Countries and Skewness

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1 Introduction

Prior research shows that there skewness is relationship between skewness and asset returns. The intuition behind this idea is related to prospect theory: investors overvalue small probabilities and undervalue large probabilities. This means that in asymetric return distributions, mispricings can be found due to the perception of risk. For example, for a given asset that has historically been negatively skewed, the resulting return for this asset should be abnormally high due to the over expectation for returns to continue to be negative. This relationship has been found to exist at the country-level returns through exchange traded funds to produce significant alpha. Our study attempts to become forward looking through predictive modeling. If the predictive models are successful, we might be able to capture excess return due to a first-mover advantage.

2 Related Literature

Various literature had significance in the inspiration of the idea described in the introduction. One paper named "Paper profits or real money? Trading costs and stock market anomalies in country ETFs" by Adam Zaremba and Laura Andreu assessed anomalies found in country-based exchange traded funds. This study was an ex-post or a backward looking study. The analysis most relevant to our analysis that was conducted in the paper, was the skewness sorted portfolios on country-based exchange traded funds. Zaremba and Andreu were able to find alpha at very significant levels when sorted on observed skewness.

Another paper, "Skewness preference across countries" by Adam Zaremba and Andrzej Nowak, indicates that assets with positively skewed returns are traded at a premium to assets with negative skewness. They show that there is a robust negative relationship between skewness and future returns in various international markets.

3 Data and Methodology

3.1 Commentary on Data Used

3.1.1 Country Exchange Traded Funds

There were 72 country-based exchange traded funds that were used in this study. These exchange traded funds are provided by the MSCI international investing index series. The coverage of many of these countries grew throughout the time between 2000 and 2019.

In Figure 1, you can observe the major periods where MSCI created new country-based exchange traded funds. These funds are market capitalization weighted with prominent companies from the country they attempt to proxy. These securities are traded and denominated in the United States Dollar. Dollar denominated instruments promote comparability from a return perspective.

Specifically, the exchange traded funds used in this analysis were the exchange traded funds from the following countries: China; United States; Canada; Japan; India; Malaysia; Indonesia; Taiwan; Korea; Thailand; Australia; United Kingdom; Brazil; Hong Kong; Greece; Philippines; Russia; Peru; Poland; New Zealand; Germany; Chile; Singapore; Hungary; Czech Republic; Spain; United Arab Emirates; Colombia; Mexico; Turkey; Israel; Egypt; Italy; France; Qatar; Pakistan; Argentina; Ireland; Sweden; Belgium;

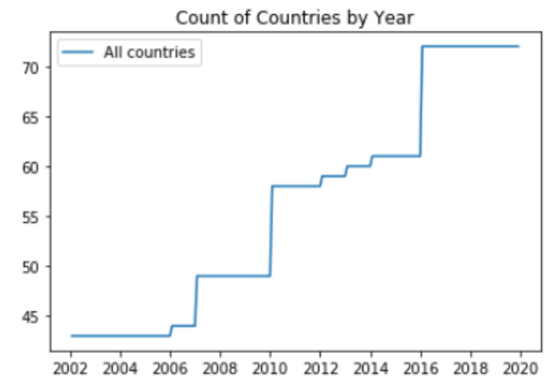

Figure 1: Count of Country Exchange Traded Funds Over Time

Switzerland; Portugal; Norway; Austria; Netherlands; Finland; Denmark; Morocco; Kuwait; SRI Lanka; Romania; Jordan; Vietnam; Bulgaria; Ukraine; Mauritius; Croatia; Serbia; Ghana; Slovenia; Kenya; Estonia; Bahrain; Botswana; Nigeria; Lithuania; Tunisia; Oman; Zimbabwe; Kazakhstan; Bangladesh; Bosnia And Herzegovina; Jamaica; and Lebanon

3.1.2 Measures of Societal Differences

This study used three measures of societal difference across countries. These measures were:

- Government Effectiveness
- Political Stability
- Voice and Accountability
- Gross Domestic Product per Capita

Government effectiveness The first measure, government effectiveness, captures the perceptions for the quality of public services, independence from political pressures, and governmental credibility to policy committment. These specific measures are very important for an environment of commerce that promotes successful business. This measure is provided by the World Bank and is measured by several underlying surveys and features of a country. For example, this index aggregates perceptions towards commerce infrastructure, public health, electricity grids, and excessive bureaucracy.

