phi)d\phi - \frac{\partial}{\partial z}(q_z)dz + \dot{q}\,dr \cdot rd\phi \cdot dz = \rho(dr \cdot rd\phi \cdot dz)c\frac{\partial T}{\partial t}. \qquad (10)$$

Substituting Eqs. (7), (8) and (9) for the conduction rates, find

$$-\frac{\partial}{\partial r}\left[-k(rd\phi \cdot dz)\frac{\partial T}{\partial r}\right]dr - \frac{\partial}{\partial\phi}\left[-k(drdz)\frac{\partial T}{r\partial\phi}\right]d\phi - \frac{\partial}{\partial z}\left[-k(dr \cdot rd\phi)\frac{\partial T}{\partial z}\right]dz$$

$$+\dot{q}\,dr \cdot rd\phi \cdot dz = \rho(dr \cdot rd\phi \cdot dz)c\frac{\partial T}{\partial t}. \qquad (11)$$

Dividing Eq. (11) by the volume of the CV, Eq. 2.20 is obtained.

$$\frac{1}{r}\frac{\partial}{\partial r}\left[kr\frac{\partial T}{\partial r}\right] + \frac{1}{r^2}\frac{\partial}{\partial\phi}\left[k\frac{\partial T}{\partial\phi}\right] + \frac{\partial}{\partial z}\left[k\frac{\partial T}{\partial z}\right] + \dot{q} = \rho c\frac{\partial T}{\partial t} \qquad <$$

PROBLEM 2.36

KNOWN: Three-dimensional system – described by cylindrical coordinates (r,ϕ,θ) – experiences transient conduction and internal heat generation.

FIND: Heat diffusion equation.

SCHEMATIC: See Figure 2.10.

ASSUMPTIONS: (1) Homogeneous medium.

ANALYSIS: The differential control volume is V = dr·rsinθdϕ·rdθ, and the conduction terms are identified in Figure 2.10. Conservation of energy requires

$$q_r - q_{r+dr} + q_\phi - q_{\phi+d\phi} + q_\theta - q_{\theta+d\theta} + \dot{E}_g = \dot{E}_{st}. \tag{1}$$

The generation and storage terms, both representing volumetric phenomena, are

$$\dot{E}_g = \dot{q}V = \dot{q}[dr \cdot r\sin\theta d\phi \cdot rd\theta] \qquad \dot{E}_{st} = \rho Vc\frac{\partial T}{\partial t} = \rho[dr \cdot r\sin\theta d\phi \cdot rd\theta]c\frac{\partial T}{\partial t}. \tag{2,3}$$

Using a Taylor series expansion, we can write

$$q_{r+dr} = q_r + \frac{\partial}{\partial r}(q_r)dr, \quad q_{\phi+d\phi} = q_\phi + \frac{\partial}{\partial\phi}(q_\phi)d\phi, \quad q_{\theta+d\theta} = q_\theta + \frac{\partial}{\partial\theta}(q_\theta)d\theta. \tag{4,5,6}$$

From Fourier's law, the conduction heat rates have the following forms.

$$q_r = -kA_r\partial T/\partial r = -k[r\sin\theta d\phi \cdot rd\theta]\partial T/\partial r \tag{7}$$

$$q_\phi = -kA_\phi\partial T/r\sin\theta\partial\phi = -k[dr \cdot rd\theta]\partial T/r\sin\theta\partial\phi \tag{8}$$

$$q_\theta = -kA_\theta\partial T/r\partial\theta = -k[dr \cdot r\sin\theta d\phi]\partial T/r\partial\theta. \tag{9}$$

Substituting Eqs. (2), (3) and (4), (5), (6) into Eq. (1), the energy balance becomes

$$-\frac{\partial}{\partial r}(q_r)dr - \frac{\partial}{\partial_\phi}(q_\phi)d\phi - \frac{\partial}{\partial\theta}(q_\theta)d\theta + \dot{q}[dr \cdot r\sin\theta d\phi \cdot rd\theta] = \rho[dr \cdot r\sin\theta d\phi \cdot rd\theta]c\frac{\partial T}{\partial t} \tag{10}$$

Substituting Eqs. (7), (8) and (9) for the conduction rates, find

$$-\frac{\partial}{\partial\theta}\left[-k[r\sin\theta d\phi \cdot rd\theta]\frac{\partial T}{\partial r}\right]dr - \frac{\partial}{\partial\phi}\left[-k[dr \cdot rd\theta]\frac{\partial T}{r\sin\theta\partial\phi}\right]d\phi$$

$$-\frac{\partial}{\partial\theta}\left[-k[dr \cdot r\sin\theta d\phi]\frac{\partial T}{r\partial\theta}\right]d\theta + \dot{q}[dr \cdot r\sin\theta d\phi \cdot rd\theta] = \rho[dr \cdot r\sin\theta d\phi \cdot rd\theta]c\frac{\partial T}{\partial t} \tag{11}$$

Dividing Eq. (11) by the volume of the control volume, V, Eq. 2.23 is obtained.

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[kr^2\frac{\partial T}{\partial r}\right] + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\phi}\left[k\frac{\partial T}{\partial\phi}\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[k\sin\theta\frac{\partial T}{\partial\theta}\right] + \dot{q} = \rho c\frac{\partial T}{\partial t}. \qquad <$$

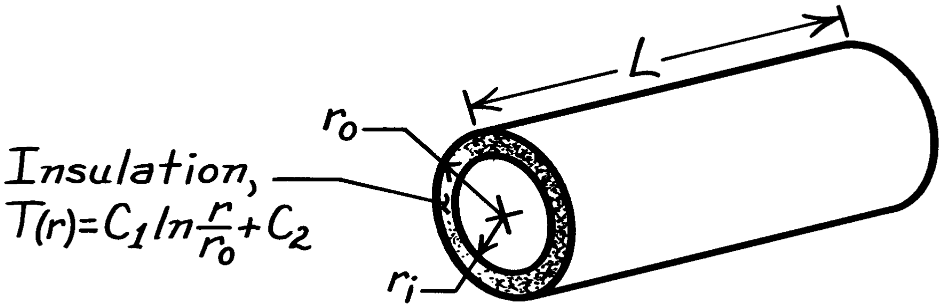
COMMENTS: Note how the temperature gradients in Eqs. (7) - (9) are formulated. The numerator is always ∂T while the denominator is the dimension of the control volume in the specified coordinate direction.

PROBLEM 2.37

KNOWN: Temperature distribution in steam pipe insulation.

FIND: Whether conditions are steady-state or transient. Manner in which heat flux and heat rate vary with radius.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in r, (2) Constant properties.

ANALYSIS: From Equation 2.20, the heat equation reduces to

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)=\frac{1}{\alpha}\frac{\partial T}{\partial t}.$$

Substituting for T(r),

$$\frac{1}{\alpha}\frac{\partial T}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{C_1}{r}\right)=0.$$

Hence, steady-state conditions exist. <

From Equation 2.19, the radial component of the heat flux is

$$q_r''=-k\frac{\partial T}{\partial r}=-k\frac{C_1}{r}.$$

Hence, q_r'' decreases with increasing $r\left(q_r''\alpha 1/r\right)$. <

At any radial location, the heat rate is

$$q_r=2\pi r L q_r''=-2\pi k C_1 L$$

Hence, q_r is independent of r. <

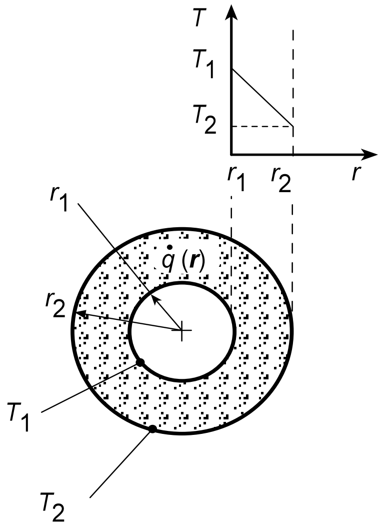
COMMENTS: The requirement that q_r is invariant with r is consistent with the energy conservation requirement. If q_r is constant, the flux must vary inversely with the area perpendicular to the direction of heat flow. Hence, q_r'' varies inversely with r.

PROBLEM 2.38

KNOWN: Inner and outer radii and surface temperatures of a long circular tube with internal energy generation.

FIND: Conditions for which a linear radial temperature distribution may be maintained.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction, (2) Constant properties.

ANALYSIS: For the assumed conditions, Eq. 2.20 reduces to

$$\frac{k}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\dot{q}=0$$

If $\dot{q}$ = 0 or $\dot{q}$ = constant, it is clearly impossible to have a linear radial temperature distribution. However, we may use the heat equation to infer a special form of $\dot{q}$ (r) for which dT/dr is a constant (call it C_1). It follows that

$$\frac{k}{r}\frac{d}{dr}\left(r\,C_1\right)+\dot{q}=0$$

$$\dot{q}=-\frac{C_1 k}{r}$$ <

where $C_1 = (T_2 - T_1)/(r_2 - r_1)$. Hence, if the generation rate varies inversely with radial location, the radial temperature distribution is linear.

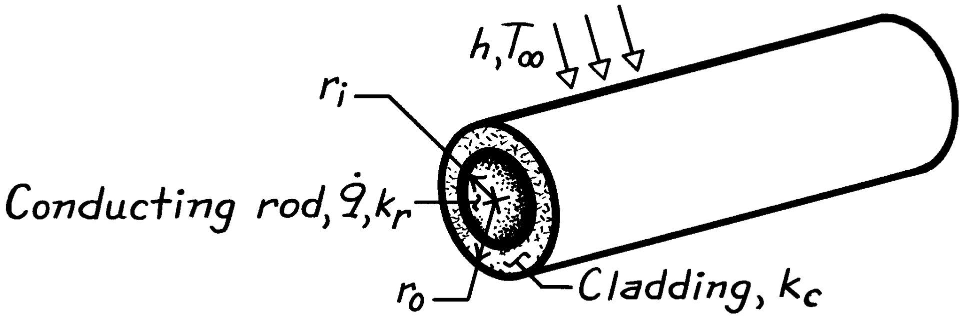
COMMENTS: Conditions for which $\dot{q} \propto (1/r)$ would be unusual.

PROBLEM 2.39

KNOWN: Radii and thermal conductivity of conducting rod and cladding material. Volumetric rate of thermal energy generation in the rod. Convection conditions at outer surface.

FIND: Heat equations and boundary conditions for rod and cladding.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in r, (3) Constant properties.

ANALYSIS: From Equation 2.20, the appropriate forms of the heat equation are

Conducting Rod:

$$\frac{k_r}{r}\frac{d}{dr}\left(r\frac{dT_r}{dt}\right)+\dot{q}=0 \qquad <$$

Cladding:

$$\frac{d}{dr}\left(r\frac{dT_c}{dr}\right)=0. \qquad <$$

Appropriate boundary conditions are:

(a) $dT_r / dr|_{r=0} = 0$ <

(b) $T_r(r_i) = T_c(r_i)$ <

(c) $k_r \frac{dT_r}{dr}|_{r_i} = k_c \frac{dT_c}{dr}|_{r_i}$ <

(d) $k_c \frac{dT_c}{dr}|_{r_o} = h\left[T_c(r_o) - T_\infty\right]$ <

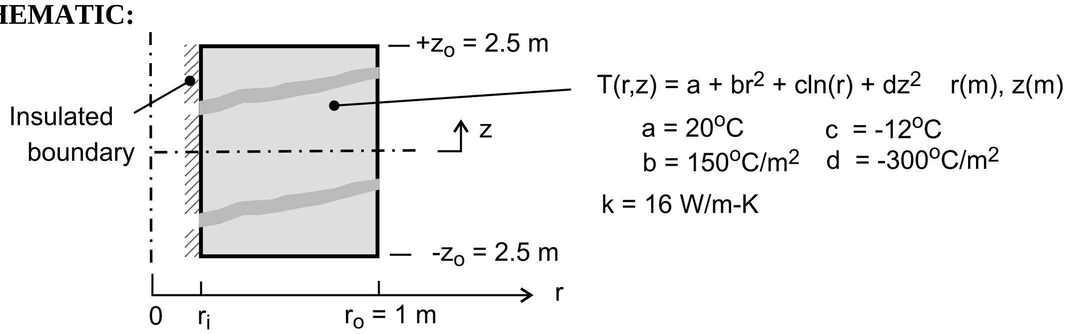
COMMENTS: Condition (a) corresponds to symmetry at the centerline, while the interface conditions at $r = r_i$ (b,c) correspond to requirements of thermal equilibrium and conservation of energy. Condition (d) results from conservation of energy at the outer surface.

