FLOW PLANNING

ASSIGNMENT 2

COSC364-19S1 INTERNET TECHNOLOGY AND ENGINEERING

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Figure 1: Traffic problems are not unique to computer networks.

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1 Problem Description

Given a network (figure 2) with X source nodes, Y transit nodes and Z destination nodes, a program was designed to generate an LP file that could be used by CPLEX to determine certain network characteristics.

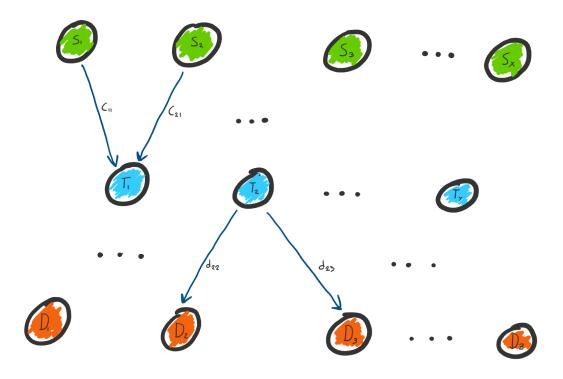


Figure 2: An example network showing nodes S_i , T_k and D_j and links c_{ik} and d_{kj} .

Traffic travelling from S_i to D_j must travel through exactly 2 transit nodes with a total demand volume of h_{ij} (equation 10). Furthermore, the load upon each transit node must be balanced.

2 Problem Formulation

This problem was solved with the use of binary variable constraints (equations 6, 7 and 9) and the minimisation of our objective function (equation 1). All normal non-negativity constraints were applied (equations 11, 12, 13 and 14).

The following network properties were solved for:

- The capacities of each link (equations 3 and 4).
- The load on each transit node (equation 5).
- The value of each flow (equations 2 and 8).

Notation:

• X is the number of source nodes.

- Y is the number of transit nodes.
- \bullet Z is the number of destination nodes.
- S_i is the *i*th source node.
- T_k is the kth transit node.
- D_j is the jth destination node.
- h_{ij} is the demand flow between S_i and D_j . This is equal to 2i + j.
- c_{ik} is the link capacity between S_i and T_k .
- d_{kj} is the link capacity between T_k and D_j .
- x_{ikj} is the decision variable associated with the path S_i - T_k - D_j .
- u_{ikj} is the binary decision variable associated with x_{ikj} . These are required because h_{ij} must be split across exactly 2 transit nodes.
- l_k is the load on T_k .

Note: Due to the limitations of the LP file format, many of the following equations must be rearranged for use in CPLEX. Most notably, there cannot be any variables on the right hand side of an equality or inequality.

2.1 Objective Function

$$\min ini_{[x,c,d,r]} r \tag{1}$$

2.2 Constraints

$$\sum_{k=1}^{Y} x_{ikj} = h_{ij} \qquad i \in \{1, \dots, X\}, j \in \{1, \dots, Z\}$$
 (2)

$$\sum_{j=1}^{Z} x_{ikj} = c_{ik} \qquad i \in \{1, \dots, X\}, k \in \{1, \dots, Y\}$$
 (3)

$$\sum_{i=1}^{X} x_{ikj} = d_{kj} \qquad k \in \{1, \dots, Y\}, j \in \{1, \dots, Z\}$$
 (4)

$$\sum_{k=1}^{Y} x_{ikj} = l_k \qquad i \in \{1, \dots, X\}, j \in \{1, \dots, Z\}$$
 (5)

$$\sum_{k=1}^{Y} u_{ikj} = 2 i \in \{1, \dots, X\}, j \in \{1, \dots, Z\}$$
 (6)

$$x_{ikj} = \frac{u_{ikj}h_{ij}}{2} \qquad i \in \{1, \dots, X\}, k \in \{1, \dots, Y\}, j \in \{1, \dots, Z\}$$
 (7)

$$\sum_{i=1}^{X} \sum_{j=1}^{Z} x_{ikj} \le r \qquad k \in \{1, \dots, Y\}$$
 (8)

$$u_{ikj} \in \{0,1\}$$
 $i \in \{1,\ldots,X\}, k \in \{1,\ldots,Y\}, j \in \{1,\ldots,Z\}$ (9)

$$h_{ij} = 2i + j$$
 $i \in \{1, \dots, X\}, j \in \{1, \dots, Z\}$ (10)

2.3 Non-Negativity Constraints

$$r \ge 0 \tag{11}$$

$$x_{ikj} \ge 0$$
 $i \in \{1, \dots, X\}, k \in \{1, \dots, Y\}, j \in \{1, \dots, Z\}$ (12)

$$c_{ik} \ge 0$$
 $i \in \{1, \dots, X\}, k \in \{1, \dots, Y\}$ (13)

$$d_{kj} \ge 0$$
 $k \in \{1, \dots, Y\}, j \in \{1, \dots, Z\}$ (14)

3 Results

LP files were generated with parameters $X = Z = 9, Y \in \{3, 4, 5, 6, 7, 8\}$. These were then processed with CPLEX, recording the time taken to solve each problem. Important data points were extracted from the CPLEX output and are listed in table 1.

