Active Filters with Control Based on the p-q Theory

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Introduction

Due the intensive use of power converters and other non-linear loads in industry and by consumers in general, it can be observed an increasing deterioration of the power systems voltage and current waveforms.

The presence of harmonics in the power lines results in greater power losses in distribution, interference problems in communication systems and, sometimes, in operation failures of electronic equipments, which are more and more sensitive since they include microelectronic control systems, which work with very low energy levels. Because of these problems, the issue of the power quality delivered to the end consumers is, more than ever, an object of great concern.

International standards concerning electrical power quality (IEEE-519, IEC 61000, EN 50160, among others) impose that electrical equipments and facilities should not produce harmonic contents greater than specified values, and also specify distortion limits to the supply voltage. Meanwhile, it is mandatory to solve the harmonic problems caused by those equipments already installed.

Passive filters have been used as a solution to solve harmonic current problems, but they present several disadvantages, namely: they only filter the frequencies they were previously tuned for; their operation cannot be limited to a certain load; resonances can occur because of the interaction between the passive filters and other loads, with unpredictable results. To cope with these disadvantages, recent efforts have been concentrated in the development of active filters.

Active Filters

There are basically two types of active filters: the shunt type and the series type. It is possible to find active filters combined with passive filters as well as active filters of both types acting together.

Fig. 1 presents the electrical scheme of a shunt active filter for a three-phase power system with neutral wire, which is able to compensate for both current harmonics and power factor. Furthermore, it allows load balancing, eliminating the current in the neutral wire. The power stage is, basically, a voltage-source inverter with only a single capacitor in the DC side (the active filter does not require any internal power supply), controlled in a way that it acts like a current-source. From the measured values of the phase voltages (v_a, v_b, v_c) and load currents (i_a, i_b, i_c), the controller calculates the reference currents ($i_{ca}*$, $i_{cb}*$, $i_{cc}*$, $i_{cn}*$) used by the inverter to produce the compensation currents (i_{ca}, i_{cb}, i_{cc}, i_{cn}). This solution requires 6 current sensors and 4 voltage sensors, and the inverter has 4 legs (8 power semiconductor switches). For balanced loads without 3rd order current harmonics (three-phase motors, three-phase adjustable speed drives, three-phase controlled or non-controlled rectifiers, etc) there is no need to compensate for the current in neutral wire. These allow the use of a simpler inverter (with only three legs) and only 4 current sensors. It also eases the controller calculations.

Fig. 2 shows the scheme of a series active filter for a three-phase power system. It is the dual of the shunt active filter, and is able to compensate for distortion in the power line voltages, making the voltages applied to the load sinusoidal (compensating for voltage harmonics). The filter consists of a voltage-source inverter (behaving as a controlled voltage source) and requires 3 single-phase transformers to interface with the power system. The series active filter does not compensate for load current harmonics but it acts as high-impedance to the current harmonics coming from the power source side. Therefore, it guarantees that passive filters eventually placed at the load input will not drain harmonic currents from the rest of the power system. Another solution to solve the load current harmonics is to use a shunt active filter together with the series active filter (Fig. 3), so that both load voltages and the supplied currents are guaranteed to have sinusoidal waveforms.

