FLOW PLANNING

ASSIGNMENT 2

COSC364-19S1 INTERNET TECHNOLOGY AND ENGINEERING

Will Cowper

ID: 81163265

Contribution: 50%

Jesse Sheehan

ID: 53366509

Contribution: 50%

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Figure 1: Traffic problems are not unique to computer networks.

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Name:	Will Cowper	Jesse Sheehan
Student ID:	81163265	5336650a
Signature:	aleyen	W.
Date:	29-5-19	29/5/19

1 Problem Description

Given a network (figure 2) with X source nodes, Y transit nodes and Z destination nodes, a program was designed to generate an LP file that could be used by CPLEX to determine certain network characteristics.

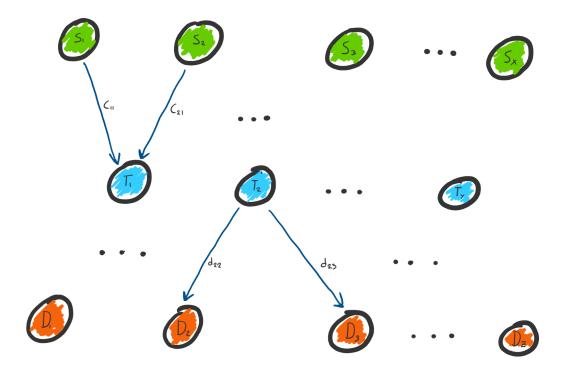


Figure 2: An example network showing nodes S_i , T_k and D_j and links c_{ik} and d_{kj} , $i \in \{1, ..., X\}, k \in \{1, ..., Y\}, j \in \{1, ..., Z\}.$

Traffic travelling from S_i to D_j must travel through exactly 2 transit nodes with a total demand volume of h_{ij} (equation 10). Furthermore, the load upon each transit node must be balanced.

2 Problem Formulation

This problem was solved with the use of binary variable constraints (equations 6, 7 and 9) and the minimisation of our objective function (equation 1). All normal non-negativity constraints were applied (equations 11, 12, 13 and 14).

The following network properties were solved for:

- The capacities of each link (equations 3 and 4).
- The load on each transit node (equation 5).
- The value of each flow (equations 2 and 8).

Notation:

- X is the number of source nodes.
- \bullet Y is the number of transit nodes.
- \bullet Z is the number of destination nodes.
- S_i is the *i*th source node.
- T_k is the kth transit node.
- D_j is the jth destination node.
- h_{ij} is the demand flow between S_i and D_j . This is equal to 2i + j.
- c_{ik} is the link capacity between S_i and T_k .
- d_{kj} is the link capacity between T_k and D_j .
- x_{ikj} is the decision variable associated with the path S_i - T_k - D_j .
- u_{ikj} is the binary decision variable associated with x_{ikj} . These are required because h_{ij} must be split across exactly 2 transit nodes.
- l_k is the load on T_k .

Note: Due to the limitations of the LP file format, many of the following equations must be rearranged for use in CPLEX. Most notably, there cannot be any variables on the right hand side of an equality or inequality.

2.1 Objective Function

$$\min_{[x,c,d,r]} r \tag{1}$$

2.2 Constraints

$$\sum_{k=1}^{Y} x_{ikj} = h_{ij} \qquad i \in \{1, \dots, X\}, j \in \{1, \dots, Z\}$$
 (2)

$$\sum_{j=1}^{Z} x_{ikj} = c_{ik} \qquad i \in \{1, \dots, X\}, k \in \{1, \dots, Y\}$$
 (3)

$$\sum_{i=1}^{X} x_{ikj} = d_{kj} \qquad k \in \{1, \dots, Y\}, j \in \{1, \dots, Z\}$$
 (4)

$$\sum_{k=1}^{Y} x_{ikj} = l_k \qquad i \in \{1, \dots, X\}, j \in \{1, \dots, Z\}$$
 (5)

$$\sum_{k=1}^{Y} u_{ikj} = 2 i \in \{1, \dots, X\}, j \in \{1, \dots, Z\}$$
 (6)

$$x_{ikj} = \frac{u_{ikj}h_{ij}}{2}$$
 $i \in \{1, \dots, X\}, k \in \{1, \dots, Y\}, j \in \{1, \dots, Z\}$ (7)

$$\sum_{i=1}^{X} \sum_{j=1}^{Z} x_{ikj} \le r \qquad k \in \{1, \dots, Y\}$$
 (8)

$$u_{ikj} \in \{0,1\}$$
 $i \in \{1,\ldots,X\}, k \in \{1,\ldots,Y\}, j \in \{1,\ldots,Z\}$ (9)

$$h_{ij} = 2i + j$$
 $i \in \{1, \dots, X\}, j \in \{1, \dots, Z\}$ (10)

