A New Interpretation of the Akagi-Nabae Power Components for Nonsinusoidal Three-Phase Situations

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***Abstract*—This note deals with a different interpretation of the power decomposition proposed by Akagi and coauthors for distorted three-phase situations. This enables us to generalize the technique to single-phase systems and polyphase systems and also to include rigorously zero sequence currents and voltages.**

I. Introduction

FOR single-phase power systems in sinusoidal situations, concepts such as active power, reactive power, active current, reactive current, power factor, etc., are well defined. Various attempts [1]-[4] have been proposed to generalize these concepts to the three-phase case with unbalanced and distorted currents and voltages.

In their so-called pq-theory, Akagi *et al.* [1] introduce the concept of instantaneous reactive power to generalize the classical reactive power concept for single-phase sinusoidal systems to the three-phase nonsinusoidal situation. Their concept is very interesting for practical purposes, in particular to analyze the instantaneous compensation of reactive power without energy storage. The concepts of the power components introduced by Akagi have been amply discussed by Ferrero and Superti-Furga [2]; they named them "Park powers" and derived many interesting new properties.

However, the theory developed still raises some conceptual problems:

- the theory is only complete for three-phase systems without zero sequence currents and voltages;
- the single-phase situation cannot be derived from the three-phase case;
- there is no generalization to systems with more than three phases; such systems have been proposed for bulk power transmission [5].

In the present note it is shown how the concepts introduced by Akagi and coauthors [1] can be reformulated. To make the paper more or less self-contained, the theory developed by Akagi is briefly recalled and discussed in Section II. In Section III another approach is given to derive the same concepts. In Section IV it is shown that the questions mentioned above are resolved by the proposed approach.

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II. A Summary of Akagi and Nabae's Power Theory

For three-phase voltages and currents without zero sequence components, the α and β components are defined by

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \tag{2}$$

where v and i denote voltages and currents, and where a, b, and c denote the three phases. The *instantaneous active power* is

$$p(t) = v_\alpha i_\alpha + v_\beta i_\beta = v_a i_a + v_b i_b + v_c i_c. \tag{3}$$

The *instantaneous imaginary power*, introduced by Akagi, is defined by

$$q(t) = v_\alpha i_\beta - v_\beta i_\alpha. \tag{4}$$

It can readily be checked that the following hold:

$$i_a^2 + i_b^2 + i_c^2 = i_\alpha^2 + i_\beta^2 \tag{5}$$

and also

$$i_a^2 + i_b^2 + i_c^2 = \frac{p(t)^2 + q(t)^2}{v_a^2 + v_b^2 + v_c^2}. \tag{6}$$

This shows that for a given voltage the energy loss in the transmission line is reduced if $p(t)$ and/or $q(t)$ decrease. Using compensators without energy storage, the instantaneous power $p(t)$ cannot change, and hence minimum line losses are obtained for zero imaginary power. Akagi's imaginary power concept hence exactly shows to what extent compensators without energy storage can be used to reduce line losses. This result is precisely the extremely interesting contribution realized by Akagi and his coauthors.

Akagi *et al.* [1] then express the powers in the three phases as

$$\begin{cases} p_a(t) = p_{ap}(t) + p_{aq}(t) \\ p_b(t) = p_{bp}(t) + p_{bq}(t). \\ p_c(t) = p_{cp}(t) + p_{cq}(t) \end{cases} \tag{7}$$

The first parts are proportional to $p(t)$; for the three phases they sum up to $p(t)$. The second parts are proportional to $q(t)$ and sum up to zero. In each phase the instantaneous power is split up into a real power component and an imaginary power component. Only the latter component can be affected by compensation without energy storage. The components p_{aq}, p_{bq}, and p_{cq} correspond to power transfer between the three phases.

If zero sequence components are present, the corresponding powers

$$p_{a0} = v_a i_0, \quad p_{b0} = v_b i_0, \quad p_{b0} = v_b i_0 \tag{8}$$

are added to the above expressions by Akagi [1]; nothing is said about the contribution to real and imaginary powers due to zero sequence currents and voltages. However, it can easily be seen that the zero sequence currents and voltages can contribute to the real power as well as to the imaginary power (by considering, e.g., the interaction of zero sequence currents and positive sequence voltages). On the other hand, Ferrero and Superti–Furga [2] add

$$p_0 = v_0 i_0 \tag{9}$$

to the instantaneous real power in each of the phases, but again do not discuss whether the zero sequence components should be taken into account for the imaginary power.

