

# FLOW PLANNING

## ASSIGNMENT 2

COSC364-19S1 INTERNET TECHNOLOGY AND ENGINEERING

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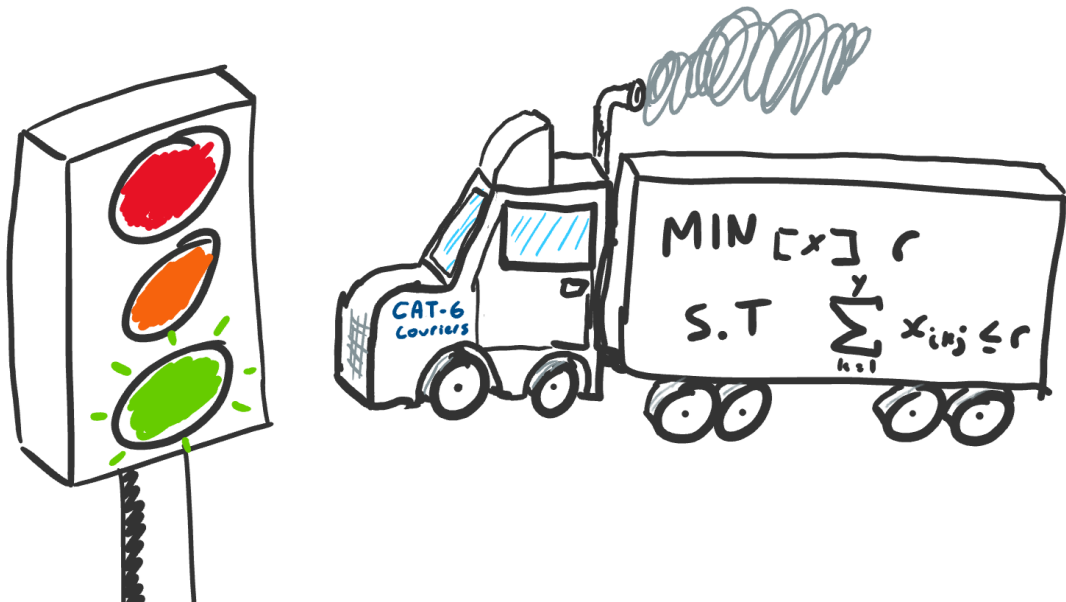


Figure 1: Traffic problems are not unique to computer networks.

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29-5-19      29/5/19

# 1 Problem Description

Given a network (figure 2) with  $X$  source nodes,  $Y$  transit nodes and  $Z$  destination nodes, a program was designed to generate an LP file that could be used by CPLEX to determine certain network characteristics.

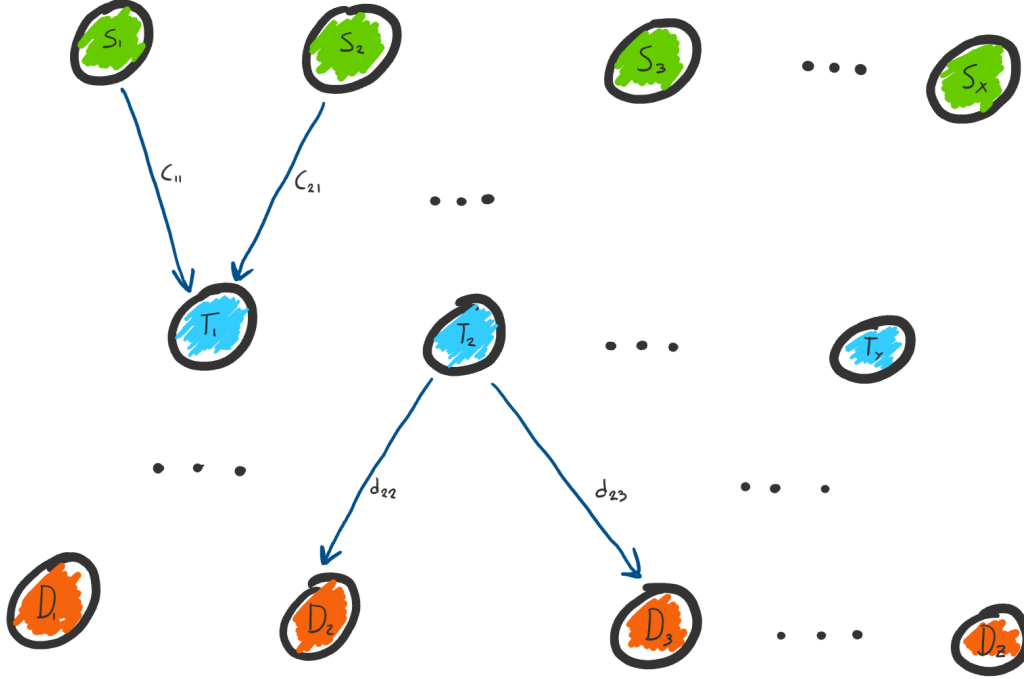


Figure 2: An example network showing nodes  $S_i$ ,  $T_k$  and  $D_j$  and links  $c_{ik}$  and  $d_{kj}$ ,  $i \in \{1, \dots, X\}, k \in \{1, \dots, Y\}, j \in \{1, \dots, Z\}$ .

Traffic travelling from  $S_i$  to  $D_j$  must travel through exactly 2 transit nodes with a total demand volume of  $h_{ij}$  (equation 10). Furthermore, the load upon each transit node must be balanced.

## 2 Problem Formulation

This problem was solved with the use of binary variable constraints (equations 6, 7 and 9) and the minimisation of our objective function (equation 1). All normal non-negativity constraints were applied (equations 11, 12, 13 and 14).

The following network properties were solved for:

- The capacities of each link (equations 3 and 4).
- The load on each transit node (equation 5).
- The value of each flow (equations 2 and 8).

