FLOW PLANNING

ASSIGNMENT 2

COSC364-19S1 INTERNET TECHNOLOGY AND ENGINEERING

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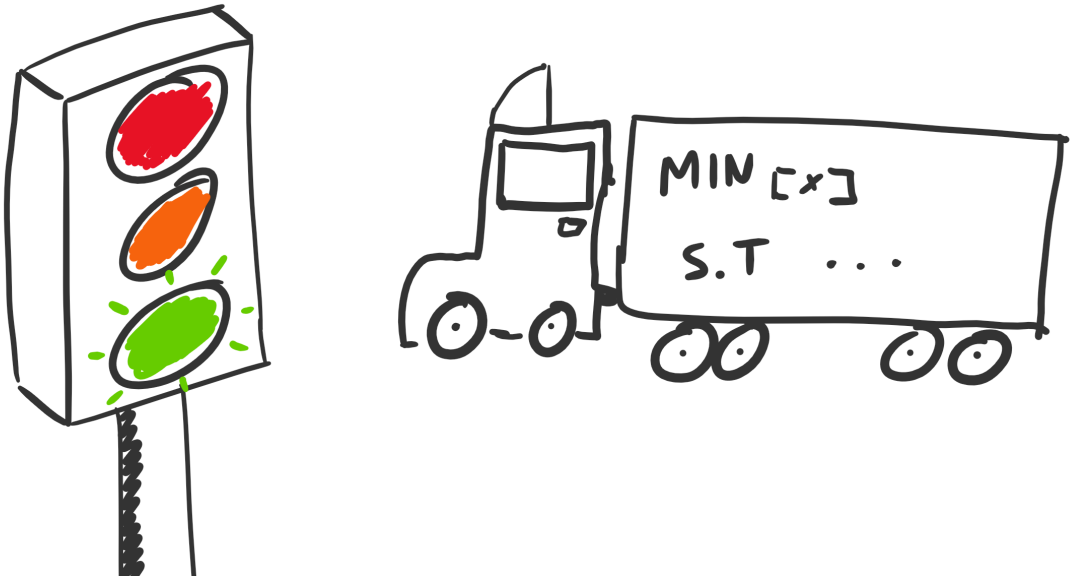

Figure 1: An artist's impression of a traffic problem outside of the Jack Erskine building (J. P. Sheehan, May 2019).

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1 Problem Description

Given a network (figure **??**) with X source nodes, Y transit nodes and Z destination nodes, a program was designed to generate an LP file that could be used by CPLEX to determine certain network characteristics.

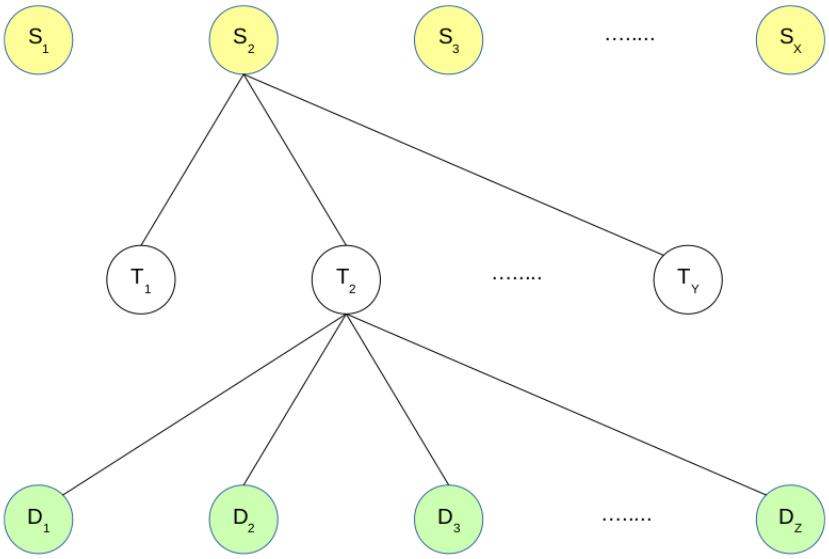

Figure 2: An example network (A. Willig, April 2019).

