Piecewise Linear Models

KOGOD SCHOOL *of* BUSINESS
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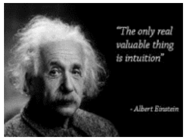
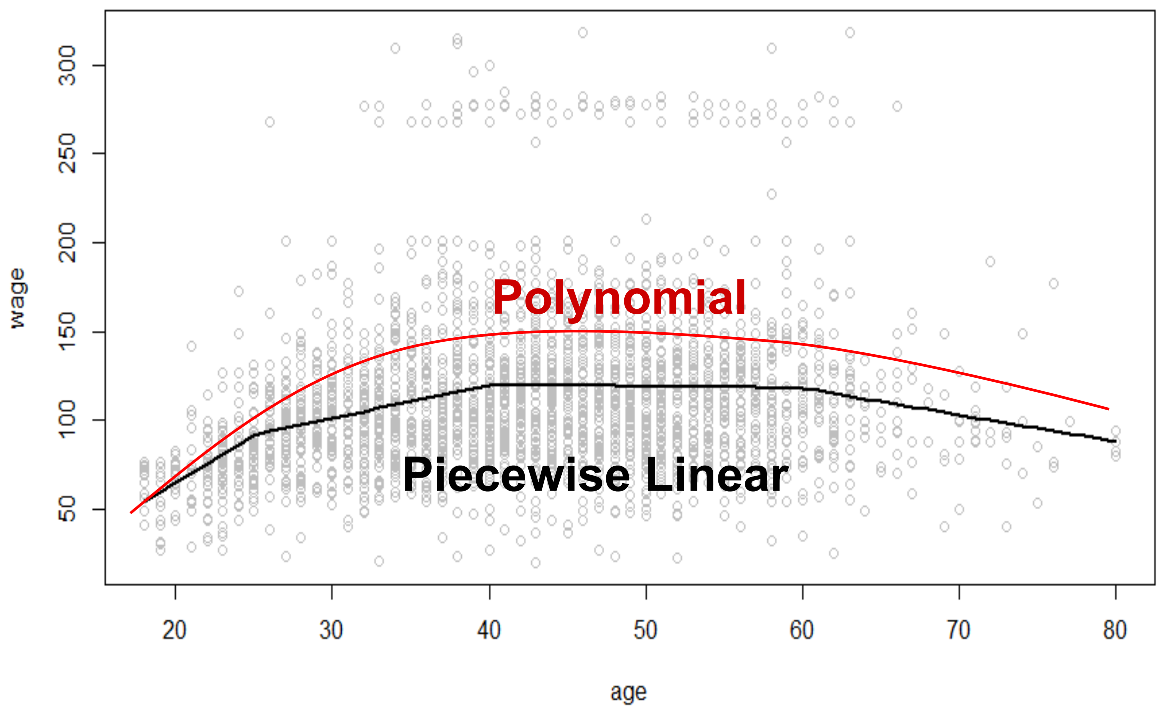


Piecewise Linear: Intuition

- Piecewise linear models are an attractive **alternative** to **polynomials**
- Rather than a curve we fit **connected linear** regression **models**
- Piecewise linear models retain the **simplicity** of linear models and are **easier** to **interpret**.
- But because the slope of the regression can change at any selected point, it can provide **better** training **fit** with **less overfitting**.