2.3 Non-Negativity Constraints

$$r \ge 0 \tag{11}$$

$$x_{ikj} \ge 0$$
 $i \in \{1, \dots, X\}, k \in \{1, \dots, Y\}, j \in \{1, \dots, Z\}$ (12)

$$c_{ik} \ge 0$$
 $i \in \{1, \dots, X\}, k \in \{1, \dots, Y\}$ (13)

$$d_{kj} \ge 0$$
 $k \in \{1, \dots, Y\}, j \in \{1, \dots, Z\}$ (14)

3 Results

LP files were generated with parameters $X = Z = 9, Y \in \{3, 4, 5, 6, 7, 8\}$. These were then processed with CPLEX, recording the time taken to solve each problem. Important data points were extracted from the CPLEX output and are listed in table 1.

\mathbf{Y}	Time (ms)	Links	Load Spread	Max. c _{ik}	Max. d _k j
3	81.6132	52	0.0	94.500000	73.500000
4	117.3323	68	0.5	8.500000	9.500000
5	285.8704	83	0.0	9.500000	9.500000
6	443.1351	101	0.0	83.000000	9.500000
7	223.422	118	1.5	9.500000	9.500000
8	987.7775	134	0.5	9.500000	9.500000

Table 1: The raw data as extracted and processed from the CPLEX output.

An analysis of these results confirms many assumptions that were made about the problem. The number of non-zero link capacities increases linearly (figure 4), the transit node load spread is very close to (if not exactly) zero for most networks (figure 5), and the amount of time taken to solve the problem increases (almost) non-linearly as the number of transit nodes increases (figure 3). It is important to note that the load on the transit nodes was only able to be balanced in three of the networks, however the load spread on the remaining three networks is very low. CPLEX has done its best to equalise the loads on each transit node but could not balance them in some cases due to the demand flows and the number of nodes.

The most obvious feature of the results is the data for the Y = 7 network. It appears to be an outlier as it takes less time to solve than the Y = 5 network and has the highest transit node load spread by a factor of 3. **Explain this behaviour!!!**

The second odd feature of the data is the highest link capacities of the Y=3 and Y=6 networks (figure 6). Also explain this behaviour!!!

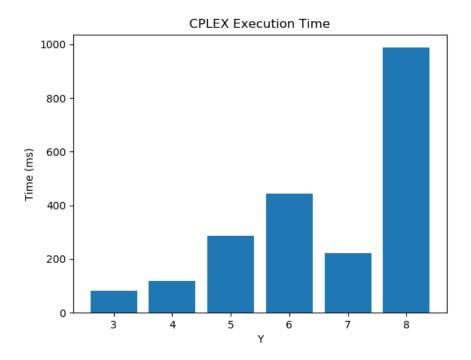


Figure 3: The time taken to execute the LP file in CPLEX for each network.

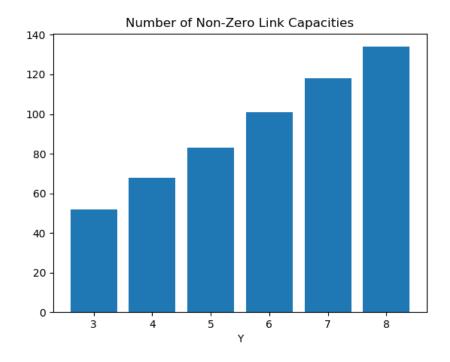


Figure 4: The number of non-zero link capacities in each network.

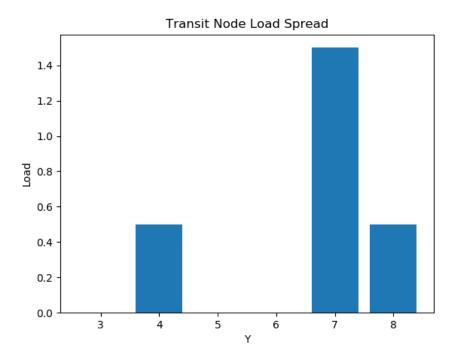


Figure 5: The amount of spread in the load for all transit nodes in each network.

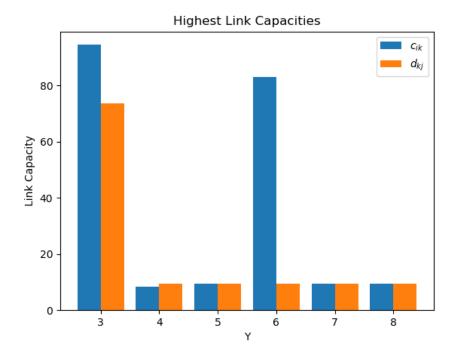


Figure 6: The highest capacity links for each network. Both the C_{ik} and D_{kj} links are listed.