The above brief discussion of the power theory introduced by Akagi and coauthors, shows the important contribution made by them, but also points to a number of conceptual problems.

- There is a one-to-one correspondence between the α and β components and the currents and voltages in the three phases; the powers $p(t)$ and $q(t)$ can hence be expressed in a function of the currents and voltages in the three phases. The question thus arises whether the introduction of the α and β components is really necessary.
- The theory is only relevant for three-phase power systems. The single-phase case cannot be derived from it as a special case. Also the extension to systems with more than three phases [5] is not straightforward.
- The theory does not take properly into account the presence of zero sequence currents and voltages.

III. A New Interpretation of pq-Theory

Let the number of phases be denoted by m. The instantaneous currents and voltages in the m phases of the line are represented by the m-dimensional vectors $i(t)$ and $v(t)$. The instantaneous power transmitted to the load is the scalar or internal product of these vectors:

$$p(t) = v(t)^T i(t) \tag{10}$$

where the superscript T denotes matrix transposition. Let $i_p(t)$ be the orthogonal projection of the vector $i(t)$ on the vector $v(t)$, with respect to the vector product $v^T i$. By definition, the vector $i_p(t)$ is proportional to the vector $v(t)$, and is such that

$$v(t)^T i(t) = v(t)^T i_p(t). \tag{11}$$

Explicitly,

$$i_p(t) = \frac{v(t)^T i(t)}{|v(t)|^2} v(t) \tag{12}$$

or

$$i_p(t) = \frac{p(t)}{|v(t)|^2} v(t) \tag{13}$$

where $|\cdot|$ denotes the length of a vector, i.e., $|v|^2 = v^T v$. Equation (12) can be derived from linear algebra by considering the projection of the vector $i(t)$ on the vector $v(t)$. Moreover, one may readily check that (11) is satisfied. The current

$$i_q(t) = i(t) - i_p(t) \tag{14}$$

is orthogonal to $v(t)$, such that

$$v(t)^T i_q(t) = 0. \tag{15}$$

Summarizing, the instantaneous current vector can be decomposed into two components:

$$i(t) = i_p(t) + i_q(t) \tag{16}$$

- the *instantaneous active current* $i_p(t)$, which is proportional to the voltage $v(t)$, and corresponds to the instantaneous power;
- the *instantaneous nonactive (or reactive) current* $i_q(t)$, which does not contribute to power transfer.

It is readily seen that these current components contain all information contained in the real and imaginary power components introduced by Akagi. The instantaneous real power is obviously equal to

$$v(t) i_p(t). \tag{17}$$

The instantaneous imaginary power can be associated with

$$|q(t)| = |v(t)| \cdot |i_q(t)|. \tag{18}$$

For three-phase systems without zero sequence components (where the currents and voltages can be represented in a plane) a sign can be attributed to the quantity $q(t)$. Indeed, then $q(t)$ can be associated with the cross product of the current and voltage vectors; this is a vector whose length is given by the above expression and which is orthogonal to the plane of the current and voltage vectors. The sign can be associated with the direction of that vector with respect to that plane. For more general cases this sign interpretation is not possible. It is straightforward to check that $i_q(t)$ corresponds to the reactive current obtained by Akagi from the imaginary power defined by means of the α and β components.

Our claim is that it is not necessary to define the real and imaginary power components, but that it suffices to define the current components $i_p(t)$ and $i_q(t)$ (the vector or its length) to obtain the same results as obtained by Akagi.

(i) From the definition it follows that

$$|i(t)|^2 = |i_p(t)|^2 + |i_q(t)|^2 \tag{19}$$

and for three phases

$$|i(t)|^2 = |i_a(t)|^2 + |i_b(t)|^2 + |i_c(t)|^2. \tag{20}$$

Moreover,

$$|i_p(t)|^2 = \frac{p(t)^2}{|v(t)|^2} \tag{21}$$

and

$$|i_q(t)|^2 = \frac{q(t)^2}{|v(t)|^2}. \tag{22}$$

Hence (19) expresses exactly the same as (6): the line losses are proportional to the sum of the squares of $p(t)$ and $q(t)$, or the sum of the squares of $|i_p(t)|$ and $|i_q(t)|$. We assume that the resistance in the neutral conductor is zero. To reduce the line losses as much as possible without altering the instantaneous power (or the instantaneous active current), that is, without using energy storage, the imaginary power $q(t)$, or equivalently the instantaneous nonactive current $i_q(t)$, should be annihilated. The magnitude of $q(t)$ or the length of $i_q(t)$ characterizes the instantaneous line loss component which can be reduced by elements without energy storage. Note that (19) not only holds for three phases, but also for an arbitrary number of phases.

