Multi-level Power Quality Assessment towards Virtual Testing of More Electric Aircraft

Yang Ji
Department of System Dynamics and Control
Institute of Robotics and Mechatronics
German Aerospace Center
Muenchnerstr. 20, Wessling, Germany
Email: Yang.Ji@dlr.de

Johann Bals
Department of System Dynamics and Control
Institute of Robotics and Mechatronics
German Aerospace Center
Muenchnerstr. 20, Wessling, Germany
Email: Johann.Bals@dlr.de

***Abstract*—This paper introduces a multi-level power quality assessment approach towards virtual testing of more electric aircraft. Three levels with increasing accuracies are rough, approximated and exact power quality assessment. The requirement to the accuracy of power quality assessment strongly depends on different analysis jobs. With the multi-level power quality assessment approach, both sufficient accuracy and minimal cost can be ensured. Advanced steady-state analysis approaches for the switched power electronic systems with regard to the exact power quality assessment are introduced. The convergence performance of the applied steady-state analysis approaches is significantly improved by estimation of the state variables at steady-state using a generalized averaging technique. The proposed approach that has been implemented in Dymola is demonstrated by a pulse width modulated buck converter.**

I. Introduction

Aiming at global energy optimization of aircrafts, the concept More Electric Aircraft becomes more interesting for the aircraft developers. Concerning the complete development process of the future more electric aircraft, large scale testing of energy systems using virtual rigs gains increased attention. The virtual testing allows quickly checking the overall performance, anticipating any problem on the hardware and extrapolating results with a real hardware test. Within the virtual testing activities, the power quality assessment is a major analysis task. Depending on analysis tasks with different objectives, power quality assessments with diverse accuracies are needed. To get a rough knowledge on the system performance, the so-called mean value averaged model is often sufficient. At this power quality assessment level, only the fundamental frequency element is considered. For the next level, the generalized averaging technique can be used to achieve selective multi-frequency model from the original topological system model. Power quality assessment taking more system harmonics into account can be subsequently performed by the multi-frequency harmonic averaged model. For this purpose, a unified harmonic state space representation valid for a wide class of power electronic systems is presented to allow an automated generation of the averaged models with selective approximation accuracy. Finally, to fulfill the highest requirement of power quality analysis e.g. for EMC study and filter design, the exact finding for the power electronic waveforms is necessary. The conventional brute-force simulation for yielding the steady-state waveforms of a large scale system containing high frequency switching components is very expensive and suffering from numerical instabilities. The problem to find the steady-state waveform can efficiently be solved by an optimization based approach or a Quasi-Newton method. In order to improve the convergence performance when applying those steady-state finding techniques, the starting points are determined by previously simulating a set of multi-resolution approximations – averaged models from the original topological model using the generalized averaging technique.

II. Unified harmonic state-space description for power electronics systems

When establishing the mathematical models of power electronic systems, diverse modeling levels can be considered depending on applications.

- behavioural model is based on equations derived from the subsystem structure and electrical circuit. A behavioural model reflects both low and high frequency dynamics including switching effects serving to perform power quality simulation and analysis.
- Functional model is derived from the behavioural model by time averaging of high frequency periodical switching waveforms. A functional model reflects only low frequency behaviour of the original system excluding the switching ripple. The functional model is typically used for control design, stability study and transient performance analysis.
- The associated functional model can be extended by using generalized averaging technique. Here, the circuit state variables are approximate by a Fourier series expansion with time-dependent coefficients. This representation results in an unified time-invariant set of differential equations, where the state variables are the coefficients of the corresponding Fourier series of the circuit state variables

It is obvious, that the generalized averaging technique plays a central role for the multi-level power quality assessment. The generalized averaged models can be directly used for rough and approximated power assessment even without transformation in the frequency domain. Furthermore, the information derived

