# Chapter 48 Outline

## Network Modeling: Estimation, Inference, Comparison, and Selection

**John P. Schoeneman**[1] **and Bruce A. Desmarais**[1]

[1]Penn State, Political Science, University Park, Pond Lab

**Introduction:** Most statistical models used in the social sciences rely on the assumption that the observations in a dataset (or at least clusters of observation) are drawn independently from a common data generating process. However, when analyzing networks it is often inappropriate to assume that observations are independent (Desmarais and Cranmer, 2017). Indeed, the network analyst is often interested in studying the ways in which observations depend upon each other (e.g., do friends influence each others' choices to vote (Bond et al., 2012), do legislators reciprocate support for legislation (Kirkland and Williams, 2014)).

**Overview:** In this chapter we will review three of the most popular statistical models that have been developed to analyze network data, along with their extensions. These include the exponential random graph model (ERGM) (Cranmer and Desmarais, 2011), the latent space model (LSM) (Dorff, Minhas and Ward, 2016), and the stochastic block model (SBM) (Latouche, Birmelé and Ambroise, 2011). We will review the application of each method to networks in which ties are binary (e.g., are two legislators on at least one committee together), count (e.g., on how many committees do two legislators both serve), and continuous (e.g., how much time do two legislators spend together in committee meetings each week). The complexity and analytical intractability of the mathematical forms of these models render exact approaches to estimation infeasible. As such, estimation in these models proceeds by sampling from a Bayesian posterior and/or approximate methods of maximum likelihood (Nowicki and Snijders, 2001; Raftery et al., 2012; Van Duijn, Gile and Handcock, 2009). We will review these estimation methods, and how they are applied to the ERGM, LSM, and SBM. One complication that arises from the variety of sampling-based and approximate approaches to inference with network models is that standard information criteria that would normally be used to select between different families of statistical models (e.g., AIC, BIC, DIC) cannot be calculated in a manner that is comparable across different families of network models. Recently, researchers have considered various methods of out-of-sample prediction in order to compare and select among network models in terms of generalization error (e.g., Desmarais and Cranmer, 2012; Dorff, Minhas and Ward, 2016), with some debate regarding whether this is an appropriate approach (Block et al., 2018). We will conduct a simulation study to evaluate the effectiveness of comparing the performance of models in terms of generalization error. We will consider and compare several measures of generalization error, and provide recommendations regarding which measures should be used with specific types of networks and models. This simulation study will be, to our knowledge, the first experiment designed to evaluate approaches to comparing network models based on generalization error.

**Organization:** This chapter will be divided into three sections. In the first section we will give an overview of the most common models used in network analysis. In the second section we will discuss the approaches to estimation that are commonly used with these models. In the last section we will conduct a simulation study with the goal of evaluating the effectiveness of measuring generalization error to assist practitioners in selecting among these models.

# 1 Introduction

Do states go to war with the enemies of their enemies? Which international trade relationships are most likely to break down over the next decade? What would happen to the Statistical inference with network data permits researchers to test hypotheses about network generation, predict network structure, and simulate networks from probabilistic models. How would the world system of military alliances rewire if the United States were to exit the North Atlantic Treaty Organization? What these questions have in common is that they address systems of international relationships (conflict, trade, and alliances)—systems that can be represented as networks in which the states constitute the nodes, and the relationships constitute the edges between nodes. Inferential network analysis represents a methodological class that can be drawn upon to answer questions like these.

Many, perhaps even most, of the empirical phenomena of interest to scholars of international relations can be represented as international networks. Paired with the availability of accessible software implementations, the conceptual appropriateness of inferential network analysis in international relations has led to widespread application over the last 10–15 years. Networks that have been studied through the use of inferential network analysis include, but are not limited to, trade (Chu-Shore, 2010; Ward, Ahlquist and Rozenas, 2013; Ward and Hoff, 2007) [CITES], conflict (Cranmer and Desmarais, 2011; Dorff and Ward, 2013; Gallop, 2016; Hoff and Ward, 2004; Ward, Siverson and Cao, 2007) [CITES], alliances (Cranmer, Desmarais and Menninga, 2012) [CITES], transnational terrorism (Asal et al., 2016; Desmarais and Cranmer, 2013; Metternich et al., 2013) [CITES], sanctioning (Cranmer, Heinrich and Desmarais, 2014; Dorff and Minhas, 2017)[CITES], IGOs (Cao, 2012; Dorussen and Ward, 2008; Lupu and Greenhill, 2017) [CITES] and NGOs (Atouba and Shumate, 2015) [CITES]. This research has led to several innovative findings. For example, [SOMETHING FROM KINNE's WORK], latent network structure can be used to forecast the evolution of civil conflict [CITE SHAHRYAR's STUFF], [CITE A FINDING FROM OLGA's WORK]. Inferential network analysis is a valuable piece in the IR scholar's toolkit.

Methods for inferential network analysis come primarily in the form of probabilistic models for networks, which are fit to data using common estimation frameworks (e.g., maximum likelihood estimation, Bayesian inference). Inferential network analysis is now far too broad of a methodological area to effectively review all of the models in any detail. In what follows we will review three modeling approaches in detail. The first model, which we refer to as the latent space model (LSM), is a class of models in which each node is attributed with a position in a latent space of features. The second model, the exponential random graph model (ERGM) is a model that can be customized to represent networks with any quantifiable characteristic (e.g., a high number of reciprocated ties, a strong degree of clustering). The third model we discuss in detail is the stochastic blockmodel (SBM). The first two models, LSM and ERGM, are commonly used in international relations. The SBM, however, has seen little use in IR. We make the case that the SMB should be used in political science as an alternative to ERGM and SBM. After presenting a review of these three models, we provide a brief overview of several approaches that we did not review in detail, but of which IR scholars should be aware. After reviewing these models we present an application in which we replicate [the analysis in PHILIPS PAPER] using each of these modelsand focus in particular on the application of SBM, since SBM is new in IR.

# 2 Model description

The three models we review in detail, the LSM, ERGM, and SBM share a common starting point in the case of dichotomous edges (i.e., the edge does or does not exist). Consider a research problem in which the researcher is interested in using covariates that can be measured on the directed pair $(i, j)$, where $i$

is the index of the potential sender of an edge, and $j$ is the index of the potential receiver of the edge, to model whether the edge $(i, j)$ exists. Denote the edge indicator $y_{i,j} = 1$ if there is an edge from node $i$ to node $j$, and 0 otherwise. Let $x_{i,j}$ be a covariate that can be measured on the directed dyad (e.g., if the nodes are states, the distance between their capital cities), or mapped to the dyad (e.g., $x_{i,j}$ is the GDP of the sender state, $i$). A standard approach to modeling the dyadic variable $y$ as a function of $x$ would be to estimate a logistic regression in which $y$ is the dependent variable and $x$ is the independent variable. The LSM, ERGM, and SBM all reduce to a logistic regression in which edge indicators are regressed on measured covariates in the case where the network components of the respective models do not contribute to the fit of the model [CITES]. In other words, logistic regression is a special case of LSM, ERGM, and SBM.

