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Budeanu and Fryze: Two frameworks for interpreting power properties of circuits with nonsinusoidal voltages and currents

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Contents The development of power theory of circuits with nonsinusoidal voltages and currents was shaped for several decades by two different approaches introduced, separately, by Budeanu and Fryze in the nineteen thirties. This paper investigates these two power theories from the present perspective and up to date comprehension of power phenomena in circuits with distorted voltages and currents. It shows the reasons for which Budeanu's power theory misinterprets power phenomena and why it does not provide fundamentals for the power factor improvement. This paper shows that a number of concepts introduced to power theory by Fryze are still alive and are very important for developing that theory, but it also has a number of serious limitations. Because of that, some issues, important theoretically, and a number of practical problems in systems with distorted waveforms were not solved within Fryze's power theory. This was the reason for further developing the power theory of systems with nonsinusoidal voltages and currents. This process is not completed yet and a lot can be learned from the history of this theory's development.

Budeanu und Fryze - Zwei Ansätze zur Interpretation der Leistungen in Stromkreisen mit nichtsinusförmigen Spannungen und Strömen

Übersicht Die Entwicklung der Leistungstheorie für Stromkreise mit nichtsinusförmigen Spannungen und Strömen wurde mehrere Jahrzehnte von zwei verschiedenen Ansätzen beeinflußt, die Budeanu und Fryze unabhängig voneinander in den 1930ern einführten. Dieser Aufsatz untersucht beide Leistungstheorien aus heutiger Sicht und mit dem derzeitigen Verständnis der Leistungser scheinungen bei verzerrten Spannungen und Strömen. Er zeigt die Gründe für die Fehlinterpretation der Erscheinungen in Budeanus Leistungstheorie auf und gibt an, warum diese nicht zur Grundlage für eine Leistungsfaktorverbesserung dienen kann. Es wird weiter gezeigt, daß einige von Fryze in die Leistungstheorie eingeführte Konzepte für die Weiternentwicklung dieser Theorie von großer Wichtigkeit sind, aber auch ernsthafte Grenzen aufweisen. Deswegen konnten einige theoretisch wichtige Fragestellungen und einige praktische Probleme in Systemen mit verzerrten Zeitfunktionen mit der Leistungstheorie von Fryze nicht gelöst worden. Dies war der Grund für eine Weiterentwicklung der Leistungstheorie für Systeme mit nichtsinusförmigen Spannungen und Strömen. Dieser Vorgang ist noch nicht abgeschlossen, wobei aus der Geschichte dieser Theorieentwicklung noch viel zu lernen ist.

1 Introduction

More than hundred years have passed since the moment which can be considered the very beginning of the development of power theory of circuits with nonsinusoidal voltages and currents. Namely, in 1892 Ch. Steinmetz concluded [1] that mercury arc rectifiers have the apparent power higher than the active one, without any phase shift between the voltage and current but only because of the current distortion. At first, after this observation, researches were concerned with interpretation and measurement [2, 3] of the power factor at distorted voltages and currents. Suddenly it occurred, that the power factor cannot be measured as the cosine value of the voltage and current phase-shift. In the twenties the quest began for a definition of the reactive power Q which would provide a power equation of circuits with nonsinusoidal voltages and currents that was similar to the power equation for circuits with sinusoidal voltages and currents, namely

$$S^2 = P^2 + Q^2, \qquad (1)$$

where S is the apparent and P is the active power of the load. Several papers on this subject were published by the end of the twenties. In 1925, Illovici suggested [4] two alternatives for the reactive power definition, namely

$$Q_I = \sum_{n=1}^{\infty} nU_nI_n \sin\varphi_n, \quad Q_I = \sum_{n=1}^{\infty} \frac{1}{n} U_nI_n \sin\varphi_n, \qquad (2)$$

where U_n, I_n and φ_n are the RMS values of the voltage and current of the n-th order harmonics and their phase-shift. These two quantities are measured by a wattmeter with the resistor in the voltage coil replaced by an inductor or by a capacitor, respectively. Unfortunately, they do not satisfy equation (1). Finally, W. Weber [6] and F. Emde [7] concluded that it is not possible to define the reactive power Q that would satisfy this equation.

In 1927 C. Budeanu developed a power equation and definitions of the reactive and distortion powers [5] which

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dominated studies on systems with nonsinusoidal voltages and currents for several decades [10, 16, 17]. Power definitions for nonsinusoidal systems are founded on Budeanu's approach even in the present American Standard [22].

Four years after Budeanu suggested his power equations, S. Fryze managed to find [8] the reactive power definition that satisfies equation (1). Furthermore, Fryze managed to do it without decomposition of the voltage and current waveforms into harmonics, i.e., in the time-domain. Although supported with the German Standard [9], Fryze's power theory was not so known as Budeanu's. However, with an increasing disappointment with Budeanu's theory, Fryze's approach effected formidably the development [12–14, 18, 23] of power theory of circuits with nonsinusoidal voltages and currents.