Traffic travelling from S_i to D_j must travel through exactly 2 transit nodes with a total demand volume of h_{ij} (equation **??**). Furthermore, the load upon each transit node must be balanced.

2 Problem Formulation

This problem was solved with the use of binary variable constraints (equations **??**, **??** and **??**) and the minimisation of our objective function (equation **??**). All normal non-negativity constraints were applied (equations **??**, **??**, **??** and **??**).

The following network properties were solved for:

- The capacities of each link (equations **??** and **??**).
- The load on each transit node (equation **??**).
- The value of each flow (equations **??** and **??**).

Notation:

- X is the number of source nodes.
- Y is the number of transit nodes.
- Z is the number of destination nodes.
- S_i is the ith source node.

- T_k is the kth transit node.
- D_j is the jth destination node.
- h_{ij} is the demand flow between S_i and D_j. This is equal to $2i + j$.
- c_{ik} is the link capacity between S_i and T_k.
- d_{kj} is the link capacity between T_k and D_j.
- x_{ikj} is the decision variable associated with the path S_i-T_k-D_j.
- u_{ikj} is the binary decision variable associated with x_{ikj}. These are required because h_{ij} must be split across exactly 2 transit nodes.
- l_k is the load on T_k.

Note: Due to the limitations of the LP file format, many of the following equations must be rearranged for use in CPLEX. Most notably, there cannot be any variables on the right hand side of an equality or inequality.

2.1 Objective Function

$$\text{minimize}_{[x,c,d,r]}\ r \tag{1}$$

2.2 Constraints

$$\sum_{k=1}^{Y} x_{ikj} = h_{ij} \qquad i \in \{1,\ldots,X\}, j \in \{1,\ldots,Z\} \tag{2}$$

$$\sum_{j=1}^{Z} x_{ikj} = c_{ik} \qquad i \in \{1,\ldots,X\}, k \in \{1,\ldots,Y\} \tag{3}$$

$$\sum_{i=1}^{X} x_{ikj} = d_{kj} \qquad k \in \{1,\ldots,Y\}, j \in \{1,\ldots,Z\} \tag{4}$$

$$\sum_{k=1}^{Y} x_{ikj} = l_{k} \qquad i \in \{1,\ldots,X\}, j \in \{1,\ldots,Z\} \tag{5}$$

$$\sum_{k=1}^{Y} u_{ikj} = 2 \qquad i \in \{1,\ldots,X\}, j \in \{1,\ldots,Z\} \tag{6}$$

$$x_{ikj} = \frac{u_{ikj} h_{ij}}{2} \qquad i \in \{1,\ldots,X\}, k \in \{1,\ldots,Y\}, j \in \{1,\ldots,Z\} \tag{7}$$

$$\sum_{i=1}^{X}\sum_{j=1}^{Z} x_{ikj} \leq r \qquad k \in \{1,\ldots,Y\} \tag{8}$$

$$u_{ikj} \in \{0,1\} \qquad i \in \{1,\ldots,X\}, k \in \{1,\ldots,Y\}, j \in \{1,\ldots,Z\} \tag{9}$$

$$h_{ij} = 2i + j \qquad i \in \{1,\ldots,X\}, j \in \{1,\ldots,Z\} \tag{10}$$

2.3 Non-Negativity Constraints

$$r \geq 0 \tag{11}$$

$$x_{ikj} \geq 0 \qquad i \in \{1,\ldots,X\}, k \in \{1,\ldots,Y\}, j \in \{1,\ldots,Z\} \tag{12}$$

$$c_{ik} \geq 0 \qquad i \in \{1,\ldots,X\}, k \in \{1,\ldots,Y\} \tag{13}$$

$$d_{kj} \geq 0 \qquad k \in \{1,\ldots,Y\}, j \in \{1,\ldots,Z\} \tag{14}$$

3 Results

LP files were generated with parameters $X = Z = 9, Y \in \{3, 4, 5, 6, 7, 8\}$. These were then processed with CPLEX, recording the time taken to solve each problem. Important data points were extracted from the CPLEX output and are listed in table **??**.

| ... your table ... |

Table 1: insert caption here, yo!

