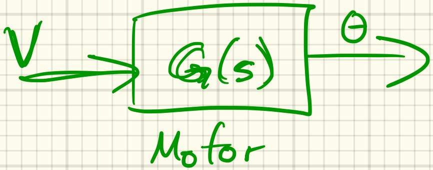
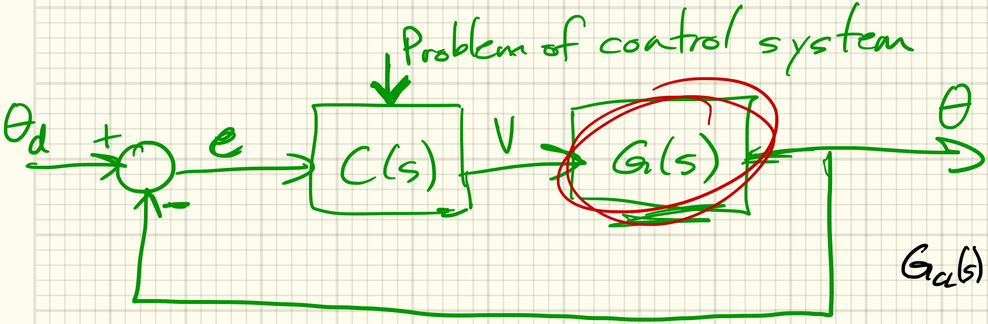
SPRING 2017

PID TUTORIAL

Why Control System?

1. We have unstable systems
2. We want to make the system behave in a particular way





$$G_{cl}(s) = \frac{\Theta(s)}{\Theta_d(s)} = \frac{C(s)G(s)}{1+C(s)G(s)}$$

This tutorial is on PID control

- P — proportional
- I — integral
- D — derivative

PID control

derivative
Integral
proportional

$$c(t) = K_p e + K_I \int_0^t e \, d\tau + K_D \dot{e}$$

assume $x(0) = 0$

$\dot{x} + 1x = u$

$sX(s) + X(s) = U(s)$

$G(s)\ \frac{X(s)}{U(s)} = \frac{1}{s+1}$

$x(t) = Ce^{-t}$

First order system

$\dot{x} - 1x = u$

$sX(s) - X(s) = U(s)$

$G(s) = \frac{X(s)}{U(s)} = \frac{1}{s-1}$

with

Proportional
$C(s) = K_p$

$x(t) = Ce^{t}$

$$G_{CL}(s) = \frac{K_p \frac{1}{s-1}}{1 + K_p \frac{1}{s-1}} = \frac{K_p}{s + (K_p - 1)}$$

Let $K_p = 2$

$x(t) = C' e^{-t}$

First order system with PD control

$$G(s) = \frac{1}{s-1} \qquad C(s) = K_p + K_d s$$

$$G_{cl}(s) = \frac{(K_p + K_d s)\left(\frac{1}{s-1}\right)}{1 + (K_p + K_d s)\frac{1}{s-1}} = \frac{K_p + K_d s}{(1+K_d)s + (K_p - 1)}$$

$$= \frac{\frac{K_p}{1+K_d} + \frac{K_d}{1+K_d}s}{s + \frac{K_p - 1}{1+K_d}} = \frac{A + Bs}{s + p}$$

$$= C_1 e^{-pt} - C_2 p e^{-pt} + \delta(t)$$

2nd Order System

$$G(s) = \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

ω_n: natural frequency

ζ: damping coefficient

$$C(s) = K_p + K_D s$$

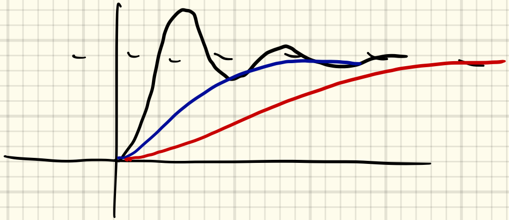
$$G_{CL}(s) = \frac{(K_p + K_D s)\left(\frac{1}{\quad}\right)}{1 + (K_p + K_D s)\frac{1}{\quad}} = \frac{K_p + K_D s}{s^2 + 2\zeta\omega_n s + \omega_n^2 + K_p + K_d s}$$

$$= \frac{K_p + K_D s}{s^2 + \underbrace{(2\zeta\omega_n + K_d)}_{2\zeta'\omega_n'} s + \underbrace{(\omega_n^2 + K_p)}_{\omega_n'^2}}$$

$K_d = -2\zeta\omega_n \Rightarrow$ undamped

Pick K_d + K_p wisely. What does that mean?

Types of Response



Zeigler-Nichols

$$G(s) = \frac{1}{s^2+2\zeta\omega_n s+\omega_n^2} \qquad C(s) = K_p$$

$$G_{cl}(s) = \frac{K_p}{s^2+2\zeta\omega_n s + (\omega_n^2+K_p)} = \frac{K_p}{(s+j\omega)(s-j\omega)}$$

1. Increasing K_p until system oscillates ($K_d=0, K_I=0$)
2. Measure (A) period of oscillation (B) Amplitude of oscillation
3. Compute good K_p, K_I, K_d