

This example solves the distributed minimization problem

$$\begin{aligned} \min J(q, u) &= \frac{1}{2} \|u - u^d\|^2 + \frac{\alpha}{2} \|q\|^2 \\ \text{s.t. } (\nabla u, \nabla \phi) &= (q + f, \phi) \quad \forall \phi \in H_0^1(\Omega) \end{aligned}$$

on the domain $\Omega = [0, 1]^2$, and the data is chosen as follows:

$$\begin{aligned} f &= \left(20\pi^2 \sin(4\pi x) - \frac{1}{\alpha} \sin(\pi x) \right) \sin(2\pi y) \\ u^d &= (5\pi^2 \sin(\pi x) - \sin(4\pi x)) \sin(2\pi y) \end{aligned}$$

and $\alpha = 10^{-3}$. Hence its solution is given by:

$$\begin{aligned} \bar{q} &= \frac{1}{\alpha} \sin(\pi x) \sin(2\pi y) \\ \bar{u} &= \sin(4\pi x) \sin(2\pi y) \end{aligned}$$

In addition the following functionals are evaluated:

$$\text{MidPoint: } u(0.5; 0.5)$$

$$\text{MeanValue: } \int_{\Omega} u$$