Introduction to
my work on ~~'Comp...~~
~~The Complicated~~ Interaction Algorithms, Complicated Nonmonotonic
Predicates, Dynamically Databases (changing)
~~Automatic~~ Programs and their
Startling Relationship to one Another & Established of Totally Complicated Structures

~~An Important Advances~~

in The Theory of Dynamic Data,
Nonmonotonic Predicates, ~~and~~ Automatic Programs,
And their relationship to one another

~~Complicated~~

~~The Theory of Predicate Algorithms,~~
~~Dynamically Changing Databases, Automatic~~
~~Computer Programs, and their Subtle~~
~~Interactions~~

The Theory of Complicated
Dynamically Changing Databases,
Predicate Returning Algorithms, Automatic
Computer Programs and their
Subtle Interactions 118

## $\boxed{7.2} \rightarrow$ The $\Pi$-Projection

This section will introduce the concept of the $\Pi$-projection. The purpose of that projection will be to help us simplify an initial $e(X,y)$ predicate expression by removing some of its atomic predicates. The concept of the $\Pi$-project is importan because it will be used is

Sections 7.3 and 7.6

The definition of

the $\Pi$-projection is somewhat

complicated. Our discussion will

therefore begin by introducing a

preliminary notation

~~because it will be used in Sections 7.3 and 7.6, the definition~~

Throughout this chapter, the symbols of $\bar{X}$, $\bar{Y}$, $X$ and $Y$ will have the same meanings as they had elsewhere in this thesis. Thus, the first two symbols will ~~the~~ denote fixed elements that belong to the respective $R_X$ and $R_Y$ relations, and the latter two symbols will ~~similarly~~

denote variables that belong

to those two relations.

Also in this chapter, a very different meaning will be attached to the symbols of $e(X,y)$ and $e(\bar{X},\bar{y})$. The

~~meaning~~ ~~The~~ former term will designate

some predicate-expression." The latter term

will denote the truth-value

which this predicate assumes when

the fixed elements of $\bar{X}$ and

$\bar{Y}$ are substituted into it.

Let $e_1$ and $e_2$ denote two

distinct predicates. In several parts

of this chapter we will wish to

know whether the equalities

of "$e_1(\bar{X},\bar{Y}) = e_2(\bar{X},\bar{Y})$" or the

"$e_1(X, y) = e_2(\bar{X}, y)$" are satisfied

These two equalities will have

very different meanings. The first

equality will be defined to hold when

the $e_1(\bar{X}, \bar{Y})$ and $e_2(\bar{X}, \bar{Y})$

terms possess the same

truth-value. The second equality

will have a different meaning

~~meaning~~ becau

"$e_1(X,Y) = e_2(\bar{X},Y)$" are satisfied

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~~meaning~~ becau

because it describes a condition between two variables rather than two fixed elements. The "$e_1(X, y) = e_2(X, y)$" equality will ~~there be~~ be defined to hold only when the $e_1$ and $e_2$ predicates are sufficiently similar to guarantee that all elements will be defined to hold whenever

the logical condition

of "$\forall x \forall y \; g_1(x, y) = g_2(x, y)$"

is satisfied.

~~must necessarily~~ be

Sections 7.3 and 7.6

will use the concepts of

$\Pi$-projections to help them

perform

The preceding concepts will be used in our definitions of the $\Pi$ and $\overline{\Pi}$ projections. The remaining parts of this section will be devoted to defining and giving examples of these two projections.

Definition 7.2.A: Let $e(X, Y)$ denote a predicate expression which is built out of the

atomic predicates of $A_1(x,y), A_2(x,y) \ldots A_l(x,y)$

Let $A_1(x,y), A_2(x,y) \ldots A_K(x,y)$

denote a subset of K of

these $l$ predicates. Also, let

$b_1 b_2 \ldots b_K$ denote a collection

of K boolean values.

The $\Pi\left(e; \underset{A_1}{\overset{b_1}{\downarrow}} \underset{A_2}{\overset{b_2}{\downarrow}} \ldots \underset{A_K}{\overset{b_K}{\downarrow}}\right)$ projection

will be defined to be that

~~predicate which is begotten by~~

~~repla~~

predicate which is identical

to $e(X,Y)$ except that

the $A_1(X,Y)$ $A_2(X,Y)$ ... $A_k(X,Y)$

atomic predicates have been replaced

by the $b_1 b_2 \ldots b_k$ boolean values.

