







Number of Factors to Extract

- P variables have P principal components, that is, P factors that can be extracted
- But the goal in factor analysis is to reduce the model dimension to M << P factors
- So, how many factors are optimal to extract for a model?
 - ➤ Use a **business rationale** to group variables into factors and do "Confirmatory Factor Analysis" (**CFA**) to verify or discard the factors
 - Use the standard threshold point around Eigenvalue = 1
 - ➤ Use a given cumulative variance explained threshold a common rule of thumb is to use enough factors to explain 70% of the variance.



Variance Explained

- It also helps to see how much variance is explained by each factor
- And to look at the cumulative variance explained by the first M factors
- Modeling a 3-Factor solution, as suggested by the Scree Plot explains 92% of the variance, so we don't need any more factors
- Even a 2-Factor solution would be fine

	PC1	PC2	PC3
SS loadings	5.47	4.17	1.41
Proportion Var	0.46	0.35	0.12
Cumulative Var	0.46	0.80	0.92
Proportion Explained			
Cumulative Proportion	0.49	0.87	1.00



Factor Loadings

- Factor analysis reports "Factor Loadings" and % of Variance Explained by each factor.
- A factor loading is a statistic that shows how much a particular variable contributes to the variance of a factor
- The idea is to group variables with high Factor Loadings F
 = 0.5 rule of thumb high factor loadings within a factor indicate that variables are highly correlated and therefore group together
- Don't group variables with light loadings $F < 0.5 \rightarrow$ low correlation among the variables \rightarrow no basis for grouping
- The next slide was covered in the PCA Regression lecture, but it duplicated here as a refresher





Illustration Hitters{ISLR} data

- Factor Loadings for a 3-Factor solution
- It is not as easy to visualize the variables to group into each factor

```
PC1 PC2
                          PC3
Hitters.AtBat
               0.0910.95
Hitters.Hits
               0.08 0.95
               0.14 0.48
Hitters.HmRun
               0.06 0.94
Hitters.Runs
Hitters.RBI
               0.18
Hitters.Walks
Hitters.CAtBat 0.98 0.1
Hitters.CHits
               0.97
                    0.15
Hitters.CHmRun
               0.84
                    0.080.45
Hitters.CRuns
               0.97
                    0.18 \ 0.03
Hitters.CRBI
               0.95
Hitters.CWalks
```





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