A Generalization of Lucas de Groot's Interpolation Theory

Linus Romer

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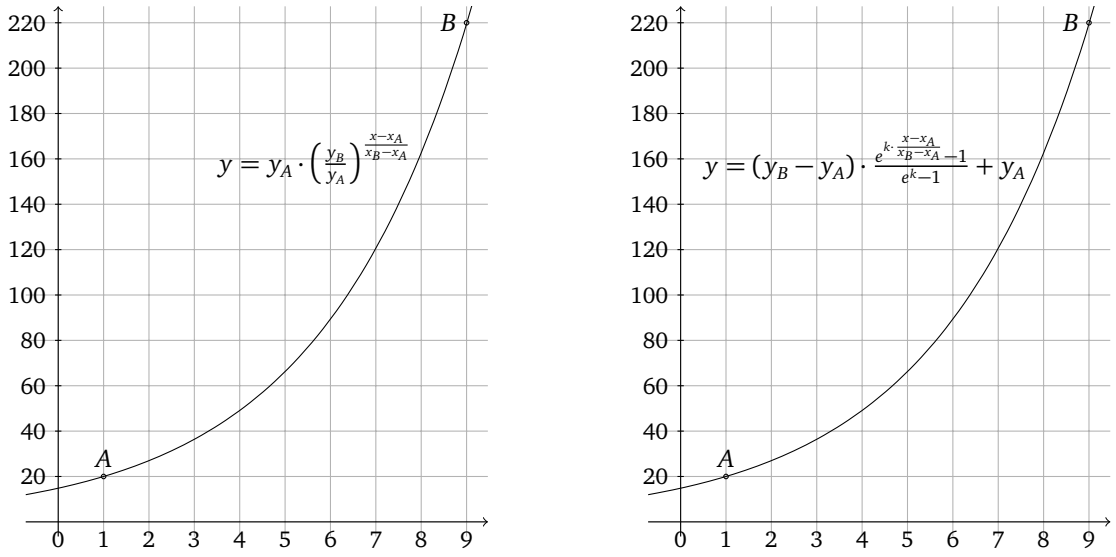
Lucas de Groot's Interpolation Theory states, that font weights of a family of fonts should progress like a geometric sequence. E.g. the first font should have the weight 20 (denoted as $A(1, 20)$ in the image below left) and the ninth font should have the weight 220 (denoted as $B(9, 220)$. In mathematical terms *Lucas de Groot's Interpolation Theory* means interpolating the points A and B by an exponential function

$$y = y_A \cdot \left(\frac{y_B}{y_A}\right)^{\frac{x-x_A}{x_B-x_A}}.$$

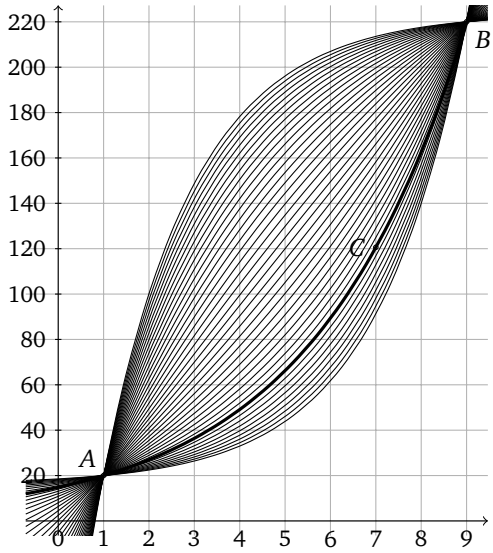


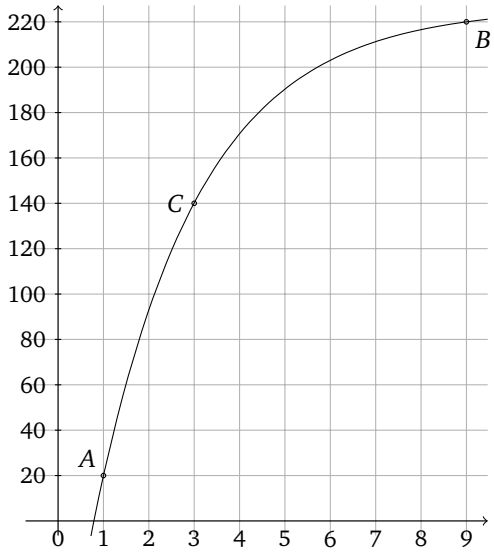
This function is a special case of a generalized set of functions

$$y = (y_B - y_A) \cdot \frac{e^{k \cdot \frac{x-x_A}{x_B-x_A}} - 1}{e^k - 1} + y_A$$

depicted in the upper right. This means, when we choose the parameter $k = \ln\left(\frac{y_B}{y_A}\right)$, the upper right function is the same as the upper left function. The great thing about this is, that

we can vary now k in order to get similar curves (for $k = 0$ we have to use a linear function in order to prevent divison by zero):





With the generalized function, we can now interpolate three points A, B and C — as long as monotony is still given (depicted in the upper right). Of course, we lose the property of geometric progression. In return, it is now possible that y becomes zero.