4 Appendix

4.1 Source Code

4.1.1 src/lp_gen.py

This script is responsible for producing a valid LP file from the given command line parameters.

```
import inspect
import functools
import sys
import os.path

__TITLE__ = "COSC-364 Assignment 2 LP Generator"
__AUTHORS__ = [("Will Cowper", "81163265"), ("Jesse Sheehan", "53366509")]

# Change these variables to alter the behaviour of the LP file generator
PATH_SPLIT = 2


def DEMAND_FLOW(i, j): return 2 * i + j


TEMPLATE = """\
\\ {}, LP Output File
\\ Written by {}
\\ Parameters: X={}, Y={}, Z={}, Split={}, Demand={}

MINIMIZE
\tr

SUBJECT TO

\t\\ DEMAND CONSTRAINTS
\t{}

\t\\ CAPACITY CONSTRAINTS FOR LINKS BETWEEN SOURCE AND TRANSIT NODES
\t{}

\t\\ CAPACITY CONSTRAINTS FOR LINKS BETWEEN TRANSIT AND DESTINATION NODES
\t{}

\t\\ OBJECTIVE FUNCTION LOAD CONSTRAINTS
\t{}
```

```
\t\\ TRANSIT NODE LOAD CONSTRAINTS
\t{}

\t\\ BINARY VARIABLE AND DECISION VARIABLE CONSTRAINTS
\t{}

\t\\ BINARY VARIABLE CONSTRAINTS (ONLY 2 ACTIVE TRANSIT NODES)
\t{}

BOUNDS

\t\\ NON-NEGATIVITY CONSTRAINTS
\tr >= 0
\t{}

BIN

\t\\ BINARY VARIABLES
\t{}

END
"""

# DEFINE SOME UTILITY FUNCTIONS


def get_lp_filename(x, y, z):
    """ Returns the filename that the LP data should be saved to. """
    return "problem_{0}_{1}_{2}.lp".format(x, y, z)


def crange(first, last):
    """ Returns a list of characters between the two characters passed in (
    inclusive).
    >>> crange('A', 'C')
    ['A', 'B', 'C']
    >>> crange('A', 'A')
    ['A']
    """
    if ord(first) > ord(last):
        raise ValueError("last must come after first")

    else:
        return [chr(i) for i in range(ord(first), ord(last) + 1)]


def repeat(obj, n):
    """ Returns a list with obj repeated n times.
    >>> repeat(1, 1)
    [1]
    >>> repeat(42, 0)
    []
    >>> repeat(5, 4)
    [5, 5, 5, 5]
    >>> repeat([1, 2], 2)
    [[1, 2], [1, 2]]
    """
    return [obj for _ in range(n)]
```

```
96
def perms(lists):
98      """ Returns all the permutations of the elements.
        >>> perms([])
100     []
        >>> perms([['a', 'b', 'c']])
102     [('a',), ('b',), ('c',)]
        >>> perms([['a', 'b', 'c'], ['x', 'y', 'z']])
104     [('a', 'x'), ('a', 'y'), ('a', 'z'), ('b', 'x'), ('b', 'y'), ('b', 'z')
        , ('c', 'x'), ('c', 'y'), ('c', 'z')]
        """
106     if len(lists) == 0:
            return []
108
        elif (len(lists) == 1):
110         return [(x,) for x in lists[0]]

112     else:
            return [(x,) + y for x in lists[0] for y in perms(lists[1:])]
114


116 def concat(permutations):
        """ Returns the permutations concatenated as strings.
118     >>> concat(perms([['a', 'b', 'c']]))
        ['a', 'b', 'c']
120     >>> concat(perms([['a', 'b', 'c'], ['x', 'y', 'z']]))
        ['ax', 'ay', 'az', 'bx', 'by', 'bz', 'cx', 'cy', 'cz']
122     """
        return [functools.reduce(lambda x, y: x + str(y), p, '') for p in
        permutations]
124


126 def get_function_source(fn):
        src = inspect.getsource(fn)
128     return src[str(src).index('return')+7:]


130
def get_lines(strings):
132     return '\n\t'.join(strings)

134 # DEFINE SOME FUNCTIONS SPECIFIC TO THE PROBLEM


136
def get_nodes(x, y, z):
138     """ Returns a tuple containing the source, transit and destination node
        ids as integers. """
        s = list(range(1, x + 1))
140     t = list(range(1, y + 1))
        d = list(range(1, z + 1))
142     return s, t, d


144
def get_demand_constraints(s, t, d):
146     """ Returns a list of demand constraints. """
        return [' + '.join(["x_{0}{1}{2}".format(i, k, j) for k in t]) + ' =
        {0}'.format(DEMAND_FLOW(i, j))
148             for (i, j) in perms([s, d])]
```

```
150
151 def get_source_transit_capacity_constraints(s, t, d):
152     """ Returns a list of capacity constraints for the links between the
        source and transit nodes. """
153     return \
154         [' + '.join(["x_{0}{1}{2}".format(i, k, j) for j in d]) +
155             ' - c_{0}{1} = 0'.format(i, k) for (i, k) in perms([s, t])]
156
157
158 def get_transit_destination_capacity_constraints(s, t, d):
159     """ Returns a list of capacity constraints for the links between the
        transit and destination nodes. """
160     return \
161         [' + '.join(["x_{0}{1}{2}".format(i, k, j) for i in s]) +
162             ' - d_{0}{1} = 0'.format(k, j) for (k, j) in perms([t, d])]
163
164
165 def get_transit_load_constraints(s, t, d):
166     """ Returns the list of transit load constraints. """
167     return [' + '.join(["x_{0}{1}{2}".format(i, k, j) for (i, j) in perms([
        s, d])]) +
168             ' - l_{0} = 0'.format(k) for k in t]
169
170
171 def get_objective_function_load_constraints(s, t, d):
172     """ Returns the list of objective function load constraints. """
