

This example solves the minimization problem

$$\begin{aligned} \min J(q, u) &= \frac{1}{2} \|u - u^d\|^2 + \frac{\alpha}{2} \|q\|^2 \\ \text{s.t. } (\nabla u, \nabla \phi) &= (f, \phi) \quad \forall \phi \in H_0^1(\Omega; \mathbb{R}^2) \end{aligned}$$

on the domain  $\Omega = [0, 1]^2$ . In addition we set the dirichlet data of the state on the boundary as follows

$$\begin{aligned} u_0(0, y) &= q_0, & u_0(1, y) &= q_1, & u_0(x, 0) &= q_2, & u_0(x, 1) &= q_3, \\ u_1 &= q_4^3. \end{aligned}$$

The data is chosen as follows:

$$\begin{aligned} f &= \begin{pmatrix} 20\pi^2 \sin(\pi x) \sin(\pi y) \\ 1 \end{pmatrix} \\ u^d &= \begin{pmatrix} \sin(\pi x) \sin(\pi y) * x \\ x \end{pmatrix} \end{aligned}$$

with  $\alpha = 10$ .