(ii) For a two-phase system the decomposition (16) is exactly the same as the decomposition obtained by Akagi:

$$i_{\alpha p}(t) = \frac{v_\alpha(t)}{v_\alpha(t)^2 + v_\beta(t)^2}\, p(t) \tag{23}$$

$$i_{\alpha q}(t) = \frac{-v_\beta(t)}{v_\alpha(t)^2 + v_\beta(t)^2}\, q(t) \tag{24}$$

$$i_{\beta p}(t) = \frac{v_\beta(t)}{v_\alpha(t)^2 + v_\beta(t)^2}\, p(t) \tag{25}$$

$$i_{\beta q}(t) = \frac{v_\alpha(t)}{v_\alpha(t)^2 + v_\beta(t)^2}\, q(t). \tag{26}$$

(iii) For three-phase systems without zero sequence currents and voltages the decomposition of the three-phase current vector according to (16) yields exactly the same result as the concepts of real and imaginary power in Akagi's paper [1]. In that paper the three-phase current is first transformed to (α, β)-components. Then these currents are split up into active and nonactive components as in (23)–(26). Finally, the two-phase quantities are again transformed to three-phase quantities. An interesting feature of this interpretation is that it clearly shows why the (α, β)-components are introduced. These components are not required for physical reasons; the (α, β)-components enable us to derive in an elegant way the projection of the current vector on the voltage vector and the component of the current vector orthogonal to the current vector. In this analysis use is made of the fact that the transformation from (a, b, c)-components to (α, β)-components is orthogonal. However, one should realize that the algorithm using (α, β)-components assumes that there are no zero sequence currents and voltages.

IV. Discussion

1. The decomposition of the current vector into a component parallel to the voltage vector and a component orthogonal to it, enables us to recover Akagi's results for three-phase systems without zero sequence components (three-wire systems).

2. The decomposition proposed in the previous section is not restricted to three phases. It is valid for any number of phases: in this way Akagi's concept of compensation without energy storage can be extended to multiphase power systems [5]. This is an interesting feature of the analysis of the present paper. As was pointed out in Section II, Akagi's concept of instantaneous imaginary power is restricted to three-phase systems. The concept of nonactive current however is valid for any number of phases.

3. The single-phase case is also contained in the analysis of Section III. This feature distinguishes it from the analysis in the original work by Akagi. Indeed if i is a scalar, then clearly i_p coincides with i, and i_q does not exist. The line losses cannot be reduced by elements without energy storage. This is a well-known fact; nevertheless, it is interesting to have it included in the theory for an arbitrary number of phases.

4. The decomposition of the current, as well as the concepts of instantaneous real and imaginary powers introduced by Akagi, clearly show that compensation for line loss reduction comprises two aspects:

(a) compensation without energy storage: this corresponds to reducing $|i_q(t)|$ or $|q(t)|$, without altering the *instantaneous* power transfer;

(b) compensation with energy storage: this corresponds to reducing the *average* loss, without altering the *average* power transfer. In other words for a given voltage vector and given *average* active power

$$P = \frac{1}{T}\int_0^T p(t)\, dt \tag{27}$$

we want to modify $i_p(t)$ or $p(t)$, such that the average loss or, equivalently

$$\frac{1}{T}\int_0^T |i_p(t)|^2\, dt \tag{28}$$

is reduced as much as possible. This is the only aspect which is relevant for single-phase power transmission.

Compensation aspect (a) is related to reducing power oscillations between phases, which do not correspond to oscillations of the total power transmitted from source to load. Compensation aspect (b) corresponds to reducing power oscillations between source and load; it is clear that this requires elements with energy storage.

However, it should be emphasized that not every oscillation of power between phases yields a contribution in the imaginary power $q(t)$. If a balanced source and a balanced load are considered in sinusoidal steady state, then there is no oscillation in the power transmitted between source and load. There is oscillation of the power between phases: one part proportional to the active power and the other part proportional to the reactive power. The former part does not yield a contribution to the imaginary power, but only the latter part does. Since only the latter part can be reduced by compensation (with or without energy storage), it is justified that only this part contributes to the imaginary power.

