Guaranteed Voronoi Diagrams of Uncertain Sites

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Jeff Sember

University of British Columbia

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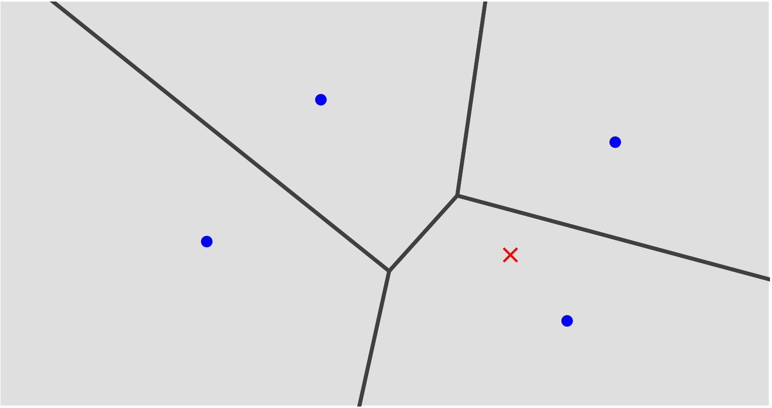
Introduction

Standard Voronoi diagram



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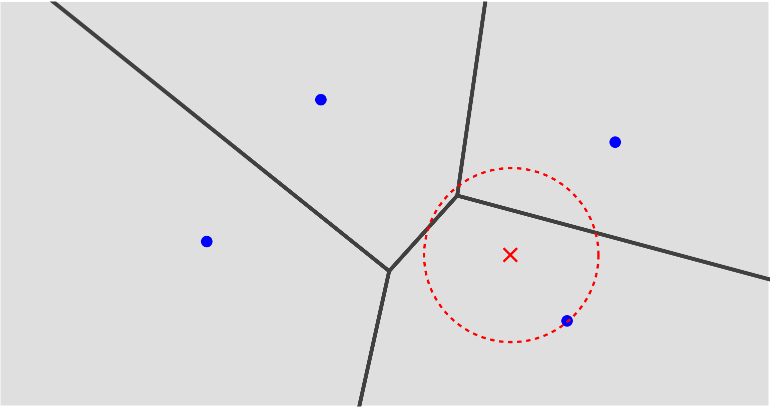
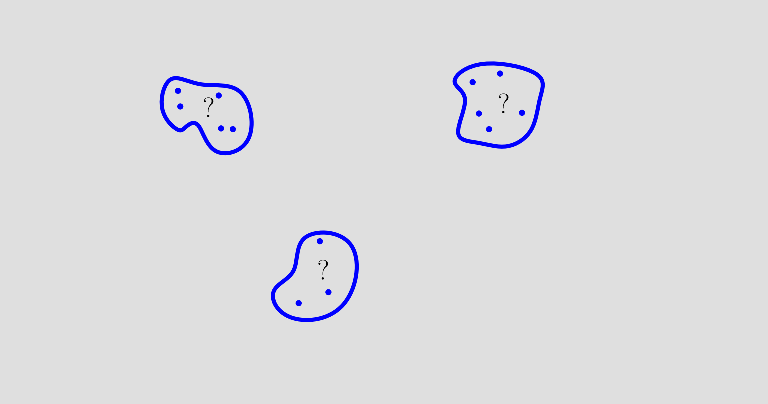


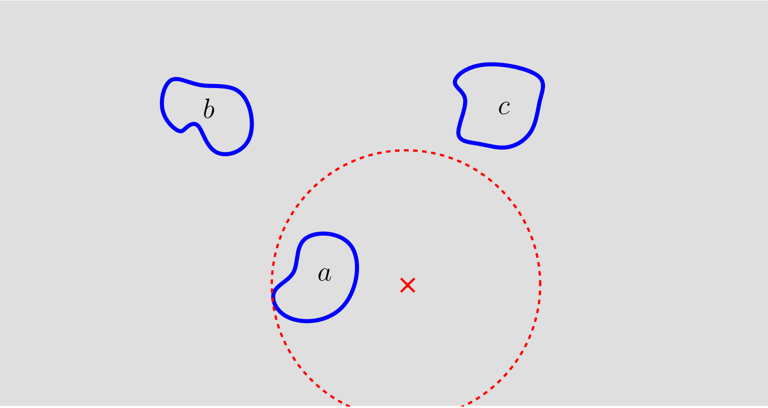
Introduction

What if exact locations of sites are unknown?



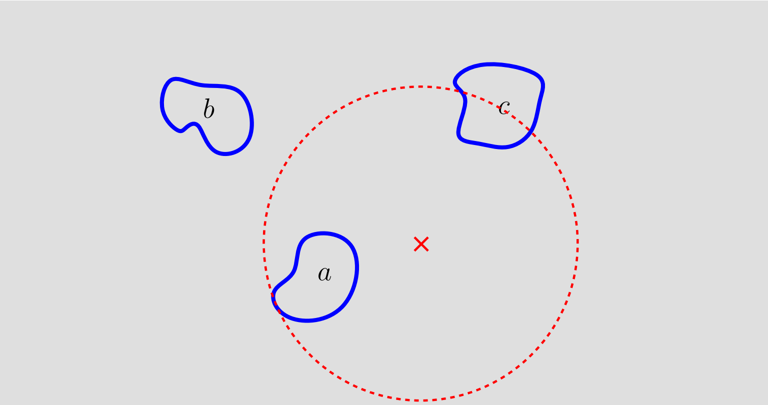
Introduction

X is guaranteed closest to *a*



Introduction

Is X closer to a than c? Than b?



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- exact location of n sites is unknown
- each site is known to lie within a region
- what can be said about arbitrary point in $\mathbb{R}^2$?
- what does such a Guaranteed Voronoi Diagram look like?

Outline

1 Properties

2 Uncertain discs

3 Uncertain polygons

4 Guaranteed Subset Voronoi Diagrams

5 Conclusion

Properties

- Uncertain regions $\mathcal{D} = \{D_1, \ldots, D_n\}$

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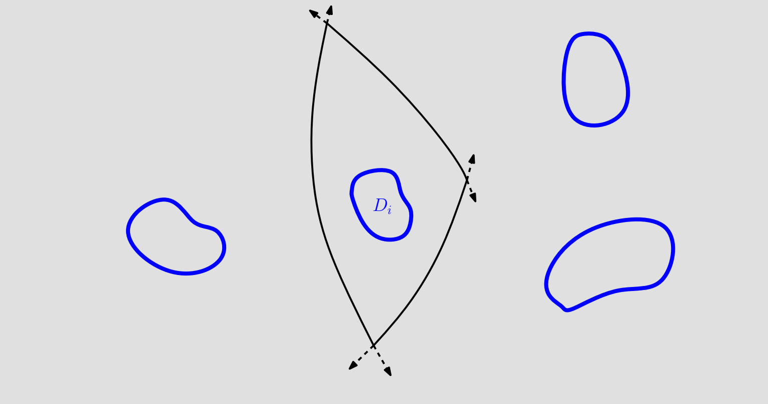
- Half plane $H(i,j)$:
  points guaranteed closer to D_i than D_j
- Edge $\langle i,j \rangle$:
  boundary of $H(i,j)$
  farthest point in D_i as close as nearest point in D_j