In the case of each model, network structure is layered on top of the simple logistic regression [PROBIT FOR AMEN?]. The particular form of network structure added varies across the models. In the LSM, each node is represented by some number of continuous-valued features (e.g., coordinates in a Euclidean space), and the probability of a tie is given by some function of the features (e.g., the Euclidean distance between nodes). We refer to the network structure incorporated in the LSM framework as 'selection', since the function defined on the latent feature serves as a partner selection function for nodes in the network. The network component under ERGM is designed to capture the prevalence of subnetworks that are theoretically interesting or otherwise distinct. Examples of these subnetworks include triangles (i.e., triples of nodes $(i, j.k)$ in which there is an edge connecting each pair of nodes), and mutual dyads (i.e., dyads in which there is an edge from $i$ to $j$ and from $j$ to $i$). In the ERGM, subnetwork prevalence is controlled through the incorporation of dependence among the edges. For example, if a network is modeled to have a relatively high number of mutual dyads, the formation of an edge from $i$ to $j$ increases the likelihood that an edge will form from $j$ to $i$. Since the basic building blocks of the network component of the ERGM reflect different forms of dependence, we refer to the network structure in the ERGM as 'dependence'. The network structure of the SBM is build on a common interest in identifying the communities, clusters, or groups that define the main blocks of ties in the networks. Under the SBM's network structure, the blocks of which two nodes are members determines whether those two nodes will form an edge. Since the network component of the SBM is focused on segmenting the nodes into a set of groups, we refer to the network structure in the SBM as segmentation.

## 2.1 Latent space modeling of networks

The notion of selection is deeply rooted in the international relations literature. The likelihood that one state initiates a militarized conflict against another state has been found to be a function of both the sender and receiver states' level of democratization [CITES], the ratio of the two states' military capabilities scores [CITE], and the geographic distance between the two states [CITES]. The probability of conflict initiation is given by a combination of a variety of functions applied to a variety of states' features. The LSM reflects this very structure—the probability of a tie depending on a combination of functions of node features—with exception being that the features inferred in the LSM are latent. That is, given one or more functions according to which the latent features affect the probability of an edge, in the LSM the nodes' feature values are inferred to be those that best explain the pattern of edges observed in the data.

## 2.2 The exponential random graph model

Theories of dependence in international relations have been developing rapidly in recent years. Example findings include that there is a strong tendency towards rapid retaliation in the issuance of international economic sanctions, SOMETHING FROM A SAOM PAPER, SOMETHING FROM ANOTHER ERGM PAPER. The ERGM opens up a new class of hypotheses that can be tested relative to regression models. In

addition to studying how covariates effect the absence/presence of edges, the ERGM permits researchers to test hypotheses about how the edges effect each other (e.g., is the enemy of an enemy a friend)? The key components in using an ERGM are to develop hypotheses regarding the ways in which edges depend on each other, and conceptualize these dependencies in terms of subnetwork structures such as dyads and triads.

## 2.3 The stochastic blockmodel

Concepts in international relations related to system polarity [CITES], [CONCEPT FROM ELIZABETH AND SKYLER], fall within the orbit of the SBM formulation. Under the SBM there is a finite number of node types or 'blocks', and each block is defined by the probabilities according to which nodes within the block form ties with each other, and with nodes in other blocks. If there are $n$ nodes in the network, the SBM reduces the complexity of interactions among those $n$ nodes, to interactions among and within a set of $b$ blocks, where $b$ is typically much smaller than $n$.

Segmentation SBM is common elsewhere, and should be used more in polisci

## 2.4 Additional methods in brief

The LSM, ERGM, and SBM reflect the core components

# 3 Model comparison and selection

Issue is that we need Bayes for both SBM and LSM, which makes AIC/BIC difficult to calibrate. Would prefer to use held out likelihood.

Challenge with ERGM is that joint likelihood cannot really be evaluated, unlike with LSM or SBM

Wang et al. (2016) provide an excellent example of doing both multiple imputation and held out edge prediction to evaluate ERGM predictive performance.

https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4743534/

# 4 Replication

| | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 |
|---|---|---|---|---|---|---|---|---|
| Edges | −0.87*** | −5.15*** | −2.50*** | −6.00*** | −5.12*** | −5.12*** | −4.58*** | −6.43*** |
| | (0.17) | (1.23) | (0.30) | (0.76) | (1.01) | (1.13) | (0.56) | (0.80) |
| Preference Similarity | 0.18** | 0.11 | −0.02 | −0.02 | 0.09 | 0.11 | −0.02 | 0.02 |
| | (0.06) | (0.07) | (0.11) | (0.12) | (0.07) | (0.07) | (0.11) | (0.13) |
| Government Alter | | 0.60** | | 0.37 | 0.61** | 0.58** | 0.67* | 0.51 |
| | | (0.19) | | (0.33) | (0.19) | (0.19) | (0.30) | (0.34) |
| Scientific Ego | | 0.06 | | 1.54*** | 0.35 | 0.07 | 1.59*** | 1.58*** |
| | | (0.22) | | (0.36) | (0.20) | (0.22) | (0.34) | (0.37) |
| Common Committee | | 0.30*** | | 0.14* | 0.34*** | 0.29*** | 0.21*** | 0.16** |
| | | (0.05) | | (0.06) | (0.05) | (0.05) | (0.05) | (0.06) |
| Scientific Communication | | 2.88*** | | | | 2.86*** | | |
| | | (0.60) | | | | (0.63) | | |
| Political Communication | | | | 2.89*** | | | | 2.94*** |
| | | | | (0.58) | | | | (0.59) |
| Interest Group Homophily | | 1.06*** | | 1.22* | 1.10*** | 1.04*** | 1.55** | 1.28 |
| | | (0.29) | | (0.59) | (0.28) | (0.30) | (0.59) | (0.68) |
| Influence Attribution | | 0.94*** | | 0.32 | 0.92*** | 0.88*** | 0.56 | 0.26 |
| | | (0.18) | | (0.31) | (0.18) | (0.20) | (0.30) | (0.32) |
| gwesp.fixed.0.1 | | 2.52* | | 0.49* | 2.48** | 2.47* | 0.61* | 0.34 |
| | | (1.06) | | (0.23) | (0.86) | (0.97) | (0.24) | (0.23) |
| gwdsp.fixed.0.1 | | −0.13** | | −0.17 | −0.13** | −0.13** | −0.13 | −0.22* |
| | | (0.04) | | (0.10) | (0.04) | (0.05) | (0.09) | (0.09) |
| Reciprocity | | 0.81** | | 1.76*** | 0.86*** | 0.82** | 1.85*** | 1.69*** |
| | | (0.25) | | (0.50) | (0.25) | (0.25) | (0.52) | (0.51) |
| nodeicov.betweenness | | | | | | 0.00 | | 0.02*** |
| | | | | | | (0.00) | | (0.00) |
| AIC | 1161.59 | 849.59 | 456.09 | 313.09 | 887.52 | 850.39 | 352.91 | 303.07 |
| BIC | 1171.13 | 902.04 | 465.62 | 365.55 | 935.21 | 907.61 | 400.60 | 360.29 |
| Log Likelihood | -578.80 | -413.80 | -226.04 | -145.55 | -433.76 | -413.19 | -166.46 | -139.54 |

