Fryze - Buchholz - Depenbrock: A time-domain power theory

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***Abstract* - Several power theories trying to explain the difference between the physically defined active power and the apparent power are known – and under discussion. This paper gives an overview of a universal time-domain power theory mainly developed by Fryze, Buchholz and Depenbrock. It is verified by theoretical considerations and explained by examples. The theory allows the assessment of loads concerning effectivity of energy transfer and the application-specific splitting of currents into desirable and undesirable components – which in turn can, e.g., be compensated.**

I. Introduction

Already in 1865 Maxwell [1] introduced the concept of phase displacement, caused by reactive elements. In 1892 Steinmetz [2] demonstrated, that in case of a nonlinear load non-active currents occur without phase displacement. A historical overview of the development of power theory in the following decades has been published by Skudelny [3] in 1979 at a special ETG[1] conference dedicated to power quantities. At that conference Depenbrock [4] published the main concepts of his extension of the work of Fryze [5, 6] and Buchholz [7] which is the subject of this paper. This so-called FBD method [8] has a firm theoretical base and is applicable to any number of conductors and any kind of current and voltage waveforms. It is also the basis of the German standard on power definition [9, 10].

Requirements concerning power theory have been published, e.g., in 1994 [11]. A discussion of such requirements and possibilities to meet them has been led within a working group associated with the "International Workshop on Power Definitions and Measurements under Nonsinusoidal Conditions", with the result being published by Ferrero [12].

This paper presents the theoretical background of the power theory and illustrates its use and meaning by suitably selected examples. It also suggests and discusses names for quantities, based on [12].

Only those publications containing basic contributions are explicitly referenced in this tutorial. Further publications are, e.g., listed on a WEB-page maintained by Ferrero [13]

II. Aim of Power Theory

First of all, a power theory has to assess the energy exchange between a source and a load at its terminals concerning effectivity. The result of this assessment is the power factor λ which is a characteristic of the load and may, therefore, not depend on any characteristics of the source. This includes the condition, that the power factor may not depend on the way in which source and load are connected. Also, the internal structure of the load is without avail.

In a second step, a power theory may provide information on how to improve the energy exchange of a given load (or a combination of several loads) by compensation. This task is application (and compensator) specific and includes, e.g., the decomposition of the load current into various components using time-domain or frequency-domain methods.

Effectivity of energy transfer, in this context, refers to supplying energy with minimal total current at a given voltage.

A generally applicable power theory – like all theories – becomes invalid if only one counter-example is given. It may, however, still be useful within a limited scope, e.g., for compensation purposes.

III. Basic Idea of the FBD Power Theory

The basic idea of Fryze is to always represent a single-phase load, concerning its active power, by a suitably calculated **equivalent active conductance** G_a fed by the same voltage as the load (Fig. 1). The current time function of this equivalent conductance defines the active current. It is always proportional to that of voltage[2].

$$i_{a1} = G_a \, u_{12} \tag{1}$$

It transfers the same energy per period as the original load current, but with minimal rms value of current. The definition is valid for all conceivable voltage waveforms.

That component of load current which is not the active current clearly defines the non-active current[3] i_{x1}, which is modelled by a suitably controlled current source because in general the circuit elements associated with this current are unknown or not of interest:

$$i_{x1} = i_1 - i_{a1} \tag{2}$$

[1] Energy Technology Group of VDE, the German Association for Electrical, Electronic & Information Technologies

[2] The instantaneous values of the active current give – strictly speaking – the instantaneous active current. In literature the term *instantaneous active current* is sometimes used for a quantity which is subsequently introduced in this paper and named *power current*.

[3] Again: The instantaneous values of this current give – strictly speaking – the instantaneous non-active current. In literature the term *instantaneous non-active current* is sometimes used for a quantity which is subsequently introduced in this paper and named *powerless current*.

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These definitions are included in the **I**nternational **E**lectrotechnical **V**ocabulary (IEV) [14].

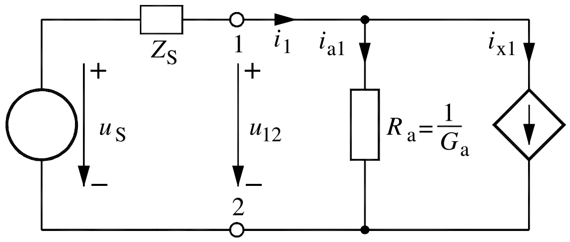

Figure 1. Equivalent circuit for load current decomposition

The time functions of active current and of non-active current are **orthogonal functions**. The time function of active current is identical with that of the voltage.

In general the equivalent circuit has nothing to do with the original structure of the load. It is not derived from the load structure but only depends on the voltage and current at the terminals of the load. The same equivalent circuit models all conceivable loads which cause the same current for a given voltage. The equivalent circuit guarantees that Kirchhoff's laws are fulfilled by the decomposed currents.

The time function associated with active power[4] results from the active current

$$p_a = u_{12}\, i_{a1} = G_a\, u_{12}^2 \quad (3).$$

This basic concept of decomposition was extended to multiple conductor systems by Buchholz and to instantaneous power and freely selectable orthogonal non-active current components by Depenbrock.

IV. BASICS DEMONSTRATED BASED ON TWO-CONDUCTOR CIRCUITS

Every electrical engineer has a basic understanding of instantaneous power, active power, reactive power and apparent power. The two-conductor (single-phase) case is perhaps the only one where all engineers agree – at least in the sinusoidal case. Already in case of nonsinusoidal voltages and currents diverse propositions are made. Before continuing with more complex multi-conductor circuits, the main properties of single-phase circuits are characterized. Periodic quantities are presupposed.

A. Sinusoidal quantities

Consider the circuit in Fig. 2 with load voltage u_{12} and current i_1 according to (4). Source voltage and source impedance are not accounted for if defining power quantities. They just complete the circuit.

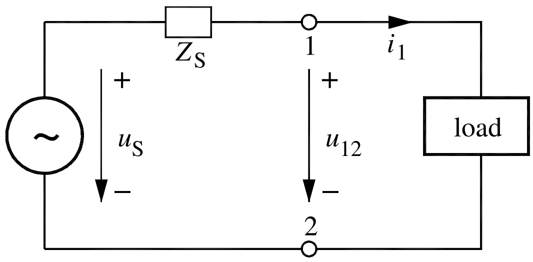

Figure 2. Single-phase example with sinusoidal quantities

$$\begin{aligned} u_{12} &= \hat{u}\cos(\omega t + \varphi_u) \\ &= \hat{u}[\cos(\omega t)\cos(\varphi_u) + \sin(\omega t)\sin(\varphi_u)] \\ i_1 &= \hat{i}\cos(\omega t + \varphi_i) \\ &= \hat{i}[\cos(\omega t)\cos(\varphi_i) + \sin(\omega t)\sin(\varphi_i)] \\ &= i_{a1}(t) + i_{q1}(t) \end{aligned} \quad (4)$$

With the phase displacement between voltage and current

$$\varphi = \varphi_u - \varphi_i \quad (5)$$

active power P_a is

$$P_a = \overline{p_a} = \overline{p} = \overline{u_{12}\, i_1} = \frac{\hat{u}\,\hat{i}}{2}\cos(\varphi) \quad (6)$$

and the equivalent active conductance, which has to transfer the same energy per period,

$$G_a = \frac{P_a}{U_{12}^2} = \frac{P_a}{\hat{u}^2/2} = \frac{\hat{i}}{\hat{u}}\cos(\varphi) \quad (7).$$

Active and non-active currents follow from (1) and (2). Only in case of sinusoidal quantities the non-active current is called reactive current i_q (see IEV, [14]).

$$\begin{aligned} i_{a1} &= u_{12}\, G_a = u_{12}\frac{\hat{i}}{\hat{u}}\cos(\varphi) \\ i_{x1} &= i_{q1} = i_1 - i_{a1} \end{aligned} \quad (8)$$

Fig. 3 shows the resulting time functions for phase angles $\varphi_u = 70°$, $\varphi_i = 30°$, $\varphi = \varphi_u - \varphi_i = 40°$

This decomposition is commonly known and agreed upon. Here an inductor could be the cause of the reactive current. Active and reactive component could, however, also result from more complex phenomena, for example, a load containing suitably controlled power electronics.

