Efficient Snap Rounding with Integer Arithmetic

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Outline

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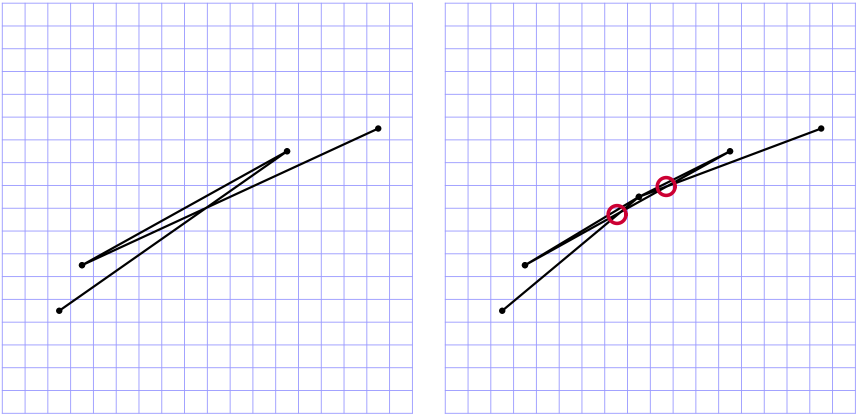
- approach: round endpoints, intersection points to integer grid
- problem: extraneous intersections

The Problem



Hobby (1996)

The Problem

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- Every pixel containing endpoint or intersection point is hot
- Every segment intersecting hot pixel is rerouted through that pixel's center

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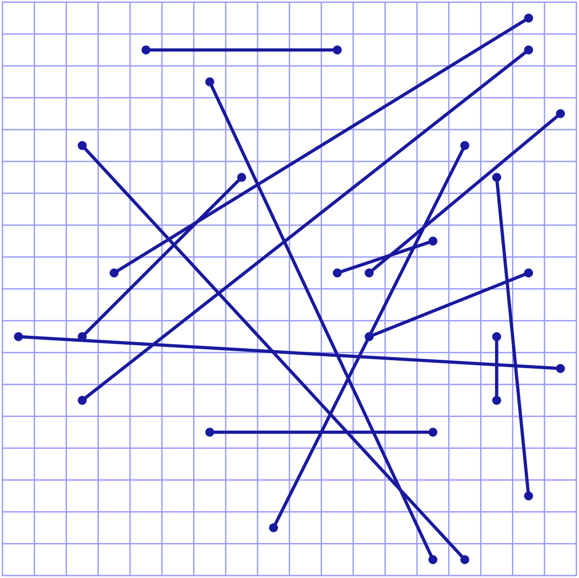

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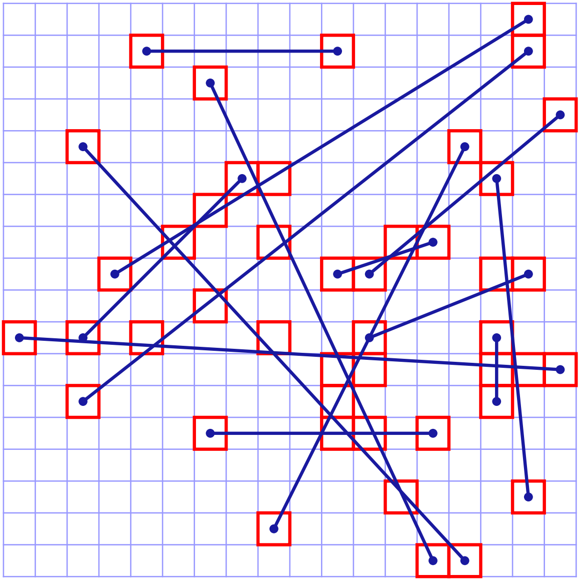

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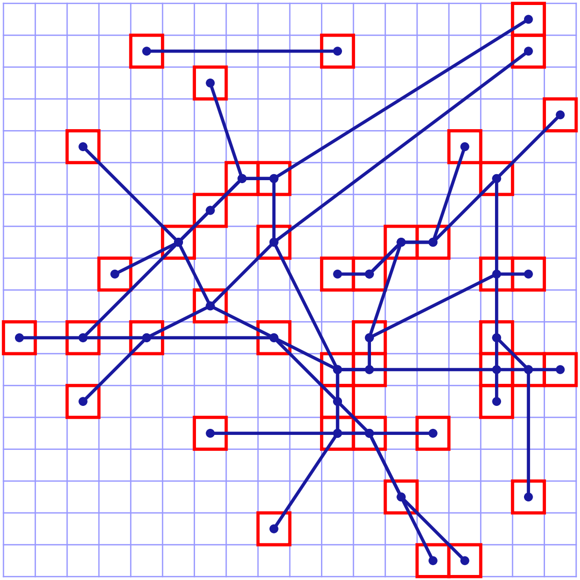

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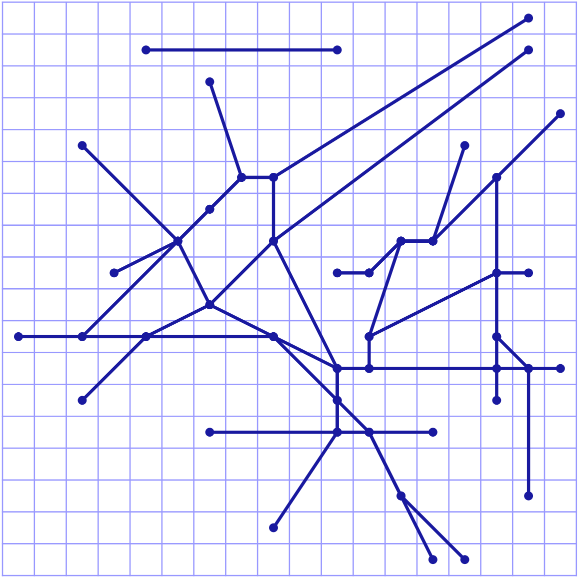

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- all rely on high precision computation (i.e., clipping segments to pixel boundaries)

Our Results

- first algorithm: generates set of snapped segments in $O(|I| + \Sigma_c \, is(c) \log n + |I_m^*|)$ time

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- first algorithm: generates set of snapped segments in $O(|I| + \Sigma_c\, is(c) \log n + |I_m^*|)$ time
- second algorithm: generates rounded arrangement of segments in $O(|I| + \Sigma_c\, is(c) \log n + |I^*| \log n)$ time

Algorithm One

- uses modified Bentley & Ottman (1979) plane sweep

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- sweep line is actually a column of half pixels (to process endpoints cleanly)
- active list, intersection events stored in a modified B+ tree
- endpoint events stored in separate priority queue

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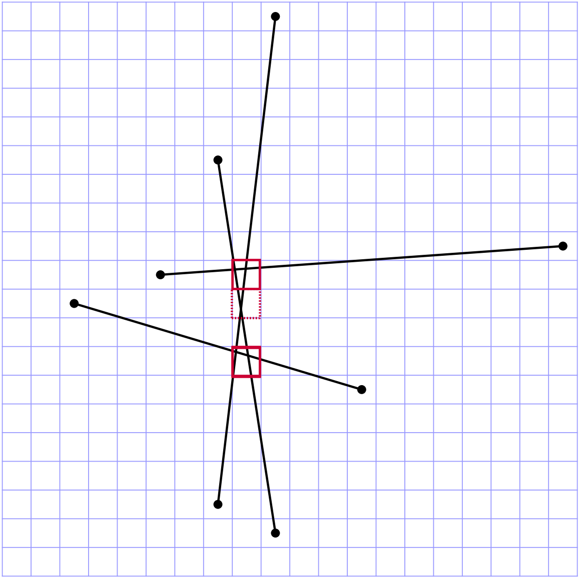
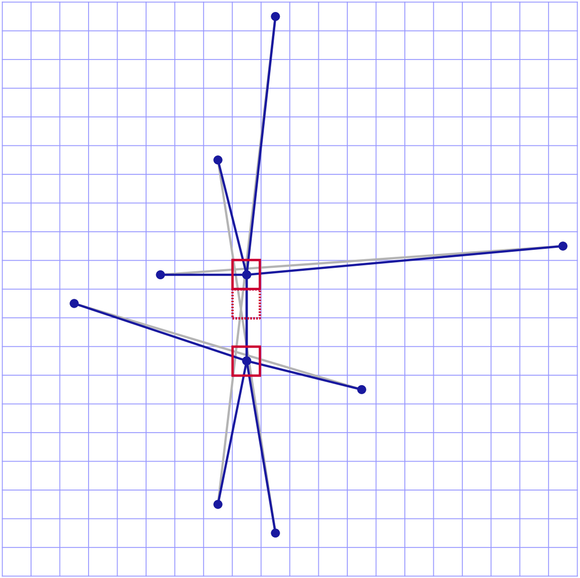
- modify snap rounding definition slightly
- hot pixel set can now be represented as hotRange: a lowest and highest hot pixel for the segment

Proof of Lemma 1

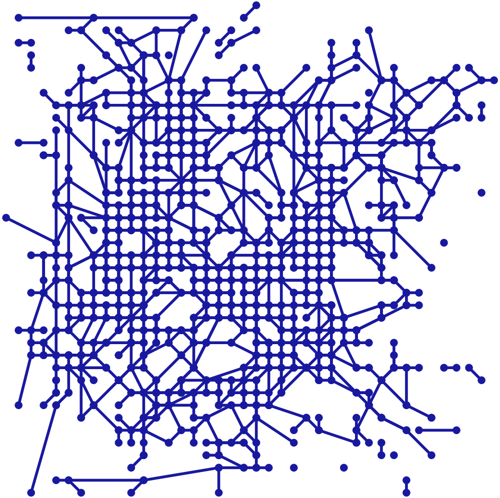


Proof of Lemma 1



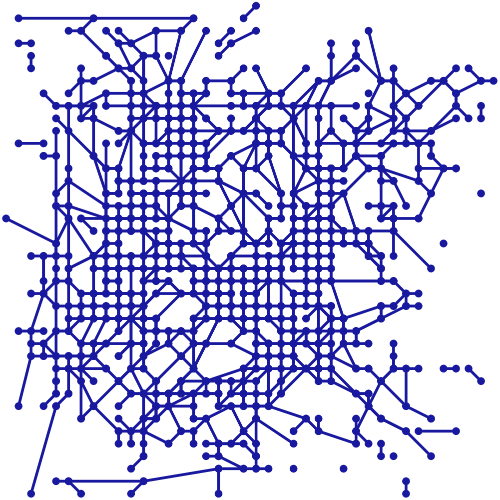
Modified Definition

An example snapped arrangement using the original definition:



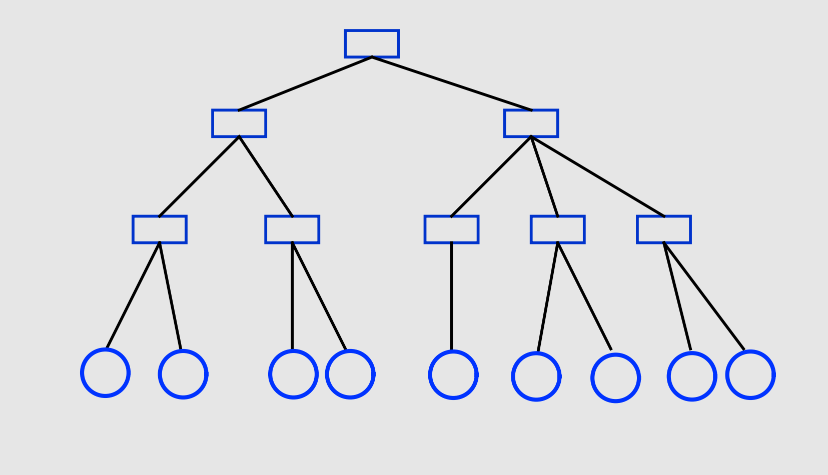
Modified Definition

The same arrangement using our definition:



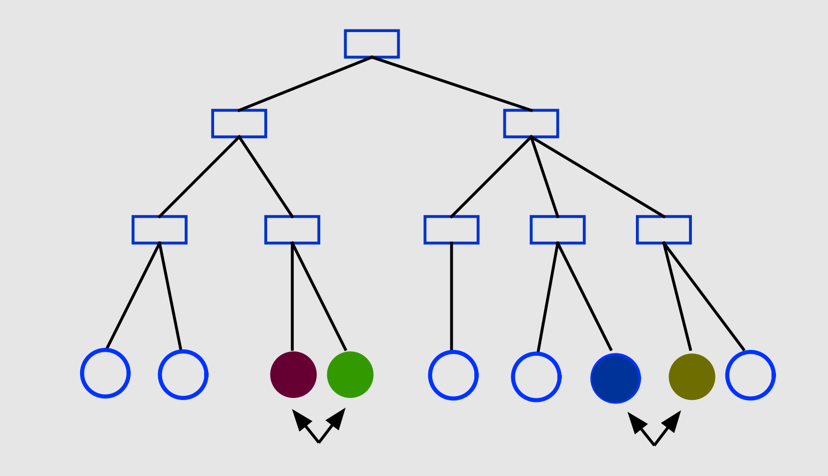
Algorithm One

Active list is leaf level of tree



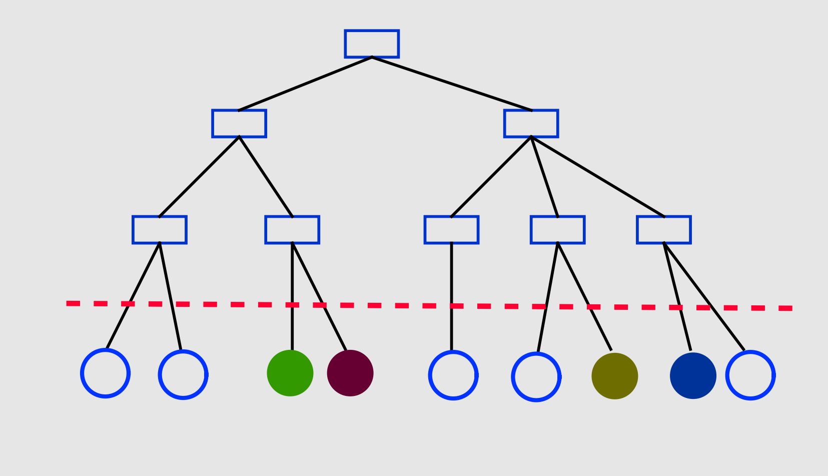
Algorithm One

Detect seed events: neighboring segments intersecting within column



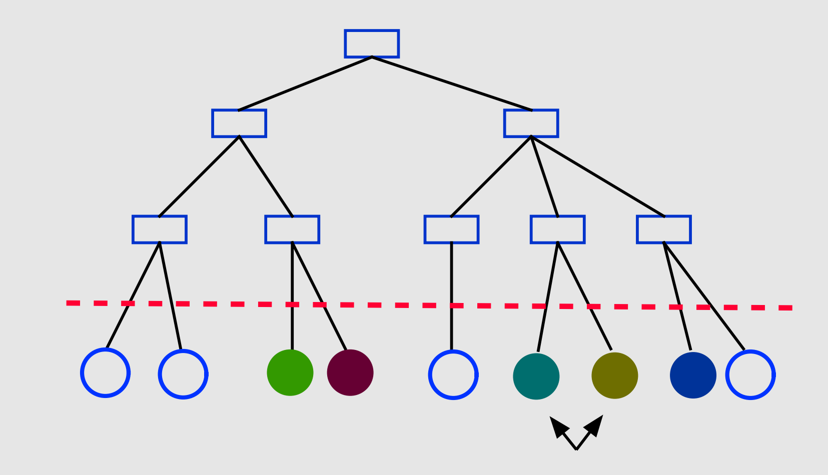
Algorithm One

Manipulate active list, ignore rest of tree data structure



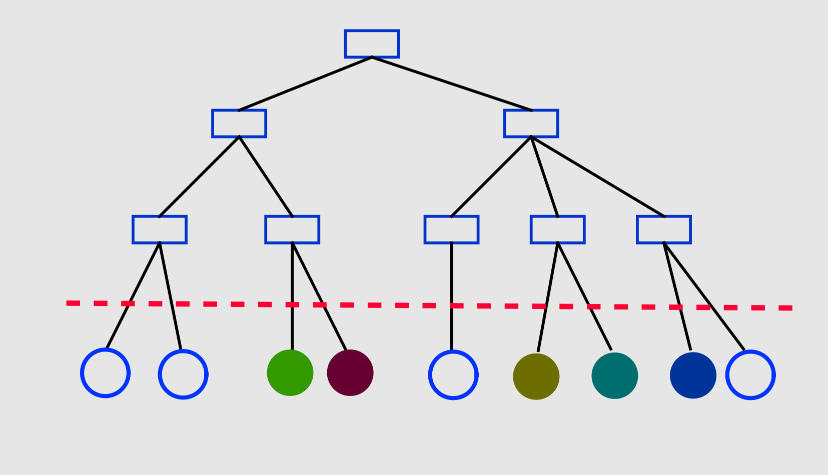
Algorithm One

Detect additional intersection events



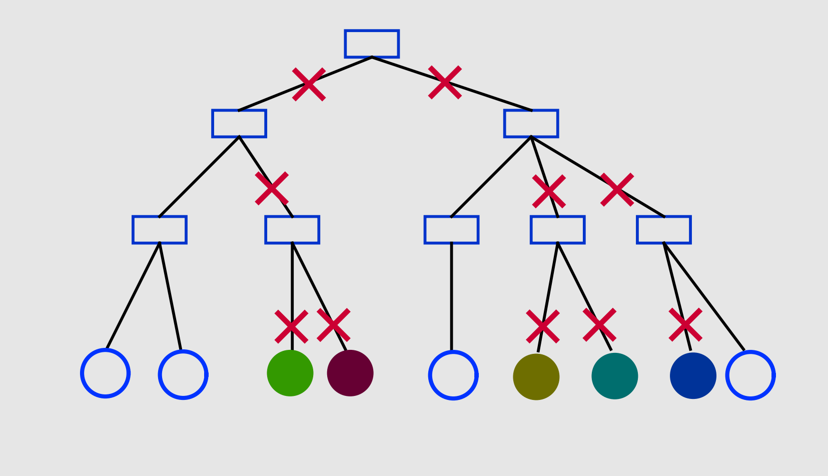
Algorithm One

Continue until order stabilizes



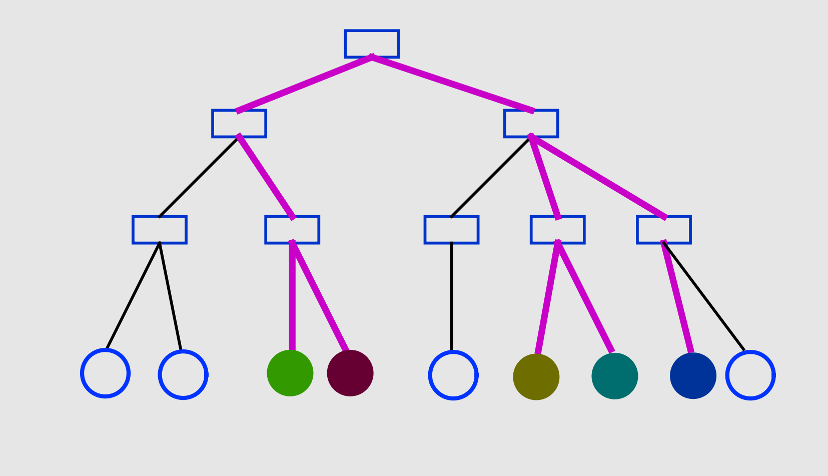
Algorithm One

Recalculate internal paths from moved segments



Algorithm One

Tree now ready for right side of sweep column



Algorithm One

At each sweep column position, three processes:

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- hot pixel process

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At each sweep column position, three processes:

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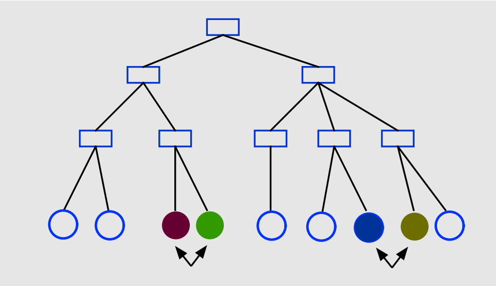
- hot pixel process
- snap process

Sweep Process

- query tree for seed events



Sweep Process

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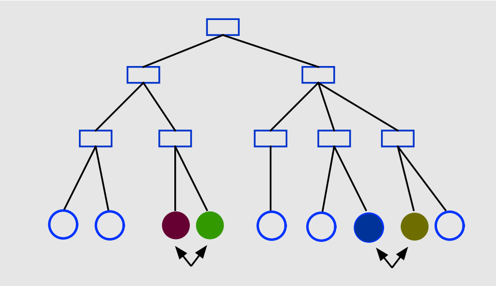

- push seed events onto a stack

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While seed event stack is not empty:

- pop event

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- add segments, old, and new neighbors to snapSet

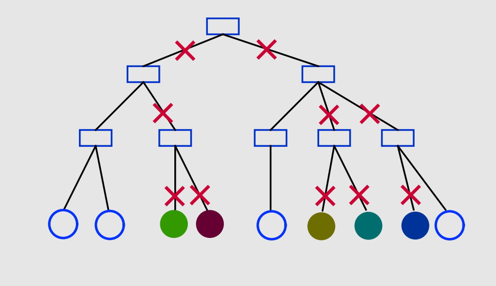
Sweep Process

While seed event stack is not empty:

- pop event
- add pixel containing intersection to each segment's hotRange
- exchange position of segments within active list
- add segments, old, and new neighbors to snapSet
- test if segments intersect new neighbors within column; if so, push event onto stack

Sweep Process

- recalculate internal paths for snapSet segments



Sweep Process

Details omitted:

- starting/stopping segments

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- dealing with vertical or zero-length segments

Hot Pixel Process

- examine set of all segments that were involved in seed events

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- construct sorted, linked list of distinct hot pixels from hotRange fields

Snap Process

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- careful use of pointers and sorted list of hot pixels allows this to be done in $O(1)$ time for each snapped fragment

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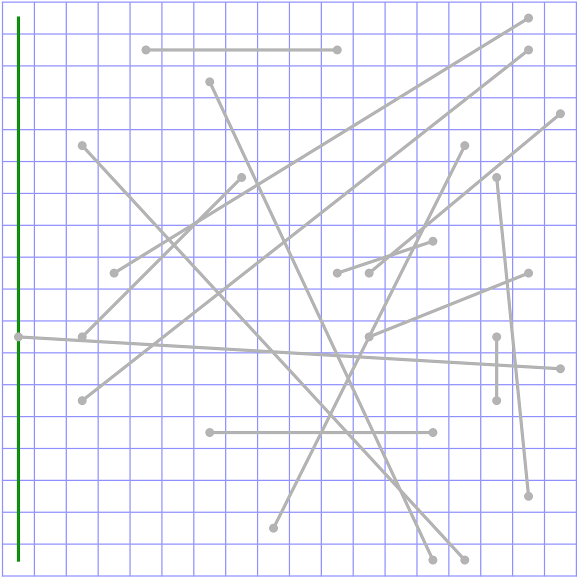

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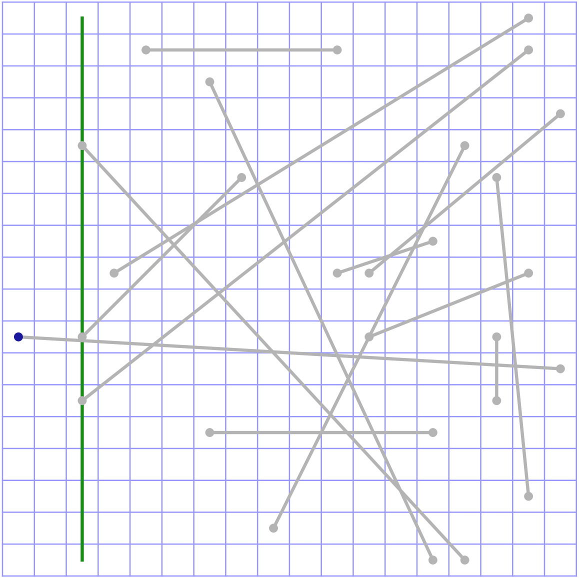