173     return [' + '.join(["c_{0}{1}".format(i, j) for i in s]) +
174             ' - r <= 0' for j in d]
175
176
177 def get_binary_and_decision_variable_constraints(s, t, d):
178     """ Returns the binary and decision variable constraints. """
179     return ['{3} x_{0}{1}{2} - {4} u_{0}{1}{2} = 0'.format(i, k, j,
        PATH_SPLIT, DEMAND_FLOW(i, j)) for (i, k, j) in perms([s, t, d])]
180
181
182 def get_binary_constraints(s, t, d):
183     """ Returns a list of binary variable constraints. """
184     return [' + '.join(["u_{0}{1}{2}".format(i, k, j) for k in t]) + ' = {}
        '.format(PATH_SPLIT)
185             for (i, j) in perms([s, d])]
186
187
188 def get_binary_variables(s, t, d):
189     """ Returns a list of binary variables. """
190     return ["u_{0}{1}{2}".format(i, k, j) for (i, k, j) in perms([s, t, d])
        ]
191
192
193 def get_non_negativity_constraints(s, t, d):
194     """ Returns a list of non-negativity constraints. """
195     return ["x_{0}{1}{2} >= 0".format(i, k, j) for (i, k, j) in perms([s, t
        , d])] + ["c_{0}{1} >= 0".format(i, k) for (i, k) in perms([s, t])] + ["
        d_{0}{1} >= 0".format(k, j) for (k, j) in perms([t, d])]
196
197
198 def generate_lp_file(title, authors, x, y, z):
```

```
    """ Returns the LP file contents as per the project specification. """
    s, t, d = get_nodes(x, y, z)

    demand_constraints = get_lines(get_demand_constraints(s, t, d))
    source_transit_capacity_constraints = get_lines(
        get_source_transit_capacity_constraints(s, t, d))
    transit_destination_capacity_constraints = get_lines(
        get_transit_destination_capacity_constraints(s, t, d))
    non_negativity_constraints = get_lines(get_non_negativity_constraints(
        s, t, d))
    objective_function_load_constraints = get_lines(
        get_objective_function_load_constraints(s, t, d))
    transit_load_constraints = get_lines(
        get_transit_load_constraints(s, t, d))
    binary_and_decision_constraints = get_lines(
        get_binary_and_decision_variable_constraints(s, t, d))
    binary_variable_constraints = get_lines(get_binary_constraints(s, t, d)
    )
    binary_variables = get_lines(get_binary_variables(s, t, d))

    return TEMPLATE.format(
        title,
        authors,
        x,
        y,
        z,
        PATH_SPLIT,
        get_function_source(DEMAND_FLOW),
        demand_constraints,
        source_transit_capacity_constraints,
        transit_destination_capacity_constraints,
        objective_function_load_constraints,
        transit_load_constraints,
        binary_and_decision_constraints,
        binary_variable_constraints,
        non_negativity_constraints,
        binary_variables)


# DEFINE SOME HELPERS FOR GETTING THE THING RUNNING

def print_version():
    print('{0} by {1}'.format(__TITLE__, get_author_string()))


def print_usage():
    print('Usage: {0} <x> <y> <z> [output directory]'.format(sys.argv[0]))


def get_problem_parameters():
    """ Returns a tuple containing the x, y and z parameters. """
    try:
        x = int(sys.argv[1])
        y = int(sys.argv[2])
        z = int(sys.argv[3])
    except:
        print_usage()
        exit(-1)
```

```
256
    if x <= 0:
258         print("Error: x must be strictly positive")
            exit(-1)
260
    if x >= 10:
262         print("Error: x must be less than ten")
            exit(-1)
264
    if y <= 0:
266         print("Error: y must be strictly positive")
            exit(-1)
268
    if y >= 10:
270         print("Error: y must be less than ten")
            exit(-1)
272
    if z <= 0:
274         print("Error: z must be strictly positive")
            exit(-1)
276
    if z >= 10:
278         print("Error: z must be less than ten")
            exit(-1)
280
    return x, y, z
282


284 def save_lp_file(filename, data):
    try:
286         f = open(filename, 'w')
            f.write(data)
288         f.close()
    except:
290         print("Error: could not save file '{0}'".format(filename))
            exit(-1)
292


294 def get_author_string():
    return ', '.join(
296         ["{0} ({1})".format(name, sid) for (name, sid) in __AUTHORS__])


298
def main():
300     print_version()
    if len(sys.argv) != 4 and len(sys.argv) != 5:
302         print_usage()
            exit(-1)
304     else:
            output_dir = '.'
306         if len(sys.argv) == 5:
                output_dir = sys.argv[4]
308
            x, y, z = get_problem_parameters()
310         data = generate_lp_file(__TITLE__, get_author_string(), x, y, z)
            filename = os.path.join(output_dir, get_lp_filename(x, y, z))
312         save_lp_file(filename, data)
            print("Success: saved as '{0}'".format(filename))
```

```
314
316 if __name__ == "__main__":
        main()
```