PROBLEM 2.40

KNOWN: Steady-state temperature distribution for hollow cylindrical solid with volumetric heat generation.

FIND: (a) Determine the inner radius of the cylinder, r_i, (b) Obtain an expression for the volumetric rate of heat generation, $\dot{q}$, (c) Determine the axial distribution of the heat flux at the outer surface, $q''_r(r_o, z)$, and the heat rate at this outer surface; is the heat rate *in* or *out* of the cylinder; (d) Determine the radial distribution of the heat flux at the end faces of the cylinder, $q''_z(r, +z_o)$ and $q''_z(r, -z_o)$, and the corresponding heat rates; are the heat rates *in* or *out* of the cylinder; (e) Determine the relationship of the surface heat rates to the heat generation rate; is an overall energy balance satisfied?

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction with constant properties and volumetric heat generation.

ANALYSIS: (a) Since the inner boundary, r = r_i, is adiabatic, then $q''_r(r_i, z) = 0$. Hence the temperature gradient in the r-direction must be zero.

$$\left.\frac{\partial T}{\partial r}\right)_{r_i} = 0 + 2br_i + c/r_i + 0 = 0$$

$$r_i = +\left(-\frac{c}{2b}\right)^{1/2} = \left(-\frac{-12°C}{2 \times 150°C/m^2}\right)^{1/2} = 0.2\ m$$ <

(b) To determine $\dot{q}$, substitute the temperature distribution into the heat diffusion equation, Eq. 2.20, for two-dimensional (r,z), steady-state conduction

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right) + \frac{\dot{q}}{k} = 0$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\left[0 + 2br + c/r + 0\right]\right) + \frac{\partial}{\partial z}\left(0 + 0 + 0 + 2dz\right) + \frac{\dot{q}}{k} = 0$$

$$\frac{1}{r}\left[4br + 0\right] + 2d + \frac{\dot{q}}{k} = 0$$

$$\dot{q} = -k\left[4b - 2d\right] = -16\,W/m \cdot K\left[4 \times 150°C/m^2 - 2\left(-300°C/m^2\right)\right]$$

$$\dot{q} = 0\,W/m^3$$ <

(c) The heat flux and the heat rate at the outer surface, r = r_o, may be calculated using Fourier's law. Note that the sign of the heat flux in the positive r-direction is negative, and hence the heat flow is *into* the cylinder.

$$q''_r(r_o, z) = -k\left.\frac{\partial T}{\partial r}\right)_{r_o} = -k\left[0 + 2br_o + c/r_o + 0\right]$$

Continued …..

PROBLEM 2.40 (Cont.)

$$q''_r\left(r_{o,}z\right) = -16\ \text{W/m}\cdot\text{K}\left[2\times 150°\text{C/m}^2 \times 1\ \text{m} - 12°\text{C}/1\ \text{m}\right] = -4608\ \text{W/m}^2$$ <

$$q_r\left(r_o\right) = A_r\, q''_r\left(r_{o,}z\right) \qquad \text{where} \qquad A_r = 2\pi r_o\left(2z_o\right)$$

$$q_r\left(r_o\right) = -4\pi\times 1\ \text{m}\times 2.5\ \text{m}\times 4608\ \text{W/m}^2 = -144,765\ \text{W}$$ <

(d) The heat fluxes and the heat rates at end faces, z = + z_o and – z_o, may be calculated using Fourier's law. The direction of the heat rate *in* or *out* of the end face is determined by the sign of the heat flux in the positive z-direction.

At the upper end face, z = + z_o: heat rate is *out* of the cylinder <

$$q''_z\left(r,+z_o\right) = -k\left.\frac{\partial T}{\partial z}\right)_{z_o} = -k\left[0+0+0+2dz_o\right]$$

$$q''_z\left(r,+z_o\right) = -16\ \text{W/m}\cdot\text{K}\times 2\left(-300°\text{C/m}^2\right)2.5\ \text{m} = +24,000\ \text{W/m}^2$$ <

$$q_z\left(+z_o\right) = A_z\, q''_z\left(r,+z_o\right) \qquad \text{where} \qquad A_z = \pi\left(r_o^2 - r_i^2\right)$$

$$q_z\left(+z_o\right) = \pi\left(1^2 - 0.2^2\right)\text{m}^2\times 24,000\ \text{W/m}^2 = +72,382\ \text{W}$$ <

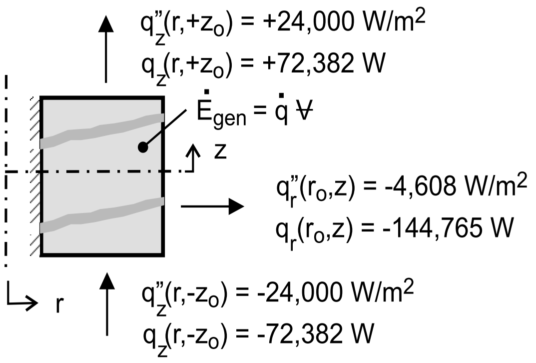
At the lower end face, z = - z_o: heat rate is *out* of the cylinder <

$$q''_z\left(r,-z_o\right) = -k\left.\frac{\partial T}{\partial z}\right)_{-z_o} = -k\left[0+0+0+2dz_o\right]$$

$$q''_z\left(r,-z_o\right) = -16\ \text{W/m}^2\cdot\text{K}\times 2\left(-300°\text{C/m}\right)\left(-2.5\ \text{m}\right) = -24,000\ \text{W/m}^2$$ <

$$q_z\left(-z_o\right) = -72,382\ \text{W}$$ <

(e) The heat rates from the surfaces and the volumetric heat generation can be related through an overall energy balance on the cylinder as shown in the sketch.



$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = 0 \qquad \text{where} \qquad \dot{E}_{gen} = \dot{q}\forall = 0$$

$$\dot{E}_{in} = -q_r\left(r_o\right) = -\left(-144,765\ \text{W}\right) = +144,765\ \text{W}$$ <

$$\dot{E}_{out} = +q_z\left(z_o\right) - q_z\left(-z_o\right) = \left[72,382 - \left(-72,382\right)\right]\text{W} = +144,764\ \text{W}$$ <

The overall energy balance is satisfied.

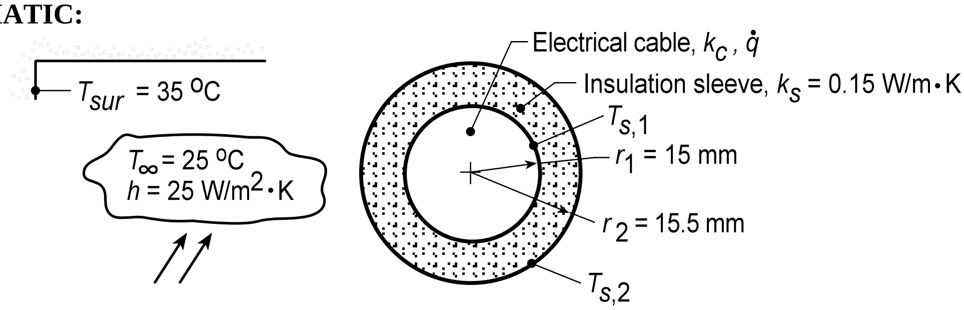
COMMENTS: When using Fourier's law, the heat flux q''_z denotes the heat flux in the positive z-direction. At a boundary, the sign of the numerical value will determine whether heat is flowing into or out of the boundary.

PROBLEM 2.41

KNOWN: An electric cable with an insulating sleeve experiences convection with adjoining air and radiation exchange with large surroundings.

FIND: (a) Verify that prescribed temperature distributions for the cable and insulating sleeve satisfy their appropriate heat diffusion equations; sketch temperature distributions labeling key features; (b) Applying Fourier's law, verify the conduction heat rate expression for the sleeve, q'_r, in terms of $T_{s,1}$ and $T_{s,2}$; apply a surface energy balance to the cable to obtain an alternative expression for q'_r in terms of $\dot{q}$ and r_1; (c) Apply surface energy balance around the outer surface of the sleeve to obtain an expression for which $T_{s,2}$ can be evaluated; (d) Determine $T_{s,1}$, $T_{s,2}$, and T_o for the specified geometry and operating conditions; and (e) Plot $T_{s,1}$, $T_{s,2}$, and T_o as a function of the outer radius for the range $15.5 \le r_2 \le 20$ mm.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, radial conduction, (2) Uniform volumetric heat generation in cable, (3) Negligible thermal contact resistance between the cable and sleeve, (4) Constant properties in cable and sleeve, (5) Surroundings large compared to the sleeve, and (6) Steady-state conditions.

ANALYSIS: (a) The appropriate forms of the heat diffusion equation (HDE) for the insulation and cable are identified. The temperature distributions are valid if they satisfy the relevant HDE.

Insulation: The temperature distribution is given as

$$T(r) = T_{s,2} + \left(T_{s,1} - T_{s,2}\right)\frac{\ln(r/r_2)}{\ln(r_1/r_2)} \tag{1}$$

and the appropriate HDE (radial coordinates, SS, $\dot{q}$ = 0), Eq. 2.20,

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

$$\frac{d}{dr}\left(r\left[0 + \left(T_{s,1} - T_{s,2}\right)\frac{1/r}{\ln(r_1/r_2)}\right]\right) = \frac{d}{dr}\left(\frac{T_{s,1} - T_{s,2}}{\ln(r_1/r_2)}\right) ?=? 0$$

Hence, the temperature distribution satisfies the HDE. <

Cable: The temperature distribution is given as

$$T(r) = T_{s,1} + \frac{\dot{q}r_1^2}{4k_c}\left(1 - \frac{r^2}{r_1^2}\right) \tag{2}$$

and the appropriate HDE (radial coordinates, SS, $\dot{q}$ uniform), Eq. 2.20,

Continued...

PROBLEM 2.41 (Cont.)

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{\dot{q}}{k_c}=0$$

$$\frac{1}{r}\frac{d}{dr}\left(r\left[0+\frac{\dot{q}r_1^2}{4k_c}\left(0-\frac{2r}{r_1^2}\right)\right]\right)+\frac{\dot{q}}{k_c}?=?0$$

$$\frac{1}{r}\frac{d}{dr}\left(-\frac{\dot{q}r_1^2}{4k_c}\frac{2r^2}{r_1^2}\right)+\frac{\dot{q}}{k_c}?=?0$$

$$\frac{1}{r}\left(-\frac{\dot{q}r_1^2}{4k_c}\frac{4r}{r_1^2}\right)+\frac{\dot{q}}{k_c}?=?0$$

Hence the temperature distribution satisfies the HDE. <

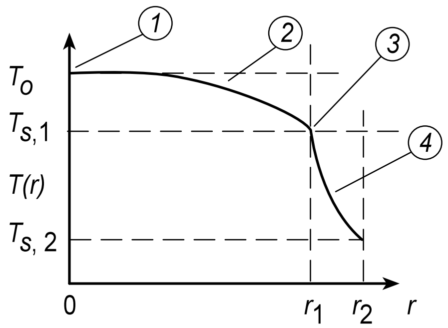
The temperature distributions in the cable, $0 \le r \le r_1$, and sleeve, $r_1 \le r \le r_2$, and their key features are as follows:

(1) Zero gradient, symmetry condition,

(2) Increasing gradient with increasing radius, r, because of $\dot{q}$,

(3) Discontinuous T(r) across cable-sleeve interface because of different thermal conductivities,

(4) Decreasing gradient with increasing radius, r, since heat rate is constant.