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from harmonic state space models is extremely important to successfully find the periodic steady-state solution i.e. the exact power quality analysis using the optimization based approach or the Quasi Newton method, which are much more efficient in contrast to conventional brute-force based simulation of the behavioural model. The classical state space averaging technique [1] has been widely used in the analysis and control design of pulse width modulated power electronics systems. Some limitations of practical consequence in consideration of DC offset performance and closed-loop stability are addressed in [2], [3], [4]. Applications of the classical state space averaging technique are subject to small ripple and sufficient high switching frequency in the object systems. To fix those drawbacks, the idea of the generalized averaging method for power electronic systems has been first introduced in [5]. Consequently, further developments and applications of the generalized averaging method have been carried out mostly by two working group in [6], [7]. The basic idea is to present the original signal $x(\tau)$ on the interval $\tau \in [t-T,t]$ by the Fourier series

$$x(\tau) = \sum_{\ell=-\infty}^{\infty} \langle x\rangle_{\ell}(t) e^{i\ell\omega\tau}$$

where $\omega = 2\pi/T$ and T is typically the shortest constant time interval of the switching components in system. The ℓth time varying complex Fourier coefficient can be obtained by

$$\langle x\rangle_{\ell}(t) = \frac{1}{T}\int_{t-T}^{t} x(\tau)e^{-i\ell\omega\tau}d\tau.$$

In spite of the quite detailed explanations with examples to the generalized averaging technique in literatures e.g. [6], [7], the readers could be easily led into the misunderstanding, that it is extremely complicated to establish the harmonic state space differential equations in terms of the time varying Fourier coefficients $\langle x\rangle_{\ell}(t)$ for power electronic systems. The reason is a missing unified state space representation which permits one to automatically apply the generalized averaging technique. It is well known, a wide class of power electronic systems e.g. DC/DC, DC/AC converters with a fixed switching time constant can be regarded as linear time periodic (LTP) systems which are written by

$$\begin{aligned} \dot{x} &= A(t)x + B(t)u \\ y &= C(t)x + D(t)u \end{aligned} \tag{1}$$

where $t \in [0,T]$ and A,B,C,D are piecewise periodic matrices such as

$$\begin{aligned} A(t+T) &= A(t), \qquad & B(t+T) &= B(t), \\ C(t+T) &= C(t), \qquad & D(t+T) &= D(t). \end{aligned}$$

The harmonic state space representation for LTP systems has been introduced in the work of [8], which is also feasible to describe a wide class of power electronic systems. The harmonic state space representations of LTP systems is derived by feeding a complex exponentially modulated periodic (EMP) signal

$$u(t) = \sum_{m=-\infty}^{\infty} u_m e^{(s+im\omega)t} \tag{2}$$

with $s \in \mathbb{C}$ to a LTP system in (1). The state response of the LTP system is also an EMP signal as

$$x(t) = \sum_{m=-\infty}^{\infty} x_m e^{(s+im\omega)t} \tag{3}$$

and the output can be described as linear combination of the state values such as

$$y(t) = \sum_{m=-\infty}^{\infty} y_m e^{(s+im\omega)t}. \tag{4}$$

Substituting the equations (2), (3) and (4) into (1) yields the following linear time invariant harmonic state space representation of LTP systems in Laplace domain for $n \in \mathbb{Z}$:

$$\begin{aligned} (s+in\omega)x_n &= \sum_{m=-\infty}^{\infty} A_{n-m}x_m + \sum_{m=-\infty}^{\infty} B_{n-m}u_m \\ y_n &= \sum_{m=-\infty}^{\infty} C_{n-m}x_m + \sum_{m=-\infty}^{\infty} D_{n-m}u_m \end{aligned} \tag{5}$$