**Notation:**

- $X$  is the number of source nodes.
- $Y$  is the number of transit nodes.
- $Z$  is the number of destination nodes.
- $S_i$  is the  $i$ th source node.
- $T_k$  is the  $k$ th transit node.
- $D_j$  is the  $j$ th destination node.
- $h_{ij}$  is the demand flow between  $S_i$  and  $D_j$ . This is equal to  $2i + j$ .
- $c_{ik}$  is the link capacity between  $S_i$  and  $T_k$ .
- $d_{kj}$  is the link capacity between  $T_k$  and  $D_j$ .
- $x_{ikj}$  is the decision variable associated with the path  $S_i$ - $T_k$ - $D_j$ .
- $u_{ikj}$  is the binary decision variable associated with  $x_{ikj}$ . These are required because  $h_{ij}$  must be split across exactly 2 transit nodes.
- $l_k$  is the load on  $T_k$ .

**Note:** Due to the limitations of the LP file format, many of the following equations must be rearranged for use in CPLEX. Most notably, there cannot be any variables on the right hand side of an equality or inequality.

Python, BASH and PowerShell scripts (section 4.1) were used to automate the process (figure 3) of generating the LP files, analysing the CPLEX data, and producing graphs.

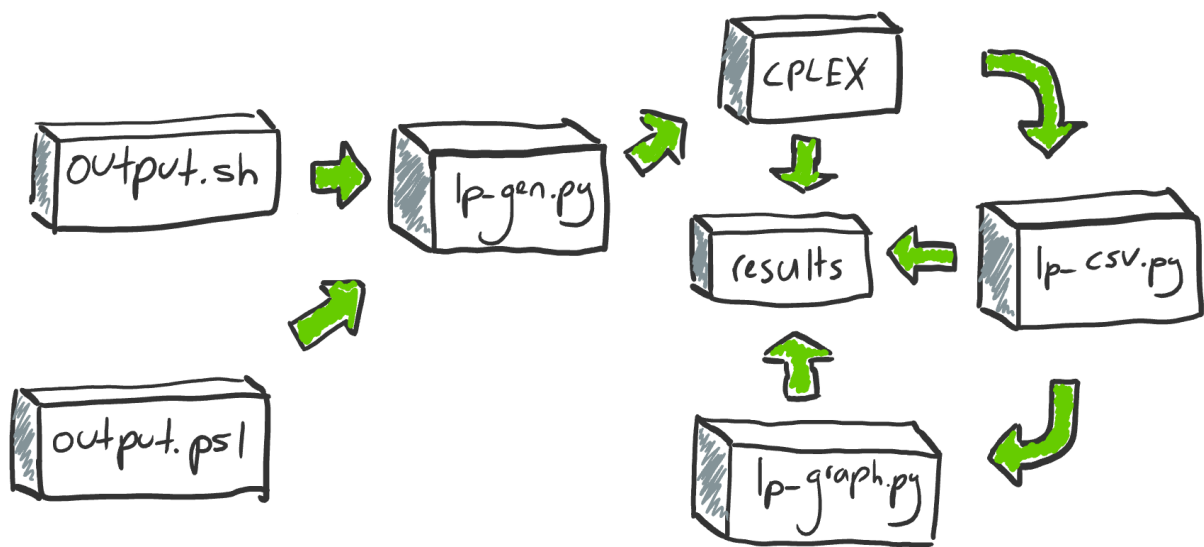


Figure 3: A graphical interpretation of the script execution used.

## 2.1 Objective Function

$$\text{minimize}_{[x,c,d,r]} r \quad (1)$$

## 2.2 Constraints

$$\sum_{k=1}^Y x_{ikj} = h_{ij} \quad i \in \{1, \dots, X\}, j \in \{1, \dots, Z\} \quad (2)$$

$$\sum_{j=1}^Z x_{ikj} = c_{ik} \quad i \in \{1, \dots, X\}, k \in \{1, \dots, Y\} \quad (3)$$

$$\sum_{i=1}^X x_{ikj} = d_{kj} \quad k \in \{1, \dots, Y\}, j \in \{1, \dots, Z\} \quad (4)$$

$$\sum_{k=1}^Y x_{ikj} = l_k \quad i \in \{1, \dots, X\}, j \in \{1, \dots, Z\} \quad (5)$$

$$\sum_{k=1}^Y u_{ikj} = 2 \quad i \in \{1, \dots, X\}, j \in \{1, \dots, Z\} \quad (6)$$

$$x_{ikj} = \frac{u_{ikj} h_{ij}}{2} \quad i \in \{1, \dots, X\}, k \in \{1, \dots, Y\}, j \in \{1, \dots, Z\} \quad (7)$$

$$\sum_{i=1}^X \sum_{j=1}^Z x_{ikj} \leq r \quad k \in \{1, \dots, Y\} \quad (8)$$

$$u_{ikj} \in \{0, 1\} \quad i \in \{1, \dots, X\}, k \in \{1, \dots, Y\}, j \in \{1, \dots, Z\} \quad (9)$$

$$h_{ij} = 2i + j \quad i \in \{1, \dots, X\}, j \in \{1, \dots, Z\} \quad (10)$$

## 2.3 Non-Negativity Constraints

$$r \geq 0 \quad (11)$$

$$x_{ikj} \geq 0 \quad i \in \{1, \dots, X\}, k \in \{1, \dots, Y\}, j \in \{1, \dots, Z\} \quad (12)$$

$$c_{ik} \geq 0 \quad i \in \{1, \dots, X\}, k \in \{1, \dots, Y\} \quad (13)$$

$$d_{kj} \geq 0 \quad k \in \{1, \dots, Y\}, j \in \{1, \dots, Z\} \quad (14)$$

### 3 Results

LP files were generated with parameters  $X = Z = 9, Y \in \{3, 4, 5, 6, 7, 8\}$ . These were then processed with CPLEX, recording the time taken to solve each problem. Important data points were extracted from the CPLEX output and are listed in table 1.

Table 1: The raw data as extracted and processed from the CPLEX output.