[4] Here also: The instantaneous values of active power give – strictly speaking – the instantaneous active power. In literature the term *instantaneous active power* is often used for a quantity which is subsequently introduced in this paper and named *collective instantaneous power*.

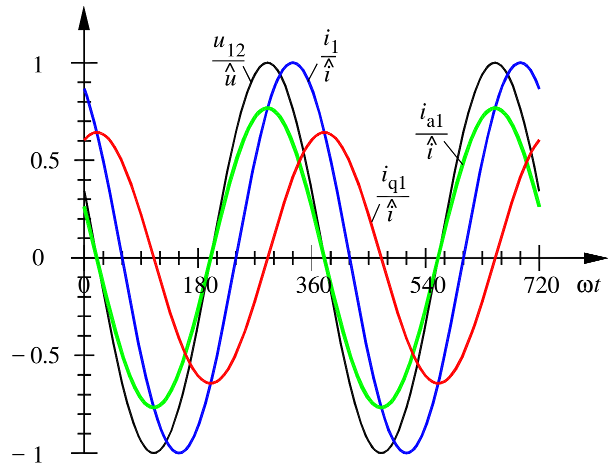

Figure 3. Decomposition into active and reactive current

There are currents – which powers are associated?

The product of voltage u_{12} and current i_1 gives instantaneous power p – which oscillates (Fig. 4). There are time intervals where instantaneous power is negative – energy flows from load to source.

The product of voltage u_{12} and active current i_a gives the instantaneous value of active power p_a. It is proportional to the square of the voltage. Therefore it oscillates, but it is always positive. Its mean value is – by definition – the same as that of instantaneous power p (6).

The product of voltage and reactive current gives the instantaneous power quantity $u_{12}\, i_{q1}$ with mean value zero. The zero mean value is obvious, because by definition the total energy during one period is already delivered by the active current. This power quantity $u_{12}\, i_{q1}$ is **not** associated with reactive power, which is defined below.

In this way we have worked through all products of voltage and current components, the results are shown in Fig 4.

Time functions are rather not very graphic. There are several well-known rms phasor representations. The one chosen here (Fig. 5) can readily be extended to nonsinusoidal or multi-conductor cases for sinusoidal quantities. In effect it associates a cosine function with one axis and a sine function with the quadrature axis. Voltage phasor **U** and current phasor **I** can be used to graphically define reactive power (Fig. 5).

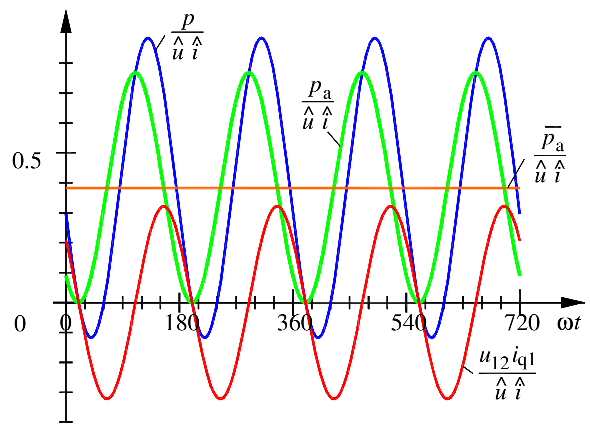

Figure 4. Time functions of power quantities related to Fig. 3

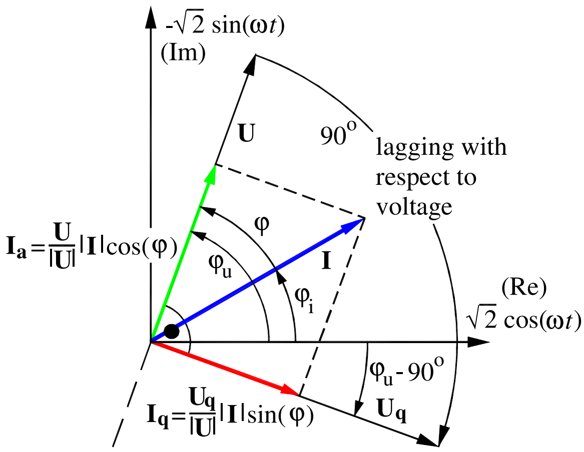

Figure 5. Rms phasor representation

Simplified, active power is given by $P_a = \mathbf{U} \bullet \mathbf{I} = \mathbf{U} \bullet \mathbf{I}_a$, while reactive power Q is the "active power" resulting from current and a fictitious voltage time function u_q described by the phasor[5] $\mathbf{U}_q$. The fictitious voltage phasor is defined by the original voltage phasor, rotated by 90° in lagging direction (clockwise, Fig. 5):

$$Q = \overline{u_q\, i} = \mathbf{U}_q \bullet \mathbf{I} = \mathbf{U}_q \bullet \mathbf{I}_q = \frac{\hat{u}\,\hat{i}}{2}\sin(\varphi) \qquad (9).$$

While the product $u_{12}\, i_{q1}$ is a physical power quantity associated with reactive current, reactive power Q has no direct link to physical power quantities. Depending on the angle φ, reactive power is positive or negative, indicating inductive or capacitive current.

Finally, the relations to apparent power $S = U_{12} I_1$ are needed. These use the orthogonality of currents and multiply by the rms value of voltage:

$$\begin{aligned}
&I_1^2 = I_{a1}^2 + I_{x1}^2 = I_{a1}^2 + I_{q1}^2 \Rightarrow \\
&U_{12}^2 I_1^2 = U_{12}^2 I_{a1}^2 + U_{12}^2 I_{x1}^2 = U_{12}^2 I_{a1}^2 + U_{12}^2 I_{q1}^2 \Leftrightarrow \qquad (10)\\
&S^2 = P_a^2 + {P_x}^2 = P_a^2 + Q^2 = \frac{\hat{u}\,\hat{i}}{2};\;\; P_x = U_{12} I_{x1} = |Q|
\end{aligned}$$

Non-active power P_x is the apparent power of a purely non-active element. Therefore it is not a signed quantity.

Sub-Conclusion:

- The definition of active current is based on an equivalent active conductance
- In the general case the time function of active power (the optimal time function of power) is not a constant, but proportional to the square of the voltage time function

[5] The mathematical background of phasor representation is indeed the use of vectors of orthogonal time functions, as indicated in Fig. 5. The inner product of these vectors (including time dependency) gives the time function of power and reactive power, the mean values of which are active and reactive power.

- The difference between current and active current is the non-active current. Under sinusoidal conditions it is called reactive current [14]
- Reactive current can be *modelled* by a reactive equivalent element (inductor, capacitor). The apparent power of such equivalent elements is equal to the absolute value of their reactive power
- Reactive power is not a physical quantity, the same applies to apparent power. However, the definitions make sense, as will be shown later
- Reactive power is a signed quantity
- Orthogonal current components (in this case with orthogonal time functions) are the key to understand active and non-active quantities

B. *Nonsinusoidal quantities*

Let us now turn to nonsinusoidal conditions, Fig. 6, selecting a simple example.