(b) Using Fourier's law for the radial-cylindrical coordinate, the heat rate through the *insulation* (sleeve) per unit length is

$$q_r' = -kA_r'\frac{dT}{dr} = -k2\pi r\frac{dT}{dr}$$ <

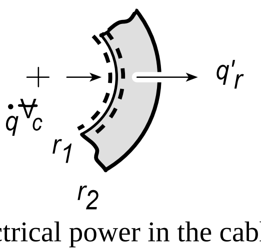
and substituting for the temperature distribution, Eq. (1),

$$q_r' = -k_s 2\pi r\left[0+\left(T_{s,1}-T_{s,2}\right)\frac{1/r}{\ln\left(r_1/r_2\right)}\right] = 2\pi k_s\frac{\left(T_{s,1}-T_{s,2}\right)}{\ln\left(r_2/r_1\right)} \quad (3)$$ <

Applying an energy balance to a control surface placed around the cable,

$$\dot{E}_{in}-\dot{E}_{out}=0$$

$$\dot{q}\forall_c'-q_r'=0$$



where $\dot{q}\forall_c$ represents the dissipated electrical power in the cable

Continued...

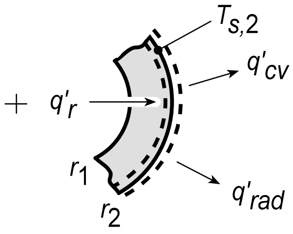
PROBLEM 2.41 (Cont.)

$$\dot{q}\left(\pi r_1^2\right) - q'_r = 0 \qquad \text{or} \qquad q'_r = \pi \dot{q} r_1^2 \tag{4}$$ <

(c) Applying an energy balance to a control surface placed around the outer surface of the sleeve,

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q'_r - q'_{cv} - q'_{rad} = 0$$



$$\pi \dot{q} r_1^2 - h\left(2\pi r_2\right)\left(T_{s,2} - T_\infty\right) - \varepsilon\left(2\pi r_2\right)\sigma\left(T_{s,2}^4 - T_{sur}^4\right) = 0 \tag{5}$$ <

This relation can be used to determine $T_{s,2}$ in terms of the variables $\dot{q}$, r_1, r_2, h, T_∞, ε and T_{sur}.

(d) Consider a cable-sleeve system with the following prescribed conditions:

r_1 = 15 mm	k_c = 200 W/m·K	h = 25 W/m²·K	ε = 0.9
r_2 = 15.5 mm	k_s = 0.15 W/m·K	T_∞ = 25°C	T_{sur} = 35°C

For 250 A with R'_e = 0.005 Ω/m, the volumetric heat generation rate is

$$\dot{q} = I^2 R'_e / \forall'_c = I^2 R'_e \Big/ \left(\pi r_1^2\right)$$

$$\dot{q} = (250\,A)^2 \times 0.005\,\Omega/m \Big/ \left(\pi \times 0.015^2\,m^2\right) = 4.42\times10^5\ W\big/m^3$$

Substituting numerical values in appropriate equations, we can evaluate $T_{s,1}$, $T_{s,2}$ and T_o.

Sleeve outer surface temperature, $T_{s,2}$: Using Eq. (5),

$$\pi \times 4.42\times10^5\ W\big/m^3 \times (0.015m)^2 - 25\,W\big/m^2\cdot K \times (2\pi\times0.0155m)\left(T_{s,2} - 298K\right)$$
$$-0.9\times(2\pi\times0.0155m)\times5.67\times10^{-8}\ W\big/m^2\cdot K^4\left(T_{s,2}^4 - 308^4\right)K^4 = 0$$

$$T_{s,2} = 395\,K = 122^\circ C$$ <

Sleeve-cable interface temperature, $T_{s,1}$: Using Eqs. (3) and (4), with $T_{s,2}$ = 395 K,

$$\pi \dot{q} r_1^2 = 2\pi k_s \frac{\left(T_{s,1} - T_{s,2}\right)}{\ln\left(r_2/r_1\right)}$$

$$\pi \times 4.42\times10^5\ W\big/m^3 \times (0.015\,m)^2 = 2\pi\times0.15\,W/m\cdot K\,\frac{\left(T_{s,1} - 395\,K\right)}{\ln(15.5/15.0)}$$

$$T_{s,1} = 406\,K = 133^\circ C$$ <

Continued...

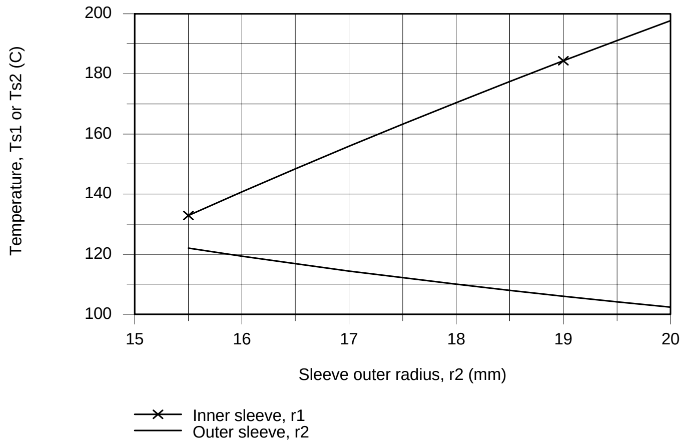
PROBLEM 2.41 (Cont.)

Cable centerline temperature, T_o: Using Eq. (2) with $T_{s,1}$ = 133°C,

$$T_o = T(0) = T_{s,1} + \frac{\dot{q} r_1^2}{4k_c}$$

$$T_o = 133^\circ C + 4.42 \times 10^5 \; W/m^3 \times (0.015\, m)^2 / (4 \times 200 \; W/m \cdot K) = 133.1^\circ C$$ <

(e) With all other conditions remaining the same, the relations of part (d) can be used to calculate T_o, $T_{s,1}$ and $T_{s,2}$ as a function of the sleeve outer radius r_2 for the range $15.5 \le r_2 \le 20$ mm.



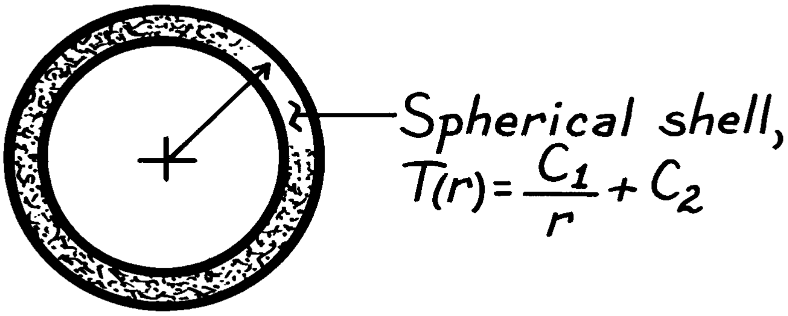
On the plot above T_o would show the same behavior as $T_{s,1}$ since the temperature rise between cable center and its surface is 0.12°C. With increasing r_2, we expect $T_{s,2}$ to decrease since the heat flux decreases with increasing r_2. We expect $T_{s,1}$ to increase with increasing r_2 since the thermal resistance of the sleeve increases.

PROBLEM 2.42

KNOWN: Temperature distribution in a spherical shell.

FIND: Whether conditions are steady-state or transient. Manner in which heat flux and heat rate vary with radius.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in r, (2) Constant properties.

ANALYSIS: From Equation 2.23, the heat equation reduces to

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)=\frac{1}{\alpha}\frac{\partial T}{\partial t}.$$

Substituting for T(r),

$$\frac{1}{\alpha}\frac{\partial T}{\partial t}=-\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{C_1}{r^2}\right)=0.$$

Hence, steady-state conditions exist. <

From Equation 2.22, the radial component of the heat flux is

$$q_r''=-k\frac{\partial T}{\partial r}=-k\frac{C_1}{r^2}.$$

Hence, q_r'' decreases with increasing $r^2\left(q_r''\alpha 1/r^2\right)$. <

At any radial location, the heat rate is

$$q_r=4\pi r^2 q_r''=4\pi k C_1.$$

Hence, q_r is independent of r. <

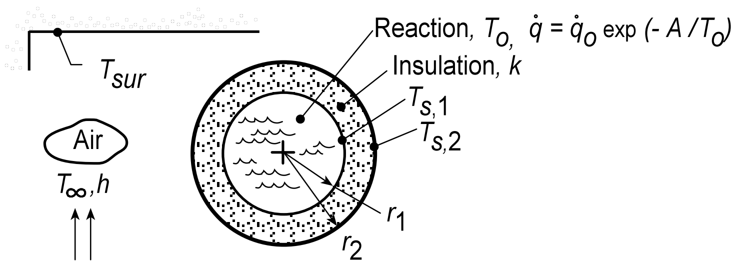
COMMENTS: The fact that q_r is independent of r is consistent with the energy conservation requirement. If q_r is constant, the flux must vary inversely with the area perpendicular to the direction of heat flow. Hence, q_r'' varies inversely with r^2.

PROBLEM 2.43

KNOWN: Spherical container with an exothermic reaction enclosed by an insulating material whose outer surface experiences convection with adjoining air and radiation exchange with large surroundings.

FIND: (a) Verify that the prescribed temperature distribution for the insulation satisfies the appropriate form of the heat diffusion equation; sketch the temperature distribution and label key features; (b) Applying Fourier's law, verify the conduction heat rate expression for the insulation layer, q_r, in terms of $T_{s,1}$ and $T_{s,2}$; apply a surface energy balance to the container and obtain an alternative expression for q_r in terms of $\dot{q}$ and r_1; (c) Apply a surface energy balance around the outer surface of the insulation to obtain an expression to evaluate $T_{s,2}$; (d) Determine $T_{s,2}$ for the specified geometry and operating conditions; (e) Compute and plot the variation of $T_{s,2}$ as a function of the outer radius for the range $201 \le r_2 \le 210$ mm; explore approaches for reducing $T_{s,2} \le 45°C$ to eliminate potential risk for burn injuries to personnel.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, radial spherical conduction, (2) Isothermal reaction in container so that $T_o = T_{s,1}$, (2) Negligible thermal contact resistance between the container and insulation, (3) Constant properties in the insulation, (4) Surroundings large compared to the insulated vessel, and (5) Steady-state conditions.

ANALYSIS: The appropriate form of the heat diffusion equation (HDE) for the insulation follows from Eq. 2.23,

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=0 \qquad (1)$$ <

The temperature distribution is given as

$$T(r)=T_{s,1}-\left(T_{s,1}-T_{s,2}\right)\left[\frac{1-(r_1/r)}{1-(r_1/r_2)}\right] \qquad (2)$$

Substitute T(r) into the HDE to see if it is satisfied:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\left[0-\left(T_{s,1}-T_{s,2}\right)\frac{0+\left(r_1/r^2\right)}{1-(r_1/r_2)}\right]\right)?=?0$$

$$\frac{1}{r^2}\frac{d}{dr}\left(+\left(T_{s,1}-T_{s,2}\right)\frac{r_1}{1-(r_1/r_2)}\right)?=?0$$ <

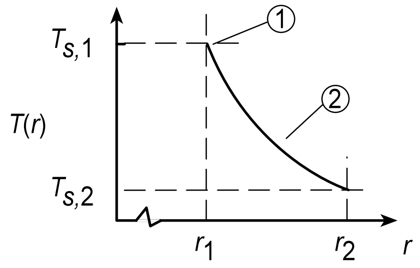
and since the expression in parenthesis is independent of r, T(r) does indeed satisfy the HDE. The temperature distribution in the insulation and its key features are as follows:

Continued...