Y	Time (ms)	Links	Load Spread	Max. $c_{ik}$	Max. $d_{kj}$
3	81.6132	52	0.0	94.500000	73.500000
4	117.3323	68	0.5	8.500000	9.500000
5	285.8704	83	0.0	9.500000	9.500000
6	443.1351	101	0.0	83.000000	9.500000
7	223.422	118	1.5	9.500000	9.500000
8	987.7775	134	0.5	9.500000	9.500000

An analysis of these results confirms many assumptions that were made about the problem. The number of non-zero link capacities increases linearly (figure 5), the transit node load spread is very close to (if not exactly) zero for most networks (figure 6), and the amount of time taken to solve the problem increases (almost) non-linearly as the number of transit nodes increases (figure 4). It is important to note that the load on the transit nodes was only able to be balanced in three of the networks, however the load spread on the remaining three networks is very low. CPLEX has done its best to equalise the loads on each transit node but could not balance them in some cases due to the demand flows and the number of nodes.

The most obvious feature of the results is the data for the  $Y = 7$  network. It appears to be an outlier as it takes less time to solve than the  $Y = 5$  and  $Y = 6$  networks and has the highest transit node load spread by a factor of 3. **Explain this behaviour!!!**

The second odd feature of the data is the greatest link capacities of the  $Y = 3$  and  $Y = 6$  networks (figure 7). The exact reason behind this behaviour is unknown but it would have something to do with these specific combinations of  $X$ ,  $Y$  and  $Z$  not being relatively prime.

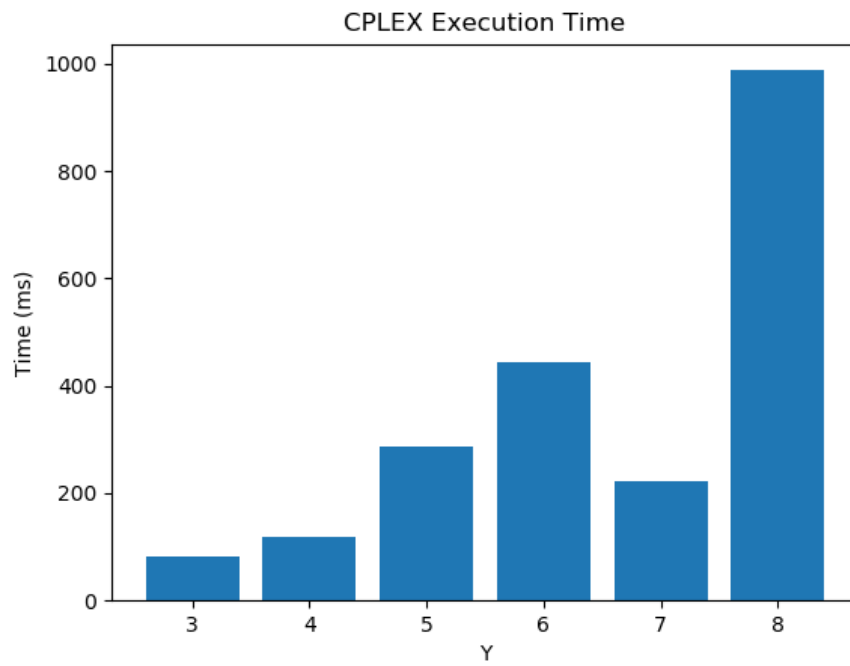


Figure 4: The time taken to execute the LP file in CPLEX for each network.

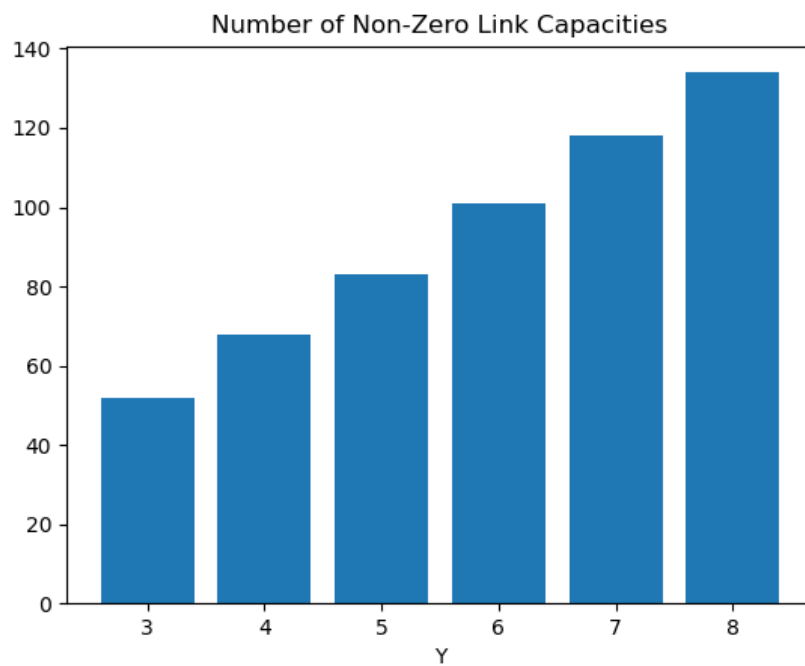


Figure 5: The number of non-zero link capacities in each network.

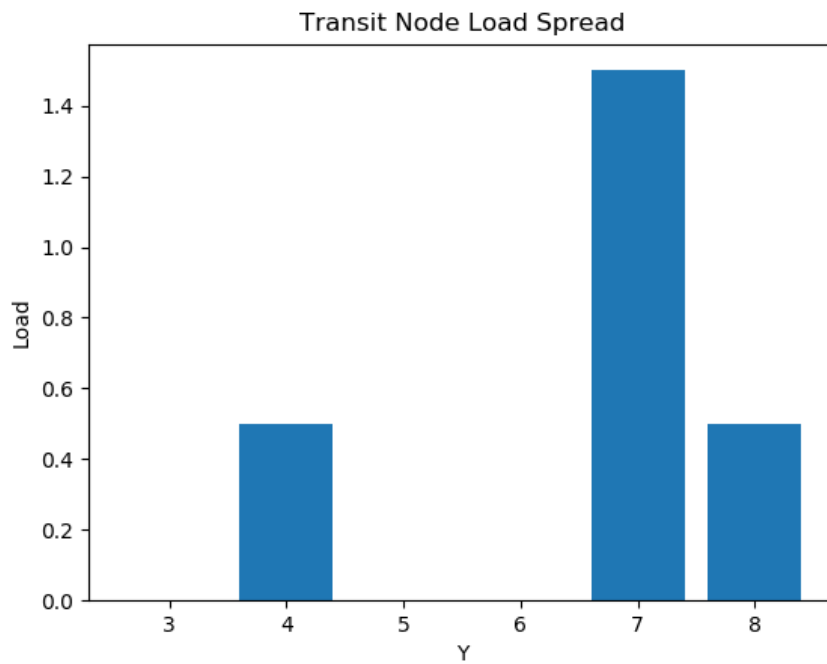


Figure 6: The amount of spread in the load for all transit nodes in each network.

