0.0.1 General problem description

This example solves the distributed minimization problem

$$\begin{aligned} \min J(q,u) &= \frac{1}{2}\|u-u^d\|^2 + \frac{\alpha}{2}\|q\|^2 \\ \text{s.t.}(\nabla u, \nabla \phi) &= (q+f,\phi) \ \ \forall\, \phi \in H_0^1(\Omega) \end{aligned}$$

on the domain $\Omega = [0,1]^2$, and the data is chosen as follows:

$$\begin{aligned} f &= \left(20\pi^2 \sin(4\pi x) - \frac{1}{\alpha}\sin(\pi x)\right)\sin(2\pi y) \\ u^d &= \big(5\pi^2 \sin(\pi x) + \sin(4\pi x)\big)\sin(2\pi y) \end{aligned}$$

and $\alpha = 10^{-3}$. Hence its solution is given by:

$$\begin{aligned} \overline{q} &= \frac{1}{\alpha}\sin(\pi x)\sin(2\pi y) \\ \overline{u} &= \sin(4\pi x)\sin(2\pi y). \end{aligned}$$

Thus the exact optimal value of the cost functional can be calculated as

$$J^* = J(\overline{q},\overline{u}) = \frac{1}{8}\Big(25\pi^4 + \frac{1}{\alpha}\Big).$$

In addition the following functionals are evaluated:

$$\text{MidPoint: } u(0.5;0.5)$$

$$\text{MeanValue: } \int_\Omega u$$

The example shows how to estimate the error in the cost functional for stationary optimization problems.