Max. c_{ik} \mathbf{Y} Time (ms) Links Load Spread Max. $d_{k,j}$ 3 81.6132 52 0.0 94.500000 73.500000 4 117.332368 8.500000 9.5000000.55 285.8704 83 0.09.5000009.5000006 443.1351 101 0.0 83.000000 9.500000 7 223.422118 1.5 9.5000009.5000008 987.7775 134 0.59.5000009.500000

Table 1: The raw data as extracted and processed from the CPLEX output.

An analysis of these results confirms many assumptions that were made about the problem. The number of non-zero link capacities increases linearly (figure 4), the transit node load spread is very close to (if not exactly) zero for most networks (figure 5), and the amount of time taken to solve the problem increases (almost) non-linearly as the number of transit nodes increases (figure 3). It is important to note that the load on the transit nodes was only able to be balanced in three of the networks. **Give a reason why.**

The most obvious feature of the results is the data for the Y = 7 network. It appears to be an outlier as it takes less time to solve than the Y = 5 network and has the highest transit node load spread by a factor of 3. **Explain this behaviour!!!**

The second odd feature of the data is the highest link capacities of the Y = 3 and Y = 6 networks (figure 6). Also explain this behaviour!!!

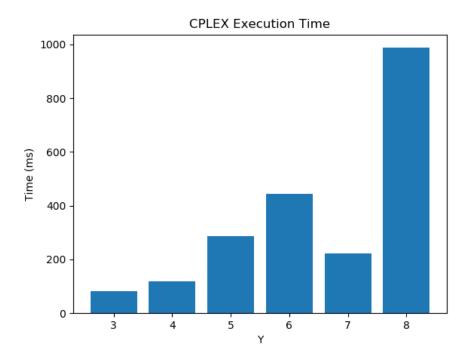


Figure 3: The time taken to execute the LP file in CPLEX for each network.

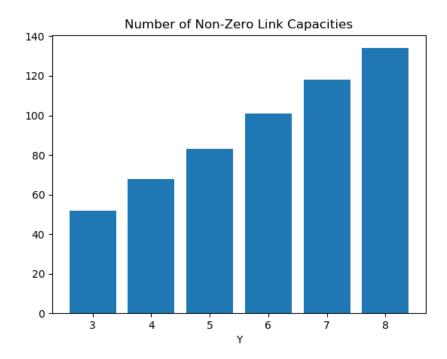


Figure 4: The number of non-zero link capacities in each network.

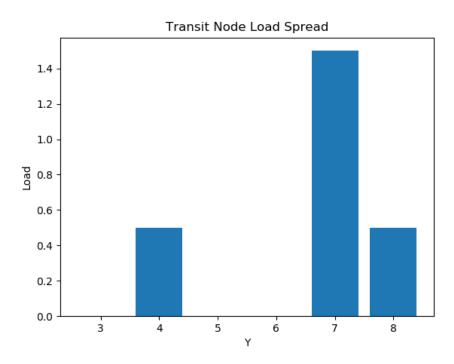


Figure 5: The amount of spread in the load for all transit nodes in each network.

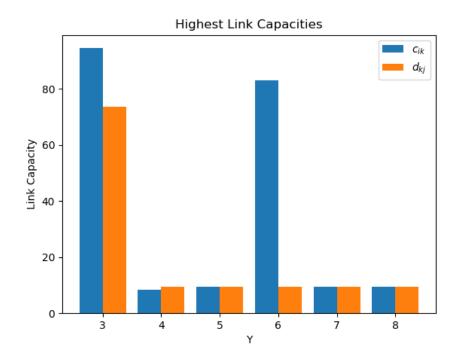


Figure 6: The highest capacity links for each network. Both the C_{ik} and D_{kj} links are listed.