Shunt active filters are already commercially available, while the series and series-shunt types are yet at prototype level.
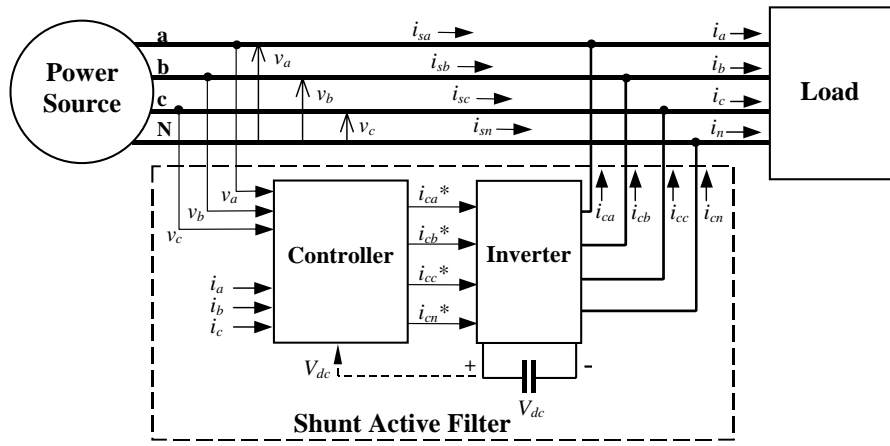
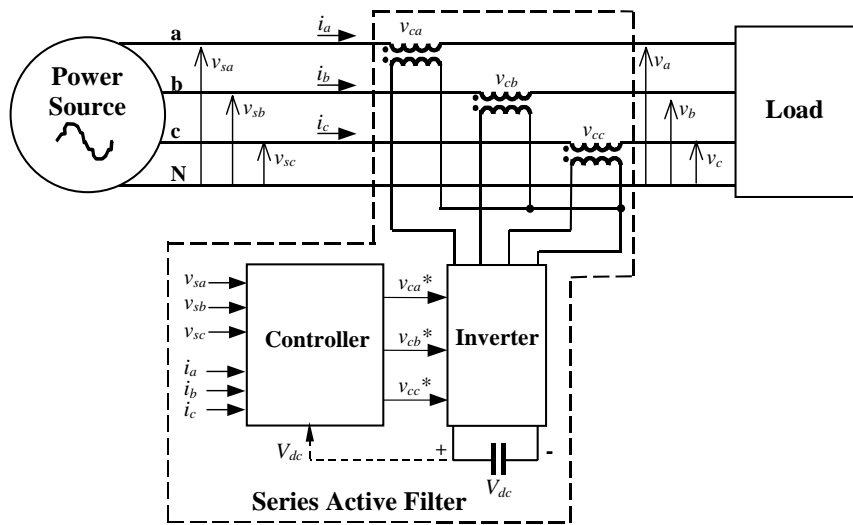Fig. 1 - Shunt active filter in a three-phase power system.

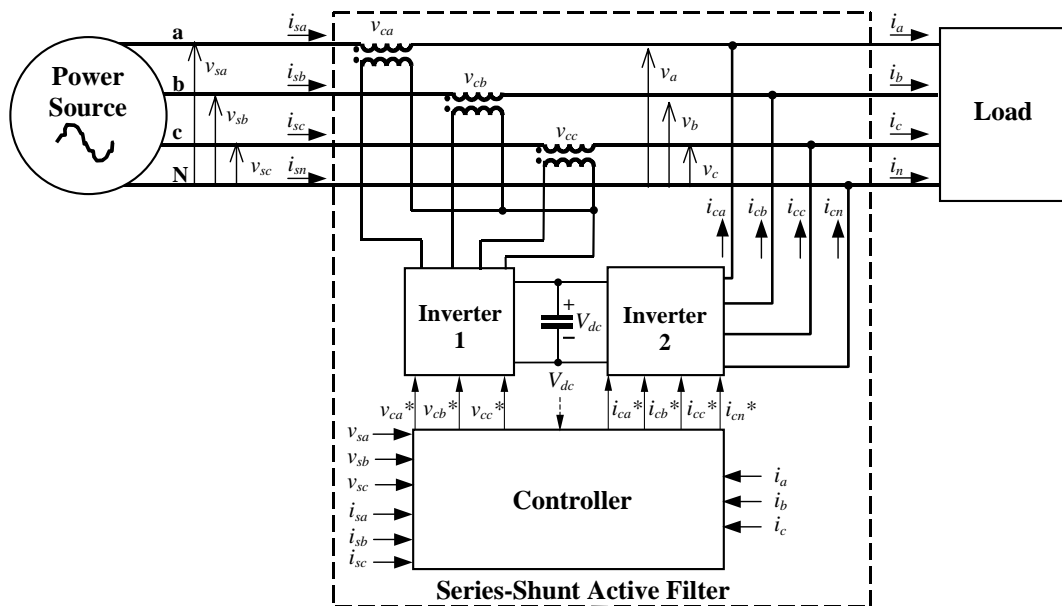Fig. 2 - Series active filter in a three-phase power system.

Fig. 3 – Series-shunt active filter in a three-phase power system.