Political stability The second measure, political stability, measures the likelihood of political instability and politically motivated violence like terrorism. It is our conjecture that business will operate differently in societies experiencing different degrees of this stability. To determine this level of political stability, the World Bank aggregates underlying data including armed conflict, social unrest, internal conflicts, and political riots.

Voice and accountability The measure of a country's voice and accountability is very important to aspects of business like free trade and the ability to express ideas. Additionally, aspects of accountability can manifest itself in the quality of corporate governance. This measure, captures abilities for citizens to select government, express ideas, and other human rights. Specifically, the World Bank aggregates various sub indices like democracy measures, freedoms of association, civil liberties, and accountability of government employees.

Gross domestic product per capita Gross domestic product per capita is a broad measure for the wealth of individuals residing in a country. It is our conjecture that companies can experience performance shocks when gross domestic product per capita suddenly diminishes. This measure, provided by the World Bank, is converted from local currency to United States Dollars.

3.1.3 Asset Pricing Models and Data Used

There are many different models that attempt to price financial securities. In this study, we will use two prominent pricing models:

- Capital Asset Pricing Model
- Fama-French Three Factor Model

Capital Asset Pricing Model The Capital Asset Pricing Model is one of the more simple and widely used models. The model assumes that returns of securities are dependent solely on its beta to the market portfolio. The model also assumes that there is no alpha or intercept in this relationship between the market portfolio returns and the asset's returns. For this study, we use the S&P 500 exchange traded fund for the market proxy.

Fama-French Three Factor Model The Fama-French Three Factor Model is a continuation on the Capital Asset Pricing Model. Again, this is a model that attempts to price securities. The differences between this model and the Capital Asset Pricing Model are the introductions of two additional factors. There is the high minus low factor which represents the performance between growth and value investing. Lastly, there is the small minus big factor which represents performance differences between small market capitalization and large market capitalization.

3.2 Commentary on Dependent Variable

This study was primarily a predictive modeling problem. For the dependent variable, we used the skewness measure. We believe that countries should exhibit various levels of idiosyncratic risk. These country specific risks, to our hypothesis, should manifest themselves in greater downside risk.
Our study included the prediction of 252-day skewness and 1,260-day skewness. The 1-year time frame should have more variation in the skewness level. On the other side, the 5-year skewness should have less variation. We believe that, with independent variables that do not suddenly change, the reduced variability would better represent the changing business landscapes. However, significant changes in a societal factor could still alter the level of skewness if the business landscape objectively changes!

3.3 Portfolio Sorting Methodology

To quantify the effect of a predictive model, we implement a dynamic model training and sorting methodology. The portfolio sorting methodology contains the following steps at each year end:

1. Train the model using all data prior to the previous year
2. Predict the level of skewness using end of year data
3. Sort portfolios on predicted skewness
4. Rebalance monthly

We will be utilizing three sorted portfolios to attempt to diversify idiosyncratic risk across exchange traded funds. While each of these country-level exchange traded funds are made up of individual companies, anything larger than three portfolios could become problematic due to the lack of traded assets within the portfolios.

3.4 Model Selection

While our initial hypothesis was that there would be a direct relationship between these societal factors and skewness, it can be argued that this complex problem is not as simple as a linear relationship. To that end, we employ ordinary least squares regression and a random forest regressor model. The ordinary least squares regression will simply be direct, linear relationship between the independent variables and the dependent variable. The random forest regressor will capture the interactions between independent variables to, in the best case scenario, improve the predictive power of the model.

3.4.1 Ordinary Least Squares Regression

For the purposes of illustration, we showcase the trained model for the year of 2018. The 2018 model is used to predict the skewness in 2019. The model is trained on all data in the study period up till 2017.

Predicting 252-day skewness for 2019 For illustrative purposes, the model at year end in 2018 was selected. This includes all available data prior to 2017.