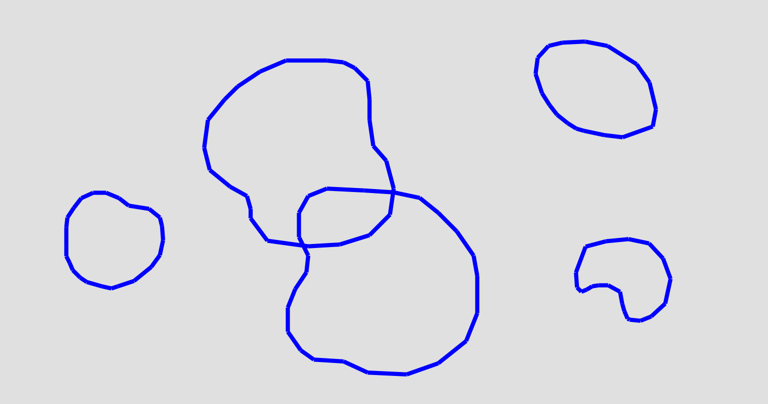
Properties

Cell for D_i : $\bigcap_{j \neq i} H(i, j)$



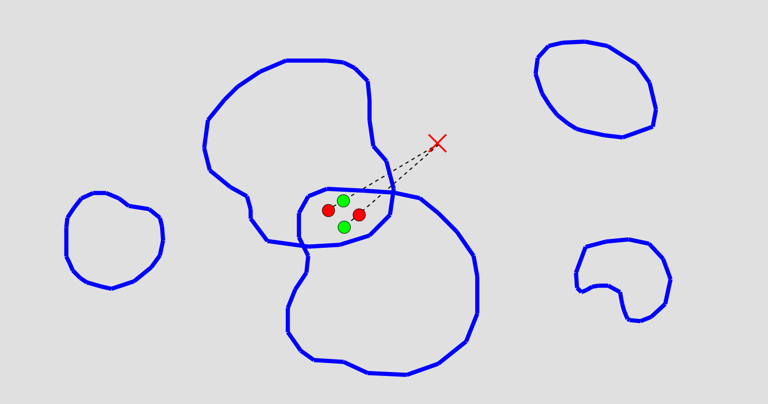
Properties

What if regions overlap?



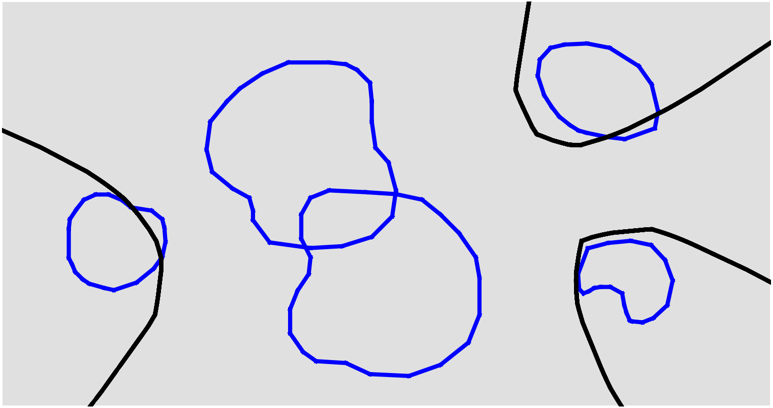
Properties

No guarantee possible



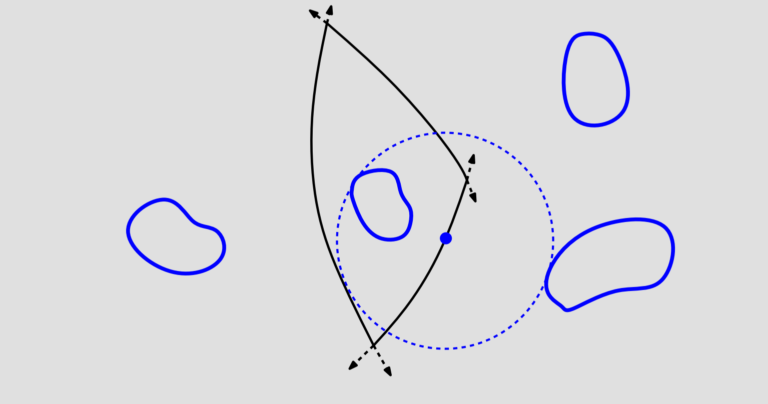
Properties

Cell for both regions is empty



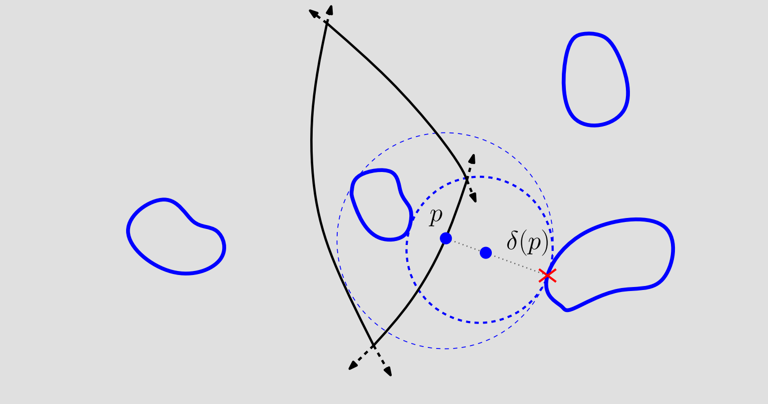
Properties

Lemma 1



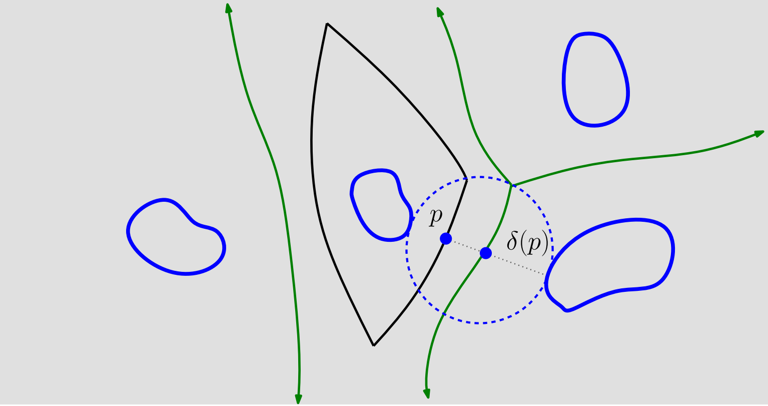
Properties

$\delta(\cdot)$ function



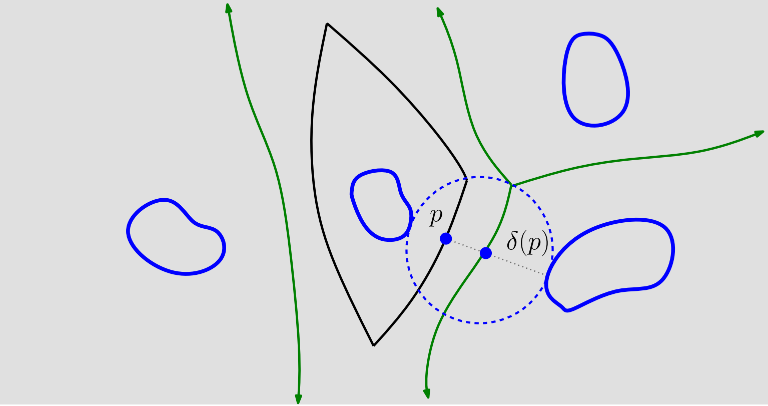
Properties

Maps points on guaranteed edges to standard edges



Properties

Lemma 3: $\delta(\cdot)$ preserves ccw order

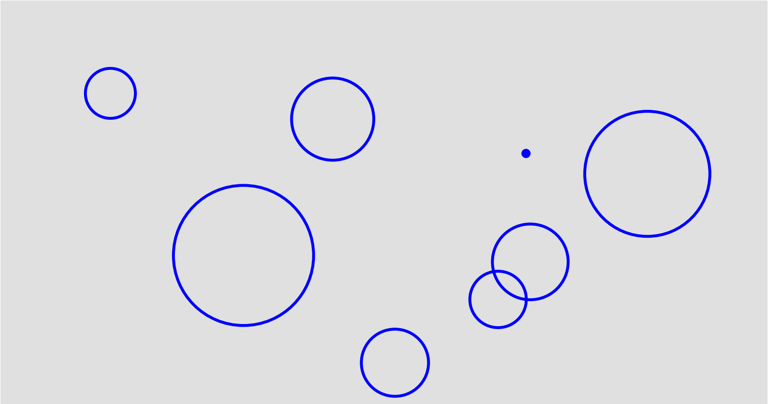


Outline

1. Properties
2. Uncertain discs
3. Uncertain polygons
4. Guaranteed Subset Voronoi Diagrams
5. Conclusion

Uncertain discs

What if uncertain regions are discs?