4 Appendix

4.1 Source Code

$4.1.1 \quad \text{src/lp_gen.py}$

This script is responsible for producing a valid LP file from the given command line parameters.

```
# lp_gen.py
  #
  # COSC364 Assignment 2
  # 30/05/2019
  # Written by Will Cowper, Jesse Sheehan
  import inspect
  import functools
  import sys
10 import os.path
  __TITLE__ = "COSC-364 Assignment 2 LP Generator"
  _AUTHORS__ = [("Will Cowper", "81163265"), ("Jesse Sheehan", "53366509")]
  # Change these variables to alter the behaviour of the LP file generator
_{16} PATH_SPLIT = 2
  def DEMANDFLOW(i, j): return 2 * i + j
20
_{22} TEMPLATE = """ \
  \\ {}, LP Output File
  \\ Written by {}
  \ \ \ \  Parameters: X=\{\}, Y=\{\}, Z=\{\}, Split=\{\}, Demand=\{\}\}
  MINIMIZE
  \ t r
28
30 SUBJECT TO
32 \ t \ \ DEMAND CONSTRAINTS
  \setminus t \{ \}
  \t\\ CAPACITY CONSTRAINTS FOR LINKS BETWEEN SOURCE AND TRANSIT NODES
  \setminus t \{ \}
  \t \setminus \t \setminus \t  CAPACITY CONSTRAINTS FOR LINKS BETWEEN TRANSIT AND DESTINATION NODES
  \setminus t \{ \}
40
  \t\\ OBJECTIVE FUNCTION LOAD CONSTRAINTS
  \setminus t \{ \}
  \t \ \ \ TRANSIT NODE LOAD CONSTRAINTS
  \setminus t \{ \}
  \t\\ BINARY VARIABLE AND DECISION VARIABLE CONSTRAINTS
  \setminus t \{ \}
```

```
50 \ \ t \ BINARY VARIABLE CONSTRAINTS (ONLY 2 ACTIVE TRANSIT NODES)
   \setminus t \{ \}
  BOUNDS
54
   \t\\ NON-NEGATIVITY CONSTRAINTS
  \langle tr \rangle = 0
56
   \setminus t \{ \}
58
  BIN
60
   \t \\ BINARY VARIABLES
  \ t { }
62
64 END
  # DEFINE SOME UTILITY FUNCTIONS
68
   def get_lp_filename(x, y, z):
       """ Returns the filename that the LP data should be saved to. """
       return "problem\{0\}\{1\}\{2\}.lp".format(x, y, z)
74
   def crange(first , last):
       """ Returns a list of characters between the two characters passed in (
76
       inclusive).
       >>> crange('A', 'C')
       ['A', 'B', 'C']
       >>> crange('A', 'A')
       [ 'A ']
80
       if ord(first) > ord(last):
82
            raise ValueError("last must come after first")
84
       else:
            return [chr(i) for i in range(ord(first), ord(last) + 1)]
86
88
   def repeat(obj, n):
       """ Returns a list with obj repeated n times.
90
       >>> repeat (1, 1)
       [1]
92
       >>> repeat(42, 0)
       >>> repeat (5, 4)
       [5, 5, 5, 5]
96
       >>> repeat ([1, 2], 2)
       [[1, 2], [1, 2]]
       return [obj for _ in range(n)]
100
102
   def perms(lists):
       """ Returns all the permutations of the elements.
104
       >>> perms([])
       106
```

```
>>> perms ([[ 'a', 'b', 'c']])
       [('a',), ('b',), ('c',)]
108
      >>> perms([['a', 'b', 'c'], ['x', 'y', 'z']])
       if len(lists) == 0:
112
           return []
114
       elif (len(lists) == 1):
           return [(x,) for x in lists [0]
116
       else:
118
           return [(x,) + y \text{ for } x \text{ in } lists[0] \text{ for } y \text{ in } perms(lists[1:])]
  def concat (permutations):
      """ Returns the permutations concatenated as strings.
      >>> concat (perms ([[ 'a', 'b', 'c']]))
124
       ['a', 'b', 'c']
      >>> concat (perms ([[ 'a', 'b', 'c'], ['x', 'y', 'z']]))
       ['ax', 'ay', 'az', 'bx', 'by', 'bz', 'cx', 'cy', 'cz']
128
      return [functools.reduce(lambda x, y: x + str(y), p, '') for p in
      permutations]
130
  def get_function_source(fn):
       src = inspect.getsource(fn)
       return src[str(src).index('return')+7:]
134
136
  def get_lines(strings):
       return '\n\t'.join(strings)
138
  # DEFINE SOME FUNCTIONS SPECIFIC TO THE PROBLEM
142
  def get_nodes(x, y, z):
       """ Returns a tuple containing the source, transit and destination node
144
       ids as integers. """
       s = list(range(1, x + 1))
      t = list(range(1, y + 1))
146
      d = list(range(1, z + 1))
       return s, t, d
148
150
  def get_demand_constraints(s, t, d):
       """ Returns a list of demand constraints. """
       return [' + '.join(["x_{0}{1}{2}".format(i, k, j) for k in t]) + ' =
      \{0\}'. format (DEMAND.FLOW(i, j))
               for (i, j) in perms ([s, d])
154
  def get_source_transit_capacity_constraints(s, t, d):
      """ Returns a list of capacity constraints for the links between the
158
      source and transit nodes. """
       return \
```

```
[' + '.join(["x_{-}\{0\}\{1\}\{2\}".format(i, k, j) for j in d]) +
160
                              ' - c_{-}\{0\}\{1\} = 0'.format(i, k) for (i, k) in perms([s, t])]
162
     def get_transit_destination_capacity_constraints(s, t, d):
164
             """ Returns a list of capacity constraints for the links between the
            transit and destination nodes. """
             return \
                      [' + '.join(["x_{-}\{0\}\{1\}\{2\}".format(i, k, j) for i in s]) +
                              ' - d_{-}\{0\}\{1\} = 0'.format(k, j) for (k, j) in perms([t, d])]
168
170
     def get_transit_load_constraints(s, t, d):
             """ Returns the list of transit load constraints. """
             return ['+'.join(["x_{-}\{0\}\{1\}\{2\}".format(i, k, j) for (i, j) in perms([
                               ' - 1_{-}\{0\} = 0'. format(k) for k in t
174
     def get_objective_function_load_constraints(s, t, d):
             """ Returns the list of objective function load constraints. """
178
             return [' + '.join(["c_{-}\{0\}\{1\}".format(i, j) for i in s]) +
