matical machinery to accomplish this

Definition 4.2 The deduction method **Tab-1.** was defined in both our articles [46, 52]. It is, essentially, a compromise between Tab and Xtab deduction, where a "Tab-1" proof for Ψ from an axiom basis α corresponds to a set of ordered pairs $(p_1, \phi_1), (p_2, \phi_2), ..., (p_k, \phi_k)$ where

1. $\phi_k = \Psi$

2. For every $j \leq k-1$, each p_j is a Tab-proof of a Rank-1* sentence ϕ_j from the union of α with the preceding Rank-1* sentences of $\phi_1, \phi_2, ..., \phi_{j-1}$.

noteworthy prods

The Rank-1* constraint, used by Item 2, is ~~quite significant~~ because it makes

lie midway in efficiency between

Tab-1 ~~less efficient than~~ Xtab ~~in the extreme worst case~~. (This is because

and Tab under extreme worst case

Tab-1, unlike Xtab, requires all-except-its-final wff ϕ_j to be Rank-1* sentences, as Footnote[4] explains.) Thus in extreme worsr cases, Tab-1 proofs from a base axiom system α can be *sharply longer* than Xtab proofs, while still being potentially *sharply more compressed* than Tab proofs. In particular, the constraint that Items (1) and (2) imposes upon the string of ordered pairs

[4]Unlike Tab-1, Xtab does not impose a similar Rank-1* requirement upon ϕ when its Law of the Ex Middle allows $\phi \vee \neg\phi$ to appear anywhere as a permissible logical axiom, for arbitrary ϕ.)

matical machinery to accomplish this

Definition 4.2 The deduction method **Tab-1.** was defined in both our articles [46, 52]. It is, essentially, a compromise between Tab and Xtab deduction, where a "Tab-1" proof for Ψ from an axiom basis α corresponds to a set of ordered pairs $(p_1, \phi_1), (p_2, \phi_2), ..., (p_k, \phi_k)$ where

1. $\phi_k = \Psi$

2. For every $j \leq k-1$, each p_j is a Tab-proof of a Rank-1* sentence ϕ_j from the union of α with the preceding Rank-1* sentences of $\phi_1, \phi_2, ..., \phi_{j-1}$.

The Rank-1* constraint, used by Item 2, is noteworthy partly because it makes Tab-1 lie midway in efficiency between Tab and Xtab under extreme worst

circumstances

case. (This is because Tab-1, unlike Xtab, requires all-except-its-final wff ϕ_j to be Rank-1* sentences, as Footnote[4] explains.) Thus under extreme worst case circumstances, a Tab-1 proof from a base axiom system α can be *sharply longer* than an Xtab proof, while still being potentially *sharply more compressed* than

counterpart

its Tab ~~proofs~~. In particular, the constraint that Items (1) and (2) imposes upon

[4]Unlike Tab-1, Xtab does not impose a similar Rank-1* requirement upon ϕ when its Law of the Excluded Middle allows $\phi \vee \neg\phi$ to appear anywhere as a permissible logical axiom, for arbitrary ϕ.)

A Stronger Result in Tableaux 2002 Conference

1) Prior JSL-2001 paper could recognize consistency by knowing that

"There is no Sem Tab Proof of 0=1 from me"

2) Stronger Tab-2002 Proceedings studies systems recognize

"There is no Sem Tab Proof of a Π_1 sentence and its negation from me"

3) Also have established above not generalize for Π_2

complexities far exceeding the level of Π_1^* sentences.

In other words, *** is technically a Π_1^* encoded axiom schema that flirts with the prospect of more complicated formulae lying inside its proof " p ", without actually crossing into this dangerous domain.

Much of the magic power of the $IS_D(PA^*)$ formalism stems from this *"Flirting without Trespassing"* effect. In particular, the self-affirming axiom system $IS_D(PA^*)$ will be able to partially evade Gödel's Second Incompleteness Theorem by **"ECHOING"** Peano Arithmetic (within a context where this "echoing" is not also an extension of Peano Arithmetic).

4.4 More Details about the "Echo Statement" $***$

A duplicate copy of the prior section's "Echo Statement" $***$ is provided below:

$$*** \qquad\qquad \forall\, p \quad \{ \ \text{HilbPrf}_{\text{PA}^*}(\ulcorner \Phi \urcorner, p) \ \Rightarrow \ \Phi \ \}$$

An interesting aspect of this statement is that one ideally wishes to be capable of of proving the existence of an integer p satisfying its clause " $\text{HilbPrf}_{\text{PA}^*}(\ulcorner \Phi \urcorner, p)$ "

the magnitude of $O(\text{Log}_2\, P)$ [illegible]

via a formal proof whose length is not much ~~longer~~ than ~~the binary length of~~

large

p ~~itself~~. It is possible to achieve this goal, but we will need some more mathe-

