

## Boolean Algebra Axioms and Theorems

**Table 2.1** Axioms of Boolean algebra

Axiom		Dual		Name
A1	$B = 0 \text{ if } B \neq 1$	A1'	$B = 1 \text{ if } B \neq 0$	Binary field
A2	$\bar{0} = 1$	A2'	$\bar{1} = 0$	NOT
A3	$0 \bullet 0 = 0$	A3'	$1 + 1 = 1$	AND/OR
A4	$1 \bullet 1 = 1$	A4'	$0 + 0 = 0$	AND/OR
A5	$0 \bullet 1 = 1 \bullet 0 = 0$	A5'	$1 + 0 = 0 + 1 = 1$	AND/OR

**Table 2.2** Boolean theorems of one variable

Theorem		Dual		Name
T1	$B \bullet 1 = B$	T1'	$B + 0 = B$	Identity
T2	$B \bullet 0 = 0$	T2'	$B + 1 = 1$	Null Element
T3	$B \bullet B = B$	T3'	$B + B = B$	Idempotency
T4		$\bar{\bar{B}} = B$		Involution
T5	$B \bullet \bar{B} = 0$	T5'	$B + \bar{B} = 1$	Complements

**Table 2.3** Boolean theorems of several variables

Theorem		Dual		Name
T6	$B \bullet C = C \bullet B$	T6'	$B + C = C + B$	Commutativity
T7	$(B \bullet C) \bullet D = B \bullet (C \bullet D)$	T7'	$(B + C) + D = B + (C + D)$	Associativity
T8	$(B \bullet C) + (B \bullet D) = B \bullet (C + D)$	T8'	$(B + C) \bullet (B + D) = B + (C \bullet D)$	Distributivity
T9	$B \bullet (B + C) = B$	T9'	$B + (B \bullet C) = B$	Covering
T10	$(B \bullet C) + (B \bullet \bar{C}) = B$	T10'	$(B + C) \bullet (B + \bar{C}) = B$	Combining
T11	$(B \bullet C) + (\bar{B} \bullet D) + (C \bullet D)$ $= B \bullet C + \bar{B} \bullet D$	T11'	$(B + C) \bullet (\bar{B} + D) \bullet (C + D)$ $= (B + C) \bullet (\bar{B} + D)$	Consensus
T12	$\overline{B_0 \bullet B_1 \bullet B_2 \dots}$ $= (\bar{B}_0 + \bar{B}_1 + \bar{B}_2 \dots)$	T12'	$\overline{B_0 + B_1 + B_2 \dots}$ $= (\bar{B}_0 \bullet \bar{B}_1 \bullet \bar{B}_2)$	De Morgan's Theorem

From Harris & Harris, Digital Design and Computer Architecture, Morgan Kaufmann, 2012.