Budeanu's and Fryze's theories co-existed for more than fifty years. There were even attempts to combine them [16] into a single power theory. Unfortunately, interpretation of power phenomena of circuits with nonsinusoidal voltages and currents remained unclear and controversial. Moreover, no progress was achieved with respect to power factor improvement with reactive compensators using these two theories. The situation changed in the eighties with an increase in the number of sources of waveform distortion, due to developments in power electronics. New attempts were made aimed at developing power theory and methods of power factor improvement in circuits with nonsinusoidal voltages and currents. Consequently, power properties of such circuits are much better understood now and there is a variety of methods for power factor improvement which go far beyond the results provided by Budeanu's and Fryze's power theories. Very often, however, in books and papers, power properties of circuits with nonsinusoidal waveforms are explained only in terms of Budeanu's or Fryze's theories. Consequently, they shape the knowledge of electrical engineers on powers in nonsinusoidal circuits, thus impairing their comprehension of power phenomena in such circuits. Often this knowledge is shaped even by only one of these theories. Therefore, it would be beneficial to discuss these theories from the present perspective. Moreover, the development of power theory is not completed yet. We may learn a lot from the history of sixty years of controversy between Budeanu's and Fryze's approaches and this can be beneficial for the further development of power theory.

2 Budeanu's power theory

Budeanu's definition of reactive power Q_B is a direct extension of that definition for systems with sinusoidal voltages and currents, where the active and reactive powers are equal to

$$P = UI\cos\varphi, \qquad Q = UI\sin\varphi. \tag{3}$$

Since the active power P in single-phase systems with nonsinusoidal voltage and currents can be calculated as

$$P = \sum_{n=0}^{\infty} U_n I_n \cos\varphi_n, \tag{4}$$

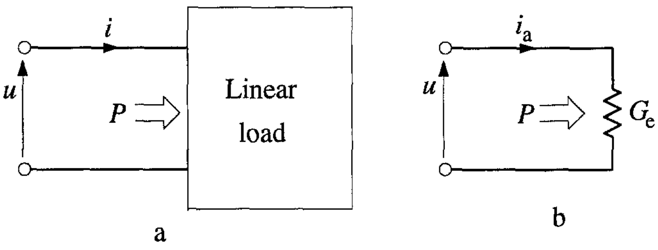

Fig. 1. **a** Linear load and **b** its equivalent circuit

where U_n and I_n are RMS values of the n-th order harmonic and φ_n is their phase-shift, Budeanu concluded that the reactive power should be defined as

$$Q_B = \sum_{n=1}^{\infty} U_n I_n \sin\varphi_n. \tag{5}$$

With such a definition, the sum of squares of the active and reactive power is less than the square of the apparent power S, defined as a formal product of the voltage and current RMS values, namely

$$S = \| u \| \| i \|, \tag{6}$$

where

$$\| i \| = \sqrt{\sum_{n=0}^{\infty} I_n^2}, \qquad \| u \| = \sqrt{\sum_{n=0}^{\infty} U_n^2}. \tag{7}$$

Consequently, Budeanu concluded that there is also another power in circuits with distorted voltages and currents, namely

$$D = \sqrt{S^2 - (P^2 + Q_B^2)}. \tag{8}$$

It was named *distortion power*.

In the eighties, mainly due to contributions by Milic, [10], Nowomiejski [16] and Fisher [17], Budeanu's power theory became a sophisticated mathematical tool, unfortunately, without any technical implications. It failed to provide theoretical fundamentals for power factor improvement. As to the author's knowledge, Shepherd and Zakikhani were the first who concluded in writing [11] that Budeanu's theory is a misconception. A detailed proof for this was published [20] by the author of this paper in 1987, exactly sixty years after Budeanu's theory was developed.

3 Fryze's power theory

Fryze's power theory is based on the supply current decomposition into the *active current*, i_a, and the *reactive current*, denoted here by i_F, namely

$$i = i_a + i_F. \tag{9}$$

The active current is the current of a resistive load, shown in Fig. 1b, which is equivalent to the original load, shown in Fig. 1a, with respect to the active power P at the same supply voltage $u(t)$.

$$i_a(t) = \frac{P}{\| u \|^2} u(t) = G_e u(t). \tag{10}$$

The conductance G_e is referred to as an *equivalent conductance* of the load. The remaining part of the supply current

$$i_F = i - i_a \ , \tag{11}$$

was assumed to be the reactive current. The scalar product of these currents

$$(i_a, i_F) = \frac{1}{T}\int_0^T i_a i_F \mathrm{d}t = \frac{1}{T}\int_0^T i_a(i - i_a)\mathrm{d}t$$
$$= \frac{P}{\| u \|^2}\frac{1}{T}\int_0^T ui\mathrm{d}t - \frac{P^2}{\| u \|^4}\frac{1}{T}\int_0^T u^2\mathrm{d}t = 0, \tag{12}$$

thus, they are mutually orthogonal. Hence, the RMS values of these currents fulfill the relationship

$$\| i \|^2 = \| i_a \|^2 + \| i_F \|^2, \tag{13}$$

and consequently, the power equation can be written as

$$S^2 = P^2 + Q_F^2, \tag{14}$$

with the reactive power, defined according to Fryze,

$$Q_F = \| u \| \| i_F \|. \tag{15}$$

Fryze's power theory, initially dominated by Budeanu's, gained increasing attention with a vanishing interest in the later one. To have a progress in the comprehension of power phenomena in nonsinusoidal circuits and to develop compensation methods it occurred that it was necessary, however, to go beyond Fryze's theory. This paper will explain the reasons for that.