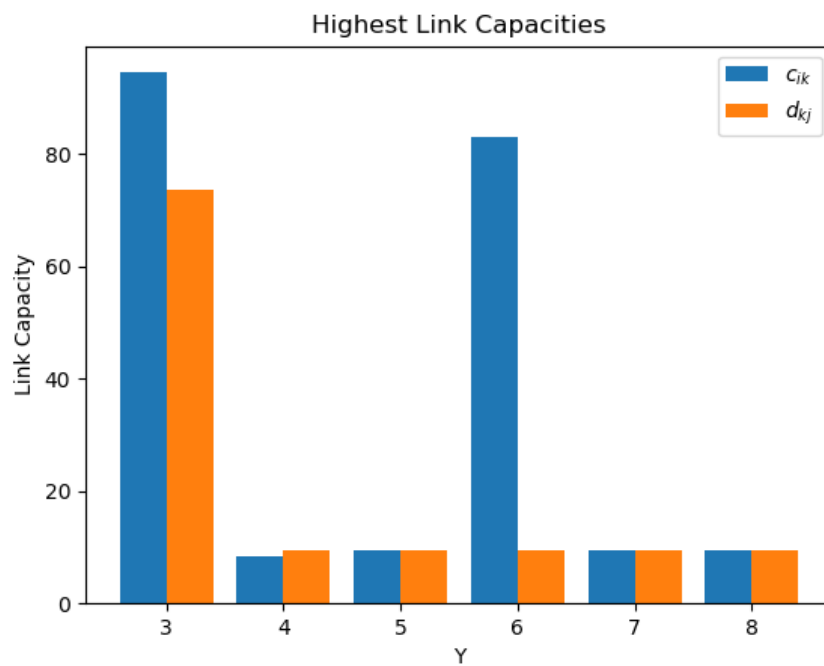


Figure 7: The highest capacity links for each network. Both the  $c_{ik}$  and  $d_{kj}$  links are listed.



## 4 Appendix

### 4.1 Source Code

#### 4.1.1 src/lp\_gen.py

This script is responsible for producing a valid LP file from the given command line parameters.

```

1 # lp_gen.py
2 #
3 # COSC364 Assignment 2
4 # 30/05/2019
5 # Written by Will Cowper, Jesse Sheehan
6
7 import inspect
8 import functools
9 import sys
10 import os.path
11
12 __TITLE__ = "COSC-364 Assignment 2 LP Generator"
13 __AUTHORS__ = [("Will Cowper", "81163265"), ("Jesse Sheehan", "53366509")]
14
15 # Change these variables to alter the behaviour of the LP file generator
16 PATH_SPLIT = 2
17
18
19 def DEMANDFLOW(i, j): return 2 * i + j
20
21
22 TEMPLATE = """\
23 \ \ {}, LP Output File
24 \ \ Written by {}
25 \ \ Parameters: X={}, Y={}, Z={}, Split={}, Demand={}
26
27 MINIMIZE
28 \ tr
29
30 SUBJECT TO
31
32 \ t \ \ DEMAND CONSTRAINTS
33 \ t {}
34
35 \ t \ \ CAPACITY CONSTRAINTS FOR LINKS BETWEEN SOURCE AND TRANSIT NODES
36 \ t {}
37
38 \ t \ \ CAPACITY CONSTRAINTS FOR LINKS BETWEEN TRANSIT AND DESTINATION NODES
39 \ t {}
40
41 \ t \ \ OBJECTIVE FUNCTION LOAD CONSTRAINTS
42 \ t {}
43
44 \ t \ \ TRANSIT NODE LOAD CONSTRAINTS
45 \ t {}
46
47 \ t \ \ BINARY VARIABLE AND DECISION VARIABLE CONSTRAINTS
48 \ t {}

```

```

50 \t\\ BINARY VARIABLE CONSTRAINTS (ONLY 2 ACTIVE TRANSIT NODES)
   \t{}
52
   BOUNDS
54
   \t\\ NON-NEGATIVITY CONSTRAINTS
56 \tr >= 0
   \t{}
58
   BIN
60
   \t\\ BINARY VARIABLES
62 \t{}
64 END
   """
66
   # DEFINE SOME UTILITY FUNCTIONS
68
70 def get_lp_filename(x, y, z):
   """ Returns the filename that the LP data should be saved to. """
72     return "problem-{{0}}-{{1}}-{{2}}.lp".format(x, y, z)
74
76 def crange(first, last):
   """ Returns a list of characters between the two characters passed in (
   inclusive).
   >>> crange('A', 'C')
78     ['A', 'B', 'C']
   >>> crange('A', 'A')
80     ['A']
   """
82     if ord(first) > ord(last):
         raise ValueError("last must come after first")
84
86     else:
         return [chr(i) for i in range(ord(first), ord(last) + 1)]
88
90 def repeat(obj, n):
   """ Returns a list with obj repeated n times.
   >>> repeat(1, 1)
92     [1]
   >>> repeat(42, 0)
94     []
   >>> repeat(5, 4)
96     [5, 5, 5, 5]
   >>> repeat([1, 2], 2)
98     [[1, 2], [1, 2]]
   """
100     return [obj for _ in range(n)]
102
104 def perms(lists):
   """ Returns all the permutations of the elements.
   >>> perms([])
106     []