../src/lp_gen.py

4.1.2 src/lp_csv.py

This script is responsible for converting the output of the CPLEX log files into a single CSV file for further processing.

```
import csv
import sys
import os.path


def csvWrite(data):
    with open(sys.argv[2], 'a') as csvFile:
        writer = csv.writer(csvFile)
        writer.writerow(data)

    return


def openFile(Y):
    with open(os.path.join(sys.argv[1], '{0}.txt'.format(Y)), 'r') as
    in_file:
        stripped = [line.strip() for line in in_file.readlines()]
        lines = [line for line in stripped if line]
        data = []
        # Y
        data.append(Y)
        # elapsed time
        data.append(max(parseFile("elapsed_", lines)))
        # no of non-zero c links
        data.append(len(parseFile("c_", lines)))
        # no of non-zero d links
        data.append(len(parseFile("d_", lines)))
        # smallest_transit_node_load
        data.append(min(parseFile("l_", lines)))
        # largest_transit_node_load
        data.append(max(parseFile("l_", lines)))
        # highest cap c network
        data.append(max(parseFile("c_", lines)))
        # highest cap d network
        data.append(max(parseFile("d_", lines)))
        print(data)
        csvWrite(data)

        return


'''Returns a list of all values that start with the given string'''


def parseFile(string, lines):
```

```
    values = []
    for line in lines:
        if line.startswith(string):
            values.append(line.split()[1])

    return values


if __name__ == "__main__":
    if len(sys.argv) != 3:
        print("Usage: {0} <input directory> <csv file>".format(sys.argv[0])
    )
        exit(-1)

    openFile(3)
    openFile(4)
    openFile(5)
    openFile(6)
    openFile(7)
    openFile(8)
```