4
Budeanu's reactive power and energy oscillation

Though the misconceptions hidden in Budeanu's power theory, noticed by Shepherd and Zakikhani [11] and discussed in detail [20] by the author of this paper, were revealed long ago, this theory is still alive. Recently published papers and books which refer to this theory as the only model of power phenomena in circuits with nonsinusoidal waveforms, seem to indicate that it flourishes in the electrical engineering community. We should not be surprised with this when we realize that it has dominated electrical engineering for over more than six decades. At least two generations of electrical engineers used it. Most of them are unaware that Budeanu's power theory is deeply erroneous.

To explain this, we have to return to the very fundamentals. The reactive power Q in single-phase circuits with sinusoidal voltages and currents is a measure of energy oscillation between the supply source and the load. At the supply voltage and load current equal to, respectively

$$u(t) = \sqrt{2}U\cos\omega_1 t, \quad i(t) = \sqrt{2}U\cos(\omega_1 t - \varphi), \tag{16}$$

the instantaneous power:

$$p(t) = \frac{\mathrm{d}}{\mathrm{d}t}W(t) = u(t)i(t) = 2UI\cos\omega_1 t\cos(\omega_1 t - \varphi), \tag{17}$$

can be expressed in the form:

$$p(t) = UI\cos\varphi(1 - \cos 2\omega_1 t) + UI\sin\varphi\sin 2\omega_1 t$$
$$= P(1 - \cos 2\omega_1 t) + Q\sin 2\omega_1 t = p_a(t) + p_b(t). \tag{18}$$

Thus, the reactive power, $Q = UI\sin\varphi$, is the amplitude of the oscillating component,

$$p_b(t) = Q\sin 2\omega_1 t, \tag{19}$$

of the instantaneous power. Electrical engineers usually extrapolate this property to circuits with nonsinusoidal voltages and currents, and because of the same name, they are inclined to interpret the reactive power Q_B as a measure of energy oscillation. Unfortunately, this is a wrong conclusion due to the following reasons. The instantaneous power of the voltage and current n-th order harmonic

$$u_n(t) = \sqrt{2}U_n\cos n\omega_1 t, \tag{20}$$

$$i_n(t) = \sqrt{2}I_n\cos(n\omega_1 t - \varphi_n), \tag{21}$$

by an analogy to a sinusoidal situation, can be expressed as

$$p_n(t) = P_n(1 - \cos 2n\omega_1 t) + Q_n\sin 2n\omega_1 t$$
$$= p_{an}(t) + p_{bn}(t), \tag{22}$$

where

$$P_n = U_n I_n\cos\varphi_n \geq 0, \quad Q_n = U_n I_n\sin\varphi_n \geq / < 0. \tag{23}$$

i.e., Q_n is the amplitude of the instantaneous power oscillation of frequency $2n\omega_1$ which could be positive, negative or zero. If the voltage and current of a linear load are distorted and composed of N harmonics, i.e.,

$$u(t) = \sum_{n=0}^{N} u_n(t), \qquad i(t) = \sum_{n=0}^{N} i_n(t), \tag{24}$$

then Budeanu's reactive power Q_B, defined with equation (3), is nothing else than the sum of amplitudes Q_n, namely

$$Q_B = \sum_{n=1}^{N} Q_n. \tag{25}$$

Each oscillating component $p_{bn}(t)$ has a different frequency, while Q_n can be positive, negative or zero, thus due to mutual cancellation of amplitudes Q_n, the oscillation of energy between the source and the load may exist even if $Q_B = 0$. Unlike the reactive power in circuits with sinusoidal voltages and currents, the reactive power defined by Budeanu cannot be interpreted as a measure of energy oscillation.

Illustration 1. The lack of any relation between Budeanu's reactive power and energy oscillation is illustrated with the circuit shown in Fig. 2. The circuit parameters were chosen such that at the supply voltage:

$$u(t) = \sqrt{2}(100\sin\omega_1 t + 25\sin 3\omega_1 t)\mathrm{V},$$

the load current is

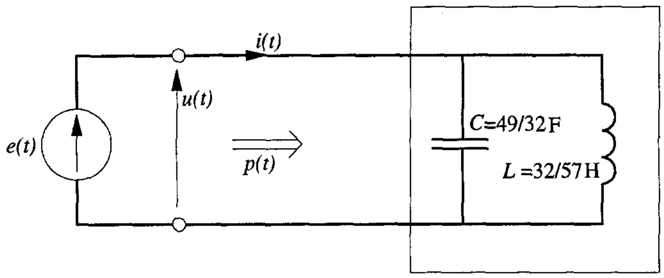

Fig. 2. Circuit with zero Budeanu's reactive power Q_B but with energy oscillation

$$i(t) = \sqrt{2}[25\sin(\omega_1 t - \tfrac{\pi}{2}) + 100\sin(3\omega_1 t + \tfrac{\pi}{2})]\mathrm{A}.$$

At such a voltage and current, the reactive power Q_B is equal to zero, since

$$Q_B = Q_1 + Q_3 = 2500 - 2500 = 0.$$

The instantaneous power $p(t)$ varies as shown in Fig. 3. Its sign changes, thus energy oscillates between the load and the source despite zero reactive power.