***$p < 0.001$, **$p < 0.01$, *$p < 0.05$

**Table 1.** ERGM Replication

|                          | Model 1          | Model 2          | Model 3          | Model 4          | Model 5          | Model 6          | Model 7          | Mo      |
|--------------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|---------|
| edges                    | 2.30             | 1.45             | −0.07            | −2.40            | 1.54             | 2.13             | −1.80            | −       |
|                          | [−0.44; 4.98]    | [−1.42; 4.07]    | [−4.04; 3.66]    | [−6.31; 1.36]    | [−1.29; 4.18]    | [−1.23; 5.17]    | [−6.13; 2.09]    | [−6.9   |
| Preference Similarity    | 0.20             | −1.59            | 0.45             | −1.77            | −1.59            | −2.30            | −1.44            | −       |
|                          | [−2.40; 3.03]    | [−4.22; 1.21]    | [−3.32; 4.44]    | [−5.33; 2.12]    | [−4.24; 1.26]    | [−5.14; 1.09]    | [−5.45; 2.91]    | [−5.3   |
| Government Alter         |                  | 0.66*            |                  | 0.68*            | 0.85*            | 0.96*            | 0.82             | 0.      |
|                          |                  | [0.66; 0.66]     |                  | [0.42; 0.97]     | [0.42; 1.30]     | [0.55; 1.43]     | [−0.02; 1.61]    | [0.43   |
| Scientific Ego           |                  | −0.01            |                  | 0.95*            | 0.25             | −0.26*           | 1.33*            | 1.      |
|                          |                  | [−0.45; 0.54]    |                  | [0.03; 1.67]     | [−0.31; 0.83]    | [−0.27; −0.25]   | [0.48; 2.19]     | [0.88   |
| Common Committe          |                  | 1.59*            |                  | 1.22             | 1.89*            | 1.74*            | 2.30*            | 1       |
|                          |                  | [1.58; 1.60]     |                  | [−0.44; 2.82]    | [1.36; 2.41]     | [1.32; 2.35]     | [0.84; 3.64]     | [−0.5   |
| Scientific Commuication  |                  | 3.02*            |                  |                  |                  | 3.35*            |                  |         |
|                          |                  | [1.73; 4.74]     |                  |                  |                  | [2.09; 4.43]     |                  |         |
| Political Commuication   |                  |                  |                  | 2.74*            |                  |                  |                  | 3.      |
|                          |                  |                  |                  | [1.47; 4.11]     |                  |                  |                  | [2.40   |
| Interest Group Homophily |                  | 1.91*            |                  | 1.10             | 2.13*            | 1.89*            | 1.86             | 1.      |
|                          |                  | [1.86; 1.95]     |                  | [−0.75; 2.82]    | [1.07; 3.24]     | [0.98; 2.65]     | [−0.09; 3.88]    | [1.11   |
| Influence Attribution    |                  | 1.40*            |                  | 0.87*            | 1.20*            | 1.28*            | 1.27*            | 0.      |
|                          |                  | [1.40; 1.40]     |                  | [0.10; 1.68]     | [0.81; 1.63]     | [0.90; 1.91]     | [0.56; 2.03]     | [0.22   |
| nodeicov.betweenness     |                  |                  |                  |                  |                  | −0.00*           |                  | 0.      |
|                          |                  |                  |                  |                  |                  | [−0.00; −0.00]   |                  | [0.00   |
| BIC (Overall)            | 1057.64          | 954.68           | 487.17           | 506.51           | 971.64           | 961.00           | 501.48           | 52      |
| BIC (Likelihood)         | 833.39           | 726.85           | 316.74           | 275.73           | 737.39           | 732.03           | 307.71           | 28      |
| BIC (Latent Positions)   | 224.25           | 227.83           | 170.43           | 230.78           | 234.25           | 228.96           | 193.77           | 24      |

* 0 outside the confidence interval

**Table 2.** LSM models

[NOT A ROBUSTNESS CHECK, AN ILLUSTRATION OF SBM]