The example load consists of a two-terminal switch and an ideal ohmic resistor R_L in series connection. Following the Fryze concept, the resistor itself is an active element. The switch is not associated with any physical power at all and therefore a non-active element. It opens and closes periodically. In this example it is open for the first 60% of the period T and closed for the remaining 40% (Fig. 7).

The active power results in

$$P_a = 0.4\,\hat{u}\,\hat{i} = 0.4\,\hat{u}^2 / R_L \qquad (11)$$

and the equivalent active conductance (7) to

$$\hat{u}^2 G_a \overset{!}{=} P_a \;\Rightarrow\; G_a = 0.4/R_L \qquad (12)$$

giving the active current as

$$i_{a1} = u_{12}\,G_a = 0.4\,u_{12}/R_L = 0.4\,\hat{i} \qquad (13).$$

The non-active current is the difference between current and active current (2). Because of nonsinusoidal conditions it must not be called reactive current. While reactive power in the sinusoidal case is a signed quantity non-active power does not carry a sign – it is always positive. Fig. 8 gives the powers of the example circuit.

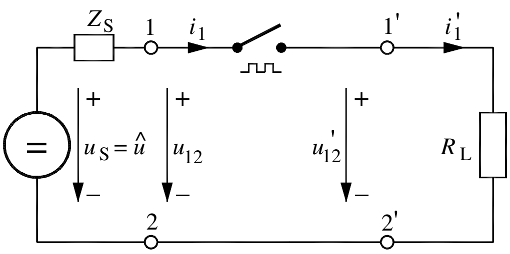

Figure 6. DC source with load consisting of switch and resistor

With the rms value of current i_1, $I_1 = \sqrt{0.4}\,\hat{i}$, apparent power is $S = \sqrt{0.4}\,\hat{u}\hat{i}$ and the power factor

$$\lambda = P_a/S = 0.4/\sqrt{0.4} = \sqrt{0.4} \approx 0.632 < 1 \text{ [6] } (14).$$

The apparent power of the equivalent current source defines the non-active power P_x

$$P_x = U_{12} I_x = \hat{u}\,\sqrt{0.4(1-0.4)}\;\hat{i} = \sqrt{0.24}\;\hat{i}\,\hat{u} \qquad (15).$$

Because of the two orthogonal time functions, i_{a1} and i_{x1}, the sum of the squares of active and non-active power gives the square of apparent power also under nonsinusoidal conditions (10).

In the general case non-active power has nothing to do with a simple phase-shift, as it could be defined in the frequency-domain. A fictitious voltage waveform associated with non-active power cannot be produced by, e.g., phase-shifting each frequency component of the load voltage by 90°.

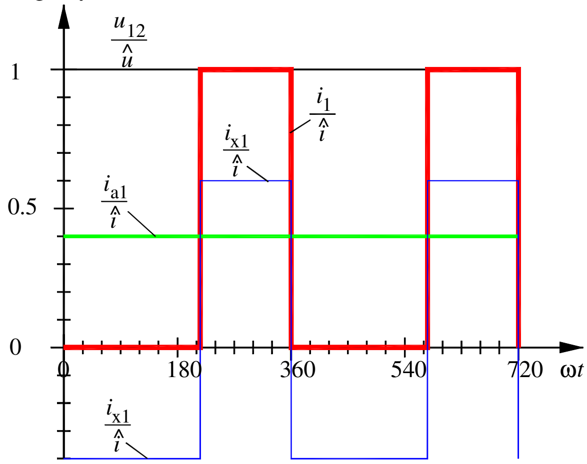

Figure 7. Switched resistor: voltage and currents

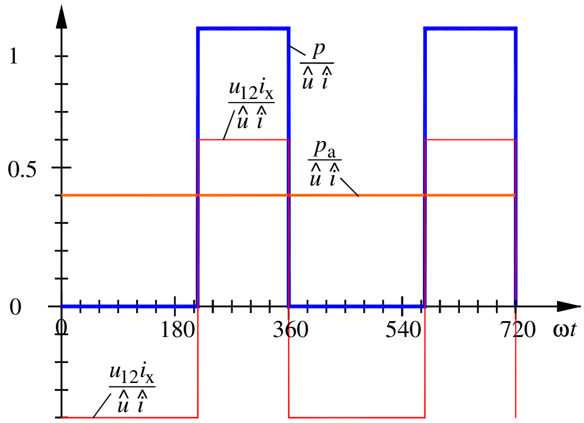

Figure 8. Switched resistor: power quantities

[6] IEEE standard 1459-2000 uses this definition, German standard DIN 40110 uses $|P_a|/S$. In case of recuperation the IEEE version becomes negative, while the German version is always between 0 (worst) and 1 (best). The difference is negligible but should be known.

Sub-Conclusion:

- A switch may cause non-active power, although energy flows in only one direction and no power at all is associated with the switch itself
- A generally accepted definition for reactive power exists only under sinusoidal conditions
- Non-active power is always positive, it can in general not be associated with reactive elements
- In the general case, no physical interpretation of non-active power is possible
- Current decomposition into active and non-active components is always possible if one period of current and voltage is known
- Active current has a clearly defined time function – the instantaneous active current. It should be avoided to give this name to other quantities
- Non-active current has a clearly defined time function – the instantaneous non-active current. It should be avoided to give this name to other quantities
- The non-active current can be computed without using non-active power or fictitious voltages

C. *Frequency-domain considerations*

Time-domain approaches and frequency-domain approaches are often seen separately. However, time-domain and frequency-domain are linked by mathematical transformations (Fourier series). Each time-domain theory automatically has its meaning in the frequency-domain – and vice versa. In some cases, frequency-domain techniques are very important, for example in connection with classical filter circuits. In other cases, frequency-domain techniques lead to equations which easily cause misinterpretations.

The following example, suggested by Depenbrock, illustrates possible misinterpretations. Consider voltage and current of a load as shown in Fig. 9.

The example shown in Fig. 9 is a prototype for several examples. Voltage and current are selected such that power is permanently zero. The whole current is obviously non-active. However, a frequency decomposition of the waveforms gives an infinite number of voltage and current harmonics. In the frequency-domain, depending on the selected symmetry of the time functions, voltages and currents seem to interact – or not.

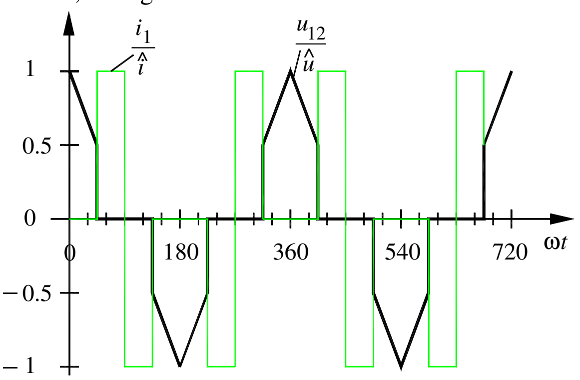

Figure 9. Nonsinusoidal current and voltage at a load

"Harmonic active power", or "harmonic reactive power", erroneously defined as

$$P_{\mathrm{a}\nu} = \hat{u}_\nu\,[\hat{i}_\nu \cos(\varphi_\nu)]\;;\; P_{\mathrm{q}\nu} = \hat{u}_\nu\,[\hat{i}_\nu \sin(\varphi_\nu)] \quad (16)$$

seem to exist, although in no time instant power occurs.