This result confirms, that association of Budeanu's reactive power with energy oscillation is a misinterpretation of power phenomena in circuits with nonsinusoidal voltages and currents.

5
Distortion power and waveform distortion

The second quantity defined by Budeanu was the distortion power D. Its name suggests that it relates a power to waveform distortion. In linear resistive circuits, where the voltage and current are not *mutually* distorted, the distortion power D is always equal to zero, irrespective of the waveform distortion. Thus distortion power D might only relate power to a mutual distortion of voltage and current.

Let us check, how the distortion power D is related to mutual distortion of the voltage and current. Budeanu's original formula (8) can be modified to the form

$$D = \sqrt{\sum_{n=0}^{N} U_n^2 \sum_{n=0}^{N} I_n^2 - \left(\sum_{n=0}^{N} U_n I_n \cos\varphi_n\right)^2 - \left(\sum_{n=1}^{N} U_n I_n \sin\varphi_n\right)^2} = \sqrt{\frac{1}{2}\sum_{r=0}^{N}\sum_{s=0}^{N} A_{rs}}. \tag{26}$$

If the load is linear, the terms A_{rs} are equal to

$$A_{rs} = U_r^2 I_s^2 + U_s^2 I_r^2 - 2U_r U_s I_r I_s \cos(\varphi_r - \varphi_s) = U_r^2 U_s^2 \mid \boldsymbol{Y}_r - \boldsymbol{Y}_s \mid^2, \tag{27}$$

where $\boldsymbol{Y}_r$ and $\boldsymbol{Y}_s$ are load admittances for frequencies $r\omega_1$ and $s\omega_1$. Thus the distortion power D can be expressed as

$$D = \sqrt{\frac{1}{2}\sum_{r=0}^{N}\sum_{s=0}^{N} U_r^2 U_s^2 \mid \boldsymbol{Y}_r - \boldsymbol{Y}_s \mid^2}. \tag{28}$$

Magnitudes $\mid \boldsymbol{Y}_r - \boldsymbol{Y}_s \mid$ are non-negative, thus distortion power D is equal to zero only if for each r, s, such that U_r and $U_s \neq 0$, admittances $\boldsymbol{Y}_r$ and $\boldsymbol{Y}_s$ are equal. Thus, if for such harmonic orders n

$$\boldsymbol{Y}_n = Y_n e^{-j\varphi_n} = \text{const.} \tag{29}$$

This is the necessary condition for the distortion power D to be equal to zero. Let as check, if this is also the necessary condition for the lack of mutual distortion of voltage and current. They are not mutually distorted but only shifted, if

$$i(t) = a\ u(t - \tau), \tag{30}$$

where a is dimensional constant and τ is the time shift. For shifted quantities, the complex RMS values of the voltage and current harmonics have to fulfill the following relationship

$$\boldsymbol{I}_n = a\boldsymbol{U}_n e^{-jn\omega_1\tau} = \boldsymbol{Y}_n \boldsymbol{U}_n. \tag{31}$$

Consequently, the voltage and current waveforms are not mutually distorted but only shifted, only if for each harmonic order n such that $U_n \neq 0$, the load admittance is equal to

$$\boldsymbol{Y}_n = a e^{-jn\varphi_1}, \tag{32}$$

with $\varphi_1 = \omega_1\tau$. Apart from resistive loads, i.e., with $\varphi_n = 0$, the necessary conditions (29) for zero distortion power D, and the necessary condition for the lack of mutual distortion between voltage and current (32) are mutually exclusive. If the load current is not distorted with respect to the supply voltage, i.e., condition (32) is fulfilled, then condition (29) is not fulfilled and the distortion power D cannot be equal to zero. Such a situation is shown below.

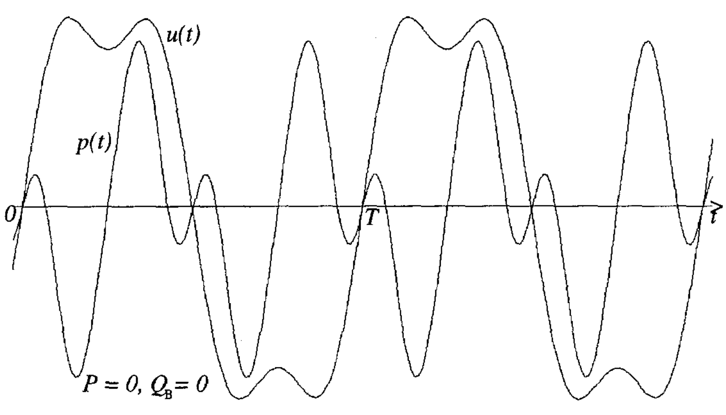

Fig. 3. Supply voltage $u(t)$ and instantaneous power $p(t)$ in the circuit shown in Fig. 2. At negative values $p(t)$ energy flows to the supply source