                                 - r \ll 0, for j in d
182
     def get_binary_and_decision_variable_constraints(s, t, d):
              """ Returns the binary and decision variable constraints. """
184
             \mbox{return $['\{3\}$ $x_{-}\{0\}\{1\}\{2\}$ - $\{4\}$ $u_{-}\{0\}\{1\}\{2\}$ = 0'.format(i, k, j, k)$} \label{eq:control_eq}
            PATH_SPLIT, DEMAND.FLOW(i, j)) for (i, k, j) in perms([s, t, d])
186
     def get_binary_constraints(s, t, d):
188
             """ Returns a list of binary variable constraints. """
             return ['+'.join(["u_{0}{1}{2}".format(i, k, j) for k in t]) + ' = {}
190
             '. format (PATH_SPLIT)
                              for (i, j) in perms([s, d])]
192
     def get_binary_variables(s, t, d):
             """ Returns a list of binary variables. """
             return ["u<sub>-</sub>{0}{1}{2}".format(i, k, j) for (i, k, j) in perms([s, t, d])
196
198
     def get_non_negativity_constraints(s, t, d):
             """ Returns a list of non-negativity constraints. """
200
             return [x_{0}_{1}] = 0, format(i, k, j) for (i, k, j) in perms([s, t]
             (i, k) = [c_{0}(1)] + [c_{0}(1)] = [c_{0}(1)] + [c_{0}(
            d_{-}\{0\}\{1\} >= 0".format(k, j) for (k, j) in perms([t, d])]
202
     def generate_lp_file(title, authors, x, y, z):
204
              """ Returns the LP file contents as per the project specification. """
             s, t, d = get\_nodes(x, y, z)
             demand_constraints = get_lines(get_demand_constraints(s, t, d))
208
              source_transit_capacity_constraints = get_lines(
                      get_source_transit_capacity_constraints(s, t, d))
210
```

```
transit_destination_capacity_constraints = get_lines(
           get_transit_destination_capacity_constraints(s, t, d))
212
       non_negativity_constraints = get_lines(get_non_negativity_constraints(
           s, t, d))
       objective_function_load_constraints = get_lines(
           get_objective_function_load_constraints(s, t, d))
       transit_load_constraints = get_lines(
           get_transit_load_constraints(s, t, d))
       binary_and_decision_constraints = get_lines(
           get_binary_and_decision_variable_constraints(s, t, d))
220
       binary_variable_constraints = get_lines(get_binary_constraints(s, t, d)
       binary_variables = get_lines(get_binary_variables(s, t, d))
222
       return TEMPLATE.format(
224
           title,
           authors,
226
           х,
           у,
           Ζ.
           PATH_SPLIT,
230
           get_function_source(DEMAND.FLOW),
           demand_constraints,
           source_transit_capacity_constraints,
           transit_destination_capacity_constraints,
234
           objective_function_load_constraints,
           transit_load_constraints,
           binary_and_decision_constraints,
           binary_variable_constraints,
238
           non_negativity_constraints,
           binary_variables)
242
  # DEFINE SOME HELPERS FOR GETTING THE THING RUNNING
  def print_version():
       print('{0} by {1}'.format(__TITLE__, get_author_string()))
246
248
  def print_usage():
       print('Usage: {0} <x> <y> <z> [output directory]'.format(sys.argv[0]))
250
252
   def get_problem_parameters():
       """ Returns a tuple containing the x, y and z parameters. """
254
       try:
           x = int(sys.argv[1])
256
           y = int(sys.argv[2])
           z = int(sys.argv[3])
258
       except:
           print_usage()
260
           \operatorname{exit}(-1)
262
       if x \ll 0:
           print("Error: x must be strictly positive")
264
           \operatorname{exit}(-1)
266
       if x >= 10:
```

```
print("Error: x must be less than ten")
268
            \operatorname{exit}(-1)
270
        if y <= 0:
            print("Error: y must be strictly positive")
272
            \operatorname{exit}(-1)
274
        if y >= 10:
            print("Error: y must be less than ten")
276
            \operatorname{exit}(-1)
        if z \ll 0:
            print("Error: z must be strictly positive")
280
            exit(-1)
282
        if z >= 10:
            print("Error: z must be less than ten")
284
            exit(-1)
286
       return x, y, z
288
   def save_lp_file(filename, data):
       try:
            f = open(filename, 'w')
292
            f.write(data)
            f.close()
       except:
            print("Error: could not save file '{0}'".format(filename))
296
            \operatorname{exit}(-1)
298
   def get_author_string():
300
       return ', '.join(
            ["{0} ({1})".format(name, sid) for (name, sid) in __AUTHORS__])
304
   def main():
       print_version()
306
        if len(sys.argv) != 4 and len(sys.argv) != 5:
            print_usage()
308
            \operatorname{exit}(-1)
        else:
310
            output_dir = '.'
            if len(sys.argv) == 5:
312
                 output_dir = sys.argv[4]
314
            x, y, z = get_problem_parameters()
            data = generate_lp_file(\_TITLE\_\_, get_author\_string(), x, y, z)
316
            filename = os.path.join(output_dir, get_lp_filename(x, y, z))
            save_lp_file (filename, data)
318
            print("Success: saved as '{0}'".format(filename))
320
  if __name__ == "__main__":
       main()
```