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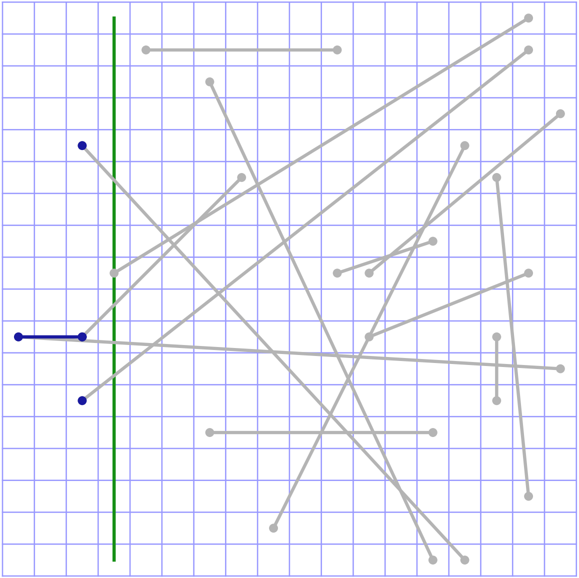

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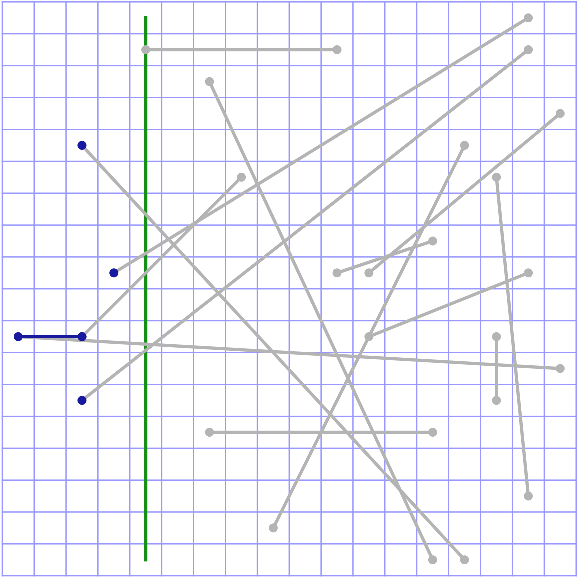

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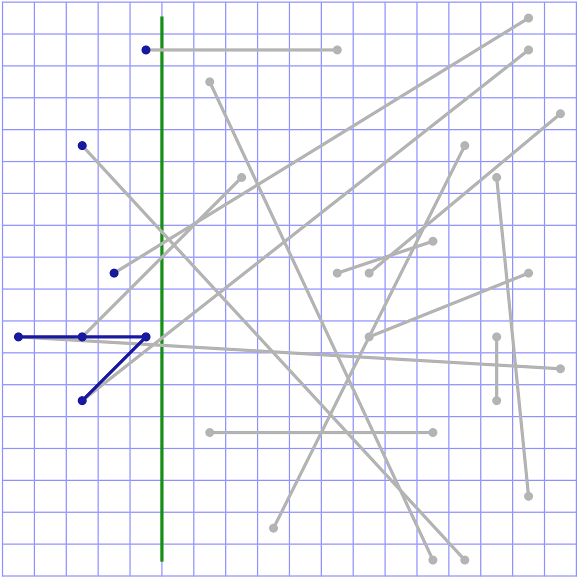

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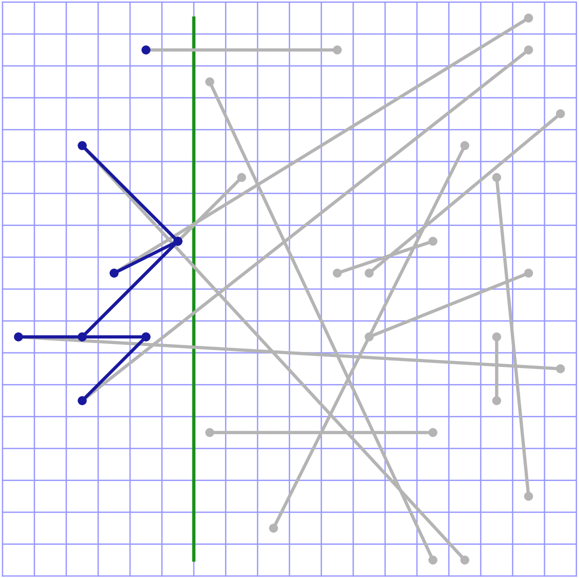

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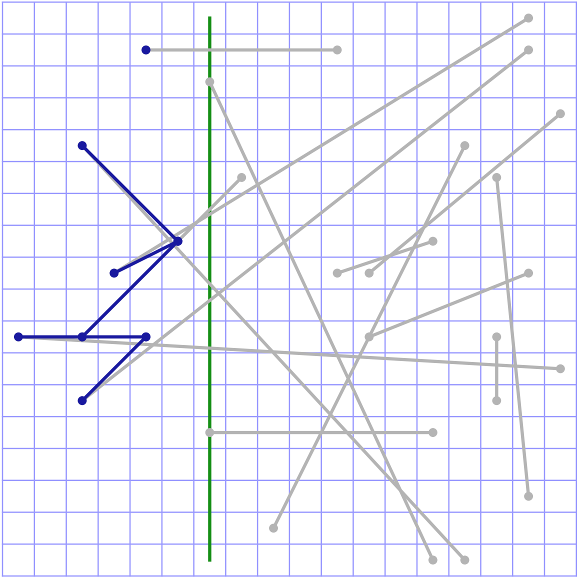

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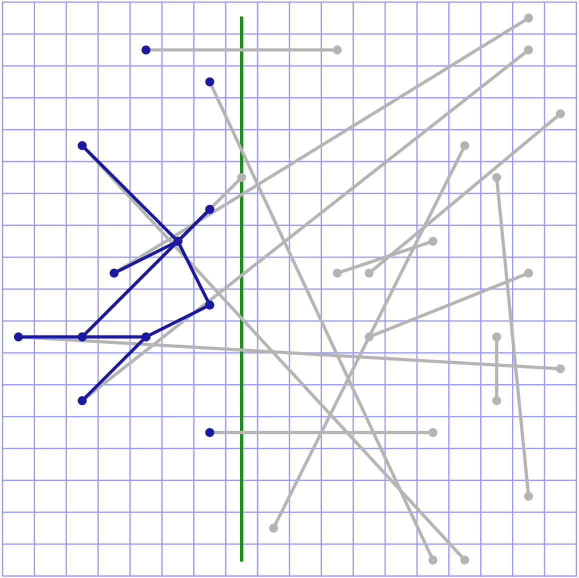

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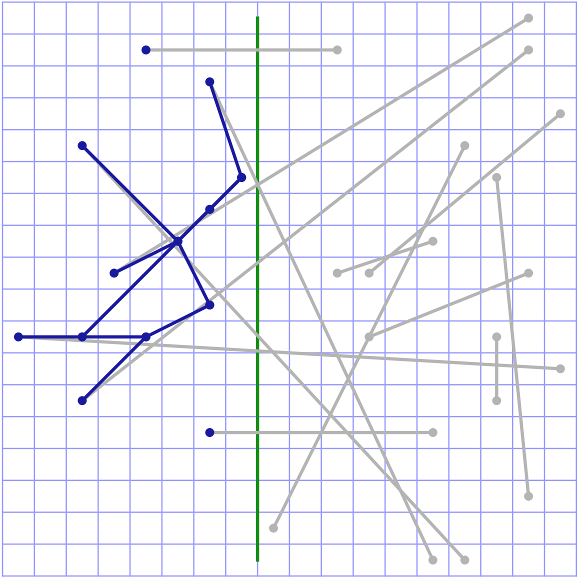

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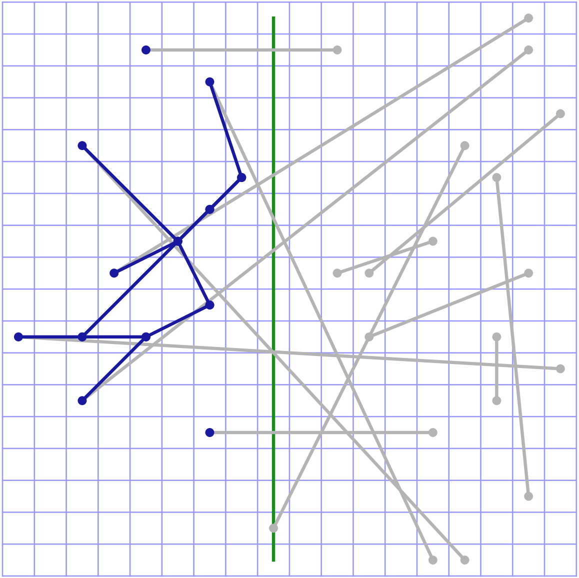

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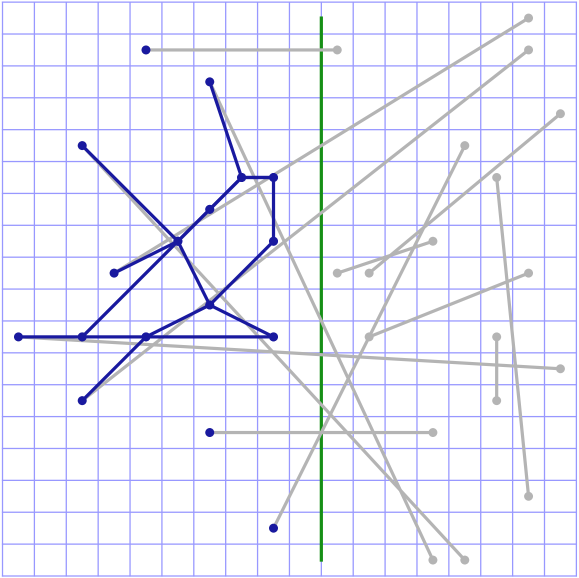

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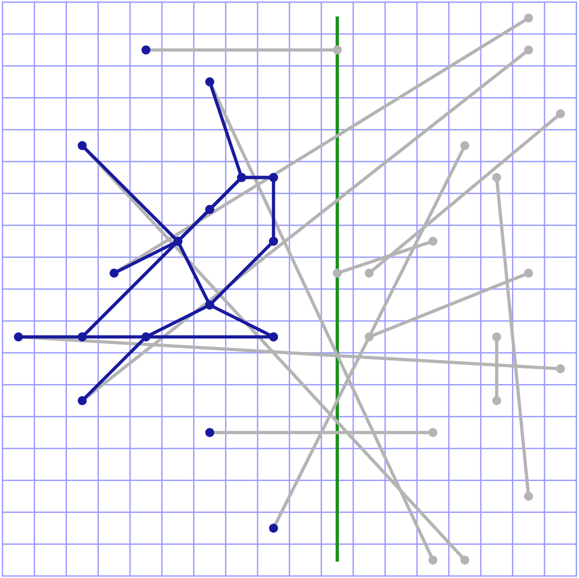

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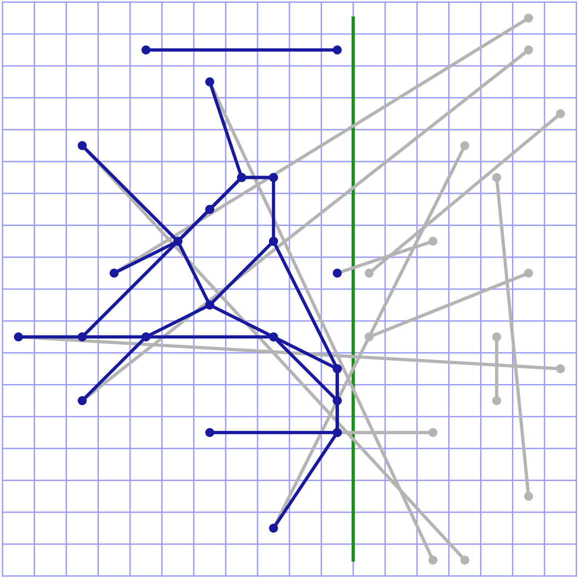

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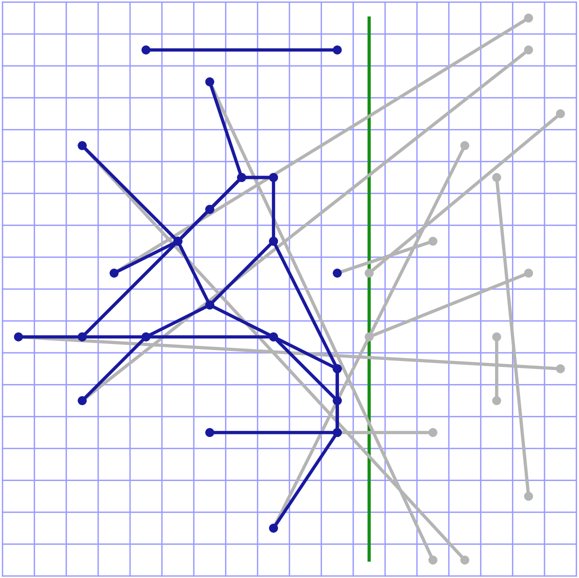