Illustration 2. Parameters of the load shown in Fig. 4 and supplied with the voltage

$$u(t) = 100\sqrt{2}(\sin\omega_1 t + \sin 3\omega_1 t)\mathrm{V},$$

with $\omega_1 = 1$ rad/s, were chosen such that condition (32) is fulfilled, namely

$$Y_1 = \frac{2}{3}\,\mathrm{e}^{-\mathrm{j}\frac{\pi}{2}}\,\mathrm{S}, \qquad Y_3 = \frac{2}{3}\,\mathrm{e}^{-\mathrm{j}3\frac{\pi}{2}} = \frac{2}{3}\,\mathrm{e}^{\mathrm{j}\frac{\pi}{2}}\,\mathrm{S}.$$

Hence, the load current is equal to

$$i(t) = \frac{2}{3}100\sqrt{2}[\sin(\omega_1 t - \tfrac{\pi}{2}) + \sin(3\omega_1 t + \tfrac{\pi}{2})]$$
$$= \frac{2}{3}100\sqrt{2}[\sin\omega_1(t - \tfrac{T}{4}) + \sin 3\omega_1(t - \tfrac{T}{4})] = \frac{2}{3}u(t - \tfrac{T}{4}),$$

thus it is not distorted with respect to the supply voltage but shifted by $T/4$. At the same time, however, the distortion power, D, is not equal to zero. It is equal to

$$D = \frac{2}{3}10\sqrt{2}\mathrm{kVA}.$$

The opposite situation, where at zero distortion power, D, the voltage and current are mutually distorted is possible, as well. This is shown below.

Illustration 3. Parameters of the load shown in Fig. 5 and supplied with the voltage

$$u(t) = \sqrt{2}(100\sin\omega_1 t + 50\sin 3\omega_1 t)\mathrm{V},$$

with $\omega_1 = 1$ rad/s, were chosen such that condition (29) is fulfilled, namely

$$Y_1 = Y_3 = 1\mathrm{e}^{\mathrm{j}\frac{\pi}{2}}\,\mathrm{S}.$$

Hence, the load current is equal to

$$i(t) = \sqrt{2}[100\sin(\omega_1 t + \tfrac{\pi}{2}) + 50\sin(3\omega_1 t + \tfrac{\pi}{2})]\mathrm{A}.$$

The voltage and current waveforms are shown in Fig. 6 They are mutually distorted. However, the distortion power, D, is equal to zero.

Consequently, the distortion power D in single-phase linear circuits has nothing in common with waveform distortion, similarly as Budeanu's reactive power has nothing in common with energy oscillation. Thus, Budeanu's power theory misinterprets power phenomena in electrical circuits.

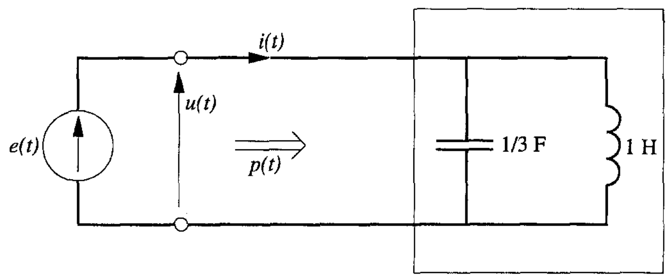

Fig. 4. Circuit with supply current not distorted with respect to voltage, but with non zero distortion power D

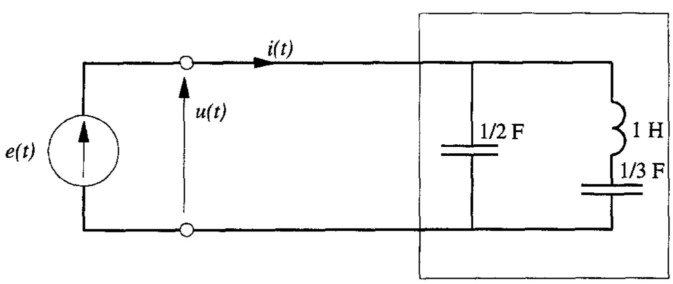

Fig. 5. Circuit with the supply current distorted with respect to voltage, but with zero distortion power D

6 Budeanu's power theory and power factor improvement

Apart from interpretation of power phenomena, theoretical fundamentals for power factor improvement is one of the main practical goals for power theory development. Unfortunately, no method of power factor improvement based on Budeanu's power theory was found.

Improvement of the power factor in systems with sinusoidal voltages and currents, is possible by compensation of the reactive power. Therefore, compensation of Budeanu's reactive power was also the primary objective for attempts aimed at improving the power factor in systems with nonsinusoidal waveforms. It was not observed, that unlike circuits with sinusoidal waveforms, there is no relation between power factor and the reactive power Q_B in circuits with nonsinusoidal waveforms. To show this, let us decompose the n-the order current harmonic of a load supply current, given with eqn. (21), as follows

