TRAFFIC PLANNING

ASSIGNMENT 2

COSC364-19S1 INTERNET TECHNOLOGY AND ENGINEERING

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May 29, 2019

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Date:	29-5-19	29/5/19

1 Problem Description

Given a network (figure 1) with X source nodes, Y transit nodes and Z destination nodes, a program was designed to generate an LP file that could be used by CPLEX to determine certain network characteristics.

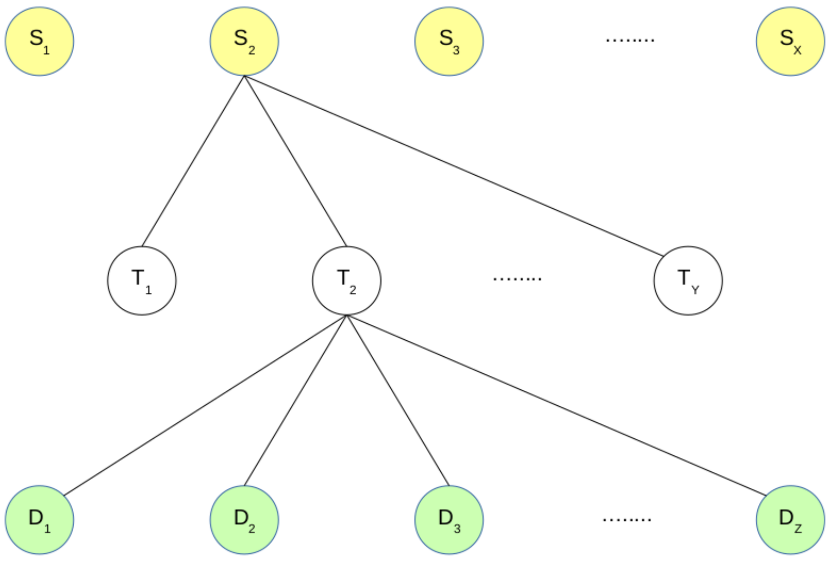

Figure 1: An example network (A. Willig, April 2019).

Traffic travelling from S_i to D_j must travel through exactly 2 transit nodes with a total demand volume of h_{ij} (equation 10). Furthermore, the load upon each transit node must be balanced.

2 Problem Formulation

This problem was solved with the use of binary variable constraints (equations 6, 7 and 9) and the minimisation of our objective function (equation 1). All normal non-negativity constraints were applied (equations 11, 12, 13 and 14).

The following network properties were solved for:

- The capacities of each link (equations 3 and 4).
- The load on each transit node (equation 5).
- The value of each flow (equations 2 and 8).

Notation:

- X is the number of source nodes.
- Y is the number of transit nodes.
- Z is the number of destination nodes.
- S_i is the ith source node.

- T_k is the kth transit node.
- D_j is the jth destination node.
- h_{ij} is the demand flow between S_i and D_j. This is equal to $2i + j$.
- c_{ik} is the link capacity between S_i and T_k.
- d_{kj} is the link capacity between T_k and D_j.
- x_{ikj} is the decision variable associated with the path S_i-T_k-D_j.
- u_{ikj} is the binary decision variable associated with x_{ikj}. These are required because h_{ij} must be split across exactly 2 transit nodes.
- l_k is the load on T_k.

Note: Due to the limitations of the LP file format, many of the following equations must be rearranged for use in CPLEX. Most notably, there cannot be any variables on the right hand side of an equality or inequality.

2.1 Objective Function

$$\text{minimize}_{[x,c,d,r]}\ r \tag{1}$$

2.2 Constraints

$$\sum_{k=1}^{Y} x_{ikj} = h_{ij} \qquad i \in \{1,\dots,X\}, j \in \{1,\dots,Z\} \tag{2}$$

$$\sum_{j=1}^{Z} x_{ikj} = c_{ik} \qquad i \in \{1,\dots,X\}, k \in \{1,\dots,Y\} \tag{3}$$

$$\sum_{i=1}^{X} x_{ikj} = d_{kj} \qquad k \in \{1,\dots,Y\}, j \in \{1,\dots,Z\} \tag{4}$$

$$\sum_{k=1}^{Y} x_{ikj} = l_k \qquad i \in \{1,\dots,X\}, j \in \{1,\dots,Z\} \tag{5}$$

$$\sum_{k=1}^{Y} u_{ikj} = 2 \qquad i \in \{1,\dots,X\}, j \in \{1,\dots,Z\} \tag{6}$$