This again proves that physical interpretations are very critical and may be misleading. The active and reactive power associated with a harmonic can, of course, be derived by developing time functions of voltage, current components or powers into harmonic series, e.g.:

$$P_{\mathrm{a}\nu} = \frac{\hat{u}_\nu}{\sqrt{2}}\frac{\hat{i}_{\mathrm{a}\nu}}{\sqrt{2}} = G_\mathrm{a}\frac{\hat{u}_\nu^2}{2} \quad (17).$$

Sub-Conclusion:

- Care is needed when interpreting frequency-domain decomposition
- It does not make sense to associate "active power" of a harmonic to the definition given in (16). Active current has a time function which can be developed into a harmonic series (17). This is the only true basis for the active power associated with a harmonic

V. Extension to Multi-Conductor Circuits

The work of Fryze was extended to multi-conductor circuits by Buchholz [7], who concentrated on periodic quantities, and Depenbrock, who treated instantaneous quantities [15] and introduced rules for application-specific decompositions [4]. This tutorial presents the up-to-date state of the theory without regard to details of contribution.

A. *Virtual star point*

Introducing a third conductor already causes lots of discussion. While most engineers and scientists take the currents of the conductors for granted, voltage definition seems to be difficult – from the point of view of the author the IEEE standard 1459 still has deficiencies in this respect.

The reason for the discussion on voltage definition is that in two-conductor circuits the voltage between the two conductors is used – nobody wishes to measure two voltages against a reference point of any kind. In three-conductor systems, however, measuring three voltages against a reference, usually earth potential or "neutral", is often suggested. This causes problems, because the selection of the reference potential determines the three voltages – but obviously not the behaviour of the load connected to the three conductors.

The logical extension of two-conductor to three-conductor systems would be to use the conductor-to-conductor voltages, which do not need an artificial reference (and, due to Kirchhoff's law, sum up to zero like the currents). However, these voltages are not directly related to the conductor currents (which also sum up to zero according to Kirchhoff's law).

The solution is to introduce the virtual star point (or true neutral point), which defines voltages which permanently and under all conditions sum up to zero. Such quantities are called zero-sum quantities in the following. They allow simple and unified mathematical treatment of any number of conductors. This star point exists for any number of conductors. It is possible, but not necessary, to create this star point. Consider an arbitrary load with voltages and currents as shown in Fig. 10:

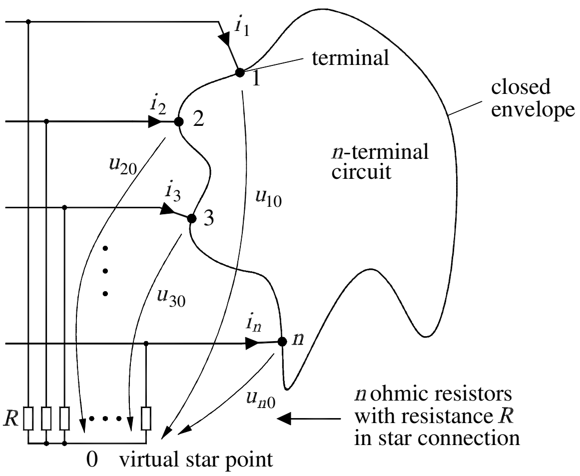

Figure 10. Arbitrary multi-conductor load (n conductors)

The virtual star point can be created by a set of n equal ohmic resistors in star connection. Each of the conductors is connected to the free end of exactly one of the resistors. The currents through the resistors sum up to zero – consequently the voltages at the resistors also sum up to zero, because they result from the currents multiplied by R.

Consider an arbitrary reference point used for voltage measurement, called r. The voltages measured against this reference point are then $u_{\nu r}; \nu = 1,2,\cdots,n$. The voltage between arbitrary reference and virtual star point is u_{r0}. The voltages between the terminals and the virtual star point can be expressed using the voltages measured against the arbitrary reference point and the voltage u_{r0}:

$$u_{\nu 0} = u_{\nu r} + u_{r0}; \nu = 1,2,\cdots,n \tag{18}$$

These voltages have to sum up to zero:

$$\begin{aligned} 0 = \sum_{\nu=1}^{n} u_{\nu 0} = \sum_{\nu=1}^{n} (u_{\nu r} + u_{r0}) = n\,u_{r0} + \sum_{\nu=1}^{n} u_{\nu r} \\ \Leftrightarrow u_{r0} = -\frac{1}{n}\sum_{\nu=1}^{n} u_{\nu r} \end{aligned} \tag{19}$$

With u_{r0} from (19) inserted into (18) the voltages with reference to the virtual star point can be derived from voltages measured against any common reference point r. If one of the conductors is selected as reference, the respective voltage is zero and only $(n-1)$ voltages have to be determined. This matches the situation concerning the currents: They also sum up to zero, so only $(n-1)$ have to be determined, too.

The same zero-sum voltages can also be calculated based on the conductor-to-conductor voltages according to

$$u_{\nu 0} = \frac{1}{n}\sum_{\mu=1}^{n} u_{\nu\mu}; \mu = 1,2,\cdots,n \tag{20}$$

This equation and further important considerations concerning multi terminal circuits are found in [16]. From the point of view of the author, (18) and (19) are easier to understand and require less calculation effort – but (20) proves that no selection of a reference point is needed – the zero-sum voltage state of an n-conductor system can also be derived directly from the conductor-to-conductor voltages.

B. Vectors of currents and voltages

For easier mathematical notation, currents and voltages are represented by vectors, where element No. ν describes the quantity of conductor ν:

$$\mathbf{i} = [i_1 \quad i_2 \quad \cdots \quad i_n]^{\mathrm{T}}; \; \mathbf{u} = [u_{10} \quad u_{20} \quad \cdots \quad u_{n0}]^{\mathrm{T}} \tag{21}$$

Please recall that currents and voltages are zero-sum quantities by definition

$$\sum_{\nu=1}^{n} i_\nu = 0; \; \sum_{\nu=1}^{n} u_{\nu 0} = 0 \tag{22}$$

This notation is valid for $n \geq 2$ (two or more conductors).

Where clear from the context, the term "vector of" is sometimes omitted in the following for sake of brevity. Vectors of quantities are shortly referenced as quantities.

C. Collective quantities

In the end, one scalar value, the power factor, should describe the quality of energy transfer caused by a load. This value has to take into account all currents and all voltages. Also, in case of more than two conductors, the load can be characterized – partly – on an instantaneous basis. For this, all instantaneous values of voltages and currents have to be treated collectively. This is accomplished by collective instantaneous values defined as the norm (the "length") of the current and voltage vectors:

$$u_\Sigma = \|\mathbf{u}\| = \sqrt{\mathbf{u}^{\mathrm{T}} \bullet \mathbf{u}} = \sqrt{\sum_{\nu=1}^{n} u_{\nu 0}^2}; \; i_\Sigma = \|\mathbf{i}\| = \sqrt{\sum_{\nu=1}^{n} i_\nu^2} \tag{23}$$

The rms value of the collective instantaneous values is called collective rms value

$$\begin{aligned} U_\Sigma = \sqrt{\frac{1}{T}\int_{t-T}^{T} u_\Sigma^2 \,\mathrm{d}t} = \sqrt{\sum_{\nu=1}^{n} U_{\nu 0}^2} \\ I_\Sigma = \sqrt{\frac{1}{T}\int_{t-T}^{T} i_\Sigma^2 \,\mathrm{d}t} = \sqrt{\sum_{\nu=1}^{n} I_\nu^2} \end{aligned} \tag{24}$$

D. Instantaneous characterization of a load

The instantaneous power of the load follows from the inner (or dot) product of the vectors of currents and voltages

$$p = \mathbf{u}^{\mathrm{T}} \bullet \mathbf{i} \qquad (25).$$

The vectors of currents $\mathbf{i}$ and voltages $\mathbf{u}$ define a plane in an n-dimensional vector space[7] including the origin[8] (Fig. 11).