PROBLEM 2.43 (Cont.)

(1) $T_{s,1} > T_{s,2}$

(2) Decreasing gradient with increasing radius, r, since the heat rate is constant through the insulation.



(b) Using Fourier's law for the radial-spherical coordinate, the heat rate through the insulation is

$$q_r = -kA_r \frac{dT}{dr} = -k\left(4\pi r^2\right)\frac{dT}{dr}$$ <

and substituting for the temperature distribution, Eq. (2),

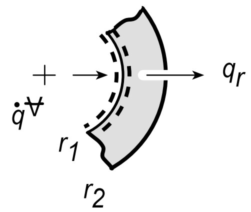
$$q_r = -k\pi r^2 \left[0 - \left(T_{s,1} - T_{s,2}\right)\frac{0 + \left(r_1/r^2\right)}{1 - \left(r_1/r_2\right)}\right]$$

$$q_r = \frac{4\pi k\left(T_{s,1} - T_{s,2}\right)}{\left(1/r_1\right) - \left(1/r_2\right)} \qquad (3)$$ <

Applying an energy balance to a control surface about the container at $r = r_1$,

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$\dot{q}\forall - q_r = 0$$



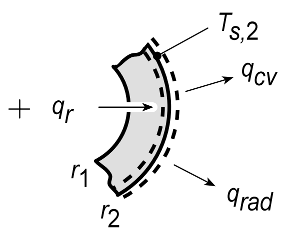
where $\dot{q}\forall$ represents the generated heat in the container,

$$q_r = (4/3)\pi r_1^3 \dot{q} \qquad (4)$$ <

(c) Applying an energy balance to a control surface placed around the outer surface of the insulation,

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q_r - q_{cv} - q_{rad} = 0$$



$$q_r - hA_s\left(T_{s,2} - T_\infty\right) - \varepsilon A_s \sigma\left(T_{s,2}^4 - T_{sur}^4\right) = 0 \qquad (5)$$ <

Continued...

where

$$A_s = 4\pi r_2^2 \tag{6}$$

These relations can be used to determine $T_{s,2}$ in terms of the variables $\dot{q}$, r_1, r_2, h, T_∞, ε and T_{sur}.

(d) Consider the reactor system operating under the following conditions:

r_1 = 200 mm	h = 5 W/m^2·K	ε = 0.9
r_2 = 208 mm	T_∞ = 25°C	T_{sur} = 35°C
k = 0.05 W/m·K		

The heat generated by the exothermic reaction provides for a volumetric heat generation rate,

$$\dot{q} = \dot{q}_o \exp\left(-A/T_o\right) \qquad q_o = 5000\,\text{W/m}^3 \qquad A = 75\,\text{K} \tag{7}$$

where the temperature of the reaction is that of the inner surface of the insulation, $T_o = T_{s,1}$. The following system of equations will determine the operating conditions for the reactor.

Conduction rate equation, insulation, Eq. (3),

$$q_r = \frac{4\pi \times 0.05\,\text{W/m}\cdot\text{K}\left(T_{s,1} - T_{s,2}\right)}{\left(1/0.200\,\text{m} - 1/0.208\,\text{m}\right)} \tag{8}$$

Heat generated in the reactor, Eqs. (4) and (7),

$$q_r = 4/3\pi\left(0.200\,\text{m}\right)^3 \dot{q} \tag{9}$$

$$\dot{q} = 5000\,\text{W/m}^3 \exp\left(-75\,\text{K}/T_{s,1}\right) \tag{10}$$

Surface energy balance, insulation, Eqs. (5) and (6),

$$q_r - 5\,\text{W/m}^2\cdot\text{K}\,A_s\left(T_{s,2} - 298\,\text{K}\right) - 0.9A_s\,5.67\times10^{-8}\,\text{W/m}^2\text{·K}^4\left(T_{s,2}^4 - \left(308\,\text{K}\right)^4\right) = 0 \tag{11}$$

$$A_s = 4\pi\left(0.208\,\text{m}\right)^2 \tag{12}$$

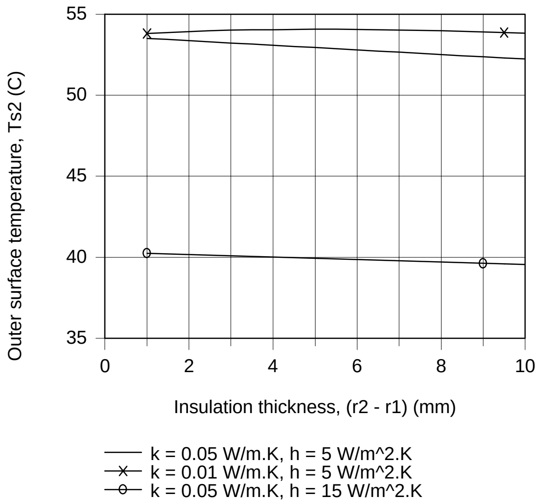
Solving these equations simultaneously, find that

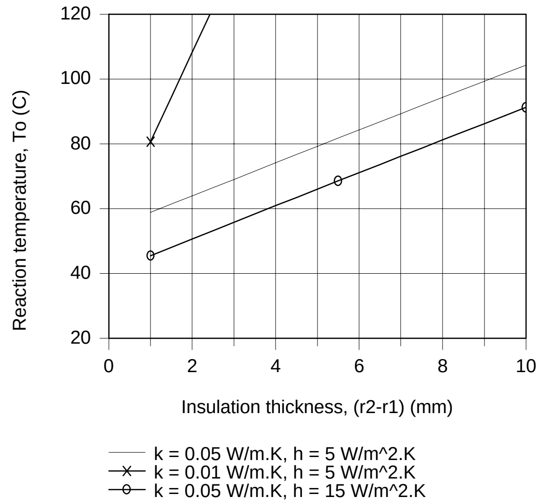
$$T_{s,1} = 94.3^\circ\text{C} \qquad T_{s,2} = 52.5^\circ\text{C}$$ <

That is, the reactor will be operating at $T_o = T_{s,1}$ = 94.3°C, very close to the desired 95°C operating condition.

(e) From the above analysis, we found the outer surface temperature $T_{s,2}$ = 52.5°C represents a potential burn risk to plant personnel. Using the above system of equations, Eqs. (8)-(12), we have explored the effects of changes in the convection coefficient, h, and the insulation thermal conductivity, k, as a function of insulation thickness, $t = r_2 - r_1$.

Continued...





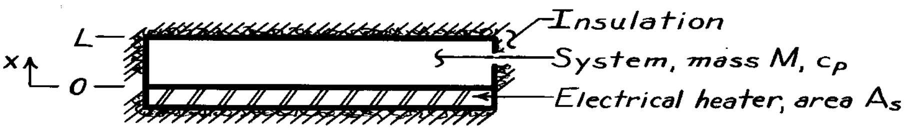
In the $T_{s,2}$ vs. $(r_2 - r_1)$ plot, note that decreasing the thermal conductivity from 0.05 to 0.01 W/m·K slightly increases $T_{s,2}$ while increasing the convection coefficient from 5 to 15 $W/m^2 \cdot K$ markedly decreases $T_{s,2}$. Insulation thickness only has a minor effect on $T_{s,2}$ for either option. In the T_o vs. $(r_2 - r_1)$ plot, note that, for all the options, the effect of increased insulation is to increase the reaction temperature. With k = 0.01 W/m·K, the reaction temperature increases beyond 95°C with less than 2 mm insulation. For the case with h = 15 $W/m^2 \cdot K$, the reaction temperature begins to approach 95°C with insulation thickness around 10 mm. We conclude that by selecting the proper insulation thickness and controlling the convection coefficient, the reaction could be operated around 95°C such that the outer surface temperature would not exceed 45°C.

PROBLEM 2.44

KNOWN: One-dimensional system, initially at a uniform temperature T_i, is suddenly exposed to a uniform heat flux at one boundary, while the other boundary is insulated.

FIND: (a) Proper form of heat equation and boundary and initial conditions, (b) Temperature distributions for following conditions: initial condition ($t \le 0$), and several times after heater is energized; will a steady-state condition be reached; (c) Heat flux at x = 0, L/2, L as a function of time; (d) Expression for uniform temperature, T_f, reached after heater has been switched off following an elapsed time, t_e, with the heater on.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) No internal heat generation, (3) Constant properties.

ANALYSIS: (a) The appropriate form of the heat equation follows from Eq. 2.15. Also, the appropriate boundary and initial conditions are:

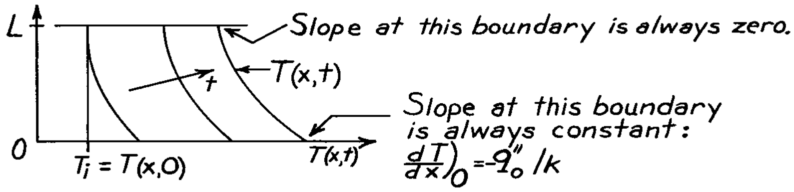
$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Initial condition: $T(x,0) = T_i$ Uniform temperature

Boundary conditions: $x = 0$ $\quad q''_o = -k\partial T/\partial x)_0$

$x = L$ $\quad \partial T/\partial x)_L = 0$

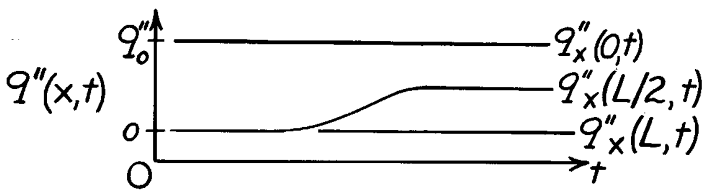
(b) The temperature distributions are as follows:



<

No steady-state condition will be reached since $\dot{E}_{in} = \dot{E}_{st}$ and $\dot{E}_{in}$ is constant.

(c)The heat flux as a function of time for positions x = 0, L/2 and L is as follows:



<

(d) If the heater is energized until $t = t_e$ and then switched off, the system will eventually reach a uniform temperature, T_f. Perform an energy balance on the system, Eq. 1.11b, for an interval of time $\Delta t = t_e$,

$$E_{in} = E_{st} \qquad E_{in} = Q_{in} = \int_0^{t_e} q''_o A_s dt = q''_o A_s t_e \qquad E_{st} = Mc(T_f - T_i)$$

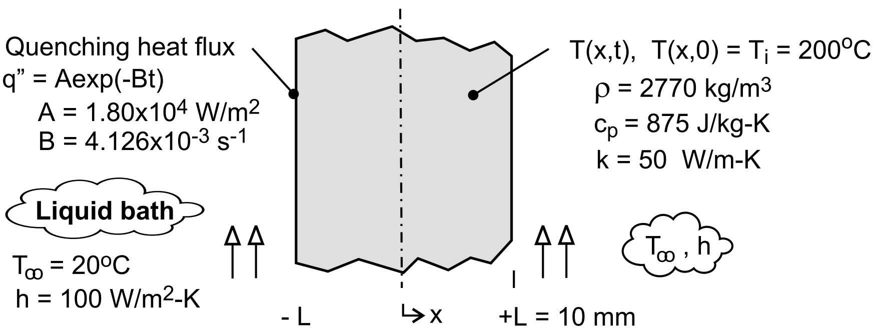
It follows that $\quad q''_o A_s t_e = Mc(T_f - T_i) \quad$ or $\quad T_f = T_i + \dfrac{q''_o A_s t_e}{Mc}.$ <

PROBLEM 2.45

KNOWN: Plate of thickness 2L, initially at a uniform temperature of T_i = 200°C, is suddenly quenched in a liquid bath of T_∞ = 20°C with a convection coefficient of 100 W/m^2·K.