../src/lp_csv.py

4.1.3 src/lp_graph.py

This script is responsible for reading the CSV file and producing several graphs.

```
import csv
import sys
import os.path
import numpy as np

try:
    import matplotlib.pyplot as plt
except:
    print("Error: could not load 'matplotlib'. Install with 'pip install
    matplotlib' and then try again.")
    exit(-1)


def get_data(data, key):
    return list(map(lambda d: d[key], data))


def get_time(data):
    return get_data(data, "time")


def get_len_nonzero_links(data):
    return list(map(lambda d: d["len_c_links"] + d["len_d_links"], data))


def get_transit_load_spread(data):
    return list(map(lambda d: d["max_load"] - d["min_load"], data))


def get_max_cap_c(data):
    return get_data(data, "max_cap_c")
```

```
32
33  def get_max_cap_d(data):
34      return get_data(data, "max_cap_d")
35
36
37  def get_Y(data):
38      return get_data(data, "Y")
39
40
41  def save_execution_time_plot(filename, data):
42      """ Saves a plot of execution time. """
43      plt.bar(get_Y(data), get_time(data))
44      plt.xlabel("Y")
45      plt.ylabel("Time (ms)")
46      plt.title("CPLEX Execution Time")
47      plt.savefig(filename)
48      plt.close()
49      print("Saved '{}'".format(filename))
50
51
52  def save_num_nonzero_links_plot(filename, data):
53      """ Saves a plot of the number of non-zero links. """
54      plt.bar(get_Y(data), get_len_nonzero_links(data))
55      plt.xlabel("Y")
56      plt.ylabel("")
57      plt.title("Number of Non-Zero Link Capacities")
58      plt.savefig(filename)
59      plt.close()
60      print("Saved '{}'".format(filename))
61
62
63  def save_transit_load_spread_plot(filename, data):
64      """ Saves a plot of the transit load spread. """
65      plt.bar(get_Y(data), get_transit_load_spread(data))
66      plt.xlabel("Y")
67      plt.ylabel("Load")
68      plt.title("Transit Node Load Spread")
69      plt.savefig(filename)
70      plt.close()
71      print("Saved '{}'".format(filename))
72
73
74  def save_highest_capacity_links_plot(filename, data):
75      """ Saves a plot of the transit load spread. """
76      width = 0.4
77      Ys = np.array(get_Y(data))
78      cs = plt.bar(Ys, get_max_cap_c(data), width, label="$C_{ik}$")
79      ds = plt.bar(Ys + width, get_max_cap_d(data), width, label="$D_{kj}$")
80      plt.xticks(Ys + width / 2, map(lambda x: int(x), Ys))
81      plt.legend(handles=[cs, ds])
82      plt.xlabel("Y")
83      plt.ylabel("Link Capacity")
84      plt.title("Highest Link Capacities")
85      plt.savefig(filename)
86      plt.close()
87      print("Saved '{}'".format(filename))
88
```

```
90 def get_data_from_csv(csv_filename):
92     """ Returns an array of dictionaries containing the CSV data. """
     with open(csv_filename, newline='') as csv_file:
94         csv_reader = csv.DictReader(csv_file, fieldnames=[
                                        "Y", "time", "len_c_links", "
    len_d_links", "min_load", "max_load", "max_cap_c", "max_cap_d"])
96         rows = []
           for row in csv_reader:
98             if csv_reader.line_num == 1:
                   continue
100            d = {}
               for key in row:
                   d[key] = float(row[key])
102            rows.append(d)
           return rows
104
106 def convert_csv_to_images(csv_filename, output_folder):
    """ Converts the data from the CSV into a set of graphs. """
108    data = get_data_from_csv(csv_filename)
       base_filename = os.path.splitext(os.path.join(
110        output_folder, os.path.basename(csv_filename)))[0]

112    save_execution_time_plot(base_filename + "_time.png", data)
       save_num_nonzero_links_plot(base_filename + "_num_nonzero_links.png",
    data)
114    save_transit_load_spread_plot(
           base_filename + "_transit_load_spread.png", data)
116    save_highest_capacity_links_plot(
           base_filename + "_highest_capacity_links.png", data)
118

120 def print_usage():
    print("Usage: {0} <csv file> <output folder>")
122

124 if __name__ == "__main__":
    if len(sys.argv) != 3:
126        print_usage()
           exit(-1)
128

    convert_csv_to_images(sys.argv[1], sys.argv[2])
```