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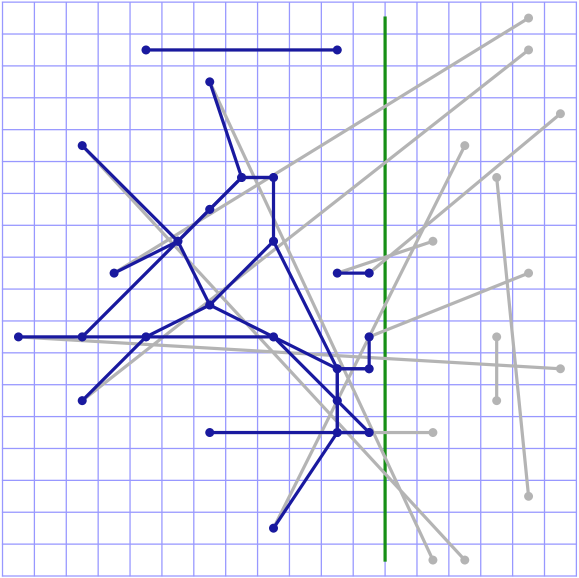

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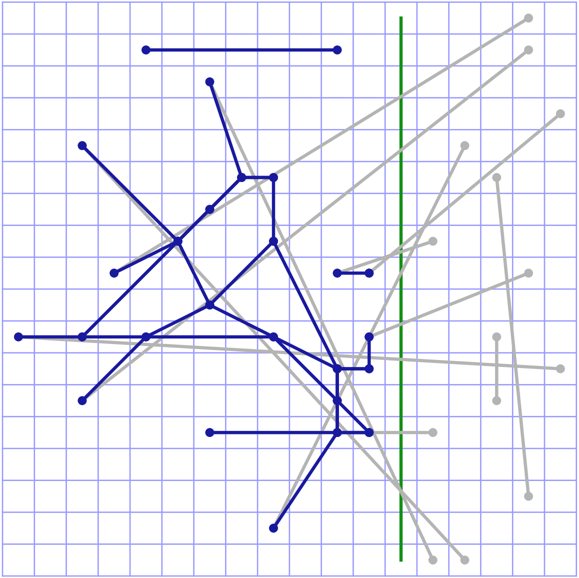

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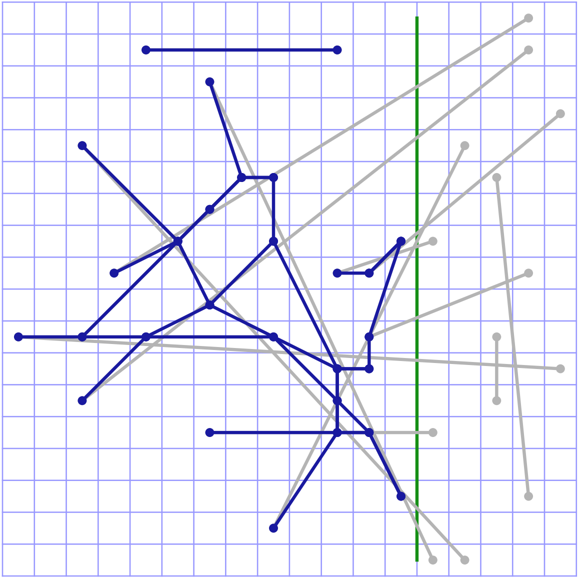

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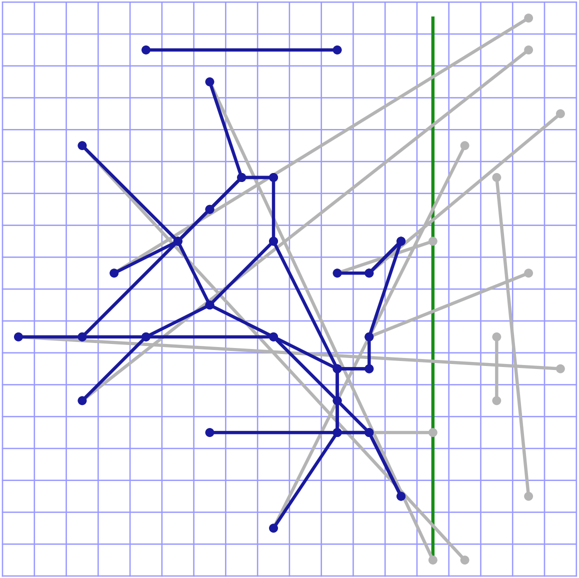

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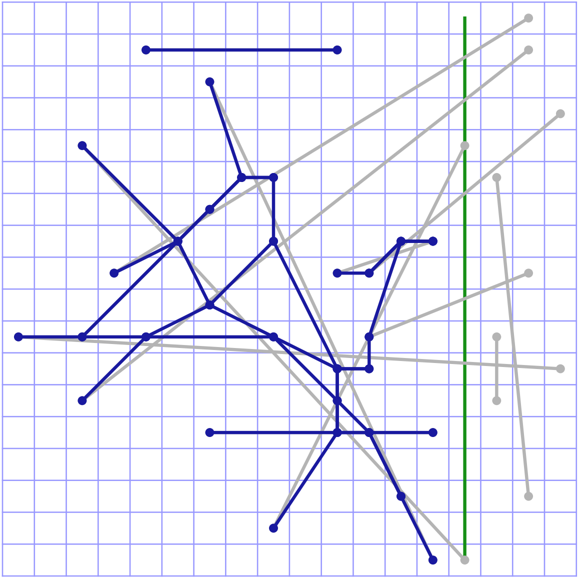

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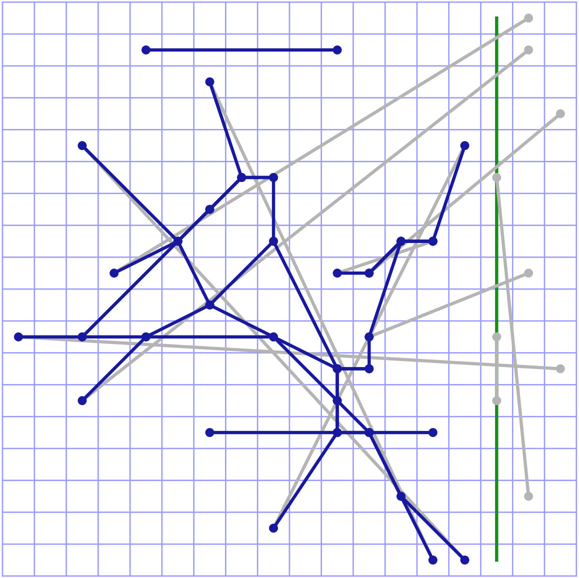

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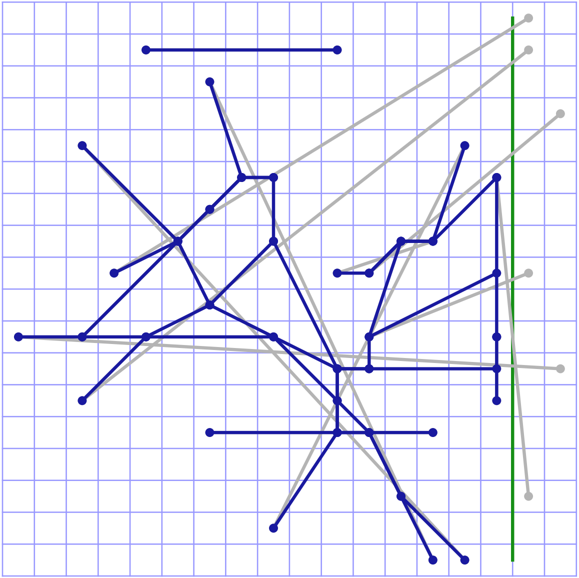

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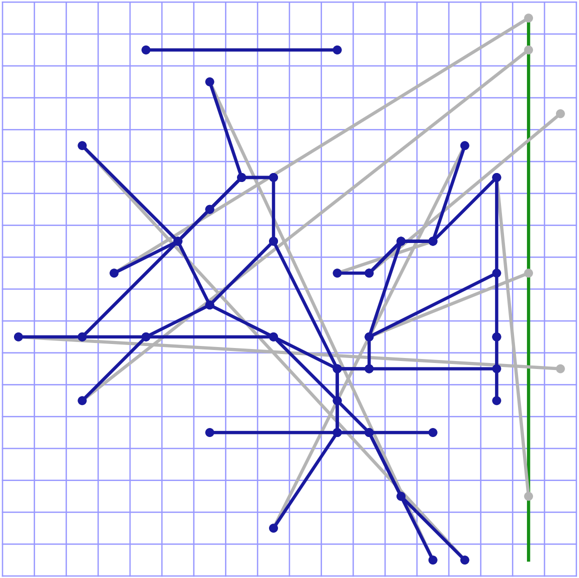

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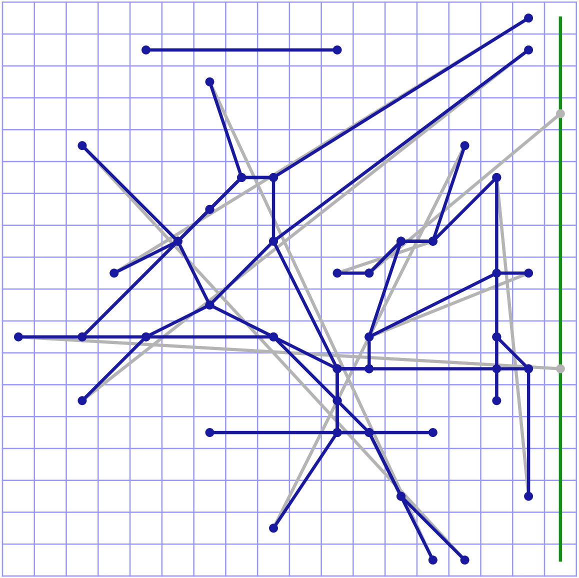

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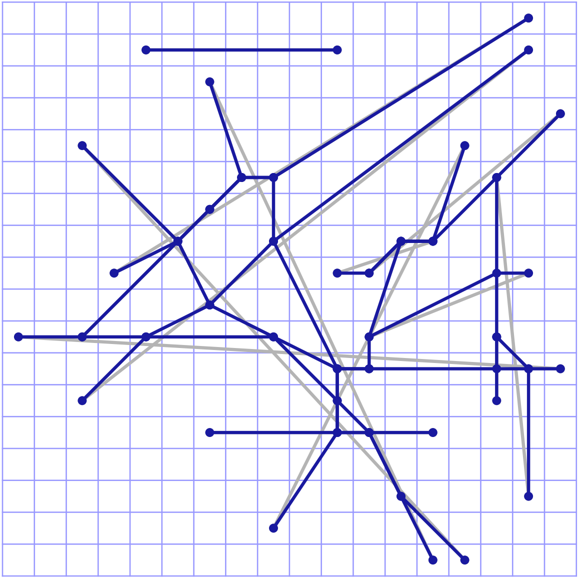

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- recalculating internal paths for each of the $O(is(c))$ segments can be done in $O(is(c) \log n)$ time
- running time for process is thus $O(|I| + \Sigma_c\, is(c) \log n)$

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- running time for this process can be included in that of the sweep process

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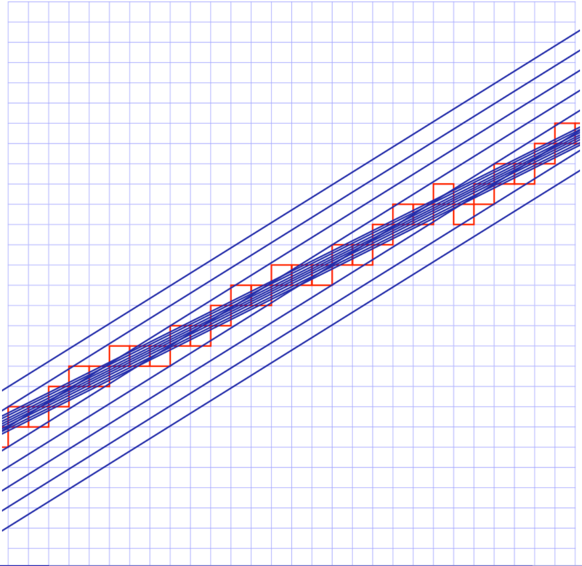
Algorithm Two: Why?