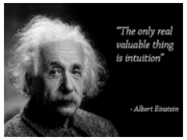


Piecewise Linear Example Wage{ISLR}

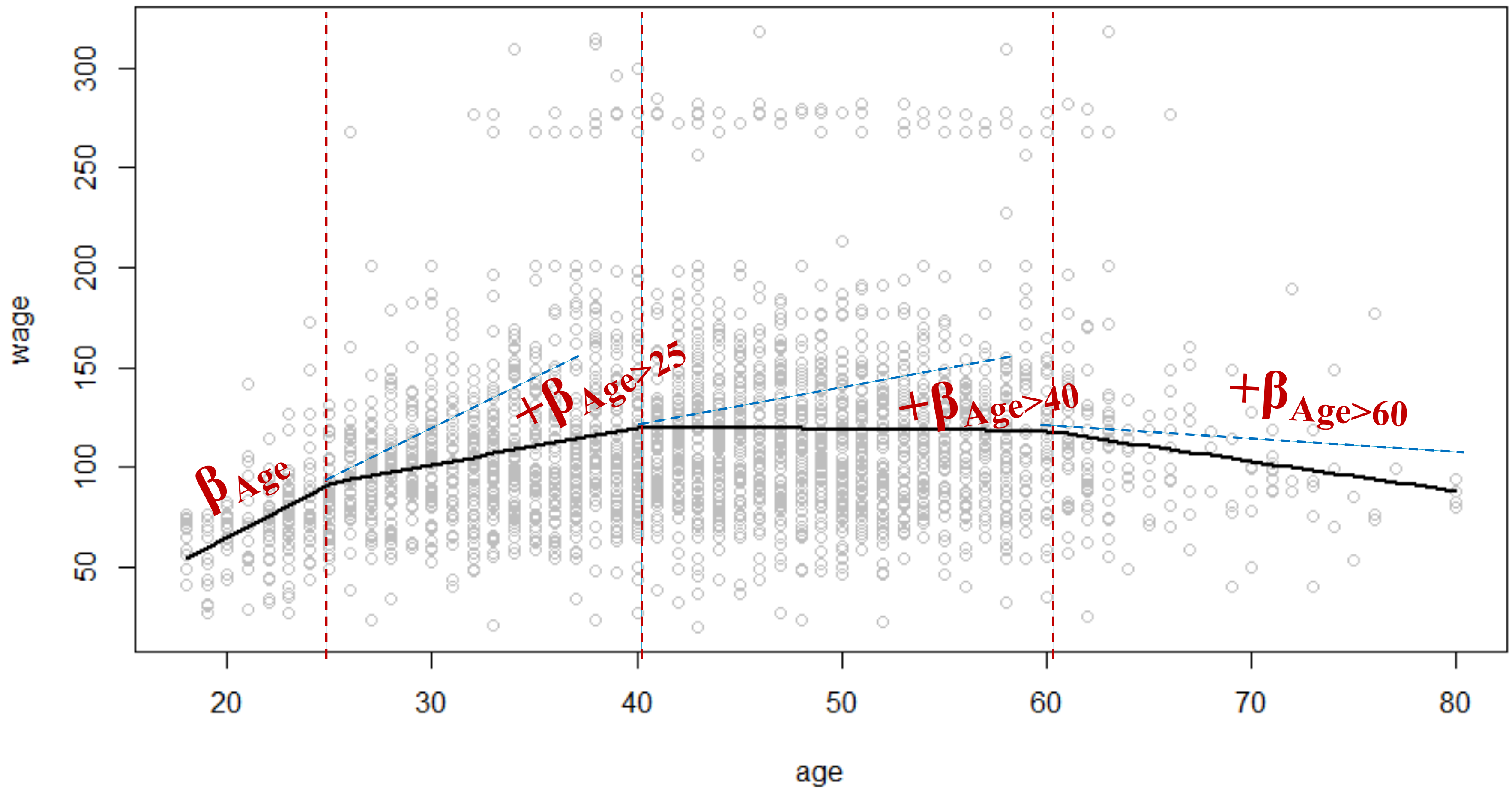
- Say, you want to predict **wage** as a function of **age**
- But you notice that the wage **patterns change** at ages 25, 40 and 60
- So, we want to fit a **piecewise linear** function that give us **4** different regression **slopes** between these particular **"knots"**
- We do this by creating **interaction dummy** variables as follows:

lm(wage ~ age + I((age-25)*(age>25)) + I((age-40)*(age>40)) + I((age-60)*(age>60)), data = Wage) →

- ➢ The coefficient for ***age*** gives the effect of age up to age 25
- ➢ The function ***I((age-25)*(age>25))*** is an **interaction** variable that takes a value of **0** (when ***age<25***) (i.e., does **not change** the **slope** of the variable ***age***
- ➢ But if ***age>25*** then it multiplies the age beyond 25 ***(age-25)*** by 1 (because ***age>25***)
- ➢ It's coefficient provides the **increment** (or **reduction**) in **slope**
- ➢ Same thing for ***(age>40)*** and ***(age>60)***



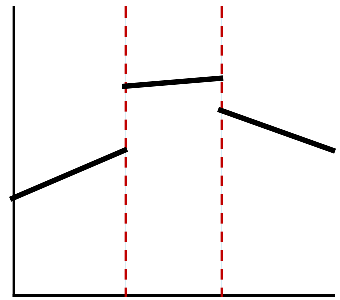
Graphical Illustration



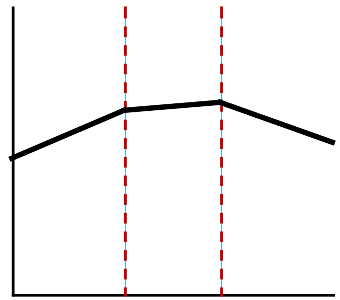
Note in this example (see R script) that β_{Age} is **(+)** but the remaining **β's** are (-) → the **effect of age** goes down **diminishes** at ages 25, 40 and 60.

Piecewise Linear vs. Linear Spline

- The only difference between **piecewise linear** and a **linear spline** is that the former does **not connect** the regression lines **at the knots** (i.e., the lines shift), whereas the latter does **connect** at the **knots**



Piecewise Linear



Linear Spline

- However, as you saw in the previous example, we can force a piecewise linear model to have connected lines at the know (i.e., a linear spline), simply by subtracting the knot value from the x values after the knot (e.g., ***age-25***). This is equivalent to shifting the Y axis to towards the right to the knot.

Tips

```
fit.piecewise=lm(y~x+
                   I((x-25)*(x>25))+
                   I((x-40)*(x>40))+
                   I((x-60)*(x>60)),
                   data=dataName)
```

→ Fits a stepwise linear model with 4 segments separated by 3 knots set arbitrarily at `x=25`, `40` and `60` respectively

The coefficient of `x` provides the slope of the regression line for `x<=25`; the next coefficient (e.g., for `x>25`) provides the "change" in slope after the `x>25` knot; and so on

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