../src/lp_graph.py

4.1.4 output.sh

This BASH script is responsible for executing the other scripts as well as timing and running CPLEX (under the Linux operating system).

```
#!/bin/bash
2 for y in 3 4 5 6 7 8
do
4    python3 src/lp_gen.py 9 $y 9 lp_files
     start=$(date +%s%N)
6    cplex -c "read lp_files/problem_9_${y}_9.lp" "optimize" "display
     solution variables -" > cplex_logs/$y.txt
```

```
    end=$(date +%s%N)
    duration=$(expr $end - $start)
    duration=$(expr $duration / 1000000)
    echo -e "\nelapsed_time: $duration ms" >> cplex_logs/$y.txt
done

python3 src/lp_csv.py cplex_logs cplex_data.csv
python3 src/lp_graph.py cplex_data.csv graphs
```

../output.sh

4.1.5 output.ps1

This PowerShell script is responsible for executing the other scripts as well as timing and running CPLEX (under the Windows operating system).

```
For ($i=3; $i -le 8; $i++) {
    python src/lp_gen.py 9 $i 9 lp_files
    $perf = Measure-Command -Expression {$data = cplex -c ("read lp_files/
    problem_9_" + $i + "_9.lp") "optimize" "display solution variables -"}
    $ms = $perf.TotalMilliseconds
    [System.IO.File]::WriteAllLines("cplex_logs/$i.txt", $data + "`
    nelapsed_time: $ms ms")
}

python src/lp_csv.py cplex_logs lp_files/cplex_data.csv
python src/lp_graph.py lp_files/cplex_data.csv graphs
```