The p-q theory

In 1983, *Akagi et al.* [1, 2] have proposed the "The Generalized Theory of the Instantaneous Reactive Power in Three-Phase Circuits", also known as instantaneous power theory , or p-q theory. It is based on instantaneous values in three-phase power systems with or without neutral wire, and is valid for steady-state or transitory operations, as well as for generic voltage and current waveforms. The p-q theory consists of an algebraic transformation (Clarke transformation) of the three-phase voltages and currents in the *a-b-c* coordinates to the *α-β-0* coordinates, followed by the calculation of the p-q theory instantaneous power components:

$$\begin{bmatrix} v_0 \\ v_\alpha \\ v_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \cdot \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \cdot \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \qquad \begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \cdot \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \qquad (1)$$

$$p_0 = v_0 \cdot i_0 \qquad \text{instantaneous zero-sequence power} \qquad (2)$$

$$p = v_\alpha \cdot i_\alpha + v_\beta \cdot i_\beta \qquad \text{instantaneous real power} \qquad (3)$$

$$q \equiv v_\alpha \cdot i_\beta - v_\beta \cdot i_\alpha \qquad \text{instantaneous imaginary power (by definition)} \qquad (4)$$

The power components p and q are related to the same α-β voltages and currents, and can be written together:

$$\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} v_\alpha & v_\beta \\ -v_\beta & v_\alpha \end{bmatrix} \cdot \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \qquad (5)$$

These quantities are illustrated in Fig. 4 for an electrical system represented in *a-b-c* coordinates and have the following physical meaning:

$\bar{p}_0$ = mean value of the instantaneous zero-sequence power – corresponds to the energy per time unity which is transferred from the power supply to the load through the zero-sequence components of voltage and current.

$\tilde{p}_0$ = alternated value of the instantaneous zero-sequence power – it means the energy per time unity that is exchanged between the power supply and the load through the zero-sequence components. The zero-sequence power only exists in three-phase systems with neutral wire. Furthermore, the systems must have unbalanced voltages and currents and/or 3rd harmonics in both voltage and current of at least one phase.

$\bar{p}$ = mean value of the instantaneous real power – corresponds to the energy per time unity which is transferred from the power supply to the load, through the *a-b-c* coordinates, in a balanced way (it is the desired power component).

$\tilde{p}$ = alternated value of the instantaneous real power – It is the energy per time unity that is exchanged between the power supply and the load, through the *a-b-c* coordinates.

q = instantaneous imaginary power – corresponds to the power that is exchanged between the phases of the load. This component does not imply any transference or exchange of energy between the power supply and the load, but is responsible for the existence of undesirable currents, which circulate between the system phases. In the case of a balanced sinusoidal voltage supply and a balanced load, with or without harmonics, $\bar{q}$ (the mean value of the instantaneous imaginary power) is equal to the conventional reactive power ($\bar{q} = 3 \cdot V \cdot I_1 \cdot sin\,\phi_1$).
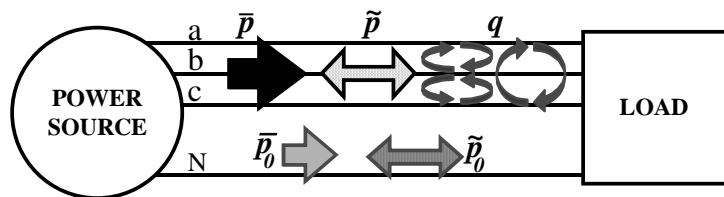

Fig. 4 – Power components of the p-q theory in *a-b-c* coordinates.

The p-q theory applied to shunt active filters

The p-q theory is one of several methods that can be used in the control active filters [3-11]. It presents some interesting features, namely:
- It is inherently a three-phase system theory;
- It can be applied to any three-phase system (balanced or unbalanced, with or without harmonics in both voltages and currents);
- It is based in instantaneous values, allowing excellent dynamic response;
- Its calculations are relatively simple (it only includes algebraic expressions that can be implemented using standard processors);
- It allows two control strategies: constant instantaneous supply power and sinusoidal supply current.