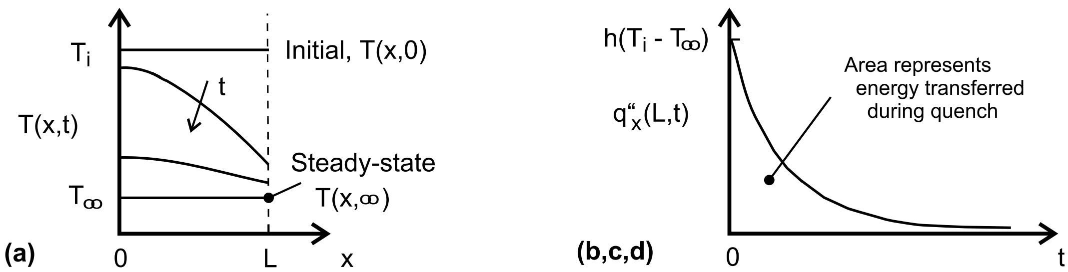
FIND: (a) On T-x coordinates, sketch the temperature distributions for the initial condition (t ≤ 0), the steady-state condition (t → ∞), and two intermediate times; (b) On $q''_x - t$ coordinates, sketch the variation with time of the heat flux at x = L, (c) Determine the heat flux at x = L and for t = 0; what is the temperature gradient for this condition; (d) By performing an energy balance on the plate, determine the amount of energy per unit surface area of the plate (J/m^2) that is transferred to the bath over the time required to reach steady-state conditions; and (e) Determine the energy transferred to the bath during the quenching process using the exponential-decay relation for the surface heat flux.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, and (3) No internal heat generation.

ANALYSIS: (a) The temperature distributions are shown in the sketch below.



(b) The heat flux at the surface x = L, $q''_x(L,t)$, is initially a maximum value, and decreases with increasing time as shown in the sketch above.

(c) The heat flux at the surface x = L at time t = 0, $q''_x(L,0)$, is equal to the convection heat flux with the surface temperature as T(L,0) = T_i.

$$q''_x(L,0) = q''_{conv}(t=0) = h(T_i - T_\infty) = 100\,W/m^2\cdot K\,(200-20)°C = 18.0\,kW/m^2 \qquad <$$

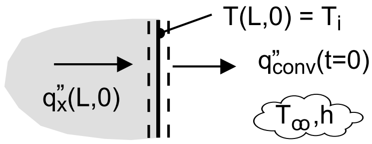
From a surface energy balance as shown in the sketch considering the conduction and convection fluxes at the surface, the temperature gradient can be calculated.

Continued …..

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q''_x(L,0) - q''_{conv}(t=0) = 0 \qquad \text{with} \qquad q''_x(L,0) = -k\left.\frac{\partial T}{\partial x}\right)_{x=L}$$

$$\left.\frac{\partial T}{\partial x}\right)_{L,0} = -q''_{conv}(t=0)/k = -18\times10^3\ \mathrm{W/m^2}/50\ \mathrm{W/m\cdot K} = -360\ \mathrm{K/m} \qquad <$$



(d) The energy transferred from the plate to the bath over the time required to reach steady-state conditions can be determined from an energy balance on a time interval basis, Eq. 1.11b. For the initial state, the plate has a uniform temperature T_i; for the final state, the plate is at the temperature of the bath, T_∞.

$$E''_{in} - E''_{out} = \Delta E''_{st} = E''_f - E''_i \qquad \text{with} \qquad E''_{in} = 0,$$

$$-E''_{out} = \rho c_p (2L)[T_\infty - T_i]$$

$$E''_{out} = -2770\ \mathrm{kg/m^3} \times 875\ \mathrm{J/kg\cdot K}(2\times0.010\ \mathrm{m})[20-200]\mathrm{K} = +8.73\times10^6\ \mathrm{J/m^2} \qquad <$$

(e) The energy transfer from the plate to the bath during the quenching process can be evaluated from knowledge of the surface heat flux as a function of time. The area under the curve in the $q''_x(L,t)$ vs. time plot (see schematic above) represents the energy transferred during the quench process.

$$E''_{out} = 2\int_{t=0}^{\infty} q''_x(L,t)\,dt = 2\int_{t=0}^{\infty} Ae^{-Bt}\,dt$$

$$E''_{out} = 2A\left[-\frac{1}{B}e^{-Bt}\right]_0^\infty = 2A\left[-\frac{1}{B}(0-1)\right] = 2A/B$$

$$E''_{out} = 2\times1.80\times10^4\ \mathrm{W/m^2}/4.126\times10^{-3}\mathrm{s^{-1}} = 8.73\times10^6\ \mathrm{J/m^2} \qquad <$$

COMMENTS: (1) Can you identify and explain the important features in the temperature distributions of part (a)?

(2) The maximum heat flux from the plate occurs at the instant the quench process begins and is equal to the convection heat flux. At this instant, the gradient in the plate at the surface is a maximum. If the gradient is too large, excessive thermal stresses could be induced and cracking could occur.

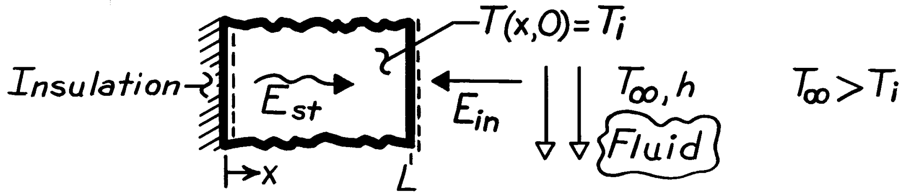
(3) In this thermodynamic analysis, we were able to determine the energy transferred during the quenching process. We cannot determine the rate at which cooling of the plate occurs without solving the heat diffusion equation.

PROBLEM 2.46

KNOWN: Plane wall, initially at a uniform temperature, is suddenly exposed to convective heating.

FIND: (a) Differential equation and initial and boundary conditions which may be used to find the temperature distribution, T(x,t); (b) Sketch T(x,t) for these conditions: initial ($t \le 0$), steady-state, $t \to \infty$, and two intermediate times; (c) Sketch heat fluxes as a function of time for surface locations; (d) Expression for total energy transferred to wall per unit volume (J/m^3).

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) No internal heat generation.

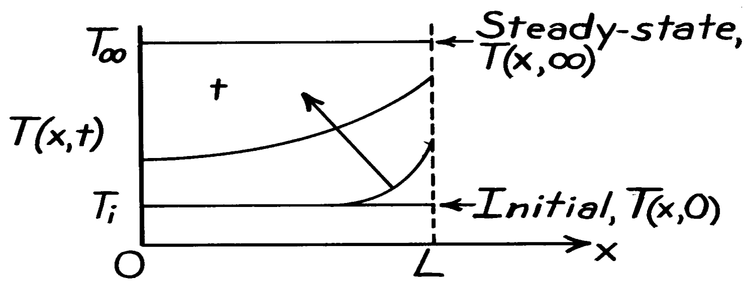
ANALYSIS: (a) For one-dimensional conduction with constant properties, the heat equation has the form,

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

and the conditions are:

$$\begin{cases} \text{Initial, } t \le 0: & T(x,0) = T_i & \text{uniform} \\ \text{Boundaries:} \quad x = 0 & \partial T/\partial x)_0 = 0 & \text{adiabatic} \\ \qquad x = L & -k\partial T/\partial x)_L = h\left[T(L,t) - T_\infty\right] & \text{convection} \end{cases}$$

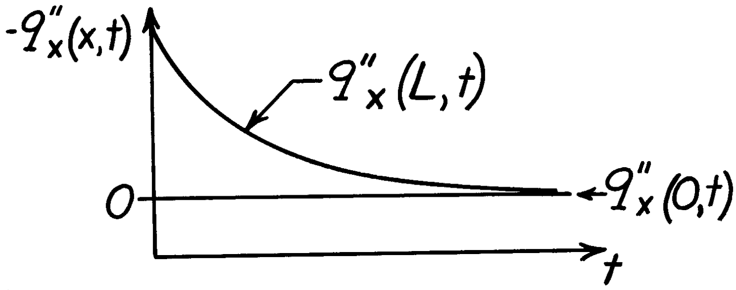
(b) The temperature distributions are shown on the sketch.



Note that the gradient at x = 0 is always zero, since this boundary is adiabatic. Note also that the gradient at x = L decreases with time.

(c) The heat flux, $q_x''(x,t)$, as a function of time, is shown on the sketch for the surfaces x = 0 and x = L.

Continued …..



For the surface at $x = 0,\ q''_x(0,t) = 0$ since it is adiabatic. At x = L and t = 0, $q''_x(L,0)$ is a maximum

$$q''_x(L,0) = h\left[T(L,0) - T_\infty\right]$$

where T(L,0) = T_i. The gradient, and hence the flux, decrease with time.

(d) The total energy transferred to the wall may be expressed as

$$E_{in} = \int_0^\infty q''_{conv} A_s dt$$

$$E_{in} = hA_s \int_0^\infty \left(T_\infty - T(L,t)\right)dt$$

Dividing both sides by A_sL, the energy transferred per unit volume is

$$\frac{E_{in}}{V} = \frac{h}{L}\int_0^\infty \left[T_\infty - T(L,t)\right]dt \qquad \left[J/m^3\right]$$

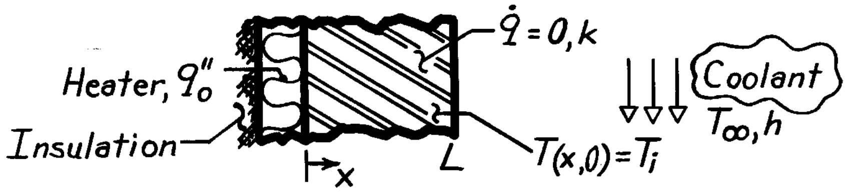
COMMENTS: Note that the heat flux at x = L is into the wall and is hence in the negative x direction.

PROBLEM 2.47

KNOWN: Plane wall, initially at a uniform temperature T_i, is suddenly exposed to convection with a fluid at T_∞ at one surface, while the other surface is exposed to a constant heat flux q''_o.

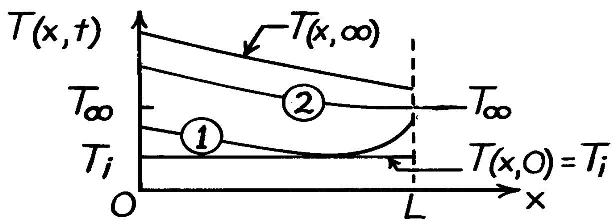
FIND: (a) Temperature distributions, T(x,t), for initial, steady-state and two intermediate times, (b) Corresponding heat fluxes on $q''_x - x$ coordinates, (c) Heat flux at locations x = 0 and x = L as a function of time, (d) Expression for the steady-state temperature of the heater, T(0,∞), in terms of $q''_o,\ T_\infty,\ k,\ h$ and L.