$4.1.2 \quad src/lp_csv.py$

This script is responsible for converting the output of the CPLEX log files into a single CSV file for further processing.

```
\# lp_csv.py
 #
 # COSC364 Assignment 2
 # 30/05/2019
  # Written by Will Cowper, Jesse Sheehan
  import csv
  import sys
  import os.path
  def csvWrite(data):
      with open(sys.argv[2], 'a', newline=',') as csvFile:
          writer = csv.writer(csvFile)
14
          writer.writerow(data)
16
  def floatmap (enumerable):
      return list (map(lambda x: float(x), enumerable))
20
  def openFile(Y):
      with open(os.path.join(sys.argv[1], '{0}.txt'.format(Y)), 'r') as
     in_file:
          stripped = [line.strip() for line in in_file.readlines()]
24
          lines = [line for line in stripped if line]
          data = []
26
          # Y
          data.append(Y)
28
          # elapsed time
          data.append(max(parseFile("elapsed_", lines)))
30
          # no of non-zero c and d links
          data.append(len(parseFile("c_", lines)) + len(parseFile("d_", lines
     )))
          # transit load spread (largest_transit_node_load -
     smallest_transit_node_load)
          data.append(max(floatmap(parseFile("l_", lines))) -
34
                       min(floatmap(parseFile("l_", lines))))
          # highest cap c network
36
          data.append(max(parseFile("c_", lines)))
          # highest cap d network
38
          data.append(max(parseFile("d_", lines)))
          csvWrite(data)
40
42
  ""Returns a list of all values that start with the given string ""
  def parseFile(string, lines):
46
      values = []
      for line in lines:
          if line.startswith(string):
               values.append(line.split()[1])
50
```

```
return values
  if = name_{-} = "-main_{-}":
      if len(sys.argv) != 3:
56
          print("Usage: {0} <input directory > <csv file >".format(sys.argv[0])
          exit(-1)
58
      # delete the CSV, otherwise we will append to it
60
      os.unlink(sys.argv[2])
      openFile(3)
      openFile(4)
64
      openFile(5)
      openFile(6)
      openFile(7)
      openFile(8)
      print ("Saved CSV data to '{}'".format(sys.argv[2]))
```