../output.ps1

4.2 Generated LP File

4.2.1 lp_files/problem_3_2_4.lp

```
\ COSC-364 Assignment 2 LP Generator, LP Output File
\ Written by Will Cowper (81163265), Jesse Sheehan (53366509)
\ Parameters: X=3, Y=2, Z=4, Split=2, Demand=2 * i + j


MINIMIZE
  r

SUBJECT TO

  \ DEMAND CONSTRAINTS
  x_111 + x_121 = 3
  x_112 + x_122 = 4
  x_113 + x_123 = 5
  x_114 + x_124 = 6
  x_211 + x_221 = 5
  x_212 + x_222 = 6
  x_213 + x_223 = 7
  x_214 + x_224 = 8
  x_311 + x_321 = 7
  x_312 + x_322 = 8
  x_313 + x_323 = 9
```

```
x_314 + x_324 = 10

\ CAPACITY CONSTRAINTS FOR LINKS BETWEEN SOURCE AND TRANSIT NODES
x_111 + x_112 + x_113 + x_114 - c_11 = 0
x_121 + x_122 + x_123 + x_124 - c_12 = 0
x_211 + x_212 + x_213 + x_214 - c_21 = 0
x_221 + x_222 + x_223 + x_224 - c_22 = 0
x_311 + x_312 + x_313 + x_314 - c_31 = 0
x_321 + x_322 + x_323 + x_324 - c_32 = 0

\ CAPACITY CONSTRAINTS FOR LINKS BETWEEN TRANSIT AND DESTINATION NODES
x_111 + x_211 + x_311 - d_11 = 0
x_112 + x_212 + x_312 - d_12 = 0
x_113 + x_213 + x_313 - d_13 = 0
x_114 + x_214 + x_314 - d_14 = 0
x_121 + x_221 + x_321 - d_21 = 0
x_122 + x_222 + x_322 - d_22 = 0
x_123 + x_223 + x_323 - d_23 = 0
x_124 + x_224 + x_324 - d_24 = 0

\ OBJECTIVE FUNCTION LOAD CONSTRAINTS
c_11 + c_21 + c_31 - r <= 0
c_12 + c_22 + c_32 - r <= 0
c_13 + c_23 + c_33 - r <= 0
c_14 + c_24 + c_34 - r <= 0

\ TRANSIT NODE LOAD CONSTRAINTS
x_111 + x_112 + x_113 + x_114 + x_211 + x_212 + x_213 + x_214 + x_311 +
  x_312 + x_313 + x_314 - l_1 = 0
x_121 + x_122 + x_123 + x_124 + x_221 + x_222 + x_223 + x_224 + x_321 +
  x_322 + x_323 + x_324 - l_2 = 0

\ BINARY VARIABLE AND DECISION VARIABLE CONSTRAINTS
2 x_111 - 3 u_111 = 0
2 x_112 - 4 u_112 = 0
2 x_113 - 5 u_113 = 0
2 x_114 - 6 u_114 = 0
2 x_121 - 3 u_121 = 0
2 x_122 - 4 u_122 = 0
2 x_123 - 5 u_123 = 0
2 x_124 - 6 u_124 = 0
2 x_211 - 5 u_211 = 0
2 x_212 - 6 u_212 = 0
2 x_213 - 7 u_213 = 0
2 x_214 - 8 u_214 = 0
2 x_221 - 5 u_221 = 0
2 x_222 - 6 u_222 = 0
2 x_223 - 7 u_223 = 0
2 x_224 - 8 u_224 = 0
2 x_311 - 7 u_311 = 0
2 x_312 - 8 u_312 = 0
2 x_313 - 9 u_313 = 0
2 x_314 - 10 u_314 = 0
2 x_321 - 7 u_321 = 0
2 x_322 - 8 u_322 = 0
2 x_323 - 9 u_323 = 0
2 x_324 - 10 u_324 = 0

```

```
    \ BINARY VARIABLE CONSTRAINTS (ONLY 2 ACTIVE TRANSIT NODES)
80  u_111 + u_121 = 2
    u_112 + u_122 = 2
82  u_113 + u_123 = 2
    u_114 + u_124 = 2
84  u_211 + u_221 = 2
    u_212 + u_222 = 2
86  u_213 + u_223 = 2
    u_214 + u_224 = 2
88  u_311 + u_321 = 2
    u_312 + u_322 = 2
90  u_313 + u_323 = 2
    u_314 + u_324 = 2
92
    BOUNDS
94
    \ NON-NEGATIVITY CONSTRAINTS
96  r >= 0
    x_111 >= 0
98  x_112 >= 0
    x_113 >= 0
100 x_114 >= 0
    x_121 >= 0
102 x_122 >= 0
    x_123 >= 0
104 x_124 >= 0
    x_211 >= 0
106 x_212 >= 0
    x_213 >= 0
108 x_214 >= 0
    x_221 >= 0
110 x_222 >= 0
    x_223 >= 0
112 x_224 >= 0
    x_311 >= 0
114 x_312 >= 0
    x_313 >= 0
116 x_314 >= 0
    x_321 >= 0
118 x_322 >= 0
    x_323 >= 0
120 x_324 >= 0
    c_11 >= 0
122 c_12 >= 0
    c_21 >= 0
124 c_22 >= 0
    c_31 >= 0
126 c_32 >= 0
    d_11 >= 0
128 d_12 >= 0
    d_13 >= 0
130 d_14 >= 0
    d_21 >= 0
132 d_22 >= 0
    d_23 >= 0
134 d_24 >= 0

136 BIN
```

```
138    \ BINARY VARIABLES
       u_111
140    u_112
       u_113
142    u_114
       u_121
144    u_122
       u_123
146    u_124
       u_211
148    u_212
       u_213
150    u_214
       u_221
152    u_222
       u_223
154    u_224
       u_311
156    u_312
       u_313
158    u_314
       u_321
160    u_322
       u_323
162    u_324

164 END
```

../lp_files/problem_3_2_4.lp