Table 1: Regression Coefficients in OLS Model in 2018

Variable	**Coef.**	**St. Err.**	**t**	**p Value**	**[0.025**	**0.975]**
Constant	0.0020	0.102	0.020	0.984	-0.198	0.202
governmentEffectiveness	-0.2228	0.112	-1.991	0.047	-0.442	-0.003
politicalStability	0.1699	0.081	2.095	0.036	0.011	0.329
voiceAccountability	-0.1274	0.077	-1.651	0.099	-0.279	0.024
GDP per capita	9.748e-06	3.28e-06	2.976	0.003	3.32e-06	1.62e-05

In Table 1, we can observe the regression coefficients for the ordinary least squares model in 2018. Of the five independent variables, three are statistically significant at the 5% confidence level. While these variables show significance, these five variables only explain 2.9% of the total variance for the 252-day skewness dependent varaible. When the model is used to predict the following year's skewness, the root mean squared error is 1.97. This is a large error as skewness in the following year had a mean of -0.238 and a standard deviation of 1.912.

Predicting 1260-day skewness for 2019 For illustrative purposes, the model at year end in 2018 was selected. This includes all available data prior to 2017.

Table 2: Regression Coefficients in OLS Model in 2018

Variable	**Coef.**	**St. Err.**	**t**	**p Value**	**[0.025**	**0.975]**
Constant	0.9010	0.347	2.598	0.010	0.220	1.582
governmentEffectiveness	-0.5230	0.352	-1.488	0.137	-1.213	0.167
politicalStability	1.0004	0.261	3.832	0.000	0.488	1.513
voiceAccountability	-0.4318	0.232	-1.865	0.063	-0.886	0.023
GDP per capita	1.312e-05	1.07e-05	1.228	0.220	-7.85e-06	3.41e-05

In Table 2, we can observe the regression coefficients for the ordinary least squares model in 2018. Of the five independent variables, two are statistically significant at the 5% confidence level. While these variables show significance, these five variables only explain 4.4% of the total variance for the 1260-day skewness dependent varaible.

Comparison of dependent variables Compared to the coefficients in Table 1, the constant became in the 5-year model very significant and the political stability feature became more significant. Again, as we increase the window of calculating skewness, skewness becomes more stable. However, after doing so, there was a marginal increase in explanatory power meaning these independent variables do not capture the variance of skewness in any significant manner. When the model is used to predict the following year's skewness, the root mean squared error is 1.534. This is a large error as skewness in the following year had a mean of -0.319 and a standard deviation of 0.7255.

3.4.2 Random Forest Regressor

After unsatisfactory results with a linear explanation of country skewness based on societal factors, we propose that there could be a hidden relationship in a non-linear space. To discover this relationship, we must first find optimal hyperparameters that find the balance between training sample error and testing sample error.

Hyperparameter tuning in 2018 on the 252-day skewness variable In the random forest regressor model, one of the most important hyperparameters is the maximum depth allowed for the tree to extend. A tree with an unconstrained depth can easily fit any training data perfectly at the cost of performance on the testing data.

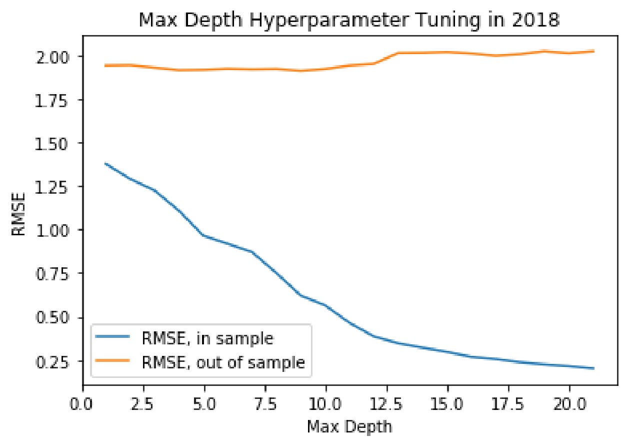

Figure 2: Tuning of the max depth hyperparameter

In Figure 2, we begin to see the root mean squared error of skewness prediction fall substantially as the depth increases in the training sample. However, the error does not drop significantly at all. This is clear that the interactions between the independent variables do not add much benefit, at least in 2018. Although the predictive power is near random, we declare that the optimal maximum depth for the random forest is at a depth of three.

Hyperparameter tuning in 2018 on the 1260-day skewness variable Again, we attempt to tune the model's maximum depth that the trees use in their splits. This time, the dependent has less variability as the window of calculation has increased by four years of daily data.

In Figure 3, we begin to see the root mean squared error of skewness prediction fall substantially as the depth increases in the training sample. However, the error does not drop significantly at all. This is clear that the interactions between the independent variables do not add much benefit, at least in 2018. Although the predictive power is near random, we declare that the optimal maximum depth for the random forest is at a depth of three.