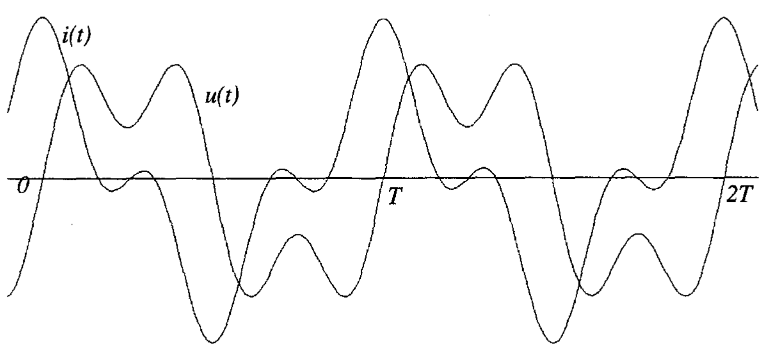

Fig. 6. Mutual distortion of the voltage and the supply current in the circuit shown in Fig. 5. with zero distortion power D

$$\begin{aligned} i_n(t) &= \sqrt{2}\, I_n \cos\,(n\omega_1 t - \varphi_n) \\ &= \sqrt{2}\, I_n \cos\varphi_n \cos n\omega_1 t + \sqrt{2}\, I_n \sin\varphi_n \sin n\omega_1 t \\ &= \sqrt{2}\, \frac{P_n}{U_n} \cos n\omega_1 t + \sqrt{2}\, \frac{Q_n}{U_n} \sin n\omega_1 t, \end{aligned} \tag{33}$$

hence its RMS value is equal to

$$\|\, i_n \,\| = \sqrt{\left(\frac{P_n}{U_n}\right)^2 + \left(\frac{Q_n}{U_n}\right)^2}. \tag{34}$$

Since harmonics are mutually orthogonal, the RMS value of the supply current can be expressed as

$$\|\, \boldsymbol{i} \,\| = \sqrt{\sum_{n=0}^{N} \|\, i_n \,\|^2} = \sqrt{\sum_{n=0}^{N} \left(\frac{P_n}{U_n}\right)^2 + \sum_{n=1}^{N} \left(\frac{Q_n}{U_n}\right)^2}. \tag{35}$$

This formula shows, that at a specified voltage, i.e., specified harmonic RMS values U_n and harmonic active powers P_n, the RMS value of the current is minimum not when Budeanu's reactive power, $\sum Q_n = Q_B$ is minimum, but when the term

$$\sum_{n=1}^{N} \left(\frac{Q_n}{U_n}\right)^2, \tag{36}$$

is equal to zero, i.e., when the reactive power Q_n of each harmonic is equal to zero. Thus compensation of Budeanu's reactive power may not result in the reduction of the supply current RMS value and improvement of the power factor. At the same time, even if the reactive power, Q_B is equal to zero, the power factor can still be improved with a *reactive compensator*. Such a situation is demonstrated in illustration 4.

Illustration 4. The circuit shown in Illustration 1 has the reactive power Q_B at the load terminals equal to zero. The supply current RMS value is equal to 103.1 A. The load admittance for the supply voltage harmonics is equal to $Y_1 = -j1/4$ S and $Y_3 = +j4$ S. If the admittance of a shunt reactive compensator is equal to $Y_1 = +j1/4$ S and $Y_3 = -j4$ S, such a compensator reduces the supply current to zero. A compensator with such admittances may have a structure and parameters as shown in Fig. 7. It reduces the RMS value of the current, however, the reactive power Q_B remains unchanged and equal to zero.

Thus, Budeanu's approach not only misinterprets power phenomena but it is also useless as a theoretical framework for power factor improvement. Unfortunately, for several decades Budeanu's power theory has provided illusions that it correctly describes power phenomena in circuits with nonsinusoidal waveforms. Because of these illusions, its impact on the comprehension of the power properties of circuits with nonsinusoidal voltages and currents in the electrical engineering community has to be evaluated, therefore, as very harmful.

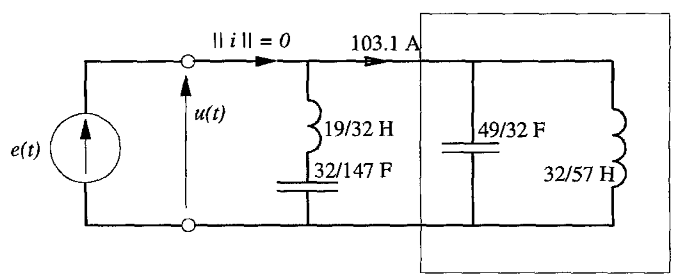

Fig. 7. Circuit with zero Budeanu's reactive power Q_B and reactive compensator

7
Interpretative features of Fryze's power theory

The possibility of describing power properties of electrical circuits directly in the time-domain suggested by Fryze's power theory is very appealing because the time-domain is usually considered as the original domain for electrical quantities, while the frequency-domain is treated rather as an artificial mathematical tool. Also, instrumentation for the time-domain is much more simple than for the frequency-domain. This was particularly crucial at the time when Budeanu's and Fryze's theories were developed. There was no instrumentation for on-line harmonic phase-shift measurement. Therefore, for almost five decades Budeanu's reactive power remained a quantity that was not measurable. The first patent for reactive power Q_B meter [15] was granted to the author of this paper in 1974, i.e., 47 years after this power was defined. Consequently, Fryze's approach has become the focus of considerable attention.

Unfortunately, Fryze's power theory has stringent limitations, both with respect to interpretative features as well as its practical implementations. First of all, it provides a very shallow interpretation of power phenomena. The idea of the *active current* is powerful, however, its interpretation as a *useful component* of the supply current is not fully convincing. The *active power* is not synonymous to *useful power*. Active power associated with harmonics is often not useful but harmful, especially in rotating machines. The reactive current i_F according to Fryze does not have interpretation other than that this is a *useless current*, which is a trivial conclusion. What is not useful is useless, of course. There is a number of distinctive power phenomena in the load which contribute to the reactive current i_F value, however, Fryze's theory does not relate this current to particular properties and parameters of the load.