5. One should clearly distinguish the instantaneous active current, defined by (13), from the active current, as defined by Fryze [3]:

$$i_{\text{act}}(t) = \frac{P}{\|v(t)\|^2}\, v(t) \tag{29}$$

where $\|v(t)\|$ denotes the r.m.s. value, i.e.

$$\|v(t)\|^2 = \frac{1}{T}\int_0^T |v(t)|^2\, dt. \tag{30}$$

Note the difference between, on the one hand, $|v(t)|$, a time-dependent quantity corresponding to the length of the vector $v(t)$, and, on the other hand, the r.m.s. value $\|v(t)\|$. The instantaneous active current vector is a current which is at any time proportional to the voltage vector and which corresponds to the same instantaneous active power as the actual current vector. The active current is a current vector with the same waveform as the voltage vector, which corresponds to the same *average* active power as the actual current vector. Reactive compensation *with energy storage* makes it possible to reduce the line loss from the value corresponding to the current i_p to the value corresponding to the current i_{act}.

6. The presence of zero sequence currents does not introduce any problems with respect to the concepts of Section III. The zero sequence currents should, in general, partly be included in the instantaneous active current and partly in the nonactive current. In general, the zero sequence currents affect the instantaneous active power as well as the instantaneous imaginary power. This is in contrast with the discussion of zero sequence components by Akagi [1] and by Ferrero and Superti-Furga [2]. If, for example, there is a zero sequence current but not a zero sequence voltage, then the zero sequence current does not yield a contribution to the instantaneous active power. However, it yields a contribution to the nonactive current and hence to the imaginary power as defined by Akagi, since it yields an increase of the line losses. On the other hand, if there is a zero sequence current as well as a zero sequence voltage, they yield a contribution to the instantaneous power transfer and should be included in the instantaneous active power. Indeed in the former case the zero sequence current is orthogonal to the voltage vector and is completely included in $i_q(t)$. In the latter case this is not so.

7. It has been shown [6] that any sinusoidal three-phase current corresponding to zero average active power can be realized by means of linear reactive elements (with energy storage). A question which remains to be addressed is which part of the current can be compensated by elements without energy storage and which part by linear reactive elements with energy storage, in order to obtain minimal line currents and line losses. Also it is interesting to investigate which is the minimal energy storing capacity required.

8. Czarnecki [3], [4] proposes a decomposition of the line current into

(a) the active current, given by (29), with the same waveform as the voltage which yields the *average* active power, and

(b) various current components corresponding to zero *average* active power which can be attributed to different physical phenomena (unbalance, energy storage, scattering, etc.). It would be interesting to analyze the relationships that exist between the current decomposition of Section III and Czarnecki's decomposition procedure.

V. Conclusion

In this paper it has been shown that the instantaneous real and imaginary powers introduced by Akagi *et al.* [1] can be interpreted nicely by a current decomposition. This viewpoint yields the following new features.

- The new interpretation is not only valid for three-phase power systems, but it can also be used in polyphase power systems. Moreover, it contains the single-phase case as a special case.
- The new interpretation is well suited to take into account zero sequence currents and voltages.

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References

[1] H. Akagi, Y. Kanazawa, and A. Nabae, "Instantaneous reactive power compensators comprising switching devices without energy storage components," *IEEE Trans. Ind. Appl.*, vol. IA-20, pp. 625-631, 1984.

[2] A. Ferrero and G. Superti-Furga, "A new approach to the definition of power components in three-phase systems under nonsinusoidal conditions," *IEEE Trans. Instrum. Meas.*, vol. 40, pp. 568-577, 1991.

[3] L. S. Czarnecki, "Orthogonal decomposition of the currents in a three-phase nonlinear asymmetrical circuit with a nonsinusoidal voltage source," *IEEE Trans. Instrum. Meas.*, vol. IM-37, pp. 30-34, 1988.

[4] L. S. Czarnecki, "Scattered and reactive current, voltage and power in circuits with nonsinusoidal waveforms and their compensation," *IEEE Trans. Instrum. Meas.*, vol. 40, pp. 563-567, 1991.

[5] J. L. Willems, "A new approach to the analysis of mixed three-phase and six-phase power systems," *Int. J. Electrical Energy and Power Systems*, vol. 11, pp. 115-122, 1989.

[6] J. L. Willems, "The compensation of reactive and unbalanced currents in three-phase power systems," *Proceedings of the 1991 International Workshop on Power Definitions and Measurements under Nonsinusoidal Conditions*, Como, Italy, Sept. 1991, pp. 113-128.