The x_n is the nth time varying complex Fourier coefficient vector relating to the nth harmonic of the state vector. Analogously, u_m is referred to the mth harmonic of $u(t)$. A_{n-m} is obtained from the Fourier expansion of the system matrix $A(t)$, the same for B_{n-m}, C_{n-m}, and D_{n-m}. Defining a Toeplitz form matrix $\mathscr{A}_k$ in terms of the Fourier series coefficient matrix of the system matrix $A(t)$ such as

$$\mathscr{A}_k = \begin{bmatrix} A_0 & \cdots & A_{-k} & \cdots & A_{-2k} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ A_k & \cdots & A_0 & \cdots & A_{-k} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ A_{2k} & \cdots & A_k & \cdots & A_0 \end{bmatrix}$$

and similar expression for $B(t)$, $C(t)$ and $D(t)$ with corresponding Toeplitz matrices, $\mathscr{B}_k$, $\mathscr{C}_k$ and $\mathscr{D}_k$, a compact harmonic state space representation of an approximated LTP systems by $k \geq 0$ harmonics can be written as

$$\begin{aligned} s\mathscr{X}_k &= (\mathscr{A}_k - \mathscr{N}_k)\mathscr{X}_k + \mathscr{B}_k\mathscr{U}_k \\ \mathscr{Y}_k &= \mathscr{C}_k\mathscr{X}_k + \mathscr{D}_k\mathscr{U}_k. \end{aligned} \tag{6}$$

We use the notations

$$\mathscr{N}_k = \operatorname{blockdiag}\left(-ki\omega I,\ \ldots,\ -i\omega I,\ 0,\ i\omega I,\ \ldots,\ ki\omega I\right)$$

with $0, I \in \mathbb{R}^{\dim x \times \dim x}$ and the complex harmonic coefficients

$$\begin{aligned} \mathscr{X}_k &= \left[x_{-k}^T, \ldots, x_0^T, \ldots, x_k^T\right]^T, \\ \mathscr{Y}_k &= \left[y_{-k}^T, \ldots, y_0^T, \ldots, y_k^T\right]^T, \\ \mathscr{U}_k &= \left[u_{-k}^T, \ldots, u_0^T, \ldots, u_k^T\right]^T. \end{aligned}$$

Equation (6) has overall complex values which is cumbersome for describing systems having real inputs signals $\mathscr{U}_{real,k}$. For the sake of simplification, the desired real representation of (6) can be derived by introducing a transformation matrix $\mathscr{T}_{\mathscr{U},k} \in \mathbb{C}^{(2k+1)\cdot\dim u \times (2k+1)\cdot\dim u}$ [9] such that

$$\mathscr{U}_{real,k} = \mathscr{T}_{\mathscr{U},k}\mathscr{U}_k$$

where the transformation matrix has the form

$$\mathscr{T}_{\mathscr{U},k} = \begin{bmatrix} \frac{1}{2}I & & & 0 & & & \frac{1}{2}I \\ & \ddots & & & & \iddots & \\ & & \frac{1}{2}I & & \frac{1}{2}I & & \\ 0 & & & I & & & 0 \\ & & -\frac{1}{2}iI & & \frac{1}{2}iI & & \\ & \iddots & & & & \ddots & \\ -\frac{1}{2}iI & & & 0 & & & \frac{1}{2}iI \end{bmatrix}$$

with $I \in \mathbb{R}^{\dim u \times \dim u}$. The real input vector $\mathscr{U}_{real}$ is described by

$$\mathscr{U}_{real,k} = \left[u_{c,k}^T, \ldots, u_{c,1}^T, u_0^T, u_{s,1}^T, \ldots, u_{s,k}^T\right]^T$$

where $u_{c,m}$, $u_{s,m}$ $(m = 1, \ldots, k)$ correspond to the real and imaginary parts of the complex Fourier coefficients u_m. The real input signal u can be reconstructed by

$$u(t) = u_0 + \sum_{m=1}^{\infty} 2u_{c,m} \cos m\omega t - \sum_{m=1}^{\infty} 2u_{s,m} \sin m\omega t. \quad (7)$$