As observed by Halperin & Packer (2002), $|I_m^*|$ can be $\Theta(n^3)$:



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- perform initial pass to find hot pixels; this is first algorithm with snap process omitted
- perform two additional passes to find segment/hot pixel intersections: vertical sweep of 'mostly horizontal' segments, horizontal sweep of 'mostly vertical' segments
- running time is $O(|I| + \Sigma_c\, is(c) \log n + |I^*| \log n)$, where $|I^*|$ is complexity of snapped arrangement

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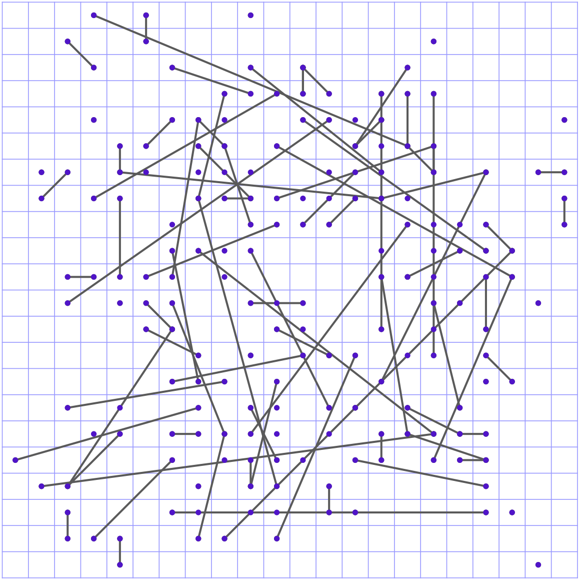

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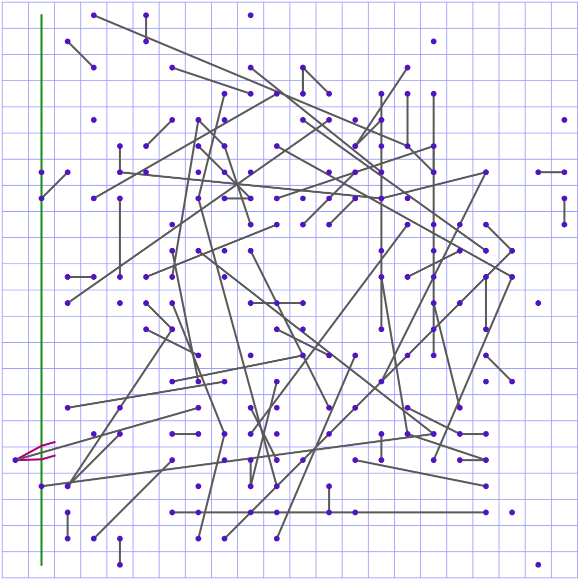

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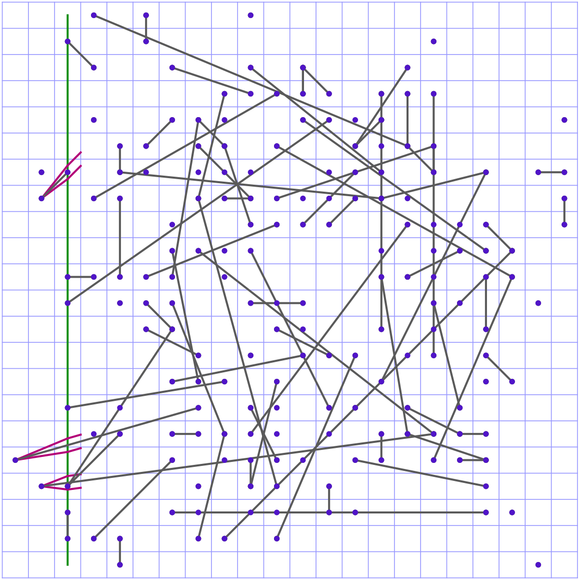

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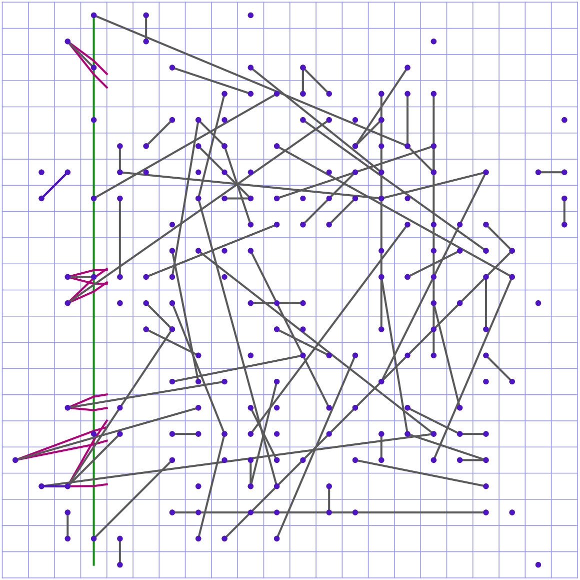

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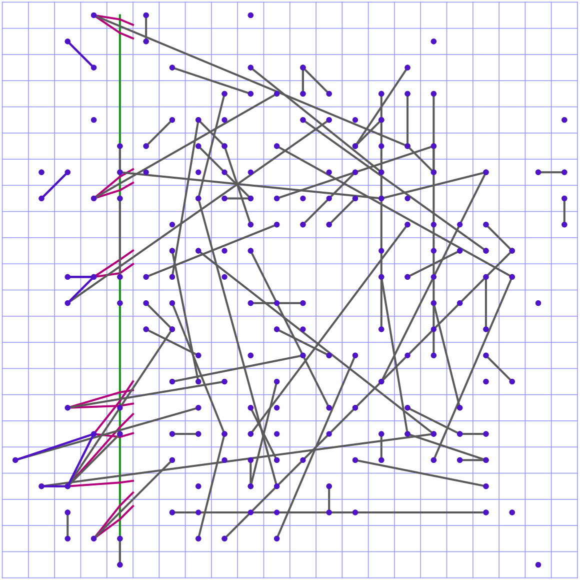

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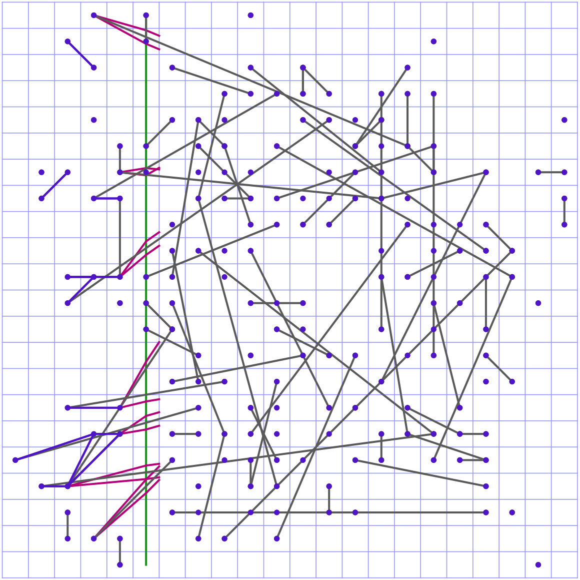

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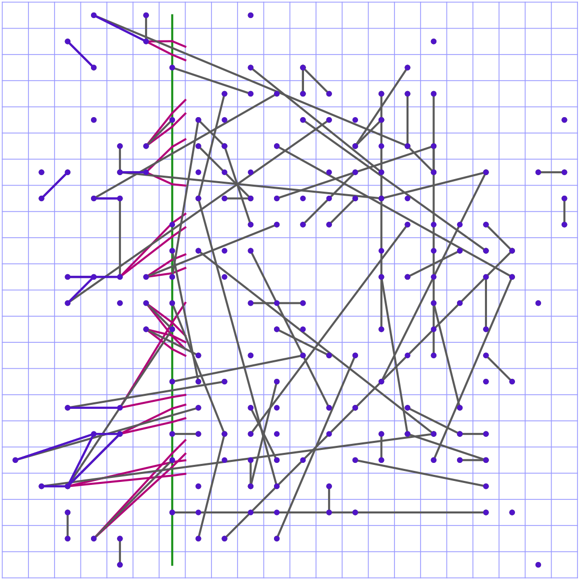

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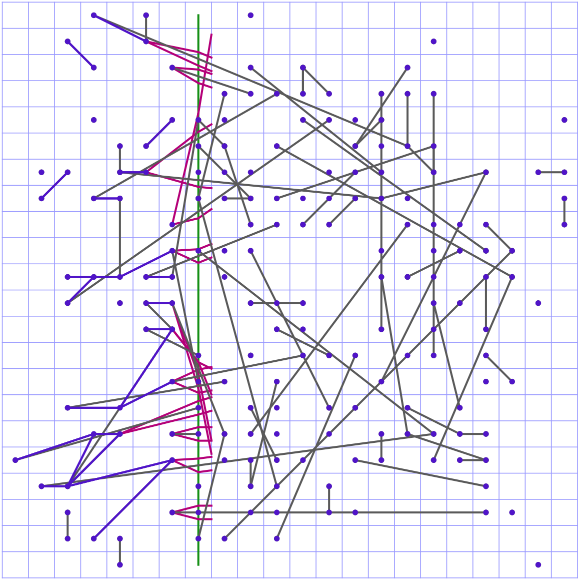

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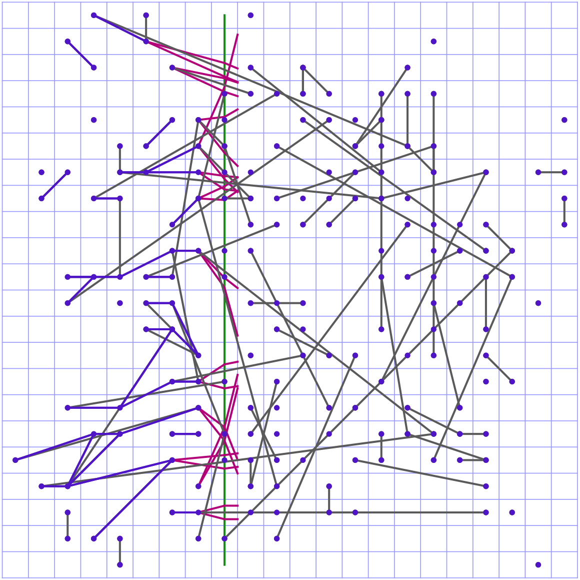

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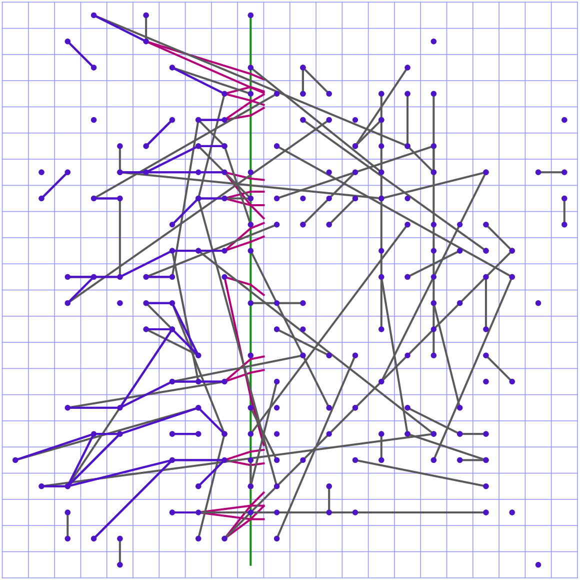

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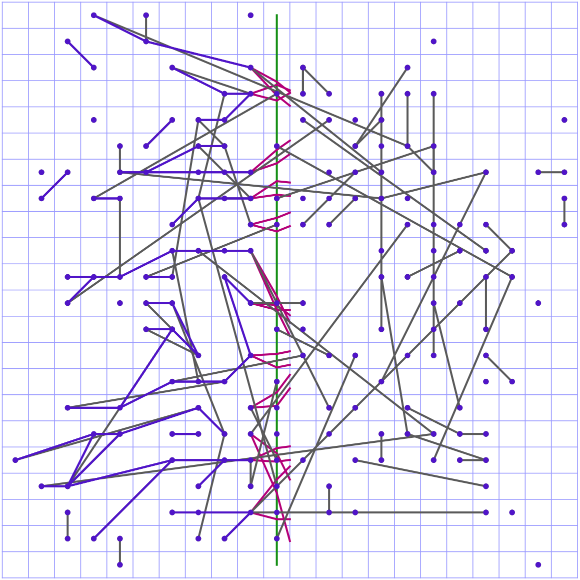