../src/lp_csv.py

$4.1.3 \text{ src/lp_graph.py}$

This script is responsible for reading the CSV file and producing several graphs.

```
# lp_graph.py
  #
2
  # COSC364 Assignment 2
4 # 30/05/2019
  # Written by Will Cowper, Jesse Sheehan
  import csv
  import sys
  import os.path
  try:
      import numpy as np
      print ("Error: could not load 'numpy'. Install with 'pip install numpy'
     and then try again.")
      exit(-1)
16
  \operatorname{try}:
      import matplotlib.pyplot as plt
  except:
      print ("Error: could not load 'matplotlib'. Install with 'pip install
      matplotlib' and then try again.")
      exit(-1)
24
  def get_data(data, key):
      return list (map(lambda d: d[key], data))
28
```

```
def get_time(data):
      return get_data(data, "time")
30
  def get_len_nonzero_links(data):
      return get_data(data, "len_links")
34
  def get_transit_load_spread(data):
      return get_data(data, "load_spread")
38
40
  def get_max_cap_c(data):
      return get_data(data, "max_cap_c")
42
  def get_max_cap_d(data):
      return get_data(data, "max_cap_d")
46
48
  def get_Y(data):
      return get_data(data, "Y")
50
52
  def save_execution_time_plot(filename, data):
      """ Saves a plot of execution time. """
54
      plt.bar(get_Y(data), get_time(data))
      plt.xlabel("Y")
plt.ylabel("Time (ms)")
56
      plt.title("CPLEX Execution Time")
58
      plt.savefig (filename)
      plt.close()
60
      print("Saved '{}'".format(filename))
62
  def save_num_nonzero_links_plot(filename, data):
64
      """ Saves a plot of the number of non-zero links.
      plt.bar(get_Y(data), get_len_nonzero_links(data))
      plt.xlabel("Y")
      plt.ylabel("")
      plt.title("Number of Non-Zero Link Capacities")
      plt.savefig (filename)
      plt.close()
      print("Saved '{}'".format(filename))
72
  def save_transit_load_spread_plot(filename, data):
      """ Saves a plot of the transit load spread. """
76
      plt.bar(get_Y(data), get_transit_load_spread(data))
      plt.xlabel("Y")
plt.ylabel("Load")
      plt.title ("Transit Node Load Spread")
80
      plt.savefig (filename)
      plt.close()
      print("Saved '{}'".format(filename))
86 def save_highest_capacity_links_plot(filename, data):
```

```
Saves a plot of the transit load spread. """
       width = 0.4
       Ys = np.array(get_Y(data))
       cs = plt.bar(Ys, get_max_cap_c(data), width, label="$c_{ik}$")
       ds \, = \, plt.\,bar\,(\,Ys \, + \, width \, , \, \, get\_max\_cap\_d\,(\,data\,) \, , \, \, width \, , \, \, label="\,\$d\_\{\,kj\,\}\,\$"\,)
       plt.xticks(Ys + width / 2, map(lambda x: int(x), Ys))
92
       plt.legend(handles=[cs, ds])
       plt.xlabel("Y")
       plt.ylabel("Link Capacity")
       plt.title("Highest Link Capacities")
96
       plt.savefig (filename)
       plt.close()
98
       print("Saved '{}'".format(filename))
100
   def get_data_from_csv(csv_filename):
       """ Returns an array of dictionaries containing the CSV data. """
       with open(csv_filename, newline='') as csv_file:
104
            csv_reader = csv.DictReader(csv_file, fieldnames=[
                                          "Y", "time", "len_links", "load_spread"
106
       , "max_cap_c", "max_cap_d"])
           rows = []
            for row in csv_reader:
                d = \{\}
                for key in row:
                    d[key] = float(row[key])
                rows.append(d)
           return rows
114
   def convert_csv_to_images(csv_filename, output_folder):
116
       """ Converts the data from the CSV into a set of graphs. """
       data = get_data_from_csv(csv_filename)
118
       base_filename = os.path.splitext(os.path.join(
            output_folder, os.path.basename(csv_filename)))[0]
       save_execution_time_plot(base_filename + "_time.png", data)
       save_num_nonzero_links_plot(base_filename + "_num_nonzero_links.png",
      data)
       save_transit_load_spread_plot(
            base_filename + "_transit_load_spread.png", data)
       save\_highest\_capacity\_links\_plot (
            base_filename + "_highest_capacity_links.png", data)
128
   def print_usage():
       print("Usage: {0} <csv file > <output folder>")
   if __name__ == "__main__":
134
       if len(sys.argv) != 3:
            print_usage()
136
            \operatorname{exit}(-1)
138
       convert_csv_to_images(sys.argv[1], sys.argv[2])
```