Analogously transforming complex state and output harmonic vectors in (6) by transformation matrices $\mathscr{T}_{\mathscr{X},k}$, $\mathscr{T}_{\mathscr{Y},k}$ with appropriate dimensions, the real harmonic state space representation of LTP systems approximated by k harmonics can be finally achieved as

$$\begin{aligned} s\mathscr{X}_{real,k} &= \mathscr{A}_{real,k}\mathscr{X}_{real,k} + \mathscr{B}_{real,k}\mathscr{U}_{real,k} \\ \mathscr{Y}_{real,k} &= \mathscr{C}_{real,k}\mathscr{X}_{real,k} + \mathscr{D}_{real,k}\mathscr{U}_{real,k} \end{aligned} \quad (8)$$

with

$$\begin{aligned} \mathscr{A}_{real,k} &= \mathscr{T}_{\mathscr{X},k}(\mathscr{A}_k - \mathscr{N}_k)\mathscr{T}_{\mathscr{X},k}^{-1}, \\ \mathscr{B}_{real,k} &= \mathscr{T}_{\mathscr{X},k}\mathscr{B}_k\mathscr{T}_{\mathscr{U},k}^{-1}, \\ \mathscr{C}_{real,k} &= \mathscr{T}_{\mathscr{Y},k}\mathscr{C}_k\mathscr{T}_{\mathscr{X},k}^{-1}, \\ \mathscr{D}_{real,k} &= \mathscr{T}_{\mathscr{Y},k}\mathscr{D}_k\mathscr{T}_{\mathscr{U},k}^{-1}. \end{aligned} \quad (9)$$

The equation (8) provides an unified formalization for the generalized averaging modeling of a wide class power electronic systems, which are approximated by k harmonics. By using symbolic toolbox e.g. Maple or MATLAB, the generalized averaging modeling can be easily computed for arbitrary harmonic approximation, once the periodic state-space description in (1) is known. As result, the approaches to derive harmonic state-space representation discussed in [5], [6], [7] can be significantly simplified. The second level power quality assessment can be directly performed by the simulation of the harmonic averaged model. Moreover, the estimation on the steady-state waveforms acquired the harmonic averaged models with different accuracy is crucial to establish an adequate setup of the advanced steady-state analysis techniques regarding the convergence performance.

III. ADVANCED PERIODIC STEADY-STATE ANALYSIS TECHNIQUE

The analysis of a steady-state solution for switched power electronic systems consists of the determination of a long-term periodic solution. We concentrate to compute solutions in the time domain with a known period $T > 0$. The electrical system is modeled by an initial value problem

$$\dot{x} = f(x,t), \quad x(0) = x_0 \quad (10)$$

with initial vales $x_0 \in \mathbb{R}^n$ of the states x. The right hand side f is a nonsmooth (probably discontinuous) function that describes the dynamics of the system including switching effects. We assume that there exists a periodic solution $x(t) = x(t+T)$ for $t \geq 0$. Consequently, the task is to find initial values x_0 such that

$$\dot{x}(t) = f(x(t),t), \quad t \in [0,T], \quad x(0) = x(T) = x_0.$$

From a mathematical point of view a system of n nonlinear equations $x(T,x_0) = x_0$ has to be solved. Numerical algorithms like Quasi-Newton methods may be used to directly treat the problem. An automated steady-state analysis approach based on Quasi-Newton methods is reported in [10]. An alternative approach is related to optimization. The periodic solution is identical to the solution of the following optimization problem:

$$\min_{x_0} z(x_0), \quad z(x_0) := \|x(T) - x_0\|_2^2. \quad (11)$$

In the field of numerical optimization a couple of algorithms are available, that have different properties and application areas [11]. We use this variety and handle the finding of a periodic solution as optimization problem. Model simulations and gradient evaluations are driven by an optimization algorithm that iteratively generates new initial values x_0 of the periodic system. The gradient computations are optional and only necessary for corresponding algorithms like Sequential Quadratic Programming (SQP) methods [11].