```

```

108 >>> perms([[ 'a', 'b', 'c ']])
110 [( 'a',), ( 'b',), ( 'c',)]
112 >>> perms([[ 'a', 'b', 'c '], [ 'x', 'y', 'z ']])
114 [( 'a', 'x'), ( 'a', 'y'), ( 'a', 'z'), ( 'b', 'x'), ( 'b', 'y'), ( 'b', 'z')
116 , ( 'c', 'x'), ( 'c', 'y'), ( 'c', 'z')]
118 """
120 if len(lists) == 0:
122     return []
124
126 elif (len(lists) == 1):
128     return [(x,) for x in lists[0]]
130
132 else:
134     return [(x,) + y for x in lists[0] for y in perms(lists[1:])]
136
138 def concat(permutations):
140     """ Returns the permutations concatenated as strings.
142     >>> concat(perms([[ 'a', 'b', 'c ']]))
144     [ 'a', 'b', 'c ' ]
146     >>> concat(perms([[ 'a', 'b', 'c '], [ 'x', 'y', 'z ']]))
148     [ 'ax', 'ay', 'az', 'bx', 'by', 'bz', 'cx', 'cy', 'cz' ]
150     """
152     return [functools.reduce(lambda x, y: x + str(y), p, '') for p in
154             permutations]
156
158 def get_function_source(fn):
160     src = inspect.getsource(fn)
162     return src[ str(src).index('return')+7:]
164
166 def get_lines(strings):
168     return '\n\t'.join(strings)
170
172 # DEFINE SOME FUNCTIONS SPECIFIC TO THE PROBLEM
174
176 def get_nodes(x, y, z):
178     """ Returns a tuple containing the source, transit and destination node
180     ids as integers. """
182     s = list(range(1, x + 1))
184     t = list(range(1, y + 1))
186     d = list(range(1, z + 1))
188     return s, t, d
190
192 def get_demand_constraints(s, t, d):
194     """ Returns a list of demand constraints. """
196     return [ ' + '.join(["x_{0}{1}{2}".format(i, k, j) for k in t]) + ' =
198 {0}'.format(DEMANDFLOW(i, j))
200             for (i, j) in perms([s, d])]
202
204 def get_source_transit_capacity_constraints(s, t, d):
206     """ Returns a list of capacity constraints for the links between the
208     source and transit nodes. """
210     return \

```

```

160         [ ' + '.join(["x-{}{}{}2".format(i, k, j) for j in d]) +
162             ' - c-{}{}1 = 0'.format(i, k) for (i, k) in perms([s, t])]

164 def get_transit_destination_capacity_constraints(s, t, d):
165     """ Returns a list of capacity constraints for the links between the
166     transit and destination nodes. """
167     return \
168         [ ' + '.join(["x-{}{}{}2".format(i, k, j) for i in s]) +
169             ' - d-{}{}1 = 0'.format(k, j) for (k, j) in perms([t, d])]

170 def get_transit_load_constraints(s, t, d):
171     """ Returns the list of transit load constraints. """
172     return [ ' + '.join(["x-{}{}{}2".format(i, k, j) for (i, j) in perms([
173         s, d])]) +
174             ' - l-{} = 0'.format(k) for k in t]

176 def get_objective_function_load_constraints(s, t, d):
177     """ Returns the list of objective function load constraints. """
178     return [ ' + '.join(["c-{}{}1".format(i, j) for i in s]) +
179             ' - r <= 0' for j in d]

182 def get_binary_and_decision_variable_constraints(s, t, d):
183     """ Returns the binary and decision variable constraints. """
184     return [ '{3} x-{}{}{}2 - {4} u-{}{}{}2 = 0'.format(i, k, j,
185         PATH_SPLIT, DEMANDFLOW(i, j)) for (i, k, j) in perms([s, t, d])]

186
188 def get_binary_constraints(s, t, d):
189     """ Returns a list of binary variable constraints. """
190     return [ ' + '.join(["u-{}{}{}2".format(i, k, j) for k in t]) + ' = {}
191     '.format(PATH_SPLIT)
192         for (i, j) in perms([s, d])]

194 def get_binary_variables(s, t, d):
195     """ Returns a list of binary variables. """
196     return ["u-{}{}{}2".format(i, k, j) for (i, k, j) in perms([s, t, d])
197 ]

198
200 def get_non_negativity_constraints(s, t, d):
201     """ Returns a list of non-negativity constraints. """
202     return ["x-{}{}{}2 >= 0".format(i, k, j) for (i, k, j) in perms([s, t,
203         d])] + ["c-{}{}1 >= 0".format(i, k) for (i, k) in perms([s, t])] + ["
204         d-{}{}1 >= 0".format(k, j) for (k, j) in perms([t, d])]

206
208 def generate_lp_file(title, authors, x, y, z):
209     """ Returns the LP file contents as per the project specification. """
210     s, t, d = get_nodes(x, y, z)

211     demand_constraints = get_lines(get_demand_constraints(s, t, d))
212     source_transit_capacity_constraints = get_lines(
213         get_source_transit_capacity_constraints(s, t, d))