As seen before, $\bar{p}$ is usually the only desirable p-q theory power component. The other quantities can be compensated using a shunt active filter (Fig. 5). As shown by *Watanabe et al.* [12, 13], $\bar{p}_0$ can be compensated without the need of any power supply in the shunt active filter. This quantity is delivered from the power supply to the load, through the active filter (see Fig. 5). This means that the energy previously transferred from the source to the load through the zero-sequence components of voltage and current, is now delivered in a balanced way from the source phases.

It is also possible to conclude from Fig. 5 that the active filter capacitor is only necessary to compensate $\tilde{p}$ and $\tilde{p}_0$, since these quantities must be stored in this component at one moment to be later delivered to the load. The instantaneous imaginary power (q), which includes the conventional reactive power, is compensated without the contribution of the capacitor. This means that, the size of the capacitor does not depend on the amount of reactive power to be compensated.
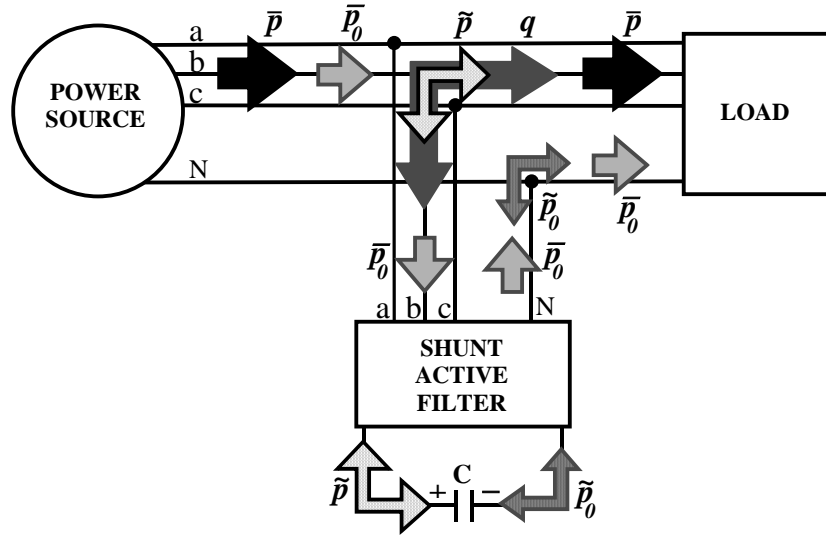

Fig. 5 - Compensation of power components $\tilde{p}$, q, $\tilde{p}_0$ and $\bar{p}_0$ in *a-b-c* coordinates.

To calculate the reference compensation currents in the α-β coordinates, the expression (5) is inverted, and the powers to be compensated ($\tilde{p} - \bar{p}_0$ and q) are used:

$$\begin{bmatrix} i_{c\alpha}* \\ i_{c\beta}* \end{bmatrix} = \frac{1}{v_\alpha^2 + v_\beta^2} \cdot \begin{bmatrix} v_\alpha & -v_\beta \\ v_\beta & v_\alpha \end{bmatrix} \cdot \begin{bmatrix} \tilde{p} - \bar{p}_0 \\ q \end{bmatrix} \tag{6}$$

Since the zero-sequence current must be compensated, the reference compensation current in the *0* coordinate is i_0 itself:

$$i_{c0}* = i_0 \tag{7}$$

In order to obtain the reference compensation currents in the *a-b-c* coordinates the inverse of the transformation given in expression (1) is applied:

$$\begin{bmatrix} i_{ca}* \\ i_{cb}* \\ i_{cc}* \end{bmatrix} = \sqrt{\frac{2}{3}} \cdot \begin{bmatrix} 1/\sqrt{2} & 1 & 0 \\ 1/\sqrt{2} & -1/2 & \sqrt{3}/2 \\ 1/\sqrt{2} & -1/2 & -\sqrt{3}/2 \end{bmatrix} \cdot \begin{bmatrix} i_{c0}* \\ i_{c\alpha}* \\ i_{c\beta}* \end{bmatrix} \tag{7}$$

$$i_{cn}* = -\left(i_{ca}* + i_{cb}* + i_{cc}* \right)$$

The calculations presented so far are synthesized in Fig. 6 and correspond to a shunt active filter control strategy for constant instantaneous supply power. This approach, when applied to a three-phase system with balanced sinusoidal voltages, produces de following results (Fig. 8):
- the phase supply currents become sinusoidal, balanced, and in phase with the voltages. (in other words, the power supply "sees" the load as a purely resistive symmetrical load);
- the neutral current is made equal to zero (even 3rd order current harmonics are compensated);
- the total instantaneous power supplied,

$$p_{3s}(t) = v_a \cdot i_{sa} + v_b \cdot i_{sb} + v_c \cdot i_{sc} \tag{8}$$

is made constant.