SCHEMATIC:



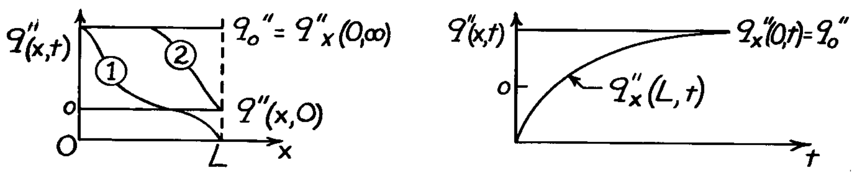
ASSUMPTIONS: (1) One-dimensional conduction, (2) No heat generation, (3) Constant properties.

ANALYSIS: (a) For $T_i < T_\infty$, the temperature distributions are



Note the constant gradient at x = 0 since $q''_x(0) = q''_o$.

(b) The heat flux distribution, $q''_x(x,t)$, is determined from knowledge of the temperature gradients, evident from Part (a), and Fourier's law.



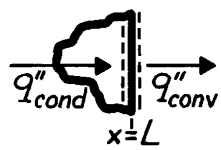
(c) On $q''_x(x,t) - t$ coordinates, the heat fluxes at the boundaries are shown above.

(d) Perform a surface energy balance at x = L and an energy balance on the wall:

$$q''_{cond} = q''_{conv} = h\left[T(L,\infty) - T_\infty\right] \quad (1), \qquad q''_{cond} = q''_o. \quad (2)$$

For the wall, under steady-state conditions, Fourier's law gives

$$q''_o = -k\frac{dT}{dx} = k\frac{T(0,\infty) - T(L,\infty)}{L}. \qquad (3)$$



Combine Eqs. (1), (2), (3) to find:

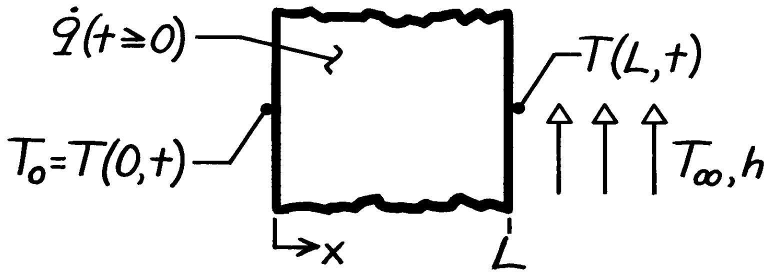
$$T(0,\infty) = T_\infty + \frac{q''_o}{1/h + L/k}.$$

PROBLEM 2.48

KNOWN: Plane wall, initially at a uniform temperature T_o, has one surface (x = L) suddenly exposed to a convection process ($T_\infty > T_o$,h), while the other surface (x = 0) is maintained at T_o. Also, wall experiences uniform volumetric heating $\dot{q}$ such that the maximum steady-state temperature will exceed T_∞.

FIND: (a) Sketch temperature distribution (T vs. X) for following conditions: initial ($t \le 0$), steady-state ($t \to \infty$), and two intermediate times; also show distribution when there is no heat flow at the x = L boundary, (b) Sketch the heat flux $\left(q''_x \text{ vs. } t\right)$ at the boundaries x = 0 and L.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) Uniform volumetric generation, (4) $T_o < T_\infty$ and $\dot{q}$ large enough that $T(x,\infty) > T_\infty$.

ANALYSIS: (a) The initial and boundary conditions for the wall can be written as

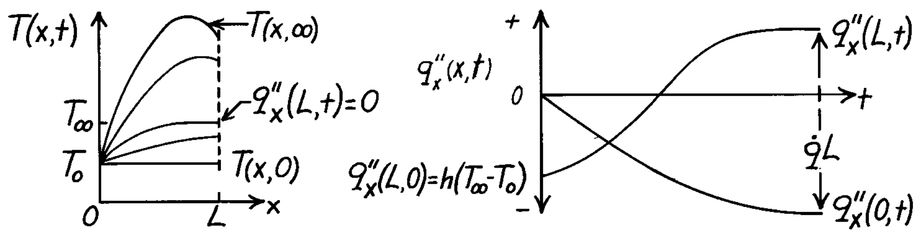
Initial ($t \le 0$): $T(x,0) = T_o$ Uniform temperature

Boundary: $x = 0 \quad T(0,t) = T_o$ Constant temperature

$$x = L \qquad -k\left.\frac{\partial T}{\partial x}\right)_{x=L} = h\left[T(L,t) - T_\infty\right] \qquad \text{Convection process.}$$

The temperature distributions are shown on the T-x coordinates below. Note the special condition when the heat flux at (x = L) is zero.

(b) The heat flux as a function of time at the boundaries, $q''_x(0,t)$ and $q''_x(L,t)$, can be inferred from the temperature distributions using Fourier's law.



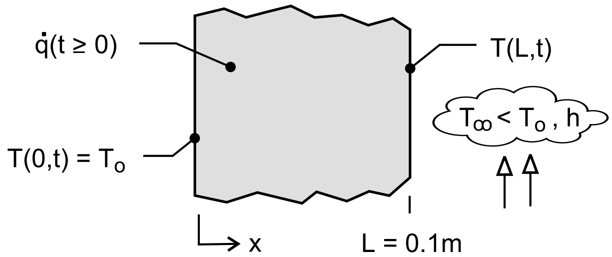
COMMENTS: Since $T(x,\infty) > T_\infty$ and $T_\infty > T_o$, heat transfer at both boundaries must be out of the wall. Hence, it follows from an overall energy balance on the wall that $+q''_x(0,\infty) - q''_x(L,\infty) + \dot{q}L = 0$.

PROBLEM 2.49

KNOWN: Plane wall, initially at a uniform temperature T_o, has one surface (x = L) suddenly exposed to a convection process ($T_\infty < T_o$, h), while the other surface (x = 0) is maintained at T_o. Also, wall experiences uniform volumetric heating $\dot{q}$ such that the maximum steady-state temperature will exceed T_∞.

FIND: (a) Sketch temperature distribution (T vs. x) for following conditions: initial ($t \le 0$), steady-state ($t \to \infty$), and two intermediate times; identify key features of the distributions, (b) Sketch the heat flux $\left(q''_x \text{ vs. } t\right)$ at the boundaries x = 0 and L; identify key features of the distributions.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) Uniform volumetric generation, (4) $T_\infty < T_o$ and $\dot{q}$ large enough that $T(x,\infty) > T_o$.

ANALYSIS: (a) The initial and boundary conditions for the wall can be written as

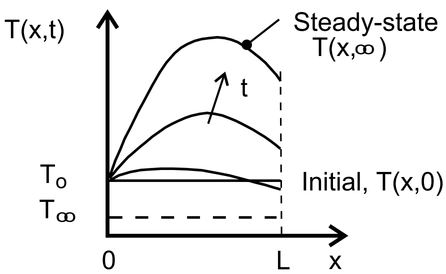
Initial ($t \le 0$): $\quad T(x,0) = T_o \quad$ Uniform temperature

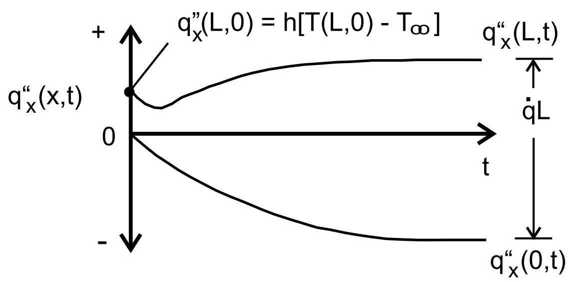
Boundary: $\quad x = 0 \quad T(0,t) = T_o \quad$ Constant temperature

$$x = L \qquad -k\left.\frac{\partial T}{\partial x}\right)_{x=L} = h\left[T(L,t) - T_\infty\right] \qquad \text{Convection process.}$$

The temperature distributions are shown on the T-x coordinates below. Note that the maximum temperature occurs under steady-state conditions not at the midplane, but to the right toward the surface experiencing convection. The temperature gradients at x = L increase for t > 0 since the convection heat rate from the surface increases as the surface temperature increases.

(b) The heat flux as a function of time at the boundaries, $q''_x(0,t)$ and $q''_x(L,t)$, can be inferred from the temperature distributions using Fourier's law. At the surface x = L, the convection heat flux at t = 0 is $q''_x(L,0) = h(T_o - T_\infty)$. Because the surface temperature dips slightly at early times, the convection heat flux decreases slightly, and then increases until the steady-state condition is reached. For the steady-state condition, heat transfer at both boundaries must be out of the wall. It follows from an overall energy balance on the wall that $+q''_x(0,\infty) - q''_x(L,\infty) + \dot{q}L = 0$.



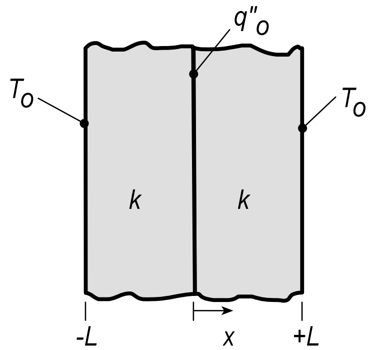


PROBLEM 2.50

KNOWN: Interfacial heat flux and outer surface temperature of adjoining, equivalent plane walls.

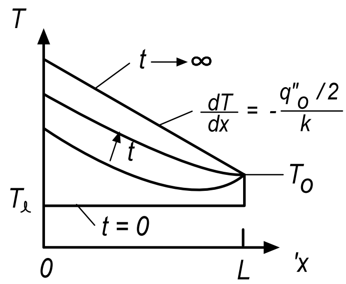
FIND: (a) Form of temperature distribution at representative times during the heating process, (b) Variation of heat flux with time at the interface and outer surface.

SCHEMATIC:

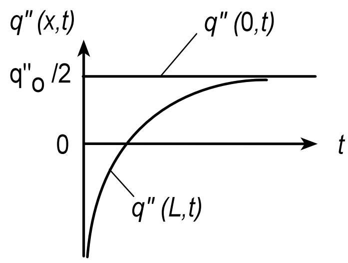


ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties.

ANALYSIS: (a) With symmetry about the interface, consideration of the temperature distribution may be restricted to $0 \le x \le L$. During early stages of the process, heat transfer is *into* the material from the outer surface, as well as from the interface. During later stages and the eventual steady state, heat is transferred *from* the material at the outer surface. At steady-state, $dT/dx = -\left(q_o''/2\right)/k = \text{const}$. and $T(0,t) = T_o + \left(q_o''/2\right)L/k$.



(b) At the outer surface, the heat flux is initially negative, but increases with time, approaching $q_o''/2$. It is zero when $dT/dx|_{x=L} = 0$.

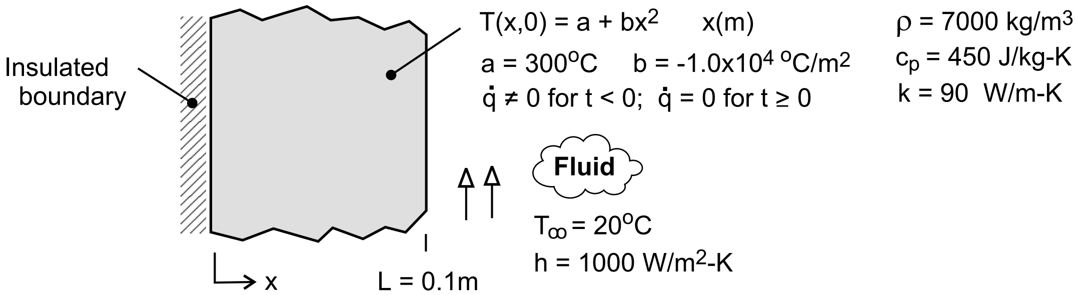


PROBLEM 2.51

KNOWN: Temperature distribution in a plane wall of thickness L experiencing uniform volumetric heating $\dot{q}$ having one surface (x = 0) insulated and the other exposed to a convection process characterized by T_∞ and h. Suddenly the volumetric heat generation is deactivated while convection continues to occur.