4 Appendix

4.1 Source Code

$4.1.1 \quad \text{src/lp_gen.py}$

This script is responsible for producing a valid LP file from the given command line parameters.

```
# lp_gen.py
  #
  # COSC364 Assignment 2
_{4} # _{30}/_{05}/_{2019}
  # Written by Will Cowper, Jesse Sheehan
  import inspect
  import functools
  import sys
10 import os.path
12 __TITLE__ = "COSC-364 Assignment 2 LP Generator"
  _AUTHORS__ = [("Will Cowper", "81163265"), ("Jesse Sheehan", "53366509")]
  # Change these variables to alter the behaviour of the LP file generator
_{16} | PATH\_SPLIT = 2
  def DEMANDFLOW(i, j): return 2 * i + j
20
22 TEMPLATE = """ \
  \\ {}, LP Output File \\ Written by {}
  MINIMIZE
  \ t r
30 SUBJECT TO
  \t\\ DEMAND CONSTRAINTS
  \t{}
  \t\\ CAPACITY CONSTRAINTS FOR LINKS BETWEEN SOURCE AND TRANSIT NODES
36 \ t { }
  \t \ \ CAPACITY CONSTRAINTS FOR LINKS BETWEEN TRANSIT AND DESTINATION NODES
  \setminus t \{ \}
40
  \t\\ OBJECTIVE FUNCTION LOAD CONSTRAINTS
42 \setminus t \{\}
  \t\\ TRANSIT NODE LOAD CONSTRAINTS
  \setminus t \{ \}
  \t\\ BINARY VARIABLE AND DECISION VARIABLE CONSTRAINTS
  \setminus t \{ \}
```

```
\t\\ BINARY VARIABLE CONSTRAINTS (ONLY 2 ACTIVE TRANSIT NODES)
   \t \{ \}
  BOUNDS
54
   \t\\ NON-NEGATIVITY CONSTRAINTS
  \langle tr \rangle = 0
56
   \ t { }
58
  BIN
60
   \t \ \ \ BINARY VARIABLES
  \setminus t \{ \}
62
64 END
  # DEFINE SOME UTILITY FUNCTIONS
68
  def get_lp_filename(x, y, z):
70
       """ Returns the filename that the LP data should be saved to. """
       return "problem_{0}_{1}_{2}.lp".format(x, y, z)
74
   def crange(first , last):
       """ Returns a list of characters between the two characters passed in (
76
       inclusive).
       >>> crange ('A', 'C')
       ['A', 'B', 'C']
       >>> crange('A', 'A')
       [ 'A ']
80
       if ord(first) > ord(last):
82
            raise ValueError("last must come after first")
84
           return [chr(i) for i in range(ord(first), ord(last) + 1)]
86
88
   def repeat (obj, n):
       """ Returns a list with obj repeated n times.
90
       >>> repeat (1, 1)
       [1]
92
       >>> repeat (42, 0)
       94
       >>> repeat (5, 4)
       [5, 5, 5, 5]
96
       >>> repeat ([1, 2], 2)
       [[1, 2], [1, 2]]
98
       return [obj for _ in range(n)]
100
   def perms(lists):
       """ Returns all the permutations of the elements.
104
       >>> perms([])
106
```

```
>>> perms([['a', 'b', 'c']])
[('a',), ('b',), ('c',)]
>>> perms([['a', 'b', 'c'], ['x', 'y', 'z']])
108
       [('a', 'x'), ('a', 'y'), ('a', 'z'), ('b', 'x'), ('b', 'y'), ('b', 'z')
       , (, c, , , x, ), (, c, , , y, ), (, c, , , z, )]
       if len(lists) == 0:
112
            return []
114
        elif (len(lists) == 1):
            return [(x,) for x in lists[0]]
116
       else:
118
            return [(x,) + y \text{ for } x \text{ in lists } [0] \text{ for } y \text{ in perms } (\text{lists } [1:])]
120
   def concat (permutations):
       """ Returns the permutations concatenated as strings.
       >>> concat (perms ([['a', 'b', 'c']]))
124
       ['a', 'b', 'c']
       >>> concat (perms ([[ 'a', 'b', 'c'], ['x', 'y', 'z']]))
       ['ax', 'ay', 'az', 'bx', 'by', 'bz', 'cx', 'cy', 'cz']
128
       return [functools.reduce(lambda x, y: x + str(y), p, '') for p in
       permutations ]
130
   def get_function_source(fn):
132
       src = inspect.getsource(fn)
       return src[str(src).index('return')+7:]
134
136
   def get_lines(strings):
       return '\n\t'.join(strings)
138
140 # DEFINE SOME FUNCTIONS SPECIFIC TO THE PROBLEM
142
   def get_nodes(x, y, z):
        "" Returns a tuple containing the source, transit and destination node
144
       ids as integers. """
       s = list(range(1, x + 1))
       t = list(range(1, y + 1))
       d = list(range(1, z + 1))
       return s, t, d
148
150
   def get_demand_constraints(s, t, d):
       """ Returns a list of demand constraints. """
       return [' + '.join(["x_{0}{1}{2}".format(i, k, j) for k in t]) + ' =
       \{0\}'. format (DEMAND.FLOW(i, j))
                for (i, j) in perms([s, d])]
   def get_source_transit_capacity_constraints(s, t, d):
       """ Returns a list of capacity constraints for the links between the
158
       source and transit nodes. """
       return \
```

```
[' + '.join(["x_{-}{0}{1}{2}".format(i, k, j) for j in d]) +
160
              ' - c_{-}\{0\}\{1\} = 0'. format(i, k) for (i, k) in perms([s, t])]
162
  def get_transit_destination_capacity_constraints(s, t, d):
164
      """ Returns a list of capacity constraints for the links between the
      transit and destination nodes. """
      return \
          168
  def get_transit_load_constraints(s, t, d):
      """ Returns the list of transit load constraints. """
      return [' + '.join(["x_{0}{1}{2}".format(i, k, j) for (i, j) in perms([
     s, d])]) +
               -1_{-}\{0\} = 0'.format(k) for k in t]
174
176
  def get_objective_function_load_constraints(s, t, d):
      """ Returns the list of objective function load constraints. """