$\equiv$

Example 7.3.B: Let $e(X,Y)$

denote the predicate expression

~~given in~~ e

given below:

(1) $e(X,Y) = \{A_1(X,Y) \text{ OR } [A_2(X,Y) \text{ AND } A_3(X,Y)]\}$

Some examples of $\Pi$-projections

are shown below:

(2) $$\Pi(e, \underset{A_2}{\overset{\text{TRUE}}{\downarrow}}) = \{A_1(X,Y) \text{ OR } [\text{TRUE AND } A_3(X,Y)]\}$$
$$= \{A_1(X,Y) \text{ OR } A_3(X,Y)\}$$

(3) $$\Pi(e, \underset{A_2}{\overset{\text{FALSE}}{\downarrow}}) = \{A_1(X,Y) \text{ OR } [\text{FALSE AND } A_3(X,Y)]\}$$

$$= \{A_1(X,Y)\}$$

~~Other examples of Π-projections~~

(4) $\Pi\left(e, \underset{A_1}{\overset{\text{FALSE}}{\downarrow}} \quad \underset{A_2}{\overset{\text{TRUE}}{\downarrow}}\right) = \{A_3(X,Y)\}$

(5) $\Pi\left(e, \underset{A_1}{\overset{\text{TRUE}}{\downarrow}} \quad \underset{A_2}{\overset{\text{TRUE}}{\downarrow}}\right) = \{\text{TRUE or } A_3(X,Y)\}$

$= \text{TRUE}$

(6) $\Pi\left(e, \underset{A_1}{\overset{\text{FALSE}}{\downarrow}} \quad \underset{A_2}{\overset{\text{FALSE}}{\downarrow}}\right) = \text{FALSE}$

≡ ~~The Π-projection will be used in the next section of this chapter. A slightly~~

Example 7.3.6: Sometimes predicate

$e(X,Y)$ will possess an

$A_i(X,Y)$ atomic predicates which

occurs in several positions

within this predicate. In this case, all

such occurances of the $A_i(X,Y)$ atomic

predicates should be replaced

by the designated boolean value

when the $\pi$-projection is

~~operations on this~~

performed. An example of

such a multiple substition can be seen

by examining equation 7. Some

Π-projection of this equation are

shown in equations 8 and 9

(7) $e(X,Y) = \{[A_1(X,Y) \text{ AND } A_2(X,Y)] \text{ OR } [(\text{NOT } A_1(X,Y)) \text{ AND } A_3(X,Y)]\}$

(8) $\Pi(e, \overset{\text{TRUE}}{\underset{A_1}{\downarrow}}) = \{A_2(X,Y)\}$

(9) $\Pi(e, \overset{\text{FALSE}}{\underset{A_1}{\downarrow}}) = \{A_3(X,Y)\}$

≡

The $\Pi$- projection

will be used in the

next section of this

chapter. A slightly

different version of this ~~chapter~~

projection will be used

in section 7.6. That section

will employ the "$\Pi$ over-line"

projection. The definition of

this $\overline{\Pi}$ projection is given

below:

Definition 7.3.D: Let $e(X,Y)$ denote

~~given again denote~~ a

a predicate expression whose

atomic predicates are once

again denoted as $A_1(X,Y), A_2(X,Y), \ldots A_L(X,Y)$

Also, let $A_1 A_2 \ldots A_K$ ~~also once again~~

denote a subset of K of those L

atomic predicates. Let us further

recall that $A_i(\bar{X}, \bar{Y})$ denotes

the truth-value which

results when the $\bar{X}$ and $\bar{Y}$

elements are substitited into the

the ~~truth values of $\bar{X}$ and $\bar{y}$~~

~~are substituted into the~~

$A_i(X,y)$ predicate. The $\overline{\Pi}(e,\bar{X},\bar{y}; A_1 A_2 \ldots A_k)$

expression will be defined to

be that predicate which is identical to $e(X,y)$ except that this predicate's $K$ distinct $A_i(X,y)$

predicates are replaced by the

$A_i(\bar{X},\bar{y})$ truth-values.