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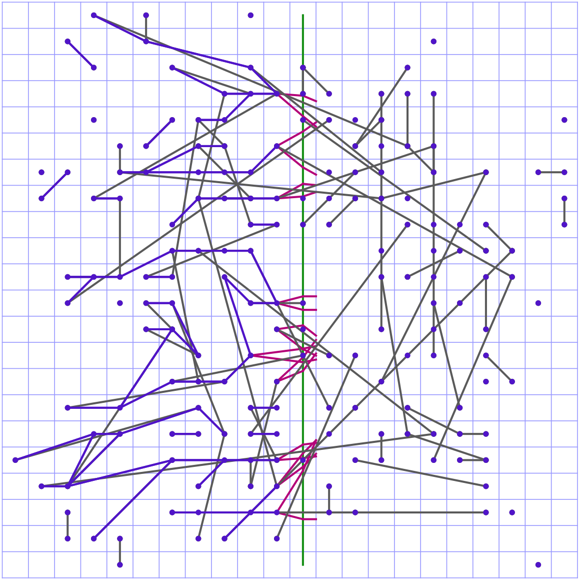

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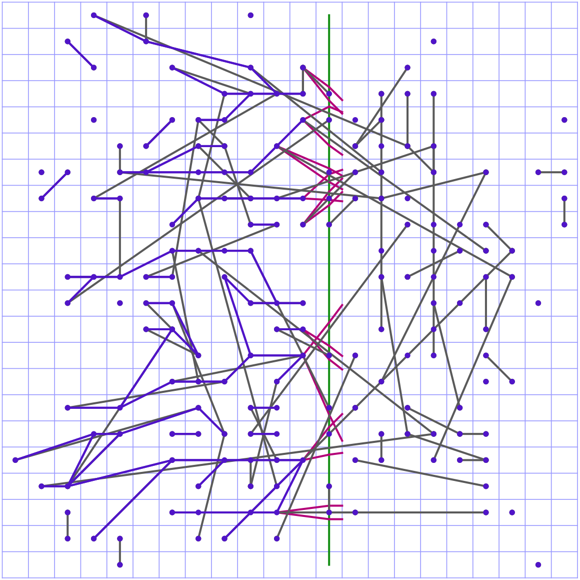

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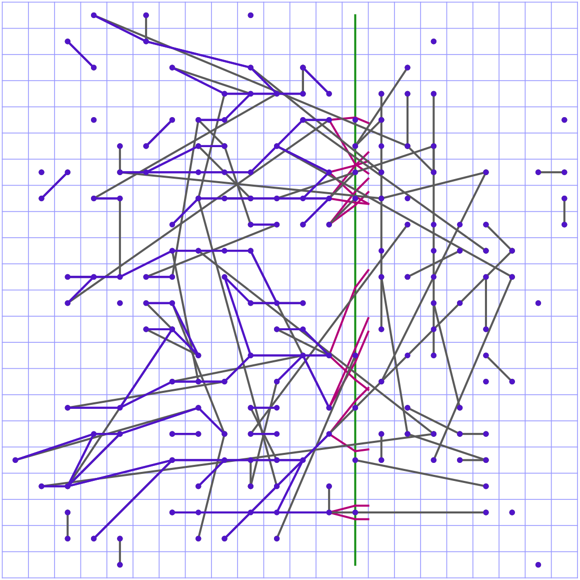

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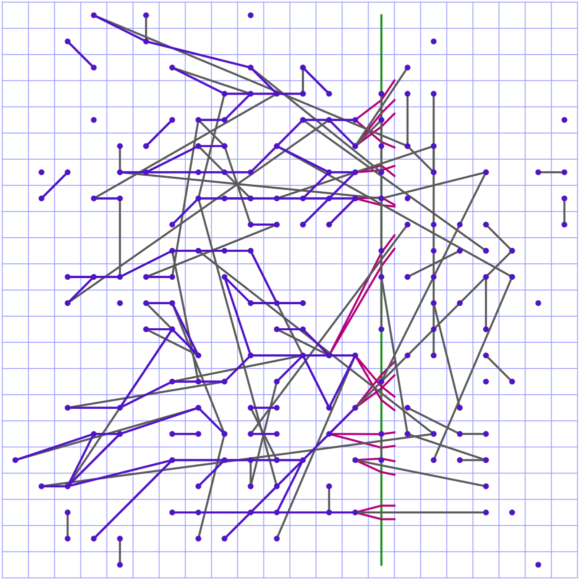

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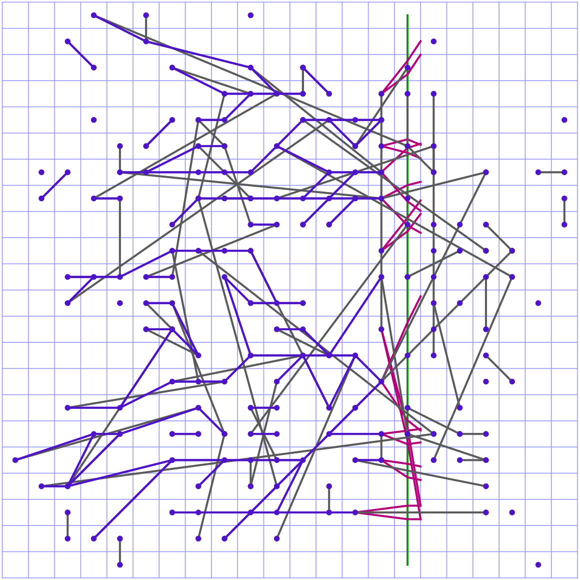

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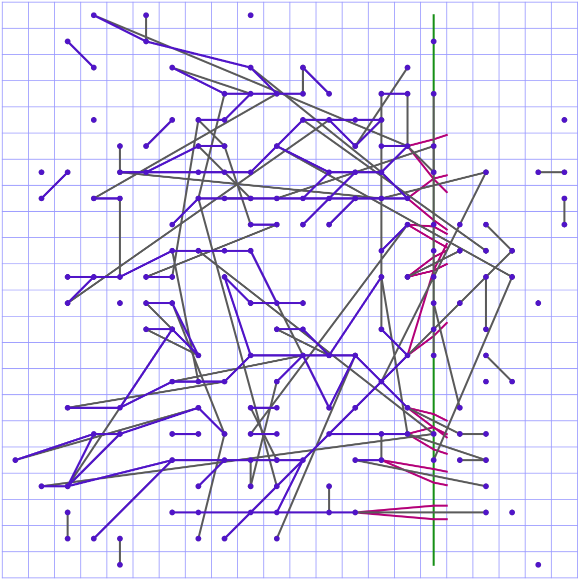

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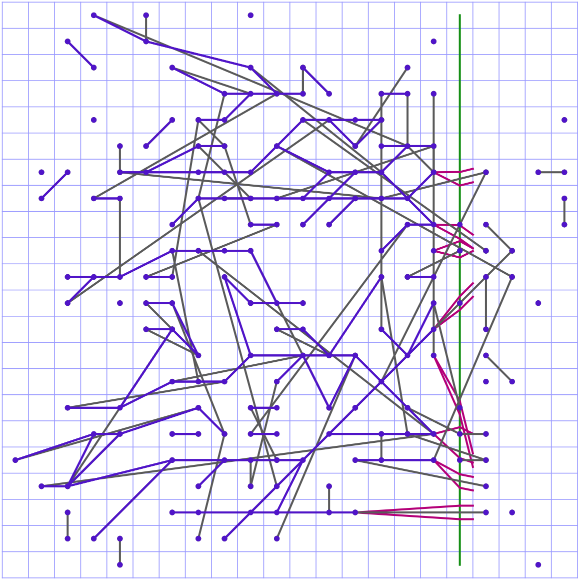

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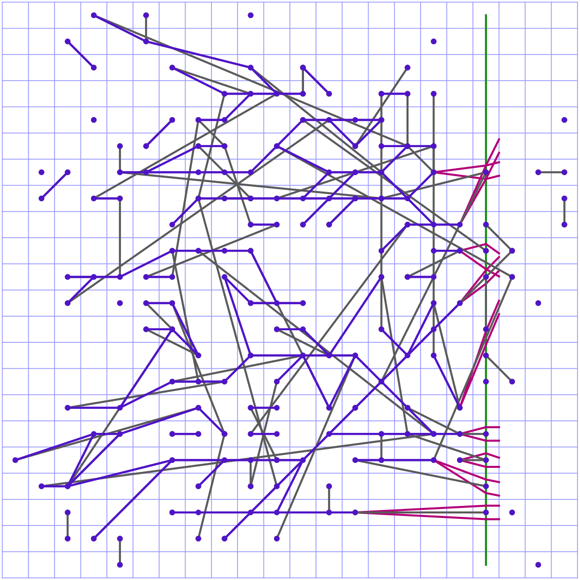

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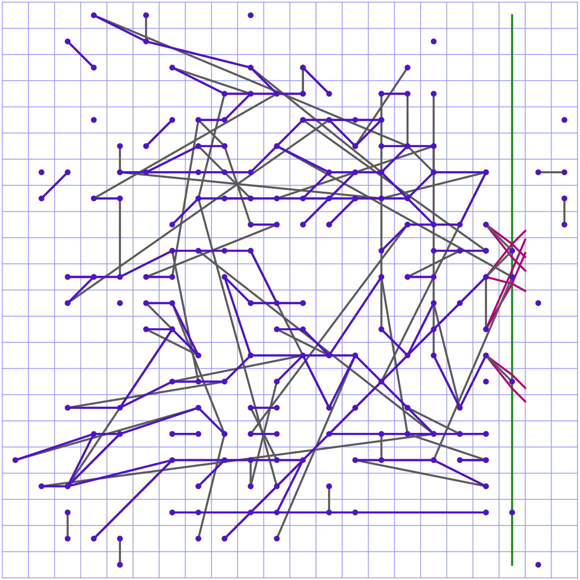

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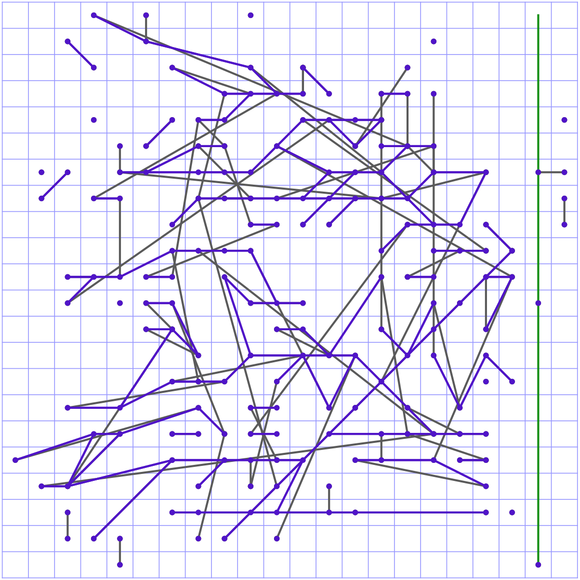

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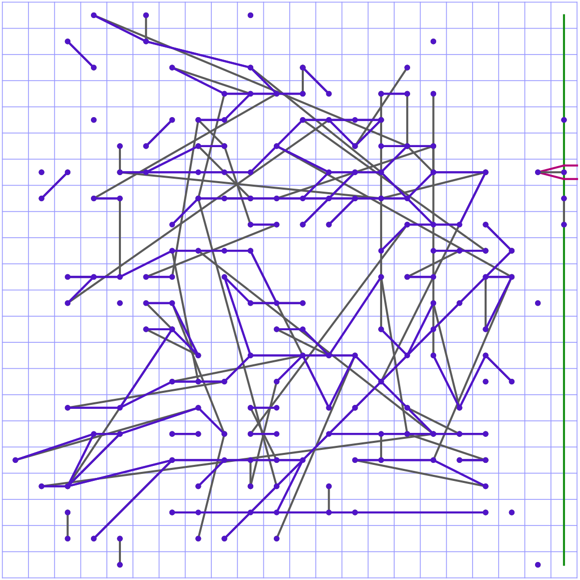