$$x_{ikj} = \frac{u_{ikj}h_{ij}}{2} \qquad i \in \{1,\dots,X\}, k \in \{1,\dots,Y\}, j \in \{1,\dots,Z\} \tag{7}$$

$$\sum_{i=1}^{X}\sum_{j=1}^{Z} x_{ikj} \leq r \qquad k \in \{1,\dots,Y\} \tag{8}$$

$$u_{ikj} \in \{0,1\} \qquad i \in \{1,\dots,X\}, k \in \{1,\dots,Y\}, j \in \{1,\dots,Z\} \tag{9}$$

$$h_{ij} = 2i + j \qquad i \in \{1,\dots,X\}, j \in \{1,\dots,Z\} \tag{10}$$

2.3 Non-Negativity Constraints

$$r \geq 0 \tag{11}$$

$$x_{ikj} \geq 0 \qquad i \in \{1,\dots,X\}, k \in \{1,\dots,Y\}, j \in \{1,\dots,Z\} \tag{12}$$

$$c_{ik} \geq 0 \qquad i \in \{1,\dots,X\}, k \in \{1,\dots,Y\} \tag{13}$$

$$d_{kj} \geq 0 \qquad k \in \{1,\dots,Y\}, j \in \{1,\dots,Z\} \tag{14}$$

3 Results

LP files were generated with parameters $X = Z = 9, Y \in \{3, 4, 5, 6, 7, 8\}$. These were then processed with CPLEX, recording the time taken to solve each problem. Important data points were extracted from the CPLEX output and are listed in table 1.

... your table ...

Table 1: insert caption here, yo!

4 Appendix

4.1 Source Code

4.1.1 src/__main__.py

```
import sys

from lp_gen import generate_lp_file
from lp_utils import get_lp_filename

__TITLE__ = "COSC-364 Assignment 2 LP Generator"
__AUTHORS__ = [("Will Cowper", "81163265"), ("Jesse Sheehan", "53366509")]


def print_version():
    print('{0} by {1}'.format(__TITLE__, get_author_string()))


def print_usage():
    print('Usage: {0} <x> <y> <z>'.format(sys.argv[0]))


def get_problem_parameters():
    """ Returns a tuple containing the x, y and z parameters. """
    try:
        x = int(sys.argv[1])
        y = int(sys.argv[2])
        z = int(sys.argv[3])
    except:
        print_usage()
        exit(-1)

    if x <= 0:
        print("Error: x must be strictly positive")
        exit(-1)

    if y < 0:
        print("Error: y must be strictly positive")
        exit(-1)

    if z <= 0:
        print("Error: z must be strictly positive")
        exit(-1)
```

```python
    return x, y, z


def save_lp_file(filename, data):
    try:
        f = open(filename, 'w')
        f.write(data)
        f.close()
    except:
        print("Error: could not save file '{0}'".format(filename))
        exit(-1)

def get_author_string():
    return ', '.join(
        ["{0} ({1})".format(name, sid) for (name, sid) in __AUTHORS__])

def main():
    print_version()
    if len(sys.argv) != 4:
        print_usage()
        exit(-1)
    else:
        x, y, z = get_problem_parameters()
        data = generate_lp_file(__TITLE__, get_author_string(), x, y, z)
        filename = get_lp_filename(x, y, z)
        save_lp_file(filename, data)
        print("Success: saved as '{0}'".format(filename))


if __name__ == "__main__":
    main()
```