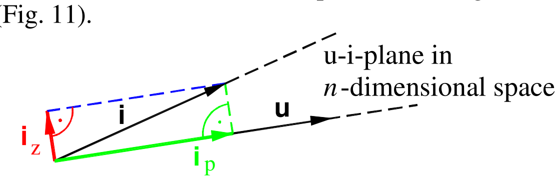

Figure 11. u-i-plane in n-dimensional space

Within this plane, the vector of currents can be decomposed into two components: The vector of power currents $\mathbf{i}_{\mathrm{p}}$ parallel (or antiparallel) to the vector of voltages $\mathbf{u}$ and the vector of powerless currents $\mathbf{i}_{\mathrm{z}}$ perpendicular to $\mathbf{u}$. In case of $n = 2$ only $\mathbf{i}_{\mathrm{p}} = \mathbf{i}$ exists, $\mathbf{i}_{\mathrm{z}}$ cannot exist ($\mathbf{i}_{\mathrm{z}} = 0$).

The inner product of two perpendicular vectors is zero. With (25) follows

$$p = \mathbf{u}^{\mathrm{T}} \bullet (\mathbf{i}_{\mathrm{p}} + \mathbf{i}_{\mathrm{z}}) = \mathbf{u}^{\mathrm{T}} \bullet \mathbf{i}_{\mathrm{p}} + \underbrace{\mathbf{u}^{\mathrm{T}} \bullet \mathbf{i}_{\mathrm{z}}}_{=0} = \mathbf{u}^{\mathrm{T}} \bullet \mathbf{i}_{\mathrm{p}} \qquad (26).$$

Only the vector of power currents, linked to the vector of voltages by the power conductance $G_{\mathrm{p}}(t)$,

$$\mathbf{i}_{\mathrm{p}} = G_{\mathrm{p}}(t)\,\mathbf{u} \qquad (27)$$

transfers energy to the load in each instant. The value of the power conductance follows from (23), (25) and (26)

$$p = \mathbf{u}^{\mathrm{T}} \bullet \mathbf{i}_{\mathrm{p}} = \mathbf{u}^{\mathrm{T}} \bullet G_{\mathrm{p}}(t)\,\mathbf{u} \Rightarrow G_{\mathrm{p}}(t) = p \big/ u_{\Sigma}^2 \qquad (28).$$

The power conductance describes the power part of an equivalent load which in total has equivalent circuit diagram shown in Fig. 12.

The perpendicular vector of powerless currents, $\mathbf{i}_{\mathrm{z}}$, follows from load current and vector of power currents

$$\mathbf{i}_{\mathrm{z}} = \mathbf{i} - \mathbf{i}_{\mathrm{p}} = \mathbf{i} - G_{\mathrm{p}}(t)\,\mathbf{u} \qquad (29).$$

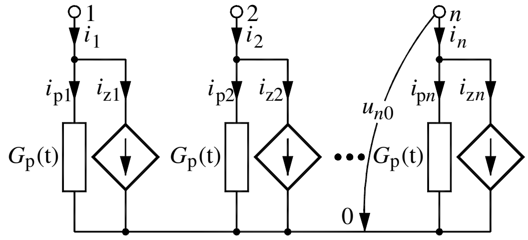

Figure 12. Equivalent circuit diagram of equivalent load

It usually causes power in each of the conductors, but the sum of all these powers is **z**ero in every instant. No resultant energy transfer occurs – this vector of currents is definitely a vector of non-active currents[9].

Power quantities are defined in analogy to the two-conductor case by making use of the mathematical orthogonality of the perpendicular vectors of currents:

$$\|\mathbf{i}\|^2 = \|\mathbf{i}_{\mathrm{p}}\|^2 + \|\mathbf{i}_{\mathrm{z}}\|^2 \Rightarrow \|\mathbf{u}\|^2\|\mathbf{i}\|^2 = \|\mathbf{u}\|^2\|\mathbf{i}_{\mathrm{p}}\|^2 + \|\mathbf{u}\|^2\|\mathbf{i}_{\mathrm{z}}\|^2$$
$$\Leftrightarrow u_{\Sigma}^2\, i_{\Sigma}^2 = u_{\Sigma}^2\, i_{\mathrm{p}\Sigma}^2 + u_{\Sigma}^2\, i_{\mathrm{z}\Sigma}^2 \Leftrightarrow s^2 = p^2 + p_{\mathrm{z}}^2 \qquad (30).$$

The quantity s is called collective instantaneous apparent[10] power. Please note: No instantaneous value of $S = U_{\Sigma} I_{\Sigma}$ exists. Straightforwardly, p_{z} is called "collective instantaneous apparent powerless power". It is always positive and not a physical quantity. These names could, of course, be discussed.

Up to now apparent power was simply accepted. It is no physical quantity, therefore a strong motivation is needed. This motivation can be given by mathematical and by practical considerations, and it starts with a motivation for the definition of s given in (30).

E. Motivation of instantaneous apparent power s

Quantities in (30) which are directly physical are $\mathbf{u}, \mathbf{i}$ and p. They are linked by the Cauchy-Schwarz inequality, stating finally

$$s^2 \geq p^2 \; ; \quad s^2 = p^2 \;\text{ if }\; \mathbf{i} = G_{\mathrm{p}}(t)\,\mathbf{u} \qquad (31).$$

Instantaneous apparent power s is the maximal value of the absolute value of instantaneous power $|p|$ under the constraint of given collective instantaneous values $\|\mathbf{u}\| = u_{\Sigma}, \|\mathbf{i}\| = i_{\Sigma}$. In other words: A given instantaneous power p at given voltages $\mathbf{u}$ is best realized by currents $\mathbf{i} = \mathbf{i}_{\mathrm{p}} = G_{\mathrm{p}}(t)\,\mathbf{u} \Leftrightarrow p_{\mathrm{z}} = 0$.

[7] An additional limitation follows from (22): The degree of freedom of the vectors is only (n-1). In consequence, for $n = 2$, both vectors are collinear, the plane is reduced to a single line.

[8] In two dimensions, the two zero-sum currents and the two zero-sum voltages are inversely equal, leading to a diagonal in the plane. In n dimensions, a "diagonal" (n-1) subspace including the origin results.

[9] It is misleading to use the name *instantaneous non-active current* vector, because total non-active currents can only follow from considering a whole period – and these total non-active currents also have instantaneous values.

[10] The characterizer "apparent" in context with power quantities indicates that these power quantities are derived using fictitious voltages and are therefore no physical quantities

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A practical motivation assumes identical resistances R_S in all conductors feeding the load[11]. Instantaneous power loss in these resistances is given by

$$p_R = \sum_{\nu=1}^{n} R_S\, i_\nu^2 = R_S\, i_\Sigma^2 \qquad (32).$$

The minimum of i_Σ^2 leads to minimal instantaneous losses. Under the conditions detailed above it is characterized by

$$\mathbf{i}_z = 0 \Leftrightarrow \mathbf{i} = \mathbf{i}_p = G_p(t)\,\mathbf{u} \Leftrightarrow p^2 = s^2 \qquad (33).$$

F. Example for loads causing powerless currents

Fig. 13 gives two examples for loads causing powerless currents.