3.4.3 Model Conclusions

Objectively poor predictive power was conveyed in the preceding sections. However, we should at least present what is considered the best model. The following table summarizes in sample and out of sample root mean squared errors for both dependent variables and both models.

In Table 3, we can compare the trade-offs for model implementation in terms of predictive error. We can observe that the random forest model has outperformed the ordinary least squares model on the basis of root

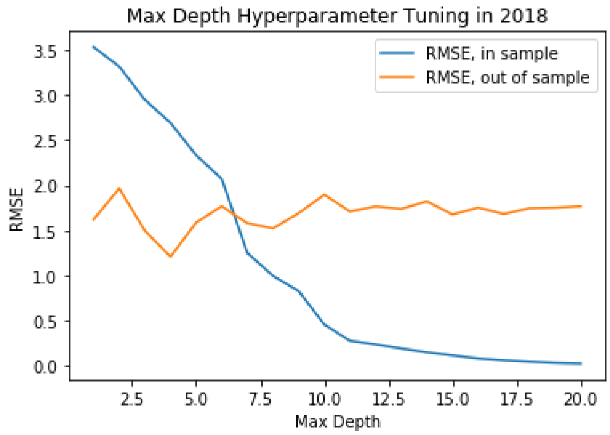

Figure 3: Tuning of the max depth hyperparameter

Table 3: Model Error Comparison in 2018

Dependent Variable	In Sample RMSE	Out of Sample RMSE	Model Type
252-day	1.376	1.970	OLS
252-day	1.107	1.913	RF
1260-day	3.515	1.5342	OLS
1260-day	2.694	1.208	RF

mean squared error for either dependent variable. This is to be expected as a random forest regressor model can learn more from interaction between independent variables. In terms of the test set, this is where we see some slight deviations. In the 252-day model, out of sample root mean squared errors are very similar. This difference in error is not very significant, however. In the 1260-day model, the random forest model is much lower than the ordinary least squares model. However, in Figure 3, the dip at a max depth of three is likely attributed to luck. Over any other year, this slight improvement would likely diminish. Therefore, the remaining analysis will continue with solely ordinary least squares regression.

4 Results

For each year in our study, we sort the each country's exchange traded fund on its predicted skewness for the following year. For example, if we predict skewness to be low for the coming year, that country will be put into a portfolio with other low-predicted skewness country exchange traded funds. In this example, we expect that the low-predicted skewness portfolio should have characteristics of a negatively skewed return distribution, like larger downside returns. These downside returns can be conveyed with a conditional value at risk statistic.

Specifically, we used a dynamic training method to make predictions for the skewness in the next year. Our model is, therefore, adaptive as new data emerges, but contains historical patterns. With the predictions that the rolling models produce, we can observe the returns of subsequently formed portfolios.

4.1 Predicting 252-day Skewness with Ordinary Least Squares

4.1.1 Model Error Throughout Time

Throughout time, we predict skewness to then sort portfolios. In Figure 4, we can observe each model's iterative error throughout time.

Clearly, we can see that the predictive power nearing the 2006-2008 period becomes much worse. It is worth mentioning that the out of sample line in Figure 4 represents the following year, which explains why

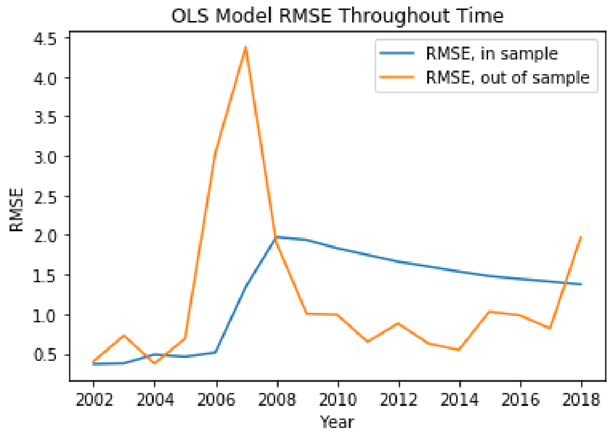

Figure 4: Model in and out of sample error throughout time

there is volatility appearing in 2005. This is actually due to the volatility experienced starting in 2006. [1]