178
      return ['+'.join(["c_{0}{1}".format(i, j) for i in s]) + '- r <= 0' for j in d]
180
182
  def get_binary_and_decision_variable_constraints(s, t, d):
      """ Returns the binary and decision variable constraints. """
184
      return ['{3} x_{0}{1}{2} - {4} u_{0}{1}{2} = 0'.format(i, k, j,
     PATH_SPLIT, DEMANDFLOW(i, j)) for (i, k, j) in perms([s, t, d])]
186
  def get_binary_constraints(s, t, d):
188
      """ Returns a list of binary variable constraints. """
      190
      '. format (PATH_SPLIT)
             for (i, j) in perms ([s, d])
192
  def get_binary_variables(s, t, d):
194
      "" Returns a list of binary variables. """
      return ["u_{0}{1}{2}".format(i, k, j) for (i, k, j) in perms([s, t, d])
196
198
  def get_non_negativity_constraints(s, t, d):
      """ Returns a list of non-negativity constraints. """
200
      (s, d) + ["c<sub>-</sub>{0}{1} >= 0".format(i, k) for (i, k) in perms([s, t])] + ["
     d_{-}\{0\}\{1\} >= 0".format(k, j) for (k, j) in perms([t, d])]
202
  def generate_lp_file(title, authors, x, y, z):
204
      """ Returns the LP file contents as per the project specification. """
      s, t, d = get\_nodes(x, y, z)
      demand_constraints = get_lines(get_demand_constraints(s, t, d))
208
      source_transit_capacity_constraints = get_lines(
          get_source_transit_capacity_constraints(s, t, d))
```

```
transit_destination_capacity_constraints = get_lines(
           get_transit_destination_capacity_constraints(s, t, d))
212
       non_negativity_constraints = get_lines(get_non_negativity_constraints(
           s, t, d))
214
       objective_function_load_constraints = get_lines(
           get_objective_function_load_constraints(s, t, d))
216
       transit_load_constraints = get_lines(
           get_transit_load_constraints(s, t, d))
       binary_and_decision_constraints = get_lines(
           get_binary_and_decision_variable_constraints(s, t, d))
220
       binary_variable_constraints = get_lines(get_binary_constraints(s, t, d)
       binary_variables = get_lines(get_binary_variables(s, t, d))
222
       return TEMPLATE. format (
           title,
           authors,
226
           х,
           у,
228
           PATH_SPLIT,
230
           get_function_source(DEMAND_FLOW),
           demand_constraints,
           source_transit_capacity_constraints,
           transit_destination_capacity_constraints,
234
           objective_function_load_constraints,
           transit_load_constraints,
           binary_and_decision_constraints,
           binary_variable_constraints,
238
           non_negativity_constraints,
           binary_variables)
240
242
  # DEFINE SOME HELPERS FOR GETTING THE THING RUNNING
244
   def print_version():
       print('{0} by {1}'.format(__TITLE__, get_author_string()))
246
   def print_usage():
       print('Usage: {0} <x> <y> <z> [output directory]'.format(sys.argv[0]))
250
   def get_problem_parameters():
       """ Returns a tuple containing the x, y and z parameters. """
254
       trv:
           x = int(sys.argv[1])
           y = int(sys.argv[2])
           z = int(sys.argv[3])
258
       except:
260
           print_usage()
           \operatorname{exit}(-1)
262
       if x \le 0:
           print("Error: x must be strictly positive")
264
           exit(-1)
266
       if x >= 10:
```

```
print("Error: x must be less than ten")
268
            \operatorname{exit}(-1)
        if y \ll 0:
            print("Error: y must be strictly positive")
272
274
        if y >= 10:
             print("Error: y must be less than ten")
276
             \operatorname{exit}(-1)
278
        if z \ll 0:
            print("Error: z must be strictly positive")
280
            exit(-1)
282
        if z >= 10:
             print("Error: z must be less than ten")
284
             \operatorname{exit}(-1)
286
        return x, y, z
288
   def save_lp_file(filename, data):
290
        try:
             f = open(filename, 'w')
292
            f.write(data)
            f.close()
294
        except:
            print("Error: could not save file '{0}'".format(filename))
296
            \operatorname{exit}(-1)
298
   def get_author_string():
300
        return ', '.join(
            ["{0} ({1})".format(name, sid) for (name, sid) in _AUTHORS__])
302
304
   def main():
306
        print_version()
        if len(sys.argv) != 4 and len(sys.argv) != 5:
             print_usage()
308
             \operatorname{exit}(-1)
        else:
             output_dir = '.
             if len(sys.argv) == 5:
312
                 output_dir = sys.argv[4]
314
            x, y, z = get_problem_parameters()
            data = generate\_lp\_file\left(\_TITLE\_\_, \ get\_author\_string\left(\right), \ x, \ y, \ z\right)
316
            filename = os.path.join(output_dir, get_lp_filename(x, y, z))
318
             save_lp_file (filename, data)
             print("Success: saved as '{0}'".format(filename))
320
   if __name__ == "__main__":
       main()
```