A classical separation of optimization methods leads to globally and locally converging algorithms. Stochastic approaches like genetic algorithms tend to find a global optimum whereas gradient based methods quickly converge in the near of the solution point. An adequate speed of convergence of a gradient based algorithm requires the reliable numerical evaluation of the objective function's gradient. Especially for dynamical systems with discontinuous right hand sides numerical finite differences (FD) of $\partial z(x_0)/\partial x_0$ may have a poor accuracy. Alternative approaches for the evaluation of gradients that integrate the sensitivity equation allow to generate more accurate results. These advanced techniques have been implemented in an enhancement of the existing Design Optimization library [12] for Modelica and Dymola within a PhD thesis [13] and within the Eurosyslib project [14]. The Optimization library is based on the optimization software MOPS [15] that provides a set of optimization algorithms to numerically solve multi-objective optimization problems. One of the main advantages of the new Optimization library is the capability to automatically handle different types of optimization tasks. One of these tasks is based on periodic solutions of Modelica models. Hence, the tool chain for the steady-state analysis is completely incorporated in Modelica / Dymola and easily enables the possibility to further enhance the tool.

The graphical user interface for the optimization of the periodic solution in Dymola provides specifications for all aspects of the optimization procedure. For a general optimization problem the main aspects are the definition of the tuners and criteria. Tuners are the parameters that can be varied by the optimization algorithm whereas the criteria assess the tuner values by means of a model. In case of the periodic system the tuners are the initial values x_0 and the criteria are the absolute values of the vector $x(T)-x_0$, computed by the numerical integration of the initial value problem (10). An objective function compresses the possible many criteria to one value to be minimized. In problem (11) the objective function is selected as the sum of squares of the criteria. Further, the tool offers the selection of different optimization methods: SQP, Quasi-Newton, Pattern search, Simplex method and Genetic algorithm. The firstly mentioned two algorithms are based on gradients of the objective function.

IV. Demonstration using a buck converter

Generally, there are three major steps to conduct the multi-level power quality assessment. First, the DC averaged model shall be simulated to point out the settling time when the steady-state arrives. Simultaneously, the rough power quality assessment mostly applicable for DC network, can be accomplished by reading the simulation result at steady state. Afterwards, more accurate averaged models e.g. the first harmonic approximation of the original topological model shall be simulated only until the settling time. The magnitudes of Fourier coefficient - harmonic state variables of time domain simulation directly illustrate the approximated power quality assessment result. For the exact power quality assessment, the optimization setup regarding start values, tuner ranges, criteria demands can be established by means of simulation results of the generalized averaged models. Fast Fourier transformation of the waveform is subsequently conducted to get the accurate frequency domain performance. In this section a buck converter including an input filter is used to demonstrate the proposed optimization based steady-state analysis approach. A controlled DC/DC buck converter is usually a critical component for system stability in an electric on-board network due to its inherent character: its controller exhibits a negative input impedance. In combination with an unsuitable input filter, the DC/DC converter begins to oscillate and becomes unstable. With respect to the stability analysis result in [16], the stability reserve can be improved by increasing the capacitance value of the input filter C_f, which leads to a longer time duration until the steady state arrives. For the sake of simplicity, only the open loop model of the buck converter in continues conduction mode is considered in this paper. The modeling of close loop DC/DC converters taking the pulse width modulation into account is addressed in [7].

A. Modeling of the buck converter

The topological Modelica model of a buck converter including an input filter is depicted in Fig. 1. By introducing the duty ratio d and switching period T_s of the pulse generator, the performance of the switcher can be defined for on as $0 < t < dT_s$ and for off as $dT_s < t < T_s$. The state space representation is written as

$$\dot{x} = A(t)x + B(t)u \tag{12}$$

where $x = [i_{L_f},\ v_{C_f},\ i_L,\ v_c]^T$, $B(t) = \left[1/L_f,\ 0,\ 0,\ 0\right]^T$, $u = V_0$ and the periodic matrix

$$A(t) = \begin{cases} \begin{bmatrix} -\frac{R_f}{L_f} & -\frac{1}{L_f} & 0 & 0 \\ \frac{1}{C_f} & 0 & -\frac{1}{C_f} & 0 \\ 0 & \frac{1}{L} & 0 & -\frac{1}{L} \\ 0 & 0 & \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} & \text{for } t \in [0, dT_s) \\ \begin{bmatrix} -\frac{R_f}{L_f} & -\frac{1}{L_f} & 0 & 0 \\ \frac{1}{C_f} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{L} \\ 0 & 0 & \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} & \text{for } t \in [dT_s, T_s) \end{cases} \tag{13}$$