../src/__main__.py

4.1.2 src/lp_utils.py

```python
import functools
import inspect


def get_lp_filename(x, y, z):
    """ Returns the filename that the LP data should be saved to. """
    return "problem_{0}_{1}_{2}.lp".format(x, y, z)


def crange(first, last):
    """ Returns a list of characters between the two characters passed in (
   inclusive).
    >>> crange('A', 'C')
    ['A', 'B', 'C']
    >>> crange('A', 'A')
    ['A']
    """
    if ord(first) > ord(last):
        raise ValueError("last must come after first")

    else:
        return [chr(i) for i in range(ord(first), ord(last) + 1)]


def repeat(obj, n):
    """ Returns a list with obj repeated n times.
    >>> repeat(1, 1)
    [1]
    >>> repeat(42, 0)
    []
    >>> repeat(5, 4)
    [5, 5, 5, 5]
    >>> repeat([1, 2], 2)
    [[1, 2], [1, 2]]
    """
    return [obj for _ in range(n)]


def perms(lists):
    """ Returns all the permutations of the elements.
    >>> perms([])
    []
    >>> perms([['a', 'b', 'c']])
    [('a',), ('b',), ('c',)]
    >>> perms([['a', 'b', 'c'], ['x', 'y', 'z']])
    [('a', 'x'), ('a', 'y'), ('a', 'z'), ('b', 'x'), ('b', 'y'), ('b', 'z')
    , ('c', 'x'), ('c', 'y'), ('c', 'z')]
    """
    if len(lists) == 0:
        return []

    elif (len(lists) == 1):
        return [(x,) for x in lists[0]]

    else:
        return [(x,) + y for x in lists[0] for y in perms(lists[1:])]


def concat(permutations):
    """ Returns the permutations concatenated as strings.
    >>> concat(perms([['a', 'b', 'c']]))
    ['a', 'b', 'c']
    >>> concat(perms([['a', 'b', 'c'], ['x', 'y', 'z']]))
    ['ax', 'ay', 'az', 'bx', 'by', 'bz', 'cx', 'cy', 'cz']
    """
    return [functools.reduce(lambda x, y: x + str(y), p, '') for p in
    permutations]

def get_function_source(fn):
    src = inspect.getsource(fn)
    return src[str(src).index(':')+2:]

def get_lines(strings):
    return '\n\t'.join(strings)


if __name__ == "__main__":
    import doctest
    doctest.testmod()
```

../src/lp_utils.py

4.1.3 src/lp_gen.py

```
from lp_utils import perms, concat, get_lines, get_function_source

# Change these variables to alter the behaviour of the LP file generator
PATH_SPLIT = 2
DEMAND_FLOW = lambda i, j: 2 * i + j

TEMPLATE = """\
\\ {}, LP Output File
\\ Written by {}
\\ Parameters: X={}, Y={}, Z={}, Split={}, Demand={}

MINIMIZE
\tr

SUBJECT TO

\t\\ DEMAND CONSTRAINTS
\t{}

\t\\ CAPACITY CONSTRAINTS FOR LINKS BETWEEN SOURCE AND TRANSIT NODES
\t{}

\t\\ CAPACITY CONSTRAINTS FOR LINKS BETWEEN TRANSIT AND DESTINATION NODES
\t{}

\t\\ OBJECTIVE FUNCTION LOAD CONSTRAINTS
\t{}

\t\\ TRANSIT NODE LOAD CONSTRAINTS
\t{}

\t\\ BINARY VARIABLE AND DECISION VARIABLE CONSTRAINTS
\t{}

\t\\ BINARY VARIABLE CONSTRAINTS (ONLY 2 ACTIVE TRANSIT NODES)
\t{}

BOUNDS

\t\\ NON-NEGATIVITY CONSTRAINTS
\tr >= 0
\t{}

BIN

\t\\ BINARY VARIABLES
\t{}

END
"""
```

```python

def get_nodes(x, y, z):
    """ Returns a tuple containing the source, transit and destination node ids as integers. """
    s = list(range(1, x + 1))
    t = list(range(1, y + 1))
    d = list(range(1, z + 1))
    return s, t, d


def get_demand_constraints(s, t, d):
    """ Returns a list of demand constraints. """
    return [' + '.join(["x_{0}{1}{2}".format(i, k, j) for k in t]) + ' = {0}'.format(DEMAND_FLOW(i, j))
            for (i, j) in perms([s, d])]


def get_source_transit_capacity_constraints(s, t, d):