$4.1.2 \quad \text{src/lp_csv.py}$

This script is responsible for converting the output of the CPLEX log files into a single CSV file for further processing.

```
# lp_csv.py
 #
 # COSC364 Assignment 2
 # 30/05/2019
  # Written by Will Cowper, Jesse Sheehan
  import csv
 import sys
  import os.path
  def csvWrite(data):
      with open(sys.argv[2], 'a', newline='') as csvFile:
          writer = csv.writer(csvFile)
14
          writer.writerow(data)
16
  def floatmap (enumerable):
      return list (map(lambda x: float(x), enumerable))
20
  def openFile(Y):
      with open(os.path.join(sys.argv[1], '{0}.txt'.format(Y)), 'r') as
     in_file:
          stripped = [line.strip() for line in in_file.readlines()]
24
          lines = [line for line in stripped if line]
          data = []
26
          # Y
          data.append(Y)
28
          # elapsed time
          data.append(max(parseFile("elapsed_", lines)))
30
          # no of non-zero c and d links
          data.append(len(parseFile("c_", lines)) + len(parseFile("d_", lines
     )))
          # transit load spread (largest_transit_node_load -
     smallest_transit_node_load)
          data.append(max(floatmap(parseFile("l_", lines))) -
                       min(floatmap(parseFile("l_", lines))))
          # highest cap c network
36
          data.append(max(parseFile("c_", lines)))
          # highest cap d network
38
          data.append(max(parseFile("d_", lines)))
          csvWrite(data)
40
42
  "". Returns a list of all values that start with the given string ""
  def parseFile(string, lines):
46
      values = []
      for line in lines:
48
          if line.startswith(string):
              values.append(line.split()[1])
50
```

```
return values
52
54
  if _-name_- = "_-main_-":
      if len(sys.argv) != 3:
56
           print("Usage: {0} <input directory > <csv file >".format(sys.argv[0])
           \operatorname{exit}(-1)
58
      # delete the CSV, otherwise we will append to it
60
      os.unlink(sys.argv[2])
62
      openFile(3)
      openFile(4)
64
      openFile(5)
      openFile (6)
66
      openFile(7)
      openFile(8)
68
      print("Saved CSV data to '{}'".format(sys.argv[2]))
70
```