Applying the unified harmonic state space representation (5)

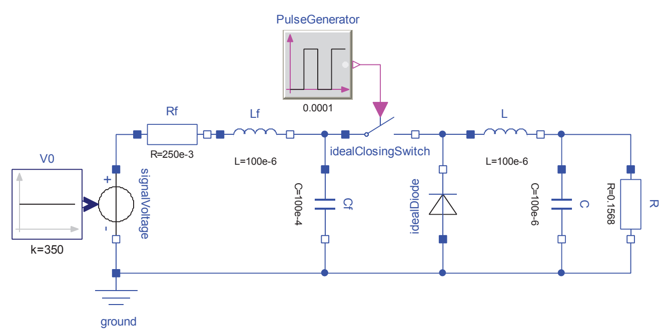

Fig. 1. Topological model of a buck converter in Modelica

with $n = 0$, the 0th order average model (DC term approximation) of the buck converter can be easily derived as

$$\frac{d}{dt}\begin{bmatrix} i_{L_f} \\ v_{C_f} \\ i_L \\ v_C \end{bmatrix} = A_0 \begin{bmatrix} i_{L_f} \\ v_{C_f} \\ i_L \\ v_C \end{bmatrix} + \begin{bmatrix} \frac{V_0}{L_f} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

with

$$A_0 = \begin{bmatrix} -\frac{R_f}{L_f} & -\frac{1}{L_f} & 0 & 0 \\ \frac{1}{C_f} & 0 & -\frac{d}{C_f} & 0 \\ 0 & \frac{d}{L} & 0 & -\frac{1}{L} \\ 0 & 0 & \frac{1}{C} & -\frac{1}{RC} \end{bmatrix}$$

Since $B(t)$ and u in (12) are constant during a period, the complex Fourier coefficients have the property $B_k = 0$ and $u_k = 0$ for $k \neq 0$. The first order harmonic state space representation with twelve state variables of the buck converter is described by

$$\begin{bmatrix} (s-i\omega)x_{-1} \\ sx_0 \\ (s+i\omega)x_1 \end{bmatrix} = \begin{bmatrix} A_0 & A_{-1} & A_{-2} \\ A_1 & A_0 & A_{-1} \\ A_2 & A_1 & A_0 \end{bmatrix} \begin{bmatrix} x_{-1} \\ x_0 \\ x_1 \end{bmatrix} + \begin{bmatrix} B_0 & 0 & 0 \\ 0 & B_0 & 0 \\ 0 & 0 & B_0 \end{bmatrix} \begin{bmatrix} 0 \\ u_0 \\ 0 \end{bmatrix}$$

where $\omega = 2\pi/T_s$, $B_0 = \left[1/L_f,\ 0,\ 0,\ 0\right]^T$, $u_0 = V_0$,

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\sin 2\pi d + i(\cos 2\pi d - 1)}{2\pi C_f} & 0 \\ 0 & \frac{\sin 2\pi d + i(\cos 2\pi d - 1)}{2\pi L} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\sin 4\pi d + i(\cos 4\pi d - 1)}{4\pi C_f} & 0 \\ 0 & \frac{\sin 4\pi d + i(\cos 4\pi d - 1)}{4\pi L} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