8
Fryze's power theory and power factor improvement

The time-domain approach of Fryze's theory, which makes the theory so attractive, is its weakness when passive methods of power factor improvement are considered. Although time- and frequency-domains are equivalent, methods of reactive compensator and filter synthesis in the time-domain have not yet been developed. Consequently, resonant harmonic filters and reactive compensators have to be designed in the frequency-domain, but Fryze's power theory rejects the concept of harmonics. Limitations of Fryze's power theory as to power factor improvement with reactive compensator is illustrated with the following example.

Illustration 5. Parameters of two loads shown in Fig. 8a and 8b were chosen such that when they are supplied from the same source of distorted voltage:

$$u(t) = 100\sqrt{2}\ (\sin\omega_1 t + \sin 3\omega_1 t)\ \mathrm{V}, \quad \omega_1 = 1\ \mathrm{rad/s}\ ,$$

these loads cannot be distinguished in terms of Fryze's power components. Both have the same active power $P = 10$ kW, reactive power $Q_F = 10$ kVA and apparent power $S = 14.1$ kVA, thus, they have the same power factor $\lambda = 0.71$. The reactive power Q_F of the load shown in Fig. 8a can be compensated entirely, which improves the power factor to $\lambda = 1$. The reactive power of the load shown in Fig. 8b cannot be compensated, however, with any shunt reactive compensator below $Q_F = 8$ kVA, so that, the power factor cannot be higher than $\lambda = 0.78$.

Fryze's power theory does not explain why the loads considered in illustration 5 have different power properties with respect to the possibility of their power factor improvement. Also, it does not enable us to design the reactive compensator needed to improve the power factor. This explanation and compensator design is based on a more advanced power theory, discussed in Ref. [18].

Compensation of the reactive current i_F with a controlled current source (CCS), connected as shown in Fig. 9, which reduces the supply current to its active component i_a is the basic idea of the operation of active harmonic filters. Fryze's decomposition, generalized for three-phase systems can, therefore, be considered as an alternative to the instantaneous imaginary power theory [19] developed for generating signals needed for active harmonic filter control. Unfortunately, Fryze's decomposition of the supply current provides correct results in such situations only if there is a unidirectional flow of active power from the supply source to the load. When there is a source of the active power also on the load side, and consequently, there is a bidirectional flow of the active power between the load and the source, then compensation of Fryze's reactive current i_F may not provide desirable results. Non-linearity of the load parameters and/or periodic switching are the most common reason for the reverse flow of active power at some harmonic frequencies. i.e., from the load to the supply source.

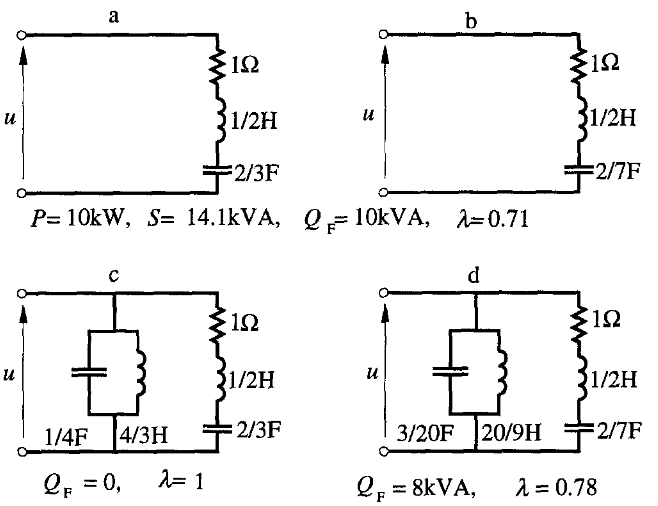

Fig. 8a–d. a,b Two loads that cannot be distinguished in terms of Frizz's powers. **c,d** The same loads with reactive compensators that improve power factor to maximum value

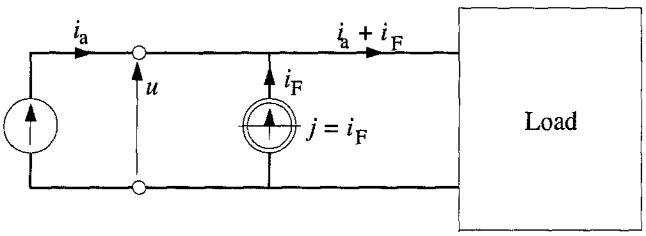

Fig. 9. Circuit with Fryze's reactive current i_F compensated by controlled current source