4.1.4 output.sh

This BASH script is responsible for executing the other scripts as well as timing and running CPLEX (under the Linux operating system).

```
#!/bin/bash
for y in 3 4 5 6 7 8
do

python3 src/lp_gen.py 9 $y 9 lp_files
start=$(date +%s%N)
cplex -c "read lp_files/problem_9_${y}_9.lp" "optimize" "display
solution variables -" > cplex_logs/$y.txt
end=$(date +%s%N)
duration=$(expr $end - $start)
duration=$(expr $duration / 1000000)
echo -e "\nelapsed_time: $duration ms" >> cplex_logs/$y.txt

done

python3 src/lp_csv.py cplex_logs cplex_data.csv
python3 src/lp_graph.py cplex_data.csv graphs
```

../output.sh

4.1.5 output.ps1

This PowerShell script is responsible for executing the other scripts as well as timing and running CPLEX (under the Windows operating system).

```
For ($i=3; $i-le 8; $i++) {
    python src/lp_gen.py 9 $i 9 lp_files
    $perf = Measure-Command -Expression {$data = cplex -c ("read lp_files/
    problem_9_" + $i + "_9.lp") "optimize" "display solution variables -"}

$ms = $perf. TotalMilliseconds
    [System.IO.File]:: WriteAllLines("cplex_logs/$i.txt", $data + "'
    nelapsed_time: $ms ms")

}

python src/lp_csv.py cplex_logs lp_files/cplex_data.csv
python src/lp_graph.py lp_files/cplex_data.csv graphs
```