FIND: (a) Determine the magnitude of the volumetric energy generation rate associated with the initial condition, (b) On T-x coordinates, sketch the temperature distributions for the initial condition ($T \le 0$), the steady-state condition ($t \to \infty$), and two intermediate times; (c) On q''_x - t coordinates, sketch the variation with time of the heat flux at the boundary exposed to the convection process, $q''_x(L,t)$; calculate the corresponding value of the heat flux at t = 0; and (d) Determine the amount of energy removed from the wall per unit area (J/m^2) by the fluid stream as the wall cools from its initial to steady-state condition.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, and (3) Uniform internal volumetric heat generation for t < 0.

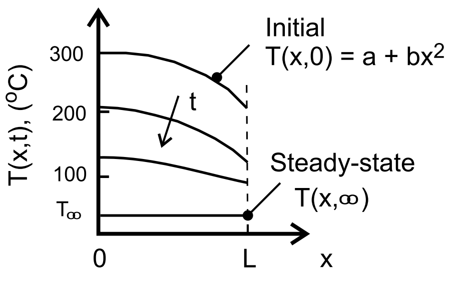
ANALYSIS: (a) The volumetric heating rate can be determined by substituting the temperature distribution for the initial condition into the appropriate form of the heat diffusion equation.

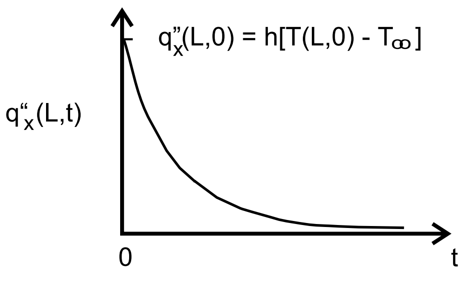
$$\frac{d}{dx}\left(\frac{dT}{dx}\right)+\frac{\dot{q}}{k}=0 \qquad \text{where} \qquad T(x,0)=a+bx^2$$

$$\frac{d}{dx}(0+2bx)+\frac{\dot{q}}{k}=0+2b+\frac{\dot{q}}{k}=0$$

$$\dot{q}=-2kb=-2\times 90\,\text{W/m}\cdot\text{K}\left(-1.0\times 10^{4}\,^\circ\text{C/m}^2\right)=1.8\times 10^6\,\text{W/m}^3 \qquad <$$

(b) The temperature distributions are shown in the sketch below.





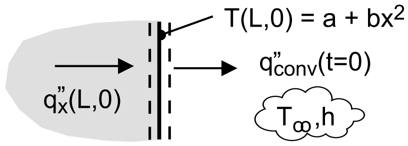
Continued …..

PROBLEM 2.51 (Cont.)

(c) The heat flux at the exposed surface x = L, $q''_x(L,0)$, is initially a maximum value and decreases with increasing time as shown in the sketch above. The heat flux at t = 0 is equal to the convection heat flux with the surface temperature T(L,0). See the surface energy balance represented in the schematic.

$$q''_x(L,0) = q''_{conv}(t=0) = h\left(T(L,0) - T_\infty\right) = 1000\,W/m^2 \cdot K\left(200-20\right)°C = 1.80\times10^5\,W/m^2 \quad <$$

where $\quad T(L,0) = a + bL^2 = 300°C - 1.0\times10^4 °C/m^2 \left(0.1m\right)^2 = 200°C.$



(d) The energy removed from the wall to the fluid as it cools from its initial to steady-state condition can be determined from an energy balance on a time interval basis, Eq. 1.11b. For the initial state, the wall has the temperature distribution $T(x,0) = a + bx^2$; for the final state, the wall is at the temperature of the fluid, $T_f = T_\infty$. We have used T_∞ as the reference condition for the energy terms.

$$E''_{in} - E''_{out} = \Delta E''_{st} = E''_f - E''_i \qquad \text{with} \qquad E''_{in} = 0$$

$$-E''_{out} = \rho c_p L\left[T_f - T_\infty\right] - \rho c_p \int_{x=0}^{x=L} \left[T(x,0) - T_\infty\right] dx$$

$$E''_{out} = \rho c_p \int_{x=0}^{x=L} \left[a + bx^2 - T_\infty\right] dx = \rho c_p \left[ax + bx^3/3 - T_\infty x\right]_0^L$$

$$E''_{out} = 7000\,kg/m^3 \times 450\,J/kg\cdot K\left[300\times0.1 - 1.0\times10^4\left(0.1\right)^3/3 - 20\times0.1\right]K\cdot m$$

$$E''_{out} = 7.77\times10^7\,J/m^2 \quad <$$

COMMENTS: (1) In the temperature distributions of part (a), note these features: initial condition has quadratic form with zero gradient at the adiabatic boundary; for the steady-state condition, the wall has reached the temperature of the fluid; for all distributions, the gradient at the adiabatic boundary is zero; and, the gradient at the exposed boundary decreases with increasing time.

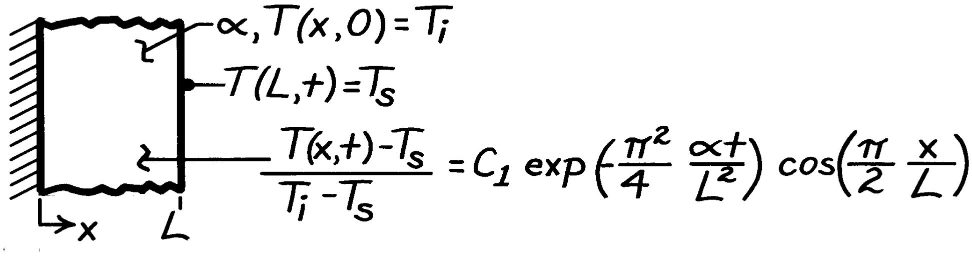
(2) In this thermodynamic analysis, we were able to determine the energy transferred during the cooling process. However, we cannot determine the rate at which cooling of the wall occurs without solving the heat diffusion equation.

PROBLEM 2.52

KNOWN: Temperature as a function of position and time in a plane wall suddenly subjected to a change in surface temperature, while the other surface is insulated.

FIND: (a) Validate the temperature distribution, (b) Heat fluxes at x = 0 and x = L, (c) Sketch of temperature distribution at selected times and surface heat flux variation with time, (d) Effect of thermal diffusivity on system response.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in x, (2) Constant properties.

ANALYSIS: (a) To be valid, the temperature distribution must satisfy the appropriate forms of the heat equation and boundary conditions. Substituting the distribution into Equation 2.15, it follows that

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$-C_1(T_i - T_s)\exp\left(-\frac{\pi^2}{4}\frac{\alpha t}{L^2}\right)\left(\frac{\pi}{2L}\right)^2 \cos\left(\frac{\pi}{2}\frac{x}{L}\right)$$

$$= -\frac{C_1}{\alpha}(T_i - T_s)\left(\frac{\pi^2}{4}\frac{\alpha}{L^2}\right)\exp\left(-\frac{\pi^2}{4}\frac{\alpha t}{L^2}\right)\cos\left(\frac{\pi}{2}\frac{x}{L}\right).$$ <

Hence, the heat equation is satisfied. Applying boundary conditions at x = 0 and x = L, it follows that

$$\frac{\partial T}{\partial x}\Big|_{x=0} = -\frac{C_1 \pi}{2L}(T_i - T_s)\exp\left(-\frac{\pi^2}{4}\frac{\alpha t}{L^2}\right)\sin\left(\frac{\pi}{2}\frac{x}{L}\right)\Big|_{x=0} = 0$$ <

and

$$T(L,t) = T_s + C_1(T_i - T_s)\exp\left(-\frac{\pi^2}{4}\frac{\alpha t}{L^2}\right)\cos\left(\frac{\pi}{2}\frac{x}{L}\right)\Big|_{x=L} = T_s.$$ <

Hence, the boundary conditions are also satisfied.

(b) The heat flux has the form

$$q''_x = -k\frac{\partial T}{\partial x} = +\frac{kC_1\pi}{2L}(T_i - T_s)\exp\left(-\frac{\pi^2}{4}\frac{\alpha t}{L^2}\right)\sin\left(\frac{\pi}{2}\frac{x}{L}\right).$$

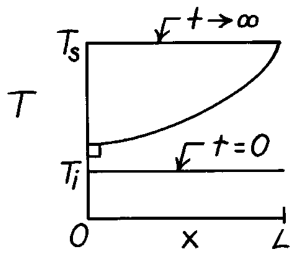
Continued …..

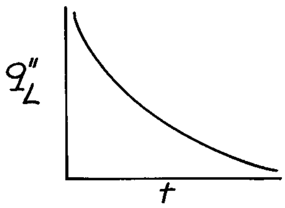
PROBLEM 2.52 (Cont.)

Hence, $q''_x(0) = 0,$ <

$$q''_x(L) = +\frac{kC_1\pi}{2L}(T_i - T_s)\exp\left(-\frac{\pi^2}{4}\frac{\alpha t}{L^2}\right).$$ <

(c) The temperature distribution and surface heat flux variations are:





(d) For materials A and B of different α,

$$\frac{\left[T(x,t) - T_s\right]_A}{\left[T(x,t) - T_s\right]_B} = \exp\left[-\frac{\pi^2}{4L^2}(\alpha_A - \alpha_B)t\right]$$

Hence, if $\alpha_A > \alpha_B$, $T\,(x,t) \to T_s$ more rapidly for Material A. If $\alpha_A < \alpha_B$, $T\,(x,t) \to T_s$ more rapidly for Material B. <

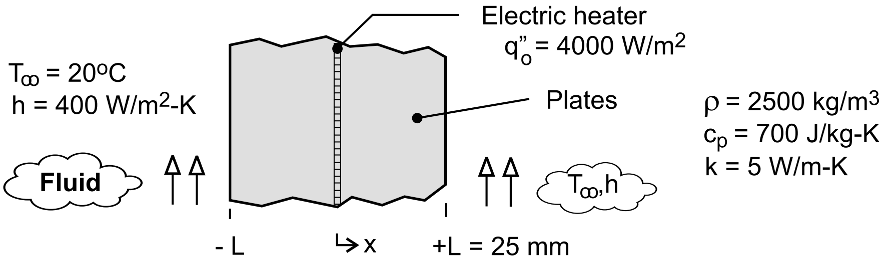
COMMENTS: Note that the prescribed function for T(x,t) does not reduce to T_i for t $\to$ 0. For times at or close to zero, the function is not a valid solution of the problem. At such times, the solution for T(x,t) must include additional terms. The solution is consideed in Section 5.5.1 of the text.

PROBLEM 2.53

KNOWN: Thin electrical heater dissipating 4000 W/m^2 sandwiched between two 25-mm thick plates whose surfaces experience convection.

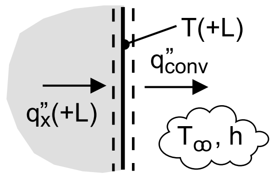
FIND: (a) On T-x coordinates, sketch the steady-state temperature distribution for -L ≤ × ≤ +L; calculate values for the surfaces x = L and the mid-point, x = 0; label this distribution as Case 1 and explain key features; (b) Case 2: sudden loss of coolant causing existence of adiabatic condition on the x = +L surface; sketch temperature distribution on same T-x coordinates as part (a) and calculate values for x = 0, ± L; explain key features; (c) Case 3: further loss of coolant and existence of adiabatic condition on the x = - L surface; situation goes undetected for 15 minutes at which time power to the heater is deactivated; determine the eventual (t → ∞) uniform, steady-state temperature distribution; sketch temperature distribution on same T-x coordinates as parts (a,b); and (d) On T-t coordinates, sketch the temperature-time history at the plate locations x = 0, ± L during the transient period between the steady-state distributions for Case 2 and Case 3; at what location and when will the temperature in the system achieve a maximum value?