A_{-1} and A_{-2} are conjugate to A_1 and A_2, respectively. Applying the transformation in (8) with the transformation matrices $\mathscr{T}_{\mathscr{X},1} \in \mathbb{C}^{12\times 12}$ and $\mathscr{T}_{\mathscr{U},1} \in \mathbb{C}^{3\times 3}$, the first order real harmonic state space representation of the buck converter can be written in

$$\dot{\mathscr{X}}_{real,1} = \mathscr{A}_{real,1}\mathscr{X}_{real,1} + \mathscr{B}_{real,1}\mathscr{U}_{real,1} \tag{14}$$

where $\mathscr{X}_{real,1} = [x_{c,1}^T,\ x_0^T,\ x_{s,1}^T]^T$, $\mathscr{U}_{real,1} = V_0$, $\mathscr{B}_{real,1} = [0,\ 0,\ 0,\ 0,\ 1/L_f,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0]^T$, and $\mathscr{A}_{real,1} = [\mathscr{A}^{*}_{real,1}, \mathscr{A}^{**}_{real,1}, \mathscr{A}^{***}_{real,1}]$ with

$$\mathscr{A}^{*}_{real,1} = \begin{bmatrix} -\frac{R_f}{L_f} & -\frac{1}{L_f} & 0 & 0 \\ \frac{1}{C_f} & 0 & -\frac{\sin(4\pi d)+4\pi d}{4C_f\pi} & 0 \\ 0 & \frac{\sin(4\pi d)+4\pi d}{4L\pi} & 0 & -\frac{1}{L} \\ 0 & 0 & \frac{1}{C} & -\frac{1}{CR} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\sin(2\pi d)}{C_f\pi} & 0 \\ 0 & \frac{\sin(2\pi d)}{L\pi} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\omega & 0 & 0 & 0 \\ 0 & -\omega & -\frac{\cos(4\pi d-1)}{4C_f\pi} & 0 \\ 0 & \frac{\cos(4\pi d-1)}{4L\pi} & -\omega & 0 \\ 0 & 0 & 0 & -\omega \end{bmatrix}$$

$$\mathscr{A}^{**}_{real,1} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\sin(2\pi d)}{2C_f\pi} & 0 \\ 0 & \frac{\sin(2\pi d)}{2L\pi} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{R_f}{L_f} & -\frac{1}{L_f} & 0 & 0 \\ \frac{1}{C_f} & 0 & -\frac{d}{C_f} & 0 \\ 0 & \frac{d}{L} & 0 & -\frac{1}{L} \\ 0 & 0 & \frac{1}{C} & -\frac{1}{CR} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\cos(2\pi d-1)}{2C_f\pi} & 0 \\ 0 & \frac{\cos(2\pi d-1)}{2L\pi} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathscr{A}^{***}_{real,1} = \begin{bmatrix} \omega & 0 & 0 & 0 \\ 0 & \omega & -\frac{\cos(4\pi d-1)}{4C_f\pi} & 0 \\ 0 & \frac{\cos(4\pi d-1)}{4L\pi} & \omega & 0 \\ 0 & 0 & 0 & \omega \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\cos(2\pi d-1)}{C_f\pi} & 0 \\ 0 & \frac{\cos(2\pi d-1)}{L\pi} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{R_f}{L_f} & -\frac{1}{L_f} & 0 & 0 \\ \frac{1}{C_f} & 0 & -\frac{\sin(4\pi d)-4\pi d}{4C_f\pi} & 0 \\ 0 & \frac{-\sin(4\pi d)+4\pi d}{4L\pi} & 0 & -\frac{1}{L} \\ 0 & 0 & \frac{1}{C} & -\frac{1}{CR} \end{bmatrix}$$