4.1.2 Sorted Portfolio Performance Throughout Time

Our sorted portfolio cumulative return can be seen in Figure

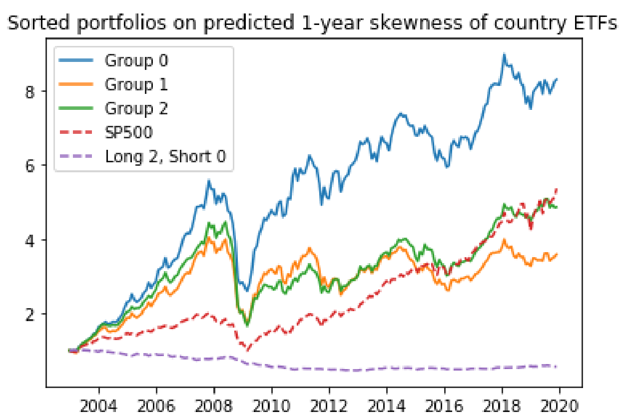

Figure 5: Performance throughout time across portfolios

In Figure 5, we can see that the three portfolios. Group 0 contains the countries with the most negative predicted skewness. Group 1 contains the countries with the most positive predicted skewness. Additionally, the S&P 500 exchange traded fund is included. Our expectations for these portfolios would be that Group 0 would have the most downside risk. From the depiction in Figure 5, that is not readily apparent.

In Table 5, we can observe that there is not a monotonic relationship with the conditional value at risk for these portfolios. We would have expected that Group 0 would have the most negative conditional value at risk. This, however, is not the case.

4.1.3 Analysis of Alpha: Capital Asset Pricing Model

Next, we attempt to see if we have found any alpha when considering the widely referenced pricing models.

In Table 6, we do not find any significant alpha at the 5% confidence level. At the 10% confidence level, we do find alpha in the group 0 portfolio. To our implementation validity, it is a good sign to see relatively

[1] To see what would have happened with the random forest regression, check the Appendix. Is this the pursuit of a model or a basis of reality?

Table 4: Monthly Conditional Value at Risk at 5% confidence

Portfolio	CVaR
Group 0	-10.01%
Group 1	-12.14%
Group 2	-12.06%
S&P 500	9.29%
Long 2, Short 0	-4.76%

Table 5: Annualized Portfolio Statistics

Portfolio	Mean Return	St. Dev. of Returns	Sharpe Ratio
Group 0	13.99%	16.72%	0.8366
Group 1	9.18%	17.89%	0.5163
Group 2	10.87%	17.00%	0.6411
S&P 500	10.94%	13.84%	0.7902
Long 2, Short 0	-3.12%	7.17%	-0.4355

stable beta coefficients as this would be expected when we are effectively sorting portfolios randomly due to poor model performance.

4.1.4 Analysis of Alpha: Fama-French Three Factor Model

Next, we attempt to see if we have found any alpha when considering the widely Fama-French Three Factor Model.

In Table 7, we do not find any significant alpha at the 5% confidence level. At the 10% confidence level, we do find alpha in the group 0 portfolio. To our implementation validity, it is a good sign to see relatively stable beta coefficients as this would be expected when we are effectively sorting portfolios randomly due to poor model performance.

4.2 Predicting 1260-day Skewness with Ordinary Least Squares

4.2.1 Model Error Throughout Time

Throughout time, we predict skewness to then sort portfolios. In Figure 6, we can observe each model's iterative error throughout time.