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) No internal volumetric generation in plates, and (3) Negligible thermal resistance between the heater surfaces and the plates.

ANALYSIS: (a) Since the system is symmetrical, the heater power results in equal conduction fluxes through the plates. By applying a surface energy balance on the surface x = +L as shown in the schematic, determine the temperatures at the mid-point, x = 0, and the exposed surface, x + L.



$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q''_x(+L) - q''_{conv} = 0 \qquad \text{where} \qquad q''_x(+L) = q''_o / 2$$

$$q''_o / 2 - h\left[T(+L) - T_\infty\right] = 0$$

$$T_1(+L) = q''_o / 2h + T_\infty = 4000\,\text{W/m}^2 / \left(2 \times 400\,\text{W/m}^2 \cdot \text{K}\right) + 20°\text{C} = 25°\text{C} \qquad <$$

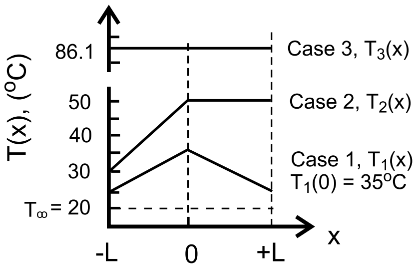
From Fourier's law for the conduction flux through the plate, find T(0).

$$q''_x = q''_o / 2 = k\left[T(0) - T(+L)\right] / L$$

$$T_1(0) = T_1(+L) + q''_o L / 2k = 25°\text{C} + 4000\,\text{W/m}^2 \cdot \text{K} \times 0.025\text{m} / \left(2 \times 5\,\text{W/m} \cdot \text{K}\right) = 35°\text{C} \qquad <$$

The temperature distribution is shown on the T-x coordinates below and labeled Case 1. The key features of the distribution are its symmetry about the heater plane and its linear dependence with distance.

Continued …..



(b) Case 2: sudden loss of coolant with the existence of an adiabatic condition on surface x = +L. For this situation, all the heater power will be conducted to the coolant through the left-hand plate. From a surface energy balance and application of Fourier's law as done for part (a), find

$$T_2(-L) = q_o''/h + T_\infty = 4000\,\mathrm{W/m^2}/400\,\mathrm{W/m^2\cdot K} + 20°\mathrm{C} = 30°\mathrm{C} \qquad <$$

$$T_2(0) = T_2(-L) + q_o'' L/k = 30°\mathrm{C} + 4000\,\mathrm{W/m^2} \times 0.025\ \mathrm{m}/5\,\mathrm{W/m\cdot K} = 50°\mathrm{C} \qquad <$$

The temperature distribution is shown on the T-x coordinates above and labeled Case 2. The distribution is linear in the left-hand plate, with the maximum value at the mid-point. Since no heat flows through the right-hand plate, the gradient must zero and this plate is at the maximum temperature as well. The maximum temperature is higher than for Case 1 because the heat flux through the left-hand plate has increased two-fold.

(c) Case 3: sudden loss of coolant occurs at the x = -L surface also. For this situation, there is no heat transfer out of either plate, so that for a 15-minute period, Δt_o, the heater dissipates 4000 W/m^2 and then is deactivated. To determine the eventual, uniform steady-state temperature distribution, apply the conservation of energy requirement on a time-interval basis, Eq. 1.11b. The initial condition corresponds to the temperature distribution of Case 2, and the final condition will be a uniform, elevated temperature $T_f = T_3$ representing Case 3. We have used T_∞ as the reference condition for the energy terms.

$$E_{in}'' - E_{out}'' + E_{gen}'' = \Delta E_{st}'' = E_f'' - E_i'' \qquad (1)$$

Note that $E_{in}'' - E_{out}'' = 0$, and the dissipated electrical energy is

$$E_{gen}'' = q_o''\Delta t_o = 4000\,\mathrm{W/m^2}(15\times 60)\mathrm{s} = 3.600\times 10^6\ \mathrm{J/m^2} \qquad (2)$$

For the final condition,

$$\begin{aligned} E_f'' &= \rho c(2L)[T_f - T_\infty] = 2500\,\mathrm{kg/m^3} \times 700\,\mathrm{J/kg\cdot K}(2\times 0.025\mathrm{m})[T_f - 20]°\mathrm{C} \\ E_f'' &= 8.75\times 10^4 [T_f - 20]\,\mathrm{J/m^2} \end{aligned} \qquad (3)$$

where $T_f = T_3$, the final uniform temperature, Case 3. For the initial condition,

$$E_i'' = \rho c\int_{-L}^{+L}[T_2(x) - T_\infty]dx = \rho c\left\{\int_{-L}^{0}[T_2(x) - T_\infty]dx + \int_0^{+L}[T_2(0) - T_\infty]dx\right\} \qquad (4)$$

where $T_2(x)$ is linear for $-L \le x \le 0$ and constant at $T_2(0)$ for $0 \le x \le +L$.

$$T_2(x) = T_2(0) + [T_2(0) - T_2(L)]x/L \qquad -L \le x \le 0$$

$$T_2(x) = 50°\mathrm{C} + [50 - 30]°\mathrm{C}\,x/0.025\mathrm{m}$$

$$T_2(x) = 50°\mathrm{C} + 800x \qquad (5)$$

Substituting for $T_2(x)$, Eq. (5), into Eq. (4)

Continued …..

PROBLEM 2.53 (Cont.)

$$E_i'' = \rho c \left\{ \int_{-L}^{0} \left[50 + 800x - T_\infty \right] dx + \left[T_2(0) - T_\infty \right] L \right\}$$

$$E_i'' = \rho c \left\{ \left[50x + 400x^2 - T_\infty x \right]_{-L}^{0} + \left[T_2(0) - T_\infty \right] L \right\}$$

$$E_i'' = \rho c \left\{ -\left[-50L + 400L^2 + T_\infty L \right] + \left[T_2(0) - T_\infty \right] L \right\}$$

$$E_i'' = \rho c L \left\{ +50 - 400L - T_\infty + T_2(0) - T_\infty \right\}$$

$$E_i'' = 2500\,\mathrm{kg/m^3} \times 700\,\mathrm{J/kg\cdot K} \times 0.025\,\mathrm{m} \left\{ +50 - 400 \times 0.025 - 20 + 50 - 20 \right\} \mathrm{K}$$

$$E_i'' = 2.188 \times 10^6\,\mathrm{J/m^2} \tag{6}$$

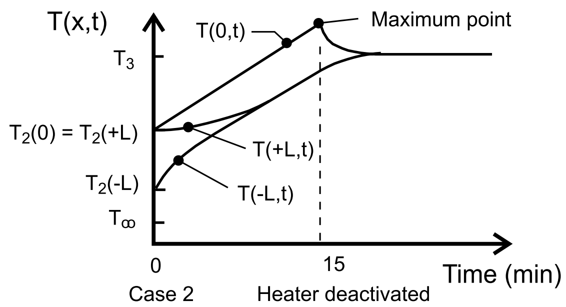
Returning to the energy balance, Eq. (1), and substituting Eqs. (2), (3) and (6), find $T_f = T_3$.

$$3.600 \times 10^6\,\mathrm{J/m^2} = 8.75 \times 10^4 \left[T_3 - 20 \right] - 2.188 \times 10^6\,\mathrm{J/m^2}$$

$$T_3 = (66.1 + 20)^\circ\mathrm{C} = 86.1^\circ\mathrm{C}$$ <

The temperature distribution is shown on the T-x coordinates above and labeled Case 3. The distribution is uniform, and considerably higher than the maximum value for Case 2.

(d) The temperature-time history at the plate locations x = 0, ± L during the transient period between the distributions for Case 2 and Case 3 are shown on the T-t coordinates below.



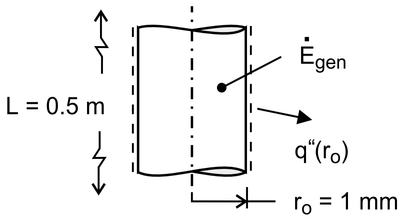
Note the temperatures for the locations at time t = 0 corresponding to the instant when the surface x = - L becomes adiabatic. These temperatures correspond to the distribution for Case 2. The heater remains energized for yet another 15 minutes and then is deactivated. The midpoint temperature, T(0,t), is always the hottest location and the maximum value slightly exceeds the final temperature T_3.

PROBLEM 2.54

KNOWN: Radius and length of coiled wire in hair dryer. Electric power dissipation in the wire, and temperature and convection coefficient associated with air flow over the wire.

FIND: (a) Form of heat equation and conditions governing transient, thermal behavior of wire during start-up, (b) Volumetric rate of thermal energy generation in the wire, (c) Sketch of temperature distribution at selected times during start-up, (d) Variation with time of heat flux at r = 0 and r = r_o.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, radial conduction, (2) Constant properties, (3) Uniform volumetric heating, (4) Negligible radiation from surface of wire.

ANALYSIS: (a) The general form of the heat equation for cylindrical coordinates is given by Eq. 2.20. For one-dimensional, radial conduction and constant properties, the equation reduces to

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\dot{q}=\frac{\rho c_p}{k}\frac{\partial T}{\partial t}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad <$$

The initial condition is $\quad T(r,0)=T_i \qquad <$

The boundary conditions are: $\quad \partial T/\partial r|_{r=0}=0 \qquad <$

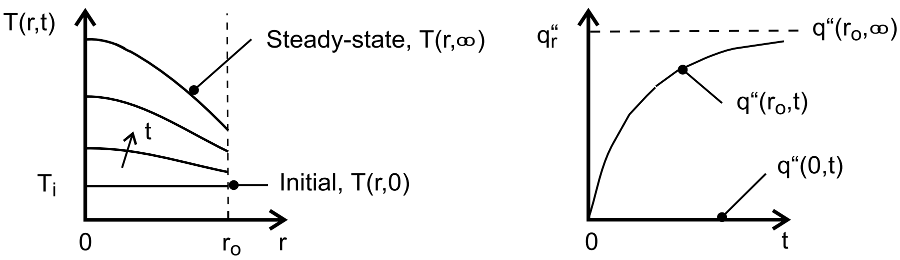
$$-k\left.\frac{\partial T}{\partial r}\right|_{r=r_o}=h\left[T(r_o,t)-T_\infty\right] \qquad <$$

(b) The volumetric rate of thermal energy generation is

$$\dot{q}=\frac{\dot{E}_g}{\forall}=\frac{P_{elec}}{\pi r_o^2 L}=\frac{500\ W}{\pi(0.001m)^2(0.5m)}=3.18\times10^8\ W/m^3 \qquad <$$

Under steady-state conditions, all of the thermal energy generated within the wire is transferred to the air by convection. Performing an energy balance for a control surface about the wire, $-\dot{E}_{out}+\dot{E}_g=0$, it follows that $-2\pi r_o L\ q''(r_o, t\to\infty)+P_{elec}=0$. Hence,

$$q''(r_o, t\to\infty)=\frac{P_{elec}}{2\pi r_o L}=\frac{500\ W}{2\pi(0.001m)0.5m}=1.59\times10^5\ W/m^2 \qquad <$$



COMMENTS: The symmetry condition at r = 0 imposes the requirement that $\partial T/\partial r|_{r=0}=0$, and hence $q''(0,t)=0$ throughout the process. The temperature at r_o, and hence the convection heat flux, increases steadily during the start-up, and since conduction to the surface must be balanced by convection from the surface at all times, $|\partial T/\partial r|_{r=r_o}$ also increases during the start-up.