Reflecting Upon the Goals of Hilbert's Mysterious Second Problem From the Perspective of Echo Theory

Dan E.Willard

State University of New York at Albany

Abstract

Section 2 of this article will document that both Gödel and Hilbert had believed that Hilbert's Consistency Program should be continued, although they both certainly were aware that Gödel's Second Incompletenss Theorem had discouraged many logicians from continuing Hilbert's investigation. This article will use the Fitting-Smullyan tableau-style formalism to explain why these presumptions by Gödel and Hilbert were quite reasonable. Moreover we will employ the distinction between the two "E-words" of *"Extension"* and *"Echo"* to clarify the

subtle mysteries that have enveloped Hilbert's puzzling Second Problem during the essentially last 120 years.

Our thesis will be simple enough to be comprehensible to a large audience at the end of a 2-semester introductory course on proof theory.

Keywords and Phrases: Gödel's Second Incompleteness Theorem, Hilbert's Second Problem, Consistency, Smullyan-Fitting Semantic Tableau Deduction, Hilbert-Frege Deduction.

Mathematics Subject Classification: 03B52; 03F25; 03F45; 03H13

LFC; 20-23

A Modified "Tunnel" Styled

~~On a 3-Part "Tripod" Styled~~ Reply to Hilbert's ~~Mysterious~~ Second Problem

that Uses the Four "E-words" of "Eschew" Encapsulate "Echo" and Expo

Dan E.Willard

State University of New York at Albany

Abstract

Hilbert's mysterious year-1900 Second Problem asked mathematicians to devise a methodology whereby Peano Arithmetic can confirm its own consistency. Gödel's famous 1931 paper showed that a fully positive reply can never be made to Hilbert's question. This article will explain how Hilbert's question is such a complicated issue that it can be better receive a 3-way styled "Tripod" reply.

We also provide substantial evidence that Gödel would likely agree with the main opinions expressed in this article.

Keywords and Phrases: Gödel's Second Incompleteness Theorem, Hilbert's Second Problem, Consistency, Smullyan-Fitting Semantic Tableau Deduction, Hilbert-Frege Deduction.

~~Elongate~~ Elongated Exhaust ~~Ext~~ Evaluate

Encapsulate

Echo

Extend

E-word

"Eschew"

"entity"

"evoke"

1 Introduction

This article is a continuation of a series of papers that began with the 1993 article [38] and continued until and through the year-2021 article [47]. This series, which included six sole-authored papers appearing in the JSL and APAL, had focused on discussing generalizations and ~~boundary case~~ partial exceptions for the Second Incompleteness Theorem. The two goals of this paper will be to explore the underlying philosophy that motivated this series and to explain how it is related to several generalizations and ~~boundary-case~~ partial exceptions to the Second Incompleteness Theorem , including some important new results formalized by Artemov [3, 4].

Our general theme will be that Hilbert's year-1900 Second Problem is too complex an issue to receive a 1-directional or even 2-directional answer. Instead, it will require a 3-directional answer, called the "Tripod Reply" to Hilbert's question. The multi-faceted response will be the focus of [illegible] of Escher's [illegible]

The first leg of this 3-part response to Hilbert's Second Problem will rest on the combination of Gödel's initial version of his Second Incompleteness Theorem and its numerous generalizations. They collectively establish that many logical formalisms, besides Peano Arithmetic (PA), are unable to corroborate their own consistency in a fully extensive respect. While these gener-

Echo
~~Expansio~~
Expanso and Exclusio
E... mphasis

About the Characterization of a Fine Line That Separates Generalizations and Boundary-Case Exceptions for the Second Incompleteness Theorem under Semantic Tableau Deduction

Dan E.Willard

State University of New York at Albany

Abstract

Our previous research showed that the semantic tableau deductive methodology of Fitting and Smullyan permits boundary-case exceptions to the Second Incompleteness Theorem, if multiplication is viewed as a 3-way relation (rather than as a total function). It is known that tableau methodologies prove a schema of theorems verifying all instances of the Law of the Excluded Middle. But if *one promotes* this schema of theorems into formalized logical axioms, then the meaning of the pronoun of "I", used by our self-referencing engine, changes quite sharply. Our partial evasions of the Second Incompleteness Theorem shall then come to a complete halt.

CITATION INFORMATION : This paper was invited by the *Journal of Logic and Computation* and will appear in print in 2021. Its shorter 19-page conference-style version had appeared in January 2020 at the Springer-sponsored LFCS meeting under a slightly different title [57].

KEYWORDS and PHRASES: Semantic Tableau deduction, Hilbert's Second Problem, Partial Revival of Hilbert's Consistency Program, Generalizations of the Second Incompleteness Theorem.

Mathematics Subject Classification: 03B10; 03B45; 03F03; 03F25; O3F30.