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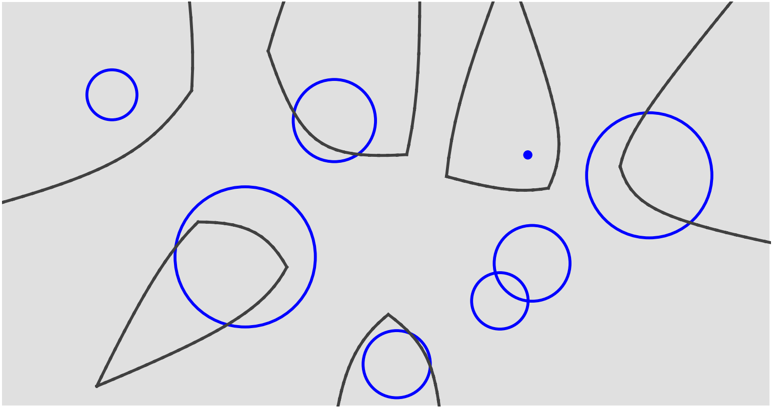

Uncertain discs

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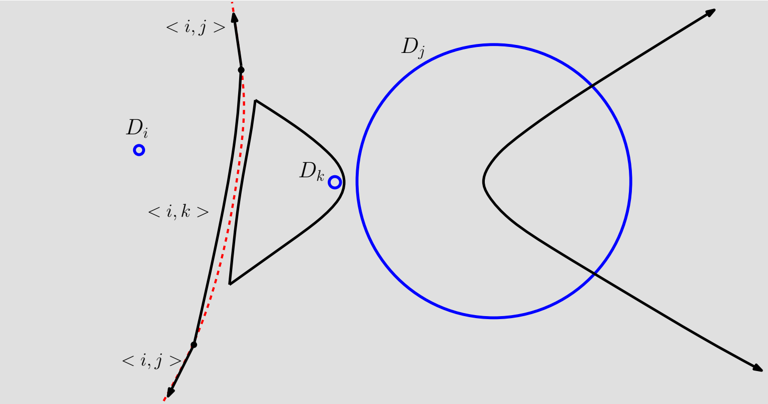
- $\langle i,j \rangle = \{\ p \mid \ \mathrm{dist}(p, S_j) - \mathrm{dist}(p, S_i) =$ constant $\}$
- $\langle i,j \rangle$ is a hyperbolic arc

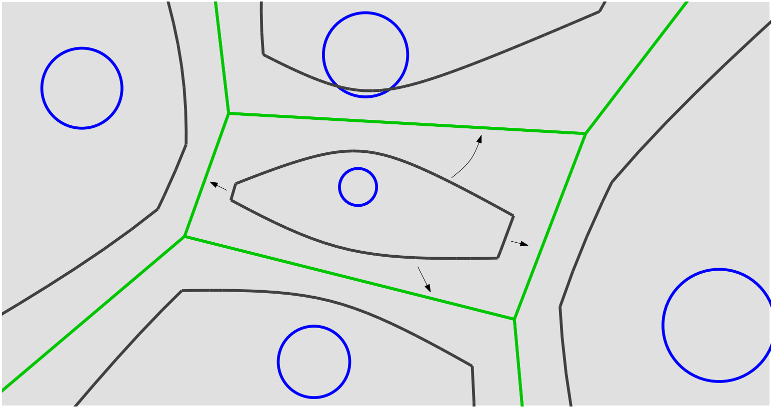
Uncertain discs

More than one edge $\langle i, j \rangle$ can appear



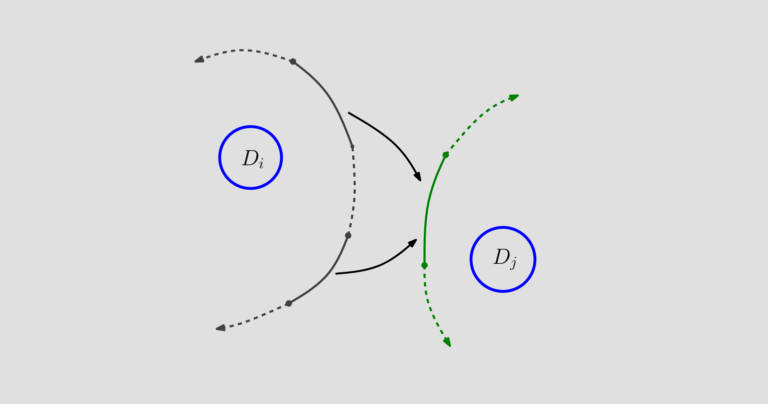
Uncertain discs: Complexity

Each edge $\langle i, j \rangle$ charged to edge of standard Voronoi diagram



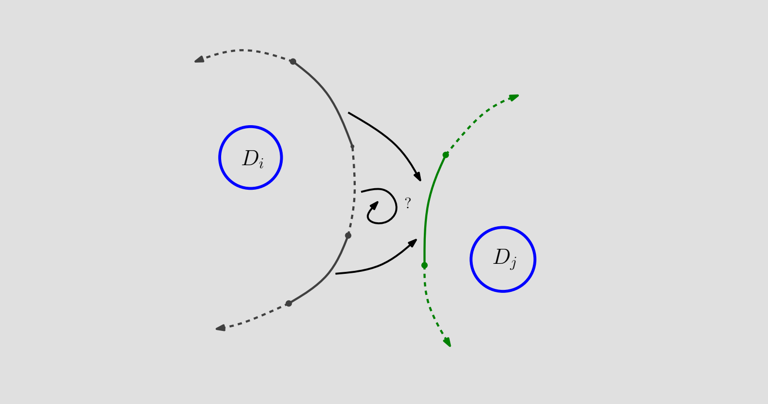
Uncertain discs: Complexity

Suppose distinct edges $\langle i, j \rangle$ charged to same standard edge...



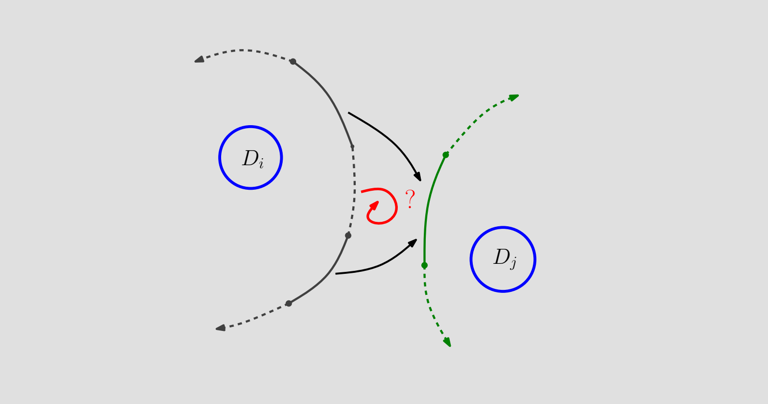
Uncertain discs: Complexity

. . . edge between them must be charged somewhere. . .