    """ Returns a list of capacity constraints for the links between the source and transit nodes. """
    return \
        [' + '.join(["x_{0}{1}{2}".format(i, k, j) for j in d]) +
            ' - c_{0}{1} = 0'.format(i, k) for (i, k) in perms([s, t])]


def get_transit_destination_capacity_constraints(s, t, d):
    """ Returns a list of capacity constraints for the links between the transit and destination nodes. """
    return \
        [' + '.join(["x_{0}{1}{2}".format(i, k, j) for i in s]) +
            ' - d_{0}{1} = 0'.format(k, j) for (k, j) in perms([t, d])]


def get_transit_load_constraints(s, t, d):
    """ Returns the list of transit load constraints. """
    return [' + '.join(["x_{0}{1}{2}".format(i, k, j) for (i, j) in perms([s, d])]) +
            ' - l_{0} = 0'.format(k) for k in t]

def get_objective_function_load_constraints(s, t, d):
    """ Returns the list of objective function load constraints. """
    return [' + '.join(["c_{0}{1}".format(i, j) for i in s]) +
            ' - r <= 0' for j in d]

def get_binary_and_decision_variable_constraints(s, t, d):
    """ Returns the binary and decision variable constraints. """
    return ['{3} x_{0}{1}{2} - {4} u_{0}{1}{2} = 0'.format(i, k, j, PATH_SPLIT, DEMAND_FLOW(i, j)) for (i, k, j) in perms([s, t, d])]


def get_binary_constraints(s, t, d):
    """ Returns a list of binary variable constraints. """
    return [' + '.join(["u_{0}{1}{2}".format(i, k, j) for k in t]) + ' = {}'.format(PATH_SPLIT)
            for (i, j) in perms([s, d])]


def get_binary_variables(s, t, d):
    """ Returns a list of binary variables. """
    return ["u_{0}{1}{2}".format(i, k, j) for (i, k, j) in perms([s, t, d])]


def get_non_negativity_constraints(s, t, d):
    """ Returns a list of non-negativity constraints. """
    return ["x_{0}{1}{2} >= 0".format(i, k, j) for (i, k, j) in perms([s, t, d])] + ["c_{0}{1} >= 0".format(i, k) for (i, k) in perms([s, t])] + ["d_{0}{1} >= 0".format(k, j) for (k, j) in perms([t, d])]

def generate_lp_file(title, authors, x, y, z):
    """ Returns the LP file contents as per the project specification. """
    s, t, d = get_nodes(x, y, z)

    demand_constraints = get_lines(get_demand_constraints(s, t, d))
    source_transit_capacity_constraints = get_lines(
        get_source_transit_capacity_constraints(s, t, d))
    transit_destination_capacity_constraints = get_lines(
        get_transit_destination_capacity_constraints(s, t, d))
    non_negativity_constraints = get_lines(get_non_negativity_constraints(
        s, t, d))
    objective_function_load_constraints = get_lines(get_objective_function_load_constraints(s, t, d))
    transit_load_constraints = get_lines(
        get_transit_load_constraints(s, t, d))
    binary_and_decision_constraints = get_lines(get_binary_and_decision_variable_constraints(s, t, d))
    binary_variable_constraints = get_lines(get_binary_constraints(s, t, d))
    binary_variables = get_lines(get_binary_variables(s, t, d))

    return TEMPLATE.format(
        title,
        authors,
        x,
        y,
        z,
        PATH_SPLIT,
        get_function_source(DEMAND_FLOW),
        demand_constraints,
        source_transit_capacity_constraints,
        transit_destination_capacity_constraints,
        objective_function_load_constraints,
        transit_load_constraints,
        binary_and_decision_constraints,
        binary_variable_constraints,
        non_negativity_constraints,
        binary_variables)
```