 $../\mathrm{src/lp_csv.py}$

4.1.3 src/lp_graph.py

This script is responsible for reading the CSV file and producing several graphs.

```
# lp_graph.py
  #
  # COSC364 Assignment 2
4 # 30/05/2019
  # Written by Will Cowper, Jesse Sheehan
  import csv
  import sys
  import os.path
10
  try:
      import numpy as np
12
  except:
      print("Error: could not load 'numpy'. Install with 'pip install numpy'
14
     and then try again.")
      exit(-1)
16
  try:
      import matplotlib.pyplot as plt
      print ("Error: could not load 'matplotlib'. Install with 'pip install
      matplotlib' and then try again.")
      exit(-1)
  def get_data(data, key):
      return list (map(lambda d: d[key], data))
26
28
```

```
def get_time(data):
      return get_data(data, "time")
30
  def get_len_nonzero_links(data):
      return get_data(data, "len_links")
34
  def get_transit_load_spread(data):
      return get_data(data, "load_spread")
38
40
  def get_max_cap_c (data):
      return get_data(data, "max_cap_c")
42
44
  def get_max_cap_d(data):
      return get_data(data, "max_cap_d")
46
48
  def get_Y(data):
      return get_data(data, "Y")
50
  def save_execution_time_plot(filename, data):
      """ Saves a plot of execution time. """
54
      plt.bar(get_Y(data), get_time(data))
      plt.xlabel("Y")
56
      plt.ylabel("Time (ms)")
      plt.title("CPLEX Execution Time")
      plt.savefig (filename)
      plt.close()
60
      print("Saved '{}'".format(filename))
62
  def save_num_nonzero_links_plot(filename, data):
64
      """ Saves a plot of the number of non-zero links.
      plt.bar(get_Y(data), get_len_nonzero_links(data))
      plt.xlabel("Y")
plt.ylabel("")
68
      plt.title("Number of Non-Zero Link Capacities")
      plt.savefig(filename)
70
      plt.close()
      print("Saved '{}'".format(filename))
72
  def save_transit_load_spread_plot(filename, data):
      """ Saves a plot of the transit load spread. """
      plt.bar(get_Y(data), get_transit_load_spread(data))
      plt.xlabel("Y")
78
      plt.ylabel("Load")
      plt.title("Transit Node Load Spread")
80
      plt.savefig (filename)
      plt.close()
      print("Saved '{}'".format(filename))
84
86 def save_highest_capacity_links_plot(filename, data):
```

```
""" Saves a plot of the transit load spread. """
       width = 0.4
       Ys = np.array(get_Y(data))
       cs = plt.bar(Ys, get_max_cap_c(data), width, label="$c_{ik}$")
90
       ds = plt.bar(Ys + width, get_max_cap_d(data), width, label="$d_{kj}$")
       plt.xticks(Ys + width / 2, map(lambda x: int(x), Ys))
       plt.legend(handles=[cs, ds])
       plt.xlabel("Y")
plt.ylabel("Link Capacity")
       plt.title("Highest Link Capacities")
96
       plt.savefig(filename)
       plt.close()
       print("Saved '{} '".format(filename))
100
  def get_data_from_csv(csv_filename):
       """ Returns an array of dictionaries containing the CSV data. """
       with open(csv_filename, newline='') as csv_file:
           csv_reader = csv.DictReader(csv_file, fieldnames=[
                                        "Y", "time", "len_links", "load_spread"
106
      , "max_cap_c", "max_cap_d"])
           rows = []
           for row in csv_reader:
               d = \{\}
               for key in row:
                   d[key] = float(row[key])
               rows.append(d)
112
           return rows
114
  def convert_csv_to_images(csv_filename, output_folder):
       """ Converts the data from the CSV into a set of graphs. """
       data = get_data_from_csv(csv_filename)
118
       base_filename = os.path.splitext(os.path.join(
           output_folder, os.path.basename(csv_filename)))[0]
       save_execution_time_plot(base_filename + "_time.png", data)
       save_num_nonzero_links_plot(base_filename + "_num_nonzero_links.png",
      data)
124
       save_transit_load_spread_plot(
           base_filename + "_transit_load_spread.png", data)
       save_highest_capacity_links_plot(
126
           base_filename + "_highest_capacity_links.png", data)
128
  def print_usage():
130
       print("Usage: {0} <csv file > <output folder>")
  if __name__ == "__main__":
134
       if len(sys.argv) != 3:
           print_usage()
136
           exit(-1)
138
       convert_csv_to_images(sys.argv[1], sys.argv[2])
```