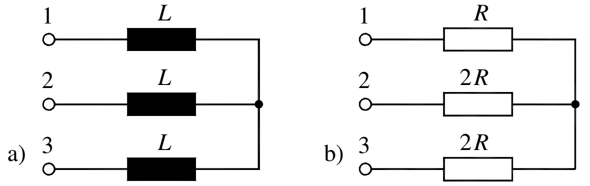

Figure 13. Example loads causing (also) powerless currents

Under balanced three-phase sinusoidal conditions three separate (not magnetically coupled) inductors (part a) lead to purely powerless currents. Three resistors with different resistances in star connection (part b) cause power and powerless currents.

This shows that powerless currents in general do not describe any physical kind of energy exchange between load branches inside the load. Also, they are in no way bound to reactive elements.

G. Compensation of powerless currents

The interesting aspect of powerless currents is that they can be computed instantaneously and compensated without any resultant energy transfer by a suitably designed shunt compensator (Fig. 14).

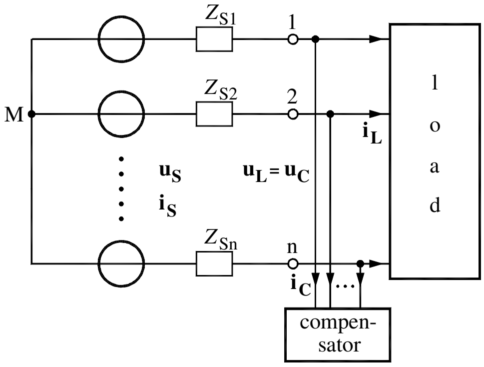

Figure 14. Shunt compensator

This can be done without any calculation of power quantities besides collective instantaneous power p combining (23), (25), (26) and (29)

$$\mathbf{i}_C = -\mathbf{i}_z = \mathbf{i}_p - \mathbf{i}_L = \frac{\mathbf{u}_L^T \bullet \mathbf{i}_L}{\mathbf{u}_L^T \bullet \mathbf{u}_L}\mathbf{u}_L - \mathbf{i}_L \qquad (34)$$

This way of compensation is very simple and low-effort, but in some applications it has severe disadvantages which should be considered [17, 18, 19, 23].

H. Characterization of a load considering a whole period

The active power (6) is the mean value, taken over a period T, of instantaneous power. In case of n conductors:

$$P_a = \frac{1}{T}\int_{t-T}^{t} p\,dt = \frac{1}{T}\int_{t-T}^{t} \mathbf{u}^T \bullet \mathbf{i}\,dt = \frac{1}{T}\int_{t-T}^{t} \sum_{\nu=1}^{n} (u_{\nu 0}\, i_\nu)\,dt \qquad (35)$$

If voltages and currents are periodic in T, active power is constant – otherwise a sliding integration interval leads to time-variant results, which are not discussed here but important for the behaviour of, e.g., compensators in case of dynamics.

Only the (time dependent) vector of active currents

$$\mathbf{i}_a = G_a\,\mathbf{u} \Rightarrow p_a = \mathbf{u}^T \bullet \mathbf{i}_a = G_a \mathbf{u}^T \bullet \mathbf{u} = G_a u_\Sigma^2 \qquad (36)$$

permanently proportional to the vector of voltages $\mathbf{u}$ with constant factor of proportion (the active conductance G_a) transfers energy optimally seen over a period. It also defines the time function p_a associated with active power. The active currents $\mathbf{i}_a$ have to cause the same active power as the currents $\mathbf{i}$:

$$\frac{1}{T}\int_{t-T}^{t} \mathbf{u}^T \bullet \mathbf{i}_a dt \overset{(36)}{=} G_a \frac{1}{T}\int_{t-T}^{t} u_\Sigma^2\,dt \overset{(24)}{=} G_a U_\Sigma^2 = U_\Sigma I_{a\Sigma} = P_a \qquad (37)$$

giving the active conductance as

$$G_a = P_a \big/ U_\Sigma^2 = \overline{p} \big/ \overline{u_\Sigma^2} \neq \overline{G_p} = \overline{p \big/ u_\Sigma^2} \text{ if } u_\Sigma \neq \text{const.} \qquad (38).$$

In general the active conductance is **not** the mean value of the power conductance – except in those cases where the collective instantaneous value of voltages is constant! For compensation purposes in nearly ideal three-phase systems with three or four conductors it may be sufficient to assume a constant value of u_Σ, which – with (36) – implies the time function of active power to be constant. In case of unsymmetrical voltages – or two-conductor circuits – this is not optimal. Clearly, compensation issues and practical assumptions have to be separated carefully from power theory.

Non-active currents $\mathbf{i}_x$ result, like (8), in

$$\mathbf{i}_x = \mathbf{i} - \mathbf{i}_a = \mathbf{i} - G_a\,\mathbf{u} \qquad (39).$$

[11] This is, of course, an approximation, but the best possible one. Consider that the load should be assessed without knowing the characteristics of the source.

The equivalent circuit associated is the same as for instantaneous decomposition (Fig. 12), but with $G_{\mathrm{p}}(t)$ replaced by G_{a} and $\mathbf{i}_{\mathrm{z}}$ by $\mathbf{i}_{\mathrm{x}}$.

We now have two types of orthogonal quantities: non-active currents and active currents are orthogonal with relation to a period (37), powerless currents and power currents because their vectors are perpendicular. The active currents certainly do not contain any powerless component – the powerless currents are included in the non-active currents, which can, according to (27) and (29), be decomposed into powerless currents and a remaining non-active component of power currents. This component of power currents is called vector of variation currents $\mathbf{i}_{\mathrm{v}}$

$$\mathbf{i}_{\mathrm{v}} = \mathbf{i}_{\mathrm{x}} - \mathbf{i}_{\mathrm{z}} = \mathbf{i} - \mathbf{i}_{\mathrm{a}} - \mathbf{i}_{\mathrm{z}} = \mathbf{i}_{\mathrm{p}} - \mathbf{i}_{\mathrm{a}} \qquad (40).$$

It is orthogonal to the active currents, because it does not contribute to active power, and collinear to the vector of voltages because it does not contain powerless currents. It causes resultant physical power (variation power) p_{v} in every time instant – but this power is a purely alternating periodic quantity with mean value zero.

Based on the orthogonal decomposition of currents

$$\mathbf{i} = \mathbf{i}_{\mathrm{a}} + \mathbf{i}_{\mathrm{x}} = \mathbf{i}_{\mathrm{a}} + \mathbf{i}_{\mathrm{v}} + \mathbf{i}_{\mathrm{z}} \;, \qquad (41)$$

eq. (23), (24) and in analogy to (30) apparent power S is:

$$\begin{gathered} I_\Sigma^2 = I_{\mathrm{a}\Sigma}^2 + I_{\mathrm{x}\Sigma}^2 = I_{\mathrm{a}\Sigma}^2 + I_{\mathrm{v}\Sigma}^2 + I_{\mathrm{z}\Sigma}^2 \Rightarrow \;(\cdot\, U_\Sigma^2) \\ U_\Sigma^2 I_\Sigma^2 = U_\Sigma^2 I_{\mathrm{a}\Sigma}^2 + U_\Sigma^2 I_{\mathrm{x}\Sigma}^2 = U_\Sigma^2 I_{\mathrm{a}\Sigma}^2 + U_\Sigma^2 I_{\mathrm{v}\Sigma}^2 + U_\Sigma^2 I_{\mathrm{z}\Sigma}^2 \\ \Leftrightarrow S^2 = P_{\mathrm{a}}^2 + P_{\mathrm{x}}^2 = P_{\mathrm{a}}^2 + P_{\mathrm{v}}^2 + P_{\mathrm{z}}^2 \end{gathered} \qquad (42).$$

Straightforwardly, P_{x} is non-active power, P_{v} apparent variation power and P_{z} apparent powerless power. While it is simple to define current components, it is very hard to find meaningful and consistent names for the power quantities. Fortunately, most of these power quantities are not really needed – the current components are sufficient.