In the case of a non-sinusoidal or unbalanced supply voltage, the only difference is that the supply current will include harmonics (Fig 9), but in practical cases the distortion is negligible.
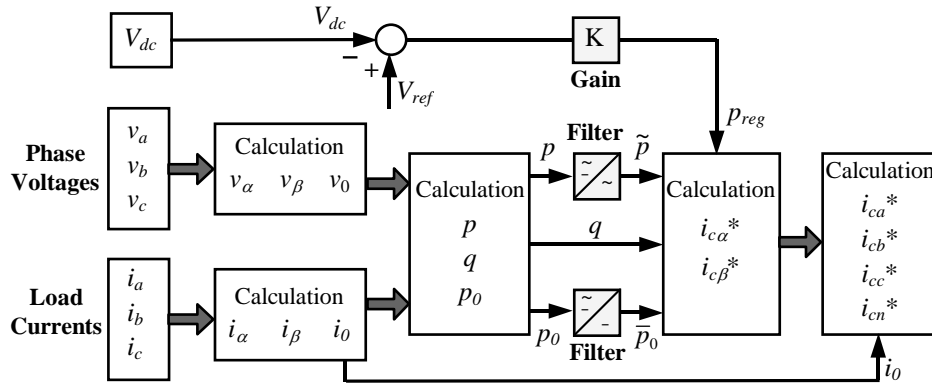

Fig. 6 – Calculations for the constant instantaneous supply power control strategy.

The sinusoidal supply current control strategy must be used when the voltages are distorted or unbalanced and sinusoidal currents are desired. The block diagram of Fig. 7 presents the calculations required in this case. When this approach is used the results, illustrated in Fig. 10, are:
- the phase supply currents become sinusoidal, balanced, and in phase with the fundamental voltages;
- the neutral current is made equal to zero (even 3rd order current harmonics are compensated);
- the total instantaneous power supplied (p_{3S}) is not made constant, but it presents only a small ripple (much smaller than before compensation).
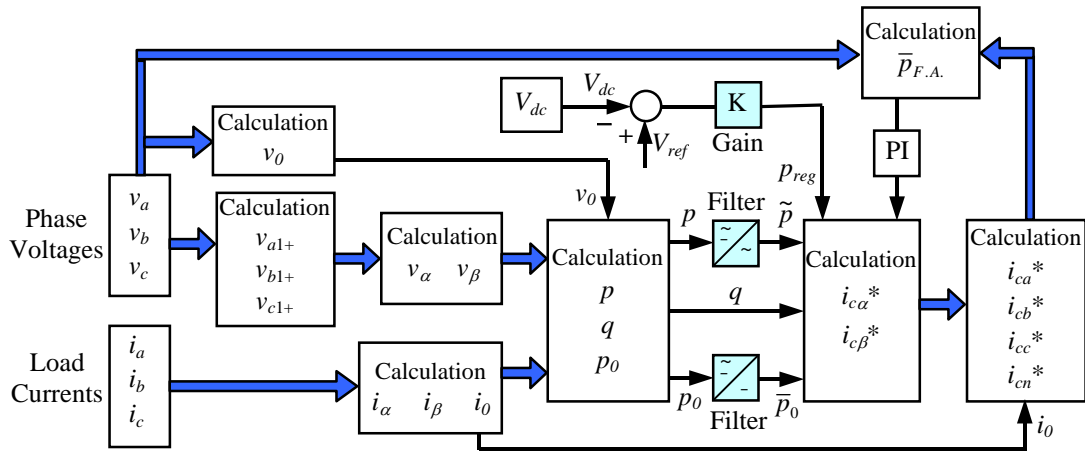
Fig. 7 – Calculations for the sinusoidal supply current control strategy.