Preference Similarity

**Table 3.** SBM covariate plot. Results from models for which DIC could not be calculated were discarded.
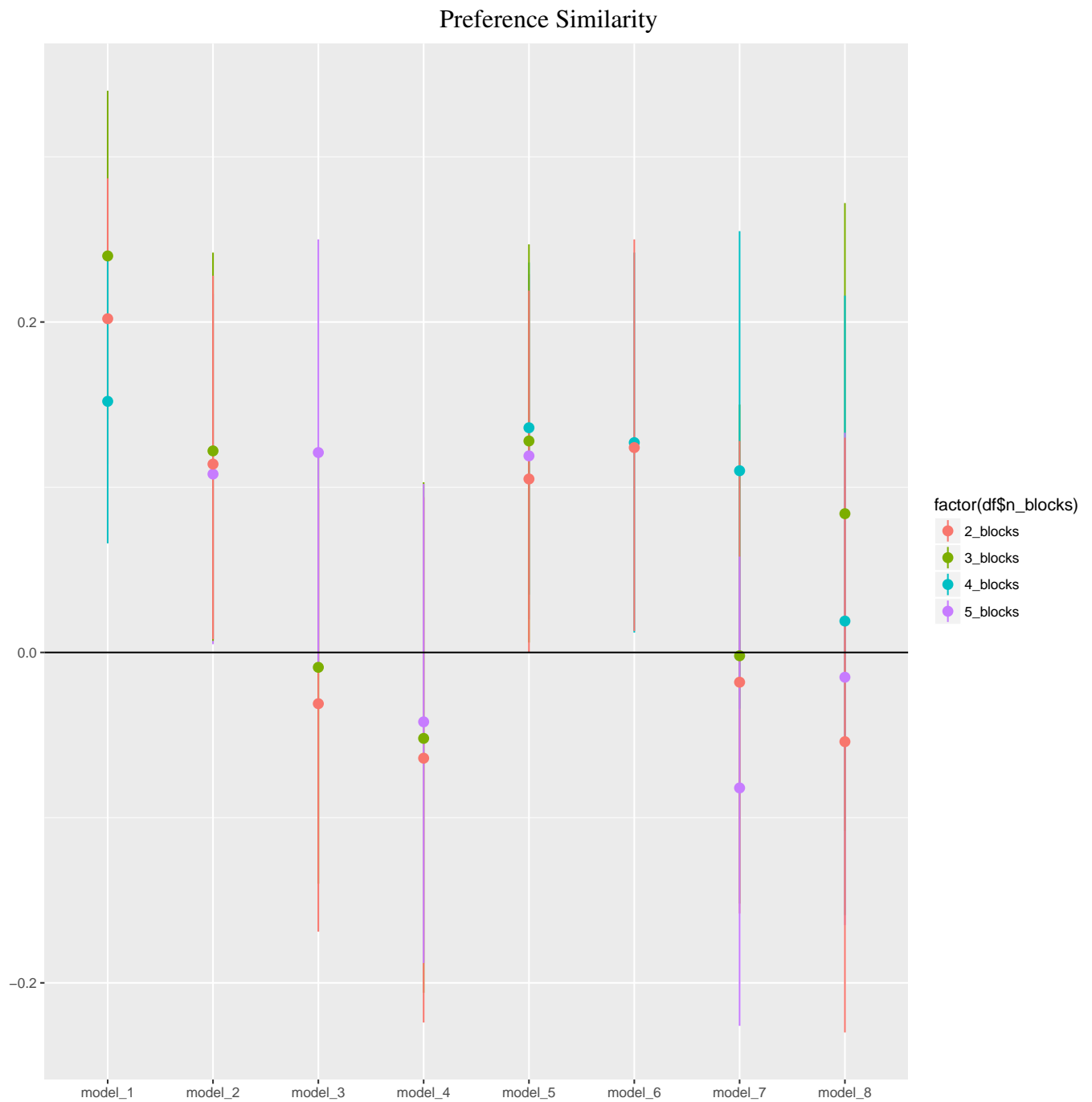