This example solves the distributed minimization problem

$$\begin{aligned}\min J(q,u) &= \frac{1}{2}\|u-u^d\|^2 + \frac{\alpha}{2}\|q\|^2 \\ \text{s.t.}(\nabla u, \nabla\phi) &= (f(q),\phi) \;\; \forall\, \phi \in H_0^1(\Omega;\mathbb{R}^2)\end{aligned}$$

on the domain $\Omega = [0,1]^2$, and the data is chosen as follows:

$$\begin{aligned}f(q) = q_0 &\begin{pmatrix} 2\pi^2 \sin(\pi x)\sin(\pi y) \\ 0 \end{pmatrix} \\ + q_1 &\begin{pmatrix} 5\pi^2 \sin(\pi x)\sin(2\pi y) \\ 0 \end{pmatrix} \\ + q_2 &\begin{pmatrix} 0 \\ 8\pi^2 \sin(2\pi x)\sin(2\pi y) \end{pmatrix}\end{aligned}$$

with $\alpha = 0$ and u^d such that the solution is given by:

$$\begin{aligned}\overline{q} &= \big(1; 0.5; 1\big) \\ \overline{u} &= \begin{pmatrix} \sin(\pi x)(\sin(\pi y) + 0.5\sin(2\pi y)) \\ \sin(2\pi x)\sin(2\pi y) \end{pmatrix}\end{aligned}$$