Uncertain discs: Complexity

...violates Lemma 3 (ccw order of δ mapping)



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- once by $\langle i, j \rangle$, once by $\langle j, i \rangle$
- standard Voronoi diagram has linear complexity [M. Sharir, '85]

Theorem 4

Guaranteed Voronoi Diagram for discs has linear complexity

Uncertain discs: Construction

- construct standard Voronoi diagram of discs
  in $O(n \log n)$ time [S. Fortune, '86]

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Theorem 6

Guaranteed Voronoi Diagram for discs
can be constructed in $O(n \log n)$ time

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1. Properties
2. Uncertain discs
3. Uncertain polygons
4. Guaranteed Subset Voronoi Diagrams
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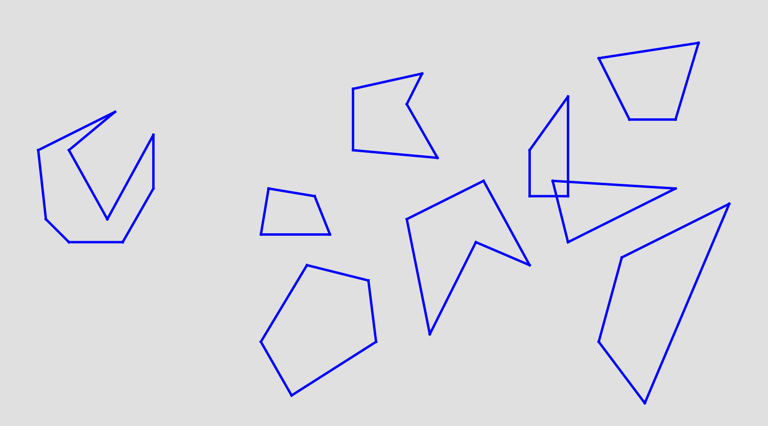
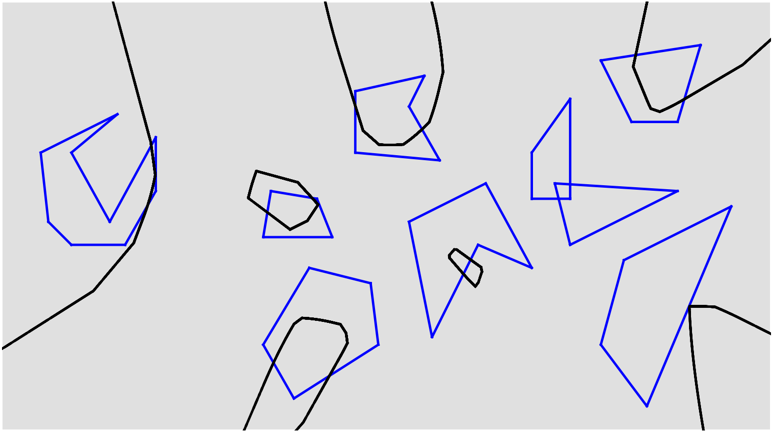
Uncertain polygons

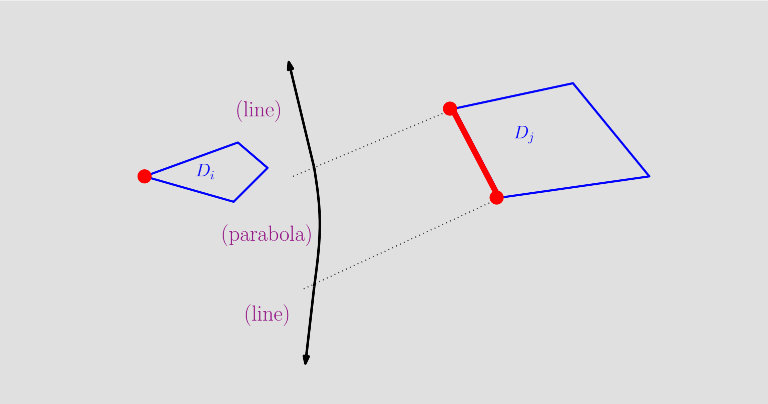


Uncertain polygons



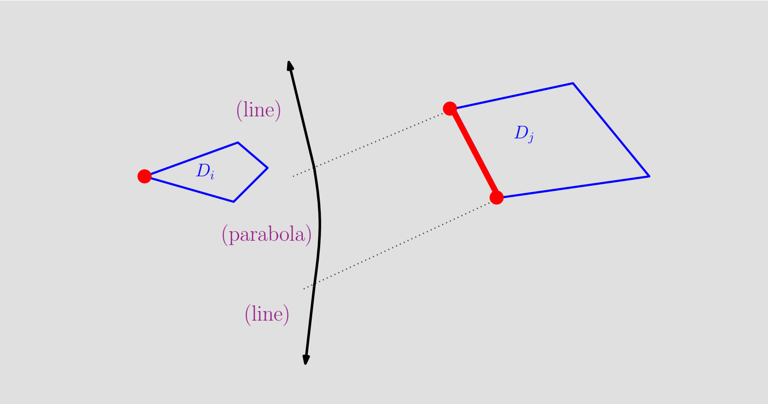
Uncertain polygons

$\langle i, j\rangle$ consists of parabolic arcs $\langle i^u, j^v\rangle$

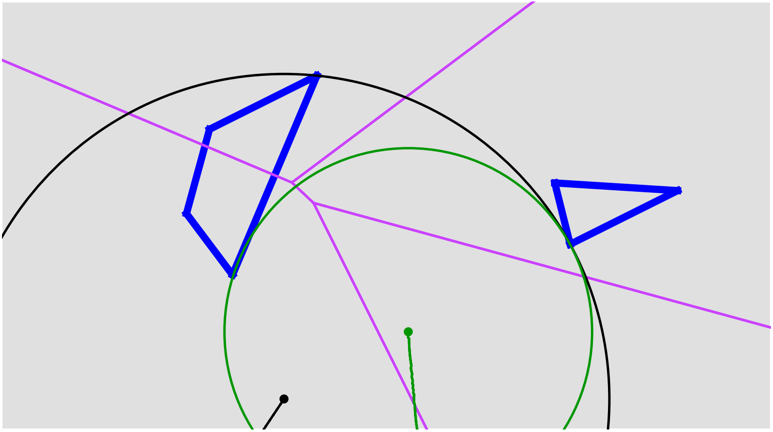


Uncertain polygons

i^u is farthest vertex of $R_{\langle i \rangle}$
j^v is nearest vertex or edge of $R_{\langle j \rangle}$



Uncertain polygons: Complexity



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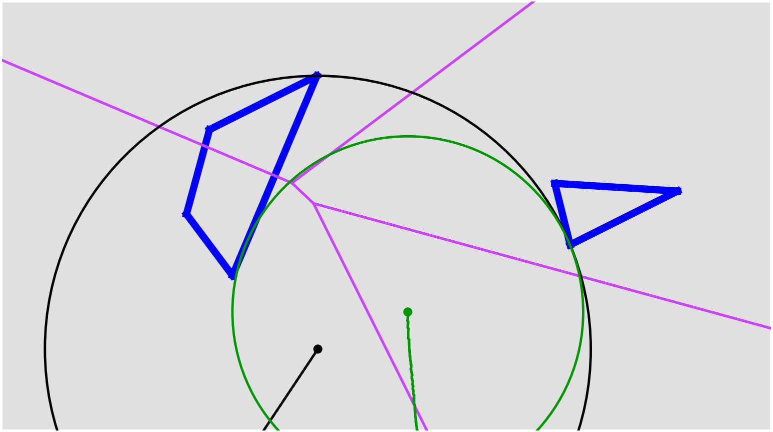

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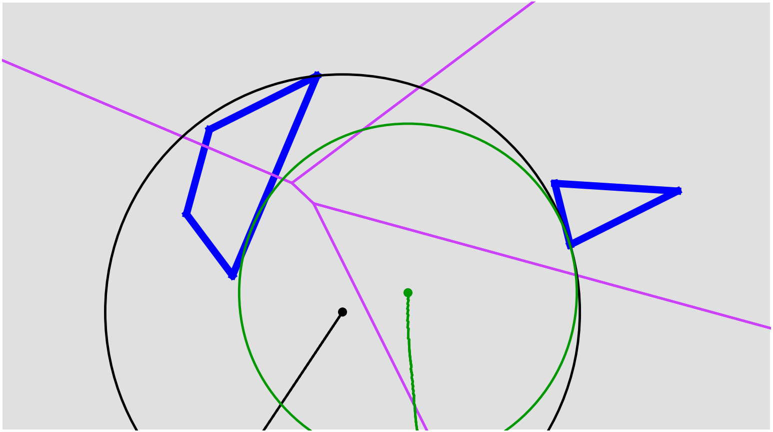