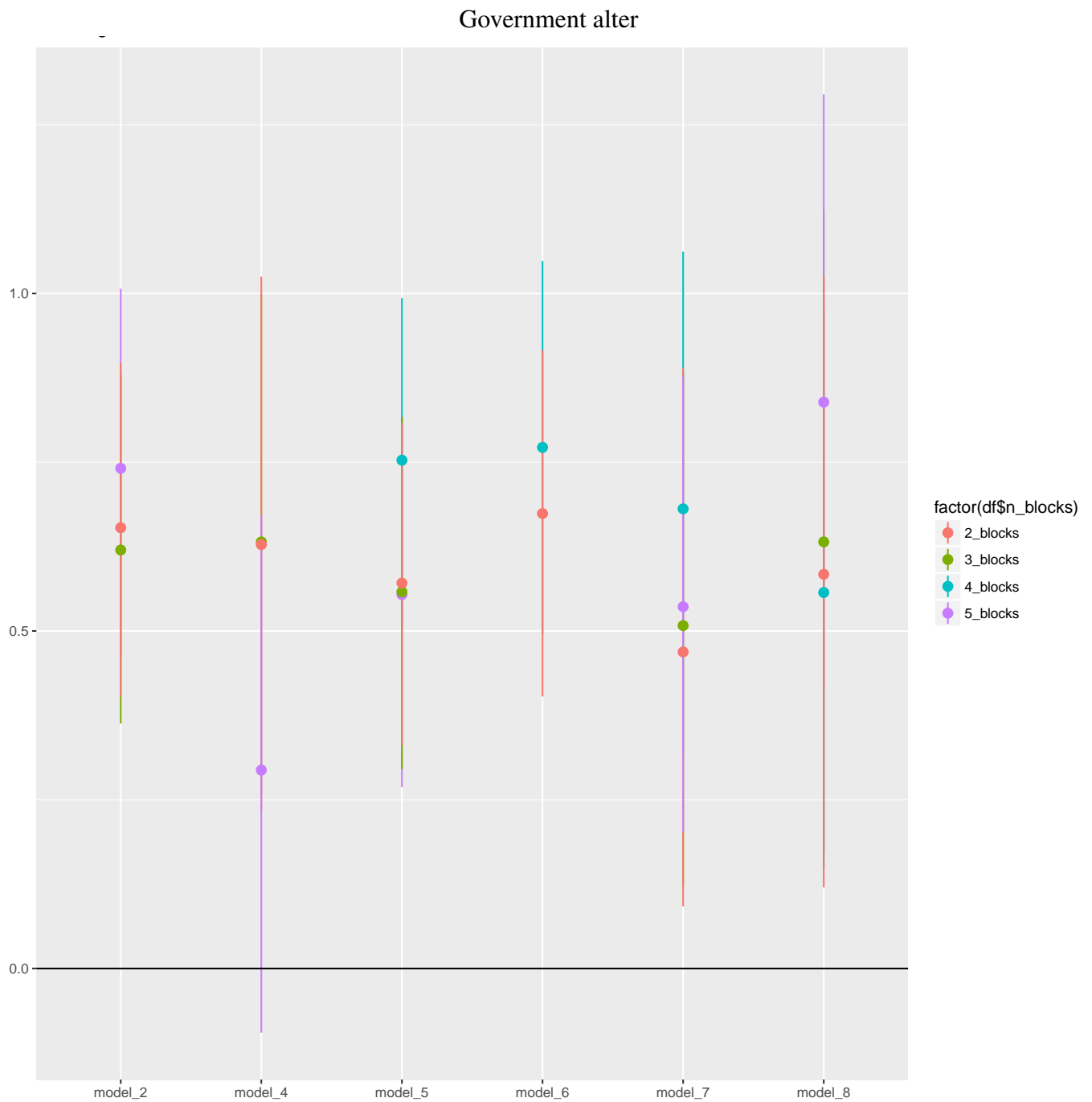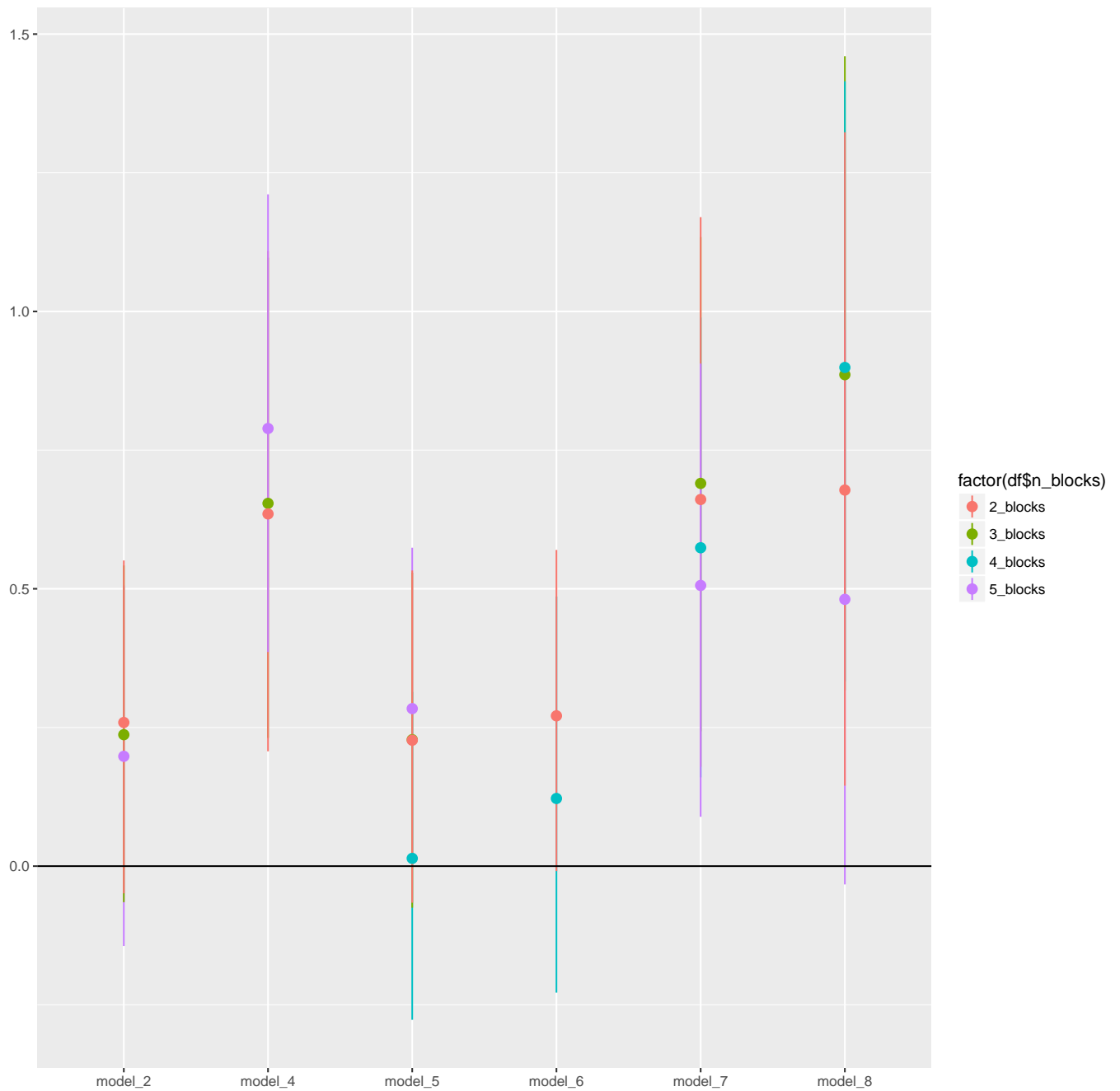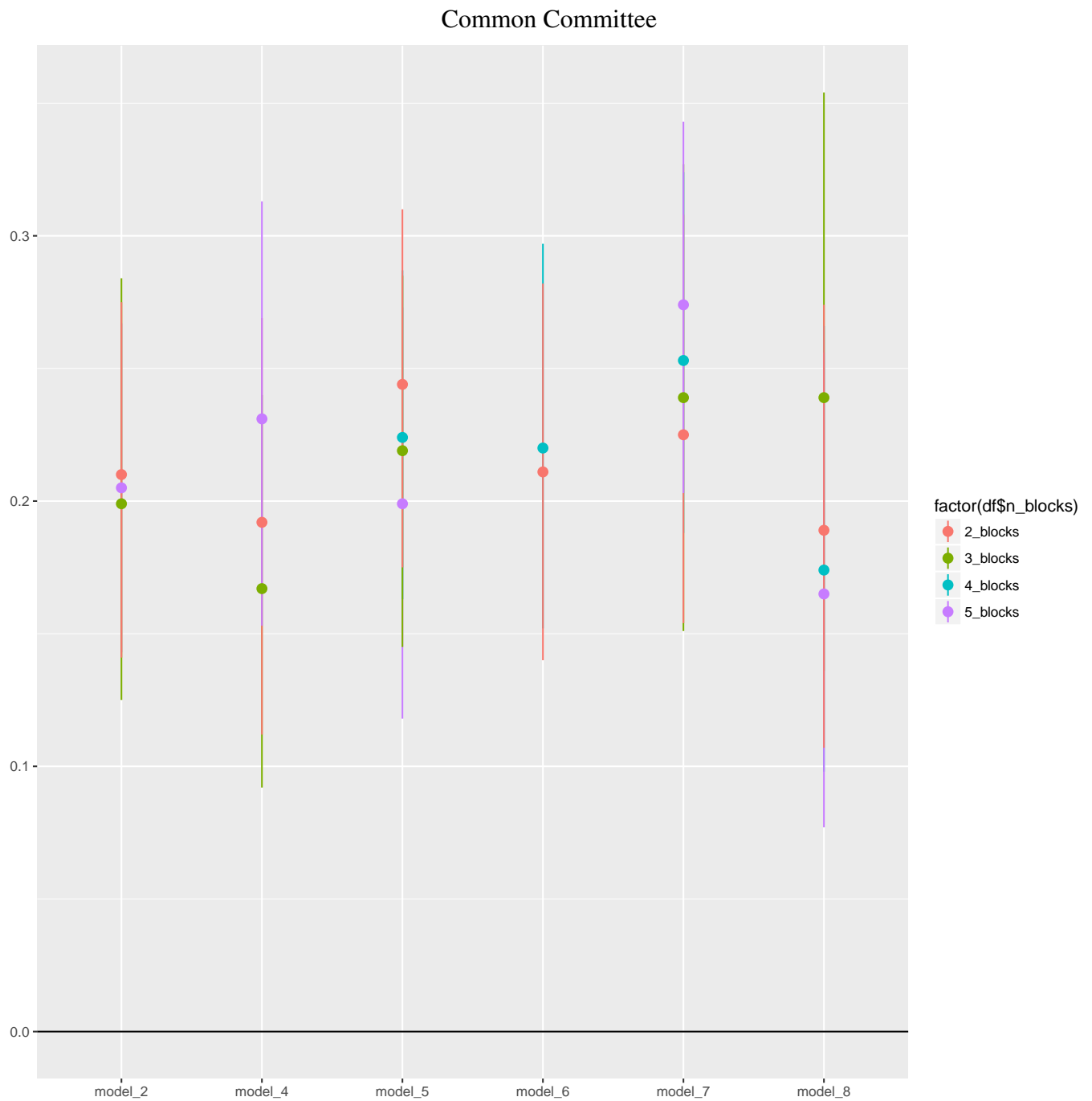
Government alter

