Matrix Notation

Some Matrix Notation

$$\overbrace{\underbrace{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \text{Etc,} \\ y_n \end{bmatrix}}_{\mathbf{Y}\ ^{(n \times 1)}} \quad \underbrace{\begin{bmatrix} x_{11}\ x_{21}\ \ldots\ x_{k1} \\ x_{12}\ x_{22}\ \ldots x_{k2} \\ x_{13}\ x_{23}\ \ldots x_{k3} \\ \text{Etc,} \\ x_{1n}\ x_{2n}\ \ldots x_{kn} \end{bmatrix}}_{\mathbf{X}\ ^{(n \times k)}}}^{\text{Data}} \Rightarrow \overbrace{\underbrace{\begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \text{Etc,} \\ \beta_n \end{bmatrix}}_{\boldsymbol{\beta}\ ^{(n \times 1)}}}^{\text{Coefficients}} \Rightarrow \overbrace{\underbrace{\begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \hat{y}_3 \\ \text{Etc,} \\ \hat{y}_n \end{bmatrix}}_{\hat{\mathbf{Y}}\ ^{(n \times 1)}}}^{\text{Prediction}} + \overbrace{\underbrace{\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \text{Etc,} \\ \varepsilon_n \end{bmatrix}}_{\boldsymbol{\varepsilon}\ ^{(n \times 1)}}}^{\text{Error}}$$

Predictors → Independent Variables

Outcomes → Dependent Variables

Matrix algebra facilitates computation, e.g.:

Predictive Model: $\mathbf{Y} = \hat{\mathbf{Y}} + \boldsymbol{\varepsilon} = \boldsymbol{\beta}\mathbf{X} + \boldsymbol{\varepsilon}$

Sum of Squared Errors: $\boldsymbol{\varepsilon}'\boldsymbol{\varepsilon}$

Regression Coefficients: $\boldsymbol{\beta} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}$

More Matrices: Covariance & Correlation

$$\overbrace{\begin{bmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} & \ldots & \sigma_{1k} \\ \sigma_{21} & \sigma_2^2 & \sigma_{23} & \ldots & \sigma_{2k} \\ \sigma_{31} & \sigma_{32} & \sigma_3^2 & \ldots & \sigma_{3k} \\ \ldots & & & & \\ \sigma_{n1} & \sigma_{n2} & \sigma_{n3} & \ldots & \sigma_n^2 \end{bmatrix}}^{\text{Covariance Matrix}} \Rightarrow \rho_{ij} = \sigma_{ij}/\sigma_i\sigma_j \Rightarrow \overbrace{\begin{bmatrix} 1 & \rho_{12} & \rho_{13} & \ldots & \rho_{k} \\ \rho_{21} & 1 & \rho_{23} & \ldots & \rho_{2k} \\ \rho_{31} & \rho_{32} & 1 & \ldots & \rho_{3k} \\ \ldots & & & & \\ \rho_{n1} & \rho_{n2} & \rho_{n3} & \ldots & 1 \end{bmatrix}}^{\text{Correlation Matrix}}$$

$\Sigma = \mathrm{Cov}(X)$ $\qquad$ $R = \mathrm{Corr}(X)$

Kogod School of Business