4.1.4 output.sh

This BASH script is responsible for executing the other scripts as well as timing and running CPLEX (under the Linux operating system).

```
#!/bin/bash
for y in 3 4 5 6 7 8

do

python3 src/lp_gen.py 9 $y 9 lp_files
    start=$(date +%s%N)

cplex -c "read lp_files/problem_9_$ {y}_9.lp" "optimize" "display
    solution variables -" > cplex_logs/$y.txt
    end=$(date +%s%N)

duration=$(expr $end - $start)
    duration=$(expr $duration / 1000000)
    echo -e "\nelapsed_time: $duration ms" >> cplex_logs/$y.txt

done

python3 src/lp_csv.py cplex_logs cplex_data.csv
    python3 src/lp_graph.py cplex_data.csv graphs
```

../output.sh

4.1.5 output.ps1

This PowerShell script is responsible for executing the other scripts as well as timing and running CPLEX (under the Windows operating system).

```
For ($i=3; $i -le 8; $i++) {
    python src/lp_gen.py 9 $i 9 lp_files
    $perf = Measure-Command -Expression {$data = cplex -c ("read lp_files/
    problem_9_" + $i + "_9.lp") "optimize" "display solution variables -"}

$ms = $perf.TotalMilliseconds
    [System.IO.File]::WriteAllLines("cplex_logs/$i.txt", $data + "'
    nelapsed_time: $ms ms")

}

python src/lp_csv.py cplex_logs lp_files/cplex_data.csv
python src/lp_graph.py lp_files/cplex_data.csv graphs
```