**Table 4.** SBM covariate plot. Results from models for which DIC could not be calculated were discarded.

**Table 5.** SBM covariate plot. Results from models for which DIC could not be calculated were discarded.
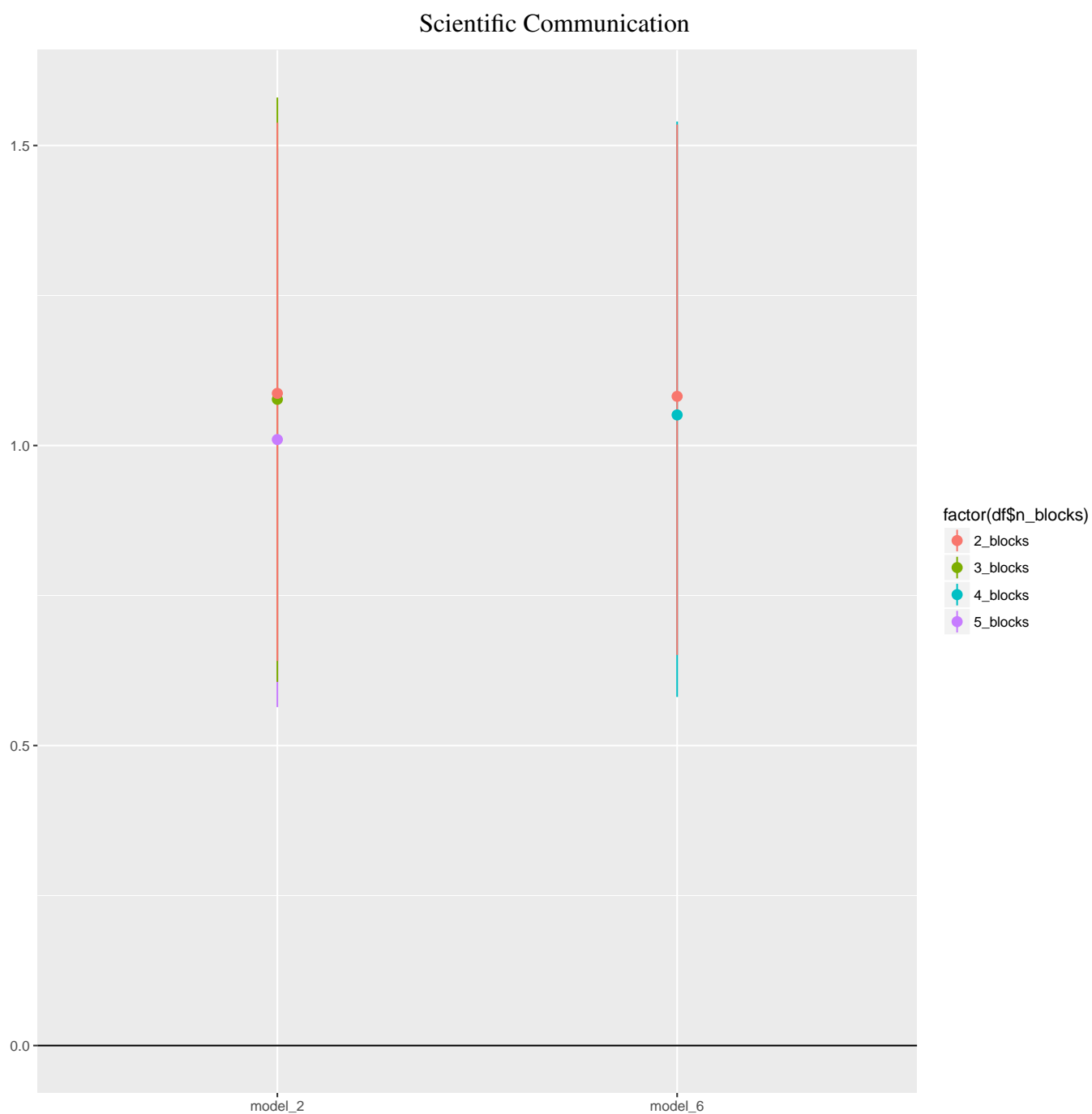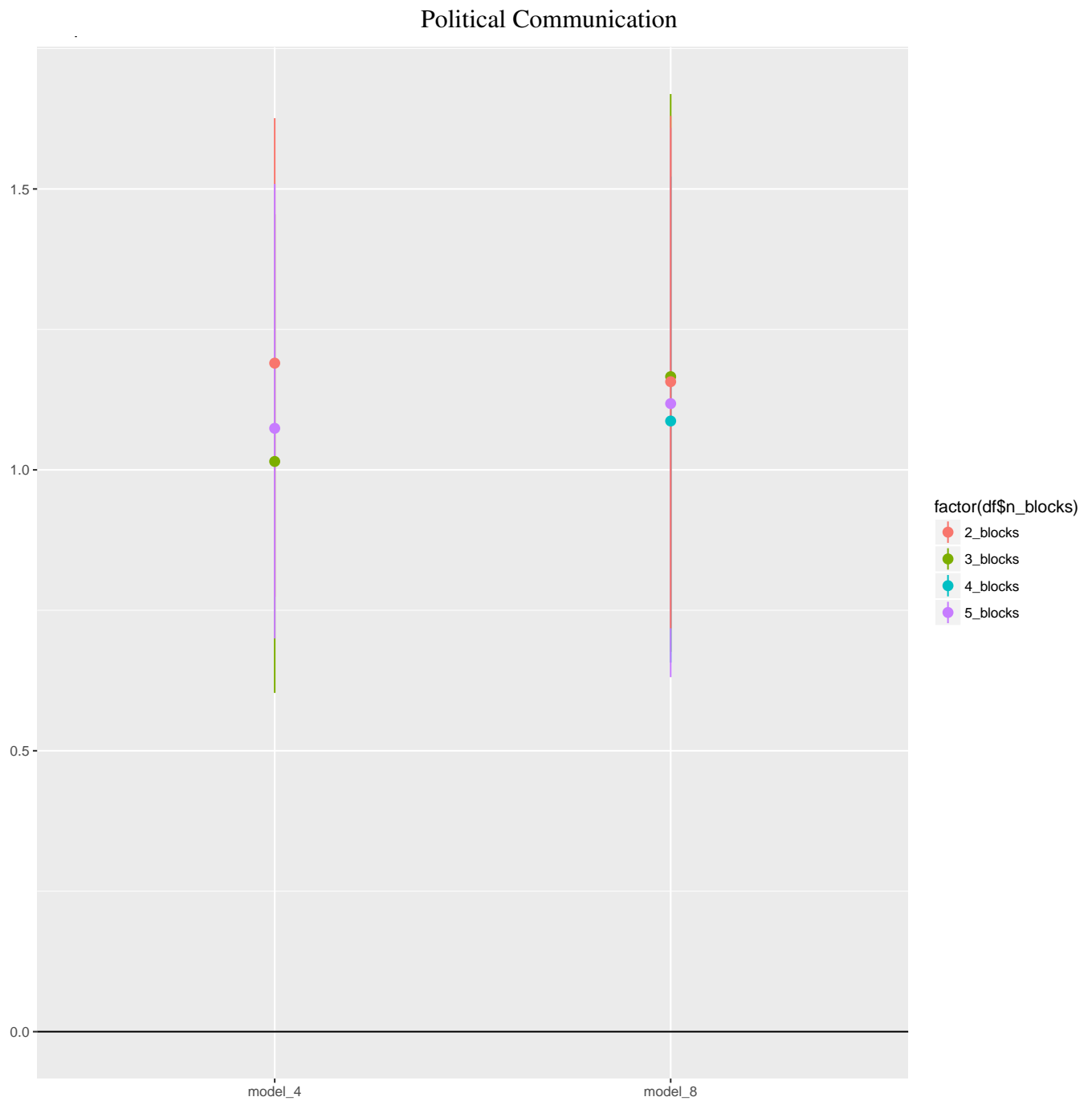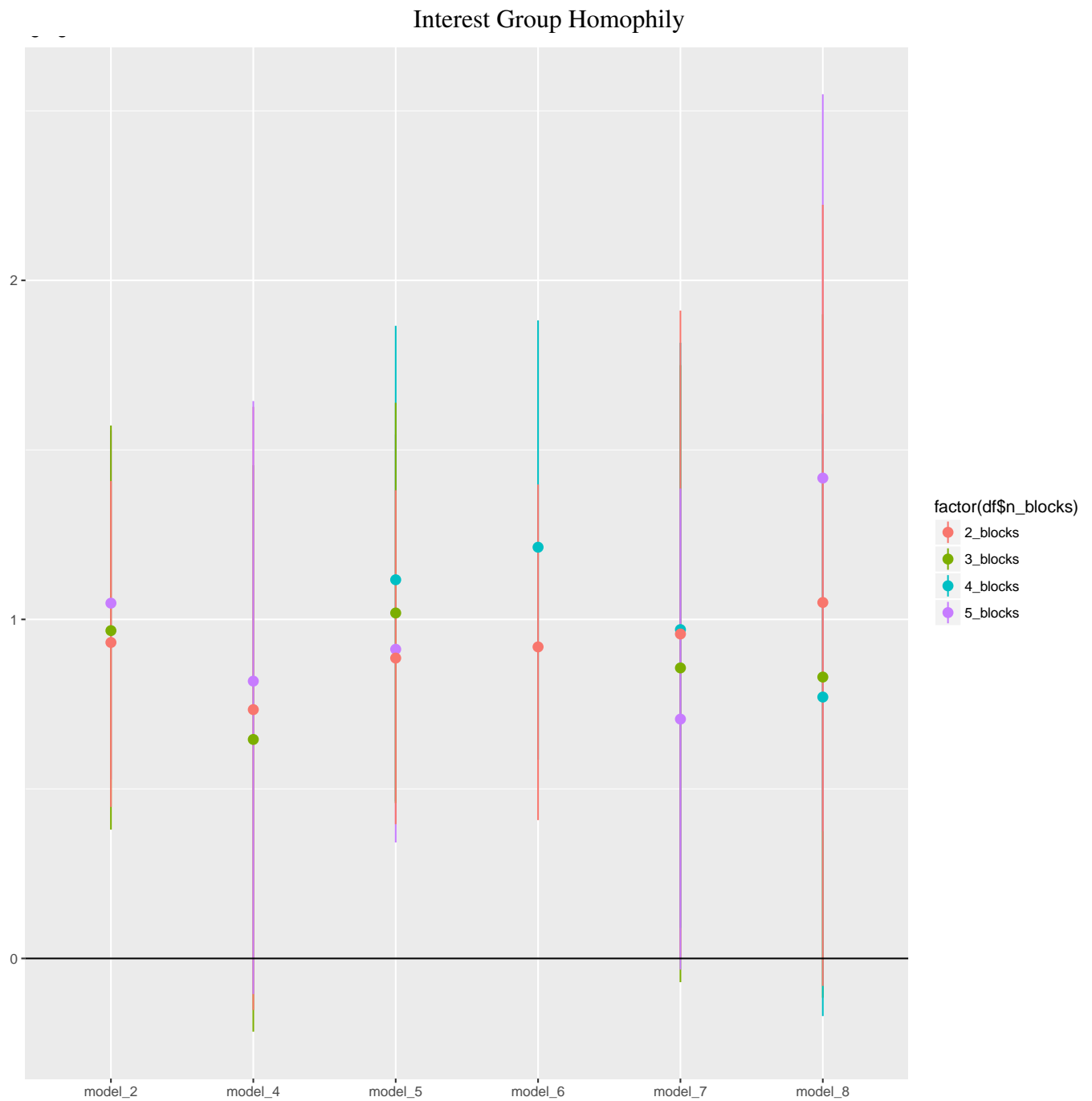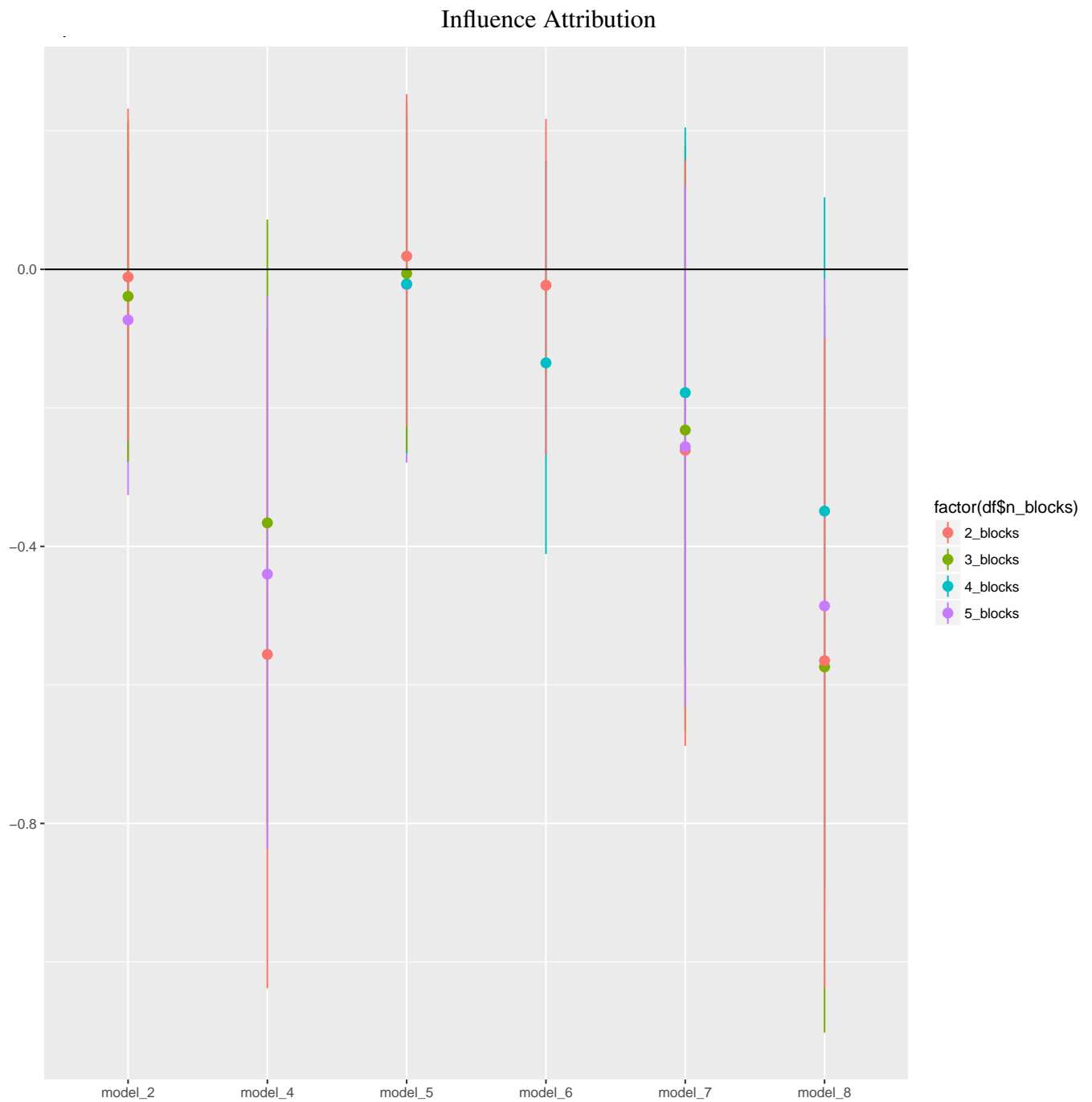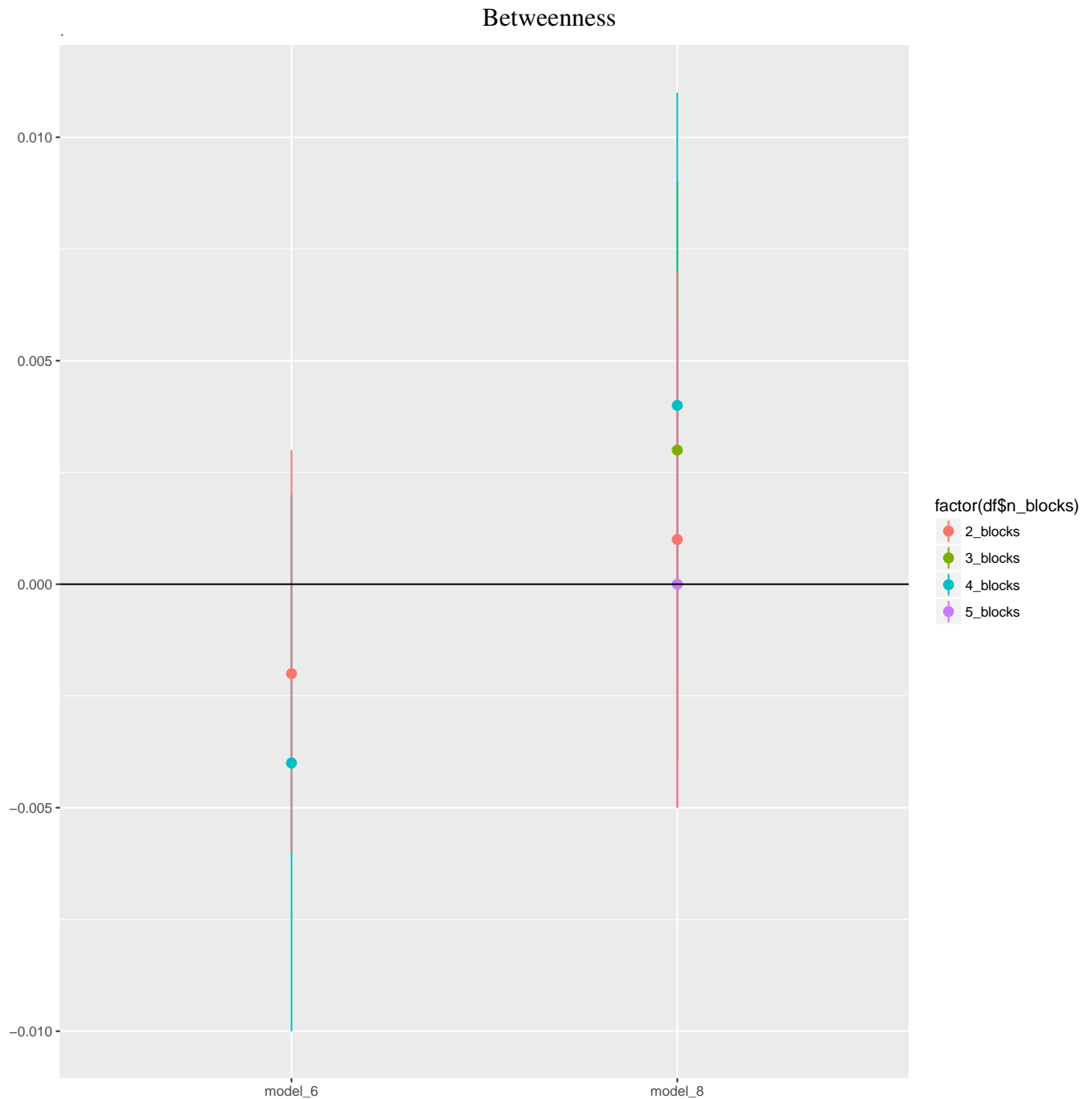
Common Committee