The practical implementation of the shunt active filter demands the regulation of the voltage at the inverter DC side (V_{dc} - the capacitor voltage) as suggested in Fig. 6 and Fig. 7, where V_{ref} is the reference value required for proper operation of the active filter inverter.

Figures 8, 9 and 10, present simulation results using *Matlab/Simulink* [14, 15] for a three-phase power system with a shunt active filter. They include the following waveforms, corresponding to two-cycles of steady-state operation: total instantaneous power at load and source, phase voltages, load and source currents (phase and neutral currents). In the cases with distorted voltages the voltage total harmonic distortion (THD) is equal to 10%, which is a higher value than what is regulated by any power quality standard.
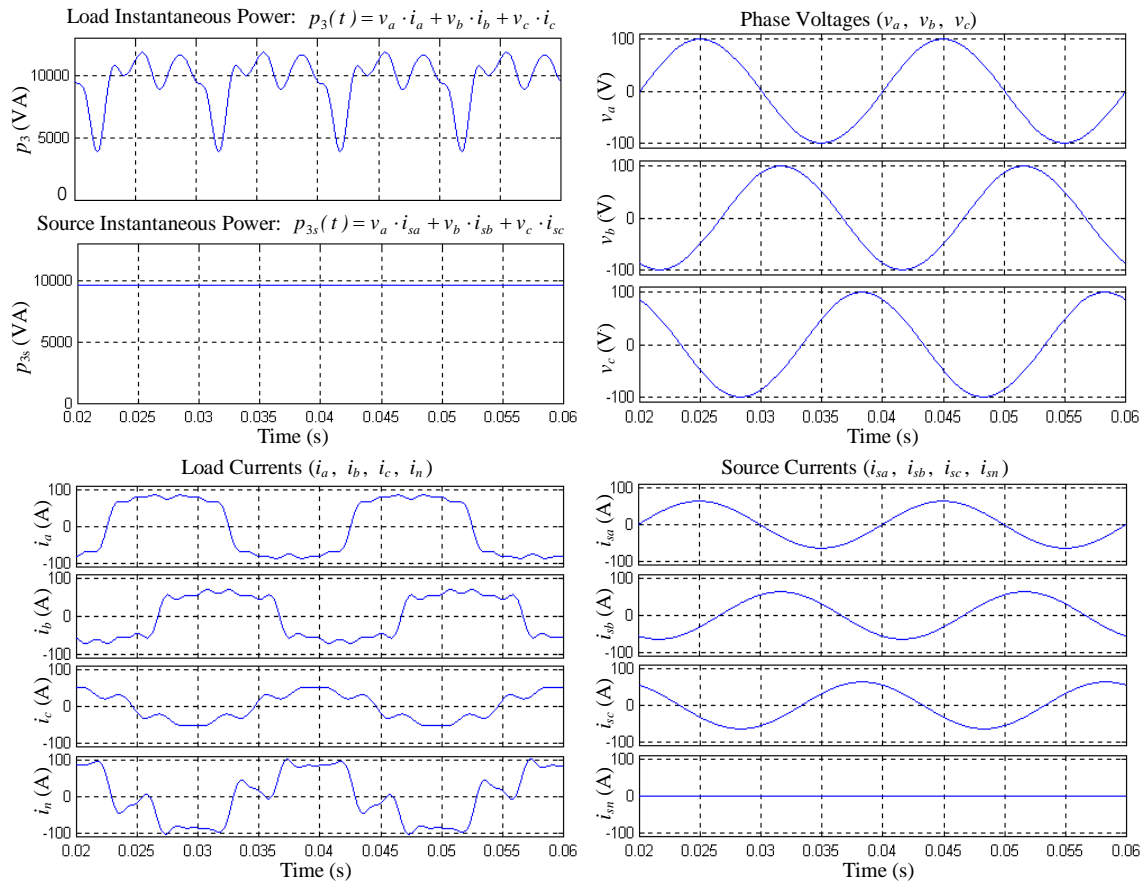