```

```

transit_destination_capacity_constraints = get_lines(
    get_transit_destination_capacity_constraints(s, t, d))
non_negativity_constraints = get_lines(get_non_negativity_constraints(
    s, t, d))
objective_function_load_constraints = get_lines(
    get_objective_function_load_constraints(s, t, d))
transit_load_constraints = get_lines(
    get_transit_load_constraints(s, t, d))
binary_and_decision_constraints = get_lines(
    get_binary_and_decision_variable_constraints(s, t, d))
binary_variable_constraints = get_lines(get_binary_constraints(s, t, d)
)
binary_variables = get_lines(get_binary_variables(s, t, d))

return TEMPLATE.format(
    title ,
    authors ,
    x,
    y,
    z,
    PATH_SPLIT,
    get_function_source(DEMAND_FLOW) ,
    demand_constraints ,
    source_transit_capacity_constraints ,
    transit_destination_capacity_constraints ,
    objective_function_load_constraints ,
    transit_load_constraints ,
    binary_and_decision_constraints ,
    binary_variable_constraints ,
    non_negativity_constraints ,
    binary_variables)

# DEFINE SOME HELPERS FOR GETTING THE THING RUNNING

def print_version():
    print( '{0} by {1}'.format(__TITLE__, get_author_string()))

def print_usage():
    print( 'Usage: {0} <x> <y> <z> [output directory]'.format(sys.argv[0]))

def get_problem_parameters():
    """ Returns a tuple containing the x, y and z parameters. """
    try:
        x = int(sys.argv[1])
        y = int(sys.argv[2])
        z = int(sys.argv[3])
    except:
        print_usage()
        exit(-1)

    if x <= 0:
        print("Error: x must be strictly positive")
        exit(-1)

    if x >= 10:

```

```

268     print("Error: x must be less than ten")
269     exit(-1)
270
271     if y <= 0:
272         print("Error: y must be strictly positive")
273         exit(-1)
274
275     if y >= 10:
276         print("Error: y must be less than ten")
277         exit(-1)
278
279     if z <= 0:
280         print("Error: z must be strictly positive")
281         exit(-1)
282
283     if z >= 10:
284         print("Error: z must be less than ten")
285         exit(-1)
286
287     return x, y, z
288
289
290 def save_lp_file(filename, data):
291     try:
292         f = open(filename, 'w')
293         f.write(data)
294         f.close()
295     except:
296         print("Error: could not save file '{0}'".format(filename))
297         exit(-1)
298
299
300 def get_author_string():
301     return ', '.join(
302         ["{0} ({1})".format(name, sid) for (name, sid) in __AUTHORS__])
303
304
305 def main():
306     print_version()
307     if len(sys.argv) != 4 and len(sys.argv) != 5:
308         print_usage()
309         exit(-1)
310     else:
311         output_dir = '.'
312         if len(sys.argv) == 5:
313             output_dir = sys.argv[4]
314
315         x, y, z = get_problem_parameters()
316         data = generate_lp_file(__TITLE__, get_author_string(), x, y, z)
317         filename = os.path.join(output_dir, get_lp_filename(x, y, z))
318         save_lp_file(filename, data)
319         print("Success: saved as '{0}'".format(filename))
320
321
322 if __name__ == "__main__":
323     main()

```

../src/lp-gen.py

#### 4.1.2 src/lp\_csv.py

This script is responsible for converting the output of the CPLEX log files into a single CSV file for further processing.

```

# lp_csv.py
#
# COSC364 Assignment 2
# 30/05/2019
# Written by Will Cowper, Jesse Sheehan

import csv
import sys
import os.path

def csvWrite(data):
    with open(sys.argv[2], 'a', newline='') as csvFile:
        writer = csv.writer(csvFile)
        writer.writerow(data)

def floatmap(enumerable):
    return list(map(lambda x: float(x), enumerable))

def openFile(Y):
    with open(os.path.join(sys.argv[1], '{0}.txt'.format(Y)), 'r') as in_file:
        stripped = [line.strip() for line in in_file.readlines()]
        lines = [line for line in stripped if line]
        data = []
        # Y
        data.append(Y)
        # elapsed time
        data.append(max(parseFile("elapsed_", lines)))
        # no of non-zero c and d links
        data.append(len(parseFile("c_", lines)) + len(parseFile("d_", lines)))
        # transit load spread (largest_transit_node_load -
        # smallest_transit_node_load)
        data.append(max(floatmap(parseFile("l_", lines))) -
                     min(floatmap(parseFile("l_", lines))))
        # highest cap c network
        data.append(max(parseFile("c_", lines)))
        # highest cap d network
        data.append(max(parseFile("d_", lines)))
        csvWrite(data)

'''Returns a list of all values that start with the given string'''

def parseFile(string, lines):
    values = []
    for line in lines:
        if line.startswith(string):
            values.append(line.split()[1])

```

```
52     return values
54
56 if __name__ == "__main__":
57     if len(sys.argv) != 3:
58         print("Usage: {0} <input directory> <csv file>".format(sys.argv[0]))
59     )
60     exit(-1)
61
62     # delete the CSV, otherwise we will append to it
63     os.unlink(sys.argv[2])
64
65     openFile(3)
66     openFile(4)
67     openFile(5)
68     openFile(6)
69     openFile(7)
70     openFile(8)
71
72     print("Saved CSV data to '{0}'".format(sys.argv[2]))
```

../src/lp\_csv.py

#### 4.1.3 src/lp\_graph.py

This script is responsible for reading the CSV file and producing several graphs.

```
# lp_graph.py
2 #
3 # COSC364 Assignment 2
4 # 30/05/2019
5 # Written by Will Cowper, Jesse Sheehan
6
7 import csv
8 import sys
9 import os.path
10
11 try:
12     import numpy as np
13 except:
14     print("Error: could not load 'numpy'. Install with 'pip install numpy'
15           and then try again.")
16     exit(-1)
17
18 try:
19     import matplotlib.pyplot as plt
20 except:
21     print("Error: could not load 'matplotlib'. Install with 'pip install
22           matplotlib' and then try again.")
23     exit(-1)
24
25 def get_data(data, key):
26     return list(map(lambda d: d[key], data))
27
28
```



```

30 def get_time(data):
    return get_data(data, "time")

32
34 def get_len_nonzero_links(data):
    return get_data(data, "len_links")

36
38 def get_transit_load_spread(data):
    return get_data(data, "load_spread")

40
42 def get_max_cap_c(data):
    return get_data(data, "max_cap_c")

44
46 def get_max_cap_d(data):
    return get_data(data, "max_cap_d")

48
50 def get_Y(data):
    return get_data(data, "Y")

52
54 def save_execution_time_plot(filename, data):
    """ Saves a plot of execution time. """
    plt.bar(get_Y(data), get_time(data))
    plt.xlabel("Y")
    plt.ylabel("Time (ms)")
    plt.title("CPLEX Execution Time")
    plt.savefig(filename)
    plt.close()
    print("Saved '{}'".format(filename))

62
64 def save_num_nonzero_links_plot(filename, data):
    """ Saves a plot of the number of non-zero links. """
    plt.bar(get_Y(data), get_len_nonzero_links(data))
    plt.xlabel("Y")
    plt.ylabel("")
    plt.title("Number of Non-Zero Link Capacities")
    plt.savefig(filename)
    plt.close()
    print("Saved '{}'".format(filename))

74
76 def save_transit_load_spread_plot(filename, data):
    """ Saves a plot of the transit load spread. """
    plt.bar(get_Y(data), get_transit_load_spread(data))
    plt.xlabel("Y")
    plt.ylabel("Load")
    plt.title("Transit Node Load Spread")
    plt.savefig(filename)
    plt.close()
    print("Saved '{}'".format(filename))

84
86 def save_highest_capacity_links_plot(filename, data):