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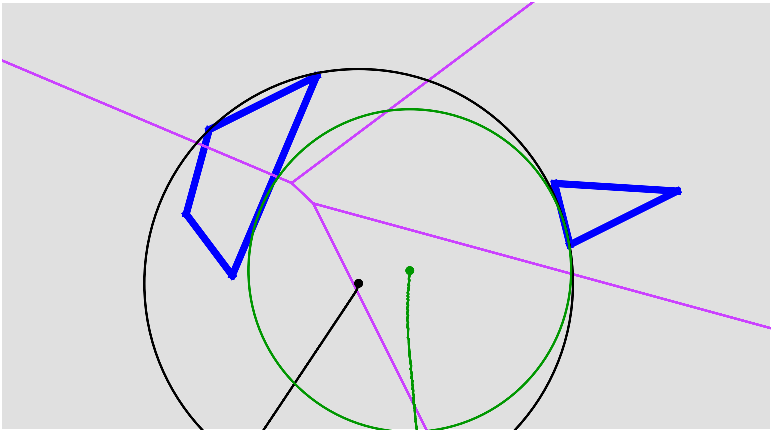

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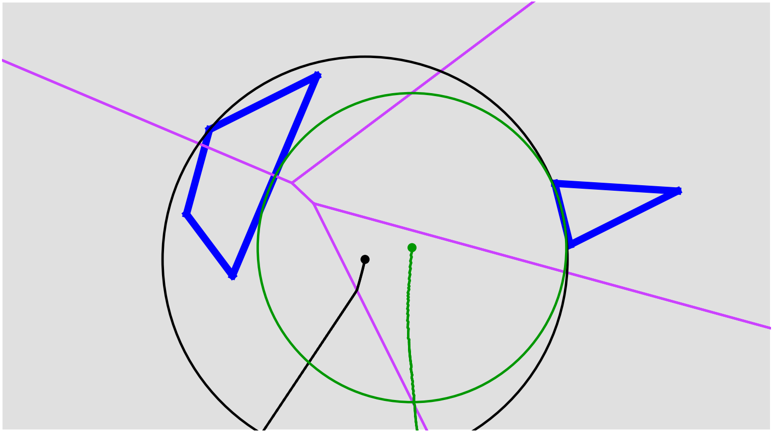

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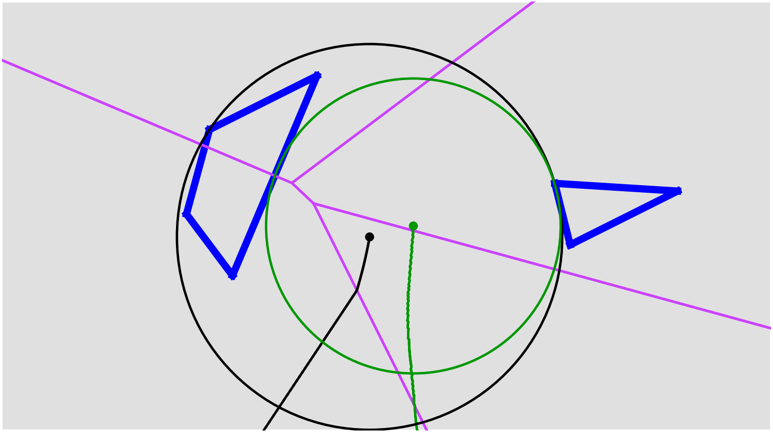

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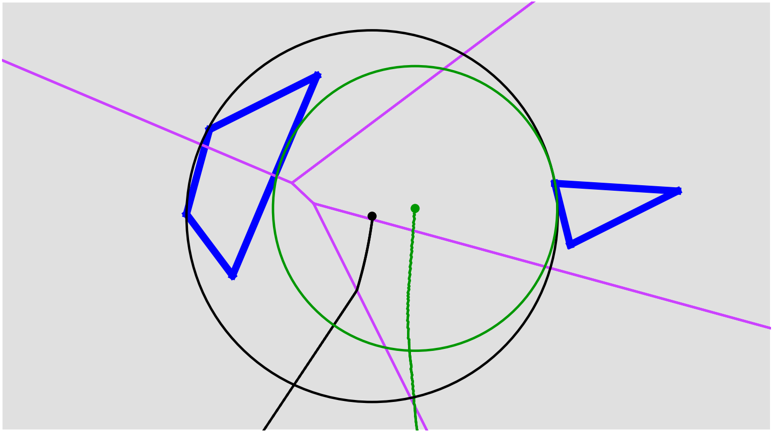

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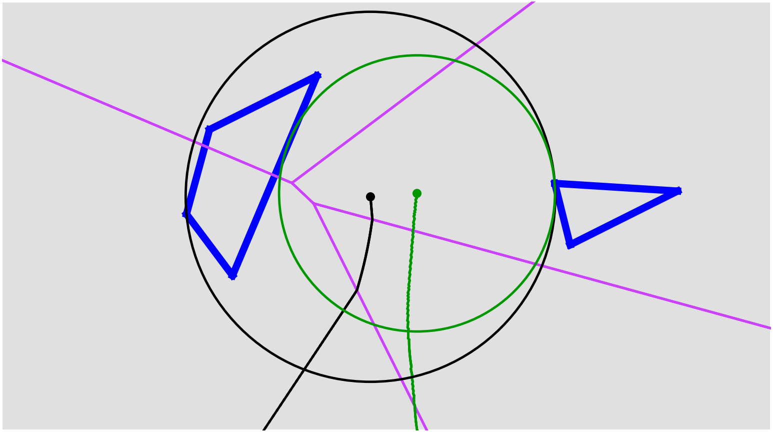

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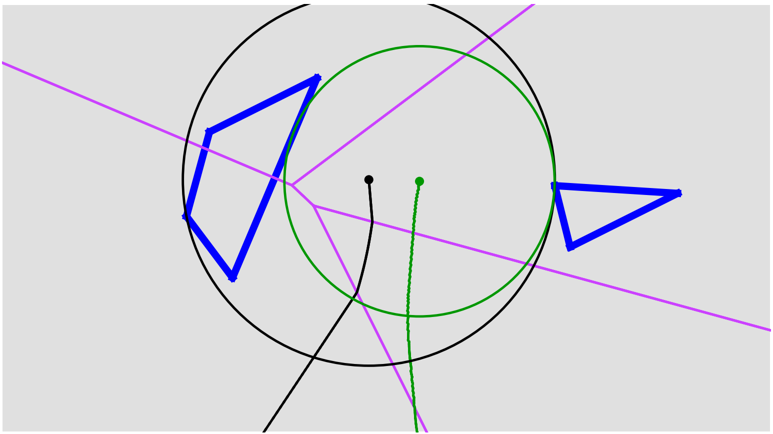

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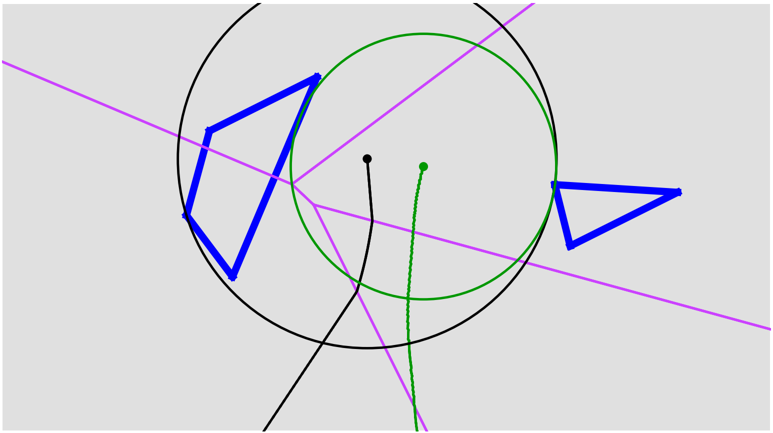