**Table 6.** SBM covariate plot. Results from models for which DIC could not be calculated were discarded.

**Scientific Communication**

**Table 7.** SBM covariate plot. Results from models for which DIC could not be calculated were discarded.

Political Communication

**Table 8.** SBM covariate plot. Results from models for which DIC could not be calculated were discarded.

**Table 9.** SBM covariate plot. Results from models for which DIC could not be calculated were discarded.

Influence Attribution

**Table 10.** SBM covariate plot. Results from models for which DIC could not be calculated were discarded.

**Table 11.** SBM covariate plot. Results from models for which DIC could not be calculated were discarded.

## 4.1 Model Comparison

**I only compare models one through four, as models five through eight are not reported in the main paper and are merely robustness tests. I included the results though above so that we could still review them and take them out later if we need too.**

1. ERGM replication

(a) Models two and four do not perfectly replicate. Most of the substantive findings remain, but in model four the significance level for common committee drops from 99.9% to 95%. There are a few other drops in significance for other covariates, but not as big. Signage remains the same for the replication across all models even when there are changes in the coefficient size or significance.

2. LSM

   (a) There are substantial differences between the ERGM results and the LSM results. For example, in model 1, LSM does not have significant correlation between preference similarity and political communication. In model 2 the point estimate for common committee is five times larger and is not significant in model 4. Influence attribution is almost two times larger in model 2 and 4. In the LSM models, intercept estimates are not significant in any model, but they are highly significant in the ERGM models. Interest group homophily is not significant in model 4.

3. SBM

   (a) Preference Similarity: In model 2, point estimates remain above 95% significance cutoff. The variables for model 2 do not absorb preference similarity measures as they do in the ERGM.
   (b) Government Alter: Point estimates are a little higher and in model 4 it is not 95% significant for five blocks.
   (c) Scientific Ego: Still statistically significant in model 4, but point estimates are less than half.
   (d) Common Committee: Point estimate is about two thirds for model 2's and slightly bigger than model 4's.
   (e) Scientific Communication: Point estimate in model 2 is about $1/3$
   (f) Political Communication: Point estimate in model 4 is about $1/3$
   (g) Interest Group Homophily: Point estimates are slightly smaller and in model 4 they are not significant at the 95% threshold.
   (h) Influence attribution: point estimates are negative and not statistically significant instead of being around one.
   (i) The number of blocks that results in the lowest DIC is not consistent across models. For the most part, models with four blocks have the lowest DIC for political communication but two blocks is best for scientific communication. Many of the institutional/relational structure variables are smaller not always significant at the same level.

## References

Asal, Victor H, Hyun Hee Park, R Karl Rethemeyer and Gary Ackerman. 2016. "With Friends Like These… Why Terrorist Organizations Ally." *International Public Management Journal* 19(1):1–30.

Atouba, Yannick C and Michelle Shumate. 2015. "International nonprofit collaboration: Examining the role of homophily." *Nonprofit and Voluntary Sector Quarterly* 44(3):587–608.

Block, Per, Johan Koskinen, James Hollway, Christian Steglich and Christoph Stadtfeld. 2018. "Change we can believe in: Comparing longitudinal network models on consistency, interpretability and predictive power." *Social Networks* 52:180–191.

Bond, Robert M, Christopher J Fariss, Jason J Jones, Adam DI Kramer, Cameron Marlow, Jaime E Settle and James H Fowler. 2012. "A 61-million-person experiment in social influence and political mobilization." *Nature* 489(7415):295.

Cao, Xun. 2012. "Global networks and domestic policy convergence: A network explanation of policy changes." *World Politics* 64(3):375–425.

Chu-Shore, Jesse. 2010. "Homogenization and Specialization Effects of International Trade: Are Cultural Goods Exceptional?" *World Development* 38(1):37–47.

Cranmer, Skyler J and Bruce A Desmarais. 2011. "Inferential Network Analysis with Exponential Random Graph Models." *Political Analysis* 19(1):66–86.

Cranmer, Skyler J, Bruce A Desmarais and Elizabeth J Menninga. 2012. "Complex dependencies in the alliance network." *Conflict Management and Peace Science* 29(3):279–313.

Cranmer, Skyler J, Tobias Heinrich and Bruce A Desmarais. 2014. "Reciprocity and the structural determinants of the international sanctions network." *Social Networks* 36:5–22.

Desmarais, Bruce A and Skyler J Cranmer. 2012. "Micro-level interpretation of exponential random graph models with application to estuary networks." *Policy Studies Journal* 40(3):402–434.

Desmarais, Bruce A and Skyler J Cranmer. 2013. "Forecasting the locational dynamics of transnational terrorism: A network analytic approach." *Security Informatics* 2(1):8.

Desmarais, Bruce A and Skyler J Cranmer. 2017. "Statistical Inference in Political Networks Research." *The Oxford Handbook of Political Networks* p. 203.

Dorff, Cassy and Michael D Ward. 2013. "Networks, dyads, and the social relations model." *Political Science Research and Methods* 1(2):159–178.

Dorff, Cassy and Shahryar Minhas. 2017. "When Do States Say Uncle? Network Dependence and Sanction Compliance." *International Interactions* 43(4):563–588.

Dorff, Cassy, Shahryar Minhas and Michael D Ward. 2016. Latent Networks and Spatial Networks in Politics. In *The Oxford Handbook of Political Networks*.

Dorussen, Han and Hugh Ward. 2008. "Intergovernmental organizations and the Kantian peace: A network perspective." *Journal of Conflict Resolution* 52(2):189–212.

Gallop, Max B. 2016. "Endogenous networks and international cooperation." *Journal of Peace Research* 53(3):310–324.

Hoff, Peter D and Michael D Ward. 2004. "Modeling dependencies in international relations networks." *Political Analysis* 12(2):160–175.

Kirkland, Justin H and R Lucas Williams. 2014. "Partisanship and reciprocity in cross-chamber legislative interactions." *The Journal of Politics* 76(3):754–769.

Latouche, Pierre, Etienne Birmelé and Christophe Ambroise. 2011. "Overlapping stochastic block models with application to the french political blogosphere." *The Annals of Applied Statistics* pp. 309–336.

Lupu, Yonatan and Brian Greenhill. 2017. "The networked peace: Intergovernmental organizations and international conflict." *Journal of Peace Research* 54(6):833–848.

Metternich, Nils W, Cassy Dorff, Max Gallop, Simon Weschle and Michael D Ward. 2013. "Antigovernment networks in civil conflicts: How network structures affect conflictual behavior." *American Journal of Political Science* 57(4):892–911.

Nowicki, Krzysztof and Tom A B Snijders. 2001. "Estimation and prediction for stochastic blockstructures." *Journal of the American statistical association* 96(455):1077–1087.

Raftery, Adrian E, Xiaoyue Niu, Peter D Hoff and Ka Yee Yeung. 2012. "Fast inference for the latent space network model using a case-control approximate likelihood." *Journal of Computational and Graphical Statistics* 21(4):901–919.

Van Duijn, Marijtje AJ, Krista J Gile and Mark S Handcock. 2009. "A framework for the comparison of maximum pseudo-likelihood and maximum likelihood estimation of exponential family random graph models." *Social Networks* 31(1):52–62.

Wang, Cheng, Carter T Butts, John R Hipp, Rupa Jose and Cynthia M Lakon. 2016. "Multiple imputation for missing edge data: A predictive evaluation method with application to Add Health." *Social networks* 45:89–98.

Ward, Michael D, John S Ahlquist and Arturas Rozenas. 2013. "Gravity's rainbow: A dynamic latent space model for the world trade network." *Network Science* 1(1):95–118.

Ward, Michael D and Peter D Hoff. 2007. "Persistent patterns of international commerce." *Journal of Peace Research* 44(2):157–175.

Ward, Michael D, Randolph M Siverson and Xun Cao. 2007. "Disputes, democracies, and dependencies: A reexamination of the Kantian peace." *American Journal of Political Science* 51(3):583–601.