```

```

88     """ Saves a plot of the transit load spread. """
width = 0.4
Ys = np.array(get_Y(data))
90     cs = plt.bar(Ys, get_max_cap_c(data), width, label="$c_{ik}$")
ds = plt.bar(Ys + width, get_max_cap_d(data), width, label="$d_{kj}$")
92     plt.xticks(Ys + width / 2, map(lambda x: int(x), Ys))
plt.legend(handles=[cs, ds])
94     plt.xlabel("Y")
plt.ylabel("Link Capacity")
96     plt.title("Highest Link Capacities")
plt.savefig(filename)
98     plt.close()
print("Saved '{}'".format(filename))
100
102 def get_data_from_csv(csv_filename):
    """ Returns an array of dictionaries containing the CSV data. """
104     with open(csv_filename, newline='') as csv_file:
        csv_reader = csv.DictReader(csv_file, fieldnames=[
106                                     "Y", "time", "len_links", "load_spread"
, "max_cap_c", "max_cap_d"])
        rows = []
108         for row in csv_reader:
            d = {}
110             for key in row:
                d[key] = float(row[key])
112             rows.append(d)
        return rows
114
116 def convert_csv_to_images(csv_filename, output_folder):
    """ Converts the data from the CSV into a set of graphs. """
118     data = get_data_from_csv(csv_filename)
    base_filename = os.path.splitext(os.path.join(
120         output_folder, os.path.basename(csv_filename)))[0]

    save_execution_time_plot(base_filename + "_time.png", data)
    save_num_nonzero_links_plot(base_filename + "_num_nonzero_links.png",
122     data)
    save_transit_load_spread_plot(
124         base_filename + "_transit_load_spread.png", data)
    save_highest_capacity_links_plot(
126         base_filename + "_highest_capacity_links.png", data)
128
130 def print_usage():
    print("Usage: {0} <csv file> <output folder>")
132
134 if __name__ == "__main__":
    if len(sys.argv) != 3:
136         print_usage()
        exit(-1)
138
    convert_csv_to_images(sys.argv[1], sys.argv[2])

```

../src/lp-graph.py

#### 4.1.4 output.sh

This BASH script is responsible for executing the other scripts as well as timing and running CPLEX (under the Linux operating system).

```
#!/bin/bash
2 for y in 3 4 5 6 7 8
do
4     python3 src/lp_gen.py 9 $y 9 lp_files
      start=$(date +%s%N)
6     cplex -c "read lp_files/problem_9-{$y}_9.lp" "optimize" "display
      solution variables -" > cplex_logs/$y.txt
      end=$(date +%s%N)
8     duration=$(expr $end - $start)
      duration=$(expr $duration / 1000000)
10    echo -e "\nelapsed_time: $duration ms" >> cplex_logs/$y.txt
done
12 python3 src/lp_csv.py cplex_logs lp_files/cplex_data.csv
14 python3 src/lp_graph.py lp_files/cplex_data.csv graphs
```

../output.sh

#### 4.1.5 output.ps1

This PowerShell script is responsible for executing the other scripts as well as timing and running CPLEX (under the Windows operating system).

```
For ($i=3; $i -le 8; $i++) {
2     python src/lp_gen.py 9 $i 9 lp_files
      $perf = Measure-Command -Expression {$data = cplex -c ("read lp_files/
      problem_9-" + $i + "_9.lp") "optimize" "display solution variables -"}
4     $ms = $perf.TotalMilliseconds
      [System.IO.File]::WriteAllLines("cplex_logs/$i.txt", $data + " '
      elapsed_time: $ms ms")
6 }
8 python src/lp_csv.py cplex_logs lp_files/cplex_data.csv
python src/lp_graph.py lp_files/cplex_data.csv graphs
```