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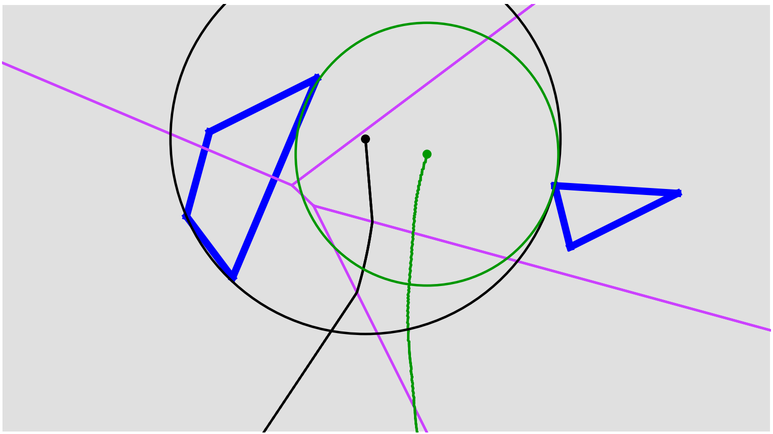

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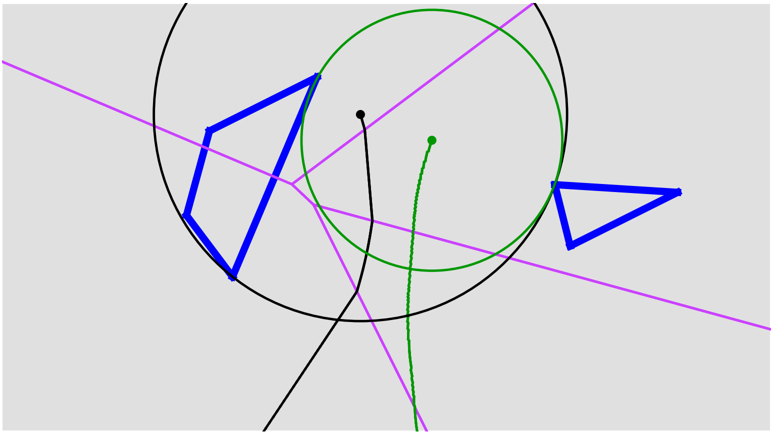

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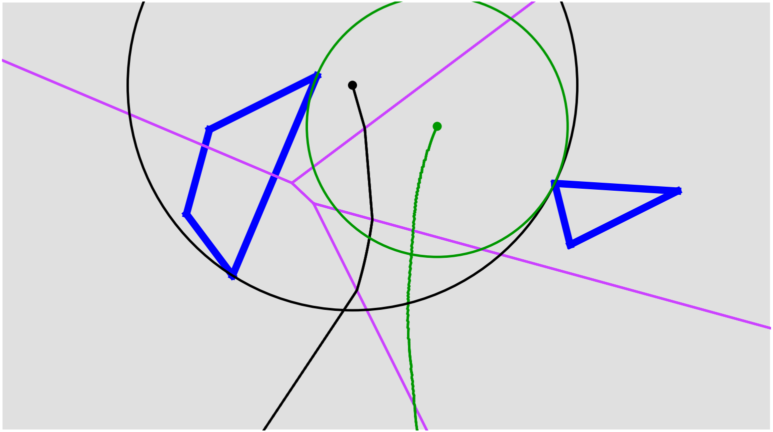

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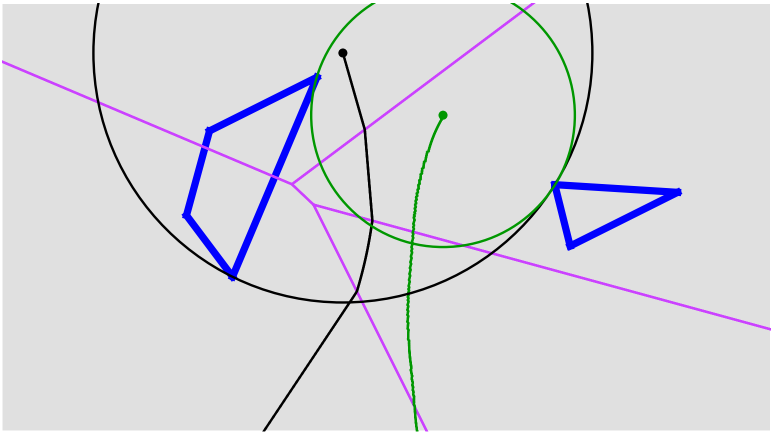

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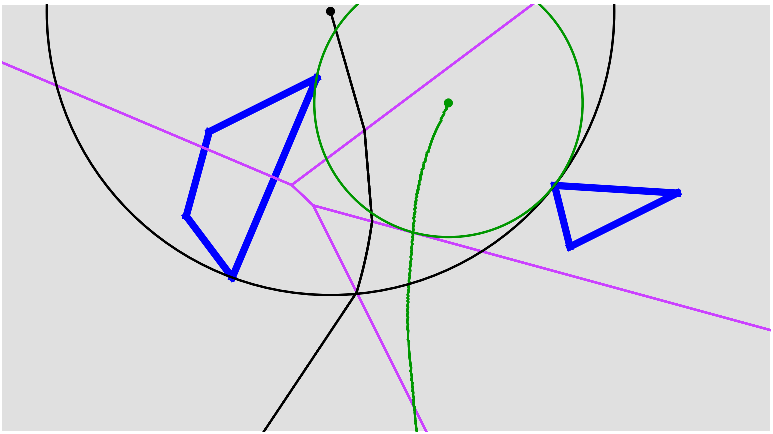

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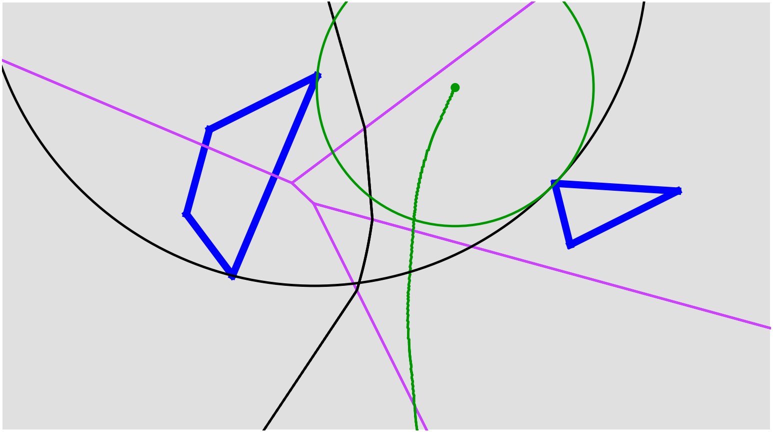

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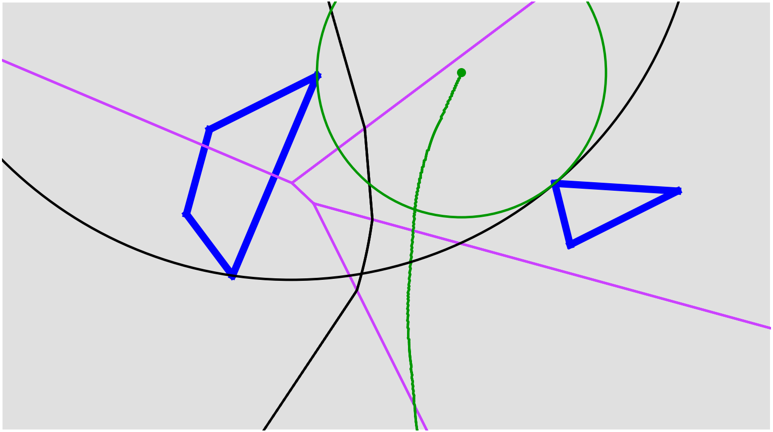