B. Result of multi-level power quality assessment

The parameters of the demonstration buck converter used for the simulation are written in Tab. I. The detailed process to calculate the optimization based steady-state waveform of the buck converter is addressed in [17]. The first subfigure in Fig.2 shows the simulation the result of output voltage of the behavioural model using the optimization based steady-state analysis approach. The functional model and first harmonic averaged model are used to achieve the DC and first harmonic simulation results depicted in the second subfigure of Fig.2. The rough and approximated power quality assessment of the output voltage (V_c) can be directly executed by reading the steady-state values of the DC (V_c_DC) and the first harmonic ($V_c\_1stHarmonic$) simulation results. The exact power quality analysis is conducted by Fast Fourier Transformation of the behavioural model simulation result. The frequency spectrum and the total harmonic distortion analysis of the output voltage are depicted in the Fig. 3. The Modelica signal analysis tool introduced in [18] is used to perform the Fast fourier Transformation and total harmonic distortion analysis. It is easy to see, the first harmonic simulation result of the steady-state in the 2nd subfigure of Fig.2 correctly reflects the first harmonic peak in the frequency spectrum.

V. Conclusion

A Multi-level power quality assessment approach for virtual testing of more electric aircraft is addressed. Depending the di-

TABLE I
PARAMETERS OF THE BUCK CONVERTER

Parameter	Description	Value	Unit
L_f	Inductance of input filter	10^{-4}	Henry
R_f	Resistance of input filter	0.25	Ohm
C_f	Capacitance of input filter	10^{-2}	Farad
L	Inductance of buck converter	10^{-4}	Henry
R	Resistance of buck converter	0.1568	Ohm
C	Capacitance of buck converter	10^{-4}	Farad
T_s	Switching period	0.0001	Second

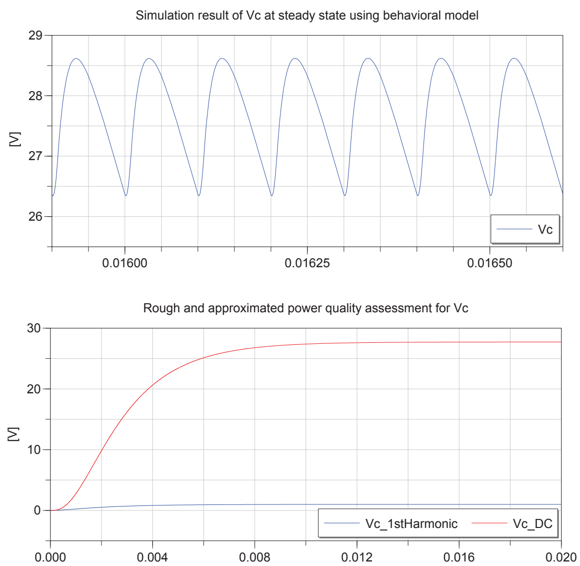

Fig. 2. Simulation results of the buck converter output voltage

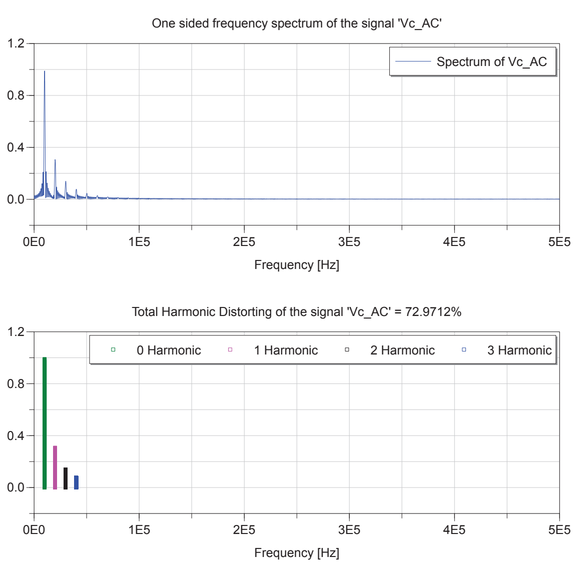

Fig. 3. Multi-level power quality assessment results of the buck converter output voltage

verse requirements of analysis jobs, power quality assessment can be conducted at different levels using advanced modeling and steady-state finding techniques. As result, both sufficient accuracy and minimal cost for the electrical power quality virtual testing of more electric aircraft can be ensured.

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