Fig. 8 – Simulation results for the constant instantaneous supply power strategy with sinusoidal voltages.
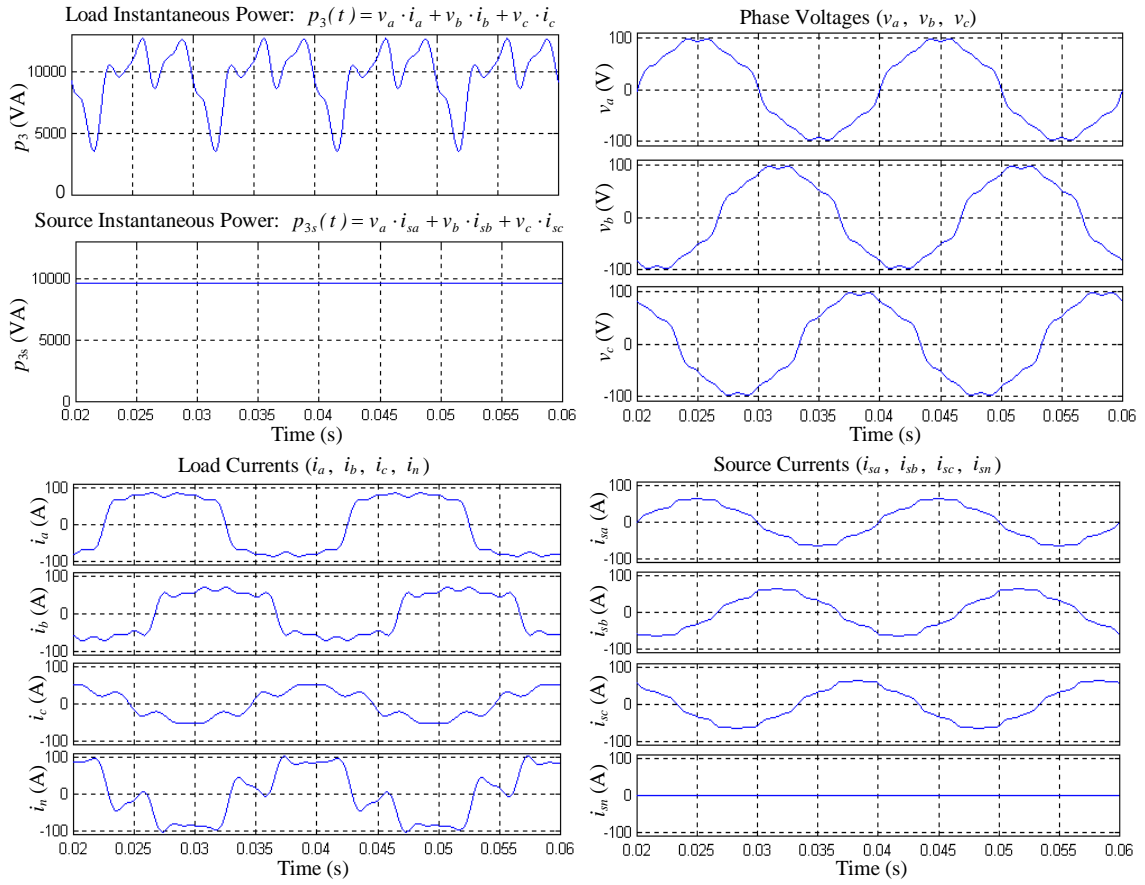
Load Instantaneous Power: $p_3(t) = v_a \cdot i_a + v_b \cdot i_b + v_c \cdot i_c$

Phase Voltages (v_a, v_b, v_c)

Source Instantaneous Power: $p_{3s}(t) = v_a \cdot i_{sa} + v_b \cdot i_{sb} + v_c \cdot i_{sc}$

Load Currents (i_a, i_b, i_c, i_n)

Source Currents (i_{sa}, i_{sb}, i_{sc}, i_{sn})

Fig. 9 - Simulation results for the constant instantaneous supply power strategy with distorted voltages.