../output.ps1

4.2 Generated LP File

4.2.1 lp_files/problem_3_2_4.lp

```
\ COSC-364 Assignment 2 LP Generator, LP Output File
\ Written by Will Cowper (81163265), Jesse Sheehan (53366509)
\ Parameters: X=3, Y=2, Z=4, Split=2, Demand=2 * i + j

MINIMIZE

r

SUBJECT TO
```

```
\ DEMAND CONSTRAINTS
               x_{-}111 + x_{-}121 = 3
12
              x_{-}112 + x_{-}122 = 4
              x_{-}113 + x_{-}123 = 5
              x_{-}114 + x_{-}124 = 6
              x_211 + x_221 = 5
              x_212 + x_222 = 6
              x_{-}213 + x_{-}223 = 7
               x_{2}14 + x_{2}24 = 8
              x_311 + x_321 = 7
20
              x_{-}312 + x_{-}322 = 8
              x_313 + x_323 = 9
              x_314 + x_324 = 10
24
              \ CAPACITY CONSTRAINTS FOR LINKS BEIWEEN SOURCE AND TRANSIT NODES
              x_1111 + x_1112 + x_1113 + x_1114 - c_111 = 0
              x_121 + x_122 + x_123 + x_124 - c_12 = 0
              x_211 + x_212 + x_213 + x_214 - c_21 = 0
28
              x_{2}21 + x_{2}22 + x_{2}23 + x_{2}24 - c_{2}2 = 0
              x_{-}311 + x_{-}312 + x_{-}313 + x_{-}314 - c_{-}31 = 0
              x_{-}321 + x_{-}322 + x_{-}323 + x_{-}324 - c_{-}32 = 0
               \ CAPACITY CONSTRAINTS FOR LINKS BEIWEEN TRANSIT AND DESTINATION NODES
              x_1111 + x_2111 + x_3111 - d_111 = 0
34
              x_1112 + x_2112 + x_3112 - d_112 = 0
              x_113 + x_213 + x_313 - d_13 = 0
36
              x_{1}14 + x_{2}14 + x_{3}14 - d_{1}14 = 0
              x_121 + x_221 + x_321 - d_21 = 0
              x_122 + x_222 + x_322 - d_22 = 0
              x_{-}123 + x_{-}223 + x_{-}323 - d_{-}23 = 0
              x_124 + x_224 + x_324 - d_24 = 0
               \ OBJECTIVE FUNCTION LOAD CONSTRAINTS
              c_{-}11 + c_{-}21 + c_{-}31 - r <= 0
44
              c_12 + c_22 + c_32 - r \le 0
               c_{-}13 + c_{-}23 + c_{-}33 - r \le 0
46
               c_{14} + c_{24} + c_{34} - r <= 0
48
               \ TRANSIT NODE LOAD CONSTRAINTS
50
              x_1111 + x_2112 + x_1113 + x_2114 + x_2111 + x_2112 + x_2113 + x_2114 + x_3111 + x_1111 + x_2111 + x_2111 + x_3111 + x_41111 + x_4111 + 
                   x_{-}312 \ + \ x_{-}313 \ + \ x_{-}314 \ - \ l_{-}1 \ = \ 0
              x_121 + x_122 + x_123 + x_124 + x_221 + x_222 + x_223 + x_224 + x_321 + x_124 + x_12
                   x_322 + x_323 + x_324 - l_2 = 0
               \ BINARY VARIABLE AND DECISION VARIABLE CONSTRAINTS
              2 x_{1}11 - 3 u_{1}11 = 0
              2 x_{1}12 - 4 u_{1}12 = 0
              2 x_{-}113 - 5 u_{-}113 = 0
              2 x_{-}114 - 6 u_{-}114 = 0
              2 x_{-}121 - 3 u_{-}121 = 0
58
              2 x_{1}22 - 4 u_{1}22 = 0
              2 x_{1}23 - 5 u_{1}23 = 0
60
              2 x_{1}24 - 6 u_{1}24 = 0
              2 x_{2}11 - 5 u_{2}11 = 0
              2 x_{2}12 - 6 u_{2}12 = 0
              2 x_{-}213 - 7 u_{-}213 = 0
              2 x_{-}214 - 8 u_{-}214 = 0
              2 x_{-}221 - 5 u_{-}221 = 0
```

```
2 x_{-}222 - 6 u_{-}222 = 0
     2 x_{2}3 - 7 u_{2}3 = 0
68
     2 x_{2}4 - 8 u_{2}4 = 0
     2 x_{3}11 - 7 u_{3}11 = 0
70
     2 x_{-}312 - 8 u_{-}312 = 0
     2 x_{3}13 - 9 u_{3}13 = 0
     2 x_{3}14 - 10 u_{3}14 = 0
     2 x_{3}21 - 7 u_{3}21 = 0
     2 x_{3}22 - 8 u_{3}22 = 0
     2 x_{3}23 - 9 u_{3}23 = 0
76
     2 x_{3}24 - 10 u_{3}24 = 0
     \ BINARY VARIABLE CONSTRAINTS (ONLY 2 ACTIVE TRANSIT NODES)
     u_{-}111 + u_{-}121 = 2
80
     u_{-}112 + u_{-}122 = 2
     u_{-}113 + u_{-}123 = 2
     u_1114 + u_1124 = 2
     u_{-}211 + u_{-}221 = 2
84
     u_{-}212 + u_{-}222 = 2
     u_{-}213 + u_{-}223 = 2
     u_{-}214 + u_{-}224 = 2
     u_{-}311 + u_{-}321 = 2
     u_{-}312 + u_{-}322 = 2
     u_{-}313 + u_{-}323 = 2
     u_{-}314 + u_{-}324 = 2
92
   BOUNDS
94
     \ NON-NEGATIVITY CONSTRAINTS
     r >= 0
96
     x_1111 >= 0
     x_1112 >= 0
     x_1113 >= 0
     x_1114 >= 0
100
     x_121 >= 0
     x_122 >= 0
     x_123 >= 0
     x_124 >= 0
104
     x_{-}211 >= 0
106
     x_212 >= 0
     x_213 >= 0
     x_214 >= 0
108
     x_{-}221 >= 0
     x_222 >= 0
110
     x_223 >= 0
     x_224 >= 0
     x_311 >= 0
     x_312 >= 0
114
     x_{-}313 >= 0
     x_314 >= 0
116
     x_321 >= 0
     x_322 >= 0
118
     x_323 >= 0
     x_324 >= 0
     c_{-}11 >= 0
     c_{-}12 >= 0
     c_{-}21 >= 0
     c_{-}22 >= 0
```

```
c_{-}31 >= 0
       c_{-}32 >= 0
126
       d_{-}11 >= 0
       d_{-}12 >= 0
128
       d_{-}13 >= 0
       d_{-}14 >= 0
130
       d_{-}21 >= 0
       d_{\,{\scriptscriptstyle -}}22 \;>=\; 0
       d_{-}23 >= 0
       d_{-}24 >= 0
134
136 BIN
       \ BINARY VARIABLES
138
       u_1111
       u\_112
       u_{-}113
       u_{-}114
142
       u_-121
       u_{-}122
144
       u_{-}123
       u_124
146
       u\_211
       u\_212
       u\_213
       u\_214
150
       u\_221
       u_{-}222
152
       u_{-}223
       u_{-}224
154
       u\_311
       u_312
       u_313
       u_314
158
       u\_321
       u_322
160
       u_323
       u_{-}324
164 END
```

 $../lp_files/problem_3_2_4.lp$