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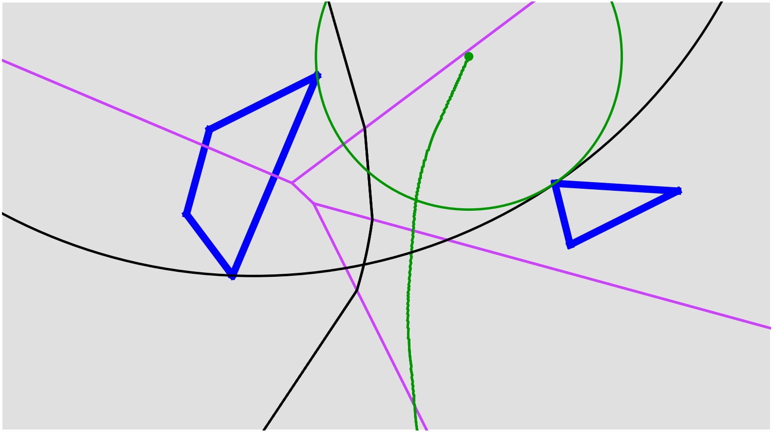

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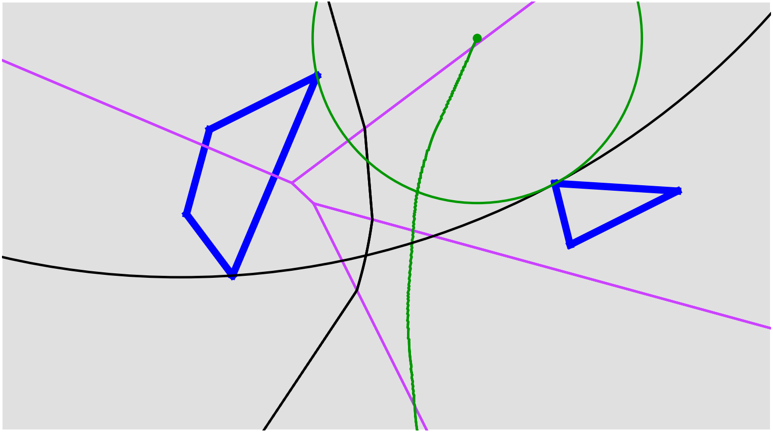

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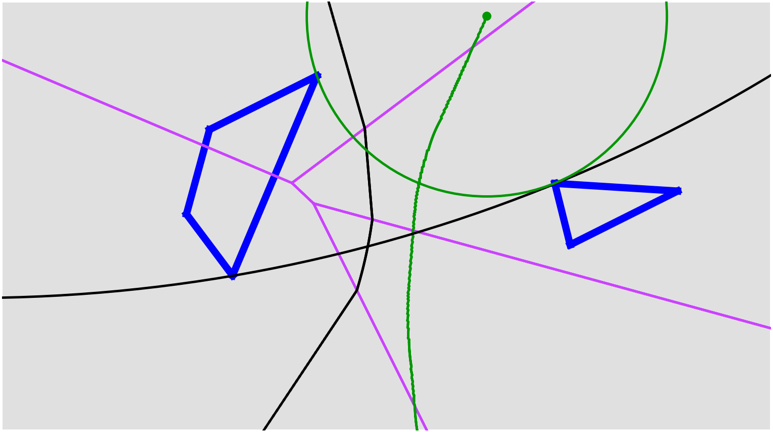

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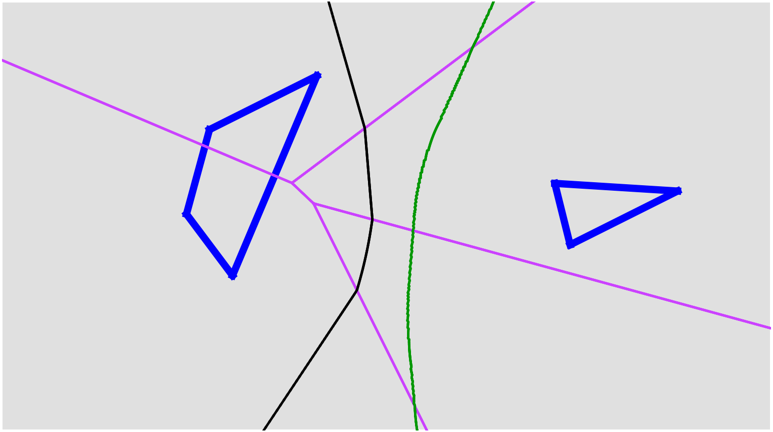

Uncertain polygons: Complexity

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Theorem 7

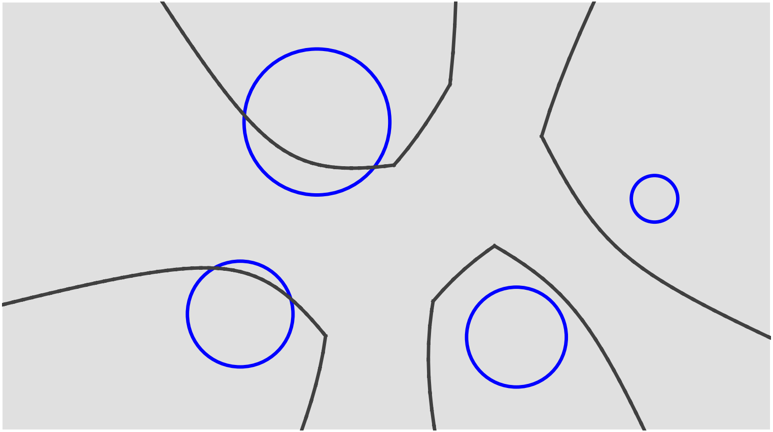
Guaranteed Voronoi Diagram for polygons has linear complexity

Outline

1. Properties
2. Uncertain discs
3. Uncertain polygons
4. Guaranteed Subset Voronoi Diagrams
5. Conclusion

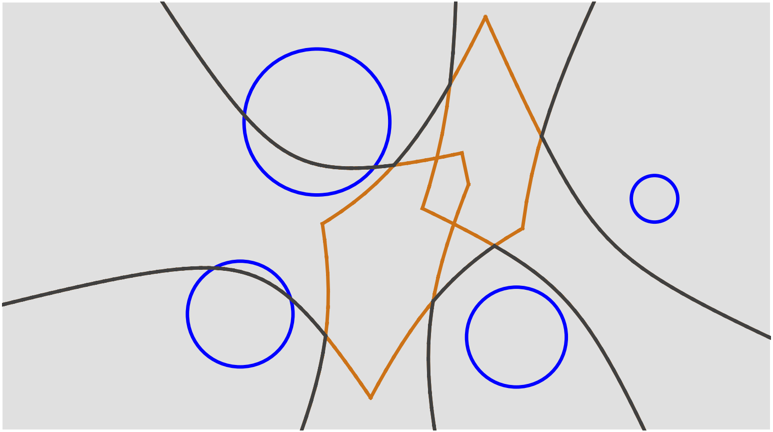
Guaranteed Subset Voronoi Diagrams: $V^{\{\}}(\mathcal{D})$