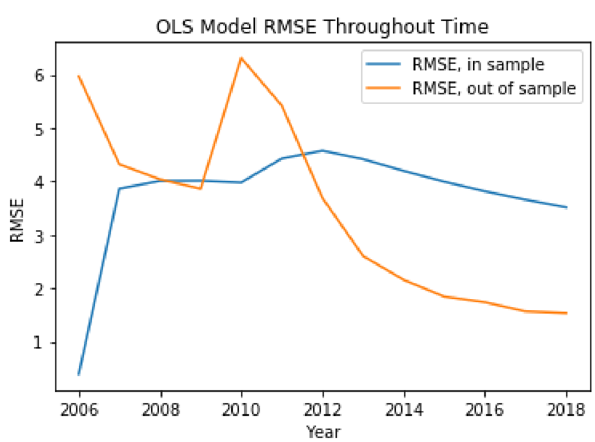

Figure 6: Model in and out of sample error throughout time

Table 6: Analysis of alpha using the Capital Asset Pricing Model

Variable	**Coef.**	**St. Err.**	**t**	**p Value**	**[0.025**	**0.975]**	**Portfolio**
Constant	0.0042	0.002	1.881	0.061	-0.000	0.009	Group 0
Mkt-RF	0.9277	0.054	17.081	0.000	0.821	1.035	Group 0
Constant	-0.0007	0.002	-0.345	0.730	-0.005	0.004	Group 1
Mkt-RF	1.0396	0.053	19.631	0.000	0.935	1.144	Group 1
Constant	0.0012	0.002	0.574	0.567	-0.003	0.005	Group 2
Mkt-RF	0.9690	0.053	18.445	0.000	0.865	1.073	Group 2
Constant	-0.0029	0.001	-1.982	0.049	-0.006	-1.57e-05	Long 2, Short 0
Mkt-RF	0.0414	0.036	1.138	0.256	-0.030	0.113	Long 2, Short 0

Table 7: Analysis of alpha using the Fama-French Three Factor Model

Variable	**Coef.**	**St. Err.**	**t**	**p Value**	**[0.025**	**0.975]**	**Portfolio**
Constant	0.0040	0.002	1.796	0.074	-0.000	0.008	Group 0
Mkt-RF	0.9758	0.060	16.336	0.000	0.858	1.094	Group 0
HML	-0.0597	0.091	-0.656	0.512	-0.239	0.120	Group 0
SMB	-0.1842	0.102	-1.811	0.072	-0.385	0.016	Group 0
Constant	-0.0007	0.002	-0.315	0.753	-0.005	0.004	Group 1
Mkt-RF	1.0857	0.057	18.919	0.000	0.973	1.199	Group 1
HML	0.1009	0.087	1.155	0.249	-0.071	0.273	Group 1
SMB	-0.2875	0.098	-2.941	0.004	-0.480	-0.095	Group 1
Constant	0.0013	0.002	0.616	0.538	-0.003	0.005	Group 2
Mkt-RF	1.0134	0.057	17.774	0.000	0.901	1.126	Group 2
HML	0.0968	0.087	1.115	0.266	-0.074	0.268	Group 2
SMB	-0.2765	0.097	-2.847	0.005	-0.468	-0.085	Group 2
Constant	-0.0027	0.001	-1.827	0.069	-0.006	0.000	Long 2, Short 0
Mkt-RF	0.0376	0.039	0.952	0.342	-0.040	0.115	Long 2, Short 0
HML	0.1565	0.060	2.603	0.010	0.038	0.275	Long 2, Short 0
SMB	-0.0922	0.067	-1.371	0.172	-0.225	0.040	Long 2, Short 0

The predictive power expressed in Figure 6 is very poor. This is due to a much smaller training period. Additionally, this training period also happend to be a time period of extreme volatility.

Due to the poor performance of the models, and a lack of data, further analysis of these models will stop here!

5 Conclusions

While previous studies showed that there is an abnormality when sorting country exchange traded funds on skewness, these studies were backward looking. Our analysis, which attempted to tie country-level skewness to societal influences resulted in poor performance of this relationship. This relationship was not discovered in a linear space nor a non-linear space where independent features could interact with one another. The results in the predicted negative-skew portfolio did result in outperformance, this can likely be attributed to luck.

6 Appendix

6.1 Random Forest RMSE Throughout Time

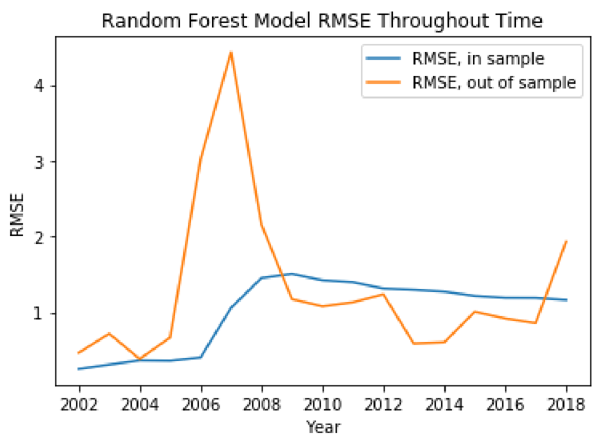

Figure 7: Random forest model in and out of sample error throughout time