**Comment:** An perhaps somewhat simpler definition of the $\overline{\Pi}(e,\bar{X},\bar{y}, A_1 A_2 \ldots A_k)$

projection can be given if

the $\Pi$ projection is used. The

~~$\overline{\overline{\Pi}}(e, \bar{X}, \bar{Y}, A_1 A_2 \ldots A_K)$ projection~~

projection will thus be defined to be that

special version of the $\Pi$ projection given in the

following equation:

$$(10)\quad \overline{\overline{\Pi}}(e, \bar{X}, \bar{Y}; A_1 A_2 \ldots A_K) = \Pi\left[e \begin{array}{cccc} A_1(\bar{X},\bar{Y}) & A_2(\bar{X},\bar{Y}) & & A_K(\bar{X},\bar{Y}) \\ \downarrow & \downarrow & \cdots & \downarrow \\ A_1(X,Y) & A_2(X,Y) & & A_K(X,Y) \end{array}\right]$$

$\equiv$

Example 7.3.D Let us calculate

the $\Pi(e, \bar{X}, \bar{Y}; "X.a > Y.b_1," "X.a > Y.b_2")$

~~for the predicate defined in~~

~~equat~~

projection for the $e(X,Y)$ predicate, defined below:

~~$e(X,Y) = \{((X.a_1 > Y.b \text{ AND } X.a_1 = 1) \text{ OR } (X.a_2 > Y.b \text{ AND } X.a_2 = 2))\}$~~

11) $e(X,Y) = \{(X.a > Y.b_1 \text{ AND } Y.b_1 > 1) \text{ OR } (X.a > Y.b_2 \text{ AND } Y.b_2 > 2)\}$

The value of this projection

~~$\overline{\Pi}(e, \overline{X}, \overline{Y})$~~

will depend on the truth-values

of the "$\overline{X}.a > \overline{Y}.b_1$" and "$\overline{X}.a > \overline{Y}.b_2$"

atomic predicates. The following table illustrates how these two truth-values govern the results of the $\overline{\Pi}(e, \overline{X}, \overline{Y}, \text{"}X.a>y.b_1\text{,"} \text{"}X.a_2>y.b_2\text{"})$ projection:

| "$X.a>Y.b_1$" truth-value | "$X.a>Y.b_2$" truth-value | Resulting $\overline{\Pi}$ Projection |
|---|---|---|
| TRUE | TRUE | $Y.b_1>1$ OR $Y.b_2>2$ |
| TRUE | FALSE | $Y.b_1>1$ |
| FALSE | TRUE | $Y.b_2>2$ |
| FALSE | FALSE | FALSE |

Various types of $\Pi$-projections will be discussed in sections 7.3 and 7.6. This concept is important because it will help us to simplify our notation in those discussions.

~~simple, Combination of well-Known~~

well-Known inverted-file techniques with

Section 3.2's special hashing

functions.

Our various database

algorithms will be designed

to process E-8 predicates by

making subroutine-calls to

the E-7 algorithms. The

last section's Π-projection

will be used by the process

which reduces ~~an~~ initial E-8 expression to

~~to reduced to a simpler~~

simpler E-7 expression. The $e_0 e_1 e_2 \ldots e_K$

predicates that are ~~introduced~~

predicates, defined below, will play

a major role in that

reduction process:

Definition 7.3.E: Let $e(X,Y)$ denote an E-1 expression which possesses the K tabular predicates of $T_1^{XY}, T_2^{XY}, \ldots, T_K^{XY}$. Let $i$ denote an integer whose value is between zero and K. Throughout this section, the symbol $e_i(X,Y)$

~~$e_i(X,Y)$ predicate will be presumed~~

will be called the i-th reduction predicate of $e(X,y)$. The definition which $e_i(X,Y)$ will assume in this section is given below:

$$(1)\quad e_i(X,y) = \Pi\left(e,\ \underset{T_1(X,y)}{\overset{\text{FALSE}}{\downarrow}}\ \underset{T_2(X,y)}{\overset{\text{FALSE}}{\downarrow}}\ \cdots\ \underset{T_i(X,y)}{\overset{\text{FALSE}}{\downarrow}}\right)$$

$$\equiv$$

Remark 7.3.F: Note that the preceding definition implied that $e_K(X,y)$ is an E-7 expression (because all K of e's tabular predicate are removed when the $e_K(X,Y)$

expression is formed). Also, Definition 7.3.E

can be easily shown to imply that (because
$e(X,Y)$ and $e_0(X,Y)$ are the same predicate).
These two facts will be used by

our E-S algorithms.

Definition 7.3.G Let $S^F_{e_i}(X)$ denote the
sum associated with the $e_i(X,Y)$ predicate. During
this section, the symbol

$Z^F_i(X)$ will be an abbreviation for

the following arithmetic