../output.ps1

4.2 Generated LP File

4.2.1 lp_files/problem_3_2_4.lp

```
\ COSC-364 Assignment 2 LP Generator, LP Output File
\ Written by Will Cowper (81163265), Jesse Sheehan (53366509)
\ Parameters: X=3, Y=2, Z=4, Split=2, Demand=2 * i + j

MINIMIZE

r

SUBJECT TO
```

```
\ DEMAND CONSTRAINTS
         x_1111 + x_121 = 3
         x_{-}112 + x_{-}122 = 4
         x_{-}113 + x_{-}123 = 5
         x_{-}114 + x_{-}124 = 6
         x_{-}211 + x_{-}221 = 5
         x_212 + x_222 = 6
         x_{2}13 + x_{2}23 = 7
         x_214 + x_224 = 8
         x_311 + x_321 = 7
20
         x_{-}312 + x_{-}322 = 8
         x_{-}313 + x_{-}323 = 9
         x_{-}314 + x_{-}324 = 10
24
         \ CAPACITY CONSTRAINTS FOR LINKS BEIWEEN SOURCE AND TRANSIT NODES
         x_1111 + x_1112 + x_1113 + x_1114 - c_111 = 0
         x_121 + x_122 + x_123 + x_124 - c_12 = 0
         x_211 + x_212 + x_213 + x_214 - c_21 = 0
         x_221 + x_222 + x_223 + x_224 - c_22 = 0
         x_311 + x_312 + x_313 + x_314 - c_31 = 0
30
         x_321 + x_322 + x_323 + x_324 - c_32 = 0
         \ CAPACITY CONSTRAINTS FOR LINKS BEIWEEN TRANSIT AND DESTINATION NODES
         x_{-}111 + x_{-}211 + x_{-}311 - d_{-}11 = 0
34
         x_1112 + x_2112 + x_3112 - d_112 = 0
         x_113 + x_213 + x_313 - d_13 = 0
36
         x_114 + x_214 + x_314 - d_14 = 0
         x_121 + x_221 + x_321 - d_21 = 0
38
         x_122 + x_222 + x_322 - d_22 = 0
         x_123 + x_223 + x_323 - d_23 = 0
         x_124 + x_224 + x_324 - d_24 = 0
42
         \ OBJECTIVE FUNCTION LOAD CONSTRAINTS
         c_{-}11 + c_{-}21 + c_{-}31 - r <= 0
44
         c_{-}12 + c_{-}22 + c_{-}32 - r <= 0
         c_{13} + c_{23} + c_{33} - r \le 0
46
         c_14 + c_24 + c_34 - r \le 0
48
         \ TRANSIT NODE LOAD CONSTRAINTS
         x_{-}111 + x_{-}112 + x_{-}113 + x_{-}114 + x_{-}211 + x_{-}212 + x_{-}213 + x_{-}214 + x_{-}311 
50
            x_{-}312 + x_{-}313 + x_{-}314 - l_{-}1 = 0
         x_{-}121 + x_{-}122 + x_{-}123 + x_{-}124 + x_{-}221 + x_{-}222 + x_{-}223 + x_{-}224 + x_{-}321 +
            x_{-3}22 + x_{-3}23 + x_{-3}24 - 1_{-2} = 0
         \ BINARY VARIABLE AND DECISION VARIABLE CONSTRAINTS
         2 x_111 - 3 u_111 = 0
         2 x_{1}12 - 4 u_{1}12 = 0
         2 x_{1}13 - 5 u_{1}13 = 0
56
         2 x_{1}14 - 6 u_{1}14 = 0
         2 x_{1}21 - 3 u_{1}21 = 0
         2 x_{-}122 - 4 u_{-}122 = 0
         2 x_{-}123 - 5 u_{-}123 = 0
60
         2 x_124 - 6 u_124 = 0
         2 x_{2}11 - 5 u_{2}11 = 0
         2 x_{-}212 - 6 u_{-}212 = 0
         2 x_{2}13 - 7 u_{2}13 = 0
64
         2 x_{2}14 - 8 u_{2}14 = 0
         2 x_{2}21 - 5 u_{2}21 = 0
```

```
2 x_{2}22 - 6 u_{2}22 = 0
     2 x_{2}3 - 7 u_{2}3 = 0
     2 x_{2}24 - 8 u_{2}24 = 0
     2 x_{3}11 - 7 u_{3}11 = 0
     2 x_{-}312 - 8 u_{-}312 = 0
     2 x_313 - 9 u_313 = 0
     2 x_314 - 10 u_314 = 0
     2\ x_{-}321\ -\ 7\ u_{-}321\ =\ 0
     2 x_{3}22 - 8 u_{3}22 = 0
     2 x_{3}23 - 9 u_{3}23 = 0
     2 x_{3}24 - 10 u_{3}24 = 0
78
      \ BINARY VARIABLE CONSTRAINTS (ONLY 2 ACTIVE TRANSIT NODES)
     u_{-}111 + u_{-}121 = 2
80
     u_{-}112 + u_{-}122 = 2
     u_{-}113 + u_{-}123 = 2
     u_1114 + u_1124 = 2
      u_{-}211 + u_{-}221 = 2
      u_{-}212 + u_{-}222 = 2
      u_{-}213 + u_{-}223 = 2
      u_{-}214 + u_{-}224 = 2
      u_{-}311 + u_{-}321 = 2
     u_{-}312 + u_{-}322 = 2
      u_{-}313 + u_{-}323 = 2
90
      u_{-}314 + u_{-}324 = 2
92
   BOUNDS
94
      \ NON-NEGATIVITY CONSTRAINTS
     r >= 0
96
     x_1111 >= 0
     x_{-}112 >= 0
98
     x_{-}113 >= 0
     x_1114 >= 0
100
     x_121 >= 0
     x_122 >= 0
102
     x_123 >= 0
     x_124 >= 0
104
      x_{-}211 >= 0
     x_212 >= 0
106
     x_{-}213 >= 0
      x_{-}214 >= 0
      x_{-}221 >= 0
      x_{-}222 >= 0
      x_{-}223 >= 0
     x_224 >= 0
     x_311 >= 0
     x_312 >= 0
114
     x_313 >= 0
     x_314 >= 0
     x_321 >= 0
     x_{-}322 >= 0
118
     x_323 >= 0
     x_324 >= 0
120
      c_{-}11 >= 0
      c_{-}12 >= 0
      c_{-}21 >= 0
      c_{-}22 >= 0
```

```
c_{-}31 >= 0
      c_{-}32 >= 0
126
      d_{-}11 >= 0
      d_{-}12 >= 0
      d_{-}13 >= 0
      d_{-}14 >= 0
130
      d_{-}21 >= 0
      d_{-}22 >= 0
      d_-23 \ >= \ 0
      d_{-}24 >= 0
134
136 BIN
      \ BINARY VARIABLES
138
      u_111
      u_1112
      u_1113
      u_1114
142
      u_1121
      u_{-}122
      u_{-}123
      u_{-}124
146
      u_-211
      u_-212
148
      u_213
      u\_214
150
      u_{-}221
      u_{-}222
152
      u_{-}223
      u\_224
154
      u\_311
      u\_312
156
      u_-313
      u_-314
158
      u_321
      u_{-}322
160
      u\_323
      u\_324
162
164 END
```

 $../lp_files/problem_3_2_4.lp$