The power factor, valid under all periodic conditions, results in

$$\lambda = P_{\mathrm{a}} / S \;\text{ (IEEE) or }\; \lambda = |P_{\mathrm{a}}| / S \;\text{ (IEV)} \qquad (43).$$

Again, (42) formally defines power quantities based on orthogonal current components. This definition is sensible – but certainly a conventional one which needs theoretical backing. This is given in the next subsection.

I. *Motivation of apparent power S*

Quantities behind (42) which are directly physical are $\mathbf{u}$, $\mathbf{i}$ and p defining the collective rms values U_Σ, I_Σ and active power P_{a}. Those are linked by the Cauchy-Schwarz inequality, expanded by integration over a period, stating finally

$$S^2 \geq P_{\mathrm{a}}^2 \;; \quad S^2 = P_{\mathrm{a}}^2 \text{ if } \mathbf{i} = G_{\mathrm{a}}\, \mathbf{u} \;\; \forall\, t \in T \qquad (44).$$

Apparent power is the maximal value of active power P_{a} under the constraint of given collective rms values U_Σ, I_Σ. In other words: A given active power P_{a} at given collective rms value of voltages U_Σ is best realized by currents $\mathbf{i} = \mathbf{i}_{\mathrm{a}} = G_{\mathrm{a}}\, \mathbf{u} \Leftrightarrow P_{\mathrm{x}} = 0$.

A practical motivation again assumes identical resistances R_{S} in all conductors feeding the load. The total energy consumed by these resistances during a period is

$$T\, P_{\mathrm{R}} = T \sum_{\nu=1}^{n} R_{\mathrm{S}}\, I_\nu^2 = T\, R_{\mathrm{S}}\, I_\Sigma^2 \qquad (45).$$

The minimum of I_Σ^2 leads to minimal total losses. Under the conditions detailed above it is characterized by

$$\mathbf{i}_{\mathrm{x}} = 0 \Leftrightarrow \mathbf{i} = \mathbf{i}_{\mathrm{a}} = G_{\mathrm{a}}\, \mathbf{u} \Leftrightarrow P_{\mathrm{a}}^2 = S^2 \qquad (46).$$

J. *Compensation of non-active currents*

Based on the current components defined – without using any power quantities besides active power P_{a} – non-active currents $\mathbf{i}_{\mathrm{x}}$ can be compensated by applying a shunt compensator as already presented in Fig. 14. In this case the compensator currents $\mathbf{i}_{\mathrm{C}}$ have to be the negative of the non-active currents:

$$\begin{aligned} \mathbf{i}_{\mathrm{C}} &= -\mathbf{i}_{\mathrm{x}} = \mathbf{i}_{\mathrm{a}} - \mathbf{i}_{\mathrm{L}} = G_{\mathrm{a}}\mathbf{u} - \mathbf{i}_{\mathrm{L}} \\ &= \frac{P_{\mathrm{a}}}{U_\Sigma^2}\mathbf{u} - \mathbf{i}_{\mathrm{L}} = \frac{\int_{t-T}^{t} \mathbf{u}^{\mathrm{T}} \bullet \mathbf{i}_{\mathrm{L}}\, \mathrm{d}t}{\int_{t-T}^{t} \mathbf{u}^{\mathrm{T}} \bullet \mathbf{u}\, \mathrm{d}t}\mathbf{u} - \mathbf{i}_{\mathrm{L}} \end{aligned} \;.\; [12] \qquad (47).$$

By sliding the integration interval along the time axis, as indicated by the chosen limits, this compensation method remains valid also in case of dynamic load changes [19].

In steady state, compensation of powerless currents does not need any energy storage – in contrast, compensation of variation currents does. Based on variation currents and the voltages, the energy variation which has to be compensated can be derived allowing to dimension the energy storage capability of the compensator. Under dynamic conditions larger energy storage may be required. Several publications cover this aspect.

Additional aspects of such a compensation are:

- If currents and voltages are known to contain no even-order harmonics, the integration interval can be halved, so that dynamic reaction is improved

[12] Eq. (46) shows a structure which is on the one hand symmetrical and similar to (34) and on the other hand resembles the calculation of Fourier coefficients. It subdivides into orthogonal functions like Fourier, but these functions are nonsinusoidal in the general case.

- The equivalent load seen by the source is purely resistive and symmetrical (Fig. 12 with $G_p(t)$ replaced by G_a , $\mathbf{i}_p$ by $\mathbf{i}_a$, $\mathbf{i}_z$ by $\mathbf{i}_x$, $\mathbf{i}_x$ being zero because having been compensated). As long because G_a is positive all conceivable harmonics, unsymmetries and other effects are damped. For negative G_a care has to be taken
- In practice such a compensator cannot be realized for arbitrary current waveforms, there are limits resulting from obtainable current slopes, dead times and further effects. Methodologies to overcome part of the effects of such limitations are proposed in literature
- Normally it is not necessary to compensate the non-active currents totally. In practice limits, usually defined for frequency components are given. In such a case a compensation based on frequency components may be advisable
- An assessment of the load is the main goal of the FBD power theory. Compensated loads can be assessed by this theory treating load and compensator together as a new load

SUBCONCLUSION FOR MULTI-CONDUCTOR CIRCUITS

- Current components, based on orthogonality and, motivated by Cauchy-Schwarz inequalities, can easily and unambiguously be defined and named for *n*-conductor circuits
- The methodologies for instantaneous components and those for periodic conditions are very similar
- No power quantities besides active power are needed to define and calculate current components for compensation
- Powers resulting from the current components can be defined to assess a load. Most of these powers are fictitious quantities without true physical meaning

VI. FROM THEORY TO METHOD

The FBD theory presented here decomposes the currents into basic orthogonal components. In addition, the FBD method as proposed by Depenbrock [8] allows further decomposition into orthogonal components, which may be defined application specific. It gives a universal framework for constructing an equivalent load circuit fulfilling Kirchhoff's laws. This ensures that the decomposition made still covers the original properties of the load and reduces the risk of mathematical misconceptions leading to decompositions which are physically impossible (e.g., sets of load currents which do not sum up to zero).

VII. TRANSFORMATION OF ZERO-SUM QUANTITIES INTO OTHOGONAL COMPONENTS

In case of three conductors, the associated zero-sum quantities are linearly dependent. A suitable transformation can be applied to constitute mutually orthogonal quantities, reducing their number by one. For example, the mutually orthogonal α- and β-component of the Clarke transformation (also known as space-vector components) completely describe the linearly independent part of three zero-sum quantities. The three zero-sum quantities are treated equally. For four conductors, the 0-component of the Clarke transformation also has to be used – but in this case the four zero-sum quantities are not treated equally. A power theory has to treat all quantities equally, therefore the Clarke transformation is not optimal for four conductors. Instead, hyper space vectors could be used [24].

Linearly independent, mutually orthogonal quantities are much easier to use when designing control schemes.

VIII. RELATION TO INSTANTANEOUS POWER THEORY (P-Q THEORY)

Based on Clarke components, Akagi et. al. [20] introduced the p-q theory. It is applied to three-conductor and four-conductor system.

A. *Three-conductor systems*

In case of three conductors, power (equivalent to instantaneous power and the vector of power currents introduced in this tutorial) and imaginary power (containing the information about the vector of powerless currents) are introduced. From the point of view of this author, the interpretation of imaginary power, motivated by energy exchange between the conductors, is questionable – see Fig. 13.