Much of $\mathbb{R}^2$ falls in 'neutral zone': no guaranteed closest site



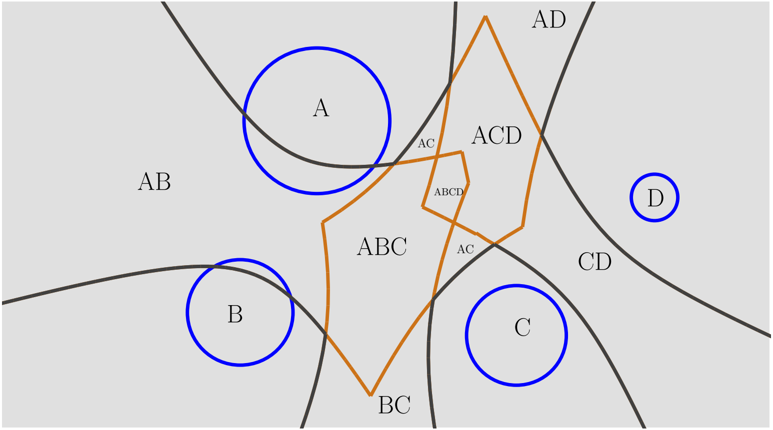
Guaranteed Subset Voronoi Diagrams: $V^{\{\}}(\mathcal{D})$

Modify guarantee to involve subsets of sites



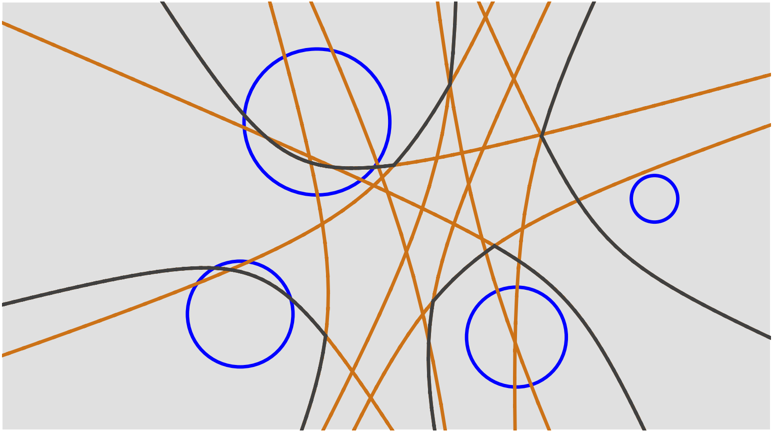
Guaranteed Subset Voronoi Diagrams: $V^{\{\}}(\mathcal{D})$

Guarantee: p closest to some $D_i \in S$ than to any $D_i \in \overline{S}$



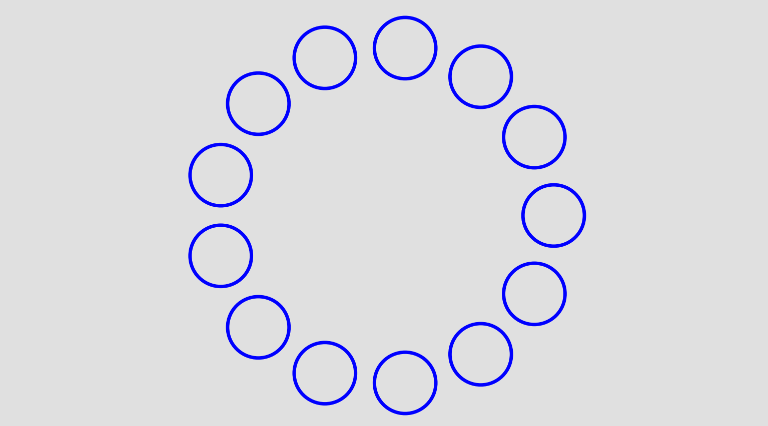
Guaranteed Subset Voronoi Diagrams: $V^{\{\}}(\mathcal{D})$

Lemma 8: Complexity is $O(n^4)$



Guaranteed Subset Voronoi Diagrams: $V^{\{\}}(\mathcal{D})$

Complexity is $\Omega(n^2)$



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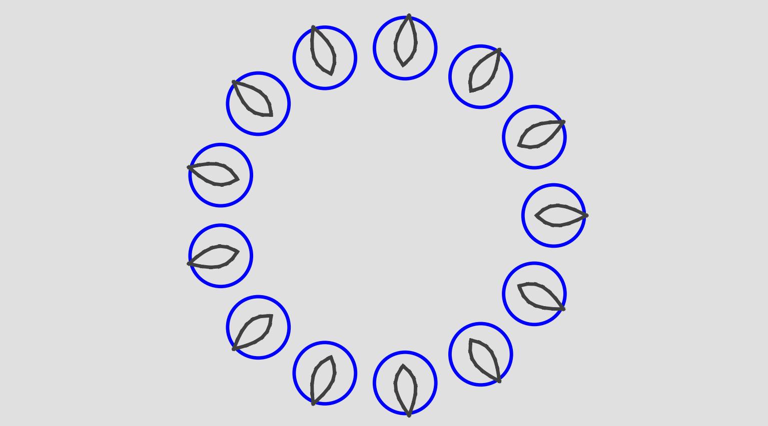

Guaranteed Subset Voronoi Diagrams: $V^{\{\}}(\mathcal{D})$

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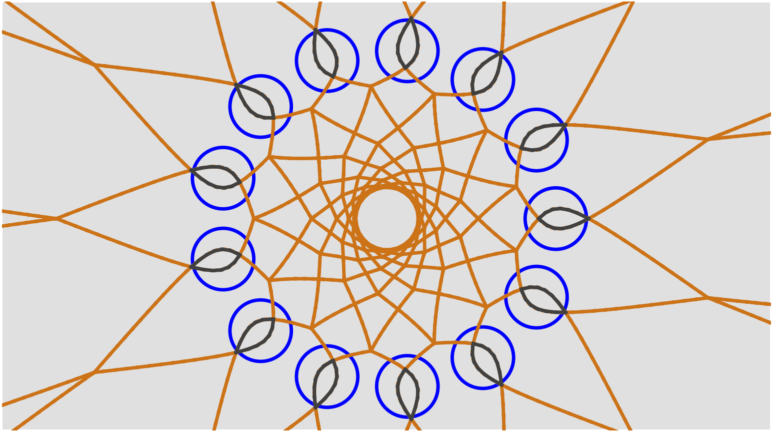

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Related work

- uncertain sites, probabilistic view: expected closest, probably closest
  [Aurenhammer et al., '91]

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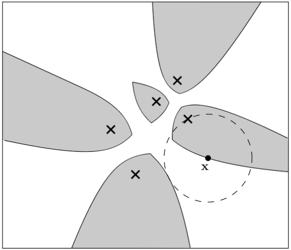
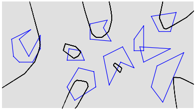
  [Aurenhammer et al., '91]
- 'neutral zone' similar to zone diagrams



[Asano et al., '07]

Future work

- efficient algorithm for generating $V(\mathcal{D})$ (polygons) 

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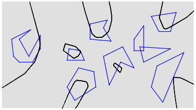
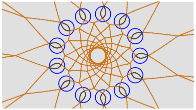
- efficient algorithm for generating $V(\mathcal{D})$ (polygons) 
- tighten bound for complexity of $V^{\{\}}(\mathcal{D})$ 

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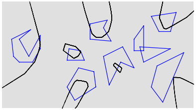
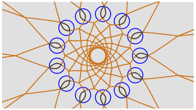
- efficient algorithm for generating $V(\mathcal{D})$ (polygons) 
- tighten bound for complexity of $V^{\{\}}(\mathcal{D})$ 
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Future work

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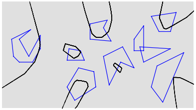

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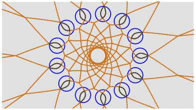

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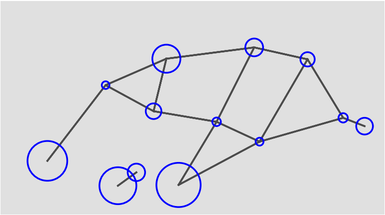

- investigate dual of $V(\mathcal{D})$

Resources

Java applet available at: www.cs.ubc.ca/~jpsember