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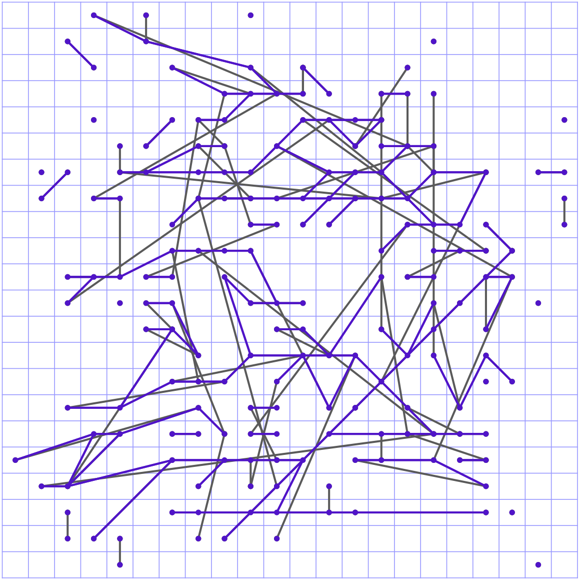

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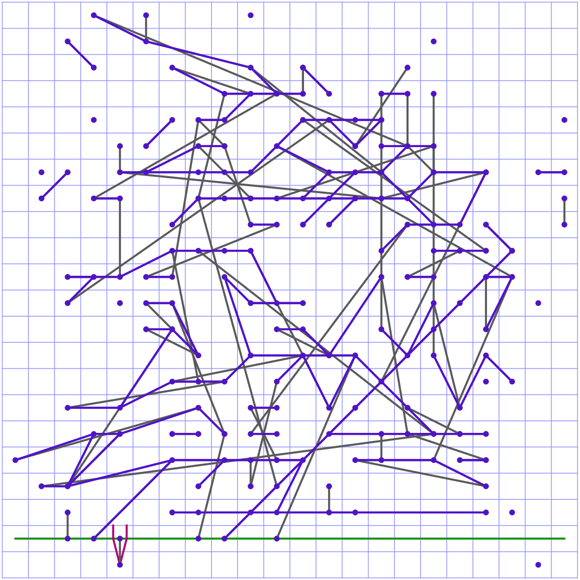

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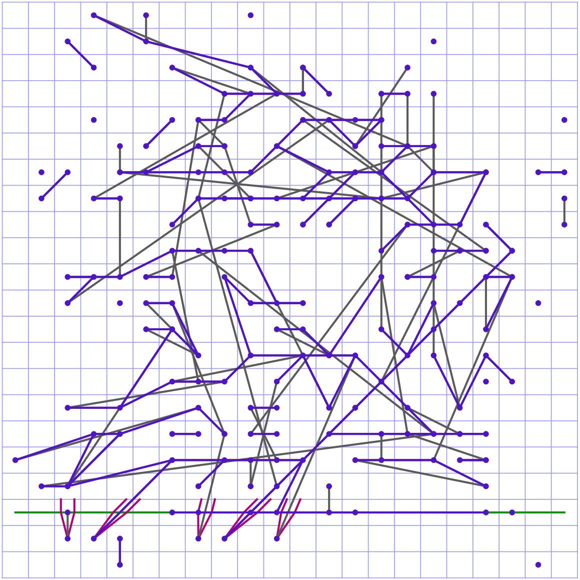

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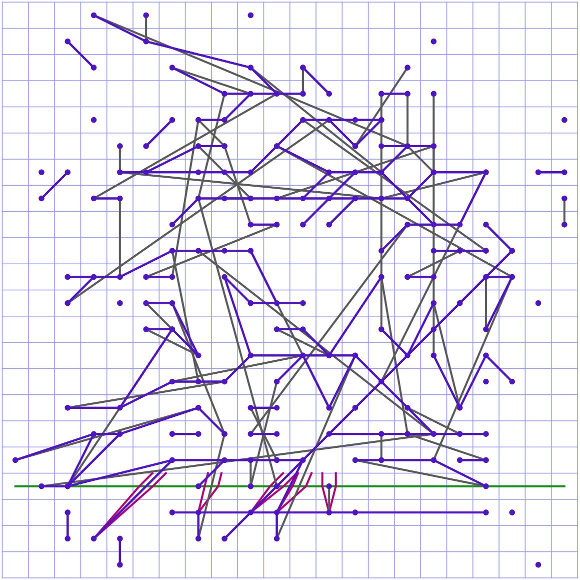

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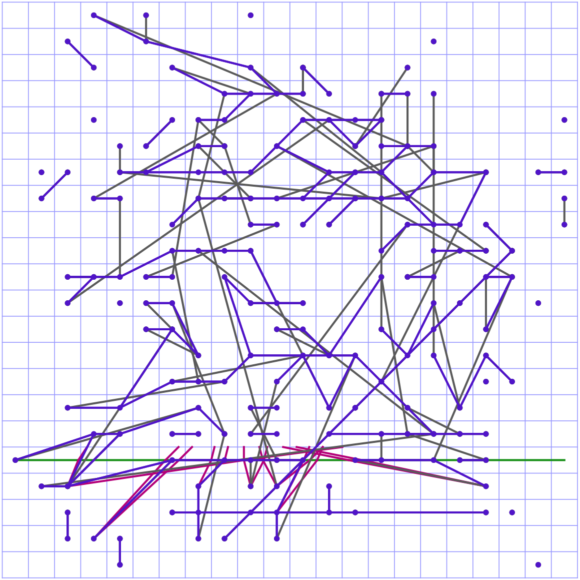

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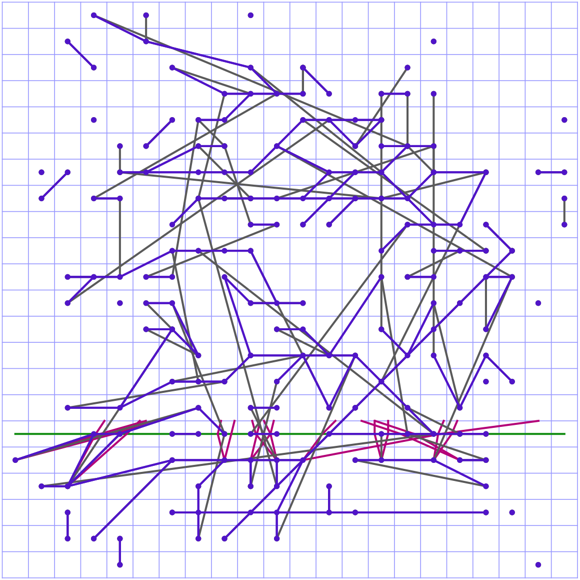

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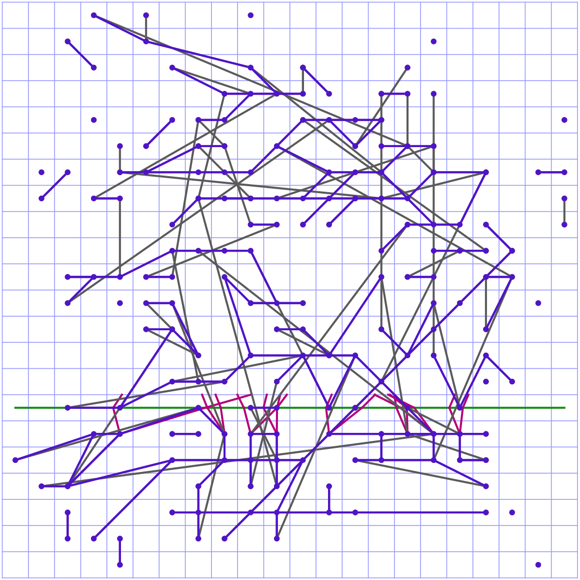

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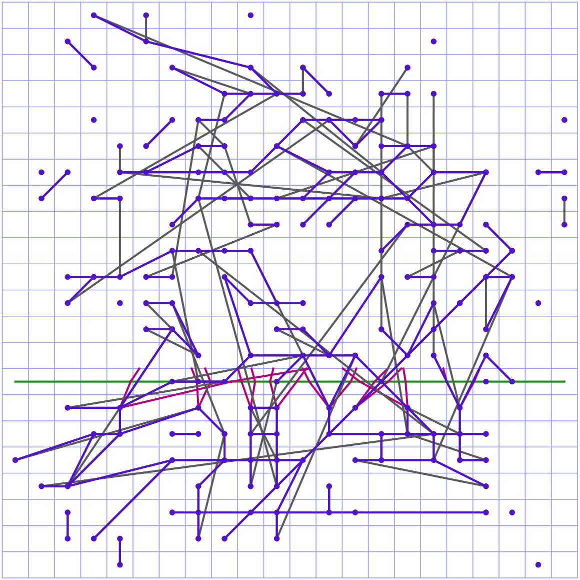

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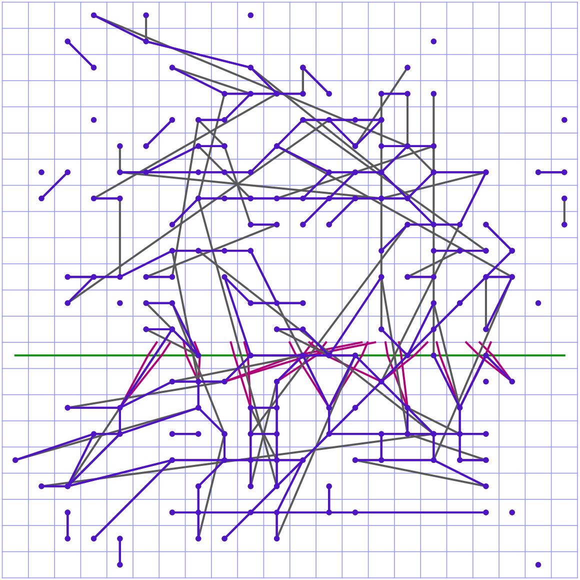

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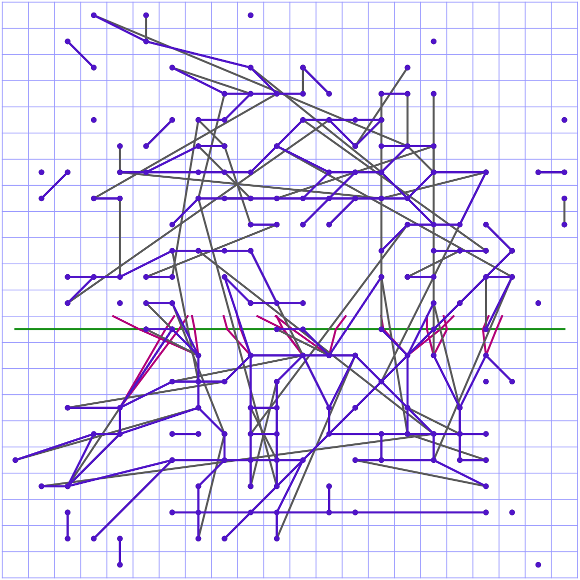

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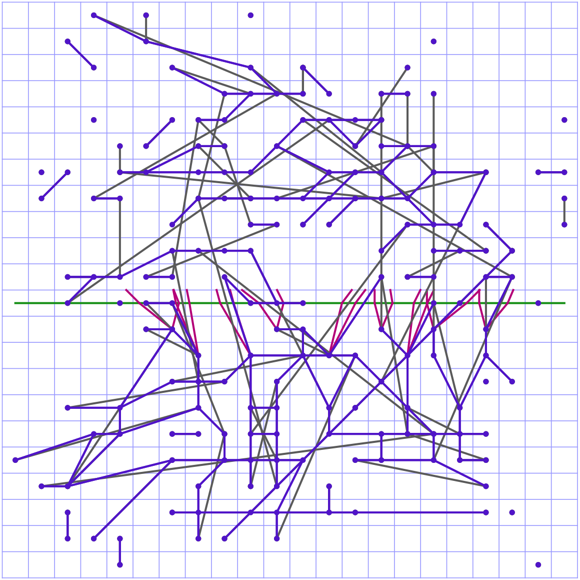

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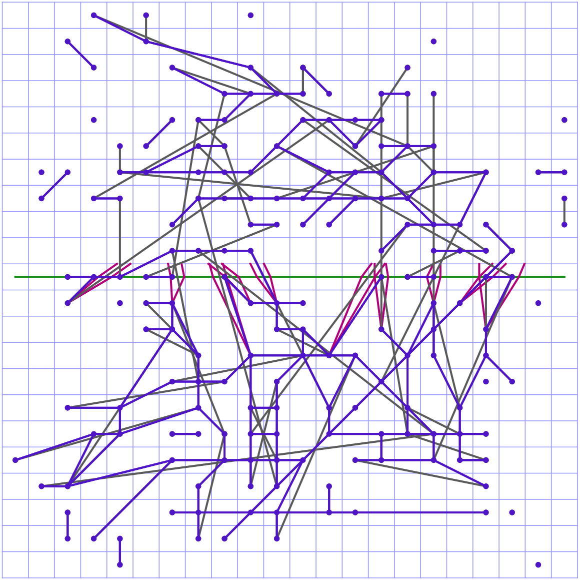