Illustration 6. If the load shown in Fig. 10 is linear and of resistance $R = 8\Omega$, without any source of current, and if this load is supplied from the source of voltage

$$e_1 = 100\sqrt{2}\ \sin\omega_1 t \quad \mathrm{V},$$

then the active power $P = 800$ W is delivered from the supply source to such a load. Let us suppose, that the second harmonic current, of the value

$$j_2 = 10\sqrt{2}\sin 2\omega_1 t\ \mathrm{A},$$

is generated in the load. Analysis of the circuit results in the following values of the voltage and current observed at the load terminals

$$i = 10\sqrt{2}\ \sin\omega_1 t + 8\sqrt{2}\ \sin 2\omega_1 t\ \mathrm{A} \quad \text{with } \| i \| = 12.806\ \mathrm{A},$$

$$u = 80\sqrt{2}\ \sin\omega_1 t - 16\sqrt{2}\ \sin 2\omega_1 t\ \mathrm{V} \quad \text{with } \| u \| = 81.584\ \mathrm{V},$$

and the active power measured at these terminal is equal to $P = 672$ W at the apparent power $S = 1045$ VA. The active power is reduced as compared to the previous value of $P = 800$ W, because 128 Watts is delivered from the load to the supply source by the second harmonic current and dissipated in the supply source resistance. According to Fryze, the equivalent conductance G_e has the value of

$$G_e = \frac{P}{\| u \|^2} = \frac{672}{81.584^2} = 0.101\ \mathrm{S}.$$

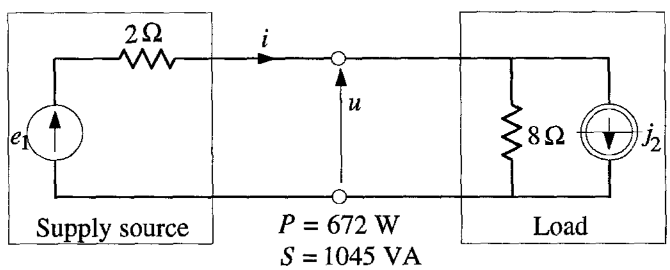

Fig. 10. Circuit with bidirectional flow of energy. At fundamental frequency flows from the supply source to the load; at the second harmonic it flows from the load to the supply source

Consequently, the active and reactive components of the supply current are equal to

$$i_a = G_e u = 8.077\sqrt{2}\ \sin\omega_1 t - 1.616\sqrt{2}\ \sin 2\omega_1 t \text{ A},$$

$$i_F = i - i_a = 1.923\sqrt{2}\ \sin\omega_1 t + 9.616\sqrt{2}\ \sin 2\omega_1 t \text{ A}.$$

Even when the current i_F is compensated, the supply current remains still distorted. Also, as it can be checked, the load active power remains unchanged, since the load voltage changes to

$$u' = e_1 - 2i_a = 83.846\sqrt{2}\sin\omega_1 t + 3.232\sqrt{2}\sin 2\omega_1 t \text{ V}.$$

If instead of compensating Fryze's reactive current i_F, however, the load generated current j_2 is compensated with a controlled current source connected at the load terminals, then the load voltage and current distortion are entirely eliminated and the active power delivered to the load increases to $P = 800$ W, while the apparent power declines to $S = 800$ VA. Compensation of Fryze's reactive current does not provide such results.

9 Comparison of Budeanu's and Fryze's impact upon power theory

Taking into account all misinterpretations of power phenomena suggested by Budeanu's power theory, discussed in detail in Sections 4, 5 and 6, it is difficult to find any real contribution of that theory to our present comprehension of these phenomena and to the methods of power factor improvement. Progress was not achieved until Budeanu's power equation, power definitions and all interpretations were abandoned. Maybe, keeping alive the frequency-domain approach, although it was not Budeanu's original concept, is the only advantage brought about by his theory.

Unlike Budeanu's theory, Fryze's approach contributed formidably to developing and to the present stage of power theory. A number of Fryze's original concepts, even if sometimes controversial or slightly modified, are still alive. Perhaps, the most important of them is treating the power equation as secondary with respect to the current equation and the attempt to explain power properties in terms of the decomposition of current into orthogonal components. Shepherd and Zakikhani [11], Depenbrock [12, 23], Kusters and Moore [13] and the author of this paper [18, 21, 24] followed this approach and it contributed to the progress of the development of power theory. Also, Fryze's concept of active current, sometimes referred to as *Fryze's current*, is a very important one, even if, as shown in Section 8, this concept could be controversial in some situations. Nevertheless, power theory develops by enriching Fryze's theory rather than by abandoning it. Only Fryze's idea that power properties should not be explained in the frequency-domain but only in the time-domain had to be abandoned. The main progress in power theory was obtained in the frequency-domain.

10 Conclusions

Development of the power theory of circuits with nonsinusoidal voltages and current was dominated for over six decades by Budeanu's power theory which eventually occurred to be a misleading theory. Distortion power has nothing in common with waveform distortion and Budeanu's reactive power is not a measure of energy oscillation between the source and the load. Moreover, there is no relation between Budeanu's reactive power and power factor.

Since two generations of electrical engineers have interpreted power properties in such circuits in terms of that theory, it exerted a negative effect on the present comprehension of power phenomena in circuits with nonsinusoidal voltages and currents. At the same time Fryze's approach contributed advantageously to the development of power theory, a number of original concepts are still alive, although some ideas remain controversial. In particular, Fryze's concept of the decomposition of current into orthogonal components implemented, however, not in the time-, but in the frequency-domain, has proved to be very fruitful for explaining power properties of circuits with nonsinusoidal voltages and currents.

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