../output.ps1

## 4.2 Generated LP File

### 4.2.1 lp\_files/problem\_3.2.4.lp

```

\ COSC-364 Assignment 2 LP Generator , LP Output File
2 \ Written by Will Cowper (81163265), Jesse Sheehan (53366509)
\ Parameters: X=3, Y=2, Z=4, Split=2, Demand=2 * i + j
4
6 MINIMIZE
   r
8
10 SUBJECT TO
   \ DEMAND CONSTRAINTS
12 x_111 + x_121 = 3
14 x_112 + x_122 = 4
16 x_113 + x_123 = 5
18 x_114 + x_124 = 6
20 x_211 + x_221 = 5
22 x_212 + x_222 = 6
24 x_213 + x_223 = 7
26 x_214 + x_224 = 8
28 x_311 + x_321 = 7
30 x_312 + x_322 = 8
32 x_313 + x_323 = 9
34 x_314 + x_324 = 10
36
38 \ CAPACITY CONSTRAINTS FOR LINKS BETWEEN SOURCE AND TRANSIT NODES
40 x_111 + x_112 + x_113 + x_114 - c_11 = 0
42 x_121 + x_122 + x_123 + x_124 - c_12 = 0
44 x_211 + x_212 + x_213 + x_214 - c_21 = 0
46 x_221 + x_222 + x_223 + x_224 - c_22 = 0
48 x_311 + x_312 + x_313 + x_314 - c_31 = 0
50 x_321 + x_322 + x_323 + x_324 - c_32 = 0
52
54 \ CAPACITY CONSTRAINTS FOR LINKS BETWEEN TRANSIT AND DESTINATION NODES
56 x_111 + x_211 + x_311 - d_11 = 0
58 x_112 + x_212 + x_312 - d_12 = 0
60 x_113 + x_213 + x_313 - d_13 = 0
62 x_114 + x_214 + x_314 - d_14 = 0
64 x_121 + x_221 + x_321 - d_21 = 0
66 x_122 + x_222 + x_322 - d_22 = 0
68 x_123 + x_223 + x_323 - d_23 = 0
70 x_124 + x_224 + x_324 - d_24 = 0
72
74 \ OBJECTIVE FUNCTION LOAD CONSTRAINTS
76 c_11 + c_21 + c_31 - r <= 0
78 c_12 + c_22 + c_32 - r <= 0
80 c_13 + c_23 + c_33 - r <= 0
82 c_14 + c_24 + c_34 - r <= 0
84
86 \ TRANSIT NODE LOAD CONSTRAINTS
88 x_111 + x_112 + x_113 + x_114 + x_211 + x_212 + x_213 + x_214 + x_311 +
   x_312 + x_313 + x_314 - l_1 = 0
90 x_121 + x_122 + x_123 + x_124 + x_221 + x_222 + x_223 + x_224 + x_321 +
   x_322 + x_323 + x_324 - l_2 = 0
92

```

---

\ BINARY VARIABLE AND DECISION VARIABLE CONSTRAINTS

$$2 \ x_{.111} - 3 \ u_{.111} = 0$$

$$2 \ x_{.112} - 4 \ u_{.112} = 0$$

$$2 \ x_{.113} - 5 \ u_{.113} = 0$$

$$2 \ x_{.114} - 6 \ u_{.114} = 0$$

$$2 \ x_{.121} - 3 \ u_{.121} = 0$$

$$2 \ x_{.122} - 4 \ u_{.122} = 0$$

$$2 \ x_{.123} - 5 \ u_{.123} = 0$$

$$2 \ x_{.124} - 6 \ u_{.124} = 0$$

$$2 \ x_{.211} - 5 \ u_{.211} = 0$$

$$2 \ x_{.212} - 6 \ u_{.212} = 0$$

$$2 \ x_{.213} - 7 \ u_{.213} = 0$$

$$2 \ x_{.214} - 8 \ u_{.214} = 0$$

$$2 \ x_{.221} - 5 \ u_{.221} = 0$$

$$2 \ x_{.222} - 6 \ u_{.222} = 0$$

$$2 \ x_{.223} - 7 \ u_{.223} = 0$$

$$2 \ x_{.224} - 8 \ u_{.224} = 0$$

$$2 \ x_{.311} - 7 \ u_{.311} = 0$$

$$2 \ x_{.312} - 8 \ u_{.312} = 0$$

$$2 \ x_{.313} - 9 \ u_{.313} = 0$$

$$2 \ x_{.314} - 10 \ u_{.314} = 0$$

$$2 \ x_{.321} - 7 \ u_{.321} = 0$$

$$2 \ x_{.322} - 8 \ u_{.322} = 0$$

$$2 \ x_{.323} - 9 \ u_{.323} = 0$$

$$2 \ x_{.324} - 10 \ u_{.324} = 0$$

\ BINARY VARIABLE CONSTRAINTS (ONLY 2 ACTIVE TRANSIT NODES)

$$u_{.111} + u_{.121} = 2$$

$$u_{.112} + u_{.122} = 2$$

$$u_{.113} + u_{.123} = 2$$

$$u_{.114} + u_{.124} = 2$$

$$u_{.211} + u_{.221} = 2$$

$$u_{.212} + u_{.222} = 2$$

$$u_{.213} + u_{.223} = 2$$

$$u_{.214} + u_{.224} = 2$$

$$u_{.311} + u_{.321} = 2$$

$$u_{.312} + u_{.322} = 2$$

$$u_{.313} + u_{.323} = 2$$

$$u_{.314} + u_{.324} = 2$$

BOUNDS

\ NON-NEGATIVITY CONSTRAINTS

$$r \geq 0$$

$$x_{.111} \geq 0$$

$$x_{.112} \geq 0$$

$$x_{.113} \geq 0$$

$$x_{.114} \geq 0$$

$$x_{.121} \geq 0$$

$$x_{.122} \geq 0$$

$$x_{.123} \geq 0$$

$$x_{.124} \geq 0$$

$$x_{.211} \geq 0$$

$$x_{.212} \geq 0$$

$$x_{.213} \geq 0$$

$$x_{.214} \geq 0$$

$$x_{.221} \geq 0$$

$$x_{.222} \geq 0$$

```
112 x_223 >= 0
114 x_224 >= 0
116 x_311 >= 0
118 x_312 >= 0
120 x_313 >= 0
122 x_314 >= 0
124 x_321 >= 0
126 x_322 >= 0
128 x_323 >= 0
130 x_324 >= 0
132 c_11 >= 0
134 c_12 >= 0
136 c_21 >= 0
138 c_22 >= 0
140 c_31 >= 0
142 c_32 >= 0
144 d_11 >= 0
146 d_12 >= 0
148 d_13 >= 0
150 d_14 >= 0
152 d_21 >= 0
154 d_22 >= 0
156 d_23 >= 0
158 d_24 >= 0
160
162 BIN
164 \ BINARY VARIABLES
166 u_111
168 u_112
170 u_113
172 u_114
174 u_121
176 u_122
178 u_123
180 u_124
182 u_211
184 u_212
186 u_213
188 u_214
190 u_221
192 u_222
194 u_223
196 u_224
198 u_311
200 u_312
202 u_313
204 u_314
206 u_321
208 u_322
210 u_323
212 u_324
214
216 END
```

../lp\_files/problem\_3\_2\_4.lp