Load Instantaneous Power: $p_3(t) = v_a \cdot i_a + v_b \cdot i_b + v_c \cdot i_c$

Phase Voltages (v_a, v_b, v_c)

Source Instantaneous Power: $p_{3s}(t) = v_a \cdot i_{sa} + v_b \cdot i_{sb} + v_c \cdot i_{sc}$

Load Currents (i_a, i_b, i_c, i_n)

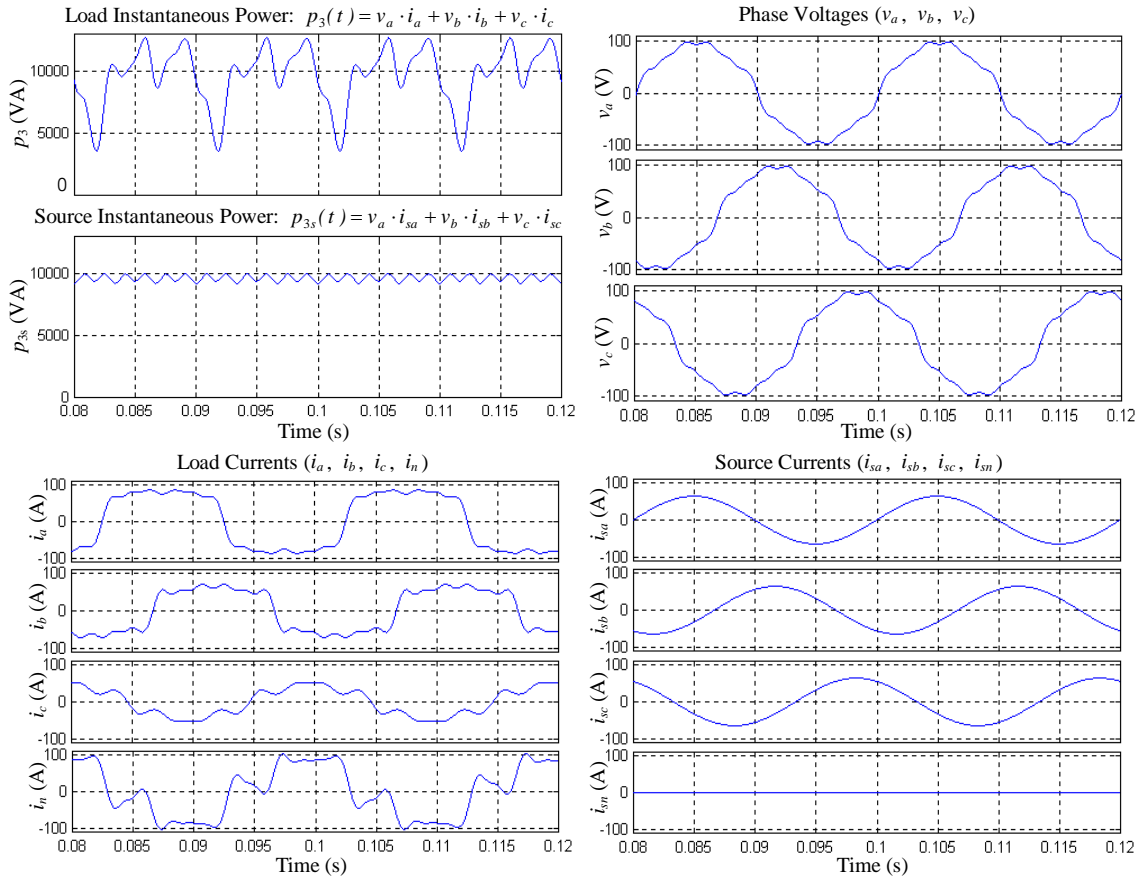Source Currents (i_{sa}, i_{sb}, i_{sc}, i_{sn})

Fig. 10 - Simulation results for the sinusoidal supply current strategy with distorted voltages.

Conclusions

Active filters are an up-to-date solution to power quality problems. Shunt active filters allow the compensation of current harmonics and unbalance, together with power factor correction, and can be a much better solution than the conventional approach (capacitors for power factor correction and passive filters to compensate for current harmonics).

This paper presents the p-q theory as a suitable tool to the analysis of non-linear three-phase systems and for the control of active filters.

Based on this theory, two control strategies for shunt active filters were described, one leading to constant instantaneous supply power and the other to sinusoidal supply current.

The implementation of active filters based on the p-q theory are cost-effective solutions, allowing the use of a large number of low-power active filters in the same facility, close to each problematic load (or group of loads), avoiding the circulation of current harmonics, reactive currents and neutral currents through the facility power lines.

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