../src/lp_gen.py

4.2 Generated LP File

4.2.1 problem_3_2_4.lp

```
\ COSC-364 Assignment 2 LP Generator, LP Output File
\ Written by Will Cowper (81163265), Jesse Sheehan (53366509)
\ Parameters: X=3, Y=2, Z=4, Split=2, Demand=2 * i + j


MINIMIZE
  r

SUBJECT TO

  \ DEMAND CONSTRAINTS
  x_111 + x_121 = 3
  x_112 + x_122 = 4
  x_113 + x_123 = 5
  x_114 + x_124 = 6
  x_211 + x_221 = 5
  x_212 + x_222 = 6
  x_213 + x_223 = 7
  x_214 + x_224 = 8
  x_311 + x_321 = 7
  x_312 + x_322 = 8
  x_313 + x_323 = 9
  x_314 + x_324 = 10

  \ CAPACITY CONSTRAINTS FOR LINKS BETWEEN SOURCE AND TRANSIT NODES
  x_111 + x_112 + x_113 + x_114 - c_11 = 0
  x_121 + x_122 + x_123 + x_124 - c_12 = 0
  x_211 + x_212 + x_213 + x_214 - c_21 = 0
  x_221 + x_222 + x_223 + x_224 - c_22 = 0
  x_311 + x_312 + x_313 + x_314 - c_31 = 0
  x_321 + x_322 + x_323 + x_324 - c_32 = 0

  \ CAPACITY CONSTRAINTS FOR LINKS BETWEEN TRANSIT AND DESTINATION NODES
  x_111 + x_211 + x_311 - d_11 = 0
  x_112 + x_212 + x_312 - d_12 = 0
  x_113 + x_213 + x_313 - d_13 = 0
  x_114 + x_214 + x_314 - d_14 = 0
  x_121 + x_221 + x_321 - d_21 = 0
  x_122 + x_222 + x_322 - d_22 = 0
  x_123 + x_223 + x_323 - d_23 = 0
  x_124 + x_224 + x_324 - d_24 = 0

  \ OBJECTIVE FUNCTION LOAD CONSTRAINTS
  c_11 + c_21 + c_31 - r <= 0
  c_12 + c_22 + c_32 - r <= 0
  c_13 + c_23 + c_33 - r <= 0
  c_14 + c_24 + c_34 - r <= 0

  \ TRANSIT NODE LOAD CONSTRAINTS
  x_111 + x_112 + x_113 + x_114 + x_211 + x_212 + x_213 + x_214 + x_311 +
    x_312 + x_313 + x_314 - l_1 = 0
  x_121 + x_122 + x_123 + x_124 + x_221 + x_222 + x_223 + x_224 + x_321 +
    x_322 + x_323 + x_324 - l_2 = 0

  \ BINARY VARIABLE AND DECISION VARIABLE CONSTRAINTS
  2 x_111 - 3 u_111 = 0
  2 x_112 - 4 u_112 = 0
  2 x_113 - 5 u_113 = 0
  2 x_114 - 6 u_114 = 0
```

```
    2 x_121 - 3 u_121 = 0
    2 x_122 - 4 u_122 = 0
    2 x_123 - 5 u_123 = 0
    2 x_124 - 6 u_124 = 0
    2 x_211 - 5 u_211 = 0
    2 x_212 - 6 u_212 = 0
    2 x_213 - 7 u_213 = 0
    2 x_214 - 8 u_214 = 0
    2 x_221 - 5 u_221 = 0
    2 x_222 - 6 u_222 = 0
    2 x_223 - 7 u_223 = 0
    2 x_224 - 8 u_224 = 0
    2 x_311 - 7 u_311 = 0
    2 x_312 - 8 u_312 = 0
    2 x_313 - 9 u_313 = 0
    2 x_314 - 10 u_314 = 0
    2 x_321 - 7 u_321 = 0
    2 x_322 - 8 u_322 = 0
    2 x_323 - 9 u_323 = 0
    2 x_324 - 10 u_324 = 0

    \ BINARY VARIABLE CONSTRAINTS (ONLY 2 ACTIVE TRANSIT NODES)
    u_111 + u_121 = 2
    u_112 + u_122 = 2
    u_113 + u_123 = 2
    u_114 + u_124 = 2
    u_211 + u_221 = 2
    u_212 + u_222 = 2
    u_213 + u_223 = 2
    u_214 + u_224 = 2
    u_311 + u_321 = 2
    u_312 + u_322 = 2
    u_313 + u_323 = 2
    u_314 + u_324 = 2

BOUNDS

    \ NON-NEGATIVITY CONSTRAINTS
    r >= 0
    x_111 >= 0
    x_112 >= 0
    x_113 >= 0
    x_114 >= 0
    x_121 >= 0
    x_122 >= 0
    x_123 >= 0
    x_124 >= 0
    x_211 >= 0
    x_212 >= 0
    x_213 >= 0
    x_214 >= 0
    x_221 >= 0
    x_222 >= 0
    x_223 >= 0
    x_224 >= 0
    x_311 >= 0
    x_312 >= 0
    x_313 >= 0
    x_314 >= 0
    x_321 >= 0
    x_322 >= 0
    x_323 >= 0
    x_324 >= 0
    c_11 >= 0
    c_12 >= 0
    c_21 >= 0
    c_22 >= 0
    c_31 >= 0
    c_32 >= 0
    d_11 >= 0
    d_12 >= 0
    d_13 >= 0
    d_14 >= 0
    d_21 >= 0
    d_22 >= 0
    d_23 >= 0
    d_24 >= 0

BIN

    \ BINARY VARIABLES
    u_111
    u_112
    u_113
    u_114
    u_121
    u_122
    u_123
    u_124
    u_211
    u_212
    u_213
    u_214
    u_221
    u_222
    u_223
    u_224
    u_311
    u_312
    u_313
    u_314
    u_321
    u_322
    u_323
    u_324

END
```

../problem_3_2_4.lp

4.3 Plagiarism Declaration