In this example we consider the following benchmark problem from plasticity theory:

$$(\Pi(\sigma(u)), \varepsilon(\varphi)) = (g, \varphi)_{\Gamma_N}. \tag{1}$$

Here $\tilde{\Omega}$ is a quadratic domain with side length 200 mm, where a circular hole with radius 10 mm around the center is cut out. Using symmetries of the domain, we restrict our actual computational domain Ω to the upper left quarter of $\tilde{\Omega}$.
In the above equation, $\varepsilon(v) := \frac{1}{2}(\nabla v + \nabla v^T)$ is the symmetric strain tensor, and the symmetric stress tensor σ is defined as

$$\sigma(v) := 2\mu\varepsilon(v)^D + \kappa \cdot tr(\varepsilon(v))I$$

where τ^D is the deviatoric part of a tensor τ, in two dimensions defined as

$$\tau^D := \tau - \frac{1}{2}tr(\tau)I.$$

The main difference with respect to the elastic case is the projection operator Π in equation (1). It is defined as follows:

$$\Pi(\tau) = \begin{cases} \tau & |\tau^D| \leq \sigma_0 \\ \sigma_0|\tau^D|^{-1}\tau^D + \frac{1}{2}tr(\tau)I & |\tau^D| > \sigma_0 \end{cases}$$

In our computations, we choose $\sigma_0 = \sqrt{\frac{2}{3}} \cdot 450$, and the above parameters μ and κ as $\mu = 80193.800283$ resp. $\kappa = 190937.589172$. The corner points of our computational domain are in anticlockwise order: $(0,0), (90,0)$, $(100,10), (100,100)$ and $(0,100)$. We prescribe homogeneous Dirichlet boundary conditions in y-direction between $(0,0)$ and $(90,0)$ (lower boundary part), homogeneous Dirichlet boundary conditions in x-direction between $(100,10)$ and $(100,100)$ (right boundary part).
The goal of our computations is to detect a subdomain in Ω where plastic behaviour occurs (compare *E. Stein (editor), Error-controlled Adaptive Finite Elements in Solid Mechanics, Wiley (2003), pp. 386 - 389*). This subdomain depends on the righthand side g in equation (1) which we write as $g = \lambda \cdot p$ with $p = 100$ and $\lambda \in [1.5; 4.5]$.