For instantaneous quantities, concerning the results, no difference between the FBD-theory and the p-q-theory exists. The equations resulting from FBD-theory are, perhaps, less complex, because transformation into power quantities and back is not needed [23].

Under periodic conditions the aim of p-q theory is to keep power constant and, normally, compensate imaginary power and the oscillating part of power $\tilde{p}$ [13]. Using the quantities of the FBD method for this p-q theory concept, optimal source currents $\mathbf{i}_{Spq}$ and compensator currents $\mathbf{i}_{Cpq}$ simply are

$$\mathbf{i}_{Spq} = \mathbf{u}\,P_a \big/ u_\Sigma^2\,;\ \mathbf{i}_{Cpq} = \mathbf{i}_{Spq} - \mathbf{i}_L \qquad (48).$$

The motivation is to load the grid optimally in the sense of what it was designed for. For compensation, this is acceptable as u_Σ is more or less constant. Under fault conditions (single conductor undervoltage) keeping power constant is not sensible, because currents become large in time intervals where u_Σ is small (47). As power theory, this approach is not useful – a theory should always deliver optimal results.

A compensation based on the equivalent active conductance (38) proposed by FBD gives optimal results under all conceivable voltage conditions.

B. *Four-conductor systems*

In p-q theory, the four-conductor case is usually reduced to a three-conductor case by totally eliminating the current in the neutral conductor and applying the three-phase version of p-q theory to the remaining three conductors. Again, in most cases and if the neutral

[13] The oscillating part of power can only be computed based on one period of time – it is not an instantaneously available quantity!

conductor is known, this solution is acceptable from a practical point of view.

From a theoretical point of view, FBD has two advantages:

- The results are independent of the names and functions of the conductors, because all conductors are treated equally
- In case of compensation, the neutral current is eliminated only if the potential of the neutral conductor is that of the virtual star point. Otherwise, a neutral current drawing energy and thus reducing the voltage between neutral conductor and virtual star point is enforced

Modifications of the p-q theory [21, 22] only modify these characteristics without eliminating them [23]. This is because the characteristics of the Clarke transformation, which does not treat all conductors equally, are not properly taken into account.

IX. Further Examples

In addition to the examples included into this paper, examples can be found in various publications. For theory, [8], [16], [23] and [24], concerning practical compensation [11], [19], [25], [26] and [27] are recommended.

X. Conclusion

This paper presents the FBD power theory which is universally applicable to systems with any number of conductors and any kind of voltage- and current waveforms. The relations concerning currents, voltages, instantaneous-, active-, non-active and apparent power quantities are given. Basis of the theory is the decomposition of currents into orthogonal components, instantaneously and with regard to a period.

As an extension, the FBD method bases on this theory, allowing for further application-specific decomposition. All quantities can always be associated with equivalent load circuits fulfilling Kirchhoff's laws.

Up to now, no example was found which contradicts the theory or the method.

Further conclusions are integrated into the text at the end of subsections.

List of Symbols

Meaning of typeface (if not stated otherwise below)

u time function (of voltage)
$\mathbf{u}$ time-dependent vector (of voltages)
U rms value (of voltage)
$\mathbf{U}$ phasor (of voltage)
G time-independent value (of conductance)
$G(t)$ time-dependent value (of conductance)

Symbols (basic typeface)

G conductance
i current
n number of conductors
p power
Q reactive power ($Q = P_q$)
R resistance
S apparent power
s instantaneous apparent power
t time
T period
u voltage
Z impedance
φ time-independent phase angle
ω time-independent angular frequency

Subscripts

1 n number of a conductor
a active
C compensator
e equivalent
i associated with current
L load (omitted if obvious)
p power
q reactive
R associated with resistance
S source
u associated with voltage
v variation
x non-active
z powerless ("zero power")
Σ collective
ν, μ numeration
ν special numeration: harmonic order

Superscripts

T transposed

Overscripts

^ peak (or maximum) value
– mean value
~ oscillating part

Miscellaneous

| | absolute value
‖ ‖ norm (of a vector)
• inner product

List of Definitions

An asterisk (*) denotes quantities which are not strictly physical.
Abbreviation: coll. = collective; inst. = instantaneous

$u_\Sigma = \lVert\mathbf{u}\rVert = \sqrt{\mathbf{u}^T \bullet \mathbf{u}}$	coll. inst. value of voltages
$i_\Sigma = \lVert\mathbf{i}\rVert = \sqrt{\mathbf{i}^T \bullet \mathbf{i}}$	coll. inst. value of currents
$U_\Sigma = \sqrt{\frac{1}{T}\int_{t-T}^{T} u_\Sigma^2\, dt}$	coll. rms value of voltages
$I_\Sigma = \sqrt{\frac{1}{T}\int_{t-T}^{T} i_\Sigma^2\, dt}$	coll. rms value of currents
$G_p(t) = p/u_\Sigma^2$	power conductance
$\mathbf{i}_p = \mathbf{u}G_p(t)$	vector of power currents
$p = \mathbf{u}\bullet\mathbf{i} = \mathbf{u}\bullet\mathbf{i}_p = u_\Sigma^2 G_p(t)$	(inst.) power; time function of power
$G_a = P_a/U_\Sigma^2$	equivalent active conductance
$\mathbf{i}_a = \mathbf{u}G_a$	vector of active currents
$p_a = \mathbf{u}\bullet\mathbf{i}_a = u_\Sigma^2 G_a$	time function associated with active power; (inst. active power)
$P_a = \overline{p} = \overline{p_a}$	active power
$\mathbf{i}_z = \mathbf{i} - \mathbf{i}_p$	vector of powerless currents

$\mathbf{u} \bullet \mathbf{i}_\mathrm{z} \equiv 0 \; \forall t$	NOT defined as quantity, always zero!
$* p_\mathrm{z} = u_\Sigma i_{\mathrm{z}\Sigma} \neq \mathbf{u} \bullet \mathbf{i}_\mathrm{z}$	coll. inst. apparent powerless power
$* P_\mathrm{z} = U_\Sigma I_{\mathrm{z}\Sigma}$	apparent powerless power
$* s = u_\Sigma i_\Sigma$	coll. inst. apparent power
....	
$\mathbf{i}_\mathrm{v} = \mathbf{i} - \mathbf{i}_\mathrm{a} - \mathbf{i}_\mathrm{z} = \mathbf{i}_\mathrm{x} - \mathbf{i}_\mathrm{z}$	vector of variation currents
$p_\mathrm{v} = \mathbf{u} \bullet \mathbf{i}_\mathrm{v} = p - p_\mathrm{a} = u_\Sigma^2 (G_\mathrm{p}(t) - G_\mathrm{a})$	time function of variation power, $\overline{p_\mathrm{v}} = 0$
$* P_\mathrm{v} = U_\Sigma I_{\mathrm{v}\Sigma}$	apparent variation power
....	
$\mathbf{i}_\mathrm{x} = \mathbf{i} - \mathbf{i}_\mathrm{a}$	vector of non-active currents
$* P_\mathrm{x} = U_\Sigma I_{\mathrm{x}\Sigma}$	non-active power
$\mathbf{i}_\mathrm{q} = \mathbf{i} - \mathbf{i}_\mathrm{a}$	vector of reactive currents (sinusoidal!)
$* Q = \overline{u_\mathrm{q}\, i}; \; P_\mathrm{x} = \lvert Q \rvert$	reactive power (sinusoidal conditions!)
....	
$* S = U_\Sigma I_\Sigma$	apparent power
$* \lambda = P_\mathrm{a} / S$ (IEEE) or $* \lambda = \lvert P_\mathrm{a} \rvert / S$ (IEV)	power factor

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