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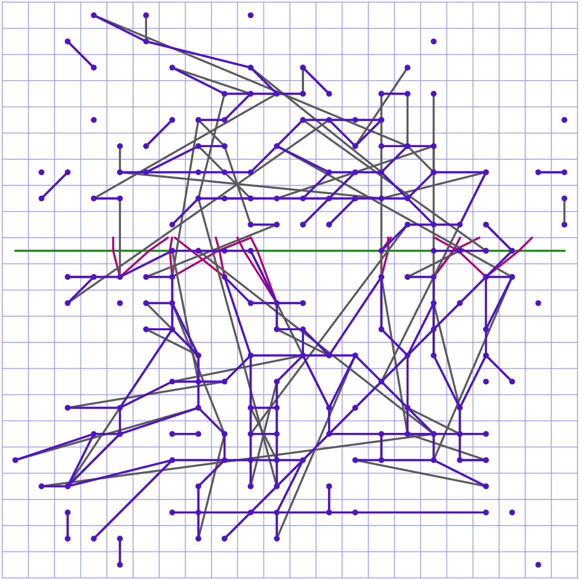

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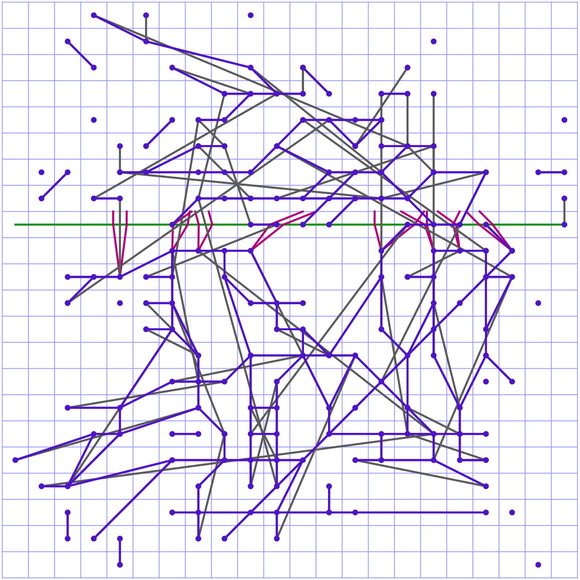

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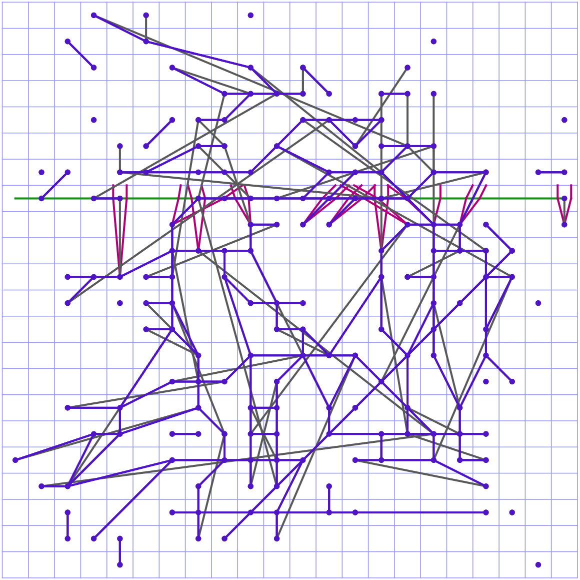

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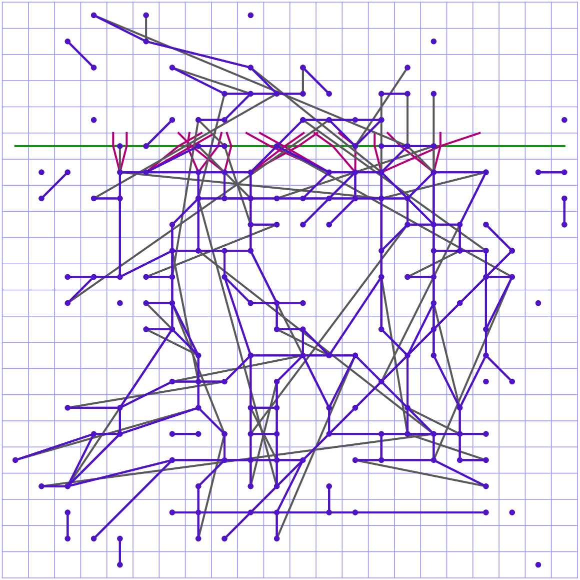

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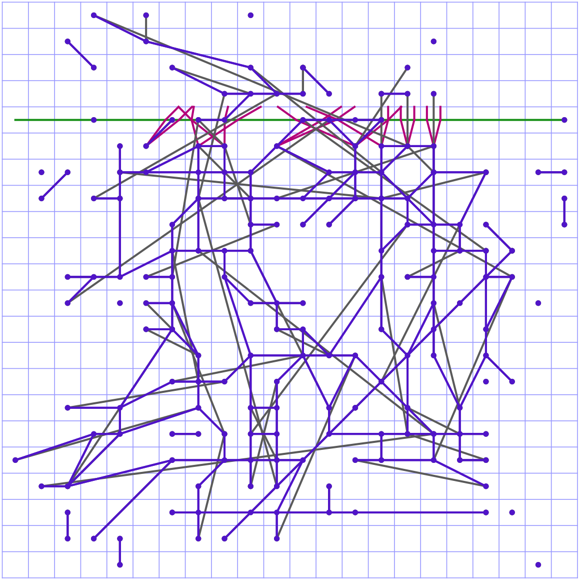

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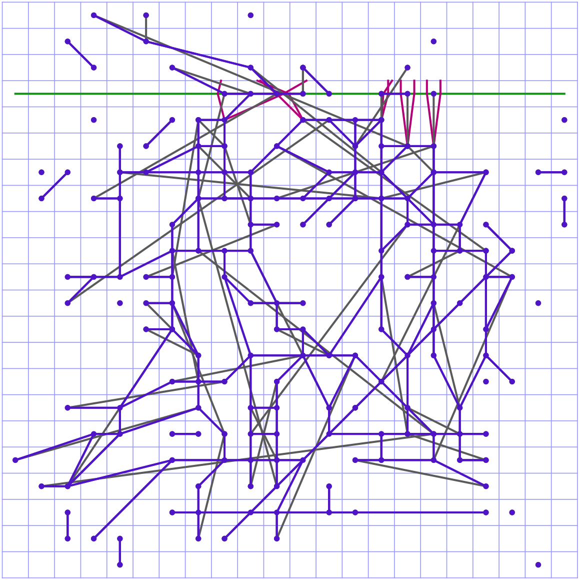

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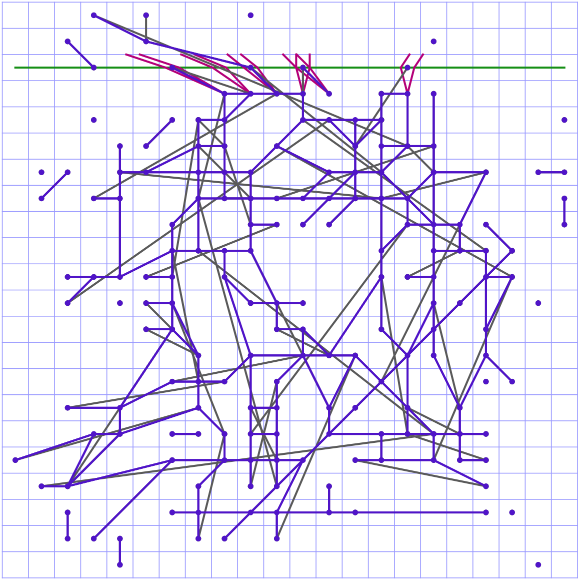

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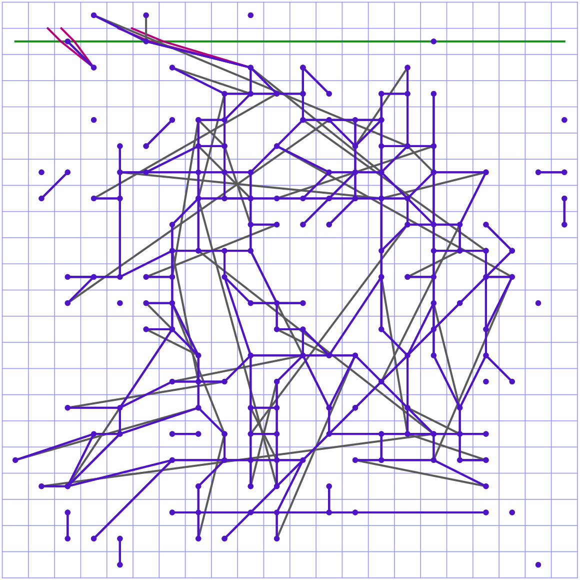

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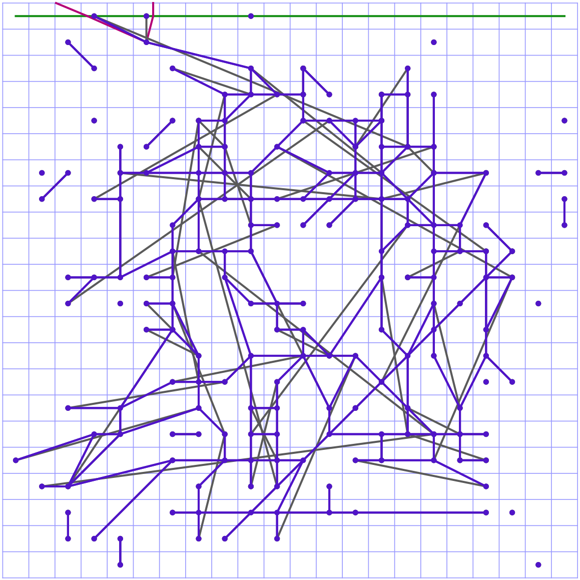

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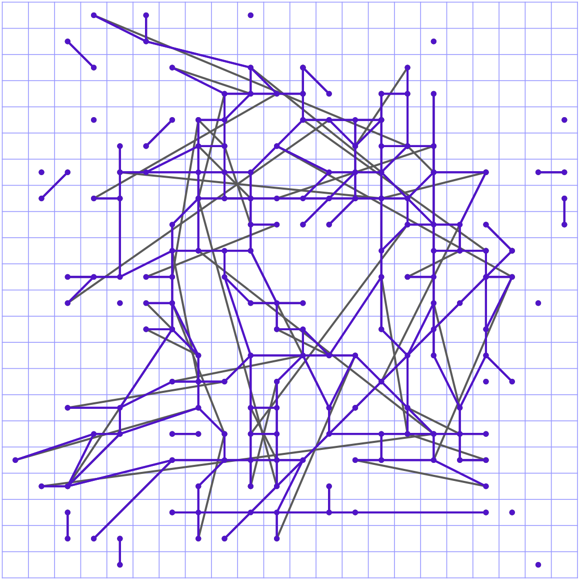

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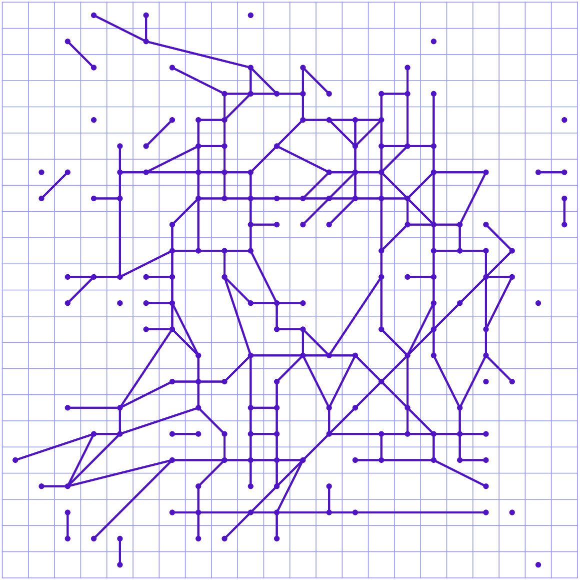

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- are based on interactions within columns of pixels ($is(c)$) instead of individual pixels ($is(h)$ or $ed(h)$)

Conclusion

An applet demonstrating both algorithms is available at

`http://www.sfu.ca/~jpsember/snap.html`